

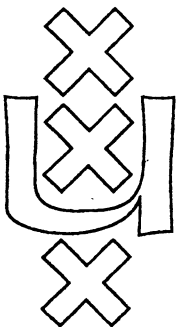
**Institute for Logic, Language and Computation**

**WHAT COMES FIRST IN DYNAMIC SEMANTICS**

**A compositional, non-representational account of natural language presupposition**

David Ian Beaver

ILLC Prepublication Series  
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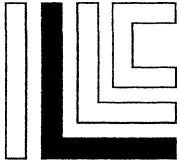


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# What Comes First in Dynamic Semantics

A compositional, non-representational account of natural language presupposition<sup>1</sup>

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## Abstract

The Context Change Potential (CCP) model of presupposition, due primarily to Karttunen and Heim, is formally elaborated and modified within a propositional dynamic logic, a first-order dynamic logic, and within a three sorted type theory. It is shown that the definitions of connectives and quantifiers can be motivated independently of the phenomenon of presupposition by consideration of the semantics of anaphora and epistemic modality (cf. the work of Groenendijk & Stokhof and Veltman), and that these independently motivated definitions provide a solution to the projection problem for presupposition. It is argued that with regard to the interaction between presupposition and quantification the solution is empirically superior to those in competing accounts. The treatment of epistemic modality is also shown to be superior to existing dynamic accounts, combining solutions to traditional modal identity problems with an adequate treatment of the behaviour of presuppositions in quantified contexts. A compositional semantics which integrates dynamic treatments of quantification, anaphora, modality and presupposition is then specified for a fragment of English. Finally, a formal model of global accommodation (cf. Lewis, Heim, van der Sandt) is defined, this model differing from previous accounts in being *non-structural*. This means that the accommodated material cannot be deduced from formal properties of the utterance alone, but is essentially dependent on world-knowledge and common sense reasoning. Thus the model provides an essentially pragmatic account in the style of Stalnaker. It is shown that the formal model provides both a general solution to the problem of the *informativeness* of presuppositions, and a specific solution to a problem within the CCP model, namely its tendency to yield inappropriately weak conditionalised presuppositions. It is argued that the pragmatic model can provide superior predictions to any purely semantic theory of presupposition, and to any theory based on a purely structural account of *accommodation* or *cancellation*.

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# Chapter 1

## Introduction

If there is no cat and no mat, then the following sentence does not make much sense:

E1 The cat is on the mat.

This is commonly explained in terms of *presupposition failure*, the sentence being said to presuppose the existence of a uniquely salient cat and a uniquely salient mat. Why is it important to distinguish the presuppositions of a sentence from other aspects of its meaning? What is the motivation for claiming that E1 presupposes that there is a cat and a mat and asserts that the first is on the second, rather than claiming that the sentence asserts the existence of a cat and the existence of a mat which it is on? There are many motivations for preferring the first division, not least being differences between the logical properties of presuppositions and assertions. It is these logical differences, to which I will come shortly, which are the prime concern of this thesis. However, it is also possible to find quite simple practical reasons for dividing off presuppositional aspects of meaning. Suppose that you make the following request to your household robot:

E2 Put the cat out!

What if the cat was already out? Should the robot comply with your request by firstly bringing the cat in and then putting it out? And if the robot happened to know that, following an unfortunate accident, whatever there had once been of a household cat was now in the municipal sewage system, how should the robot respond? Should it, perhaps, roll down to the local pet shop and buy a new cat which it could then bring in and put out? What if the robot happened to know that the cat had just had kittens, so that the definite description “the cat” did not uniquely refer, even within the restricted local domain of the household? Should the robot begin by drowning all but one of the animals so that the uniqueness condition was satisfied?

It seems obvious that these three situations, that in which the cat is already out, that in which there is no cat, and that in which there is more than one cat, all violate the assumptions implicit in the original instruction. You would want your robot to be able to distinguish between your assumptions about what is already the case, and your desires about what ought to be the case. If the assumptions appear to be valid, then the robot should attempt to comply with your wishes, but otherwise the robot should somehow signal the inappropriacy of the request. The robot’s calculation of your assumptions is by no means trivial, since your assumptions are only partly signaled by the presuppositions of your utterances. In general, the calculation requires a judicious mixture of linguistic knowledge and common sense. Most of this thesis will concern the linguistic component of the robot’s task, but, in chapter 5, I will return to the issue of common sense.

The range of constructions which have at one time or another been called presuppositional is large, and it is by no means clear to what extent they form a uniform class. In this thesis I will focus on just two types of construction, definites, which I take to include not only definite descriptions, but also pronouns and proper names, and *factives*. To give an example of this latter class, the attitude verb “realize” is said to be *factive*, and presuppose the truth of its propositional complement:

E3 Anna realizes that Bertha is hiding.

Whilst I will occasionally mention presuppositional constructions other than definites and factives, the formal theory as such will be limited to just these two.

## 1.1 The Projection Problem

Utterances of either (E3) or its negation (E4) would tend to implicate that Bertha was hiding:

E4 Anna does not realize that Bertha is hiding.

It is this characteristic behavior of presuppositions under negation which most clearly marks them out as different from ordinary entailments. For example an utterance of (E5) would also lead to the conclusion that Bertha was hiding, but this would not be the case with an utterance of (E6). So (E5) is said to entail but not presuppose that Bertha is hiding.

E5 Bertha is hiding in the attic.

E6 Bertha is not hiding in the attic.

The Projection Problem for Presuppositions, as first discussed by Langendoen and Savin [LS71], is normally conceived of as the problem of predicting the presuppositions of (utterances of) complex sentences in terms of the presuppositions of (utterances of) their parts. I am not happy with such a formulation of the projection problem. It suggests that there is an *a priori* means of identifying “the presuppositions of complex sentences”, and that the theorist’s job merely consists in accounting for this presuppositional data.

There are several tests for presupposition, as discussed, for instance, in [vdS92]. However, it is fair to say that these tests are at their best in determining the presuppositions of simple sentences. They are unreliable or difficult to apply to complex sentences, and have no application at all to discourses containing more than one sentence.

For example, behaviour under negation makes a good test for presuppositions in relatively simple sentences, but the only candidate for the negation of, for instance, the English conditional is the “it’s not the case” construction. Yet it is by no means obvious that the presuppositional behaviour of this construction will parallel that of a sentence internal negation. Besides, linguistic judgements become highly unstable with respect to sentences like “it’s not the case that perhaps Bertha is in the cellar and if Anna realizes that Bertha is not hiding in the attic then Bertha is hiding in the cupboard.” Similar difficulties apply to all the other tests for presupposition that have been suggested in the literature — none of them is really effective for compound sentences.

But the conclusion of this short methodological diatribe is not a plea for the abandonment of the term *presupposition*. On the contrary, I find the notion of presupposition very useful at an intuitive level, and will often refer to presuppositions, presuppositional constructions, and presupposition projection. But I will not take the raw data to consist of sentences and their presuppositions, and then present a theory which predicts for each sentence its presuppositions. In a sense I will not specifically be addressing the Projection Problem for Presuppositions. Instead, I will consider that old and familiar projection problem of which presupposition is but a tiny part, namely the problem of predicting the *meanings* of complex sentences in terms of the *meanings* of their parts, or the problem of *compositionality* as it is best known.

I will attempt to follow a long standing tradition of philosophers and semanticists. The data will consist of implications between sentences of natural language, and to account for the data I will define logics which yield these implications as entailments between formulae, combined with a general way of translating from natural language into the logic. An obvious advantage of using implications as the raw data is that it makes perfect sense to talk of a complex discourse having a particular sentence as an implication, whereas, as I have said, there is no test for presuppositions which applies to multiple sentence texts. In summary, the projection problem for presuppositions forms just one part of a much larger projection problem, and not a very clearly defined part. I will address the larger, better defined problem, but in the process the sub-problem, whatever the details of its definition, should be covered.

Some may object, saying that the *defeasibility* of presuppositions makes them qualitatively different from other sorts of entailment. I do not believe that defeasibility would show up any inherent short-comings in the methodology I have described. A suitably broad conception of *logic* can certainly encompass



defeasible entailment, and indeed dynamic formalisms closely related to those developed in this thesis are increasingly seeing application to non-monotonic reasoning.

So, what are the special demands that presuppositional constructions place on a compositional theory of meaning? I will now present some simple, illustrative data, more complicated cases being considered later in the thesis. Firstly we would want to predict that both (E3) and (E4) above implicate that Bertha is hiding. We would also want to predict that utterance of (E7), in which a presuppositional construction is embedded in the antecedent of a conditional, would implicate that Bertha was hiding. Similar predictions seem warranted for embedding under the modality “might”, as seen in (E8), and for iterative embedding of presuppositional constructions as in (E9).

E7 If Anna realizes that Bertha is hiding, then she will find her.

E8 Anna might realize that Bertha is hiding.

E9 Bertha regrets that Anna realizes that she is hiding.

However, complex sentences do not uniformly preserve the presuppositions of their parts. For instance, an utterance of (E10) would not implicate that Bertha was in the attic, in spite of the occurrence of “Bertha is in the attic” as the complement of a factive verb. Similarly, neither (E11) nor (E12) seems to implicate that Bertha is hiding at all, although they both involve a factive verb with complement “Bertha is hiding”.

E10 If Bertha is not in the kitchen, then Anna realizes that Bertha is in the attic.

E11 If Bertha is hiding, then Anna realizes that Bertha is hiding.

E12 Perhaps Bertha is hiding and Anna realizes that Bertha is hiding.

## 1.2 The Context Change Potential model

By the Context Change Potential (CCP) model of presupposition, I mean the formal model developed primarily by Lauri Karttunen [Kar73, Kar74] and Irene Heim [He83]. The integration of Karttunen’s work into Montague Grammar by Karttunen and Peters [KP79] is also relevant, whilst the theory has also been influenced by the work of Stalnaker [St74] and Lewis [Le79], although their contributions are less technical. The theories found in [Gaz79] and [vdS92] also rely on context change, but as these theories are not normally formulated in terms of CCP’s, I will reserve the term CCP for accounts in the Karttunen/Heim style.

In the CCP model the principal notions governing the behavior of presupposing constructions are contextual satisfaction and contextual incrementation. The key assumptions are:

- A context satisfies (supports, entails) a set of propositions.
- An informative utterance augments the context of interpretation, increasing the set of propositions satisfied. A sentence is said to have a certain Context Change Potential, by which is meant the ability of particular utterances of the sentence to augment (update, increment) the context of utterance.
- If a sentence presupposes something, then an utterance of the sentence can only be interpreted in contexts which satisfy the propositions that are presupposed.
- When a complex sentence is uttered, some of the parts of the sentence may be interpreted in local contexts which differ from the global context in which the entire utterance is uttered.

Under this conception the Presupposition Projection Problem becomes instead a problem of determining the local contexts of interpretation of the parts of a sentence in terms of the global context of utterance. Thus in [Kar73] the conditional is defined to have the truth conditions of material implication and heritage conditions given by: “Context X satisfies-the-presuppositions-of ‘If A then B’ just in case (i) X satisfies-the-presuppositions-of A, and (ii) X [augmented with] A satisfies-the-presuppositions-of B.” Time for some examples.

E13 Bertha is small.

E14 Anna regrets that Bertha is small.

E15 Bertha is small and Anna regrets it.

The single sentence in (E13) has a CCP that may be thought of as a function, namely a function from incoming contexts in which it may not be established that Bertha is small, to outgoing contexts which differ minimally from the incoming context by supporting this proposition.

The factivity of the main verb in (E14) means that the sentence is standardly taken to presuppose that Bertha is small. In terms of the CCP model, the sentence can only be felicitously uttered in contexts which already support the proposition that Bertha is small. Thus the CCP of the sentence behaves like a partial function. The sub-domain of contexts on which the function is defined contains only those contexts where it is established that Bertha is small. The range contains only contexts which additionally support the proposition that Anna regrets that Bertha is small.

On the other hand (E15) is standardly taken to entail rather than presuppose that Bertha is small. In the Context Change model this derives from the treatment of conjunction. A context may be augmented by a conjunctive sentence by firstly updating with the first conjunct, and then updating the resulting intermediate context with the second conjunct. Thinking of the CCP's of the conjuncts as partial functions, the CCP of a conjunction becomes a functional composition of the CCP's of the conjuncts. Since, in the case of (E15), the range of the first conjunct's CCP is precisely the subset of contexts for which the CCP of the second conjunct is defined, the composition of the two CCP's will be a total function on the domain of contexts. In other words, (E15) may be felicitously uttered in any context, and so the model predicts that the sentence as a whole has no presuppositions. The effect of the sentence is simply to update any context firstly with the information that Bertha is small, and then with the information that this bothers Anna.

### 1.3 Overview of the thesis

The last few years have seen a shift of emphasis in the study of semantics. The traditional job of the natural language semanticist involved relating sentences to truth conditions, or to functions from certain contextual parameters to truth conditions. However, much recent work has concentrated on how the process of understanding itself helps to determine the relevant contextual parameters, and thus to determine the truth conditions. To some researchers, truth conditions have become secondary, the primary object of study being the way in which context changes during language processing. In other words, there has been a shift from a static conception of meaning, through a contextually sensitive but still essentially static conception, leading (finally?) to a radically dynamic view. At the same time as this philosophical shift has occurred, there has also been a tendency for semanticists to import formal approaches to modelling dynamics from the discipline of theoretical computer science. The influx of new ideas and methods is leading to significant advance in the analysis of natural language.

The CCP model of presupposition is a *dynamic* account of meaning *par excellence*, but its genesis preceded many of the recent technical advances in dynamic semantics. In this thesis, I will elaborate and defend the CCP model, attempting to show that by taking advantage of recent technical developments, most of the outstanding problems with the model can be overcome. Furthermore, it will be shown how dynamic theories of presupposition, anaphora, quantification and epistemic modality can be integrated into a single compositional grammar fragment. The reason for attempting this integration is twofold. Firstly I believe that the essential details of the theory of information underlying the CCP model can be motivated independently of the study of presupposition, and that this can only be demonstrated within an integrated theory that incorporates both presupposition and various other dynamic phenomena. Secondly, it is worth providing an integrated theory with wide empirical coverage simply to show that this can be done, and thus that the various theories are compatible.

The remainder of this thesis will run as follows. Chapter 2 introduces the reader to dynamic semantics, firstly showing how the context sensitivity of epistemic modality can be modelled in a simple propositional system, and how this same system can be adapted to account for the simple presupposition projection data from §1.1. In chapter 3 a dynamic semantics for predication, quantification and anaphora within a first-order language, ABLE, is developed, and in chapter 4 this semantics is further refined. The

refinement is shown to solve some difficult problems occurring in the interaction between quantification and modality on the one hand, and quantification and presupposition on the other. In chapter 5 I show how the semantics developed for ABLE can be used as the basis of a compositional analysis of English, within a system I refer to as Kinematic Montague Grammar (KMG). It is then demonstrated that an adequate treatment of presupposition must involve a complex interaction between world knowledge and compositionally derived meaning. A formal model based on an extension to KMG is developed, and it is shown that this model has the potential to account for empirical observations which are problematic for other theories of presupposition. Finally, in chapter 6 I draw some general conclusions, and mention some avenues for future research.

## Chapter 2

# Two Birds and One Stone

### 2.1 Aim

Two apparently disparate aspects of natural language meaning, presupposition and epistemic modality, can be tackled using a single, suitably dynamic, theory of information. The current chapter has two main objectives. Firstly it should serve as an introduction to the treatment of presupposition within a dynamic framework, and thus to the more ambitious developments in the remainder of the thesis. Secondly the chapter is intended to show that the underlying assumptions behind the Context Change model of presupposition, which was developed in the seventies and early eighties and is thus one of the first instances of a *dynamic* theory of natural language meaning, can be independently motivated through a consideration of epistemic modality.

In the remainder of the chapter I will weave backwards and forwards between the topics of presupposition and epistemic modality. Beginning with presupposition, in § 2.2 I focus on a problematic methodological issue concerning the Context Change model. In § 2.3 I jump to the semantics of epistemic modality, presenting some data and suggesting informally how a dynamic semantics could improve over more traditional static accounts of modality. A formal presentation of a dynamic system appropriate to the treatment of the data in § 2.3 is given in § 2.4. Finally, in § 2.5, it is shown that a minor extension to this system provides us with an account of presupposition, and we see how the account fares with the data presented in the introductory chapter to the thesis.

### 2.2 Descriptive versus Explanatory Adequacy

One objection to Karttunen's account was that the CCP's of complex sentences are defined arbitrarily and with no independent motivation. In early formulations the meanings of lexical items included separate specifications of truth conditions and heritage conditions, where by *heritage* conditions, I mean the rules which say how presuppositions will be projected.

Consider the [Kar73] definition for the conditional mentioned in §1.2, in which the truth conditions are as for material implication and the heritage conditions are given by: "Context X satisfies-the-presuppositions-of 'If A then B' just in case (i) X satisfies-the-presuppositions-of A, and (ii) X [augmented with] A satisfies-the-presuppositions-of B." Since the truth conditions are specified independently of the heritage conditions, it would be possible to imagine a child mistakenly learning the correct truth conditions of the conditional but the wrong heritage conditions<sup>1</sup>. This does not seem plausible, and we would clearly prefer a theory in which the heritage conditions were not specified separately, but somehow derived from the truth conditions and other general principles.

[He83] attempted to rescue Karttunen's approach by showing how truth conditions could generally be derived from appropriate specifications of Context Change Potential. On this basis she claimed that the CCP model had at least the explanatory adequacy of its competitors, such as Gazdar's theory. However Mats Rooth (as cited in [Heim 90]) and Scott Soames [So86] have noted that whilst the correct truth conditions derive from the CCP's for connectives that Heim specifies, this would also be the case for

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<sup>1</sup>This is similar to the argument in [He83].

a number of other specifications of the CCP, and some of these alternative specifications would give incorrect heritage conditions. So we could still imagine a child learning CCP's for some connectives that gave the correct truth conditions but incorrect heritage conditions. On this basis, Heim (eg. in [Heim 90]) has been forced to retract her claim.

However, I believe that the CCP model can be saved from the quagmire of non-explanatory ignominy. The key observation is that the recent work of Frank Veltman [Ve91] on the semantics of epistemic modal operators relies on a strikingly similar underlying notion of context change to that utilised by Heim. And indeed this is hardly surprising given that both have taken inspiration from the same philosophical well-springs, for instance from the work of Robert Stalnaker and David Lewis.

I will borrow from Veltman's work to show how the context sensitivity of words like "might" and "must" motivates a dynamic semantics. None of the alternative CCPs for connectives that have been suggested by Rooth and Soames would be compatible with this semantics, and it is hard to imagine how a relevantly different dynamic semantics could still get the facts right about the meanings of the epistemic modalities.

I will then show how a simple extension to the logic developed in 2.3 — in fact the addition of a single unary operator — produces a system with all the presupposition inheritance properties we would expect of a CCP model. In the process, the connection between presupposition and the epistemic modalities, and also the logic of presupposition itself, will become transparent.

## 2.3 Hide and Seek with Epistemic Modalities

Imagine the following situation, which is very like an example considered in [Ve86]. The difference is that he had misplaced his marbles, whereas I have lost a number of women.

- Anna is seeking Bertha, Clothilde and Daisy, and for our benefit she is recording her thoughts on a small portable cassette recorder.
- Anna has searched almost everywhere, and she knows that the only remaining hiding places are the cupboard (which is not in the attic) and the attic (which is not in the cupboard.)
- Only one person fits in the cupboard.
- Anna, having heard some noises, knows that somebody is in the cupboard.

Let us consider what we would expect to find on Anna's tape, restricting our attention to discourses involving interesting mixtures of connectives and epistemic modalities, as they might occur when Anna tells us what she has found out. Firstly, look at the following two examples involving conjunctions, which I take to include sentence sequencing as well as the particle "and", the word "perhaps", which I take to mark epistemic possibility, and "must" which seems to act as a sort of epistemic necessity operator:

**E16** Perhaps it is Bertha in the cupboard and ... it is Clothilde in the cupboard. Got you! So Bertha must be in the attic.

**E17?** It is Clothilde in the cupboard and ... perhaps it is Bertha in the cupboard. So Bertha must be in the attic.

It is quite plausible that we might find (E16), but by contrast it is hard to imagine an occurrence of the discourse in (E17). The reason for this is clear. As Anna learns about where everybody is hiding, she gradually eliminates possibilities. So what is possible at one point may not be possible after the addition of new information. However, the reverse does not hold. So long as Anna has not been in anyway deluded, and provided she is suitably cautious in her reasoning process and does not make any unwarranted eliminations of possibilities, then the addition of new information can never increase the number of open possibilities.

The only significant difference between examples (E16) and (E17) seems to be in the ordering of conjuncts, and this will motivate the definition of an asymmetric conjunction. We will also need to define epistemic operators that are sensitive to the local context of interpretation. Thus the meaning of a sentence "A and B.", or a sequence of sentences "A. B. ", will be expressed as an update with A

followed by an update with B. An occurrence of “perhaps A” at a particular point in a discourse will mean that *at that point in the discourse* the possibility of A remains, and we will take “must A” to mean that at the phrase’s point of occurrence, the possibility of A being false has been excluded.

Next, consider a case involving a conditional:

**E18 ?Daisy might be in the cupboard. So if Daisy is not in the cupboard, then she might be in the cupboard.**

We could not account for this data by interpreting the conditional as material implication, and taking “perhaps” and “must” to be standard, say S5, modal possibility and necessity operators using the same modal accessibility relation. Under such *static* assumptions the consequents of conditionals would be evaluated with respect to the same context (i.e. the same possible world) as the conditionals as a whole. The intuitively invalid argument in (E18) would be valid in the standard picture, since if it was possible that Daisy was in the cupboard, then any conditional with an expression of this possibility in the consequent would be true.

**E19 Clothilde is in the attic. Now although Bertha might be hiding in the cupboard, and might be hiding in the attic, I conclude that if Daisy is not hiding in the cupboard then Bertha must be hiding there, and if Daisy is hiding in the cupboard then Bertha must be hiding in the attic.**

By contrast (E19) is a valid argument, but would appear invalid on the standard picture. Standardly, (E19) would come to imply that:

- i. There are accessible worlds where Bertha is in the cupboard,
- ii. There are accessible worlds where Bertha is in the attic,
- iii. Daisy is in the attic implies that Bertha is in the cupboard in all accessible worlds,
- iv. Daisy is in the cupboard implies that Bertha is in the attic in all accessible worlds.

Suppose that Daisy was in the cupboard. Then (iv) could only be true if Bertha was in the cupboard in all accessible worlds, which contradicts (ii). On the other hand, suppose that Daisy was not in the cupboard. Then (iii) could only be true if Bertha was in the attic in all accessible worlds, which contradicts (i). The relevant cases being thus exhausted, the discourse as a whole would appear logically inconsistent. However, this is at odds with our intuitions, for it seems that (E19), although somewhat convoluted, is a perfectly reasonable thing for Anna to say.

Such examples support an analysis of the conditional as an assertion of the consequent under the assumption of the antecedent: this will be stated more formally below. Thus, concerning the conditional in (E19), the assumption that Daisy was in the cupboard, would exclude the possibility that she was not there, and the conditional would not be supported by the given assumptions. In (E19) whilst Anna is still open to the possibility that Bertha is in the cupboard, under the additional assumption that Daisy is in the cupboard she is prepared to assert that Bertha is not in the cupboard. Similarly, whilst she is open to the possibility that Bertha is in the attic, under the assumption that Daisy is in the attic it would have to be the case — since she would have no other open possibilities — that Bertha is in the cupboard.

I will not discuss the meaning of negation in detail. Suffice it to say that we must define a negation consistent with our picture of an agent gradually eliminating possibilities. It must predict that in a case like (E20), when we hear Anna telling us the negation of “Daisy is in the cupboard”, we need no longer consider alternatives where Daisy is in the cupboard. And, considering examples like (E21) — I leave the reader to invent some more, or search for counter-examples, according to taste — it should predict that epistemic possibility and necessity behave as logical duals.

**E20 ?Daisy is not in the cupboard. Perhaps Daisy is in the cupboard.**

**E21 Daisy might not be in the cupboard. So it’s not the case that Daisy must be in the cupboard.**

## 2.4 Update Logic (UL)

To meet the requirements of the data presented above, I will define a logic along the lines of the “first example” given in [Ve86]. The significant difference will be syntactic, in that I will allow arbitrarily deep embeddings of the epistemic modalities whereas Veltman prefers to keep his modalities near the surface.

For the moment I will restrict myself to a propositional language defined over some set of atomic propositions, such as the proposition that Bertha is in the cupboard and the proposition that she is in the attic. I will let  $\mathcal{P}$  stand for the set of atomic propositions.

Unlike in the Tarskian scheme, where semantics concerns itself with determining the truth or falsity of propositions, the main concern of Update Logic is the potential of a proposition to change an agent’s information state. An information state will be identified with the range of open possibilities an agent has with regard to her knowledge of reality. Each open possibility, or *possible world*, will provide a complete picture of reality. To this end a proposition will be identified with a set of possible worlds, intuitively the set of worlds in which the proposition is true, and an information state will be a set of possible worlds.

**Definition D1 (Models for UL)** *A model of UL is a pair  $\langle W, F \rangle$  where  $W$  is a set of possible worlds and  $F$  is an interpretation function mapping propositional constants to sets of worlds.*

**Definition D2 (Information in UL)** *An information state (context) in UL is a subset of  $W$ . Thus the minimal information state is  $W$  itself, which will also be written  $\overline{1}$ , and the maximal information state is the empty set of worlds, also written  $\perp$ . Non-maximal information states will be called **consistent**.*

**Definition D3 (Syntax of UL)** *The sentences of an Update Logic restricted to the propositions in  $\mathcal{P}$  are formed in the usual way from the atomic propositions in  $\mathcal{P}$ , the unary operators NOT, MIGHT, MUST and the binary connectives AND and IMPLIES. We will use  $p, q$  as metavariables over atomic propositions, and  $\phi, \psi$  as metavariables over arbitrary sentences of UL.*

The above definitions seem to assume worlds as ontological primitives. However, Veltman’s system has a more syntactic flavour, in that worlds are not basic but identified with sets of atomic propositions. To see how a set of atomic propositions can be equated with a possible world think of the atomic propositions in the set as those which are true in that world, and those not in the set as false in that world. Later, when we work through some examples, it will be useful to view worlds in this syntactic light, and the following definition gives a method of constructing the appropriate models:

**Definition D4 (Term Models for UL)** *A term model for UL over the atomic propositions  $\mathcal{P}$  is a pair  $\langle W, F \rangle$  where  $W$  is the powerset of  $\mathcal{P}$  and  $F$  is a function such that if  $p \in \mathcal{P}$  and  $w \in W$  then  $w \in F(p)$  iff  $p \in w$ .*

Now we are in a position to define the semantics of UL. The meaning of an expression  $\phi$  of UL, written  $\llbracket \phi \rrbracket$  will be defined as a relation, written in infix notation, between two information states, intuitively an input and an output state. In general we should think of this relation as holding between a given pair of states just in case when we are in the first state the new information could leave us in the second state:

**Definition D5 (Update)** *If  $\sigma \llbracket \phi \rrbracket \tau$  then  $\tau$  is said to be an **update** of  $\sigma$  with  $\phi$ .*

In fact, in the following definition all UL formulae will denote relations which are total functions on the domain of information states. I have diverged superficially from Veltman by specifying the semantics relationally rather than functionally: this will become important later.

**Definition D6 (Semantics of UL)** For all models  $\mathcal{M}$  and information states  $\sigma, \tau$ , the relation  $[\cdot]_{\text{UL}}^{\mathcal{M}}$  (sub- and super-scripts omitted where unambiguous) is given recursively by:

- (1)  $\sigma[\text{atomic}] \tau$  iff  $\tau = \{w \in \sigma \mid w \in F(p)\}$
- (2)  $\sigma[\phi \text{ AND } \psi] \tau$  iff  $\exists v \sigma[\phi]v[\psi] \tau$
- (3)  $\sigma[\text{NOT } \phi] \tau$  iff  $\exists v \sigma[\phi]v \wedge \tau = \sigma \setminus v$
- (4)  $\sigma[\phi \text{ IMPLIES } \psi] \tau$  iff  $\sigma[\text{NOT } (\phi \text{ AND } (\text{NOT } \psi))] \tau$
- (5)  $\sigma[\text{MIGHT } \phi] \tau$  iff  $\exists v \sigma[\phi]v \wedge$   
 $(v \neq \perp \rightarrow \tau = \sigma) \wedge$   
 $(v = \perp \rightarrow \tau = \perp)$
- (6)  $\sigma[\text{MUST } \phi] \tau$  iff  $\exists v \sigma[\phi]v \wedge$   
 $(v = \sigma \rightarrow \tau = \sigma) \wedge$   
 $(v \neq \sigma \rightarrow \tau = \perp)$
- (7)

Let us consider the clauses of definition D6 individually:

- (1) **Atomic Propositions** The base case of the recursion says that to update an information state with an atomic proposition, you must remove all those worlds in that state which are incompatible with the new proposition, and what remains is the outgoing state.
- (2) **Conjunction** The meaning of a conjunction is defined as a relational composition between the meanings of the conjuncts. This definition corresponds to the informal analysis above suggesting that to update with a conjunction, you should update with the first conjunct, and then with the second.
- (3) **Negation** This is defined in terms of a set complement operation. We find those worlds in the input state which are compatible with the negated proposition, and the output state is what remains after removing these worlds from the input.
- (4) **Implication** Implication is defined using a standard equivalence, and it is the fact that a dynamic conjunction is used within that equivalence that gives the implication its dynamic flavour. In particular, the consequent is only evaluated in the context set up by a previous assertion of the antecedent.
- (5) **Epistemic Possibility** There are two cases to be considered in the definition of the MIGHT-operator, which corresponds to Veltman's "might" operator. Either the propositional complement of the MIGHT corresponds to one of the open possibilities in the incoming information state (which is established by attempting to update with the argument and checking that the result is not the absurd state) in which case the outgoing state is identified with the incoming one, or else the complement is already falsified by the incoming state, in which case the result is absurdity.
- (6) **Epistemic Necessity** Again there are two relevant cases. Either adding the complementary proposition would not remove any worlds from the incoming state, in which case the complementary proposition "must" hold in the input state and the outgoing state is again identified with the incoming one, or else the complement would remove some worlds. In this case the complement is not yet established, it is clearly false that the complement "must" be true in the incoming state, and the final result is absurdity.

There are several notions of entailment that can be appropriate to a dynamic logic like UL, and for discussion the reader is referred to [Ve91]. The definition below says that a sequence of UL premises entails a conclusion just in case the relational composition of the meanings of the premises has in its range only fixed points of the conclusion. In other words, once we have updated any information state with all the premises, updating with the conclusion would add no new information.



**Definition D7 (Entailment in UL)**

$$\phi_1, \dots, \phi_n \models \psi \quad \text{iff} \quad \forall \sigma_0, \dots, \sigma_n \\ \sigma_0[\phi_1]\sigma_1[\phi_2] \dots [\phi_n]\sigma_n \rightarrow \sigma_n[\psi]\sigma_n$$

The following clause gives a derivative notion of entailment against a particular background of assumptions:

**Definition D8 (Contextual Entailment in UL)** *If  $\sigma$  is an information state, then:*

$$\phi_1, \dots, \phi_n \models_{\sigma} \psi \quad \text{iff} \quad \forall \sigma_1, \dots, \sigma_n \\ \sigma[\phi_1]\sigma_1[\phi_2] \dots [\phi_n]\sigma_n \rightarrow \sigma_n[\psi]\sigma_n$$

**Examples**

We will now consider some simple-minded translations of examples (E16 – E21) above. We will confine ourselves to an update language restricted to the six atomic propositions  $bC, cC, dC, bA, cA$  and  $dA$ , which concern who is hiding where. For instance  $bC$  is the proposition that Bertha is hiding in the Cupboard, and  $dA$  is the proposition that Daisy is hiding in the Attic. In the translations below I have ignored the presuppositional component of the it-clefts in some of the examples, and have also ignored the propositional content of “Got you!”. Further, I have treated discourses of the form “A. So B.” and “A. I conclude that B.” as consisting of two parts, an assertion of the content of A, and a *meta-level* assertion that A entails B in the context  $\sigma$  of the given assumptions. In the following discussion, the context  $\sigma$  will correspond to the hide-and-seek situation described at the beginning of §2.3.

Firstly the cases motivating the asymmetric definition of conjunction:

- E16 a. Perhaps it is Bertha in the cupboard and ... it is Clothilde in the cupboard. Got you! So Bertha must be in the attic.  
 b.  $\text{MIGHT}bC \text{ AND } cC, (\text{MIGHT}bC \text{ AND } cC \models_{\sigma} \text{MUST}bA)$
- E17 a. ?It is Clothilde in the cupboard and ... perhaps it is Bertha in the cupboard. So Bertha must be in the attic.  
 b.  $cC \text{ AND } \text{MIGHT}bC, (cC \text{ AND } \text{MIGHT}bC \models_{\sigma} \text{MUST}bA)$

In explaining the contrast between these two examples (and also for the discussion of the remaining examples), we will need to consider what would happen to our information state as we heard them playing on Anna’s cassette recorder. It will be helpful to construct the relevant information states using the term model over the above six atomic propositions. A world will be a subset of the six atomic propositions, and an information state will be a set of such worlds.

However, since we already know that one person cannot be in two places at once, and that each person is in at least one place, our initial information state need not contain surreal possible worlds like  $\{bA, bC, cA, cC\}$ , which would depict Bertha and Clothilde as being in both the cupboard and the attic, and Daisy as being nowhere. Furthermore we know that only one person fits in the cupboard, so we can eliminate possible worlds like  $\{bC, cC, dC\}$ , which would paint a picture of a very crowded cupboard indeed. And one more piece of information: somebody is in the cupboard. The only three possible worlds compatible with all this information are  $w_1 = \{bA, cA, dC\}$ ,  $w_2 = \{bA, cC, dA\}$  and  $w_3 = \{bC, cA, dA\}$ , and if we initially have just this information, our information state will be  $\{w_1, w_2, w_3\}$ .

Now consider the effect of updating this information with the formulae in (E16b). The reader should verify that the only possible sequence of information states starting with  $\{w_1, w_2, w_3\}$  is:

$$\text{E16 c. } \{w_1, w_2, w_3\}[\text{MIGHT}bC]\{w_1, w_2, w_3\}[cC]\{w_2\}[\text{MUST}bA]\{w_2\}$$

On the other hand, the only possible sequence of states resulting from an update with the formulae in (E17b), and starting from the same initial state is:

$$\text{E17 c. } \{w_1, w_2, w_3\}[cC]\{w_2\}[\text{MIGHT}bC]\perp[\text{MUST}bA]\perp$$

Thus the oddity of (E17) arises because updating a state which does not allow for the possibility of  $bA$  with the proposition  $\text{MIGHT } bA$  yields a contradictory information state.

Regarding the conditionals in (E18) and (E19), we see that the contextual entailment in the first is not valid, since in the context of  $\{w_1, w_2, w_3\}$ ,  $\llbracket (\text{NOT } dC) \text{ IMPLIES MIGHT } dC \rrbracket$  is not a fixed point, but the contextual entailment in the second is valid since  $\{w_1, w_3\}$  is a fixed point of:

$$\llbracket ((\text{NOT } dC) \text{ IMPLIES MUST } bC) \text{ AND } (dC \text{ IMPLIES MUST } bA) \rrbracket$$

Note that although  $\llbracket (\text{NOT } dC) \text{ IMPLIES MIGHT } dC \rrbracket$  made an invalid conclusion to the argument in (E18), the account does not in general predict any oddity for assertions of this form. Saying “If Daisy is not in the cupboard then she might be in the cupboard” is just seen as a long-winded way of saying that Daisy is in the cupboard. Though this seems odd, it is arguable that it does not reflect a problem with the semantics of  $\text{MIGHT}$  *per se*. In UL updating with any conditional for which the consequent cannot be validated is equivalent to denying the antecedent, and this seems to reflect ordinary language usage in cases where the conclusion and antecedent are unrelated, as with the ironic use of sentences like “If this logic is classical, then I’m a Dutchman!” However, this type of use of a conditional becomes particularly strange when the antecedent and consequent are semantically related, as in the sentence “If this logic is classical then *modus tollens* does not hold in it.” Whilst this last sentence is, after some effort, interpretable, it is certainly not natural, and in this respect it is comparable to the earlier “if Daisy is not in the cupboard then she might be in the cupboard”. I would suggest that it is merely the complexity of the inference in these cases that makes them odd, and not any basic logical incoherence.

- E18 a. ?Daisy might be in the cupboard. So if Daisy is not in the cupboard, then she might be in the cupboard.
- b.  $\text{MIGHT } dC, (\text{MIGHT } dC \models_{\sigma} (\text{NOT } dC) \text{ IMPLIES MIGHT } dC)$
- c.  $\{w_1, w_2, w_3\} \llbracket \text{MIGHT } dC \rrbracket \{w_1, w_2, w_3\}$   
 $\llbracket (\text{NOT } dC) \text{ IMPLIES MIGHT } dC \rrbracket \{w_1\}$
- E19 a. Clothilde is in the attic. Now although Bertha might be hiding in the cupboard, and might be hiding in the attic, I conclude that if Daisy is not hiding in the cupboard then Bertha must be hiding there, and if Daisy is hiding in the cupboard then Bertha must be hiding in the attic.
- b.  $cA, (cA \text{ AND MIGHT } bC \text{ AND MIGHT } bA \models_{\sigma} ((\text{NOT } dC) \text{ IMPLIES MUST } bC) \text{ AND } (dC \text{ IMPLIES MUST } bA))$
- c.  $\{w_1, w_2, w_3\} \llbracket cA \rrbracket \{w_1, w_3\} \llbracket (\text{MIGHT } bC) \text{ AND MIGHT } bA \rrbracket \{w_1, w_3\}$   
 $\dots \llbracket ((\text{NOT } dC) \text{ IMPLIES MUST } bC) \text{ AND } (dC \text{ IMPLIES MUST } bA) \rrbracket \{w_1, w_3\}$

We have already considered some examples involving negation, so it should by now be clear to the reader why (E20) is anomalous. The statement that “Daisy is not in the cupboard” removes any alternatives in which Daisy was in the cupboard, and the following assertion that there still remains the possibility of Daisy being in the cupboard leads to absurdity.

- E20 a. ?Daisy is not in the cupboard. Perhaps Daisy is in the cupboard.
- b.  $(\text{NOT } dC) \text{ AND MIGHT } dC$
- c.  $\{w_1, w_2, w_3\} \llbracket \text{NOT } dC \rrbracket \{w_2, w_3\} \llbracket \text{MIGHT } dC \rrbracket \perp$

However, (E21) adds something new to the discussion, since it involves a negation outscoping an epistemic modality:

- E21 a. Daisy might not be in the cupboard. So it's not the case that Daisy must be in the cupboard.
- b.  $\text{MIGHT NOT } dC, (\text{MIGHT NOT } dC \models_{\sigma} \text{NOT MUST } dC)$
- c.  $\{w_1, w_2, w_3\} \llbracket \text{MIGHT NOT } dC \rrbracket \{w_1, w_2, w_3\} \llbracket \text{NOT MUST } dC \rrbracket \{w_1, w_2, w_3\}$

That (E21) is consistent follows from the fact that the first and second sentences of (E21) translate into equivalent formulae of UL. This, of course, is just one example of a more general equivalence, namely that  $\text{MIGHT}$  and  $\text{MUST}$  are logical duals:

**Fact F1** For any formula  $\phi$  and informationstates  $\sigma, \tau$ :

$$\sigma[\text{MUST}\phi]\tau \text{ iff } \sigma[\text{NOT}(\text{MIGHT}(\text{NOT}\phi))]\tau$$

*Proof:* From the definitions of NOT and MIGHT it can be seen that:

$$\begin{aligned} \sigma[\text{MIGHT}(\text{NOT}\phi)]\tau \text{ iff } & \exists v \sigma[\phi]v \wedge \\ & (v \neq \sigma \rightarrow \tau = \sigma) \wedge \\ & (v = \sigma \rightarrow \tau = \perp) \end{aligned}$$

Using the definition of NOT once more we obtain:

$$\begin{aligned} \sigma[\text{NOT}(\text{MIGHT}(\text{NOT}\phi))]\tau \text{ iff } & \exists v \sigma[\phi]v \wedge \\ & (v \neq \sigma \rightarrow \tau = \perp) \wedge \\ & (v = \sigma \rightarrow \tau = \sigma) \end{aligned}$$

But this is just the definition of MUST.

To understand how examples like (E21) constrain the definition of negation, we need only consider alternative possible definitions which would be consistent with the classical picture of negation, but not preserve the logical duality of the dynamic modal operators. For instance, we could have defined negation by:

**Definition D9 (Pointwise Negation)**

$$\sigma[\#\phi]\tau \text{ iff } \tau = \{w \in \sigma \mid \{w\}[\phi]\perp\}$$

This negation is *pointwise* in that it looks at the individual worlds in the incoming state, and checks which ones are incompatible with the negated proposition. Using such a negation would not have affected examples (E16 – E20), since it is easily verified that it is equivalent with the earlier negation provided the negated proposition contains no epistemic modalities. But the entailment in (E21) would not have held, since the conclusion would no longer be a fixed point in the context set up by the premise, as is seen from the following sequence of updates:

$\{w_1, w_2, w_3\}[\text{MIGHT}\#\text{dC}]\{w_1, w_2, w_3\}[\#\text{MUST}\text{dC}]\{w_2, w_3\}$ . In fact we would have the unlikely equivalence:  $\#\text{MIGHT}\phi \equiv \#\text{MUST}\phi \equiv \#\phi$ . The original definition of negation, in which the negated proposition is evaluated with respect to the entire incoming context rather than just its parts, is clearly preferable.

## 2.5 A Presupposition Operator

The dynamic account above leads to a straightforward characterization of the CCP notion of presupposition. A context can only be updated with a sentence if the presuppositions of the sentence are already satisfied in the context. More formally:

**Definition D10 (Satisfaction)** A context  $\sigma$  satisfies a formula  $\phi$  iff  $\models_{\sigma} \phi$  (or equivalently  $\sigma[\phi]\sigma$ .)

**Definition D11 (Admission)** A context  $\sigma$  admits (can be updated with) a formula  $\phi$  iff there is a context  $\tau$  such that  $\sigma[\phi]\tau$ .

**Definition D12 (Presupposition)** A formula  $\phi$  presupposes a formula  $\psi$  iff for all contexts  $\sigma$ , if  $\sigma$  admits  $\phi$  then  $\sigma$  satisfies  $\psi$ .

In these terms, the formulae of UL carry no non-trivial presuppositions, since every context can be updated with any formula of UL. This is because the meanings of UL formulae define the equivalent of total functions on the domain of contexts. However, I will now extend UL with a single unary operator which allows us to restrict the incoming contexts for which an update is defined. In the resulting Partial Update Logic (PUL), some formulae will define the equivalent of partial functions on the domain of contexts.

**Definition D13 (Models for PUL)** *As for UL.*

**Definition D14 (Information in PUL)** *As for UL.*

**Definition D15 (Syntax of PUL)** *As for UL but with an additional unary operator,  $\partial$ , “the presupposition operator”.*

**Definition D16 (Semantics of PUL)** *As for UL but with the following additional clause:*

$$\sigma[\partial\phi]\tau \text{ iff } \tau = \sigma \wedge \sigma[\phi]\sigma$$

**Definition D17 (Entailment and Contextual entailment in PUL)** *As for UL.*

The presupposition operator  $\partial$  is reminiscent of the modal operator **MUST** defined previously. Given any formula  $\phi$  which itself contains no presuppositions, both  $\partial\phi$  and **MUST** $\phi$  have the same fixed points as  $\phi$ . That is, for all states  $\sigma$ , if  $\sigma[\phi]\sigma$  then  $\sigma[\partial\phi]\sigma$  and  $\sigma[\mathbf{MUST}\phi]\sigma$ . However, the two operators differ with respect to the non-fixed points of  $\phi$  — that is the states  $\sigma$  such that there is a state  $\tau \neq \sigma$  for which  $\sigma[\phi]\tau$ . The presupposition operator will not define a transition for such points. That is, if  $\sigma$  is a non-fixed point of  $\phi$  then  $\sigma$  does not *admit*  $\partial\phi$ . On the other hand, the necessity operator does define a transition for the non-fixed points: for any state  $\sigma$  which is not a fixed point of  $\phi$ , we have that  $\sigma[\mathbf{MUST}\phi]\perp$ . So the  $\partial$ -operator is importantly different from the **MUST**-operator in that all contexts admit **MUST** $\phi$ , whereas no consistent context for which updating with **MUST** $\phi$  would yield an absurd state admits  $\partial\phi$ .

I will now show how the presupposition operator can be used to reproduce the CCP treatment of presupposition as it concerns the examples from section 1.1. Consider (E3a) together with the suggested translation in (E3b):

- E3**
- a. Anna realizes that Bertha is hiding.
  - b.  $\partial bih$  AND  $cb\_a\_bih$
  - c.  $\partial bih$  AND  $cb\_a\_bih \models bih$

I have assumed two atomic propositions in this translation, *bih*, the proposition that *Bertha Is Hiding*, and *cb\_a\_bih*, the proposition that *Anna has Come-to-Believe that Bertha is Hiding*. The translation is given as a conjunction of the presupposition that Bertha is hiding together with the assertion that Anna has come to believe this. I have used a similar translation scheme for (E4):

- E4**
- a. Anna does not realize that Bertha is hiding.
  - b. NOT ( $\partial bih$  AND  $cb\_a\_bih$ )
  - c. NOT ( $\partial bih$  AND  $cb\_a\_bih$ )  $\models bih$

In these translations I have been intentionally naive with respect to the lexical semantics of “realizes”, and I would not wish to defend a general strategy of dividing the meaning of a mentalistic factive verb into one presupposed proposition and one asserted proposition about someone’s mental state. The same strategy seems particularly problematic in the case of the verb “regret”, a verb so intrinsically intensional that it is almost impossible to isolate a purely mental component for it in ordinary English. The best I could manage was the strange circumlocution “negative vibes arising from belief”. But for the moment it will be helpful to assume this division of meaning, as it will make the logical behaviour of presuppositions transparent. In [Be93b] I consider what aspects of my assumptions about lexical semantics are actually critical to the analysis of presupposition.

Crucially, both the entailments in (E3c) and (E4c) are valid in Partial Update Logic. If a formula *presupposes* (in the technical sense of definition D12) that Bertha is hiding, then (i) the formula entails that Bertha is hiding, and (ii) the negation of the formula entails that Bertha is hiding. In fact a negated formula always carries precisely the same presuppositions (i.e. the set of propositions which are *presupposed* in the above technical sense) as its positive counterpart. Thus PUL preserves the characteristic behaviour of presuppositions under negation.

Since an understanding of this behaviour is essential to the remainder of the thesis, I will go through the entailments in (E3c) and (E4c) in detail. Given a PUL term model restricted to the two propositions

$bih$  and  $cb\_a\_bih$ , information states will be subsets of the following four worlds:

$$\begin{aligned} A &= \{bih, cb\_a\_bih\} \\ B &= \{bih\} \\ C &= \{cb\_a\_bih\} \\ D &= \emptyset \end{aligned}$$

Firstly let us consider the denotation of the first sub-formula in (E3b),  $\partial bih$ . From definition D6, the meaning of  $bih$  is given by:

$$(8) \quad \sigma[bih]\tau \text{ iff } \tau = \{w \in \tau \mid bih \in w\}$$

Definition D16 allows us to calculate from this the denotation of  $\partial bih$ :

$$(9) \quad \sigma[\partial bih]\tau \text{ iff } \tau = \sigma \text{ and } \sigma[bih]\sigma \text{ iff } \tau = \sigma \text{ and } \forall w \in \sigma \text{ } bih \in w$$

This relation is equivalent to a set of pairs of states, where each state is expressed in terms of the four worlds  $A - D$ :

$$(10) \quad \llbracket \partial bih \rrbracket \equiv \{ \langle \{A, B\}, \{A, B\} \rangle, \langle \{A\}, \{A\} \rangle, \langle \{B\}, \{B\} \rangle, \langle \perp, \perp \rangle \}$$

Utilising definition D6 again, we can calculate the denotation of the whole formula:

$$(11) \quad \begin{aligned} \sigma[\partial bih \text{ AND } cb\_a\_bih]\tau &\text{ iff } \exists v \sigma[\partial bih]v \llbracket cb\_a\_bih \rrbracket \tau \\ &\text{ iff } \forall w \in \sigma \text{ } bih \in w \text{ and} \\ &\quad \tau = \{w \in \sigma \mid cb\_a\_bih \in w\} \end{aligned}$$

Again this can be written as an equivalent set of pairs:

$$(12) \quad \llbracket \partial bih \text{ AND } cb\_a\_bih \rrbracket \equiv \{ \langle \{A, B\}, \{A\} \rangle, \langle \{A\}, \{A\} \rangle, \langle \{B\}, \perp \rangle, \langle \perp, \perp \rangle \}$$

It can be easily verified that the formula entails both  $bih$  and  $cb\_a\_bih$ , since both of the possible output states (i.e. the right-hand members of the pairs in (12))  $\{A\}$  and  $\perp$  are fixed points of  $bih$  and  $cb\_a\_bih$ . Similarly we can calculate the denotation of the formula in (E4b):

$$(13) \quad \sigma[\text{NOT}(\partial bih \text{ AND } cb\_a\_bih)]\tau \text{ iff } \exists v \sigma[\partial bih \text{ AND } cb\_a\_bih]v \text{ and } \tau = \sigma \setminus v$$

Expressing this as a set of pairs, in (14) below, shows that the only information states that can result from updating with (E4b) are  $\{B\}$  and  $\perp$ :

$$(14) \quad \llbracket \partial bih \text{ AND } cb\_a\_bih \rrbracket \equiv \{ \langle \{A, B\}, \{B\} \rangle, \langle \{A\}, \perp \rangle, \langle \{B\}, \{B\} \rangle, \langle \perp, \perp \rangle \}$$

These two output states are once again fixed points of  $bih$ , so the entailment that  $bih$  is preserved. Indeed, all the possible input states (i.e. the left-hand members of the pairs in 14) are also fixed points of  $bih$ , which shows that the formula in (E4b) *presupposes*  $bih$  as well as entailing it. The same holds for (E3b) — all the possible inputs of the formula are fixed points of  $bih$ . However, (E4b) differs from (E3b) in that the output states of (E4b) are not fixed points of  $cb\_a\_bih$ , but of  $\text{NOT } cb\_a\_bih$ . Thus (E4a) is seen to presuppose that Bertha is hiding, and assert that Anna has not come to believe this.

Before proceeding to the remaining examples, a comment is in order about the translations in (E3b) and (E4b). Both of these translations involved an asymmetric conjunction, and derivation of the correct presuppositional behaviour depended crucially on the ordering of the conjuncts. This seems unnatural, for it is not obvious why there should be any preferred ordering of these conjuncts which essentially derive from the lexical semantics of a single verb rather than from any surface ordering of lexical items.

However, it is quite possible to introduce a second, *static* conjunction into PUL:

**Definition D18 (Static Conjunction)**

$$\sigma[\phi \& \psi]\tau \quad \text{iff} \quad \exists \rho, v \sigma[\phi]\rho \text{ and } \sigma[\psi]v \text{ and } \tau = v \cap \rho$$

If the dynamic conjunctions in (E3b) and (E4b) were replaced with this static conjunction, the same presuppositional behaviour would result, and the ordering of the conjuncts would be irrelevant. With this additional connective, a sensible strategy might be to translate surface occurrences of “and”, “but” and sentence sequencing in terms of the dynamic conjunction, and to make all other conjunctions static. My reasons for not pursuing this strategy here are pedagogical — one type of conjunction is enough for current purposes.

Let us now consider some more examples from section 1.1. The entailments in (E7c), (E8c) and (E9c) show that in PUL if a formula contains a presupposed proposition embedded within the antecedent of a conditional, or within an operator of epistemic possibility, or within another presuppositional construction, then the formula as a whole will entail the presupposed proposition. Thus the PUL analysis correctly predicts that all the three examples entail that Bertha is hiding.

In the translations, *awfb* is the proposition that *Anna Will Find Bertha*, and  $\text{NVB\_B\_}(\partial\_bih \& cb\_a\_bih)$  is the proposition that *Bertha has Negative Vibes* arising from her *Belief* that Anna realizes that Bertha is hiding:

- E7 a. If Anna realizes that Bertha is hiding, then she will find her.  
 b.  $(\partial\_bih \text{ AND } cb\_a\_bih) \text{ IMPLIES } awfb$   
 c.  $(\partial\_bih \text{ AND } cb\_a\_bih) \text{ IMPLIES } awfb \models bih$
- E8 a. Anna might realize that Bertha is hiding.  
 b.  $\text{MIGHT}(\partial\_bih \text{ AND } cb\_a\_bih)$   
 c.  $\text{MIGHT}(\partial\_bih \text{ AND } cb\_a\_bih) \models bih$
- E9 a. Bertha regrets that Anna realizes that she is hiding.  
 b.  $\partial(\partial\_bih \text{ AND } cb\_a\_bih) \text{ AND NVB\_B\_}(\partial\_bih \& cb\_a\_bih)$   
 c.  $\partial(\partial\_bih \text{ AND } cb\_a\_bih) \text{ AND NVB\_B\_}(\partial\_bih \& cb\_a\_bih) \models bih \text{ AND } cb\_a\_bih$

As a final illustration of PUL, we consider two examples where an embedded presupposition is not projected. Example (E10) shows the standard weak predictions of the CCP model with respect to presuppositions embedded in the consequent of a conditional. As shown in (E10c), the conditional in (E10a) does not entail that Bertha is in the attic. We only have the weaker entailment shown in (E10d), that if Bertha is not in the kitchen then she is in the attic. Note that whilst this behaviour seems appropriate for the conditional in (E10a), similar CCP predictions for other conditionals have often been criticized. In a later chapter it will be shown how this aspect of the CCP model can be defended.

- E10 a. If Bertha is not in the kitchen, then Anna realizes that Bertha is in the attic.  
 b.  $(\text{NOT } bC) \text{ IMPLIES } (\partial bA \text{ AND } cb\_a\_ba)$   
 c.  $(\text{NOT } bC) \text{ IMPLIES } (\partial bA \text{ AND } cb\_a\_ba) \not\models bA$   
 d.  $(\text{NOT } bC) \text{ IMPLIES } (\partial bA \text{ AND } cb\_a\_ba) \models (\text{NOT } bC) \text{ IMPLIES } bA$

Any PUL information state will admit (E12b), so that the sentence as a whole carries no presupposition. The reason should by now be familiar. The second clause is evaluated in the context set up by previous evaluation of the first clause. Since updating with the sub-formula *bih* results in a context containing only worlds in which Bertha is happy, and since the sub-formula corresponding to the second clause,  $\partial\_bih \text{ AND } cb\_a\_bih$  is defined on all such contexts, the whole formula will be admitted by any incoming context. And if a formula is admitted by any context, then it has no presuppositions.

- E12 a. Perhaps Bertha is hiding and Anna realizes that Bertha is hiding.  
 b.  $\text{MIGHT}(bih \text{ AND } \partial\_bih \text{ AND } cb\_a\_bih)$   
 c.  $\text{MIGHT}(bih \text{ AND } \partial\_bih \text{ AND } cb\_a\_bih) \not\models bih$

## The N-bird Problem

It should be clear that the theory of interpretation underlying the CCP model of presupposition can be independently motivated in terms of extraneous semantic phenomena. I have considered only one of these phenomena, namely the behaviour of epistemic modality. However, the treatment of epistemic modality is far from being the only non-presuppositional motivation for a dynamic semantics. A far better established motivation is the treatment of donkey and discourse anaphora, and in the following chapters I make some attempt at the harder “3-bird” problem, combining a treatment of presupposition and modality with a Groenendijk & Stokhof-style treatment of anaphora. An account of presupposition along the lines I have sketched also has potential for a dynamic treatment of focus, as shown in [Krif92]. But the search must continue for the *semanticist’s stone*, that single theory of information with which we could knock any arbitrary collection of problems in the theory of Natural Language meaning straight out of the sky.

## Chapter 3

# A Bit Like English

### 3.1 Introduction

The system (ABLE) to be described in this chapter and the next, which will form the basis of the fragment to be defined in chapter 5, brings together ideas from many sources. The account of presupposition, is a further development of that in [Be92], which is based on the earlier work of Heim [He83] and Karttunen (eg. [Kar73]); the theory of anaphora descends from that of Kamp [Kam81], Heim [He82], Groenendijk and Stokhof [GS91a] and Dekker [De92]; the dynamic approach to quantification is based on the work of Dekker [De92], Chierchia [Ch92] and Groenendijk and Stokhof [GS91a, GS91b]; and the account of epistemic modality, which extends Veltman's [Ve91] *might* operator as incorporated in UL and PUL systems of chapter 2 to the predicate level, arose from collaboration with those of the above who are Amsterdam colleagues<sup>2</sup>.

I will follow [Mu90] in preferring type theory over **IL** as a formalism appropriate to the embedding of dynamic semantics in an otherwise Montagovian theory of meaning. **IL** is a *designer* logic. Montague's aim was to build a formalism that reflected his Fregean view of meaning, and intertwined ideas from modal logic and type theory so as to reflect that view. Thus the underlying formalism Montague created is inextricably tied to the application Montague had in mind. For that very reason, any attempt to model a qualitatively different account of meaning using **IL** is fraught with problems. More particularly, in the last decade or so, much effort has gone into theories of meaning which are *partial* and/or *dynamic*, but the theory of meaning Montague had in mind was both *total* and *static*.

For this reason, it has been generally recognized that **IL** was not handed down on stone tablets, and is open to modification or replacement. Thus Muskens [Mu90] has introduced a variant of type theory to model partiality, and Groenendijk and Stokhof [GS91b] have utilised a variant of **IL** developed by Janssen in order to model dynamics. However, I think it is clear that whilst some alternative to **IL** is required, the needless multiplication of semantic formalisms is to be avoided. Fortunately, one does not have to look far in order to find an existing formalism adequate to my purposes: classical type theory, apart from having a much cleaner logic than **IL**, is well suited to modelling the dynamics of natural language.

To be more precise, the advantage of type theory over **IL** in the remainder of this thesis is as follows. I will want to reason formally about information and information states, and possible worlds will be involved in the specification of these states. However, in **IL** it is difficult to reason explicitly about possible worlds, since *intensional* objects are cloaked by special syntactic restrictions which prevent the use of objects of type *s* (i.e. possible worlds). In type theory, there is no restriction as to which types can be represented syntactically by constants and variables, and the full apparatus of functional abstraction and application is available over all types. Thus, in type theory, it is possible to be explicit about possible worlds (and other aspects of information states) where in **IL** one would have to use *ad hoc* and round-about trickery.

This chapter will be taken up with an initial definition of the semantics for a language A Bit Like English, or ABLE. ABLE is a first order language in the tradition of DPL, EDPL and KPL<sup>3</sup>. That

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<sup>2</sup>Thus Dekker, Groenendijk, Stokhof and Veltman.

<sup>3</sup>These abbreviations are for, respectively, the Dynamic Predicate Logic of Groenendijk and Stokhof [GS91a], the



is to say, it is a dynamic logic based around a language sufficiently close to English that those with imagination and faith can easily believe that formulae are compositionally derivable from the English sentences they are supposed to represent.

**Definition D19 (Syntax of ABLE)** *Given a set of predicates  $\mathcal{P}$  consisting of unary predicates  $\mathcal{P}^1$ , binary predicates  $\mathcal{P}^2$  and attitude predicates  $\mathcal{P}^a$ , a language  $\mathcal{L}_{\text{ABLE}}^{\mathcal{P}}$  is given by recursion over the following set of rewrite rules, where all brackets are optional:*

$$\begin{aligned}
\text{DM} &\implies 1 \mid 2 \mid \dots \\
\text{DET} &\implies \text{SOME} \mid \text{THE} \mid \text{EVERY} \mid \text{NO} \mid \text{MOST} \mid \text{FEW} \mid \text{EXACTLY-ONE} \\
\text{FORM} &\implies \mathcal{P}^1.\text{DM} \mid \mathcal{P}^2.\text{DM}.\text{DM} \mid \mathcal{P}^a.\text{DM}.\text{(FORM)} \mid \\
&\quad (\text{DM IS DM}) \mid (\text{DET}.\text{DM}.\text{FORM}.\text{FORM}) \mid \\
&\quad (\text{FORM AND FORM}) \mid (\text{FORM OR FORM}) \mid (\text{FORM IMPLIES FORM}) \mid \\
&\quad (\text{NOT FORM}) \mid (\text{MIGHTFORM}) \mid (\text{MUSTFORM})
\end{aligned}$$

In the following sections of this chapter I will firstly discuss some general and meta-theoretical considerations, before tackling the various basic components of ABLE one at a time. In chapter 4 it will be shown how this basic apparatus can be applied to the study of presupposition and epistemic modality. Those who lack imagination or faith will hopefully be appeased by chapter 5, where it will be shown how ABLE can be utilised in the definition of a compositional grammar fragment.

## 3.2 Some Metatheory

Throughout the chapters 3 – 5, type theory will play much the same role as IL does in PTQ: it will be the vehicle for the formal expression of meaning. In chapters 3 and 4 it will be used to specify the semantics of ABLE, and in chapter 5 it will be used to specify the semantics of English. ABLE itself being a formal language, it is unusual to be so explicit about the semantic meta-language, since the logician’s *meta-lingua franca* — if the collision of greek and latin may be excused — which consists of a rather ill-defined mixture of first (or higher) order logic, (naive) set theory, and a sprinkling of technical-ese (“where  $x$  does not occur free in the formula  $\phi$ ”), typically suffices. For the moment I must beg the reader’s indulgence: my motivation for having used type theory to give the semantics of ABLE will be made clear in chapter 5.

**Definition D20 (The Metalanguage)** *Ty<sub>3</sub> is a three sorted type theory along the lines of Gallin’s Ty<sub>2</sub><sup>4</sup> [Gal75], which itself is a reformulation of Russell’s Theory of Types, having the normal apparatus of abstraction, function application, existential and universal quantification over objects of every type, and standard truth functional connectives, as well as a number of distinguished constants to be introduced in the remainder of the thesis. The symbol “.”, which will be used left-associatively, will denote function application.*

*The types are given by the category TYPE in the following recursion, in which  $d, e, w$  and  $t$  are, respectively, the types of discourse referents, individuals, possible worlds and truth values:*

$$\begin{aligned}
\text{BASIC} &\implies d \mid e \mid w \mid t \\
\text{TYPE} &\implies \text{BASIC} \mid \langle \text{TYPE}, \text{TYPE} \rangle
\end{aligned}$$

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Eliminative Dynamic Predicate Logic of Dekker [De93], and finally my Kinematic Predicate Logic [Be92]

<sup>4</sup>Also see [GS84] for an application of Ty<sub>2</sub>, and yet another motivating discussion.

**Definition D21 (Models)** A Model  $\mathcal{M}$  for  $Ty_3$  is a tuple  $\langle W, E, \|\cdot\| \rangle$  where  $W$  and  $E$  are non-empty. Each type  $\alpha$  is associated with a domain  $\mathcal{D}_\alpha$ , with  $\mathcal{D}_w = W$ ,  $\mathcal{D}_a = \mathbf{N}$  (the natural numbers),  $\mathcal{D}_e = E$ ,  $\mathcal{D}_t = \{\text{true}, \text{false}\}$  and domains for the complex types being built up recursively from the basic domains such that for all types  $\alpha$  and  $\beta$ , the domain  $\mathcal{D}_{(\alpha, \beta)}$  is the set of functions from  $\mathcal{D}_\alpha$  to  $\mathcal{D}_\beta$ . The interpretation function  $\|\cdot\|$  is a mapping from constants of type  $\alpha$  to elements of the domain  $\mathcal{D}_\alpha$ , with the distinguished constants  $\top$  and  $\perp$  of type  $t$  mapped onto true and false respectively, and any discourse marker  $i$  (where  $i$  must be in  $\mathbf{N}$ ) mapped to  $i$ .

Rather than independently defining models for ABLE, I will simply let  $Ty_3$  models also be ABLE models. Using  $Ty_3$  models for a simple first order language might be felt to be overkill, but will be formally adequate.

ABLE formulae are to be interpreted dynamically, as functions from information states to information states. Following [St79] and [Ve91], factual information will be encoded in terms of shrinking sets of possible worlds. At a given point in a conversation the information state of a participant will be partially characterised by a set of worlds, intuitively those worlds which are compatible with everything that has been established up until that point.

A conversational participant must do a certain amount of bookkeeping in order to keep track of what is being talking about. For current purposes it will be assumed that what is being talked about — the topics of conversation — are individuals or groups of individuals, and not, for instance, properties or propositions. Further, as the reader might have expected, it will be assumed that each participant keeps track of the conversation using a set of *discourse markers*. On encountering a new discourse topic, for instance introduced by an indefinite noun phrase, a conversational participant assigns a discourse marker to that topic.

I will take a discourse *referent* or *topic* to be a public entity, something shared by all the conversational participants: the reader should see [De93] for some indication of how a system like the one developed in his paper lends itself to an account of discourse referents as public objects. By contrast I will treat discourse *markers* as personal record-keeping devices private to each conversational participant. In fact, discourse markers, as the reader may have gathered from the above definition of  $Ty_3$  models, will simply be natural numbers. Thus the privacy of discourse markers amounts only to the absence of any assumption that different participants use the same markers.

The relationship between discourse markers and the objects in the model is mediated by what I will call an *extended sequence*, a simple development of the notion of a *sequence* employed by Heim: where no ambiguity is introduced I will use the terms *sequence* and *extended sequence* interchangeably. A Heimian sequence is a partial function from discourse markers to objects. An extended sequence is just a total function from discourse markers to sets of objects, and this set will be referred to as the *sequence valuation* of the discourse marker by the sequence.

**Definition D22 (Extended Sequences)** Any object of type  $\langle d, \langle e, t \rangle \rangle$  is an *extended sequence*, and the type of such an object will be abbreviated as  $\sigma$ .

The sequence valuation of an unused marker is the empty set, and other markers are mapped onto singleton or larger sets according to whether they represent one or many objects. Strictly I should not talk of sets of objects, since the meta-theory being used is usually interpreted using functions rather than sets, but I find that some things are best understood in terms of functions, and some in terms of sets. So I will continue to talk about sets of this or that, but when I do so I will usually mean not *sets of this or that* but rather *the characteristic function corresponding to a set of this or that*, that is a function from the domain of *this* or *that* into the domain of truth values. I will be similarly loose in my use of the word relation, so that for example functions from *this* into functions from *that* to truth values might be called relations between *this* and *that* according to my mood at time of writing.

Extended sequences can be equated with the *states* of Jeroen Groenendijk and Martin Stokhof's Dynamic Montague Grammar (DMG) [GS91b] or Reinhard Muskens' Logic of Change (LoC) [Mu90], since in these systems the only function of an information state is to keep track of the values of discourse markers. A first difference to note is that in DMG and LoC, *states* are total assignment functions, whereas here extended sequences are to be interpreted as partial assignment functions (ie. a marker being mapped onto the empty set is interpreted as equivalent to the marker not being in the domain of the function) albeit encoded in a space of total functions. A second significant difference is that in

both DMG and LoC states rather than discourse markers are ontologically primary: there is a basic domain of states in the models (type  $s$ ), and discourse markers are interpreted as functions from states to individuals (type  $\langle s, e \rangle$ ), although in DMG this is the *intensional* type corresponding to a discourse marker, and its *extension* simply has type  $e$ ).

The motivation for states being basic in DMG and LoC does not appear to be philosophical but technical, and stems from consideration of anaphora to objects of types other than that of individuals, although neither DMG nor LoC provides any treatment of non-individual-typed anaphora. If discourse markers are basic, then, in order to keep track of information about high-typed objects in a completely general way, the notion of a state has to be complicated so as to provide mappings from discourse markers to the high-typed domains. Indeed, if we really want the potential to keep track of arbitrarily typed objects, then there will be no single type of state up to the job, since the type hierarchy is infinite but individual types are finite.

The solution suggested by Theo Janssen [Ja84], which is to assume a basic type of states, and make discourse markers do the work, is quite general. In order to keep track of objects of some type  $\alpha$ , simply use discourse markers having (intensional) type  $\langle s, \alpha \rangle$ . That such an elegant sleight of the mathematical hand should yield an immediate and general solution to the problem seems to good to be true, and, indeed, it is. A number of technical difficulties arise, most of which can be solved using some relatively simple constraints on the structure of the space of information states, but the most serious of which requires rather unappetising (although not unreasonable) restrictions to be placed on the typed models. The problem is that in a standard (generalised) model for type theory (or Montague's IL or Janssen's DIL) there will not in general be enough states to store information about arbitrarily typed domains. A simple example is the domain of objects of type  $\langle s, t \rangle$ : in a non-trivial model, the cardinality of this domain must be greater than the cardinality of the domain of states,  $s$ . Thus for any discourse marker  $d$  of type  $\langle s, \langle s, t \rangle \rangle$ , there will always be an object  $S$  of type  $\langle s, t \rangle$  such that there is no state  $s$  having the property  $d(s) = S$ . Clearly this is undesirable, since the whole point of discourse markers is that they can stand for any given object in the relevant domain. The solution offered by Janssen [Ja90] involves the construction of canonical models. Although there is a sense in which discourse markers will still not be able to take arbitrary values, this becomes irrelevant for most purposes, since for any assignment of a value to a discourse marker expressible in the language, there will always be an appropriate state in a canonical model.

But is such a mathematically general approach really motivated for the analysis of natural language? I know of no argument to the effect that there are anaphoric expressions in natural language of arbitrary type. I can conceive of only a handful of relevant types, and any claim for the existence of anaphora other than to individuals, properties or propositions stands in need of defence. Yet even if the number of anaphoric domains was thus limited, some might still prefer Janssen's approach, since otherwise an information state combining separate mappings of discourse markers into even three distinct domains would have a somewhat messy type.

However, such ugliness cannot not really be removed: Janssen's technique merely serves to sweep the mess under the meta-theoretical carpet. Consider the case where we are only interested in anaphoric reference to individuals. Whereas if discourse markers are assumed to be basic, the ordinary definition of a typed model will guarantee that the functional type of states contains enough states to allow for arbitrary mappings of discourse markers onto the relevant entities, if states are assumed to be basic there is no such guarantee. As mentioned above, special postulates have to be used in order to force the state space to have the right structure. And what is the right structure? Well, for every discourse marker and individual, there must be a state which takes the marker onto the entity. In other words, the state space has a structure equivalent to the functional space from the domain of markers to the relevant domain of entities (although this equivalence need not be an isomorphism, since there could be extra *irrelevant* states, and thus several states corresponding to the same function from markers to entities). So the space of states must have a certain sort of structure, and the only question is whether that structure is encoded in type theory (or whatever) as is done here, or as a restriction on type models, as in DMG and LoC.

A final consideration regarding this issue is the fact that in English, and I suppose this to be a universal fact about natural languages, not only individual anaphora, but also kind anaphora, VP anaphora and propositional anaphora can be mediated by noun-phrases, typically pronominal. Thus the NP "one" can be anaphoric to a kind, and the NP "it" can be anaphoric to a VP ("Do it!") or a proposition ("Believe

it!”). In order to analyse such anaphora some type shifting operation might be required for relating properties and propositions to their individual correlates. And if, at the point of anaphoric resolution, the individual correlate of a previously mentioned property or proposition is invoked, then why not simply assume that the anaphoric information concerned the individual correlate all along? On the other hand, in property theories the whole question is commonly avoided, since in such theories there is commonly no assumption of a typed ontology: properties, propositions and physical entities are all just different types of individual. One way or another, I think it possible, and even sensible, to defend an account of anaphoric information in which no types other than that of individuals (or sets of them) are involved.

Since, technical justification aside, I have given an essentially psychological explanation of the nature of discourse markers, and have now defined a class of mappings from discourse markers into some other part of the  $Ty_3$  model, the reader could legitimately ask whether I also intend models to have psychological significance. In order to avoid being sidetracked into a runaway philosophical discussion of the ontological foundations of model theoretic semantics, I will give a simple answer: yes, models must have psychological significance. This thesis, in common with the vast bulk of the recent literature on dynamic semantics, has pretensions towards presenting an account of language processing, for processing considerations are the primary philosophical motivation for the choice of a dynamic framework. But the account can only make sense as a processing model if the human mind can be said to maintain a structure homomorphic to that given in the earlier, formal definition of a model. In short, I do not know what models really are, but I am forced to claim that everybody has one.

Now — apologies for bringing the reader back from philosophical wonderland with such a bump — I will define some distinguished constants for talking about extended sequences. Note that throughout this thesis I will assume that the  $Ty_3$  quantifiers  $\forall$  and  $\exists$  outscope connectives, so that for instance  $\exists x \phi \wedge \psi$  will mean  $\exists x (\phi \wedge \psi)$ . In case of ambiguity I will use round brackets to demarcate the scope of quantifiers, but the scope of lambda abstracts will always be indicated with square brackets.

### Meaning Postulate MP1

- (1)  $\odot_\sigma = \lambda D_d \lambda x_e [\perp]$
- (2)  $domain_{\langle \sigma, \langle d, t \rangle \rangle} = \lambda f_\sigma \lambda D_d [\exists x_e f.D.x]$
- (3)  $\succeq_{\langle \sigma, \langle \sigma, t \rangle \rangle} = \lambda f_\sigma \lambda g_\sigma [\forall D_d domain_{\langle \sigma, \langle d, t \rangle \rangle}.g.D \rightarrow g.D = f.D]$

This meaning postulate, and those that follow, should be thought of as constraints which  $Ty_3$  models should obey if they are to suit the purpose of defining a semantics for ABLE. A model which obeys all the meaning postulates to be given I will call a *suitable* model. Thus in any suitable model  $\odot_\sigma$  is interpreted as the empty sequence, that is the function mapping every discourse marker onto the empty set of individuals, and  $domain_{\langle \sigma, \langle d, t \rangle \rangle}$  is a function from any sequence onto the set discourse markers in its domain. Thus we have that  $domain.\odot = \lambda D_d [\perp]$ . The constant  $\succeq_{\langle \sigma, \langle \sigma, t \rangle \rangle}$  can be thought of as a binary relation: using infix notation,  $f \succeq g$  can be read as “ $f$  is an extension of  $g$ ”, meaning that for all discourse markers in the domain of sequence  $g$ , the two sequences give the same interpretation, but  $f$  may also assign values to some additional discourse markers.

Since infix form often makes binary relations more perspicuous, I will adopt the following convention: if  $+$  is a binary operation of type  $\langle \alpha, \langle \alpha, t \rangle \rangle$ , where  $\alpha$  is any type except  $t$ , and if  $A$  and  $B$  are of type  $\alpha$ , then  $A + B$  will be used to mean  $+.A.B$ . Furthermore, iteration will be allowed, such that if  $C$  is also of type  $\alpha$  then  $+.A.B \wedge +.B.C$  may be abbreviated as  $A + B + C$ . Sometimes the notation  $A\{+\}B\{+\}C$  will be used to show that  $+$  is being treated as an infix operator. But beware! Although this is the standard convention of iteration within physical science, where statements like “ $x \geq y \geq z$ ” are normal, it is non-standard in logic. The logician’s favourite binary relations are relations between truth-denoting objects, and “ $\phi \vee \psi \vee \chi$ ” for example, although ambiguous, would never be used to mean “ $(\phi \vee \psi) \wedge (\psi \vee \chi)$ ”; this is the reason for the exceptive clause saying that  $\alpha$  cannot be  $t$ . For the truth-functional connectives in  $Ty_3$ , as for the ABLE connectives, the standard logical conventions will be assumed.

Information states are defined as mappings from possible worlds onto sets of extended sequences. Ignoring for a moment the differences between my definition of an extended sequence and other authors’ definitions of sequences, assignments or partial assignments, there remains only a superficial difference between making information states into sets of pairs of worlds and sequences as in [He82], or into functions

from worlds to sets of extended sequences as here, or into functions from worlds to total assignments as in [EC92]. The additional slight variation on Heim's original notion of a context is of course just a by-product of the use of functional type theory as a meta-language. In fact I will sometimes prefer to talk of a pair of a world  $w$  and an extended sequence  $f$  being contained in some state  $I$ , rather than saying that  $I$  maps  $w$  to a set containing  $f$ , and I will sometimes talk of one such pair being an extension of another, meaning that each pair involves the same possible world, but that the sequence in one pair is an extension of the sequence in the other pair.

**Definition D23 (Information States in ABLE)** *Any object of type  $\langle w, \langle \sigma, t \rangle \rangle$  is an information state, and the type of such objects will be abbreviated as  $\iota$ .*

It would be natural at this point to explore the structure of the space of ABLE information states in detail. Rather than doing this, I refer the reader to the discussion in [De93], in which the algebra of a closely related state space, that of Dekker's EDPL, is examined more fully. However, I will introduce some constants which make ABLE information easier to manipulate:

### Meaning Postulate MP2

- (1)  $\cap_{\langle \iota, \langle \iota, \iota \rangle \rangle} = \lambda I_i \lambda J_i \lambda w_w \lambda f_\sigma [I.w.f \wedge J.w.f]$
- (2)  $\cup_{\langle \iota, \langle \iota, \iota \rangle \rangle} = \lambda I_i \lambda J_i \lambda w_w \lambda f_\sigma [I.w.f \vee J.w.f]$
- (3)  $\setminus_{\langle \iota, \langle \iota, \iota \rangle \rangle} = \lambda I_i \lambda J_i \lambda w_w \lambda f_\sigma [I.w.f \wedge \neg J.w.f]$
- (4)  $\sqsubseteq_{\langle \iota, \langle \iota, \tau \rangle \rangle} = \lambda I_i \lambda J_i [\forall w \forall f J.w.f \rightarrow I.w.f]$
- (5)  $\overline{\top}_\iota = \lambda w_w \lambda f_\sigma [f_\sigma = \odot_\sigma]$
- (6)  $\perp_\iota = \lambda w_w \lambda f_\sigma [\perp]$
- (7)  $\omega\text{-set}_{\langle \iota, \langle w, t \rangle \rangle} = \lambda I_i \lambda w_w [\exists f_\sigma I.w.f]$
- (8)  $p\text{-domain}_{\langle \iota, \langle d, t \rangle \rangle} = \lambda I_i \lambda D_d [\exists w_w \exists f_\sigma I.w.f \wedge \text{domain}_{\langle \sigma, \langle d, t \rangle \rangle}.f.D]$
- (9)  $t\text{-domain}_{\langle \iota, \langle d, t \rangle \rangle} = \lambda I_i \lambda D_d [\forall w_w \forall f_\sigma I.w.f \rightarrow \text{domain}_{\langle \sigma, \langle d, t \rangle \rangle}.f.D]$

The interpretation of the first four constants, for which infix notation will be used, should be obvious: thinking of states as sets of world-sequence pairs,  $\cap$ ,  $\cup$ ,  $\setminus$  and  $\sqsubseteq$  are just the standard set-theoretic operators. The constants  $\overline{\top}$  and  $\perp$  represent respectively the *zero* information state, which may be thought of as the established common ground at the beginning of a conversation, and the *absurd* information state, which is reached whenever an information state is updated with contradictory propositions. The function  $\omega\text{-set}$  associates with each information state a set of possible worlds, intuitively those worlds which are compatible with all the information up to that point in the discourse. Thus  $(\omega\text{-set}.\overline{\top})$  is the set of all possible worlds, and  $(\omega\text{-set}.\perp)$  is the empty set of worlds — for there are no possible worlds that are compatible with contradictory information.

The constants  $p\text{-domain}_{\langle \iota, \langle d, t \rangle \rangle}$  and  $t\text{-domain}_{\langle \iota, \langle d, t \rangle \rangle}$  are analogues of  $\text{domain}_{\langle \sigma, \langle d, t \rangle \rangle}$ , which was introduced above. Given a state  $I$ ,  $p\text{-domain}.I$  denotes the set of discourse markers which are at least partially defined in  $I$ . If  $D$  is a discourse marker, then  $p\text{-domain}.I.D$  will hold just in case there is some world associated with a sequence which has  $D$  in its domain. The total domain of  $I$ , the set of discourse markers which have a value in every world-sequence pair in  $I$ , is given by  $t\text{-domain}.I$ . This thesis will mostly concern itself with totally-defined discourse markers, so that regarding most information states which arise in examples, the partial and total domains will be identical. However, the possibility of partially defined discourse markers do arise.

Following Heim, I will call the denotation of an ABLE formula a *context change potential*:

**Definition D24 (Context Change Potentials)** *Any object of type  $\langle \iota, \langle \iota, t \rangle \rangle$  is a context change potential (CCP), and the type of such objects will be abbreviated as  $\pi$ .*

ABLE formulae are thus relations between information states, and have denotations of the form  $\lambda I \lambda J [p_\iota]$ . Such expressions have the by now obvious interpretation that  $\lambda I$  is an abstraction over possible input states, and  $\lambda J$  over possible outputs. If an ABLE formula has denotation  $F$  and it holds for some  $I$  and  $J$  that  $I\{F\}J$ , we say that in state  $I$  the formula provides an *update* to state  $J$ . In fact it will hold that no ABLE formula denotes more than one update from a given input state, but it will

be argued that expressions of natural language should be thought of as having such relational meanings, for the ambiguity and underspecificity of natural language often means that there is more than one way in which a given expression could be used to update an information state.

In effect this non-determinism will be built into the translation from natural language into ABLE formulae, so that ABLE could be viewed as, in that favourite phrase of the last few decades “a disambiguated language of logical form”. However, as you might expect given the Montagovian pretensions of this thesis, there will be no claim that ABLE bears a special relationship, like identity for example, to any language of mental representation. The motivation for putting the non-determinism into the translation function, or more properly the translation *relation*, is methodological. Keeping ABLE denotations deterministic permits the definition of a relatively clean logic over the ABLE language, and thus facilitates the process of turning what I think is the semanticist’s primary source of data, namely natural language entailments, into intuitions about how natural language expressions must be translated. So we now turn to the problem of defining a notion of entailment for a language which claims to describe not facts about the world, but information change in agents. Consider a notion of entailment used in [Ve91]:

A formula entails another if after updating any state with the first, updating with the second adds no new information.

In ABLE, as in Heim’s File Change Semantics and Dekker’s Eliminative Dynamic Predicate Logic, there are two ways in which information can grow. Firstly extra constraints can be learnt concerning the interpretation of predicates and whatever discourse markers are in use, and secondly new discourse markers may be added. It seems that it is the first type of information and not the second which is relevant to our intuitive notion of natural language entailment, as is shown in the following example discourse<sup>5</sup>:

- E22 a. There are three frogs, and exactly two are in the water.  
 b. Therefore one of the frogs is out of the water.  
 c. Obviously, it is not swimming.

The argument from (a) to (b) is valid, since in any world in which the first is true the second will also be true. However, (b) also introduces a new discourse referent. This is shown by considering (a) followed directly by (c), a strange discourse indeed. It is clear that (a) does not by itself license the pronoun in (c). However, the full discourse of (a)+(b)+(c) is quite natural. Thus (b) augments the context set up by (a) even though (b) is entailed by (a), and more generally we must conclude that an entailed sentence can introduce certain types of new information. A suitably modified version of Veltman’s entailment, which is essentially that used in [GS91a] and [De92], is thus:

A formula entails another if after updating with the first, updating with the second adds no new information except for the possible introduction of new discourse referents.

The formalisation of this notion hinges on the possibility of differentiating between different types of information. The following postulate defines a notion of *closure* with respect to anaphoric potential. The anaphoric closure of a CCP  $F$ , written  $\downarrow F$ , is a purely eliminative CCP: it may remove some of the states in the input, but it will not introduce any new discourse markers. Given states  $I$  and  $J$ , and a CCP  $F$ , the formula  $I\{\downarrow F\}J$  will be true if and only if some state  $K$  can be obtained by updating  $I$  with  $F$ , and the output,  $J$ , is that subset of the world-sequence pairs in the input,  $I$ , which have extensions in  $K$ :

### Meaning Postulate MP3

$$\begin{aligned} \downarrow_{\langle\pi,\pi\rangle} &= \lambda F_{\pi}\lambda I_i\lambda J_i; [\exists K_i I\{F\}K \wedge \\ &\quad J = \lambda w_w\lambda f_{\sigma} [I.w.f \wedge \exists g_{\sigma} g \succeq f \wedge K.w.g]] \end{aligned}$$

Such a notion of closure does not make sense for arbitrary CCPs. For instance, if a CCP were to denote a *downdate* (a loss of information), such that the  $w$ -set of the output could be a strict superset

<sup>5</sup>This discourse is reminiscent of Partee’s *marble* examples.

of the  $\omega$ -set of the input, then the anaphoric closure of the CCP would not preserve this property: the anaphoric closure of a CCP always denotes an update, whereby the output  $\omega$ -set is a (not necessarily strict) subset of the input  $\omega$ -set. Thus, to be sure that the notion of anaphoric closure is appropriate to ABLE CCPs, in Appendix A the following fact will be proved:

**Fact F2 (Eliminativity)** *For any ABLE formula,  $\phi$ , and states  $I$  and  $J$ ,  $I \llbracket \phi \rrbracket J$  if and only if  $J$  contains only extensions of world-sequence pairs in  $I$ .*

If updating a context with the closure of some CCP would have no effect, then the context will be said to *satisfy* the CCP:

#### Meaning Postulate MP4

$$\text{satisfies}_{\langle \iota, \langle \pi, \iota \rangle \rangle} = \lambda I \lambda F [I \{ \downarrow F \} I]$$

Infix notation will be used for relations between states and CCPs, producing formulae like  $I$  *satisfies*  $F$ . It is now simple to define a binary relation *entails* which holds between two CCPs just in case any update with the first produces a state which *satisfies* the second, and in terms of this constant to define an entailment relation holding directly between ABLE formulae, as opposed to their denotations.

#### Meaning Postulate MP5

$$\text{entails}_{\langle \pi, \langle \pi, \iota \rangle \rangle} = \lambda F_{\pi} \lambda F'_{\pi} [\forall I_{\iota} \forall J_{\iota} \\ I \{ F \} J \rightarrow J \text{ satisfies } F']$$

#### Definition D25 (Entailment in ABLE)

$$\phi \models_{\text{ABLE}} \psi \quad \text{iff} \quad \llbracket \phi \rrbracket \{ \text{entails} \} \llbracket \psi \rrbracket \text{ is valid on} \\ \text{the class of suitable models.}$$

This thesis is largely concerned with the entailments of presupposing formulae, and it is to the notion of presupposition that we now turn. The denotations of ABLE formulae are functions from input states to sets of output states, and there may be some input states which are mapped onto the empty set of output states. For such inputs, the ABLE formula provides no update. Those states for which an ABLE formula does provide an update will be said to *admit* the formula, and in terms of this property *presupposition* is defined. One ABLE formula *presupposes* another when the only states from which the first formula provides an update are those in which the second is satisfied:

#### Meaning Postulate MP6

- (1)  $\text{admits}_{\langle \iota, \langle \pi, \iota \rangle \rangle} = \lambda I_{\iota} \lambda F_{\pi} [\exists J_{\iota} I \{ F \} J]$
- (2)  $\text{presupposes}_{\langle \pi, \langle \pi, \iota \rangle \rangle} = \lambda F \lambda F' [\forall I_{\iota} I \text{ admits } F \rightarrow I \text{ satisfies } F']$

**Definition D26 (Presupposition)** *An ABLE formula  $\phi$  presupposes an ABLE formula  $\psi$  if and only if  $\text{presupposes}_{\langle \pi, \langle \pi, \iota \rangle \rangle} \cdot \llbracket \phi \rrbracket \cdot \llbracket \psi \rrbracket$  is true in every suitable model.*

The ABLE notions of presupposition and entailment are logically independent, in the sense that not all presuppositions of a formula are entailments, and not all entailments are presuppositions. However, for the class of non-modal formulae (i.e. the sub-language of ABLE not involving MIGHT), the presuppositions of a formula will form a strict subset of the entailments.

To finish this section I will introduce one last meta-theoretic notion, *consistency*. MP7 says that a state  $I$  is *consistent-with* a CCP  $F$  just in case it is possible to update  $I$  with  $F$  and not end up in the absurd information state. A *consistent* formula is just one for which there is some state which it is *consistent-with*.

#### Meaning Postulate MP7

- (1)  $\text{consistent-with}_{\langle \iota, \langle \pi, \iota \rangle \rangle} = \lambda I_{\iota} \lambda F_{\pi} [\exists J_{\iota} I \{ F \} J \wedge \neg (J = \perp)]$
- (2)  $\text{consistent}_{\langle \pi, \iota \rangle} = \lambda F_{\pi} [\exists I_{\iota} \text{ consistent-with}_{\langle \pi, \langle \iota, \iota \rangle \rangle} \cdot I \cdot F]$

Symbol	Type	Interpretation
$d$	$d$	discourse markers
$D$	$d$	variables over discourse markers
$a, b, c$	$e$	individual constants
$x, y, z$	$e$	variables over individuals
$A, B, C$	$\varepsilon$ $= \langle e, t \rangle$	group constants
$X, Y, Z$	$\varepsilon$	variables over groups
$w$	$w$	variables over worlds
$f, g, h$	$\sigma$ $= \langle d, \langle e, t \rangle \rangle$	variables over extended sequences
$I, J, K$	$\iota$ $= \langle w, \langle \sigma, t \rangle \rangle$	variables over information states
$F$	$\pi$ $= \langle \iota, \langle \iota, t \rangle \rangle$	variables over the denotations of ABLE formulae (CCPs)
$\mathcal{P}$	$\rho$ $= \langle d, \pi \rangle$	variables over dynamic properties
$\mathcal{Q}$	$\langle \rho, \langle \rho, \pi \rangle \rangle$	variables over dynamic generalized quantifiers

Table 1: Types of Meta-variables

**Definition D27 (Consistency)** An ABLE formula  $\phi$  is **consistent** if and only if  $\text{consistent}_{\langle \pi, t \rangle} \cdot \llbracket \phi \rrbracket$  is true in every suitable model. An ABLE formula  $\phi$  is **consistent with an information state  $I$**  if and only if  $\text{consistent-with}_{\langle \pi, \langle \iota, t \rangle \rangle} \cdot \llbracket \phi \rrbracket \cdot I$  is true in every suitable model.

The following sections will be concerned with the definition of the semantics of ABLE formulae in terms of  $\text{Ty}_3$ . Having introduced all the major types of objects to be used, sequences, states and so forth, the presentation can be simplified by ceasing to decorate every variable and constant with its type, and instead using a simple set of typing conventions: these are given in Table 1. For all the symbols given in the table, it will be assumed that the same symbol with subscripted numbers or superscripted dashes is of the same type.

### 3.3 Predication and Identity

What will be presented in this section is not so much a theory of predication as a place for such a theory to go. For example, a unary ABLE predicate can combine with an ABLE discourse marker to produce an ABLE formula. So I will simply assume that for every unary ABLE predicate there is a corresponding  $\text{Ty}_3$  constant which has the type of a function mapping the denotation of an ABLE discourse marker to the denotation of an ABLE formula. ABLE discourse markers correspond directly to  $\text{Ty}_3$  discourse markers, that is to say objects of type  $d$ , and ABLE formulae denote CCPs, which have the type  $\pi$ . It will be assumed that any  $P^1$  in the set of unary ABLE predicates is also in the set of  $\text{Ty}_3$  constants of type  $\langle d, \pi \rangle$ , and constants of this type will be called *dynamic unary predicates*. Similarly, every binary ABLE predicate will correspond to a *dynamic binary predicate* of type  $\langle d, \langle d, \pi \rangle \rangle$ , and every ABLE attitude predicate will correspond to a *dynamic attitude predicate* of type  $\langle d, \langle \pi, \pi \rangle \rangle$  — a function from discourse markers to a function from the denotation of an ABLE formula to the denotation of an ABLE formula. The clauses for the semantics of ABLE predications are thus trivial:



**Definition D28 (Semantics of Predication in ABLE)**

$$\begin{aligned} [P^1.i] &= P^1.i \\ [P^2.i.j] &= P^2.i.j \\ [P^a.i.\phi] &= P^a.i.[\phi] \end{aligned}$$

It is arguable that such a *minimalist* approach to the interpretation of predications leaves a bit too much unsaid, for ABLE predicates are intended in the first place to correspond to the predicates of natural language, and there are many functions in the denotation spaces of the dynamic predicate constants which have no intuitive interpretation in terms of natural language. As things stand, it is possible for an ABLE predication to denote, for example, an information downgrade, or the introduction of every prime numbered discourse marker into the context, or perhaps some sort of complement operation leaving only worlds in the  $\omega$ -set of the output which were not in the  $\omega$ -set of the input. To exclude such possibilities I will put some constraints on the denotations of dynamic predicates, although the job of creating particular entries within the remaining denotation space I see as the role of the lexical semanticist, and I will do no more in that respect than give a couple of examples.

The following set of meaning postulates radically restricts the behaviour of ABLE predications by relating the dynamic predicate constants to objects of lower, static types such as might be found in a more conventional Montague grammar. The traditional, Fregean intension of a one place predicate is, of course, a function from worlds to a function from individuals to truth values, which in the current framework would be an object of type  $\langle w, \langle e, t \rangle \rangle$ . Similarly, the Fregean intension of a two-place predicate can be correlated with a  $Ty_3$  object of type  $\langle w, \langle e, \langle e, t \rangle \rangle \rangle$ . Given that ABLE concerns not only individuals (type  $e$ ) but also groups of individuals (type  $\varepsilon$ ), it will come as no surprise that in the first of the following clauses dynamic unary predicate constants are related to objects of type  $\langle w, \langle \varepsilon, t \rangle \rangle$ , whilst in the second clause dynamic binary predicate constants are related to objects of type  $\langle w, \langle \varepsilon, \langle \varepsilon, t \rangle \rangle \rangle$ . I will call objects of these types static unary predicates and static binary predicates, respectively.

The first postulate says that for every dynamic unary predicate constant there must be some static unary predicate, such that whenever the CCP obtained by application of the constant to a discourse marker provides an update from some input state, the input state must have the discourse marker in its domain, in which case the output state can be calculated in terms of the static predicate. In particular, the output must be the set of world-sequence pairs in the input such that the extension of the static predicate at the world includes the sequence valuation of the discourse marker.

The second postulate follows the first closely, and presumably requires no further explanation. The third postulate concerns dynamic attitude predicates, but it does not simply relate them to Fregean intensions of attitude verbs, the reason being that it is difficult to find a single such postulate appropriate to the needs of both factive and non-factive attitude verbs. Instead, for any given formula which serves as the propositional complement of a dynamic attitude predicate, the combination of the dynamic attitude predicate and the denotation of the formula is related to a static unary predicate. In effect the translations of complete verb phrases involving attitude verbs, like “realises that she is surrounded” and “doubts that Shakespeare will ever write another best-seller”, are constrained to behave like intransitive verbs. Note the use in the following postulates of the type  $\varepsilon$ , which is simply an abbreviation for  $\langle e, t \rangle$ :

**Meaning Postulate MP8**

If  $P^1$  is a dynamic unary predicate constant then:

$$\begin{aligned} \exists V_{\langle w, \langle \epsilon, t \rangle \rangle} \quad & \forall I \forall J \forall D \quad I\{P^1.D\}J \rightarrow \\ & t\text{-domain}.I.D \wedge \\ & J = \lambda w \lambda f [I.w.f \wedge V.w.(f.D)] \end{aligned}$$

If  $P^2$  is a dynamic binary predicate constant then:

$$\begin{aligned} \exists V_{\langle w, \langle \epsilon, \langle \epsilon, t \rangle \rangle \rangle} \quad & \forall I \forall J \forall D \forall D' \quad I\{P^2.D.D'\}J \rightarrow \\ & t\text{-domain}.I.D \wedge t\text{-domain}.I.D' \wedge \\ & J = \lambda w \lambda f [I.w.f \wedge V.w.(f.D).(f.D')] \end{aligned}$$

If  $P^a$  is a dynamic attitude predicate constant, and  $F$  of type  $\pi$  is the denotation of some ABLE formula, then:

$$\begin{aligned} \exists V_{\langle w, \langle \epsilon, t \rangle \rangle} \quad & \forall I \forall J \forall D \quad I\{P^a.D.F\}J \rightarrow \\ & t\text{-domain}.I.D \wedge \\ & J = \lambda w \lambda f [I.w.f \wedge V.w.(f.D)] \end{aligned}$$

On the basis of these postulates some general characteristics of unary and binary ABLE predications can be given. If an ABLE formula  $\phi$  is of the form  $P^1.i$  or  $P^2.i.j$  then the following will hold:

**Partiality** There may be some states from which  $\phi$  does not produce an update. For instance  $\phi$  will only provide an update from states which have the discourse markers in  $\phi$  in their domain. That is to say:

$$\exists I \neg(I \text{ admits } [\phi])$$

**Determinism**  $\phi$  is functional, so that for every input state there is at most one possible output state.

Formally:

$$\forall I \forall J (I[\phi]J \rightarrow \neg(\exists K I[\phi]K \wedge \neg(K = J)))$$

**Distributivity** An update with  $\phi$  can be calculated pointwise on the individual world-sequence pairs in the input, a property which is discussed in [GS90]. Thus if  $\langle\langle w, f \rangle\rangle$  denotes the *singleton* information state having only one sequence-world pair in it, namely the pair consisting of  $w$  and  $f$ , then for any states  $I$  and  $J$ :

$$I[\phi]J \rightarrow J = \lambda w \lambda f [\exists K \exists w' \exists f' I.w'.f' \wedge \langle\langle w', f' \rangle\rangle[\phi]K \wedge K.w.f]$$

**Eliminativity** If  $I[\phi]J$  then  $J$  contains only a subset of the world-sequence pairs in  $I$ . This property, which, like distributivity, is discussed in [GS90], excludes the possibility of  $\phi$  being a downdate, and also excludes the possibility of  $\phi$  introducing new discourse markers. If  $I$  and  $J$  are states then:

$$I[\phi]J \text{ iff } \forall w \forall f J.w.f \rightarrow I.w.f$$

**Relevance** Only the discourse markers mentioned in  $\phi$  are relevant to the calculation of an update with  $\phi$ . More formally, if  $I - k$  denoted a state differing from  $I$  only by the sequences in  $I$  being *shrunk* so as not to give a value to  $k$ , then we would have for any states  $I, J$  and  $K$ :

$$(I[\phi]J \wedge (I - k)[\phi]K) \rightarrow J = \lambda w \lambda f [I.w.f \wedge \exists g f \succeq g \wedge K.w.g]$$

**Alpha-invariance** The names of the discourse markers in  $\phi$  do not matter, except to the extent that they determine sequence valuations. Let us say that  $\phi[i/k]$  denotes the formula obtained by substituting  $k$  for  $i$  in  $\phi$ , and that  $I[i/k]$  denotes the information state obtained by *swapping* the values given to  $i$  and  $k$  in every sequence in  $I$ . Then if  $k$  is not mentioned in  $\phi$  and is not in the domain of the state  $I$  we would have that for any states  $I$  and  $J$ :

$$I[\phi]J \leftrightarrow I[i/k][\phi[i/k]]J[i/k]$$

These six properties could, of course, have been stated as meaning postulates in the first place, and would have replaced the above postulate MP8. However, I have not attempted any proof that this alternative would yield precisely the same denotation space for the dynamic predicate constants.

To illustrate the working of ABLE predication, I will firstly consider the definition of ABLE predicates MALE<sup>u</sup>, FEMALE<sup>u</sup>, ANIMATE<sup>u</sup>, NEUTER<sup>u</sup>, SINGULAR<sup>u</sup> and PLURAL<sup>u</sup>. We begin with a simple theory about the meaning of the predicates, expressed in terms of simple static predicates:

### Meaning Postulate MP9

$$\begin{aligned}
\forall w \forall x \text{ male}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x &\rightarrow \neg \text{female}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x \\
\forall w \forall x \text{ female}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x &\rightarrow \neg \text{male}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x \\
\forall w \forall x \neg (\text{neuter}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x) &\rightarrow (\text{male}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x \vee \text{female}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x) \\
\forall w \forall x \text{ male}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x &\rightarrow \text{animate}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x \\
\forall w \forall x \text{ female}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x &\rightarrow \text{animate}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x \\
\forall X \text{ singular}_{\langle \epsilon, \tau \rangle} . X &\rightarrow \exists x (X . x \wedge \forall y (X . y \rightarrow x = y)) \\
\forall X \text{ plural}_{\langle \epsilon, \tau \rangle} . X &\rightarrow \exists x \exists y (X . x \wedge X . y \wedge \neg (x = y))
\end{aligned}$$

I take it that these postulates do not require much explanation, save for noting that I allow objects to be both neuter and sexed. This might be appropriate in the case of animals, for instance, since we can refer to an animal as “it” even though it has a sex. Of course, in many languages it would be completely inappropriate to conflate semantic and grammatical gender. For such a language it would be more sensible to class the discourse markers themselves as having gender, number and so on, so that the number and gender of the discourse referents would be irrelevant for the purposes of the grammar. There are several ways in which ABLE could be extended so as to allow for this possibility, for instance by using predicates like *female*<sub>*d,t*</sub> to provide a permanent sortal structure on the domain of discourse markers, or by using predicates like *female*<sub>*w,⟨d,t⟩*</sub> to make the sortal categorization of discourse markers contingent; in this case an information state could be updated with the fact that a given discourse marker fell into a particular grammatical category. I will not pursue these possibilities any further here since for the fragment of English with which I will be concerned, there seems to be a systematic relation between grammatical and semantic categorizations. In the following postulate, dynamic unary predicates are defined in terms of the static predicates introduced above. Note that the first postulate defines *distributive* predicates, so that a group of individuals can only be animate if all the members of the group are animate. It would, however, make no sense for the predicates SINGULAR<sup>u</sup> and PLURAL<sup>u</sup> to be distributive.

### Meaning Postulate MP10

If  $P^1$  is one of the dynamic unary predicate constants MALE<sup>u</sup>, FEMALE<sup>u</sup>, NEUTER<sup>u</sup> and ANIMATE<sup>u</sup> then:

$$\begin{aligned}
P^1 &= \lambda D \lambda I \lambda J [t\text{-domain}.I.D \wedge \\
&\quad J = \lambda w \lambda f [I.w.f \wedge \forall x (f.D.x \rightarrow P_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x)]]
\end{aligned}$$

If  $P^1$  is either of the dynamic unary predicate constants SINGULAR<sup>u</sup> and PLURAL<sup>u</sup> then:

$$\begin{aligned}
P^1 &= \lambda D \lambda I \lambda J [t\text{-domain}.I.D \wedge \\
&\quad J = \lambda w \lambda f [I.w.f \wedge P_{\langle \epsilon, \tau \rangle} . (f.D)]]
\end{aligned}$$

Identity, although not introduced as one of the predication clauses syntactically, behaves as a two place predicate, and would obey a version of postulate MP8, above. The following semantic clause makes an ABLE identity statement *i is j* provide an update only if both discourse markers are in the domain of the input state, in which case the output is the set of world-sequence pairs from the input such that the sequence valuation of the two discourse markers is identical. Note that identity is effectively a *collective* predicate, since the condition for identity of two groups is not that every member of the first group is equal to every member of the second, but that the two groups consist of the same set of individuals.

### Definition D29 (Semantics of Identity in ABLE)

$$\begin{aligned}
[[i \text{ is } j]] &= \lambda I \lambda J [t\text{-domain}.I.i \wedge t\text{-domain}.I.j \wedge \\
&\quad J = \lambda w \lambda f [I.w.f \wedge f.i = f.j]]
\end{aligned}$$

So far we have only seen ABLE predicates which provide an update on a context whenever the context has the predicated discourse markers in its domain. However, a predicate may involve more complex

presuppositions. Presuppositions may be stated as meaning postulates explicitly stating what formulae must be satisfied by the input state of a predication in order for the predication to provide an update. For instance the following postulate would restrict the predicate  $\text{WALK}^u$  so that it could only apply to markers the values of which were already established to be animate entities:

$$\forall I \forall D (\exists J I\{\text{WALK}^u.D\}J) \leftrightarrow I \text{ satisfies } (\text{ANIMATE}^u.D)$$

All the presuppositional examples considered in the second chapter of this thesis concerned factive verbs, like “realize” and “regret”. To ensure that the corresponding ABLE predicates  $\text{REGRET}^a$  and  $\text{REALIZE}^a$  have appropriate presuppositional properties, it is simply stipulated that they have the property of *factivity*, defined below:

**Definition D30 (Factivity)** *A dynamic attitude predicate denoted by the constant  $\text{ATT}^a$  is factive if*

$$\forall I \forall D \forall F \quad I \text{ admits } \text{ATT}^a.D.F \leftrightarrow \\ t\text{-domain}.I.D \wedge I \text{ satisfies } F$$

**Meaning Postulate MP11** *The dynamic attitude predicate constants  $\text{REGRET}^a$  and  $\text{REALIZE}^a$  are factive.*

In what follows, it will often be useful to ignore presupposition altogether. For this purpose a class of distributive, and almost presupposition-free predicates is now introduced:

**Definition D31 (Simple Predicates)**

*A dynamic unary predicate denoted by the constant  $P^u$  is simple if there is a constant  $P$  of type  $\langle w, \langle e, t \rangle \rangle$  such that:*

$$\forall I \forall J \forall D \quad I\{P^u.D\}J \leftrightarrow \\ (t\text{-domain}.I.D \wedge \\ J = \lambda w \lambda f [I.w.f \wedge \forall x (f.D.x \rightarrow P.w.x)])$$

*A dynamic binary predicate denoted by the constant  $P^b$  is simple if there is a constant  $P$  of type  $\langle w, \langle e, \langle e, t \rangle \rangle \rangle$  such that:*

$$\forall I \forall J \forall D \forall D' \quad I\{P^b.D.D'\}J \leftrightarrow \\ (t\text{-domain}.I.D \wedge t\text{-domain}.I.D' \wedge \\ J = \lambda w \lambda f [I.w.f \wedge \\ \forall x \forall y ((f.D.x \wedge f.D'.y) \rightarrow P.w.x.y)])$$

The reason for calling the simple predicates *almost* presupposition free is that there remains a presupposition that the predicated discourse markers are in the domain of the input. However, if this condition is met in some input state, then a predication involving a simple predicate is guaranteed to provide an update. Note that on the above definition, all the predicates  $\text{MALE}^u$ ,  $\text{FEMALE}^u$ ,  $\text{ANIMATE}^u$  and  $\text{NEUTER}^u$  are simple.

### 3.4 Connectives

The definitions of the connectives given below are natural generalisations of those presented in the first part of this thesis, and furthermore differ little from the definitions found in [De92] and [Be92], which in turn are close to those of [He83].

Conjunction is defined as relational composition, and negation is defined as a set complement operation, although the anaphoric closure of the negated formula is taken so as to avoid problems which would be caused by the introduction of discourse markers in the negated formula. Implication is defined using the same standard equivalence as in chapter 2, and disjunction is also defined using a standard equivalence.

**Definition D32 (Semantics of Connectives in ABLE)**

$$\llbracket \phi \text{ AND } \psi \rrbracket = \lambda I \lambda J [\exists K I \llbracket \phi \rrbracket K \llbracket \psi \rrbracket J]$$

$$\begin{aligned} \llbracket \text{NOT } \phi \rrbracket &= \lambda I \lambda J [\exists K I \downarrow \llbracket \phi \rrbracket K \wedge \\ &J = I \setminus K] \end{aligned}$$

$$\llbracket \phi \text{ IMPLIES } \psi \rrbracket = \llbracket \text{NOT } (\phi \text{ AND NOT } \psi) \rrbracket$$

$$\llbracket \phi \text{ OR } \psi \rrbracket = \llbracket \text{NOT } (\text{NOT } \phi \text{ AND NOT } \psi) \rrbracket$$

**3.5 Determiners**

The treatment of determiners to be given is related to earlier dynamic accounts such as Heim’s, Chierchia’s or Groenendijk and Stokhof’s. I hope I will be able to demonstrate that within a dynamic setting an analysis of determiners is possible which begins to parallel that of [BC81] in its uniformity, and yet encompasses a much broader view of meaning than is found in their by now standard account of generalized quantifiers.

Historically we could attribute to Russell the first serious attempt at a uniform analysis of determiners. But although the tradition of quantificational analysis extending from his work has fared well in the treatment of relatively exotic determiners, it has faced much criticism closer to home, in particular regarding the treatment of the humblest determiners of all, “the” and “a”. The two best known philosophical challenges to the uniform quantificational analysis of determiners, namely that of Strawson with respect to definites, and that of Geach with respect to the interaction of indefinites and definites, have both been met with apparently non-quantificational solutions in the work of Heim and Kamp in the early eighties. It is not until [GS91a], building on [Ba87] and [Ro87], that the possibility of bringing indefinites back into the quantificational fold — given a suitably broad conception of quantification — became readily apparent. [Ch92] showed a way to extend the approach developed in [GS91a] and [GS91b] to a wider range of determiners, and the current work continues in much the same direction, attempting to bring out the similarities between the dynamic analyses not only of indefinites and paradigmatically quantificational determiners, but also of definites.

**3.5.1 Indefinites**

The semantics of the ABLE determiner SOME will follow the standard dynamic analysis, which holds that the meaning of an indefinite resides in its ability to introduce a new referent into the discourse context. When an indefinite is used in a conversation the speaker may or may not intend to refer to a particular object which he or she has in mind. But regardless of whether the indefinite is being used *specifically* or *non-specifically*, it will not generally be the case that the other conversational participants are able to pinpoint a particular object to which the indefinite refers, and thus each participant’s information state must leave the reference of the indefinite phrase underspecified. In fact, given that in this chapter an information state is being conceived of only as a model of the common ground, and that other aspects of a participant’s knowledge are being ignored, even the speaker’s information state must leave the reference of an indefinite noun-phrase underspecified. The process of updating an information state so as to incorporate an underspecified referent begins with the assignment of a completely underspecified value to a previously unused discourse marker, which is defined in terms of the constants *add* and *+*:

**Meaning Postulate MP12**

$$(1) \quad \textit{add} = \lambda D \lambda f \lambda g \left[ \exists x g = \lambda D' \lambda y \left[ \begin{array}{l} D' = D \rightarrow x = y \wedge \\ (\neg D' = D) \rightarrow f.D'.y \end{array} \right] \right]$$

$$(2) \quad + = \lambda D \lambda I \lambda J [\neg \textit{p-domain}.I.D \wedge \\ J = \lambda w \lambda f [\exists g I.w.g \wedge g\{\textit{add}.D\}f]]$$

A  $Ty_3$ -formula  $f\{add.i\}g$  says that sequences  $f$  and  $g$  agree on the values they assign to all discourse markers apart from  $i$ , and that  $g$  maps  $i$  to a set containing one, arbitrary object. The constant  $+$  is defined in terms of  $add$ , and says something similar at the level of information states instead of sequences. A formula  $I\{+.i\}J$ , “ $J$  is an *arbitrary extension* of  $I$  with a value for  $i$ ”, means that  $i$  is not in the domain of state  $I$ , and that any pair of a world and a sequence in  $J$  differs from some pair in  $I$  only by the sequence being extended with some arbitrary valuation for  $i$ .

In ABLE, determiner clauses are of the form DET.DM.FORM.FORM. I will call the discourse marker at the head of such a clause the *determined* marker, and, following standard conventions, I will call the first subformula the *restrictor* and the second the *scope*. The interpretation of a formula  $SOME.i.\phi.\psi$  can be thought of procedurally as an instruction to perform the following sequence of modifications to an input state: arbitrarily extend the input with a value for the determined marker, then add firstly the information in the restrictor, and secondly the information in the scope. This is captured by the following definition:

**Definition D33 (Semantics of SOME: first version)**

$$\llbracket SOME.i.\phi.\psi \rrbracket = \lambda I \lambda J [\exists I_{in} \exists I_{res} I\{+.i\}I_{in}[\phi]I_{res}[\psi]J]$$

In (E23) a simple example of an indefinite sentence and its ABLE translation is given. The translation defines (the relational equivalent of) a partial function from input states which give no valuation for the determined marker 1, to output states containing only world-sequence pairs which are defined on 1 and map it onto an individual from the domain which is a walking woman in the given world. The output pairs are thus extensions of elements of that subset of the input pairs involving a world where there is a walking woman.

- E23 a. A woman is walking.  
 b.  $SOME.1.(WOMAN^u.1).(WALK^u.1)$

### 3.5.2 Definites

In [Be92] I analysed definite descriptions in a similar way to [He83]. It was assumed that a definite description presupposed the existence of an object which satisfied the description. However, the analysis I will present for the ABLE determiner THE will be closer to the treatment given in Heim’s thesis [He82]. The difference is as follows: in her thesis Heim argues not that a definite presupposes the existence of an object satisfying the description, but that it presupposes there to be a salient discourse marker whose value satisfies the description. Thus a formula  $THE.i.(WOMAN^u.i).(WALK^u.i)$  will provide an update only from those contexts in which the determined marker is established to refer to a woman.

The analysis has the advantage, as will be seen later, that it can yield a uniform treatment of various sorts of definite noun phrase — the semantics of proper names and pronouns will all be defined in terms of the semantics of THE. On the other hand, there are many uses of definite descriptions for which the analysis given here will be inadequate, for instance when the referent has been introduced but the fact that it satisfies the given description has to be inferred, or when the referent has not been explicitly introduced but only implicitly made salient by its relevance to other topics of conversation. I believe, in agreement with views expressed by Heim, that such uses of definites should be dealt with by supplementing the theory with an account of *accommodation* — the process whereby a context which does not satisfy the presuppositions of a sentence is adjusted so as to allow update with that sentence. In chapter 5 I will indicate how such a mechanism may be formalised.

The following definition restricts input states from which a formula  $\llbracket THE.i.\phi.\psi \rrbracket$  provides an update to those which both have the determined marker in their domain and support the proposition in the restrictor. If these conditions are met then the output state is simply the input updated sequentially with the restrictor and the scope.

**Definition D34 (Semantics of THE)**

$$\llbracket THE.i.\phi.\psi \rrbracket = \lambda I \lambda J [t\text{-domain}.I.i \wedge I \text{ satisfies } [\phi] \wedge \exists K I[\phi]K[\psi]J]$$

Examples E24–E26, which provide an inane continuation to E23, illustrate the effects of the above definition:

- E24 a. The woman is talking.  
 b. THE.1.(WOMAN<sup>u</sup>.1).(WALKING<sup>u</sup>.1)
- E25 a. Butch is barking.  
 b. THE.2.(NAMED-BUTCH<sup>u</sup>.2).(BARK<sup>u</sup>.2)
- E26 a. She regrets that he is barking.  
 b. THE.1.(FEMALE<sup>u</sup>.1 AND SINGULAR<sup>u</sup>.1).  
 (REGRET<sup>a</sup>.1.(THE.2.(MALE<sup>u</sup>.2 AND SINGULAR<sup>u</sup>.2).(BARK<sup>u</sup>.2)))

The scene was set, the reader will recall, by the introduction of a walking woman, for whom a corresponding ABLE discourse marker 1 was introduced. Such a context can be updated with E24b, which provides an update from any input state in which 1 is determined to be a woman, to that subset of input world-sequence pairs in which the referent of 1 is talking. Occurrences of discourse markers outside of the syntactic scope of the quantificational determiner that introduced them, as in E24b, will sometimes be referred to as being *unquantified*. E25b does not provide an update from all possible outputs of the conjunction of E23b and E24b, but only from those in which the unquantified marker 2 is totally defined, and only takes *butch* values. Supposing this condition — which will be turned to in a moment — is met, E25 adds the information that 2 is barking, and thus provides an appropriate input to E26b, which is only defined on contexts in which 1 is female, 2 is a barking male. Of course, this input will only be fully appropriate on the assumption that being named Butch entails masculinity.

### Meaning Postulate MP13

- (1) NAMED-BUTCH<sup>u</sup> is simple
- (2)  $\forall w \forall x \text{ named-butch}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x \rightarrow \text{male}_{\langle \omega, \langle \epsilon, \tau \rangle \rangle} . w . x$

Failing the existence of such a postulate, accommodation would be needed, enabling the addition of the information that 2 was male as and when this was presupposed. Accommodation should also provide the answer to the problem of satisfying the presupposition of E25, that the discourse marker 2 corresponded to Butch. However, as will be seen in chapter 4, even without describing a general mechanism of accommodation, presupposing formulae like E25b can still play an interesting role in the logic of ABLE

To end this section on definites, here are some obvious abbreviations for ABLE formulae:

$$\begin{aligned} \text{SHE} . i . \phi &= \text{THE} . i . (\text{SINGULAR}^u . i \text{ AND FEMALE}^u . i) . \phi \\ \text{HE} . i . \phi &= \text{THE} . i . (\text{SINGULAR}^u . i \text{ AND MALE}^u . i) . \phi \\ \text{IT} . i . \phi &= \text{THE} . i . (\text{SINGULAR}^u . i \text{ AND NEUTER}^u . i) . \phi \\ \text{THEY} . i . \phi &= \text{THE} . i . (\text{PLURAL}^u . i) . \phi \end{aligned}$$

### 3.5.3 Dynamic Generalized Quantifiers

According to the standard generalized quantifier analysis of [BC81], the quantificational determiners of natural language correspond to a binary quantifier relations, a binary quantifier relation being able to combine with some set to produce a generalized quantifier, which is a property of sets. With which sets should the quantifier relations combine? The restrictor and scope of the quantifier appear to be properties of individuals, so that the sets are simply the sets of individuals in the extensions of the properties. But here the classic problem of donkey anaphora arises. Consider the following Geachian examples:

- E27 a. Every farmer owns a donkey.  
 b. EVERY . i .  
 (FARMER<sup>u</sup> . i).  
 (SOME . j . (DONKEY<sup>u</sup> . j) . (OWNS<sup>b</sup> . i . j))
- E28 a. Every farmer who owns a donkey beats it.

- b. EVERY.*i*.  
 (FARMER<sup>u</sup>.*i* AND SOME.*j*.(DONKEY<sup>u</sup>.*j*). (OWNS<sup>b</sup>.*i*.*j*.)  
 (IT.*j*.(BEATS<sup>b</sup>.*i*.*j*))

The generalized quantifier analysis for (E27) will be familiar: the quantifier relation corresponding to *every* must hold between the set of farmers and the set of donkey owners. Equally familiar will be the difficulties that the analysis faces with (E28). The denotation of the restrictor is obvious — it is just the property of being a farmer who owns a donkey. But the scope does not seem to denote an absolute property: the property of being a *beater of it* must be relativised to some interpretation for the pronoun *it*.

This problem concerns the *internal dynamism* (cf. [GS91a]) of quantificational constructions, by which is meant the fact that the dynamic effects of updating with the restrictor of a quantifier help determine the context of evaluation for the scope. To begin with I will present an analysis which makes quantifiers internally dynamic, but makes them *externally static*. The final ABLE semantics, however, will make quantifiers both internally and externally dynamic, thus both allowing for internal anaphoric links within the quantificational construction, and allowing whole quantificational constructions to introduce discourse markers so that they may support later anaphoric reference.

To introduce internally dynamic quantifiers, it will be helpful to consider a restricted set of worlds: in this way the relevant information states become more easily visualisable. In figure 1 the characteristics of eight individuals (*a – h*) in four different worlds (W1 – W4) are pictured. It should be clear that in world W2, for instance, one farmer who owns three donkeys beats only one of them, whilst the two other farmers own one donkey each, and in each case beat it. Note that I am also assuming the predicates FARMER<sup>u</sup>, DONKEY<sup>u</sup>, OWNS<sup>b</sup> and BEATS<sup>b</sup> to be *simple* (i.e. both presupposition free and distributive) in the sense of section 3.3 above, thus allowing the denotation of a predication involving one of these to be calculated in terms of underlying lower typed predicates like *farmer*<sub>(w,(e,t))</sub>: it is the denotation of these lower typed predicates which is pictured in figure 1.

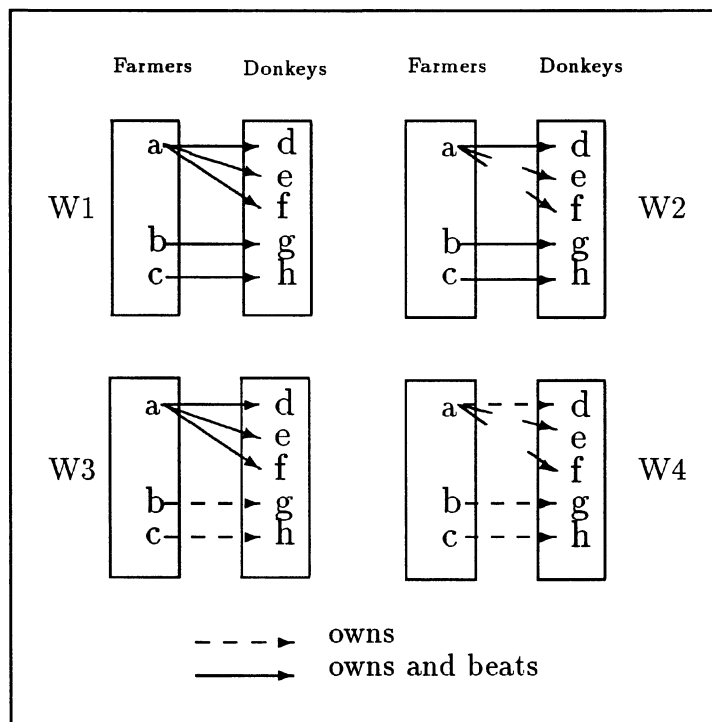


Figure 1: Some Worlds

Suppose that we wish to update the information state which has only these four worlds in its  $\omega$ -set, and which has no discourse markers in its domain, with the formula in (E28b), the ABLE representation



of the classic donkey sentence. We begin the analysis of the universal in the same way as for an indefinite, calculating the effect of introducing the determined marker  $i$  into the input context, and then updating with the restrictor followed by the scope. Let us call the information state in which the discourse marker  $i$  has been initialised *init*, the state reached after updating with the restrictor *res*, and the state reached after updating with the scope *sco*.

Given that, for the moment, it is being assumed that the complete quantificational construction introduces no discourse markers, the output state must contain a subset of the world-sequence pairs in the input, namely all those world-sequence pairs which are compatible with the information in the quantificational formula. To discern whether a given world-sequence pair from the input should be in the output, it is necessary to look at the extensions of that pair in the contexts *res* and *sco*, and check what values the determined discourse marker takes in these extensions.

Let us represent sequences in such a way that, for example, a sequence in which the discourse marker  $i$  is mapped onto the single individual  $a$ , and the discourse marker  $j$  is mapped onto the group containing  $b$  and  $c$  becomes:  $\langle i \mapsto \{a\}, j \mapsto \{b, c\} \rangle$ . Thus the example input state, which contains no information about discourse markers, simply consists of the following set of pairs:

$$\left( \begin{array}{l} W1 \\ W2 \\ W3 \\ W4 \end{array} \begin{array}{l} \langle \rangle \\ \langle \rangle \\ \langle \rangle \\ \langle \rangle \end{array} \right)$$

The four worlds pictured in fig.1 do not vary with respect to the denotation of the underlying predicates *farmer*, *donkey* and *owns*: in each world there are five ways in which  $i$  and  $j$  can be mapped respectively onto a farmer and a donkey which that farmer owns. This means that update of the input state with the restrictor introduces only five different sequences, so that *res* consists of each of the four worlds paired with each of the five sequences:

$$\left( \begin{array}{l} W1 \\ W1 \\ W1 \\ W1 \\ W1 \\ W2 \\ W2 \\ \vdots \end{array} \begin{array}{l} \langle i \mapsto \{a\}, j \mapsto \{d\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{e\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{f\} \rangle \\ \langle i \mapsto \{b\}, j \mapsto \{g\} \rangle \\ \langle i \mapsto \{c\}, j \mapsto \{h\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{d\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{e\} \rangle \\ \vdots \end{array} \right)$$

For a given world-sequence pair  $wf$  from the input and a given *res*, let us call the set of sequences which extend  $f$ , and are paired with  $w$  in *res*, the *restrictor sequence set* of  $wf$ . Formally the restrictor sequence set will be given by the expression:  $\lambda h [h \succeq f \wedge res.w.h]$ . A similar expression can be used to determine the *scope sequence set* of a given world-sequence pair from the input.

The state *sco*, which includes not only information about farmer-donkey ownership, but also about cruelty to animals, associates different sets of sequences with each world:

$$\left( \begin{array}{l} W1 \\ W1 \\ W1 \\ W1 \\ W1 \\ W2 \\ W2 \\ W2 \\ W3 \\ W3 \\ W3 \end{array} \begin{array}{l} \langle i \mapsto \{a\}, j \mapsto \{d\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{e\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{f\} \rangle \\ \langle i \mapsto \{b\}, j \mapsto \{g\} \rangle \\ \langle i \mapsto \{c\}, j \mapsto \{h\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{d\} \rangle \\ \langle i \mapsto \{b\}, j \mapsto \{g\} \rangle \\ \langle i \mapsto \{c\}, j \mapsto \{h\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{d\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{e\} \rangle \\ \langle i \mapsto \{a\}, j \mapsto \{f\} \rangle \end{array} \right)$$

Here W1 is the only world which is paired with the same sequences as in *res*, all three farmers being hideously cruel in this world. By contrast, each of W2 and W3 remain paired with only three of the

original five sequences, whilst the utopian W4 is no longer even in the  $\omega$ -set of *sco*: there are no beatings in W4.

One way to define internally dynamic quantifiers would be to preserve in the output those world-sequence pairs from the input for which the restrictor sequence set and scope sequence set are satisfied the given quantifier relation. Thus, in the current example concerning the quantifier *every*, only the pair  $W1\langle \rangle$  would be in the output, since this is the only pair for which every extension in *resis* also in *sco*. However, as is shown in [Ro87], this sequence counting approach will not work in general. Consider the following simple variation on E28:

E29 a. Most farmers who own a donkey beat it.

- b. MOST.*i*.  
 FARMER<sup>u</sup>.*i* AND (SOME.*j*.(DONKEY<sup>u</sup>.*j*). (OWNS<sup>b</sup>.*i*.*j*)).  
 (IT.*j*.(BEATS<sup>b</sup>.*i*.*j*))

Intuitively this sentence should be true only in worlds W1 and W2, since in both of the other worlds less than half of the farmers are malign. However, counting sequences would leave not only W1 and W2 in the output, but also W3. In this world, one farmer happens to own and beat three donkeys whilst the other two farmers do not beat their single donkeys, which means that three out of the five sequences from *res* are preserved in *sco*, and, 3/5 being more than half, this in turn means that the quantifier relation *most* is satisfied. The conclusion to be drawn is that the quantification involves counting farmers and not sequences.

It is easy to calculate the relevant set of individuals from a given restrictor sequence set or scope sequence set. Suppose that for some pair *wf* the restrictor sequence set is  $G_{res}$ , and the determined marker is *i*. Then the set of values of *i* in *res* extensions of *wf* will be:  $\lambda x [\exists h G_{res}.h \wedge h.i.x]$ , and similarly for the scopal values.

In the example we have been looking at, each of the four world-sequence pairs in the input will yield the same set of values for the determined marker in the restrictor, namely the set:  $\{a, b, c\}$ . The scopal values will of course differ. The input pairs  $W1\langle \rangle$  and  $W2\langle \rangle$  yield values  $\{a, b, c\}$ , whilst  $W3\langle \rangle$  yields the set  $\{a\}$ , and  $W4\langle \rangle$  yields the empty set. Clearly using these sets will be an improvement in the case of E29, since it will only hold for the pairs  $W1\langle \rangle$  and  $W2\langle \rangle$  that over half the members from the restrictor extensions are still in the scope extensions.

There are other consequences of the decision to count individuals rather than sequences, to which we will come shortly. Firstly let us see what the upshot of the above discussion is in terms of a general semantics for internally dynamic quantificational determiners. In the definition below, *init*, *res* and *sco* are assumed to be variables of type  $\iota$ , and have the same interpretation as in the above discussion. The output state *J* is obtained by calculating for each *wf* in the input state the corresponding restrictor and scope sequence sets ( $G_{res}$  and  $G_{sco}$ , which are each of type  $\langle \sigma, t \rangle$ ), then calculating from these the sets of values taken by the determined marker in the restrictor and the scope (these sets being given by  $X_{res}$  and  $X_{sco}$ , each of type  $\varepsilon = \langle e, t \rangle$ ), and finally checking whether the relevant underlying quantifier relation holds between two sets. The definition we arrive at is essentially the same as that of [Ch92], although tailored to ABLE's needs:

**Definition D35 (Non-existential Determiners - First Version)**

If  $\mathcal{D}$  is one of EVERY, MOST, FEW or NO, and  $\mathcal{D}'$  is a corresponding quantifier relation of type  $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ , then:

$$\begin{array}{l}
 [\mathcal{D}.i.\phi.\psi] = \lambda I \lambda J \\
 \left[ \begin{array}{l}
 \exists \text{init} \exists \text{res} \exists \text{sco} \\
 I\{+.i\}\text{init}[\phi]\text{res}[\psi]\text{sco} \wedge \\
 \\
 J = \lambda w \lambda f \\
 \left[ \begin{array}{l}
 \exists G_{\text{res}} \exists G_{\text{sco}} \exists X_{\text{res}} \exists X_{\text{sco}} \\
 I.w.f \wedge \\
 G_{\text{res}} = \lambda h [h \succeq f \wedge \text{res}.w.h] \wedge \\
 G_{\text{sco}} = \lambda h [h \succeq f \wedge \text{sco}.w.h] \wedge \\
 X_{\text{res}} = \lambda x [\exists h G_{\text{res}}.h \wedge h.i.x] \wedge \\
 X_{\text{sco}} = \lambda x [\exists h G_{\text{sco}}.h \wedge h.i.x] \wedge \\
 \mathcal{D}'.X_{\text{res}}.X_{\text{sco}}
 \end{array} \right]
 \end{array} \right]
 \end{array}$$

The following postulate ensures that the underlying quantifier relations have appropriate denotations. The definitions for EVERY' and NO' should be clear. I have made the simplifying assumption that MOST' means *more than half*, and FEW' means *less than half*. The definition for MOST' says that two sets of individuals  $X$  and  $Y$  stand in the relation MOST' if and only if there is no function with domain  $Y \setminus X$  (i.e. the set containing only members of  $Y$  that are not in  $X$ ) and range  $X \cap Y$ : this implies that the cardinality of  $X \cap Y$  is greater than the cardinality of  $X \setminus Y$ , and thus that more than half of the elements of  $X$  are in  $Y$ . The definition for FEW' runs along similar lines.

**Meaning Postulate MP14**

$$\begin{array}{l}
 \text{EVERY}' = \lambda X \lambda Y [\forall x (X.x \rightarrow Y.x)] \\
 \text{NO}' = \lambda X \lambda Y [\neg \exists x (X.x \wedge Y.x)] \\
 \text{MOST}' = \lambda X \lambda Y [\neg \exists F_{(e,e)} \\
 \quad \forall x ((X.x \wedge Y.x) \rightarrow \exists y (X.y \wedge \neg Y.y \wedge F.y = x))] \\
 \text{FEW}' = \lambda X \lambda Y [\neg \exists F_{(e,e)} \\
 \quad \forall x ((X.x \wedge \neg Y.x) \rightarrow \exists y (X.y \wedge Y.y \wedge F.y = x))]
 \end{array}$$

Before going on to consider externally dynamic quantifiers, we will briefly consider how the account given so far bears on the problem of *weak* and *strong* readings for donkey sentences. In the case of E28, the strong reading — that obtained on Lewis's case-quantification analysis, in Kamp's DRT and in Heim's FCS — is the reading where every donkey owning farmer beats every donkey he owns. This is *strong* in the sense that it entails the so-called *weak* reading, where every donkey owning farmer beats at least one of the donkeys he owns. There is, to the best of my knowledge, as yet no conclusive evidence as to which reading is appropriate for E28, or whether, indeed, both readings are present. However, intuitions are clearer with regard to donkey sentences involving right downward monotone quantifiers such as “no”:

E30 a. No farmer who owns a donkey beats it.

b. NO.*i*.

$$\begin{array}{l}
 (\text{FARMER}^u.i \text{ AND SOME}.j.(\text{DONKEY}^u.j).(\text{OWNS}^b.i.j)). \\
 (\text{IT}.j.(\text{BEATS}^b.i.j))
 \end{array}$$

Regarding this example it is difficult to justify the existence of a weak reading — the reading where no farmer beats all of his donkeys although some farmers may beat some of their donkeys. It seems that the existence of a single example of a farmer owning a donkey but not beating it would be enough to falsify the statement in E30.

In general the definition in D35 yields *existential* readings. Thus E28 comes to have the weak reading where every donkey-owning farmer beats at least one donkey, and E30 comes to have the strong reading

(which as I have indicated seems to be the only plausible one) where no farmer beats any of his or her donkeys. However, both [Ch92] and [Kan93] indicate that it is best to allow for the possibility of right upward monotone quantifiers having both strong and weak readings. To this end I note that we at least have the option of making the upward monotone quantifiers ambiguous, simply by introducing two new ABLE quantifiers EVERY\* and MOST\*. I leave it to the reader to verify that the following definitions would introduce strong right upward monotone determiners:

**Definition D36 (Strong upward monotone determiners)**

$$\begin{aligned} \llbracket \text{EVERY}^* .i.\phi.\psi \rrbracket &= \llbracket \text{NO}.i.\phi.(\text{NOT } \psi) \rrbracket \\ \llbracket \text{MOST}^* .i.\phi.\psi \rrbracket &= \llbracket \text{FEW}.i.\phi.(\text{NOT } \psi) \rrbracket \end{aligned}$$

One final point about the limits of the current strategy for defining internally dynamic quantifiers: as discussed by Chierchia, the strategy is only appropriate for *conservative* quantifiers. This property holds of a quantifier, roughly speaking, if in calculating the truth of a quantificational statement, it is only necessary to consider how many of the individuals satisfying the restrictor also satisfy the scope, and it is irrelevant whether or not individuals not satisfying the restrictor satisfy the scope. Thus in evaluating whether it is true that “every girl is hiding”, it is irrelevant whether or not boys and other non-girls are hiding. It seems possible to maintain that this property holds for all English determiners: the only putative exception with which I am familiar is the word “only”. However, it is quite plausible that “only” is not a determiner at all, since, from the point of view of a naive syntactician, it has a quite different distribution than would be expected. For instance, “only” can modify not only nouns, but also noun-phrases, as in “only me”, as well as commonly occurring outside of noun-phrases. On the other hand, even if “only” is not a determiner, this does not excuse us from the task of giving it a semantics. But my feeling is that this job belongs within a more general theory of the dynamics of focus-sensitive constructions (see, for instance, [Krif92]) and not here.

We now move on to the external dynamics of quantifiers. Consider the following examples:

- E31 a. Most farmers own a donkey.  
 b. MOST.*i*. (FARMER<sup>u</sup>.*i*).  
       (SOME.*j*.(DONKEY<sup>u</sup>.*j*). (OWN<sup>b</sup>.*i*.*j*))
- E32 a. They ride to the pub.  
 b. THEY.*i*. (RIDE-TO-PUB<sup>u</sup>.*i*)
- E33 a. Most of the donkeys are alcoholic.  
 b. MOST.*k*. (THE.*j*. (DONKEY<sup>u</sup>.*j* AND PLURAL<sup>u</sup>.*j*). (OF<sup>b</sup>.*i*.*j*))  
       (ALCOHOLIC<sup>u</sup>.*k*)

The fact that E31a–E33a constitute a coherent discourse shows that it is a simplification to assume that quantificational determiners have no anaphoric potential. It seems that E31a introduces at least two new topics of conversation: the set of donkey owners, and the set of donkeys that people own. In fact, there may well be other new topics, such as the set of all farmers and the set of all donkeys, but I will ignore these, although it seems plausible that the approach that will be described could be extended appropriately.

To make matters more tangible, the desired dynamic effects of E31b can be relativised to the example input state used earlier, consisting of the four worlds from fig.1 paired with empty sequences. Given this input, the following update would leave (in each world) *i* mapped to the set of all farmers, and *j* mapped to the set of donkeys owned by farmers:

$$\left\{ \begin{array}{l} W1 \\ W2 \\ W3 \\ W4 \end{array} \right\} \left\{ \begin{array}{l} \langle \rangle \\ \langle \rangle \\ \langle \rangle \\ \langle \rangle \end{array} \right\} \llbracket \text{E31b} \rrbracket \left\{ \begin{array}{l} W1 \\ W2 \end{array} \right\} \left\{ \begin{array}{l} \langle i \mapsto \{a, b, c\}, j \mapsto \{d, e, f, g, h\} \rangle \\ \langle i \mapsto \{a, b, c\}, j \mapsto \{d, e, f, g, h\} \rangle \end{array} \right\}$$

In the earlier definition of internally dynamic quantifiers, D35, extensions of a given input pair compatible with both the restrictor and scope were collected in the scope sequence set. Let us say

that some input pair  $wf$  has a scope sequence set  $G_{sco}$ , and that  $wf$  would have been in the output state according to the earlier definition of internally dynamic quantifiers, D35. Then the output of an externally dynamic quantifier might be expected to contain the pair consisting of  $w$  together with the sequence  $g$  defined as follows:

$$g = \lambda D \lambda x [\exists h G_{sco}.h \wedge h.D.x]$$

This formula defines  $g$  as a function which maps each discourse marker onto the set of all values which it is assigned by the member sequences of the scope sequence set: we can call this the *scope sequence union*. However, although this approach would lead to an acceptable definition for E31b above, it would not be appropriate for right downward monotone quantifiers. For instance, the input world-sequence pairs which satisfy a quantification with the determiner “few” may sometimes be associated with empty scope sequence sets since, on standard assumptions, there being few farmers who own a donkey is compatible with there being none at all. Thus calculating output sequences from scope sequence sets alone will sometimes yield only empty output sequences, and so fail to preserve the anaphoric information present in the input state.

One strategy for avoiding this problem would be to use different definitions for the semantics of upward and downward right monotone quantifiers. However, a single definition for both upward and downward monotone quantifiers is possible, provided the output sequence corresponding to a given input pair is calculated in terms of a combination of the input sequence and the scope sequence set. For a given input pair  $wf$ , having a scope set  $G_{sco}$ , the relevant output sequence associated with  $w$  will be calculated using the following formula:

$$g = \lambda D \lambda x \left[ \begin{array}{l} f.D.x \vee \\ \exists h G_{sco}.h \wedge h.D.x \end{array} \right]$$

This makes the output sequence map each discourse marker onto the set of values which it is assigned either by the initial sequence  $f$ , or by member sequences of the scope sequence set. The problem is thus avoided, since if the scope sequence set is empty, the output sequence will be the same as the input, and if the scope sequence set is nonempty, then the output sequence will just be the scope sequence union, as before. We thus finally arrive at a semantics for non-existential determiners which incorporates both internal and external dynamism:

**Definition D37 (Non-existential Determiners - Second Version)**

If  $\mathcal{D}$  is one of EVERY, MOST, FEW or NO, and  $\mathcal{D}'$  is a corresponding quantifier relation of type  $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ , then:

$$\begin{array}{l} [[\mathcal{D}.i.\phi.\psi] = \lambda I \lambda J \\ \left[ \begin{array}{l} \exists init \exists res \exists sco \\ I\{+.i\}init[\phi]res[\psi]sco \wedge \\ \\ J = \lambda w \lambda g \\ \left[ \begin{array}{l} \exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco} \\ I.w.f \wedge \\ G_{res} = \lambda h [h \succeq f \wedge res.w.h] \wedge \\ G_{sco} = \lambda h [h \succeq f \wedge sco.w.h] \wedge \\ X_{res} = \lambda x [\exists h G_{res}.h \wedge h.i.x] \wedge \\ X_{sco} = \lambda x [\exists h G_{sco}.h \wedge h.i.x] \wedge \\ \mathcal{D}'.X_{res}.X_{sco} \wedge \\ g = \lambda D \lambda x \left[ \begin{array}{l} f.D.x \vee \\ \exists h G_{sco}.h \wedge h.D.x \end{array} \right] \end{array} \right] \end{array} \right] \end{array}$$

In the recent semantic literature, indefinites have typically been distinguished from other quantificational determiners by virtue of their anaphoric potential. Having given, in D37, a definition that forms externally dynamic ABLE determiners from arbitrary quantifier relations, an obvious question arises: is the same definition appropriate for indefinites? Consideration of the following two classic examples shows that although extending D37 to include indefinites opens up a promising vista, it is not tenable:

E34 a. ?There is a doctor in London. He is Welsh.

- b.  $\text{SOME}.i.(\text{DOCTOR}^u.i).(\text{IN-LONDON}^u.i)$   
 $\text{AND HE}.i.(\text{WELSH}^u.i)$

E35 a. If a farmer owns a donkey then he beats it.

- b.  $\text{SOME}.i.(\text{FARMER}^u.i).(\text{SOME}.j.(\text{DONKEY}^u.j).(\text{OWNS}^b.i.j))$   
 $\text{IMPLIES HE}.i.(\text{IT}.j.\text{BEATS}^b.i.j)$

It has been argued, in [Ev77], that E34a is odd or incoherent, and that this oddity arises from the fact that the first sentence does not uniquely pick out a single individual to which the singular pronoun in the second sentence can refer. The judgement that the discourse is incoherent is not clear cut: there is at least one reading of E34a on which it is coherent, namely the so-called *specific* reading, as would be more obvious if the noun phrase “a doctor” were replaced with “a certain doctor”. However, let us assume for the moment that E34a is indeed incoherent, and consequently that explaining this incoherence would be a desirable goal for a semantic theory. Extending D37 to include the determiner SOME, and translating E34a in the obvious way as E34b, would immediately satisfy this desideratum. The translation of the first sentence would no longer introduce a discourse marker which ranged over alternative individual doctors in each world, but instead would introduce a marker assigned to the set of all London doctors in each world. Since an ABLE formula  $\text{HE}.i.\phi$  carries the presupposition that  $i$  is established to be both male and singular, presupposition failure would follow unless the input state guaranteed that there was only one London doctor, and that this individual was male.

Regarding E34a, then, the extension of D37 to indefinites would have acceptable, or even desirable results. However, the same strategy would have significant effects for the treatment of quantificational donkey sentences, and disastrous effects for the treatment of conditional donkey sentences like E35a. In the case of quantificational donkey sentences like the prototypical E28a, the important consequence would be the introduction of a presupposition that every farmer had at most one donkey. But the translation of E35a would carry a far stronger uniqueness presupposition, namely that there was only one farmer, and that that this farmer had only one donkey. In effect Lewis’s insight that the donkey conditional can be seen in terms of *case-quantification* would have been lost, since under such an analysis there could only be one case. Furthermore, it does not seem that an alternative semantics for the pronouns in E35a could save the analysis of indefinites. The translation of “a farmer owns a donkey” would introduce markers corresponding to the complete set of farmers and the complete set of donkeys owned by farmers, and it would not preserve information about which farmers owned which donkeys. Clearly extensive further modifications would be needed in order to provide a compositional semantics for the conditional which entailed only that farmers beat their own donkeys.

Thus it seems that it would be wiser to stick with the original semantics for indefinites given in D33 than to use the same semantics as for the non-existential determiners. However, it remains somewhat disconcerting that D33 bears so little resemblance to D37, and I think it reasonable to wonder whether the ABLE determiner SOME, as defined in D33, in any sense qualifies as a quantifier. I will now show that D33 can be reformulated in more obviously quantificational terms, and this reformulation will have the advantage that it is also appropriate to other related quantifiers like EXACTLY-ONE.

In terms of internal dynamics, the following definition for the existential quantifiers SOME and EXACTLY-ONE is identical to D38, which defined the semantics of EVERY, MOST, FEW and NO. However, the external dynamics is quite different. Instead of collecting together members of the scope sequence set to produce group referents, as in D37, the following definition puts all members of relevant scope sequence sets directly into output world-sequence pairs:

**Definition D38 (Existential Determiners: second version)**

If  $\mathcal{D}$  is SOME or EXACTLY-ONE, and  $\mathcal{D}'$  is a corresponding quantifier relation of type  $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ , then:

$$\begin{array}{l}
 [\mathcal{D}.i.\phi.\psi] = \lambda I \lambda J \\
 \left[ \begin{array}{l}
 \exists \text{init} \exists \text{res} \exists \text{sco} \\
 I\{+.i\} \text{init}[\phi] \text{res}[\psi] \text{sco} \wedge \\
 \\
 J = \lambda w \lambda g \\
 \left[ \begin{array}{l}
 \exists f \exists G_{\text{res}} \exists G_{\text{sco}} \exists X_{\text{res}} \exists X_{\text{sco}} \\
 I.w.f \wedge \\
 G_{\text{res}} = \lambda h [h \succeq f \wedge \text{res}.w.h] \wedge \\
 G_{\text{sco}} = \lambda h [h \succeq f \wedge \text{sco}.w.h] \wedge \\
 X_{\text{res}} = \lambda x [\exists h G_{\text{res}}.h \wedge h.i.x] \wedge \\
 X_{\text{sco}} = \lambda x [\exists h G_{\text{sco}}.h \wedge h.i.x] \wedge \\
 \mathcal{D}'.X_{\text{res}}.X_{\text{sco}} \wedge \\
 G_{\text{sco}}.g
 \end{array} \right]
 \end{array} \right]
 \end{array}$$

I leave it to the reader to verify that the new definition is equivalent to D33 with regard to the determiner SOME. We can now say in precisely what sense the definition for SOME is quantificational: the output of a formula  $\text{SOME}.i.\phi.\psi$  is determined in terms of an underlying quantifier relation. In particular, the set of input pairs of which extensions survive in the output of such a formula can be determined by checking whether the relation  $\text{SOME}'$  holds between the set of values taken by  $i$  after update with the restrictor and the set of values taken by  $i$  after update with the scope. It remains only to specify the quantifier relations relevant to the determiners SOME and EXACTLY-ONE:

**Meaning Postulate MP15**

$$\begin{array}{l}
 \text{SOME}' = \lambda X \lambda Y [\exists x (X.x \wedge Y.x)] \\
 \text{EXACTLY-ONE}' = \lambda X \lambda Y [\exists x (X.x \wedge Y.x \wedge \forall y ((X.y \wedge Y.y) \rightarrow x = y))]
 \end{array}$$

## Chapter 4

# Presupposition and Modality in ABLE

In this chapter the dynamic framework developed so far will be adapted to allow for an adequate treatment of presupposition and epistemic modality. I will begin, in §4.1, by detailing the projection behaviour of presuppositions in the system as so far defined, showing some advantages of the approach, but also revealing a problem in the interaction between presupposition and quantification. The problem is one familiar from Heim's development of the CCP model, as found in her 1983 paper [He83]. §4.2 will be spent ignoring the problem, and instead concentrating on the semantics of epistemic modality. It will be shown that essentially the same definition of the *might* operator as was given for the system UL, in chapter 2, is also appropriate for ABLE, and that this definition leads immediately to a satisfying treatment of modal identity. However, it will also be shown that difficulties occur when quantifiers outscope the *might* operator, and this will motivate some tinkering with definitions from chapter 3. Having dealt appropriately with the semantics of epistemic modality, we will return, in §4.3, to presupposition, only to find that the earlier problems concerning the interaction of presupposition and quantification have miraculously vanished. In the remainder of the chapter some further extensions of ABLE will be discussed: a presuppositionally adequate semantics for disjunction will be given (thus countering a claim of Soames [So86] that this would be impossible in the CCP model), and a presuppositional way of accounting for some modal subordination phenomena will be offered.

### 4.1 Presupposition projection in ABLE

It will now be shown that, regarding the projection of presuppositions, the system developed in chapter 3 is comparable with Heim's 1983 account<sup>6</sup>. It will also be shown, in §4.1.2, that the ABLE treatment of proper names and definites, combined with ABLE's presupposition projection properties, lead to DRT-like anaphoric accessibility.

#### 4.1.1 Projection from connectives

With regard to the connectives, ABLE manifests much the same presupposition projection behaviour as PUL, or any other CCP account. This is most obviously seen by considering ABLE translations of the simple examples from chapter 1, for which the following abbreviations will be useful:

$$\begin{aligned} \text{BERTHA}.D.\phi &= \text{THE}.D.(\text{NAMED-BERTHA}^u.D).\phi \\ \text{ANNA}.D.\phi &= \text{THE}.D.(\text{NAMED-ANNA}^u.D).\phi \\ \text{bih} &= \text{BERTHA}.7.(\text{HAPPY}^u.7) \end{aligned}$$

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<sup>6</sup>That is to say, the system is comparable with the more formally developed aspects of Heim's 1983 account, since including Heim's informal description of a mechanism of *accommodation* would lead to a quite different set of predictions. In chapter 5, I will consider ways in which the process of accommodation might be formalised.



We begin with simple cases of presuppositions embedded under negations or in the antecedents of conditionals:

- E36 a. Bertha is hiding  
 b. *bih*
- E3 a. Anna realizes that Bertha is hiding  
 b. ANNA.3.(REALIZES<sup>a</sup>.3.*bih*)
- E4 a. Anna does not realize that Bertha is hiding.  
 b. NOT (ANNA.3.(REALIZES<sup>a</sup>.3.*bih*))
- E7 a. If Anna realizes that Bertha is hiding, then she will find her.  
 b. (ANNA.3.(REALIZES<sup>a</sup>.3.*bih*)) IMPLIES (FIND<sup>b</sup>.3.7)

It is easily shown that all of E3b, E4b and E7b *presuppose* (and also entail) E36b. Indeed, these are just instances of the following general property:

**Fact F3** *If  $\phi$  presupposes  $\psi$ , then NOT  $\phi$ ,  $\phi$  AND  $\chi$  and  $\phi$  IMPLIES  $\chi$  all presuppose  $\psi$ .*

*Proof: Simple. Firstly show by inspection of the clause for NOT (AND, IMPLIES) that if I admits NOT  $\phi$  (etc.) then I admits  $\phi$ . Thus the set of contexts which admit  $\phi$  is a superset of those which admit NOT  $\phi$  (etc.), and if all contexts which admit  $\phi$  satisfy  $\psi$  (the condition for  $\phi$  to presuppose  $\psi$ ), then it must be the case that all contexts which admit NOT  $\phi$  (etc.) satisfy  $\psi$ .*

Next a case of one presuppositional construction embedded within another:

- E9 a. Bertha regrets that Anna realizes that she is hiding.  
 b. BERTHA.7.(REGRETS<sup>a</sup>.7.(ANNA.3.(REALIZES<sup>a</sup>.3.*bih*)))

E9b presupposes E36b, and once again this is an instance of a more general principle:

**Fact F4** *If  $\phi$  presupposes  $\psi$ , and  $\psi$  presupposes  $\chi$ , then  $\phi$  presupposes  $\chi$ .*

*Proof: The result follows directly from the definitions of presupposition and satisfaction. If  $\phi$  presupposes  $\psi$  then every context which admits  $\phi$  must satisfy  $\psi$ , but  $\psi$  can only be satisfied in contexts which admit it, and all these contexts satisfy  $\chi$ . Thus every context which admits  $\phi$  satisfies  $\chi$ .*

What really marks the CCP model out from other theories is the occurrence of conditionalized presuppositions, as in the following case:

- E10 a. If Bertha is not in the kitchen, then Anna realizes that Bertha is in the attic.  
 b. (BERTHA.7.(IN-KITCHEN<sup>u</sup>.7)) IMPLIES  
 (ANNA.3.(REALIZES<sup>a</sup>.3.(IN-ATTIC<sup>u</sup>.7)))  
 c. (BERTHA.7.(IN-KITCHEN<sup>u</sup>.7)) IMPLIES  
 (IN-ATTIC<sup>u</sup>.7)

As might be expected, E10b presupposes the conditional E10c, i.e. that if Bertha is not in the kitchen, then she's in the attic. Conditionalized presuppositions are probably the single most controversial aspect of the CCP model, and the paradigmatically CCP behaviour of ABLE in this respect, as witnessed by the following fact, will be explored in more detail in chapter 5.

**Fact F5** *If  $\phi$  presupposes  $\psi$ , then  $\chi$  AND  $\phi$  and  $\chi$  IMPLIES  $\phi$  each presuppose  $\chi$  IMPLIES  $\psi$ .*

*Proof: It suffices to show that under the assumption that I admits ( $\chi$  AND  $\phi$ ) or I admits ( $\chi$  IMPLIES  $\phi$ ) it*

follows that  $I$  satisfies  $[\chi \text{ IMPLIES } \psi]$ . Here is the reasoning for the conjunctive case, that for implication being similar:

$I \text{ admits } (\chi \text{ AND } \phi)$	assumption
$\exists J I[\chi]J \wedge J \text{ admits } [\phi]$	defns. of AND, <i>admits</i>
$\exists J I[\chi]J \wedge J \text{ satisfies } [\psi]$	ass., defn. of <i>presupposes</i>
$\exists J I[\chi]J \downarrow [\psi]J$	defn of <i>satisfies</i>
$\exists J I[\chi]J [\text{NOT } \psi] \perp$	defn. of NOT
$I[\chi \text{ AND NOT } \psi] \perp$	defn. of AND
$I[\text{NOT } (\chi \text{ AND NOT } \psi)]I$	defn. of NOT
$I[\chi \text{ IMPLIES } \psi]I$	defn. of IMPLIES
$I \text{ satisfies } [\chi \text{ IMPLIES } \psi]$	defn. of <i>satisfies</i>

### 4.1.2 The Projection of Proper Names

The projection of presuppositions from embedded contexts, combined with the fact that proper names presuppose the presence of an appropriately named discourse marker, leads to proper names having very special anaphoric properties. Consider the following example:

- E37 a. If Butch is happy, then he is barking. He is happy. Therefore he is barking.
- b. THE.2.(NAMED-BUTCH<sup>u</sup>.2).(HAPPY<sup>u</sup>.2) IMPLIES  
THE.2.(MALE<sup>u</sup>.2 AND SINGULAR<sup>u</sup>.2).(BARK<sup>u</sup>.2)
- c. THE.2.(MALE<sup>u</sup>.2 AND SINGULAR<sup>u</sup>.2).(HAPPY<sup>u</sup>.2)
- d. THE.2.(MALE<sup>u</sup>.2 AND SINGULAR<sup>u</sup>.2).(BARK<sup>u</sup>.2)
- e. E37b,E37c  $\models_{\text{ABLE}}$  E37d

On its most obvious reading, that in which all the pronouns refer to Butch, E37a is an intuitively valid argument involving one application of *modus ponens*. Correspondingly, the ABLE entailment in E37e holds — that is to say, the translations of the first two sentences (i.e. E37b,c) entail the translation of the third (i.e. E37d). Yet E37b carries the same presupposition as E25b (“Butch is barking”), namely that 2 is established to refer to an individual named Butch. So even without specifying ways in which contexts satisfying the presuppositions of a formula may be generated, the formula can still appear in the premises of arguments. This is because the notion of entailment given in D25 does not depend on the premises being defined on all contexts, but only says that for all *suitable* contexts which satisfy the presuppositions of the premises, updating with the premises yields a state which satisfies the consequent.

It may surprise some readers that the ABLE translations of E37 can sustain the anaphoric linkage between the proper name Butch, which occurs within the antecedent of a conditional, and later pronouns beyond the scope of the conditional. In effect, ABLE manifests accessibility conditions for proper names and other definites reminiscent of Kamp’s Discourse Representation Theory. The fact that a presuppositional analysis for proper-names can account for their distinctive anaphoric properties is discussed in [Ze92], but this awkward detail of Kamp’s work had generally been ignored in earlier reformulations of DRT, such as [GS91b] and [Mu90].

Looked at from a slightly different angle, the projection behaviour of definites in ABLE tells us something about the relationship between DRT and File Change Semantics. The special mechanism for dealing with accessibility of names appears to be one of the few ways in which DRT and FCS differ significantly. Yet the ABLE analysis of definites, which I have shown leads to DRT-like anaphoric accessibility conditions, is basically that found in the original manifestation of FCS, namely Heim’s thesis. Thus FCS contains the core of an idea that not only reproduces, but also explains the special behaviour of definites and names in DRT. One can imagine adding an ABLE-like notion of entailment to FCS, something along the lines of “one sentence entails another if after updating a file set with the file change potential (FCP) of the first, updating with the FCP of the second has no effect”. I would speculate that if someone were to take the trouble to do this, they could provide a more concrete demonstration that DRT-like behaviour of definites can be derived within FCS.

### 4.1.3 Quantificational Projection: A Problem

The following example shows that a problem which appeared in Heim's 1983 paper has resurfaced:

- E38 a. A man discovered that he owned a priceless Modigliani.  
 b.  $\text{SOME}.i.(\text{MAN}^u.i).(\text{DISCOVER}^a.i.(\text{OAPM}^u.i))$
- E39 a. Every man owned a priceless Modigliani.  
 b.  $\text{EVERY}.j.(\text{MAN}^u.j).(\text{OAPM}^u.j)$

After updating with the restrictor of E38b, a state would be reached in which  $i$  was known to be a man. Assuming that the dynamic attitude predicate  $\text{DISCOVER}^a$  is constrained to be *factive*, this state could only be updated with the scope of E38b if the formula  $\text{OAPM}^u.i$  was satisfied. However, this formula would only be satisfied if all the values of  $i$  were already established to correspond to priceless Modigliani owners. Thus a state could only be updated with E38b if in that state every man was established to be a priceless Modigliani owner. Thus E38b *presupposes* E39b. The problem is that E38a certainly does not presuppose (or entail) E39a.

Heim's account also predicts overly strong, universal presuppositions from existential sentences, and for essentially the same reason: quantified markers represent *arbitrary objects*. Thus at the point where the factive verb in E38b is reached, the discourse marker plays the role of an *arbitrary* man, and updating can only continue if it is established that any arbitrary man satisfies the factive's presupposition. The provisional semantics for ABLE's determiner's presented in chapter 3 is uniform in this respect, so that the semantics for all the quantificational determiners is based on the same approach as for indefinites, which can be summed up as "add an arbitrary object, and check what effect updating with the restrictor and scope will have". Thus the occurrence of universal presuppositions is not restricted to indefinites alone, as is shown by the following examples:

- E40 a. Every woman regrets that she is married  
 b.  $\text{EVERY}.i.(\text{WOMAN}^u.i).(\text{REGRET}^a.i.(\text{MARRIED}^u.i))$
- E41 a. No woman regrets that she is married  
 b.  $\text{NO}.i.(\text{WOMAN}^u.i).(\text{REGRET}^a.i.(\text{MARRIED}^u.i))$
- E42 a. Every woman who regrets that she is married is sane  
 b.  $\text{EVERY}.i.(\text{WOMAN}^u.i \text{ AND } \text{REGRET}^a.i.(\text{MARRIED}^u.i)).(\text{SANE}^u.i)$
- E43 a. Every woman is married  
 b.  $\text{EVERY}.i.(\text{WOMAN}^u.i).(\text{MARRIED}^u.i)$

According to the chapter 3 semantics, each of E40b, E41b and E42b presuppose E43b. This may be defensible for E40a, though I will later argue that even in this case the universal presupposition is inappropriate, it seems harder to justify the presence of such a strong presupposition for E41a and E42a. Intuitively, E41a is true just in case there are no married women who regret being married, and E43a is true just in case every married woman who regrets being married is sane. Such truth conditions are, of course, compatible with the existence of unmarried women.

The following fact summarises the problem with the *arbitrary object* analysis of quantification, showing that, as things stand, whenever a presupposing construction is bound within the scope of a quantifier, a universal presupposition arises:

**Fact F6** *If  $\phi$  presupposes  $\psi$ ,  $D$  is any quantificational determiner (i.e. not THE), and true of type  $\pi$  is interpreted as the trivial CCP  $\lambda I \lambda J [I = J]$ , then under the temporary definitions D38 and D37 the following hold:*

1.  $D.i.\phi.\chi$  presupposes  $\text{EVERY}.i.\text{true}.\psi$ , and
2.  $D.i.\chi.\phi$  presupposes  $\text{EVERY}.i.\chi.\psi$

*Sketch of proof for (1): inspection of definitions D38 and D37 shows that if  $I$  admits  $\llbracket \mathcal{D}.i.\phi.\chi \rrbracket$  then  $\exists J I\{+i\}J \wedge J$  admits  $\llbracket \phi \rrbracket$ , and thus that  $\exists J I\{+d\}J \wedge J$  satisfies  $\llbracket \psi \rrbracket$ . By the definition of satisfaction it follows that  $(+i) \circ \llbracket \psi \rrbracket$  (where  $\circ$  is relational composition) is satisfied in  $I$ , and thus that  $(+i) \circ \text{true} \circ \llbracket \psi \rrbracket$  is satisfied in  $I$ , from which the result follows by the semantics of EVERY. The reasoning is similar for (2).*

The above problem motivates an alteration to the semantics of quantification, but, as will be seen in the next section, such a change can to some extent be motivated by independent considerations.

## 4.2 Epistemic modality

### 4.2.1 Modal Identity Problems

In this section I will provide a semantics for the unary ABLE operator MIGHT, extending the development of Veltman's [Ve91] account of epistemic modality in the first part of this thesis, which concerned a propositional language, to the first-order language of ABLE. In fact the PUL definition (D6) requires little modification, except for adjustment to the  $\text{Ty}_3$  format of ABLE semantic clauses:

**Definition D39 (Epistemic Modalities in ABLE)**

$$\begin{aligned} \llbracket \text{MIGHT} \phi \rrbracket &= \lambda I \lambda J \exists K I[\phi]K \wedge \\ &\quad (\neg(K = \perp) \rightarrow J = I) \wedge \\ &\quad (K = \perp \rightarrow J = \perp) \\ \llbracket \text{MUST} \phi \rrbracket &= \llbracket \text{NOT}(\text{MIGHT}(\text{NOT} \phi)) \rrbracket \end{aligned}$$

This definition for MIGHT, which clearly preserves the intuitions behind the operator in Veltman's original work and in the UL/PUL systems from chapter 2 of this thesis, is essentially that used in [De92, De93]. In discussing the general properties of these operators, it is helpful to introduce Veltman's notion of a *test*. Tests are a special class of formulae which have, in terms of ABLE, the following property:

**Definition D40 (Tests)** *A formula  $\phi$  is a test if and only if:*

$$\forall I \forall J I[\phi]J \rightarrow (J = I \vee J = \perp)$$

This means that updating with a test is either uninformative, the output state being the same as the input state, or *over-informative*, the output being the absurd state. It is now possible to get a formal grasp on the interpretation of ABLE's epistemic modalities:

**Fact F7** *The MIGHT-operator defines a consistency test. Thus for any non-absurd state  $I$  which admits an ABLE formula  $\phi$ :*

1.  $\text{MIGHT} \phi$  is a test.
2.  $I[\llbracket \text{MIGHT} \phi \rrbracket I]$  iff  $I$  consistent-with  $\llbracket \phi \rrbracket$
3.  $I[\llbracket \text{MIGHT} \phi \rrbracket \perp]$  iff  $\neg I$  consistent-with  $\llbracket \phi \rrbracket$

*Proof of (1): Simply observe that if a pair  $I, J$  are in the denotation of  $\text{MIGHT} \phi$ , then by the definition of MIGHT there must be some state  $K$  for which  $I[\phi]K$  holds, and since  $K$  either is or is not the absurd state, at least one of the two conditional antecedents in the definition must be met, from which it follows that at least one of the consequents holds, namely that  $J = I$  or  $J = \perp$ , but this is the condition for test-hood.*

*Proof of (2) and (3). The condition for  $I[\llbracket \text{MIGHT} \phi \rrbracket I]$  is that updating with  $\phi$  does not lead to the absurd state, and this is also the condition for consistency of  $\phi$  with  $I$ . On the other hand, the condition for  $I[\llbracket \text{MIGHT} \phi \rrbracket \perp]$  is that updating with  $\phi$  does lead to the absurd state, and this is both necessary and sufficient for consistency to fail.*

The following lemma helps to clarify the interpretation of the MUST-operator:

**Lemma L1** *For any state  $I$  which admits a formula  $\phi$ :*

$$I \text{ satisfies } \llbracket \phi \rrbracket \text{ iff } \neg I \text{ consistent-with } \llbracket \text{NOT } \phi \rrbracket$$

*Proof:* By the definition of satisfaction,  $I$  satisfies  $\llbracket \phi \rrbracket$  iff  $I \llbracket \phi \rrbracket I$ , and by the definition of negation this holds iff  $I \llbracket \text{NOT } \phi \rrbracket \perp$ , which in turn, by the definition of consistency, holds iff  $\neg I$  consistent-with  $\llbracket \text{NOT } \phi \rrbracket$ .

**Fact F8** *The MUST-operator defines a test for satisfaction. Thus for any non-absurd state  $I$  which admits an ABLE formula  $\phi$ :*

1.  $\text{MUST}\phi$  is a test.
2.  $I \llbracket \text{MUST}\phi \rrbracket I$  iff  $I$  satisfies  $\llbracket \phi \rrbracket$
3.  $I \llbracket \text{MUST}\phi \rrbracket \perp$  iff  $\neg I$  satisfies  $\llbracket \phi \rrbracket$

*Proof of (1):* Since for any  $\phi$ ,  $\text{MIGHT}\phi$  is a test, it follows that  $\text{MIGHT NOT } \phi$  is also a test, and so if a pair  $I, J$  is in the denotation of  $\text{MIGHT}(\text{NOT } \phi)$ , then  $J = I$  or  $J = \perp$ . From the semantics of negation it follows that if  $I \llbracket \text{MIGHT}(\text{NOT } \phi) \rrbracket I$  then  $I \llbracket \text{NOT}(\text{MIGHT}(\text{NOT } \phi)) \rrbracket \perp$ , and if  $I \llbracket \text{MIGHT}(\text{NOT } \phi) \rrbracket \perp$  then  $I \llbracket \text{NOT}(\text{MIGHT}(\text{NOT } \phi)) \rrbracket I$ , and since this is exhaustive, the result follows.

*Proof of (2):*

$$\begin{aligned} I \text{ satisfies } \phi & \text{ iff } \neg(I \text{ consistent-with } \llbracket \text{NOT } \phi \rrbracket) && \text{lemma L1} \\ & \text{ iff } I \llbracket \text{MIGHT}(\text{NOT } \phi) \rrbracket \perp && \text{defn. of consistency} \\ & \text{ iff } I \llbracket \text{NOT}(\text{MIGHT}(\text{NOT } \phi)) \rrbracket I && \text{defn. of negation} \end{aligned}$$

*Proof of (3):* Similar to (2).

I will not discuss the MUST operator in detail here, but the reader is invited to verify that it behaves appropriately with respect to the hide-and-seek examples from chapter 2. Even without examining the applications of the MUST operator, it is at least comforting that the dual of a consistency test should be a test for satisfaction, and, on the basis of this interpretation, it might be natural to think of MUST as corresponding not to a standard English modal, but as one of the sentential operators which could be called *argument connectives*, for example “so” or “therefore”. It is also comforting that the definitions of the modalities preserve the standard CCP presupposition inheritance properties of modals:

**Fact F9** *If  $\phi$  presupposes  $\psi$ , then (1)  $\text{MIGHT}\phi$  and (2)  $\text{MUST}\phi$  also presuppose  $\psi$ .*

*Proof:* From the definition of MIGHT it can be seen that  $I$  admits  $\text{MIGHT}\phi$  only if  $I$  admits  $\phi$ , from which the first result follows by the definition of presupposition. The same result for MUST follows from the proof of (1), the duality of the modalities, and the inheritance properties of NOT (Fact F3).

To see that in general this projection behaviour is appropriate — counter-examples being discussed in chapter 5 — recall the following example from chapter 1, which was argued to presuppose that Bertha is hiding:

E8 Anna might realize that Bertha is hiding.

Although there remain some problems, to which we will turn shortly, the account of epistemic possibility already has significant applications, as is shown by examples E44–E47:

- E44 a. It is possible there is a happy farmer, but, then again, it is possible that there are no happy farmers.  
 b.  $\text{MIGHT}(\text{SOME}.i.(\text{FARMER}^u.i).(\text{HAPPY}^u.i)) \wedge \text{MIGHT}(\text{NO}.i.(\text{FARMER}^u.i).(\text{HAPPY}^u.i))$
- E45 a. No farmer is happy.  
 b.  $\text{NO}.i.(\text{FARMER}^u.i).(\text{HAPPY}^u.i)$
- E46 a. Perhaps the spy is the president.

b.  $\text{MIGHT}.\langle \text{THE}.i.\langle \text{SPY}^u.i \rangle.\langle \text{THE}.j.\langle \text{PRESIDENT}^u.j \rangle.(i = j) \rangle \rangle$

E47 a. The spy is not the president.

b.  $\text{THE}.i.\langle \text{SPY}^u.i \rangle.\langle \text{THE}.j.\langle \text{PRESIDENT}^u.j \rangle.\langle \text{NOT}(i = j) \rangle \rangle$

If my proposal for research into depression amongst farmers were to include E44a, and if, having obtained funding and done the relevant fieldwork, I were later to produce a report including E45a, I think I would feel justified in taking a certain pride in the fruitfulness of my research. In spite of an initial dearth of data, I would have made an important discovery about the distribution of agriculture-related emotional disorders. On the other hand, if my research proposal were to include E45a, and my final conclusions included E44a, I think DARPA would be justified in questioning whether I deserved further funding. For, quite apart from the limited military potential of my results, it would seem that my proposals tended to be rather unreliable. The conclusion that some farmer might be happy is simply inconsistent with the previous assertion that no farmer is happy.

The contrast between E44a followed by E45a on the one hand, and the E45a followed by E44a on the other, is mirrored by the fact that the ABLE formula E44b AND E45b is consistent, whereas the formula E45b AND E44b is not. This would hold if definition D39 was used just as it will with the final definition for the semantics of MIGHT to be given below. The predictions with regard to examples E46 and E47 are similarly sensible on either definition. In particular, the formula E46b AND E47b is consistent, but swapping the order of the two conjuncts would yield an inconsistent formula. The success of ABLE in dealing with this last pair of examples shows that a serious problem with another recent attempt to combine DPL and update semantics, namely the Dynamic Modal Predicate Logic (DMPL) of van Eijk and Ceparrello [EC92], has been circumvented. In DMPL it would make little sense to give a definition like D39. The reasons are technical: the DMPL notions of entailment and negation require that DMPL formulae can be calculated pointwise over assignment functions (recall the property of *distributivity* introduced in section 3.3). However, D39 cannot be calculated pointwise over assignment functions (sequences) as consistency of a formula is evaluated with respect to the full set of world-sequence pairs in the input. To give a flavour of the DMPL semantics for epistemic possibility, it must be translated into ABLE terms. This is not difficult given the technical proximity of DMPL to ABLE, although fans of DMPL may take some time to convince themselves that the following definition really is close to that given in [EC92]:

**Definition D41 (Epistemic Possibility: DMPL version)**

$$\begin{aligned} \llbracket \text{MIGHT} \phi \rrbracket &= \lambda I \lambda J [\exists K I \downarrow \llbracket \phi \rrbracket K \wedge \\ &J = \lambda w \lambda f [I.w.f \wedge \exists w' K.w'.f]] \end{aligned}$$

This definition can be seen as being in terms of equivalence classes of world-sequence pairs in the input: each class contains only pairs involving the same sequence. Given some input state  $I$ , the (anaphorically closed) output of  $\text{MIGHT} \phi$  will be the union of all the equivalence classes for which at least one world-sequence pair would survive in an update with  $\phi$ . Put another way, a given world-sequence pair in the input will survive just in case there is some relevant world where the values that the sequence gives to discourse markers are consistent with  $\phi$ . Unfortunately the use of a definition like D41 means that in DMPL the following entailment is valid:

$$\exists x \exists y \diamond (x = y) \models x = y$$

Thus if the above examples were to be translated into DMPL with “the spy” as  $x$  and “the president” as  $y$ , then the validity of the pattern above would mean that learning that the spy might be the president would be equivalent to learning that the spy was the president. Furthermore, there is no obvious alternative translation which would solve the problem. For instance, translating the definite descriptions as constants would not help, since in DMPL updating with  $\text{MIGHT}(a = b)$  for separate constants  $a$  and  $b$  always yields absurdity. Besides this, the problem is not restricted to definite descriptions but applies also to pronominals:

E48 a. The first lady is not spying

- b.  $\text{THE}.j.(\text{FL}^u.j).(\text{NOT SPYING}^u.j)$
- E49 a. I can see a woman in the Whitehouse  
 b.  $\text{SOME}.i.(\text{WOMAN}^u.i).(\text{I-SEE-IN-W}^u.i)$
- E50 a. She might be spying.  
 b.  $\text{SHE}.i.\text{MIGHT}(\text{SPYING}^u.i)$
- E51 a. However, she might be the first lady.  
 b.  $\text{SHE}.i.(\text{THE}.j.(\text{FL}^u.j).\text{MIGHT}(i \text{ IS } j))$

This discourse consisting of E48a–E51a seems perfectly consistent, and, indeed, the formula (E48b AND E49 AND E50 AND E51) is consistent in ABLE. Yet there is a single discourse referent which might be spying, and also might be the first lady, even though the first lady is apparently above suspicion. Thus the sub-formula  $\text{MIGHT}(\text{SPYING}^u.i)$  in E50b should not — as would be the case in DMPL — remove sequences which map  $i$  onto the first lady, even though such sequences considered in isolation lead to values which are inconsistent with the assertion of spyness.

It should be realised that identity problems like those found in DMPL are not new, and not essentially dynamic in nature: many static systems of modal predicate logic have comparable properties. Indeed Kripke has argued persuasively [Krip72] that identity is a non-contingent property, so that if two things are identical then they are necessarily so. Given the logical duality of necessity and possibility, it immediately follows that things which might not be identical are not identical, and thus that things which might be identical are identical. However, Kripke’s arguments concern metaphysical contingency within total logics. In this setting we may wonder, along a parallel line to that taken by Kripke, which pairs of objects could falsify the open formula  $\diamond(x = y) \rightarrow x = y$ : substituting distinct values for  $x$  and  $y$  into the implication would falsify the antecedent, whereas substituting one value for both  $x$  and  $y$  would validate the consequent. But, if the formula is not falsifiable, then  $=$  is not metaphysically contingent.

The power of the Kripkean argument for non-metaphysically contingent identity does not, however, extend into the epistemic realm. For although it is impossible to simultaneously locate a pair of objects in the world which would obviously falsify the implication, it seems that the objects of thought and discourse, which may not even have a location in the exterior world, behave differently. There is manifestly no contradiction in positing that the objects referred to by two different descriptions, perhaps even in different languages, are in fact identical, nor is there necessarily any certainty about the question of whether a person seen in one guise is the same person as was separately seen in another guise. This is why it is consistent, albeit unwise in the wrong company, to assert that “the spy might be the president”, and still allow that they may later be discovered to be different.

In summary, whilst DMPL could well be an interesting object of logical investigation, it lacks any intuitive interpretation. In most respects the  $\diamond$  operator behaves as like Veltman’s MIGHT: learning what *is* the case restricts what merely *might* be. However, with regard to questions of identity the  $\diamond$  behaves not as a *Veltmanesque* epistemic modality but as a *Kripkean* metaphysical modality: learning that things *might* be identical determines that *are* identical. The reason is technical. An epistemic operator must be sensitive to all aspects of a referent’s identity, but DMPL’s entailment (and also negation) is not compatible with such a sensitive operator. On the other hand, the entailment used in ABLE or in Dekker’s EDPL (from which much of the inspiration for ABLE derives), is compatible with the definition of an identity-sensitive operator. Finally, and without going into details, I should point out the *sequence semantics* for DPL introduced by Vermeulen [Ver92] provides yet another route to an identity-sensitive modality, but, as with DMPL and ABLE, would also be technically compatible with a DMPL-style insensitive operator.

### 4.2.2 The dynamics of quantifying-in.

Unfortunately, the definition of MIGHT given in D39, does not interact appropriately with quantificational determiners. For example, it is clear that E52a and E53a express quite different propositions, but the formula E52b differs from E53b only in terms of its anaphoric potential. That is to say, E52b and E53b are satisfied by exactly the same set of states.

- E52 a. Most politicians might be spying.  
 b.  $\text{MOST}.i.(\text{POLITICIAN}^u.i)(\text{MIGHT}(\text{SPYING}^u.i))$
- E53 a. Some politician might be spying.  
 b.  $\text{SOME}.i.(\text{POLITICIAN}^u.i)(\text{MIGHT}(\text{SPYING}^u.i))$

The definition for the semantics of ABLE's quantificational determiners is based on the principle of adding a discourse marker which in different sequences takes each single object as its value, and then checking what new information about the possible value of the marker is obtained by updating with the restrictor and scope. Thus the privileged marker stands proxy for the entire set of objects onto which it is mapped by different sequences associated with the various intermediate information states involved in updating with a quantificational formula. However, the interpretation of discourse markers outside of the scope of their introductory quantifier is quite different. An unquantified marker may also have several alternative valuations, but the interpretation is essentially disjunctive rather than conjunctive. Rather than standing proxy for the entire set of objects which it can take as values, an unquantified marker is understood as standing for just one of the alternative values: the presence of alternative values represents not multiplicity but underspecificity.

This difference in interpretation becomes crucial when quantifying into a modal context. MIGHT must check separately for consistency of its argument with regard to each of the alternative values for the quantified markers. Thus in E52b and E53b, where  $i$  is a quantified discourse marker, the effect of updating with  $\text{MIGHT}(\text{SPYING}^u.i)$  should be to remove all those sequences which map  $i$  onto an object which is known not to be spying. However, the same sub-formula should have a quite different effect when it occurs in E50b, above.

In order to define a notion of epistemic possibility which preserves the positive traits of that given in D39, and also gives a sensible account of the behaviour of quantified markers which distinguishes appropriately between E52b and E53b, the semantics of ABLE quantification will be altered. Note, however, that this is not the only way to proceed: in an earlier version of this chapter (appearing as [Be93b]), a special mechanism was added for keeping track of quantified markers, and the semantics of MIGHT was altered so as to interact differently with quantified and unquantified markers. Here I will formulate things more along the lines taken by my colleagues, Groenendijk, Stokhof and Veltman, in recent talks [GSV93]. The motivation, however, differs somewhat from theirs, in that my reason for following their formulation is not simply that it eliminates the above-mentioned problem with epistemic modality, but that it will also lead to an elegant solution of some longstanding problems in the treatment of presupposition.

As I have said, under the semantics above, a quantified marker at some point in a formula stands proxy for the entire set of individuals which are compatible with the formula up to that point. So, although the semantics of MIGHT is only appropriate when multiplicity of interpretations represents underspecificity of reference, this is not the case with a quantified marker. The solution is to alter the semantics of quantification so that rather than considering all the different possible values of the quantified marker simultaneously, they are considered one at a time. By evaluating the restrictor and scope of a quantification separately for each value of the quantified marker, operators in the restrictor and scope become insensitive to the multiplicity of different values. To this end, a special way of interpreting a formula is defined. Given an ABLE formula,  $\phi$ , and a discourse marker,  $d$ ,  $[\phi]_d$  will define that relation between information states such that after splitting up the first state with respect to each different value of the marker  $d$ , and updating each of the resulting states using the normal interpretation  $[\phi]$ , the outputs can be recombined to yield the second state.  $[\phi]_d$  is defined using a constant *distribute*, where *distribute.F.d* will be referred to as the distributive re-interpretation of the CCP  $F$  with respect to the marker  $d$ .

In the definition below, *distribute* is defined in two stages. Firstly a function *unfold* is given, where *unfold.F.D* defines a relation between two information states holding just in case there is a value of  $d$  such that after restricting the input to sequences which give that valuation, updating with  $F$  yields the output. Here, the process of *restricting* an information state with respect to values taken by some discourse marker is easily defined: if  $I$  is a state and  $d$  is a marker, then the restriction of  $i$  with respect to the value  $X$  for  $d$  is given by  $\lambda w \lambda f [I.w.f \wedge f.d = X]$ . The constant *distribute* is then defined such that a formula *distribute.F.d* denotes a CCP having as inputs only states which admit *unfold.F.d*, and



having as output the union of the outputs of  $\llbracket \phi \rrbracket_a$ . Here, given an input  $I$ , the union of the outputs of a CCP  $F$  is just  $\lambda w \lambda f [\exists K I\{F\}K \wedge K.w.f]$ .

### Meaning Postulate MP16

- (1)  $unfold = \lambda F \lambda D \lambda I \lambda J [\exists X (\lambda w \lambda f [I.w.f \wedge f.d = X])\{F\}J]$
- (2)  $distribute = \lambda F \lambda D \lambda I \lambda J [I \text{ admits } (unfold.F.D) \wedge$   
 $J = \lambda w \lambda f [\exists K I\{unfold.F.D\}K \wedge K.w.f]]$

### Definition D42 (Distributive Interpretation of ABLE Formulae)

$$\llbracket \phi \rrbracket_a = distribute.\llbracket \phi \rrbracket.d$$

The definition allows for a relatively conservative alteration to the quantifiers. It can be seen from the following two properties that the use of  $\llbracket \phi \rrbracket_a$  in place of  $\llbracket \phi \rrbracket$  will only have an effect in a very restricted range of circumstances:

**Fact F10** *If  $\phi$  does not contain the discourse marker  $d$  then  $\llbracket \phi \rrbracket_a = \llbracket \phi \rrbracket$*

*Sketch of proof:* ABLE predicates have the property of relevance (cf. §3.3), which is easily shown (by induction over formula complexity) to be preserved by all ABLE's operators. Thus we need only consider states  $I, J$  which do not have  $d$  in their domain. For such states it is clear that  $I[\llbracket \phi \rrbracket]J \leftrightarrow I[\llbracket \phi \rrbracket_a]J$  (Note that the definition of  $\llbracket \phi \rrbracket_a$  is compatible with the input state only giving  $d$  the value  $\lambda x [\perp]$ .)

**Fact F11** *If  $I$  admits  $\llbracket \phi \rrbracket$  and  $\phi$  does not contain any occurrences of the epistemic modality, MIGHT, then  $I[\llbracket \phi \rrbracket]J \leftrightarrow I[\llbracket \phi \rrbracket_a]J$*

*Sketch of proof:* ABLE predicates have the property of distributivity (cf. §3.3), and this can be shown (another induction) to be preserved by ABLE's connectives and determiners. Thus we need only consider singleton states (those with only one world-sequence pair), and for these the result is trivial, since distributing over a single world-sequence pair has no effect.

Having given a definition of  $\llbracket \phi \rrbracket_a$ , amending the semantics of quantifiers is easy. Where before the passage from input to output was mediated by a sequence of updates  $I\{+.i\}init[\llbracket \phi \rrbracket]res[\psi]sco$ , we simply replace the occurrences of  $\llbracket . \rrbracket$  by  $\llbracket . \rrbracket_a$  to get the new definitions, given below for completeness:

### Definition D43 (Non-existential Determiners - Final Version)

*If  $\mathcal{D}$  is one of EVERY, MOST, FEW or NO, and  $\mathcal{D}'$  is a corresponding quantifier relation of type  $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ , then:*

$$\llbracket [\mathcal{D}.i.\phi.\psi] \rrbracket = \lambda I \lambda J \left[ \begin{array}{l} \exists init \exists res \exists sco \\ I\{+.i\}init[\llbracket \phi \rrbracket_a]res[\psi]_a.sco \wedge \\ \\ J = \lambda w \lambda g \\ \left[ \begin{array}{l} \exists f \exists G_{res} \exists G_{sco} \exists X_{res} \exists X_{sco} \\ I.w.f \wedge \\ G_{res} = \lambda h [h \succeq f \wedge res.w.h] \wedge \\ G_{sco} = \lambda h [h \succeq f \wedge sco.w.h] \wedge \\ X_{res} = \lambda x [\exists h G_{res}.h \wedge h.i.x] \wedge \\ X_{sco} = \lambda x [\exists h G_{sco}.h \wedge h.i.x] \wedge \\ \mathcal{D}'.X_{res}.X_{sco} \wedge \\ g = \lambda D \lambda x \left[ \begin{array}{l} f.D.x \vee \\ \exists h G_{sco}.h \wedge h.D.x \end{array} \right] \end{array} \right] \end{array} \right]$$

**Definition D44 (Existential Determiners: final version)**

If  $\mathcal{D}$  is SOME or EXACTLY-ONE, and  $\mathcal{D}'$  is a corresponding quantifier relation of type  $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ , then:

$$\begin{array}{l}
 \llbracket \mathcal{D}.i.\phi.\psi \rrbracket = \lambda I \lambda J \\
 \left[ \begin{array}{l}
 \exists \text{init} \exists \text{res} \exists \text{sco} \\
 I\{+.i\}\text{init}\llbracket \phi \rrbracket, \text{res}\llbracket \psi \rrbracket, \text{sco} \wedge \\
 \\
 J = \lambda w \lambda g \\
 \left[ \begin{array}{l}
 \exists f \exists G_{\text{res}} \exists G_{\text{sco}} \exists X_{\text{res}} \exists X_{\text{sco}} \\
 I.w.f \wedge \\
 G_{\text{res}} = \lambda h [h \succeq f \wedge \text{res}.w.h] \wedge \\
 G_{\text{sco}} = \lambda h [h \succeq f \wedge \text{sco}.w.h] \wedge \\
 X_{\text{res}} = \lambda x [\exists h G_{\text{res}}.h \wedge h.i.x] \wedge \\
 X_{\text{sco}} = \lambda x [\exists h G_{\text{sco}}.h \wedge h.i.x] \wedge \\
 \mathcal{D}'.X_{\text{res}}.X_{\text{sco}} \wedge \\
 G_{\text{sco}}.g
 \end{array} \right]
 \end{array} \right]
 \end{array}$$

What is the reason for the definedness condition, *I admits (unfold.F.D)* in postulate MP16? It is easy to see that with no such condition some rather strange effects would arise. For instance, if the formula  $F$  was the denotation of a simple predication  $P(d')$  for some discourse marker  $d'$  not in the domain of the input, then *unfold.F.D* would have no outputs, and the effect of taking the *union* of the outputs would be to produce the absurd state. Whereas, for such an input,  $\llbracket P(d') \rrbracket$  would not define an output,  $\llbracket P(d') \rrbracket_a$  would define an update to the absurd state. Thus undefinedness would cease to propagate through formulae involving distributive updates, leading to a rather strange inconsistency, and consequently strange logic. So some definedness condition is needed. However, it would be possible to use a stronger definedness condition, say insisting not just that there is at least one value of the distributed marker for which there is an update, but requiring that for every value an update was defined. Such a condition would still be sensible as far as the interaction of quantification and modality was concerned, but would lead to different predictions regarding the projection of presuppositions from quantified contexts. In fact, such a strong definedness condition would yield the same predictions for presupposition projection as were obtained with the earlier definitions of quantification in chapter 3. However, in the next section it will be shown that whereas the earlier definitions of quantification give the same undesirable presupposition projection effects as plagued the system in [He83], the new definitions provide quite attractive projection properties.

### 4.3 Presupposition in ABLE revisited

Fact F6, which says that universal presuppositions arise from the chapter 3 definitions of quantification, would fail under the ammended definitions. To see this it suffices to give a counter-example, so constructing an information state which admits E38b but does not satisfy E39b will be adequate:

- E38 a. A man discovered that he owned a priceless Modigliani.  
 b. SOME. $i$ .(MAN $^u$ . $i$ ). (DISCOVER $^a$ . $i$ . (OAPM $^u$ . $i$ ))
- E39 a. Every man owned a priceless Modigliani.  
 b. EVERY. $j$ .(MAN $^u$ . $j$ ). (OAPM $^u$ . $j$ )

Let us assume MAN $^u$  and OAPM $^u$  to be simple, and DISCOVER $^a$  to be factive. Here is a definition of an appropriate information state in a simplified model:

- The state  $I$  contains the single world sequence pair  $w, \langle \rangle$
- The domain  $D_e$  consists of only two individuals,  $a$  and  $b$

- $w$  is a man's world (i.e. both  $a$  and  $b$  are men)
- In  $w$ ,  $a$  owns a priceless Modigliani and has discovered that he owns it, whilst  $b$  does not own a priceless Modigliani. Note that ownership of a priceless Modigliani is assumed to be a basic property, so that the object corresponding to the Modigliani can be ignored.

That  $I$  does not satisfy E39b should be clear, since in  $I$  there is a man who does not own a priceless Modigliani. Inspection of the semantics for existential determiners (D44) shows that the condition for  $I$  to admit E38b is that there exist states  $J, K, L$  such that:

$$I\{+.i\}J[\text{MAN}^u.i]_i K[\text{DISCOVER}^a.i.(OAPM^u.i)]_i L$$

It is easy to find the appropriate values for  $J$  and  $K$ .  $J$  must consist of two world sequence pairs mapping  $i$  onto  $a$  and  $b$ , respectively. Furthermore, since both  $a$  and  $b$  are men, we should expect the update with  $[\text{MAN}^u.i]_i$  not to remove any world-sequence pairs from  $J$ : since  $\text{MAN}^u$  is simple,  $J$  must admit  $[\text{MAN}^u.i]_i$ , and F11 means that we can ignore the fact that  $\text{MAN}^u.i$  is evaluated distributively with respect to  $i$ . Thus:

$$J = K = \{w : \langle i \mapsto a \rangle, w : \langle i \mapsto b \rangle\}$$

By the definition of distributive update (D42),  $L$  is calculated by taking the union of all the states  $M$  (provided that there is at least one state  $M$ ) such that:

$$K\{\text{unfold}.\text{DISCOVER}^a.i.(OAPM^u.i)]_i\}M$$

Since of the two values for  $i$ , only  $a$  corresponds to an individual satisfying the factive presupposition of  $\text{DISCOVER}^a.i$ , there will only be one appropriate  $M$ :

$$M = \{w : \langle i \mapsto a \rangle\}$$

The existence of this single value for  $M$  is enough to guarantee that E38b is admitted in  $I$ , as required. Further, it can be seen that  $L$  will take the same value, i.e. the singleton set containing the pair of  $w$  and a sequence mapping  $i$  onto the individual  $a$ , and, by inspection of the semantics for SOME, that this will also be the output of the whole formula E38b.

So given that Fact F6 is no longer appropriate, what are the presuppositions of quantificational formulae involving a bound presuppositional expression? The following fact shows that under the new semantics for quantifiers, although the universal presuppositions associated with the chapter 3 definitions have gone, there remain at least existential constraints:

**Fact F12** *If  $\phi$  presupposes  $\psi$ ,  $\mathcal{D}$  is any quantificational determiner (i.e. not THE), and true of type  $\pi$  is interpreted as the trivial CCP  $\lambda I \lambda J [I = J]$ , then under the final definitions D43 and D44:*

1.  $\mathcal{D}.i.\phi.\chi$  presupposes  $\text{SOME}.i.\text{true}.\psi$ , and
2.  $\mathcal{D}.i.\chi.\phi$  presupposes  $\text{SOME}.i.\chi.\psi$

*Sketch of proof for (1): From D43 and D44,  $\mathcal{D}.i.\phi.\chi$  is admitted by any state for which:*

$$\exists \text{init} \exists \text{res} \exists \text{sco} \ I\{+.i\} \text{init}[\phi]_i \text{res}[\chi]_i \text{sco}$$

*Given the assumption that  $\phi$  presupposes  $\psi$ , it follows that a necessary condition for the formula to be admitted by  $I$  is that there is some state  $\text{init}$  such that:*

$$I\{+.i\} \text{init} \wedge \text{init admits } [\phi]_i$$

*From the definition of distributive update, it follows that:*

$$\text{init admits } [\phi]_i \text{ iff } \text{init admits } (\text{unfold}.\text{DISCOVER}^a.i.\phi)]_i$$

Using the definition of *unfold* gives us:

$init\ admits\ [\phi]$  iff  $\exists X ((\lambda w \lambda f [init.w.f \wedge f.d = X])\ admits\ [\phi])$

But by assumption  $\phi$  presupposes  $\psi$ , so:

$init\ admits\ [\phi]$  iff  $\exists X ((\lambda w \lambda f [init.w.f \wedge f.d = X])\ satisfies\ [\psi])$

Thus for  $I$  to admit  $\mathcal{D}.i.\phi.\chi$ , it must be the case that there is some individual in a singleton set  $X$  such that if  $I$  is extended with the marker  $i$ , and is then restricted by removing all the world sequence pairs where  $i$  is not mapped onto  $X$ , the resulting state satisfies  $\psi$ . This existential requirement is enough to force the required condition, that  $SOME.i.true.\psi$  is satisfied in the input, but  $I$  omit the remaining details. The proof of (2) would be similar.

So whilst E38b does not presuppose E39b, it does presuppose E54b:

- E54 a. Some man owned a priceless Modigliani.  
b.  $SOME.j.(MAN^u.j).(OAPM^u.j)$

Similar results are obtained for the other quantificational examples considered earlier, and it will now be argued that these results are not only reasonable, but represent an advance over previous analyses. Firstly a case with a presupposition in the restrictor of a quantifier:

- E42 a. Every woman who regrets that she is married is sane  
b.  $EVERY.i.(WOMAN^u.i\ AND\ REGRET^a.i.(MARRIED^u.i)).(SANE^u.i)$
- E55 a. Some woman is married  
b.  $EVERY.i.(WOMAN^u.i).(MARRIED^u.i)$

As pointed out earlier, the prediction of a universal presupposition for E42 is quite clearly inappropriate. On the current analysis, E42b presupposes E55b, and this existential presupposition is certainly to be preferred. Although there can be no question that analyses<sup>7</sup> which predict a universal presupposition for this case are inappropriate, there must remain a question as to whether the existential analysis is correct. I think it quite possible to defend a view that such sentences should have no presuppositions, but only implicate that there are some individuals which satisfy the restrictor conditions, and thus implicate that there are some individuals which satisfy the restrictor's presuppositions. It could be that variations on the definition of distributive update used within the semantics of quantifiers would lead to such results, but in this thesis I will not attempt any further development along these lines.

Much more controversial are cases where a presupposition occurs in the scope of a quantificational construction. Unfortunately, there is, to my knowledge, no intellectually rigorous analysis of such sentences in the existing presuppositional literature, and most authors have been happy to advocate whatever claims about the data pose fewest problems for their own theory. There is thus no consensus concerning the presuppositions of the following examples, the first being from §4.3 above:

- E40 a. Every woman regrets that she is married  
b.  $EVERY.i.(WOMAN^u.i).(MARRIED^u.i)$
- E56 a. Every man discovered that he owned a priceless Modigliani  
b.  $EVERY.i.(MAN^u.i).(DISCOVER^a.i.(OAPM^u.i))$

I find the lack of consensus surprising, since there is no reason why standard presuppositional tests should not be applied in order to resolve the issue. I will only consider one such test, embedding under negation:

- E57 a. Not every woman regrets that she is married.  
b.  $NOT\ (EVERY.i.(WOMAN^u.i).(MARRIED^u.i))$

<sup>7</sup>Eg. [He83] without her suggested addition of a mechanism for accommodation.

- E58 a. Not every man discovered that he owned a priceless Modigliani.  
 b. NOT (EVERY.*i*.(MAN<sup>u</sup>.*i*).(DISCOVER<sup>a</sup>.*i*.(OAPM<sup>u</sup>.*i*)))

I believe E57a to be true: there are women who do not regret that they are married. Given that not every woman is married, it is clear that such a view contradicts any claim that E57a entails that every woman is married. Thus, if it is accepted that E57a is the negation of E40a, then E40a cannot presuppose that every woman is married. My own feeling is that E58a is a slightly strange thing to say unless you have previous reason to think that there have been some discoveries of Modigliani ownership. However, I think it indisputable that the sentence could be true in worlds where some, but not all, men had discovered that they owned Modigliani's. This is sufficient to show that E56 does not presuppose universal Modigliani ownership.

These observations are completely in line with the predictions obtained within ABLE: none of the above four examples carry universal presuppositions. However, this is not the end of the story. My intuition is that, knowing that not all women are married, E40a is a slightly strange thing to say, and, given that not every man owns a Modigliani, E56 is an even stranger thing to say. However, even knowing that not all women are married, E41 does not seem in the least odd. My intuition is that it is simply false, since there are married women who regret being married:

- E41 a. No woman regrets that she is married  
 b. NO.*i*.(WOMAN<sup>u</sup>.*i*).(REGRET<sup>a</sup>.*i*.(MARRIED<sup>u</sup>.*i*))

Similarly, E59 is quite compatible with there being some non-Modigliani owners<sup>8</sup>:

- E59 a. No man discovered that he owned a priceless Modigliani  
 b. NO.*i*.(MAN<sup>u</sup>.*i*).(DISCOVER<sup>a</sup>.*i*.(OAPM<sup>u</sup>.*i*))

In ABLE these contrasts are explained straightforwardly. Although none of the above regretful-wife examples are predicted to presuppose that every woman is married, E40 is predicted to entail that every woman is married, although this is not the case with E57 or E41. Likewise, none of the Modigliani-discovery examples are predicted to presuppose that every man owns a Modigliani, although E56 is predicted to entail this proposition. Thus, what seems to me rather a strong contrast between occasions in which “every” and “no” are appropriate<sup>9</sup>, is explained not in terms of any difference in presupposition, but in terms of entailments.

It may seem that the contrasts are too vague to merit the sharp semantic division which ABLE predicts. However, I believe that the apparent blurriness of the contrasts is explained solely by the failure to specify the context of utterance in sufficient detail, which means that regarding the above examples it is not *a priori* clear exactly which sets of individuals are being talked about. The contrasts become much stronger when more of the preceding context of utterance is given:

E60 Ten girls were playing hide-and-seek.

E61 Every girl discovered that she could hide in the attic.

E62 No girl discovered that she could hide in the attic.

Suppose that Clothilde, one of the ten girls mentioned in E60 is too fat to fit through the trap-door to the attic, and thus cannot possibly hide there. It seems that in this case the discourse consisting of E60 followed by E61 is not appropriate. On the other hand, E60 followed by E62 could conceivably be true. The contrast is very sharp. Once again, it will be explained quite readily in ABLE, since E61 will entail that every girl could hide in the attic, but E62 will not entail this.

These contrasts point towards a problem with some types of account of presupposition and quantification, namely those accounts where bound presuppositions in either the restrictor or scope of a quantificational construction are able to trigger restriction of the quantificational domain. The first proposal

<sup>8</sup>Some would doubtless go further, and say that E59 is compatible with their being no Modigliani owners. As mentioned above, I think this is a tenable viewpoint, although I will not attempt to adjust ABLE's semantics accordingly.

<sup>9</sup>Hans Kamp drew attention to this contrast at a recent DYANA workshop in Amsterdam

of this sort is that of Heim [He83], who described a mechanism she referred to as *local accommodation*, which could be used to repair quantificational (and other) contexts in order to avoid presupposition failure<sup>10</sup>. Van der Sandt [vdS92] has presented a more fully developed proposal along the same lines. Both Heim's and van der Sandt's theories would allow the presuppositions in the consequent of either E61 or E62 to trigger restriction of the quantificational domain to the set of girls who could fit into the attic. Thus, on either of these accounts, E61 would be true just in case every girl who could hide in the attic discovered that she could, and E62 would be true just in case no girl who could hide in the attic discovered that she could. Neither account has anything to say about why E61 could follow E60 in the given situation, but E62 could not.

These examples show that presuppositions in the scope of a quantification do not automatically trigger domain restriction, and that any theory of accommodation which says they do, is wrong. However, it does not follow from the fact that some women are single that it is never correct to say that every woman regrets that she is married. Rather, the contexts in which this can be appropriately uttered are those in which marriage and the set of married women are already salient, and the previous salience of the set of married women licences domain restriction. I will not attempt to describe formally a mechanism whereby previous context could trigger domain restriction, but it is clear that the basic apparatus is present in ABLE to enable such a development. In particular, the fact that plural referents can be generated means that it would be straightforward to add a clause to the semantics of quantificational determiners to allow quantification to be restricted to a set demarcated by some existing plural referent. Neither would this be an *ad hoc* addition to the system: it is uncontroversial that some mechanism of quantificational domain restriction is present in natural language, and that domain restriction is heavily dependent on previous context. Yet to fully validate my conclusions, rigorous empirical work would be needed. One possible, which is not to say easy, approach would be as follows:

1. Obtain a huge, semantically annotated corpus.
2. Mark the presuppositional constructions.
3. Search for presuppositional constructions occurring bound within the scope of a quantifier.
4. Pick out the cases where there must be domain restriction relating to the conditions imposed by the presupposition.
5. Check whether in any of these cases, the domain restriction could not be naturally explained in terms of the salience of the relevant set in the previous context.
6. If there are such cases, vote for the Heim/van der Sandt ticket. If not, vote for me.<sup>11</sup>

In conclusion, ABLE offers a semantic solution to many problems that arise when quantifiers and presuppositions interact, and a solution that is to some extent independently motivated by a consideration of the semantics of epistemic modality. However, there remain many outstanding problems of both an empirical and technical nature. I have indicated that I have doubts about whether even a prediction of an existential presupposition is appropriate in the above cases. But I will finish this section by noting yet another potential problem. The reader may like to verify that in the above cases where a presupposition is bound by a quantifier, ABLE predicts not only an existential presupposition, but a presupposition that there is at least one rigid designator that has the presupposed property. Thus for E41 to be admitted, there would have to be an element of the domain of individuals such that in every world in the input that individual was married. This technical artefact has little obvious empirical support, and it is clear to me that further work is required<sup>12</sup>.

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<sup>10</sup>One of the main reasons why such a mechanism was needed in her system was to avoid the problem of universal presuppositions from existential sentences. However, as Heim notes, invoking *local accommodation* to remove the strong presuppositions brings problems, and Soames [So86] makes it clear that her attempted solution to these problems is not adequate. In ABLE, as I have discussed, the problem of universal presuppositions from existential sentences is solved *semantically*, and without reference to *local accommodation*, so that the problems with the *local accommodation* account are circumvented.

<sup>11</sup>No running mate has yet come forward.

<sup>12</sup>In fact the slight differences in the version of this work presented in [Be93b] mean that the system given there does not suffer from the rigid-designator problem, so clearly we should not give up hope!

## Chapter 5

# Lets Get Real !

The language ABLE is all very well as a means for developing intuitions about the dynamics of determiners, presuppositions and so forth, but ABLE is only a bit like English. The aim of this chapter is to develop a more realistic model of the process of natural language interpretation, in particular providing answers to the following two questions:

1. How can the meaning of English sentences be derived compositionally?
2. How does world knowledge constrain utterance interpretation?

### 5.1 Kinematic Montague Grammar

In chapters 3 and 4, ABLE was set up as an independent language, which merely happened to have its semantics defined in terms of another formal language, namely  $Ty_3$ . The choice of type theory as a meta-language certainly did not enhance readability, a fact of which the reader is presumably painfully aware, and it is probably of little consolation that using type theory in this way did not enhance *writability* either. But now we come to the pay-off.

The pay-off is that it is straightforward to embed ABLE in  $Ty_3$ , as a mere sub-language, and then use the apparatus of type theory to relate expressions of ABLE to the meanings of expressions of English. I say *straightforward*, but what I really mean is that Montague did the hard work, by showing how (a modified form of) type theory can be used to specify the semantics of natural language.

#### 5.1.1 ABLE in $Ty_3$

In this section it will be shown how to define  $Ty_3$  constants AND, NOT, IMPLIES, THE, EVERY, NO, MOST, FEW, EXACTLY-ONE, IS, MIGHT and MUST: the embedding of ABLE into  $Ty_3$  enriched with these constants will then be unproblematic. Consider the definitions for the semantics of the ABLE connectives, given originally in chapter 3:

$$[[\phi \text{ AND } \psi]] = \lambda I \lambda J [\exists K I [[\phi]] K [[\psi]] J]$$

$$[[\text{ NOT } \phi]] = \lambda I \lambda J [\exists K I \downarrow [[\phi]] K \wedge J = I \setminus K]$$

$$[[\phi \text{ IMPLIES } \psi]] = [[\text{ NOT } (\phi \text{ AND NOT } \psi)]]$$

These definitions can be used as the basis of three  $Ty_3$  constants AND, NOT, and IMPLIES, of types  $\langle \pi, \langle \pi, \pi \rangle \rangle$ ,  $\langle \pi, \pi \rangle$  and  $\langle \pi, \langle \pi, \pi \rangle \rangle$ , respectively, where AND and IMPLIES will be used in infix form. In the following postulate  $\phi$  and  $\psi$  are taken to be  $Ty_3$  variables of type  $\pi$ :

**Meaning Postulate MP17**

$$\text{AND} = \lambda\phi\lambda\psi\lambda I\lambda J [\exists K I\{\phi\}K\{\psi\}J]$$

$$\text{NOT} = \lambda\phi\lambda I\lambda J [\exists K I\{\downarrow\phi\}K \wedge J = I \setminus K]$$

$$\text{IMPLIES} = \lambda\phi\lambda\psi [\text{NOT}(\phi \text{ AND NOT } \psi)]$$

The general recipe for producing an appropriate  $\text{Ty}_3$  constant from an ABLE semantic clause runs as follows: firstly wipe out all the semantic brackets (but replacing formulae  $[[\phi]]_D$  with *distribute. $\phi$ .D*), and then remove the argument expressions one at a time from left to right in the *definiendum*, adding them from right to left together with a preceding lambda at the beginning of the *definiens*. Rather than giving the by now obvious definitions for the remaining constants, the reader is referred to appendix B.

Given the enrichment of  $\text{Ty}_3$  with these constants, it can be seen that any sentence of ABLE is also a sentence of  $\text{Ty}_3$ . Furthermore, since the postulates restricting the interpretation of the  $\text{Ty}_3$  constants are based on the same definitions as for the original ABLE semantics, it is clear that for any formula,  $\phi$ , in the ABLE fragment of  $\text{Ty}_3$ , the following property will hold:

$$[[\phi]] = \phi$$

**5.1.2 Indexed English in  $\text{Ty}_3$** 

It will now be shown how indexed trees of English can be given a direct interpretation in type theory. Trees may be represented as bracketed strings, such that E63b is a tree for the indexed English discourse in E63a:

- E63 a.  $A_1$  woman realises  $John_2$  owns  $a_3$  donkey.  $She_1$  is angry.
- b.  $(((((a\ 1)\ woman)(realises\ ((John\ 2)\ (owns\ ((a\ 3)\ donkey))))))o)$   
 $((she\ 1)\ is-angry)$
- c.  $[t]SOME.1.(WOMAN^u.1)(REALISES^a.1.($   
 $THE.2.(NAMED-JOHN^u.2).(SOME.3.(DONKEY^u.3).(OWNS^b.2.3)))$   
 $AND\ (SHE.1.(IS-ANGRY^b.1))$

Note that E63b is the tree not for a sentence of indexed English, but for a discourse, the  $o$  representing a sentence sequencing operator. If words of English are defined to be constants of the right types, and if some simple notation conventions are used, such trees become formulae of type theory. We will now see how such constants can be defined, and, moreover, in a way which yields intuitively reasonable meanings — at least intuitive for those who accept that the meaning of E63a is given by the ABLE formula E63c.

Firstly, let us say that simple nouns and intransitive verbs correspond directly to the dynamic unary predicate constants that were used for ABLE's semantics, which were constants of type  $\langle d, \pi \rangle$ :

**Meaning Postulate MP18**

$$woman = WOMAN^u$$

$$donkey = DONKEY^u$$

$$is-angry = IS-ANGRY^u$$

Propositional verbs, of course, correspond to ABLE's dynamic attitude predicate constants, except that the ABLE attitudes combined firstly with the subject marker and then with the propositional argument, whereas it will be more natural here to assume that an attitudinal predicate combines firstly with a proposition to form a verb phrase. Thus an argument swap is required:



**Meaning Postulate MP19**

$$realises = \lambda F \lambda D [\text{REALISES}^a.D.F]$$

In ABLE, determiners combined with a discourse marker and two propositions to form a proposition. Clearly a determiner of indexed English ought to combine with a discourse marker and two dynamic properties. Variables  $\mathcal{P}$  and  $\mathcal{P}'$ , of type  $\rho = \langle d, \pi \rangle$ , will be used to range over dynamic properties, of which dynamic unary predicates are examples.

**Meaning Postulate MP20**

$$\begin{aligned} a &= \lambda D \lambda P \lambda P' [\text{SOME}.D.(P.D).(P'.D)] \\ the &= \lambda D \lambda P \lambda P' [\text{THE}.D.(P.D).(P'.D)] \end{aligned}$$

Noun phrases now correspond to  $\text{Ty}_3$  formulae of type  $\langle \rho, \pi \rangle$ , and variables  $\mathcal{Q}$  and  $\mathcal{Q}'$  will range over objects of this type. For instance *a.1.woman* is a formula of type  $\langle \rho, \pi \rangle$ . A transitive verb should combine with an noun phrase to form a verb phrase, so transitive verbs must have the type  $\langle \langle \rho, \pi \rangle, \rho \rangle$ . This means that in order to define constants for transitive verbs in terms of ABLE's dynamic binary predicate constants, which have type  $\langle d, \rho \rangle$ , a little type raising is necessary:

**Meaning Postulate MP21**

$$owns = \lambda \mathcal{Q} \lambda D [\mathcal{Q}.(owns^b.D)]$$

The sequencing operator  $\circ$  is to be interpreted as conjunction:

**Meaning Postulate MP22**

$$\circ = \text{AND}$$

All that remains for the above example are pronouns and proper names. As with ABLE, these may be interpreted in terms of the determiner *the*:

**Meaning Postulate MP23**

$$\begin{aligned} john &= \lambda D [the.D.NAMED-JOHN^u] \\ she &= \lambda D [the.D.(\lambda D [\text{SINGULAR}^u.D \text{ AND FEMALE}^u.D])] \end{aligned}$$

Let us adopt the following notation conventions for  $\text{Ty}_3$ :

**Reversability:** If  $X$  is of type  $\alpha$  and  $Y$  is of type  $\langle \alpha, \beta \rangle$  then  $(XY) = (YX) = Y.X$

**Parsimony:** For any  $\text{Ty}_3$  expressions  $X$ ,  $Y$  and  $Z$ , if  $X(YZ)$  is a sentence of type theory, and  $(XY)Z$  is not, then  $XYZ$  will be understood as equivalent to  $X(YZ)$ , and *vice versa* if  $(XY)Z$  is a sentence of type theory but  $X(YZ)$  not. If neither or both of these bracketings are sentences of type theory, then the unbracketed string will not be understood as a sentence of type theory.

**Indexing:** If  $X$  is of type  $\langle d, \alpha \rangle$  and  $Y$  is of type  $d$ , then  $X_Y = X.Y$ .

Given the first of these conventions, E63b can be taken as a well-formed sentence of type theory. More importantly, given the above postulates, we have that E63b = E63c. Here is a small part of the proof, the rest being left up to the reader:

$$\begin{aligned}
a &= \lambda D \lambda P \lambda P' [\text{SOME}.D.(P.D).(P'.D)] \\
(a\ 3) &= \lambda P \lambda P' [\text{SOME}.3.(P.3).(P'.3)] \\
donkey &= \text{DONKEY}^u \\
((a\ 3)\ donkey) &= \lambda P' [\text{SOME}.3.(\text{DONKEY}^u.3).(P'.3)] \\
owns &= \lambda Q \lambda D [Q.(\text{OWNS}^b.D)] \\
(owns\ ((a\ 3)\ donkey)) &= \lambda Q \lambda D [Q.(\text{OWNS}^b.D)]. \\
&\quad (\lambda P' [\text{SOME}.3.(\text{DONKEY}^u.3).(P'.3)]) \\
&= \lambda D [(\lambda P' [\text{SOME}.3.(\text{DONKEY}^u.3).(P'.3)]).(\text{OWNS}^b.D)] \\
&= \lambda D [\text{SOME}.3.(\text{DONKEY}^u.3).(\text{OWNS}^b.D.3)] \\
john &= \lambda D [\text{the}.D.\text{NAMED-JOHN}^u] \\
the &= \lambda D \lambda P \lambda P' [\text{THE}.D.(P.D).(P'.D)] \\
john &= \lambda D \lambda P' [\text{THE}.D.(\text{NAMED-JOHN}^u.D).(P'.D)] \\
(john\ 2) &= \lambda P' [\text{THE}.2.(\text{NAMED-JOHN}^u.2).(P'.2)] \\
((John\ 2)\ (owns\ ((a\ 3)\ donkey))) &= \lambda P' [\text{THE}.2.(\text{NAMED-JOHN}^u.2).(P'.2)]. \\
&\quad (\lambda D [\text{SOME}.3.(\text{DONKEY}^u.3).(\text{OWNS}^b.D.3)]) \\
&= \text{THE}.2.(\text{NAMED-JOHN}^u.2). \\
&\quad (\text{SOME}.3.(\text{DONKEY}^u.3).(\text{OWNS}^b.2.3))
\end{aligned}$$

The last two of the above notation conventions allow even more English-like structures to be interpreted in type theory. The convention of *parsimony* means that brackets are only needed where they disambiguate, and the *indexing* convention allows indices to be subscripted. Given these conventions, “(a<sub>1</sub> woman realises John<sub>2</sub> owns a<sub>3</sub> donkey o) she<sub>1</sub> is-angry”, for example, becomes a Ty<sub>3</sub> formula.

Next I will consider a classic example from the presupposition literature:

E64 a. Somebody managed to succeed George V on the throne of England.

The infamous E64a appeared in an endnote to Karttunen and Peter’s “Conventional Implicatures in Montague Grammar” [KP79], a paper which brought Montagovian standards of rigour to the analysis of presupposition. Unfortunately, these standards have not become the norm. In the first part of that paper it is argued that the various constructions which had up to that time been uniformly labelled as “presuppositional” do not in fact fall into a single linguistic category, and that many supposedly presuppositional constructions are better thought of as examples of Gricean implicatures of various sorts. In the second part of the paper an extension of Montague Grammar is presented which is intended to account for the behaviour of one of these sorts of Gricean implicature, so called *conventional implicature*. In fact all of the constructions which have been used as examples of presupposition triggers in this thesis would be labelled conventional implicatures by Karttunen and Peters. Thus, from a technical point of view it seems reasonable to ignore the terminological disagreement and temporarily identify their notion of conventional implicature with the notion of presupposition developed in this thesis: this will facilitate the comparison of the predictions of their theory with those of the theory developed here. The basis of Karttunen and Peters’ extension to Montague Grammar is the division of meaning into two components, which for current purposes can be labelled as *assertive* and *presuppositional*, with rules of composition being separately specified for each component.

The fact that assertions and presuppositions are specified independently means that the ordinary assertive meaning of an expression places no constraints on what the presupposition might be, so that at first sight their approach appears to be of great generality. However, it is this very independence of assertion and presupposition which ultimately must force us to reject such an approach, for it makes it impossible to specify scope and binding relations between the assertion and presupposition. This is essentially what Karttunen and Peters observed with respect to E64a. They assume that the control verb “manage” carries a presupposition (or conventional implicature) to the effect that the subject of the verb has difficulty in achieving whatever is specified by the verbs infinitival complement. Thus, for example, “I managed to complete my thesis” should presuppose that I had difficulty in doing so. That

“manage” displays classically presuppositional behaviour can be seen from the fact that the “difficulty” inference tends to survive from certain embedded contexts. For instance “I *might* manage to complete my thesis” still seems to suggest that I would find completion difficult. Karttunen and Peters argue that it is natural to infer from E64a that the person who succeeded George V had difficulty in doing so, and that it is the fact that the successor did not in fact have any difficulty which makes the sentence seem odd to those with a little historical knowledge. Thus it seems that it is presupposed that some individual had difficulty, and asserted of the same individual that he eventually succeeded, so that there must be some binding relation between the presupposition and the assertion. To repeat: such a binding relation cannot obtain in Karttunen and Peters’ system, which yields a completely independent presupposition that somebody had difficulty George V, and assertion that somebody succeeded George V. As Karttunen and Peters observe, there are many people who would have found it difficult to succeed George V, so that the predicted presupposition is trivially satisfied, and no account of the example’s oddity is provided.

In this thesis I have attempted to develop an integrated account of presuppositional and assertive aspects of meaning, indeed even arguing that much of the peculiar behaviour of presupposition can be predicted from the study of extraneous semantic phenomena. Within such an integrated account, it is possible to describe binding relations between presuppositions and assertions, or *vice versa*. Let us consider a translation into type theory of (the relevant aspects of) E64a:

E64 b. *Somebody<sub>6</sub> managed to succeed George-V<sub>5</sub>*

Of course, this does not look much like a formula of type theory, but if the right constants are defined, then, according to the above notational conventions, it will be. There is a minor problem in treating this example, in that ABLE does not contain the equivalent of control verbs, although there is no obvious reason why ABLE should not be extended in this respect. For the moment it will suffice to make the crass assumption that control verbs are interpreted in terms of underlying dynamic attitude predicates. The following postulate restricts the denotation of an ABLE attitude predicate  $\text{MANAGED}^a$  such that  $\text{MANAGED}^a.D.F$  can only provide an update if it is established that the proposition  $F$  “is problematic for” the individual represented by  $D$ , in which case the output is calculated by simply updating with the proposition  $F$ .

#### Meaning Postulate MP24

$$\forall I \forall J \forall D \forall F I \{ \text{MANAGED}^a.D.F \} J \leftrightarrow \\ t\text{-domain}.I.D \wedge I \text{ satisfies } \text{PROBLEMATIC-FOR}^a.D.F \wedge I \{ F \} J$$

The definition of the constants appearing in E64b is now straightforward. The noun phrase *somebody*, which has the same type as for a name or pronoun, is defined in terms of the ABLE determiner  $\text{SOME}$  and a unary predicate  $\text{PERSON}^u$ , such that *somebody<sub>7</sub>* is equivalent to  $a_7$  *person*:

#### Meaning Postulate MP25

$$\text{somebody} = \lambda \mathcal{P} \lambda D [\text{SOME}.D.(\text{PERSON}^u.D).(\mathcal{P}.D)]$$

The constants *George-V*, *succeed* and *to* do not require much thinking about, names and transitive verbs having been dealt with above, and *to* being assumed semantically trivial:

#### Meaning Postulate MP26

$$\text{George-V} = \lambda D \lambda \mathcal{P} [\text{THE}.D.(\text{NAMED-GEORGE-V}^u.D).(\mathcal{P}.D)] \\ \text{to} = \lambda \mathcal{P} [\mathcal{P}] \\ \text{succeed} = \lambda \mathcal{Q} \lambda D [\mathcal{Q}.(\text{SUCCEED}^b.D)]$$

Finally, *managed* of type  $(\rho, \rho)$  is defined in terms of the attitude predicate  $\text{MANAGED}^a$ , so that for any marker  $D$  and dynamic property  $\mathcal{P}$ , *managed.D.P* defines the same CCP as  $\text{MANAGED}^a.D.(\mathcal{P}.D)$ :

#### Meaning Postulate MP27

$$\text{managed} = \lambda \mathcal{P} \lambda D [\text{MANAGED}^a.D.(\mathcal{P}.D)]$$

Given these postulates, it is easily verified that the formulae in E64b and E64c denote the same CCP:

$$\text{E64 c. } \text{SOME.6.}(\text{PERSON}^u.6).(\text{MANAGED}^a.6. \\ (\text{THE.5.}(\text{NAMED-GEORGE-V}^u.5).(\text{SUCCEED}^b.6.5)))$$

ABLE formulae of this form were discussed in §4.3, and, on the basis of that discussion, it is clear that E64c will (only) be admitted by contexts in which at least one individual is established to find his succession to the throne problematic, and it will provide an update to a context where such an individual actually has succeeded George V<sup>13</sup>

Whilst the above *ad hoc* translations of indexed English discourses into type theory serve for illustrative purposes, it is, at least on methodological grounds, more useful to specify general procedures for the interpretation process. This will make the theory easier to extend or attack, according to taste. Here, then, is an *official* definition of the syntax of the language of indexed, bracketed English (Indexed Bracklish):

**Definition D45 (Syntax of Indexed Bracklish)** *Given sets of names, nouns, intransitive, transitive and attitude verbs, occupying the categories Name, N, IV, TV, and AV respectively, and that the category  $i$  consists of the integers, the language of indexed bracklish is defined by the following rewrite rules:*

$$\begin{aligned} \text{det} &\Rightarrow \text{the} \mid \text{a} \mid \text{exactly one} \mid \text{every} \mid \text{no} \mid \text{most} \mid \text{few} \\ \text{PN} &\Rightarrow \text{he} \mid \text{she} \mid \text{it} \mid \text{they} \\ \text{NP} &\Rightarrow ((\text{det } i) \text{ N}) \mid ((\text{det}' (N (\text{who VP}))) \mid (\text{PN } i) \mid (\text{Name } i)) \\ \text{VP} &\Rightarrow \text{IV} \mid (\text{TV NP}) \mid (\text{AV } S) \\ \text{S} &\Rightarrow (\text{NP VP}) \mid ((\text{NP do(es)n't}) \text{ VP}) \mid (\text{NP (might VP)}) \mid (\text{perhaps } S) \\ &\quad \mid ((\text{if } S) \text{ S}) \mid (\text{S and } S) \\ \text{D} &\Rightarrow \text{S} \mid (\text{D } (\circ \text{D})) \end{aligned}$$

Below I present a schema for defining constants for the terminal symbols of a language of Indexed Bracklish: all underlined expressions are to be understood as replaceable by constants representing elements of the relevant categories.

**Definition D46 (Schema for constants of Indexed Bracklish)**

$$\begin{aligned} \underline{n} &= \underline{\text{N}}^u \\ \underline{iv} &= \underline{\text{IV}}^u \\ \underline{tv} &= \lambda Q \lambda D [\underline{\text{Q}}(\underline{\text{TV}}^u)] \\ \underline{av} &= \lambda F \lambda D [\underline{\text{ATTITUDE}}^a.D.F] \\ \underline{det} &= \lambda D \lambda \mathcal{P} \lambda \mathcal{P}' [\underline{\text{DET}}.D.(\mathcal{P}.D).(\mathcal{P}'.D)] \\ \underline{name} &= \lambda D \lambda \mathcal{P} [\underline{\text{THE}}.D.(\underline{\text{NAMED-NAME}}^u.D)(\mathcal{P}.D)] \\ \underline{pronoun} &= \lambda D \lambda \mathcal{P} [\underline{\text{THE}}.D.(\underline{\text{CONDITIONS ON D}}^u)(\mathcal{P}.D)] \\ \underline{who} &= \lambda \mathcal{P} \lambda \mathcal{P}' \lambda D [\mathcal{P}.D \text{ AND } \mathcal{P}'.D] \\ \underline{might} &= \lambda \mathcal{P} \lambda D [\underline{\text{MIGHT}}.(\mathcal{P}.D)] \\ \underline{perhaps} &= \lambda F [\underline{\text{MIGHT}}.F] \\ \underline{and} &= \lambda F \lambda F' [F \text{ AND } F'] \\ \underline{\circ} &= \lambda F \lambda F' [F \text{ AND } F'] \\ \underline{if} &= \lambda F \lambda F' [F \text{ IMPLIES } F'] \\ \underline{do(es)n't} &= \lambda \mathcal{P} \lambda Q [\underline{\text{NOT}}(Q.\mathcal{P})] \end{aligned}$$

The only definitions introduced which are not of types familiar from the earlier example derivations, are those for *who*, *might*, *perhaps*, *don't* and *doesn't*. The first of these, *who*, is defined as the conjunction

<sup>13</sup>In fact I have reservations about the basic analysis of “managed” advocated by Karttunen and Peters, and have used more or less the same analysis simply in order to bring out the difference in the binding mechanisms of the two theories. The point that the Karttunen and Peters theory does not yield an adequate account of binding could have been made with the Modigliani ownership examples which I considered only, and I refrained from doing this only because they themselves do not consider these examples. My own preference would be for analysing the presuppositions of “managed” not in terms of a simple proposition about the subject having had difficulty, but in terms of the hearer’s expectations. However, I will not attempt to justify this here.

of two properties. The reason for including both *perhaps* and *might* is one of convenience, in that the discussion of epistemic modality in §4.2 involves both examples where the subject noun phrase appears within the scope of a modality, and examples where the subject takes scope over the modality. It seems reasonable to assume that the sentential operator *perhaps* always takes wide scope, and this is reflected in the above definition. However, whereas the “might” of English can appear in either scope configuration, I have assumed here that *might* is semantically just a verb phrase modifier, so that the subject takes wide scope. The reader may like to verify that the indexed bracketish sentence *((most 1) politicians) (might be-spying)* (cf. E52) is given the meaning discussed in §4.2. In contrast, the constants *don't* and *doesn't* are both defined so as to take wide scope over the subject noun phrase in the interpretation of Indexed Bracketish. There is no fundamental reason why *might* should not be defined similarly, such that it also took wide scope. However, to extend the interpretation of Indexed Bracketish with a systematic way of accounting for such scope ambiguities, for instance using type polymorphism approach as in [Em90] or [He89], would take us beyond the scope of this thesis.

## 5.2 A Plea for Common Sense

### 5.2.1 Mutual Ignorance

What is the common ground? What, for example, is the common ground between you and me? To answer this question, you need to know something about my beliefs. But what do you know about my beliefs? Can you specify even one proposition which you are sure I believe in?<sup>14</sup>

In short, no participant in a conversation knows what the common ground is. This truism ought to be rather disturbing to those who would associate the information states of dynamic semantics with the common ground of the conversational participants, a conflation which I always found attractive. But if states are identified with the common ground, and nobody knows what the common ground is, then the states could not be states *of* anybody, and a state which is not a state of something or someone, is not of much use to anything or anyone.

I would like to be *realistic* about information states. But this desire for realism is not merely a philosophical *penchant*. It reflects the obvious application for a theory of linguistic information, namely computational implementation of a natural language understanding system. If information states can be realistically interpreted as approximations to those aspects of the psychological state of a hearer or reader which are relevant to understanding, then the theory relating natural language to those states can be seen as an abstract specification of the machine states of an automated natural language processor. I will not be happy with any definition of an information state that is inconsistent with the thesis that states are in the hearer's head.<sup>15</sup>

Of course, the intuition that information states be *associated* with some notion of common ground need not force us to accept that the relationship is one of identity. In what follows, I will try to show that once the proper relationship between information states and the common ground is established, the very intangibility and unknowability of the common ground can become a source of inspiration rather than despair.

Consider the case of a doctoral thesis: it is doubtful that there is any determinate notion of a common ground between author and readers. Unless the examiners recommend that every copy be burnt, the thesis will remain for an indefinite number of years in a dusty university archive. There always remains

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<sup>14</sup>A tautology, perhaps? As it happens, I never accept tautologies, although I am quite partial to contradictions. You too? Well, then we certainly have a lot in common.

<sup>15</sup>It is important to separate *psychological realism* from *representationalism*, and at the same time to realise just how small the difference between them is. The difference is one of quantity, not quality. I am not an anti-representationalist, by which I mean that I would not wish to argue that humans do not use representations when they process language. On the contrary, given a suitably broad conception of representation, the hypothesis that understanding language involves representations becomes no more than a statement that humans are intensional agents. However, I believe that the data afforded us by natural language puts only very general constraints on the nature of representation. In particular, a claim that, say, anaphoric data forces us into a strong form of linguistic representationalism, *à la DRT*, seems to me untenable. To be a psychological realist about a theory is to claim that it provides an abstract description of mental representations. Representationalism can be a disease that occurs when a psychological realist comes to think that he or she has discovered the ultimate nature of mental representation. On the other hand, representationalism could also be considered as programming rather than linguistics: it is the process of turning an abstract specification of an information state into a more concrete data structure.

the possibility that somebody will accidentally stumble across the thesis, and begin to read. But if the common ground is not only unknowable by the author, but arguably completely indeterminate, then the candidate, as with any author, is forced to make assumptions about the knowledge of the eventual readers. These assumptions may even end up partially determining who the readership is. Thus, although the real common ground is indeterminate, an operational definition could be given: the common ground is (modulo rewrite requests of the examiners) whatever the candidate decides it will be.

Unfortunately, this operational definition is of only limited assistance to the reader of a doctoral thesis, for, as has already been established, the reader knows nothing about the author with any certainty. But the operational definition does tell us about the nature of the problem facing the readers: the readers do not know what the author assumed about the common ground, but they know that if they had information corresponding to that assumed common ground, then they could safely update this state with the content of the thesis. In effect, readers are forced to reason abductively from the presuppositions of the text to calculate the assumptions of the writer.

Those familiar with Stalnaker's work [St74] will recognize his theory of pragmatic presupposition in my description up to this point of the difficulties facing those writing and reading theses — my operational definition of the common ground coincides with his definition of *speaker's presuppositions*. Now, however, I want to ask a question that will take us beyond Stalnaker's account. Given that the readers never know what information the author assumed them to have before they started reading, how can we represent the information that a reader has after reading an arbitrary initial chunk of thesis?

In terms of KMG, the obvious answer would run as follows. The relevant aspect of a reader's information state at any point is an object of type  $\iota$ , such that at the beginning of the thesis the reader has information corresponding to the minimal state  $\bar{\top}$ , and later states are obtained by updating this state successively with the content of the thesis. Such an answer is clearly naive, for as soon as the reader reached a presupposition which had not been previously justified in the text, the rules of KMG would give no indication as to how to continue updating. Could the reader simply have started with a more informed state, and updated that? Yes, of course, but which one? Unfortunately, no single information state is appropriate, if the term *information state* is taken simply to mean the objects of type  $\iota$  introduced in the previous chapters. An example should clarify the point.

Suppose that as a reader you do not know whether it was Montague or Kahlisch, or perhaps even Carnap who first started using the term *pragmatics* to describe what is now commonly called *indexical semantics*. But you guess that the author of the thesis will know which it was, and furthermore is quite likely to take for granted that readers also know. Thus, as far as you know, the author may choose to use the presuppositional phrase "the well-known fact that Carnap first used the term 'pragmatics' to refer to what we now call indexical semantics". If you had simply assumed that the author's model of the common ground involved Carnap having made the introduction, then you could safely update with a sentence containing this phrase. On the other hand, you know that the author may use the same phrase with "Kahlisch" or "Montague" substituted for "Carnap". If you happen to choose the right assumption you will have no problem, but  $\iota$ -typed information states leave no room for sitting on the fence. For if you simply choose to begin reading the thesis with a disjunctive state in which it is not established who actually was responsible, then as soon as you come across the use of one of the above presuppositional phrases you will be sunk: your information state will not satisfy the presupposition, and, at least according to the theory of the last few chapters, there will be no way to continue updating.

Thus we are faced with a conundrum. If we interpret an information state realistically, in terms of the information the reader might conceivably have about the writer, then KMG will often fail to provide an update when it clearly should. On the other hand, interpreting information states as the assumed common ground of the writer is of no use in providing a realistic model of how a reader's information state develops, since a reader cannot know in advance what the writer has assumed. Conclusion? The notion of updating developed in this thesis so far is incorrect. Could the definition of an information state be preserved, but an alternative notion of update be used? If so, then the new notion will have to be defined with great caution, for I have gone to considerable lengths in this thesis to motivate a theory of information independently of presuppositional data. Thus, there is little leeway for making major changes to the relations between contexts defined by English sentences, without actually reformulating the definition of a context. Neither is it possible simply to weaken the constraints on the incoming context due to presuppositional constructions. For instance, it would not be appropriate to alter the definition of factivity such that the incoming context was only constrained to be consistent with the presupposition

rather than to satisfy it, whilst making the output of a factive verb satisfy the presupposition. This would yield the right entailment patterns for unembedded factives, but would destroy projection properties: the negation of a factive would no longer entail the truth of the factive complement.

### 5.2.2 Information Sets

The Carnap/Kahlsh/Montague story above is suggestive. Suppose that you maintained in parallel a number of  $\iota$ -typed states, such that in some Carnap redefined pragmatics, in others it was Kahlsh or Montague, and perhaps in some it was not established that any of them redefined anything. Then you could try updating each state separately with the content of the thesis, and see which structures survived. That way, whatever assumptions the author had made concerning the redefinition of pragmatics, at least one state would satisfy the assumptions, and would thus produce an output. This idea is easily formalised. If states are to be maintained in parallel, then updating must be framed in terms of sets of states, where each state in the set corresponds to a possibly correct model of the speaker's assumptions about the common ground:

**Definition D47 (Information Sets)** *An information set is any object of type  $\langle \iota, \tau \rangle$ , and variables  $S, T$  will range over objects of this type.*

Now a new set-update operator,  $+$ , can be defined. This operator maps information sets onto functions from CCPs to information sets, and is thus of type  $\langle \langle \iota, \tau \rangle, \langle \pi, \langle \iota, \tau \rangle \rangle$ . If  $S$  is an initial information set, then the set-update of  $S$  with a CCP  $\phi$  is denoted by  $S + \phi$ , which is simply the set of outputs obtained by updating (in the KMG sense) the elements of the set  $S$  with  $\phi$ :

**Meaning Postulate MP28**

$$+ = \lambda S \lambda \phi \lambda I [\exists J S.J \wedge J\{\phi\}I]$$

To make sense of information sets, it helps to relate them to familiar logical notions. The following postulate defines a constant *supports*. Support is an analogous notion to satisfaction, but defined at the level of information sets rather than states:

**Meaning Postulate MP29**

$$\text{supports} = \lambda S \lambda \phi [\forall I S.I \rightarrow I \text{ satisfies } \phi]$$

In terms of *supports*, a notion of entailment can be defined. The following fact shows that defining  $\phi$  to entail  $\psi$  as “updating the *minimal* information set with  $\phi$  yields an information set supporting  $\psi$ ” just yields the earlier definition of entailment from chapter 3. Naturally, a *minimal* information set is actually maximal, in the sense that it contains every possible information state.

**Fact F13 (Support-based Entailment)** *If  $\phi$  and  $\psi$  are eliminative CCPs, then:*

$$\phi \models \psi \text{ iff } (\lambda I [\top] + \phi) \text{ supports } \psi$$

*Proof:* By the definition of *supports* and  $+$ ,  $(\lambda I [\top] + \phi) \text{ supports } \psi$  iff every possible output of  $\phi$  satisfies  $\psi$ , but this is just the definition of entailment in MP5.

As it happens yet another standard dynamic notion of entailment in terms of information sets reduces to the same old definition:

**Fact F14 (Another support-based Entailment)** *If  $\phi$  and  $\psi$  are eliminative CCPs, then:*

$$\phi \models \psi \text{ iff } \forall S \forall S' S + \phi = S' \rightarrow S' \text{ supports } \psi$$

*Proof:* The definition of  $+$  shows that update is distributive across the component states of an information set (i.e. if  $S$  is the union of  $S_1, \dots, S_n$  then  $S + \phi$  is the union of  $S_1 + \phi, \dots, S_n + \phi$ ) so that it suffices to consider singleton information sets. But for these the requirement  $\forall S \forall S' S + \phi = S' \rightarrow S' \text{ supports } \psi$  reduces to  $\forall I \forall I' I\{\phi\}I' \rightarrow I' \text{ satisfies } \psi$ , which again is just the earlier definition of entailment in MP5<sup>16</sup>

<sup>16</sup>Both of these support based notions of entailment correspond to definitions proposed by Veltman [Ve91], who also observes that they collapse for distributive systems.

Since lifting the semantics up to the level of information sets has no effect on entailment, it is clear that the presupposition projection properties established in chapter 4 will be maintained. The only difference is that whereas at the level of states, presuppositions can cause undefinedness, at the level of information sets a CCP always yields an update. In fact, F13 combined with the fact that (non-modal) presuppositions of CCPs are also entailments, shows that updating the minimal information set with a CCP which presupposes some (non-modal) proposition will always yield a set which supports that proposition.

Let us see how information sets help with a simple example, based on the earlier “who redefined pragmatics?” story. Firstly, here are some constants of indexed bracklish and some relevant states:

- All constants are defined using the schema in D46, *carnap*, *montague* and ‘*pragmatics*’ as names, *redefine*=*redefined* as a TV, *realised-that* as a factive AV, and *didn’t* using the same semantics as for *doesn’t*.
- $A$  satisfies (*carnap*<sub>1</sub> redefined ‘*pragmatics*’<sub>2</sub>)
- $A$  satisfies (*montague*<sub>3</sub> *didn’t* redefined ‘*pragmatics*’<sub>2</sub>)
- $B$  satisfies (*montague*<sub>3</sub> redefined ‘*pragmatics*’<sub>2</sub>)
- $B$  satisfies (*carnap*<sub>1</sub> *didn’t* redefined ‘*pragmatics*’<sub>2</sub>)
- $C = A \cup B$
- $A\{\textit{montague}_3 \textit{didn't realise-that carnap}_1 \textit{redefined 'pragmatics'}_2\}A'$

Neither  $B$  nor  $C$  admits the proposition *montague*<sub>3</sub> *didn’t realise-that carnap*<sub>1</sub> redefined ‘*pragmatics*’<sub>2</sub>. However, representing an information set containing iotas  $I, J, \dots$  as  $(I, J, \dots)$ , we have the following update:

$(A, B, C) + \textit{montague}_3 \textit{didn't realise-that carnap}_1 \textit{redefined 'pragmatics'}_2 = (A')$

Thus an information state which does not support the proposition that Carnap redefined pragmatics can still be updated with a sentence in which the proposition is presupposed. More generally, by being realistic about what a hearer knows of the speaker’s assumptions about the common ground, an obvious difficulty with the CCP model has been resolved. Once it is recognised that the states of ABLE and KMG semantics are not to be identified with the common ground, but are to be thought of as possible models of the common ground, the awkward fact that presuppositions can be informative becomes unproblematic.

### 5.2.3 Information Orderings

Information sets provide a model of an agent’s uncertain knowledge of the common ground, but there is good reason to consider more sophisticated models. It is intuitively clear that not all assumptions that a writer/speaker might make are equally plausible. For instance, it is more plausible that Carnap redefined pragmatics than that Carnap’s dog redefined pragmatics, and it seems reasonable that a reader’s model of the writer’s assumptions about the common ground should reflect this difference. But it is not appropriate merely to assume that some states are in the context and some not, since there is no obvious place to draw the line. If it is implausible that Carnap’s dog redefined pragmatics, is it plausible that Carnap’s grandmother redefined pragmatics?

One can imagine various more structured notions of context than simple sets of states that might be used to encode the difference in plausibility of different assumptions. For example we might consider adding a type  $p$  of probabilities to the type theoretic setup, and defining a context to be a function of type  $\langle \iota, p \rangle$  — something like a *vague* set. The more plausible states would be mapped onto higher probabilities. Such a notion of context would allow very fine-grained distinctions between states to be made: in fact, although the vague set analysis seems intuitive to me, I do not have applications in mind which would require quite such a fine grain. It will suffice if a context provides a measure of the relative plausibility of different states, such as to answer to questions like “is state  $I$  more plausible than state  $J$ ”. In other words, what is needed is some sort of ordering over states:



**Definition D48 (Information Ordering)** Any object  $O$  of type  $\pi$  which has the property of transitivity is an information ordering. An initial information ordering is additionally reflexive.

$$\begin{aligned} \text{Reflexivity:} \quad & \forall I \quad I\{O\}I \\ \text{Transitivity:} \quad & \forall I \forall J \forall K \quad (I\{O\}J \wedge J\{O\}K) \rightarrow I\{O\}K \end{aligned}$$

An *information ordering*  $O$  is a set of pairs of states, such that if a pair of states  $I$  and  $J$  is in the ordering relation, represented as  $I\{O\}J$ , then both  $I$  and  $J$  could correspond to the generator's assumptions, and  $I$  is at least as plausible as  $J$ . Note that information orderings have the same type as CCPs, which are also relations between states. However, orderings and KMG CCPs are quite different slices of  $\pi$ . For instance, whereas KMG CCPs are never transitive relations, orderings are by definition always transitive, and whilst the only pairs in the denotation of KMG CCPs are those in which the output is more informative than the input, there is no corresponding restriction on information orderings.

The use of orderings to represent default information of various kinds is becoming well-established, as in the work of Velman [Ve91], although Veltman's *expectation frames* involve orderings over possible worlds rather than over  $\iota$ -typed states. It is clear that their must be many more states than there are worlds, and thus that information orderings provide a more fine-grained notion of preference than expectation frames. Indeed, it is clear that any ordering over worlds can be expressed using an ordering over states, if we simply associate the sub-part of an information ordering containing only single-world states with the corresponding expectation frame. I will not make any attempt to account for how preference orderings over states are formed, but simply take as given that the common sense of an agent provides him or her with such an ordering. It is clear that an underlying ordering over worlds could be used to induce a (very partial) ordering over states, and thus that mechanisms like those discussed by Veltman to provide orderings over worlds could be reinterpreted so as to generate information orderings. We could then add some additional general preference criteria, like specifying that if two states differ only by one having a larger domain of discourse markers than the other, then the smaller-domained state is *a priori* more plausible.

Updating can be defined along similar lines to the information-set based definition, above, using a constant  $*$ , of type  $\langle \pi, \langle \pi, \pi \rangle \rangle$ . The update operation proceeds by taking all pairs of states in the denotation of the initial ordering, and trying to update each element of the pair with the CCP. If both states can be successfully updated, then the resulting states are paired in the output ordering. Thus if one state is initially at least as plausible as another, and both states admit the CCP, then in the final ordering the output from the first state will be at least as plausible as the output from the second.

#### Meaning Postulate MP30

$$* = \lambda O \lambda \phi \lambda I \lambda J [\exists I' \exists J' I'\{O\}J' \wedge I'\{\phi\}I \wedge J'\{\phi\}J]$$

A reader who had forgotten or never knew who redefined pragmatics, and was taken in by my story, might now be surprised by the fact that it was Bar Hillel who first defined pragmatics to be indexical semantics. Yet even if you were surprised by this fact, or surprised that I knew it, the surprise does not mean that you had any difficulty updating with the previous sentence. This is what would be expected if the structures which you updated were like information orderings, and the mechanism of update was something like the  $*$  operation: even apparently unlikely alternatives should be represented somewhere in an information ordering, albeit rather low down.

However, orderings need not *contain* every state, in the sense that for some states  $I$  it will neither be the case that  $\exists J I\{O\}J$  nor that  $\exists J J\{O\}I$ . In particular, an ordering produced by updating with a CCP will only contain states which are possible outputs of the CCP, and it need not even contain all of those. Thus, an information ordering registers two kinds of information. Firstly it says which states are possible models of the speaker's assumptions, and secondly it says what the preferences are amongst those states. Since an initial ordering is *reflexive*, which means that every state is at least as plausible as itself, it is clear that every state will be contained in an initial ordering. The set of states which a non-initial ordering registers as being possible models of the speaker's assumptions will be precisely those which the ordering says are at least as plausible as themselves:

#### Meaning Postulate MP31

$$\text{contains} = \lambda O \lambda I [I\{O\}I]$$

The constant *contains* relates information orderings to information sets: if  $O$  is an information ordering, then *contains*. $O$  is an information set. Thus *contains* can be used to apply the above definition of *support* to information orderings: an ordering  $O$  supports a CCP  $\phi$  iff (*contains*. $O$ ) supports  $\phi$ . Clearly this possibility combined with F13 and F14 shows that some natural notions of entailment in terms of information orderings would correspond to the original definition in terms of  $\iota$ -typed states. It is also clear that the potential informativeness of presuppositions is accounted for in just the same way as with information sets. But, as will be demonstrated shortly, there is more to the processing of presuppositions than can be naturally explained in terms of strict entailment, and this is where the additional structure of information orderings comes into play.

In an information ordering, there are some *optimal* states, in the sense that they are at least as plausible as all the other states which the ordering contains. Sometimes it will be the case that the optimal states are also minimal, by which I mean that all the non-preferred states are extensions of a preferred state. In this case, the set of propositions supported by the set of optimal states will be just the same as the set of propositions supported by the entire set of states which the ordering contains. However, it may also be the case that the optimal states are non-minimal. In that case, there will be propositions supported by the optimal states that are not supported by the ordering as a whole. If updating some ordering  $O$  with a CCP  $\phi$  leads to an ordering in which the set of optimal states supports another CCP  $\psi$ , then I will say that relative to  $O$ ,  $\phi$  *implicates*  $\psi$ . The formal definitions of *optimal* and *implicates* are straightforward:

#### Meaning Postulate MP32

$$\begin{aligned} \textit{optimal} &= \lambda O \lambda I [ \textit{contains} . O . I \wedge \forall J [ J \{ O \} I \rightarrow I \{ O \} J ] \\ \textit{implicates} &= \lambda O \lambda \phi \lambda \psi [ ( \textit{optimal} . ( O * \phi ) ) \textit{supports} \psi ] \end{aligned}$$

Let us consider a case where strict entailment seems too weak:

E65 If I go to London, my sister will pick me up at the airport.

As shown in 4.1, in ABLE (and the same clearly holds for KMG) presuppositions in the consequent of a conditional yield conditional presuppositions. Thus the CCP corresponding to E65 presupposes that if I go to London then I will have a sister. The occurrence of these weak, conditionalized presuppositions is one of the most strongly criticized aspects of the CCP model (see e. g. [Gaz79]). It seems intuitively obvious that somebody hearing 4.1 would conclude not only that if the speaker were to go to London he would have a sister, but that the speaker actually has a sister. In terms of information orderings, this inference pattern might lead us to come to a certain conclusion about a hearer's typical information ordering. For the inference to go through, some states in which it was definitely established that the speaker had a sister would have to be more plausible than all states in which the existence of a sister was conditionalised on the speaker's journeying to London. Here, by saying that one state is *more plausible than* another, I mean that the first is at least as plausible as the second, but not *vice versa*. Relative to such an ordering, 4.1 would *implicate* that the speaker had a sister, since updating the ordering with 4.1 would lead to an ordering in which the optimal states all satisfied the existence of a sister.

But why should it be the case that states in which the speaker has a sister are more plausible than certain states in which this proposition is conditionalised? This is a difficult question to answer completely, but I can imagine some potential lines of explanation. In the first place, note that although the CCP corresponding to E65 will only conditionally presuppose the existence of a sister, it will unconditionally presuppose the presence of a discourse marker in the input state. The input state will admit the CCP just in case in the world-sequence pairs where the speaker goes to London, the discourse marker is constrained to be the speaker's sister. Suppose that the speaker's immediate family were generally assumed to correspond to rigid designators, and suppose further that there were a general principle to the effect that states where assumed discourse markers corresponded to rigid designators were more plausible than those where assumed markers are non-rigid. In that case, of all the states which contained a marker that corresponded to the speaker's sister in any world-sequence pair, those in which the marker corresponded to the speaker's sister in every world-sequence pair would be most plausible, and this would lead to the observed inference pattern.

Whether or not the reader finds this convincing, it should be clear what I am aiming for: I would like to justify the particular inference pattern relevant to E65 in terms of more general principles. In

fact, I think that the above markers-are-rigid line of reasoning is still not general enough, for there are many cases not involving the assumption of a new marker but where KMG's prediction of a conditional presupposition seems too weak. Consider the following example:

**E66** If Charles turns up, then everybody will be amazed that both Charles and Diana are here.

Any state in which it is established that if Charles turns up then both Charles and Diana will be here would admit the above sentence. However, in a situation where it is known that Charles and Diana generally try to avoid each other, there is a clear tendency to come to a stronger conclusion on hearing E66, namely that Diana is already here<sup>17</sup>. For this result to be predicted, it would have to be the case that a state where Diana's presence is established is more plausible than every state where Diana's presence is conditional on Charles' arrival. The source of such orderings must be connected with the nature of explanation and justification, in particular the fact that if Charles and Diana habitually avoid each other, then Charles' arrival does not help explain Diana's presence. On the other hand, it is clear that Charles' arrival does help explain (or even entails) Charles' presence, so it is not surprising that E66 does not implicate that Charles is definitely here.

When reading each of the following two examples, ask yourself whether there actually is any more hot water:

**E67** If Jane takes a bath, Bill will be annoyed that there is no more hot water.

**E68** If Jane wants a bath, Bill will be annoyed that there is no more hot water.

There is a clear contrast between these two examples. An utterance of E67 does not suggest that there actually is no more hot water, but only that if Jane takes a bath, there will be no more hot water. On the other hand, E68 suggests strongly that there is no more hot water. Put another way, E67 is compatible with the standard CCP prediction of a conditional reading, but E68 is not. It is clear that there exist information orderings that would lead to precisely these predictions. Any ordering satisfying the following two conditions would suffice:

1. At least one state in which it is established that there is no hot water is more plausible than all states in which it is not known whether there is hot water, but in which it is known that if Jane wants a bath then there will be no hot water.
2. A state in which it is not known whether or not there is hot water but in which it is established that if Jane has a bath then there will be no more hot water must be at least as plausible as all states where it is definitely established that there is no hot water.

The general question is, why would it be reasonable to expect information orderings to have such properties? My answer to this question is on the one hand both simple and obvious, and on the other hand both awkward to implement and incompatible with many contemporary theories of presupposition. Many linguists will surely find it unpalatable. The answer is: common sense.

Let me expand on this. The contrast between E67 and E68 results from our ability to find a commonsensical explanation of the lack of hot water in terms of somebody having taken a bath, but in our inability to fully explain a lack of hot water in terms of somebody simply wanting a bath. The simple assumption that there is a finite amount of relevant hot water — about a bathful — is sufficient to allow justification of their being no more hot water in situations where Jane has just taken a bath. However, the same simple assumption would not suffice in the case of E68, and a number of other assumptions would be needed, such as the assumption that if Jane wants a bath then she will definitely take one. Thus it is the relative plausibility of assumptions not explicitly mentioned in the text of the example sentences that determines what is implicated. Here are three more examples which illustrate the same point:

**E69** If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

<sup>17</sup>If the word "both" in the example is stressed, then the stronger conclusion seems more likely regardless of our knowledge about Charles and Diana. However, I have nothing to say about the role of intonation, and my remarks concern a neutrally intoned version of the sentence, or, if there is no such thing as neutral intonation, at least a version with no sharp focal stress.

E70 If John has an exam today, Mary will notice that he is smoking more than usual.

E71 If Jane goes into the cave without a light, she will be annoyed that she cannot see much.

In all of these cases, the standard CCP prediction of a conditional presupposition seems unobjectionable. These predicted presuppositions are, respectively, that if Spaceman Spiff lands on Planet X then his weight will be higher than it would be on Earth, that if John has an exam today then he is smoking more than usual, and that if Jane goes into the cave without a light then she will not be able to see much. I would argue that in all three cases, what makes the conditional presuppositions reasonable is the possibility of finding a deeper explanation. Thus, in the first case, the reason why the conditional presupposition makes sense is that it can be justified in terms of common sense physics. We suppose that Planet X is a planet, but we do not know whether Planet X is a big planet or a little planet. The simple assumption that Planet X is big (some readers may have more sophisticated views on the relationship between planets and their gravitational field strength) is sufficient to tell us that if Spiff lands on X then his weight will be higher than it would be on Earth. I leave it to the reader to construct explanations for the implications of the other two examples. Of course, it might be that the reader deems E70 to implicate that John actually is smoking more than usual, or perhaps thinks E71 is most naturally uttered in a situation where Jane is known to be blind. Yet this would by no means be problematic. On the contrary, it would be grist to my mill, as it indicates something that is quite natural within the current account, namely that the reader's plausibility criteria differ somewhat from my own.

#### 5.2.4 Theories lacking Common Sense

Many theories of presupposition fail to account for the pragmatic nature of presuppositional justification. The phenomena I have described mitigate against any purely *semantic* theory of presupposition, and against any theory of presupposition which depends on a *structural* account of *accommodation* or *cancellation*. I will now justify this claim, explaining the italicised terms as I go.

By a *semantic* theory of presupposition, I mean one in which presupposition is accounted for in terms of truth conditions, and in which the truth conditions are calculated compositionally. Neither of the sentences "Jane wants a bath" nor "Jane takes a bath" is logically related to the sentence "there is no hot water", by which I mean that there are no entailment relations between these sentences. From a purely semantic point of view, there is thus no relevant difference between E67 and E68, and no way to explain the contrast. By far the lions share of the existing literature on presupposition concerns semantic approaches, usually depending on some sort of multi-valued logic: none of these approaches is compatible with my claim that the contrast between E67 and E68 is not naturally explained in terms of truth conditions but in terms of plausibility<sup>18</sup>.

Stalnaker's theory of pragmatic presupposition was a radical departure from traditional semantic theories of presupposition, and it is in the context of Lewis's discussion of Stalnaker's account [Le79] that the term *accommodation* was introduced. Accommodation is the process whereby hearers repair their view of the common ground<sup>19</sup> to enable updating, when otherwise presupposition failure would result. My generalisation of updating to information sets and information orderings can be seen as an attempt to formalise Lewis's intuitive notion of accommodation. However, the approach developed here contrasts sharply with the formalisation of Lewis' accommodation developed by Heim [He83], and with the more recent DRT version of van der Sandt [vdS92]. Both Heim and van der Sandt's accounts provide *structural* notions of accommodation: one might caricature them as involving *move- $\alpha$*  at the level of logical form. Let me explain.

Heim's informal characterisation of accommodation could apply equally to the formalisation she gives or to the account that I have presented. She says:

Suppose [a sentence] S is uttered in a context c which doesn't admit it... simply amend the context c to a richer context c', one which admits S and is otherwise like c, and then proceed to compute c' [updated with] S instead of c [updated with] S.

<sup>18</sup>The philosophical literature on multi-valued approaches to presupposition is so large that it is hard to know exactly which theory to cite as being inadequate. For the sake of concreteness, I will mention that my criticisms apply to van Frassen's account [vF75], which is a paradigmatic example of a rigorously developed semantic theory of presupposition.

<sup>19</sup>Lewis refers not to the *hearer's view of the common ground* but to the *conversational scoreboard*.

However, the slightly more formal characterisation of accommodation which she then gives involves an extra ingredient. She talks not only of there being contexts which admit S and contexts which do not, but of a specific proposition which S presupposes. She suggests that the mechanism of adjusting a context so as to admit a S consists of adding this proposition to the context. According to her description, this will apply equally if S occurs in an embedded context. Thus if c admits R but does not admit a sentence “if R then S”, and S presupposes the proposition  $\alpha$ , then one way of proceeding is to add  $\alpha$  to c, and then update this context with “if R then S”. This is the alternative that she refers to as *global accommodation*: in effect the presupposition  $\alpha$  is moved (and conjoined) to the front of the sentence, to yield “ $\alpha$  and if R then S”, and an update is then performed with the resulting formula. Heim is also suggests that a presupposition may be added to some of the intermediate contexts involved in calculating an update, what she calls *local accommodation*. This has much the same effect as allowing alternative landing sites for  $\alpha$  at the level of logical form. Thus if update with “if R then S” fails, she would offer three alternative ways of updating, corresponding to the sentences: “ $\alpha$  and if R then S”, “if ( $\alpha$  and R) then S”, and “if R then ( $\alpha$  and S)”. In Van der Sandt’s account, which I will not describe in detail here, the *move- $\alpha$*  flavour is even more obvious than in Heim’s sketchily described theory, except that in van der Sandt’s account  $\alpha$  is moved around within the discourse representation structures of Kamp’s DRT.

I can now state what I mean by a *structural* account of accommodation. A structural account is one in which any given presuppositional construction has a single presupposed proposition, and accommodation associated with a sentence containing that construction consists of adding this proposition to some relevant context. Here the relevant contexts are the initial context, and some set of intermediate contexts which can be specified for each sentence type.

The problem that I see for a purely *structural* account of accommodation is as follows: it is not possible to predict on structural grounds alone exactly what should be accommodated. In general, the exact accommodated material can only be calculated with reference to the way in which world knowledge and plausibility criteria interact with the meaning of a given sentence.

Consider E66, the Charles-and-Di example, above. Here the relevant presupposition trigger is the phrase “amazed that both Charles and Diana are here”. Since “amazed” is factive, we obtain the presupposition that both Charles and Diana are here. However, I have argued that it is appropriate to globally accommodate not that both Charles and Diana are here, but only that Diana is here. I would not wish to claim that no structural account of accommodation could lead to this result, but certainly no existing such account does, and either Heim’s or van der Sandt’s theories would require apparently *ad hoc* modifications. Furthermore, the possibility of accommodating non-globally introduces as many problems as it solves. Accommodating that both Charles and Diana are here in the antecedent of the conditional might yield a meaning like “If Charles turns up and both Charles and Diana are here, then everyone will be amazed that both Charles and Diana are here”. This was the most charitable version of accommodation into the antecedent which I could manage, and yet it is clear that it does not yield a sentence corresponding to the meaning of E66. Accommodating into the consequent yields a somewhat more plausible meaning, something like: “If Charles turns up then both Charles and Diana are here and everyone will be amazed (by that).” I think there are occasions of use of E66 where this meaning would be reasonable. However, the implication that Diana is here would remain unexplained.

The most glaring weak point of a structural account of accommodation concerns the fact that there is no way for it to produce the conditional readings which I have argued are appropriate for many of the above cases. This is ironic, since the occurrence of such readings in the CCP account without an accommodation mechanism has traditionally been taken as the most serious problem with the CCP model. I will consider in detail just one of the above cases where I have advocated a conditional reading:

E69 If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

The “fact that” construction in this case triggers the presupposition that Spiff’s weight is higher than it would be on Earth. Structural accounts of accommodation suggest that this proposition should be globally accommodated. However, this result is simply wrong: as discussed above, it is not normal to conclude from E69 that Spiff’s weight is higher than it would be on Earth. Indeed, it seems natural for this sentence to be uttered under conditions where Spiff is hanging about in space, and completely weightless. Can non-global accommodation save the structural account? Accommodation into the antecedent produces something like “If Spaceman Spiff’s weight is higher than it would be on Earth and

he lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.” I do not think this is a possible meaning of E69. Accommodation into the consequent appears to improve, yielding (after charitable adjustment of tense) “If Spaceman Spiff lands on Planet X, his weight will be higher than it would be on Earth and he will be bothered by the fact that his weight is higher than it would be on Earth.” This provides a reasonable meaning for E69, and offers hope that if only some way could be found of removing the two incorrect readings, the structural account might be saved. Unfortunately, a very slight variation on E69 produces an example where the structural account produces four incorrect (or, at the very least, non-preferred) readings, and completely fails to yield the preferred reading:

**E72 It is unlikely that if Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.**

The preferred reading of this sentence is still one involving the conditional implication, i.e. if he lands on Planet X, Spiff’s weight will be higher than it is on Earth, and quite natural assumptions about the dynamics of the “it is unlikely” construction would lead to the model presented here making the same presuppositional predictions for this example as for E69. But in this case the structural account no longer yields the right reading after accommodation into the consequent of the conditional. This would yield “It is unlikely that if Spaceman Spiff lands on Planet X, his weight will be higher than it is on Earth and he will be bothered by it.”, which does not imply that if he lands on Planet X, Spiff’s weight will be higher than it is on Earth. On the contrary, there is even a slight suggestion from this sentence that if he lands on Planet X his weight probably will not be higher than it is on Earth, which is clearly inappropriate.

With regard to the other examples involving conditional presupposition readings from conditional sentences, similar considerations apply. In all cases, the closest a structural account gets to predicting a conditional reading is by accommodation into the consequent of the original conditional. However, in each case the example can be embedded under an operator of probability or possibility, and in the resulting sentences, there is no way for a structural account to yield the relevant conditional implication. Thus the above argument concerning the lack of an appropriate structural-accommodation treatment of E69 and E72 could be repeated with regard to E67 and E73, E70 and E74 and E71 and E75.

**E73 Probably, if Jane takes a bath, Bill will be annoyed that there is no more hot water.**

**E74 Perhaps if John has an exam today, Mary will notice that he is smoking more than usual.**

**E75 It is possible that if Jane goes into the cave without a light, she will be annoyed that she cannot see much.**

I should make it clear that although I have strongly criticised the Heim and van der Sandt accounts, both theories have a great deal to offer, and are certainly superior, from a purely empirical viewpoint, to any other existing account. Most importantly, I do not think it possible to defend a theory of presupposition which does not involve some account of local accommodation, which allows solutions to (amongst many others) the following traditional presuppositional riddles:

**E76 The King of France is not bald because there is no King of France**

**E77 Either the King of Buganda will open parliament or the President of Buganda will.**

In the first case, local accommodation of the existence of a King of France (and an appropriate discourse marker) within the scope of the first negation would allow for the fact that the sentence does not implicate that there is a King of France<sup>20</sup>. In the second case, local accommodation of the existence of a King of Buganda in the first disjunct and the existence of a President in the second would allow the sentence to be used in cases where it was not established whether Buganda was a monarchy or a republic. However, accepting that there is such a thing as local accommodation is only part of what is needed. I believe that local accommodation is a highly constrained process, and there is still no adequate theory of precisely what the constraints are.

<sup>20</sup>But see [vdS] for an alternative view of how this type of example should be treated.

I will now consider a related theory, the *cancellation* account of Gazdar [Gaz79]. Gazdar's theory begins with a primitive relation of *pre-supposition* between sentential clauses and propositions. The pre-suppositions (or *potential presuppositions*) of a compound sentence consist of the set of all the pre-suppositions of the sentential sub-parts of the sentence. With each sentence are also sets of entailments and *im-plicatures*. The updating of a context, viewed as a set of propositions, with a sentence, begins with the addition of the entailments of the sentence (and closure under logical consequence), and then proceeds by adding as many of the im-plicatures as possible without creating an inconsistent context, and finally as many of the presuppositions as possible without creating an inconsistent context. The presuppositions of a sentence in some context are just those pre-suppositions which get added in the update process. In this model of the update process, some pre-suppositions fail to be added to the context: these are said to be *cancelled*.

Gazdar's theory is a dynamic theory, in that, at least at the level of discourses, the interpretation of a sequence of sentences is interpreted as a sequence of updates. However, presupposition is not viewed, as it is in this thesis, as placing constraints on the input context in the update process. Indeed, although Gazdar's theory is dynamic, pre-suppositions place additional constraints not on the input but on the output context, so perhaps it would be better to describe the cancellation account as being not a theory of *presupposition* but of *postsupposition*.

In a sense *cancellation* is the inverse of *global accommodation*. Heim [He83], after suggesting her enhancement of the CCP model with an account of accommodation, makes the following observation:

Note that by stipulating a *ceteris paribus* preference for global over local accommodation, we recapture the effect of [Gazdar's] assumption that presupposition cancellation occurs only under the threat of inconsistency.

I find this stunning. With one short remark buried in a terse paper Heim offers a simple synthesis between the two antitheses of 1970's presupposition theory, namely the Kartunnen derived model which her paper uses as its base, and Gazdar's cancellation account. In a stroke this shows a way to eliminate the bulk of existing counter-examples to the CCP model, and introduces a way of thinking about Gazdar's theory that preserves his insight that default reasoning is involved in the processing of presuppositions, whilst restoring the intuition that, in some sense, presuppositions are to do with *what come first*, with definedness conditions on the input rather than preferences on the output.

However, the fact that Gazdar's model would be empirically comparable to the CCP model enhanced with a structural account of accommodation is not entirely good news. For it means that the above criticisms of structural accounts of accommodation can be applied equally well to Gazdar's model. For all its claims to be a *pragmatic* theory, Gazdar's account does not allow much latitude in determining the set of pre-suppositions of a given sentence: we could say that the potential presuppositions are determined *structurally*. Just as with the Heim and van der Sandt accounts, Gazdar's theory fails to predict conditional presuppositions when they are needed. Regarding the Spaceman-Spiff example (the same would apply to any of the above conditional-reading examples), and in the absence of any prior knowledge to the effect that at time of utterance Spiff was weightless, Gazdar's theory would yield the unconditionalised conclusion that Spiff's weight is higher than it would be on Earth:

E69 If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

Given a previous context which determined that Spiff was weightless, which would cause cancellation of the pre-supposition that Spiff's weight is higher than it would be on Earth, Gazdar's theory would yield more reasonable results. The fact that in his theory all pre-suppositions of a sentential clause are also entailments means that "he will be bothered by the fact that his weight is higher than it would be on Earth" not only pre-supposes but also entails that Spiff's weight is higher than it would be on Earth. This means that after cancellation, the sentence would yield a meaning (cf. the above discussion of the Heim and van der Sandt theories) equivalent to "If Spaceman Spiff lands on Planet X, his weight will be higher than it would be on Earth and he will be bothered by the fact that his weight is higher than it would be on Earth." However, once again, this is no general solution, firstly because Gazdar's theory makes the wrong prediction in the absence of definite prior knowledge of Spiff's weightlessness, and secondly because, just as with the Heim and van der Sandt accounts, it completely fails to predict the right conclusion with respect to the embedded version of E69, namely E72:

E72 It is unlikely that if Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on Earth.

In summary, let me repeat that all purely semantic theories of presupposition, and all purely structural accommodation or cancellation based theories of presupposition are doomed to failure, for they lack common sense.

### 5.3 Non-determinism and local accommodation, suggestions for future research.

Although PUL, ABLE and KMG each involve a relationally specified semantics, none of them take full advantage of that denotation space. In particular, in none of these systems is it possible for a formula to be non-deterministic, in the sense that for a given input there is more than one output. Yet I can conceive of a number of potential applications for such non-determinism. One such application was presented in the version of KMG appearing in [Be93b], in which fully relational CCPs are used to model DRT style non-deterministic pronoun resolution, and on this basis a semantics is given for a fragment of unindexed English. Thus in the system presented there, given an input state which has two discourse markers in its domain, both of which are established to correspond to singular females, update with the relational CCP corresponding to “she is walking” could yield either of two output states, one in which the first discourse marker was asserted to be walking, and one in which the second was. This is an attractive approach to pronoun resolution, since the assumption that syntax somehow determines pre-indexed logical forms always struck me as unsatisfying. It is clear that syntax imposes strong constraints on coreference, but it is equally clear that syntax does not completely determine the issue.

Non-determinism could also be used in the modelling of local accommodation, which, as I indicated in chapter 5, I feel to be an essential component of an adequate theory of presupposition. The trouble is that it is clearly inappropriate to allow arbitrary local accommodation, so the question is, how can it be constrained. One way of constraining it is by restricting what can be accommodated structurally, as in the Heim and van der Sandt models of accommodation. However, their accounts are on the one hand still too unconstrained — witness the inappropriate readings which local accommodation was shown to generate in the discussion of quantification and presupposition in chapter 4 and in the discussion of the role of common sense in chapter 5 — and on the other hand are too constrained. A structural account of local accommodation is too constrained because it cannot possibly account for the phenomenon of bridging:

E78 If I go to a wedding then the rabbi will get drunk

This example has a reading where the rabbi is understood to be somehow related to the wedding mentioned in the antecedent, rather than being a globally salient rabbi. The nature of the conceptual bridge that can be built between a wedding and a rabbi is essentially non-structural, and relies on world-knowledge rather than any detail of the form of the sentence. Only a common-sense reasoning based approach like that developed in the previous section count account for the phenomenon of bridging.

Let me indicate briefly how we might introduce a non-structural notion of local accommodation using a radical (in terms of traditional semantic theory rather than AI natural language systems) alteration to the simple PUL system from chapter 2. Suppose that we replaced the PUL definition of a context as a set of worlds with a notion of a context as a pair of a set of worlds and a cost<sup>21</sup>: the cost will be used to constrain local accommodation. For a context  $\sigma$ , let us say that  $\sigma_0$  is a set of worlds and  $\sigma_1$  is a cost. The cost is supposed to represent the implausibility of a given accommodated piece of information, so that the more implausible the propositions that have to be accommodated, the higher the cost. Let us then say that there is some relation of *plausible extension* between contexts  $\mapsto$ , is determined by world knowledge, and which has at least the following properties:

1. If  $\sigma \mapsto \tau$ , then  $\tau_0 \subseteq \sigma_0$   
Plausible extension cannot cause a downgrade.

<sup>21</sup>cf. Sperber and Wilson’s [SW84] use of *relevance*, which is sort of processing cost .



2. If  $\sigma \mapsto \tau$ , and  $\tau_0 \subset \sigma_0$  then  $\tau_1 > \sigma_1$ .  
Plausible extension increases the cost.
3. If  $\sigma \mapsto \tau$  then there is no  $v$  such that  $v_0 = \tau_0$  but  $v_1 \neq \tau_1$  and If  $\sigma \mapsto \tau$ , then  $\tau_0 \subseteq \sigma_0$  and  $\tau_1 \geq \sigma_1$   
Extending some initial state with any given proposition has only one cost.
4.  $\forall \sigma \sigma \mapsto \sigma$   
Extending with a tautology (i.e. not at all) is free.

Here are the first few clauses showing what the semantics might look like:

$$\begin{aligned}
\sigma \llbracket p_{\text{atomic}} \rrbracket \tau & \text{ iff } \tau_1 = \sigma_1 \wedge \tau_0 = \{w \in \sigma_0 \mid w \in F(p)\} \\
\sigma \llbracket \phi \text{ AND } \psi \rrbracket \tau & \text{ iff } \exists v \sigma \llbracket \phi \rrbracket v \llbracket \psi \rrbracket \tau \\
\sigma \llbracket \text{NOT } \phi \rrbracket \tau & \text{ iff } \exists v \exists v' \sigma \rightsquigarrow v \llbracket \phi \rrbracket v' \\
& \wedge \tau_1 = v'_1 \wedge \tau_0 = \sigma_0 \setminus v'_0 \\
\sigma \llbracket \phi \text{ IMPLIES } \psi \rrbracket \tau & \text{ iff } \sigma \llbracket \text{NOT } (\phi \text{ AND } (\text{NOT } \psi)) \rrbracket \tau
\end{aligned}$$

For atomic propositions the cost of the input is simply identified with the cost of the output, whilst conjunction and implication are defined in the normal way. The interesting clause is that for  $\text{NOT } \phi$ , which allows for plausible extension before update with  $\phi$ .

Normally, the fact that not accommodating anything is cheaper than accommodating something will mean that the cheapest update with a given formula is equivalent to the standard PUL update. However, in some circumstances, failure to accommodate would yield presupposition failure. Consider the following example:

**E79 Perhaps Bertha is hiding. However, if Anna discovers that Bertha is not hiding, then she will be upset.**

The input context to the second sentence will not satisfy the proposition that Bertha is not hiding, and thus updating can only proceed if extra information is accommodated into the antecedent of the conditional, a possibility which arises from the definition of implication in terms of the new negation. On quite reasonable assumptions about the nature of plausible extension, the cheapest update with the sentence will be one corresponding to the local accommodation into the antecedent of the proposition that Bertha is not hiding.

This is a far from complete story, but it does suggest at least the possibility that the theory of meaning described in this paper could be extended with an account of local accommodation. Personally, I would be happier with an account that did not involve postulation of explicit costs and relied instead on a preference ordering mechanism like that invoked to describe global accommodation in chapter 5. However, in the absence of any detailed proposal, I think it wise to keep an open mind, with there being clear potential for solutions using orderings, costs, or probabilities. Given recent formal convergence in the field of non-monotonic reasoning, it seems likely that a solution depending on any one of these might eventually be recast in terms of either of the others.

## Chapter 6

# Conclusion

In [Gaz79], Gazdar concludes of Karttunen's "plugs, holes and filters" account of presupposition, the theory which formed the basis of the CCP model:

The... theory... has had a long and distinguished career. First formulated in 1971, published in 1973, modified and reconceptualized in 1974, and formalized and reterminologized in 1975. But the theory as of 1978 is in poor shape, enmeshed in its own epicycles, beset by counterexamples and constantly in need of "conversational implicatures" to unclog the filters and explain the leakage from its plugs. The time for euthanasia has arrived.

Fifteen years later, the CCP model is alive and kicking. Indeed, it seems to me that, as of 1978, Karttunen's account of presupposition was not a theory five years past its prime, but one fifteen years ahead of its time. Much the same could be said of Karttunen's even earlier account of discourse referents [Kar76]. For it is only in the last few years that the technical methods have been developed with which to give adequate expression to the essentially dynamic model that Karttunen was advocating.

The combination of Heim's work with the further developments in this thesis have provided solutions to many of the problems with Karttunen's account. Yet the resulting theory is by no means just an *ad hoc* collection of repairs. On the contrary, as I have gone to great lengths to demonstrate, the same analysis of internal sentence dynamics which is appropriate to the requirements of quantification, anaphora and epistemic modality, is also at the heart of the CCP theory.

In particular, the developments in this thesis can be summed up as follows:

- A theory of quantification has been presented which accounts for both internal and external dynamic properties of generalised quantifiers, together with a theory of anaphora that exploits the properties of the quantifiers to yield a treatment of singular and plural donkey and discourse anaphora.
- A theory of epistemic modality has been described which accounts for classic modal identity problems and for the logic of modality within quantified contexts.
- A solution has been given for the projection problem for presuppositions arising within compound and quantified sentences, and it has been argued that the results with respect to quantified sentences improve on those found in the existing literature.
- The accounts of quantification, anaphora, modality and presupposition have been integrated within a compositional semantics for a fragment of English.
- It has been shown how global accommodation can be formalised within an account which is both dynamic and pragmatic, by using preference orderings over information states, and it was demonstrated that this resolves both the general problem of how presuppositions can be informative, and one particular problem with the CCP model concerning the prediction of overly weak conditional presuppositions from compound sentences. It was argued that all purely semantic theories of presupposition, and all theories of presupposition involving structural accounts of accommodation or cancellation make systematically incorrect predictions by failing to account for the importance of common sense in determining exactly what is accommodated.

Having shown that the greater part of the CCP theory can be independently motivated in terms of extraneous semantic phenomena, what remains that is specific to a theory of presupposition is just that it really is a theory of **presupposition**. If you convert the intuitive idea that presuppositions come *before* other aspects of meaning into the technical claim that presuppositions are constraints on input contexts in dynamic semantics, what results, broadly, is the CCP model. Thus presupposition is what comes first in dynamic semantics.

# Appendix A

## Loose Ends

In this appendix I will firstly clarify the relationship between ABLE and PUL, and then give some general properties of ABLE.

### A.1 PUL and ABLE

It will now be demonstrated that ABLE preserves the intuitions of the propositional system, PUL, introduced in chapter 2. Although it would be possible to define a direct embedding of PUL in ABLE, this would be messy, and instead I will introduce a slight variant of ABLE,  $ABLE+\partial$ . This variant is just ABLE but with the addition of atomic propositions and a  $\partial$  operator:

**Definition D49 ( $ABLE+\partial$ )** • *ABLE+* $\partial$  has the same syntax as ABLE, but with two additional classes of formulae, firstly a set of atomic formulae,  $\mathcal{P}$ , and secondly formulae of the form  $\partial\text{FORM}$ .

- For every  $p \in \mathcal{P}$ , there is a corresponding  $\text{Ty}_3$  constant  $p$  of type  $\langle w, t \rangle$ .
- $\llbracket p \rrbracket = \lambda I \lambda J [J = \lambda w \lambda f [I.w.f \wedge p.w]]$
- $\llbracket \partial\phi \rrbracket = \lambda I \lambda J [I = J \wedge I \text{ satisfies } \phi]$
- Remaining semantic clauses and definition of entailment, which will be written  $\models_{ABLE+\partial}$ , are as for ABLE.

It can now be shown that regarding the intersection of the PUL and  $ABLE+\partial$  languages, PUL and  $ABLE+\partial$  entailment are identical. The proof will be sketchy in places, but then the result only concerns the internal consistency of this thesis, and is presumably not of general interest. Note that in this section  $\sigma$  will not be used as an abbreviation for the type of ABLE extended sequences, but as a variable over PUL information states.

#### Lemma L2

1. If  $p \in \mathcal{P}$ , then  $\forall I$   $I$  admits  $\llbracket p \rrbracket$ . That is to say, the atomic formulae have no presuppositions.
2. If  $p \in \mathcal{P}$ , then  $\downarrow \llbracket p \rrbracket = \llbracket p \rrbracket$ . That is, atomic formulae do not introduce new discourse markers.
3. If  $\phi$  is a formula of PUL and of  $ABLE+\partial$ , then  $\downarrow \llbracket \phi \rrbracket = \llbracket \phi \rrbracket$ . That is, the PUL fragment of  $ABLE+\partial$  does not introduce discourse markers.
4. If  $\phi$  is a formula of PUL and of  $ABLE+\partial$ , then for states  $I, I', J, J'$  such that  $\omega\text{-set}.I = \omega\text{-set}.I'$  and  $\omega\text{-set}.J = \omega\text{-set}.J'$ , it holds that:

$$I[\llbracket \phi \rrbracket]J \text{ iff } I'[\llbracket \phi \rrbracket]J'$$

*Proofs of (1) and (2) follow directly from the ABLE+ $\partial$  semantics for atomic propositions above and the definitions of admits and  $\downarrow$ . Proof of (3) follows by inspection of the semantic clauses in the PUL fragment of ABLE+ $\partial$ , and verification that, in each case, if the subformulae introduces no discourse markers, then neither does the compound formula. (4) follows from (3) and another induction over formulae of ABLE+ $\partial$ .*

**Definition D50 (Information Injection)** *It is simple to define a mapping from PUL models to ABLE models: we simply identify the set of worlds,  $W$ , in a PUL model with the domain  $\mathcal{D}_w$  in the ABLE ( $\text{Ty}_3$ ) model, and the PUL interpretation function with the ABLE ( $\text{Ty}_3$ ) interpretation function. This allows the definition of an injective mapping,  $+$ , from PUL information states to ABLE information states. In the following, the variable  $\omega$  of type  $\langle w, t \rangle$  stands for a PUL information state:*

$$\omega^+ = \lambda w \lambda f [\omega.w \wedge f = \odot]$$

**Lemma L3 Semantic Correspondence** *For any formula  $\phi$  in the PUL fragment of ABLE+ $\partial$ , and any two PUL information states  $\omega_1, \omega_2$ ,*

$$\omega_1 \llbracket \phi \rrbracket_{\text{PUL}} \omega_2 \text{ iff } \omega_1^+ \llbracket \phi \rrbracket_{\text{ABLE}+\partial} \omega_2^+$$

*Proof: An induction over formula complexity. I will consider only a few cases:*

**Conjunction** *It must be shown that if the result holds for  $\phi$  and  $\psi$ , then:*

$$\omega_1 \llbracket \phi \text{ AND } \psi \rrbracket_{\text{PUL}} \omega_2 \text{ iff } \omega_1^+ \llbracket \phi \text{ AND } \psi \rrbracket_{\text{ABLE}+\partial} \omega_2^+$$

*This follows trivially from the fact that in both systems conjunction is defined as relational composition.*

**Negation** *The semantic clauses for PUL and ABLE+ $\partial$  run as follows:*

$$\begin{aligned} \sigma \llbracket \text{NOT } \phi \rrbracket_{\text{PUL}} \tau & \text{ iff } \exists v \sigma \llbracket \phi \rrbracket_{\text{PUL}} v \wedge \tau = \sigma \setminus v \\ \llbracket \text{NOT } \phi \rrbracket_{\text{ABLE}+\partial} & = \lambda I \lambda J [\exists K I \downarrow \llbracket \phi \rrbracket_{\text{ABLE}+\partial} K \wedge \\ & \quad J = I \setminus K] \end{aligned}$$

*By lemma L2, we are only interested in  $\phi$  for which  $\downarrow \llbracket \phi \rrbracket_{\text{ABLE}+\partial} = \llbracket \phi \rrbracket_{\text{ABLE}+\partial}$ . Thus we can ignore the closure operation in the ABLE+ $\partial$  semantics. It remains only to show that for any three PUL information states  $\sigma, v$  and  $\tau$ ,  $\sigma \setminus v = \tau$  iff  $\sigma^+ \setminus v^+ = \tau^+$ , which is clear from the definitions of  $+$  and  $\setminus_{\langle \iota, \langle \iota, \iota \rangle \rangle}$ .*

**Epistemic Possibility** *Here are the definitions:*

$$\begin{aligned} \sigma \llbracket \text{MIGHT } \phi \rrbracket_{\text{PUL}} \tau & \text{ iff } \exists v \sigma \llbracket \phi \rrbracket v \wedge \\ & \quad (v \neq \perp \rightarrow \tau = \sigma) \wedge \\ & \quad (v = \perp \rightarrow \tau = \perp) \\ \llbracket \text{MIGHT } \phi \rrbracket_{\text{ABLE}+\partial} & = \lambda I \lambda J \exists K I \llbracket \phi \rrbracket K \wedge \\ & \quad (\neg(K = \perp) \rightarrow J = I) \wedge \\ & \quad (K = \perp \rightarrow J = \perp) \end{aligned}$$

*It suffices that  $+$  maps the PUL absurd information state onto the ABLE absurd information state.*

The following fact sums up the close relation between the logics of PUL and ABLE+ $\partial$ :

**Fact F15** *If  $\phi$  and  $\psi$  are formulae of PUL involving no atomic presuppositions except for those in  $\mathcal{P}$ , then:*

$$\phi \models_{\text{PUL}} \psi \text{ iff } \phi \models_{\text{ABLE}+\partial} \psi$$

*Proof:* It must be shown that for formulae not introducing discourse markers, the two notions of entailment are equivalent. For single premise arguments, the PUL notion of entailment (defn. D7) may be written:

$$\phi \models_{PUL} \psi \quad \text{iff} \quad \forall \omega_0, \omega_1 \\ \omega_0 \llbracket \phi \rrbracket_{PUL} \omega_1 \rightarrow \omega_1 \llbracket \psi \rrbracket_{PUL} \omega_1$$

In  $ABLE+\partial$ , (collapsing the two parts of the definition in chapter 3) entailment is defined as:

$$\phi \models_{ABLE+\partial} \psi \quad \text{iff} \quad \forall I \forall J \\ I \llbracket \phi \rrbracket_{ABLE+\partial} J \rightarrow J \text{ satisfies } \llbracket \psi \rrbracket_{ABLE+\partial}$$

From lemma L2.4, we know that if  $I'$  and  $J'$  have an empty domain, and  $\omega\text{-set}.I = \omega\text{-set}.I'$  and  $\omega\text{-set}.J = \omega\text{-set}.J'$ , then:

$$I \llbracket \phi \rrbracket_{ABLE+\partial} J \quad \text{iff} \quad I' \llbracket \phi \rrbracket_{ABLE+\partial} J'$$

It follows that in considering  $ABLE+\partial$  we only need to consider states with an empty domain, so that the above definition is equivalent to:

$$\phi \models_{ABLE+\partial} \psi \quad \text{iff} \quad \forall \omega_0 \forall \omega_1 \\ \omega_0^+ \llbracket \phi \rrbracket_{ABLE+\partial} \omega_1^+ \rightarrow \omega_1^+ \text{ satisfies } \llbracket \psi \rrbracket_{ABLE+\partial}$$

Given lemma L3, we have that for sentences of PUL:

$$\omega_0 \llbracket \phi \rrbracket_{PUL} \omega_1 \quad \text{iff} \quad \omega_0^+ \llbracket \phi \rrbracket_{ABLE+\partial} \omega_1^+$$

Thus it is only necessary to prove that:

$$\omega_1 \llbracket \psi \rrbracket_{PUL} \omega_1 \quad \text{iff} \quad \omega_1^+ \text{ satisfies } \llbracket \psi \rrbracket_{ABLE+\partial}$$

The definition of satisfaction from chapter 3 is:

$$\text{satisfies} = \lambda I \lambda F [I \{ \downarrow F \} I]$$

From lemma L2.2 and another application of the semantic correspondence lemma, the result follows.

## A.2 Properties of ABLE

The main facts demonstrated in this section are:

1. All ABLE formulae are eliminative. Without this property the definitions of satisfaction and entailment would not be appropriate.
2. The non-modal presuppositions of an ABLE formula are also entailments.

**Definition D51 (Extension)** A state  $I$  is an extension of a state  $J$  iff :

$$\forall w \forall f I.w.f \rightarrow (\exists g f \succeq g \wedge J.w.g)$$

**Definition D52 (Eliminativity)** A CCP  $F$  is eliminative iff for any states  $I$  and  $J$ , if  $I \{ F \} J$  then  $J$  is an extension of  $I$ .

**Fact F16** All ABLE formulae are eliminative.

*Proof:* An induction over ABLE formulae. The most difficult cases are the quantificational determiners, which I will leave until last. Firstly, the postulate on the denotation of ABLE predicates, MP8, forces the output to be a subset of the input, from which eliminativity follows. Conjunction is defined as relational composition, and it is clear that if two CCPs are eliminative then their composition will be. Negation is defined such that the output is the input minus some other set, which means that once again the output must be a subset of the input. The output of a definite determiner, if defined, is the input updated with the scope condition, so it suffices that, by the induction hypothesis, the scope condition will be eliminative.

Since tests are eliminative, the output either being the input or the absurd state, facts D40 and F8 guarantee that the epistemic modalities are eliminative.

The definition of the semantics of existential determiners, D44, is of the following form:

$$\llbracket \mathcal{D}.i.\phi.\psi \rrbracket = \lambda I \lambda J \left[ \begin{array}{l} \dots \wedge \\ J = \lambda w \lambda g \\ \left[ \begin{array}{l} \exists f \exists sco \exists G_{sco} \dots I.w.f \wedge \dots \wedge G_{sco} = \lambda h [h \succeq f \wedge sco.w.h] \wedge \\ \dots \wedge G_{sco}.g \end{array} \right] \end{array} \right]$$

Thus we have that for existential determiners  $\mathcal{D}$ , if  $I[\mathcal{D}.i.\phi.\psi]J$ , then:

$$\forall w \forall g [J.w.g \rightarrow \exists f \exists sco \exists G_{sco} I.w.f \wedge \\ G_{sco} = \lambda h [h \succeq f \wedge sco.w.h] \wedge \\ G_{sco}.g]$$

From this it follows that:

$$\forall w \forall g [J.w.g \rightarrow \exists f I.w.f \wedge g \succeq f]$$

This is the required result.

Similarly, it follows from definition D43 that if  $\mathcal{D}$  is a non-existential determiner and  $I[\mathcal{D}.i.\phi.\psi]J$ , then:

$$\forall w \forall g [J.w.g \rightarrow \exists f \exists sco \exists G_{sco} I.w.f \wedge \\ G_{sco} = \lambda h [h \succeq f \wedge sco.w.h] \wedge \\ g = \lambda D \lambda x \left[ \begin{array}{l} f.D.x \vee \\ \exists h G_{sco}.h \wedge h.D.x \end{array} \right]$$

Once again, it is straightforward to verify that  $g$  must be an extension of  $f$ , and the result follows. This completes the proof.

**Definition D53 (Singleton)**

$$\langle\langle w, f \rangle\rangle = \lambda w' \lambda g [w' = w \wedge f = g]$$

**Definition D54 (Distributivity)** A CCP  $F$  is distributive iff

$$\forall I \forall J I\{F\}J \leftrightarrow J = \lambda w \lambda f [ \\ \exists g I.w.g \wedge \langle\langle w, g \rangle\rangle\{F\}\langle\langle w, f \rangle\rangle]$$

**Fact F17** The non-modal formulae of ABLE are distributive.

*Sketch of Proof:* Another induction on (non-modal) formula complexity, starting with the fact that MP8 guarantees that ABLE predications are interpreted distributively.

**Definition D55 (Persistence)** A CCP  $F$  is persistent iff

$$(I \text{ satisfies } F \wedge I; J \text{ is an extension of } I) \rightarrow J \text{ satisfies } F$$

**Fact F18** Non-modal ABLE formulae are persistent.

*Proof:* ABLE predicates have the property of relevance, so that their denotation is only sensitive to the values the input state gives to the predicated markers. Thus if an ABLE predication is satisfied in a state, then adding new discourse markers to the state will produce another state in which the predication

is satisfied. Since predications are distributive, it holds that if a predication  $F$  is satisfied in some state  $I$ , then it must be satisfied in every subset of  $I$ . It follows that ABLE predications are persistent.

Inspection of the remaining semantic clauses show that they all preserve relevance, so that if any ABLE formula is satisfied in a state then adding new discourse markers will yield another state which satisfies the formula. Since all non-modal formulae are distributive, it follows that all non-modal formulae are persistent.

**Fact F19** *If  $\phi$  is a non-modal formula and  $\psi$  presupposes  $\phi$ , then  $\psi \models \phi$ .*

*Proof:* Since  $\phi$  is non-modal, it must be persistent, so that if it is satisfied in a state then it must be satisfied in any extension of the state. Since  $\psi$  presupposes  $\phi$ , it follows that every state that admits  $\psi$  satisfies  $\phi$ . Since all ABLE formulae are eliminative, it follows that an output of  $\psi$  must be an extension of the input. Thus if an input of  $\psi$  satisfies  $\phi$ , then the output must also satisfy  $\phi$ . This yields the required result, since every output of  $\psi$  satisfying  $\phi$  is the condition for  $\psi \models \phi$ .



# Appendix B

## ABLE in $\text{Ty}_3$

The following postulate defines  $\text{Ty}_3$  constants for all the **ABLE** operators, as well as the pronominal abbreviations:

### Meaning Postulate MP33

- (1) **IS** =  $\lambda D \lambda D' \lambda I \lambda J [t\text{-domain}.I.D \wedge t\text{-domain}.I.D' \wedge J = \lambda w \lambda f [I.w.f \wedge f.D = f.D']]$
- (2) **AND** =  $\lambda F \lambda F' \lambda I \lambda J [\exists K I\{F\}K\{F'\}J]$
- (3) **NOT** =  $\lambda F \lambda I \lambda J [\exists K I\{F\}K \wedge J = \lambda w \lambda f [I.w.f \wedge \neg(\exists g g \succeq f \wedge K.w.g)]]$
- (4) **IMPLIES** =  $\lambda F \lambda F' \lambda I \lambda J [\exists I' \exists I'' I\{F\}I'\{F'\}I'' \wedge J = \lambda w \lambda f [I.w.f \wedge \forall g (g \succeq f \wedge I'.w.g) \rightarrow (\exists h h \succeq g \wedge I''.w.h)]]$
- (5) **OR** =  $\lambda F \lambda F' \lambda I \lambda J [\exists K \exists K' I\{F\}K \wedge I\{F'\}K' \wedge J = \lambda w \lambda f [I.w.f \wedge \exists g g \succeq f \wedge (K.w.g \vee K'.w.g)]]$
- (6) **THE** =  $\lambda D \lambda F \lambda F' \lambda I \lambda J [t\text{-domain}.I.D \wedge I\{\text{satisfies}\}F \wedge \exists K I\{F\}K\{F'\}J]$
- (7) **SHE** =  $\lambda D \lambda F [\text{THE}.D.(\text{SINGULAR}^u.D \text{ AND FEMALE}^u.D).F]$
- (8) **HE** =  $\lambda D \lambda F [\text{THE}.D.(\text{SINGULAR}^u.D \text{ AND MALE}^u.D).F]$
- (9) **IT** =  $\lambda D \lambda F [\text{THE}.D.(\text{SINGULAR}^u.D \text{ AND NEUTER}^u.D).F]$
- (10) **THEY** =  $\lambda D \lambda F [\text{THE}.D.(\text{PLURAL}^u.D).F]$
- (11) **MIGHT** =  $\lambda V_{(\delta, \tau)} \lambda F \lambda I \lambda J [\exists K I\{F\}K \wedge J = \lambda w \lambda f [I.w.f \wedge \exists w' \exists g K.w'.g \wedge g \equiv_V f]]$

(12) If  $\mathcal{D}$  is one of EVERY, MOST, FEW or NO then:

$$\mathcal{D} = \lambda D \lambda F \lambda F' \lambda I \lambda J \left[ \begin{array}{l} \exists \text{init} \exists \text{res} \exists \text{sco} \\ I\{+.D\} \text{init}\{F\} \text{res}\{F'\} \text{sco} \wedge \\ \\ J = \lambda w \lambda g \left[ \begin{array}{l} \exists f \exists G_{\text{res}} \exists G_{\text{sco}} \exists X_{\text{res}} \exists X_{\text{sco}} \\ I.w.f \wedge \\ G_{\text{res}} = \lambda h [h \succeq f \wedge \text{res}.w.h] \wedge \\ G_{\text{sco}} = \lambda h [h \succeq f \wedge \text{sco}.w.h] \wedge \\ X_{\text{res}} = \lambda x [\exists h G_{\text{res}}.h \wedge h.d_i.x] \wedge \\ X_{\text{sco}} = \lambda x [\exists h G_{\text{sco}}.h \wedge h.d_i.x] \wedge \\ \mathcal{D}'.X_{\text{res}}.X_{\text{sco}} \wedge \\ g = \lambda D \lambda x \left[ \begin{array}{l} f.D.x \vee \\ \exists h G_{\text{sco}}.h \wedge h.D.x \end{array} \right] \end{array} \right] \end{array} \right]$$

(13) If  $\mathcal{D}$  is SOME or EXACTLY-ONE, then:

$$\mathcal{D} = \lambda D \lambda F \lambda F' \lambda I \lambda J \left[ \begin{array}{l} \exists \text{init} \exists \text{res} \exists \text{sco} \\ I\{+.D\} \text{init}\{F\} \text{res}\{F'\} \text{sco} \wedge \\ \\ J = \lambda w \lambda g \left[ \begin{array}{l} \exists f \exists G_{\text{res}} \exists G_{\text{sco}} \exists X_{\text{res}} \exists X_{\text{sco}} \\ I.w.f \wedge \\ G_{\text{res}} = \lambda h [h \succeq f \wedge \text{res}.w.h] \wedge \\ G_{\text{sco}} = \lambda h [h \succeq f \wedge \text{sco}.w.h] \wedge \\ X_{\text{res}} = \lambda x [\exists h G_{\text{res}}.h \wedge h.d_i.x] \wedge \\ X_{\text{sco}} = \lambda x [\exists h G_{\text{sco}}.h \wedge h.d_i.x] \wedge \\ \mathcal{D}'.X_{\text{res}}.X_{\text{sco}} \wedge \\ G_{\text{sco}}.g \end{array} \right] \end{array} \right]$$

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