The Game of Interrogation

Abstract

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In this talk we give a logical analysis of the following language game:

Definition 1 (The Game of Interrogation) Interrogation is a game for two players: the *interrogator* and the *witness*. The rules of the game are as follows:

A. The interrogator may only raise issues by asking the witness non-superfluous questions.
B. The witness may only make credible non-redundant statements which exclusively address the issues raised by the interrogator.

The game of interrogation can be looked upon as a logical idealization of the process of cooperative information exchange. The elements of the rules can be linked to elements of the Gricean Cooperation Principle: The requirement that the witness makes credible statements is related to the Maxim of Quality; that the statements of the witness should be non-redundant, and the questions of the interrogator non-superfluous, relates to the Maxim of Quantity; and that the witness should exclusively address the issues raised by the interrogator is a formulation of the Maxim of Relation.

The logical analysis takes the form of developing logical notions which enable us to judge whether the game was played in accordance with the rules. Relative to a suitable language, and a semantic interpretation for that language, the logic of interrogation has to provide us with logical notions by means of which we can arbitrate the game.

As a language for the game of interrogation, we use a simple query-language, a language of first order predicate logic enriched with simplex interrogatives:

Definition 2 (Query-Language) Let PL be a language of predicate logic.
The *Query Language* QL is the smallest set such that:

i. If $\phi \in PL$, then $\phi \in QL$;
ii. If $\phi \in PL$, $\vec{x}$ a sequence of n variables $(0 \leq n)$, then $?\vec{x}\,\phi \in QL$.

We can represent the proceedings of a game of interrogation as a sequence of formulae in QL. Given the strict casting, we do not have to indicate who said what: The interrogatives are uttered by the interrogator, the indicatives by the witness.

Next we state the semantic interpretation of the language. We do so in two steps. First we give a standard denotational semantics, and in terms of that we define the context change potentials of the formulae of the language. For the indicative part of the language, we provide a standard truth definition. The denotational semantics for the interrogatives is a so-called partition semantics.

Definition 3 (Semantics of Questions)
$\|?\vec{x}\,\phi\|_{w,g} = \{v \in W \mid \forall \vec{e} \in D^n\colon \|\phi\|_{v,g[\vec{x}/\vec{e}]} = \|\phi\|_{w,g[\vec{x}/\vec{e}]}\}$.

The denotation of in an interrogative in a world w consists of the set of worlds where the question has the same (true and complete) answer.

To be able to define context change potentials, we introduce a notion of structured context which is suitable for specifying the context change of both interrogatives and indicatives.

Definition 4 (Structured Contexts)
A *context* C is a symmetric and transitive relation on the set of possible worlds W.

Intuitively, if two worlds are contextually related, it is not an issue whether the actual world is like the one or like the other. If two worlds are disconnected it is an issue whether the actual world is like the one or the other. For ease of notation, we write $w \in C$ as short for $\langle w, w \rangle \in C$. If $w \notin C$, this means that this world is excluded by our information.

The context change potentials of sentences and sequences thereof are defined as follows. ($\phi!$ ranges over the indicatives of the language, $\phi?$ over the interrogatives.)

Definition 5 (Context Change Potentials)
i. $C[\phi!] = \{\langle w, v \rangle \in C \mid \|\phi!\|_w = \|\phi!\|_v = 1\}$;
ii. $C[\phi?] = \{\langle w, v \rangle \in C \mid \|\phi?\|_w = \|\phi?\|_v\}$;
iii. For $\tau = \phi_1; \ldots; \phi_n$, $C[\tau] = C[\phi_1] \ldots [\phi_n]$.

Indicatives provide data by eliminating worlds from the context. Interrogatives raise issues by disconnecting worlds in the context.

Given a suitable language for the game and a semantics interpretation for that language, the logic of the game of interrogation should provide us with logical notions which enable us to arbitrate whether the game was played according to the rules. Relative to a (possible empty) sequence of sentences τ in QL, we define whether a sentence ϕ is pertinent after τ. The notion of pertinence has three elements: Consistency, non-entailment, and licensing. Consistency is defined in a rather standard way:

Definition 6 (Consistency) ϕ is *consistent with* τ iff $\exists C\colon C[\tau][\phi] \neq \emptyset$.

The notion of consistency will be used to deal with the qualitative aspects, the credibility of utterances. The notion of entailment is also standard:

Definition 7 (Entailment) $\tau \models \phi$ iff $\forall C\colon C[\tau] = C[\tau][\phi]$.

The notion of (non-)entailment will be used to deal with qualitative aspects, the non-superfluousness of questions, and the non-redundancy of statements. The only new logical notion, is the notion of licensing:

Definition 8 (Licensing) τ *licenses* ϕ iff $\forall C, w, v\colon \langle w, v \rangle \in C[\tau]$ & $w \notin C[\tau][\phi] \Rightarrow v \notin C[\tau][\phi]$.

Licensing forbids the elimination of some world from the context, in case some world which is contextually related to it, remains. Licensing is used to arbitrate relational aspects, it enables us to judge whether the statements of the witness exclusively address the issues raised by the interrogator. The overall arbitration of the game of interrogation is taken care of by the notion of pertinence.

Definition 9 (Pertinence) ϕ is *pertinent after* τ iff
i. ϕ is consistent with τ (*Quality*)
ii. ϕ is not entailed by τ (*Quantity*)
iii. ϕ is licensed after τ (*Relation*)

We will discuss some logical properties of the notion of pertinence, and will illustrate its use in discussing some linguistic examples, which intend to show that the notion of pertinence can help to explain certain structural semantic facts about natural language.