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DYNAMIC MONTAGUE GRAMMAR

a first sketch

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1 Introduction

The system of dynamic predicate logic (henceforth, *DPL*), developed in Groenendijk & Stokhof [1989] as a compositional alternative for Kamp's discourse representation theory (henceforth, *DRT*), shares with the latter the restriction of being a first-order system. In the present paper, we are mainly concerned with overcoming this limitation. We shall define a dynamic semantics for a typed language with λ-abstraction which is compatible with the semantics *DPL* specifies for the language of first-order predicate logic. We shall propose to use this new logical system as the semantic component of a Montague-style grammar (referred to as 'dynamic Montague Grammar, *DMG*), which will enable us to extend the compositionality of the *DPL*-treatment of donkey-sentences and cross-sentential anaphora to the subsentential level. Furthermore, we shall extend this analysis also in this sense that we shall add new, dynamic interpretations for logical constants which in *DPL* were treated in a static fashion. This will substantially increase the descriptive coverage of *DMG*, although it also presents some difficulties of its own.

One of the motives behind the development of *DPL* was a concern for compatibility of frameworks. The days of the hegemony (or tyranny, if you will) of Montague grammar (*MG*) as *the* paradigm of model-theoretic semantics for natural language, are over. And rightly so, since the development of alternatives, such as *DRT*, clearly has been inspired by limitations and shortcomings of *MG*. These alternatives have enabled us to cover new ground, and they have brought us a new way of looking at our familiar surroundings. Yet, every 'expanding' phase is necessarily followed by one of 'contraction', in which, again, attempts are made to unify the insights and results of various frameworks. The present paper is intended as a contribution to such a unification.

It may seem conservative, though, to 'fall back' so to speak to the *MG*-framework. For several reasons, we think that this is not the case. For one thing, a lot of adequate work has been done in the *MG*-framework which simply has not yet been covered in any of the alternatives. Some of it can be transferred in an obvious way, and thus offers no real challenge, but that does not hold for all of it. For example, the elegant and uniform treatment of various intensional phenomena that *MG* offers, will be hard to come by in some of the other frameworks. Also, we are convinced that the capacities of *MG* have not been exploited to the limit, that sometimes an analysis is carried out in a rival framework simply because it is more fashionable. But the most important reason of all concerns compositionality. The framework of *MG* is built on this principle, which, for philosophical and methodological reasons, we think is one of the most central principles in natural language semantics. Consequently, confronting other frameworks with *MG*, trying to unify them, is also putting things to the test. Can this analysis be cast in a compositional mould? But also, what consequences does compositionality have in this case? (And, to be sure, some of the latter may cause us to reject it after all.)

So, we feel there is ample reason for the undertaking of this paper. But for those who are not really inspired by such grandiose considerations, we may add that, after all, in *DRT*, too, one would want to overcome the limitations of a first-order system in some way. Well, this paper offers one.

We shall proceed as follows. In section 2, we introduce, along the lines of Janssen [1986], a system of dynamic intensional logic (henceforth, *DIL*), and indicate how it can be used as a means to represent the meanings of natural language expressions. Section 3 is devoted to a translation of *DPL* into *DIL*, showing that the former's capacities to deal with various phenomena in a compositional way, carry over to the latter. Then we turn, in section 4, to the task of defining a fragment and of showing that it can be interpreted in a rigorously compositional manner. Section 5 discusses some empirical shortcomings of *DPL* which *DMG* inherits, and sketches some extensions of *DIL* which will enable us to handle these cases. The resulting semantics is not without problems of its own, though, and these are discused as well. Appearances not withstanding, the *DMG*-system, as developed up to that point, is extensional, so we also discuss the incorporation of intensionality. By way of illustration, we indicate how a compositional treatment of 'modal subordination' phenomena could be arrived at in the enriched system.

Throughout, we shall suppose the reader has a working knowledge of *MG*, and of *DPL*, though we trust that, but for sections 2 and 3, the latter is not necessary for an understanding of this paper. Finally, we will not go into the relationship with the original work in the area of fiscourse representation (Kamp [1981,1983], Heim [1982,1983], Seuren [1986]), or with other attempts at a compositional formulation (Barwise [1987], Rooth [1987], Zeevat [1989]). For some discussion, see Groenendijk & Stokhof [1988,1989].

2 Dynamic intensional logic

2.1 Variables and discourse markers

If we extend *DPL* to a language which has a type structure and contains λ-abstraction, the problem immediately arises that now we are dealing with a system in which, at least from the viewpoint of natural language semantics, variables have two functions, instead of just one. If we use predicate logic to translate natural language sentences, variables are used to represent quantificational expressions, and anaphoric relations between such expressions and pronouns. (We disregard free occurrences of variables, since we don't see a proper use for them from the natural language point of view. But if one insists, they may be interpreted as having yet a third, viz., a referring, function.) However, once we switch to the use of a system that allows us to translate really compositionally, a different use of variables appears. For, λ-abstracts, and the variables occurring therein, play a different role: they are used to represent non-sentential, functional expressions, such as complex nouns and verbs. In such constructs, unless they originate from a quantificational expression, variables don't have a direct natural language counterpart. So, whatever reason there was to treat our logical quantifiers and variables dynamically that originates in the meaning of their natural language counterparts, does not automatically extend to these cases.

Consider the following example. The complex common noun phrase *farmer who owns a donkey* can be represented in a language with λ-abstraction as $\lambda x[\mathtt{farmer}(x) \wedge \exists y[\mathtt{donkey}(y) \wedge \mathtt{own})(x, y)]]$. The occurrences of y derive directly from the quantificational expression *a donkey* as it occurs as the direct object of *own*. In the dynamic semantics of *DPL*, the values assigned to it by the various input assignments of a formula in which this expression occurs, depending on the nature of the entire formula, may be 'passed on' to the corresponding output assignments. Thus, the existential quantifier may bind variables which are outside its scope, and that is the way in which we account, in a compositional manner, for the fact that the indefinite term may be the antecedent of pronouns in sentences to come. Now consider the occurrences of x. These do not have a natural language counterpart, and, at first sight at least, a dynamic approach to them seems to make no sense. From the viewpoint of natural language semantics, we might say that such variables are really artefacts, which as a consequence do not partake in any dynamic meaning of natural language expressions.

The point can perhaps be appreciated more clearly if we consider, instead of the usual indirect interpretation, the *direct* approach, i.e., interpretation without translation in a logical language. Stating the meaning of *farmer who owns a donkey*, for example as part of describing that of the sentence *Every farmer who owns a donkey, beats it*, in a direct way, we would assign a dynamic meaning to the indefinite term *a donkey*. This carries over to the property which we associate with the entire phrase *farmer who owns a donkey*, but that is all. Being a property itself isn't anything dynamic, at least not as far as dynamics goes in the context of the present discussion, i.e., as far as anaphoric relations between quantificational expressions and pronouns are concerned. All we need to make sure is that the dynamics of the indefinite term is not blocked, i.e., we must allow properties to be dynamic in the secondary sense of being transparent to any intrinsically dynamic constituents they may have.

Returning to the perspective of indirect interpretation again, we notice that this argument extends to quantificational expresssions in general. Although *some* occurrences of the existential quantifier need to be interpreted dynamically, viz., those which are used to represent the dynamic meaning of indefinite terms, it does not follow that *all* of them need to be handled this way. And in fact, once we transcend the simple, 'all in one fell swoop' way of translating that is our only choice if we use a first-order system, and we start doing some really compositional translating, we encounter lots of situations in which we would want to use the quantificational apparatus of our system in the standard, nondynamic way. The case of the λ-operator is a particularly striking one, since it seems to defy any reasonably intuitive dynamic interpretation, at least in its ordinary uses such as the one indicated above. But also for the quantifiers, it is easy to come up with examples in which their ordinary interpretation is the most appropriate one. A welter of examples is provided by all kinds of instances of 'lexical decomposition' which takes place in the translation process.

To be sure, in a system like *DPL*, the ordinary, static meaning of quantifiers (and connectives) can be imitated, e.g., by the use of the closure operator $\diamondsuit$. So, the line of reasoning just sketched, does not constitute an argument that is decisive in any respect. Yet, it does suggests, rather strongly, that our analysis will gain in perspicuity, at least, if not in adequacy, if we make a distinction in our logical language between ordinary cases of the use of variables and quantifiers, which are abstraction and ordinary quantification, and dynamic cases, which are those where we want to represent the dynamic semantics of quantificational structures of natural language and anaphoric relations.

This is indeed the line we shall take in what follows. We shall use variables and quantifiers in the ordinary, static way, and add to our logical language two new syntactic categories. One is that of so-called 'discourse markers', the other is that of so-called 'state switchers'. Both are borrowed from Janssen [1986], where they are put to a different use. So, we free our usual quantificational apparatus from the task of dealing with the dynamic 'passing on' and 'picking up' of referents. As a consequence, assignments, of values to variables, are no longer the semantic locus in which the dynamics resides. The dynamics is now dealt with by means of the discourse markers and state-switchers, which are interpreted in terms of a new semantic parameter, that of a 'state'.

The general idea is the following. Discourse markers are used in the translations of indefinite terms and pronouns. Part of the meaning of the indefinite term is to assign a value, one that is a witness of the term, to the discourse marker. Also, provisions are taken to pass on this value, to further occurrences of the same discourse marker. This is taken care of, among other things, by the state switcher. The denotations of discourse markers are determined with reference to a state, and the state switcher will allow us to pick out certain states. More specifically, it will allow us to identify states in which discourse markers have certain values, by switching from the current state to such a state. Finally, the meanings of sentences will be constructed in such a way that they determine that subsequent sentences are interpreted with respect to the most recently identified state. This they bring about by being themselves a kind of descriptions of states, as we shall see in section 2.3.

Before turning to a statement of the system, and to its applications, a few words of warning are in order.

First of all, as we already indicated above, we do not claim that the distinction between variables and discourse markers, quantifiers and state switchers, assignments and states, is really forced upon us. As far as we can see, there is no principled obstacle for a fusion of them, i.e., there is no reason to believe that we can not deal with the dynamics of natural language by means of a typed system with λ-abstraction in which it is the variables and quantifier which do the job. But we do feel, rather strongly, that the system to be given shortly, in which these distincions are made, is perspicuous, easy to learn and to use, and, very important, easy to extend.

That brings us to the second point. The system is referred to, rather unfortunately, as a system of dynamic *intensional* logic. However, at least in the main body of this paper, intensionality in the ordinary sense of the word plays no role. The role of the states, which act as a parameter in the notion of interpretation, is strictly confined to giving the value of discourse markers, and nothing else. Yet, we do use intensional terminology, such as 'proposition' and the like, in connection with them, introduce the $^\wedge$-operator to express abstraction over them, and so on. This may be confusing, but our reason for using the intensional vocabulary is that, in a later stage, we want to incorporate also ordinary intensionality in the system. And we want to do so by extending the notion of a state so as to also play the role of a possible world (point in time, and so on).

Finally, we restrict our use of discourse markers initially rather severely, to just expressions which refer to individuals. Of course, in a later stage, we want to allow also discourse markers over other types of objects. The restriction is motivated by a wish to avoid certain technical complications, at least initially, so as to not bother the reader with formal details which might draw her attention away from the main points we want to make.

2.2 Syntax and semantics of *DIL*

The system of dynamic intensional logic (henceforth, *DIL*) is a variant of the system of intensional typetheory *IL*, which is used in Montague's *PTQ*.

The types of *DIL* are the same as those of *IL*:

Definition 1 (Types) T, the set of types, is the smallest set such that:

1. $e, t \in T$
2. If $a, b \in T$, then $\langle a, b\rangle \in T$
3. If $a \in T$, then $\langle s, a\rangle \in T$

As usual, the syntax takes the form of a definition of ME_a, the set of meaningfull expressions of type a. Given sets of constants, CON_a, and variables, VAR_a, for every type a, and a set of discourse markers DM, the definition runs as follows:

Definition 2 (Syntax)

1. If $\alpha \in CON_a \cup VAR_a$, then $\alpha \in ME_a$
2. If $\alpha \in DM$, then $\alpha \in ME_e$
3. If $\alpha \in ME_{\langle a,b\rangle}, \beta \in ME_a$, then $\alpha(\beta) \in ME_b$

4. If $\phi, \psi \in ME_t$, then $\neg\phi, [\phi \wedge \psi], [\phi \vee \psi], [\phi \rightarrow \psi] \in ME_t$
5. If $\phi \in ME_t, \nu \in VAR_a$, then $\exists\nu\phi, \forall\nu\phi \in ME_t$
6. If $\alpha, \beta \in ME_a$, then $\alpha = \beta \in ME_t$
7. If $\alpha \in ME_a, \nu \in VAR_b$, then $\lambda\nu\alpha \in ME_{\langle b,a\rangle}$
8. If $d \in DM, \alpha \in ME_e, \beta \in ME_a$, then $\{\alpha/d\}\beta \in ME_a$
9. If $\alpha \in ME_a$, then $^\wedge\alpha \in ME_{\langle s,a\rangle}$
10. If $\alpha \in ME_{\langle s,a\rangle}$, then $^\vee\alpha \in ME_a$
11. Nothing is in ME_a, for any type a, except on the basis of a finite number of applications of 1–10

New with respect to *IL* are clauses 2 and 8. According to 2, discourse markers are expressions of type e. Clause 8 introduces the state switchers: they turn an expression of arbitrary type a into an expression of the same type. The idea is that the new expression will be evaluated with respect to the state determined by the state switcher. Finally, notice that our ordinary intensional operators, such as $\Box$, $\Diamond$, and tense operators, do not occur. The $^\wedge$- and $^\vee$-operators are present, and express abstraction over, and application to states, respectively.

Let us now turn to the semantics. Starting from two disjunct, non-empty sets D and S of individuals and states respectively, D_a, the domain corresponding to type a, is defined in the familiar fashion:

Definition 3 (Domains)

1. $D_e = D$
2. $D_t = \{0, 1\}$
3. $D_{\langle a,b\rangle} = D_b^{D_a}$
4. $D_{\langle s,a\rangle} = D_a^S$

As will become apparent shortly, D_a can not always be interpreted as the domain in which all expressions of type a are interpreted, the exception being D_e, since discourse markers, although expressions of type e, will be interpreted as 'individual concepts', i.e., as functions from S to D_e.

A model M is a triple $\langle D, S, F\rangle$, where D and S are as above, and F is a function which interprets the constants and the discourse markers. Specifically, if $\alpha \in CON_a$, then $F(\alpha) \in D_a$, and if $\alpha \in DM$, then $F(\alpha) \in D^S$. Further, G, the set of assignments, is the set of all functions g such that if $\nu \in VAR_a$, $g(\nu) \in D_a$. So, except for the discourse markers, all basic expressions are assigned extensional interpretations.

In order for the state switchers to be given their proper interpretation, we formulate two postulates which *DIL*-models should satisfy, which define certain properties of states:

Postulate 1 (Distinctness) If for all $d \in DM$: $F(d)(s) = F(d)(s')$, then $s = s'$

Postulate 2 (Update) For all $s \in S$, $d \in DM$, $\mathbf{d} \in D$ there exists an $s' \in S$ such that:

1. $F(d)(s') = \mathbf{d}$; and
2. for all $d' \in DM$, $d' \neq d$: $F(d')(s) = F(d')(s')$

Since the interpretation of the constants does not depend on the state, these two postulates guarantee that, for each state s, discourse marker d and object $\mathbf{d}$, there exists a unique state s' which differs from s at most in this respect that the denotation of d in s' is $\mathbf{d}$. We refer to this state as $\langle d \leftarrow \mathbf{d} \rangle s$. This notation reminds one, of course, of the notation $g[\nu/\mathbf{d}]$ for assignments, and one might well ask why we do not define states as assignments of values to discourse markers, instead of introducing them as primitive objects and then force them to behave as such. The answer is that, at least as far as the present system is concerned, we could indeed proceed in that way. However, as we remarked above, in a later stage we want to incorporate ordinary intensional aspects of interpretation as well, and one way to go about doing that would be to extend the role of states in such a way that they also act as possible worlds.

Now we state the semantics by defining the notion $[\![\alpha]\!]_{M,s,g}$, the interpretation of α with respect to M, s, and g, as follows:

Definition 4 (Semantics)

1. $[\![c]\!]_{M,s,g} = F(c)$, for every constant c
   $[\![\nu]\!]_{M,s,g} = g(\nu)$, for every variable ν
2. $[\![d]\!]_{M,s,g} = F(d)(s)$, for every discourse marker d
3. $[\![\alpha(\beta)]\!]_{M,s,g} = [\![\alpha]\!]_{M,s,g}([\![\beta]\!]_{M,s,g})$
4. $[\![\neg\phi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = 0$
   $[\![\phi \wedge \psi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = [\![\psi]\!]_{M,s,g} = 1$
   $[\![\phi \vee \psi]\!]_{M,s,g} = 1$ iff $[\![\phi]\!]_{M,s,g} = 1$ or $[\![\psi]\!]_{M,s,g} = 1$
   $[\![\psi \rightarrow \phi]\!]_{M,s,g} = 0$ iff $[\![\phi]\!]_{M,s,g} = 1$ and $[\![\psi]\!]_{M,s,g} = 0$
5. $[\![\exists\nu\phi]\!]_{M,s,g} = 1$ iff there is a $\mathbf{d} \in D_a$ such that $[\![\phi]\!]_{M,s,g[\nu/\mathbf{d}]} = 1$, where a is the type of ν
   $[\![\forall\nu\phi]\!]_{M,s,g} = 1$ iff for all $\mathbf{d} \in D_a$ it holds that $[\![\phi]\!]_{M,s,g[\nu/\mathbf{d}]} = 1$, where a is the type of ν
6. $[\![\alpha = \beta]\!]_{M,s,g} = 1$ iff $[\![\alpha]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g}$
7. $[\![\lambda\nu\alpha]\!]_{M,s,g} =$ that function $h \in D_a^{D_b}$ such that $h(\mathbf{d}) = [\![\alpha]\!]_{M,s,g[\nu/\mathbf{d}]}$ for all $\mathbf{d} \in D_b$, where a is the type of α, and b the type of ν
8. $[\![\{\alpha/d\}\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,\langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s,g}$

9. $[\![{}^\wedge\alpha]\!]_{M,s,g}$ = that function $h \in D_a^S$ such that $h(s) = [\![\alpha]\!]_{M,s,g}$ for all $s \in S$

10. $[\![{}^\vee\alpha]\!]_{M,s,g} = [\![\alpha]\!]_{M,s,g}(s)$

The notions of truth, validity, entailment and equivalence are defined in the usual way. We call ϕ *true with respect to* M, s *and* g iff $[\![\phi]\!]_{M,s,g} = 1$. We say that ϕ is *valid* iff ϕ is true with respect to all M, s, and g, and we write $\models \phi$. We set ϕ *entails* ψ iff for all M, s, and g, $[\![\phi]\!]_{M,s,g} = 1 \Rightarrow [\![\psi]\!]_{M,s,g} = 1$, and we write $\phi \models \psi$. Finally, we call arbitrary expressions α and β of the same type *equivalent* iff $\models \alpha = \beta$.

All clauses of definition 4 are completely standard, with the exception of 2 and 8. Clause 2 expresses, as was already announced above, that the interpretation of discourse markers is dependent on the state parameter. In clause 8, the semantics of the state switcher is given. The interpretation of $\{\alpha/d\}\beta$ with respect to a state s is arrived at by interpreting β with respect to a state s' which differs at most from s in that the denotation of the discourse marker d in s' is the object that is the denotation of the expression α in s. The two postulates on *DIL*-models guarantee that s' is unique, and, hence, that we can interpret state-switchers in this way. State switchers in fact operate in accordance with their name: they switch the state with respect to which an expression is evaluated to some uniquely determined, possibly different state. With a few exceptions, a state switcher $\{\alpha/d\}$ behaves semantically like the corresponding syntactic substitution operator $[\alpha/d]$, as is evident from the observations reported in fact 3 below.

First, we define the class of *intensionally closed* expressions as follows:

Definition 5 (ICE)

1. $c, \nu \in ICE$, for every constant c and variable ν
2. ${}^\wedge\alpha \in ICE$, for every well-formed expression α
3. everything constructed solely from elements of ICE by means of functional application, connectives, quantifiers and/or λ-abstraction, is again an element of ICE

Notice that there are intensionally closed expressions which contain discourse markers, for example ${}^\wedge d$, and also that there are expressions which are not intensionally closed which do not contain them, for example ${}^\vee p$, where p is a variable of type $\langle s, t\rangle$.

It is easy to check that the following holds:

Fact 1 If $\alpha \in ICE$, then $[\![\alpha]\!]_{M,s,g} = [\![\alpha]\!]_{M,s',g}$, for all $s, s' \in S$

Now we observe the following fact concerning state switchers:

Fact 2 If $\beta \in ICE$, then $\{\alpha/d\}\beta$ is equivalent with β

It is easy to see that this holds: if $[\![\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,s',g}$, for all $s, s' \in S$, then certainly $[\![\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,\langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s,g}$, whence $[\![\{\alpha/\}\beta]\!]_{M,s,g} = [\![\beta]\!]_{M,s,g}$.

The following sums up the behaviour of state switchers in the various syntactic environments:

Fact 3 (Properties state switchers)

1. $\{\alpha/d\}c$ is equivalent with c, for every constant c
   $\{\alpha/d\}\nu$ is equivalent with ν, for every variable ν
2. $\{\alpha/d\}d$ is equivalent with α
   $\{\alpha/d\}d'$ is equivalent with d', for all $d' \in DM : d \neq d'$
3. $\{\alpha/d\}(\beta(\gamma))$ is equivalent with $\{\alpha/d\}\beta(\{\alpha/d\}\gamma)$
4. $\{\alpha/d\}(\phi \wedge \psi)$ is equivalent with $\{\alpha/d\}\phi \wedge \{\alpha/d\}\psi$ (analogously for negation and the other connectives)
5. $\{\alpha/d\}\exists\nu\phi$ is equivalent with $\exists\nu\{\alpha/d\}\phi$, if ν does not occur freely in α (analogously for $\forall\nu\phi$)
6. $\{\alpha/d\}(\beta = \gamma)$ is equivalent with $\{\alpha/d\}\beta = \{\alpha/d\}\gamma$
7. $\{\alpha/d\}\lambda\nu\beta$ is equivalent with $\lambda\nu\{\alpha/d\}\beta$, if ν does not occur freely in α
8. $\{\alpha/d\}\{\alpha'/d'\}\beta$ is equivalent with $\{\alpha'/d'\}\{\alpha/d\}\beta$
9. $\{\alpha/d\}{}^\wedge\beta$ is equivalent with ${}^\wedge\beta$
10. $\{\alpha/d\}{}^\vee\beta$ is *not* equivalent with ${}^\vee\beta$

Clauses 1 and 9 are instances of fact 2 above. In clause 2, the effect of application of a state switcher to a discourse marker is decribed. As for 10, it should be noticed that what is meant is that the equivalence does not hold for *all* β, although for some it does. A simple case is ${}^\wedge c$, for $\{\alpha/d\}{}^{\vee\wedge}c$ is equivalent with ${}^{\vee\wedge}c$. An example of a case in which the equivalence does not hold will be discussed shortly.

According 3–8, the state switcher may always be pushed inside an expression, with the exception of those which begin with the extension operator ${}^\vee$. State switchers distribute over function-argument structures (3), conjunctions, disjunctions, and implications (4), and identities (6). Also they may be pushed over negation (4), over the quantifiers (5) and the λ-operator (7), provided no binding problems occur, and over state switchers (8). In combination with 1, 2 and 9, this allows us to 'resolve' state switchers in a large number of cases, though not in all. Generally, an occurrence of a state switcher in an expression may be moved inward until one of three cases obtains. It hits the corresponding discourse marker, in which case it is resolved in accordance with 5. It may end up in front of a constant, or a variable, or a different discourse marker, in which case it disappears. Or it may strand in front of an expression without a further move being possible. In this case we are dealing with an expression of the form ${}^\vee\beta$. As is stated in 10, $\{\alpha/d\}{}^\vee\beta$ can *not* always be reduced further. This role of 'stranding site' for state switchers that the extension operator $^\vee$ plays, will prove to be of prime importance in what follows. The following example illustrates why the reduction in question does not hold generally.

Consider the expresssion $\lambda p^\vee p$, in which p is a variable of type $\langle s, t\rangle$. The expression as a whole is of type $\langle\langle s,t\rangle, t\rangle$, and it denotes a set of propositions. With respect to a state s, the denotation of $\lambda p^\vee p$, $[\![\lambda p^\vee p]\!]_{M,s,g}$, is the set of those propositions $\mathbf{p}$ such that $\mathbf{p}$ is true in s. If we apply a state-switcher $\{\alpha/d\}$ to this expression, we get $\{\alpha/d\}\lambda p^\vee p$, which is equivalent to $\lambda p\{\alpha/d\}^\vee p$. In a state s, this expression denotes the set of propositions which are true in $\langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s, g$, the unique state which differs at most from s in that the denotation of the discourse marker d is the denotation of the expression α in the original state s. The latter set of propositions is only identical to the set denoted by our original expression $\lambda p^\vee p$ in s if $s = \langle d \leftarrow [\![\alpha]\!]_{M,s,g}\rangle s, g$. But of course, that need not be the case.

We end this short overview of the *DIL*-system by noticing that two familiar facts, one concerning the interaction between $^\wedge$ and $^\vee$, and the other concerning λ-conversion, go through:

Fact 4 $^{\vee\wedge}\alpha$ is equivalent with α

Fact 5 $\lambda\nu\alpha(\beta)$ is equivalent with $[\beta/\nu]\alpha$, if:

1. all free variables in β are free for ν in α
2. $\beta \in ICE$, or there is no occurrence of ν in γ which is in the scope of $^\wedge$

In the following two sections, we show how the *DIL*-system can be used to give the meanings of natural language sentences. First, we discuss what type of *DIL*-object would be a proper one to be associated with sentences, and next, we show how translation which express this type of semantic object, can be obtained

2.3 Interpreting and translating sentences

In *MG* an (indicative) sentence is translated into an expression of type t: the extension of a sentence is a truth value, and its intension a proposition. This is the explication that *MG* gives of the idea that the meaning of a sentence can be identified with its truth conditions. We shall see that this static notion of meaning is retained in *DMG*, but as a secondary one, which can be derived from the primary notion, which is essentially richer.

The general starting point of theories of dynamic semantics is that the meaning of a sentence resides in its information change potential. Sentences carry information, and the best way that this information can be characterized is by showing how it *changes* information, for example the information of someone who processes the sentence. This idea is quite general, and one way to capture part of it is by looking upon the meaning of a sentence as something which tells us which propositions are true after its contents have been processed. We may assume that this processing takes place in some situation, so that we can regard the extension of a sentence in a situation as those propositions which are true after the sentence has been processed in that situation. Notice that the situation in which the sentence is processed, may itself contribute information, i.e., which set of propositions results, will in general also depend on the situation in which a sentence is processed. This informal description clearly displays the dynamic character of this view on meaning. And it also makes clear that we need a fairly complex type of objects if we are to give a formal explication of it.

At this point, we should remark, perhaps superfluously, that the present theory, like *DPL*, *DRT*, and others, accounts for just one aspect of this notion of meaning as information change potential. Here, 'information' is information about referents (of discourse markers) only. Analogously, a proposition in *DIL* is a function from states to truth values, and, as was already noticed earlier, at least for the moment, this has nothing to do with intensionality in the ordinary sense. So no account is given of those aspects of meaning which concern updating of partial information about the world. In fact, in the present context the world can only be equated with the model in which interpretation takes places, which leaves no room for partiality of information. However, as was already pointed out earlier, the organization of *DIL* deliberately anticipates the incorporation of intensionality. If we can make states also play the part of possible worlds, then the scheme to be explained in what follows will indeed explicate to a considerable extent, and in uniform way, the notion of meaning as information change potential.

It is in keeping with this that sentences are translated in *DMG* into expressions of type $\langle\langle s,t\rangle,t\rangle$. In other words, in a state they denote a set of propositions. The meaning of a sentence is an object of type $\langle s,\langle\langle s,t\rangle,t\rangle\rangle$, i.e., a relation between states and propositions, or, equivalently, a function from states to sets of propositions.

Let us try to make this a bit more clear by exploiting the following analogy. The extension of a sentence in *DMG* is an object of type $\langle\langle s,t\rangle,t\rangle$, and such an object may be viewed as a generalized quantifier over states, in much the same way as objects of type $\langle\langle e,t\rangle,t\rangle$, the kind of semantic objects that function as the translations of quantified terms such as *the woman*, or *a man who was smoking*, are generalized quantifiers over individuals. A generalized quantifier over individuals denotes a set of properties (sets) of individuals, and a generalized quantifier over states denotes a set of properties (sets) of states, i.e., a set of propositions.

This means that we may paraphrase the extensions of sentences in more or less plain English, using such phrases as *a state which...*, *the state such that...*, and so on. A characteristic example is the following:

(1) a state which differs from s at most in this respect that the denotation of the discourse marker d is an object which belongs to the set of men and

to the set of objects which walk in the park

Now, the extension of which English sentence is this the paraphrase of? First of all, we notice that (1) does not identify a state. It is an existentially quantified term, and it denotes a set of sets of states, the smallest elements of which are singleton sets, each containing a different state. Specifically, for every object $\mathbf{d}$ which is a man and which walks in the park, $\{\langle d \leftarrow \mathbf{d}\rangle s\}$ is such a singleton. Secondly, we draw attention to the role played by the state s, remarking that everything which is true in s and which does not concern the denotation of d, remains true in each of the new states. So, the transition from s to this set of propositions characterizes a change of information about the denotation of this discourse marker d.

Let us now look at the following sentence:

(2) A man walks in the park

Assume that indefinite terms are interpreted using discourse markers. What change of information does (2) bring about, if we interpret it with respect to some state s? Clearly, no unique state results, any state in which the relevant discourse marker denotes a man who walks in the park and which is like s in all other respects, will do. So, with regard to a fixed state s, the processing of a sentence like (2) should result in a set of states, each representing, so to speak, a possible way in which (2) could be true, given s. This set is of course no other than the union of the minimal elements of the set of sets denoted by (1). So, (1) is indeed a proper characterization of the information carried by (2) in s. And if we abstract over s in (1), we get a proper characterization of the information change potential, i.e., the meaning, of (2).

Of course, natural language sentences themselves don't contain discourse markers, nor do they refer to states explicitly, so we can't look upon (1) as the extension of (2). Rather, it is a paraphrase of the extension of its translation in *DIL*, and from (1) we can infer some characteristics of this translation. It will contain the discourse marker d, and it will, among other things, consist of assertions that express that the extension of d has the properties of being a man and of walking in the park: $\mathtt{man}(d) \wedge \mathtt{walk\_in\_the\_park}(d)$. The existential quantification that is part of the meaning of the indefinite term, will appear as an ordinary existential quantifier over elements of the domain: $\exists x$. A state switcher will relate the values of x to the discourse marker d: $\exists x\{x/d\}[\mathtt{man}(d) \wedge \mathtt{walk\_in\_the\_park}(d)]$. Now, this is not the actual translation of (2), since a very important element is still missing. (This is already obvious from the fact that this expression is of type t, and not of the required type $\langle\langle s,t\rangle,t\rangle$. Notice also that it is equivalent with $\exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x)]$.) What is not yet accounted for is the possibility inherent in (2) that the indefinite term serve as an antecedent for pronouns in following sentences. This means that its translation should be able in some way to pass on the information it carries about the discourse marker to other formulas, something in which the state switcher can be expected to play a crucial role, of course. Actually, how this is to be done is no other question than how sequences of sentences are to be interpreted in this scheme. Before we address that question explicitly, however, two other observations are in order.

The first is that not all natural language sentences will involve 'state switching'. Consider (3):

(3) He whistles

Suppose, again, that we interpret this with respect to some state s. Given some prior determination of which object the pronoun *he* refers to, we check whether s satisfies the condition that this object whistles. If it does, we continue to regard s as a possible state, if it doesn't, we discard it, i.e., we no longer take it into account. Notice that no other states than s itself are involved in this process. Information change potential and state switching are not to be confused: the change of information (about referents of discourse markers) takes place at the level of sets of states, and it may, but need not, involve state switching, which takes place at the level of states. Nevertheless, we can describe what goes in the same format we used for (2). Consider (4):

(4) the state s, and the denotation of the disourse marker d in s is an object which whistles

If we translate the pronoun *he* in (3) by means of the discourse marker d, (4) can be regarded as giving the extension of (3) relative to s: (4) denotes the ultrafilter generated by s, if s satisfies the condition stated; and it denotes the empty set otherwise. Clearly, this neatly characterizes the information change brought about by (3) which we described informally above.

The second observation concerns the following. Above, we have said that the notions of the extension and the intension of a sentence with which we are familiar from the standard, static semantics, are retained as derivative notions. Now we show how. Our informal characterization of the dynamic meaning of a sentence was that the extension of a sentence in a state s consists of those propositions which are true after the sentence is processed in s. If the sentence can be succesfully processed, the resulting set of propositions will be non-empty, and should contain the tautologous proposition. A proposition can also be interpreted as a predicate over states, and the tautologous proposition corresponds to the predicate 'is a (possible) state'. If we apply the extension of a sentence in a state s to this predicate, what we get is an assertion which is either true or false in s. Consider (1). If we apply it to the predicate 'is a possible state', we end up with an assertion which can be phrased as follows:

(5) There is a state which differs from s at most in this respect that the denotation of the discourse marker d is an object which belongs to the set of men and to the set of objects which walk in the park

Since our postulates guarantee that to each state s a unique state s' corresponds which differs at most from s in that the denotation of the discourse marker d has certain properties, this means that (5) expresses the familiar truth conditions of (2): the bit about s, s', and d may be dropped, since it is guaranteed to hold, and we end up with:

(6) There exists an object which belongs to the set of men and to the set of objects which walk in the park

As is evident from (6), the truth or falsity of sentence (2) is compeletely independent of the denotations of discourse markers, and hence is also state independent. It depends solely on the model. But this does not hold for all sentences. An example is provided by (3). If we apply (4), the paraphrase of the extension of this sentence, to the predicate 'be a (possible) state', the following assertion results:

(7) The denotation of the discourse marker d in s is an object which whistles

In this case, the truth conditions are state dependent, since the truth or falsity of (7) depends on the value of the discourse marker d in s. This is, of course, nothing but a reflection of the fact that (3) contains the pronoun *he*, which makes its truth or falsity dependent on the state in which it is interpreted. For it is the state which determines the referent of the pronoun, by fixing the denotation of the discourse marker by means of which we translate it.

From these considerations two conclusions may be drawn. First, we can extract the ordinary truth conditions from the dynamic interpretation of a sentence by applying the latter to the tautologous proposition. And second, these truth conditions may be state dependent.

Now, we return to the question of how sequences of sentences will be interpreted, and, in the wake of that, how the meanings of sentences are able to 'pass on' information to sentences to follow. We described the extension of a sentence in a state as a set of propositions, viz., those which are true after the sentence has been processed in this state. As we have seen above, this may have resulted in a state switch, but it need not have. Consider sentence (3), and the paraphrase of its extension in s, (4). Instead of (4), we could also have used:

(8) the set of propositions $\mathbf{p}$ such that the denotation of the disourse marker d in s is an object which whistles and $\mathbf{p}$ is true in s

This paraphrase is equivalent with (4): it characterizes s if the condition on d is fulfilled, and the empty set otherwise. Suppose we now process the following sentence, assuming it to be about the same individual as (3), i.e., interpreting it using the same discourse marker:

(9) He is in a good mood

Like (3), this sentence involves no state switch, as (10) shows:

(10) the set of propositions **p** such that the denotation of the disourse marker d in s is an object which is in a good mood and **p** is true in s

Suppose (3) and (9) are both true in s. This implies that starting from s and interpreting, first (3), and next (8), we end up in s, having checked that it satisfies the conditions formulated by (3) and (8). In other words, the extension in s of the sequence:

(11) He whistles. He is in a good mood

can be paraphrased as:

(12) the set of propositions **p** such that the denotation of the disourse marker d in s is an object which whistles and which is in a good mood and **p** is true in s

In getting to (12) from (8) and (9), we do something like the following. We construct from (9) a proposition which contains its truth contional content (and something else to be explained later one), and to this proposition (8) is applied. This may look like a mere trick, but in fact it is completely in accordance with the description we have given of the extension of a sentence in a state as consisting of those propositions which are true in that state after the sentence has been processed. For a following sentence in a sequence, of course, claims to be just that: a sentence which is true in the state with respect to which it is interpreted, which is the state which results after processing its predecessor.

This means that we can also look upon the propositions which form the extension of a sentence as something giving the truth conditional contents of its possible continuations. This is not particularly useful if we just consider sentences which do not involve state switching. But in case a sentence does change the state with respect to which interpretation takes place, the abstraction over the contents of possible continuations gives us the means to pass on this information, i.e., to force the contents of sentences to come to be determined with respect to the changed state. Consider (2), and its paraphrase (1), again. The latter can also be phrased as:

(13) the set of propositions **p** such that there is a state s' which differs from s at most in this respect that the denotation of d is an object which is a man and which walks in the park and **p** is true in s'

The abstraction is now over propositions which are true in s', and this means that the truth conditional contents of sentences following (2) are to be determined with respect to s', and not with respect to the original s. If we first process (2), and then (3), the content of the latter will be determined with respect to s', a state in which d refers to a man which walks. And this means that (3) will be interpreted as saying of this individual that he whistles. Thus we get an account of the anaphoric relation between *a man* and *he* in:

(14) A man walks in the park. He whistles

Two remarks are in order at this point. First, we notice once again that the change brought about by a sentence like (2) is relevant only for those sentences which follow it, which contain the discourse marker d. For what is changed is the state parameter, which gives information about referents of discourse markers, not the situation which is being described, or, to put it differently, with respect to which sentences are evaluated. This remains fixed, being, in fact, embodied in the model in which interpretation takes place.

Second, we described the interpretation of a sequence of two sentences as the result of the application of the extension of the first sentence to a proposition constructed from that of the second sentence which 'contains its truth conditional content'. If we are interested in just the sequence itself, the truth conditional content, which can be acquired in the way described above, would suffice. However, we want to interpret longer sequences, too, and, more important, we want to interpret them in a compositional, step-by-step manner. And that is the reason why we need something more than just the truth conditional content of a sentence. For, unless we are sure that the sentence we are interpreting as part of a sequence is the last one, we need an 'angle' for the following sentences to hook on to, a 'place-holder' proposition for which the content of the following sentence can be substituted, which in its turn will contain another such place-holder, and so on. In other words, we need to make sure that if we calculate the extension in state s of a sequence consisting of a sentence ϕ followed by a sentence ψ, the result is again something which can be combined with another sentence χ. The extensions of sentences being functions, this means that sequencing of sentences is interpreted as intensional function composition. We apply the extension of the second sentence ψ to an arbitrary proposition, and abstract over the state. The result is a proposition which asserts the truth conditional content of ψ and the truth of this arbitary proposition. To this we apply the extension of the first sentence ϕ, which gives us the assertion of the truth conditional content of ϕ in conjunction with that of ψ and of the truth of the arbitrary proposition. If we abstract over the latter again, what we get is the set of propositions $\mathbf{p}$ such that $\mathbf{p}$ is true in the situation which results if first ϕ is processed in s, and next ψ is processed in the situation which results from that. Less informally, but more in accordance with the actual way in which sequences of sentences are translated in *DMG*:

(15) those propositions $\mathbf{p}$ such that after the processing of ϕ in s it holds that $\mathbf{p}$ holds after the processing of ψ

Notice that the truth conditions of a sequence can be defined just as above: if the tautologous proposition is an element of this set of propositions, the sequence of the sentences ϕ and ψ is true in s.

We end this section by showing in some more detail what actual translations of sentences into *DIL*-expressions look like. This may serve as a further illustration of what was said above, and it may also help the reader to appreciate a peculiar characteristic of *DIL* which definitely distinguishes it from *DPL*. The latter is a system in which a dynamic semantics is given for the language of first-order logic. *DIL*, on the other hand, has a completely static semantics. It contains some relatively new features, the discourse markers and state switchers, but these two are interpreted statically, not dynamically. So, how can we expect *DIL* to be put to the same use as *DPL*, which was to represent the dynamic meanings of natural language expressions? The answer is, of course, that *DIL* allows us to formulate on the object language level what in *DPL* was formulated in the meta-language, viz., dynamic interpretations. This feature makes it possible to describe in *DIL* dynamic notions using a static (meta-)vocabulary, but of course at a cost: the language is intrinsically more complicated. Whether this is a virtue, or a vice, or does not really matter, is perhaps largely a matter of taste. As will become apparent shortly, *DIL*-translations are more complicated than *DPL*-translations, even disregarding the complexity that stems from it being a higher-order language. On the other hand, *DIL*-expressions tend to provide very transparant representations of the dynamic meanings of the sentences they are translations of, whereas in *DPL* one has constantly to bear in mind the dynamic semantics of what look like ordinary, familiar expressions.

Now we show how object language formulae can be obtained in a rather straightforward way by 'formalizing' so to speak the 'plain English' we have been using sofar.

If we use a logical notation for (1), the paraphrase of the extension in s of (2), we get the following formula of our meta-language:

(16) $\lambda \mathbf{p} \in D_{\langle s,t \rangle}[\exists s' \in S \exists \mathbf{d} \in D[[\![d]\!]_{s'} = \mathbf{d} \ \& \ [\forall d' \in DM : d' \neq d \Rightarrow [\![d']\!]_s = [\![d']\!]_{s'}] \ \& \ \mathtt{man}(\mathbf{d}) \ \& \ \mathtt{walk\_in\_the\_park}(\mathbf{d}) \ \& \ \mathbf{p}(s')]]$

Given our postulates we know that the s' in question is unique (for a fixed d) and that we may denote it as $\langle d \leftarrow \mathbf{d} \rangle s$, which means that we can reduce (16) to:

(17) $\lambda \mathbf{p}[\exists \mathbf{d} : \mathtt{man}(\mathbf{d}) \ \& \ \mathtt{walk\_in\_the\_park}(\mathbf{d}) \ \& \ \mathbf{p}(\langle d \leftarrow \mathbf{d} \rangle s)]$

A *DIL*-formula which denotes this in state s is:

(18) $\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x) \wedge \{x/d\}{}^{\vee}p]$

In a similar fashion we can write the interpretation of (3), as it was paraphrased in (4), as:

(19) $\lambda \mathbf{p}[\mathtt{whistle}([\![d]\!]_s) \ \& \ \mathbf{p}(s)]$

A *DIL*-formula which has this denotation in state s is:

(20) $\lambda p[\mathtt{whistle}(d) \wedge {}^{\vee}p]$

Now consider the denotation of a sequence of two sentences ϕ and ψ as described above in (15). Using a logical notation, we may write:

(21) $\lambda \mathbf{p}[[\![\phi]\!]_S(\lambda s'[[\![\psi]\!]_{s'}(\mathbf{p})])]$

A corresponding *DIL*-formula is:

(22) $\lambda p[\phi({}^\wedge\psi(p))]$

If we substitute for ϕ and ψ the translations (18) and (20) of the two sentences (2) and (3) which make up sequence (14), the result is the following:

(23) $\lambda p[\lambda p\exists x[\mathtt{man}(x)\wedge\mathtt{walk\_in\_the\_park}(x)\wedge\{x/d\}{}^\vee p]({}^\wedge\lambda p[\mathtt{whistle}(d)\wedge{}^\vee p](p))$

After some conversion, we get:

(24) $\lambda p\exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x) \wedge \{x/d\}{}^{\vee\wedge}[\mathtt{whistle}(d) \wedge {}^\vee p]]$

After ${}^{\vee\wedge}$-elimination and using the semantic properties of the state switcher, this reduces to:

(25) $\lambda p\exists x[\mathtt{man}(x) \wedge \mathtt{walk\_in\_the\_park}(x) \wedge \mathtt{whistle}(x) \wedge \{x/d\}{}^\vee p]$

The essential feature of this representation of the sequence of sentences (14) is that the binding of the indefinite term in the first sentence is passed on, by means of the state-switcher, to the second. Notice also that the fact that $\{x/d\}{}^\vee p$ can not be reduced plays an essential role here.

3 *DPL* and *DIL*

In this section we show that *DIL* is at least capable of providing an account for the same phenomena that *DPL* can handle. We do so by providing a translation of *DPL* into *DIL* which preserves meaning in a yet to be explicated way.

The discussion of the examples in the previous section, contains nearly all the ingredients we need for a definition of a translation function $\sharp$ which takes *DPL*-formulas into *DIL*-expressions of type $\langle\langle s,t\rangle,t\rangle$. Atomic formulae, existential quantification and conjunction have been treated. We only need to add negation, since the other logical operators are definable in *DPL* in terms of these.

Definition 6 (Translation $\sharp$ of *DPL* into *DIL*)

1. $\sharp x_n = d_n$, $d_n \in DM$
2. $\sharp c = c$, a constant of type e
3. $\sharp P^n = P$, a constant of type $\langle e_1 \ldots \langle e_n, t\rangle \ldots\rangle$
4. $\sharp Pt_1 \ldots t_n = \lambda p[\sharp P(\sharp t_1)\ldots(\sharp t_n) \wedge {}^\vee p]$
5. $\sharp[\phi \wedge \psi] = \lambda p\sharp\phi({}^\wedge(\sharp\psi(p)))$
6. $\sharp\exists x_n\phi = \lambda p\exists x\{x/d_n\}(\sharp\phi(p))$
7. $\sharp\neg\phi = \lambda p[\neg\sharp\phi({}^\wedge\mathtt{true}) \wedge {}^\vee p]$

Clause 1 of this definition assumes that the sets VAR and DM are equinumerous, and it maps *DPL*-variables onto discourse markers of *DIL*. Clauses 2 and 3 translate individual constants and predicate constants in a straightforward way. Atomic *DPL*-formulae are mapped onto expressions of the type we have considered in the previous section. Since they are not dynamic, their *DIL*-translations contain no state switchers. The interesting clauses are 5 and 6, since they concern the 'truly', i.e., the externally dynamic logical constants from *DPL*: conjunction and the existential quantifier. Conjunctions are translated according to the scheme of intensional function composition which was defined above for the translation of sequences of sentences. In the translation of an existentially quantified formula the state switcher makes its appearance, producing a formula of the kind we encountered above. Finally, the expression **true** in clause 7 is a constant which refers to 1. Obviously, $[\![{}^\wedge\mathbf{true}]\!]_{M,s,g} = S$, for all M, s, and g, so ${}^\wedge\mathbf{true}$ is the tautologous proposition referred to in the previous section.

What is the relation between a *DPL*-formula ϕ and its *DIL*-translation $\sharp\phi$? Take a *DPL*-model M^{DPL} and a *DIL*-model M^{DIL}, based on the same set D, and with the interpretation functions F chosen so as to coincide on the individual constants and predicate constants. Let π be a $1-1$-correspondence between VAR^{DPL}, the set of *DPL*-variables, and DM, the set of discourse markers. Then π induces a $1-1$-correspondence Π between G^{DPL}, the set of *DPL*-assignments, and S, as follows:

$$\Pi(g) = s \Leftrightarrow g(x) = (F(\pi(x)))(s)$$

Now we can state the following fact:

Fact 6 $\langle h, k\rangle \in [\![\phi]\!]_M^{DPL} \Leftrightarrow \{\Pi(k)\} \in [\![\sharp\phi]\!]_{M,\Pi(h),g}^{DIL}$

We now introduce a few notation conventions which will make the transition from *DPL* to *DIL* even more straightforward. Also, these conventions later on will facilitate the representation of translations of natural language expressions in *DMG*, by providing an easy to read format.

Definition 7 ($\uparrow$) $\uparrow\phi = \lambda p[\phi \wedge {}^\vee p]$, where ϕ is an expression of type t

The operator $\uparrow$ can be viewed as a type-shifting operation. If ϕ is true in state s, $\uparrow\phi$ denotes the set of all true propositions in s, if ϕ is false in s, then $\uparrow\phi$ denotes the empty set. The meaning of ϕ and $\uparrow\phi$ are hence one-to-one correllated. $[\![\uparrow\phi]\!]_{M,s,g}$ typically gives us the denotation of a sentence which does not have truly dynamic effects. For such sentences, it holds that their denotation in state s is always either the empty set, or the set of all true propositions in s. The translation of the atomic *DPL*-formula Px, $\lambda p[P(x) \wedge {}^\vee p]$ according to definition 6, can now be written more succinctly as $\uparrow P(x)$.

Definition 8 ($\downarrow$) $\downarrow\Phi = \Phi({}^\wedge\mathbf{true})$, where Φ is an expression of type $\langle\langle s,t\rangle,t\rangle$

An expression Φ of type $\langle\langle s,t\rangle,t\rangle$ is typically the kind of expression that functions as the translation of sentences in *DMG*, and as translation of *DPL*-formulae. In our discussion of the interpretation of sentences in section 2.3, we saw that appplication to the predicate 'is a possible state' of a generalized quantifier over states which gives the extension of a sentence, results in a proposition which represents the usual truth conditions of that sentences. The expression $^\wedge\mathtt{true}$ is a representation in *DIL* of this predicate 'is a possible state'. Application of the operator $\downarrow$ to the dynamic translation of a sentence hence results in a formula which represents its truth conditions. The formula $\downarrow\Phi$ is true in state s iff Φ can be succesfully processed in s. Its negation $\neg\downarrow\Phi$ boils down to the assertion that Φ can not be succesfully processed in s, that after processing Φ no possible state results.

If we look at the translation of *DPL*-negation, we can now, using our two notation conventions, represent it as $\uparrow\neg\downarrow\natural\phi$. This means that the result of negation is a static expression which either denotes the set of all true propositions in s, viz., in case ϕ can not be processed succesfully in s, or the empty set, in case ϕ can be so processed in s. So, the static character of negation in *DPL* has been taken over in our *DIL*-translation. We shall abbreviate $\uparrow\neg\downarrow$ even more:

Definition 9 ($\sim$-negation) $\sim\Phi = \uparrow\neg\downarrow\Phi$ $(= \lambda p[\neg\Phi(^\wedge\mathtt{true})\wedge{}^\vee p])$

The double negation $\sim\sim\Phi$ corresponds to $\uparrow\downarrow\Phi$ ($\sim\sim\Phi = \uparrow\neg\downarrow\uparrow\neg\downarrow\Phi = \uparrow\neg\neg\downarrow\Phi$ $= \uparrow\downarrow\Phi$). Since $\uparrow\downarrow\Phi$ is not equivalent with ϕ, it follows that $\sim\sim\Phi$ is not equivalent with Φ. (Cf. the fact that $^\wedge{}^\vee\alpha$ is not equivalent with α. But, like $^\vee{}^\wedge\alpha = \alpha$, we have $\downarrow\uparrow\phi = \phi$.) Any dynamic effects that Φ may have, are cancelled in $\uparrow\downarrow\Phi$. This means that we can expect $\uparrow\downarrow\Phi$ to be equivalent with Φ only if no such effects are present, i.e., if Φ has a static interpretation.

By way of example, consider the expression $\lambda p[\phi\wedge\{x/d\}^\vee p]$ of type $\langle\langle s,t\rangle,t\rangle$. Using definition 8, we can rewrite its 'lowering' $\downarrow\lambda p[\phi\wedge\{x/d\}^\vee p]$ to type t as $\lambda p[\phi\wedge\{x/d\}^\vee p](^\wedge\mathtt{true})$, which reduces to $\phi\wedge\{x/d\}\mathtt{true}$, by means of λ-conversion and $^\vee{}^\wedge$-elimination. Since $\mathtt{true}$ is a constant, and hence state independent, a further reduction to $\phi\wedge\mathtt{true}$ is possible, an expression which is equivalent to ϕ. If we raise this expression of type t again to one of type $\langle\langle s,t\rangle,t\rangle$ by applying the operator $\uparrow$ to it, we get $\uparrow\phi$, which, by definition 7, can be written as $\lambda p[\phi\wedge{}^\vee p]$. But the latter expression is not equivalent to our original $\lambda p[\phi\wedge\{x/d\}^\vee p]$, from which fact we may conclude that $\uparrow\downarrow\Phi$ is not equivalent with Φ.

The operator $\downarrow$ may be looked upon as a kind of closure operator, one which closes off a (piece of) text (cf. the $\Diamond$-operator in *DPL*). It reduces the meaning of a sentence to its truth-conditional content, 'freezing' so to speak any dynamic effects it may have had. The point is that once closed off, a piece of text remains that way, even if it is raised again to the higher type by means of $\uparrow$.

Now, we introduce some more notational conventions. First, ;-conjunction:

Definition 10 (Dynamic ;-conjunction) $\Phi\,;\Psi = \lambda p[\Phi(^\wedge(\Psi(p)))]$

The expression $\Phi\,;\Psi$ represent the dynamic conjunction, or sequence, of two sentences. The translation of a *DPL*-conjunction $\phi \wedge \psi$ can now be written as $\sharp\phi\,;\sharp\psi$.

Next, existential quantification:

Definition 11 (Dynamic $\mathcal{E}$-quantifier) $\mathcal{E}d\Phi = \lambda p \exists x\{x/d\}(\Phi(p))$, where d is a discourse marker and x a variable of type e

Using this definition, we can write the translation of a *DPL*-formula $\exists x_n \phi$ more perspicuously as $\mathcal{E}d_n\sharp\phi$.

Using the above definitions, each *DPL*-formula which is build using atomic formulae, negation, conjunction and existential quantification only, is mapped to a corresponding *DIL*-formula which mirrors its syntactic construction quite directly. If we add the following notation conventions for the other connectives and for the universal quantifier, we extend this characteristic to all formulae:

Definition 12 ($\Rightarrow$-implication) $\Phi \Rightarrow \Psi = \sim[\Phi\,;\sim\Psi]$
$(= \lambda p[\neg(\phi(^\wedge\neg(\Psi(^\wedge\mathtt{true})))) \wedge {}^\vee p])$

Definition 13 (or-disjunction) Φ **or** $\Psi = \sim[\sim\Phi\,;\sim\Psi]$
$(= \lambda p[[\Phi(^\wedge\mathtt{true}) \vee \Psi(^\wedge\mathtt{true})] \wedge {}^\vee p])$

Definition 14 ($\mathcal{A}$-quantifier) $\mathcal{A}d\Phi = \sim\mathcal{E}d\sim\Phi$
$(= \lambda p[\forall x\{x/d\}\Phi(^\wedge\mathtt{true}) \wedge {}^\vee p])$

The qualification 'dynamic' does not appear in these definitions, since, as will become clear below, these notions of implication, disjunction, and the universal quantifier, are, like their *DPL*-counterparts, not truly, i.e., externally dynamic. The same holds for the notion of negation defined above. In section 5.2, we give dynamic definitions of these constants, too.

Fact 6 guarantees that the various properties of the logical constants which characterize the dynamic semantics of *DPL*, carry over to *DIL*. More specifically, the following two equivalences hold:

$$\mathcal{E}d\Phi\,;\Psi = \mathcal{E}d[\Phi\,;\Psi]$$

$$\mathcal{E}d\Phi \Rightarrow \Psi = \mathcal{A}d[\Phi \Rightarrow \Psi]$$

The first equivalence forms the basis of an account of cross-sentential anaphora, and the second is of paramount importance for a compositional interpretation of donkey-sentences.

Also, various non-equivalences of *DPL* reappear in *DIL*. For example:

$$\mathcal{E}d\sim\Phi \neq \sim\mathcal{A}d\Phi$$

However, as is to be expected, these formulas do have the same truth conditional content, as is apparent from the following equivalence:

$$\downarrow\mathcal{E}d\sim\Phi = \downarrow\sim\mathcal{A}d\Phi$$

So, although the dynamic properties of these constants differ, at the truth conditional level the existential and the universal qunatifier are related in the usual way.

The following equivalences will enables us to replace dynamic operators by their static counterparts at the level of truth-conditional content, thus allowing us to write down more orthodox translations containing only the usual operators in some cases:

$$\downarrow\uparrow\phi = \phi$$

$$\sim\uparrow\phi = \uparrow\neg\phi$$

$$\uparrow\phi ; \uparrow\psi = \uparrow[\phi \wedge \psi]$$

$$\uparrow\phi \Rightarrow \uparrow\psi = \uparrow\phi \rightarrow \psi$$

$$\uparrow\phi \textbf{ or } \uparrow\psi = \uparrow[\phi \vee \psi]$$

In view of the fact that it is possible to translate *DPL* into *DIL*, keeping *DPL*'s characteristic logical properties intact, we may conclude that all phenomena that could be dealt with using *DPL*, can also be dealt with using *DIL*. But our motive for switching from *DPL* to *DIL* was that we wanted to be able to account for the phenomena in question, notably cross-sentential anaphora and donkey-sentences, in a fully compositional way, using the resources of a typed system with λ-abstraction. That will be the subject of the next section.

One final remark to conclude this section. If we compare *DPL* with *DIL*, and, for the sake of comparison, equate assignments and states, we notice that the interpretations of sentences in *DIL* are more complex objects that they are in *DPL*. In the latter system, the meaning of a sentence is taken to be a relation between assignments (states), i.e., a function from assignments (states) to sets of assignments (states). But in *DIL*, sentence meaning is an object of a more complex type, viz., a function from states (assignments) to sets of sets of states (assignments).

In our translation of *DPL* into *DIL* the richer structure of the notion of sentence meaning has not yet been used. However, as we shall see in section 5.2, the additional complexity does pay off, in this sense that it enables us to deal with a wider range of phenomena than *DPL* is able to cope with.

4 Dynamic Montague grammar

In this section we shall outline how the expressions of a small fragment of English can be assigned a compositional dynamic interpretation through translation into *DIL*.

4.1 A small fragment

The fragment has as its basic categories the usual IV (intransitive verb phrases), CN (common noun phrases), and S (for (sequences of) sentences). Derived categories are of the form A/B, A and B any category. Employed in the fragment are NP (= S/IV, noun phrases), Det (= NP/CN, determiners), and TV (= IV/NP, transitive verb phrases). Furthermore, we shall make use of three syncategorematic constructies: sentence sequencing; formation of (indicative) conditionals; and formation of restictive relative clauses.

Next, we define a function f from categories to DIL-types:

Definition 15 (Category to type assignment)

1. $f(S) = \langle\langle s,t\rangle,t\rangle$; $f(CN) = f(IV) = \langle e,\langle\langle s,t\rangle,t\rangle\rangle$
2. $f(A/B) = \langle\langle s,f(B)\rangle,f(A)\rangle$

The relation with the ordinary MG-types will be obvious: t is switched for $\langle\langle s,t\rangle,t\rangle$.

We might also want to make our category-type relationship more flexible, allowing for example S to be associated with t, $\langle\langle s,t\rangle,t\rangle$,and perhaps other types as well, all of which are related systematically by means of a small set of type-shifting principles. But in order not to introduce too much novelties at the same time, we stick, for the time being at least, to the orthodox, rigid category-type association.

Among the syntactic rules of the fragment we find some rather obvious rules of functional application corresponding to the derived categories we employ. We won't spell these out here, but just notice that the corresponding semantic rules exhibit the usual pattern of the translation of the functional expression being applied to the intension of the translation of its argument. Thus, letting α' denote the translation of α, as usual: if α is of category A/B, and β is of category B, then the result of applying functional application to them translates as $\alpha'({}^\wedge\beta')$. By the way, we notice, just once more, that the intension operator abstracts over states, i.e., over assignments of values to variables, and not over possible worlds or other, more familiar parameters.

Let us now turn to some examples of translations of basic expressions. In what follows x is a variable of type e, P and Q of type $\langle s,\langle e,\langle\langle s,t\rangle,t\rangle\rangle\rangle$, and $\mathcal{P}$ of type $\langle s,\langle\langle s,\langle\langle e,\langle\langle s,t\rangle,t\rangle\rangle\rangle,\langle\langle s,t\rangle,t\rangle\rangle\rangle$; **man** and **walk** are constants of type $\langle e,t\rangle$, and **love** of type $\langle e,\langle e,t\rangle\rangle$; the d_i are discourse markers.

Definition 16 (Translations of basic expressions)

1. $man \rightsquigarrow \lambda x \uparrow \mathtt{man}(x)$
2. $walk \rightsquigarrow \lambda x \uparrow \mathtt{walk}(x)$
3. $love \rightsquigarrow \lambda \mathcal{P} \lambda x[{}^\vee\mathcal{P}({}^\wedge\lambda y \uparrow \mathtt{love}(y)(x))]$
4. $a_i \rightsquigarrow \lambda P \lambda Q \mathcal{E} d_i[{}^\vee P(d_i)\,;{}^\vee Q(d_i)]$

5. $every_i \rightsquigarrow \lambda P \lambda Q \mathcal{A} d_i[{}^{\vee}P(d_i) \Rightarrow {}^{\vee}Q(d_i)]$

6. $he_i \rightsquigarrow \lambda Q[{}^{\vee}Q(d_i)]$

7. $John_i \rightsquigarrow \lambda Q[\{\mathtt{j}/d_i\}{}^{\vee}Q(d_i)]$

A few remarks. The translations 1–3 are remarkably like those we are familiar with from standard *MG*, the only difference being the occurrence of the operator $\uparrow$, which makes sure that a formula such as $\mathtt{man}(x)$, which is of type t, is raised in the appropriate way to $\lambda p[\mathtt{man}(x) \wedge {}^{\vee}p]$, of type $\langle\langle s,t\rangle,t\rangle$. As we have seen above, raising such a static formula makes no real difference, it just changes the type. The possible denotations of $\mathtt{man}(x)$ and $\lambda p[\mathtt{man}(x) \wedge {}^{\vee}p]$ are one-to-one related. So, for basic expressions of category *CN*, *IV*, and *TV*, the *DIL*-translations do not differ in any essential way from the usual ones. By the way, we also could produce the former from the latter by means of just one type-shifting rule.

As for determiners, things are different. If we look at 4 and 5 above, the translations, thanks to our notation conventions, look pretty much like what we are used to, the main differences being that dynamic connectives occur in the place of static ones, and that we use dynamic quantifiers, introducing discourse markers, instead of the ordinary, static quantifiers. The most important point is, however, that we need to chose a particular discourse marker in the translation, which is indicated by the index that occurs in the determiner itself. So, the present approach assumes that we do not translate sentences as such, but indexed structures, i.e., sentences in which determiners, pronouns, and proper names, are marked with indices. The function of the indexing mechanism is, of course, to stake out possible anaphoric relationships among constituents: if a pronoun is to be related anaphorically to an *NP*, a necessary (but not sufficient) condition is that both carry the same index.

In itself this is not entirely a new thing to do in *MG*. In more orthodox versions of *MG*, anaphoric relations are dealt with by means of quantification rules in the syntax. This involves deriving a structure which contains indexed syntactic variables, and subsequently 'quantifying in' one or more *NP*'s. And if an *NP* and a pronoun are to be related anaphorically, the underlying structure needs to contain (at least) two variables with the same index.

It may be remarked at this point that the quantification rules of orthodox *MG* serve two purposes. Beside being the mechanism with which anaphoric relations are dealt with, they are also used to account for scope ambiguities. The first role is taken over in *DMG* by the state-switchers and discourse markers. And the second role, too, can be played by a different mechanism, viz., that of type-shifting rules (see Hendriks [1988]), which means that in principle quantification rules can be dispensed with altogether.

As is obvious from 7 above, translating a proper name also involves choosing an appropriate discourse marker in the translation. The value of this discourse marker is set to the individual that is denoted by the associated constant. Notice that this way of going about only deals with anaphoric relations. That proper names (like definite descriptions) sometimes also bear kataphoric relations to pronouns is not accounted for.

Finally, we turn to the translation rules which correspond to the three syncategorematic constructions we have introduced in our fragment:

Definition 17 (Translation of syncategorematic constructions)

1. Sentence sequencing: $\phi\ .\ \psi \rightsquigarrow \phi' \,;\psi'$
2. Conditional sentences: *If* ϕ, *then* $\psi \rightsquigarrow \phi' \Rightarrow \psi'$
3. Restrictive relative clauses: $\alpha_{CN} + \beta_S \rightsquigarrow \lambda x[\alpha'(x)\,;\beta'(x)]$

Again, it may be noticed that these translations are exactly like their ordinary counterparts but for the occurrence of dynamic logical constants.

By way of illustration we shall work out a few examples in detail in the next section. In doing so, we shall emphasize the reduction of the structurally simple and familiar translations that the rules provide, to 'basic' *DIL*expressions, which exhibit the dynamic aspects of interpretation most clearly.

4.2 Examples

A man walks. He talks

The first example which we discuss, is the sequence of sentences *A man walks. He talks*, a slightly simplified version of our example (14). If we want to obtain a translation in which the pronoun in the second sentence is anaphorically related to the indefinite term occurring in the first one, we have to assign both the same index: A_1 *man walks.* He_1 *talks.* Functional application and some standard reduction produce the following translation of the first sentence:

$$\mathcal{E}d_1[\uparrow \mathtt{man}(d_1)\,;\uparrow \mathtt{walk}(d_1)]$$

Since we know that $\uparrow\phi\,;\uparrow\psi$ is equivalent with $\uparrow[\phi \wedge \psi]$, this can be reduced to:

$$\mathcal{E}d_1 \uparrow[\mathtt{man}(d_1) \wedge \mathtt{walk}(d_1)]$$

Now we eliminate $\mathcal{E}d_1$ using definition 11:

$$\lambda p\exists x[\{x/d_1\}(\uparrow[\mathtt{man}(d_1) \wedge \mathtt{walk}(d_1)](p))]$$

Replacement of $\uparrow[\mathtt{man}(d_1) \wedge \mathtt{walk}(d_1)]$ by $\lambda p[\mathtt{man}(d_1) \wedge \mathtt{walk}(d_1) \wedge {}^{\vee}p]$ and application of λ-conversion gives us:

$$\lambda p\exists x[\{x/d_1\}[\mathtt{man}(d_1) \wedge \mathtt{walk}(d_1) \wedge {}^{\vee}p]$$

Finally, we move the state switcher inwards, using the equivalences stated in fact 3 of section 2.2, replacing occurrences of d_1 by x. The state switcher strands on the last conjunct ${}^{\vee}p$:

$$\lambda p\exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p]$$

The ordinary first-order translation of *A man walks* can be deduced from this translation as follows. Using the operator $\downarrow$, we reduce this formula, which is of type $\langle\langle s,t\rangle,t\rangle$, to one of type t:

$$\downarrow \lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p]$$

which reduces to:

$$\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p]({}^{\wedge}\mathtt{true})$$

Of course, this formula no longer represents the full dynamic meaning of the sentence in question, but only its static meaning, i.e., its truth conditional content. This is easy to see, since λ-conversion, ${}^{\vee\wedge}$-elimination, and the state independent meaning of the constant **true** guarantee that this formula is equivalent to the usual translation:

$$\exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x)]$$

Let us now turn to the second sentence in the sequence, *He talks.* This second sentence has as its reduced translation:

$$\uparrow \mathtt{talk}(d_1)$$

The translation rule for sentence sequences tells us that the translation of the entire sequence is:

$$\mathcal{E}d_1[\uparrow \mathtt{man}(d_1)\,;\uparrow \mathtt{walk}(d_1)]\,;\uparrow \mathtt{talk}(d_1)$$

We know we can write this directly as:

$$\mathcal{E}d_1[\uparrow \mathtt{man}(d_1)\,;\uparrow \mathtt{walk}(d_1)\,;\uparrow \mathtt{talk}(d_1)]$$

But, for the sake of illustration, we work this out in detail. First, we take the dynamic conjunction of the reduced representation of the first sentence and that of the second sentence:

$$\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p]\,;\uparrow \mathtt{talk}(d_1)$$

Eliminating the ;-conjunction, we get:

$$\lambda q[\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}{}^{\vee}p]({}^{\wedge}(\uparrow \mathtt{talk}(d_1)(q)))]$$

Application of λ-conversion and ${}^{\vee\wedge}$-elimination gives:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}(\uparrow \mathtt{talk}(d_1)(q))]$$

Notice that λ-conversion is allowed in this case because the argument is intensionally closed. Elimination of $\uparrow$ and another application of λ-conversion gives:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}[\mathtt{talk}(d_1 \wedge {}^{\vee}q]]$$

Moving the state switcher inside the conjunction we get as a final result:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \mathtt{talk}(x) \wedge \{x/d_1\}^\vee q]$$

Again, if we apply this formula to $^\wedge\mathtt{true}$, we get the ordinary first-order translation of this sequence. However, we did obtain this result in a completely compositional way.

If a man walks, he talks

As our second example, we choose the conditional sentence which consists of the two sentences which constitute our first example. Using the reductions made above, and the translation rule for conditional sentences, we get:

$$\lambda p \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \{x/d_1\}^\vee p] \Rightarrow \lambda p[\mathtt{talk}(d_1) \wedge {}^\vee p]$$

We rewrite this using the definition of $\Phi \Rightarrow \Psi$ as $\sim [\Phi\,; \sim \Psi]$. First we take a look at the negation of the consequent:

$$\sim \lambda p[\mathtt{talk}(d_1) \wedge {}^\vee p]$$

Using the definition of $\sim \Phi$ as $\lambda p[\neg\Phi(^\wedge\mathtt{true}) \wedge {}^\vee p]$ and some standard reductions, this can be rewritten as:

$$\lambda p[\neg\mathtt{talk}(d_1) \wedge {}^\vee p]$$

Taking this result in ;-conjunction with the antecedent, we arrive at:

$$\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \neg\mathtt{talk}(x) \wedge \{x/d_1\}^\vee q]$$

All we need to do now, is form the negation of this subresult. Rewriting the $\sim$-negation leads to:

$$\lambda p[\neg[\lambda q \exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \neg\mathtt{talk}(x) \wedge \{x/d_1\}^\vee q](^\wedge\mathtt{true})] \wedge {}^\vee p]$$

The constant **true** can be eliminated, and the final result then is:

$$\lambda p[\neg\exists x[\mathtt{man}(x) \wedge \mathtt{walk}(x) \wedge \neg\mathtt{talk}(x)] \wedge {}^\vee p]$$

Notice that the state switcher has disappeared altogether. So, should we continue this conditional with a sentence containing the pronoun he_1, the latter would not be anaphorically related to the indefinite term in the conditional. This seems to borne out by the facts, for a sequence such as:

(26) If a man walks, he talks. He waves

can not be interpreted in such a way that the pronoun in the second sentence is anaphorically linked to the indefinte term in the first sentence. (However, see section 5.1 for some more discussion of this matter.)

Finally, we remark that the translation given above is equivalent with:

$$\lambda p[\forall x[[\mathtt{man}(x) \wedge \mathtt{walk}(x)] \rightarrow \mathtt{talk}(x)] \wedge {}^{\vee}p]$$

Consequently, the truth conditional content of our example is represented in the standard way, which we obtain by application of the $\downarrow$-operator:

$$\forall x[[\mathtt{man}(x) \wedge \mathtt{walk}(x)] \rightarrow \mathtt{talk}(x)]$$

It is easily checked that the sentence *Every man who walks, talks*, which has its direct translation:

$$\mathcal{A}d_1[[\uparrow \mathtt{man}(d_1)\,;\uparrow \mathtt{walk}(d_1)] \Rightarrow \uparrow \mathtt{talk}(d_1)]$$

reduces to the same formula, and that hence the equivalence of this sentence with our example is accounted for.

Every farmer who owns a donkey, beats it

Although the previous example has already shown how donkey-type anaphora are dealt with in *DMG*, we shall also show how the classic donkey-sentence *Every farmer who owns a donkey, beats it* is derived. We use the following indexing: *Every*$_1$ *farmer who owns a*$_2$ *donkey, beats it*$_2$.

Applying some reductions, we get as the translation of *own a*$_2$ *donkey*:

$$\lambda x \mathcal{E}d_2[\uparrow \mathtt{donkey}(d_2)\,;\uparrow \mathtt{own}(d_2)(x)]$$

Using our translation rule for restricte relative clauses, we get as translation of the complex CN *farmer who owns a donkey*:

$$\lambda x[\uparrow \mathtt{farmer}(x)\,; \mathcal{E}d_2[\uparrow \mathtt{donkey}(d_2)\,;\uparrow \mathtt{own}(d_2)(x)]$$

The reduced translation of the entire subject term *every*$_1$ *farmer who owns a*$_2$ *donkey* then is:

$$\lambda Q \mathcal{A}d_1[[\uparrow \mathtt{farmer}(d_1)\,; \mathcal{E}d_2[\uparrow \mathtt{donkey}(d_2)\,;\uparrow \mathtt{own}(d_2)(d_1)]] \Rightarrow {}^{\vee}Q(d_1)]$$

The intransitive verb phrase *beat it*$_2$ translates simply as:

$$\lambda x[\uparrow \mathtt{beat}(d_2)(x)]$$

Combining the translation of the subject term with that of the verb phrase by functional application, gives the following translation for the sentence as a whole:

$$\mathcal{A}d_1[[\uparrow \mathtt{farmer}(d_1)\,; \mathcal{E}d_2[\uparrow \mathtt{donkey}(d_2)\,;\uparrow \mathtt{own}(d_2)(d_1)]] \Rightarrow \uparrow \mathtt{beat}(d_2)(d_1)]$$

The translation of the previous example has shown that an occurrence of $\mathcal{E}d_i$ in the antecedent of a conditional, can be replaced by an occurrence of $\mathcal{A}d_i$ which has scope over the conditional as a whole. This means that we get:

$$\mathcal{A}d_1\mathcal{A}d_2[[\uparrow \mathtt{farmer}(d_1)\,;\uparrow \mathtt{donkey}(d_2)\,;\uparrow \mathtt{own}(d_2)(d_1)] \Rightarrow\uparrow \mathtt{beat}(d_2)(d_1)]$$

The occurrences of $\uparrow$ can moved outwards, changing the dynamic connectives into static ones:

$$\mathcal{A}d_1\mathcal{A}d_2 \uparrow [[\mathtt{farmer}(d_1) \wedge \mathtt{donkey}(d_2) \wedge \mathtt{own}(d_2)(d_1)] \rightarrow \mathtt{beat}(d_2)(d_1)]$$

Elimination of $\uparrow$ results in:

$$\mathcal{A}d_1\mathcal{A}d_2\lambda p[[[\mathtt{farmer}(d_1) \wedge \mathtt{donkey}(d_2) \wedge \mathtt{own}(d_2)(d_1)] \rightarrow \mathtt{beat}(d_2)(d_1)] \wedge {}^{\vee}p]$$

Now we use the equivalence of $\mathcal{A}d\Phi$ with $\lambda p[\forall x\{x/d\}\Phi({}^{\wedge}\mathtt{true}) \wedge {}^{\vee}p]$, and reduce this, through a number of standard reductions to:

$$\lambda p[\forall x\forall y\{x/d_1\}\{y/d_2\}[[\mathtt{farmer}(d_1) \wedge \mathtt{donkey}(d_2) \wedge \mathtt{own}(d_2)(d_1)] \rightarrow \\ \mathtt{beat}(d_2)(d_1)] \wedge {}^{\vee}p]$$

Moving the two state-switchers inwards gives us as a final result:

$$\lambda p[\forall x\forall y[[\mathtt{farmer}(x) \wedge \mathtt{donkey}(y) \wedge \mathtt{own}(y)(x)] \rightarrow \mathtt{beat}(y)(x)] \wedge {}^{\vee}p]$$

Notice that the last conjunct ${}^{\vee}p$ was *not* in the scope of the two state-switchers. Consequently, these have disappeared, and so have the two discourse markers. Hence, the terms in this sentence will not be able to bear any anaphoric relations to pronouns in sentences to come. This seems to be borne out by the facts. (But, again, we refer to the next section for some discussion.)

This example concludes our exposition of the fragment. What we have shown in detail is that the two central phenomena of cross-sentential anaphora and donkey-sentences, can be treated in *DMG* an adequate and completely compositional way. This means that *DMG* is indeed a semantic theory that unifies important insights from *MG* and *DRT*.

5 Shortcomings and extensions

In this section we shall show that potentially *DMG* is more than just the sum of *MG* and *DRT*.

5.1 Shortcomings

We recall that negation in *DPL* is not really dynamic at all. In much the same way as negation in *DRT* turns a *DRS* into a condition, negation in *DPL* results in a formula which does not have any dynamic effects. And this property also characterizes the $\sim$-negation of *DMG*. For, as we have seen above, once a sentence is negated, all anaphoric links between terms occurring in the sentence and pronouns 'outside' are blocked.

Now, it seems that this is corroborated by the facts. Consider the following example:

(27) No man walks in the park. He whistles

It is not possible, indeed, to interpret the pronoun in the second sentence as being anaphorically linked to the term in the first one.

However, certain observations can be made which point in a different direction. Consider double negation. As we have seen above, double negation in *DIL*, as in *DPL* does not restore any dynamic effects a sentence may have had. Yet, (28) and (29) do seem to express the same thing, at least as far as anaphoric relations are concerned:

(28) It is not the case that John doesn't own a car. It is red and it is parked in front of his house

(29) John owns a car. It is red and it is parked in front of his house

A straightforward solution would be to take define negation in such a way that a sentence and its double negation become fully equivalent, i.e., that they do not just have the same truth conditions but also the same dynamic properties. That means trying to find another, truly dynamic version of negation. It should be noted that the introduction of such a notion would allow us to deal with the equivalence of (28) and (29), but also that it would leave unexplained the fact that such a dynamic negation evidently may not be used in all cases, witness the example of (27).

This is indeed the line we shall pursue. It may be worthwhile, though, to briefly point out another line of reasoning, which does not strike us as altogether untenable. It seems possible to argue that in (28) the pronoun *it* is not directly anaphorically related to the indefinite term *a car* in the preceding sentence, but only indirectly, mediated through a unique object, the existence of which can be inferred from the previous discourse and the context. Uniqueness is supposed to be all important here. For the example in question, uniqueness can be supposed to be guaranteed by the context, viz., by some kind of default-assumption that asociates a unique car with every (?) person. That uniqueness is necessary may be argued by pointing out other examples, such as the following:

(30) It is not the case that John doesn't own a book. It is lying on his desk.

Observe that an interpretation of the pronoun *it* in the second sentence as being anaphorically linked to the indefinite term *a book* in the first one, seems possible only if we assume that we are talking about a specific book, say the text-book used in a course, of which, again by default, we may assume that every participant has a unique copy. In a neutral context, however, an anaphoric interpretation does not seem to be possible, and this can be explained by pointing out that a default assumption that associates a unique book with each person is rather unrealistic.

Although an account along these lines is attractive, it remains to be seen whether it will stand up in the end. Clearly, it needs to be investigated further. However, we shall now turn to the other, more direct way of tackling these problems. It consists in trying to give dynamic definitions of the other logical constants. But, as we shall see, such an approach also is not without problems of its own.

The static character of negation in *DPL* is reflected in that of disjunction, implication and the universal quantifier, which are defined in the usual way in terms of it, and conjunction and the existential quantifier. That disjunction, implication and the universal quantifier are static is thus due to the static character of negation. In *DIL* we have followed the same approach, of course with the same effects.

In these cases, too, positive empirical evidence may be adduced. For example, consider the sequence:

(31) Every man walks in the park. He whistles

The pronoun *he* in the second sentence can not be interpreted as an anaphor which is bound by the universal term *every man* in the first sentence. And the static character of the universal quantifier nicely accounts for this. However, there are certain kinds of discourses in which anaphoric relations of the indicated kind are possible. Consider:

(32) Every player chooses a pawn. He puts it on square one

Now, of course, we can give a translation of this sequence in *DPL*, and hence also in *DIL*, which has the intended interpretation. However, such a translation can not be gotten in a compositional manner. The following *DPL*-formula expresses the right meaning:

(33) $\forall x[\mathtt{player}(x) \rightarrow [\exists y[\mathtt{pawn}(y) \wedge \mathtt{choose}(y)(x)] \wedge \mathtt{put\_on\_square\_one}(y)(x)]]$

The existential quantifier binds the rightmost occurrence of the variable y allright, but this translation is not the dynamic conjunction of the translations of the two sentences in the sequence, hence this analysis is not adequate. Of course, *DMG* doesn't fare better in this respect. Here, too, we need to re-analyze our translation of the first sentence while translating the second one, in order to obtain an adequate translation of the entire sequence. Such a process of re-analysis, however, constitutes a flagrant violation of compositionality.

Compositionality dictates that we translate sequence (32) by forming the dynamic conjunction of the translations of the two component sentences, which gives us:

(34) $\forall x[\mathtt{player}(x) \rightarrow [\exists y[\mathtt{pawn}(y) \wedge \mathtt{choose}(y)(x)]] \wedge \mathtt{put\_on\_square\_one}(y)(x)]$

And this leaves us with the task of finding a truly, i.e., externally dynamic interpretation of the universal quantifier which will make (34) equivalent with (33).

Again, we should remark that if we find such an interpretation, we can account for these facts, but that we cannot by the same token explain why example (31) is out.

With regard to the latter example, we notice that if we use a plural, instead of a singular pronoun in the second sentence, we get a discourse that sounds more acceptable:

(35) Every man walks in the park. They (all) whistle

An adequate translation would be:

(36) $\forall x[\mathtt{man}(x) \rightarrow [\mathtt{walk\_in\_the\_park}(x) \wedge \mathtt{whistle}(x)]]$

And this formula has the same structure as (33).

Of course, one may object that other factors have entered the discussion here, since we have introduced plural pronouns. And one may argue that the pronoun *they* is not linked to the term *every man* but to the set of men that is being introduced by the *CN* occurring in this term. For, like (35), (37), too, allows an anaphoric interpretation of the plural pronoun in the second sentence:

(37) No man walks in the park. They (all) stayed home

There seems to be no way to link *they* anaphorically to *no man*, but establishing such a connection with the *CN man* seems straightforward.

Notice, however, that a translation which has the same structure as (33) and (36) would serve as well:

(38) $\forall x[\mathtt{man}(x) \rightarrow [\neg\mathtt{walk\_in\_the\_park}(x) \wedge \mathtt{home}(x)]]$

In this case, too, the second sentence appears as a new conjunct in the consequent of the first sentence. This suggest that the anaphoric link in (37) is not between *no man* and *they*, but between this pronoun and the 'hidden', 'deep' occurrence of *every man* in the first sentence.

On the basis of these examples it is hard to decide whether it is the full term of just the *CN* that provides the antecedent. But examples like the following may be helpful here:

(39) Many Irishmen do not take a holiday. They rather stay at home

(40) Few Irishmen take a holiday. They rather stay at home

Unless we take refuge in some kind of generic interpretation of the second sentence in these examples, the pronoun *they* must be taken to refer, not to the set of Irishmen, which is associated with the CN, but with the many Irishmen that do not take a holiday. Again, this involves the assumption that on some 'deep' level *many* occurs in *few*.

Whatever the case may be, there exist also other remarkable cases, this time involving disjunction:

(41) Either there is no bathroom here, or it is in a funny place

This disjunction is equivalent with the conditional:

(42) If there is a bathroom here, it is in a funny place

Now, (42) is a donkey-type conditional, which can easily be accomodated in *DPL* and in *DRT*. Not so, however, the equivalent donkey-disjunction (41). It is the static character of negation, which is inherited by disjunction, which causes the problem.

5.2 Extending the dynamics

At the end of section 3, we noticed that the translation of *DPL* into *DIL* uses the dynamic potentials of the latter only to a limited degree. And the same holds, of course, for the *DMG*-fragment that we have discussed. *DIL* can accomodate, besides the dynamic $\mathcal{E}$, also a truly, i.e. externally dynamic $\mathcal{A}$, the definition of which runs as follows:

Definition 18 (Dynamic $\mathcal{A}$-quantifier) $\mathcal{A}d\Phi = \forall x[\{x/d\}(\Phi(p))]$

It is interesting to note that if we take the static closure $\uparrow\!\downarrow \mathcal{A}d\Phi$ of $\mathcal{A}d\Phi$ thus defined, we end up with the old definition 14:

$$\uparrow\!\downarrow \mathcal{A}d\Phi = \lambda p[\forall x[\{x/d\}(\Phi(^\wedge\mathbf{true}))] \wedge {}^\vee p]$$

So, the old, static version of the universal quantifier can be gotten out of the new one, just given, by means of a standard static closure. In what follows we will continue to use $\mathcal{A}$, which now stands for the newly defined quantifier, and we will refer to $\uparrow\!\downarrow \mathcal{A}$ as $\mathcal{A}_\downarrow$.

The connection between the dynamic existential quantifier $\mathcal{E}$, defined in definition 11, and the dynamic universal quantifier $\mathcal{A}$, just given, can be expressed in the usual way, using negation, if we give negation, too, a truly, i.e., externally dynamic interpretation. The following interpretation is the standard notion of complementation:

Definition 19 (Dynamic $\sim$-negation) $\sim\Phi = \lambda p \neg(\Phi(p))$

Here, too, it holds that the old definition of $\sim\Phi$ can be gotten out of the new one, by means of its standard static closure $\uparrow\!\downarrow\!\sim\Phi$. Also, we reserve $\sim$ for the newly defined notion, and write the old one as $\sim_\downarrow$.

As was remarked above, we are now able to state the interdefiniablility of $\mathcal{E}$ and $\mathcal{A}$ in the usual way:

$$\mathcal{E} \sim \Phi \Leftrightarrow \sim \mathcal{A}\Phi$$

Notice also that for $\sim$, unlike for $\sim_{\downarrow}$, double negation holds:

$$\sim\sim \Phi \Leftrightarrow \Phi$$

Also, we may define in terms of the externally dynamic notion of negation and the dynamic notion of ;-conjunction, dynamic notions of implication and disjunction:

Definition 20 (Dynamic $\Rightarrow$-implication) $\Phi \Rightarrow \Psi = \sim [\Phi\,; \sim \Psi]$
$(= \lambda p \neg \Phi(^\wedge \neg (\Psi(p)))$

Definition 21 (Dynamic either...or...-disjunction) **either** Φ **or** $\Psi =$ $\sim [\sim \Phi\,; \sim \Psi](= \lambda p \Phi(^\wedge \neg (\Psi(p)))$

The old version of implication can again be gotten by static closure of the new one. For disjunction, this does not hold (which is why we introduce a new phrase to denote this notion). Beside the notion just defined, there is another obvious candidate, which is union on the level of type $\langle\langle s, t\rangle, t\rangle$. It is the static closure of this notion which is equivalent to the old one:

Definition 22 (Dynamic or-disjunction) Φ **or** $\Psi = \lambda p[\Phi(p) \vee \Psi(p)]$

In a similar fashion, we may define an alternative notion of conjunction, which comes down to intersection:

Definition 23 (Dynamic and-conjunction) Φ **and** $\Psi = \lambda p[\Phi(p) \wedge \Psi(p)]$

The dynamic notions of conjunction and disjunction which 'underly' the dynamic universal and existential quantifiers, are these notions **or** and **and**.

The reader may now have lost oversight over the various notions introduced and their interrelations, so we sum up what is available systematically. We have defined the following logical operators: $\mathcal{E}$, $\mathcal{A}$, $\sim$, ; , $\Rightarrow$, **either**... **or**..., **or**, and **and**. A minimal set which allows us to derive all of these, by means of standard definitions, should contain: one of the quantifiers $\mathcal{E}$, or $\mathcal{A}$; $\sim$-negation; one of the connectives ; , $\Rightarrow$, or **either**... **or**...; and one of the connectives **or**, or *and*. Each of these logical constants can be turned into a static one by means of closure with $\uparrow\downarrow$. For $\mathcal{A}$, $\sim$, $\Rightarrow$, and **or**,what we get are the notions defined in section 3.

Now, we turn to the problematic examples which we have discussed in section 5.1, and indicate briefly how they can be handled using this extended dynamic semantics. We do not work everything out in detail, we just mention some logical facts which will make our claim plausible.

First of all, as was already observed above, the law of double negation holds for $\sim$-negation, and this makes it possible to treat cases like (28).

Also, we notice that the following holds:

$$[\Phi \Rightarrow \Psi] \,;\, \Upsilon \Leftrightarrow \Phi \Rightarrow [\Psi \,;\, \Upsilon]$$

This means that a sequence consisting of a conditional followed by another sentence, can be interpreted in such a way that the sentence following the conditional, is interpreted as a conjunct of the consequent of the conditional. Notice that the old interpretation of such a sequence is also still available, since we may also use the static closure $\Rightarrow_{\downarrow}$ of $\Rightarrow$ in the translation of a conditional sentence.

Further, the following equivalence holds:

$$\mathcal{A}d\Phi \,;\, \Psi \Leftrightarrow \mathcal{A}d[\Phi \,;\, \Psi]$$

In conjunction with the previous fact, this implies that we have:

$$\mathcal{A}d[\Phi \Rightarrow \Psi] \,;\, \Upsilon \Leftrightarrow \mathcal{A}d[\Phi \Rightarrow [\Psi \,;\, \Upsilon]$$

And on this fact an account can be based of examples like (32) and (35).

Also, the following equivalences hold:

$$\textbf{either } \Phi \textbf{ or } \Psi \Leftrightarrow \sim \Phi \Rightarrow \Psi$$

$$\textbf{either } \sim\mathcal{E}d\Phi \textbf{ or } \Psi \Leftrightarrow \mathcal{E}d\Phi \Rightarrow \Psi \Leftrightarrow \mathcal{A}d[\Phi \Rightarrow \Psi]$$

This means that the extended dynamic semantics is also able to deal with donkey-disjunctions like (41). It should be noted, by the way, that this notion of disjunction is like for implication in this respect, that sentences which follow it, are in fact conjoined with its second argument:

$$\textbf{either } \Phi \textbf{ or } \Psi \,;\, \Upsilon \Leftrightarrow \textbf{either } \Phi \textbf{ or } [\Psi \,;\, \Upsilon]$$

This is needed for a treatment of a case like:

(43) Either there is no bathroom here, or it is in a funny place. In any case, it is not on the ground floor

So, the **either**... **or**...-disjunction enables us to account for anaphoric relations between terms in the first disjunct and pronouns in the second, and between terms in the second disjunct and pronouns in subsequent sentences. The **or**-disjunction is different in this respect. This notion lets quantifiers in either disjunct bind variables in subsequent formulae, but it does not allow for any binding relations between the disjuncts themselves. In the **or**-disjunction, sentences which follow are in fact treated as conjoined with each of the disjuncts:

$$[\Phi \textbf{ or } \Psi] \,;\, \Upsilon \Leftrightarrow [\Phi \,;\, \Upsilon] \textbf{or } [\Psi \,;\, \Upsilon]$$

The following sequence exhibits this pattern:

(44) A professor or an assistant professor will attend the meeting if the university board. He or she will report to faculty

So, there seems to be a role to play for **or**-disjunction as well. The notion of **and**-conjunction follows suit:

$$[\Phi \textbf{ and } \Psi]\,;\Upsilon \Leftrightarrow [\Phi\,;\Upsilon] \textbf{ and } [\Psi\,;\Upsilon]$$

Disregarding the fact that plurals should be treated differently, this enables us to give a straightforward account of examples like (45), which are parallel to example (35), which involved universal quantification:

(45) John and Harry walk in the park. They whistle

Notice that we get the required interpretation if we use in the translations of *John* and *Harry* the same discourse marker, and we also use this discourse marker in the translation of the pronoun in the second sentence. So, we must assume the following indexing:

(46) John_1 and Harry_1 walk in the park. They_1 whistle

The same holds for example (44), which involves disjunction. In order to get the right interpretation, we need to attach the same index to each of *a professor*, *an assistant professor*, and *he or she.*

In our discussion of the various examples in section 5.2, we have already indicated that the extended dynamic semantics, though able to account for them, is not without problems.

We remark that a static treatment of the universal quantifier and implication is not unmotivated. There certainly are empirical facts which support it. On the other hand, as we have observed above, there are also facts which point in a different direction. This poses a problem, of course, but one which need not make us feel too uncomfortable. Evidently, the extended dynamic semantics developed above gives us the means to deal with both kinds of cases. For, we are not committed to the use of the dynamic constants is all cases, there is always the possibility to use the static closure of a constant. So the problem that remains is to find out which textual and contextual factors determine which interpretation of these constants we must choose in a particular case. But clearly this is a well-defined, by and large empirical question which further investigation may very well answer.

Negation, however, presents us with more difficulties. We observe that the following holds:

$$\sim\Phi\,;\Psi \Leftrightarrow\sim[\Phi\,;\Psi]$$

This equivalence predicts that if we translate natural language negation as $\sim$-negation, an occurrence of negation in a natural language sentence extends to all sentences that follow it. Since there are no facts to support this, it seems that we are always forced to use the static closure of a negation. But why?

The phenomenon of negation taking wide scope over following sentences in a sequence, also occurs with embedded negations:

$$\mathcal{A}d[\Phi \Rightarrow \sim \Psi]\,;\Upsilon \Leftrightarrow \mathcal{A}d[\Phi \Rightarrow [\sim \Psi\,;\Upsilon]] \Leftrightarrow \mathcal{A}d[\Phi \Rightarrow \sim [\Psi\,;\Upsilon]]$$

This predicts a wrong interpretation for example 37:

(37) No man walks in the park. They are at home

We only get the right interpretation for this sequence if we take the negation in the consequent statically. For evidently static negation does not validate the equivalences above. But, here, too, the question immediately arise: why doe we *have* to interpret negation statically?

Notice that this problem with negation differs from the one observed above with the universal quantifier and implication. In the latter case, there are observations which support a static interpretation, and there are facts which call for the dynamic one. As for negation, however, all natural language facts seem to point toward the static interpretation, yet we need the dynamic definition of negation to get the dynamic meanings of other logical constants in a regular way.

Two ways are open to us. First of all, we may take the observations just made as strong suggestions that negation as complementation is simply too powerful a notion of dynamic negation. Something in between the static notion and this one, which is 'too dynamic', seems to be called for.

Another way of approaching the matter is to try to find an explanation of why we have to choose static negation. One such explanation could start out from the following observation. The formula $\sim \Phi$ has a stronger propositional content than $\sim \Phi\,;\Psi$, since the latter is the same as $\sim [\Phi\,;\Psi]$. If we assume that in the course of processing a text, the information it conveys is updated after every sentence, then a problem arises if we interpret negation dynamically. For, if we first process $\sim \Phi$ and next Ψ, the information conveyed by the sequence $\sim \Phi\,;\Psi$ turns out to be weaker than the information we already updated with after processing $\sim \Phi$. So, this implies that we need to correct information with which we have already updated, since it turns out to be too strong.

Of course, we do not want to deny that retraction of information occurs. A simple example is provided by the sequence *John will attend. Unless he is ill*, where the *untill* with which the second sentence starts, explicitly indicates that retraction of information takes place. However, in the case at hand such a mechanism do not seem to be involved. For we can hardly interpret negation in natural language as signalling that if the sentence in which it occurs, is followed by another one, the information which the former conves, will always be retracted.

Armed with this observation, we may argue for the existence of a 'metarule' which governs the choice of a dynamic or a static interpretation of logical constants. The rule would state that a static interpretation is obligatory if a dynamic interpretation would imply that on continuation of the text, weaker information would be conveyed than has been conveyed thus far.

A more subtle version of this rule, which would take into account that there are grades of dynamics, would force us to choose for static negation in the consequent of an implication: it would force the interpretation $\Phi \Rightarrow\sim_{\downarrow}\Psi$ instead of $\Phi \Rightarrow\sim\Psi$, since the propositional content of $\Phi \Rightarrow [\sim_{\downarrow}\Psi\,;\Upsilon]$ is at least as strong as that of $\Phi \Rightarrow\sim_{\downarrow}\Psi$, whereas $\Phi \Rightarrow [\sim\Psi\,;\Upsilon]$ is at most as strong as $\Phi \Rightarrow\sim\Psi$.

Although an appeal to such meta-rules of interpretation may offer an empirically adequate account, we feel that having a notion of negation which makes such an appeal superfluous, and which accounts in a straightforward way for what the other approach only explains indirectly, is to be preferred.

Clearly, the matters discussed in this section stand in need of further investigation. But they also make clear that *DMG*, and the version of *DIL* it uses, provide us with interesting possibilities of giving a compositional account of aspects of dynamic interpretation and anaphoric relations, which are outside the scope of orthodox *MG* and of *DRT*.

5.3 Incorporating intensionality

Another direction in which the *DMG*-system needs to be extended has to do with the incorporation of the ordinary notion of intensionality. Also, we might want to extend the set of discourse markers to include also discourse markers of other types than e. Both extensions can be defined, but they give rise to some technical complications. In this section we shall confine ourselves to merely argueing very briefly that both kinds of extensions are needed without going into the technical details. The actual implementation of what follows we defer to another paper.

The phenomenon of so-called 'modal subordination' (see Roberts [1987,1989]) is very well suited for this purpose. Consider the following example:

(47) A tiger might come in. It would eat you first

The meaning of this sentence can be paraphrased as follows:

(48) Possibly(a tiger comes in) and necessarily(if a tiger comes in, it eats you first)

One way to get a compositional interpretation of (47), is to assume that its first sentence introduces a semantic object that corresponds to the meaning of the sentence *A tiger comes in*, and that the second sentence contains an anaphor, triggered by the modal *would*, which picks up this object.

If that is correct, the meaning of *would* Ψ would be:

$$\lambda p[\Box[{}^{\vee}D_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle} \Rightarrow \Psi] \wedge {}^{\vee}p]$$

Here, D is a discourse marker of the same type as that of the meaning of a sentence.

The fact that the operator *possibly* introduces a referent, can be accounted for by defining a dynamic counterpart of λ-abstraction, as follows (d and x are of the same, arbitrary type):

$$\ell d\beta = \lambda x\{x/d\}\beta$$

Application of a dynamic λ-term to an argument means that in the process of conversion the discourse marker gets bound to the argument. And this makes it possible to 'pass on' the referent of the argument to sentences to come.

Now we translate *possibly* as:

$$\ell D\lambda p[\Diamond \downarrow {}^{\vee}D \wedge {}^{\vee}p]$$

and *Possibly,* Φ becomes:

$$\ell D\lambda p[\Diamond \downarrow {}^{\vee}D \wedge {}^{\vee}p]({}^{\wedge}\Phi)$$

The translation of the entire sequence is simply the dynamic conjunction of the two schematic translations we have obtained sofar. If we make some plausible assumptions about the semantics of this extension of *DIL*, it will turn out that such a conjunction, provided we choose identically indexed discourse markers, is an adequate representation of (47).

To see that this is so, notice that the above translation of *Possibly,* Φ reduces to:

$$\{{}^{\wedge}\Phi/D\}\lambda p[\Diamond \downarrow {}^{\vee}D \wedge {}^{\vee}p]$$

which in turn is equivalent to:

$$\lambda p[\{{}^{\wedge}\Phi/D\}\Diamond \downarrow {}^{\vee}D \wedge \{{}^{\wedge}\Phi/D\}{}^{\vee}p]$$

From this expression we want to be able to derive:

$$\lambda p[\Diamond \downarrow \Phi \wedge \{{}^{\wedge}\Phi/D\}{}^{\vee}p]$$

For this transition to be valid, it should hold that $\{{}^{\wedge}\Phi/D\}\Diamond \downarrow {}^{\vee}D$ is equivalent to $\Diamond \downarrow \Phi$. In *DIL* as we have defined it sofar, however, this is not guaranteed. For if we interpret modal operators as unrestricted quantifiers ranging over states, $\{\alpha/d\}\Diamond\phi$ is equivalent to $\Diamond\phi$, since $\Diamond\phi$ is an intensionally closed expression.

But the interpretation we want for $\Diamond$, is not that of an existential quantifier over states, of course, but that of a quantifier ranging over possible worlds. This means that the semantics of *DIL* really needs to be extended to incorporate possible worlds, so that interpretation can be relativized to possible worlds as well. Another way of putting this is to say is that we need a richer notion of a state, one which can play the role, both of a possible world in the classical sense, and that of a state in the dynamic sense. The latter option is elegant and parsimonious, and it is with an eye to it that we introduced states the way we did in the first place. It calls for some intricate definitions, however, so for simplicity's sake we now suppose that we introduce possible worlds as a separate parameter, and interpret with respect to pairs of possible worlds and states. In such a set-up, a state switcher should take us from a pair $\langle w, s\rangle$ to a pair $\langle w, s'\rangle$. And the modal operator $\Diamond$ should go from $\langle w, s\rangle$ to $\langle w', s\rangle$. Going about this way means that $\Diamond\phi$ is not an (entirely) intensionally closed expression any longer, which means that $\{\alpha/d\}\Diamond\phi$ would be equivalent with $\Diamond\{\alpha/d\}\phi$, and that the reduction of $\lambda p[\{^\wedge\Phi/D\}\Diamond \downarrow {}^\vee D \wedge \{^\wedge\Phi/D\}{}^\vee p]$ to $\lambda p[\Diamond \downarrow \Phi \wedge \{^\wedge\Phi/D\}{}^\vee p]$ can be carried out.

Conjunction of the latter with $\lambda p[\Box[{}^\vee D_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle} \Rightarrow \Psi] \wedge {}^\vee p]$, and some obvious reductions, gives:

$$\lambda p[\Diamond \downarrow \Phi \wedge \{^\wedge\Phi/D\}[\Box \downarrow [{}^\vee D \Rightarrow \Psi] \wedge {}^\vee p]]$$

And, interpreting $\Box$ along the same lines as $\Diamond$, this is equivalent with:

$$\lambda p[\Diamond \downarrow \Phi \wedge \Box \downarrow [\Phi \Rightarrow \Psi] \wedge {}^\vee p]]$$

which indeed gives correct interpretation of the sequence (47), analyzed along the lines of (48).

Although working out just one example in a sketchy way constitutes no real proof, of course, it still seems reasonable to conclude that an extension of *DIL* with discourse markers of other types than just *e* and enriched with possible worlds in some form or other, will allow us to deal with notoriously problematic cases like (47) in a fully compositional way. And that is another indication that *DIL*, and *DMG*, are more than just the sum of orthodox *MG* and *DRT*.

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