The Nature of Quantum Information

MSc Thesis *(Afstudeerscriptie)*

written by

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Abstract

The field of quantum information is rapidly developing and gaining increasing attention, due to its significant advancements in different areas such as quantum computing and quantum information theory and its relevance for a wide range of disciplines. However, there is a philosophical dimension to quantum information that is still relatively unexplored. In this thesis we will analyse the nature of quantum information, based on qualitative approaches to information. We will not aim to give a unique answer to the question 'What is quantum information?', but we will provide a range of perspectives on the topic. We consider different approaches to classical information that take into account qualitative properties of information such as informational content, aboutness, situatedness and its connection to knowledge. We will also consider the overarching themes of taking information as range and information as correlation. We will examine how these accounts of information need to be adjusted to be applicable to quantum information, in order to identify the differences between classical and quantum information. We conclude that classical information could be viewed as a subcategory of quantum information under specific circumstances, but that both types of information are indispensable. Furthermore, we suggest possible areas for future research.

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Contents

1 Introduction

The field of quantum information is a relatively young research area that started off in the 1970s and 1980s, when the first ideas arose to combine quantum mechanics and information theory, in order to design and use quantum systems for information processing techniques. In a short period of time, the area developed into a very important area of research that gains much attention from various fields such as computer science, physics, philosophy, but also cryptography and neuroscience. As Nielsen and Chuang (2010, p.xix) point out in their seminal work *Quantum Computation and Quantum Information*, due to the rapid development of the field, remarkable advancements have been achieved in various aspects. These are for instance significant developments in hardware, such as large-scale quantum computers, and in quantum information theory, leading to numerous communication protocols and a deepened understanding of quantum communication channels.

As seen from this kind of progress, the topic of quantum information has mainly been approached from a technical standpoint, with its roots in engineering and with the main goal to utilise quantum phenomena to solve current computational problems. But despite the attention for the field, there are many philosophical aspects to quantum information that remain relatively unexplored. Quantum information presents an intriguing and enlightening subject of study, which could possibly lead to insights into the puzzles in the foundations of quantum mechanics (Timpson, 2013, p.2). It has even been argued that information should be taken as a fundamental category, from which a description of the physical world, quantum or classical, would arise (Bub, 2005; Wheeler, 1989; Zeilinger, 1999). The new objective would then be to give information-theoretic constraints, a description of how information evolves and what principles it adheres to, from which (quantum) physics can be deduced (Clifton et al., 2003). Furthermore, there is a lack of consensus on fundamental questions such as ‘What is quantum information?’ and ‘How is it different from classical information?’. There exist diverse positions, ranging from those who consider it to be similar to classical information, but encoded in quantum systems (Caves and Fuchs, 1996; R. B. Griffiths, 2002; Lombardi, Holik, et al., 2016; Timpson, 2013), to those who take it as an entirely distinct form of information (Jozsa, 2004), to some who question the existence of quantum information altogether (Duwell, 2003).

In this thesis, we will delve into the nature of quantum information, examining its properties and possible differences from classical information, by reflecting on our comprehension and interaction with information. We take a philosophical approach and we will provide a range of perspectives on the topic. We aim to explore the adjustments to our understanding of the notion of information when considering the principles of quantum mechanics. To do so, it is imperative to first establish an understanding of the concept of information in general.

1.1 What is information?

First of all, one might ask: what is ‘information’? This is not an easily answered question, since the term is being used in a myriad of contexts, both in everyday life and in academia. As Capurro and Hjørland (2003, p.356) remark, the notion of information has been used in almost all scientific disciplines. On one hand, we have physics, in which information is tightly linked to entropy or the physical organisation of the state of the world, on the other hand, we have biology, in which we consider for instance the information stored in DNA our cells. The term ‘information’ carries a different meaning in these different contexts and it is therefore doubtful whether it is possible to provide an overarching definition or unifying theory of information (Adriaans and van Benthem, 2008, p.17; Floridi, 2004, p.40).

The term ‘information’, as it is often used in colloquial speech or general everyday use, again refers to various ideas and often it is not made precise what the conceptual distinctness of these uses is (Timpson, 2008, p.22). It can refer to the content of a cognitive state of an agent. As Enßlin et al. (2019, p.1) put it: “What is information? A short answer could be: Anything that changes our minds and states of thinking!” We talk about getting across information from one person to another, learning new information, or understanding the information from a message (a text or an image). In this sense, it is a *subjective* notion, as the informational content an agent can learn from a message or signal depends on their background knowledge. But, ‘information’ can also refer to the linguistic meaning contained in a text. Or it can refer to any kind of meaningful data stored in a medium, such as code in a computer or environmental information, such as the number of rings in a tree trunk. This type of information still exists and can be meaningful even if there is no agent to witness it (Floridi, 2016, p.398). Therefore, if we consider these notions of information, it does not necessarily need to be subjective or agent dependent, they take information as something *objective*. Capurro and Hjørland (2003, p.396) writes: “In our view, the most important distinction is that between information as an object or a thing (e.g., number of bits) and information as a subjective concept, information as a sign; that is, as depending on the interpretation of a cognitive agent.” Thus, these different definitions seem to provide a main distinction between different concepts of information.

1.1.1 Categorising types of information

A distinction along the same lines is proposed by Adriaans and van Benthem (2008, p.11). They put forward three categories, which represent clusters of how information is approached in research. First, there is *Information-A*, which refers to agents gaining knowledge about the world through information transfer, observation, communication and logical deduction. In this setting, information is approached in a qualitative manner: the goal is to model or understand what information agents have access to. The idea of *aboutness* is important here, as the information is always about something or some situation in the world and this is taken into account. This is opposed to *Information-B*, where the focus is on measuring information quantitatively. In this setting, the content or meaning of the information is not relevant, it is about the communication of messages from a source to a receiver and measuring the information in these messages in terms of uncertainty via a probabilistic account. The most used formal account of this type of information is information theory as described by Shannon (1948). Lastly, one can also measure information quantitatively by considering the algorithmic complexity of some string of code, to reflect the amount of information. In Kolmogorov complexity, the information is then defined as the length of the shortest program on a universal Turing machine that can compute this algorithm. This last definition of information, referred to as *Information-C*, will not be considered in this thesis. More details about this can be found in general textbook introducing these topics, such as the one by Li and Vitányi (1997).

Where Adriaans and van Benthem (2008) are mainly focused on the use of the concept of information in different academic research, Timpson (2013, p.2) makes the distinction on a more general level. He separates *everyday information* from *technical information.* We will consider this categorisation in depth in Section 2.1, as it provides a running thread throughout the rest of the thesis.

1.1.2 Information and knowledge

In general, the term information is derived from the verb ‘to inform’ (Timpson, 2013, p.11). What is provided or transferred when someone is informed of something is what we call ‘information’. This shows its close connection to the concept of knowledge and to the cognitive states of agents. We will see this connection in several accounts of information that we will cover in this thesis.

When we refer to ‘knowledge’, there is not one accepted definition in the philosophy literature for this.[1] We will stick to the often-used definition proposed by Plato of knowledge, who defined it to be ‘justified true belief’ (van Benthem and Martinez, 2008, p.232; Haddock, 2010, p.195). This characterisation of knowledge has often been disputed in the literature (de Grefte, 2023). Nevertheless, the condition of knowledge to be true is one that most adjustments of this definition of knowledge have: knowing some proposition p also requires that p must be true or factual. This is an aspect of knowledge that is relevant to our analysis of information.

1.1.3 Information carriers and systems

As broad as different conceptions of information are, equally broad are the different types of carriers of information. In this thesis, we will therefore consider various carriers of information on an abstract level. This ranges from humans or general intelligent agents to objects such as books or physical particles such as atoms or photons, to abstract signals. We will speak often about classical or quantum systems. Let us clarify this notion. In general, a system can be defined as a collection of elements that interact with one another and in this way form a cohesive whole. We can distinguish between classical and quantum systems, where a quantum system consists of a collection of objects on the macroscale, such as atoms or molecules, which exhibit quantum behaviour which can be described by the theory of quantum mechanics.

A system can be seen as a part of physical reality. Often systems are taken to be an idealised notion of a defined part of reality. A system usually can be in various states and has certain ways of evolving into different states. We can distinguish between open and closed systems. *Open systems* interact with their environment or other systems to some extent. On the other hand, *closed systems* are not influenced by anything external and they do not exchange any matter or energy with their environment. In reality, all systems, except for the entire Universe, experience some level of interaction. Still, we can approximate certain systems can as closed (Nielsen and Chuang, 2010, p.82).

Now that we have a general idea of the notion of information and some clarification of concepts out of the way, we will describe in more detail what the aim of this thesis is.

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[1] For an overview of different accounts of knowledge see (Pritchard, 2018).

1.2 Main question

As discussed above, the field of quantum information revolves around a technical notion of information, which can be seen as Information-B in the categorisation from Adriaans and van Benthem (2008). Thus, this is a quantitative approach to information, similar to classical information theory. This goes a long way toward the aim of engineering and the task-oriented approach to quantum information, to research what can be achieved with quantum systems that cannot be achieved with classical ones (Timpson, 2008, p.2). However, in this thesis, we are interested in a qualitative approach to information, focusing on epistemic, semantic and everyday notions of information and considering qualitative properties of information such as aboutness, situatedness and its connection to knowledge. These aspects of information have been extensively considered in the classical case, but we aim to extend this to quantum information.

The objective is not to provide a new theory or framework but to review existing qualitative theories for classical information and examine how they can be adapted and applied to to quantum information. In doing so, we will explore the differences between classical and quantum information, identify potential challenges to compare them and provide a general overview of the topic. We will not provide a unique answer to the question of what quantum information is, but we hope to contribute to a better understanding by considering a range of perspectives. We will explore the territory, as this discussion received very little attention in the literature, and focus on the first steps towards understanding quantum information from a qualitative point of view. The only qualitative definition of quantum information provided in the literature that is known to the author is the one by Timpson (2013, p.60), which is also discussed by Duwell (2019, p.3). However, these are still qualitative definitions in a technical approach to quantum information. Therefore, our central question is: What can we learn about quantum information by considering similar qualitative approaches as to classical information? In addition to this, we aim to identify the main questions and topics that need to be reviewed in order to better understand the nature of quantum information. We will consider aspects such as the physicality and locality of information, information carriers and the effects of interaction with information.

There exist direct strategies that attempt to derive philosophical or foundational insights from quantum information theory, for instance by taking information as a physical entity and primitive in quantum mechanics (Brukner and Zeilinger, 2003; Bub, 2005; Svozil, 2000). However, this thesis is concerned with indirect approaches. These are approaches that seek to deepen the understanding of quantum theory by considering information-theoretic constraints and using quantum information theory to examine possible constraints on physical laws (Timpson, 2013, p.8). Thus, by taking such an indirect approach, our main goal is to explore and analyse the nature of quantum information, examining its qualitative properties and differences from classical information. We will not provide a definitive definition of quantum information, as this has not been given for classical information either, but aim to contribute to a deeper understanding by considering existing approaches to classical information and see if and how similar versions apply to quantum information. We hope to pave the way for further investigation into quantum information and its philosophical implications, by identifying relevant questions and topics.

1.3 Outline of this thesis

This thesis begins by introducing and examining frameworks and theories for semantic information and qualitative approaches in classical information. We will consider the general distinction by Timpson (2013) between an everyday notion and a technical notion of information, as this is a distinction we will come back to frequently. We will briefly touch upon what it means for information to be physical. We continue with considering several semantic accounts of information that focus on the content or the meaning of information, as opposed to technical accounts of information such as the one used by Shannon (1948). We follow the distinction made by Adriaans and van Benthem (2008, p.17) in taking information as range or information as correlation as the overarching theme for these approaches. We will consider the work of Floridi (2016) and more formal approaches such as a quantitative account of semantic information (Carnap and Bar-Hillel, 1952), epistemic logic (Hintikka, 1962), Dretske's account of information flow (1981) and situation theory (Barwise and Perry, 1983). We will highlight important elements or principles of these accounts, which we will later use in our comparison with quantum information.

In Chapter 3 we provide a general introduction to quantum information. We compare bits and qubits, the units of classical and quantum information respectively. We consider the different properties of qubits that distinguishes them from classical bits, such as superposition and entanglement. We will also discuss the phenomenon of quantum non-locality and how this was derived from the principles of quantum mechanics. Furthermore, we consider six no-go theorems, which put constraints on the possibilities of interaction with and manipulation of quantum information. Subsequently, we look at the idea of giving an information-theoretic characterisation of quantum mechanics in terms of constraints on information as proposed by Clifton et al. (2003). Lastly, we analyse quantum teleportation, as it provides an example of a quantum information protocol and makes use of the entanglement of qubits, combining both classical and quantum information channels.

In Chapter 4 we start with a comparison of classical and quantum information based on the principles discussed in the preceding chapter. We look specifically at different aspects of information, such as information carriers, where we compare bits and qubits. These are not only used as units of information but denote two-level classical or quantum systems. We discuss the need for a physical carrier of information in which we reflect on how this could be the case in quantum teleportation. Next, we consider states of classical and quantum systems, with the most important characteristic being that quantum states need not be distinguishable. This is taken as a fundamental difference between quantum and classical information. We look at the interaction with information through the measurement of properties of systems, and which information is accessible or inaccessible for agents, which again highlights important differences between classical and quantum systems. Finally, the distinction between classical and quantum correlations is discussed, which lies at the root of quantum information being non-local, another difference from classical information.

Chapter 5 continues by investigating how the theories of classical information, which were discussed in Chapter 2, might be applied to quantum information. We will first do this on the basis of two main pillars: the creation and erasure of information. We will reflect on the no-go theorems from Chapter 3 and see what these constraints might teach us about the nature of quantum information, namely that it is conserved in closed systems. Furthermore, since quantum information theory is concerned with a technical notion of quantum information, we will consider if there is an everyday notion as a counterpart. We will investigate what would be necessary for a semantic account of quantum information and in doing so discuss the impact of different quantum interpretations on the notion of quantum information. We will come back to the general themes of taking information as range and information as correlation and see if these are applicable to quantum information, by considering specific aspects of the formalisms discussed in Chapter 2.

At the end of Chapters 4 and 5 we will provide a summary reflecting on the most important findings of these chapters. Chapter 6 will build on that and briefly conclude this thesis. We also consider some ideas for further research.

2 Classical information

2.1 Two general categories of information

As discussed in the introduction, there are different ways in which the notion 'information' is used. In these various contexts, the term 'information' seems to refer to distinct concepts. According to Timpson (2013, p.2), the most important distinction that should be made in all these different notions of information is between the idea of everyday information versus technical information. In this section, we will consider Timpson's account of information and this distinction in more depth. We would like to point out his account of information and this distinction here, as it provides a basis that is useful for the rest of this thesis.

Firstly, Timpson (2013, p.10) points out that information is an abstract mass noun since it is uncountable and refers to an abstract concept, rather than a concrete physical object or substance. This is important to keep in mind both for the everyday and technical sense of information. According to Timpson (2013, p.82) confusion in the reasoning about information and information transfer, such as in the case of quantum teleportation, which we will discuss in Section 3.6, often stems from the fact that it is not clear that information is an abstract noun, and thus it represents an intangible idea. This means that information does not have a location in space and time. Furthermore, it is a mass noun, meaning that it is not countable. Therefore, it can very well be compared in its use with other abstract mass nouns such as 'truth'.

A second general observation by Timpson (2013, p.12) about the term 'information' and how it is used is to draw a distinction between possessing information and containing it. The first is about having knowledge and by acquiring information one can come to possess it. Containing information is often used synonymously with containing knowledge. This can be said about for instance books, which give a way to provide knowledge. Following Timpson, it boils down to a category distinction: "to possess information is to have a certain ability, while for something to contain information is for it to be in a certain state (to possess certain occurrent categorical properties)" (Timpson, 2013, p.13).

In accordance with Timpson, there is an important distinction to be made based on the contexts that the term 'information' are used. On one hand, there is the use of information in colloquial speech and everyday communication and on the other hand the use of information in a technical and engineering setting. In these two different contexts, the use of the notion of 'information' does not refer to the same concept and thus should not be viewed as the same thing. The first type of information is what Timpson (2013, p.2) calls the everyday concept of information. This is referring to how the term 'information' is used in everyday language. This concept is derived from the verb "to inform", as information is what is getting communicated when someone is informed of something. In this context, we can clearly see the strong ties this notion of information has with the concept of knowledge. Namely, to inform someone of something is to provide them with new knowledge. Furthermore, the everyday use of the concept of information is often about language and meaning. Timpson (2013) remarks:

> "With information in the everyday sense, a characteristic use of the term is in phrases of the form: 'information about p', where p might be some object, event, or topic; or

> in phrases of the form: 'information that q'. Such phrases display what is often called intentionality. They are directed towards, or are about something (which something may, or may not, be present)." (Timpson, 2013, p.12)

This aboutness of the information is an important aspect of this type of information that we will consider in greater depth when we look at information as correlation. Everyday information usually also revolves around the concept of a person or other cognitive agent, who is able to receive, read, code or decode and use the information. So, everyday information can for instance refer to the information in sentences that are spoken or written, it can be visual cues in images that are sent or the way certain things or people behave in our physical world. The common factor is this: information has the power to inform, to cause a change (in mental state) by interpreting what is being sensed (by our senses or a computer sensor) in some way. This goes to show that everyday information has to carry meaning and that it usually is subjective: the meaning that is ascribed to it by some person might not be the same as by another person after learning this piece of information. This is because one interprets new information, based on their background knowledge. Thus, everyday information refers to a type of information that is semantic (it carries meaning) and is generally subjective.

The second type of information is what Timpson (2013, p.2) refers to as a technical notion of information. This type of information, which Timpson calls 'information_t', is a physically defined quantity introduced for specific purposes and is argued about in a mathematical or physical manner. A technical notion of information is designed with the goal of providing insight into abstract notions of structure, such as complexity or function. This is the kind of information often used in sciences, ranging from algorithmic information in computer sciences to biological information in DNA. Timpson did not provide a precise definition for the everyday concept of information, but he defines technical information, $information_t$, as follows:

> "Information_t is what is produced by an information_t source that is required to be reproducible at the destination if the transmission is to be counted a success." (Timpson, 2013, p.22)

As Timpson (2013, p.22) already remarks, this is a very general definition, but it is so by choice. It needs to be applicable to different kinds of technical information, where there are different aims and interests of what it is exactly that is communicated or transmitted. This definition of technical information is the kind of information that Shannon's Mathematical Theory of Communication (MTC) is about (Shannon, 1948). Shannon's information theory is one of the most used frameworks to study transmission, communication and storage of information and quantify information in this technical sense.[2] It proved to be a very influential theory of information, not only for information in the technical sense, but it has also been used as a base for semantic accounts of information, such as the one by Dretske (1981), which we will discuss in Section 2.3.2.

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[2] We will not discuss Shannon's work in more depth in the current thesis, but apart from the original paper (Shannon, 1948), more information about this can be found in any textbook about information theory, such as (MacKay, 2003; Nielsen and Chuang, 2010).

2.1.1 Comparison of everyday and technical information

Let us compare the two concepts in a bit more depth. First, Timpson (2013) points out that, contrary to what is often thought, the notion of technical information as it is used in MTC is not merely a quantitative notion:

> "Shannon's analysis does provide us with a notion of what is produced (pieces of information$_t$), but it certainly does not in general quantify information in the everyday sense." (Timpson, 2013, p.43).

Timpson means that even though Shannon may not have provided some clear definition of what information is, it can be defined in terms of how it is communicated or transferred, which is exactly what Timpson aims to do in his definition of information$_t$. Furthermore, even though information theory is about quantifying information, we cannot apply this to quantify semantic or everyday information. These two notions of information are very different, which Timpson (2008, p.23) emphasises. Technical information seems to have no or very limited links to the semantic or epistemic concepts of everyday information, as the goal is not to describe information states or knowledge of agents, but rather to provide a way to talk about information in communication protocols and encoding of information over channels. Therefore, technical information refers to a type of information that is not based on meaning, so not semantic and is often understood as objective, rather than agent-dependent. Thus, it is clear that there are different notions of information that are seemingly incompatible.

But is there a way to find common ground between these two different notions of information? Both can be used to reason about something containing information. However, also in this respect, there is an important difference. Shannon's information theory (Shannon, 1948) can be helpful in identifying and quantifying the correlations that are present between some A and B. It can be used to describe objectively the type of correlation (using probability theory). However, this is not enough to provide an explanation as to why some object or agent A contains or possesses information about B (in the everyday sense), since it does not provide an inferential explanation. For this we need the concept of knowledge, which is not considered in a technical notion of information.

Another similarity between everyday information and technical information is that both notions refer to an abstract concept. As discussed above, Timpson (2013, p.3) stresses it is exactly this that often causes confusion in arguing or reasoning about information. It does not have a location in space and time. This is quite clear for the concept of everyday information, since even though it is generally accepted that information needs a physical carrier (Enßlin et al., 2019, p.1), such as a book or a brain, it is not the case that this carrier is the information itself. Timpson (2013) argues that also in the case of information$_t$, it should strictly be viewed as an abstract noun, even though this notion of information does not follow from the verb "inform" or any use of the term information in everyday language, as we will see below.

Consider the following situation: we have two agents Alice and Bob. Alice says to Bob: "I am fascinated by quantum computers". Now Bob gained new knowledge about Alice. This can be seen as a transfer of information in the everyday sense. But what is this information? How is the information related to the specific sentence Alice uttered? To understand this we need to distinguish between the *proposition*, the *sentence type* and the *sentence token*. Clearly, the sentence spoken by Alice contains linguistic meaning, so it is used to express a proposition, to get across the meaning behind the words. The specific utterance in that moment from Alice to Bob is what is called the sentence token. If Alice would repeat exactly the same sentence to Bob it would be a different sentence token, however, it would be of the same sentence type. All sentence tokens (whether it be spoken, written or communicated in any other way) are instantiations of a sentence type. Alice could also have picked different words to get the same message across to Bob, and then the sentence type would be different, but the proposition remains the same. Bob would then learn the same information, and thus we can conclude that everyday information is the proposition. As Timpson (2013, p.18) points out, sentence tokens are concrete things in spacetime, they can be a pattern of sound waves or a sentence written on a page, but propositions and sentence types on the other hand are abstracta. So, how does this work for technical information? In the case of MTC, "[t]he fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point." (Shannon, 1948, p.379). That is, to produce another token of the same type at the other end of the communication system. In this case, the piece of information$_t$ that is tried to get across is the sequence produced by the source. And to be more precise, it is the sequence type, not the sequence token, that is the piece of information$_t$, since we could relabel the message and still get across the same information. Shannon (1948) also mentions:

> "The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have meaning; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem." (Shannon, 1948, p.379)

This goes to show that the proposition of the message is irrelevant, as opposed to the case of everyday information. Hence, we see that the two notions of information really are distinct.

2.1.2 Physicality of information

From the explanation above, it follows why also technical information should be understood as an abstract noun, even though it is often claimed to be physical. This can be seen from the famous claim by Landauer (1991, p.23) that "information is physical". It is not immediately clear how this claim should be interpreted and this leaves room for different interpretations of the claim, as it is not clearly defined what is meant by 'physical'. If we talk about the physicality of information, we mean that information can be described using natural science or specifically physics and adheres to these laws.

This notion of 'physical' should not be confused with the idea of information being a physical substance, something tangible in our universe. Neither is it about information needing a physical carrier, which we will discuss in Section 4.1. Clearly, Timpson (2013, p.10) does not consider information as a physical substance, as he stresses it is an abstract noun. We also see this from the following: If we consider information$_t$ as quantifying information, namely as the compressibility of a source in a communication system such as is done in the MTC by Shannon (1948), then this is a physical property. But at the same time, the information is something abstract. If we consider information$_t$ as pieces of information, then by the argument above, we see that these are types of sequences or sentences, which are also abstracta. Thus, technical information is abstract and at the same time physical. But, as Timpson (2013, p.68) argues, claiming that some physically defined term such as information$_t$ is physical does not provide any interesting insight.

Furthermore, physics is not applicable to the notion of everyday information, since this concept usually depends on the cognitive states of agents, the subjective interpretation in reading/understanding/(de)coding the information and it is grounded in language. Claiming then that everyday information is physical, amounts to claiming that mental states and semantic attributes are reducible to physical terms, which is an ongoing discussion in the philosophy of mind. Thus, according to Timpson (2013, p.68), everyday information cannot be seen as physical, and it is, similarly to technical information, something abstract. This difference, technical information being physical whereas everyday information is not, is another aspect that highlights the distinctness of these types of information.

2.2 Semantic information

The general distinction between everyday information and technical information that Timpson (2013) suggests can be specified further. As we have seen in the previous section, Timpson did not provide a definition for everyday information, but only specified the qualities or properties that some kind of everyday information usually has. The most important being that this type of information is semantic, it carries *meaning*, and is intentional, it is *about* something. In the literature, different specifications and definitions of semantic information have been proposed. We will consider the most prominent ones in this section, starting with the account of Floridi (2004, 2016). This account, together with the one from Carnap and Bar-Hillel (1952) have a focus on information in natural language. We will also consider the account by Dretske (1981), which focuses more on the information in signs in the physical world, which is why Floridi (2011, p.92) refers to this type of information as *environmental information*). Even though this is a theory of semantic information, it is based on the technical account of information in the MTC by Shannon (1948). The variety of approaches to theories of semantic information that we consider in this section will provide us with an overview of qualitative approaches to classical information, which we can later compare with quantum information.

2.2.1 Floridi's account of semantic information

According to Floridi (2016, p.45), there is a distinction between *semantic content* and *semantic information.* The first refers to data with well-formed syntax (structure) and semantics (meaning). If the semantic content is either true or false, then it is called *factual.* This is opposed to for instance instructional semantic content, such as the sentence: "Bring me the book", which is not a sentence that can be true or false. Floridi (2016, p.45) argues that factual semantic content is not yet information, since it does include statements that are false, or "misinformation". When Alice would tell Bob a statement that is not true, then Alice did not inform Bob of something. Therefore, this statement should not be counted as information. Timpson (2013, p.12) agrees with this argument by Floridi, as he states that falsehoods count as misinformation and misinformation should not be seen as a type of information. What is counted as information depends on the receiving agent and their background knowledge or beliefs. This determines whether some statement or fact informs them.

Thus, taking all the above into account, Floridi (2016, p.45) gives the following definition for *factual semantic information*:

> "p qualifies as factual semantic information if and only if p is (constituted by) well-formed, meaningful and veridical data."

Let us unpack this definition. First, what is meant by data here exactly? Floridi points out that information cannot be dataless, but it can consist of a single datum, which is "reducible to just a lack of uniformity between two signs" (Floridi, 2004, p.43). Thus, this can be a bit that takes the value 0 or 1, or a switch that is flipped up or down, etc. These data can be combined in such a way that they obey structure and are well-formed, in order to derive some meaning from it. This can be the grammar of a language, but can also be some set of rules for another type of communication. The data must also be truthful, or as Floridi asserts in the definition, veridical. He uses this term because the data can be of various forms, it can be in the form of sentences or some patterns, images, maps, formulae, etc., and in those cases, the term "veridical" is preferred over "truthful".

Floridi (2016, p.46), similarly to Timpson (2013), highlights the incompatibility of semantic information with information in Shannon's sense (technical information). There are two main differences. Firstly, Floridi's account of semantic information aims to analyse information as semantic content, to answer when and how something is information or how information can be about something else. Secondly, it aims for a connection to related concepts of information or forms of epistemic phenomena, such as the relation of information and knowledge and the communication of information between agents via messages.

2.3 Formalising semantic information

In the literature on semantic information, different approaches have been proposed to provide a formalisation. We will discuss the most influential ones in this section. Some of these approaches provide a more quantitative account, such as the one by Carnap and Bar-Hillel (1952), whereas others are focused on qualitative aspects of semantic information, such as the possible worlds approach used in epistemic logic by Hintikka (1962). Altogether, a general distinction can be drawn between approaches, as is done by van Benthem and Martinez (2008, p.217). They distinguish between logical approaches that take *information as range* and approaches that take *information as correlation.* In both cases, the topic of interest is the dynamics of information: how information behaves and how information flows between agents, objects or signals.

In the information-as-range approach, the main idea is that one can characterise an informational state with the range of possible configurations of the state of the world that correspond to the information available at that informational state. When learning new information, this range of possibilities is reduced, which denotes the reduction of uncertainty about the actual state of the world. On the other hand, in the information-as-correlation approach, the main focus is on how information flows in a structure of components that are systematically correlated. This approach is useful to highlight the 'aboutness' of information (every piece of information is about something) and its 'situatedness' (how information relates to a particular setting in which the informational signal occurs). In the coming sections, we will consider these two main approaches in more depth and examine several accounts of semantic information that fit with these approaches.

2.3.1 Information as range

The earliest account of semantic information has been provided by Carnap and Bar-Hillel (1952). They were inspired by the work of Shannon (1948) and aimed to give a quantitative account of semantic information, which is not possible with the MTC as Shannon proposed it. At the basis of their account lies the idea that the amount of semantic information contained in a sentence or proposition is inversely dependent on the likelihood of the sentence or proposition being true. This means that the more likely for a proposition to be true, the less information it carries. The other way around: the less likely that the proposition is true, the amount of information contained in it is high.

Carnap and Bar-Hillel (1952, p.8) disagree with Floridi (2016) on the matter of 'false information'. Following their probabilistic account, a contradictory sentence has the highest informational value. This is for the following reason: "A self-contradictory sentence asserts too much; it is too informative to be true" (Carnap and Bar-Hillel, 1952, p.8). A tautology, on the other hand, has no informational value. It does not contain any new information, since it will always be true and this is known. Floridi (2016, p.47) argues this leads to a problem, which he calls the Bar-Hillel-Carnap Paradox: if some agent Alice tells Bob something that is never true, a contradiction, then Bob would be maximally informed by this statement according to the account of Carnap and Bar-Hillel, which clearly is a counter-intuitive conclusion. This paradox would be avoided by adopting Floridi's account of semantic information, in which both contradictions and tautologies are completely uninformative, and their degree of information is 0. Even with possible objections such as the one from Floridi (2016), the account of semantic information that was proposed by Carnap and Bar-Hillel (1952) was very influential and proves to be relevant till the present day. This is because it highlights an important principle regarding information and information flow that forms the groundwork of different theories of classical information, both in the technical and everyday sense (Adriaans and van Benthem, 2008, p.17).

Namely, the account of semantic information from Carnap and Bar-Hillel (1952) has an important similarity with the MTC as proposed by Shannon (1948). This is the following overarching principle of information: there is an inverse relationship between the information contained by a proposition and the likelihood of the proposition being true. This is known as the *Inverse Relationship Principle* (IRP). This principle lies at the basis of a logical approach to information, which is referred to as 'information as range' (van Benthem, 2005, p.1; van Benthem and Martinez, 2008, p.217). Here, the main idea is that the larger an agent's range of options for what the real world might look like, the less information the agent has, and the other way around. Based on this idea Hintikka (1962) proposed epistemic and doxastic logic, the most used logic for modelling agents' knowledge and beliefs and their updates in a wide variety of disciplines (from philosophy to computer science). Epistemic and doxastic logic uses possible world semantics to reflect this common-sense relation between information and uncertainty. An important feature of this logic is that it models how the range of possible worlds for each agent will change after encountering information, via observations or communication. This reflects the dynamic process of learning information.

Epistemic logic

Let us consider a multi-agent epistemic logic with a possible world semantics in more depth, to see how the idea of information as range is modelled here. The information that epistemic logic usually describes is knowledge and is therefore *hard information*: it is unrevisable and factive. This is opposed to *soft information*, which is information that is revisable if new information is encountered, such as the agent's attitudes and beliefs (van Benthem and Martinez, 2008, p.236). Epistemic logic, due to its relational character, can also be used to describe different kind of information flow, such as information flow between distributed systems (Baltag and Smets, 2010, p.3007). In that case, it does not describe knowledge of agents, but rather information contained in a system. We will describe the syntax and semantics of epistemic logic in the style of Fagin et al. (1995) and van Ditmarsch et al. (2015) (in these sources much more details about epistemic and doxastic logics can be found). Take a finite set Ag, consisting of agents $a, b, c, \ldots$. To describe the knowledge of these agents we use the following formal language (extending propositional logic): let At be the set of all primitive propositions, which we label $p, q, r, \ldots$, Boolean operators $\neg$ ('not') and $\wedge$ ('and'), from which we define the other Boolean operators $\vee$ ('or') and $\rightarrow$ ('implies') and $\leftrightarrow$ ('if and only if') as usual. The language also includes the modal operator K. The propositions can stand for any primitive information about the world, so we can let p stand for "it is raining in Amsterdam". We then use the K operator to express the knowledge of agents about these statements. For instance: $K_b p$ would mean 'Agent b knows that it is raining in Amsterdam' and $K_a(K_b p \vee K_b \neg p)$ stands for 'Agent a knows that agent b knows whether it is raining in Amsterdam or not'. We use Greek letters $\varphi, \psi, \chi, \ldots$ to denote well-formed formulas.

This describes the syntax of the logic, but in order to derive meaning we need a formal model to evaluate the truth value of some formula on this model. This is given by Kripke structures. In propositional logic, a valuation V provides the truth value for each proposition. Such a valuation thus reflects a certain situation, or the state of the world, by assigning 'true' or 'false' to statements such as "it is raining in Amsterdam". To derive the truth value of any formula, the valuation is extended using an inductive definition for each Boolean operator. Following Hintikka (1962), to model the knowledge and beliefs of agents, the agents consider a set of situations to be possible, reflected by having a set of worlds and a different valuation for the primitive propositions in each world. It is said that an agent a then knows some formula φ if φ is evaluated true in each world that a considers possible.

A Kripke frame $F = (S, R^{\mathsf{Ag}})$ is defined as follows:

- Ag is a set of agents
- S is a non-empty set of states $s_1, \ldots s_n$
- R^{Ag} is a function, creating for each agent $a \in \mathsf{Ag}$ an accessibility relation $R_a \subseteq S \times S$ between states

Then by adding a valuation for each primitive proposition in each state to the graph that is given by the Kripke frame F, we get a Kripke model $M = (S, R^{\mathsf{Ag}}, V^{\mathsf{At}})$, with:

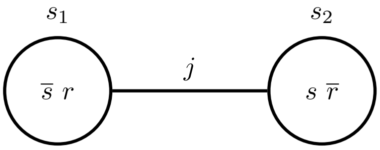
- $V^{\mathsf{At}} : S \rightarrow (\mathsf{At} \rightarrow \{ \textit{ true, false } \})$ is a function that, for each state $s \in S$ and each primitive propositions $p \in \mathsf{At}$ determines the truth value $V^{\mathsf{At}}(s)(p)$ of p is in that state s

To derive the truth value of any formula φ, the valuation is extended using an inductive definition for each Boolean operator, exactly as in propositional logic, but we also need to add a truth function for the K operator. If a formula *phi* is true in model M, state s, we write $M, s \models \varphi$, and we define this inductively as follows:

- $M, s \models p$ iff $V(s)(p) = \ true$ for $p \in \mathsf{At}$
- $M, s \models \psi \wedge \chi$ iff $M, s \models \psi$ and $M, s \models \chi$
- $M, s \models \neg\psi$ iff $M, s \not\models \psi$
- $M, s \models K_a\psi$ iff $M, t \models \psi$ for all $t \in S$ such that sR_at (different notation for $(s, t) \in R_a$)

Now to see how the concept of information as range holds in epistemic logic, let us consider an example. This example models the flow of hard information and shows that eliminating possibilities (possible states) corresponds to a gain of information. If an agent has a very large range of worlds she considers possible, this means that she has less knowledge, and thus hard information, about the actual state of the world. The information is encoded in 'state spaces' that shrink as agents learn more. This information is of type Information-A as defined by Adriaans and van Benthem (2008, p.11), since it is relative to agents and it revolves around acquiring information through dynamic events such as observation, communication or inference.

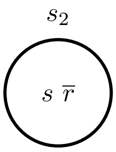
Take for example a weatherman called John, who knows that in Amsterdam it is either rainy or sunny and it is never the case that it is rainy and sunny at the same time. We can model this situation using epistemic logic as follows: let j denote the agent John, the r denotes the proposition 'it is raining in Amsterdam' and s denotes 'it is sunny in Amsterdam'. Then we have that the formula $K_j((s \vee r) \wedge \neg(s \wedge r))$ must be true, which says that John knows it is rainy or sunny in Amsterdam, but not both. This formula is true in both states in the following model:



The two states of the model s_1 and s_2, the information states, represent the two possible states of the world: one in which it is raining and one in which it is sunny in Amsterdam. John cannot distinguish between these two information states, since there is a lack of information to do so. Both of these states could be representing the real world, as far as he knows. This is why we have that the states s_1 and s_2 are related by R_j. In epistemic logic, the accessibility relation R is an equivalence relation, meaning that it is reflexive, transitive and symmetric. This is to reflect the properties that are ascribed to knowledge, such as veridicality of knowledge, meaning that if an agent knows some fact, this fact must be true. This is why we require the relation R to be reflexive. Furthermore, we have positive introspection: when an agent knows something, she is aware of her knowledge, so she knows that she knows it. This is why the relation needs to be transitive. Lastly, we have negative introspection. This means that the agent is aware of what they do now know: if she does not know something, she knows that she does not know. This requires the relation to be

Euclidian and because it is also reflexive, it follows that it is also symmetric.

Now imagine John is currently in a room without any windows, but because he does not hear the rain ticking on the roof, he can infer that it is not raining at this moment. This is new hard information that he receives and we can update our model accordingly:



John now knows that it is not raining, thus s_2 reflects the status of the actual world and thus state s_1 can be eliminated from his knowledge base. We now have that it holds that $K_j \neg r$, meaning John knows that it is not raining. It also holds that $K_j s$, meaning John knows that it is sunny, since we have for all states t such that $s_1 R t$ (which is in this case only s_1 itself), it holds that $t \models K_j s$. Thus, as we can see from this example, the reduction of the range of possibilities corresponds to a gain of information for agent John, who now knows that it is sunny, without having looked out of the window.

As seen by the example above, epistemic logic thus precisely formalises the main idea of the information-as-range paradigm. Namely, there is a direct correlation between the amount of information an agent possesses and the range of possible configurations of what the world can look like the agent considers. This correlation is described by the inverse relationship principle (IRP). Epistemic logic is an approach to semantic information, or information as it is used in the everyday sense, since it approaches information in a qualitative manner. However, the IRP is also found in technical approaches to information, such as the MTC by Shannon (1948), as this theory revolves around modelling the reduction of ranges of uncertainty in a quantitative approach. Therefore, the information-as-range approach can be seen as an overarching theme in theories of information (Adriaans and van Benthem, 2008, p.17).

2.3.2 Information as correlation

The second major approach to semantic information as presented by van Benthem and Martinez (2008, p.217) is 'information as correlation'. Central to this approach is the focus on how information in a structured information environment behaves. The aim is to be able to analyse in an abstract way how certain components of distributed systems or situations can have dependencies. Often, one part of such a structured environment, through systematic connections, can carry information about some other part. This semantic feature is referred to as the 'aboutness' of information, which was earlier discussed[3] in the distinction between different approaches to information by Adriaans and van Benthem (2008) and Timpson (2013). The aboutness of information can be captured by *constraints* that correlate different situations. These constraints can be seen as information channels between different situations or information environments (van Benthem and Martinez, 2008, p.221). Furthermore, the *situatedness* of information is important. This means that the informational content of an informational signal can depend on the particular setting in which it occurs.

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[3] See Section 2.1.

The same signal can carry different information in different situations. In general, the information in the information-as-range paradigm is not agent dependent, as the correlation between situations is there even when there is no agent to observe it (Adriaans and van Benthem, 2008, p.18). This is in contrast with the information-as-range approach, where the information is usually taken to be relative to agents and their knowledge.

The MTC by Shannon (1948) can be seen as the starting point of the information-as-correlation approach. In his theory, Shannon considered communication systems consisting of a source and a receiver, that are connected by a noisy channel. His theorems are about the correlation between these two information sites. He provided measures for the source's information rate and the information-carrying capacity of the channel. His account is a purely quantitative account and is not concerned with the content of the information. However, for the goal of our thesis, we are interested in semantic or qualitative accounts of information as correlation. These accounts are based on the ideas of Shannon's MTC, but consider the qualitative aspects of information flow.

First, we will consider the account information proposed by Dretske (1981), starting the information-as-correlation approach applied to semantic information. Then we will consider situation semantics, a formal framework to capture this idea of information as correlation, as proposed by Barwise and Perry (1983) and further worked out by Devlin (1991).

Dretske's view of information

In *Knowledge and the Flow of Information*, Dretske (1981) develops an account of information that is based on Shannon's information theory (MTC). Dretske aims to provide a philosophical account of a theory of information, rather than a technical one. As Dretske (1981, p.40) points out, Shannon's MTC is a theory of signal transmission, signals which carry information, but it is not a theory of information itself. Shannon's information theory mainly focuses on quantifying information, the amount of information in signals, without looking at the specific information they get across. However, information is not only about the signals but also about the content they carry, which is semantic. Information refers not only to the vehicle of communication but also to what is communicated. Nevertheless, Shannon's theory of information is still useful, according to Dretske, since MTC puts some constraints on what a signal can carry and thus on what information is.

We should also be careful not to equate the term 'information' with 'meaning' in a semantic account of information. Clearly, the two terms are closely connected. It might seem that the semantic aspects of information that are not captured in a technical account boil down to its meaning. However, even though information, in the everyday sense of the word, is semantic, it is not the case that all information carried by a sign is identical to the meaning of the sign. Dretske (1981, p.42) argues that the meaning one assigns to a sign or piece of information is agent-dependent: what one can learn from a signal, depends on the background knowledge one already has. Opposed to this, Dretske views information as something objective and existing independent of agents: the information a signal carries is what it *in principle* is capable of learning someone. This is thus independent of the background knowledge of specific agents or someone to observe it. In Dretske's account, information is capable of generating knowledge. This information is only incidentally related to the meaning of the signal (Dretske, 1981, p.44). Due to taking information as an objective commodity Dretske (1981, p.47) argues, similarly to Floridi (2016), that there cannot be false information. If Alice utters a false statement f to Bob, without Bob knowing that it is false, then the words in sentence f contain meaning for Bob. However, this meaning is not the information that f carries, since f cannot inform B about the state of the world, it cannot yield knowledge. This is because of the requirement of knowledge to be true[4]. This is only one example to show that the meaning and information carried by a signal can be different and thus the two terms should not be used synonymously. The concept of 'meaning' is a core concept in the fields of philosophy of language, linguistics and metaphysics and we cannot do this discussion justice in the brief section that we devoted to it here. This is just to point out that also in a semantic account of information, such as the one from Dretske, information can be taken as an objective commodity.

The objective character of information in Dretske's account is highlighted by the way he starts his book:

> "In the beginning there was information, the word came later". (Dretske, 1981, p.vii)

This goes to show, that in his theory of information, he stipulates that it is some commodity that exists independently of cognitive agents. It is not something that is dependent on interpretation by an intelligent agent. It does not become information only when it is assigned a meaning or significance by such an agent. Instead, Dretske aimed to provide a naturalistic account of cognition by explaining cognitive attitudes in terms of information flow. He views information as something external to cognitive agents, existing in natural signs, that could be described in terms of laws. This way, he explains how the mind, if we see it as a purely physical system, can occupy states that have a meaning. Later, this view of information having its own semantics, independent of agents or observers, is referred to as *environmental information* (Floridi, 2011, p.91-92) or natural information (Piccinini and Scarantino, 2016, p.27). An example of this type of information is the rings in the trunk of a tree. This provides information about the age of the tree and does so independently of an agent to interpret it. Or the information that smoke carries about the presence of fire. Due to this information existing in nature and the law-based theory that Dretske developed for it, and the strong distinction between the concepts of meaning and information, it might seem that this is not an account of semantic information. However, the main purpose of Dretske's theory is to capture the essence of information and its relation to knowledge. The transmission of information can be described by law-based theories and approached quantitatively, but the semantic aspects of information are due to the intentionality that is inherent to information transmission (Lombardi, 2005, p.32). Therefore, Dretske's theory uses both concepts from Shannon (1948), but also focuses on truth, knowledge and the aboutness of information and thus provides a semantic account of information.

The distinction between 'information', as it is defined by Dretske, and 'meaning', can indicate that Shannon's MTC might still be able to tell us something about information, even when it does not consider meaning at all. Dretske (1981) summarises:

> "Information is a commodity that, given the right recipient, is capable of yielding knowledge. What we can learn, in terms of both content and amount, is limited by

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[4] See Section 1.

> the information available. It is, moreover, a commodity that can be transmitted, received, exchanged, stored, lost, recovered, bought, and sold. In terms of what it can do, and what can be done to it, this is the sort of thing we are referring to when we talk about information. What we ask of information theory is that it tell us, exactly, what this thing is." (Dretske, 1981, p.47)

Dretske thus aims to use the MTC as a base for his theory of semantic information. Adopting MTC as a theory for semantic information leads to an immediate problem: Shannon's MTC is about the average amount of information (entropy) generated at a source or transmitted by a source. But, according to Dretske (1981, p.47), one cannot average the content of a particular message, only the amount learned from it. This leads to a problem when we want to take into account the meaning, reference or truth of specific messages. Since, for a theory of semantic information, we need to consider the information content of particular messages and signals (Dretske, 1981, p.48). Thus, if we want to use information theory, we need to find a way to not consider only averages, but to talk about information contained in particular messages or signals. Therefore, Dretske continues by using the formulas provided by MTC in a different way than intended by Shannon, to get the amount of information contained in a single message or signal, instead of only averaging over them. This is done by considering the surprisal value $I(s_a) = log1/p(s_a)$ of a particular event or state of affairs s_a. The information carried by a signal r_a about s_a is then given by $I_s(r_a) = I(s_a) * E(r_a)$, where $E(r_a)$ is the *equivocation* of the signal, which is the amount of uncertainty about whether the signal r_a correctly indicates whether event s_a occurred. These formulas provide an absolute measure of the amount of information, but only if all probabilities are known. This is often not the case, since one would need to know the probability of the event and also all alternative possibilities and their probabilities, which are very complex to establish in real-life examples. But these formulas can also be used to make a comparison, between "the amount of information generated by the occurrence of an event, and the amount of information a signal carries about that event" Dretske (1981, p.54). In this way, they can provide insight into how information behaves.

Main principles of Dretske's semantic information

Dretske (1981, p.57) then outlines several conditions on the definition of information in a semantic theory of information. First of all, he introduces the *Xerox principle*, which according to him, any theory of semantic information should preserve:

1. Xerox principle: If A carries the information that B, and B carries the information that C, then A carries the information that C.

The Xerox principle is essential for the flow of information. We can then combine this principle, with the idea that we can compare amounts of information, without needing to know the absolute numerical measure. This leads to the following condition: if some signal carries the information that s if F, then it must be the case that the amount of information about s that is carried by the signal is equal to the amount of information generated by s being F. So, for instance: if s's being F generates 5 bits of information, then if the signal carries 3 bits of information, it cannot carry the information that s if F. If it would, then either the Xerox principle is violated, or one needs to accept the idea that informational content can be transmitted, no matter how little (as long as it is greater than 0) information is conveyed. As Dretske (1981, p.63) objects to both these ideas, he starts his semantic theory of information by proposing the following conditions. Suppose a signal carries the information that s is F. Then:

> 2. The signal carries as much information about s as would be generated by s's being F.

Thus, Shannon's MTC tells us that the amount of information carried by a signal about a source creates an upper bound on what information it can carry. If the equivocation is zero, this means that condition 2 is satisfied. If the equivocation is larger than 0, the signal carries less information about s than the amount that would be generated by s's being F. From MTC it does not follow that the signal cannot carry any information about a source at all if the equivocation is not 0, since the equivocation is dependent on how we decide on possibilities at the source. If we have more possibilities (so make events more specific) then the equivocation of potential signals increases, but this also increases the surprisal value $I(s)$, since each of the events is less likely to happen. This means that overall, the amount of transmitted information is unchanged. Furthermore, Dretske (1981, p.64) proposes condition 3 to enforce that information is truthful:

> 3. s is F.

And lastly, another condition is introduced, since conditions 2 and 3 together are not sufficient to make sure that a signal with the right amount of information, actually carries the information that s is F. It could carry the 'wrong' bits of information, namely bits that carry the information that s is G, while still having the same amount of bits as would be generated by s's being F. Therefore, we add the following constraint (Dretske, 1981, p.64):

> 4. The quantity of information the signal carries about s is (or includes) that quantity generated by s's being F (and not, say, by s's being G).

The problem with this constraint is that it is formulated rather vaguely, as we do not have a clear definition of what we mean by one quantity being or including another quantity when we consider more than a numerical comparison. However, this constraint is necessary to enforce that the signal carries the information that s is F. To satisfy these three constraints in one definition for *informational content*, Dretske (1981) proposes the following:

> "A signal r carries the information that s is F = The conditional probability of s's being F, given r (and k), is 1 (but, given k alone, less than 1)." (Dretske, 1981, p.65)

This definition tells us that it is required to have a lawful regularity between the information (that s is F) and the signal (r), in order for the signal to carry the information. The variable k stands for how the background knowledge of the receiving agent about the possibilities that exist at the source (s), determines the information that the agent can get from the signal. As an example: if Alice knows that s is either F or G, then a signal that carries the information that s is not F, carries for Alice the information that s is G, whereas another agent with different background knowledge might not be able to get this information from the same signal s. This does not undermine the idea that information is an objective commodity, since it does not mean that the information that is carried by a signal is relative. It only relativises what information each agent can subtract from a signal. We need to condition both on the signal and on this relative knowledge of possibilities at the source in order to establish whether there is informational content. So for instance: the fact that my phone is ringing (which is the signal r) carries the information that I (s) am being called by someone (F) if and only if the conditional probability of me (s) being called by someone (F), given that my phone rings (r) and that I understand what this sound means (k), is 1.

D'Alfonso (2014, p.308) summarises the three main reasons for Dretske to require the conditional probability to be 1, and not some other value. First, Dretske aims for the *conjunction principle* to hold, and in that case, the probability cannot be lower than 1. It is defined as follows:

> 5. Conjunction principle: if a signal r carries the information that A and the information that B, then it also carries the information that $A\&B$.

If A and B are independent and both have a probability of 0.9 instead of 1, then the probability of $A\&B$ is only 0.81, so the signal would not carry the information that $A\&B$. This is not how we want our definition of informational content to behave, so we need the conjunction principle to hold and therefore need a conditional probability of 1. Secondly, as is discussed above, the Xerox principle must hold. This principle tells us that carrying information is transitive, needs to hold. Again, if the conditional probability threshold is set to some value lower than 1 then this principle does not hold. Thirdly, if a signal would carry information even if the conditional probability is less than 1, then you would not have the link between knowledge and information. In order for something to be knowledge, it needs to be factive and thus have a probability of 1. Therefore, information can only be knowledge if it is definite to occur.

Usually, signals do not carry one piece of information but a variety, due to the nesting of information. For instance, if I hear my telephone ring, I gain from this acoustic signal the information that someone is calling me, but it also tells me that a radio wave signal has travelled from a cell tower to the chip in my phone, it tells me that my phone is charged and not on mute, etc. Therefore, we do not speak of *the* informational content of a signal, since there is different information content that can be subtracted from it. Pieces of information can be related to each other, but still be separate pieces of information. Dretske (1981, p.71) defines nesting as follows:

> The information that t is G is nested in s's being F = s's being F carries the information that t is G.

This definition of nested information, together with the definition of informational content, shows that if some signal r carries the information that s is F, it also carries all the information that is nested in s's being F. It thus seems that the information content carried by a signal quickly overflows, since there is so much nested information. However, the requirement of the existence of a lawful regularity, also causes the signal to fail to carry a vast amount of information. Dretske highlights that correlation should not be confused with informational relations. A correlation can be based on pure coincidence, rather than a lawful regularity based on natural laws of logical principles. So, even if the properties F and G are correlated perfectly, this does not mean that s being F carries any information about s being G or the other way around. This can be better understood if we consider two communication systems. One system is between agent A and B and the other is between agent C and D. Imagine that by sheer coincidence agent A always sends out exactly the same message at the same time to agent B as agent C does to agent D. This means, assuming that the channels are completely reliable and no other messages are sent, that the messages that agent A sends out and the messages that agent D receives are perfectly correlated. However, it is clear that agent D does not receive any information from agent A, since they operate in completely separated communication systems. Hence, a perfect correlation is not enough to establish a flow of information.

Analog and digital information

The above concludes our summary of Dretske's account of semantic information. Before we continue with situation semantics, we will look at one distinction between types of information that Dretske (1981, p.137) puts forward, which is also used by Devlin (1991, p.18). Dretske differentiates between information being carried by a signal in *digital* and *analog* form. Usually, these terms are used to denote the coding of information of a discrete or continuous variable property. However, Dretske assigns a different meaning, namely:

> "I will say that a signal (structure, event, state) carries the information that s is F in *digital* form if and only if the signal carries no additional information about s, no information that is not already nested in s's being F."(Dretske, 1981, p.137)

This means that when a signal only carries the specific information that s is F and nothing else, it carries this information in digital form. All the information that is nested in s is F will clearly also be carried by the signal and hence it can carry more general information. For instance, if there is a signal carrying the information that s is a square, and nothing else, then it carries this information in digital form, while also carrying the information that s is a rectangle, simply because this is nested in s being a square. Then, on the other hand:

> "If the signal does carry additional information about s, information that is not nested in s's being F, then I shall say that the signal carries this information in *analog* form."(Dretske, 1981, p.137)

This means that when a signal carries other information than s is F, which could be more specific or determinate, that is not nested in s's being F, it carries the information that s is F in analog form. For instance, if we have a signal that carries the information that s is a red square, it carries the information that s is a square in analog form. However, every signal carries information in both analog and digital form, it depends on which information is carried by the signal we consider. When we pick the most specific information, so in our example that s is both red and a square, this is the information about s carried in digital form. This concludes our brief analysis of the theory of semantic information proposed by Dretske. In the next section, we will see that this digital/analog distinction is related to the notion of an infon in situation theory, which is a different theory of semantic information based on correlations.

Situation Theory

The inception of the logical framework of situation theory was the book *Situations and Attitudes* by Barwise and Perry (1983). They aimed to provide a logical approach to information as correlation, able to express what information about one site is available at another site, which is separated in some sense, such as in terms of space, time or perspective. The situation theory and situation semantics proposed by Barwise and Perry, are heavily inspired by the work from Dretske (1981). Barwise and Perry (1983, p.xiii) write: "Our emphasis on information rests heavily on the work of Dretske", by which they refer to the way Dretske put the notion of information central in his theory. By taking information as an objective commodity that exists in the world and in nature independent of observers, he created a framework to talk about this type of environmental information.

Barwise and Perry created situation semantics focusing on information flow in natural language. Within their work, they first laid the groundwork for situation theory, as the underlying mathematical framework of situation semantics. Since situation semantics is focused on natural language, it is not relevant to the goal of this thesis. Hence, we will only consider situation theory as developed by Barwise and Perry (1983). We also consider the extension of situation theory as proposed by Devlin (1991), who introduces the notion of an *infon* as a fundamental discrete item of information. Devlin emphasises that the goal of situation theory is not to answer the question 'What is information?', but rather provide a science of information, a way to talk about it qualitatively. Since it is something intrinsic to our universe, we should approach it similarly to how we approach topics in physics. Thus, we should start with an empirical study of information and based on this formulate a mathematical framework, instead of starting with formal mathematics and trying to express information flow in it. To this end, he takes information as the underlying concept of the entire theory by starting from an ontological definition of information, which is different to the approach of Dretske (1981), who focuses mainly on information flow in his theory (Devlin, 1991, p.18). Similar to the view of Dretske (1981), information is taken as something that can be described by certain laws or rules, which Devlin calls *constraints*. These constraints link various types of situations. An example of a constraint is:

Smoke means fire

Which is a statement to express a lawlike relation between situations in which there is smoke and situations in which there is fire. These lawlike relations are everywhere in reality, and it is exactly these relations that allow a situation to carry information about some other situation. With this in mind, let us now take a closer look at the formalities of situation theory.

The most important aspect of situation theory, what sets it apart from other logical frameworks, is the introduction of *situations* in the ontology of the theory. Barwise and Perry (1983, p.7) describe these situations, of which reality consists, as "individuals having properties and standing in relations at various spatiotemporal locations". It is important to note that we are always in some situation, since they correspond to any part or activity in the world. Barwise and Perry continue by developing a theory of meaning in terms of relations between situations. The goal of the theory is to provide a classificatory scheme: "a system of abstract objects that allow us to describe the meaning both of expressions and of mental states in terms of the information they carry about the external world." (Barwise and Perry, 1983, p.7). To describe or abstract information in the environment, we need to be able to distinguish objects. This is what Devlin calls *individuation.* It allows us to see a table as one object, instead of a collection of molecules without a beginning and end. It is the way an agent carves up the world, and it does not need to be aware of it. This structure is necessary to give a classificatory scheme that Barwise and Perry refer to. Devlin calls this a *scheme of individuation.* Since there is not one way to carve up the world, different schemes can be used to examine the same reality. This causes an important difference with Dretske's account of information, in which information is taken as an objective commodity. Due to schemes of individuation being agent-dependent, and these are fundamental for deriving the building blocks of the arguments in situation theory, information is not described in an objective, agent-independent manner. The theory consists of the following building blocks, which together are needed for such a scheme of individuation:

- Individuals: these are items that are individuated as 'objects'. They are denoted by $a, b, c, \ldots$
- Relations: there are various *properties* (which are simply 1-ary relations) that can hold or fail to hold of individuals, or relations that can hold between them. We denote relations with $P, Q, R, \ldots$. Each relation comes with a certain fixed and finite number of argument places.
- Locations: this includes both locations in space and time. For spatial locations we use $l, l_0, l_1, \ldots$ and for temporal ones we use $t, t_0, t_1, \ldots$.

These building blocks can be invariant across situations, as a factor that remains the same. Therefore, they are referred to as *uniformities.* Individuals are uniformities since the same individuals are involved in different events, which are different situations. Spacial locations are uniformities since different events can take place over time at the same location. Similarly, times are uniformities, since different events at different locations happen at the same time. Relations between or properties of objects or individuals are uniformities since they can hold over a prolonged time in different events.

Using set-theoretic notions, Barwise and Perry conceive abstract situations, which can overlap with real-life situations, but do not have to. If the abstract situation corresponds with a real-life situation, it is *actual.* If it does not, but it classifies real-life situations, then it is *factual.* Situations that get something wrong about real-life situations are *non-factual.* The uniformities described above are classified in *situation-types.* Abstract situations consist of a location and situation-types. A situation-type is a partial function from a sequence containing an n-ary relation and n individuals to a truth value of 0 or 1. This way, we can model the situation-type s in which John reads a book and Mary does not read a book as:

$$s : \{\langle\langle r, J, b\rangle, 1\rangle, \langle\langle r, M, b\rangle, 0\rangle\}$$

Where we have r standing for the binary relation x reads y, J stands for the object John, M for Mary, and b stands for the object book. This sequence is in the situation-type, hence it gets assigned truth value 1.

Next to the situation-types there is a function from locations to situation-types, which is called a *course of events.* This function tells us what is happening, in terms of situations, at which location. It is written as a set e consisting of triples $\langle l, y, i\rangle$, where l is a location,

y is a situation-type written as a sequence and i is the truth value 1 or 0. If a course of events is defined at only one location, it is called a *state of affairs.* This is denoted by a pair $s = \langle l, s_0 \rangle$, where l is a location and s_0 is a situation-type.

It is important to note that situations or situation-types are partial, they do not describe the whole state of the world. This is also an aspect in which they differ from the states in possible world semantics described in Section 2.3.1. In that case, the valuation function assigns for each states a truth value to every possible proposition. Barwise and Perry rejected possible world semantics, in order to have properties and relations as primitive, instead of a mathematical construction in the theory. This allowed for viewing situations as pieces of reality, since they are built up by objects and properties (Zalta, 1993, p.5). One cannot give an extensional definition of a situation, since it is impossible to include everything that is in a situation. Devlin (1991, p.31) asserts that situations cannot be defined in terms of other objects, but that they are taken as a fundamental part of the ontology of the theory. This means that in his description of situation theory, also situations themselves can fill the argument roles of relations. Similarly to objects or individuals, what constitutes a situation depends on the scheme of individuation of agents, and thus is subjective and relative to an agent (Devlin, 1991, p.30).

Infons

The key distinction between the work of Barwise and Perry (1983) and Devlin (1991) is that the latter introduces the notion of an *infon.* These infons are in essence the same as the course of events or state of affairs in the situation theory of Barwise and Perry (1983), but allow for a broader range of types of arguments. The main difference is that infons can also take parameters as arguments, instead of only concrete objects or individuals. In this way, they are abstractions over concrete state of affairs. An infon is taken to be the basic informational unit of situation theory. They are defined as follows: For an n-ary relation P and objects, situations or parameters $a_1, \ldots, a_n$, it is denoted as:

$$\langle\langle P, a_1, \ldots, a_n, i \rangle\rangle$$

The truth value i, called the *polarity* of the infon, is 1 if the objects $a_1, \ldots, a_n$ stand in relation P and 0 if not. Devlin defines a set of infons to be an abstract situation. This goes to show that infons are taken to be fundamental objects of the theory, and Devlin compares their status to numbers in mathematics. Infons have an absolute nature, which does not depend on the representation of the information, but is relative to the ontology of an agent (Devlin, 1991, p.21). However, they do not need to have physical existence. Infons are a way of formalising the objective commodity that information is according to Dretske (1981, p.47). Furthermore, infons are not true or false in themselves, they are true or false with regard to a situation. For a situation s and infon σ, we write

$$s \models \sigma$$

if the infon σ is made factual by s. This is referred to as a *proposition.* Thus, infons are not mere syntactic representations, but are semantic objects (Devlin, 1991, p.23). They can correspond to elements of reality and they can be true or false with respect to situations in reality. But they can also exist without corresponding to a concrete situation, and still be informational.

As Dretske (1981) also pointed out: different agents extract different information from the same signal or source. Therefore, Devlin (1991, p.15) argues that which information an agent gathers from a situation, depends on the constraints to which the agent is attuned. There are two stages of acquiring information by a cognitive agent. First, there is perception: the agent gets access to the information in the environment by means of some sensor. Secondly, there is cognition: the acquisition of specific pieces of information from the available mass of information. This last step requires the conversion from analog to digital information, in the way that Dretske (1981, p.137) defined these notions, as we have discussed in Section 2.3.2. Thus, if we put it in Dretske's terminology, one can view an infon as the digitalization of information. This is because both of these notions correspond to information in its most fundamental form. However, in Devlin's account, the infons are given in terms of agent-dependent arguments, whereas Dretske aims to describe correlations that exist independent of agents. Furthermore, due to their semantic nature, Floridi (2005, p.353) connects the notion of infons to his conception of data, since both are taken to be the fundamental objects of which information consists. Thus, we see that we can draw links between these different accounts of semantic information, albeit having different approaches.

3 Quantum information

In Chapter 2 we considered different approaches and theories of classical information. In the case of classical information, the physical carrier of the information is some classical system, that obeys the laws of classical physics. The information can be encoded using classical properties, such as having some electrical current or not. The amount of information that is associated with such a binary alternative is referred to as a *bit*. A bit can take the values 1 or 0, which represent the two well-defined states of the classical physical system. In information theory, bits also signify an amount of information, since they are used as the unit for the measure of Shannon entropy. We can also think of Shannon information as an approach to quantify the physical resources that are necessary to store classical information, since we know that classical information is embodied in the state of some physical system. Furthermore, in classical computing, one can transform information using logic operations. These operations are realised by physical devices to manipulate the value of one or multiple bits and are referred to as logic gates. In this way, we process classical information using classical computing devices.

Quantum information is different from classical information, in the way the information is carried. Quantum information is generally understood as the information that is encoded to properties of physical quantum systems, which obey the laws of quantum physics. The information is now encoded in the states of quantum systems (Timpson, 2013, p.60). A *quantum state* is a complete description of the physical system, by providing a probability distribution for every possible measured quantity on the system. The unit of quantum information is referred to as a quantum bit or a *qubit*. There are tools to manipulate the information, by manipulating the state of these qubits. One can use quantum gates, which are quantum circuits operating on qubits. Quantum information behaves very differently compared to classical information, due to the nature of the quantum systems carrying it. In this chapter, we will first consider the main principles of quantum information and computation, to see how this difference comes about. We will provide a brief overview of the basic concepts and principles of quantum mechanics[5], in a non-formal fashion, to get a better understanding of quantum systems. We will follow standard notation in the field of quantum information theory and base our explanation on the seminal work of Nielsen and Chuang (2010).

Before we start, there is an important remark to make regarding quantum interpretations. The theory of quantum mechanics is one based on mathematical formalisms and has thus far been proven to be correct based on a myriad of experiments over the past century. Nevertheless, there are different views of how this mathematical theory exactly corresponds to reality. There are many fundamental questions about which the formalism itself is silent. These are for instance: what elements of quantum mechanics exist in reality, whether the theory is local or non-local, whether there are hidden variables or not, whether the wave function collapses upon measurement or not and what the role of an observer is. Different interpretations provide different solutions to these questions and there is no consensus about which interpretation should be adopted. In this thesis, we take the Copenhagen interpretation as our guideline[6], which amongst physicists is the most widely accepted interpretation

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[5] For a general introduction to quantum mechanics or a more formal approach we refer the reader to the introductory book on quantum mechanics by D. J. Griffiths and Schroeter (2018).

[6] In his book, Freire (2022) provides a collection of papers regarding many different quantum interpreta-

(D. J. Griffiths and Schroeter, 2018, p.5). We will briefly mention other interpretations throughout the coming chapters, whenever they are relevant, to give some idea of how different quantum interpretations influence our conception of what quantum information is. Especially when we discuss a semantic approach to quantum information in Section 5.3.2 we will also compare the Copenhagen interpretation with subjective interpretations.

3.1 Qubits

The framework of quantum information theory is analogous to the one from classical information theory and can be seen as an extension of it (Timpson, 2013, p.45). As described above, the term 'qubit' refers to a two-level quantum system, meaning that it is a system with two distinguishable physical quantum states. This notion was coined by Schumacher (1995, p.2739). A qubit is the standard unit of quantum information given in terms of von Neumann entropy, analogous to how bits are the basic unit of classical information given in terms of Shannon entropy (Duwell, 2019, p.3). Where a classical bit can take the values 1 and 0, a qubit can take the values $|0\rangle$ and $|1\rangle$. These correspond to the elementary pure states of a two-state quantum system acting as a quantum information source. This notation is called bra-ket notation or Dirac notation and is the standard notation for states in quantum physics and quantum information theory. This is because these states can be described by vectors in a two-dimensional complex Hilbert space. The states $|0\rangle$ and $|1\rangle$ form an orthogonal basis in this vector space and are called the *computational basis states.*[7] Even though qubits are physical, since they represent states of a quantum system, they are approached as mathematical objects with the aim of constructing a general theory of quantum information that is independent of a specific system of implementation. Therefore, many different two-state quantum systems can be used to carry information. An example is the spin of an electron or of an atom nucleus, which is a type of intrinsic angular momentum, which can be in two possible states: 'spin up' and 'spin down'. It can also be the polarisation of a photon, which have a left or right circular polarisation. Or an atom with an excited or unexcited energy state. All of these examples can be described with the formalism of quantum information theory.

The definition of quantum information given by Schumacher (1995) is a quantitative one. Namely, it considers qubits as a measure of information produced by a quantum source (Duwell, 2019, p.3). The von Neumann entropy of a quantum source, which can be measured in qubits, can be seen as the amount of quantum information produced by such source (Timpson, 2013, p.60). In this thesis we are concerned with qualitative approaches to (quantum) information, and therefore we require a qualitative definition of quantum information. We are not necessarily interested in an amount of quantum information, but rather what a piece of quantum information *is.* According to Timpson (2013, p.61), for a technical notion of quantum information we can define this in the same fashion as we did for classical information. Recall the definition of information$_t$. If we apply this to the quantum case, we get that technical quantum information, *quantum information*$_t$, can be defined as what is being produced by a quantum source. Thus, this will be a sequence of

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tions. Chapter 3 in the book of Jaeger (2009) provides a brief general overview of several interpretations and Lewis (n.d.) provides a non-technical overview of different quantum interpretations on an introductory level. Faye (2019) gives a more elaborate introduction and overview of the Copenhagen interpretation specifically.

[7] We will not get into the technicalities of this formalism in this thesis, but for reference, one can look at the standard work by Nielsen and Chuang (2010).

quantum states. In the simplest case the source produces individual pure states, and in a richer notion of a quantum source it can also produce parts of an entangled system in mixed state, which we will discuss in more depth in Section 3.2. Again, like in the case of classical information[8] we consider not the specific sequence token as the information, but rather the sequence type. Either way, it can be seen as analogous to classical information$_t$, but not exactly the same due to the nature of the states in the sequences (which we will discuss below):

> "[T]he information$_t$ produced by the source —quantum information$_t$, now— will be specified by specifying what sequence (type) was produced. These sequences will clearly be of a different, and more interesting, sort than those produced by a classical source. (One might say that with classical and quantum information$_t$, one was concerned with different types of type!)" (Timpson, 2013, p.61)

This is a qualitative notion of quantum information (Duwell, 2019, p.3). However, it is a very general definition and does not tell us much about the nature of the information. Therefore, we will look at different properties and principles regarding quantum information sources in this thesis. Furthermore, it only concerns technical information. As we discussed in Section 2.1, this does not cover all concepts of information, as it is not concerned with information in an epistemic, semantic or everyday conception. This will be considered in Chapter 5.

3.1.1 Superposition

The most important difference between classical and quantum states, so between classical bits and qubits, is that qubits may not only exist in the states $|0\rangle$ and $|1\rangle$, but can also exist in an arbitrary *superposition* of these states. According to a collapse interpretation of quantum mechanics, such as the Copenhagen interpretation, the superposition of states exists until the qubit is observed. Namely, a measurement of an observable causes the superposition to collapse to one of the orthogonal basis states. We will have a closer look at this dynamic in Section 3.3. The superposition is written as

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{1}$$

where α and β denote complex numbers. We have that $\mid \alpha \mid^2$ and $\mid \beta \mid^2$ denote the probabilities of observing the corresponding state after measurement and therefore it must be the case that $\mid \alpha \mid^2 + \mid \beta \mid^2 = 1$. Thus, the qubit can be in a continuous number of states, for all different values of α and β. There is no parallel in classical information theory for this property.

Due to this property of superposition, it seems that one can encode an arbitrarily large amount of classical information in one qubit, since it can be in the $|0\rangle$ state and $|1\rangle$ state simultaneously and thus be in an infinite number of possible states. However, this should be approached with care. There is a difference between the information that can be stored and processed and the information that can be accessed. Timpson (2013, p.47) distinguishes between what he calls *specification information* and *accessible information.* Consider a message consisting of a sequence of n systems, for which each of these systems is prepared in a state selected from a finite set of possible states. In the classical case, this could simply be a string of n bits, that are either in state 1 or 0. We can then examine the amount of

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[8] See Section 2.1.

information that is required to prepare or specify the sequence of states, the specification information. In the classical case, this would be the same as the amount of information that can be acquired from the sequence, the accessible information. This is because the classical states, being either 1 or 0, are completely distinguishable. We can simply read off or measure the information from the message and given this, we can specify the message since we know the identity of its sequence of states.

In the case of quantum information, this is more complicated. As mentioned above, qubits can be prepared in continuously many states, and these states need not be orthogonal. This means that for each qubit in the sequence, they do not need to be prepared in a state with the same orthogonal basis vectors. Due to the peculiarities of measurement on quantum systems (to which we come back later), the state of each qubit collapses from a superposition to one of its basis states. Thus, we would not be able to perfectly determine the state of each qubit in the sequence. This means that the required specification information can be without limit. However, for the accessible information, this is not the case, since we need to measure the state of the qubits to access this information. Only in the case where each of the qubits is prepared in one out of two orthogonal fixed states, we do have that these two values are the same. We would be able to identify and specify the sequence of states. In that case, we would also be gaining one classical bit of accessible information per qubit as accessible information. But more often, the system consists of qubits in non-orthogonal states, and then it is not possible to perfectly distinguish between these states. Thus, there is no way to decode all the information that is contained in these states. The accessible information is then the maximum amount of information that can be recovered through quantum measurement (Schumacher, 1995, p.2739). Finally, it is important to note that these notions of specification and accessible information both are classical notions and they are only relevant in the context of transmitting classical information by encoding it in quantum systems. If we consider a closed quantum system, without performing any measurements, it seems that all continuous variables that describe the state of the qubits remain there. Thus, Nielsen and Chuang (2010, p.16) conclude that Nature carries this somehow as 'hidden information'.

3.2 Entanglement and quantum states

Until now we have discussed systems containing single qubits, but a system can also consist of multiple qubits. If we consider a system containing two qubits, it has four computational basis states, which are denoted by $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. As is the case for single qubits, a pair of qubits can also be in a superposition of these states, written as

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \tag{2}$$

where we have again have the following requirement: $\sum_{x\in\{0,1\}^2} |\alpha_x|^2 = 1$. Given a pair of qubits, one can also perform measurement only on one of the qubits, which alters the possible states that the pair of qubits can be in.

One of the most remarkable properties of quantum systems containing multiple qubits is that the measurement outcomes of individual qubits can be perfectly correlated, albeit them being spatially separated. A quantum state of a pair or group of qubits is said to be *entangled* when it is not possible to describe the quantum state of the individual qubits independently of the state of the other qubits. Entanglement of a quantum state can thus be seen as the multiple members of the quantum state existing in a single quantum state. This means that when the state of one of the qubits is changed, for instance by performing a measurement on it, this will cause an instantaneous change in the state of the entangled qubits. A measurement thus affects the entangled system as a whole. When the quantum state is not entangled, it is possible to factor the quantum state into the individual states. The state is then called *separable*. In that case, we can write them as a product state:

$$|\Psi\rangle_{AB} = |\varphi\rangle_A |\psi\rangle_B \tag{3}$$

where A, B denote the distinct subsystems.

3.2.1 Maximally entangled states

Important examples of two-qubit states are the *Bell states* or EPR pairs. These are four possible simple quantum states of two qubits that are maximally entangled. The Bell states are defined as follows:

$$\begin{aligned} \left|\Phi^+\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ \left|\Phi^-\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\ \left|\Psi^+\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ \left|\Psi^-\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{aligned} \tag{4}$$

This means that if we take for example the state $|\Phi^+\rangle$, the qubits are entangled in such a way that whenever the first qubit is measured to be in state $|0\rangle$, the second qubit must be as well. And similarly, whenever the first qubit is in state $|1\rangle$, the second qubit will also be in state $|1\rangle$ when measured. The probability to find the qubits to be in state $|0\rangle$ or state $|1\rangle$ is 0.5 for both cases. Without knowing that these qubits are correlated, if some agent Alice would measure the first qubit and some other agent Bob measures the second qubit, they would get a seemingly random outcome. It is only when Alice and Bob would communicate about their outcomes, that they can find that these measurement outcomes are perfectly correlated (if they in principle could repeat the experiment an infinite amount of times).

The correlation between the qubits in the Bell state above can also remain if Alice and Bob would decide to measure in a different basis. They can for instance measure in the $|+\rangle, |-\rangle$ basis. This is a basis that is orthogonal to the computational basis $|0\rangle, |1\rangle$. The states $|+\rangle, |-\rangle$ are superpositions of the computational basis states and can be seen as 'halfway' between the $|0\rangle$ and $|1\rangle$ states:

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \tag{5}$$

If Alice and Bob decide to measure in the $|+\rangle, |-\rangle$ basis, then they would still find that the qubits are perfectly correlated. This is due to the fact that the Bell states above can be rewritten in terms of these $|+\rangle$ and $|-\rangle$ states as follows:

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}(|{+}{+}\rangle + |{-}{-}\rangle) \tag{6}$$

However, a measurement of qubits in both the $|+\rangle, |-\rangle$ basis and the computational basis is incompatible, as they disturb each other. In general in quantum mechanics, two observables, which represent physical quantities, are *compatible* if you can measure one, followed by a measurement on the other observable and if you would then measure the first again, you are guaranteed of getting the same result in this final measurement as in the first. This means that these observables allow for complete sets of simultaneous eigenfunctions (D. J. Griffiths and Schroeter, 2018, p.107). It is an important difference with classical quantities that certain quantum observables may not be measurable simultaneously. We will come back to this observation, as it is important in the history of understanding quantum entanglement and non-locality. It may now seem as if it is impossible to break the entanglement, but this is not the case. It is in fact very easy: the moment we observe one entangled subsystem, so for instance perform a measurement on one of the qubits in the Bell state $|\Phi^+\rangle$, the entanglement is broken.

3.2.2 Pure and mixed states

In order to talk about subsystems of entangled states we need some different terminology. Up until now, we have only considered *pure states*, namely the basis vectors and superpositions thereof. Pure quantum states can be written as a sum of basis states, such as:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \tag{7}$$

which is a single vector in a two-dimensional complex Hilbert space. However, when we want to describe the quantum state of a part of a system containing multiple qubits, we cannot do so using pure states. When we write out the quantum state for a pair of qubits, as we did in (2), then if we only want to describe the quantum state of the first qubit, we need to do that using a *mixed state*. The qubit is not in a definite pure state, but rather in a mixture of possible pure states, all with a weighted probability. This is why a mixed state is described by a statistical mixture, which is denoted by a density matrix, as opposed to a vector which was the case for pure states. These probabilities originate from the amplitudes of the superposition in the pure state. A density matrix ρ is defined as:

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i| \tag{8}$$

where $\langle\psi_i|$ is the adjoint of the state $|\psi_i\rangle$ and p_i is the probability corresponding to the state $|\psi_i\rangle$. Since these are probabilities, it must hold that for each $p_i \geq 0$ and that

$$\sum_i p_i = 1$$

Pure states can thus be seen as limit cases of mixed states, in which for one i we have $p_i = 1$ and the rest of the probabilities are 0. In the case of mixed states in quantum mechanics, the different probabilities p_i are arising from quantum uncertainty, and are therefore quantum probabilities. But one could also have mixed states in classical physics with probabilities from a different origin of uncertainty.

The notion of quantum entanglement, similar to quantum superposition, does not have a parallel in the realm of classical information. We thus have that in the case of classical information, it holds that ‘the whole is the sum of its parts’, whereas in quantum information this does not need to be the case (Baltag and Smets, 2010, p.3007). The power of entanglement becomes clear when we consider common quantum information communication protocols, such as quantum teleportation or superdense coding.[9] Intuitively, one can imagine how sharing a pair of systems that are in an entangled state might enable agents to communicate in a way that is not possible classically, exactly because these systems have global properties that are irreducible to local properties. The other way around also holds: one cannot create an entangled system from two separable systems by performing local operations on them. The entanglement should already exist before the agents separate their subsystems and use them to communicate. Qubits can only become entangled when they interact with each other or are products of some physical process, such as atomic decay. Thus, starting with a classical communication channel, one is not able to use the properties of quantum entanglement.

3.3 Measurement, time evolution and decoherence

In the previous section, we mentioned quantum measurement, but we have not specified what we mean by that. The idea of quantum measurement is to test or manipulate a quantum system in order to get a numerical result. We cannot simply read off a numerical value, as we can often do in the case of classical systems, since quantum predictions are generally probabilistic. In the framework of quantum information theory, measurement is an operation on a qubit, to gain a classical bit of information. The measurement causes the state of the qubit to project onto the basis states in which one measures, which are generally the computational basis states $|0\rangle$ and $|1\rangle$, but can also be any other orthonormal pair of states. For any two orthonormal basis states $|a\rangle$ and $|b\rangle$, we can express an arbitrary state as the linear combination:

$$\alpha\,|a\rangle + \beta\,|b\rangle \tag{9}$$

We cannot simply examine the qubit to determine the values of α and β, we can only perform a measurement. If we then measure a qubit in this state with respect to the $|a\rangle\,,|b\rangle$ basis, we would get the classical result a with probability $|\alpha|^2$ and b with probability $|\beta|^2$. This is a generalisation of what we saw in (1). This projection onto basis states is random, and therefore it is not possible to predict with certainty to what state the qubit collapses in advance. This is the reason for fundamental indeterminacy in quantum mechanics, which we do not have in classical mechanics. If we leave measurement out of the picture, and we consider a closed, isolated quantum system, we can perfectly predict its state change. The evolution of the system can be described with unitary transformations. These are reversible and describe the state changes of the system over time. In quantum information and computation, these unitary transformations are denoted by *quantum gates*. These gates can thus be used to manipulate the state of a qubit, analogously to how logic gates manipulate bits in classical computation. The Hadamard gate takes for instance a $|0\rangle$ state to a $|+\rangle$ state and a $|1\rangle$ state to a $|-\rangle$ state. Many more examples of quantum gates can be found in the handbook *Quantum Computation and Quantum Information* (Nielsen and Chuang, 2010, p.xxx). Thus, unitary transformations and measurements are the two different ways a quantum system can evolve.

3.3.1 Different kinds of quantum measurements

For the evolution of a quantum system, we do not only distinguish between unitary evolution and measurement, but also differentiate between different kinds of quantum measurements.

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[9] We will consider the quantum teleportation protocol in some depth in Section 3.6.

As the type of measurement in an experimental setting highly depends on the choice of the physical system to implement the qubits, we will not get into the details of this. However, it is possible to give a distinction of general types of measurement, as is done by Pauli (1980, p.75). He categorises measurements of the first kind and measurements of the second kind. Measurements that are of the first kind affect the quantum state of a system less than measurements of the second kind. Namely, a measurement of the first kind only projects the state of the system onto an eigenstate of the measured observable, but leaves the rest of the system unchanged. In this way, a subsequent measurement of the same observable will lead to the same measurement outcome. Whereas a measurement of the second kind changes the state of the system in such a way that repeated measurements will have different outcomes. This effect of the measurement on the system is called *back action* and can be such that is in a controllable fashion and the value of the observed quantity before the measurement can be uniquely inferred. This way, a measurement of this type still leads to new information. It does not only matter what the measured observable is to determine the type of the measurement, but also how the measurement is performed exactly. A measurement of momentum can be of the first kind if there is a certain amount of time in between subsequent momentum measurements, as this allows for the free evolution of the position of the system, which is the complementary observable. But a momentum measurement is of the second kind if one measures again immediately (Pauli, 1980, p.12; Braginsky et al., 1980, p.548). Furthermore, there is a last, very large, group of measurements which is not regarded by Pauli (1980). These are measurements that cause an uncontrollable change to the state of the system or annihilate the system completely. This is often the case for measurements on photons, as it is very hard to measure anything without absorbing them in the process (Distante et al., 2021, p.253603-1).

3.3.2 Decoherence

As we discussed, performing a quantum measurement will in general bring about a state change of the quantum system. It is a peculiar process, about which much is still unclear in the present day. Possible quantum mechanics interpretations account for the mysteries around quantum measurement in different ways. This is the so-called *measurement problem*. It refers to the question of whether the wave function, which describes the state of a quantum system as a superposition of different eigenstates, collapse occurs upon measurement, and if so, how exactly the wave function collapses to a single basis state. We will not get into the details of different quantum mechanics interpretations. More on that can be found for instance in the overview provided by Freire (2022). What is important to note is that regardless of the quantum interpretation one adopts, quantum measurement is an irreversible process. This is due to the strong coupling between the measuring instrument and the system being measured. In the classical case, there is often a clear distinction between the measured and the measuring system. Namely, the system does not change upon measurement and repeating the measurement would give the same outcome. This is not the case for quantum mechanics, as the measurement instrument interacts and gets entangled with the measured system. The entanglement of the quantum system with the environment is called *decoherence*. Quantum decoherence can be seen as information loss from the system to its environment, through a non-unitary, therefore irreversible, dynamic. It seems to cause the transition from quantum mechanics to classical mechanics (D. J. Griffiths and Schroeter, 2018, p.462).

For a perfectly closed and isolated quantum system, there are no possible interactions with the environment. According to the laws of quantum mechanics, a system like this will maintain its quantum coherence, which refers to its wave-like properties. The state of each qubit or object is described by a wave function, which can be slit into separate waves. If there is a definite phase relation between different states of the system, so these waves operate in a coherent manner, the system is displaying quantum coherence. Such a perfectly isolated system would thus maintain its quantum properties indefinitely. However, systems like this do rarely exist in the real world, since any quantum system interacts with its environment. This means that the system will lose its coherence, as components of the wave function obtain phases from their surroundings. This loss of quantum coherence is what is meant by quantum decoherence. Especially on a macroscopic scale, objects interact with a large environment, causing them to lose their quantum behaviour to this environment, as the system becomes a mixture of semi-classical states. And on the microscopic scale, to investigate a quantum system is to interfere with it, also causing quantum decoherence. Therefore, it plays an important role in our understanding of quantum systems and the transition from quantum to classical. Quantum decoherence gives an account of why macroscopic objects do not display quantum behaviour such as being in a superposition of states. More information on the topic of decoherence from a philosophical perspective, can be found in the work of Zurek (2006).

3.4 Non-locality

In our everyday understanding of the world and in most physical theories, we assume that the world is made up of separate objects that exist individually. These objects can be macroscopic objects, such as a chair or desk, but can also be taken to be microscopic ones, such as the molecules of which these macroscopic objects exist, or even the atoms that make up these molecules. This is captured by the *principle of realism*, which asserts that objects and their properties are physically existing, independent of the mind of an observer. So, the physical universe in which we live is an objective reality and does not exist only in our mind or when we observe it. We can thus assume that the chair and the desk are existing objects in spacetime, unrelated to whether one looks at them or not.

A second assumption that we make is that all objects adhere to the concept of locality: an object can only be influenced by its direct surroundings. This concept applies to warm tea that cools off by transferring its heat to the surrounding air molecules or a ball that moves because it is pushed or picked up by someone who exerts force by touching. The state of these objects only changes, because they are interacting with their direct surroundings. In general, these objects and their behaviour or the relations between them are described by local theories of physics. These theories follow the *principle of locality.* This principle states that for a physical object to have an influence on another physical object, these objects must interact by having something that mediates the action in the spacetime between them. It cannot be the case that a warm cup of tea causes the air on the other side of the earth to heat up, this heat, in the form of movement of the air molecules, can only be transferred if these air molecules physically collide and thus mediate this energy.

Local theories in classical physics also account for interactions that at first glance might seem to be non-local. Take for example gravity. How can the sun attract the earth, while they are being such a large distance apart? At first, the theory of gravity as it was defined by Newton indeed described gravity as 'action at a distance'. According to his ideas, gravity was defined as the attraction (through empty space) of any particle of matter to another. However, due to the way gravitation is explained by the general theory of relativity by Einstein, we see it still obeys the principle of locality. Gravity is therein taken to be the result of the curvature of spacetime. Rather than having two objects attracting each other, they distort spacetime itself and this distortion results in what is perceived as an attractive force. Thus, in this theory, it is a gravitational field that mediates the action of one object to another. The disturbances propagate through the field via local interactions. Therefore, it is also not an instantaneous interaction, since the gravitational wave travels at the speed of light in a vacuum c. This is a physical constant and upper limit of how fast matter, energy or information can travel through space. Thus, gravity obeys this principle of locality.

3.4.1 EPR experiment

With this in mind, it is not strange that it was first assumed that also quantum mechanics adhered to the principles of *local realism*. However, This was until Einstein, Podolsky and Rosen (1935) put forward their famous paper, in which they provide a thought experiment, known as the EPR experiment. They argue that the description of physical reality that is provided by quantum mechanics in its current form, by the wave function, is incomplete. We provide a slightly simplified version of the argument, which is closer to the argument provided by Bohm (1951, [p.614–623), but it follows the same main ideas (for a full analysis and background of the argument, see (Redhead, 1987)). The argument is based on several assumptions. Let us first clarify what is meant by a complete theory. Einstein et al. (1935, p.777) define the condition of completeness as follows: "every element of the physical reality must have a counterpart in the physical theory." So, for the formalism of quantum mechanics to be complete, it must be the case that it can fully describe reality without anything that needs to be added to it. To use this condition of completeness, we also need some definition of what elements of reality entail. This is given by the following sufficient *criterion of reality*:

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." (Einstein et al., 1935, p.777).

This seems to be a very reasonable criterion, as it amounts to inference to the best explanation: the best and simplest explanation as to why we measure some value at time t is that there exist some element of reality having this value in accordance with the measurement outcome at time t. Lastly, they assume that there cannot be a reasonable description of reality, that does not follow the principle of locality, which in this case means that elements of physical reality belonging to a certain system cannot be affected by measurements performed on a different system, where these systems are space-like separated from each other.

Now the main strategy of the argument (in this form, there are many different forms possible) is to show that if we assume that the formalism of quantum mechanics is correct, and we also assume the criterion of reality and principle of locality, then it follows that non-commuting quantities must have definite values at the same time. Since the quantum formalism does not allow for this, and thus cannot simultaneously describe these elements of reality, it must be incomplete. This is the argument in more detail:

1. Suppose that the formalism of quantum mechanics is correct and that we have the criterion of reality and principle of locality as described above.
2. Let there be an entangled pair of qubits, that are space-like separated in the following state:

   $$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right) = \frac{1}{\sqrt{2}}\left(|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B\right)$$

   where the basis states are defined as in Section 3.1.
3. Suppose the formalism of quantum mechanics is complete.
4. We know that the qubits are perfectly correlated. Thus, suppose we would measure qubit A in the computational basis $|0\rangle, |1\rangle$ at time t_0 and find that qubit A is in state $|0\rangle$. We then know by the formalism of quantum mechanics with certainty that qubit B would be in state $|1\rangle$ for all time $t \geq t_0$.
5. Since we assumed the principle of locality, we get that also before time t_0 we have that qubit B must be in state $|1\rangle$, since
6. But we could also have chosen to measure qubit A in the $|+\rangle, |-\rangle$ basis. The qubits are then still perfectly correlated, as discussed in Section 3.2. Suppose we would have done this and found that qubit A was in state $|+\rangle$ at time t_0.
7. Then again, following the formalism of quantum mechanics we know that qubit B would be in state $|-\rangle$ for all time $t \geq t_0$. But by the principle of locality, qubit B must then also have been in state $|-\rangle$ before t_0.
8. By the criterion of reality, we can thus infer that both qubit B being in state $|1\rangle$ and $|-\rangle$ at $t < t_0$ are elements of physical reality and thus must be described by the formalism of quantum mechanics by our assumption of completeness.
9. However, these are incompatible measurements, since they require to measure in a different orthogonal basis. Hence, by the formalism of quantum mechanics, a qubit cannot be in both eigenstates at the same time.
10. Hence, we reach a contradiction.

From this, Einstein et al. (1935, p.780) concluded that the wave function does not provide a complete description of the physical reality, and thus the formalism of quantum mechanics is incomplete under the assumption of locality. Therefore, a way to complete the formalism would be by introducing *hidden variables* to which an observer has no access. If one had access to these hidden variables, they would provide the information necessary to get the determinate values for all elements of physical reality, such as the ones described in the EPR experiment setup. Einstein, Podolsky and Rosen viewed the assumption of locality, not only as reasonable, but as a necessary requirement for a candidate theory, since it is so general. Due to its generality, it is difficult to imagine any empirical consequences from it that are testable (Maudlin, 2011, p.7).

3.4.2 Bell's theorem

The assumption of locality changed when Bell (1964) was able to show that the predictions made by the formalism of quantum mechanics about the behaviour of separated pairs of systems, cannot be accounted for by any local physical theory, even with hidden variables. This is known as *Bell's theorem.* With this theorem, Bell proved a contradiction between the empirical predictions that follow from the formalism of quantum mechanics, combined with the assumption that the principle of locality holds and the assumption that there is "no conspiracy" of nature. This is the idea that any observer is free to choose the measurement settings and this choice is statistically independent of any physical process relevant to the outcome prior to the measurement. It simply asserts the existence of free will in choosing the measurement settings, rather than that nature has predetermined everything. Since the predictions of the formalism of quantum mechanics are validated by empirical evidence and the "no conspiracy" assumption is a reasonable assumption to make, it is commonly accepted that Bell's proof can be taken as a proof of quantum non-locality (Bokulich and Jaeger, 2010, p.67).

The gist of Bell's reasoning is as follows.[10] Bell (1964, p.199) showed that the statistical correlations between the measurement outcomes of certain chosen quantities on the subsystems (qubits) as described above in the EPR experiment cannot be perfect correlations simultaneously if we rely on any hidden variables to describe some predetermined outcome. Bell showed that there is an upper limit on a certain measure of correlation, for any local hidden variable theory assuming "no conspiracy". This was done by a relatively simple probabilistic argument. These types of upper limits are referred to as *Bell's inequalities.* One specific Bell inequality is the CHSH inequality, which tells us that classical correlation (having local hidden variables) has a maximum value of 2. However, the formalism of quantum mechanics predicts that these perfect simultaneous correlations exist and this is what follows from empirical evidence in many different experiments. These perfectly correlated quantum systems, which are for instance described by the maximally entangled Bell states in Section 3.2, exceed this upper limit on the measure of correlation given by Bell's inequality. In this case, the correlation measure can take a maximum value of $2\sqrt{2}$, exceeding the upper limit given by the CHSH inequality. Thus, the measurement correlations that exist in Bell states are stronger than correlations that can possibly exist in classical systems. This result was the first hint that the processing of quantum information allowed for things beyond what is possible in processing classical information (Nielsen and Chuang, 2010, p.117). In short, if we assume the principle of locality to hold, together with the assumption of "no conspiracy", we get an upper limit on correlation as given by the Bell inequalities. However, since the predictions of quantum mechanics exceed this upper limit, one of our assumptions must be flawed.

If we take the outcome of Bell's inequality together with the results from the EPR experiment, we reach the conclusion that quantum mechanics must be non-local. Bell (1981) summarised this argument as follows :

> "The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we

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[10] For the aims of this thesis, we will not consider the proof in depth, as all the details can be found in (Bell, 1964; Maudlin, 2011; Redhead, 1987).

> do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we *cannot* dismiss intervention on one side as a causal influence on the other."(Bell, 1981, p.51)

The EPR experiment has shown us that if we want to preserve locality, we must accept the incompleteness of the formalism of quantum mechanics, and introduce hidden variables. But Bell then showed that any local hidden variable theory is incompatible with the predictions of quantum theory. Hence, it follows that we need to reject local theories altogether (if we want to hold on to the criterion of reality and "no conspiracy"). This result caused a shift in the worldview of physicists, as it showed how different the behaviour of the quantum world is compared to their commonsense intuitions based on the classical world (Nielsen and Chuang, 2010, p.117).

Finally, it should be noted that non-locality provides an explanation for the effects of quantum entanglement, but these are different notions. Entanglement is what follows from the mathematics in the formalism of quantum mechanics and is therefore an algebraic concept, whereas non-locality is the theorisation of the consequences of quantum entanglement and depends on the chosen interpretation of quantum mechanics. In the present day, quantum entanglement is a generally accepted concept, whereas the idea of quantum non-locality is still causing debate. Are there other explanations possible for entanglement without accepting non-locality? Indeed, we could also reject the criterion of reality, which was an assumption on the proof of Bell's theorem. This is done if we adopt specific quantum interpretations, such as the many-worlds interpretation as proposed by Everett (1957). In that case, we can retain locality (Tipler, 2014). This goes beyond the scope of this thesis.

3.5 Principles of quantum information

In this section, we will outline the main information-theoretic principles of quantum information. First, there are several no-go theorems that provide constraints on how we can transfer, manipulate, create or delete quantum information. Each no-go theorem outlines the impossibility of a certain physical situation. These theorems are of great influence, not only on the theoretical side, but also on the experimental development around quantum information. Second, we will look at the information-theoretic characterisation of quantum theory as proposed by Clifton et al. (2003), with which they aimed to show that it is possible to take information as the fundamental concept underlying the physical description of quantum mechanics. Overall, these principles give insight into what quantum information is and how it might differ from classical information, which we will discuss in more depth in Chapter 5.

3.5.1 No-go theorems

In the development of the field of quantum information theory, over time several 'no-go theorems' have been proposed and proved. These theorems put restrictions on what is possible in the computation and communication of quantum information in the framework of quantum information theory. A thread through these no-go theorems is the fact that quantum mechanics is *linear*. This refers to the fact that the observables in quantum mechanics, which denote physical quantities, are required to be linear operators. An operator O on a Hilbert space $\mathcal{H}$ is said to be linear if the effect of the operator on a linear combination of vectors is the same as the linear combination of the effect of the operator on each of the vectors of this linear combination taken separately (Timpson, 2008, p.6). All unitary operators are linear.

We will provide a brief description of the no-go theorems, as summarised by Sharma et al. (2020) and Pathak (2013), to give a general idea of the main principles of quantum information. We will consider six no-go theorems, as listed below:

1. No-cloning theorem:
   This theorem tells us that it is not possible to copy an arbitrary unknown pure quantum state, by creating a second qubit in an identical state using unitary evolution and leaving the original qubit untouched. The no-cloning theorem in its first form was proposed by Park (1970) and later independently proved by others, such as Wootters and Zurek (1982). It can be seen as the most important out of the no-go theorems (Pathak, 2013, p.124), as it in principle tells us that it is not possible to create quantum information and if it could be violated, this would have many implications also for the other no-go theorems.[11]

   The no-cloning theorem prohibits the existence of some universal cloning machine for quantum states. This machine would take as input an arbitrary pure quantum state $|\psi\rangle$ and some standard pure state $|s\rangle$ and performs some unitary transformation U to the product state $|\psi\rangle|s\rangle$, leaving $|\psi\rangle$ intact. The result of this transformation would need to be the product $|\psi\rangle|\psi\rangle$. Suppose the cloning machine now takes a different input state $|\varphi\rangle$ and copies this state as well: $U|\varphi\rangle|s\rangle = |\varphi\rangle|\varphi\rangle$. Then it must also be able to copy the state $|\chi\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle + |\varphi\rangle)$, which is a superposition of the states $|\psi\rangle$ and $|\varphi\rangle$, as $|\chi\rangle$ is also a pure state. However, due to the linearity of quantum mechanics, the result of $U|\chi\rangle|s\rangle$ would be the entangled state $\frac{1}{\sqrt{2}}(|\psi\rangle|\psi\rangle + |\varphi\rangle|\varphi\rangle)$, instead of the product state $|\chi\rangle|\chi\rangle$.

   In general, any cloning machine that would be able to clone multiple arbitrary states would only be able to do so for states that are orthogonal to each other. Suppose we have for two arbitrary pure states $|\psi\rangle$ and $|\varphi\rangle$:

   $$U|\psi\rangle|s\rangle = |\psi\rangle|\psi\rangle$$

   $$U|\varphi\rangle|s\rangle = |\varphi\rangle|\varphi\rangle$$

   Then taking the inner product of these equations leads to:

   $$\langle\psi \mid \varphi\rangle = (\langle\psi \mid \varphi\rangle)^2$$

   This equation is only satisfied in the case that $|\psi\rangle$ and $|\varphi\rangle$ are identical states, or if $\langle\psi \mid \varphi\rangle = 1$, which means $|\psi\rangle$ and $|\varphi\rangle$ are orthogonal. Thus, the existence of a cloning device for arbitrary pure states using unitary transformation would lead to

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[11] This will be discussed in depth in Section 5.1.

a contradiction in the framework of quantum information theory. Hence, it is only possible to clone a known quantum state, or to clone quantum states from a given set if and only if all states of this set are mutually orthogonal. But then, these qubits behave like classical bits, as they do not display their quantum advantages such as superposition. This also explains why cloning is possible in the case of classical information (Nielsen and Chuang, 2010, p.530)

2. No-broadcast theorem:
   This theorem is a generalisation of the no-cloning theorem to mixed quantum states. It was first proved by Barnum et al. (1996). With the term 'broadcasting' it is meant to get across a quantum state to two or more recipients. In order to communicate the same arbitrary quantum state to multiple receivers, one would also need to clone the state. It is allowed to transport a quantum state to several places, one after the other (via the quantum teleportation protocol). However, the no-broadcast theorem tells us that it is not possible to broadcast an arbitrary quantum state, so to send out multiple copies of the same state to different receivers.

3. No-deletion theorem:
   Pati and Braunstein (2000) proved that also the time reversal dual theory of the no-cloning theorem holds for quantum information. This is the no-deletion theorem, which tells us that given two copies of some qubits in an arbitrary pure quantum state, there is no physical operation to delete the quantum state of one of them by transforming the qubit into a blank state using unitary transformation. This would be desirable in order to store new information on it. The impossibility of deleting arbitrary states is different from classical bits, which can always be copied and deleted. This shows that quantum information behaves differently on a fundamental level.

4. No-teleportation theorem:
   This theorem has a slightly confusing name, as there is a standard protocol in quantum information theory which is referred to as 'quantum teleportation'. With this protocol, one transfers quantum information from sender to receiver, and in doing so, the quantum state at the sender is destroyed, in order to create a replica of that state at the receiver[12]. However, this no-go theorem states that it is impossible to "teleport" a quantum state/qubit, by simply converting the quantum state into a sequence of classical bits, which then can be sent over a classical channel, to have a receiver reconstruct the original quantum state by using these bits. It is thus not possible to wholly convert arbitrary qubits into classical bits. This theorem was first proved by Gruska and Imai (2001). If this type of "teleportation" of a quantum state over a classical channel would be possible, this would be a direct violation of the no-cloning theorem. For suppose one could create a sequence of classical bits to convert quantum states, then these classical bits can be copied and then converted back to quantum states and hence one could copy these arbitrary quantum states.

5. No-communication theorem:
   This theorem, which was proved by Peres and Terno (2004), states that it is not possible to use shared quantum states for communication, whether it is faster than light or not. It is also often referred to as the no-signalling theorem. It disallows transmitting classical bits using prepared quantum states, whether these are pure or mixed and

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[12] See Section 3.6.

entangled or separable. Assume that we prepare two qubits in an entangled state and we have two agents Alice and Bob, which were not involved in the preparation of the quantum state of these qubits, who each take one qubit of this shared state with them to a different location. Alice and Bob are spatially separated and do not know anything about the initial state of the system. Then the question is: is there any way for Alice to perform measurements or actions on her qubit such that this would be detectable by observation of the other qubit that is with Bob (or vice versa)? It turns out the answer to this question is negative. This is because even though Alice can measure her qubit and this causes also Bob's qubit to collapse, there is no way for Bob that Alice has performed this measurement. Namely, Bob would have to measure his own qubit for this, and without knowing the outcome of Alice's measurement (via some other communication channel), there is no way for Bob to tell whether his outcome is in any way correlated with Alice's measurement outcome. This is due to the fact that the probabilities for Bob's outcomes due to quantum uncertainty are identical to his lack of knowledge expressed probabilistically (Sharma et al., 2020, p.9).

The no-communication theorem thus tells us that there is no way of signalling information to another observer by performing any action on a part of the system. Hence, there is no way to communicate classical information, using this shared quantum state for these distinct observers, even when the state is entangled. This shows that even though it seems that faster-than-light communication would be possible using quantum entanglement due to its instantaneous measurement effects, this is not the case.

6. No-hiding theorem:
   This theorem, proved by Braunstein and Pati (2007), states that if quantum information in an entangled bipartite quantum system is dispersed in the environment (via decoherence), then this information is not lost, but remains in the universe. Most importantly, they show that the information cannot be hidden in the correlation between the quantum system and the environment. This is different from classical systems, in which the information can be fully transformed into correlations between systems. An example of this is the encryption of information using a one-time pad cipher. One has a bit string and uses a random key bit string to encode this bit string by flipping its bits or not. Then the encoded bit string contains no information about the original string, as it cannot be distinguished from a random bit string. This means that all information is now hidden in the correlation between the encoded string and the secret key. In the case of quantum information, such hiding of information in correlations is not possible. The only way to completely "hide" an arbitrary quantum state is by moving it to a different subsystem of an entangled system (Braunstein and Pati, 2007, p.080502-1).

   In general, the no-cloning theorem prohibits the creation of information and the no-deletion theorem prohibits losing information, so it seems quantum information must be preserved, which can be captured in a law of *conservation of information* (Horodecki et al., 2005b, p.2042; Roncaglia, 2019, p.1285).[13] According to Braunstein and Pati, "quantum information hiding is equivalent to its erasure, whereas classical information hiding is fundamentally distinct from erasure." (Braunstein and Pati, 2007, p.080502-

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[13] More on this will be discussed in Section 5.1.2 and 5.2.

3). The no-hiding theorem thus also forbids the erasure of quantum information, but in a different form than what is prohibited by the no-deletion theorem. No-hiding tells us that the wave function describing the quantum state can move to a subspace of the Hilbert space of the environment with unitary time evolution and is thus conserved.

The theorem was tested experimentally by Samal et al. (2011). They randomised a qubit, by bringing it from a pure state to a random mixed state. Then, they recovered the information from one of the two extra qubits (the environment), using local unitary transformations, to reconstruct the original pure state of the initial qubit. This means, that no information was 'hiding' in the correlation between the qubits. This was the first experiment to demonstrate that quantum information is conserved.

3.5.2 CBH theorem

The six no-go principles listed above, do not give a minimal characterisation of quantum information. It is clear that we do not need all of them, as some are direct consequences of others, such as how the no-teleportation theorem follows from the no-cloning theorem. It is possible to give information-theoretic constraints such that these capture the essence of quantum information, and even provide an information-theoretic characterisation of quantum theory in general. This has been shown by Clifton et al. (2003). They first formulated three fundamental constraints, also in the form of no-go theorems, which we will list below. Then, they aimed to show that these three constraints can be taken as information-theoretic 'laws of nature' from which one can derive quantum theory. They do so by constructing a mathematical theory, which they show to be a minimal and sufficient mathematical abstraction of a physical theory that includes all variations of a quantum theory. Then they show that these three proposed information-theoretic constraints correspond to three physical characterisations expressed in their mathematical framework which in the most general sense defines what it means to be a quantum theory.

Let us first consider the three information-theoretic constraints (Clifton et al., 2003, p.1562):

- "the impossibility of superluminal information transfer between two physical systems by performing measurements on one of them;
- the impossibility of perfectly broadcasting the information contained in an unknown physical state; and
- the impossibility of unconditionally secure bit commitment."

The first principle corresponds to the no-communication theorem as described in Section 3.5.1. This is a straightforward and generally accepted theorem, since if communication or signalling of (classical) information via entangled states would be possible, this would violate special relativity or it would require information to travel backwards in time (R. B. Griffiths, 2002; Timpson, 2013). The second principle corresponds to the no-broadcasting theorem described in Section 3.5.1. Since this principle is the generalisation of no-cloning to mixed states, the no-cloning principle is implicit in this constraint. One can take mixed states with all probabilities p_i to be 0 except for one of the probabilities. In that case, it just corresponds to a pure state. Hence, in that case, the no-broadcasting theorem corresponds to the no-cloning theorem.

The third principle is regarding *bit-commitment* (Halvorson, 2004b; Timpson, 2013). This is a specific protocol in which two agents communicate, Alice and Bob, who do not necessarily trust each other. The aim of this protocol is to provide a two-stage way for agents to communicate a bit value, but without knowing this value immediately. So, Alice provides Bob with a bit value, so 0 or 1, in such a way that Bob cannot know the value of this bit immediately. Bob only knows with certainty that Alice has committed to a certain value, but not to which one. This is the first phase, the "commit phase". Only in the "opening/revelation phase" does Bob get to know the value of the bit that Alice committed to, after Alice provided Bob with further information to do so. However, Bob is guaranteed that upon learning the value of the bit, this is indeed the initial value that Alice committed to. If this would not be the case, and one of the agents can cheat, we have an insecure bit-commitment protocol. Then, either Alice can still change the value of the bit after having committed to one, or Bob can get to know the bit value before Alice provided the extra information to do so.

Now the third information-theoretic constraint by Clifton et al. (2003) tells us that it cannot be the case that any bit commitment protocol for quantum information is perfectly secure. For classical information, we also have that bit-commitment is not unconditionally secure, since there must be some bias in the encrypted information that Alice provides to Bob about the committed bit value. But in the case of quantum information, it turns out that bit commitment also is not perfectly secure, but for other reasons. Alice can now make use of the so-called *EPR cheating strategy* and in this way circumvent committing to a bit value, before sending this information to Bob in the commitment phase. In this strategy, Alice prepares a system in an entangled state, such as one of the Bell states.[14] Then Alice sends out one qubit of this entangled pair to Bob and then she can wait until after the commitment phase, so in the revelation phase, with performing a measurement on her half of the system. This causes a change in the state of the subsystem that is at Bob and hence a change in the bit value that Alice committed to. It turns out that this EPR cheating strategy can always be applied in a bit commitment protocol for quantum information (Bub, 2001, p.7).

Information at the foundation of physics

The important achievement of Clifton et al. (2003) was to show that these three information-theoretic constraints correspond to physical characteristics that are necessary and sufficient to characterise a general quantum theory. For this goal, they first formulated these characteristics in the setting of C^*-algebraic theories, which are a certain kind of complex algebras, that can be used to describe both classical and quantum theories.[15] Clifton et al. (2003, p.1567) argue therefore that this class of algebraic theories is general enough to have as a baseline assumption. Then they continue to show that the three proposed information-theoretic constraints correspond to properties of the algebra. Any C^*-algebraic theory satisfying the information-theoretic constraints, must have the following. Firstly, it must have commuting algebras of observable belonging to different space-like separated physical systems (referred to as kinematic independence). This corresponds to the first

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[14] See Section 4.

[15] For an introduction to this type of algebraic theories see Murphy (1990).

information-theoretic constraint of the no-communication theorem. Secondly, it must have a non-commutative algebra of observables for individual physical systems, this corresponds to the no-broadcasting constraint. Thirdly, the last information-theoretic constraint of no secure bit commitment corresponds to the constraint that the algebras allow for entangled states between space-like separated systems, and thus non-locality in the physical world. Of this last proof, one direction was later completed by Halvorson (2004b).

Clifton et al. (2003, p.1563) then argue that these physical characteristics are defining a quantum theory in the most general way. Hence, the information-theoretic constraints corresponding to them are necessary and sufficient constraints for a theory to be a theory of quantum mechanics. Based on their achievement, Clifton et al. advocate that we should embrace the idea of quantum mechanics as a theory in which the notion of quantum information is central, rather than a focus on the mechanics of waves/particles:

> "The fact that one can characterize quantum theory ... in terms of just a few simple information-theoretic principles ... lends credence to the idea that an information-theoretic point of view is the right perspective to adopt in relation to quantum theory. Notice, in particular, that our derivation links information-theoretic principles directly to the very features of quantum theory—noncommutativity and nonlocality— that are so conceptually problematic from a purely physical/mechanical point of view. We therefore suggest substituting for the conceptually problematic mechanical perspective on quantum theory an information-theoretic perspective. That is, we are suggesting that quantum theory be viewed, not as first and foremost a mechanical theory of waves and particles ..., but as a theory about the possibilities and impossibilities of information transfer." (Clifton et al., 2003, p.1563)

In a follow-up paper, Bub (2005) (one of the contributors to the CBH theorem), takes this idea even further by stating:

> "just as the rejection of Lorentz's theory in favour of special relativity (formulated in terms of Einstein's two principles) involved taking the notion of a field as a new physical primitive, so the rejection of Bohm's theory in favour of quantum mechanics— characterized via the Clifton–Bub–Halvorson (CBH) theorem in terms of three information-theoretic principles—involves taking the notion of quantum information as a new physical primitive." (Bub, 2005, p.542-543)

Bub argues for the explanation of physics in terms of information, by taking it as a physical primitive. This is in line with the idea of reducing physics to information as proposed by Wheeler (1989, p.355), with his famous hypothesis of "it from bit". With this he means that every "it", objects in the universe, particles, fields, should be derivable from binary choices, "bits". Bub's proposition for taking quantum information as a physical primitive is less radical, but fits with this general line of thinking that all of physics can be derived from information. This is not the same thing as the idea of information being physical, or reducible to physics, as briefly discussed in Section 2.1. We will come back to this discussion in Section 4.1.

However, we would like to point out that there has been a discussion as a response to their achievement and this reasoning in the literature, mainly about the choice of Clifton et al. (2003) of C^*-algebras as the mathematical framework. Spekkens (2007) and Timpson (2004,2013) question whether the starting point of C^*-algebraic theories is indeed general enough for an initial assumption or whether it is too close to the intended purpose to start with. It turns out, that the assumptions involved with taking the C^*-algebras as starting point, rule out a class of possible theories of quantum mechanics. The question that remains is whether we should take a mathematical framework general enough to also encompass these types of theories, or whether the assumptions made by Clifton et al. (2003) are reasonable initial assumptions. After all, one needs certain background assumptions or constraints on theory construction in order to derive any sensible theory of quantum mechanics. This is in line with the response from Halvorson (2004a) to certain "toy theories" of quantum mechanics that have been proposed, which do not validate the premises of the C^*-algebraic framework, such as the one by Spekkens (2007). Even though these few proposed toy theories might challenge the ideas of Clifton et al. (2003), it is clear that in general, the CBH theorem proves the importance of considering quantum information to get a better understanding of quantum theory.

3.6 Quantum teleportation

In this section, we will provide an introduction to a common quantum communication protocol. As we have discussed in the previous section, quantum states have properties, such as superposition and entanglement, which make it possible to perform certain information-processing tasks that would be unachievable using only classical states (Jaeger, 2009, p.224). The protocol for quantum teleportation is a good example to examine, as it shows these extra possibilities that the use of quantum information provides. Central to the protocol is the use of entanglement of quantum states. Even though there is so much unclear about quantum entanglement and there is no consensus in the literature on how to interpret the non-locality of the theory that seems to go hand in hand with it (Jaeger, 2009; Lombardi, Holik, et al., 2016; Timpson, 2013), it has been put to good use as a communication-theoretic resource. The remarkable aspect of entangled quantum systems is that not all of their global properties are reducible to local ones. And this also goes the other way around: there are no local actions that can be performed on one system, to cause it to increase the amount of entanglement with some other system. These distinctive global properties can be utilised in information communication protocols, although not in a direct way. As the no-communication theorem tells us, agents cannot use entangled states to communicate information to each other by performing any measurement on their part of the entangled system. So, let us analyse the teleportation protocol, to see how this entanglement is used in practice.

The main goal of the quantum teleportation protocol is to transfer a quantum state from one quantum system to another spatially separated quantum system. This protocol combines the use of both quantum and classical information in order to 'teleport' a quantum state. Essentially, starting with a system in an unknown quantum state, some agent Alice can transit an unknown quantum state to agent Bob. In the process, this state is destroyed at her end and re-appears as an intact quantum state at Bob's end. To do so, agents Alice and Bob must share a pair of qubits in a maximally entangled state (such as one of the Bell states as described here 4). It is the power of quantum entanglement, providing a correlation stronger than classically possible, that permits this teleportation. The quantum teleportation protocol, which was first proposed by Bennett et al. (1993), can be seen as the first protocol dealing with the transmission of quantum information, instead of using quantum tools to transmit classical information (Timpson, 2008, p.16).

3.6.1 The teleportation protocol

Let us consider the standard quantum teleportation protocol. We will give a global description of the steps, without going into the details of the exact state changes. For this, we refer the reader to the original paper by Bennett et al. (1993), or to Timpson (2013, p.75) or Nielsen and Chuang (2010, p.26). We start with two agents, Alice and Bob. Alice has access to a quantum system in an unknown state $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$ (with α and β being unknown amplitudes), that she wants to transmit to Bob. Let us call this system 1. Furthermore, before separating, Alice and Bob both took one qubit from an entangled pair being in Bell state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Let us refer to these as system 2, with Alice, and system 3, with Bob. Therefore, we can describe the general system, consisting of these three subsystems as follows (using subscripts for clarity):

$$|\varphi\rangle_1 \left|\Psi^-\right\rangle_{23} = \frac{1}{\sqrt{2}} \left(\alpha|0\rangle_1 + \beta|1\rangle_1\right) \left(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3\right).$$

As is clear from the above, system 1 can be written factored out and hence is separable with respect to the entangled qubits. Thus, system 3, which is with Bob, is independent of system 1. Therefore, Bob does not have access to any information about the state of system 1 at this point, as no measurement on system 3 would provide any insight at this stage. To this end, Alice must first cause the two systems 1 and 2 at her end to get entangled. Therefore, she performs a joint measurement on these systems, in the Bell operator basis, which is the set $\{|\Psi^+\rangle, |\Psi^-\rangle, |\Phi^+\rangle, |\Phi^-\rangle\}$. To perform this joint measurement, A first applies a CNOT quantum gate to systems 1 and 2, which changes the state of system 2, based on the state of system 1 and causes the entanglement between systems 1 and 2 to arise. She then performs a Hadamard gate on system 1, rotating the state of the qubit, and then she can measure both systems in the Bell operator basis (Nielsen and Chuang, 2010, p.27). If she would not do such joint measurement and only perform a measurement on system 1, the state of that system would change, but this would not affect the state of the entangled systems 2 and 3. But because of the above procedure, she causes the three systems to get correlated in such a way, that the qubit at Bob will also be projected into one of four possible pure states, depending on the measurement outcome of Alice of this joint measurement. This is because systems 2 and 3 were maximally correlated to begin with.

However, Bob is not aware that this measurement has happened nor of the state change of the entangled system. Hence, Alice must communicate this to Bob via a classical channel. The remarkable aspect of this quantum teleportation protocol is that due to the choice of measurement basis of Alice, the possible states of system 3 are now related in a rather simple way to the initial state $|\varphi\rangle$ that was to be teleported. So, Alice needs to communicate what her measurement outcome is, and based on this outcome, Bob can perform a unitary transformation on her part of the system, to cause the state of the system to change to the unknown state $|\varphi\rangle$. Alice needs to communicate 2 bits of classical information, to inform Bob of the exact transformations that are necessary to flip the state to the correct basis. After Bob has received these instructions and performed these unitary transformations, the resulting state of system 3 is now an exact replica of the initial state $|\varphi\rangle$ of system 1. On the other end, Alice is left with systems 1 and 2 being in one of the entangled states $\left|\Psi_{12}^{(\pm)}\right\rangle$

or $\left|\Phi_{12}^{(\pm)}\right\rangle$. Hence, Alice has no access anymore to the initial unknown state $|\varphi\rangle$, it has disappeared at her end.

3.6.2 Remarks about quantum teleportation

The reason that the protocol is named 'teleportation' is because Alice managed to transmit the state $|\varphi\rangle$ to Bob, without sending anything through space to Bob that is related to this state $|\varphi\rangle$. Namely, only two classical bits have been communicated over a classical channel, of which their value does not depend on the parameters α and β of state $|\varphi\rangle$ (since these are also unknown to Alice). Furthermore, there is no trace of the unknown state in the region of space near Alice, and the state has been created intact at Bob. It therefore seems as if the state disappeared and reappeared somewhere else (Timpson, 2013, p.75). If teleportation as such is possible, this seems to be a contradiction with the idea that there can be no communication faster than light. However, if we consider the protocol carefully, we see that in order to complete the teleportation, also the communication of information over a classical channel is required. Namely, Bob is not aware when Alice has performed the measurement and there is no way for Bob to know by performing any actions on their part of the system, as the no-communication theorem tells us. Even if so, Bob still needs the information from Alice based on her measurement send over a classical channel, to possibly transform the state of their qubit. The speed of information on such a classical channel is bounded by the speed of light and thus, the protocol of quantum teleportation cannot be faster than this (Nielsen and Chuang, 2010, p.28).

A second worry one might have about the protocol is whether it violates the no-cloning principle. At first sight, it appears that one can create a copy of an unknown quantum state at a different location which would be a violation of this no-go theorem. However, since the initial state of the qubit in system 1 gets irretrievably lost in the process, it cannot be seen as a copying procedure (Nielsen and Chuang, 2010, p.28). The state of this qubit ends up being one of the computational basis states upon measurement by Alice. Thus, Alice no longer has access to this state and therefore the state is not copied, but rather transmitted. Therefore, it does not cause a violation of the no-cloning principle.

A third remark is that there are alternative protocols for quantum teleportation, as described by Garcia-Escartin and Chamorro-Posada (2011, p.10). These protocols do not make use of a classical channel, so at first glance it seems that only entanglement is enough for the teleportation of information. However, in all these alternative protocols, one needs to perform joint measurements on all three qubits involved in the protocol, which requires them to all be at the same spatial location. This means that by using one of these protocols, one can no longer transport a quantum state to a spatially separated location, which seems to be exactly the purpose and the mystery of a protocol for teleportation. So, even though these alternative protocols do not make use of a classical channel, they also do not allow for teleportation in the sense of a quantum state disappearing and reappearing at a spatially separated location.

In conclusion, what the teleportation protocol shows is that in terms of resources, one shared maximally entangled pair of qubits, together with the communication of two classical bits is at least equal to the communication of one qubit (Nielsen and Chuang, 2010, p.28). This proves that entanglement is a very powerful tool in the communication of information.

But at the same time, having only access to entangled qubits is not enough to communicate information over a distance. There is a need for both a classical channel and entangled pair of qubits.

4 Comparison of classical and quantum information

In the preceding chapters, we have reviewed the basics and fundamental principles of both classical information, via a qualitative approach, and quantum information, mainly based on quantum information theory. In the current chapter, we will combine these ideas and see how the two concepts of information relate to each other. We aim to draw parallels between the different concepts of information and point out the important distinctions or reasons why some aspects of one are not applicable to the other. We will do this first on the most fundamental level, considering different aspects of information such as the carrier of information, the way it behaves when interacting with it, the relation between physics and information and how information is localised. Based on this comparison, we will compare principles of classical and quantum information in the next chapter.

4.1 Information carriers

As we have seen in our discussion of the basics of quantum information in Section 3.1, the unit of quantum information is the 'qubit'. This is analogous to the case of classical information, in which the basic unit of information is the 'bit'. There is however an important difference that should be pointed out between the use of the notions 'bit' and 'qubit' that is not analogous. Initially, in the field of information theory and communication engineering, the term 'bit' was only used as a unit for the measurement of information. In this context, it refers to the logical state that has two distinct values (often denoted 1 and 0).

> "The choice of a logarithmic base corresponds to the choice of a unit for measuring information. If the base 2 is used the resulting units may be called binary digits, or more briefly bits ... A device with two stable positions, such as a relay or a flip-flop circuit, can store one bit of information." (Shannon, 1948, p.379)

As is shown by this excerpt of Shannon, in the literature, the term did not refer to the actual physical realisation of this logical system. Namely, this could be any object or system that has two different, distinct states. The possibilities for such two-state systems are endless, such as an electrical circuit having an electric current or not, but also a light being switched on or off or a cat being dead or alive. All of these examples are able to store one bit of information, by being in state 1 or state 0 (which are assigned to the physical state by convention). Since a bit was a unit of information chosen by convention, it was not tied to the nature of information itself.

With the rise of the qubit, the term was used in the literature to denote two things: It refers to the fundamental unit of quantum information, analogous to bits, but it also refers to the simplest physical two-level quantum system (Jaeger, 2009, p.190). These two-level quantum systems are physical realisations of this fundamental unit of quantum information, such as the spin of an electron or the polarisation of a photon. The idea of a qubit referring to something physical can be traced back to Schumacher (1995) when he coined the term in his paper "*Quantum Coding*":

> "For our elementary coding system we choose the two-level spin system, which we will call a "quantum bit" or qubit. The qubit will be our fundamental unit of quantum

> information" (Schumacher, 1995, p.2744)

This ambiguous usage of the term qubit in the literature following Schumacher causes an inclination to think about (quantum) information as having a physical carrier, as the notion is referring both to the unit of information and to the system carrying one such unit. Consequently, the usage of the notion of a bit also changed, as is clarified by Duwell (2003):

> "First, "bit" refers to a unit of information that quantifies the uncertainty of two equiprobable choices. Second, "bit" also refers to a system that can be in one of two discrete states."(Duwell, 2003, p.486)

It is for this reason that Caves and Fuchs (1996, p.3) suggested using the term 'c-bit' to denote the minimal classical physical system that carries one bit of information, and 'q-bit' for the quantum physical two-state system, to avoid the conflation of different uses of the term 'bit'. After all, it is remarkable how 'bit', first used as a theoretically neutral unit of information, became tied to the notion of classical information, whereas the term 'qubit' is strictly connected to quantum information. These changes in use and meaning show that quantum information was approached as a completely new type of information, distinct from classical information, based on the information carriers (a classical two-state system versus a quantum two-state system) being different types of systems. The question is now if the original notion of a 'bit' was not general enough to be used in a theoretically neutral manner as a unit of information. This would imply that quantum information is indeed fundamentally different in nature compared to classical information and therefore requires a different unit of information.

4.1.1 Physical carriers of information

If we consider classical information, as discussed in Chapter 2, it is clear the communication of classical information requires some physical information carrier or signal. This is the general conception of information in the technical sense:

> "Physicists and engineers unquestioningly assume that the transmission of information between two points of the physical space always needs an information-bearing signal, that is, a physical process propagating from one point to the other." (Lombardi, 2005, p.34)

Based on this idea, that information always needs some physical representation bearing the information, Landauer (1991, p.23) argues that "information is physical" and therefore adheres to physical laws. Looking at the notion of information in Shannon's theory of communication, the information is communicated over a physical channel, which is a medium such as a wire, used to transmit a signal carrying the information (Shannon, 1948, p.380). If we consider the everyday notion of information, then we consider information being carried by a concrete medium adhering to physical laws which contains the information such as a book, a brain or a computer (Timpson, 2008, p.12-13). Important to note here is that the need of having a physical carrier is different than saying that information itself is physical. This is argued against by Timpson.[16] Also in the case of semantic information,

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[16] See Section 2.1.

there seems to be a need for some physical implementation of information-carrying signals. Floridi (2011, p.42) writes about semantic information: “Most people agree that there is no information without (data) representation.” Both in the case of information in the framework of environmental information by Dretske (1981) or situation theory (Devlin, 1991), the information is carried by some signal. Usually, this is understood as some sort of signal travelling through physical space from transmitter to receiver, bearing the information.

If we consider quantum information, the observation described above about information always having a physical carrier is not obviously applicable in the same way. Take for instance the quantum teleportation protocol described in Section 3.6. In this protocol, how exactly does the information go from Alice to Bob? In fact, an entire quantum state is transferred, and to specify such a quantum state using classical information one would need an infinite amount of information (Nielsen and Chuang, 2010, p.26). Namely, to specify exactly the quantum state out of the infinite possible states in a continuous Hilbert space, one would need two real numbers. Hence, how can this enormous amount of continuous information be transferred, while Alice only communicates two classical discrete bits over a channel? It is clear that the only reason this is possible is because of the quantum correlation between the entangled pair that both Alice and Bob have access to, but this correlation cannot be used to send information directly as we know from the no-communication theorem. Thus, it seems that there is no information-bearing physical signal travelling from Alice to Bob apart from the classical signal carrying two bits of information.

Therefore, the question above has led to a debate in the literature, with a wide range of possible solutions (Bokulich and Jaeger, 2010). Quite controversially, Penrose (1998, p.1928) argues for a ‘reverse-time channel’ through which quantum information can travel backwards in time, if there is access to a link via an entangled pair of qubits. He proposes that there is an information channel that ‘proceeds into the past’, linking Alice’s subsystem to the source of the EPR pair and then forward in time from the EPR to Bob’s subsystem (Penrose, 1998, p.1928). This would be the only possible physical connection between Alice and Bob spatially separated subsystems. Jozsa (2004, p.80) agrees with this idea that quantum information needs to propagate backwards in time to Alice and argues that this approach, though not intuitive, would be completely consistent with quantum theory. Therefore, Jozsa views quantum information as a distinct type of information, separate from classical information, which cannot propagate backwards in time.

A different approach is adopted by Deutsch and Hayden (2000, p.1765), who argue that the two bits that are sent over a channel from Alice to Bob carry locally hidden information. Due to the measurement process of Alice to determine the values of these two bits, they interfered with the entangled systems 1, 2 and 3 and consequently, the bits carry the information from this total system that survives the effects of decoherence. For this reason, Deutsch and Hayden (2000, p.1769) argue that these bits are not actually classical bits, but carry a qubit’s worth of information although in an inaccessible manner. The entanglement then provides a key, to locally get access to this information. This explanation is therefore opposed to the standard interpretation of quantum teleportation (as it is described in Section 3.6), where the classical bits function as a key to get access to the correct quantum state and the entanglement is seen as a mysterious non-material channel carrying the information. As becomes clear from their proposals, both Penrose (1998) and Deutsch and Hayden (2000) still search for a physical connection, a way for a physical signal to carry the information from Alice to Bob. However, Timpson (2013, p.83) argues that there is no need for such a physical carrier altogether, by regarding it as an abstract noun. We have discussed in Section 2.1 with regards to classical information, but this is also Timpson's view of quantum information:

> "The account I will go on to provide of the nature of quantum information is ontologically deflationary. We should not take the view that information in general, nor quantum information in particular, is any kind of physical substance or stuff—even if a very nebulous and aethereal one—as the writings of some authors might lead us to suppose. But neither should we take the nihilist view that quantum information does not exist. The middle way—the right way—is to pay careful attention to the logical status of the concept of information. It proves essential to recognize that 'information' is an abstract noun: then we can see clearly what information talk is doing, both in the quotidian and in the quantum context." (Timpson, 2013, p.3)

This does not mean that there is no transmission of information, but he emphasises that there does not have to be a continuous path in space-time over which the information travels, since information is no substance or entity in space-time itself. The only thing that is required in the transmission of a piece of information in such a protocol is that Bob can produce a different token of the same type as the original token that was at Alice. However, Timpson does not provide an explanation for how this token could be produced correctly. He claims to have reduced the puzzle to the question of which physical processes are involved in teleportation, which would be dependent on the choice of quantum interpretation (Timpson, 2013, p.86). But even if we view information as referring to something abstract rather than a concrete substance, we still need an explanation for the causal relation between Alice's and Bob's measurement outcomes. But if we want to avoid non-intuitive solutions, such as the ones provided by Deutsch and Hayden (2000), Jozsa (2004), and Penrose (1998), we have to let go of the requirement of a physical path in space-time for a signal carrying information.

All in all, there is no consensus on the role of information in the quantum teleportation phenomenon. But the seeming lack of a physical channel or a physical carrier for a great part of the information that is transmitted in the teleportation protocol can be seen as a basis for a fundamental distinction between classical and quantum information. Lombardi et al. (2014) summarises:

> "A defender of the physical view might retort that the difference between information that requires a carrier signal through space, which takes a finite amount of time, and information that can be transmitted without such a signal is radical enough to be the basis for distinguishing two kinds of information: classical information (which requires a carrier signal for transmission) and quantum information (which does not)." (Lombardi et al., 2014, p.22)

If we adopt such a distinction between classical and quantum information, this means that their main difference arises from the possibility of entangled states in quantum systems. Due to this possibility of entanglement, there is a non-local information exchange. There is no need for an information carrier to travel over a path in space-time, as the information is not localised in one place, as is the case for classical information. We will come back to this idea in Section 4.5. This leads to the question of whether it is only the property of entanglement that is possible for quantum systems and lacking for classical systems, which causes a new notion of information, or whether there are other distinctions based on which quantum information behaves differently.

4.2 Classical and quantum states

If we consider concrete physical systems that carry information, then we see that the same type of system might carry either classical or quantum information. We can take for instance a photon and use its polarisation to encode a qubit of quantum information, but we can also use it to encode classical information (Marinescu and Marinescu, 2012, p.222). So, if it is not the physical object itself that determines the type of information connected to it, then what is? It has to be the distinctive physical properties and processes of the system being utilised to carry the information. To be more precise, it is due to the possible physical states that the system can be in, which can be used to represent information.

As we have discussed in Section 3.1, the main distinction between bits and qubits, if we take the notions as referring to the physical systems, is that the state of the qubits can be not only the orthogonal states $|0\rangle$ and $|1\rangle$, but also any superposition of these states. This means that it can be in a large number (actually infinite) of non-distinguishable states (Nielsen and Chuang, 2010, p.15). What is meant by non-distinguishable here is that there is no measurement that one can perform on a set of quantum states to reliably (with a probability of 1) distinguish these states. Namely, suppose you take the following set of quantum states: $|1\rangle$ and $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$. Then, suppose we have two qubits, each in one of the states above, but we do not know the states. Now, if we want to distinguish them, we need to perform a measurement, to gain any knowledge about the states. But, if we measure these qubits in the computational basis, we will get that the qubit in state $|1\rangle$, will give a measurement outcome of 1, with certainty. However, the other qubit, which is in a superposition of the basis states, will yield the measurement outcome 0 with probability $\frac{1}{2}$ and 1 with probability $\frac{1}{2}$. Hence, we would only be able to distinguish these states of the qubits if we would get the outcome 0. In the other case, we would get 1 for both and there is no way for us to tell them apart.

This is opposed to the classical case, in which we can always distinguish between different states reliably, since a measurement outcome on the same state will always yield the same result. These classical states are defined as disjoint subsets in a phase space, so have no overlap between possible states. According to Caves and Fuchs (1996, p.226) and Bub (2007, p.576), this is what causes the fundamental difference between classical and quantum information. As Caves and Fuchs (1996) summarises:

> "Quantum information refers to the distinctive information-processing properties of quantum systems, which arise when information is stored in or retrieved from nonorthogonal quantum states. More information is required to prepare an ensemble of nonorthogonal quantum states than can be recovered from the ensemble by measurements. Nonorthogonal quantum states cannot be distinguished reliably, cannot be copied or cloned, and do not lead to exact predictions for the results of measurements. These properties contrast sharply with those of information stored in the microstates of a classical system." (Caves and Fuchs, 1996, p.226)

Timpson (2013) agrees with Caves and Fuchs (1996), and highlights this as one of the main distinctions between classical information and quantum information, at least used in the technical sense, as he writes:

> "There's a further option which will mark an important point of contrast with the quantum case: the varying outputs of a classical Shannon information$_t$ source are always —in principle at least— distinguishable one from another: one can tell whether the output was an x_1 or an x_2, for example." (Timpson, 2013, p.23)

Thus, Lombardi et al. (2014) points out that based on this distinction there is a rough division in all research to information:

> "... the widespread way of sorting the concepts involved to the field of information into two groups: classical-orthogonal-distinguishable and quantum-non-orthogonal-indistinguishable." (Lombardi et al., 2014, p.17)

What Lombardi means by classifying 'classical-orthogonal-distinguishable' as one group is that classical information states are orthogonal when taken as subsets of a phase space. According to Bub (2007, p.576), this can be seen analogously to orthogonal subspaces of a Hilbert space. Quantum states can occupy non-orthogonal subspaces of a Hilbert space, which results in these states being not reliably distinguishable, as described above, but they can also occupy orthogonal subspaces. Therefore, it seems that if we restrict a quantum system to solely be in orthogonal pure states, it would behave classically in terms of the distinguishability of states. The distinctive features of quantum information seem to follow from the possibility of encoding it in non-orthogonal quantum states. Thus, it is often concluded that classical information can be viewed as a special subcategory of quantum information (Marinescu and Marinescu, 2012, p.50; Lombardi, Holik, et al., 2016, p.23; Bub, 2007, p.576; Caves and Fuchs, 1996, p.31; Timpson, 2008, p.4).

This argument based on the distinguishability of states would imply that all information can be seen as quantum information. Only in special cases, in which the quantum states are pure and orthogonal we would perceive them as behaving classically. In that case, all information is quantum information. Similarly to how classical physics can be seen as a special case of quantum mechanics (Nielsen and Chuang, 2010, p.51), classical information would then refer to a specific case of quantum information, namely when only considering information carriers in orthogonal states.

4.3 Interaction with information

Now we have considered information carriers and their possible states, we continue the comparison between classical and quantum information based on interaction with information. If we consider the category of information that Adriaans and van Benthem (2008, p.11) refer to as 'Information-A'[17], then in order for an agent to learn new information about its environment, they need to make some kind of observation. In the realm of classical physics, these observations are done by some kind of sensor or measuring system, whether this is the human eye or a physical meter. In general, the goal of measurement in physics is to get

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[17] See Section 1.

a quantification for certain attributes of physical objects, by comparing a specific physical quantity with a known standard quantity of that nature.

Classical measurements are measurements that concern classical objects, which are objects on the macroscale (Jacobs, 2003, p.2). Although some measurements are such that they will change or demolish the measured object or system, in principle, a classical measurement does not change the state of the system. Namely, if one would have access to two identical classical systems, then performing the same measurement on both will lead to the same outcome. For quantum measurements, this is often not the case, as we have discussed in Section 3.3. This is due to the fact that quantum systems can also be in a state of superposition, which upon measurement will collapse to a different state. Hence, if we have two identical quantum systems, such as two qubits prepared in a $|+\rangle$ state. Now, if we measure them both in the computational basis, this can lead to one qubit collapsing to state $|0\rangle$ and the other qubit collapsing to state $|1\rangle$. The qubits are not in identical states anymore after measurement. This shows that simply the act of measuring changes the state of the quantum systems. Therefore, in order for any observer to gain new information about some quantum system, they will interact with the system in a way that differs from the classical case.

The fact that quantum measurements change the state of the measured system is one fundamental difference between classical and quantum measurement, but there are other peculiar features of quantum measurement that distinguish it from classical measurement, as we have discussed in Section 3.3. An important one is the predictability of measurement outcomes. In the case of classical physics, if one would have access to the full knowledge of a system before measurement, then one would be able to predict the correct measurement outcome with certainty. The reason that a classical measurement yields knowledge for an agent/observer is due to the lack of knowledge from that agent about certain aspects of the system. Therefore, the knowledge each agent gains from the same classical measurement can be different, it depends on the knowledge about the system that the agent already possessed beforehand. This is also the case for quantum measurements, but on top of this ‘classical’ uncertainty, there is the fundamental uncertainty of the state of a system. This means that even if an agent has maximal knowledge of a system, they are still unable to perfectly predict measurement outcomes. This is due to the above-described effect of measurement on a quantum system, as the act of measuring can bring about a state change. Furthermore, the *uncertainty principle* by Heisenberg (1927) disallows the simultaneous knowledge of values for certain physical quantities of a quantum system with perfect accuracy. This is the case if one wants to predict the value of observables that are incompatible[18], since they correspond to quantum mechanical operators that do not commute, such as the position and momentum of a particle.

Thus, we can draw a distinction between indeterminacy and ignorance or lack of knowledge about a measurement outcome. In the classical case, having maximal knowledge about the state of the system means that there is no ontic indeterminacy. If we adopt the notion of ‘information’ such as it is used by Dretske (1981)[19], we take information to be objective and independent of an observer. It is the commodity that in principle is capable of yielding knowledge. Hence, using this definition of information, a classical measurement does not

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[18] See sections 3.2 and 3.3.

[19] See Section 2.3.2.

yield new information, it only reduces the ignorance of the agent performing the measurement. This is opposed to quantum systems, in which even if one has maximal knowledge about the quantum state, the measurement outcome remains unpredictable. Therefore, a quantum measurement leads to new information.

Similarly to the case of quantum entanglement[20], for which it was first thought that quantum theory was incomplete and there must be some hidden variables to complete the theory, one could ask whether there is no possible extension of quantum theory that could account for this new information that can be gained by future measurements. This has been answered negatively by a proof provided by Colbeck and Renner (2011). They show even more generally that, assuming that the universe is not completely causally deterministic and thus measurements can be chosen freely, there exists no possible extension (whether it is with hidden variables or something else) of quantum theory that would improve the predictability of future measurement outcomes compared to quantum theory itself. This proves that the unpredictability of quantum measurement outcomes is a fundamental feature of the nature of quantum systems, rather than a matter of ignorance.

The idea that only quantum measurement leads to an information gain aligns with the ideas from Caves and Fuchs (1996). They also provide a method to quantify the amount of gained information from a quantum measurement, even when having maximal information about the state prior to measurement. It seems as if a quantum state must contain an arbitrary amount of information, as each new measurement in an incompatible basis must lead to an information gain. This is in contrast with classical systems, where an agent would acquire new knowledge by making increasingly precise measurements, allowing them to make perfect predictions on the scale of what is measured before, but having unpredictability on a smaller scale. In the quantum case, making new measurements does not lead to more predictability necessarily, as due to the incompatibility of measurements, some physical quantities might be more predictable and others become less predictable after the measurement. It is for this reason that Caves and Fuchs (1996) conclude the following:

> “The information gathered from repeated measurements on quantum systems is indeed drawn from an inexhaustible well, but it is a well of potentialities, not actualities.” (Caves and Fuchs, 1996, p.26)

Thus, in classical physics, the probabilities of each of the possible outcomes of a measurement are unknown to an agent, due to ignorance. One of the possible alternatives is occurring or will occur upon measurement, the agent is simply not aware, due to a lack of knowledge, which one it is. In quantum physics, the probabilities of possible measurement outcomes are due to a mix of both ignorance and ontic indeterminacy. One of the possible measurements will occur if an agent performs measurement in that specific measurement basis, hence they are not actual, but potential, as the system is not necessarily in that state yet. This realisation of potential states is what creates new information. Therefore, this can be seen as a main difference in interaction with classical and quantum states as information carriers.

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[20] See Section 3.2.

4.3.1 Reversibility of measurements

In the Copenhagen interpretation, due to the interaction with a measuring device and the quantum system, the act of measurement or observing is an irreversible process (Faye, 2019; Freire, 2022, p.537). This is reflected in the collapse of the wave function, which is irreversibly reduced to an eigenstate of the particular observable. An interesting thesis is discussed by Zurek (2018, p.6), who argues that it is the retention of the information that is gained by quantum measurements by some observer, that causes the measurement to be irreversible relative to this observer. Zurek shows in a brief information-theoretic argument, that in the case of classical physics, in which Newtonian dynamics can be used to describe a system's state change, the information that an observer holds does not affect the in-principle reversibility of measurements made. However, for quantum measurements, when some observer acquires new information through measurement and communicates, copies or stores this in some form, this causes the inability to restore the overall system consisting of the observer/measuring device and measured system to a pre-measurement state.

Braginsky et al. (1980, p.547) already briefly touched upon this thesis, so this idea is not new. Braginsky et al. (1980) highlight that a measurement that leads to an unpredictable disturbance of the state of the particle is a direct consequence of information distraction. If this information is stored somewhere in the universe, outside of the particle's wave function, this causes the inability to reverse the measurement. For this one would need to reinsert all the information back into the system and leave no trace behind. Zurek (2018, p.11) argues that this is an entirely different reason for irreversibility than the decoherence of the system, which can be seen as a loss of information to the environment, which is usually suggested as the main reason for the irreversibility of quantum measurement. If we adopt the view of Caves and Fuchs (1996) about information gain in quantum measurements, which is different from the knowledge gain or diminishing of ignorance in classical measurements, this fits the argument of Zurek (2018). As we argued above, if we adopt the notion of information as proposed by Dretske (1981), we conclude that there is no new information gained in classical measurements. This can be seen as an explanation as to why they are reversible. Whereas for quantum measurements this is not the case, since they lead to an increase in information. This information would need to be reinserted in order for it to be reversible, which seems impossible in practice.

4.3.2 The relation between measurement types and information gain

We could argue that there is a relation between the accuracy of the measurement, which is connected to the change it brings about in the state of the system (Braginsky et al., 1980, p.547), and the amount of information gained from the measurement (in which we take information again as an objective, observer-independent notion). If we consider for instance a system consisting of multiple qubits which are not entangled, then one could perform a measurement that determines the spin in a certain direction for one of the qubits. This causes a state change in the system concerning only this specific qubit. But, the measurement outcome also does not provide accurate information about the state of the system as a whole. In order to learn more about this, one could perform a measurement that evaluates the spin of all the qubits in the system. This increases the information gained from the measurement, but causes a bigger disturbance of the system, as the observable describing the measured quantity gets projected to eigenstates for each of the qubits.

In the case of a multipartite entangled system, any measurement one performs on one part of the system will cause a change to the entire entangled system. This is because if one measures, one causes a collapse of the wave function of the entire system, as discussed in Section 3.2. However, they also lead to an increase in information for all parts of the system, albeit this information might not be available to an observer. This is for example the case for Bob in the teleportation protocol, who only gets access to the information change by Alice's measurement, after he knows in which basis to measure. Bob's subsequent measurement does not change the state of the system anymore, but it also does not lead to an increase in information. The measurement only leads to an increase in knowledge for Bob but does not change what one in principle could get to know about the system, which is the information, following Dretske (1981). Either way, the type of quantum measurement or interaction with a quantum system thus plays an important role in describing the information flow.

As we have discussed in Section 3.3, there are different kinds of quantum measurements. We have distinguished between measurements of the first kind, in which repeated measurement leads to the same measurement outcome, and measurements of the second kind, which change the system in such a way that the measurement outcomes of repeated measurement are not the same (Pauli, 1980, p.75). In the case of measurements of the first kind, we can distinguish further by looking at quantum nondemolition measurements. These kinds of measurements are of the least disturbing type, as they require choosing an observable of which the uncertainty does not grow due to free evolution of the state, thus the predictability of the measured outcome of this observable remains as precise (Braginsky et al., 1980, p.548). Hence, a subsequent measurement is guaranteed to give the same outcome as the first. This has two main advantages: firstly, one can track the evolution of an observable in the system without any back action caused by measurements. Secondly, one can perform subsequent measurements that do not have perfect sensitivity, but by being able to repeat the measurement, gain a higher overall sensitivity (Distante et al., 2021, p.253603-1). There is extensive experimental research on quantum nondemolition measurements (Braginsky et al., 1980; Distante et al., 2021), but is often very difficult to realise in practice. Due to the nature of quantum nondemolition measurements, they can be seen as the most "classical" of all types of quantum measurements, as they affect the state of the system in a minimal way and repeated measurements are leading to the same results.

In general, we can understand quantum measurement as going from quantum information to classical information: we go from fundamental uncertainty about a state to a precise numerical result for a specific quantity. The other way around, so generating quantum information from classical information, is called *preparation* (Marinescu and Marinescu, 2012, p.49). This shows that there is a way to switch between the two based on interaction with a system. We could convert classical information to quantum information and back to classical information. However, this cannot be achieved while making sure that the information is indistinguishable from the information we started with. This is exactly because quantum measurement is a probabilistic process which alters the state of the system and is an irreversible process. Thus, Marinescu and Marinescu (2012, p.50) argue that there is a qualitative difference between classical and quantum information. They can have the same physical information carriers, such as photons or electrons, but the physical processes diverge. As discussed in Section 4.2, they conclude that classical information can be seen as a particular form of quantum information. This is exactly because for orthogonal quantum states, if some quantum system is in one of these states, this can in principle be measured without altering the state. For instance, take the computational basis states $|0\rangle$ and $|1\rangle$, which upon a measurement in the computational basis "collapse" to themselves and thus do not change. Therefore, they have a classical counterpart.

Jacobs (2003, p.2) agrees with the view of classical measurement as a subcategory of quantum measurement and provides a probabilistic account that is a unified description of both. Taking classical measurements as a subset of quantum measurements is similar to the idea of taking classical states as a subset of quantum states, discussed in Section 4.2. However, if we adopt this view, this leaves us with the question of how we can explain information gained from quantum measurements as something different than the increase in knowledge from classical measurements if they are both of the same kind. After all, if we take information to be an observer-independent notion, then following the above reasoning we seem to get a contradiction. If classical measurement is a subset of quantum measurement, why would performing a quantum measurement lead to an information increase, whereas a classical measurement does not? This type of question needs to be considered if one takes the approach of merging classical and quantum information.

4.4 Accessible and inaccessible information

Let us recall what we discussed in Section 3.1. We distinguished between accessible information and specification information, a distinction as presented by Timpson (2013, p.47). To briefly recall what these notions mean: the specification information is the information that one needs to prepare a system, whether it is classical or quantum, in a specific state. The accessible information is the information that is encoded in the system and that is thus accessible via measurement. We established that in the case of classical information carriers, these two notions of information coincide. But, when we consider classical information encoded in qubits, there is a difference between the two types of information. The specification information of a qubit can in principle be limitless, due to the possibility of superposition of states. This is not true for the accessible information, which depends on what information can be gained through measurement. There is a bound on the accessible information, due to Holevo (1973), which establishes what is the maximum amount of information that can be known about a quantum state and thus that can be obtained by measuring. The Holevo theorem tells us that this upper bound is the logarithm with base 2 of the number of orthogonal states of the system. In the case of a qubit, there are two orthogonal states $|0\rangle$ and $|1\rangle$ and thus the maximum amount of accessible information is 1 bit. So, in transforming classical to quantum information, when we prepare a sequence of qubits, we cannot encode more than a single bit of classical information per qubit. And in decoding, so yielding classical information from qubits through measurement, we can also not exceed this one bit per qubit. This is in agreement with the idea described by Marinescu and Marinescu (2012, p.50) that one can transform between the two types of information through measurement and preparation, as discussed in Section 4.3.

4.4.1 (In)accessible information in quantum teleportation

With these notions of accessible and specification information in mind, we can revisit the quantum teleportation protocol. In transferring a quantum state from Alice to Bob, it seems as if an in principle infinite amount of classical information is transferred, as this would be necessary to specify the exact quantum state to an unlimited accuracy. However, Timpson (2013, p.79) points out that even though this specification information is unboundedly large, most of this information is inaccessible to Bob. Namely, the information that Bob can access is only via measurement on the state of the transmitted system and this corresponds to the accessible information described above. When Bob performs a measurement on the teleported qubit, he can at most retrieve one bit of information out of this. Thus, this is in correspondence with the Holevo bound. Only if Bob had access to multiple identical states, he would be able to gain more information about the exact state of the qubit premeasurement. Based on this observation, Timpson (2013, p.80) concludes that the quantum teleportation protocol does not give rise to a paradox. Only if Bob would have been able to yield more than two bits of information from the transferred system, it would mean that there was faster-than-light communication of information, which would be in contradiction with the theory of relativity.

However, this reflection by Timpson does not provide a true explanation of how the information carried in the state of the system is transmitted. The accessible information is only a small fraction of the total information carried by the system. So, even though the accessible information might not exceed the amount of information sent over the classical channel in the protocol, the specification information still does. This problem would only be solved if we would adopt the view that in order for information to be transmitted to some agent at a location, it needs to be accessible by that agent. This view is referred to by Timpson (2013, p.81) as 'the attitude of the *conservative classical quantity surveyor*'. Then, in order to transmit the entire quantum state from Alice to Bob, Bob would need to have access to so many copies of the same state, that they could identify the exact original state via measurement. However, in order to get all these states to Bob, Alice would need to carry out the quantum teleportation protocol for so many states, and ultimately in doing so send an infinite number of classical bits to Bob. But then these bits could have also been used to specify the quantum state directly, as now there is no real difference between the amount of classical information sent and the specification information of the transmitted quantum state. So we can conclude that on this account, there seems to be no added value of entanglement for transmitting information in the communication protocol.

As we have seen, there is no consensus on exactly how and how much information is transmitted in the teleportation protocol. Generally speaking, we are certain that in the process there is one qubit's worth of information transmitted, if we stick purely to quantum information. Only when we want to convert this to classical information, we would need an unlimited number of bits as specification information. This information is not accessible to Bob, hence we can question whether this amount of information has really been transmitted, or whether it even makes sense to specify the amount of transmitted quantum information in terms of classical bits. After all, the state that is at Bob after teleportation has a certain specification information. But this number of bits is only required if one actually wants to specify the state. This does not mean that this classical information needs to be encoded or contained in the system directly. All in all, the explanation of the teleportation protocol does rely on our interpretation of quantum information and its relation to classical information.

4.4.2 Preparation information and missing information, a different approach

Going back to the general idea of separating accessible and specification information: the separation of two types of information in storing and retrieving information in quantum states is also described by Caves and Fuchs (1996, p.14), but they make a different distinction. They also take on one hand the specification information (or preparation information as they call it), to be the information that is necessary to prepare a quantum system in a particular state, given a system with several possible states. On the other hand, there is *missing information*, which is the information that one needs to give a maximal description of the system. This information might be acquired through measurement. In the case of a classical system, the preparation information and missing information correspond. Namely, a measurement determines what is the state of the system and this is enough to provide a maximal description of the system. In the quantum case, they are not the same. If the possible states of the system are non-orthogonal, it is impossible to distinguish them perfectly by measurement, due to the intrinsic unpredictability of quantum mechanics. Upon measurement the wave function randomly collapses to one of the possible states and the system is in one single basis state. A measurement can provide maximal information about the system in this post-measurement state, as the measurement causes the system to be in the measured state with certainty. But this state need not be in the possible pre-measurement states, hence it is impossible to figure out in what state the system was prepared originally. Therefore, the missing information cannot be larger than the specification/preparation information. Clearly, the choice of measurement basis influences how much information you acquire about the system. Therefore, to get a unique measure for missing information, Caves and Fuchs (1996, p.15) define the missing information as the minimum amount of information one needs to gain from a measurement of all possible measurement bases to provide a maximal description of the system. If one already knows the exact state of the system, the missing information is 0. Lastly, Caves and Fuchs (1996, p.19) also provides a notion of accessible information, defined as the ‘useful’ part of information one can gain through measurement to determine the prepared state of a system. In general, the accessible information must be less than or equal to the missing information.

The main difference between the concepts of preparation and missing information described by Caves and Fuchs (1996) versus specification and accessible information described by Timpson (2013), seems to be the subjectivity of the notions. Even though both of them do not explicitly treat this topic, the distinction between these types of information provided by Timpson (2013) seems to be agent independent. The accessible information is the information one can in principle acquire through measurement, solely depending on the preparation of the system instead of on the knowledge state of the agent. This is fitting with the notion of information in the framework of Dretske (1981), which we also used in the previous section. On the other hand, the notions provided by Caves and Fuchs (1996) depend on the knowledge state of the agents. Specifically, the missing information becomes 0 if the agent is already fully aware of the state of the system. Therefore, this notion of information is subjective and would fit with an agent-dependent account of information, such as the information-as-range approach discussed in Section 2.3.1. Thus, even though at first glance these two conceptual interpretations may seem very similar, the underlying assumptions of information being objective or subjective are fundamentally different.

The important common idea in the accounts from Timpson (2013) and Caves and Fuchs (1996) is the clear distinction between classical and quantum information as two different types of information. If we consider merely the quantum case, there is no point in considering the missing or accessible information, as these notions regard the encoding or decoding of classical information in quantum systems. If there is no source of classical information, the quantum information is simply measured in qubits (in the sense of a unit of information). Then, one qubit (in the sense of a two-level quantum system) carries exactly one qubit (unit of information) worth of information (Timpson, 2008, p.5).

To conclude this section, we argue that we can make a general distinction between bits and qubits as information carriers. In the case of a bit, all information is in principle accessible and in the case of a qubit, this is different. It seems that we are only able to access the 'classical part' of the information carried by a two-level quantum system, which becomes available to us upon measurement. It is exactly the quantum properties such as superposition and entanglement that cause certain information to be inaccessible by some local observer and these properties vanish upon measurement. If we have qubits that are only in orthogonal states, we get that the specification information becomes equal to the accessible information, as we can read out all information carried in the states by measurement if we know the correct basis to measure in. If we have qubits in non-orthogonal states, we cannot do this, as we cannot simply distinguish the states by measurement (Caves and Fuchs, 1996, p.27). In that case, the specification information is much larger than the accessible information. Therefore, we see that is possible to encode all information carried by classical bits in qubits, as these would simply occupy orthogonal states. But the other way around is not so obvious. It only seems possible if we have access to an infinite number of bits to specify the exact state of a qubit, or an entangled pair of qubits, as we learned from the teleportation protocol. The question is whether we can say that this specification information is really contained by a qubit and thus whether it is meaningful to make a comparison in this way.

4.5 Locality of information

As we have discussed in Section 2.1, information is not a physical substance, regardless of whether we adopt the everyday or technical definition of the word (Timpson, 2013). It can therefore not be localised in space and we cannot consider its location in the same way as one would do for the location of particles or objects. Therefore, the question 'where is information located?' seems meaningless. Still, there are a few things we can say about both classical and quantum information and their locality.

First of all, what do we mean when we say that something 'contains information' if the information is not physical? Let us recall the distinction we made between type and token, which was also discussed in Section 2.1. We can see the distinction between them as types being kinds of things and tokens being concrete instantiations of these types. According to Timpson (2013, p.24), an information source produces tokens and in order to communicate information, the aim is to create a token of the same type as the original token of the information source, at a different destination. These types are abstracta: they are not physical substances, things existing in the actual universe. Hence, if we agree with Timpson to view pieces of information as being certain types rather than tokens of some type, then clearly we cannot say that these are located in some carrier or system. Following Duwell (2008, p.205), when we say that classical information is 'contained in a system', this can only indicate that some system is correlated to an information source. Classical information transfer is then establishing correlations between the different ends of a communication system. If a system is correlated to a source, the pieces of information, which are the types produced by a source, can then be reproduced at a destination. Duwell (2008, p.205) distinguishes ‘transmitting information’ from ‘communicating information’. The first is established by correlations and the latter is due to the reproduction of information at a different location. So even though this item of classical information is not actually contained in a system, as it is not a physical substance and cannot be contained in anything, we can talk about systems being correlated with an information source, and these systems are bound to certain locations. Now we will analyse what is meant exactly by these correlations.

4.5.1 Comparison of classical and quantum correlations

Correlations are everywhere in nature. As we discussed in Section 3.2, it turns out that we can make a distinction between classical and quantum correlations, based on the phenomena that occur in quantum systems that do not occur in classical systems. The most important one is quantum entanglement.[21] It is clear from Bell’s theorem (1964) that between quantum systems there are certain degrees of correlation that one could never achieve using classical systems, exactly due to the possibility of entanglement. Entanglement seems to force us to accept quantum theory as a non-local theory: instantaneous effects can occur between distant correlated systems. But we can question whether this can also happen in the classical world. It seems to be that this is not the case, at least not in the same way as it happens in quantum correlations.

Consider the following example. Suppose we have two balls, one blue and one red and we put them in two different boxes. Now we would randomly pick one of them to leave behind on Earth and travel with the other to the moon. If we then open the box on the moon and find out that we have a blue ball there, we instantly know that the red ball must be on Earth. These are distant events and might seem non-locally correlated since the information is attained instantly. However, in this case, the colour of the ball in the box is like a local hidden variable, which becomes unhidden upon opening the box. Since Bell showed that quantum theory cannot be completed with hidden variables, the instantaneous effect of quantum correlations is fundamentally different from the classical story described above. Let us consider in some depth what this difference entails, by looking at the properties of quantum correlations.

Maudlin (2011, p.21) sums up the three most important aspects of what he calls the “quantum connection”, which refers to what is usually called entanglement, as described in Section 3.2. It is the connection between qubits (as in the Bell states), where the outcome of measurements on one part of an entangled system not only depends on the intrinsic physical state of that subsystem, but also on the result of a measurement on the other subsystem, as these are perfectly correlated. According to Maudlin (2011, p.22), there are several remarkable features of quantum correlations that set them apart from other types of correlations. These are the following:

- It is unattenuated:
  This means that it does not matter how far we separate entangled qubits, they remain in this state. It is not comparable to a classical physical force or interaction such as gravity, which weakens as physical bodies are further away from each other. The correlation in measurement outcomes is not stronger or weaker depending on the distance.

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[21] See Section 3.2.

- It is discriminating:
  A quantum connection or entangled state can be described as a private agreement between qubits. If we have two entangled photons amidst thousands of other (possibly also entangled) photons, a measurement on one photon of this entangled pair will only influence the measurement outcome of its correlated twin, rather than on any of the other photons surrounding it. The quantum connection depends on the shared history of qubits, as they must have interacted or originated from the same source in the past to gain this type of connection. This is also very different from a classical interaction or force such as gravity, which affects similar objects that are similarly situated in the same manner.

- It is instantaneous:
  From the experiments that have been done so far, we can conclude that the correlation between results of measurement performed on qubits that are in an entangled state is instantaneous, and thus faster than the speed of light in a vacuum, often denoted by the physical constant c. This seems to suggest that information between these qubits travels faster than c, which by Einstein's theory of special relativity is set to be the upper limit at which any kind of signal carrying information can travel through space. Even though for many classical theories it was also thought at first that certain forces were instantaneous, this later turned out to not be the case. Consider again the example of gravity as described above. In this sense, it seems that quantum entanglement violates a fundamental law of the universe.

We should approach this last feature of quantum correlations with some care. It is clear that no relativistic theory allows for faster-than-light communication, as this would require a signal that carries information to travel faster than light. It seems that since quantum correlations are instantaneous, it contradicts this principle from relativity. However, the correlation between two space-like separated entangled qubits does not allow for communication of the type necessary to violate this law. This is due to the inherent randomness of the possible measurement outcomes of the quantum state of the qubits. Imagine a pair of entangled qubits. If one performs a measurement on the first of them, this will yield a random result. If another agent then performs a measurement on the second qubit, this will yield a correlated measurement outcome. But since the first measurement outcome is completely random, this cannot be used to communicate anything to the agent performing a measurement on the second qubit. If one would change the state of the qubit, before measuring to make sure to get a specific measurement outcome, one would break the entanglement and thus lose the means of communicating anything, as the measurement outcomes are not correlated anymore. The confusion of quantum non-locality arises from the projection of our classical intuitions about locality onto the quantum case. It is important to note that the qubits in an entangled pair should not be viewed as separate objects that can instantaneously interact over long distances, but rather as a single quantum object that is described by a joint wave function. So, in the Copenhagen interpretation, the measurement causes this wave function of the entangled state to collapse to a certain value with certainty. But, this does not mean that anything physical is moving or changing. Hence, there is no force or information signal that is travelling faster than the speed of light.

Finally, note that the properties of quantum correlations described above are reflected in the information-theoretic constraints of the CBH theorem (Clifton et al., 2003).[22] The

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[22] See Section 3.5.2.

first constraint forbids superluminal communication of information. The third constraint requires the possibility of entangled states, which causes quantum information to be non-local, as it is not contained by one part of the system.

4.5.2 Occurrence of correlations

Now that we have a better idea of the difference between classical and quantum correlations, let us consider when each type of correlation occurs. It might seem obvious that any classical system, so systems on the macroscale, can only involve classical correlations, whereas quantum correlations apply to quantum systems. However, not every quantum system exhibits these properties necessary for stronger-than-classical correlations. Namely, if we take a quantum system in a separable state[23], then they contain no entanglement, and thus only classical correlations can occur between subsystems. These separable states can be pure or mixed quantum states and can be jointly prepared by multiple distant agents using local operations and classical communication. If correlations occur between observables of the subsystems of some system with multipartite separable states, these correlations are classical (Jaeger, 2009, p.21). This is exactly because we can describe these correlations locally since we can provide a description of the entire state based on products of the state of each (space-like separated) subsystem. Thus, following Jaeger (2009), we could simulate the local measurement outcomes on such states by a local hidden-variable theory, such as was the case for the example with the blue and red balls given above. This means that it is possible to provide common-cause explanations for the occurring correlations between separate subsystems.

On the other hand, quantum correlations can occur between entangled quantum partite systems. The correlations between different subsystems can violate the Bell inequality and are thus different from classical correlations, which cannot exceed this limit. Jaeger (2009, p.22) points out that for pure bipartite entangled quantum states, quantum correlations occur with certainty, whereas for mixed entangled quantum states, it can still be possible to describe the state as a combination of product states and thus describe the correlations locally. This shows that the difference between when classical or quantum correlations occur can be rather subtle. Finally, supposing we have an entangled quantum system in which the subsystems are quantum correlated, what can we say about the location of the quantum information carried by the system? Similarly to classical information, quantum information is not a physical substance (Timpson, 2013, p.82). Hence, talking about some system containing or transferring information should not be understood as containing or transferring a substance, such as water through pipes. In the case of classical information, we explained the 'location' of information in terms of correlations. A system that is correlated to an information source contains information, in the sense of the information type based on the token produced by the information source. In the case of quantum correlations, we cannot use this definition as it is. Exactly because entanglement comes into play. In the case of quantum correlations, the systems produced by a local information source are part of an overarching entangled system. Then if we consider a system merely correlated to the signals produced by this local information source, this system cannot exhibit the effects of the entanglement and reproduce this at a different location (Duwell, 2008, p.212).

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[23] See Section 3.2.

In general, it is very difficult to give an account of the location or locality of quantum information. This is due to the great variety of possible processes of quantum information transfer and the differences in underlying explanations. As we have seen, the possibility of entanglement complicates a definition based on mere correlations between system and source. On top of this, there is an ongoing debate in the literature on how information can be said to transfer in quantum information protocols such as quantum teleportation.[24] In many of these explanations, there is no continuous path in space-time which would guide the information from source to destination and it is questioned whether it is even sensible to consider information as having a location altogether. Duwell (2003, p.491; 2008, p.212) argues that if one wants to retain a notion of the location of information in these cases, they might view it as being dispersed across the systems that are involved in the teleportation protocol. The information is therefore not locatable to a specific place or correlation and thus can be said to be non-local. This is a different notion of non-locality than discussed in Section 3.2, which refers to the violation of quantum theory as a local realistic theory. What we mean here is non-locality as a property of information. Suppose we have two spatially separated agents Alice and Bob who both have access to a subsystem in a shared state. Now they want to extract information from their part of the system, using local operations. Also, they are allowed to communicate over a classical channel with each other. If the information is non-local, this results in that each of the agents will not be able to extract the same amount of information from the system, compared to if they and their subsystems were at the same location. Therefore, Horodecki et al. (2005a) define non-locality as follows:

> "The term *nonlocality* means here that distant parties can do worse than parties that are together, despite the fact that they can communicate classically." (Horodecki et al., 2005a, p.2).

This shows that there is part of the total information of a system which is inaccessible at specific locations. Thus, the difference between the location of classical information being local and therefore reducible to correlations between system and source, and quantum information as being (partly) non-local, can be seen as one of the main distinctions between classical and quantum information. But at the same time also quantum systems can have classical correlations.[25]

4.6 Summary of Chapter 4

In this chapter we have compared quantum and classical information, based on several aspects of information. In Section 4.1, we have established that the 'bit' was first a theoretically neutral unit of information. It became tied to classical information only after the introduction of the 'qubit', demonstrating that quantum information is perceived as a new notion of information. There are some ways in which we can distinguish between the two. First, we have argued that classical information, both in a technical and non-technical sense, requires a physical carrier. In the case of quantum information, due to this possibility of entanglement, it seems that there is a possibility of non-local information exchange (at least when it is combined with using classical information channels) which does not require

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[24] See Section 3.6.

[25] It is for this reason that in the field of quantum information theory, there are propositions to separate the classical and entangled parts of a system and to provide measures of classical correlations in quantum systems (Henderson and Vedral, 2001).

a physical carrier. This leads to the question if it is the possibility for quantum states to be entangled which causes quantum information carried in such states to be different from classical information. This has been expanded in Section 4.2 to the general distinction between classical and quantum systems based on the possible states they can be in. It has been argued that if we restrict quantum system to states that correspond to orthogonal subspaces in a Hilbert space, then they behave classically. Thus, it turns out that the fundamental difference is not the information carrier (systems being classical or quantum), but the way in which the states of the system carrying the information are utilised. If we consider a system that has non-distinguishable states, then it carries quantum information. In this reasoning, classical information would be a subcategory of quantum information.

In a similar way we considered measurements in Section 4.3, since measurements play an important role for the access or gain of information. We have argued that for a classical measurement no new information (if we take it to be an objective and agent-independent notion) is generated, since having maximal information about the state of the system means that there is no ontic indeterminacy. In the classical case we only have epistemic indeterminacy, what we referred to as 'ignorance'. For quantum measurements, there is fundamental uncertainty about the measurement outcome, even when one would have maximal knowledge about a system. A measurement causes a realisation of a potential state of the system and this is what creates information. The retention of this new information by the observer that performed a measurement is what leads to a quantum measurement being irreversible relative to the observer. Furthermore, we proposed a relation between the amount by which a system is affected or changed by the measurement and the amount of information gained from it. In line with this, we can view classical measurements as subcategory of quantum measurements, namely as measurements that, in principle, do not change the state of the system. Furthermore, we established that in the case of classical information carriers (systems of which the states that can only be described by orthogonal subspaces), accessible information and specification information coincide. But, in case we have classical information encoded in qubits, the accessible information is less than the specification information. In general, we can only access the 'classical part' of the information carried by a qubit, if we take it to denote a two-level quantum system. This becomes available upon measurement.

Lastly, in Section 4.5, we distinguished between classical and quantum correlations. We have seen that quantum correlations are instantaneous, unattenuated, discriminating. These type of correlations only occur in entangled quantum systems. We considered the locality of information and pointed out that in the classical case, the 'location' of information can be given in terms of the correlations between a system and an information source. We argued that it is not possible to use this definition for quantum information, due to the different nature of quantum correlations. This leads to the information being dispersed across the entangled systems and thus being non-local.

5 Comparison of principles of classical and quantum information

In the previous chapter, we analysed different aspects and fundamental properties of both classical and quantum information. We have compared and contrasted these notions, to see if they come down to strictly different notions of information, or whether they could be viewed as of the same kind. To extend this analysis, we will now compare the notions by considering their qualitative differences and the information-theoretic principles that we have discussed in chapters 2 and 3. Based on this analysis we will come to our concluding remarks.

5.1 Creation of information

As we have discussed in Section 3.5, the no-cloning and no-broadcast theorems prohibit the possibility to duplicate an unknown state $|\psi\rangle$ of a quantum system. Specifically, the first tells us we cannot copy an unknown pure state of a qubit. Thus, there does not exist a unitary transformation U such that we have:

$$U(|\psi\rangle\,|s\rangle) = |\psi\rangle\,|\psi\rangle \tag{10}$$

where $|\psi\rangle$ is an arbitrary pure quantum state and $|s\rangle$ is some standard pure state. The no-broadcast theorem generalises this to mixed quantum states, so it is impossible to copy any arbitrary quantum state. Hence, we can never end up with multiple systems in the same state, but we are allowed to transfer an unknown state to another quantum system (which is done by the teleportation protocol) or swap an unknown state from one system to another system (using a SWAP gate (Pathak, 2013, p.144)). These two no-go theorems follow directly from the linearity of quantum theory, and the proofs are rather straightforward. In this section, we will see that also from an informational point of view, it is intuitive that the cloning of quantum states should fail, as it amounts to the creation of information. Namely, two copies of the same quantum state contain more information than the information contained in a single copy (Jozsa, 2004, p.80). This is in contrast with classical states, which can be copied without limit and for which having copies does not lead to an increase in information.

Let us recall that often it is not possible to perfectly distinguish quantum states since they can be non-orthogonal to each other or they can be in a state of superposition. Hence, it is not always possible to determine in which state exactly a given quantum system is prepared. We must at least know in what basis to measure. This basis is a certain orthogonal set of states onto which the unknown state can be projected to get a reliable measurement outcome. Thus, in general, without knowing this, it is impossible to determine an unknown state of a single quantum system. According to Timpson (2013, p.51), this impossibility of establishing the quantum state of a single system is logically equivalent to the impossibility of cloning states. This can be seen as follows: suppose we could determine an unknown state of a given system, then we could create a device which would generate exactly a copy of this state, by preparing another qubit in this state. We have then created a copy and there is no limit on the number of times this process can be repeated, so on the number of copies. For the other direction, suppose we can clone an arbitrary quantum state. Now in order to exactly determine the state of a given system, we would just clone the state of this system until we have sufficiently many systems that are all prepared in the same state. We could then perform different measurements on each of these systems, such that each measurement increases the knowledge we have about the identity of the original unknown state. Hence, the impossibility of cloning turns out to be a principal property of quantum information, as it is equivalent to the fundamental uncertainty of states of systems that lies at the heart of quantum theory.

When we compare the state spaces of bits and qubits, which are all the possible states of the system, we establish one main difference between those. Qubits can be in a state of superposition, whereas bits are always in one of two distinguishable possible states.[26] This entails that qubits can be in a very large number of distinct but non-distinguishable states. This leads to the possible difference between accessible and specification information of quantum systems, whereas these notions always coincide in classical systems.[27] The possibility or impossibility to clone information states is another important feature based on which we can distinguish classical and quantum systems. In the case of bits, in the sense of two-state systems carrying classical information, we are free to copy their value without any restriction on the number of copies. This also holds for unknown bit values. This is in direct contrast with the no-cloning and no-broadcast theorems for quantum information. Following Shen et al. (2011), the fact that classical information can be copied is because of the possibility of perfect measurement of classical states. A perfect measurement means that the measurement does not alter the state of the measured system and can distinguish between states of the system perfectly. Shen et al. (2011, p.17) define the process of cloning an unknown state as the combination of first measuring the unknown state and then preparing another system in this measured state. Since we do not have perfect measurements in quantum mechanics, we cannot follow this procedure for copying/cloning as measuring and preparing. This difference has also been pointed out by Timpson (2013, p.50-51). If it would not be possible to have perfect measurements of classical states, classical cloning would also not be possible. As Shen et al. (2011, p.17) briefly show, cloning by a linear mapping is also not possible for linear classical states. Thus, the difference between information being cloneable or not seems not directly dependent on it being classical or quantum, but rather on the possible measurement of it. This, again, boils down to the possible states of the system carrying the information and whether these states are (non-)orthogonal, as discussed in Section 4.2.

5.1.1 Connection between no-cloning and other principles

To get a better understanding of the information-theoretic principles surrounding the (im)possibility of cloning states, let us consider what the consequences are if quantum cloning, or more general cloning of non-orthogonal states, would be possible. First of all, we have seen that this would mean we could distinguish between two quantum states with unlimited precision. Namely, we could clone any unknown state to get as many copies as desired to perform in principle infinite measurements. As a result, we would get a violation of the Holevo bound. As in this case, any transferred unknown quantum state can be determined precisely by the receiver. So, rather than having an upper limit on the accessible information of 1 bit per qubit, the receiver can now access a lot more information from a single-qubit channel (Jaeger, 2009, p.207). But how much information exactly? If quantum cloning is

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[26] See Section 1.

[27] See Section 4.4.

possible it would mean that we would be able to transfer an infinite amount of classical information using one qubit. This might seem contradictory, but this is due to the fact that in principle we can encode a bit string of an arbitrary length in a quantum state (Nielsen and Chuang, 2010, p.15). That is to say, because of the possibility of superposition, the number of possible states of a qubit is infinite and thus for each arbitrary sequence of bits, we can assign a quantum state. After transferring the qubit, the receiver can then recover all classical information by cloning the quantum state, as described above. It is only because cloning is not allowed, that this information cannot be extracted, as the superposition collapses and seems irretrievable lost upon measurement.

Secondly, if cloning would be possible, this means that the accessible information and specification information of a quantum system become the same (Timpson, 2013, p.50[28]. This can be seen from the observation above, that one can identify unknown quantum states given the possibility of cloning. Suppose some agent Alice sends one of two non-orthogonal states to Bob. Bob is unaware of which state. Given the possibility of cloning, Bob could distinguish between the two non-orthogonal states with certainty, as they can identify these states with arbitrary precision. In this way, Bob can reliably determine which state was sent and thus access all sent information in the system, instead of only gaining classical information after performing a single measurement. Hence, the accessible information is the same as the specification information. Nielsen and Chuang (2010, p.531) point out that also the other direction holds, namely that the accessible information is less than the specification information as a consequence of the no-cloning theorem. Thus, we see an equivalence between the no-cloning theorem and the distinction between accessible and specification information. This equivalence highlights that the impossibility of cloning is intertwined with informational aspects of quantum theory.

Lastly, the possibility of cloning would lead to a violation of the no-communication theorem. This theorem prohibits faster-than-light communication using shared quantum states.[29] This can be seen from the following reasoning. Suppose we have a pair of qubits in an entangled state, such as a Bell state. Now, two agents Alice and Bob that are spatially separated both have access to one qubit of this pair. If Alice performs a measurement on her qubit, the total state of the system will collapse. Therefore, also the state of the qubit of Bob will collapse to a single basis state, as described in Section 3.2. Now if Bob is able to clone this qubit, by the steps described earlier in this section, he would be able to determine with certainty what the state of the qubit is. Given that the two qubits are correlated, Bob could determine what measurement was performed by Alice. Hence, this can be used to communicate information instantaneously even when spatially separated, which would mean that there is superluminal information transfer. It is thus because of the impossibility to clone non-orthogonal states, that we do not reach a contradiction between quantum theory and the theory of relativity on faster-than-light information transfer (Fan et al., 2014, p.14). These constraints have shown to be fundamental principles in an information-theoretic characterisation of quantum theory by the CBH theorem.[30] The impossibility of cloning corresponds to principle 2, and the impossibility of superluminal information transfer is established by principle 1 from the CBH theorem. The scenario described above thus indicates that there are strong links between the information-theoretic constraints of the

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[28] See Section 4.4

[29] See Section 3.5.1.

[30] See Section 3.5.2.

CBH theorem.

In conclusion, if cloning quantum states, or specifically non-orthogonal states, would be possible, this has far-reaching consequences. Therefore, it is no surprise that the no-broadcast theorem, a more general form of the no-cloning theorem, is one of the three principles of the CBH theorem. The goal of the theorem is to characterise any quantum theory in terms of information-theoretic constraints. The impossibility of cloning non-orthogonal quantum states seems to be at the root of many other properties of quantum information. Furthermore, the last observation even points out a relation between two of the CBH information-theoretic constraints. Namely, the impossibility to copy quantum states is a necessary condition for the impossibility of faster-than-light information transfer. Thus, we have argued that constraint 2 of the CBH is a necessary, but maybe not sufficient, condition of constraint 1.

5.1.2 Information increase by cloning

A possible explanation for the strong relationship between the impossibility of copying quantum states and other principles regarding quantum information is that cloning of quantum states can be seen as the creation of information. On the contrary, the cloning of classical states or orthogonal quantum states does not lead to such an information increase. Namely, as discussed above, copying values of classical bits does not lead to a gain in information. An agent cannot get to know anything new by having more copies of the same classical state at their disposal. Whereas the cloning of quantum states would lead to the possibility to determine the quantum state with unlimited precision, something that is not possible without cloning. Also, the fact that the accessible information would then be equal to the specification information, whereas before it was strictly less than that, suggests that there is some kind of information gain.

Horodecki et al. (2005b) embrace this idea that cloning quantum information states leads to an increase of information, although cloning classical states does not lead to such information increase. In this paper, Horodecki et al. aim to show that general principles of no-cloning and no-deletion in any theory with linear dynamics follow if it is the case that two copies contain more information than one copy. They explicitly distinguish between a subjective notion of information, in which it represents knowledge, and an objective notion, in which it can be treated as a property of a physical system (Horodecki et al., 2005b, p.2042). As they take the view that any consistent description of nature must reflect its reality, they assume information to be physical and thus objective. Then they argue that it is reasonable to postulate the principle of conservation of information, in analogy to many other physical quantities such as energy. The conservation of information is then one of the fundamental laws of nature.

In the rest of the paper, Horodecki et al. (2005b) show that if we assume the conservation of information, then cloning is not allowed for any theory with linear dynamics in which multiple copies contain more information than one copy. What is meant by this, can be seen by an example. In the classical case, one can have an information source that outputs bit value 1 if the answer to a certain question is 'yes' and 0 if the answer is 'no'. Now suppose the source would output 11 and 00 for 'yes' and 'no'. In that case, the source is equally informative, since the bit values 1 and 0 were already distinguishable, so this does not change when outputting 11 or 00. But in the quantum case, we would have that giving more copies of the same quantum state as an output of an information source, leads to an increase in its informativeness. This is exactly due to the reason that the quantum states often are not perfectly distinguishable to begin with and thus can be better distinguished given more copies to work with. Since it is indeed the case for quantum mechanics that multiple copies contain more information than one copy, as is shown by a brief argument based on the von Neumann entropy of the input and output states, Horodecki et al. (2005b, p.2047) conclude that cloning of quantum states is not possible if we assume the conservation of information to hold. Horodecki et al. (2005b) have thus reached the same conclusion as by the standard proof of the no-cloning theorem, as established by (Pathak, 2013, p.124) and (Nielsen and Chuang, 2010, p.532). However, they did so via a different route instead of deriving it from the unitarity of quantum mechanics. They take a general approach of first considering only the conservation of information and then applying the insights to the theory of quantum mechanics specifically. Thus, in their approach the notion of information is central.

All in all, in this section we have seen that the no-cloning theorem is tightly connected to different aspects of (quantum) information. The principle highlights an important distinction between classical information in the form of bit values, which can in principle be copied without restrictions, and quantum information in quantum states of qubits, which can often not be copied. We have seen that the impossibility of cloning is equivalent to the impossibility of the identification of quantum states and the impossibility of faster-than-light communication. Furthermore, if cloning of quantum states would have been possible, this would mean that the creation of information is possible, as multiple copies contain more information than one copy. As Horodecki et al. (2005b, p.2046) remark, what is important here is the quantum nature of the objects that are to be cloned, by which they specifically refer to states being non-orthogonal. Classical inputs, which consist only of orthogonal states, do not have this property. This illustrates again that classical information might be taken as a subcategory of quantum information which only concerns orthogonal states, as we discussed in Section 4.2.

5.2 Erasure of information

As we discussed in the previous section, the cloning of quantum states amounts to the creation of information. This naturally leads to the question of whether the deletion of quantum states then would mean a decrease in information. Following the argumentation of Horodecki et al. (2005b, p.2045-2046) this would indeed be the case. If we assume that information is conserved, then the fact that no increase of information is allowed would lead to the no-cloning principle, whereas the prohibition of a decrease of information would lead to the principle of no-deletion. The no-deletion principle, as presented in Section 3.5.1, tells us that it is impossible to erase an unknown quantum state, which means that there is no unitary transformation U that maps an unknown quantum state $|\psi\rangle$ to a blank state (Pati and Braunstein, 2000, p.165):

$$U(|\psi\rangle|\psi\rangle\,|A_0\rangle) = |\psi\rangle|\Sigma\rangle\,|A_\psi\rangle \tag{11}$$

In this notation of the mapping, the states $|A_0\rangle$ and $|A_\psi\rangle$ are the initial and final states respectively of some auxiliary qubit that is necessary for the transformation. U is a unitary transformation and $|\Sigma\rangle$ denotes the blank state, which is some standard qubit state. In the case that $|\psi\rangle$ is a known quantum state, the no-deletion theorem does not apply and it is always possible to have a unitary transformation that transforms it into a blank state.

An important note should be made here, as the no-deletion theorem applies only to closed quantum systems.[31] Closed quantum systems are systems that do not interact in any way with other systems. They exchange no matter or energy with their environment, so the evolution of the system over time can be described strictly by unitary transformations (Nielsen and Chuang, 2010, p.81). This also solves the apparent contradiction between the impossibility of information being deleted from a quantum system and the result of a measurement on a quantum, in which information is erased or lost. Namely, if we consider a quantum system and measure its position, followed by a measurement of its momentum, the information about its position will be deleted following Heisenberg's (1927) uncertainty principle. The same holds for any pair of non-commuting operators of quantum observables.

On the other hand, for open quantum systems, it is possible to delete information from the system. The act of measurement gives rise to the measured quantum system becoming an open system. This is because the information that is in the system is then transferred to the mind of the observer (Shen et al., 2010, p.487). In an open quantum system, information can be deleted and thus the system becomes vulnerable to destruction. This does not contradict the law of conservation of information that is postulated by Horodecki et al. (2005b) since we can view the measured system together with the measurement apparatus and observer as a new system, in which the total amount of information remains the same, it is only redistributed.

5.2.1 The role of measurement in creation and erasure of information

Even though both the impossibility to create and to erase information in a closed quantum system stem from the conservation of information, there are noteworthy differences. For an open quantum system, perfect deletion of a quantum state turns out to be possible, but cloning of states is still not allowed. This can be a consequence of the role of measurement. The deletion process, in both classical and quantum systems, can be seen as a combination of measurement and transformation (Shen et al., 2011, p.19). Given a system in an unknown state, whether this is a bit state or a qubit state, one first needs to know the state. This can be achieved through measurement. Subsequently, one can transform this state into a blank state, given some transformation and in this way delete the original state. In the case of quantum systems, these transformations are unitary transformations (Shen et al., 2011, p.19). These transformations are therefore reversible, but the measurement that was performed to get to know the state of the quantum system is not. This is because the measurement might change the state of the system, as the state of the system collapses to one of the eigenstates of the measured observable. Hence, quantum states in an open system can be perfectly deleted and deleted quantum states of open systems cannot be recovered. This is fundamentally different from deleted classical states, which in principle could be recovered (Shen et al., 2011, p.19). In general, the process of deleting a state by first measuring it and subsequently transforming it is the same in the case of classical and quantum states. However, as Shen et al. (2010, p.487) points out, the difference is that in the classical case the initial state is known by the measuring agent, whereas in the quantum case, this state the agent is only aware of the quantum state after the measurement.

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[31] See section 1.1.3.

A quantum measurement thus means a change in the state of the system. In the case of deleting states, this is not relevant as the state of the system needs to be transformed anyways. However, in the case of cloning a quantum state, it is very relevant, as it means that measurement does not leave the state to be copied intact. A measurement can also not provide full information about an unknown quantum state, the specification information that is necessary to construct the state, but only provides the accessible information. This means that performing a measurement is not helpful in cloning a quantum state. This shows the incomparable role of measurement in creating and deleting information. Thus, in order to clone a state, the state must already be known by an agent. This is another important difference with cloning classical information states, as a measurement here provides full information about an unknown state. In that case, the state is also not altered and then one can prepare exact copies of the state.

Thus, the possibility of deleting an unknown state seems not to be dependent on a system being classical or quantum, but rather on a system being open or closed. In the case of a closed classical system, there is also no possibility to have a linear mapping that deletes an unknown quantum state. In a closed system, there is no possibility of measurement that causes information to be deleted or lost, such as is the case for incompatible measurements. On the other hand, it is possible to erase information in both a classical and quantum open system via the method of measuring and transforming the state. But, as Shen et al. (2010, p.487) conclude, the most important difference might be that classical states are not 'unknown' in the same way as quantum states can be unknown, since it is always possible to gain full information about them via measurement.

In general, the prohibition to delete and create information, entailed by the no-deletion and no-cloning theorems, follows from the idea that information needs to be conserved. In the classical case, when we consider orthogonal states, multiple copies do not contain more information than one copy and deleted information is recoverable. This is why copying or deleting states is not prohibited. In the case of non-orthogonal states, information contents of multiple states are different, which leads to constraints on the manipulation of these states. All in all, we have seen that the notion of information underlies the fundamental workings of the quantum formalism, and the role of information is tightly connected to measurement. For this reason, Horodecki et al. (2005b, p.2047) conclude that the laws of nature might be seen as constraints on information processing. This close link is in agreement with the idea of Clifton et al. (2003). They proposed the CBH theorem to show that it is possible to provide an information-theoretic characterisation of quantum theory. The fact that creating and erasing information is intertwined with different fundamental aspects of quantum theory, as we have seen in this and previous sections, reinforces this thesis.

5.3 Relating semantic information and quantum information

5.3.1 Technical vs. everyday information

Up until now, we have mainly considered quantum information in the technical sense, such as it is often used in quantum information theory. But the conception of information is broader than this. The distinction that was proposed by Timpson (2008, p.11), between everyday information and technical information, is one that could also be applied to quantum information. As discussed in Section 2.1, the prime difference between these notions of information is that the notion of information in the everyday use of the word is tied to meaning. Information carries meaning and is about something. It is strongly connected to knowledge and language. On the other hand, information in the technical sense of the word refers to how the concept is used in mathematical or physical theories, such as in information theory by Shannon (1948) or the extension of it to quantum information theory. In these settings, the focus is on quantifying information often using a probabilistic approach, correlating states of systems. In this context, the intentionality (or the 'aboutness') of the information or the meaning of such states is not relevant.

But how does this division of everyday information and technical information relate to the notions of semantic and non-semantic information? Timpson barely mentions influential accounts of semantic information, such as the ones we discussed in Section 2.2.[32] Because of this lack of attention, it seems that Timpson does not consider the standard division that is often made between semantic and non-semantic information. Based on some passages in his work, such as the following, it is clear that Timpson somehow equates the notion of everyday information with semantic information, or at least views the latter as a subcategory of the former:

> "We need to distinguish between the everyday semantic and epistemic notion of information and the technical notions of information$_t$ theories." (Timpson, 2013, p.71)

As is pointed out by Lombardi, Fortin, and López (2016, p.212), by strictly separating between technical information and everyday information, Timpson (2013, p.150) does not allow for a technical concept of semantic information. After all, the notions that are usually linked to semantic information, such as meaning, knowledge and intentionality, are connected by Timpson (2013, p.12) to everyday information. This is in contrast with the approaches such as the ones by Dretske (1981), Barwise and Perry (1983) and Barwise and Seligman (1997). The general aim here is to provide a technical framework for semantic information, taking into account knowledge, meaning and reference. But, this distinction between technical and everyday information and the prohibition of the concepts to be intertwined also goes the other way. Such a distinction would mean that the concept of knowledge is strictly separated from the technical definition of information. From the following passage, it becomes clear that this is indeed what Timpson means:

> "The everyday notion is a semantic and an epistemic concept linking centrally to the notions of knowledge, language, and meaning; to that of a person (language user) who might inform or be informed. The Shannon concept, by contrast, we saw to be concerned with the behaviour of various physical systems characterized abstractly, at a level at which no semantic properties are in play, nor even any epistemic ones." (Timpson, 2013, p.43)

It is thus evident that Timpson does not allow for the notion of technical information to have a link with knowledge. However, the notion of knowledge or link to knowledge is taken as a fundamental principle in many accounts of information, both everyday and technical, and in the philosophy of information in general (Lombardi et al., 2017, p.20). In fact, information is more than once defined in terms of knowledge. This can be seen for instance

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[32] With the exception of the work of Dretske (1981), which is discussed quite extensively (Timpson, 2013, p.38).

in the account proposed by Dretske (1981)[33], information is defined as "a commodity that, given the right recipient, is capable of yielding knowledge" (Dretske, 1981, p.47). And, as we discussed, Dretske's account of information is based exactly on the MTC by Shannon (1948), often taken as the standard account of technical information, also by Timpson (2013, p.11) himself. This is not the only example of a link between technical information and knowledge. Also in the context of physics, it is not uncommon to find the notion of knowledge connected to information. Sometimes the notions are even equated, such as is done by physicist Zeilinger (1999): "We have knowledge, i.e., information, of an object only through observation." (Zeilinger, 1999, p.633). And surely, the concept of information used in physics is of the technical kind according to Timpson (2013, p.2). Therefore, this strict separation between these concepts as suggested by Timpson, might not be so clear-cut.

In the context of quantum physics, measurement plays an essential role in the description of the theory.[34] Quantum measurements are the method through which an observer can gain knowledge about a system. Furthermore, we have seen that the implications of measurement are closely intertwined with the manipulation or erasure of information in a quantum system. And not only measurement is directly linked to the notion of knowledge, as a possible conception of quantum states is by an epistemic interpretation:

> "A quantum state summarizes our knowledge about a quantum system at a given moment in time, it allows us to describe what we know, as well as, what we do not know, about the system." (Marinescu and Marinescu, 2012, p.22)

It is therefore not so easy to dismiss the role of knowledge in describing quantum theory or quantum information and adopt a purely technical approach, as is attempted by Timpson (2013, p.12). The distinction between epistemic aspects of information and technical information is not as sharp as Timpson is suggesting it is. This can for instance also be seen from the fact that also experimental physicists are concerned with knowledge gained from quantum measurements. Nagali et al. (2012) are considering sequential partial measurements that lead to a maximal knowledge gain for an observer. They analyse this based on physical experiments, to gain insight into the dynamics of different measurements related to the information that can be extracted from them and the disturbance they cause to the system. This research is thus a technical approach to a topic related to an epistemic notion of information.

This is opposed to the view of Timpson (2013, p.150), who argues against any connection between an epistemic and information-theoretic/technical conception of quantum information. He points out that because of the strict distinction between everyday information and technical information, one cannot get a deeper understanding of the foundations of quantum mechanics by considering a semantic or epistemic concept of information. Even though it might seem obvious that quantum measurement is a key feature of the theory and measurement is simply a transfer of information, this is not the right "kind" of information. According to Timpson, expecting that deeper understanding will follow given that we have a developing field in quantum information theory, depends on confusion between these different types of information. Quantum information theory might provide insight into quantum information but it is only related to a technical notion of information, as it is not about an epistemic or semantic conception of information.

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[33] See Section 2.3.2.

[34] See Section 3.3.

This leaves us with the question of what approach would be applicable if we want to examine the epistemic or semantic notion of information related to quantum theory. If quantum information theory is strictly about technical information, is there some account of everyday information related to quantum information? The flow of information in measurement can be modelled in the classical case using many different approaches, such as epistemic logic in an information-as-range approach, or situation theory in an information-as-correlation approach. But in the case of quantum measurements, it is not obvious that we can simply use these approaches as well since, depending on the chosen quantum interpretation, a measurement causes an ontic change to the system. How should we model the flow of information or knowledge in this case? It seems that currently there does not exist any approach to 'everyday quantum information', that deals with knowledge, intentionality or meaning in the context of quantum information.

It is not surprising that the notion of everyday information, in the sense of how the term 'information' is used in current everyday speech and writing, is not applicable to quantum information. After all, the rules of information flow and mechanics of quantum theory are in many aspects against the intuitions we have based on the classical macroscopic world. Based on these intuitions, our everyday concept of information is established. We do not encounter microscopic objects to which quantum theory adheres in our everyday life, hence this is not incorporated in the notion of everyday information. It could be that with the rapid rise of technological advances in the field of quantum computing, such as the development of quantum internet (Cacciapuoti et al., 2020), the notion of quantum information becomes more used in daily life. It may be that then a notion of everyday quantum information develops. However, this does not seem to be the case now. Even though there is no everyday notion of information, we should consider the relationship between quantum information and epistemic or semantic aspects of information, as we do not equate this things in the same way as Timpson suggests. It is clear from the example of information flow in quantum measurement, that quantum information is obviously somehow connected to knowledge, but that this cannot be modelled with a technical notion of quantum information, as argued for by Timpson (2013, p.150). We should thus consider a different approach to provide a framework to approach this.

5.3.2 Semantic quantum information

From our discussion so far, it is clear that there is a tight link between quantum information and the quantum state of a quantum system such as a qubit. The quantum information is said to be contained in a quantum state[35] and it is even argued that the term 'quantum information' is referring to nothing more than the 'quantum state' and thus these terms are used synonymously (Duwell, 2003, p.498). In any case, it is important that we need clarity on what is referred to when considering quantum states, in order to consider meaning in the context of quantum information. This would be the first step in a semantic approach to quantum information, as we need an understanding of how quantum states are related to physical states or situations in the actual world in order to evaluate meaning or truth.

In quantum mechanics and quantum information theory, quantum states describe the probabilities of each possible measurement outcome on a quantum system. Quantum states

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[35] See Section 4.2.

are thus approached in a functional way, they are defined as purely mathematical objects, as is clear from the postulates of quantum mechanics:

> "Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the system's state space." (Nielsen and Chuang, 2010, p.80)

Or, more specifically for the theory of quantum mechanics, as is phrased in the 'quantum state postulate' by Marinescu and Marinescu (2012):

> "A state is a complete description of a physical system. In quantum mechanics, a state $|\psi\rangle$ of a system is a vector—in fact, a direction (ray) in the Hilbert space $\mathcal{H}_n$." (Marinescu and Marinescu, 2012, p.24)

But how do these mathematical objects described by the postulates, these state vectors or rays in a Hilbert space, relate to the physical world? This is a large debate amongst physicists and philosophers (Friederich, 2011; Gömöri and Hofer-Szabó, 2021; Leifer, 2014; Luc, 2023; Maroney, 2012; Pusey et al., 2012). The metaphysical question is whether they correspond to something in the real world or not. So, can they be ascribed an ontic existence, are they related to something with ontic existence or are they purely epistemic and not related to anything real? One possible answer to this is given by the criterion of reality[36], as formulated by Einstein et al. (1935, p.777). This criterion, if we accept it, tells us that if we are in an epistemic position to predict a measurement outcome of a physical quantity without changing the system, then there is an element in physical reality that corresponds to it and thus a certain ontology must be adopted. Gömöri and Hofer-Szabó (2021) summarises that "the RC [criterion of reality] can be taken as a general inference pattern from the epistemic to the ontic" (Gömöri and Hofer-Szabó, 2021, p.13442). In this sense, there is an objective reality, independent of subjective agents, to which the description of states in the theory of quantum mechanics connects. But it does not mean that the quantum state itself needs to be understood as something existing in reality.

If we require this to be the case, so we adopt the view that demands the quantum state or wave function to be part of reality, then this is a purely ontic understanding of quantum states. This is argued for by Pusey et al. (2012), who present a theorem that given some assumptions, shows that if the quantum state is interpreted as only representing information about an objective physical state, rather than being a real physical state itself, this leads to a contradiction with the predictions of quantum theory. In the case of an ontic interpretation of pure quantum states, the state vector or wave function ψ can even be taken to correspond to a real physical wave. This is what was initially argued for by physicists de Broglie and Schrödinger, who viewed it in a similar vein to waves in classical field theory (Leifer, 2014, p.68). This approach, in which the wave function ψ is something that is taken to exist in reality, is often called the *ψ-ontic* approach (Leifer, 2014, p.71). This realist way of thinking about the wave function is compatible with realist quantum interpretations such as the many-worlds interpretation by Everett (1957) or the de Broglie–Bohm theory (Bohm, 1952).

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[36] See Section 3.4.

On the other hand, in the early days of quantum theory, Bohr, Heisenberg and Pauli adopted a statistical interpretation of the wave function (Leifer, 2014, p.68), based on Born's rule, which relates probabilities of measurement outcomes to the amplitude of the wave function (Marinescu and Marinescu, 2012, p.82). The collapse of the wave function is then not required to be a process that changes physical reality, but rather an effect of the acquisition of information. This could mean that there still is a change in reality, but the theory is agnostic about this. This last line of thinking is what is often accepted within the Copenhagen interpretation and interpretations that are derived from that such as Quantum Bayesianism (QBism) by Caves et al. (2002) or relational quantum mechanics (RQM) by Rovelli (1996) (Leifer, 2014, p.68). These latter quantum interpretations have a purely epistemic interpretation of the quantum state. They have a subjective or observer-dependent flavour. These accounts reject the criterion of reality and accept quantum states as states describing an agent's knowledge. In general, this interpretation of quantum states or wave functions is referred to as *ψ-epistemic* (Leifer, 2014, p.71).

Within the ψ-epistemic approach, there is an important distinction to be made. One can view the quantum state or wave function as epistemic and also not require any underlying ontology to exist. This means that there is no requirement for the existence of a deeper part of reality that is not in quantum theory itself. Within this approach, one can have for instance an anti-realist or instrumentalist interpretation of quantum theory or science in general (Leifer, 2014, p.72). Defenders of such approaches view quantum theory and specifically quantum states as a tool to predict measurement outcomes or observations, but do not deem the theory as describing how reality is, and might even be agnostic about the existence of such a reality altogether (van Fraassen, 1980, p.72). This entails that the quantum states can be viewed as subjective and therefore quantum probabilities can be taken as Bayesian probabilities (Caves et al., 2002, p.1). On the contrary, there is a realist ψ-epistemic approach, which views the wave function not as something to exist in reality, but still requires some underlying ontology to exist. The wave function simply is not part of that ontology, but it still represents our knowledge of reality. This is the idea adopted in the well-known realist ψ-epistemic approach by Spekkens (2007), in which quantum states can be seen as states of incomplete knowledge (Spekkens, 2007, p.032110-1).

As we see in this brief overview, there are many different approaches possible in interpreting quantum states and their connection to reality in order to derive a semantic account or quantum (information) theory. Since we approached quantum information in this thesis from the view of the Copenhagen interpretation, these above-mentioned approaches and further discussion of them are outside the scope of this thesis. We mentioned them here to point out that adopting a different interpretation of quantum theory influences the concept of quantum information, especially when we consider semantic aspects. However, even amongst philosophers and physicists that accept the Copenhagen interpretation, there is a debate about the reality of the quantum state or wave function. This is because the Copenhagen interpretation is not a clearly formulated interpretation, but is based on an accumulation of ideas of different physicists over time and has many different versions (Faye, 2019, Marinescu and Marinescu, 2012, p.139). Therefore, it is difficult to identify how the Copenhagen interpretation exactly fits with these above-described interpretations of the reality of quantum states and quantum theory. In many descriptions of the interpretation, we would adopt a statistical interpretation of the wave function (Ballentine, 1970, p.358, Faye, 2019), but it is not evident whether this would lead to a ψ-ontic or ψ-epistemic conception of the quantum state (Hubert, 2022; Maroney, 2012; Oldofredi and López, 2020). It thus seems that this metaphysical discussion about the connection between the theory of quantum mechanics, on which quantum information is based, and reality is one that needs to be developed further in order to say something about this.

Let us, therefore, consider how a semantic approach in classical information was taken and try if we can say something about a semantic approach to quantum information based on this. As discussed in Section 2.2.1, a prominent account of semantic information is given by Floridi (2011). Floridi's definition of semantic information is specified according to a few properties: it is well-formed, meaningful, and veridical (truthful) data. The difficulty with this definition is that the requirement for information to be truthful leads to an assertion of a physical reality in which the truth value of some piece of data can be determined objectively. This means that when we consider quantum information to be contained in a quantum state or to be equal to a quantum state, this rules out the purely epistemic interpretation of quantum states, described by the ψ-epistemic interpretation above.

The advantage of taking a purely epistemic approach to quantum states is that it would resolve the problems of explaining what wave function collapse upon measurement is and it would also explain away the mysterious effects of non-locality in entangled pairs of qubits. This is because the essence of the epistemic approach is that quantum states reflect subjective information of agents, and thus different agents can ascribe different states to a quantum system. For this reason, it is a popular approach. But at the same time, taking the quantum state to reflect an agent's knowledge about a system, means that it is required to be factive. This is due to the commonly accepted definition of knowledge as 'justified, true belief' or another definition along similar lines in which truth or factivity is central.[37] Even if we do not take the quantum state to be knowledge, but a more general idea of semantic information, this would fall in the category of everyday information. About this Timpson writes:

> "However, once we have the everyday concept of information in play, we need to recognize that the term 'information' is, just as the term 'knowledge' is, factive. ... And the major difficulty that this presents for those wishing to understand the quantum state of an individual system as information is that this factivity entails just the sort of objectivity that the invocation of information was originally intended to bypass." (Timpson, 2013, p.148)

This is in agreement with the following argument by Luc (2023):

> "[B]ecause knowledge is factive, any state that represents someone's knowledge about a physical system thereby also represents something about the physical system itself, so there is no such thing as "mere knowledge"." (Luc, 2023, p.1)

Therefore, Luc disagrees with the distinction between epistemic and ontic quantum states that is described above. Quantum states should then be viewed as both epistemic and ontic and the only reasonable question to consider would be if the change of quantum state, in case a measurement is performed, is ontic or epistemic. Thus, based on these arguments,

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[37] See Section 1.

it seems that it is not fruitful to take an epistemic approach to quantum states, such as is done by a ψ-epistemic approach. Namely, taking the quantum state to represent some kind of knowledge or lack of knowledge, or the information one has, leads to the enforcement of the objectivity of quantum states. This is because of the factivity of this type of information: things in reality then need to be as they are known. This is opposed to the subjective understanding that is aimed for, which would result in certain benefits such as resolving the measurement problem. It seems that the quantum state can thus not merely represent information (in the agent-dependent concept such as discussed in Section 2.3.1).

Thus, in considering an approach to semantic information in the context of quantum information, we circle back to the discussion of the interpretation of quantum states and their connection to reality, because of the tight link that appears to exist between the definition of quantum information and quantum state. It can be for this reason that the discussion about semantic quantum information has yet to begin in the literature. Much attention has been devoted to the technical aspects of quantum information, but very little literature currently exists on everyday or semantic approaches to quantum information. From the analysis above it is clear that applying the ideas from classical semantic information to the quantum case is not straightforward. Therefore, this is a topic that deserves more awareness and attention from both physicists and philosophers.

5.4 Information as range and quantum information

In Section 2.3.1, we have reviewed the paradigm of 'information as range', which was identified by Adriaans and van Benthem (2008, p.17) as one of the central themes in different accounts of information. In their book *Philosophy of Information*, they do not consider quantum information but only focus on different aspects of classical information. Therefore, in the current section, we will analyse if the approach of taking information as range also applies to quantum information.

At the heart of the information-as-range approach lies the idea that the more information one agent has about the world or some part of it, the fewer different configurations they consider possible. This is what is also captured by the IRP, which tells us that there is an inverse relationship between the information contained by a proposition and the likelihood of the proposition being true. If we then consider a possible world semantics, a certain proposition will be true in some possible worlds. This is exactly the range of possibilities that support the proposition. The ones in which it is false do not support this proposition. Now imagine we have some probability distribution over the space of all possibilities. Then it follows that the more worlds support the proposition, then the likelier that it is true. And then, following the IRP, the less information it carries. So, for a proposition to be more informative, it will be supported in less possible worlds. And thus if the agent knows the truth of some proposition, or multiple propositions, that are more informative, they will consider fewer possible worlds as valid options for describing the state of the actual world.

If an agent accumulates information about the state of the world or about a certain situation, this means that they will learn the truth value of increasingly more propositions regarding this situation or world state. Following the idea of information-as-range described above, this means that for classical information, the range of possible configurations of the world that the agent deems possible is diminishing. In this approach, gaining more information always means a reduction of the range of worlds that the agent considers possible. Upon acquiring new information in the form of a truth value of a certain proposition, the range of possible worlds will remain the same, only if all worlds that set the truth value of the proposition to be the opposite of what the agent learned were already disregarded. However, in the case of quantum information, this does not work so straightforwardly. This is because acquiring new information, which is usually done via quantum measurement, changes the state of the world in many cases (this is discussed in Section 3.3). So when one measures a quantum system with respect to a certain basis, the state of the system will collapse to one of these basis states upon measurement. Furthermore, when one subsequently measures observables that are associated with non-commuting operators, then the measurement of the second observable will destroy the information about the value of the first observable, due to the uncertainty principle (Marinescu and Marinescu, 2012, p.42). Thus, precise knowledge of the value of both of these physical properties of a system is forbidden in quantum mechanics. This way, it is an epistemological statement, as it is a limitation about what we can *know* about a quantum system. But the uncertainty principle is often also understood to be an ontological statement (Hilgevoord and Uffink, 2016). It is not possible to prepare a system such that it has precise values for both incompatible physical properties. A measurement of one property brings in existence a change to the state of the system. This idea lies at the core of the Copenhagen interpretation. These properties cause quantum measurements to be fundamentally different from classical measurements.

About the difference between classical and quantum measurements, Zurek (2018) writes:

> “Measurements reset initial conditions relevant for an observer’s evolution in a manner that is tied to the choice of what is measured ... Quantum measurements (more generally, ‘quantum jumps’) undermine one of the foundational principles of the classical, Newtonian dynamics: There, consecutive measurements just narrowed down the bundle of the possible past trajectories consistent with the observer’s knowledge. ... Quantum measurement derails evolution, resetting it onto the track consistent with its outcome. The loss of distinction between initial conditions and dynamical laws is tied to the enhanced role of information in the quantum Universe: Information is not just a passive reflection of the deterministic trajectory dictated by the dynamics (as was imagined in the classical, Newtonian settings) but is acquired in a measurement process that changes the state of both the measured object and of the measuring apparatus (or of an agent/observer).” (Zurek, 2018, p.2)

Zurek uses this as an explanation as to why quantum measurements are fundamentally irreversible, whereas classical measurements are not.[38] But for us, the relevant takeaway from the above is that we can understand this ‘bundle of the possible past trajectories consistent with the observer’s knowledge’ as the range of possible worlds for this observer or agent. Then, exactly because learning new quantum information via quantum measurements can bring about an ontic change to the state of the system, this acquired information is not a passive reflection of the state of the world, as is the case for information acquired through classical measurements. Thus, in the quantum case, the agent cannot narrow down its range of possible worlds but might end up with a completely different range of possible worlds after measurement. Hence, we are not sure that we can update the range of possible worlds

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[38] This was discussed in Section 4.3.1.

by simply disregarding the worlds that do not fit with the measurement result and still have an accurate description of what the state of the world looks like amongst the leftover possible worlds. Quantum information thus does not cumulate in the same way as classical information does, and the intuitions we have about information update and information flow that lead to the information-as-range approach are based on this latter process. This is why the approach of information-as-range with the IRP as the main principle does not always hold in the case of quantum information.

5.4.1 Epistemic logic and entanglement

The standard formalisation of the analysis of knowledge is given by epistemic logic (Hintikka, 1962). As we have seen in Section 2.3.1, epistemic logic can also be seen as a neutral approach to capture the idea of information-as-range, taking information as a semantic range of possibilities (van Benthem, 2011, p.21). This is because epistemic logics are relational models, in which knowledge or information can be represented via relations. Epistemic logics are often used to model the flow of classical information between different agents. But because of its spatial features, it can also be used to model information flow between distributed systems (Baltag and Smets, 2010, p.3006). However, it only deals with classical correlations between these systems. In the case of quantum information, we also need to be able to model quantum correlations which are caused by the entanglement of subsystems of multipartite quantum systems.[39] In this case, the information carried by different parts of a system, the subsystems, does not need to be independent and they can carry information about each other even when spatially separated. Thus, the total information in such an entangled system can be more than the "sum" of information carried by the subsystems. Standard epistemic logic is not equipped to model this. Therefore, Baltag and Smets (2010) propose an adjustment of standard epistemic logic. This results in the Logic of Correlated Knowledge (LCK), which is a generalisation of epistemic logic to model quantum information flow between multipartite quantum systems.

We will not present the formal framework of LCK here, but refer the reader to the original paper by Baltag and Smets (2010) for this. The main idea is that the concept of entanglement or non-locality of information in a complex system is captured in a relational manner by introducing a modal operator K_I, for each agent (subsystem) I. This is a knowledge operator, but in LCK the notion of knowledge modelled is an implicit one. The agents used in this logic do not need to represent intelligent agents or observers as is usually the case in epistemic logics, but rather parts of a complex physical system in the form of subsystems or locations. The 'knowledge' of these agents then refers to the information carried by this part of the system, which is the knowledge or information an intelligent agent or observer could in principle get to know. Therefore, one can interpret the proposition $K_I p$ as meaning that a subsystem I carries the information that p holds. Thus, this is a form of implicit knowledge in the system that would be potentially available to some intelligent agent at a specific location. Therefore, LCK can also be seen as a spatial logic, modelling the relations between different locations of a multipartite quantum system or another complex system.

In the standard epistemic logic approach to modelling knowledge of groups of agents or subsystems, the implicit knowledge of a group is taken to be the *distributed knowledge* of the group. Distributed knowledge is the information or knowledge that is acquired by

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[39] See Section 3.2.

putting together all the knowledge of the individual agents or parts of the group or system and closing this under logical consequence. This means that in this case, the implicit knowledge of the group is simply the 'sum of its parts'. However, the type of correlation between entangled subsystems leads to the total information carried by the system to be more than the sum of its parts. This difference between classical and quantum correlations was discussed in Section 4.5.1. However, for agents to recover this information from the system, they need to be able to correlate the results of their observations or measurement outcomes. This type of observation is what is referred to by Baltag and Smets (2010, p.3007) as *joint observation*, and allowing for this type of observation or information extraction is what leads to the implicit knowledge of a group to be more than just the distributed knowledge. This information implicitly carried by an entangled system is modelled in LCK by requiring additional properties for the knowledge operator to hold. Namely, for non-fully separated systems I (so subsystems I that are part of an entangled system), it is required that:

> "the information K_I carried by a quantum (sub)system I is not the "sum" $DK_{i \in I}$ [distributed knowledge of all subsystems] of the information carried by its i-component systems. In other words: quantum epistemic frames are "non-classical"." (Baltag and Smets, 2010, p.3015)

Thus, by allowing for joint observations, LCK extends the general epistemic logic used for modelling group knowledge, in which the implicit knowledge or information carried by the system is equal to the distributed knowledge carried by subsystems. This is what makes LCK suitable for modelling the information in systems with quantum correlations.

In conclusion, we have seen that the information-as-range approach in its current form does not apply to quantum information. The information-as-range approach revolves around information update, and the process of information update is different for classical and quantum information This is due to the properties of quantum measurements, which, if we adopt the Copenhagen interpretation, diverge drastically from those of classical measurements. The main cause is that measurements of a quantum system can cause a change to the state of the system, by the collapse of the state or by measurements of incompatible quantities. A quantum measurement therefore often changes the outcome of subsequent measurements or changes the value of previously measured outcomes. Therefore,the observer cannot accumulate knowledge in the same way as in the classical case, which is vital in the information-as-range approach. However, there are specific quantum measurements which leave the system intact. This is the case if we only consider quantum systems in pure orthogonal pure states that are amongst the basis states of the measurement operator. Then, the quantum measurement and its effect are comparable to classical measurements and this could be modelled with a theory in the information-as-range approach. Furthermore, we have seen that we can capture an important aspect of quantum information, namely entanglement or quantum correlation, in the framework of epistemic logic, as is shown by the LCK (Baltag and Smets, 2010). Epistemic logic formalises the main ideas of taking information as range, but it needs to be adjusted to account for a quantum setting.

5.5 Information as correlation and quantum information

As we have seen in the previous section, correlations play a significant role in distinguishing quantum information from classical information, due to the possibility of systems being quantum entangled. Therefore, let us now turn to the general approach of taking information as correlation, which was discussed in Section 2.3.2, and see how this applies to the setting of quantum information. The information-as-correlation approach is an overarching theme in several accounts of semantic information such as the account for environmental information by Dretske (1981) and situation theory by Barwise and Perry (1983). In this section we will highlight some of the important aspects of these accounts of semantic information and see if and how they translate to quantum information.

5.5.1 Dretske's theory of information and quantum information

The general theory of information proposed by Dretske (1981) seems to be a promising account from which to consider quantum information.[40] It is derived from the ideas of information theory by Shannon (1948), which is also the basis for quantum information theory. Dretske (1981, p.47) takes information to be an objective commodity, and approaches it as something that exists in the world, independent of agents to observe it.

The first main principle of Dretske's theory of information is what he calls the Xerox principle.[41] The main idea of this principle is that information remains preserved when it is duplicated or copied to another signal, at least in the case when the copy carries all the correct information about the original vehicle signal (Dretske, 1981, p.58). This is also why the principle is named after a Xerox photocopier, since if one were to make copies of a sheet of paper A containing the information B, then these copies should also contain the information B by carrying the information A. An example of the Xerox principle would be the photons coming from the sun carrying the information that the sun is in the sky and the fact that the sun is in the sky carrying the information that it is daytime. Then the photons from the sun carry the information that it is daytime. This shows that the property of carrying certain information is transitive.

If we now turn to the case of quantum information, we need to take into account the no-cloning principle.[42] This principle tells us that it is not possible to copy an unknown quantum state, as was proven by Wootters and Zurek (1982):

> "But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? ... We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems." (Wootters and Zurek, 1982, p.802)

Therefore, the Xerox principle as it is defined by Dretske does not seem to apply to quantum information, at least not in the way intended by Dretske. Namely, for any type of quantum system that can be taken as a signal carrying quantum information, we have that there can be no way in which its information can be copied to or carried by another signal (at least not when the state is unknown), as this would lead to an increase of information.[43] This is not to say that the Xerox principle is necessarily false for quantum information. It seems

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[40] See Section 2.3.2.

[41] See 1.

[42] See Section 3.5.1.

[43] See Section 5.1.

that we cannot satisfy the antecedent of the statement, and hence it cannot be falsified. It only shows that it loses its originally intended purpose of enforcing information preservation in the process of copying information-carrying signals.

Another important principle, that is not specifically named by Dretske himself but identified in Dretske's theory by D'Alfonso (2014, p.307), is the conjunction principle (2.3.2). Dretske (1983, p.57) takes it as an "unacceptable result" if a theory of information allows for the following: some signal to carry the information that s is F and to carry the information that s is G, but not the information that s is F and G. If we then turn again to quantum systems as information-carrying signals, we run into a problem with the Heisenberg uncertainty principle.[44] This fundamental theory about measurement outcomes in quantum measurements tells us that one cannot simultaneously measure the value, with a certain precision, of observables that correspond to non-commuting operators. This means that we can have a quantum system of which we can first measure the position. Then we would have the information that system s is in position x. Subsequently, we measure the momentum and then we have the information that system s has momentum p. However, we cannot say that therefore s has both these properties at the same time. This is due to the operators being non-commutative, causing our second measurement to change the value of the first measured property. Hence, in this case, the conjunction principle of Dretske does not hold.

From the evaluation of the above principles, it seems that the theory of information proposed by Dretske does not fare well when applied to quantum information. But, there is one particular aspect in which it could be useful in modelling quantum information flow. As we discussed in Section 4.1, in both technical and everyday or semantic approaches to information, it is generally required for information to have a physical representation or a physical carrier. These information-carrying signals can be anything from photons or electrons to books or smoke. As we discussed, the need for a physical carrier of information leads to difficulties when interpreting quantum non-locality.[45] It is interesting how Dretske allows for a different perspective on this aspect of information in his proposed theory. He defines an information link between two spatially separated situations or events to exist whenever there is a dependency relation, that does not need to be a causal link:

> "From a theoretical point of view, however, the communication channel may be thought of as simply the set of dependency relations between s and r. If the statistical relations defining equivocation and noise between s and r are appropriate, then there is a channel between these two points, and information passes between them, even if there is no direct physical link joining s with r." (Dretske, 1981, p.38)

These dependency relations between situations/events s and r can be due to them having a mutual history. Then, without any causal link between them, since there is nothing at s that influences anything at r since they are spatially separated and have no physical link, they can still carry information about each other (Dretske, 1981, p.39). This is in contrast with the general approach in physical sciences, where there is some kind of physical link required in order to have a flow of information. In case we then want to maintain an informational link between entangled subsystems, this leads to counterintuitive or complicated solutions to the questions raised by non-locality, such as information propagating backwards in time

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[44] See Section 4.3.

[45] See Section 3.4 and 4.1.

(Penrose, 1998, p.1928).[46] Therefore, it is generally understood that there is no informational channel, and the connection cannot be used to communicate information. However, if we adopt the semantic approach to information proposed by Dretske (1981), we can have a framework to consider the informational link that might exist between entangled multipartite quantum systems. In that case, information is viewed as a semantic item, capable of yielding knowledge (Lombardi, 2005, p.35).

5.5.2 Situation theory and quantum information

Situation theory[47] as proposed by Barwise and Perry (1983) can be seen to be part of the philosophy of language or as a tool for linguistics, as it provides the mathematical framework for situation semantics to analyse natural language. However, it does so from a perspective of information and for this reason, we can use it as a general approach to analyse how certain contexts or situations influence and allow for information flow (Devlin, 2006, p.602). In this section, we will examine if the framework of situation theory would also be useful in describing the flow of quantum information. Hence, we will review two particular aspects of situation theory, namely the notion of an infon and the way situation theory treats the locality of information, and see how these apply to quantum information.

Infons

In situation theory, Devlin (1991, p.38) takes the infon to be the fundamental unit of information. Infons describe whether objects or situations stand in certain relations to each other or not. In this way, they allow for talking about a "piece of information", without needing a physical carrier for this information. However, due to the infon's inherent polarity in its definition, they only allow for rigid descriptions of objects standing in relations or not. In the world of quantum mechanics, objects having certain properties or relations or not are in some cases fundamentally indeterminable. We will take as example the famous double-slit experiment in quantum mechanics, to show how the infon fails to capture this aspect of quantum information.

The double-slit experiment is an experiment to demonstrate the wave-particle duality in quantum mechanics. We will not describe the experiment in detail here, as this can be found in any textbook about quantum mechanics.[48] But the general setup of the experiment is as follows. On one end there is a light source, such as a laser beam and on the other end, there is a photographic plate. In between the source and the detector is a plate in which there are two open slits. Then, the photons from the light source can travel from the source through the slits and end up detected by the photographic plate on the other side. A schematic overview of this experiment is given in Figure 1.(a) below. It was first performed by Young (1804) and it proved the wave character of visible light. This is concluded from the experiment because when the photons pass through the slits, they generate an interference pattern of light and dark bands on the photographic plate. Even if the photons are sent one by one, they interfere with themselves and cause such an interference pattern. However, each photon is found to be absorbed at a discrete spot on the photographic plate, which shows that they portray characteristics of both waves and particles. Later, the experiment

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[46] See Section 4.1.

[47] See Section 2.3.2.

[48] see for instance (D. J. Griffiths and Schroeter, 2018, p.7-8).

was repeated in many different forms and setups, also with other matter such as electrons. Feynman et al. (1965, p.3-5) were the first to describe the thought experiment with the addition of detectors to observe through which of the slits an electron passes. In that case, the particles do not generate an interference pattern anymore, as they are being detected to go through one of the slits specifically. The double-slit experiment, in all its variations, thus highlights central puzzling aspects of quantum mechanics.

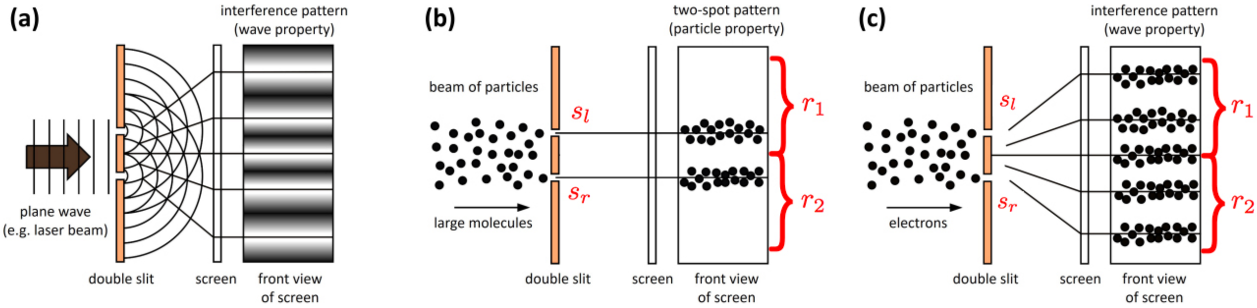

Figure 1: Schematic overview of the double-slit experiment performed with (a) photons (b) large molecules and (c) electrons. The figure shows the resulting pattern on the photographic plate, showing that photons and electrons interfere and thus have wave characteristics, whereas large enough molecules do not. Adapted from (Aydin, 2020, p.17).

We could perform a classical version of the double-slit experiment, by taking large enough objects such that they do not exhibit quantum properties, such as tennis balls or large enough molecules. A double-slit experiment with large molecules is shown in Figure 1.(b). We let the molecules pass through the slits and be detected on the other side. In this case, there would not arise an interference pattern, since the large molecules do not have wave characteristics. We can thus for each molecule determine, based on the place it is detected in the end, what its trajectory was and whether it went through the left or right slit. The molecules will simply end up right behind the slit they passed through. We can capture this information using infons, since we can express this as a relation between objects or situations. We can translate this experiment to the framework of situation theory as follows (as is also marked in Figure 1.(b)):

- Situation s: the performance of the double-slit experiment
- Situation s_l: the molecule passes the left slit
- Situation s_r: the molecule passes the right slit
- Temporal location t: time of slit passing
- Spatial location l: location of measurement set up
- Object r_1: detection of the molecule in region 1 (behind left slit)
- Object r_2: detection of the molecule in region 2 (behind right slit)

- Relation R: "leading to", an 4-ary relation

Then using these, the following are infons:

$$\sigma_1 = \langle\langle R, s_l, r_1, t, l; 1\rangle\rangle$$

$$\sigma_2 = \langle\langle R, s_r, r_2, t, l; 1\rangle\rangle.$$

The first infon carries the information that if the molecule passes the left slit at time t and location l, then this leads to the detection of the molecule in region 1, which is the region on the photographic plate behind the left slit. Similarly, the other infon carries the information that if the molecule passes the right slit, it leads to detection in region 2. We also have:

$$\sigma_3 = \langle\langle R, s_l, r_2, t, l; 0\rangle\rangle$$

$$\sigma_4 = \langle\langle R, s_r, r_1, t, l; 0\rangle\rangle.$$

These infons carry the information that the molecule going through the left slit does not lead to detection in the right region and vice versa. All these infons are made factual by situation s, the execution of the experiment:

$$s \models \sigma_1, \sigma_2, \sigma_3, \sigma_4.$$

If we now turn to the case in which we perform the experiment with microscopic objects small enough to show quantum properties, such as photons or electrons instead of large molecules, it goes differently. Then the effects of quantum mechanics come into play, and we have that objects interfere with themselves, due to their wave characteristics. This is shown in Figure 1.(c). We then replace the denotation of the following (again marked in Figure 1.(c)):

- Situation s_l: the electron passes the left slit
- Situation s_r: the electron passes the right slit
- Object r_1: detection of the electron in region 1 (left side of the plate)
- Object r_2: detection of the electron in region 2 (right side of the plate)

If we then try to formulate the infons, we cannot assign the polarity 1 or 0 to them:

$$\sigma_1 = \langle\langle R, s_l, r_1, t, l; ?\rangle\rangle$$

$$\sigma_2 = \langle\langle R, s_r, r_2, t, l; ?\rangle\rangle$$

such that we have:

$$s \models \sigma_1, \sigma_2.$$

This is because of the properties of quantum mechanics[49], which allow the electrons to be in a superposition of both going through the left slit and through the right slit. Therefore, we cannot assign a truth value of 1 to either of the infons, since we cannot say that the electron went through one of the slits depending on the region of the photographic plate it was detected in. It is not possible to define a clear relationship between the result of the experiment and the trajectory of the electron. Due to the interference of the electrons

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[49] See Chapter 3.

with themselves (if we shoot them one by one) and each other, we cannot conclude that an electron took a specific trajectory. But this does not mean that there is no information. The electrons took some kind of trajectory, and this provides some kind of information, but this cannot be captured with the binary-choice setting of the polarity of infons. The definition of infons in the work of Devlin (1991) does not allow for indeterminacy, since not both the infon and its dual can be made factual by a situation (van Benthem and Martinez, 2008, p.241). However, in quantum theory and therefore in quantum information, there is a fundamental indeterminacy. This property of quantum mechanics is what sets it apart from classical mechanics and our intuition about the classical world, and thus the behaviour of classical information:

> "From these two basic ideas alone – indefiniteness and the superposition principle – it should be clear already that quantum mechanics conflicts sharply with common sense. If the quantum state of a system is a complete description of the system, then a quantity that has an indefinite value in that quantum state is objectively indefinite; its value is not merely unknown by the scientist who seeks to describe the system. Furthermore, since the outcome of a measurement of an objectively indefinite quantity is not determined by the quantum state, and yet the quantum state is the complete bearer of information about the system, the outcome is strictly a matter of objective chance – not just a matter of chance in the sense of unpredictability by the scientist. Finally, the probability of each possible outcome of the measurement is an objective probability. Classical physics did not conflict with common sense in these fundamental ways." (Shimony, 1988, p.47)

Thus, the quantum indeterminacy that is described here, which only disappears upon measurement is not an epistemic uncertainty, but a fact of nature. If we want situation theory to describe quantum information, we need to be able to allow for this in the formal framework. One could conceive an object that has a similar status as the infon, which describes the fundamental unit of quantum information but has slightly different properties, which allow for superposition and entanglement.

Locality in situation theory

We have discussed in Section 4.5, that one major aspect in which quantum information and classical information differ is that quantum information is (partly) non-local, whereas classical information is not. The non-locality of quantum information arises if we consider entangled multipartite quantum systems. We have seen that due to the properties of quantum correlations, a subsystem can carry information about another spatially separated subsystem. This property cannot be used to transfer information directly, but it can certainly play a role in information transfer.[50] It thus seems that the total information carried by the system is dispersed across the subsystems (Duwell, 2003, p.491). In this case, we mean the implicit information (similar to the notion of implicit knowledge, used in LCK in Section 2.3.1), which is the potential information carried by a system that an agent could in principle get to learn. This non-locality can be seen as a property of the information itself and not of its carrier. The system and its parts can be clearly locatable, but the information that is carried by it might not be. However, the physicality of the system provides constraints to the location of the quantum information. The information

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[50] See for instance the teleportation protocol, discussed in Section 3.6.

can only be recovered at locations at which there is direct access to a part of the system or that have an informational link (a channel such as a telephone line between agents) to a part of the system. It cannot be the case that one can get information about a system from some separable different system, as for systems to become entangled, it is necessary that they were at the same location at some point in the past. Therefore, we can argue that the locality of the physical carrier puts a constraint on the locality of the information.

Inherent to situation theory is the anchoring of (spatial) locations to information. They are one of the fundamental building blocks of infons, together with individuals, relations, temporal locations, situations and parameters. Spatial and temporal locations are taken to be part of the ontology of the theory (Devlin, 1991, p.25). Still, the way the location argument is defined by Devlin (1991) allows for a lot of freedom:

> "(Spatial) locations will be denoted by l, l', l_0, l_1, l_2, etc. They are not necessarily like the 'points' of mathematical spaces, though they may be so; locations can have spatial extension. Thus a location / may be either a point in space or a region of space. (Usually a connected region, though I do not demand this restriction.) This, of course, endows the collection of all locations with a fairly complex structure: one location may be a point within another, two (regional) locations may overlap in space, and so on." (Devlin, 1991, p.23)

This above excerpt shows that in situation theory we are not restricted to connecting an infon to a single particular spatial location, but can also take several regions in space that are not spatially connected. These regions can correspond to the subsystems of an entangled system. Since we have argued that the locality of the quantum information carried by an entangled system is restricted by the locations of the subsystems, we can take these spatial regions corresponding to these locations as arguments for the location of the infon. This aspect makes situation theory possibly a good candidate for modelling quantum information in entangled quantum systems. However, Devlin (1991) does not specify in more detail how this can be denoted within the framework. This could be interesting to consider in future research, as well as how temporal locations are related to quantum information in connection with quantum measurement and how this can be captured in situation theory.

5.6 Summary of Chapter 5

In sections 5.1 and 5.2, we explored the creation and erasure of information. We noted that it is intuitive that the cloning of quantum states should fail, as cloning quantum states would lead to an increase in information, unlike classical states, which can be copied without limit. It turns out that this impossibility of cloning is logically equivalent to the impossibility of determining the quantum state of a system. Furthermore, we saw that if cloning were possible, it would allow for the transfer of an infinite amount of classical information using just one qubit, the accessible and specification information of quantum systems would coincide and it would enable faster-than-light communication, violating the no-communication theorem. The no-cloning and no-deleting theorems can be derived from the principle of conservation of information. In the classical case, considering systems which can only be described by orthogonal states, multiple copies do not contain more information than a single copy, and deleted information can in principle be recovered. However, in the quantum case, copying or deleting states is prohibited due to the conservation of information. The no-deletion theorem applies specifically to closed quantum systems. Whereas for open quantum systems, we pointed out that information can disappear through subsequent measurements.

Timpson (2013) distinguishes between everyday and technical information. He seems to equate everyday information to semantic information and epistemic information, thereby disallowing technical information to have a semantic or epistemic component. In Section 5.3, we argued against this view, showing a link between information and knowledge also in technical accounts of information, such as information in physics or formal semantic information derived from Shannon's MTC. We discussed that despite this, currently there is no approach to everyday quantum information. We argued that a semantic approach to quantum information would require an understanding of the connection between quantum states and reality. This is a topic of ongoing debate in the literature with on one hand ψ-realistic, and on the other hand ψ-epistemic approaches, also based on the chosen quantum interpretation. We concluded that physicists and philosophers need to explore a broader approach to quantum information beyond technical aspects, as it has received only limited attention thus far.

In Section 5.4, we examined the applicability of the information-as-range approach to quantum information. We concluded that this approach in its current form as used in classical scenarios, is generally not suitable because learning more quantum information does not necessarily decrease the range of possible states considered by an agent, which is the core idea of information-as-range. Opposed to classical measurements, acquiring information through quantum measurements does not lead to a passive reflection of the current state of the world. Namely, quantum measurements can bring about ontic changes to the system's state, which results in a completely different range of possible worlds, and this is where we get in trouble with the information-as-range approach. We also considered the Logic of Correlated Knowledge, an epistemic logic for modelling quantum information flow between multipartite quantum systems. The 'knowledge' in this logic is not the knowledge of an agent, but rather the information contained in a system accessible at a certain location. Baltag and Smets (2010) extended general epistemic logic to accommodate for joint observations. This leads to a logic suitable for modelling the property of quantum entanglement. The logic reflects that a similar approach to 'information-as-range' for classical information is possible in the quantum case, but only if it is adapted to take into account that the total information in the system is more than the sum of its parts, which is the information accessible from subsystems.

Lastly, when considering information as correlation in Section 5.5, we looked at Dretske's account of information and situation theory in the context of quantum information. The Xerox principle, a fundamental principle in Dretske's account, does not apply to quantum information in the way intended by Dretske. This is because due to the no-cloning principle copying an arbitrary piece of information carried by a quantum system to another system is not possible. Also, the conjunction principle in Dretske's account is challenged by the uncertainty principle in quantum mechanics. However, one advantage of Dretske's account lies in its ability to describe informational links that may exist between entangled multipartite quantum systems without a physical connection. When we considered situation theory, we pointed out that infons in their current form are not suitable to model quantum information, due to their inherent polarity which cannot accommodate superposition. Furthermore, we saw that situation theory does allow for information to be distributed over spatially separated parts of a system and hence an adapted version of it might be a good candidate to model the non-locality of quantum information.

6 Conclusion

The main aim of this thesis was to investigate quantum information from a broader perspective than only a technical or quantitative one, which is adopted in quantum information theory. We have discussed a qualitative definition of quantum information that has been given in the literature, but concluded that this definition only concerns a technical notion of information and does not tell us much about the philosophical nature of quantum information and the way it might differ from classical information. Therefore, we considered various approaches to classical information which take into account qualitative aspects of information, such as informational content, aboutness, situatedness or its relation to knowledge. We pointed out several limitations and advantages of using similar approaches as these existing accounts of information to model quantum information. We also considered the overarching themes of taking information as range or information as correlation. We have argued that both of these approaches, although generally presented as theoretically neutral approaches, do not straightforwardly apply to quantum information, due to its features that are different from classical information.

We have seen that these features of quantum information are born from the possibility of quantum states to be described by non-orthogonal subspaces causing them to be indistinguishable. The essence of this characteristic is that it leads to a fundamental indeterminacy. This level of indeterminacy is different from information carried by classical states, as in that case maximal knowledge of the state means that there is no ontic indeterminacy. The possible non-orthogonality of quantum states is what distinguishes quantum information from classical information, as it provides the possibility for states to be in superposition and systems to be entangled. This also leads to various characteristics of quantum information, including the impossibility of cloning and deleting quantum states, the conservation of information and the link between measurement and information increase.

We have argued that classical information could be seen as a subcategory of quantum information if we restrict information-carrying systems to occupy only orthogonal states, which are completely distinguishable. In that case, classical information and quantum information are not fundamentally different types of information but depend on the states of the information carriers which determine the behaviour. This does not mean that one notion of information should replace the other, as both seem to be indispensable. This can be seen from our discussion of the quantum teleportation protocol. In this sense, we can draw a parallel with the debate in the literature about quantum logic and classical logic. Quantum logic was presented by Birkhoff and von Neumann (1936) as a different type of logic, which could provide a better propositional structure to consider quantum mechanics. Later, it was proposed that quantum logic should replace classical logic as it would be a more suitable logic to describe reality. This view was endorsed by Putnam (1969) in his famous paper: *'Is Logic Empirical?'*. However, these ideas have been criticised since (Maudlin, 2011, p.184). Although quantum logic might be a valuable addition to the logic toolbox, it did not replace classical logic. In the same way, we suggest that we should not aim for quantum information to be viewed as a replacement of classical information.

The topic of quantum information calls for further investigation by physicists and philosophers to develop a comprehensive understanding of quantum information beyond its technical aspects. We should explore how we can expand our semantic, epistemic and everyday approaches to information to incorporate quantum information. We have seen that taking an information-theoretic perspective can provide insight into an explanation for physical phenomena, such as the irreversibility of measurement. On the other hand, the choice of interpretation of quantum mechanics has profound implications for our concept of information, as is clear from our discussion of semantic quantum information or possible explanations of information flow in quantum teleportation. Therefore, this field of research requires an interdisciplinary approach.

Possible directions of future research are:

- Reviewing the effects of adopting different quantum interpretations on the concept of (quantum) information. Especially subjective approaches such as QBism or relational quantum mechanics might lead to different explanations, since there the role of the observer and its relation to information is different.
- Examining the connection between information and entropy from the perspective of thermodynamics or statistical mechanics. As is clear from Landauer's principle (Rolf Landauer, 1961), there is a strong connection between these topics and this is also relevant to the idea that information is conserved. The conservation of information plays an important role in our distinction of classical and quantum information.
- Investigating what we can learn from various information interpretations of quantum mechanics (such as the ones proposed by Svozil (2000) and Brukner and Zeilinger (2003)). These approaches take information as a primitive notion. It would be interesting to see how these approaches relate to the discussion in this thesis.

We have not considered these topics in this thesis due to limitations of space and time, but they could certainly be of importance to the comprehension of (quantum) information.

References

Adriaans, P., & van Benthem, J. (2008). Introduction: Information Is What Information Does. In P. Adriaans & J. van Benthem (Eds.), *Philosophy of Information* (pp. 3–26). Elsevier. https://doi.org/10.1016/B978-0-444-51726-5.50006-6

Aydin, A. (2020). *Quantum Shape Effects* (Doctoral dissertation). Istanbul Technical University.

Ballentine, L. E. (1970). The Statistical Interpretation of Quantum Mechanics. *Reviews of Modern Physics*, *42*, 358–381. https://doi.org/10.1103/RevModPhys.42.358

Baltag, A., & Smets, S. (2010). Correlated Knowledge: An Epistemic-Logic View on Quantum Entanglement. *International Journal of Theoretical Physics*, *49*(12), 3005–3021. https://doi.org/10.1007/s10773-010-0411-5

Barnum, H., Caves, C. M., Fuchs, C. A., Jozsa, R., & Schumacher, B. (1996). Noncommuting Mixed States Cannot Be Broadcast. *Physical Review Letters*, *76*, 2818–2821. https://doi.org/10.1103/PhysRevLett.76.2818

Barwise, J., & Perry, J. (1983). *Situations and Attitudes*. MIT Press.

Barwise, J., & Seligman, J. (1997). *Information Flow: The Logic of Distributed Systems*. Cambridge University Press. https://doi.org/10.1017/CBO9780511895968

Bell, J. S. (1964). On the Einstein Podolsky Rosen Paradox. *Physics Physique Fizika*, *1*, 195–200. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195

Bell, J. S. (1981). Bertlmann's Socks and the Nature of Reality. *Journal de Physique, Colloque*, *42*, 41–62. https://doi.org/10.1051/jphyscol:1981202

Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., & Wootters, W. K. (1993). Teleporting an Unknown Quantum State Via Dual Classical and Einstein-Podolsky-Rosen Channels. *Physical Review Letters*, *70*, 1895–1899. https://doi.org/10.1103/PhysRevLett.70.1895

van Benthem, J. (2005). Information as Correlation vs. Information as Range: A Proposal for Identifying and Merging Two Basic Logical Traditions.

van Benthem, J. (2011). Logical Dynamics of Information and Interaction. In *Logical Dynamics of Information and Interaction* (pp. 21–44). Cambridge University Press. https://doi.org/10.1017/CBO9780511974533.003

van Benthem, J., & Martinez, M. (2008). The Stories of Logic and Information. In P. Adriaans & J. van Benthem (Eds.), *Philosophy of Information* (pp. 217–280). Elsevier.

Birkhoff, G., & von Neumann, J. (1936). The Logic of Quantum Mechanics. *Annals of Mathematics*, *37*(4), 823–843. https://doi.org/10.2307/1968621

Bohm, D. (1951). *Quantum Theory*. Prentice-Hall.

Bohm, D. (1952). A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I. *Physical Review*, *85*, 166–179. https://doi.org/10.1103/PhysRev.85.166

Bokulich, A., & Jaeger, G. (2010). *Philosophy of Quantum Information and Entanglement*. Cambridge University Press.

Braginsky, V. B., Vorontsov, Y. I., & Thorne, K. S. (1980). Quantum Nondemolition Measurements. *Science*, *209*(4456), 547–557.

Braunstein, S. L., & Pati, A. K. (2007). Quantum Information Cannot Be Completely Hidden in Correlations: Implications for the Black-Hole Information Paradox. *Physical Review Letters*, *98*, 080502. https://doi.org/10.1103/PhysRevLett.98.080502

Brukner, Č., & Zeilinger, A. (2003). Information and Fundamental Elements of the Structure of Quantum Theory. In L. Castell & O. Ischebeck (Eds.), *Time, Quantum and*

Information (pp. 323–354). Springer. https://doi.org/10.1007/978-3-662-10557-3_21

Bub, J. (2001). The Quantum Bit Commitment Theorem. *Foundations of Physics*, *31*(5), 735–756. https://doi.org/10.1023/A:1017597528026

Bub, J. (2005). Quantum Mechanics is About Quantum Information. *Foundations of Physics*, *35*(4), 541–560. https://doi.org/10.1007/s10701-004-2010-x

Bub, J. (2007). Quantum Information and Computation. In J. Butterfield & J. Earman (Eds.), *Philosophy of Physics* (pp. 555–660). Elsevier. https://doi.org/10.1214/11-STS378

Cacciapuoti, A., Caleffi, M., van Meter, R., & Hanzo, L. (2020). When entanglement meets classical communications: Quantum teleportation for the quantum internet. *IEEE Transactions on Communications*, *68*(6), 3808–3833. https://doi.org/10.1109/TCOMM.2020.2978071

Capurro, R., & Hjørland, B. (2003). The Concept of Information. *Annual Review of Information Science and Technology*, *37*, 343–411. https://doi.org/10.1002/aris.1440370109

Carnap, R., & Bar-Hillel, Y. (1952). *An Outline of a Theory of Semantic Information* (tech. rep.). M.I.T. Research Laboratory in Electronics.

Caves, C. M., & Fuchs, C. A. (1996). Quantum Information: How Much Information in a State Vector? In A. Mann & M. Revzen (Eds.), *The Dilemma of Einstein, Podolsky and Rosen - 60 Years Later. Annals of the Israel Physical Society* (pp. 226–257). Institute of Physics.

Caves, C. M., Fuchs, C. A., & Schack, R. (2002). Quantum Probabilities as Bayesian Probabilities. *Physical Review A*, *65*(2), 6. https://doi.org/10.1103/PhysRevA.65.022305

Clifton, R., Bub, J., & Halvorson, H. (2003). Characterizing Quantum Theory in Terms of Information-Theoretic Constraints. *Foundations of Physics*, *33*(11), 1561–1591. https://doi.org/10.1023/A:1026056716397

Colbeck, R., & Renner, R. (2011). No Extension of Quantum Theory Can Have Improved Predictive Power. *Nature Communications*, *2*(1). https://doi.org/10.1038/ncomms1416

D'Alfonso, S. (2014). The Logic of Knowledge and the Flow of Information. *Minds and Machines*, *24*(3), 307–325. https://doi.org/10.1007/s11023-013-9310-x

Deutsch, D., & Hayden, P. (2000). Information Flow in Entangled Quantum Systems. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *456*(1999), 1759–1774. https://doi.org/10.1098/rspa.2000.0585

Devlin, K. (1991). *Logic and information*. Cambridge University Press. https://doi.org/10.1016/0160-9327(92)90011-d

Devlin, K. (2006). Situation Theory and Situation Semantics. In D. M. Gabbay & J. Woods (Eds.), *Logic and the Modalities in the Twentieth Century* (pp. 601–664). North-Holland. https://doi.org/https://doi.org/10.1016/S1874-5857(06)80034-8

Distante, E., Daiss, S., Langenfeld, S., Hartung, L., Thomas, P., Morin, O., Rempe, G., & Welte, S. (2021). Detecting an Itinerant Optical Photon Twice without Destroying It. *Physical Review Letters*, *126*(25), 253603. https://doi.org/10.1103/PhysRevLett.126.253603

van Ditmarsch, H., van der Hoek, W., Halpern, J. Y., & Kooi, B. (Eds.). (2015). *Handbook of Epistemic Logic*. College Publications.

Dretske, F. (1981). *Knowledge and the Flow of Information*. MIT Press. https://doi.org/10.2307/2184492

Dretske, F. (1983). Précis of Knowledge and the Flow of Information (2010/02/04). *Behavioral and Brain Sciences*, *6*(1), 55–63. https://doi.org/10.1017/S0140525X00014631

Duwell, A. (2003). Quantum Information Does Not Exist. *Studies in History and Philosophy of Science Part B - Studies in History and Philosophy of Modern Physics*, *34*(3), 479–499. https://doi.org/10.1016/S1355-2198(03)00041-8

Duwell, A. (2008). Quantum Information Does Exist. *Studies in History and Philosophy of Science Part B - Studies in History and Philosophy of Modern Physics*, *39*(1), 195–216. https://doi.org/10.1016/j.shpsb.2007.03.004

Duwell, A. (2019). What is Quantum Information? *Philosophy Compass*, *14*(4), 1–9. https://doi.org/10.1111/phc3.12572

Einstein, A., Podolsky, B., & Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, *47*, 777–780. https://doi.org/10.1103/PhysRev.47.777

Enßlin, T. A., Jasche, J., & Skilling, J. (2019). The Physics of Information. *Annalen der Physik*, *531*(3). https://doi.org/10.1002/andp.201900059

Everett, H. (1957). “Relative State” Formulation of Quantum Mechanics. *Reviews of Modern Physics*, *29*(3), 454–462. https://doi.org/10.1103/RevModPhys.29.454

Fagin, R., Halpern, J. Y., Moses, Y., & Vardi, M. (1995). *Reasoning About Knowledge*. MIT Press.

Fan, H., Wang, Y.-N., Jing, L., Yue, J.-D., Shi, H.-D., Zhang, Y.-L., & Mu, L.-Z. (2014). Quantum Cloning Machines and the Applications. *Physics Reports*, *544*(3), 241–322. https://doi.org/https://doi.org/10.1016/j.physrep.2014.06.004
Quantum Cloning Machines and the Applications

Faye, J. (2019). Copenhagen Interpretation of Quantum Mechanics. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Winter 2019). Metaphysics Research Lab, Stanford University.

Feynman, R. P., Leighton, R. B., & Sands, M. (1965). *The Feynman Lectures on Physics, Vol. III: Quantum Mechanics*. Addison-Wesley.

Floridi, L. (2004). Information. In L. Floridi (Ed.), *The Blackwell Guide to the Philosophy of Computing and Information* (pp. 40–61). Blackwell Publishing. https://doi.org/10.1002/9780470757017

Floridi, L. (2005). Is Semantic Information Meaningful Data? *Philosophy and Phenomenological Research*, *70*(2), 351–370. https://doi.org/10.1111/j.1933-1592.2005.tb00531.x

Floridi, L. (2011). The Philosophy of Information. https://doi.org/10.1093/acprof:oso/9780199232383.001.0001

Floridi, L. (2016). *The Routledge Handbook of Philosophy of Information*. Routledge.

van Fraassen, B. C. (1980). The Scientific Image. https://doi.org/10.1093/0198244274.001.0001

Freire, O., Jr. (2022). *The Oxford Handbook of the History of Quantum Interpretations*. Oxford University Press. https://doi.org/10.1093/oxfordhb/9780198844495.001.0001

Friederich, S. (2011). How to Spell Out the Epistemic Conception of Quantum States. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, *42*(3), 149–157. https://doi.org/https://doi.org/10.1016/j.shpsb.2011.01.002

Garcia-Escartin, J. C., & Chamorro-Posada, P. (2011). Equivalent Quantum Circuits.

Gömöri, M., & Hofer-Szabó, G. (2021). On the Meaning of EPR’s Reality Criterion. *Synthese*, *199*(5-6), 13441–13469. https://doi.org/10.1007/s11229-021-03382-3 de Grefte, J. (2023). Knowledge as Justified True Belief. *Erkenntnis*, *88*(2), 531–549. https://doi.org/10.1007/s10670-020-00365-7

Griffiths, D. J., & Schroeter, D. F. (2018). *Introduction to Quantum Mechanics* (3rd ed.). Cambridge University Press. https://doi.org/10.1017/9781316995433

Griffiths, R. B. (2002). The Nature and Location of Quantum Information. *Physical Review A - Atomic, Molecular, and Optical Physics*, *66*(1), 15. https://doi.org/10.1103/PhysRevA.66.012311

Gruska, J., & Imai, H. (2001). Power, Puzzles and Properties of Entanglement. In M. Margenstern & Y. Rogozhin (Eds.), *Machines, Computations, and Universality* (pp. 25–68). Springer, Berlin, Heidelberg. https://doi.org/10.1007/3-540-45132-3_3

Haddock, A. (2010). Knowledge and Justification. In D. Pritchard, A. Millar, & A. Haddock (Eds.), *The Nature and Value of Knowledge: Three Investigations*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199586264.003.0009

Halvorson, H. (2004a). On Information-Theoretic Characterizations of Physical Theories. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, *35*(2), 277–293. https://doi.org/10.1016/j.shpsb.2004.02.001

Halvorson, H. (2004b). Remote Preparation of Arbitrary Ensembles and Quantum Bit Commitment. *Journal of Mathematical Physics*, *45*(12), 4920–4931. https://doi.org/10.1063/1.1812827

Heisenberg, W. (1927). Über den Anschaulichen Inhalt der Quantentheoretischen Kinematik und Mechanik. *Zeitschrift für Physik*, *43*(3), 172–198. https://doi.org/10.1007/BF01397280

Henderson, L., & Vedral, V. (2001). Classical, Quantum and Total Correlations. *Journal of Physics A: Mathematical and General*, *34*(35), 6899–6905. https://doi.org/10.1088/0305-4470/34/35/315

Hilgevoord, J., & Uffink, J. (2016). The Uncertainty Principle. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Winter 2016). Metaphysics Research Lab, Stanford University.

Hintikka, J. (1962). *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Ithaca: Cornell University Press.

Holevo, A. S. (1973). Bounds for the Quantity of Information Transmitted by a Quantum Communication Channel.

Horodecki, M., Horodecki, P., Horodecki, R., Oppenheim, J., Sen, A., Sen, U., & Synak-Radtke, B. (2005a). Local Versus Nonlocal Information in Quantum-Information Theory: Formalism and Phenomena. *Physical Review A - Atomic, Molecular, and Optical Physics*, *71*(6), 1–25. https://doi.org/10.1103/PhysRevA.71.062307

Horodecki, M., Horodecki, R., Sen, A., & Sen, U. (2005b). Common Origin of No-Cloning and No-Deleting Principles – Conservation of Information. *Foundations of Physics*, *35*(12), 2041–2049. https://doi.org/10.1007/s10701-005-8661-4

Hubert, M. (2022). Is the Statistical Interpretation of Quantum Mechanics ψ-Ontic or ψ-Epistemic? *Foundations of Physics*, *53*(1), 16. https://doi.org/10.1007/s10701-022-00651-0

Jacobs, K. (2003). On the Properties of Information Gathering in Quantum and Classical Measurements. *arXiv: Quantum Physics*.

Jaeger, G. (2009). *Entanglement, Information, and the Interpretation of Quantum Mechanics*. Springer-Verlag Berlin Heidelberg. https://doi.org/10.1007/978-3-540-92128-8

Jozsa, R. (2004). Illustrating the Concept of Quantum Information. *IBM Journal of Research and Development*, *48*(1), 79–85. https://doi.org/10.1147/rd.481.0079

Landauer, R. (1991). Information is Physical. *Physics Today*, *44*(5), 23–29. https://doi.org/10.1063/1.881299

Leifer, M. S. (2014). Is the Quantum State Real? An Extended Review of ψ-ontology Theorems. *Quanta*, *3*(1), 67–155. https://doi.org/10.12743/quanta.v3i1.22

Lewis, P. J. (n.d.). Interpretations of Quantum Mechanics [Accessed on 16/03/2023]. In *The Internet Encyclopedia of Philosophy.*

Li, M., & Vitányi, P. (1997). *An Introduction to Kolmogorov Complexity and Its Applications.* https://doi.org/10.1007/978-1-4757-2606-0

Lombardi, O. (2005). Dretske, Shannon's Theory and the Interpretation of Information. *Synthese*, *144*(1), 23–39. https://doi.org/10.1007/s11229-005-9127-0

Lombardi, O., Fortin, S., Holik, F., & López, C. (2017). *What Is Quantum Information?* Cambridge University Press. https://doi.org/10.1017/9781316494233

Lombardi, O., Fortin, S., & López, C. (2016). Deflating the Deflationary View of Information. *European Journal for Philosophy of Science*, *6*(2), 209–230. https://doi.org/10.1007/s13194-015-0128-7

Lombardi, O., Holik, F., & Vanni, L. (2014). What is Quantum Information? [Preprint], 1–27.

Lombardi, O., Holik, F., & Vanni, L. (2016). What is Quantum Information? *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, *56*, 17–26. https://doi.org/10.1016/j.shpsb.2016.10.001

Luc, J. (2023). *States vs. Changes of States: A Reformulation of the Ontic vs. Epistemic Distinction in Quantum Mechanics* (Vol. 53). Springer US. https://doi.org/10.1007/s10701-022-00662-x

MacKay, D. (2003). *Information Theory, Inference and Learning Algorithms.* Cambridge University Press.

Marinescu, D. C., & Marinescu, G. M. (2012). *Classical and Quantum Information.* Academic Press, Elsevier. https://doi.org/10.1016/C2009-0-64195-7

Maroney, O. J. E. (2012). How Statistical Are Quantum States?, 1–5. http://arxiv.org/abs/1207.6906

Maudlin, T. (2011). *Quantum Non-Locality and Relativity: Metaphysical Intimations of Modern Physics* (3rd ed.). Wiley-Blackwell. https://doi.org/10.1002/9781444396973

Murphy, G. J. (1990). *C*–Algebras and Operator Theory* (G. J. Murphy, Ed.). Academic Press. https://doi.org/https://doi.org/10.1016/B978-0-08-092496-0.50004-1

Nagali, E., Felicetti, S., De Assis, P. L., D'Ambrosio, V., Filip, R., & Sciarrino, F. (2012). Testing Sequential Quantum Measurements: How Can Maximal Knowledge Be Extracted? *Scientific Reports*, *2*, 1–5. https://doi.org/10.1038/srep00443

Nielsen, M. A., & Chuang, I. L. (2010). *Quantum Computation and Quantum Information: 10th Anniversary Edition.* Cambridge University Press. https://doi.org/10.1017/CBO9780511976667

Oldofredi, A., & López, C. (2020). *On the Classification Between ψ-Ontic and ψ-Epistemic Ontological Models* (Vol. 50). Springer US. https://doi.org/10.1007/s10701-020-00377-x

Park, J. L. (1970). The Concept of Transition in Quantum Mechanics. *Foundations of Physics*, *1*(1), 23–33. https://doi.org/10.1007/BF00708652

Pathak, A. (2013). *Elements of Quantum Computation and Quantum Communication* (1st ed.). CRC Press. https://doi.org/10.1201/b15007

Pati, A. K., & Braunstein, S. L. (2000). Impossibility of Deleting an Unknown Quantum State. *Nature*, *404*(6774), 164–165. https://doi.org/10.1038/404130b0

Pauli, W. (1980). *General Principles of Quantum Mechanics* (P. Achuthan & K. Venkatesan, Eds.). Springer Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-61840-6

Penrose, R. (1998). Quantum Computation, Entanglement and State Reduction. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, *356*, 1927–1939.

Peres, A., & Terno, D. R. (2004). Quantum Information and Relativity Theory. *Reviews of Modern Physics*, *76*(1), 93–123. https://doi.org/10.1103/revmodphys.76.93

Piccinini, G., & Scarantino, A. (2016). Computation and information data. In L. Floridi (Ed.), *The Routledge Handbook of Philosophy of Information* (pp. 23–29). Routledge.

Pritchard, D. (2018). *What Is This Thing Called Knowledge?* (4th ed.). Routledge. https://doi.org/10.4324/9781351980326

Pusey, M. F., Barrett, J., & Rudolph, T. (2012). On the Reality of the Quantum State. *Nature Physics*, *8*(6), 475–478. https://doi.org/10.1038/nphys2309

Putnam, H. (1969). Is Logic Empirical? In R. S. Cohen & M. W. Wartofsky (Eds.), *Boston studies in the philosophy of science* (pp. 216–241). Springer Netherlands. https://doi.org/10.1007/978-94-010-3381-7_5

Redhead, M. (1987). *Incompleteness, Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics.* Oxford University Press. https://doi.org/10.2307/2185497

Rolf Landauer. (1961). Irreversibility and Heat Generation in the Computing Process. *IBM Journal of Research and Development*, *5*(3), 183–191. https://doi.org/10.1147/rd.53.0183

Roncaglia, M. (2019). On the Conservation of Information in Quantum Physics. *Foundations of Physics*, *49*(11), 1278–1286. https://doi.org/10.1007/s10701-019-00304-9

Rovelli, C. (1996). Relational Quantum Mechanics. *International Journal of Theoretical Physics*, *35*(8), 1637–1678. https://doi.org/10.1007/BF02302261

Samal, J. R., Pati, A. K., & Kumar, A. (2011). Experimental Test of the Quantum No-Hiding Theorem. *Physical Review Letters*, *106*, 080401. https://doi.org/10.1103/PhysRevLett.106.080401

Schumacher, B. (1995). Quantum Coding. *Physical Review A*, *51*(4), 2738–2747. https://doi.org/10.1103/PhysRevA.51.2738

Shannon, C. E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal*, *27*(4), 379–423, 623–656. https://doi.org/10.1002/j.1538-7305.1948.tb00917.x

Sharma, K. K., Gerdt, V. P., & Gerdt, P. V. (2020). Milestone Developments in Quantum Information and No-Go Theorems. *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, *12563*, 510–525. https://doi.org/10.1007/978-3-030-66471-8_39

Shen, Y., Hao, L., & Long, G. L. (2010). A Comparative Study of Quantum and Classical Deletion. *Communications in Theoretical Physics*, *53*(3), 486–488. https://doi.org/10.1088/0253-6102/53/3/16

Shen, Y., Hao, L., & Long, G. L. (2011). Why Can We Copy Classical Information? *Chinese Physics Letters*, *28*(1), 17–19. https://doi.org/10.1088/0256-307X/28/1/010306

Shimony, A. (1988). The Reality of the Quantum World. *Scientific American*, *258*(1), 46–53. http://www.jstor.org/stable/24988945

Spekkens, R. W. (2007). Evidence for the Epistemic View of Quantum States: A Toy Theory. *Physical Review A*, *75*(3), 032110. https://doi.org/10.1103/physreva.75.032110

Svozil, K. (2000). The Information Interpretation of Quantum Mechanics. *arXiv: Quantum Physics*.

Timpson, C. G. (2008). Philosophical Aspects of Quantum Information Theory. In D. Rickles (Ed.), *The Ashgate Companion to Contemporary Philosophy of Physics* (1st ed., pp. 196–261). Routledge. https://doi.org/10.4324/9781315612676

Timpson, C. G. (2013). *Quantum Information Theory and the Foundations of Quantum Mechanics* (1st ed.). Oxford University Press.

Timpson, C. G. (2004). *Quantum Information Theory and the Foundations of Quantum Mechanics* (Doctoral dissertation). University of Oxford.

Tipler, F. J. (2014). Quantum Nonlocality Does not Exist. *Proceedings of the National Academy of Sciences of the United States of America*, *111*(31), 11281–11286. https://doi.org/10.1073/pnas.1324238111

Wheeler, J. A. (1989). Information, Physics, Quantum: The Search for Links. *Proceedings III International Symposium on Foundations of Quantum Mechanics*, 354–368.

Wootters, W. K., & Zurek, W. H. (1982). A Single Quantum Cannot be Cloned. *Nature*, *299*(5886), 802–803. https://doi.org/10.1038/299802a0

Young, T. (1804). I. The Bakerian Lecture. Experiments and calculations relative to physical optics. *Philosophical Transactions of the Royal Society of London*, 1–16. https://doi.org/10.1098/rstl.1804.0001

Zalta, E. N. (1993). Twenty-Five Basic Theorems in Situation and World Theory. *Journal of Philosophical Logic*, *22*, 385–428. https://doi.org/10.1007/BF01052533

Zeilinger, A. (1999). A Foundational Principle for Quantum Mechanics. *Foundations of Physics*, *29*, 631–643. https://doi.org/10.1023/A:1018820410908

Zurek, W. H. (2018). Quantum Reversibility Is Relative, or Do Quantum Measurements Reset Initial Conditions? *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *376*(20170315). https://doi.org/10.1098/rsta.2017.0315

Zurek, W. H. (2006). Decoherence and the Transition from Quantum to Classical — Revisited. In B. Duplantier, J.-M. Raimond, & V. Rivasseau (Eds.), *Quantum Decoherence. Progress in Mathematical Physics, vol 48.* (pp. 2–25). Birkhäuser Basel. https://doi.org/10.1007/978-3-7643-7808-0_1