Construction by Description in Discourse Representation

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Abstract

This paper uses classical logic for a simultaneous description of the syntax and semantics of a fragment of English and it is argued that such an approach to natural language allows procedural aspects of linguistic theory to get a purely declarative formulation. In particular, it will be shown how certain construction rules in Discourse Representation Theory, such as the rule that indefinites create new discourse referents and definites pick up an existing referent, can be formulated declaratively if logic is used as a metalanguage for English. In this case the declarative aspects of a rule are highlighted when we focus on the model theory of the description language while a procedural perspective is obtained when its proof theory is concentrated on. Themes of interest are Discourse Representation Theory, resolution of anaphora, resolution of presuppositions, and underspecification.

1 Introduction

In this paper we want to argue that important parts of the DRT construction algorithm (Kamp, 1981; Kamp and Reyle, 1993), which is highly procedural, can in fact be given a purely declarative, logical formulation. In particular, procedural rules such as 'introduce a new discourse referent into the universe of the DRS,' or 'identify discourse referent u with an accessible referent' have natural declarative formulations within the 'Logical Description Grammars' of (Muskens, 2001), an approach that uses logic for simultaneous description of syntax and semantics. Presuppositions can also be accounted for within such a logical set-up and we implement a rudimentary presupposition mechanism, which allows the 'binding' of presuppositions (van der Sandt, 1989; van der Sandt, 1992) and a form of global (but no intermediate or local) accommodation. Earlier logical approaches to discourse semantics typically could not account for procedural aspects of the construction algorithm. For example, in Dynamic Predicate Logic (Groenendijk and Stokhof,

1991) and related formalisms (including (Muskens, 1996)) it is necessary to assume that syntactic input to the semantic component comes pre-indexed, with co-indexation between antecedents and dependent elements representing anaphoric linking.

The question of declarativity versus procedurality is related to the question which roles the main branches of logic can play in linguistic theory. There is a natural tendency to associate linguistic semantics with model theory and linguistic syntax with proof theory. Montague's work is a prime example of the first association, while the connection between natural language syntax and proof theory is apparent from the widespread use of context free grammars in syntactic theory, from the derivations in early transformational approaches, and, even more explicitly, from Lambek's categorical calculi (Lambek, 1958; Moortgat, 1997). However, it is also fruitful to make cross-combinations. This becomes apparent, for example, when we turn to the 'model-theoretic syntax' of (Blackburn, 1993; Blackburn, Gardent and Meyer-Viol, 1993; Rogers, 1996; Blackburn and Meyer-Viol, 1996) and others. Model-theoretic syntax studies the model theory of syntactic structures such as trees and feature structures. In a similar spirit (Kurtonina, 1995) defines various forms of Kripke semantics for the language of Lambek categorial grammar, and works out the model theory and correspondence theory of such systems, thus also giving a model-theoretic twist to an enterprise whose main focus is on natural language syntax. Conversely, proof-theoretic approaches to natural language semantics also abound. Some accounts are purely proof-theoretic, such as the approach of (Ranta, 1994), which is based on Martin-Löf's intuitionistic type theory. Other applications of proof theory to linguistic semantics seek to supplement existing model-theoretic accounts. As an example of the latter the type-shifting proposals (Partee and Rooth, 1983) and (Hendriks, 1988) can be mentioned. These proposals have a definitely proof-theoretic flavour and are related to the (undirected) Lambek Calculus.

Once we are confronted with such obviously fruitful crossings of the border, any idea we might have had of an exclusive alignment of natural language syntax with proof theory and a similar alignment of model theory with linguistic semantics is thrown into doubt. There are procedural aspects of meaning and declarative aspects of form. The most natural way to deal with the former is by using proofs and the latter are best dealt with using models. In fact, what we want to emphasize here is the duality of such declarative and procedural aspects. We take the perspective of (Muskens, 2001) and will essentially use logic (classical type logic) as a metalanguage for the description of natural language (here: a fragment of English). This introduces an orthogonality between, on the one hand, the proof theory and model theory of the describing logic, and, on the other, the syntax and semantics of the fragment described. Our aim is to develop a combined logic of form and meaning; when focussing on its proof theory we get its procedural aspects, if its model theory is focused on, its declarative aspects are highlighted. This, of course, is the normal duality of proof and structure in logic.

A Logical Description Grammar (LDG) will be a logical theory $\mathcal{G}$, set up with the intention that whenever we have a logical sentence $\mathcal{I}_T$ stating some simple properties of a text T, the combined theory $\mathcal{G}+\mathcal{I}_T$ describes the syntax and semantics of T. We will assume here that the models of $\mathcal{G}+\mathcal{I}_T$ contain trees decorated with semantic values; a function (or, in fact, family of functions) σ will associate values with tree nodes. The logic talks about natural language semantics *and* natural language syntax. In its models we get semantically annotated trees (this in addition to the usual objects and relations) and in its proof theory we can reason about these.

As was argued extensively in (Muskens, 2001), *underspecification* naturally falls out of this view. Since a description $\mathcal{G}+\mathcal{I}_T$ can have more than one model, the syntax and semantics of T may remain underspecified. In particular, it can be shown that, with the right set-up, certain stages in the reasoning process that takes $\mathcal{G}+\mathcal{I}_T$ as its point of departure are closely analogous to the Underspecified Discourse Representation Structures of (Reyle, 1993). There is no need to postulate a separate structural level of the latter, we get UDRSs for free if the hearer's simultaneous reasoning about the syntax and semantics of an input expression is modeled. We conclude that the description view on Discourse Representation explains why UDRSs emerge. No stipulation of such structures is necessary.

The organization of this paper is as follows. In the next section we start with defining and explaining the 'Logical Description Grammars' of (Muskens, 2001); it will be discussed how input descriptions $\mathcal{I}_T$ are obtained and an axiomatization of some syntactic and semantic concepts is given. Section 3 interrupts the definition of LDGs and consists of a treatment of Discourse Representation Structures within type logic that, although it is related to that of (Muskens, 1996), offers a notion of DRSs more fine-grained than the one developed there. Section 4 continues setting up the LDG theory, providing a lexicon and a worked example of a hearer's reasoning with the theory. Section 5 explains how some of the details of the DRT machinery obtain a declarative treatment. It is shown how presuppositions can be bound or accommodated, how discourse referents are resolved, how discourse referents connected with proper names land up in the main DRS

Variable	Type	Variable	Type
x, y, z	e	i, j	s
k	ν	c	$st \times \pi t$
v	π		

Table 1: Variables used in this paper will have types as indicated.

and how underspecified DRSs can be identified with certain descriptions. The paper ends with a conclusion and an appendix gives a small grammar fragment.

2 LDG: Input Descriptions and Axioms

In this section and in section 4 we will give an overview of the Logical Description Grammars (LDGs) developed in (Muskens, 1995; Muskens, 1999; Muskens, 2001).[1] We describe a system that is closely related to the one given in the last of these papers, but make some choices of design that slightly deviate from those that were made there. For more details and motivation the reader is nevertheless referred to (Muskens, 2001).

We work in classical type logic with ground types e (entities), ν (tree nodes), l (tree labels), π (registers), and s (states). Since we will have occasion to use many variables and constants in these types and in complex types composed out of these primitive ones, it will be convenient to have typographical conventions of the form: 'whenever v is used, it will be a variable of type π.' Table 1 gives an overview of such typographical conventions used for variables and Table 2 gives a similar overview for constants.

It will be expedient to have *pairing* and *projection* in the logic. We will assume that whenever A is a term of type α and B a term of type β, $\langle A, B\rangle$ is a term of type $\alpha \times \beta$. Also, whenever A is a term of type $\alpha \times \beta$, $\mathsf{fst}(A)$ will be a term of type α (denoting the first element of the denotation of A) and $\mathsf{snd}(A)$ a term of type β (denoting the second element). These terms will have the obvious semantics.

[1] As far as we are aware (Muskens, 1995), presented at the Prague session of the Prague-Teinach Workshop in February 1995, was the first paper to extend the syntactic Description Theory of (Marcus, Hindle and Fleck, 1983; Vijay-Shankar, 1992; Rambow, Vijay-Shanker and Weir, 1995) to semantics. The paper also gave a solution to the question (crucial, but often overlooked) of how semantic *binding* should be dealt with in underspecified semantics.

Constants	Type	Constants	Type
r, 0, 1, ...	ν	$\prec$, $\lhd^+$	$(\nu \times \nu)t$
$>$, PEDRO, HAS, ...	νt	V	$(\pi \times s)t$
lex	νt	E	$(e \times e)t$
pn, *cn*, *tv*, ...	νt	W	et
dp, *vp*, *d*, ...	l	*Pedro*, *sleeps*, ...	$(e \times e)t$
ℓ	νl	*loves*, *has*, ...	$(e \times e \times e)t$
α^+, α^-	$\nu\nu$	B	τ
u, o, w	$\nu\pi$	Γ	$\nu\tau$
σ^β	$\nu\beta$		

Table 2: Some constants used in this paper and their associated types. Here β varies over types and τ is an abbreviation of $\pi t \times (st \times \pi t)t$ (the type of DRSs).

2.1 Input Descriptions

Descriptions in our approach will consist of two parts. The first part will be a grammar $\mathcal{G}$ consisting of certain axioms and a lexicon. This part is not supposed to vary and models a hearer's grammatical knowledge. The second part will be the description $\mathcal{I}_T$ of an observed text T. This input description can be obtained in the following way. Suppose that the text that is to be described starts with (1).

(1) Pedro has a mule

Discourse participants reason about the trees that are possibly connected with a given discourse. The input description only concerns the lexical elements of such trees. We shall assume that lexical elements can be words or discourse relations,[2] including a special unary discourse relation signifying that the discourse has started. Upon hearing (1) a discourse participant can draw certain conclusions. First, she may conclude that the discourse has

[2]We are simplifying matters considerably here. In a fully fledged discourse model, we would distinguish between discourse connectives, such as 'and', 'but', 'because' and 'either ... or', and the discourse relations which they denote or partly specify, such as *sequencing*, *contrast*, *cause*, and *disjunction*. The lexicon of the grammar would contain discourse connectives rather than discourse relations. In order to account for texts in which a discourse relation is not explicitly specified through a discourse connective, the grammar can be assumed to allow for 'implicit' or nonlexical anchors of discourse relations, as in e.g. (van Leusen, 2003). Alternatively, punctuation may be taken to function as lexical anchor for implicit discourse relations, as in e.g. (Webber, Knott and Joshi, 2001).

indeed started and that the start element, which she may give a name, say 0, must therefore be present. She draws the conclusion in (2a). Here '$>$' stands for the predicate 'is the start element' and $\prec$ denotes precedence in trees. Secondly, there is a lexical element 'Pedro', which came just after the start. So the tree must have a lexical node labeled 'Pedro'. Our hearer gives it an arbitrary node name, say 1, and concludes (2b), in which $\prec_1$ is immediate precedence (i.e. $n \prec_1 n'$ abbreviates $n \prec n' \wedge \neg\exists k[n \prec k \wedge k \prec n']$).

(2) a. $>(0) \wedge \neg\exists k\, k \prec 0$

b. $\textsc{pedro}(1) \wedge 0 \prec_1 1$

c. $\textsc{has}(2) \wedge 1 \prec_1 2$

d. $\textsc{a}(3) \wedge 2 \prec_1 3$

e. $\textsc{mule}(4) \wedge 3 \prec_1 4$

In a similar fashion the hearer adds (2c)–(2e) to her stock of beliefs, after which she may perhaps hypothesize that the end of the discourse has been reached already, hypothetically adding (3). (Addition of such an 'end' statement will make interpretation of the text heard sofar possible, as we will see below.)

(3) $\neg\exists k\, 4 \prec k$

Of course, the discourse may in fact continue, say with (4).

(4) He feeds it

Assuming that our hearer recognizes that this is a new sentence and not a continuation of previous material, she will now stipulate the existence of a discourse relation. In this paper the only binary discourse relation that will be introduced is simple *sequencing*, but see (Duchier and Gardent, 2001; Webber et al., 2001; van Leusen, 2003) for more extended treatments of discourse relations in grammar systems closely related to the present one. The set of descriptions is now continued as follows.

(5) a. $\textsc{seq}(5) \wedge 4 \prec_1 5$

b. $\textsc{he}(6) \wedge 5 \prec_1 6$

c. $\textsc{feeds}(7) \wedge 6 \prec_1 7$

d. $\textsc{it}(8) \wedge 7 \prec_1 8$

The hypothesis in (3) can no longer be upheld, of course. But at this point it can be replaced by the hypothesis in (6).

(6) $\neg \exists k\, 8 \prec k$

The input description associated with a given text T will consist of the conjunction of all descriptions collected in this way, including the hypothesis that no material follows T. For example, the input description for *'Pedro has a mule. He feeds it.'* will be the conjunction of the logical sentences in (2), (5), and (6).

2.2 Axioms

The knowledge modelled by a grammar $\mathcal{G}$ can be separated into lexical knowledge (to be discussed in section 4) and general grammatical knowledge. We will assume here that the latter contains at least the following.

- Knowledge about linguistic trees;
- Knowledge about the 'anchoring' of tree nodes; anchoring provides a mechanism that makes lexical items combine;
- Knowledge about semantics.

For each of these forms of knowledge we provide axioms.

Axioms for Trees Axioms $\mathcal{A}1$–$\mathcal{A}5$ below (see also (Cornell, 1994; Backofen, Rogers and Vijay-Shankar, 1995)) make the binary relations $\lhd^+$ and $\prec$ behave like proper dominance and precedence in linguistic trees, with r as root ($k_1 \lhd^* k_2$ will be an abbreviation of $k_1 \lhd^+ k_2 \vee k_1 = k_2$). Nodes in such trees may be *labeled* with a labeling function ℓ. For example, we may want to say that node n is labeled dp by stating $\ell(n) = dp$. Axiom 6 rules out that label names such as dp and vp corefer. Instantiations of this axiom scheme will be sentences like $dp \neq vp$ and $ap \neq pp$, etc. Axiom $\mathcal{A}7$, lastly, requires lexical nodes to be leaf nodes. i.e. nodes that do not properly dominate any other node.

$\mathcal{A}1$ $\forall k\; r \lhd^* k$

$\mathcal{A}2$ $\lhd^+$ and $\prec$ are strict partial orders

$\mathcal{A}3$ $\forall k_1 k_2\, [k_1 \prec k_2 \vee k_2 \prec k_1 \vee k_1 \lhd^+ k_2 \vee k_2 \lhd^+ k_1 \vee k_1 = k_2]$

$\mathcal{A}4$ $\forall k_1 k_2 k_3\, [[k_1 \lhd^+ k_2 \wedge k_1 \prec k_3] \rightarrow k_2 \prec k_3]$

$\mathcal{A}5$ $\forall k_1 k_2 k_3\, [[k_1 \lhd^+ k_2 \wedge k_3 \prec k_1] \rightarrow k_3 \prec k_2]$

$\mathcal{A}6$ $c_1 \neq c_2$, if c_1 and c_2 are distinct label names

$\mathcal{A}7$ $\forall k_1 k_2\, [\mathit{lex}(k_1) \rightarrow \neg k_1 \lhd^+ k_2]$

The Anchoring Axiom The next axiom plays an essential role in the mechanism that combines lexical elements into larger units. The idea is that every tree node must be licensed 'from below' by a lexical element. Each node should also be licensed 'from above' by a lexical element. A node that is licensed 'from below' can be thought of as 'produced' by the lexical element, while nodes licensed 'from above' need to be 'consumed'. Typical examples of the latter are argument nodes. The idea essentially stems from Categorial Grammar, where arguments have a negative occurrence in an element's category and must match with positive occurrences of other categories. (For the producer / consumer metaphor, see Linear Logic (Girard, 1987), which is closely related to Categorial Grammar.)

If a node k' licenses a node k 'from below' we write $\alpha^+(k) = k'$; that k' licences k 'from above' is expressed by $\alpha^-(k) = k'$. $\alpha^+(k)$ is called the *positive anchor* of k; $\alpha^-(k)$ its *negative anchor*. The axiom states that all positive and negative anchors must be lexical.

$\mathcal{A}8$ $\forall k\, [\mathit{lex}(\alpha^+(k)) \wedge \mathit{lex}(\alpha^-(k))]$

The effect of this axiom will become apparent in section 4.

Semantic Axioms Three kinds of axioms will be needed for our semantics. There will be axioms for the mechanism of *binding*, axioms for *worlds*, and an axiom that plays a role in assigning *local contexts* to tree nodes. The notion of a local context derives from (Karttunen, 1974). Here it will be a Discourse Representation Structure. The local context of a node should well be distinguished from the semantics of that node.

$\mathcal{A}9$, $\mathcal{A}10$ and $\mathcal{A}11$ implement a version of the axioms for binding in (Muskens, 1991; Muskens, 1996; Muskens, 2001). We write $V(v,i)$ for 'the value of register v in state i' and if $\delta_1, \ldots, \delta_n$ are terms of type π, we write $i[\delta_1 \ldots \delta_n]j$ for $\forall v\, [(v \neq \delta_1 \wedge \ldots \wedge v \neq \delta_n) \rightarrow V(v', i) = V(v', j)]$ (i.e. 'i and j

differ at most in $\delta_1, \ldots, \delta_n$').[3] The first axiom says that, in each state, each register can be updated selectively, i.e. its value can be set to any x while the values of other registers can remain unchanged. This axiom makes states and registers essentially behave as assignments and variables in predicate logic. (The last remark is fleshed out in (Muskens, 2001).)

$\mathcal{A}9$ $\forall i \forall v \forall x\, \exists j\, [i[v]j \wedge V(v,j) = x]$

$\mathcal{A}10$ $\forall k k'\, \rho(k) \neq \rho'(k')$, $\quad$ if $\rho, \rho' \in \{u, o, w\}$ are distinct

$\mathcal{A}11$ $\forall k_1 k_2\, [\rho(k_1) = \rho(k_2) \rightarrow k_1 = k_2]$, $\quad$ if $\rho \in \{u, o, w\}$

We let u, o, and w be functions from nodes to registers. Axiom $\mathcal{A}10$ ensures that the images of these functions are disjoint, while $\mathcal{A}11$ requires each to be injective. In this way we make sure that fresh registers come with each node. This in turn will make it easy to let indefinites be associated with new discourse referents. The values of u will typically be associated with *new* referents, while the values of o are referents that belong to the 'background' of the discourse. The values of w are registers that can store *worlds*. In this paper we will only use $w(r)$, the w value of the root.[4] A notational convention that we find useful is to write arguments of u, o, and w as subscripts, e.g. u_3 instead of $u(3)$, w_r instead of $w(r)$ etc.

While states correspond to assignments and registers correspond to variables, the notion of a possible world of course corresponds to the technical notion of a *model*. We will make some use of possible worlds here, but will not change the logic—a step often associated with their introduction. Instead we will consider possible worlds to be objects of type e, i.e. we will simply take them to be (abstract) entities. The predicate letter W will be used for the predicate 'is a world' and E will be an existence predicate, so that $E(x, y)$ stands for 'object x exists in world y' and $\lambda x\ E(x, y)$ is y's domain. In our semantics we shall make use of a (finite) set $\mathcal{L}$ of predicate letters ({*Pedro, has, mule, feeds,* ...}) all of whose arguments are of type e and whose last arguments are to be interpreted as worlds. E.g. $has(x, y, z)$ should be read as 'x has y in world z'. The following axiom scheme requires that last arguments indeed are worlds and, somewhat rigorously, demands

[3] v must be chosen as the first variable in some fixed ordering which is not free in $\delta_1, \ldots, \delta_n$.

[4] But in extensions containing modal operators it would be natural to use other values of w as well.

that all other arguments of an $\mathcal{L}$ relation denote objects in this world's domain.[5]

$\mathcal{A}12$ $\forall x_1 \ldots x_n\, y\, [R(x_1, \ldots, x_n, y) \rightarrow (W(y) \wedge E(x_1, y) \wedge \ldots \wedge E(x_n, y))]$, for each $R \in \mathcal{L}$

While $\mathcal{A}9$ in a sense required there to be enough states present for the binding mechanism to work, the next axiom scheme puts similar requirements on the sets of individuals and worlds. Since we want to use our logic as a metalanguage for English, it would be nice to have a notion of entailment around at the level of the logic (of course we already have a notion of entailment at the 'metametalevel'—the present level of description). Entailment is obtained by quantification over models, or, since the latter's role is played by worlds at the logic's level, by quantification over worlds: an entailment holds if the conclusion is true in all worlds in which the premises are true. But this requires all worlds to be available; otherwise counterexamples to an entailment might be missed. A requirement that really ensures all or all countable worlds to be present in all models does not seem to be formulable with finitary and first order means and since we do not wish to use too heavy artillery we therefore make do with a requirement that at least all *finite* worlds be present; for our natural language application this seems a very reasonable approximation.[6]

This will be ensured in the following way. Define an $\mathcal{L}$-*atom* to be an atomic formula $R(t_1, \ldots, t_n)$ with $R \in \mathcal{L}$. An $\mathcal{L}$-*literal* is an $\mathcal{L}$-atom or the negation of an $\mathcal{L}$-atom. The following axiom scheme may be instantiated by any number of variables x_1, …, x_n and any conjunction of $\mathcal{L}$-literals $\varphi(x_1, \ldots, x_n, y)$ satisfying the conditions stated.

$\mathcal{A}13$ $\exists x_1 \ldots x_n y\, [W(y) \wedge x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge \ldots \wedge x_{n-1} \neq x_n \wedge \forall x\, [E(x, y) \leftrightarrow (x = x_1 \vee \ldots \vee x = x_n)] \wedge \varphi(x_1, \ldots, x_n, y)]$, where $\varphi(x_1, \ldots, x_n, y)$ is a conjunction of $\mathcal{L}$-literals with at most x_1, …, x_n and y free that does not contain both an $\mathcal{L}$-atom and its negation.

[5] This will serve present purposes. That the requirement needs to be fine-tuned can be seen from such classic examples such as 'worship', where the object argument need not exist. The axiom also will need to be adapted if accessibility relations between worlds are considered, something we shall not do in this paper.

[6] Compare (van Benthem, 1986), where it is argued that, in order to keep complexity down, natural language semantics should concentrate on finite models.

This axiom scheme will have instantiations such as, say,

$$\begin{aligned}&\exists x_1x_2x_3y[W(y)\wedge x_1\neq x_2\wedge x_1\neq x_3\wedge x_2\neq x_3\\&\wedge\forall x\,[E(x,y)\leftrightarrow(x=x_1\vee x=x_2\vee x=x_3)]\\&\wedge\,has(x_1,x_2,y)\wedge\neg has(x_2,x_3,y)\wedge mule(x_2,y)\wedge hay(x_3,y)]\ .\end{aligned}$$

Thus every finite world is stipulated to exist, as we can easily produce an exhaustive description in this way (a so-called 'diagram'). Note that this does not only ensure the existence of many worlds but also the existence of a multiplicity of states. Since, according to $\mathcal{A}9$, each register of a state can be selectively updated with type e values, and since worlds are type e objects, each state will have many variants differing only from it in the value of the register w_r.[7]

We turn to the axiom about local contexts. The local context of a node k will be a Discourse Representation Structure $\Gamma(k)$ that can be computed from the local contexts and semantics of other nodes. $\mathcal{A}14$ states that the local context of any non-S or non-T node (T is the category of texts) equals the local context of its mother (if it has one). Here $k_1 \lhd k_2$ abbreviates $k_1 \lhd^+ k_2 \wedge \neg\exists k[k_1 \lhd^+ k \wedge k \lhd^+ k_2]$, i.e. $\lhd$ denotes the immediate dominance relation.

$\mathcal{A}14$ $\forall k_1k_2\,[[k_1 \lhd k_2 \wedge \ell(k_2)\neq s\wedge \ell(k_2)\neq t]\rightarrow \Gamma(k_2)=\Gamma(k_1)]$

How the local contexts of S nodes and T nodes are to be computed will be regulated in the lexicon.

3 A Fine-grained Compositional DRT

In this section we will give some definitions that will make key concepts of Discourse Representation Theory available. The definitions are based upon the semantic axioms that were given above and the DRT concepts that result will be needed in our further discussion of Logical Description Grammars and it is for this reason that we interrupt the treatment of the latter.

The approach here will have much in common with that of (Muskens, 1996), but will also differ from it in an important respect. What the two approaches have in common is that both are based on a transcription of truth conditions for DRT, carried out within an axiomatic extension of type logic (minor variations of the binding axioms that are relevant here are also

[7] States that have worlds in u or o registers are not formally excluded, but are of no relevance. We will also not bother to formally exclude that worlds are elements of the domains of (other) worlds.

present in (Muskens, 1996)). The difference between the two treatments results from the fact that while in (Muskens, 1996) the DRT truth conditions that were transcribed were those of (Groenendijk and Stokhof, 1991, definition 26), here we will use a variant of the semantics in (Zeevat, 1989). The reason for this difference in treatment lies not only in our wish to emphasize the flexibility of the 'transcription' approach, but also in the fact that in this way we get a more fine-grained DRT semantics (in the sense that we get stronger requirements on the identity of DRSs), which suits our present purposes better.

Some abbreviations introducing set-theoretic notation will come in handy.

Definition 1 Let $A_1, \ldots, A_n$ be terms of some type α and let X be the first variable of that type not free in these terms, then

$$\{A_1, \ldots, A_n\} \quad \text{abbreviates} \quad \lambda X\,[X = A_1 \vee \ldots \vee X = A_n]\ .$$

Furthermore, let A and B be terms of some type $(\alpha_1(\cdots(\alpha_n t)\cdots))$ and let $X_1, \ldots, X_n$ be variables such that each X_i is of tye α_i. Then

$$\begin{aligned} A \cup B &\quad \text{abbreviates} \quad \lambda X_1 \ldots X_n\,[AX_1 \ldots X_n \vee BX_1 \ldots X_n]\ ,\\ A \cap B &\quad \text{abbreviates} \quad \lambda X_1 \ldots X_n\,[AX_1 \ldots X_n \wedge BX_1 \ldots X_n]\ ,\\ A \subseteq B &\quad \text{abbreviates} \quad \forall X_1 \ldots X_n\,[AX_1 \ldots X_n \to BX_1 \ldots X_n]\ . \end{aligned}$$

In (Zeevat, 1989) the semantic value of a Discourse Representation Structure K is a pair consisting of (a) a set of discourse referents (the universe of K) and (b) a set of assignments. The latter consists of all those assignments that verify all conditions in K. This could easily be transposed to the present set-up by letting the first part of any DRS K consist of a set of registers (type πt) and its second part of a set of states (type st), so that the type of DRSs would be $\pi t \times st$. However, we opt for a variant of this approach and will deviate from Zeevat's treatment in two respects. Firstly, we will follow (Visser, 1994) in keeping track of the *free* discourse referents in any DRS or condition (see also (Van Eijck and Kamp, 1997)). This can be done by letting conditions be pairs of (a) sets of states and (b) sets of registers (those that are free in the condition). The type of a condition will then become $st \times \pi t$.

The second deviation from Zeevat is that we will let the second part of a DRS K consist of a *set of* conditions rather than of a single condition representing their conjunction. Having sets of conditions around without conjoining their members embodies the hypothesis that the language system sometimes addresses one of these in isolation. In (van Leusen, 2003) it is argued, for example, that the effect of a *correction* can be a selective downdate of a DRS, with some conditions disappearing while others remain. With these modifications the type of DRSs becomes $\pi t \times (st \times \pi t)t$, which we will often abbreviate as τ. The alternative formalization will give stronger identity criteria on DRSs.

Given that the free referents of a condition form its second element, the free referents of any DRS K can easily be computed: these are the ones that are free in some condition of K but are not in the universe of K. A DRS will be *proper* if it has no free referents. These notions are made available to the logic with the following definitions.

$$\begin{array}{rll} \mathit{free}(K) & \text{abbreviates} & \lambda v \exists c\, [\neg \mathsf{fst}(K)(v) \wedge \mathsf{snd}(K)(c) \wedge \mathsf{snd}(c)(v)] \\ \mathit{proper}(K) & \text{abbreviates} & \neg \exists v\, \mathit{free}(K)(v) \end{array}$$

The two following notions play a key role in the algebra of Discourse Representation Structures.

Definition 2 Let K and K' be terms of type τ.

$$\begin{array}{rll} K \oplus K' & \text{abbreviates} & \langle \mathsf{fst}(K) \cup \mathsf{fst}(K'), \mathsf{snd}(K) \cup \mathsf{snd}(K') \rangle \\ K \sqsubseteq K' & \text{abbreviates} & \mathsf{fst}(K) \subseteq \mathsf{fst}(K') \wedge \mathsf{snd}(K) \subseteq \mathsf{snd}(K') \end{array}$$

We will typically be interested in type τ objects $\langle \{\delta_1, \ldots, \delta_n\}, \{\gamma_1, \ldots, \gamma_m\} \rangle$ with a finite universe $\{\delta_1, \ldots, \delta_n\}$ and a finite set of conditions $\{\gamma_1, \ldots, \gamma_m\}$. Such DRSs we prefer to write as $[\delta_1 \ldots \delta_n \mid \gamma_1, \ldots, \gamma_m]$, in a way reminiscent of the usual 'box' notation for DRSs.

While we essentially follow the Zeevat approach to the semantics of Discourse Representation here, the dynamic treatment of (Groenendijk and Stokhof, 1991; Muskens, 1996) is still available. The *Groenendijk-Stokhof interpretation* of a DRS K can be defined as the relation $gs(K)$ between states i and j such that i differs from j at most as far as the universe of K is concerned and j verifies all conditions in K. In other words, $gs(K)$ can be taken to abbreviate

$$\lambda ij (\forall v\, [\neg \mathsf{fst}(K)(v) \rightarrow V(v,i) = V(v,j)] \wedge \forall c (\mathsf{snd}(K)(c) \rightarrow \mathsf{fst}(c)(j))) \ .$$

On the finite DRSs we are interested in, this leads to the more readable equation

$$gs([\delta_1 \ldots \delta_n \mid \gamma_1, \ldots, \gamma_m]) = \lambda ij\ (i[\delta_1 \ldots \delta_n]j \wedge \mathsf{fst}(\gamma_1)(j) \wedge \ldots \wedge \mathsf{fst}(\gamma_m)(j)) \ .$$

Note that the natural algebraic operations on DRSs are different from those on their Groenendijk-Stokhof interpretations. On the former the essentially

Boolean $\oplus$ rules; the latter is a relational algebra with relational composition as one of its natural operations.

A DRS K is *true* in a state i if there is a j, differing from i at most as far as the universe of K is concerned, such that j verifies all conditions in K. We will write $true(K)$ for $\lambda i \exists j \; gs(K)(i)(j)$. $true(K)$ therefore denotes the domain of $gs(K)$ and

$$true([\delta_1 \ldots \delta_n \mid \gamma_1, \ldots, \gamma_m]) = \lambda i \exists j \, (i[\delta_1 \ldots \delta_n]j \wedge \mathsf{fst}(\gamma_1)(j) \wedge \ldots \wedge \mathsf{fst}(\gamma_m)(j)) \; .$$

The notion of DRS truth immediately leads to a notion of DRS consequence: K' is said to *follow from* K if $\mathcal{A}1, \ldots, \mathcal{A}14 \models true(K) \subseteq true(K')$, i.e. if K' is true in all states i in which K is true, in any model of the axioms.[8] This notion can be relativised to any description $\mathcal{D}$ by saying that K' *follows from* K *given* $\mathcal{D}$ if $\mathcal{D}, \mathcal{A}1, \ldots, \mathcal{A}14 \models true(K) \subseteq true(K')$. It will be seen shortly how a similar notion can be obtained on the level of the describing logic.

Until now we have not paid much attention to the internal structure of DRSs, but the next definition gives notation for conditions.

Definition 3 Let $R \in \mathcal{L}$, let $\delta_1, \ldots, \delta_n$ be terms of type π (discourse referents), and let K and K' be terms of type τ (DRSs). Then

$$\begin{array}{rll}
R\{\delta_1, \ldots, \delta_n\} & \text{abbreviates} & \langle \lambda i.R(V(\delta_1, i), \ldots, V(\delta_n, i)), \{\delta_1, \ldots, \delta_n\}\rangle \\
\delta_1 \textbf{ is } \delta_2 & \text{abbreviates} & \langle \lambda i.V(\delta_1, i) = V(\delta_2, i), \{\delta_1, \delta_2\}\rangle \\
\textbf{not } K & \text{abbreviates} & \langle \lambda i.\neg true(K)(i), free(K)\rangle \\
K \Rightarrow K' & \text{abbreviates} & \textbf{not } (K \oplus [\; \mid \textbf{not } K']) \\
K \textbf{ or } K' & \text{abbreviates} & \langle true(K) \cup true(K'), free(K) \cup free(K')\rangle
\end{array}$$

Note that $R\{\delta_1, \ldots, \delta_n\}$ predicates R of the *values* of $\delta_1, \ldots, \delta_n$ in some state, not of these registers themselves. We shall write $R\{\delta, w_r\}$ as $w_r\colon R\delta$ and $R\{\delta_1, \delta_2, w_r\}$ as $w_r\colon \delta_1 R \delta_2$.

Let us consider the sublanguage of type $st \times \pi t$ and type τ terms that is given by the following Backus-Naur form, and dub it the *DRT sublanguage*. Here the δ range over π terms that have the form $\rho(n)$, where $\rho \in \{u, o, w\}$ and n is a node name. The R are taken from $\mathcal{L}$.

$$\begin{array}{rcl}
\gamma & ::= & R\{\delta_1, \ldots, \delta_n\} \mid \delta_1 \textbf{ is } \delta_2 \mid \textbf{not } K \mid K \Rightarrow K' \mid K \textbf{ or } K' \\
K & ::= & [\delta_1 \ldots \delta_n \mid \gamma_1, \ldots, \gamma_m]
\end{array}$$

[8] This gives an existential interpretation to all discourse referents that are present in the top level of a DRS. In section 5 below we shall consider a revised notion of truth, easily definable in terms of the present one, in which some of the top level referents get a referential interpretation.

It is useful to know that there are simple truth-preserving translations from the DRT sublanguage to predicate logic. The following one is taken from (Muskens, 1996) (but see also (Kamp and Reyle, 1993)).

Definition 4 Let $\dagger$ be a function that injectively maps each δ to a variable of type e. The function **tr** from $st \times \pi t$ terms in the DRT sublanguage to predicate logical formulas and the function **wp**, which takes a pair consisting of a τ term and a predicate logical formula and yields a predicate logical formula, are defined as follows.

$$\begin{aligned}
\mathbf{tr}(R\{\delta_1, \ldots, \delta_n\}) &= R(\delta_1^\dagger, \ldots, \delta_n^\dagger) \\
\mathbf{tr}(\delta_1 \textbf{ is } \delta_2) &= \delta_1^\dagger = \delta_2^\dagger \\
\mathbf{tr}(\textbf{not } K) &= \neg\mathbf{wp}(K, \top) \\
\mathbf{tr}(K \Rightarrow K') &= \neg\mathbf{wp}(K, \neg\mathbf{wp}(K', \top)) \\
\mathbf{tr}(K \textbf{ or } K') &= \mathbf{wp}(K, \top) \vee \mathbf{wp}(K', \top) \\
\mathbf{wp}([\delta_1 \ldots \delta_n \mid \gamma_1, \ldots, \gamma_m], \chi) &= \exists\delta_1^\dagger \ldots \delta_n^\dagger\, [\mathbf{tr}(\gamma_1) \wedge \ldots \wedge \mathbf{tr}(\gamma_m) \wedge \chi]
\end{aligned}$$

For any formula φ and state variable i, let φ^i be the result of substituting $V(\delta, i)$ for $\delta^\dagger$, for each $\delta^\dagger$ that is free in φ. The following holds.

Theorem 1 *Let K be a τ term in the DRT sublanguage and let i be an arbitrary type s variable. Let Γ contain all statements $\delta \neq \delta'$, for every pair δ, δ' of syntactically different discourse referents in K. Then*

$$\Gamma, \mathcal{A}1, \ldots, \mathcal{A}14 \models \mathbf{wp}(K, \top)^i \leftrightarrow \mathit{true}(K)(i)$$

The disequality statements are needed: If, say, 12 and 9 corefer, it follows that $[u_{12} \mid w_r\colon \mathit{mule}\ u_9] = [u_9 \mid w_r\colon \mathit{mule}\ u_9]$, but the translation would not preserve this. In practice this will be no limitation at all. For a proof of the theorem, use induction on complexity in the DRT sublanguage construction to show that, given the axioms and disequalities, $\mathbf{wp}(K, \chi)^i \leftrightarrow \exists j\, [gs(K)(i)(j) \wedge \chi^j]$ and $\mathbf{tr}(\gamma)^i \leftrightarrow \gamma(i)$, for all K and γ. This can be shown using the methods of (Muskens, 1996; Muskens, 2001).

In the next section we will set up a logical mechanism that will assign DRSs or λ-terms involving DRSs to nodes as their semantic values. At many levels the special discourse referent w_r for worlds will be free in such DRSs. However, it will be ensured that w_r is always present in the universes of the 'local contexts' of these nodes, and indeed in the global context, or *background* B, that we associate with the start element '$>$' (these will be proper DRSs). We will want to put (presuppositional) constraints on contexts and typically these constraints can take two forms. One possibility is to require that a context *contains* certain material. For example, for lexical nodes k that carry the proper name 'Pedro', we will demand that $[o_k \mid w_r\!: Pedro\ o_k] \sqsubseteq B$. Another possibility is that a context is required to bear a certain *logical relation* to another DRS K, where K typically does not have w_r in its universe. Below we will have a brief look at Karttunen's theory of presuppositions, which provides an example, as it requires that presuppositions *follow from* their local context.[9]

These considerations make it necessary that logical relations such as entailment and consistency can be expressed or approximated *at the level of the describing logic*, but this has essentially been taken care of in the previous section. Suppose that K is a DRS with w_r in its universe, while K' does not have w_r in its universe. Then the condition $K \Rightarrow K'$ quantifies over possible worlds. This is best illustrated with an example: let us take K to be $[w_r\, o_1 \mid w_r\!: mule\ o_1]$, 'there is a mule', and let K' be $[u_2 \mid w_r\!: farmer\ u_2,\ w_r\!: u_2\ owns\ o_1]$, 'some farmer owns the mule', so that K' should not be made to follow from K. Then, for arbitrary i,[10]

$$\mathbf{tr}(K \Rightarrow K')^i = \forall y \forall x_1\, [mule(x_1, y) \rightarrow \exists x_2\, [farmer(x_2, y) \wedge owns(x_2, x_1, y)]]\ .$$

In view of $\mathcal{A}12$ this is equivalent with

$$\begin{aligned}&\forall y[W(y) \rightarrow \forall x_1\, [(mule(x_1, y) \wedge E(x_1, y)) \rightarrow \\ &\qquad \exists x_2\, [E(x_2, y) \wedge farmer(x_2, y) \wedge owns(x_2, x_1, y)]]]\ ,\end{aligned}$$

from which it is seen that indeed quantification over worlds is involved. Note that $\mathcal{A}13$ immediately provides a singleton-domain counterexample to this purported strict implication and in general it will give finite counterexamples if there are such.

Conditions $K \Rightarrow K'$ are of type $st \times \pi t$ and we will typically want to state conditions on the t level. The following definition adds the necessary quantification over states / assignments.

Definition 5 Let K and K' be terms of type τ.

$$K \models\!\!\!\!\!\prec K' \quad \text{abbreviates} \quad \forall i\ \mathsf{fst}(K \Rightarrow K')(i)$$

[9]A second example that springs to mind are the consistency and informativity (non-entailment) requirements in Van der Sandt's theory of presupposition. An extension of our theory that would allow for local and intermediate accommodation would also make it possible to give a declarative version of Van der Sandt's theory. A third example are the felicity constraints associated with discourse relations in (van Leusen, 2003), which are also formulated in terms of entailment and consistency. It turns out that while some discourse relations require consistency and informativity, others, such as *denial* and *confirmation*, impose opposite constraints.

[10]Here $K \Rightarrow K'$ is closed. This will be the typical situation in applications.

4 LDG: Lexical Descriptions and Reasoning

We return to our exposition of the mechanics of LDG. The main themes will be the LDG *lexicon* and an explanation of the kind of *reasoning* that LDG allows.

4.1 Lexical Descriptions

There will be two kinds of lexical description, *classifying* descriptions and *elementary tree* descriptions. Of these, the elementary tree descriptions carry most information, but the classifying descriptions play a useful role in connecting elementary tree descriptions with open class words.

Classifying Descriptions In (7) classifying descriptions for the open class words *Pedro, has,* and *mule* are given. (7a) states that whenever a node k carries the word *has*, it must be classified as a transitive verb (tv) and its semantics $\sigma^{\pi(\pi\tau)}(k)$ is $\lambda v'\lambda v[\ \mid w_r\!: v\ has\ v']$. (If α is a type, we will let σ^α denote a function from tree nodes to objects of type α.) The description for *mule* in (7b) is similar, but the word is classified as a common noun (cn) and its semantics has a different type.

(7) a. $\forall k[\textsc{has}(k) \to (tv(k) \wedge \sigma^{\pi(\pi\tau)}(k) = \lambda v'\lambda v[\ \mid w_r\!: v\ has\ v'])]$

b. $\forall k[\textsc{mule}(k) \to (cn(k) \wedge \sigma^{\pi\tau}(k) = \lambda v[\ \mid w_r\!: mule\ v])]$

c. $\forall k[\textsc{pedro}(k) \to (pn(k) \wedge \sigma^{\pi}(k) = o_k \wedge [o_k \mid w_r\!: Pedro\ o_k] \sqsubseteq B)]$

The classifying description for *Pedro* in (7c), lastly, is of a slightly different nature. Here the semantics is set to the discourse referent o_k, but an additional requirement enforces that the *global context* or *background* of the discourse, for which we use the type τ constant B, contains the DRS $[o_k \mid w_r\!: Pedro\ o_k]$. This in fact implements the usual DRT requirement that discourse referents connected with names must be present in the universe of the main DRS and that any descriptive material connected with such discourse referents must likewise be available globally.

Elementary Tree Descriptions Let us now turn to the second kind of lexical descriptions, elementary tree descriptions. The idea behind these is based upon work in Tree Adjoining Grammars (Joshi, Levy and Takahashi, 1975; Schabes, 1990) and in particular upon the so-called D-Tree Grammars of (Vijay-Shankar, 1992; Rambow et al., 1995).[11] An elementary tree description states what must be true in any acceptable structure if some lexical node has certain given properties.

(8) $\forall k[tv(k) \rightarrow \exists k_1 k_2 k_3 k_4 k_5[\ell(k) = v \wedge \ell(k_1) = s \wedge \ell(k_2) = dp \wedge$
$\ell(k_3) = vp \wedge \ell(k_4) = vp \wedge \ell(k_5) = dp \wedge \Delta(k_1, k_2, k_3) \wedge \Delta(k_4, k, k_5) \wedge$
$k_3 \lhd^* k_4 \wedge \sigma(k_4) = \sigma(k)(\sigma(k_5)) \wedge \sigma(k_1) = \sigma(k_3)(\sigma(k_2)) \wedge$
$k \stackrel{+}{\hookleftarrow} \{k, k_1, k_4\} \wedge k \stackrel{-}{\hookleftarrow} \{k, k_2, k_3, k_5\}]]$

For example, (8), which uses the abbreviations in Definition 6 below, is an elementary tree description saying that whenever a tree contains a transitive verb, it must have certain other properties as well. The transitive verb must be labeled 'V', it must have a dominating node labeled 'VP', which in turn dominates a node labeled 'DP', etc. The description also contains anchoring information and semantic information.

Definition 6

$$\begin{array}{rll} \Delta(k, k', k'') & \text{abbreviates} & k \lhd k' \wedge k \lhd k'' \wedge k' \prec k'' \\ t \stackrel{+}{\hookleftarrow} \{t_1, \ldots, t_n\} & \text{abbreviates} & \forall k(\alpha^+(k) = t \leftrightarrow (k = t_1 \vee \ldots \vee k = t_n)) \\ t \stackrel{-}{\hookleftarrow} \{t_1, \ldots, t_n\} & \text{abbreviates} & \forall k(\alpha^-(k) = t \leftrightarrow (k = t_1 \vee \ldots \vee k = t_n)) \end{array}$$

In general, an elementary tree description will have the form

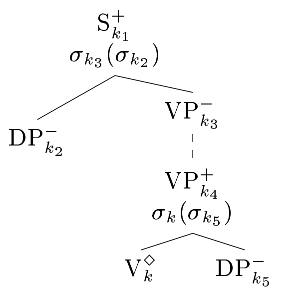
$$\forall k[cond(k) \rightarrow \exists k_1 \ldots k_n[\chi(k_1 \ldots k_n) \wedge k \stackrel{+}{\hookleftarrow} \{k'_1, \ldots, k'_p\} \wedge k \stackrel{-}{\hookleftarrow} \{k''_1, \ldots, k''_q\}]]$$

where *cond* is some condition, χ is a conjunction of formulas, and $\{k'_1, \ldots, k'_p\} \cup \{k''_1, \ldots, k''_q\} = \{k, k_1, \ldots, k_n\}$. The condition *cond* can be a predicate like *tv* or *cn*, associated with a whole class of elementary trees, it can be one of the discourse elements $>$ or SEQ, but also any of the predicates associated with words (such as PEDRO, HAS, etc.).

A Simple Graphical Notation Even with the abbreviations of Definition 6 in place elementary tree descriptions such as the one in (8) are unwieldy. It is therefore expedient to change to a graphical notation that is easier to work with even if it is somewhat less precise. The picture in (9) is an alternative representation of the description in (8).

[11] For the relation between Tree Adjoining Grammars, D-Tree Grammars and the present format, see (Muskens, 2001).

(9) *tv:*



The conventions that were followed in obtaining (9) from (8) are the following.

1. Nodes are labeled as usual and node names (variables or constants) are given as subscripts.

2. Solid lines stand for immediate dominance ($\lhd$) relations. Left-right ordering between sisters or between terminal nodes stands for precedence ($\prec$). Dashed lines stand for dominance ($\lhd^*$).

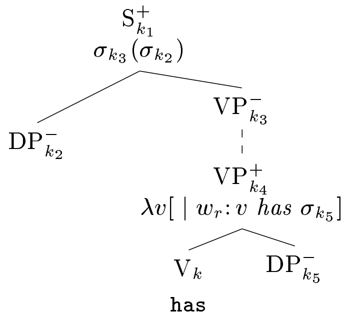
3. Nodes which are positively but not negatively anchored to the lexical node in an elementary tree description are marked $+$, while nodes which are negatively but not positively anchored are marked $-$. Nodes which are anchored both ways are saturated and unmarked. The situation that a node is not anchored at all will never occur and the anchor itself may be marked with a $\diamond$. It is understood that $k \overset{+}{\hookleftarrow} \{k_1, \ldots, k_n\}$ is asserted if k is the lexical node and $k_1, \ldots, k_n$ are all the nodes that are marked $+$ or are unmarked, and, similarly, that $k \overset{-}{\hookleftarrow} \{k_1, \ldots, k_n\}$ is part of the depicted description if $k_1, \ldots, k_n$ are all nodes that are marked $-$ or unmarked.

4. The semantic value $\sigma^{\alpha}(k)$ of a node k may be written under it and any other information may be written into the tree as well. Superscripts on σ will often be dropped. Type ν arguments may be written as subscripts and we typically write σ_k for $\sigma(k)$. Conventions to also write Γ_k for $\Gamma(k)$, and ρ_k for $\rho(k)$ ($\rho \in \{u, o, w\}$) were already introduced.

Since the official representations of elementary tree descriptions can always be reconstructed from more user-friendly pictorial representations like the one in (9), we will switch to the latter entirely.

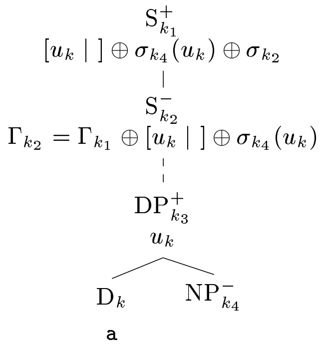
The description in (9) covers the whole class of transitive verbs. If it is conjoined with the information that k carries the lexeme *has* and with (7a) we get the picture in (10). Note that the semantics of VP_{k_4} now has a more complete specification.

(10)



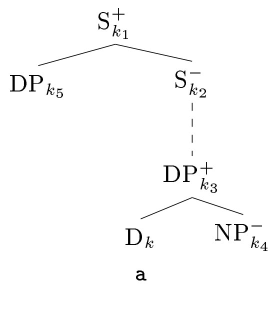
For closed class words it may often be expedient to list elementary tree descriptions in this form—with their anchoring words attached. Here, for example is an entry for the indefinite *a*.

(11)



The idea here is that the DP that is projected from the indefinite translates as a discourse referent u_k and that the declaration of that discourse referent $[u_k \mid\ ]$ and its restriction $\sigma_{k_4}(u_k)$ are quantified-in at some higher S level. S_{k_1} should be compared to the place where adjunction of the DP takes place after Quantifier Raising. In fact, if we require this S to properly dominate an additional DP, as in (12) (where semantic information has been left out), we obtain trees that are very close to LF trees in generative grammar (with DP_{k_3} carrying all syntactic information and DP_{k_5} possibly carrying semantic information).

(12)



Since the additional DP plays no role in the current set-up it will be suppressed.

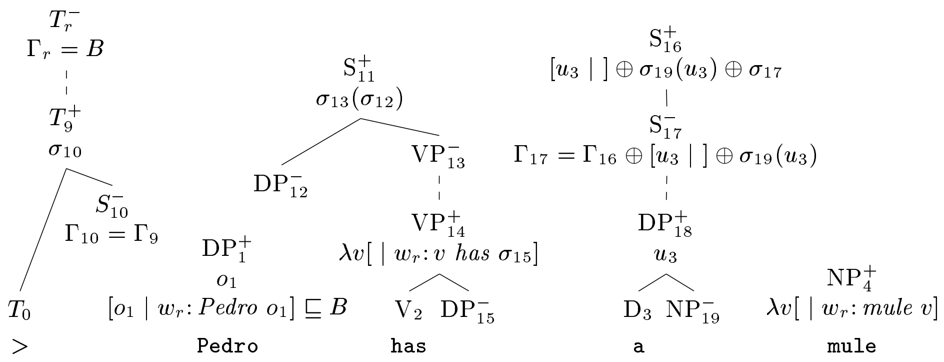
Note that Γ_{k_2}, the local context of the lower S node, is set to be the merge of the local context of the higher S node and the 'restrictor' material of the semantics of that higher node in (11). The idea is that unresolved pronouns may seek a referent in this local context, and that presuppositions may likewise resolve to it.

Some more examples of elementary tree descriptions are given in the Appendix.

4.2 Reasoning from Input Descriptions

Let us continue the discussion of the example we gave in the first parts of this paper. Our hearer has judged the statements in (2) true on the basis of her observations and has formed the temporary hypothesis that (3). She can now do some reasoning. First, she may, using universal instantiation, combine her new information with the elementary tree descriptions in her lexicon, substituting 3 for k in (11), for example. Next, she can take (fresh) witnesses for the existential quantifiers in her elementary tree descriptions, substituting, say, 16 for k_1, 17 for k_2, 18 for k_3, and 19 for k_4 in the result and making similar substitutions in the other descriptions. The result will be something like (13) (for elementary tree descriptions for the start symbol, proper names and common nouns, see the Appendix).

(13)

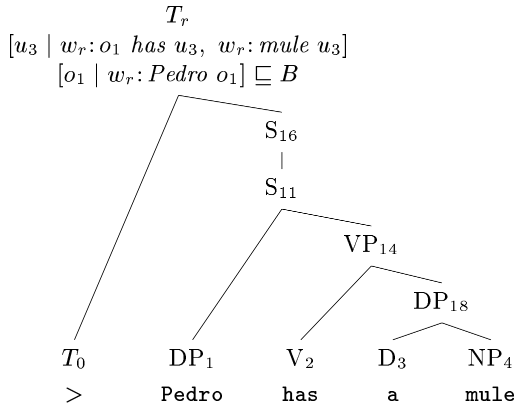


Note that since the $\prec$ relation must be linear on the lexical elements and the existing information precludes that there are elements preceding or following the given lexical elements, or are interspersed between them, all lexical elements must be one of 0, 1, 2, 3, and 4. From the anchoring information in the descriptions in (13) it can be concluded that $\{0, 1, 2, 3, 4, 9, 11, 14, 16, 18\}$ are the nodes that are positively anchored to one of the lexical nodes and that, since every node must be positively anchored, all nodes belong to this set. That these nodes must be pairwise distinct also follows from the tree descriptions. In a similar way, now using the information about negative anchoring, it can be concluded that $\{0, 2, 3, r, 10, 12, 13, 15, 17, 19\}$ likewise comprise the whole type ν domain and are pairwise distinct. This means that these two sets must be equal and that there must be a way to 'pair off' positively and negatively marked nodes and identify the pairs. The only way to do this that is in accordance with the tree axioms is given in (14).

(14) $r = 9 \wedge 10 = 16 \wedge 11 = 17 \wedge 1 = 12 \wedge 13 = 14 \wedge 15 = 18 \wedge 4 = 9$

If our hearer adds this inferred information to her description of the situation, the picture in (15) results.

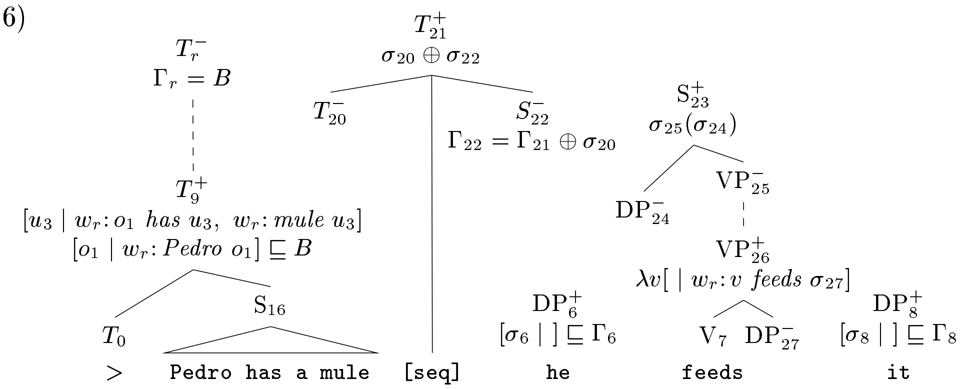
(15)



Note that in (15) not only the positively and negatively marked nodes have been 'clicked together' in a process that is somewhat reminiscent of chemical binding, but that the identifications also allowed some semantic computation. The semantics of r can be computed to be $[u_3 \mid w_r\colon o_1\ \mathit{has}\ u_3,\ w_r\colon \mathit{mule}\ u_3]$ and there still is a constraint $[o_1 \mid w_r\colon \mathit{Pedro}\ o_1] \sqsubseteq B$ on the background B. These two pieces of information are typical of the kind of information that will be collected during the processing of any discourse. The semantics of the root, σ_r, will be interpreted as an *update* of the background, while other information *constrains* the background. In section 4 we will put some general requirements on the original background B (such as properness), but the basic idea is that the background must largely be constructed or negotiated by the discourse participants. It will be only partially constrained by what is said during a conversation and it will therefore remain largely underspecified. The meaning of the discourse given a certain background B will be identified with $B \oplus \sigma_r$, what was presupposed updated with what was said. In the present example this discourse meaning is $B \oplus [u_3 \mid w_r\!: o_1\ \mathit{has}\ u_3,\ w_r\!: \mathit{mule}\ u_3]$.

Our hearer now has inferred certain information from (2) plus (3). But of course, (3) was hypothetical and upon hearing more she may replace it with (5) plus (6). In that case the following picture emerges (here we have already combined much of the material that was also in the earlier description).[12]

(16)



The picture features an elementary description for the discourse relation 'seq', which extends a given discourse (category T) with a subsequent sentence or discourse.[13] Semantically, sequencing results in the merge of the semantic values of its arguments. Note that the local context of the right sister of the sequencing relation is set to that of its mother node updated

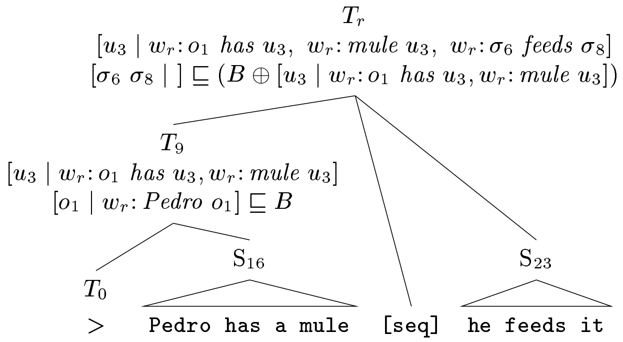
[12] The question becomes important how much of a hearer's reasoning is independent from hypothetical 'end' statements such as (3) or (6). This independent part can be asserted categorically and can monotonically be transferred to the next phase of the reasoning process. Although this matter is important, we will not go into it here.

[13] In general, it may be assumed that discourse relations take discourse constituents as their arguments, and that sentences (or clauses, as in (Webber et al., 2001)) figure as minimal discourse constituents. Since discourse relations may vary in both the syntactic structure they select and the semantic values they assign, various different elementary descriptions would be provided in a fully fledged discourse grammar. We will not pursue this here; the coverage of our little grammar is limited to the discourse relation 'seq', and the sentence connectives 'if ... then' and 'or' (see the appendix). In the absence of other types of discourse relation in the grammar, the elementary structure of 'seq' presented above only allows for left branching tree structures at discourse level. Intuitively, this means that the sequencing relation continues the description of the whole of the situation described in the preceding discourse. Further research will have to show whether this analysis is appropriate for sequencing (it is certainly not appropriate for other discourse relations, such as *concession* or *contrast*). Right branching discourse structure may be obtained if we allow the discourse relation to attach to sentences as well as discourses in its left argument.

with the semantics of the left sister. Furthermore, note that the semantics of pronouns is not specified. While the indefinite *a* is associated with a discourse referent u_3 (which resulted from instantiating k by 3 in (11)), the constraint on pronouns is that $[\sigma_k^\pi \mid\,] \sqsubseteq \Gamma_k$ (where k is the pronoun's node). This statement requires their semantics (a register) to be present in the universe of their local context.

Again there is a unique way for the + and − nodes to click together and the following picture results. Here, with the help of $\mathcal{A}14$ and lexical information about Γ, the local context of each of the pronouns has been computed to be $B \oplus [u_3 \mid w_r\!: o_1 \textit{ has } u_3,\ w_r\!: \textit{mule } u_3]$, the discourse meaning of the previous chunk of discourse.

(17)



The next section will contain further discussion of the semantic constraints that have now been collected.

5 Further DRT Construction as Inference from Input Descriptions

Above we have seen how a hearer's reasoning from an input description resulted in the piecewise compositional construction of parts of a Discourse Representation Structure. Even though the grammar and each input description have a strictly declarative formulation, inferences that take these as their point of departure, being inferences, have a procedural and computational character. In this section it will be seen that it is possible for a hearer to do further inference using the kind of description that was arrived at in (17) and that such further reasoning results in the kind of highly procedural steps posited by the DRT construction algorithm.

In the following the notion of *background* will continue to play an important role. This background B should be thought of as consisting of whatever is presupposed during the conversation in question. Obviously, what is presupposed should be highly underspecified, as it is open for negotiation between discussion participants. The background will be constrained by presuppositions that arise in the discourse and there are also a few general constraints. These are summed up in $\mathcal{A}15$: First, we take it that the background is *proper*. Modulo the naming of discourse referents, it should be possible for a background to have arisen out of previous discourse (with original background $[w_r \mid]$). Since discourse generates only proper DRSs, backgrounds too should be proper. A second constraint is that the universe of the background does not contain registers that are values of u, as the latter will be reserved for discourse referents that arise during conversation. A last requirement on the background is that its universe contains w_r as an element.

$\mathcal{A}15$ $\mathit{proper}(B) \wedge \neg\exists k\, \mathsf{fst}(B)(u_k) \wedge \mathsf{fst}(B)(w_r)$

Discourse referents that are present in the universe of the background arguably should be interpreted *referentially* rather than *existentially*. The referent w_r, which stands for the world of evaluation, may serve as an illustration, for *Pedro has a mule* should not be interpreted as 'there is some world in which Pedro has a mule' but as 'Pedro has a mule in *this* world.' The discourse referent corresponding to *Pedro* should also be interpreted as given and similarly should deictic pronouns and perhaps even definite descriptions whose referent is accommodated. A notion of truth that treats all background referents referentially will result if the previous definition of truth is revised in the following way: K is said to be *true in* i *with respect to background* B if $\mathit{true}(\langle \mathsf{fst}(K) - \mathsf{fst}(B), \mathsf{snd}(K)\rangle)(i)$ holds, i.e. referents present in the background's universe will not be interpreted existentially but will receive their interpretation from the state i. It will be assumed that discourse participants interpret the discourse with respect to some state i_0 that they take the discourse to be *about*[14] and we will often be interested in the truth of some DRS K in i_0 with respect to a given background.

To illustrate this further, let us consider the DRS $K =$

$$[w_r\, o_1\, u_3 \mid w_r\!: \mathit{Pedro}\ o_1,\ w_r\!: o_1\ \mathit{has}\ u_3,\ w_r\!: \mathit{mule}\ u_3]$$

[14] Austin (1961): "A statement is said to be true when the historic state of affairs to which it is correlated by the demonstrative conventions (the one to which it 'refers') is of a type with which the sentence used in making it is correlated by the descriptive conventions." In the present context, i_0 (or its sequence of values) can be compared to Austin's 'historic state of affairs'. Presumably, our states are somewhat richer than Austin's states of affairs, for besides a possible world they contain values for all kinds of referents. On Austinian propositions, see also (Barwise and Perry, 1983).

and assume that w_r and o_1, but not u_3, are members of the universe of background B. If, in general, we agree, for readability, to write δ^0 for $V(\delta, i_0)$, we can express the truth of K in i_0 with respect to B as

$$\begin{aligned}&\mathbf{wp}([u_3 \mid w_r\colon \mathit{Pedro}\ o_1,\ w_r\colon o_1\ \mathit{has}\ u_3,\ w_r\colon \mathit{mule}\ u_3], \top)^{i_0} = \\ &\quad \exists x_3\, [\mathit{Pedro}\ (o_1^0, w_r^0) \wedge \mathit{has}\ (o_1^0, x_3, w_r^0) \wedge \mathit{mule}\ (x_3, w_r^0)]\end{aligned}$$

Here the values of o_1^0 and w_r^0 depend on the state i_0, which functions much as an *external anchor* in the sense of (Kamp and Reyle, 1993, pp. 246–248).[15]

With the notion of background now elucidated, let us see how reasoning from input descriptions can result in a process familiar from DRT: While indefinites create fresh referents, pronouns will pick up old referents (or must get a deictic interpretation). The material connected with proper names is relegated to the main Discourse Representation Structure. Relatedly, presuppositions may either get bound or must be accommodated. At present, we can only account for *global* accommodation, but we consider it possible for a treatment such as the present one to be extended to also cover local and intermediate accommodation. We will discuss these points in turn, ending with a reminder that the theory gives a natural account of underspecification, so that we are in fact working in an *Underspecified* Discourse Representation Theory, such as the one pioneered in (Reyle, 1993).

Proper names land up in the main DRS. The elementary tree description for *Pedro* contained the statement in (18a), requiring the semantic material connected with the name to be in the background.

(18) a. $[o_1 \mid w_r\colon \mathit{Pedro}\ o_1] \sqsubseteq B$

b. $[w_r\, o_1 \mid w_r\colon \mathit{Pedro}\ o_1] \sqsubseteq B$

c. $\sigma_r = [u_3 \mid w_r\colon o_1\ \mathit{has}\ u_3,\ w_r\colon \mathit{mule}\ u_3,\ w_r\colon \sigma_6\ \mathit{feeds}\ \sigma_8]$

d. $[w_r\, o_1\, u_3 \mid w_r\colon \mathit{Pedro}\ o_1,\ w_r\colon o_1\ \mathit{has}\ u_3,\ w_r\colon \mathit{mule}\ u_3,\ w_r\colon \sigma_6\ \mathit{feeds}\ \sigma_8]$
$\sqsubseteq B \oplus \sigma_r$

In fact, using $\mathcal{A}15$, this can be strengthened slightly to (18b). Using (18c) (which was found in (17)), the hearer can now derive (18d). The discourse

[15] While (Kamp and Reyle, 1993) use anchors only for proper names, our use is wider as we let i_0 interpret all referents in the discourse's background. There is some room for fine-tuning here and it may also be held that some referents from the background are referential while others are interpreted existentially.

meaning $B \oplus \sigma_r$ must contain (and therefore entail) a certain DRS containing the material connected with the name. It should be clear that this is in fact a form of global accommodation.

Indefinites create fresh referents, but pronouns pick up old referents. That indefinites create fresh referents is a property they share with many other expressions. Their creation is a by-product of the perpetual creation of fresh node names. The referent u_3 connected with the indefinite *a* in (17) was created in this way. The moment that it was established that there was some node 3 carrying the lexeme *a* the referent u_3 also sprung into existence. Note that, by the injectivity of u, u_3 cannot corefer with a referent created at any other node.[16] Since, by $\mathcal{A}15$, u_3 can not be an element of the universe of B and since, by the same axiom, B is proper, there can also be no condition in B in which u_3 occurs. The referent is therefore truly fresh to the discourse.

The process whereby pronouns get bound by referents that already exist takes a little more care to explain. In (19a) a constraint is shown that was collected in (17). It resulted from the requirement on pronouns that $[\sigma_k^\pi \mid\,] \sqsubseteq \Gamma_k$ (with k the node carrying the pronoun). This was required of the *he* and *it* nodes and in both cases the local context Γ_k could be computed to be $B \oplus [u_3 \mid w_r\!:o_1\ \mathit{has}\ u_3, w_r\!:\mathit{mule}\ u_3]$. Using (18b) and the idempotency of $\oplus$ it is seen that in fact (19b) must hold.

(19) a. $[\sigma_6\ \sigma_8 \mid\,] \sqsubseteq B \oplus [u_3 \mid w_r\!:o_1\ \mathit{has}\ u_3, w_r\!:\mathit{mule}\ u_3]$

b. $[\sigma_6\ \sigma_8 \mid\,] \sqsubseteq B \oplus [w_r\, o_1\, u_3 \mid w_r\!:\mathit{Pedro}\ o_1,\ w_r\!:o_1\ \mathit{has}\ u_3, w_r\!:\mathit{mule}\ u_3]$

This puts constraints on what σ_6 and σ_8 are, but there are still many possibilities for these referents to resolve. In principle this is what we want, as linguistic information typically underspecifies how pronouns should resolve. However, the constraints collected thus far leave open too many possibilities and therefore more constraints are needed.

In order to illustrate this, let us concentrate on σ_8 and see what values it can take. One possibility is that the pronoun *it* has no linguistic antecedent and that σ_8 is to be identified with some register that was in the universe of the background B but is not mentioned in the discourse. We think this possibility is in fact welcome and corresponds to a reading of the text in which *it* refers to an object that is somehow salient (for example, as a result

[16] Of course the *value* of another referent may be identical with the value $V(u_3, i)$ of u_3 at any given state i. The difference between identity of referents and identity of their values is crucial to the present discussion.

of pointing) but that was not introduced by linguistic means. We identify this with the deictic use of the pronoun.

Technically, there is also the possibility that σ_8 is in fact equal to w_r. This possibility is an artefact and a consequence of our choice to have one type of register for worlds and individuals. We exclude it by postulating that no node can have w_r as its semantics.

$\mathcal{A}16$ $\neg\exists k\ \sigma_k^\pi = w_r$

A fuller treatment should perhaps comprise a more general type or kind distinction of the registers involved, so that $\mathcal{A}16$ would fall out as a consequence.

Of the remaining possibilities, the identification $\sigma_8 = o_1$ should be excluded on linguistically more interesting grounds. Since *Pedro*, has masculine gender, it cannot antecede *it*, which is neuter. Since this does not follow from the theory we have set up thus far, and since it is of course just one manifestation of a phenomenon that really pervades language, we must add a general mechanism for feature constraints. In fact this is very easy and (Muskens, 2001) shows how it can be done on the basis of the first-order feature theory of (Johnson, 1991). We refer to the discussion in (Muskens, 2001) for details. Using the notation of that paper, a general requirement to the effect that coreferring nodes should agree (on number and gender) could be formulated as follows.

(20) $\forall kk'\,[\sigma_k^\pi = \sigma_{k'}^\pi \to \forall f\,[arc(\boxed{k}, \text{AGR}, f) \to arc(\boxed{k'}, \text{AGR}, f)]]$

This could be added as an axiom, together with the three axioms (axiom schemes) for features in (Muskens, 2001). Note the mixed semantic / syntactic character of (20). This is a case of genuine mutual constraint between syntax and semantics.

With a mechanism for features such as the one described in place, there are two possibilities left for the denotation of σ_8: $\sigma_8 = u_3$ or $\sigma_8 \notin \{w_r, o_1, u_3\}$. Similarly the hearer can deduce that either $\sigma_6 = o_1$ or $\sigma_6 \notin \{w_r, o_1, u_3\}$. In fact the following four possibilities remain.

(21) a. $[w_r\, o_1\, u_3 \mid w_r{:}\, \textit{Pedro}\ o_1,\ w_r{:}\, o_1\ \textit{has}\ u_3,\ w_r{:}\, \textit{mule}\ u_3,\ w_r{:}\, o_1\ \textit{feeds}\ u_3]$
$\sqsubseteq B \oplus \sigma_r$

b. $[w_r\, o_1\, u_3\, \sigma_8 \mid w_r{:}\, \textit{Pedro}\ o_1,\ w_r{:}\, o_1\ \textit{has}\ u_3,\ w_r{:}\, \textit{mule}\ u_3,\ w_r{:}\, o_1\ \textit{feeds}\ \sigma_8]$
$\sqsubseteq B \oplus \sigma_r \wedge [\sigma_8 \mid\ ] \not\sqsubseteq [w_r\, o_1\, u_3 \mid\ ]$

c. $[w_r\, o_1\, u_3\, \sigma_6 \mid w_r\colon Pedro\ o_1,\ w_r\colon o_1\ has\, u_3,\ w_r\colon mule\, u_3,\ w_r\colon \sigma_6\ feeds\, u_3]$
$\sqsubseteq B \oplus \sigma_r \wedge [\sigma_6 \mid\ ] \not\sqsubseteq [w_r\, o_1\, u_3 \mid\ ]$

d. $[w_r\, o_1\, u_3\, \sigma_6\, \sigma_8 \mid w_r\colon Pedro\ o_1,\ w_r\colon o_1\ has\, u_3,\ w_r\colon mule\, u_3,\ w_r\colon \sigma_6\ feeds\, \sigma_8]$
$\sqsubseteq B \oplus \sigma_r \wedge [\sigma_6 \mid\ ] \not\sqsubseteq [w_r\, o_1\, u_3 \mid\ ] \wedge [\sigma_8 \mid\ ] \not\sqsubseteq [w_r\, o_1\, u_3 \mid\ ]$

The first possibility corresponds to the preferred reading, the second to the case where *it* was taken deictically, the third to a deictic reading of *he*, and the fourth to the case where both pronouns pick up an extralinguistic referent. The hearer must now make a choice as to what was meant on the basis of what she knows or may assume is in the background, i.e. further narrowing down of the possibilities may be a result of pragmatic reasoning. Note that each four of the possibilities offers suitable input for the translation that was given in Definition 4 and that in each case all relevant discourse markers can be shown to be disequal (the precondition to Theorem 1). In each case certain conclusions can be drawn from the assumption that $B \oplus \sigma_r$ is true in i_0 with respect to background B. These conclusions are, respectively

(22) a. $\exists x\, [Pedro\ (o_1^0, w_r^0) \wedge has\ (o_1^0, x, w_r^0) \wedge mule\ (x, w_r^0) \wedge feeds\ (o_1^0, x, w_r^0)]$,

b. $\exists x\, [Pedro\ (o_1^0, w_r^0) \wedge has\ (o_1^0, x, w_r^0) \wedge mule\ (x, w_r^0) \wedge feeds\ (o_1^0, \sigma_8^0, w_r^0)]$,

c. $\exists x\, [Pedro\ (o_1^0, w_r^0) \wedge has\ (o_1^0, x, w_r^0) \wedge mule\ (x, w_r^0) \wedge feeds\ (\sigma_6^0, x, w_r^0)]$,

d. $\exists x\, [Pedro\ (o_1^0, w_r^0) \wedge has\ (o_1^0, x, w_r^0) \wedge mule\ (x, w_r^0) \wedge feeds\ (\sigma_6^0, \sigma_8^0, w_r^0)]$.

While in this example only linguistically acceptable pronoun resolutions were left, other suitability constraints are necessary in other cases. One set of constraints that immediately springs to mind is that of the syntactic Binding Theory (Chomsky, 1981). The Binding Theory is stated in terms of the following tree geometric notions.

(23) a. Node k *c-commands* node k' if every branching node dominating k dominates k'.

b. A node is *bound* if it is coindexed with a c-commanding node.

c. A node is *free* if it is not bound.

It is clear that these notions can be made available in our logic if we read 'k and k' are coindexed' as $\sigma^\pi(k) = \sigma^\pi(k')$. The Binding Theory itself consists of the following three principles.

(A) An anaphor is bound in its governing category

(B) A pronominal is free in its governing category

(C) An R-expression is free

Here 'anaphor' should be read as 'reflexive or reciprocal', while the category of 'pronominals' includes non-reflexive pronouns and 'R-expressions' are DPs that are not pronouns. The syntactic literature contains much discussion about the correct definition of 'governing category', but a rough approximation is that the governing category of a node k is the lowest k' properly dominating k that is labelled DP or S. Again it is obvious that principles (A), (B), and (C) can be formalized as additional constraints within the present theory.

Note that the constraints we have discussed are all stated in a single describing logic, even though they came from syntax and semantics, two separate levels of the grammar. In fact, the resolution of pronouns obviously is also constrained by pragmatic demands and these may be formalizable as well. The description approach allows us to integrate theories from various levels and a syntactic theory such as the Binding Theory, if adopted, has immediate relevance for the construction of Discourse Representation Structures.

Presuppositions may get bound or be globally accommodated. The approach to presuppositions that will follow is not in any sense new but is given to show that some aspects of existing theories that seem exclusively procedural may nonetheless receive a declarative treatment. We focus on definite descriptions such as *the man* or *that book*, but the treatment in principle applies equally to other presupposition triggers. We will follow (Heim, 1982) in assuming that definites generally must either pick up a discourse referent that is already present in their local context or must accommodate such a discourse referent. Definite descriptions, in other words, are much like pronouns in this respect. In (24a), where an elementary tree description for the definite *the* is given, this aspect is captured by the requirement $[\sigma_k \mid\,] \sqsubseteq \Gamma_{k_1}$, which forces σ_k to either unify with an existing discourse referent, or to be included in the universe of the background in a now familiar way. However, definite descriptions, unlike pronouns, also carry descriptive material that is contributed by their common nouns.[17] This material, it is widely assumed,

[17] Clearly, we do not wish to exclude the possibility that pronouns also carry (semantic gender) presuppositions.

is of a presuppositional character. We follow (Karttunen, 1974) in requiring that presuppositions must be entailed by the local context of the point where they are triggered. In (24a) this is mirrored in the requirement that $\Gamma_{k_1} \models\!\!\!< \sigma_{k_2}(\sigma_k)$.

(24) a.

$$\begin{array}{c} \mathrm{DP}^{+}_{k_1} \\ \sigma_k \\ [\sigma_k \mid\ ] \sqsubseteq \Gamma_{k_1} \\ \Gamma_{k_1} \models\!\!\!< \sigma_{k_2}(\sigma_k) \end{array}$$

$$\begin{array}{cc} \mathrm{D}_k & \mathrm{NP}^{-}_{k_2} \\ \texttt{the} & \end{array}$$

b.

$$\begin{array}{c} \mathrm{DP}^{+}_{7} \\ \sigma_6 \\ [\sigma_6 \mid\ ] \sqsubseteq \Gamma_7 \\ \Gamma_7 \models\!\!\!< [\ \mid w_r{:}\, \mathit{mule}\ \sigma_6] \end{array}$$

$$\begin{array}{cc} \mathrm{D}_6 & \mathrm{NP}_8 \\ \texttt{the} & \texttt{mule} \end{array}$$

In (24b) the result of combining (24a) with a description of a common noun is shown. Here the process of taking arbitrary witnesses has resulted in certain instantiations for k, k_1 and k_2.

With an elementary tree description for *the* as in (24a), a text such as the one in (25a) can be assumed to have been uttered in a context that does not contain any reference to mules at all. The relevant local context for *the mule* is given in (25b) and it is clear that, if σ_6 is identified with u_3, the required entailment holds. This is a case where a presupposition is 'caught' or 'bound' by previous linguistic material. A case where presuppositional material must be accommodated is given in (25c). Here the discourse referent, say σ_2, must be assumed to be already present in the background B and, since $[\ \mid w_r{:}\, \mathit{sun}\ \sigma_2]$ must be entailed by B, the latter must also contain sufficient material for this to be the case. A simple way for B to satisfy this requirement is if in fact $[\ \mid w_r{:}\, \mathit{sun}\ \sigma_2] \sqsubseteq B$.

(25) a. Pedro has a mule. The mule is happy.

b. $B \oplus [w_r\, o_1\, u_3 \mid w_r{:}\, \mathit{Pedro}\ o_1,\ w_r{:}\, o_1\ \mathit{has}\ u_3,\ w_r{:}\, \mathit{mule}\ u_3]$

c. The sun is shining brightly

d. If the weather turns out to be good, the king will be overjoyed

That the material that is in fact accommodated is not always the *weakest* DRS such that the entailment requirement is met with is shown in (25d). Here it would be sufficient to assume that the background contains, say, σ_9, plus a condition that can be paraphrased as 'if the weather turns out good σ_9 is king' (for local contexts involved in a conditional, see the Appendix). But clearly this is not what is accommodated in ordinary contexts, where plain 'σ_9 is king' is preferred. We agree with (Beaver, 2001) that accommodation, while constrained by requirements that derive from properties of the linguistic system, is also a matter of common sense reasoning. The linguistic requirement for a felicitous utterance of (25d) is that B entails 'if the weather turns out good σ_9 is king', but there are many ways in which this requirement can be met, including the possibility that B contains 'σ_9 is king'. The present theory underspecifies these possibilities. Common sense reasoning is needed to pick out the most likely of them. While we have nothing to contribute to the theory of common sense reasoning, we want to point out that from the perspective of the present theory interaction and interdependency between linguistic reasoning from input descriptions and other forms of human reasoning is to be expected. The grammar is a reasoning mechanism and grammatical and other modes of reasoning can easily interact.

(26) a. If John marries a woman, the unlucky female will be unhappy

b. $B \oplus [w_r\, o_1\, u_3 \mid w_r\colon \mathit{John}\ o_1,\ w_r\colon o_1\ \mathit{marries}\ u_3,\ w_r\colon \mathit{woman}\ u_3]$

c. $[\ \mid w_r\colon \mathit{female}\ u_3,\ w_r\colon \mathit{unlucky}\ u_3]$

d. $[u_9 \mid w_r\colon \mathit{woman}\ u_9] \Rightarrow [\ \mid w_r\colon \mathit{female}\ u_9]$

e. $[u_9 \mid w_r\colon o_1\ \mathit{marries}\ u_9] \Rightarrow [\ \mid w_r\colon \mathit{unlucky}\ u_9]$

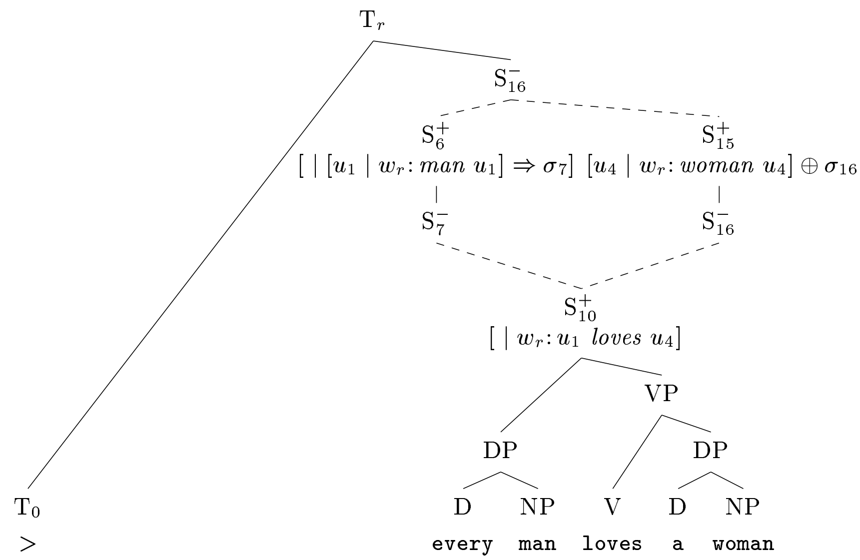
In (25) we encountered some examples in which the material associated with a definite description could either be unified completely with material already in the linguistically generated part of the local context of that description or had to be accommodated in its entirety. For good measure we also give an example where some material (the discourse referent) is unified with existing material, while other material must be presumed to be in the background. Consider (26a). The local context of *the unlucky female* in this sentence is given in (26b) and it is clear that its discourse referent can be taken to be equal with u_3. If this identification is made, (26c) must be entailed by (26b) and the common sense reasoning component must abduce one or more plausible conditions such that, if these conditions are assumed to be in B, the desired entailment holds. Clearly, one condition that can easily be assumed to be in B is (26d), 'women are females', and a more contentful assumption is (26e), 'whoever is married by John is unlucky'. If these assumptions are made, the required entailment holds.

Syntactic and semantic underspecification are inherent in the formalism. In the previous sections we have seen that the formalism presented in this paper essentially underspecifies linguistic structures. The description that is derived from a given input may be compatible with the fact that the discourse referent associated with a pronoun is identified with one previously mentioned referent or with another. It may also be compatible with the accommodation of one plausible condition that will cause entailment of a given elementary presupposition by its local context or with another. This underspecification is an essential feature of the formalism. Since descriptions are the vehicle for linguistic representation and since descriptions may have more than one model, more than one structure may correspond to a given input.

This underspecification is not restricted to anaphora and presuppositions but is a property that pervades the grammar. In (27b) a description is shown that can be derived from an input description for (27a) (see also (Muskens, 1995; Muskens, 2001)). The description underspecifies two scope possibilities, one in which $7 = 10$ and the existential outscopes the universal, and one in which $16 = 10$ and the scoping is reversed. It may be worthwile to note that the upper part of this description is closely analogous to the relevant Underspecified DRS in the theory of (Reyle, 1993).

(27) a. Every man loves a woman

b.



In other words, the perspective chosen here automatically provides us with the underspecification of structures that in standard DRT has to be added as an extra. For examples of underspecification of syntactic structure, see (Muskens, 2001).

6 Conclusion

The descriptions perspective on syntax and semantics advocated in this paper leads to a considerable change in the overall logical architecture of linguistic theory. Nevertheless it remains compatible with most existing ideas in mainstream linguistics and can even be argued to make for a smoother integration of those existing ideas. The overall logical architecture is changed because the theory focuses on descriptions of linguistic objects (including meanings and the form-meaning relation) rather than on those linguistic objects themselves; it also describes the syntactic and semantic levels of the grammar in parallel and does not follow the more common pipelined architecture in which syntactic forms must be produced before interpretation can take place. Thus room is created to account for an interdependency of syntax and semantics. On the other hand, the view that language users represent language by means of descriptions rather than by means of structures does not necessarily lead to any drastic revision of our understanding of what these structures are. In this paper combinations of standard Discourse Representation Structures and standard phrase structure trees were taken to be the objects of description. The theory obviously is also compatible with many other choices, but no radical departure from common practice is needed or indeed desired in this respect.

That the architecture of the grammar may lead to a smooth integration of existing theories may be illustrated using the example of the relation between antecedents and their dependents. Clearly, there are purely syntactic constraints on this relation, such as those given by the Binding Theory or the requirement that antecedents and dependents should concur in certain agreement features. On the other hand, Discourse Representation Theory holds that there are semantic, accessibility, constraints on the relation as well. Moreover, it is clear that the set of possible antecedent-dependent resolutions is narrowed down further by pragmatics. How are these constraints communicated between the various levels of the grammar? On a purely structural view one level of the grammar can only send *structures* to any other level, which means that a constraint or set of constraints can only be communicated by sending exactly those structures that satisfy the set of constraints. This leads to a *filtering* or *generate-and-test* procedure: The grammar first gives indexes to all DPs in a random way. Then for all resulting structures it is tested whether the Binding Theory and other syntactic constraints are satisfied. Structures that pass the tests are sent to the semantic component where further filtering is done, etc.

While such a generate-and-test perspective on grammatical processing may be acceptable for certain logically possible grammars, its wastefulness makes it unacceptable as a model of the real grammatical processing that takes place in the mind. A theory of grammatical competence that holds that only linguistic *structures* are involved in linguistic representation essentially predicts that the grammar is unprocessable. This is because the number of structures that need to be tested grows non-polynomically as a function of the length of the input. Under such circumstances testing becomes undoable.

In contrast, on the descriptions account constraints are communicated directly between the various levels of the grammar. As an example of this we have seen that adding axioms for the Binding Theory to our grammar $\mathcal{G}$, or adding the requirement in (20) (plus axioms for features) immediately narrows down the range of structures that satisfy the theory and as a consequence may also directly narrow down the range of Discourse Representation Structures that are associated with a given input. A highly desired form of modularity results, for it is possible to study syntactic and semantic constraints in their own right but it is also possible to combine them and to study their joint effects.

The view on linguistic processing advocated in this paper has been the following. Language users are in possession of a grammar $\mathcal{G}$ embodying their linguistic knowledge. This grammar essentially is a theory about which linguistic structures are possible. Language users are also in possession of general reasoning faculties that are not linguistic in nature. Some reasoning is explicit and concious, other reasoning may be implicit and subconcious. Upon being confronted with a text T, the language user can form an input description $\mathcal{I}_T$ of that text and can start to reason about what was said (and what was meant) on the basis of the conjunction of this description with $\mathcal{G}$, perhaps in further conjunction with extralinguistic knowledge. In general, the syntactic and semantic constraints in $\mathcal{G}$ need not narrow down the range of structures satisfying $\mathcal{I}_T$ to a single one, a considerable degree of underspecification may remain and what was said may not be completely determined.

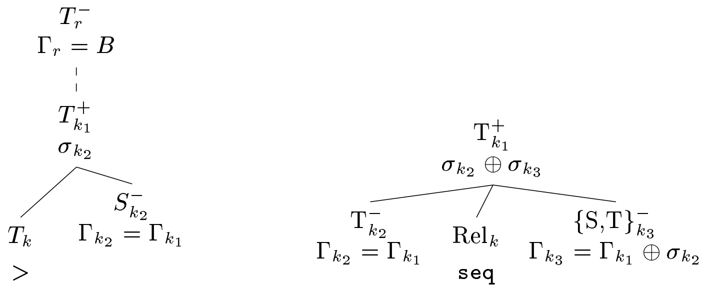
Some of this underspecification may be of the 'don't care' kind: the language user may be able to draw all conclusions that he desires without further filling in the details. But in many cases further specification is necessary and should come from pragmatic considerations, general constraints on discourse, interpretational constraints that are contributed by intonation, and so on. For an attempt to make some of these further constraints explicit and an integrated treatment of the resolution of anaphora and presupposition with the interpretation of focus and felicity conditions on discourse contributions, see (van Leusen, 2003).

A A Fragment of English in a Description Grammar for Discourse

For further illustration we specify a tiny fragment of English in the format of the description grammar proposed in the paper. For the lexemes of closed word classes (sentence and discourse connectives, determiners, and pronouns) we present the elementary and classifying descriptions in a single picture; for the lexemes of open word classes (names, nouns, and verbs) we specify the elementary and classifying descriptions separately, picking one or two prototypical lexemes to instantiate the classifying descriptions. The fragment covers sentences and discourses such as 'Pedro has a mule. He feeds it. He doesn't beat it.'; 'If Pedro has a mule then he feeds it.'; 'Pedro has a mule. Every man feeds it.'; 'Pedro beats the mule, or he feeds it'.

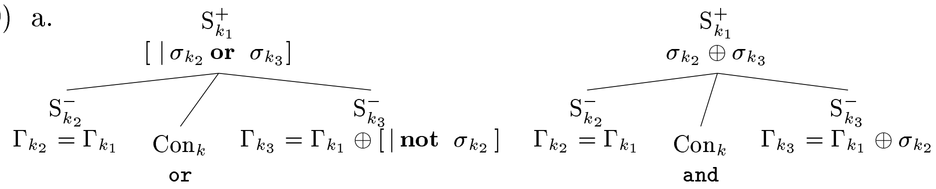
Elementary Descriptions for Some Closed Word Class Items Let us start with giving elementary tree descriptions for the two discourse relations that were considered in this paper. The $\{S, T\}$ notation in the description of the sequencing relation abbreviates the disjunction $\ell(k_3) = s \vee \ell(k_3) = t$. This disjunction allows for a more general way of grouping as was allowed in the body of the paper.

(28)

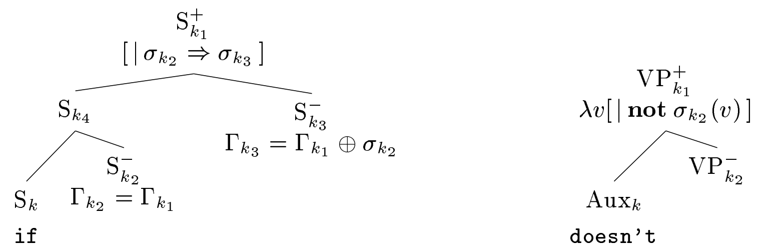


A next series of descriptions are those for sentential connectives in (29). Note that the description for *or* updates the local contexts of k_2 and k_3 in an asymmetric way. This is one of the possibilities considered in (Karttunen, 1974). For the other possibility, set $\Gamma_{k_2} = \Gamma_{k_1} \oplus [\,|\,\mathbf{not}\ \ \sigma_{k_3}\,]$.

(29) a.



b.



The determiners *a, no* and *every* receive descriptions in (30). Note that the local context of k_2 gets the same value in each case: the local context of the mother S plus the restrictor.

(30)

$$\begin{array}{c} \mathrm{S}^{+}_{k_1} \\ [u_k \mid\,] \oplus \sigma_{k_4}(u_k) \oplus \sigma_{k_2} \\ | \\ \mathrm{S}^{-}_{k_2} \\ \Gamma_{k_2} = \Gamma_{k_1} \oplus [u_k \mid\,] \oplus \sigma_{k_4}(u_k) \\ \vdots \\ \mathrm{DP}^{+}_{k_3} \\ u_k \\ \mathrm{D}_k \quad \mathrm{NP}^{-}_{k_4} \\ \texttt{a} \end{array} \qquad \begin{array}{c} \mathrm{S}^{+}_{k_1} \\ [\, \mid \textbf{not}\ ([u_k \mid\,] \oplus \sigma_{k_4}(u_k) \oplus \sigma_{k_2})] \\ | \\ \mathrm{S}^{-}_{k_2} \\ \Gamma_{k_2} = \Gamma_{k_1} \oplus [u_k \mid\,] \oplus \sigma_{k_4}(u_k) \\ \vdots \\ \mathrm{DP}^{+}_{k_3} \\ u_k \\ \mathrm{D}_k \quad \mathrm{NP}^{-}_{k_4} \\ \texttt{no} \end{array} \qquad \begin{array}{c} \mathrm{S}^{+}_{k_1} \\ [\, \mid ([u_k \mid\,] \oplus \sigma_{k_4}(u_k)) \Rightarrow \sigma_{k_2}] \\ | \\ \mathrm{S}^{-}_{k_2} \\ \Gamma_{k_2} = \Gamma_{k_1} \oplus [u_k \mid\,] \oplus \sigma_{k_4}(u_k) \\ \vdots \\ \mathrm{DP}^{+}_{k_3} \\ u_k \\ \mathrm{D}_k \quad \mathrm{NP}^{-}_{k_4} \\ \texttt{every} \end{array}$$

The last two closed class descriptions are those for the definite determiner *the* and the pronoun *he*. Other definites and other pronouns get very similar descriptions.

(31)

$$\begin{array}{c} \mathrm{DP}^{+}_{k_1} \\ \sigma_k \\ [\sigma_k \mid\,] \sqsubseteq \Gamma_{k_1} \\ \Gamma_{k_1} \approx\!\!\!\!| \ \sigma_{k_2}(\sigma_k) \\ \mathrm{D}_k \quad \mathrm{NP}^{-}_{k_2} \\ \texttt{the} \end{array} \qquad \begin{array}{c} \mathrm{DP}^{+}_{k_1} \\ \sigma_k \\ [\sigma_k \mid\,] \sqsubseteq \Gamma_{k_1} \\ | \\ \mathrm{D}_k \\ \texttt{he} \end{array}$$

Lexical Descriptions for Open Word Class Items The following descriptions exemplify classifying descriptions for proper names, common nouns, intransitive and transitive verbs.

(32) a. $\forall k\,[\textsc{pedro}(k) \rightarrow (pn(k) \wedge \sigma_k^{\pi\tau} = \lambda v\,[\ |w_r\!: Pedro\ v])]$

b. $\forall k\,[\textsc{mary}(k) \rightarrow (pn(k) \wedge \sigma_k^{\pi\tau} = \lambda v\,[\ |w_r\!: Mary\ v])]$

c. $\forall k\,[\textsc{man}(k) \rightarrow (cn(k) \wedge \sigma_k^{\pi\tau} = \lambda v\,[\ |w_r\!: man\ v])]$

d. $\forall k\,[\textsc{mule}(k) \rightarrow (cn(k) \wedge \sigma_k^{\pi\tau} = \lambda v\,[\ |w_r\!: mule\ v])]$

e. $\forall k\,[\textsc{sleeps}(k) \rightarrow (iv(k) \wedge \sigma_k^{\pi\tau} = \lambda v\,[\ |w_r\!: sleeps\ v])]$

f. $\forall k\,[\textsc{dies}(k) \rightarrow (iv(k) \wedge \sigma_k^{\pi\tau} = \lambda v\,[\ |w_r\!: dies\ v])]$

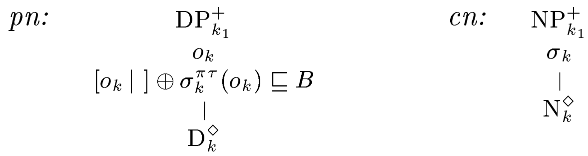
g. $\forall k[\textsc{has}(k) \rightarrow (tv(k) \wedge \sigma^{\pi(\pi\tau)}(k) = \lambda v' \lambda v\,[\ \mid w_r\!: v\ has\ v'])]$

h. $\forall k[\textsc{feeds}(k) \rightarrow (tv(k) \wedge \sigma^{\pi(\pi\tau)}(k) = \lambda v' \lambda v\,[\ \mid w_r\!: v\ feeds\ v'])]$

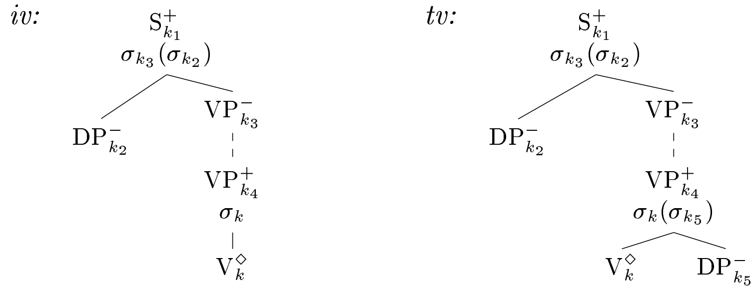
The treatment of proper names is slightly different from the one in (7d) inthe main text. Most semantic information has been taken out of the classifying description and has been moved to the elementary tree description in (33) below. That part of the information that is different for each name is communicated using a $\pi\tau$ predicate term.

The last four elementary trees are for proper names, common nouns, and verbs.

(33)



(34)



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