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# Proceedings of the ILLC Workshop on Logic and Games. Held in Amsterdam, November 19-20, 1999

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# Preface

This volume consists of the abstracts of presentations given at the ILLC Workshop on Logic and Games, held in Amsterdam on 19-20 November 1999. The aim of the workshop was to bring together researchers from various groups interested in the relations between logic and game theory and hopefully to initiate more useful interactions between these groups. We specifically invited presentations in areas such as: logical analysis of games (e.g. modelling knowledge, belief and information flow in games), logic games (e.g. model comparison games, semantic evaluation games, independence-friendly logics), game logics (e.g. extensions of program logics and modal logics to investigate the structure of games in general) and the role of language and logical definability in games.

The workshop was initiated by Johan van Benthem and Marc Pauly in the context of their course on Logic and Games at the University of Amsterdam. Organized by ILLC, the workshop was funded by the Spinoza project "Logic in Communication". The abstracts were selected by a committee consisting of Johan van Benthem, Marc Pauly and Alexandru Baltag. The editors wish to thank all the people involved in the organization of the workshop and the publication of this volume, especially Johan van Benthem, Ingrid van Loon, Marco Vervoort, Marjan Veldhuis, and all the speakers at the workshop. Furthermore, we want to thank the Spinoza project "Logic in Communication" for funding this event.

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# Invited Lectures

**Five Essays on::**

# Economics and Language

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This is an extended version of my Churchill Lectures delivered in Cambridge, England in May 1996. Part of the material overlaps with the Schwartz Lecture delivered at Northwestern University in May 1998.

**Acknowledgements**

This book emerged from the kind invitation of the fellows of Churchill College to deliver the Churchill Lectures at 1996. It is a pleasure for me to use this opportunity to thank Frank Hahn, whose support and encouragement during my career has been unforgettable.

Three people agreed on very short notice to provide their written comments and to publish them in the book: Tilman Borgers, Bart Lipman and Johan van Benthem. I thank them wholeheartedly for their willingness to do so. The idea to add comments to the book should be credited to Chris Harrison, the Cambridge University Press editor of this book.

Several people have commented on parts of the text in its various stages. In particular, I would like to thank Bart Lipman, Martin Osborne and Ran Spiegler, who were always ready to help.

My thanks to Nina Reshef and Sharon Simmer, who helped me with language editing; the remaining errors are my own.

Finally, I would like to acknowledge the financial support given me by the Israeli Science Foundation. I am also grateful to The Russell Sage Foundation and New York University, which hosted me during the period when much of the writing was completed.

# Chapter 0: Economics and Language

The psychologist Joel Davitz once wrote: "I suspect that most research in the social sciences has roots somewhere in the personal life of the researcher, though these roots are rarely reported in published papers." (Davitz, 1976). The first part of this statement definitely applies to this book. Though I am involved in several fields of economics and game theory, all my academic research has been motivated by my childhood desire to understand the way that people argue. In high school, I wanted to study logic which I thought would be useful in political debates or in legal battles against evil once I fulfilled my dream of becoming a solicitor. Unfortunately, I became neither a lawyer nor a politician and I have since come to understand that logic is not a very useful tool in these areas in any case. Nonetheless, I continued to explore formal models of game theory and economic theory, though not in the hope of predicting human behavior, not in anticipation to predict the stock market prices, and without any illusion about the ability of capturing all of reality in one simple model. I am simply interested the reasoning decision making behind and in the arguments people bring in debates. I am still puzzled and even fascinated, by the magic of the links between the formal language of mathematical models and natural language. This brings me to the subject of this lecture - "Economics and Language".

## 0.1. Economics and Language

The title of these lectures may be misleading. Although the caption "Economics and Language" is a catchy title, it is too vague. It encompasses numerous subjects most of which will not be touched on here. This series of lectures will briefly address five issues which fall under this general heading. The issues can be presented in the form of five questions::

- Why do we tend to arrange things on a line and not on a circle?
- How is it that the utterance "be careful" is understood by the listener as a warning and not as an invitation to a dance?
- How is it that the statement "it is not raining very hard" is understood to mean "it is rainy but not very hard"?
- Does the textbook utility function $\log(x_1+1)x_2$ make sense?
- Is the use of the word "strategy" in game theory rhetorical?

All the issues discussed in these lectures lie somewhere between economic theory and the study of language. Two questions spring to mind:

Why would economic theory be relevant to linguistic issues? Economic theory is an attempt to explain regularities in human interaction and the most fundamental non-physical regularity in human interaction is natural language. Economic theory carefully analyses the design of social systems; language is, in part, a mechanism of communication. Economics attempts to explain social institutions as regularities deriving from the optimization of certain functions; this may be applicable to language as well. In these lectures I will try to demonstrate the relevance of economic thought to the study of language by presenting several "economic-like" analyses to address linguistic issues.

Why would economic theory be a relevant subject of research from the point of view of language? Because economic agents are human beings for whom language is a central tool in the process of making decisions and forming judgments. And because, the other important "players" in Economic Theory, namely ourselves, the economic theorists, use formal models but these are not simply mathematical models; their significance derives from their interpretation which is expressed using daily language terms.

## 0.2 Outline of the Lectures

The book deals with five independent issues organized into two groups:

**Part I** is entitled "Economics of Language" and comprises the core of this book. In Part I, methods taken from economic theory are used to address questions regarding natural language. The basic approach is that language serves certain functions from which the properties of language are derived.

In **Chapter 1,** I assume that language is the product of a "fictitious optimizer" who operates behind a "veil of ignorance". The substantive issue studied in this chapter is the structure imposed on binary relations in daily language. The designer chooses properties of binary relations that will serve the users of the language. The three parts of the chapter discuss three distinct targets of binary relations:

(1) To enable the user of the relation to point out nameless elements.

(2) To improve the accuracy with which the vocabulary spanned by the relation approximates the actual terms to which the user of the language is referring.

(3) To facilitate the description of the relation by means of examples.

It will be shown that optimization with respect to these three targets explains the popularity of linear orderings in natural language.

In **Chapter 2,** we discuss the evolutionary development of the meaning of words. The analytical tool used is a variant of the game theoretic notion of evolutionary stable strategy. Complexity considerations are added to the standard notion of evolutionary stable equilibrium as an additional evolutionary factor.

In **Chapter 3**, I touch on pragmatics, the topic furthest from traditional economic issues that is discussed in these essays. Pragmatics searches for rules that explain the difference in meaning between a statement made in a conversation and one when it is stated in isolation. Grice examined such rules in the framework of a conversation in which the participants are assumed to be cooperative. Here, game-theoretical analysis will be used to explain a certain phenomenon found in debates.

**Part II** is entitled Language of Economics and includes two essays:
**Chapter 4** deals with the Language of Economic Agents. The starting point of the discussion is that decision makers, when making deliberate choices, often verbalize their deliberations. This assumption is especially fitting when the "decision maker" is a collective but also has appeal when the decision maker is an individual. Tools of mathematical logic are used to formulize the assumption. The objective is to analyse the constraints on the set of preferences which arise from natural restrictions on the language used by the decision maker to verbalize his preferences. I demonstrate in two different contexts that the definability constraint severely restricts the set of admissible preferences.

**Chapter 5** focuses on the rhetoric of game theory. Much has been written on the rhetoric of economics in general; little, however, has been written on the rhetoric of game theory. The starting point of the discussion is that an economic model is a combination of a formal model and its interpretation. Using the Nash bargaining solution as an illustration I will first make the obvious claim that differences in models which seem equivalent result in significant differences in the interpretation of their results. The main argument of the chapter is more controversial. I will argue that the rhetoric of game theory is misleading in that it creates the impression that game theory is more "useful" than it actually is and that a better interpretation would make game theory much less relevant than is usually claimed in the applied game theory literature.

Though the book covers several distinct issues under the heading of "economics and language", it by no means covers all the issues that might be subsumed under this rubric. For example, I do not discuss the (largely ignored) literature labeled the "economics of language" which was surveyed recently in a special issue of the International Journal of the Sociology of Language (see Grin (1996)). Grin (1996) defines the "economics of language" as "a paradigm of theoretical economics and uses the concepts and tools of economics in the study of relationships featuring linguistic variables; it focuses principally, but not exclusively on those relationships in which economic variables play a part". This body of research does indeed revolve around traditional "economic variables" and related issues such as "the economic costs and benefits of multi-language society", "language-based inequality", and "language and nationalism". However, despite the similar headings, those issues are very far from my interests as expressed in this book.

### 0.3 One more Personal Comment

While browsing through the literature in preparation for these lectures, I came across a short article written by Jacob Marshack entitled the "Economics of Language" (Marschak (1965)). The article begins with a discussion between engineers and psychologists regarding the design of the communication system of a small fighter plane. Following the discussion Marschak states: "The present writer... apologizes to those of his fellow economists who might prefer to define their field more narrowly, and who would object to... identification of economics with the search of optimality in fields extending beyond, though including, the production and distribution of marketable goods." He then continues: "Being ignorant of linguistics, he apologizes even more humbly to those linguists who would scorn the designation of a simple dial-and-buttons systems a language." I don't feel that any apology is due to economists... but I do feel a sincere apology is owed to linguists and philosophers of language. Although I am quite ignorant in those areas, I hope that these essays have presented some interesting ideas for the study of language.

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# Semantics of Informational Independence

Gabriel Sandu

October 21, 1999

First-order languages of imperfect information ($IF$ first-order languages) have been introduced by Hintikka and Sandu [2]. These languages contained independent quantifiers and connectives of the form

$$(\exists x/W),\ (\forall x/W),\ (\vee/W),\ (\wedge/W)$$

where $W$ is a set of variables ($x \notin W$), the values of which the quantifiers and the connectives on the left side of the slash are supposed to be independent of. The notion of a variable being free or bound is the same as in ordinary first-order logic. In the formula $(\exists x/W)\varphi$ the variables in $W$ are free. In Hintikka and Sandu [3] only sentences receive a semantic interpretation, and this is done in terms of games $G(\varphi, A)$ of imperfect information played with an $IF$ first-order *sentence* $\varphi$, and a model $A$. The game $G(\varphi, A)$ is played by two players, *Eloise* ($\exists$) and *Abelard* ($\forall$). The standard example which goes back to Henkin (1961) is with $\varphi$ of the form $\forall x \exists y \forall z (\exists w/\{x\}) Rxyzw$. In this case the game can be pictured like this:

$$\begin{array}{ccc} \exists: & b & d/a \\ \hline \forall: & a & c \end{array}$$

Here $a, b, c, d$ are individuals from the domain $\mid A \mid$ of the model, '$d/a$' means that when choosing the individual $d$ *Eloise* did not "know" which individual $a$ *Abelard* chose earlier. The notion of "not knowing" is made explicit in the strategies of the players: they are defined only on earlier "known" choices of the opponent. In other words, a strategy for *Eloise* in the game consists of two functions

$$f, g : \mid A \mid \to \mid A \mid$$

The truth of a sentence $\varphi$ in a model $A$, $A \models \varphi$, is defined as the existence of a winning strategy for Eloise in the game $G(\varphi, A)$. In our particular example this means

$$A \models \varphi \Leftrightarrow A \models \exists f \exists g \forall x \forall z Rxf(x)zg(z)$$

The Hintikka-Sandu interpretation of $IF$ first-order sentences in terms of games of imperfect information is clearly not compositional. Subformulas of a sentence, like for instance, $\exists w/\{x\}Rxyzw$, did not receive a semantical interpretation.

There exist at this moment three semantics for $IF$ first-order sentences. Each of them is compositional and agrees with the Hintikka-Sandu interpretation on sentences. Each of them interprets an $IF$ first-order *formula* with respect to a set of assignments, and not with respect to a an assignment, as in ordinary first-order logic. Thus the key concept for all of them is

$$A \models_X \varphi$$

where $X$ is a set of assignments. We regard an assignment $f$ as a function $f : V \rightarrow \mid A \mid$, where $V$ is a set of natural numbers.

The first to give such a semantics was Hodges [4]. The second-one is given by Caicedo and Krynicki [1], and the third one by Väänänen [6]. It is straightforward to show that the three interpretations are equivalent.

The basic idea in each of them should be clear from the following few clauses:

**Definition 1** *(a)* $A \models_X \varphi \Leftrightarrow \forall x \in X(A \models \varphi(f(i_1), ..., f(i_m)))$, *with* $\varphi(v_{i_1}, ..., v_{i_m})$ *an atomic or the negation of an atomic formula.*

*(b)* $A \models_X \varphi \wedge \psi \Leftrightarrow A \models_X \varphi$ *and* $A \models_X \psi$

*(c)* $A \models_X \varphi \vee \psi \Leftrightarrow A \models_{X_0} \varphi$ *and* $A \models_{X_1} \psi$ *for some* $X_0, X_1$ *such that* $X = X_0 \cup X_1$.

*(d)* $A \models_X \exists v_n/\{v_{i_1}, ..., v_{i_k}\}\psi \Leftrightarrow (\exists F : X \rightarrow A)(A \models_{X[F,n]} \psi$ *and* $F$ *is* $\{i_1, ..., i_k\}-$*uniform.*

Here $X[F, n]$ is the set which is obtained from $X$ by adding to each of its sequences $f$ $F(f)$ as its $n$'th element. For $F : X \rightarrow \mid A \mid, X \in Pow(A^V)$, and $T \subseteq V$,we say that $F$ is $T-$uniform, if for all $f, g \in X$ we have

$$(\forall n \in V \backslash T)(f(n) = g(n) \Rightarrow F(f) = F(g))$$

(The presentation here is from Väänänen [6], but the same idea is present in Hodges [5] and Caicedo and Krynicki [1].)

The present paper is devoted to extend the interpretation above to modal propositional $IF$-languages. In these languages we shall have modal operators of the form

$$(\Diamond_n/\{W\}), (\Box_n/\{W\})$$

where $W$ is a set of natural numbers. Here are some examples of $IF$ modal sentences:

$$\Box_1\Box_2(\Diamond_3/\{1,2\})p, \ \Box_1(p \to \Box_2(\Diamond_3/\{1\})q)$$

The game interpretion for such *sentences* is completely analoguous to that for $IF$ first-order sentences. The game is played on a model $M = (T, (R_i)_{i<n}, V)$, with $T$ a set of possible worlds, $R_i$ a binary relation on $T$, and $V$ a valuation function, and starting in a possible world $w \in T$. The choices of *Abelard* and *Eloise* consists of possible worlds standing in the relevat accessibility relations. The interesting new element is to define a compositional semantics for these languages. This will be done *via* the concept

$$M \models_X \varphi$$

where $X$ is a set of "histories" in $M$.

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# Contributed Talks

# Spaces to Play: Topo-games

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Logical games may provide a useful paradigm to analyze—and also enjoy—topology. We have been investigating Ehrenfeucht-Fraïssé style model comparison games for topological models (topological spaces equipped with a valuation function) of modal languages.

## 1. Topo-Games: the rules

Spoiler and Duplicator play over two topological models $\langle X, O, \nu\rangle$, $\langle X', O', \nu'\rangle$ starting from two given points $x \in X$, $x' \in X'$, which we call *current* points, for a given number of rounds $n$. We refer to such game as $TG(X, X', n, x, x')$. Intuitively, Spoiler is trying to prove that the two points are 'topologically' different, while Duplicator is doing the opposite. Spoiler starts by choosing a model containing the current point in that model. Duplicator replies by an open set in the other space also containing the current point. The round is not over yet, as Spoiler has now to pick a point within Duplicator's open. The new current point of that model. Duplicator replies by picking a corresponding point in Spoiler's open. The new current point of that model. The first round has thus ended. By these sequences of rounds, the two players are constructing sequences of related points. If these points always agree pairwise in all atomic propositions, Duplicator has won, otherwise Spoiler has. Winning strategies (*w.s.*) and infinite games are defined as usual.

## 2. The languages

The underlying language we use is the modal logic S4 with its original topological interpretation, [Tar38]. In this setting, every formula represents a region of a topological space and the box is interpreted as the interior operator:

$$M, x \models \Box\varphi \text{ iff } \exists o \in O : \; x \in o \wedge \forall y \in o : \; M, y \models \varphi$$

E.g., $\Diamond\Box p$ denotes the closure of the interior of the set $p$. Of interest is the second order truth definition for the modal operator. We find two nested quantifications: one over sets and one over points. The first one is reflected in the first half of a round of a $TG$ game: choosing an open set. While the second one is for the second nested quantifier: choosing a point in the open set. As for Ehrenfeucht-Fraïssé games, a notion of adequacy is available.

**Theorem 1 (Adequacy)** Duplicator has a winning strategy in $TG(X, X', n, x, x')$ iff $x$ and $x'$ satisfy the same formulas of modal rank up to $n$.

From the proof of this theorem—[AvB99]—one extracts an effective method for building winning strategies for both players.

## 3. An example

In the figure on the next page, we have three example games: the two same spoons are played upon. We view the spoons as subsets $p$ of different copies

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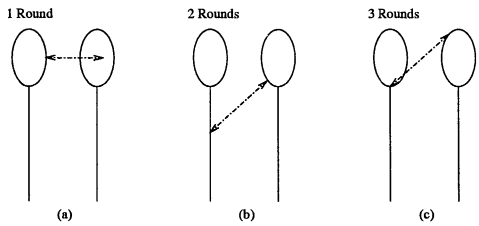


of the space $\mathbb{R}^2$. The starting points are different though. (a) The leftmost game starts by comparing a point on the frontier of the spoon with an interior point of the other spoon. Spoiler can win this game in one round, since $\Box p$ is true of the starting point of the right spoon, and its negation $\Diamond\neg p$ on the left. (b) In the central game, a point on the handle is compared with a point on the boundary of the container of the spoon. Again, Spoiler has a *w.s.* in the two round game: $\Diamond\Box\neg p$ holds on the starting point on the left spoon, but not of the starting point of the right one. (c) Finally, in the game on the left, the junction point between handle and container is related with a boundary point of the container. Spoiler has a *w.s*, since $\Diamond\Box p \wedge \Diamond(p \wedge \neg\Diamond\Box p)$ is true of the starting point on the left, but not on the right.

## 4. Game extensions

We defined, together with a family of languages of increasing expressive power, a family of *TG* games, [AvB99]. One extension is in terms of infinite games, for which we have that if Duplicator has a winning strategy in the infinite round game, then the two points are bisimilar. Another expressive power extensions goes towards globality (there are no starting points). Finally, we have defined several extensions towards geometry (e.g., in addition to opens we consider segments in the rounds of a game).

## 5. Fields of application

The definition of *TG* games is not only interesting from a merely game theoretic point of view. Its definition has brought new insights in logic, topology, and computer vision. Logic: more on interpretations of modal logics different from Kripke semantics. Topology: *w.s.* (and the related concept of bisimulation) have a strong connection with homeomorphism in topology and the correspondence can be refined to give a modal analysis of continuous mappings. Furthermore, bisimulation provides means to transfer information across spaces (e.g., connectedness is a bisimulation preserved property). Computer vision: an abstract take on languages to describe spatial patterns.

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# A Logic for Games

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The goal of this paper is to introduce a modal logic for sequential games with incomplete information. The logic is a mixture of dynamic and epistemic logics, designed to capture relevant game-theoretic notions, e.g. *perfect information*, *Nash equilibrium*, *subgame perfect equilibrium*, *knowledge of the best strategy* etc. I consider it as a first step towards a full modal formalization of the basic notions of game theory. Important notions e.g. *randomization, mixed strategies, beliefs* and *rationality* are missing from the present approach; a logical treatment of these notions would involve us in the complexities of *probabilistic modal logic.*

The main originality of the present approach consists in giving the semantics in terms of a Kripke structure formed of *pairs of states* (in a game) and *strategy profiles.* This is the basic feature of the logic. There are various natural choices for the language to be considered on this structure, and in further work I plan to explore some more of them. In this paper I chose to consider only the following: an *epistemic logic*, modelling player's knowledge of their current state and strategy, and a *dynamic logic* with programs constructed (in the usual manner) from basic *actions (moves) for each player i*, from *test* actions and from a deterministic *next-step action* (induced by the strategy profile at the given state).

The *notion of game* I consider here is an enrichment of the standard notion of game with imperfect information. Namely, as observed by Bonano and others, the standard notion does not provide a full semantics for epistemic logic, due to the fact that *each player's information sets consist only of states at which he/she has to move.* In order to talk about player's knowledge in other states, one needs to extend the information sets to obtain a full partition of the set of all the states. Bonano observed this extending can be done in many different ways, and he proposed some natural solutions to this problem. In the following, I take for granted that such a standard extension is given, and I identify a game with its extension. Consequently, for a given finite set $N$ of *players* and a given set $A$ of *actions*, a *game* is a tree $G = (S, P, \sim_i, u_i)_{i\in N}$, where $S \subseteq A^* \cup A^\omega$ is set of *states*, i.e. (finite or infinite) sequences of actions, $P$ is the*player function* from the nonterminal states to the set of players, $\sim_i$ are equivalence relations on $S$ and $u_i$ are functions from the terminal states to some linearly ordered set (usually taken to be $R$). We denote by $s \to^a s'$ the fact that $s' = sa$, and by $s \to s'$ the fact that $s \to^a s'$ for some action $a$; similarly, we put $s \to^a$ if we have $s \to^a s'$ for some state $s'$, and $s \to$ if we have $s \to^a$ for some action $a$. In the above definition, a *non-terminal state* is simply a state $s$ such that $s \to$. The initial state is $s_0 = \emptyset$ (the empty sequence). A game is assumed to satisfy suitable conditions: first, S has indeed to be *a tree*; secondly, if $s \sim_i s'$ then $s \to^a$ iff $s' \to^a$. In the following, we shall also assume all our games to be *finite*. As a consequence, the set of all utility

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scores $\{u_i(s) : s \in S\}$ is finite. Moroever, we fix a finite set $U$ of utility scores, and we consider only games having scores in $U$. A *strategy for player $i$* is a function $\sigma_i : \{s \in S : P(s) = i\} \to A$, satisfying the following conditions: $(s, \sigma_i(s)) \in S$; and if $s \sim_i s'$ then $\sigma_i(s) = \sigma_i(s')$. A *strategy profile* is a tuple $\sigma = (\sigma_i)_{i \in N}$ of strategies for each player. A strategy profile determines a unique function (also denoted by $\sigma$) from the set of non-terminal states into $A$, defined by: $\sigma(s) =: \sigma_i(s)$ if $P(s) = i$. I will denote by $\Sigma = \Sigma_G$ the set of all the strategy profiles of the game $G$. Our logical language consists of a set $Prog$ of *programs* and a set $For$ of *formulas*. The programs are formed from *tests* $?\varphi$ (as usually), *basic action labels* - one for each action $a \in A$, and a new action label $next$, called the *next-step action*. We allow the usual dynamic-logic constructions for programs: composition ;, union (non-deterministic choice) $\cup$ and Kleene star $*$. We have *atomic sentences* $p_i$, one for each possible utility score $p \in U$ and player $i \in N$. The meaning of $p_i$ will be "the current state is a terminal state at which player $i$'s score is greater or equal to $p$". We also have an atomic sentece $i$ for each player $i$, to denote the fact that it is player $i$'s turn to move. Besides these, we allow propositional variables $P, Q, \ldots$, whose meaning will depend on the model. Denote by $AtProp$ the set of all the atomic propositions, both variables and constants. Formulae are built from atomic sentences, using propositional connectives, dynamic modalities $[\pi]\varphi$ ("after program $\pi$ is executed, $\varphi$ is true") for each program $\pi$, and epistemic modalities $K_i\varphi$ ("player $i$ knows that $\varphi$"). For expressing important game properties involving *common knowledge*, we might have to add later the common-knowledge operator $K_I\varphi$ ("$\varphi$ is common knowledge among all the members of the group $I$"). The semantics is given in terms of pairs $(state, strategy - profile)$, as announced. Namely, we associate to the game $G$ a Kripke frame $K_G = (S \times \Sigma, R_\pi, R_{next}, R_i)_{\pi \in Prog, i \in N}$, where the frame relations are defined by:

$$(s,\sigma)R_a(s',\sigma') \text{ iff } s \to^a s', \sigma = \sigma';$$
$$(s,\sigma)R_{next}(s',\sigma') \text{ iff } s \to^{\sigma(s)} s', \sigma = \sigma';$$
$$(s,\sigma)R_i(s',\sigma') \text{ iff } s \sim_i s', \sigma_i = \sigma'_i;$$

while $R_{\pi\cup\pi'}, R_{\pi;\pi'}$ and $R_{\pi^*}$ are defined as usual in dynamic logic semantics. A *model* $M_G = (K_G, V)$ on this frame is given by a valuation $V : AtProp \to \mathcal{P}(S \times \Sigma)$, such that: $V(p_i) = \{(s,\sigma) : u_i(s) = p\}$, $V(i) = \{(s,\sigma) : P(s) = i\}$. Truth and validity are defined as usually. We use the following abbrebiations for special programs: $\epsilon_i =: (?i; \bigcup_{a \in A} a) \cup (?\neg i; next)$, and $\epsilon =: \bigcup_{i \in N} \epsilon_i$. With this definition, we obtain the following characterizations:

1. $\sigma$ is a Nash equilibrium profile in the game $G$ iff the pair $(s_0, \sigma)$ satisfies the following sentence
$$\theta^G_{Nash} \quad =: \quad \bigwedge_{i \in N} \bigwedge_{p \in U} (< \epsilon_i^* > p_i \to < next^* > p_i);$$

2. $\sigma$ is a subgame perfect equilibrium iff the pair $(s_0, \sigma)$ satisfies the sentence $[\epsilon^*]\theta^G_{Nash}$;

3. $G$ is a perfect information game iff the schema $\bigwedge_{a \in A, i \in N}([a]P \to [a]K_iP)$ is valid on the frame $K_G$.

## Dynamic Semantics Meets Game Theory, by Paul Dekker

*Abstract* In his 1997 Linguistics and Philosophy paper, Gabriel Sandu compared the dynamic semantic and the game-theoretical treatment of anaphora, and argued in favor of the latter. In this paper we take up Sandu's findings and show that principles from Game Theoretical Semantics instead can be used to motivate and improve procedures from Dynamic Semantics.

*Introduction* Dynamic Semantics is a branch of linguistics originating from the model-theoretic and referentially-based work of Montague and his followers. Key feature of a dynamic semantics is that it systematically takes into account certain pragmatic aspects of interpretation. Next to that of the (truth-conditional) *content* of an utterance, some versions focus on the *update* which an utterance of a sentence may bring about in the information state of a hearer; others instead focus on the *support* or evidence which a speaker may have to motivate an utterance.

In Dekker 1999 I have shown that, when we restrict our attention to a fragment of natural language which can be modeled by a language of predicate logic, the three notions interact elegantly. Content of, update with, and support for the utterance of predicate logical formulas can be defined separately, and anyone of these notions can be defined in terms of another. Besides, exchange of information can be shown to be safe. If an utterance is supported, then the update which the utterance brings about does not introduce any misinformation which the interlocutors did not have before the exchange.

The sketched outlook upon first order interpretation hinges upon two assumptions. First, indefinite noun phrase are interpreted as a certain kind of free variables, and, second, the terms are required to be used with referential intentions. Thus, support for the use of indefinite (and other) terms can be seen to relate to the game-theoretical notion of a witness. The dynamic semantic formulation of these assumptions is attractive because it does not require the existence (knowledge, disposal) of such witnesses, since it suffices to model the reliance upon or commitment to their existence.

*Principles of Dynamic Interpretation* If, as we hold, indefinite noun phrases associate with holes in (open) propositions, sentential operators have to be adjusted to cope with them, preferably in a motivated fashion. For conjunction this is easy. The dynamic semantic notion of conjunction can be understood to acknowledge the order of indefinite terms (referential intentions) in a discourse. The possible values of these expressions are ordered, and the information about them is conjoined. Thus, dynamic conjunction can still be understood to involve intersection, basically. (As proved in Dekker 1999)

The interaction of indefinites with other sentential connectives and operators (negation, implication, modalities) is less well understood. According to the standard dynamic semantic definition of these operators, they all induce an existential closure of the open positions deriving from indefinite terms. Thus, e.g., the dynamic semantic negation of $\phi$ corresponds to the classical negation of the existential closure of $\phi$.

Sofar it has remained unclear what motivates precisely these closures. It has been claimed that the involved operators introduce anaphoric islands, inaccessible for subsequent anaphoric pronouns. But, as Sandu observes, this is dubious in many cases. The operators often appear to be anaphoric peninsulae. Besides, in the cases where we do find existential closure, the rationale of this is unclear.

*Game Theoretical Motivation* In my talk I argue that further pragmatic principles, and principles from game theoretical semantics can be used to motivate these dynamic semantic principles of interpretation, as well as further qualifications of them. In the first place, intuitive rules of conversation can be seen to conspire against referential ('wide scope') readings of indefinites in a variety of (downward entailing) contexts. In the second place, when we think, game-theoretically, of negation as switching the burden of proof, the *support* for a negated formula can be spelled out in terms of evidence against the *update* with that formula. Thus, the following definition suggest itself:

(1) $\sigma \models \neg\phi$ iff $[\![\phi]\!](\sigma) = \bot$ (state $\sigma$ supports $\neg\phi$ iff the update with $\phi$ is absurd)

Under this interpretation, indefinites under a negation do not (need to) come with referential intentions, because they are not the speaker's responsibility. We see, furthermore, that the information which is required to support a negated formula, equals precisely the information which the formula is said to convey upon its standard dynamic semantic definition.

The support rule for $\neg$ can also be invoked to define support for an implication: $(\phi \rightarrow \psi) = \neg(\phi \wedge \neg\psi)$. Thus, we find that:

(2) $\sigma \models (\phi \rightarrow \psi)$ iff $[\![\phi]\!](\sigma) \models \psi$, that is, iff $\forall e$: if $e \in [\![\phi]\!](\sigma)$ then $\exists c$: $ce \in [\![\psi]\!]([\![\phi]\!](\sigma))$

This definition can be seen to capture the game-theoretical insight that support for a conditional consists in a procedure to transform a proof of the antecedent into a proof of the consequent. In (2) the update function $[\![\bullet]\!](\sigma)$ on $\sigma$ itself turns evidence for $\phi$ into possible support for $\psi$. We furthermore find that the information required to support an implication, equals the content of the implication upon its standard dynamic semantic definition.

*Functional Dependencies* Proof or support for a conditional can be substantiated further, if the supporting state disposes of a function which actually turns evidence for the antecedent into proof (witness) of the consequent. This function can be indicated by a linking function $l(\rightarrow)$:

(3) $\sigma \models_l (\phi \rightarrow \psi)$ iff $[\![\phi]\!](\sigma) \models_{l(\rightarrow)(n(\phi))} \psi$, that is, iff $\forall ce$: if $e \in \sigma$ and $ce \in [\![\phi]\!](\sigma)$ then $e_{l(\rightarrow)}(c)ce \in [\![\psi]\!]([\![\phi]\!](\sigma))$

Upon this definition indefinites in the consequent of an implication are still associated with referential intentions, be it that they may be functionally dependent upon possible values of indefinites in the antecedent.

(4) If a book is printed with Elsevier it has a table of contents.

A state $\sigma$ supports this implication under a link $l$ if it disposes over a witness-function (indicated by $e_{l(\rightarrow)}$), which associates possible books printed with Elsevier with their tables of contents. Interestingly, this function can be referred back to, if we quantify again over the same set of books:

(5) In some books it is at the end.

This example is supported iff, in any case considered possible in $\sigma$, in some of the Elsevier books the associated table of contents is at the end of the book.

*Further Dependencies* More in general, a referential interpretation of indefinites can be maintained, in an upward entailing context, when we make them functionally dependent upon governing quantifiers or operators. Thus, we can dismiss with existential closures under, e.g., $\forall x$ and $B_x$, in the following way:

(6) $\sigma \models_{l,g} \forall x\phi$ iff $\forall d$: $\sigma \models_{l',g[x/d]} \phi$ with $[l'(y)]_e = e_{l(y)}(d)$ (for $y \in D(\phi)$)

(7) $\sigma \models_l B_x\phi$ iff $\forall e \in \sigma$: $\mathcal{R}_{e_v,e_x} \models_{l'} \phi$ with $[l'(y)]_e = e_{l(y)}(e_y)$ (for $y \in D(\phi)$)

Upon this definition, we get a dynamic Skolem equivalence between

(8) $\forall x \exists y \phi(x,y)$ and $\exists f \forall y \phi(x, f(x))$

such that the witnessing Skolem function $f$ is accessible for subsequent pronouns. As predicted, dependent indefinites also provide suitable antecedents for subsequent pronouns which figure in similar contexts:

(9) Harvey courts a girl at every convention. She always comes to the banquet with him. (Karttunen 1968)

(10) If every man is given a gun, then some man will fire it. (Sandu 1997)

(11) Mary thinks there is a burglar in the house. She thinks he came in through the chimney.

(12) A woulf might come in. He would eat you first.

Support for (9) may draw from a function associating conventions with girls Harvey dates, and support for (11) relates to witnesses of Mary's, which represents burglars in her belief worlds.

Evidently, the qualified definitions of $\rightarrow$, $\forall$ and $B_a$ require more structure in the supporting information states then we had before, but the definitions are more transparent themselves since no underlying existential closure is required.

*Conclusion* In this paper we have given a further elaboration of the idea that indefinite noun phrases, like other terms, are used with referential intentions. The basic idea is that the use of these expressions is supported by witnesses in the information states of the speaker, and this idea can be used to motivate the dynamic semantic notion of conjunction as a 'pragmatically' infected form of intersection. Next we have called upon principles of game-theory to motivate and improve dynamic semantic notions of implication, quantification and modalization. The notion of role-switches or turn taking, and the call for witness-functions, provides the key to understanding (and improving the formulation of) the dynamics of these operators.

# The semantics of actions in knowledge games

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## 1 Knowledge games

A *knowledge game* is defined by a dealing of cards over players, a set of possible actions (or moves), an order protocol to determine who is next to move, and a procedure to determine if a player has won. Cards do not change hands during a play of the game. A *state of the game* is defined by a given dealing of cards and an action sequence, initially empty.

A *dealing* of cards is a function from the set of cards $C$ to the set of players $P$. The *initial state of the game* can be represented by the dealing of cards. The initial state space $S_s$ corresponding to a dealing $s$, is the subset of $P^C$ where all players have the same number of cards as in $s$. Dealings $s$ and $s'$ are indistinguishable from each other for player $p$ if they agree on his cards. This induces a structure $(\sim_p)_{p\in P}$ on $S_s$. $\sim$ is the transitive closure of all $\sim_p$. $\sim$ is the universal relation on $S_s$.

Let $c_p$ be the proposition corresponding to player $p$ holding card $c$. The initial state of the game given a dealing $s$ corresponds to the pointed Kripke model $(\langle S_s, (\sim_p)_{p\in P}, V\rangle, s)$ with $\forall s' \in S_s : V_{s'}(c_p) = 1 \Leftrightarrow s'(c) = p$.

A pointed multimodal $S5$ model is called a *knowledge state.* A knowledge state for a set of agents $B$ is called a *$B$ knowledge state.* Every state of a knowledge game is a knowledge state. A game action is a knowledge state transforming operation.

## 2 Uniform knowledge programs

An action transforming a knowledge state can be described by a knowledge program. We define the class of uniform knowledge programs (UKP). Many actions in knowledge games [Ditmarsch 99] can be modelled as *deterministic* uniform knowledge programs. This includes all game actions in Cluedo, such as showing a card (only) to another player given a request for one of three cards. The class of uniform knowledge programs (UKP) is a subclass of the class of uniform partial knowledge programs (UPKP). Uniform partial knowledge programs are formed by tests, sequential execution, nondeterministic choice, and the 'learning with choice' operation '!': $\pi \,!_B\, \pi'$ stands for '$\pi$, and group $B$ learns ($\pi$ or $\pi'$)' – a group learns a nondeterministic choice between two programs, of which the first one is executed.

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We define: $\pi\ !_B\ \pi' \in$ UPKP, if $\pi, \pi' \in$ UPKP, and $|\pi| = |\pi'|$, and $|\pi| \subset B$. Here $|\pi|$ is the *size* of a program: the group of agents 'involved' in it, inductively defined such that $|\pi\ !_B\ \pi'| = B$. So we require that group $B$ can *only* learn programs concerning subgroups *strictly* contained in it. The constraint $|\pi| = |\pi'|$ on program constructs is why we have called our programs *uniform.* The class of uniform knowledge programs (UKP) given a set of agents $P$ is the subclass of UPKP with size $P$.

The interpretation of a UKP program $\pi$ is a relation $[\![\pi]\!]$ between knowledge states. As $\pi$ is composed of partial uniform knowledge programs, we introduce the concept of *partial interpretation* $[\![\pi]\!]^{|\pi|}$ of a program. The partial interpretation of a program is its interpretation in a *restricted knowledge state.* The restriction of a $P$ knowledge state $s = (\langle S, (\sim_a)_{a\in P}, V\rangle, s)$ to the set of agents $B \subset P$ is the $B$ knowledge state $s{\restriction}B := (\langle S, (\sim_a)_{a\in B}, V\rangle, s)$.

The idea behind the interpretation of 'public learning with choice' $\pi\ !\ \pi'$ (if $B = P$ in $!_B$ we delete the index) is the following: 'First' we *partially* interpret program $\pi \cup \pi'$. This results in a finite set $S$ of alternative (nonsimilar) $|\pi|$ knowledge states. 'Then' group $|\pi|$ 'chooses' one of $S$ from those that can be reached by executing $\pi$. 'Last', we construct a $P$ knowledge state from $S$. As players in $|\pi|$ know what they have chosen, there is no $|\pi|$ access between worlds from different alternatives in $S$. For the players in $P\backslash|\pi|$ however, we add $P\backslash|\pi|$ access between *any* previously (to the execution of $\pi\ !\ \pi'$) indistinguishable worlds, whether or not from the same state in $S$!

We will present the semantics of UKP programs, and several examples.

Our approach is strongly motivated by [Baltag et al. 99] and [Gerbrandy 99]. A related approach is found in [Baltag 99]. A simple translation $\triangle$ embeds UKP programs into Gerbrandy's DEL programs. Its essential clause is $(\pi\ !_B\ \pi')^\triangle := \pi\ ; U_B(\pi \cup \pi')$. We now have found a much desired class of DEL programs under whose execution the class of $S5$ models is closed.

## 3 Knowledge programs

Some actions in knowledge games are not 'uniform': a group might learn about *different* subgroups. Also, some actions in knowledge games have parts that had best be seen to take place *simultaneously.* An example of both is the action where a player (simultaneously) shows a card to a player and one of his other cards to another player. We are currently extending UKP with parallel execution of programs and with non-uniform learning.

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# Games as Acceptors

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Every systematic effective procedure which constructs a (two person) game from some input data characterizes a formal language. This language consists of those input strings for which the resulting game is won by the first player. By this procedure we can ascribe to games a role which they so-far have rarely played in the literature: *Games as Acceptors*.

Truth is that this idea is neither new nor unknown. In the late seventies and early eighties researchers in Computation and Complexity Theory have established a group of characaterizations of the class PSPACE where games play an essential role. This very robust class can be characterized in terms of space complexity on reasonable sequential models, as the polynomial time bounded class on Ultra Parallel but Uniform Machines of various sorts (as expressed by the *Parallel Computation Thesis*), as the polynomial time bounded class on alternating squential devices, which establishes a link with machine computation games played by two opponents both controling the machine, and by the expressive power of Quantified Propositional Logic. These characterizations and their proofs represent one of the nicest achievements in Computation Theory around 1980.

However, it is quite possible that the strength of these results is also responsible for the fact that the connection between games and other computational models has not been investigated further. The general known fact that (finite Combinatorial Complete Information two-peron games in general show up a PSPACE complete problem to characterize the winning instances) has discouraged people from looking further.

In fact there are many other modes of computation which can be related to games. A prime example is the realm of nondeterministic computation, where in fact games have appeared in the literature which capture NP by having an NP-complete decision problem of being a winning game or not. However, most of these games are single player solitaire games, like the famous tiling game (in fact one of the first problems shown to be NP-complete by Levin). Even Deterministic computation can be modelled by a game, be it one which no human ever will be interested to play: like filling out your I-1040 tax form the game offers no choice to the player and already at the start it has been decided whether you win or loose.

Aside from an extension in this downward direction (towards more simple games) there are other topics which ask for being investigated from the perspective of games as acceptors.

It is well known in the realm of machine based computation there is nothing like the Church-Turing Paradise we experience in recursion Theory. We have a stable complexity theory centered on the Turing Machine model, but extending this stability (as expressed by the Invariance Thesis) towards alternative models like register machines, von-Neuman like computer models like the RAM and pointer based models like the SMM has turned out to be non trivial; several less suitable definitions have found a strong foothold in the literature.

In the realm of Parallel models the lack of stability is even larger. There is a class of parallel models based on exponential growth potential and uniformity which obeys the parallel computation thesis in a rather strict way; outside this realm chaos prevails, and each model seems to acquire its own complexity characterization (either above or below PSPACE).

On the side of the games the lack of formal foundations has its consequences. Games have been investigated for 50 years by economists but their emphasis is on the strategic form which eliminates all structural information available in the extensive form. The games corresponding to computational activities in general will be given in an extensive form. There seems to be a lack of understanding about the correct model for extensive games in general (the analogue of the Turing Machine for extensive games still must be invented), and moreover the question of equivalence between games (when are two games the same) seems far from settled.

The purpose of the proposed program to investigate the relation between games and computational models in computer science is two-fold. It should increase insight on both sides of the border.

In computer science, particularly in the area of interactive protocols and zero-knowledge exchange of information, models of computational interaction have been described which by nature are games, where features like probabilistic moves and incomplete information become relevant. Yet these games exhibit some strange feature. The winning conditions in these games are rather extraordinary: a winning starting position supports a winning strategy in a standard way, but a lost position must remain lost even if the player has the possibility of performing a number of cheats and illegal moves (up to full Byzantine behavior). This yields games where the nature of the reduction turning the game into an acceptor has obtained a *promise problem* character. It would be interesting to investigate to what extent these interactive protocols can be expressed in terms of more regular games, but also whether this promise feature can play a role in game theory as well.

Then there are acceptance modes investigated in computer science where it is far from evident whether a game is involved or not, notwitstanding the fact that these acceptance notions are defined in terns of structural properties of the full computation tree of a machine. The *Leaf languages* as investigated by the Würzburg school represent a prime example. It would be interesting to investigate whether in general a game can be invented corresponding to this leaf language recognition mechanism. If so what is the game; if not, does this fact shed some light on this acceptance mechanism in the sense that the limitation of the use of this mechanism representable by games correlates with the degree of naturalness of this mechanism.

On the side of the games the challenge is to investigate whether concepts from computation theory can be invoked to provide for a more robust and invariant concept of extensive games supporting structural equivalence notions and closely tight in with complexity theory.

At this stage this problem area has generated more questions than answers. In my presentation I will exhibit the landscape which was uncovered during the 1980-ies and indicate where the loose ends are located.

# The Game of Interrogation

Abstract

**Jeroen Groenendijk**

In this talk we give a logical analysis of the following language game:

**Definition 1 (The Game of Interrogation)** Interrogation is a game for two players: the *interrogator* and the *witness*. The rules of the game are as follows:

A. The interrogator may only raise issues by asking the witness non-superfluous questions.
B. The witness may only make credible non-redundant statements which exclusively address the issues raised by the interrogator.

The game of interrogation can be looked upon as a logical idealization of the process of cooperative information exchange. The elements of the rules can be linked to elements of the Gricean Cooperation Principle: The requirement that the witness makes credible statements is related to the Maxim of Quality; that the statements of the witness should be non-redundant, and the questions of the interrogator non-superfluous, relates to the Maxim of Quantity; and that the witness should exclusively address the issues raised by the interrogator is a formulation of the Maxim of Relation.

The logical analysis takes the form of developing logical notions which enable us to judge whether the game was played in accordance with the rules. Relative to a suitable language, and a semantic interpretation for that language, the logic of interrogation has to provide us with logical notions by means of which we can arbitrate the game.

As a language for the game of interrogation, we use a simple query-language, a language of first order predicate logic enriched with simplex interrogatives:

**Definition 2 (Query-Language)** Let $PL$ be a language of predicate logic.
The *Query Language* $QL$ is the smallest set such that:

i. If $\phi \in PL$, then $\phi \in QL$;
ii. If $\phi \in PL$, $\vec{x}$ a sequence of $n$ variables $(0 \leq n)$, then $?\vec{x}\,\phi \in QL$.

We can represent the proceedings of a game of interrogation as a sequence of formulae in $QL$. Given the strict casting, we do not have to indicate who said what: The interrogatives are uttered by the interrogator, the indicatives by the witness.

Next we state the semantic interpretation of the language. We do so in two steps. First we give a standard denotational semantics, and in terms of that we define the context change potentials of the formulae of the language. For the indicative part of the language, we provide a standard truth definition. The denotational semantics for the interrogatives is a so-called partition semantics.

**Definition 3 (Semantics of Questions)**
$||?\vec{x}\,\phi||_{w,g} = \{v \in W \mid \forall \vec{e} \in D^n\colon ||\phi||_{v,g[\vec{x}/\vec{e}]} = ||\phi||_{w,g[\vec{x}/\vec{e}]}\}$.

The denotation of in an interrogative in a world $w$ consists of the set of worlds where the question has the same (true and complete) answer.

To be able to define context change potentials, we introduce a notion of structured context which is suitable for specifying the context change of both interrogatives and indicatives.

**Definition 4 (Structured Contexts)**
A *context* $C$ is a symmetric and transitive relation on the set of possible worlds $W$.

Intuitively, if two worlds are contextually related, it is not an issue whether the actual world is like the one or like the other. If two worlds are disconnected it is an issue whether the actual world is like the one or the other. For ease of notation, we write $w \in C$ as short for $\langle w, w\rangle \in C$. If $w \notin C$, this means that this world is excluded by our information.

The context change potentials of sentences and sequences thereof are defined as follows. ($\phi!$ ranges over the indicatives of the language, $\phi?$ over the interrogatives.)

**Definition 5 (Context Change Potentials)**
i. $C[\phi!] = \{\langle w, v\rangle \in C \mid ||\phi!||_w = ||\phi!||_v = 1\}$;
ii. $C[\phi?] = \{\langle w, v\rangle \in C \mid ||\phi?||_w = ||\phi?||_v\}$;
iii. For $\tau = \phi_1; \ldots; \phi_n$, $C[\tau] = C[\phi_1] \ldots [\phi_n]$.

Indicatives provide data by eliminating worlds from the context. Interrogatives raise issues by disconnecting worlds in the context.

Given a suitable language for the game and a semantics interpretation for that language, the logic of the game of interrogation should provide us with logical notions which enable us to arbitrate whether the game was played according to the rules. Relative to a (possible empty) sequence of sentences $\tau$ in $QL$, we define whether a sentence $\phi$ is pertinent after $\tau$. The notion of pertinence has three elements: Consistency, non-entailment, and licensing. Consistency is defined in a rather standard way:

**Definition 6 (Consistency)** $\phi$ is *consistent with* $\tau$ iff $\exists C\colon C[\tau][\phi] \neq \emptyset$.

The notion of consistency will be used to deal with the qualitative aspects, the credibility of utterances. The notion of entailment is also standard:

**Definition 7 (Entailment)** $\tau \models \phi$ iff $\forall C\colon C[\tau] = C[\tau][\phi]$.

The notion of (non-)entailment will be used to deal with qualitative aspects, the non-superfluousness of questions, and the non-redundancy of statements. The only new logical notion, is the notion of licensing:

**Definition 8 (Licensing)** $\tau$ *licenses* $\phi$ iff $\forall C, w, v\colon \langle w, v\rangle \in C[\tau]$ & $w \notin C[\tau][\phi] \Rightarrow v \notin C[\tau][\phi]$.

Licensing forbids the elimination of some world from the context, in case some world which is contextually related to it, remains. Licensing is used to arbitrate relational aspects, it enables us to judge whether the statements of the witness exclusively address the issues raised by the interrogator. The overall arbitration of the game of interrogation is taken care of by the notion of pertinence.

**Definition 9 (Pertinence)** $\phi$ is *pertinent after* $\tau$ iff
i. $\phi$ is consistent with $\tau$ (*Quality*)
ii. $\phi$ is not entailed by $\tau$ (*Quantity*)
iii. $\phi$ is licensed after $\tau$ (*Relation*)

We will discuss some logical properties of the notion of pertinence, and will illustrate its use in discussing some linguistic examples, which intend to show that the notion of pertinence can help to explain certain structural semantic facts about natural language.

# Subgame Perfect Nash Equilibria in Dynamic Logic

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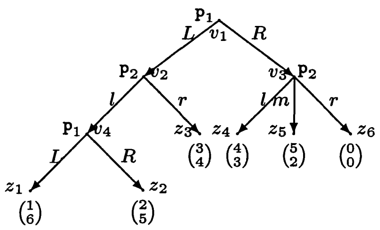
Multi-agent environments comprise decision makers whose deliberations involve reasoning about the expected behaviour of other agents. Our research concerns the development of specification languages for such multi-agent systems. As such our particular interest is with concepts of rational choice as they have been studied and formalised within game theory as well as with their integration in a qualitative and logical framework. Here we are concerned with the incorporation of the important game-theoretical notion of a subgame perfect Nash equilibrium in a modified version of Propositional Dynamic Logic ($PDL$). Extensive forms of games are linked up to Kripke frames. This enables us to characterize those frames in which the strategic choices of the agents correspond to Nash-optimal solutions by means of a formula scheme of our logical language. The present analysis is restricted to generic and finite games with perfect information and no chance moves. Furthermore, the focus will be on qualitative, rather than on quantitative features.

In extensive form games are represented as trees, the internal nodes of which are each assigned to a player. As the edges of the game tree represent the possible moves the players can make, a strategy $\sigma$ for a player comprises complete instructions how to move at each of the nodes assigned to him/her. Given strategies $\sigma_1, \ldots, \sigma_n$, one for each of the players $p_1, \ldots, p_n$, a strategy profile $\bar{\sigma} = \langle \sigma_1, \ldots, \sigma_n \rangle$ determines a unique outcome of the game. In strategic form an $n$-person game is represented by setting all strategies of the respective players against one another in an $n$-dimensional matrix. For an example, see figure 1:



| $p_1 \backslash p_2$ | $ll$ | $lm$ | $lr$ | $rl$ | $rm$ | $rr$ |
|---|---|---|---|---|---|---|
| $LL$ | 1\6 | 1\6 | 1\6 | 3\4 | 3\4 | 3\4 |
| $LR$ | 2\5 | 2\5 | 2\5 | 3\4 | 3\4 | 3\4 |
| $RL$ | **4\3** | 5\2 | 0\0 | **4\3** | 5\2 | 0\0 |
| $RR$ | **4\3** | 5\2 | 0\0 | **4\3** | 5\2 | 0\0 |

*Figure 1*

A strategy profile $\bar{\sigma} = \langle \sigma_1, \ldots, \sigma_n \rangle$ is in Nash-equilibrium if for each player $p_i$, $\bar{\sigma}$ guarantees him/her the highest payoff *given the strategies of the other players*, i.e. for each $\sigma_i' \neq \sigma_i$, the strategy profile $\langle \sigma_1, \ldots, \sigma_i', \ldots, \sigma_n \rangle$ does not grant $p_i$ a higher pay-off than $\bar{\sigma}$ does. A strategy profile is in subgame perfect Nash equilibrium if it is in Nash-equilibrium for all its subgames.

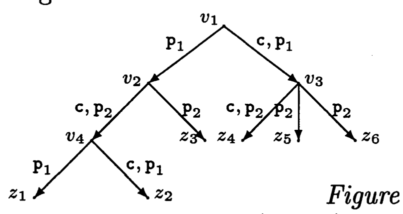
*Paul Harrenstein is sponsored by the CABS-project and Technical University, Delft.

In establishing a Nash-optimal strategy profile one will be confronted by certain fixed point concerns. When focussing on extensive forms these can be resolved by the backward induction algorithm. Backward induction defines a relation (the backward induction relation) on the game tree, which will here be denoted by $\lessdot$.

Apart from the payoff structure and the players, a game in extensive form $\mathcal{G}$ is a Kripke frame $\mathcal{F}$ ($\mathcal{G} \simeq \mathcal{F}$). The nodes correspond to states and the edges define the accessibility relation. Our approach is based on the idea that each player, $\mathtt{p}$, can be conceived of as a non-deterministic atomic program, interpreted as the transitions that correspond to the edges emanating from the nodes assigned to $\mathtt{p}$ in the game. Furthermore, a choice program $\mathtt{c}$ will be introduced to the language. This program is meant to reflect the strategy profile under consideration. In order to account for the (generic) payoff structure of the game for each player $\mathtt{p}$, a total order is imposed on the terminal states such that $v <_{\mathtt{p}} v' \iff \mathtt{u}(\mathtt{p})(v) < \mathtt{u}(\mathtt{p})(v')$, where $\mathtt{u}(\mathtt{p})(v)$ is the payoff $\mathtt{p}$ receives whenever the game terminates in node $v$. In order to gain expressive power with respect to this order, a new operator $\boxminus$ is added to the language with the following semantics:

$M, s \models \boxminus_{\mathtt{p}}\varphi :\iff$ for all leaf-nodes, $z, z'$, such that $sR^*z$ & $sR^*z' : M, z \models \varphi$ & $M, z' \not\models \varphi \implies z' <_{\mathtt{p}} z$

As such, given the strategy profile $\langle RR, ll\rangle$, the game of figure 1 corresponds to the following frame:



$z_6 <_{\mathtt{p}_1} z_1 <_{\mathtt{p}_1} z_2 <_{\mathtt{p}_1} z_3 <_{\mathtt{p}_1} z_4 <_{\mathtt{p}_1} z_5$
$z_6 <_{\mathtt{p}_2} z_5 <_{\mathtt{p}_2} z_4 <_{\mathtt{p}_2} z_3 <_{\mathtt{p}_2} z_2 <_{\mathtt{p}_2} z_1$

*Figure 2*

In figure 1 the strategy profile $\langle RR, ll\rangle$ is in subgame perfect Nash-equilibrium. In figure 2 the corresponding $\mathtt{c}$ program coincides with the the backward induction relation of the corresponding game. Although some restrictions are imposed on the $\mathtt{c}$ program — it should be deterministic, irreflexive and defined on each internal node — this is not always the case. Our objective is to characterize those frames for which the $\mathtt{c}$ program coincides with the backward induction relation of the corresponding game by means of a formula scheme of the augmented language $L$.

In order to achieve this goal let for each $\mathtt{P}' \subseteq \mathtt{P}$, $\alpha_{\mathtt{P}'}$ be the complex progam:

$\mathtt{do}\ [\langle \mathtt{p}_1\rangle\top \to \mathtt{p}_1 \square \ldots \square \langle \mathtt{p}_n\rangle\top \to \mathtt{p}_n \square \bigvee_{\mathtt{q}_i \notin \mathtt{P}'} \langle \mathtt{q}_i\rangle\top \to \mathtt{c}]\ \mathtt{od}$,

where $\mathtt{P}' = \{\mathtt{p}_1, \ldots, \mathtt{p}_n\}$. Each run of the program $\alpha_{\mathtt{P}'}$ executes $\mathtt{p}$ for a $\mathtt{p} \in \mathtt{P}$ where possible, and $\mathtt{c}$ otherwise, and terminates if a final state is reached. Accordingly, $\alpha_{\{\mathtt{p}\}}$ corresponds to the non-deterministic program that terminates in exactly those states that $\mathtt{p}$ can guarantee the game to terminate in by varying his strategy provided the other players remain true to their strategs as encoded in $\mathtt{c}$. The program $\alpha_\emptyset$ consists of a repeated execution of $\mathtt{c}$ until it reaches a final state.

Finally, the desired correspondence result can be obtained:

**Theorem 1** For each game $\mathcal{G}$ and each Kripke structure $\mathcal{F}$ such that $\mathcal{G} \simeq \mathcal{F}$:

$R_{\mathtt{c}} = \lessdot \iff$ for all $\mathtt{p} \in \mathtt{P}$: $\mathcal{F} \models (\langle \mathtt{p}\rangle\top \wedge \boxminus_{\mathtt{p}}\varphi \wedge \langle \alpha_{\{\mathtt{p}\}}\rangle\varphi) \to [\alpha_\emptyset]\varphi$

This theorem establishes that some model checking settles the question whether, when operating in circumstances that can be described as a perfect information game, agents behave as if they use backward induction in their reasoning.

# Informational independence

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**Introduction** In game theoretical semantics the truth of a formula is determined by a game between two players, $\exists$loise who tries to verify the formula, and $\forall$belard to refute it. He chooses on $\wedge$ and $\forall$, she on $\vee$ and $\exists$. An extension of such games, introduced by J. Hintikka, is IF logic: independence friendly logic. It contains the quantifier $\exists(y/x)$ which means that $\exists$loise has to choose $y$ independent of $x$, and $\psi(\vee/x)\theta$ which means that she has to choose a subformula has independent of $x$. A formula is true if $\exists$loise has a winning strategy. Hodges has given a compositional interpretation for the logic: trump semantics. His interpretation gives results that are not in accordance with intuitions concerning independence of information. Therefore an alternative interpretation will be proposed.

**Examples** Two examples (over **N**) which illustrate the aims of IF logic are:

1. $\forall x \exists (y/x)[x = y]$

When $\forall$belard has chosen, $\exists$loise does not know the value of $x$, and therefore there it may happen that she selects another value. Hence $\exists$loise has no winning strategy, so the formula is 'not true'. The variant $\forall x \exists (y/x)[y \leq x]$ is true: choose $y := 0$ ('choose $y$ to become 0').

2. $\forall x_1 \exists y_1 \forall x_2 \exists (y_2/x_1)[x_1 < y_1 \wedge x_2 < y_2]$

The choice of $y_2$ is independent of $x_1$, but may depend on $x_2$. The formula is true (on **N**). A more familiar representation of this example is by means of a branching quantifier:

$$\begin{pmatrix} \forall x_1 & \exists y_1 \\ \forall x_2 & \exists y_2 \end{pmatrix} [x_1 < y_1 \wedge x_2 < y_2]$$

**Problems** Some examples which illustrate the problems of trump semantics are:

1. $\forall x [x \neq 2 \vee (\exists y/x)[x = y]]$

The strategy which $\exists$loise follows in trump semantics is to choose $y := 2$. Her strategy for the disjunction is: if $x$ happens to be different from 2, she chooses the left side of the disjunction (which then is true), and otherwise she chooses the right hand side (which then becomes true by her choice for $y$). So $\exists$loise wins because she deduces from the context, when $\exists x/y$ is played, that $x = 2$. Since she is supposed *not* to use the value of $x$, this is cheating.

2. $\forall x [(\exists y/x)[x \neq y] \vee (\exists y/x)[x \neq y]]$

If you cannot find a $y$ independent of $x$, you also cannot find it if you have to do that twice. But in trump semantics that is not the case. If $x = 3$, then $\exists$loise chooses left, and there she always chooses 4. Otherwise $\exists$loise chooses right, and there 3. In both cases the chosen disjunct is true. So $\phi$ is true.
This example shows a strange property of trump semantics: $\phi \vee \phi$ is not in all contexts equivalent with $\phi$.

**Discussion** The examples given above illustrate that, although in trump semantics the strategy for $(\exists y/x)$ and $\vee/x$ indeed is not based directly on the value of $x$, any information about the value of $x$ which can be deduced from other sources may be used: the value of other variables, the form of other parts the formula, or strategies used in other parts of the formula. For this reason trump semantics is not a formalization of 'informational independence', but of 'incomplete information'. It is not impossible that Hodges would agree with this opinion, because the title of his paper is 'Compositional semantics for a language of imperfect information', and *not* '... for a language of informational independence'.

**The idea** The basic idea of our approach is that subformulas are considered as subgames. So the notion 'game' is generalized to arbitrary formulas. A game is a pair consisting of a formula (maybe with free variables), and a dependency set, which for each $x$ indicates which $y$'s depend on it. Such a game can be played on its own. If the formula has free variables either a initial position (initial values for the free variables) is set up by the players, or it is given by previous moves in a larger game. Then the game is played.

$\exists$loise may have a strategy which describes how she will react on moves of $\forall$belard. A strategy for $\exists y$ is a function that yields a value for $y$ which has as argument the start position of the subgame, and one for $\exists y/x$ is such a function without the value of $x$ as argument. Analogously for $\vee$ and $\vee/x$. After application of such a strategy, a new subgame is entered.

∃loise may have a strategy for a subgame which guarantees her a win. Such a winning strategy can, of course, not depend on variables that do not occur in the subgame. Also the larger context in which the subformula occurs, is irrelevant: the strategy must be winning for the subgame as such. A winning strategy for $\exists y/x$ must be one that works for all values of $x$, provided the values of $y$'s that depend on $x$ are adapted.

A winning strategy for an entire game is formed by a combination of winning strategies for its subgames. One might describe the situation as follows: associated with a (sub)game there is a shelf of plans, and when a subgame is entered in a certain initial position, a plan is taken from the shelf which fits on that position, and it is followed until a new subgame is entered. This concept of a winning strategy means that not all conceivable strategies are available; only those which are built from strategies that would also work if the game was played in isolation or as subgame of a larger game. So in comparison with Hodges' approach, less strategies are possible.

Maybe it is useful to emphasize the distinction with other interpretations of 'independence':

1. It is not assumed that there are players who forget a value for a variable and may remember it later.
2. It is not assumed that there are teams of players in which for each new variable a new player is introduced who gets only partial information
3. It is not based on equivalence classes of information sets within the choice tree of a given game.

One might say that in the present approach equivalence classes are introduced among the information sets in *all* games in which the subgame arises.

**Examples** Some examples illustrate the kind of analysis arising with the subgame:

1. $\exists y/x\,[x \neq y]$

No variables depend on $x$, hence the dependency set is empty. Let us assume that this game is played in start position $x = n$. First ∃loise has to select a value for $y$. Would there be a winning strategy for her? A strategy for $\exists y/x$ is a function that does not depend on $x$, there are no other variables in the domain, so the strategy is a constant function. A candidate is $y := n + 1$, which is winning in the given start position. But would it also be winning for other values of $x$? The answer evidently is 'No': for $x = n + 1$ the strategy $y := n + 1$ looses. Since there is no choice which works for all values of $x$, there is no winning strategy for ∃loise in the given start position. Neither there is a winning strategy in other start positions.

2. $\forall x[\exists y/x\,[x \neq y]]$

Also here the dependency set is empty, and no start values have to be given. The game starts with ∀belard choosing a value for $x$, say $x := n$. Then the subgame $\exists y/x\,[x \neq y]$ is played, from start position $x = n$. As we have seen before, there is no winning strategy for this subgame. Hence there is no winning strategy for the entire game. The formula is not 'true'.

3. $\forall x[\exists y/x\,[x \neq y] \vee \exists y/x\,[x \neq y]]$

First ∀belard chooses, say $x = n$. Next ∃loise has to choose $L$ or $R$. If she chooses $L$ then she has to play a subgame for which she has no winning strategy. The same of she chooses $R$. Hence she has no winning strategy for the entire game. So the formula is not true.

It may be instructive to consider the difference with Hodges' approach. There in the left game the strategy $y := 4$ was followed. Since ∃loise plays that subgame only when $x = 3$, she wins that subgame. But $y := 4$ is not a winning strategy for the subgame because it does not work for all values of $x$. Therefore it can, in our approach, not be part of a winning strategy in the entire game. So the difference with Hodges' approach is that he does not require that a winning strategy is built from winning strategies for subgames.

4. $\forall x_1 \exists y_1 \forall x_2\ \exists y_2/x_1\,[x_1 < y_1 \wedge x_2 < y_2]$

First we consider a subgame: $\exists y_2/x_1\,[x_1 < y_1 \wedge x_2 < y_2]$. The dependency information we have here, is that $y_1$ depends on $x_1$. Assume this subgame to be played from a position where $x_1 = n, y_1 = n + 1$, and $x_2 = m$. There is a strategy that gives ∃loise a win: let $y_2 := x_2 + 2$. This strategy is allowed because $y_2$ does not depend on $x_1$ as is required by $\exists y_2/x_1$. Furthermore, if $x_1$ has another value (say $p$), then the same strategy brings ∃loise a winning position, provided the value of $y_1$ is changed to $p + 1$. This gives ∃loise a winning strategy that can be used for the entire game. After ∀belard's choice for $x_1$, she chooses $y_1 := x_1 + 1$ , and after his choice for $x_2$, she follows the just describes winning strategy for the subgame.

**Conclusion** Informational independence can be formalized by a quantification over a class of games.

# Probability in Dynamic Epistemic Logic

Barteld P. Kooi*

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## 1 Introduction

Many of those who are currently working in the field of dynamic epistemic logic have an interest in game theory. However, probability has not been incorporated into this logic yet, although it is an important aspect of game theory. In this article I wish to make a first step in combining the two. My main inspiration comes from the work in probabilistic epistemic logic in [HT93] and the work in dynamic epistemic logic in [BMS98, Ger99].

## 2 Probabilistic Epistemic Kripke Models

The standard model in the semantics of epistemic logic is a multimodal Kripke model. Probability can be added to these models in a similar fashion as Halpern et al. added probability to their models for multi-agent systems. The basic probabilistic notion they use is that of a probability space, which can be used to model an agent's information about probabilities. Analogous to the role possible worlds play in the accessibility relations, the sample spaces of these probability spaces are sets of possible worlds.

I provide a general definition of probabilistic epistemic Kripke models with which we can interpret the language of probabilistic epistemic logic. This contains formulas like $(\mathrm{Pr}_a(\phi) \leq \alpha)$, which can be read as 'the probability $a$ assigns to $\phi$ is less than or equal to $\alpha$.' However, given these models, it is still not immediately clear how one should model a specific situation (if, for example, an agent cannot distinguish between two different probability distributions). I provide an approach on how to construct a probabilistic epistemic Kripke model from two Kripke models: one for the *nonprobabilistic* information (i.e. propositional and epistemic information) and another for the *probabilistic* information. It is often easier to think about these domains of information separately. These models can be multiplied, yielding a *probabilistic epistemic* Kripke model.

## 3 Updates

Then I turn my attention to performing updates these models. The updates I consider are restricted to tests, because other updates cannot be modeled quite

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as elegantly, though new developments in this area are forthcoming. The idea for well-founded semantics for subgroup updates with tests is by Baltag [BMS98] (he calls these updates announcements) and I add probability to it. The only addition to the definition of the accessibilty relation of the update program that is needed is saying what happens to the probability spaces. The short answer is that exactly the same happens to them as what happens to the accessibility relations. Those worlds where $\phi$ does not hold are no longer considered. So the new sample spaces can only contain worlds where $\phi$ holds. The new probability assigned to an event is its old conditional probability given that it is in the new sample space.

## 4 The Monty Hall Dilemma

Finally I will discuss the famous puzzle called the Monty Hall Dilemma. Suppose you have won a quiz and the quiz master offers you a choice of three doors. Behind one of the doors is a car, behind the other two there are goats. You pick a door, say door number one, and the quiz master who knows where the car is has to open a door containing a goat. Say he opens door number three and then asks you whether you would like to switch to door number two. The question is whether it is to your advantage to switch. Well, in this case the probability the car is behind door number two is two thirds and therefore you should switch. In this puzzle probability, knowledge and information change all play a role; the probabilistic answer to the question whether it is to your advantage to switch, the knowledge the quiz master has about the location of the car, and the information change that is caused by the quiz master opening a door. These three aspects can be analysed with probabilistic dynamic epistemic logic.

## 5 Questions for Further Research

There are many interesting issues that still have to be dealt with. For example, what is the best way to model ignorance about probabilities? How can you model an update with a sentence that has probabiliy zero, but is not logically inconsistent? I discuss these and other issues.

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# Values of questions, and Q-A games

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Why do we ask questions? Because we want to have some information. But why this particular kind of information? Because only information of this particular kind is helpful to resolve the *decision problem* that the agent faces. In this paper I argue that questions are asked because theirs answers help to resolve the questioners decision problem. By relating questions to decision problems I show (i) how we can measure the values of questions, and (ii) how with the help of these values and certain other assumptions we might model the strategic deliberations of participants in a question-answer dialogue game.

## Values of completely answered questions

In Savage's (1954) statistical decision theory a distinction is made between states of the world, acts, and consequences; states of the world together with acts determine the consequences, and the relevant expectation is unconditional expected utility. If we assume that the utility of doing action $a$ in world $w$ is $U(w, a)$, we can say that the *expected utility* of action $a$, $EU(a)$, with respect to probability function $P$ is $EU(a) = \sum_w P(w) \times U(w, a)$.

Let us now assume that the agent faces a *decisison problem*, i.e. he wonders which of the alternative actions in $A$ he should choose. A decision problem of an agent can be modeled as a triple, $\langle P, U, A\rangle$, containing (i) the agents probability function, $P$, (ii) his utility function, $U$, and (iii) the alternative actions he considers, $A$. You might wonder why we call this a decision *problem*; should the agent not simply choose the action with the greatest expected utitility? Yes, he should, if he *chooses now*. We might say that the *value* of choosing now is the expected value of the act with the greatest expected utility (where $i$ varies over the actions in $A$): $U(\text{Choose now}) = max_i \sum_w P(w) \times U(w, a_i)$.

But now suppose that the agent doesn't have to choose now, but can try to receive some useful information by first *asking a question*. Suppose that the other participant of the dialogue answers this question by giving answer $C$, and that, as a good Bayesian, the agent himself updates his probability state by conditionalizing on $C$. Then we can say that the value of making an informed decision conditional on learning $C$ is the expected utility conditional on $C$ of the act that has highest expected utility: $U(\text{Learn C, choose later}) = max_i \sum_w P(w/C) \times U(w, a_i)$.

It should be obvious that when $C$ is non-trivial with respect to $P$, the utility of choosing after learning the answer is always greater than the utility of choosing before learning. Let us call this difference the *utility of the answer*, a value that is always dependent on a decision problem. What we are after, however, is something different; we want to know what the *utility of a question* is. But here we face a problem, because even if we assume that the questioner will accept the given answer, the questioner doesn't know which answer this other participant will give. It turns out, fortunately, that we can determine the utility of a question easily when we assume that a question *partitions* the state space. First, we can determine the value of an informed decision; this value is the present expectation of expected value of the choice made after learning the answer (where $k$ varies over the alternative answers):

$$U(\text{Learn answer, choose later}) = \sum_k P(C_k) \times max_i \sum_w P(w/C_k) \times U(w, a_i)$$

In the decision theory of Savage this value is equal to $\sum_k max_i \sum_{w \in C_k} P(w) \times U(w, a_i)$, which, as shown by Skyrms (1990),[1] is on certain natural assumptions always at least as great as our earlier $U(\text{Choose now})$. But now we can also determine the *value of the question*. This value, usefulness, or relevance, of a question, given a decision problem, can be defined as the difference between $U(\text{Learn answer, choose later})$ and $U(\text{Choose now})$, which is a natural measure, if we assume that learning via questioning is cost-free.

[1] Skyrms, B. (1990), *The Dynamics of Rational Deliberation*, Cambridge Mass.: Harvard University Press.

## The question-answer game

Suppose now that an agent faces a decision problem, which question should he then ask? The answer seems obvious; the question that gives rise to the highest value with respect to that decision problem. Notice that if two questions both give rise to partitions, we can say that the first partition, $P$, is a *refinement* of the other partition, $Q$, iff $\forall X \in P : \exists Y \in Q : X \subseteq Y$. It should be obvious that the question that gives rise to a more refined partition has always a value that is at least as high as the other one.

But notice that this fact is based on two non-trivial assumptions; (i) asking a question is always cost-free, in particular, the questioner doesn't mind to make his intentions fully explicit and never tries to hide them, and (ii) the respondent will always answer the question by giving the *complete* answer; it is assumed that the respondent is fully informed and fully cooperative. Let us now give up these assumptions, i.e. let us assume that the value of a question might decrease if it makes explicit your intentions, and that the questioner has to take into account the fact that, for whatever reasons, the respondent might give a partial answer to a question even though he knows the complete answer. If we now can determine the values of the strategies of giving full or partial answers by the respondent, and if we can determine the value of a question for the questioner in case the respondent gives only a partial answer, we end up with a game between questioner and respondent.

Suppose, for instance, that John, the questioner, is in love with Mary, but, everything else being equal, prefers not to make this clear to Bill, the respondent. That is, if Bill would give a complete true answer, he would prefer to ask the question *Who will come to the party?* to asking explicitly *Will Mary come?* Let us say that being explicit has a cost of $\epsilon$. Bill, on the other hand, doesn't mind to give a full answer to the question *Will Mary come?*, but prefers to give a partial answer to the question *Who will come?* Thus, both players have two strategies, and let us assume that the game is one with perfect information.

What then are the payoffs? Assume that there only 2 relevant individuals over which the *wh*-word varies, Mary and Sue, and that it is assumed that at least one of the two will come to the party. In that case, the question *Who will come?* has three complete answers: *Only Mary*, *Only Sue*, and *Mary and Sue*. Assuming that all worlds have equal probability, that the value of an uninformed decision is 3, and that the utility of the best action in each world where the questioner knows whether Mary will come is 6, it's easy to see that the value of *Who will come?* is $(1/3 \times 6) + (1/3 \times 6) + (1/3 \times 6) - 3 = 6 - 3$, if Bill gives a complete answer, and that the value of asking the explicit question *Will Mary come?* is $(1/2 \times 6) + (1/2 \times 6) - \epsilon - 3 = 6 - \epsilon - 3$. To complete the payoff function for John, we have to determine the value of *Who will come?* in case Bill gives a partial answer. Let us also limit ourselves to the following two partial answers *Mary will come*, $M$, and *Sue will come*, $S$. Note that the latter (mention-some) answers, unlike the earlier complete answers, will not partition the state space. It seems reasonable then to determine the value of the first question in these circumstances as the difference between the following utility and 3:

$$
\begin{aligned}
U(Q_p) \quad = \quad & P(M) \times max_i \textstyle\sum_w P(w/M) \times U(w, a_i) && (= U(\text{Learn M, choose later})) \\
& + P(S) \times max_i \textstyle\sum_w P(w/S) \times U(w, a_i) && (= U(\text{Learn S, choose later})) \\
& - P(M \wedge S) \times max_i \textstyle\sum_w P(w/(M \wedge S)) \times U(w, a_i) && (= U(\text{Learn M} \wedge \text{S, choose later}))
\end{aligned}
$$

On the assumption that the act with the highest value in case John doesn't know whether Mary will come is 3 (in case Bill answers *Sue*), the value of $Q_p - 3$ will be $(2/3 \times 6) + (2/3 \times 3) - (1/3 \times 6) - 3 = 4 - 3$, which is lower than the value of the same question in case Bill gives a complete answer, and higher or equal to the explicit question *Will Mary come?* just in case $\epsilon \geq 2$. If we call the difference between the value of *Who will come?* in case Bill gives a complete or partial answer the *risk* of asking an implicit answer, we might say that John should be explicit in his questioning just in case the *cost* of asking the explicit question, $\epsilon$, is *smaller* than the *risk* of asking the implicit question, 2, which confirms our intuition about the situation.

To make the description of the game complete, we have to know whether Bill prefers the strategy of giving a complete answer to *Who will come?* in case John actually asks *Will Mary come?* or not. If not, then depending on whether the cost of being explicit for John is greater than the risk of being implicit, the game will have either one or two Nash-equilibria.

# Dialogue Games and Connexive Logic

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**Extended Abstract:**

In the first part of this talk we show that, because of the flexibility of such frameworks and their ability to capture dynamical aspects of argumentations, approaches to logic based on dialogue games[1] are fruitful tools for the study of non-classical logics. In the second part dialogue games for so-called connexive logic will be presented in more detail.

## 1 Non-Classical Logics and Dialogues[2]

The set of rules defining the games of *Dialogical Logic* is devided into structural rules, regulating the general course of the games, and particle rules, determining for each logical particle how the corresponding formulas might be attacked and defended. Several non-classical logics can be obtained by slightly changing the set of structural rules against a fixed background of particle rules.[3] For example, the formulation of dialogue games for free logics requires the addition of a structural rule restricting the Proponent's use of constants for attacking or defending quantifiers, while the formulation of paraconsistent logics asks for the addition of a structural rule restricting the Proponent's possibilities of attacking negations. As the changes of the set of rules for different non-classical logics are independent of each other, it is easy to combine these logics. For example, we obtain a combination of paraconsistent and free logics by adding both structural rules mentioned above[4].
Another possibility of formulating non-classical logics within a dialogue games framework is to add appropriate rules that allow to check metalogical constraints also on the object language level of the games. For example, considerations on relevance could be treated as follows: If in a dialogue game one player suspects that one of his opponent's moves is redundant, he might show this by taking over his opponent's position in a subsection and winning this position without making the move in question. We follow a similar strategy (of building metalogical constraints into the object language level of the games) when presenting dialogue games for connexive logic in the next part.

[1]We will focus on the so-called *Dialogical Logic* inaugurated by Paul Lorenzen and Kuno Lorenz, but most of the arguments presented should also be valid for similar approaches.

[2]Most of the arguments presented in this part are discussed at greater length in H. Rückert, 'Why Dialogical Logic?' (to appear in H. Wansing (ed.), *Essays on Non-Classical Logic*, Oxford University Press, in preparation).

[3]This strategy follows the so-called *Došen's Principle* which is well known in *Display Logic* (cf. K. Došen, 'Sequent Systems and Groupoid Models I', *Studia Logica*, 47 (1988), 353-389).

[4]For a detailed examination of combinations of paraconsistent, free and intuitionistic logics see S. Rahman, 'On Frege's Nightmare' (to appear in H. Wansing (ed.), *Essays on Non-Classical Logic*, Oxford University Press, in preparation).

## 2 Dialogue Games for Connexive Logic[5]

Many of the discussions about conditionals can best be put as follows: can those conditionals that involve an entailment relation be formulated with in a formal system? The grounds for the failure of the classical approach to entailment have usually been that they ignore the *meaning connection* between antecedent and consequent in a valid entailment. One of the first theories in the historyof logic about meaning connection resulted from the stoic discussions on tightening the relation between the If- and the Then-part of conditionals, which in this context was called $\sigma\upsilon\nu\alpha\rho\tau\eta\sigma\iota\zeta$(connection). This theory gave a justification for the validity of what we today express through the formulae $\neg(a \Rightarrow \neg a)$ and $\neg(\neg a \Rightarrow a)$, which we call the first Boethian and the first Aristotelian thesis of connexivity, respectively. Hugh MacColl and, more recently, Storrs McCall searched for a formal system in which the validity of these formulas could be expressed. Unfortunately neither system is very satisfactory. Connexive logic is not an extension of classical logic: $(a \rightarrow \neg a) \vee (\neg a \rightarrow a)$ is a classical tautology. In this part we introduce dialogue games including the formulation of a new connexive If-Then ("$\Rightarrow$"), which allow the Proponent to develop (formal) winning strategies not only for the above mentioned connexive theses but also for $(a \Rightarrow b) \Rightarrow \neg(a \Rightarrow \neg b)$ and $(a \Rightarrow b) \Rightarrow \neg(\neg a \Rightarrow b)$, which we call the second Boethian and the second Aristotelian thesis of connexivity, respectively.
As we understand it, MacColl's reformulation of the meaning connections implicit in traditional hypotheticals comprises the following conditions for the connexive If-Then:

1. The If-part should be contingent or not inconsistent. In other words, the If-part should not yield a redundant Then-part by producing an inconsistency.
2. The Then-part should not yield a redundant If-part. That is, the Then-part should not be tautological.

These two (metalogical) conditions can be readily expressed by means of the two new dialogical operators **V** and **F** and thus be build into the object language level. Stating $\mathbf{V}A$ commits to defend $A$ in a subsection of the dialogue game, whereas stating $\mathbf{F}A$ commits to defend $\neg A$. In both cases the defender is allowed to state atomic formulas whenever he needs them in the subsection; thus, his partner has to play *formally* (the relations between the different sections of a dialogue game are regulated by appropriate structural rules). The new connexive If-Then then reads:

$$A \Rightarrow B := (A \rightarrow B) \wedge (\mathbf{V}A) \wedge (\mathbf{F}B)$$

The formulation of the corresponding strategy tableaux systems has to take care of the possible changes of the formal duty by refining the tableaux rules as well as the closing rules.
Our dialogical approach to connexive logic has as an immediate consequence that uniform substitution does not hold anymore ($a \Rightarrow a$ is connexively valid, but $(b \Rightarrow b) \Rightarrow (b \Rightarrow b)$ is not). But by defining the new concept of singular logical form a very restricted form of uniform substitution can still be maintained, allowing in fact only the renaming of the atomic formulas. By losing uniform substitution it also seems as if the formulation of a calculus becomes impossible, nevertheless a tableaux system can be formulated.

[5]The dialogical connexive logic presented in this part has been developed jointly with PD Dr. Shahid Rahman (Universität des Saarlandes, Germany). See S. Rahman / H. Rückert, 'Dialogical Connexive Logic' in S. Rahman / H. Rückert (ed.), *New Perspectives in Dialogical Logic* (special issue of *Synthese*, in preparation).

# Reasoning about the Power of Coalitions

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Social Choice Theory (see e.g. [3]) has developed a general model for coalitional power which allows one to analyze e.g. whether or to what extent a group of voters can force a certain alternative to be chosen. The central notion employed is that of an *effectivity function* (see [5]): Given a finite set of agents $N$ and a set of alternatives $A$, an effectivity function $E : \mathcal{P}(N) \rightarrow \mathcal{P}(\mathcal{P}(A))$ associates with every coalition of agents the sets of alternatives which the coalition can force, i.e. $X \in E(C)$ if the coalition $C$ can force the alternative chosen to lie in $X$.

The concept of an effectivity function is static in the sense that it specifies sets of abstract alternatives which coalitions can bring about irrespective of the situation at hand. By taking the view of possible-worlds semantics, we relativize effectivity functions to states of the world. Secondly, we take the alternatives to be possible worlds again, yielding a dynamic model of coalitional power in which coalitions of agents can restrict the navigation through the state space in various ways.

Formally, the language of $N$*-agent Coalition Logic* is a multi-modal logic where the modalities are indexed by coalitions $C \subseteq N$, and formulas $\varphi$ are of the form

$$\varphi := \quad \bot \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid [C]\varphi$$

where $p \in \Phi_0$ is an atomic proposition. $[C]\varphi$ expresses that coalition $C$ is effective for $\varphi$. A coalition model for the set of agents $N$ is a triple $\mathcal{M} = (S, E, V)$ where $S$ is a nonempty set of states (the universe), $V : \Phi_0 \rightarrow \mathcal{P}(S)$ is the usual valuation function for the propositional letters, and

$$E : S \rightarrow (\mathcal{P}(N) \rightarrow \mathcal{P}(\mathcal{P}(S)))$$

is the *global effectivity structure* of the model: For every state $s \in S$, $E(s)$ is an effectivity function. Coalition models are essentially minimal models with one neighborhood relation (see [2]) for every coalition.

Depending on the type of interaction the agents are involved in, different requirements will have to be imposed on the global effectivity structure of the models. To give one example, it seems reasonable in most situations to assume that if a coalition $C$ is effective for $X$, any larger coalition $C' \supseteq C$ will also be effective for $X$. Yet, it could be the case that a new member of the larger coalition can undermine the effectiveness of some members of $C$, so that $C'$ turns out to be less effective than $C$.

A very general model of multi-agent interaction is that of a strategic game (see [6]). A play of such a game $G = (N, \{\Sigma_i | i \in N\}, o, S)$ consists of each player $i \in N$ choosing an action (or strategy) $a_i \in \Sigma_i$. The outcome of the game is then determined by the function $o : \Pi_{i \in N}\Sigma_i \rightarrow S$ which associates with every action profile an outcome state $s \in S$. Given such a strategic game, a coalition $C$ is said to be $\alpha$-*effective* for a set of outcomes $X \subseteq S$ iff $C$ has a joint strategy such that for all joint strategies of the remaining players $\overline{C}$, the outcome will be in $X$. This notion gives rise to an effectivity function $E_G^\alpha$ which can be associated to every strategic game $G$.

If we want to use our coalition logic to reason about $\alpha$-effectivity in strategic games, we need to first characterize which effectivity functions are $\alpha$-effectivity functions of some strategic game. Generalizing results from [4, 7], one can show that an effectivity function $E : \mathcal{P}(N) \to \mathcal{P}(\mathcal{P}(S))$ is the $\alpha$-effectivity function of a strategic game iff it satisfies five ***playability*** conditions, the central one being ***superadditivity***: For all $X_1, X_2, C_1, C_2$ such that $C_1 \cap C_2 = \emptyset$, $X_1 \in E(C_1)$ and $X_2 \in E(C_2)$ imply that $X_1 \cap X_2 \in E(C_1 \cup C_2)$. Imposing these playability conditions on the global effectivity structure of the coalition models, we can see every state as linked to a strategic game which can be played at that state, yielding a new state depending on the strategies or actions chosen by the players. Thus, playable coalition models are general action models, where transitions to successor states are not determined by the agents individually, but by the actions of all the agents together.

The playability conditions can easily be translated into modal formulas which can serve as the axioms of a deductive calculus for Coalition Logic. Due to the fact that effectivity functions are essentially modal neighborhood relations, one can prove soundness and completeness of this axiomatization via an adapted version of the standard canonical model construction.

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**On the logical foundations of game theory**
(Abstract)

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## 1. Introduction

During the last 15 years there has been growing interest among game theorists in epistemic conditions for game-theoretic solution concepts. Most of the work in this area has more or less explicitly employed some version of Kripke-style epistemic logic. Actually, most game theorists do not work with the syntactic formulations of epistemic logics, but instead with information-structures. An information structure, however, can be viewed as a Kripke model, and as the relation between Kripke models and normal modal systems are well known to logicians, I do not need to go into that here.

Instead, I will use the weakest normal system K (in fact, the multi-agent version thereof) to explain a problem with this kind of logic which I believe can only be adequately resolved by moving beyond "normal" modal systems. In the second part of the paper I will therefore suggest an epistemic logic which resolves the problem in what I believe to be a satisfactory way.

## 2. The logic $K_\Gamma$

For a given finite extensive game of perfect information (PI) $\Gamma$, we define the logic $K_\Gamma$ as follows: atomic formulas are: move formulas a, b, c, ... one for each move of $\Gamma$, and preference formulas $a \succ_i b$ ... where a, b are any moves, and i is a player of $\Gamma$. Wffs are made up from these atomic formulas in the usual way by applications of negation, conjunction, and belief operators $B_i$. The axioms of $K_\Gamma$ are the usual ones of multi-agent K plus $\Gamma$-specific axioms describing the rules of the game $\Gamma$ and the preferences of the players according to the payoff function of $\Gamma$. For the simplest nontrivial example of a PI game, $\Gamma_0$, which has just one player 1, who has to choose between moves a and b, whereof he prefers the former, the $\Gamma$-specific axioms are $(a \vee b) \wedge \neg(a \wedge b)$ and $a \succ_1 b$. The rules of inference are modus ponens and epistemization (which may be applied to all the axioms including the $\Gamma$-specific ones).

## 3. The problem of self-knowledge of rationality and options

Within $K_\Gamma$, we give a sufficient condition for the backward induction play of $\Gamma$ which can be shown to be weaker than the one of Aumann (1995). In this abstract, we explain our condition only for the one-player example $\Gamma_0$ (described above), which suffices to explain the problem we seek to solve in this paper.

As the player may have false beliefs in $K_{\Gamma 0}$, his choice of $b$ – contrary to his preference – may be due to his belief that $a$ is not possible. This motivates a condition we call *relative* rationality:

(RR) $\neg B_1 \neg a \Rightarrow \neg b$

As $a \succ_1 b$ is an axiom of $\Gamma_0$, this says that the player will not take action b if he considers the preferred action possible. Clearly, RR $\Rightarrow$ a does not hold in $K_{\Gamma 0}$. However, it seems natural to add the assumption that the player does consider $a$ possible. We call this assumption Possibility of Backward Induction moves:

(PBI) $\neg B_1 \neg a$

For our simple example, RR $\wedge$ PBI $\Rightarrow a$ is trivially a theorem of $\Gamma_0$. (For the general case, an analogous, but more elaborate theorem holds.) However, a problem arises from the fact that it seems natural and in line with the usual informal assumptions of game theory to assume of *all* moves that they are considered possible, and that there is mutual (or even common) belief in rationality and the structure of the game. Clearly, $B_1($ RR $\wedge \neg B_1 \neg a \wedge \neg B_1 \neg b)$ is inconsistent in $\Gamma_0$: The player can infer from what he believes that what he considers possible will not be the case.

## 2. The logic $L_\Gamma$

To resolve the above problem we suggest an epistemic logic which has a sequence of belief operators $B^0$, $B^1$, $B^2$, ... for each player, corresponding to the temporal sequence of the player's states of belief. Limiting ourselves (in this abstract) to the one-player case again, we consider the axiomatic system (for which we also provide a belief-set semantics, similar to the autoepistemic logics of Moore, 1985, and Konolige, 1988) with the following axiom schemes:

(A1) $\varphi$ , whenever $\varphi$ is a propositional calculus tautology or a $\Gamma$-specific axiom;
(A2) $B^t(\varphi)$ , whenever $\varphi$ is a propositional calculus tautology or a $\Gamma$-specific axiom;
(A3) $B^t(\varphi) \wedge B^t(\psi) \Rightarrow B^t(\varphi \wedge \psi)$;
(A4) $B^t(\varphi) \Rightarrow B^{t+1}(\varphi)$;
(A5) $B^t(\varphi) \Rightarrow B^{t+1}(B^t(\varphi))$;
(A6) $\neg B^t(\varphi) \Rightarrow B^{t+1}(\neg B^t(\varphi))$;

the sole rule of inference being modus ponens. Among other properties of this logic $L_\Gamma$, we show that a delayed version of the epistemization rule holds.

## 3. A Solution to the Problem

Within $L_\Gamma$, the problem explained above can be easily resolved: Writing ($RR^0$) for $\neg B^0 \neg a \Rightarrow \neg b$, one can verify that $\neg B^0 \neg a \wedge \neg B^0 \neg b \wedge B^0(RR^0)$ is consistent, and so is $B^1(RR^0 \wedge \neg B^0 \neg a \wedge \neg B^0 \neg b)$. These formulas can be naturally taken to describe a situation where *initially* the player considers both options possible and himself to be rational, *and then*, on reflection, recognizes that he will not take b, while remembering that he initially considered both options possible.

A multi-agent version of $L_\Gamma$ can be used to reconstruct both the backward induction argument and that it may fail if the players have insufficient knowledge about each other's reasoning processes.

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