

The Informational View of Technologies in the Scientific
Practice

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written by

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Abstract

How do people come to know things? This is the core question of this thesis that looks into the ways humans engage in their epistemic practices. The easy answer: With epistemic technologies. In this thesis, I argue that the notion of technology should be broadened to include our conceptual devices alongside our material culture. This has a profound consequence on our conception of logic and mathematics, here understood as conceptual technologies that shape our inferential practices.

I rely on the insights from scientific and logico-mathematical practices to develop and defend the view put forth. Also, my aim is to show that the best framework to analyse the epistemic import of such technologies is that of the philosophy of information (PI). In light of this, I use the main PI's method – levels of abstraction – to model the process of knowledge production, mainly in logic and mathematics. The upshot of this thesis is having a method for investigating the epistemic roles and values of conceptual (eg. negation, validity) and material (eg. microscope, telescope) tools in science. This can serve as a new framework for an empirical study of scientific and logico-mathematical practices.

Contents

1	Introduction	1
2	Practices and Technologies	3
2.1	What are technologies?	3
2.2	Cognitive Technologies in Logic and Mathematics	15
2.3	Logico-mathematical Practices in Science	22
2.3.1	Concepts and Practices	23
2.3.2	The Evidence Limit	24
2.3.3	The Social Character of Practices	26
3	The Informational View of Technologies	31
3.1	The Informational Turn	31
3.1.1	The Method of Levels of Abstraction	32
3.1.2	Two Case Studies	34
3.2	The Epistemic Value of Technologies	47
4	The Technological View of Logico-Mathematical Practices	50
4.1	Epistemology of Logic and Mathematics	50
4.2	Logico-Mathematical Niche	57
4.3	The Philosophy of Logico-Mathematical Practices	60
5	Conclusion	63

1 Introduction

The thesis investigates the roles technologies play in epistemic practices. By that, I mean to investigate how the tools we create help us form beliefs, test hypotheses and produce knowledge. To do so, an overall goal of the thesis is to examine the epistemic aspects of given scientific practices systematically. More specifically, I will look at the practices of logic and mathematics to identify how the tools that are used in those practices improve the way we can come to know things. Still, as I will argue, this view can be generalized to include scientific practice more broadly. So, the examples of tools that I use include concepts such as negation and various inference schemata, but also tools such as a microscope and a telescope.

Nevertheless, the most significant focus is given to logico-mathematical practices since their products may be understood as tools that guide the development of inferential practices in general, thus directly shaping the development of science and its methods. To illustrate, the way we conceive of the concept of consequence determines what we can infer from a set of premises. In a sense, it determines the boundaries of the space of admissible conclusions, and as such, it determines the way agents can engage in an inferential practice. This, in turn, has a direct influence on the production of scientific knowledge in general.

Keeping such a goal in mind, the thesis aims at reaching several subgoals. Firstly, it argues for the acceptance of the view that logic and mathematics can be understood as cognitive technologies. This view is put forth and defended in 2.1 and 2.2. Secondly, I will show that if one accepts such a view of logic and mathematics, then their practices are shaped by the objects they create - instead of looking at logic and mathematics as the atemporal study of concepts that are somehow 'given'. This conclusion is drawn already in 2.2, but the consequences this has on the nature of the scientific practice are explored in 2.3. More specifically, in 2.3, I will argue that, for instance, logical concepts shape our inferential practices, but since such practices are crucial for the practice of science, they shape the way we produce the scientific knowledge too. Next, I aim to show that an informational framework can easily accommodate such a view of logico-mathematical practices, which is developed in Section 3. There, I will use the method of levels of abstraction to explicate the informational character of technologies - with case studies of the microscope and of the logical negation. I will use this analysis to highlight commonalities between such, seemingly, different tools, which will motivate the need for a new epistemology of logic and mathematics, as argued for in Section 4. Moreover, I will demonstrate in Section 4 how such a framework

can help us illuminate some of the traditional issues in the intersection of philosophy of science, logic and mathematics - namely the indispensability of mathematics for natural sciences and the tension between the atemporal character of the subject of logic and the temporal character of its tools. I will conclude the thesis with an outline of the consequences of the new understanding of scientific practices with respect to technologies and highlight the main issues to be further explored.

2 Practices and Technologies

Section 2 introduces the main concepts and issues and delivers the first results of this thesis. In a sense, it explains what do I mean by the title of this thesis. So, I start by answering the question of what technologies are and tie this notion closely to that of practices. Furthermore, I introduce the conceptual machinery used to formulate the informational view of those technologies. These two goals are executed in Subsection 2.1. Then, in Subsection 2.2, my aim is to show the benefits of understanding logic and mathematics as instances of such technologies. Finally, I argue in Subsection 2.3 that such understanding of logico-mathematical technologies can help us make sense of their role in the broader scientific practice.

2.1 What are technologies?

This subsection has two main aims: to motivate a specific view of technologies and to introduce the main concepts that will later be used to formulate the new framework for investigating the epistemic roles of technologies. For the motivating part, I will argue that we ought to expand the notion of technologies to include concepts - alongside material instruments, if we want to make sense of human practices. To achieve that, I introduce the notion of affordances, which will be the main conceptual tool for formulating the new framework.

Roles and kinds of technologies The impact of technologies on the agents that use them is profound. From the stone knapping techniques of our distant ancestors and crows shaping sticks into hooks, all the way to our reliance on traffic signs and mobile apps to solve everyday problems - both human and non-human animals use, alter and eliminate parts of their surroundings to save time and energy in doing various tasks or to be able to perform activities otherwise impossible to do. This phenomenon of world-changing to accommodate easier task fulfilling is examined from a plenitude of disciplines (archaeology, anthropology, and cognitive science being the most obvious ones) [Floridi, 2011][Malafouris, 2013][Russo, 2016][Heidegger, 1977][Proffitt et al., 2016]. Particularly interesting, in the context of this thesis, are the cases in which technologies accommodate better understanding - e.g. a diagram, better orientation - e.g. a map, better memory - e.g. a grocery list, and other tasks involving various cognitive processes.

Such usefulness of technologies for our practices highlights an important aspect of how human cognition works, namely cognitive processes are closely

tied with and influenced by the environment agents are situated in, i.e. the way they understand their environment. This contrasts the received view common for the western understanding of human thought and experience, which relies on the Cartesian idea that our conscious mind operates independently of its material aspects. This view was introduced to contemporary philosophy of cognition by Putnam [1967]. In his 1967 paper, Putnam argues that we should take a functionalist perspective on cognition. In functionalism, since the same function can be implemented in a variety of material substrates, the body does not have a genuine role in constituting cognitive processes. In such a view, cognitive processes start with our senses through which we receive information from the environment. After perceiving the environment, a computational process starts with the formation of judgements about the received information by running it through an internalized database of (innate) recognizable patterns. Finally, such processes enable higher cognitive functions, such as logic, reason, and abstraction which evaluate the importance of the given stimuli. This position was championed by Fodor, as seen in Fodor [1975], Fodor [1981] and his other works.

The view emerging in the contemporary cognitive science takes a different route, by taking the enactive approach, introduced in Varela et al. [1991] in which cognition is grounded in the sensorimotor dynamics of the interactions between an organism and its environment [Stewart et al., 2010]. The cognitive processes are not seen as internal computational and representational processes. Instead, cognition is viewed through the prism of a coupled mind-body-environment system [Menary, 2013] [Fabry, 2018]. There, researchers start from a seemingly trivial fact that the human brain inhabits a body, and that this coupled brain-body system is situated spatially and temporally situated, as well as embedded in the social world. The brain and the body together facilitate the processes in the mind, where the mind is constrained by the corporeal architecture for its modes of functioning. But, in this coupled system, technologies should also have their place since tools influence in what relationship we are with the environment, how we can interact with it and what can we get out of it. So, if our cognition is shaped by the fact of our embodiment, I claim that it is also shaped by the fact of our technological status. In other words, since the way we experience the environment is closely tied with the practices we engage in in the world, then the tools that we use to perform those practices, at least indirectly, influence the way an embodied mind operates.

This interest in studying technologies can be captured with a claim that disciplines such as cognitive archaeology, anthropology and others want to investigate: Technologies shape minds and histories [Malafouris, 2013]. Let

us unpack this. By ‘technologies shape minds’ I mean that the use of technologies shapes the way we interact, understand and reason about the world [Novaes, 2012][Russo, 2016][Russo, 2012]. Just consider what writing did for the way agents interact with one another [O’Looney et al., 1989], what a microscopes and telescopes did for the way humans understand their position in the world [Baird, 2004], or what machine learning did for the way humans reason about their surroundings [Leonelli, 2010]. Writing enabled spatially and temporally distant communication, great memory capacities, it facilitated learning, but also a different kind of historical understanding of one’s own culture, etc. If one considers microscope and telescope, both technologies opened up completely new worlds to humans. The drop of water was as alive as a city, which radically changed the way we conceive of our environment and the way we think about what is possible. On the other hand, the telescope reshaped the way we think about our own species by putting it into a perspective of a vast universe, placing us on the outskirts of one of many galaxies. Finally, the computing revolution changed the way we organize and retrieve knowledge, but also, by outsourcing task such as medical diagnosis, the way we reason about the phenomena we encounter.

In a nutshell, technologies are artefacts that shape the way agents experience both the world and themselves. Consequentially, technologies that a species develops and uses modifies the way that species develops. Consider the examples of a beaver’s dam, ape’s ant stick, and, again, human writing. Each technology influenced the survival chances of the species as a whole, but also the way each individual created their own living space - humans are not special with respect to that. But, humans are still special in a certain way; namely, technology also influences the way humans understand their future, as well as history. Just consider all the utopian and dystopian visions of near and distant future brought about by new technologies, as seen in pop culture. Even some academics make predictions based on a premise of technological determinism [Russo, 2018] - the idea that technology necessarily leads us to a certain scenario; for instance Negroponte [1995] - an architect investigating the benefits of the digital in design, talks about the end of censorship and the flourishing of free speech, while an economist and social theorist Rifkin [2004] predicts a catastrophic future for our species due to work automation. Furthermore, technology also changes the way we understand our past. Consider the way genome sequencing changed the way we think of our past, placing us in the wider story of evolutionary processes, with human taking away by themselves their special place of ‘the owners of the world due to divine decision’.

Let us examine the development of writing to illustrate the point of the

role of technologies even further. The first writing system we know of is the cuneiform system, which dates back to around 3000 BC and was pictorial in its origin [Fabry, 2018]. Through the generations, the cuneiform system was cumulatively refined to accommodate an accurate representation of abstract ideas and relations. This development was not driven by some innate cognitive mechanisms, nor through some random processes [Fabry, 2018]. Instead, the writing system was revamped in an effort to shape economic practices of cultures that used it, i.e. the representation of abstract ideas and relations was needed for the advancement of trading and easier organization of the social life. Note that, in line with the embodied view of cognition, the cognitive capacities and biases, as well as the body of agents, constrained the properties that the symbolic system can have [Menary, 2013] [Fabry, 2018]. Just consider the arrangement of lines and spacing between both words and letters. Such writing norms were not intrinsic properties of the writing systems, but a set of properties imposed by the agents to afford easier representation of economic and social transactions - be it of goods or information.

Technologies are tied to practices All this points to the effects of technologies on the way agents act and think. In other words, technologies are artefacts that shape practices - be it that they create them the way a microscope enabled the study of the microworld, modify them the way digital technologies changed distant communication [Russo, 2018] or completely eliminate them the way the professionalization of chemical instrument making gave a fatal blow to the practice of alchemy [Beretta, 2014]. But, technology-human relation is a two-way street. Technologies shape what we do and how we can do it, and we design technologies to improve our current practices. This feedback loop characterized by mutual constraining and enabling of technology and its user will be a recurring theme in the rest of the thesis. Given the characterization of technology adopted, we can say that an artefact that does not have a role in some practice is not a technology at all. Let's entertain this thought for a moment. To illustrate this claim consider an agent that possesses two sources of information - oracle A that can instantly and correctly answer any question about a field X and oracle B that gives an answer 'Yes' to any question it is asked about the field X. It is reasonable to expect that the oracle A would be regarded as an 'out of this world' *tool* that can solve many open questions in field X. It would be ridiculous, on the other hand, to characterize oracle B as any kind of tool, since we can have no use of it whatsoever. So, it seems that the use in the actual practice is what makes something suitable for being called a tool, i.e. a useless artifact is not a tool.

Let us think of the relationship between usefulness and being a tool some more. Of course, for a given practice, there can be tools that can improve it more than some other tools. Moreover, there could be tools that cannot improve any practice of importance for us. But, such tools are not transferred to other agents and die out soon. That is why we can, in principle, say that there is no tool which is not useful for some practice. Nevertheless, let us see what kind of tools could fall into the group of useless tools, as they might tell us something about why other tools are successful. First, consider the Rosetta stone. The Rosetta stone has three inscriptions of a decree - in Ancient Egyptian hieroglyphs, Demotic script, and Greek script. Obviously, it had an important purpose in the time it was made. Yet, it could have happened that, at one point in time, there were no humans that knew either of those scripts and at that point of time the Rosetta stone would have been completely useless (unless one gives it an aesthetic function, but the point remains). These two examples - oracles and the Rosetta stone - point us in a direction explored in Section 3, where the concept of information is introduced to explain the notion of technology.

To make a concrete example from the history of logic - consider the connective tonk (here symbolized by \clubsuit), formulated in Prior [1960], which has the following introduction and elimination rules:

$$\begin{array}{l}
 \varphi \\
 \hline
 \mathbf{I} \\
 \varphi \clubsuit \phi \\
 \varphi \clubsuit \phi \\
 \hline
 \mathbf{E} \\
 \varphi
 \end{array}$$

As Prior notes, such a connective, if accepted as logical and, consequentially, its rules being truth-preserving, we can prove that φ implies ϕ for any φ and ϕ . If we accept this, we are lead to trivialism. It is precisely because this connective would lead us to trivialism; there is no practical use of such connective. A practice is shaped by the constraints that are imposed on it. In other words, if there is nothing to constrain an activity, that activity cannot be a practice.

Here, a concept from ecological psychology might become useful to think about why the notions of technology and practice are coupled. In the foundational work of ecological psychology, Gibson [1979] created a useful concept with the aim of explaining how people experience and act in the environments they are situated in - 'affordances'. Affordances refer to the cognitive understanding of the opportunities that the environment offers to an agent.

In other words, Gibson's concept of affordance has to do with the features of the environment that suggest to the agent how it is to be used. For instance, a doorway offers us to walk through it. We can think of it as if something about the space or the object or the structure signals to us how we might engage with it. To illustrate the importance of the concept of affordance for the methodology of cognitive sciences consider the case of architecture. We experience buildings and landscapes in a way that is embodied, i.e. our experience of the environment is determined less by its formal composition and more by how we apprehend its affordances [Goldhagen, 2017]. As Goldhagen illustrates in her work on enactive architecture, the feeling of control a child has over its home environment is inversely related to the number of people per square foot who live in that area. Consequently, children that develop in environments in which they do not *feel* that they have control over that environment have a diminished sense of autonomy, agency and efficiency. So, children raised in the slums, for instance, not only enjoy fewer opportunities, but they are also less capable of taking advantage of the opportunities available to them. In terms of affordance, we can say that such children not only are developed in an affordance-impooverished environment, but they actually do not develop sufficiently the cognitive tools needed for identifying those affordances. The importance of such cognitive tools is further exemplified in a study discussed in Goldhagen [2017], where the learning progress of pupils in classrooms was investigated, and the researchers identified environmental parameters — color, choice, complexity, flexibility, light, and connectivity — that significantly affect learning. So, cognition - in the embodied and situated sense - influences how we perceive the affordances of the environment and the affordances of the environment influence back the cognitive processes that are going on in the brain. So, I claim that to understand how we come to know things, we ought to consider what are the technologies (buildings, writing systems and concepts) with which we build our understanding of the world, i.e. with which we engage in epistemic practices in general.

The connection between the material culture and cognition is of special interest for archaeologists due to methodological reasons. They want to be able to infer something about cognition, only by looking at the remains of the material culture - archaeological artifacts. To enable that kind of inference, Malafouris developed the material engagement theory. The material engagement theory is an alternative ontology of the mind created to accommodate for the inferences about cognition based on the material culture. As such, it also falls under the umbrella term of the extended cognition theories as, not only does thinking rely on the resources of our perceptual and motor systems, but the artifacts and other organisms can be actual components of the

cognitive processes. Building on this, Malafouris, in motivating his theory, echoes Merleau Ponty's blind man's stick example to stress the role of the cane in the perceptual system of the user, where user's actual neural patterns change to accommodate the artifact. Merleau-Ponty [1962] takes the example of the blind man's stick to show that when learning to use the tool, on one side the blind man acquires motor skills breaking the barrier between himself and the stick, but also acquires perceptual skills, as he learns to perceive his environment through the stick. Furthermore, the material culture enables us to interact with our own cognition in the same way how writing a word enables us to think about the written word. Material culture thus enables - not only the improvement of our cognitive abilities (e.g. writing improving memory capabilities) but also the creation of new abilities; such as the metacognition.

To illustrate this point further, consider the analysis of technologies in Simondon [1980]. For Simondon, a 20th century French epistemologist, technology is not constituted by a single object - a single tool or machine. Instead, it is constituted by relations between the parts of the tool, relations between a tool and the agent use them, and finally by relations between a tool and its material environment. So, Simondon's notion of technology is made up of a particular tool together with the practices it is used for, as well as the conceptual apparatus that made its design possible. Hence, Simondon argues against a utilitarian or merely instrumental view of technologies, by acknowledging their active character in forming practices agents can participate in.

Moreover, by technologies taking part in the practices, they are also being the subject of change. Take, for instance, the changes in scientific concepts. Initially, such concepts enable us to think and investigate matters we weren't able to examine before. Yet, in the course of epistemic production, those concepts are also being changed, because they are being placed in relations with new epistemic artifacts created as a consequence of the scientific practice.

The same things hold for the material technologies which also, in a sense, evolve together with the practices they shape. This way of thinking about the role of technology is prominent in the field of the philosophy of technoscience, where technology is taken to have a genuine epistemic role in the production of knowledge [Russo, 2016][Russo, 2012]. The main change philosophy of technoscience introduced with respect to the way we view technologies concerns the relation between technologies and its users. Specifically, technoscience starts with the fact that the epistemic agents, e.g. scientists, are situated in the network of interactions among both technologies and other agents and the practices they perform are embodied in the sense of them being performed by embodied agents and the practices are embodied

in the technologies. Hence, practices and technologies are coupled in their use.

But, I claim that the relation between technologies and practices also goes the other way around too. That is to say, there cannot be a practice, if there are no tools - in the broad sense presented above, to perform it. Even singing and dancing require something artefactual - agent made - that enable it, for one cannot dance or sing if one does not have a form to follow, i.e. to conform to, no matter if the agent performing it is a bird or a human.

The point of the last example is to show that a technology need not be material. If we accept that a technology is an artefact that shapes practices, the space of technologies becomes much wider than ordinarily understood. Just consider concepts, e.g. the way the concept of GDP shapes economic practices or how marriage shapes social ones. The claim here is that there are non-material things that fulfil the purpose of practice shaping. This is far from a trivial claim because we then need to account for the exact import those tools have, e.g. what is the epistemic import of such tools in the scientific practice. In other words, it has concrete and profound consequences of the way we understand various practices, especially those that involve epistemic concepts such as justification, proof, and truth. If concepts are tools for shaping practices, then the way we construe epistemic tools has consequences of the way we perform epistemic practices [Allo, 2017]. Suddenly, the things that we thought of as our objects of investigation - say knowledge - becomes a tool for performing epistemic practices with a goal of shaping the way one understands her environment.

The theoretical pay-off of such a broad understanding of technologies is twofold. Firstly, it provides us with a new way to account for the production of new technologies. For instance, in the economics and management literature, the production of new technologies is attributed to exceptional individuals [Gartner, 1988] that produce them in a *deus ex machina* fashion, or completely by chance [Arthur, 1989] or through purely external pressures [Chlebna and Simmie, 2018]. By acknowledging that to create new technologies we need to have some tools in stock (cognitive and material), we get a clearer picture that takes the cognitive and material aspects of technology production seriously. Secondly, it provides us with a new way of thinking about the epistemic aspects of our practices, where tools for thought are taken on par with tools for material engagement. Then, by looking at the commonalities between the two, we can get new insights into their epistemic import in our lives.

So, when we talk about practices, we can also talk about cognitive practices - practices performed with the use of cognitive tools. Cognitive prac-

tices are patterns of activity spread out across a population [Menary, 2013]. Agents learn and acquire these practices during development, making these practices collaborative in nature. In other words, cognitive practices are real activities that take place in the social environment and they can be carried out individually or in collaborative groups with the use of cognitive tools. We can think of those practices as active processes with which we think and engage in cognitive tasks. Because of the social nature of cognitive tools and practices, Menary calls our mind ‘enculturated’. He defined enculturation as the acquisition of cognitive practices during ontogeny. Of course, to perform those practices, we have to have tools for them. Hence, we can say that enculturation is the process of acquisition of cognitive tools and the acquisition of cognitive tools transforms our overall cognitive capacities. In terms of Gibson’s affordances, we can say that the process of enculturation is the process of learning to identify and exploit affordances in the social environment, i.e. the structured socio-cultural environment provides humans with epistemic resources for the completion of cognitive tasks. This environment is called by philosophers of cognition Fabry [2018] and Menary [2013] a ‘cognitive niche’.

One might object that, if we construe such a broad notion of technology, we risk making it vacuous. But, as I will show in the following sections, the inclusion of cognitive tools in the family of technologies enriches our understanding of the role technology plays in the production of knowledge. So, except for the theoretical pay-offs listed above, it enables us to delve deeper in the investigation of how humans come to know things and interact with their environment.

Technologies and construction To sum up, human cognition should be examined from an embodied and situated point of view because it is the coupled mind-body-environment system that influences the way we come to know and understand. Furthermore, the way we experience our body and environment is influenced by the practices we perform with that body and in that environment. Those practices are performed with tools; some of which are cognitive, but they all have a cognitive aspect of it. So, the way we experience the world is the result of this interaction between our tools and the environment. Because of this, the way we perceive and understand the environment is not representational - it is not descriptive of our environment; instead, it is actively shaped by the tools we use so we can better take advantage of our environment. We construct the way we come to know and understand. This is the main tenant of the constructionist epistemology - to know is to make [Floridi, 2011].

A few points on constructionism. Consider the notion of affordance again.

The affordances we identify depend on the tools we have for their identification. So, the ‘epistemic space’ that we can explore is constrained by the tools we can use for and in epistemic practices. Floridi [2011] calls this constraining of affordance - information, i.e. information is an artifact we construct based on what we can epistemically afford. More concretely, Floridi understands *semantic* information as well-formed, meaningful and truthful data. In other words, information is data that we constructed from the data we acquired and then semanticized, i.e. we placed it in the right network or relations with other information we have. To use an analogy, information is like a map, which is well-formed and truthful about the location it maps. This raises two important points. First, information is truthful about something in the same way a map is truthful. But neither is ‘true’ - a map is not true, it is truthful in the sense of useful for a specific practice. Secondly, it cannot be false - a false map is simply not a map, it is a set of scribbles on a paper or a computer screen. Analogously, false information is not information at all - information encapsulates truth. So, knowledge, according to constructionism, encapsulates truth because it consists of information.

Since, in Floridi’s philosophy of information, knowledge encapsulates information, i.e. knowledge builds on it, and information is constructed by the agent, knowledge is constructed too. More precisely, for Floridi [2011], knowledge is made by putting information in the *correct* network of relations - one can say it is perspectival. This is not a widely accepted fact. Although this whole thesis can be seen as a motivation for such a view, a few points are in order. The basic tenet of constructionism is that only what is constructible can be known. In a minimal sense, this means that we should at least be able to conceptually construct the object of our interest to be able to think about it in the first place. If one accepts the view I defended, by which concepts are tools for shaping our thinking and inferential practices, even in this minimal sense everything we know is constructed based on those tools for manipulation of our practices.

The manipulations of environment and acquisition of information can be fine-tuned through learning and reinforcement, i.e. by learning to use new tools we learn to construct new information. In a sense, these new ways for manipulation are adaptations to the environment, which gives them a basic kind of normativity [Menary, 2013]. This normativity of how to use a tool to take on an affordance instantiates agents’ sensitivity to the salient environmental variables - affordances. For Menary [2013], this is what allows for the possibility of intentional directedness. The tool and our use enable epistemic production, so we can think of knowledge as distributed throughout the agent-technology environment.

So, agents do not experience their environment holistically, nor are they merely passive recipients of information. Instead, agents assess the usefulness of various components of the environment, i.e. what it affords, while either acting or formulating goals in preparation to act, i.e. employing the right tools for the actions they want to perform. For constructionism, this is true even if the action we undertake is observation - an observation also involves the construction of information given the tools (eyesight, the concept of a tree, of a bird...) and practices (bird watching) we want to engage in. As an example of a general constructionist theory of vision, consider Biederman's recognition-by-components theory. According to that theory, when we look at something, we see it by composing that object in our mind by using geons - 2D and 3D objects such as triangles, cylinders, etc. (40 distinct geons in total). Hence, we do not 'see' a thing outside or have a copy of it in our mind. Instead, we create a picture that we can understand given the cognitive resources that we can operate with. Geons facilitate our rapid comprehension of the myriad form-based cues that the world throws our way and, naturally, given what tools are at our disposal there is a space of possibilities we cannot transgress without a change in our cognitive and conceptual apparatus. Furthermore, from the things that we can capture with our tools, some of them are more salient than others. These two components influence the way we engage in a certain practice then. We shall return to this, but for now we should keep in mind to ideas: we use tools to perform practices and tools can be cognitive and material, and knowledge production is a practice performed by epistemic tools.

This shift is the central thread of this thesis and I aim to explore it by looking at the prime example of epistemic practices - logic and mathematics. But, given the unified view of technologies I defend here, my claim is that by looking at the specifics of logic and mathematics, we can also get valuable insights for investigating the role of technologies in the wider scientific practice. The investigation of the role of technologies in the scientific practice is not new, but it has gained much traction in recent years. Yet, these developments have focused mostly on the understanding of technologies as material culture that either instrumentally or more deeply transforms the practice of science. [J van Benthem, 2007][Novaes, 2012][Ihde, 1991] So, the technologies of interest were constrained to the scientific instruments and methods and how they foster interaction between the practitioners of science and the production of the scientific knowledge and explanations. What I aim to show is that the tools of logic and mathematics have the same epistemic impact on our practices, so we need to include non-material tools in our reflections of technologies.

The trend of examining the interaction between technologies and practitioners was already evident in Bacon's writing. For him, a scientist is not merely a contemplator of nature nor a moralist [Rossi, 2011][Russo, 2012]. Instead, scientists are involved in an endeavour of understanding natural processes which enabled her to reshape them and direct them. Furthermore, a scientist is a member of a community that is held together by common practices. Hence, a discipline is defined and shaped by the practices used to reach some goal of the discipline, instead of by the thought and talk of individual 'solitary geniuses'. Some have extended this view of what shapes the knowledge producing practices to even include things such as formal languages [Novaes, 2012] or institutions [Chlebna and Simmie, 2018]. But, the view that there are non-material, in the sense of cognitive, technologies remains either unexamined or constrained to scenarios not involving the scientific practice [Hayes, 2018] - with the most famous exception being Carnap's work on the explication [Carnap, 1950].

In [Carnap, 1950] explication is a process where an informal concept, be it a concept that we use in everyday life such as *bold* and *adult* or in a more constrained contexts such as the scientific practice the concepts of *gene* and *atom*, is given a more exact and rigid formulation. Carnap's example of *fish* can illustrate the point. He notices that the everyday concept of *fish* does not coincide with the zoologists' notion of fish enveloped by the concept *piscis*. He points out that the reason why zoologists have artificially redesigned the notion of fish was to better accommodate the layman's notion of fish in the scientific theory. In other words, for the actual practice of zoology, the everyday concept wasn't suitable for the wider theoretical aims of the theory. Hence, the concept was changed so that it expands the explanatory reach and power of the theory, i.e. to improve the practices of zoological research. More precisely, in Carnap [1950], we see a normative aspect of concept creation, where a concept should be changed if it is more fruitful for the formulation of laws. Hence, concepts do play an important epistemic role in the production of scientific knowledge and their study, as well as of the ways we can modify them, is an important part of the study of science. Moreover, if we accept the relationship between practices and technologies argued for before, it is clear that we can build on the Carnapian tradition and claim that concepts really are technologies and that this inclusion is beneficial not only philosophically, but also for the actual practice of science.

In examining the role of technologies we have constrained ourselves to the scientific practice as a prime example of epistemic practices, so we can explore the epistemic aspects of technologies in a more structured environment, i.e. the philosophy of the scientific practice is both conceptually well-developed

and it has many well-studied case studies we can rely on. For the same reason, I will rely on a specific case study within the scientific practice to motivate the existence and importance of cognitive technologies - logico-mathematical practices. One might object that the epistemology of logic and mathematics is special when compared to those of sciences. Hence, an inference from the scientific practice (i.e. the practice of natural sciences) to those of logico-mathematical practices is a mistake. But, this is precisely the view that I reject. Here, I am very much in line with the anti-exceptionalist view of logic and mathematics, in line with Williamson [2017], Hjortland [2017], and others.

For an anti-exceptionalist, one is justified in believing a logical proposition p by being justified in believing a logical theory L containing p . But, we are justified in believing in L because it better accommodates the relevant data, and possesses more relevant theoretical virtues, than other available theories [Hjortland, 2017]. In a sense, the theory choice in logic and in sciences are very much in line. In the context of this thesis, a further connection is that both are shaped by artifacts their practitioners make. So, the insights from the scientific practice can be valuable for our understanding of logico-mathematical practices.

In 2.1, I have motivated and argued for a particular understanding of the notion of ‘technology’ - artifacts that shape practices. Then, I have introduced the main concept with which I aim to explain how the ‘shaping’ of practices work - affordances. Finally, I argued that such notion of technologies envelops cognitive tools alongside material instruments and that such understanding of technologies has a notable theoretical pay-off across disciplines aiming to account for human practices - social, epistemic, and any other.

2.2 Cognitive Technologies in Logic and Mathematics

In this Subsection, I motivate the understanding of logic and mathematics as cognitive technologies, i.e. the view that the logical and mathematical objects are tools that govern (shape, improve, and correct) the logico-mathematical practices.

Formal languages as tools Here, I rely on the insights from the constructionist epistemology [Floridi, 2011] [Russo, 2012] and the study of logico-mathematical practices [Allo, 2017][Novaes, 2012][D’Agostino, 2016]. The former provides us with the necessary conceptual apparatus to formulate the role of an epistemic agent - logicians and mathematicians - and artefacts

they produce (mathematical objects) as well as tools for their expression and manipulation (formal languages) - in the production of scientific knowledge. The latter gives us a variety of examples where we can observe how the nature of tools agents use impacts the cognitive abilities of those agents, as well how both the tools and the activities of agents shape the way a discipline transforms over time.

I start with a simple observation. The logico-mathematical practices differ depending on whether they are done with or without formal languages, for formal languages radically changed the way mathematicians' and logicians' daily research proceeds, but it also enabled them to achieve results otherwise unattainable [Allo, 2017][Novaes, 2012]. This is due to the two characteristic of formal languages identified by Novaes [2012] - computability and de-semantification - which enable researchers to go beyond their cognitive capacities. More specifically, computability of formal languages allows for automated treatment of the reasoning process, enabling practitioners to tackle computationally hard problems through the use of automatised sensorimotor manipulations. De-semantification feature, on the other hand, enables practitioners to look at problems from a de-contextualized perspective thus blocking so-called computational biases that are responsible for the discrepancy between everyday and deductive reasoning.

To exemplify, consider the paradigmatic example provided by Stanovich [2003], in his work on psychology and education, where a group of participants were given the following text:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

The participants had to rank a number of statements about Linda such as:

Linda is a teacher in an elementary school and Linda works in a bookstore and takes Yoga classes.

The fallacy the participants committed concerned the following two statements:

a) *Linda is a bank teller and is active in the feminist movement* and b) *Linda is a bank teller.*

In this case, a predominant ranking was a) above b) - in other words, that the probability of a) was higher than that of b). But, notice that a) is a conjunction of b) and another statement (that she is a feminist). Because of the conjunction, the probability of b) is at least as high as that of a), as b) cannot

be false if a) is, while the other way around is possible. Of course, avoiding such fallacies can be done through de-semantification, which is the claim of both Stanovich [2003] and Novaes [2012]. The de-semantification part is precisely what we have done here. Instead of thinking about the meaning of claims a) and b), we took into consideration the syntactic differences between the two.

But we should not stop here. Formal languages can be considered technologies because they shape inferential practices, i.e. by developing new formal tools agents are able to take part in practices they were unable to take part in, for instance, Aristotle's categorical notion of quantifiers was inapt for expressing mathematical proofs, while the contemporary unary concept of quantifier enables their formal representation, which had an impact on the research programmes developed in logic and mathematics [Uzquiano, 2018].

To clarify, humans do have some mathematical capacity even in the absence of formal languages - we can complete basic arithmetical tasks from an early age - moreover other species exhibit a capacity for understanding numerosity concepts too. Nevertheless, the kind of mathematical capacities that formal systems afford concern the manipulation of numbers as part of a public, social system of mathematics with its own representations for numbers, functions, relations and other mathematical objects. We can understand such representational systems in two different ways. They can be understood as abstract systems of representation or as inscriptional schemes that can be embodied on a paper or on a computer. These representational systems possess norms such as combinatorial rules - to compute the product of two numbers we write them out and apply the partial products algorithm. Such manipulations of inscriptions are normative as there are right and wrong ways to manipulate them - as exemplified by the score on our mathematics exams. Except for the fact that there are norms for computing, there are also norms for the different representation depending on in what kind of practice we are engaging in - for instance, the pie charts to represent a variety of different quantitative relationships [Menary, 2013]. Nevertheless, such inscriptions are not the only tools that shape those kinds of practices. There are tools that have at least as much of an impact on the logico-mathematical practices, as the material formal languages do. Those tools are logico-mathematical concepts.

Concepts as tools Take, for instance, the concept of negation. Nowadays, the dominant view of negation in logic is negation as a contradictory forming operator. But, this view originated only (relatively) recently with Frege, when he defined negation as a function that maps a truth-value into the

opposite truth-value [Horn and Wansing, 2017]. This view was ‘canonized’ by Russell and Whitehead in their *Principia Mathematica* where negation is seen as the syntactical counterpart of contradiction. But Aristotle, for instance, took the negation and contradiction as not only separate, but also the latter notion was purely semantic, while the former syntactic [Speranza and Horn, 2010]. It is due to this different understanding of what the concept of negation is, that produced radically different ways the logicians created formal languages. Even now the biggest debate between the classical and paraconsistent logicians is precisely the way we should understand the concept of negation and from it how we should organize formal languages and not the other way around [Priest et al., 2018].

The lesson from this is that we should not look at concepts such as negation as given, i.e. as a sort of natural objects that we can study. Instead, we need to approach them as concepts that shape the way we use logic. This point can be illustrated already with just looking at the history of negation in the Western world. The idea that there is no ‘real negation’ out there becomes even more palpable when one considers the logico-mathematical concepts that show up in other cultures such as in India. For the purposes of this thesis, we will focus on the Anglo-American approaches to negation - most prominently the American Plan and the Australian Plan for negation [Berto and Restall, 2019].

To illustrate the difference between the American and Australian Plan, we have to take a short tour of semantics for formal languages. Given model-theoretic treatments that evaluate formulas at points, Meyer and Martin [1986] distinguished those which take the relationship between formulas and points to be two-valued and those which take it to be many-valued. For example, a two-valued approach would say that for some formula A and every possible world x , A is either true or false in x . In a four-valued approach, for instance, A can be true, false, both or neither. The former, two-valued approach, is characteristic of the Australian Plan, while the four-valued approach is predominant in the American Plan for negation. The initial commitment for each of the Plans results in a radically different conception of what negations is. In the Australian Plan, negation is understood as an exclusion-expressing device, i.e. a linguistic device used to say that two things are incompatible. In the American Plan, negation is a contradictory forming operator, i.e. an operator used to say that a thing and its negation constitute a contradiction. Furthermore, only because of this one commitment - how to understand negation, the resulting formal systems of the two Plans are radically different in their complexity, expressibility and usefulness. But, is there a party that is right, while the other one is wrong? Furthermore, given

that when the two parties say negation, but mean different things, could it be that they are simply talking about two different tools?

Given the way I conceived of technologies in section 2.1, we should say that negation is a tool for performing some practice. But what practice would that be? Negation is basic for our grasp of language, communication and implication, logical reasoning, mathematical proof strategies - to name a few. Of course, neither of the plans aim at capturing exactly this kind of all-encompassing negation. Instead, they are trying to model it with formal tools, approximate it if you will. But, if the negation is a tool, no matter how basic it is for our understanding of the world, it is designed and not discovered. As such, there is a true negation and the true account of it, as much as there is a true piece of art. There are definitely good ones and bad ones, but there is no 'platonic' negation that we can capture with formal tools. There is a cognitive tool (negation) and a material tool (formal account of the negation) that we are calibrating with accordance to a specific practice that we want to engage in with it.

The example of the concept of negation is just one motive for looking not only at the produced material culture as an example of technology but also to take the cognitive artifacts as types of technologies, i.e. logic and mathematics as cognitive technologies [Allo, 2017]. Once we digest this, the concepts of validity, consistency, possibility, are seen as cognitive artifacts with the purpose of improving and shaping the practice of logic, contrary to the received view where logicians are giving a description of what these concepts are within logic [Allo, 2017]. To clarify, the concepts in logic and mathematics are not merely tools for calculation or computation. When Allo [2017] claim that they are *cognitive devices*, he means that logic and mathematics are sciences concerning themselves with design strategies for creating tools that can improve our conceptual apparatus and, consequently, our cognitive abilities by shaping the way we engage in inferential practices.

Furthermore, the logico-mathematical practices are viewed as dynamic activities that concern themselves with agency and interaction, instead of solely with proof and computation. [J van Benthem, 2007] This is so, since proof and computation become technologies used to transfer epistemic artifacts such as justification to peers, making those practices inherently social. Moreover, under this understanding, the subject matter of logico-mathematical practices is not restricted to inferences, but also to the way we ask and answer questions, as well as communicate them - these can be called inferential practices since they are essentially the practices of transferring epistemic artifacts. The idea of transferability will come in later when we discuss the epistemic values of technologies. Nevertheless, a few points are in order. As

I've pointed out before, transferability as an important feature of artifacts and knowledge is present already with Bacon, since, for him, the feature or 'real' science was the fact that it can be taught to others and it can be done by a community of researchers. In other words, the hermit knowledge that relied on obscure (non-transferable) practices, wasn't scientific knowledge. This idea is present also in Massimi [2018] and Novaes [2012]. For Massimi, a philosopher working on the scientific perspectivalism, it is the ability to express one's perspective in the other - translatability over perspectives - that guarantees the successfulness of a research programme. Moreover, in Massimi [2018], this translatability is what 'keeps' realism about science alive. For Novaes [2012], it is precisely the ability to transfer the reasons for accepting the claim (steps in a proof in the case of logic and mathematics) to another agent that ensures the deductive standards of logico-mathematical practices.

All this makes logic and mathematics a constructionist activity. The aim of agents is not to capture and represent some phenomena, but to create new tools that can be used in inferential practices, thus enabling the production of new knowledge. [Floridi, 2011][Russo, 2016] Moreover, the tools they create can be transferred to other domains, since the tools for inferential practices are used in the scientific practice no matter the discipline. This position has to be delineated from two positions with similar names - strong epistemological constructivism and mathematical constructivism. The first one states that 'truth' and 'reality' are socially constructed [Steup, 2018]. Note how constructionism *a la* Floridi, holds that information is that which is afforded and constrained by the data acquired (where the acquisition is further constrained by the capabilities of the agent). Hence, not anything goes. Truth and reality as we know it are relational to the agent constructing knowledge and the process of construction is highly constrained.

More interestingly, constructionist view of logic and mathematics also diverges from mathematical constructivism. Take an example of natural numbers in intuitionism. According to Brouwer [1981], natural numbers emerge from the perception of the movement of time and the falling apart of a life-moment into two parts. In other words, from the human ability to perceive the passage of time, we can experience two distinct things - the past and the present point, from which we generalise the idea of 'two'. From here, we build the rest of the numbers. To go along this illustration, for a constructionist, it is not the case that we build the notion of 'two' from what we experience. Instead, we design the concept so that we can organize the perceptual data. It is not the case that we intuit some form; instead, we inscribe the form on to the environment. To make it clearer, consider the notion of a 'technosci-

entist' as introduced by [Russo, 2012]. A technoscientist is an agent whose ability to come to know the world depends on their modes of modelling it. In other words, a technoscientist is actively trying to organize the world in an effort to make it manageable. In this sense, we construct numbers so that we can interact with the environment and organize it according to our needs and not because we intuit numbers from other data. The way we experience the world (humans tend to see their environment in a discrete fashion; filled with discrete objects) affords the construction of certain concepts. But their actual construction is done by the agent and for the agent. Given such understanding of epistemic agents, logicians and mathematicians are in the business of designing concepts that best serve our practices of managing the environment - they are technoscientists *par excellence*.

Let us make a step back. I explained that the constructionist epistemology maintains that an agent knows something if they can model it and situate the resulting information into the right network of relations [Floridi, 2011]. What this means for us now is that the concepts, i.e. cognitive technologies, we use determine the salient features of phenomena the agents can identify, i.e. the sociological, cultural and technological factors that influence the construction of cognitive technologies provide affordances to be taken up by agents for the purpose of knowledge production - as such, knowledge is of a nonrepresentational character, since it does not reflect 'the world outside', but the way we conceptualize our environment. We can relate such understanding of knowledge production with the notion of virtual knowledge introduced in Wyatt et al. [2012]. Virtual knowledge is a concept that highlights the importance of the epistemic infrastructure for interpreting claims as genuine pieces of knowledge, i.e. the way we increasingly organize our research practices is to support possible new directions in knowledge production, instead of the current practices. In that sense, the notion of infrastructure that envelops funding schemes, instruments, networks, etc., coincide with the idea of epistemic affordance making capacities of ecological factors. We can generalize this fact and understand virtual knowledge as affording knowledge production in general, i.e. that it creates the space of possibility [Record, 2013] for the production, while the situatedness of agents that are in some relation to the tools used determines the saliency of certain parts of that space.

Let's make this more concrete. The variety of cognitive technologies that we produce for the purposes of inferential practices are constrained by the cultural and sociological factors, as well as the current technological inventory. For instance, the cultural and social atmosphere (highly interconnected societies and sharp economic growth) together with current material

(electronics) and cognitive technologies (algorithmic thinking) enabled the Fourth industrial revolution or Digital revolution which in turn fundamentally changed the way we understand, interact and produce knowledge and commodities in the world [Floridi, 2011]. [Russo, 2018] The digital revolutionized our inferential practices as Big Data became the new buzz word for cutting edge research in disciplines from biomedicine to humanities. So, our technologies influenced the way our environment evolved and now we are creating new technologies to make that environment more manageable for us. That is why we are creating new technologies for inferential practices that are based on datasets, which are infeasibly large for a human brain to process alone. But, we can design different strategies for tackling that problem and then use it to shape the way we benefit from that data no matter the particulars of the epistemic situation.

This last point stresses the way the tools of logic and mathematics can be transferred to other domains. This is not new. For example, mathematics has long been seen as indispensable for physics [Azzouni, 1994] in the same way how logic was seen indispensable for linguistics [Martin, 1966]. But, it is the way we think of this indispensability that changes. Here, logicians and mathematicians produce tools that modify inferential practices and it is these practices that transform the saliency other disciplines can detect in their own fields of interest. That is how we come to the next issue I want to examine: The influence that modifying inferential practices has on the production of scientific knowledge.

In 2.2, I have argued that the extended notion of technologies from 2.1 includes logico-mathematical concepts. Understood as such, concepts from logic and mathematics are seen as tools we use to shape our inferential practices. Furthermore, such understanding enables us to account for the social, dialogical, and temporal character of specific concepts.

2.3 Logico-mathematical Practices in Science

After motivating the technological view of logic and mathematics, in this Section I turn my focus to other fields that rely on the tools provided by logicians and mathematicians. First, I will tie the notions of concepts and practices more closely. Next, I will use the new conceptualization of technologies to account for their role in transgressing the evidence limit. Finally, I will show how technologies and knowledge production are inherently social.

Formal methods are given a special epistemic value by the scientific com-

munity in general and a significant part of scientific knowledge was produced by using them. In other words, the logico-mathematical technologies shaped the inferential practices of the wider scientific community. So, in a way, the reach of the scientific investigations is, at least partially, bounded by the cognitive technologies produced by logicians and mathematicians in their respective field.

I start by considering the role that particular logico-mathematical concepts play in our epistemic practices. Next, I aim to generalize the insights on logico-mathematical practices identified in the previous subsection (2.2) on to the wider notion of scientific practices, i.e. I show how the cognitive and material tools of logic and mathematics are used in other sciences. Consider the so-called evidence limit (explored in 2.3.2) - the fact that all possible evidence one can gather in natural sciences is bounded in size by LHC on one side and Hubble telescope on the other. Yet, the tools from logic and mathematics enable us to posit objects infinitely small (string theory) and infinitely large (multiverse theory) as genuine objects of the scientific study. Furthermore, I also point to the fact that the technological view of logic and mathematics can be generalized to the whole of science, i.e. that the role of technologies designed for particular scientific practices - e.g. microscope in biology - shapes those practices and play a genuine role in the production of scientific knowledge.

2.3.1 Concepts and Practices

Consider logical theories. Logical theories are essentially theories about validity, i.e. they answer the question *Is this a good argument?* As such, these theories contain statements, often generalizations, which state some conclusions regarding the validity of a set of arguments. For instance, *a valid argument preserves truth and some propositions can be both true and false.* What is important to note, the laws of a logical theory need not be their theorems. This is because logic is not a mere formalism. Instead, it is a tool that aims at differentiating between ‘good’ and ‘bad’ arguments. In the context of this thesis, a logical theory is a tool for shaping inferential practices by imposing laws of good argumentation.

So, what in particular does a logical theory aim at explaining. Priest [2016] takes a logical theory to be a theory that explains our intuitions about the validity or vernacular inferences. So, analogously to sciences, it is in the business of observing which arguments are considered good in a specific context, experimenting with different formalizations and, based on it, making predictions about which arguments will be regarded as good; which is formu-

lated in the argument schemata, e.g. modus ponens. Under this conception, the method of theory choice in logic is abductive, as logicians are trying to find the best explanation of our observation of good and bad arguments. Given the science's apparent reliance on logic and mathematics, depending on which concept of validity - and other concepts for that matter - the best theories in logic and mathematics are using to express what good arguments are, the sciences will follow, i.e. they will ground their inferential practices on the concepts championed by those that engage in theory construction that shapes those practices.

Consider now a specific concept in a logical theory - that of negation. Depending on what theory of negation we choose, different inferences are possible. It is not irrelevant if, for instance, a negation is a single function that maps a truth value of a proposition to an opposite value or if the proposition can have both truth values, i.e. a negation is not a function or there are several functions that constitute it. This is so because depending on which concept of negation we incorporate in our theory, we can construe different explanations of a phenomena or make different predictions - both in the case of logico-mathematical, but also in the case of the scientific practice. We will return to the concept of negation in Section 3.

2.3.2 The Evidence Limit

As noted before, the investigation of the ways technologies construct our understanding of knowledge and reality was done by a number of scholars coming from various background be it philosophers, sociologists or natural scientists. Consider Ihde [1991] who, in his work on the philosophy of technology, stressed the fact that science is embodied in the technologies used in the scientific practice and those technologies make the production of knowledge possible. In a sense, the persistent division of knowledge-that and knowledge-how, which stems from the ancient division between *episteme* and *techne* [Russo, 2012] becomes blurred. If technology and the activity of an agent have genuine roles in the production of knowledge, then the two are not only mixed, but we can ask if the division should remain, since the *techne* part becomes constitutive of the production of knowledge - previously thought as solely being in the realm of *episteme*.

The scientific method relies on the testability of scientific hypothesis and the instruments constrain the evidential space in which we can perform those tests. But, the received view in the mainstream philosophy of science that looks at technologies as merely instrumental for the production of knowledge [Russo, 2012], but also the dominant view in the philosophy of science in

practice which grants the genuine role in the production of knowledge solely to the scientific material culture (see [Ihde, 1991], [Malafouris, 2013]), i.e. scientific instruments, faces the challenge of the ‘evidence limit problem’. The evidence limit connects with what Record [2013] calls epistemic possibility. For instance, the possibility of agents gaining knowledge depends on whether they are capable of acquiring relevant evidence. This capability of collecting evidence depends on issues ranging from ethics to technology. Hence, the technology that agent possess influence the space of potentially acquirable evidence.

Consider the case of fundamental physics. The goal of creating the ‘theory of everything’, i.e. a fundamental theory unifying and explaining all forces of nature requires us to study the environment in ever more smaller sections - first we examined the atoms within matter, then the protons and neutrons within atoms, then protons, neutrons and electrons within atoms, then quarks within those protons and neutrons. [?] The instrument that gave us the smallest section we can experimentally probe in is the Large Hadron Collider (LHC). On the other hand, a physicist also tries to understand the fundamental nature of the Universe by looking into gradually bigger and bigger things - once they were bound by their eyesight, later with terrestrial telescopes and now with large observatories and telescopes in space. Moreover, for some large phenomena, it is not even in principle possible to investigate them. For instance, anything in the distance farther away from us than light could travel in the age of the Universe cannot be observed in principle since the light cannot be emitted back to us. Yet, physicists have been working on theories far beyond the reach of our instruments for decades now, with string theory at the smallest and multiverse theory on the largest end of the spectrum.

Deeply profound philosophical questions arise if we take these facts seriously. In particular, what is the epistemic character of lines of scientific reasoning that are not directly based on the empirical data and yet are included in the family of scientific theories? Understanding such episodes in the scientific practice is essential for understanding the notions of empirical confirmation and discovery in empirically testable and untestable parts of scientific practice. Note that, what provides us with the ability to construct and have scientific theories that are not based on direct empirical data are the inferential practices that ‘safeguard’ the scientific practice from the ‘explosion of theories’. In other words, it is the technologies that the logicians and mathematicians design that enable physicist to step out of the evidence limit, while in the same time block the possibility of any theory positing something about a phenomena not in our evidence reach, i.e. not everything

goes.

The crucial point of this is not that there is necessarily a problem with the evidence limit and the theories outside of its reach. Instead, those problems emerge if and only if we do not grant the technologies that govern our inferential practices a genuine role in the production of scientific theories, knowledge and understanding. But, if we do grant them such a status, then we ought to revisit their role in the production of scientific knowledge that is within the reach of evidence too - one cannot be selective in granting the genuine epistemic status to technologies; if they have it sometimes, they have it always.

This brings about an important consequence on our understanding of what it means to be a scientist. On one side, an agent involved in the scientific practice is a technoscientist. In other words, a scientist produces the objects of technology and uses them to construct scientific knowledge - in a sense, a scientist ought to be an epistemologist. But, given the wider understanding of what technologies are, i.e. by including the concepts in the repertoire of technologies, a scientist is also a conceptual engineer at least as much as a philosopher or a logician. It is precisely the inferential practices that highlight this point the most. Without the engineering of the cognitive technologies, the theories beyond the reach of the evidence limit would be impossible. Moreover, without such engineering, even the inferential practices used to test theories whose evidence falls into the evidence reach would be impossible as there would be no standards nor vocabulary to form and test theories.

2.3.3 The Social Character of Practices

As explored, conceiving of science in terms of its practices makes us reconsider the role of technologies - in the widest sense - in the production of knowledge. But, as I will now argue, it also has another consequence since, firstly, the inferential practices are inherently social, and secondly, the logico-mathematical tools are cognitive technologies and epistemic artifacts designed to foster the transfer of other epistemic artifacts to other agents. Hence, if one accepts the view of the practitioners of science as technoscientists (making both material and cognitive technologies), the right epistemology of science ought to be social.

This idea was present already with Bacon's critique of the hermetic tradition [Rossi, 2011]. For him, a discipline is defined and shaped by the practices used to reach some goal of the discipline, instead of by the thought and talk of individual 'solitary geniuses'. His idea of 'digestion of experience', i.e. the idea that both data collection and their interpretation through theory is

needed, captures this idea of transferability of knowledge and the role that the social character of our practices has in the production of knowledge.

Such an idea was recently developed by Massimi [2018] where she relies on the capacity of human agents to align their intentions, where the direction of intentionality corresponds to a perspective a scientist has and the common referent of the multitude of intentional stances to the realist common scientific core (be it concepts, methods or standards). So, Massimi's understanding of the scientific practice and its history relies on the feature of human hypersociality – the historicist critique of scientific rationality eliminated the idea of a lone reasoner, pointing to the plurality and versatility of social factors that influence the production of scientific knowledge.

Interestingly, she uses the same historicist critique of individualistic understanding of agents to block the relativistic understanding of scientific endeavours, as relativism thrives only under the individualistic conceptions of the scientific rationality. So, by accepting the critique, she rejects the objectivist view of science, thus coming to a position that allows for the mindful world – human agents actively construct a perspective through which they view a phenomena, and a worldly mind – the perspectives we construct are constrained by the relations, social and other, we engage in. In a sense, she saves a realist conception of science by relying on the social features of the scientific practice.

In light of the constructionist epistemology adopted, we can rephrase this as 'a technoscientist can never be seen as a sole agent'. Instead, she is always involved in the process of constructing the environment she is situated in. This stems from the fact that our understanding of the environment is dependent on the cognitive technologies we create and use to make sense of it. But the process of construction is a social one, as they are created for the transfer of information between agents. This is particularly clear in the case of the scientific practice. There, without the employment of cognitive technologies, especially those that shape inferential practices, one would not be able to engage in the construction of scientific theories and explanations, and consequentially knowledge. But, one can use those technologies precisely because one is situated in the multi-agent setting that fosters the construction of technologies aimed at transferability of information. The difference between the view developed here and that of Massimi being that she uses the sociality to save realism, while we use it here as evidence for constructionism.

To unpack the inherent sociality of knowledge we turn to the studies of the scientific progress and creativity in Wagenknecht [2015], exploring the social epistemology of research groups, and Nersessian [2008], which takes the perspective from cognitive sciences. Wagenknecht proposes a distinction

between two different forms of epistemic dependence that occur in the scientific research — opaque and translucent. A scientist is opaquely dependent upon another colleague’s labor if she does not possess the expertise necessary to carry out independently the piece of scientific labor her colleague is contributing. In other words, if a scientist does not possess conceptual or material tools, or knowledge about how to use them, for performing a certain epistemic practice, she is in an opaque dependence relation towards a collaborating scientist that can use those tools. On the other hand, if the scientist does possess the necessary expertise, but still relies on another person’s labour, then her dependence is translucent, i.e. she could perform those practices but in the interest of saving time and energy she doesn’t. As Wagenknecht notes, many actual dependence relations, are neither entirely opaque nor translucent. Yet, the important point here is that in both cases it is on the basis of trust that the collaborating scientists can enter relations of epistemic dependence. Nevertheless, there are further mechanisms, mainly dialogical, that supplement trust. Scientists fine-tune their attitudes of trust towards collaborators through dialoguing practices, eliciting explanations and probing understanding. Because of the existence of these relations, every collaborative research is inherently social, because only groups of scientists to provide a scientific justification for collaboratively formulated knowledge.

But, I want to make another step forward in the social direction. Given how we concluded that collaborative knowledge is social, we can say that if an individual scientist is unable to provide justification for the knowledge that her research out-putted without relying on other agents, then all scientific knowledge is inherently social. To produce that knowledge, a scientist uses concepts and instruments to observe, hypothesize, experiment and explain. Yet, most of those tools were not made by the scientist. They were passed on institutionally, culturally and through whatever other routes possible. Moreover, any research is always situated in a given spatial and temporal point. It seems impossible for a scientist to provide justification for every single concept, theory, instrument and idea without once failing to give full justification. A scientist will always rely on previous conclusions of what not to investigate or what might be fruitful. She will also rely on whatever mathematics or instruments are currently deemed the best for a particular purpose without reviewing every possible route to knowledge. That would be cognitively and temporally infeasible for a human being. Hence, a scientist always relies epistemically on the environment constituting her scientific niche and the knowledge she produces will be inherently social.

This point is also captured by Nersessian’s ‘Creating Scientific Concepts’

[Nersessian, 2008]. In it, novel concepts do not emerge from the minds of their originators alone. Instead, they are products of a long and organic process of conceptual innovation that takes place across not only individual but also disciplinary, boundaries. What she is acknowledging here, and what is mostly pushed aside in the popular image, is the historicity of concepts. It is precisely this historicity that serves as a witness of the existence of dependency relations between scientists not only synchronically, but also diachronically. Those concepts, or tools in general, are then shaping the scientific practice, as much as those practices are shaping them.

As an example, let us turn back to formal languages and logical concepts. These tools are used since they are regarded as more precise and unambiguous. For these reasons, these tools are taken to be suitable for the formulation of scientific theories. So, the way we construe formal languages and logical concepts influence the way we can construct scientific theories and consequentially knowledge. But, precision and unambiguity are matters of expressibility, which is a social, or more precisely dialogical, phenomena aimed at the construction of transferable knowledge [Novaes, 2012]. Here, we can see that the improvement of tools for the formulation of scientific theories actually means improvement of tools for expressing the theory for the purpose of easier interaction between practitioners. In other words, technologies are designed to improve the social aspects of the practice of science. But, this is precisely the way to improve the scientific practice, as shown by Wagenknecht [2015], because it provides an interface for employing epistemic vigilance to check the appropriateness of claims we are epistemically dependent on.

To recap, the multi-agent shift brings about a change in our understanding of the processes involved in the production of knowledge. Firstly, an idea of a sole reasoner/scientist is identified as too impoverished to account for the actual epistemic practices – social epistemological considerations are in order. Secondly, by bringing the concept of construction in the center of our understanding of epistemic artefacts, we have to account for the epistemic character of the tools that enabled us their construction. If we accept that for a technoscientist to know is to construct, it follows that a tool for knowledge construction is whatever facilitates the interaction with the environment. Hence, concepts are as much of epistemic tools as other technologies, because they enable us to organize, and interfere with the environment. Therefore, every constructing agent is a technoscientist (strictly speaking) and a philosopher (as conceptual engineer).

Consider the concepts of validity and negation again. What is a good argument certainly is not a matter of an individual, i.e. an individual cannot

decide what it is and then be persuasive in arguing with others. Coming up with what arguments are good and generalizing to a certain structure which preserves the ‘goodness’ of arguments is a dialogical affair. But, the creation of a concept of validity was not done at a single point nor in a vacuum. Concepts, as Nersessian tells us, emerge and evolve through time and space and the way they shape our practices, in the case of validity our inferential practices, is determined by the context brought about other at that time available tools. But, then those tools, by shaping our concept of validity, also had an influence on the epistemic artifacts we can construct, i.e. they brought some epistemic value to the process of construction.

When it comes to negation the point is even clearer since the negation is constrained by our notion of validity. For instance, if we observe that a modus ponens type arguments are good, then we say that such arguments are valid to account for our intuitions and to shape our future inferential practices. But, if we interpret a conditional as the boolean material implication, we can encounter instances of schemata that we might not want to accept as an instance of a good argument. For instance, this example due to McGee(1985):

If a Republican wins the election, then if it’s not Reagan who wins it will be Anderson.

A Republican will win the election.

Therefore, if it’s no Reagan who wins it will be Anderson.

What we might do then is to deny exclusivity or exhaustivity of truth and falsity, or maybe even both. Not only can the choice of a concept of negation be connected with the choice of another logical concept, but also with other commitments such as a theory of truth, vagueness, natural sciences or other theoretical or practical considerations. All this has to do with how a concept not only shapes a specific practice it was designed for, but a more general inferential practice it shapes which can be transferred to other domains of human activity.

In 2.3, I have shown many interesting aspects of technologies - they are grounded in practices, enable knowledge production, and are inherently social. I rely on the notion of affordances to account for these aspects and build on this to characterize technology as informational.

3 The Informational View of Technologies

In Section 2, I have argued that the informational notion of affordances can help us understand the role of technologies in our practices. Now, my goal is to use the method of levels of abstraction to capture such affordance-based nature of technologies. This goal is achieved in Subsection 3.1, where I use the method to analyse specific instances of technologies. In the following subsection 3.2, I argue that technologies can have genuine epistemic roles in the production of knowledge, i.e. I present what their epistemic values are.

3.1 The Informational Turn

Here I motivate the use of the concept of information to systematize the insights gathered in Section 2. Note, the use of the concept of information for examining logic and mathematics is not entirely new. Some philosophers of information have already thought about the notion of mathematical information [D’Agostino, 2016], as well as the possibility of conceiving of logic in terms of levels of abstraction [Allo, 2017]. Here, I shall use this as my stepping stones for actually defining logico-mathematical practices in informational terms with the method of levels of abstraction. Furthermore, I will analyse the workings of a microscope and of several logical negations, with the aim of identifying commonalities between the two - in the virtue of both being informational technologies for the shaping of practices.

The method of LoA will enable us to go beyond talking in terms of logico-mathematical objects and practices and start thinking in terms of informational modelling. In other words, it will provide us with a unifying framework for analysing such practices in terms of the information each particular technology affords us to produce. The upshot of this approach is that we will also be able to step away from thinking about humans as exclusive epistemic agents, since the informational framework can examine the practices by putting the ability to process information at the centre. Such a view considers how both cognitive and material technologies shape the production of knowledge, without any need to differentiate the two kinds of technologies (they are both seen as informational technologies) and without any need to address what scientists do (as they can only do what their tools let them do).

To achieve this, we turn to the main method of the philosophy of information - the method of levels of abstraction (LoA). The method is used to explicate the epistemic and ontic commitments an agent takes in generating a

theory of some system [Floridi, 2011]. Here, a system simply means whatever we are building a theory of. Method of LoA is important for the philosophy of information for two reasons. First, the explication of the commitments of a theory is crucial for an analysis of a theory, but also for construing ameliorative strategies for improving a theory. Secondly, by identifying on what LoA an agent operates, i.e. what features are more relevant/salient for that agent, we can see how informative the generated theory is. In other words, what distinctions can be used to analyse some system. Here, we use this method for analysing what kinds of distinctions, and consequently what kind of information, can particular technologies make.

After introducing the method in 3.1.1 with the help of some toy examples, in 3.1.2 we turn to two case studies - microscope and negation - where we will analyse LoAs they operate on and how it affects the theory of the system/phenomena they are used to capture. I start by examining the microscope as an informational technology. In doing so, I will highlight the similarities of the practice of microscopy with that of logic, motivating the unified view of technologies in epistemic practices in general. So, the analysis of the microscope is used to identify the commonalities between the practices of natural sciences on one side and the practices of logic on the other. Then, I turn to the concept of logical negation as a case study of a technology for the practice of logic. Finally, in 3.2, I will identify what kind of epistemic values can technologies bring in the process of constructing scientific knowledge.

3.1.1 The Method of Levels of Abstraction

The method of levels of abstraction is a method used in the philosophy of information [Floridi, 2011] to construct a model of a system, through the specification of the perspective and purpose a technoscientist takes before examining a phenomena. As such, it can capture the way technologies constrain the production of knowledge by specifying the things a particular set of technologies allows us to identify and intervene in. This should remind us of the discussion in 2.1 where we saw how technologies constrain the space of possibilities of action. In other words, the notion of observables is used to capture the concept of affordances. Each technology affords certain actions or, as Record [2013] puts it, open a certain spectrum of epistemic possibilities. Agents then use a tool to extract some information from that space of possibility. To recap, observables and predicates that constrain them correspond to the notion of affordance and the set of observables - a level of abstraction, tells us what the space of epistemic possibilities the tool affords is.

Let us introduce the method.

- A *typed variable* is a uniquely-named conceptual entity (the variable) and a set, called its type, consisting of all the values that the entity may take. Two typed variables are regarded as equal if and only if their variables have the same name and their types are equal as sets. A variable that cannot be assigned well-defined values is said to constitute an ill-typed. variable

I shall write $x:X$ to mean that x is a variable of type X .

- An *observable* is an interpreted typed variable, that is, a typed variable together with a statement of what feature of the system under consideration it represents. Two observables are regarded as equal if and only if their typed variables are equal, they model the same feature and, in that context, one takes a given value if and only if the other does.
- An observable is called *discrete* if and only if its type has only finitely many possible values; otherwise it is called analogue.

So, we can use the notion of observable to say things such as: if you want to engage in the practice of selling car insurance you should take into account the age of the client, her past insurance policy, type of the car and so on. Those are the observables that enable you to make an informed decision in the first place, but also constrain you with respect to the possible policies that you can offer to the client. More precisely, a type variable explicitly states the measurement technique, while an observable states how a measurement should be interpreted. To take an example from modal logic, we can think of nodes as type variables. But, they can be interpreted in different ways - as metaphysical worlds, computer states, positions on a chess board and so on. These all make different observables from the same type variable, thus generating a different theory in modal logic.

- A *level of abstraction* (LoA) is a finite but non-empty set of observables. No order is assigned to the observables, which are expected to be the building blocks in a theory characterised by their very definition. A LoA is called discrete (respectively, analogue) if and only if all its observables are discrete (respectively, analogue); otherwise it is called hybrid.

So, the appropriate insurance selling level of abstraction is a set of all relevant observables. Note that there is no ‘one true’ level of abstraction for selling car insurance, nevertheless, some sellers can be more successful than others due to the way they designed their level of abstraction. In the same way, there is no ‘one true’ telescope, yet one can improve it for a specific task, by designing it in such a way that it makes certain features of the

diffracted light more salient to the user. Finally, there is no ‘one true’ way of interpreting what the nodes are, but depending on what we want to model, we pick the most appropriate interpretation.

- *The behaviour of a system*, at a given LoA, is defined to consist of a predicate whose free variables are observables at that LoA. The substitutions of values for observables that make the predicate true are called the system behaviours. A moderated LoA is defined to consist of a LoA together with the behaviour at that LoA.

We want to constrain our observables to accommodate the use of a tool in the actual world, as well as to express affordable constraining nature of tools in an additional way - as opposed to merely listing the observables. For instance, in modal logic, depending on how we interpret what nodes are, we can constrain the accessibility relation in different ways to force nodes to behave in a particular manner - the relation can be universal or nodes can represent epistemic states constrained by what agents can perceive, or constrained by the space of the game being modelled.

3.1.2 Two Case Studies

Case study 1: Informational View of Natural Sciences - Microscope

Microscope and information A microscope is a natural example of a technology in science. Moreover, it is easy to use it to illustrate the instrumentalist view of technologies in science. Humans, due to the limitations of their eyes can only see things of a certain size. The microscope then is a tool to expand this limit of what humans can see. But, as Hacking [1983] shows, microscopic and macroscopic vision are not the same. ‘To see with eyes’ is different from ‘to see with microscope’ on several points. Firstly, purely physically, the images of objects we investigate with the microscope are not delineated by means of the ordinary laws of refraction, but that of diffraction - contrary to macroscopic vision. Secondly, from a cognitive perspective, if we accept the situated and embodied view of agents and practices, looking with eyes and with a microscope is different with respect to the entities involved in the practice, i.e. the situations are different in terms of how the practice is embodied and through what it is distributed. Finally, pragmatically, as Hacking points out, one has to learn to use a tool to be able to see with a tool - ‘not be able to tell a dust particle from a fruit fly’s salivary gland until he has started to dissect a fruit fly’ [Hacking, 1983]. Given that the processes of learning to see are radically different between the two, we

should not think of a microscope as a tool to see in any ordinary sense of the word. In terms of the levels of abstraction, the two ‘seeing’ diverge with respect to what they afford and constrain, i.e. what distinctions can be made and in terms of what.

Let us unpack this with the concepts introduced in Section 2. As I mentioned, the emerging paradigm in both the philosophy of cognition and the cognitive sciences takes cognitive processes to be constituted by the coupled mind-body-environment systems. In the context of this thesis, I stress the genuine epistemic role of technologies (broadly construed to include the tools of logic and mathematics) in those processes. In the case of the microscope, this technology is not seen as having solely an instrumental role in scientific research - as argued by Hacking. Instead, the microscope is actively involved in the epistemic processes going on in scientific research. Moreover, other technologies, namely concepts, are also involved in the same processes, as they influence the way agents approach and use the microscope to generate knowledge. Hence, the production of knowledge in microscopy is a result of the interplay between various kinds of technologies. So, the epistemic products of the scientific practice are not ‘things out there’, nor ‘things representing something out there’. Instead, they are artifacts best fitting our current epistemic practices. Similarly to the way humans imposed norms for writing systems to afford easier representation of economic practices, in the case of the microscope humans have imposed norms of conduct to afford easier representation of the practice of microscopy. Note the way I use ‘representation’ here. To represent some practice means to make it more tangible and manageable for agents - no matter what the state of affairs ‘out there’ is. To illustrate, the concept of ‘GDP per capita’ is used to represent the economic practices of different countries so that it is easier to understand and manage economic practices. Nevertheless, a high GDP per capita does not mean a high living standard for the inhabitants of that country, as the actual state of affairs can be that of extreme inequality.

Let us connect this point with our method of LoA. Depending on the LoA a certain technology operates on is fine-grained or abstract, we can produce more or less knowledge, i.e. we can construct more or less information from the resulting model. For instance, the ‘GDP per capita’ is more fine-grained than ‘GDP’ and lets us produce more information about the target country. More generally, a given LoA provides a quantified commitment to the kind and amount of information that can be constructed from the system. In other words, the choice of a LoA predetermines the type and quantity of information that can be contained in the model. In the case of a microscope, the technology of microscopy that we have predetermines the type and quan-

tity of information we can find in the model of the phenomena we construct. Moreover, the concepts agents have to interact with the microscope further constraints and affords the production of certain information. This captures Hacking's point that one needs to know how to use a microscope to 'see' with it.

This interplay between constraining and affording is evident in logic too. In the practice of logic, this amounts to trading expressiveness for complexity. For instance, first-order logic is expressively weaker than second-order logic, because it can make fewer distinctions about the system. Yet, its complexity level is appropriate for recursive enumerability of valid consequence and, consequently, it affords the completeness theorem. So, informativeness is not equal to successfulness or elegance. Instead, the successfulness of a tool is measured in the balance it achieves between the constraints imposed on it and the affordances it provides.

In the practice of microscopy, we have two important sources of affordances and constraints we ought to consider - a scientist, i.e. the cognitive technologies she uses, and a microscope. As noted, the scientist already operates with a set of cognitive tools acquired through previous interaction with other agents e.g. through education and social relations. For this, what is especially important are the concepts learned through scientific education, e.g. concept of cell, kingdom, species, light wavelength. Naturally, following Menary [2013], we can say that the practice of microscopy is enculturated - designed for a collaborative approach to investigating the microworld.

I claim that the same holds for the concept of negation. It is a tool, a lexicalized concept, that evolved and was passed on culturally so that future generations can make use of it for making certain distinctions and inferences. Here, we can find another reason for striking a balance between expressiveness and complexity - transferability and learnability of tool use. A similar point is made in linguistics by Szymanik and Steinert-Threlkeld [forthcoming] about the acquisition of quantifiers. Their thesis is that the quantifiers that are lexicalized and shared across generations and cultures are those which exhibit properties that make them easier to learn by new language users. In this case, for instance, a quantifier that does not strike the balance between informativeness and complexity - eg. let Q be a quantifier denoting 'All $n-2$ ', where n are the objects being quantified over - are not lexicalized, and consequentially they are not passed on. This coincides with the general point about technologies - acquisition of technologies raises as the balance of affording and constraining is better for engaging in some practice. We will return to this in the next section on the epistemic values of technologies.

So, what happens when an agent uses a microscope? The scientist takes the microscope and looks through the oculars beginning the interaction. By interaction, I mean the interplay between the levels of abstraction the scientist and the microscope operate on. She tries to construct a model of the phenomena from the perceptual data she receives. In other words, the scientist selects a set of observables she wants to capture through the microscope so that she can model the perceptual data and construct some information based on that. On the other hand, the data with which the scientist can operate with is constrained by the setting of the microscope, i.e. by the set of observables of the microscope. For example, a biologist operates with concepts of cell, flagella and protist based on which she can ‘make sense’ of the data she receives. On the other hand, the microscope, for instance, constraints the data by the diffraction limit, i.e. microscopes have a theoretical limit of differentiating anything smaller than the wavelength of light [Shannon and Ford, 2018]. This diffraction limit is one of the observables of microscopes level of abstraction.

Here, it becomes clear that, in terms of the influence on the constructibility of information about the model of the phenomena, the scientist and the microscope are on a par, analogously to the way a formal system and a practitioner of it are on a par. A concept - such as negation or validity for instance, constraints the data which a practitioner can include in the evaluation of, say, an argument or a logical theory. For example, consider counterpossible reasoning. The standard conditional logics - that had success with counterfactuals, entail that any counterfactual whose antecedent is impossible is vacuously true [Berto and Jago, 2018], which is unsatisfactory. This happens because the LoA of those conditional logics cannot differentiate between those counterpossible cases. If we do want to make those distinctions, we need to generate a more informative theory by adopting a more fine-grained LoA - in the same way how, for example, we would make new distinctions by using a fluorescence microscope which makes different data more salient to the user.

This idea of grouping humans and technologies under the same epistemic umbrella is not new to the philosophers of information, as both have an active role in the construction of information. That is why in Floridi [2011] we encounter the notion of an inforg. For Floridi, an inforg is an informational organism that actively participates in the shaping of its environment - an infosphere. Initially, Floridi introduced this concept to explicate the ethical aspects of designing and intervening in this informationally understood environment. This is where [Russo, 2016] comes in and extends this notion to knowledge production too. Since knowledge is a constructionist enterprise

and inforgs have the capacity to design their environment, inforgs are not only ethical but also epistemic agents.

So, we can say that, in the case of microscopy, we have two inforgs interacting, each with its own set of observables. The key notion here is that of interaction. It is not the case that each inforg simply retains its level of abstraction. Instead, they can actively shape it to enable the production of new information. For instance, if the scientist is not satisfied with the data provided by the microscope, they can start *setting-experimenting*, which is essentially calibrating her observables, as well as the observables of the microscope, so that the microscope provides different data to the scientist and the scientist to make different data more salient. Moreover, the microscope, by changing the data it provides, also changes the observables the scientist takes, as the initial scientist's observables were selected with some level of uncertainty since the scientist did not know what the observed phenomena is. So, each inforg influences the set of observables the other takes, thus changing the model they can create, which in turn changes the information that can be constructed from those models. Once the satisfactory calibration of observables is achieved, the scientist can construct some information from the model and presents it as the result of the microscopy study. That information is the basis for what we call scientific knowledge. This analysis was done in a specific method of science - that of observation - but we claim that it also holds in others, such as experimentation. Again, experimentation involves calibration of observables between a scientist and the experimental set-up. Here, we echo Hacking's notion of constructing the phenomena. The interaction between the scientist and the microscope is driven by creating a stable phenomena which can be modelled so that a certain body of information can be created about it.

If we accept such an interpretation of the scientific practice, science can be understood as world-building, instead of world-describing. This follows from the fact that inforgs acquire information by interacting with other inforgs through the process of affordance-making - here described as the calibration of the observables. Note that this does not imply a sort of relativism. Not everything goes, since the set of possible information that can be constructed is constrained by the set of possible observables that an inforg can take [Floridi, 2011]. What it does imply is a sort of relationism. The fact that we can describe the world in terms of strings and multiverses, UV light and human-spectrum light, lifeless physics and lifefull biology, etc. is due to the fact that we can change our set of observables thanks to the technologies we can interact with. So, there is no one true description of the world, but there are appropriate and inappropriate models, as well as correct and incorrect

placement of information in the wider network of semantic artifacts. We shall return to this point in the next section.

Microscope’s LoA Now, let’s capture the processes described with the method of LoA. First, what is the LoA of a microscope? For the present purposes, I will present the LoA of the compound light microscope. First, let us identify the observables that constrain the workings of a microscope.

As Shannon and Ford [2018] describe, the microscope works by illuminating a specimen with the light from the condenser, which is diffracted by the details in the object plane, i.e. the smaller the detail, the wider the angle of diffraction. The structure of the object can be captured as a sum of sinusoidal components, where the spatial frequency is defined as the reciprocal of the distance between adjacent peaks in the sinusoidal function. Each spatial frequency component produces diffraction at a specific angle depending on the wavelength of light. The image that the microscope produces is actually produced by the interference between the diffracted waves collected by the objective. Note though, the objective never manages to collect all of the diffracted waves. To sum up, what the microscope captures is the interference of the subset of light waves diffracted by the specimen. The measure of the ability to collect these waves is called the numerical aperture (N.A.). So, the N.A. is the sine of half the angle of the cone of light from each point of the object that can be accepted by the objective multiplied by the refractive index (R.I.: bending of a ray of light when passing from one medium into another) of the medium in which the object is immersed. In other words, N.A. is the ability to discern detail. R.I., on the other hand, is the ability of the medium to help the objective discern details, e.g. a medium denser than air will produce a shorter wavelength of light than air, improving the resolution of the microscope. In other words, the limit of resolution (or the limit of saliency) is constrained by the wavelength of light and the N.A.. These are our candidates for observables but, for now, let us move on.

The microscope was not built to work alone. It is a tool for humans to use. Naturally then, the way a microscope constraints what can be detected is also influenced with the way human body constraints the design of the microscope. For instance the depth of focus decreases as N.A. increases. But, for a scientist to have any utility from a small depth of focus, the microscope itself has to provide the user with smooth and stable motion, otherwise, in the case of shaking, for instance, the benefits of the focus are lost. Moreover, if we consider the actual use of the light microscope as described in usage-manuals¹, a scientist ought to ‘adjust the interpupillary distance of

¹[The Microscope Store, 2019]

the oculars to suit you comfortably' and 'turn the coarse focus control until a clear image is obtained'. Next, a scientist should 'try experimenting with various settings to get the most effective view of your specimen' and 'remember when adjusting the focus that the objective should never touch the specimen'. We can see from these instructions that it is expected from scientists that they will have to 'work as they go' as the process of microscopy is not uniform with respect to, even a class of, specimens. Instead, the process of microscopy is a dynamic one which manifests itself in constant experimenting with the settings until 'the right view' is found. It should also be noted that the right view depends on the fact we want to establish, which is not always clear what it is - as we will see soon in the case of hemimastigotes.

On the other hand, the human body can afford a simpler design in some cases. For instance, objectives do not produce a flat image surface, but since the eye can accommodate for the field curvature no interventions into the design are needed. Even though these considerations are interesting in their own right, I will abstract away from them for the present purposes to preserve the simplicity of the presentation.

So, the compound light microscope has the following level of abstraction:

- Compound Light Microscope

- Type variable

WL which is a continuous variable

$N.A.$ is a maximal half-angle

$R.I.$ is a real number

- Observable

we interpret: WL as the wavelength of light, and $N.A.$ as the numerical aperture which is the product of $N.A.$ and $R.I.$

- Predicate(s) constraining the observable(s)

$R.I.$ of vacuum is 1, i.e. the speed of light in a vacuum is the same as the phase velocity of light in vacuum. For example, since the rate at which the wave 'moves' (i.e. its phase velocity) through water is slower than in vacuum, the $R.I.$ of water is greater than that of vacuum, namely 1.333.

Now, given what we saw about how the microscope works, we can fully appreciate the quote from Hacking [1983] where he says:

'The microscopist has far more amazing tricks than the most imaginative of armchair students of the philosophy of perception.'

The practice of microscopy really is much more than simply looking through a lens. It is essentially a process of modelling given the data constrained by the apparatus. In other words, the design of the microscope actively participates in the production of information - a microscope is an inforg. But, as I mentioned before, a user is as much uncouplable from the technology, as the technology is from the user. There are cognitive technologies involved in the production of knowledge in microscopy, let alone the technical advancements and the embodiment constrains on the design. For instance, as reported in Lax et al. [2018] when researchers observed hemimastigote microbes with a microscope, they worked in the confounds of their technologies - material and cognitive. The problem was that they tried to place those organisms among one of the eukaryote groups, based on the features that they ‘saw’. Depending on what features a researcher would make most salient, different classifications were made. As it turns out, the hemimastigotes belong to a distinct supergroup [Lax et al., 2018]. The point here is that the concepts we use to classify these organisms have an active role in how we ‘see’ things through the microscope. In other words, new scientific advancements can be made simply by introducing new concepts and not solely by introducing new instruments. This might seem as trivial, but this point goes against the received view in the philosophy of science and the only way to explain such impact of concepts is to grant them a status of technologies with genuine epistemic values.

What the method of LoA provides us is the way to explicate what exactly influences technologies ability to produce information, i.e. what is the relationship between the affording and the constraining elements of a particular technology and what are its consequences for the theory generated from that LoA. This will prove to be very useful for examining logical concepts, as the analysis of LoA will enable us to investigate a single and compare a plurality of concepts on different levels of commitment, i.e. what are the building blocks of the theory (type variables), how is the data interpreted and explained (observables) and what behaviour is expected (predicates constraining observables).

Case study 2: Informational View of Logic - Negation

I have already introduced the American and the Australian Plan for negation in Subsection 2.2 and raised some issues that they encounter individually, as well as some issues that they have with each other. What I do now is analysing a number of negations in terms of their respective LoAs which will enable me to say something about the resulting theories of negation.

Let us examine several types of negation on offer today. We start with a modal conception of negation as developed in Berto [2015].

a) Negation as Incompatibility [Berto, 2015]

Negotiation as Incompatibility developed by Berto [2015] is an example following the Australian Plan. Australian Plan semantics for negation is based on two ideas. First, negation is an exclusion-expressing device and we utter negations to express incompatibilities and second, because incompatibility is modal, negation is a modal operator as well [Berto and Restall, 2019].

An interesting move in this account of negation is done with respect to the way the negation is defined. In Berto [2015], negation is such a basic and primitive concept that it cannot have an exact explication in the form of a reduction. Berto traces this back to situations where humans had to make choices, decisions. The feature of choices is the fact that different choices are incompatible - one cannot choose all. Negation then is a way to lexicalize this situation in which an agent has to exclude the possibility of one of the choices. So, it is this relation of (in)compatibility of choices that grounds the use of negation, but grounding here means explaining and not defining in terms of. So, the observables we need to express this involve the (in)compatibility relation and its relata - in this case possible worlds, but we can call it situations, updates, points and so on. Let's use the method of LoA to explicate how the theory of negation as incompatibility is built up.

- Type variable:
 - r which is a two placed relation xRy
 - o which is a set of objects
- Observable:
 - (In)compatibility
 - Worlds
- Predicate(s) constraining the Incompatibility observable(s):
 - $A \vDash B \Rightarrow nB \vDash nA$ (Contraposition)
 - $xR_Ny \Rightarrow yR_Nx$ (Double Negation Introduction)
 - $xR_Ny \Rightarrow y \sqsubseteq x$ (Law of Excluded Middle)
 - $\forall x \exists y (xR_Ny \wedge (yR_Nz \Rightarrow z \sqsubseteq x))$ Double Negation Elimination
 - $xR_Ny_1 \wedge xR_Ny_2 \Rightarrow y_1 \sqsubseteq z \wedge y_2 \sqsubseteq z \wedge xR_Nz$ (De Morgan entailment)

- Explaining negation in terms of incompatibility

Finally, based on this level of abstraction we can generate our theory, or definition in this case, of negation as : $\neg a$ is true in w iff, in all w' compatible with w , a fails to be true.

Of course, just the two observables do not do the trick. We need to constrain the relation somehow, to force it to behave as we would expect incompatibility to behave in the world. Here, Berto resorts to his intuitions of how such a relation should work. For instance, Berto maintains that incompatibility is symmetric, i.e. if a is incompatible with b , so is b with a . As a result of such constraints on the relation, the theory generated by this LoA will feature double negation introduction to our formulas. But, consider the following case presented by De and Omori [2017]. An active construction site next to me is incompatible with me enjoying a book. Yet, me enjoying a book is compatible with an active construction site. It seems that this case goes against Berto's intuition and that negation really is not symmetric. The problem here is that a formal feature (negation introduction) was already chosen to be present in the logical theory before we went through the features of the compatibility relation that is supposed to explain negation. In other words, we cannot determine with which predicates we will constrain our observables, before we determine which observables we will include in our level of abstraction.

Before we move on, I want to return to one point raised in Berto and Restall [2019]. This is that the negation is a *device* that was *created* by the agents to enable themselves to perform certain tasks. For Berto, the task is to claim that something is the case, which rules out something else in the world. Restall, on the other hand, is more sympathetic to the idea that negation is used for signalling what the utterer's commitment is. Nevertheless, for both, negation is an important tool that changed the way we perform linguistic and inferential practices. This is due to the fact, according to Berto and Restall [2019], that humans were able to lexicalize the incompatibility relation thus enabling them to explicitly state their stances and commitments. But, even though this program is pluralist in the sense of understanding and defining of the negation, Negation as Incompatibility programme claims that there is such thing as the 'true' negation, i.e. any negation that does not comply to the inferential rules set by the incompatibility relation, cannot be called negation. In other words, Berto [2015] does not make a commitment on the interpretation of type variable and accepts any negation as long as there are no changes to the predicates constraining the Incompatibility observable.

b) Dialetheist Negation as Contradictory Forming [Priest, 2006]

For Priest [2006] there can be many theories of negation, but there is only one such thing as a negation that we can make a theory of. Then, the acceptable theories of negation are constrained by the actual properties of real negation. For Priest, the theory of negation is the theory of contradictoriness relationships. A reader will notice how different such a view of negation is when compared to the one held by Bert and Restall. This is because the main concept that grounds Berto's negation is compatibility, while for Priest is a truth value. Naturally, the resulting theory of negation will be very different too. Let us look at the dialetheist negation.

- Type variable:

a which is a singleton set $\{\varphi\}$

t which is a set $\{T, F\}$

r which is a relation aRt

- Observable:

we interpret: a as an assertion, t as truth value and r as a relation 'truth value is'

- Predicate(s) constraining the observable(s):

$$\neg \forall a (a \wedge \neg a \Rightarrow a')$$

- • Defining negation in terms of truth and falsity

Based on this, we can define negation as: $\neg a$ is true iff a is false; $\neg a$ is false iff a is true.

So, the dialethic negation is primarily characterized by the relation 'truth value is'. That is where the core of Priest's understanding of negation lies, namely, this relation does not have to connect each assertion to one and only one truth value, i.e. it is not a function. But, contradiction is the property of a pair of propositions which cannot both be true and cannot both be false at the same time - or at least that is the history of the concept of contradiction [Novaes, 2007]. So, Priest's notion of negation rests on the idea of paraconsistency which means that in some circumstances two propositions that are contradictory according to classical logic can be held true at the same time. But, if this is what Priest takes contradiction to be, then there is a question about what exactly is he talking about. If Priest changes the very definition of contradiction, and consequently negation, is it still the

same logical object as the one classical logicians are talking about? This problem is called *the Quineian challenge* [Novaes, 2007]. Quine claimed that any deviation from classicality implies ‘changing of the subject’, i.e. when a ‘deviant’ logician changes the definition of a negation, it is simply not *about* the thing the classical logician talked about; it is not about ‘the real negation’. Although this discussion goes beyond the scope of this thesis, a short note is in order. Given the view of negation as a technology, the ‘changing of the subject’ is here seen as changing of the LoA, with the system that provides the data which the LoA constraints stay the same.

If we compare the approach from Priest [2006] with that of Berto [2015] we can see two things. The first thing concerns the observables of their respective levels of abstractions. In Berto [2015], we can easily see that the aim is to ground the concept of negation in a certain practice - that of expressing (in)compatibility for various social and other purposes. So, Berto and Restall aim at showing that a concept is used to perform a certain practice and from it they want to see how the grounding relation is constrained - including the problematized property of symmetry. In Priest [2006] on the other hand, the explication of the concept of negation starts with claims of exclusivity of the predicates true and false - the predicates constraining the relation of ‘truth value is’. From this, Priest argues what negation actually is. Note, a paraconsistent logician might disagree with the way negation was presented - as an unrestricted relation. Instead, one might opt to constrain the relation with multiple functions, which enables a proposition to be evaluated differently in terms of truth value by different functions.

c) Classical Negation as Contradictory Forming (Frege, Russell)

- Type variable
 - a which is a singleton set $\{\varphi\}$
 - t which is a set $\{T, F\}$
 - r which is a relation aRt
- Observable
 - we interpret: a as an assertion, t as truth value and r as a relation ‘truth value is’
- Predicate(s) constraining the observable(s)
 - $r =_{Def} r(a) : Truthvalue$, i.e. $r : a \rightarrow t$ (n is not any kind of relation, but one with a unique outcome - function)

- Defining negation in terms of truth and falsity

In other words, a negation works as follows: $\neg A$ is true iff A is not true.

Frege's conception of negation is that the negation is a function that maps a truth-value into the opposite truth-value. As we can see, the observables of the classical conception of negation as contradictory forming operator and those of the dialetheist conception are the same. The only difference between the two approaches is the fact that the classicists have introduced a constraint on the 'truth value is' relation so that it can only have one unique result, i.e. they constrained it to be a function. So, in a sense, the classicist understanding of negation is present in the dialetheist account too. This should not be a surprise as the dialetheist negation was created precisely as a response to the classical account. Paraconsistent logicians often claim that the classical logic cannot discriminate between good and bad theories nor can it account for the inconsistency of various formal systems once they are axiomatized [Priest, 2006][Novaes, 2007]. This, according to them, is due to the fact that classical logic is too coarse-grained to detect important features of those systems. In other words, the constraining of the relation observable does not have the theoretical payoff needed to employ it.

So, what is the value of the analysis in terms of LoA? First, we can see what are the structural differences, in terms of LoA, between different conceptions of negation. As noted earlier, this tells us why the theories generated by these LoAs are different. Type variables tell us in what kind of measurement are we grounding the explicated notion. What this essentially tells us is what are the epistemic commitments of a theory, i.e. what is the structure of knowledge a theory can possibly produce. When we interpret those type variables, i.e. make observables out of them, we are taking metaphysical commitments. It does not change the structure of produced knowledge whether a relation stands between real worlds of fictional worlds. But, it makes a difference with respect to the way we ground our explanation of a system/phenomena. Finally, by constraining observables, we are making predictions that it will behave in a certain way. For instance, by constraining the truth value relation to be a function - and claiming that classical logic is true, we are making a prediction that we will not encounter cases where contradictory propositions hold. Of course, in such theory building, practitioners need to make some difficult choices. To return to the informativeness of classical logic again, because of the LoA it is generated from, it treats every contradiction in the same way. In other words, it cannot produce valuable information about paraconsistent systems, because it does not have the

observables needed to identify them (or make salient). To be able to do this, it needs to be able to make further distinctions, something paraconsistent logic prides itself of being able to do. Note though, more information does not mean a better system. If we can make distinctions between everything, we lose the ability to communicate efficiently because the complexity of the resulting theory is too high.

3.2 The Epistemic Value of Technologies

I rely on the informational view of technologies put forth in the last subsection to examine the epistemic values these technologies have. As I will argue, we can identify three kinds of epistemic values that technologies can import in the process of knowledge production. Those epistemic values are: the productive value, the procedural value, and the engineering value of technologies.

The examples of logic-mathematical technologies, as well as scientific instruments such as the microscope, have shown how these technologies shape the production of scientific knowledge. By ‘shape’, I mean how technologies enable and constrain the construction of information about a modelled phenomena. As noted before, we ought to grant technologies a genuine role in the production of scientific knowledge to account for what is actually going on in the scientific practice. I call this the *productive epistemic value* of technology. A technology T has a productive epistemic value if T actively participates in the affording and constraining of what information we can construct about the modelled phenomena. For instance, a telescope makes certain far away objects more salient, thus having a productive epistemic value in astronomy. Without it, it would be very hard to make major advancements. Note that not everything that *enables* enables production has a productive value. For instance, my lunch and the digestive system enable me to produce knowledge and without it I would be unable to do so. But, neither actively interacts with other technologies with the aim of knowledge production. To echo Simondon [1980], my digestive system is not in the right network of relations that enable knowledge production. In other words, the productive value of technologies is seen by the influence it has on the LoA generating the theory we base our knowledge on. There are other necessary conditions for acquiring knowledge, e.g. food, but that do not influence the observables we include or the behaviour that constraints a certain LoA.

Interestingly, not all technologies are made with the aim of improving the results of the epistemic processes, i.e. the improvement of epistemic practices

is possible in other ways than solely improving the production of new scientific knowledge. Just consider the role of technologies in the transferability of information production, i.e. as technologies have a genuine role in the production of knowledge, then they also have a crucial role in the reproduction of knowledge too; placing the study of the role of technology in science in the centre of the discussion on reproducibility of science, open science and metascience. I call this the procedural epistemic value of technology. As an example let us return to the microscope. One of the important features of microscopy is taking pictures of the observed phenomena, i.e. photomicrography. Photomicrography has been done since the 19th century on film [Renfo, 2015], while for the last few decades digital images prevail. An interesting fact is that the quality of the digital images falls short of those made on film, i.e. there is no increase in accuracy in the new technique. Yet, the new technology radically lowered the costs, shortened the time consumed and simplified the process of imaging. In other words, there was an increase in the procedural value of the microscopy. Following the work on deliberation in Peter [2013], I claim that this procedural value is of an epistemic character as it enabled easier transferability of knowledge and reproducibility of imaging. Reproducibility is a genuine epistemic value as it fosters greater learnability of microscopy and consequently greater epistemic caution of scientists since a larger number of other scientists can exhibit their epistemic vigilance through the reproduction of the same knowledge producing process. Here, the feature of sociality becomes evident again, as discussed in 2.3.2.

The third and last epistemic value I will consider is motivated by the apparent special epistemic status of logic and mathematics. Since they are the disciplines whose products shape the inferential practices of other sciences, it is no wonder they are seen as special. Their products have, what I call, the engineering value of technology, as they enable new ways a scientific practice can be done - just consider the benefits of formalization, as well as of conceptual enrichment, again. But, logic and mathematics are not exclusive members of such type of endeavours. Essentially, any practice whose product can be used to shape the inferential practices of scientific disciplines has this engineering value. This value is epistemic, as it enables new ways to produce knowledge. In a sense, logico-mathematical practices can have an engineering epistemic value for science, because the tools they produce can have a productive epistemic role in the scientific practices.

In the vocabulary of the method of LoA, disciplines such as logic and mathematics are investigating new ways to formulate LoAs of the same systems and, in doing so, discover new ways of designing LoAs that might be useful in other contexts, i.e. that can be applied to systems they weren't in-

tended for initially. Given my emphasis that these disciplines are in an effort of balancing expressiveness and complexity, it should not be surprising that this can be useful in theory construction in science, i.e. that the products of logic and mathematics are highly transferable to other domains of knowledge production.

Furthermore, logic and mathematics are not the only such disciplines. In principle, any discipline is able to design a LoA that can be highly transferable or useful in some other field. Yet, due to their specific focus on different conceptualizations of the same phenomena, some disciplines are particularly successful in designing them. Take an example of metaphysics. Traditionally understood, metaphysics has been rejected by the scientific community at large, but also by the philosophical - especially among the philosopher of information. Yet, one can defend metaphysics on the grounds of the engineering value of the subdiscipline, as its products can be seen as technologies for the conceptualization of the objects of the scientific study - take for example the concept of an atom or that of a multiverse. What metaphysicians do is look at concepts and analyse their theoretical payoff in different settings. Instead of looking at metaphysics as investigating what there really is, we can look at metaphysics as the study of the variety of LoAs that we can adopt. The question whether the universe is discrete or continuous, if time is eternal or not and so on are matters of adopted LoAs and not of *What there really is?* [Floridi, 2011]. When thought about in such a way, we can say that metaphysics has an important role of producing and testing new conceptual tools that could be transferred to other fields, from science to art, with a goal of enabling the construction of new, previously unattainable, information by affording new distinctions or by eliminating old ones.

In 3.2, I have argued that technologies have at least three epistemic values that they import in our epistemic practices. As I have argued, technologies do not have merely an instrumental value for our practices - e.g. improving the accuracy of our beliefs. Instead, they can foster the production of new information, they can achieve greater learnability and transferability of information, and they can produce novel tools for improving practices across disciplines.

4 The Technological View of Logico-Mathematical Practices

In Section 4, we contemplate the consequences of the informational view on technologies on our understanding of science and its practice. Here, we build on the method of levels of abstraction and the notion of the model seen as the end product of modelling by the philosophy of information, i.e. model as artifacts generated by adopting a LoA. Based on the informational framework put forth in Section 3, I argue for the relationist conception of scientific knowledge and the engineering understanding of the scientific practice. In other words, in Subsection 4.1, I deal with the question of what is the epistemology of logic and mathematics. Also, I look at the question of apparent indispensability of mathematics for natural sciences. We use the insights from Section 3 to create a new answer for that traditional question in philosophy of mathematics and science by referring to the inference shaping role of mathematics, concluding that although it is not indispensable, given how our scientific practices are organized, this is the most effective way to go about our scientific practices. Then, in 4.2, I formulate the new understanding of the practices of logic and mathematics - and science in general for that matter, by which they are instances of epistemic niche construction. Given that, in 4.3, I outline further research that can be done on the roles and values of technologies in our epistemic practices.

4.1 Epistemology of Logic and Mathematics

Our main question in this subsection concerns the way we come to know truths of logic and mathematics and how this connects to the technological view of respective practices. Given the way we conceived of technologies - their scope, role, and kind - there is an apparent tension between thinking of logic as a study of unchangeable relations (say validity) and thinking of logic as a tool that was shaped by an for humans. The tension lies in the atemporal status of logical truths and the temporality of artifacts we use to 'retrieve them'. In other words, The problem is that logic seems to deal with necessary, eternal truths. So the history, or the context of discovery, of how we come to discover these truths is not relevant for its contemporary use. In the context of this thesis, this amounts to the point that the study of logico-mathematical technologies has nothing interesting to say to actual practitioners of logic and mathematics. My aim is to argue against such a view.

All too human We have seen many philosophers arguing that technologies have a much more interesting role in science, if we look at the scientific practice [Hacking, 1983] [Russo, 2012] [Ihde, 1991]. That is why I took the same route in this thesis, but to argue the same point about logic and mathematics. In this case, we can look at what the truths of logic mean for human practices and what difference do they make. I have noted in Section 3 that the choice of formal systems are connected to the balance they achieve between expressibility and complexity. But, the concepts of logic and mathematics do not only emerge in practices of logic and mathematics we know today, nor are they always created with a specific goal in mind. Some concepts, such as that of a number for instance, are designed to deal with our everyday practices. So, the way we experience the world and interact with it also influences the way we conceive of logico-mathematical concepts. For instance, not only do we discover only a subset of the infinitely many truths about logic, but we also design new tools and methods for selecting that subset.

Theoretical physicist Rovelli [2016], when making a similar point to argue for a type of contingency of mathematics, draws an analogy between the activity of mathematics and Michelangelo's characterization of sculpturing. For Michelangelo, sculpturing amounts to 'finding' a sculpture in a block of marble, as opposed to the process of carving out. Rovelli argues that it is trivially true that the sculpture was in the block of marble before the act of sculpturing since any sculpture is a subset of the block it was made in/out of. But, the key point is in 'any sculpture' and in the fact that the sculptor chooses which of the subsets will she 'find'. Analogously, we create those concepts which are useful for us in terms of their expressive power, while remaining usable given our cognitive boundaries. Further on, he argues that we deal only with interesting and useful mathematics and in doing so, we are often unable to respect the norms of mathematical perfection [Rovelli, 2016]. That is why, for instance, logicians and mathematicians have aspired to understand - semanticize - the theories they formulate, instead of merely building a useful formal system. Here we can see the philosophical value of the historical perspective on the scientific and the logico-mathematical practice, as it can be informative regarding why, for instance, logic and its concepts became relevant for us and things we do. So, not only can we learn something about logic and mathematics by looking at how we interact with the world, but we can also learn something about our cognitive and sensory abilities, by looking at the concepts we are able to create and comprehend.

But, in Section 2, I have also argued that technologies have a genuine role in the production of knowledge. So, it is not the case that they merely help us select a set of true claims from some pool of independent facts -

help us see the sculpture in the piece of marble. In a minimal sense, concepts as technologies enable our interpretation and understanding of our environment. So, in a sense, they shape the way we understand some facts. Note though, this does not mean that there are no agent-independent facts. Instead, we can claim that logic is not a practice that makes possible the description of such facts, and that we construct logical concepts to shape our specific practices. It seems that both the view that logic does not have anything to do with reality and the view that it discovers eternal truths about reality - are unsatisfactory. The latter seems to be too egoistic on one side, as humans are seen as somehow privileged in obtaining these truths, while at the same time we cannot explain how it is possible that we can obtain those truths in the first place. The former simply seems too costly. Not only have logic and mathematics been tremendously successful and it drove progress in numerous fields - from physics to computer science, but it also looks and feels like it is talking about eternal truths and abstracting from the content - think of the inference schemata. But, scientists use that mathematics which is available and convenient without considering whether their experiments confirm mathematics or whether mathematics has a realist/platonic interpretation. Furthermore, as I will argue, there is a way out of this tension and it lies precisely in the human ability to construct knowledge and environment. Here, contrary to the tradition of relegating the human factor from epistemic practices, e.g. Descartes radical scepticism due to our human nature, the human element is crucial for saving the successfulness, while maintaining artificialness, of logic. Consider the following schemata:

$$\begin{aligned} &\phi \vee \psi \\ &\phi \rightarrow \theta \\ &\psi \rightarrow \theta \\ &\theta \end{aligned}$$

Not a lot of people would argue that this is not a valid inference. But does the conclusion follow from the premises independently of an agent's understanding? This is the question posed both by Floridi [2011] and D'Agostino [2016]. Well, to infer 4. we need to see that it follows from 2. and 3.. To see that, we need to consider what if ϕ is the case or what if ψ is the case. But, we do not have that information in the proof [D'Agostino, 2016]. Premise 1. is not helpful, since it does not tell us anything about just ϕ or just ψ . As Rosen puts it, the disjuncts do not make the disjunction true, but the disjunction is true in virtue of the disjuncts [Rosen, 2011]. What is needed to get to the conclusion is to imagine/construct a scenario in which one is the case and infer θ and then repeat this for 3. This does not happen in the proof and this information are not contained in it. The essential part of the proof

rests on the fact that there is some agent performing it [Floridi, 2011]. That means that, if there weren't any agents that would give some extra input to a deductive proof, it simply should not follow through. This also makes intuitive sense when considering the dialogical perspective on deduction put forth in Novaes [2012], because, there also, the proof assumes there being *an arguer* and *a sceptic*.

LoA and laws To see how the method of LoA can help us make sense of such an understanding of logic, let us look at a toy example as discussed in [Allo, 2017]. Let LoA_1 have the observables represented as the atomic propositions p and q , and LoA_2 the atomic propositions p , q , and r . LoA_2 is both more discerning than LoA_1 because it can distinguish between two possible ways in which p and q can jointly be true with respect to r , as well as actually express how the two ways in which p and q can jointly be true differ with respect to the truth of r . As Allo [2017] notes, the question of how a set of conceptual resources (observables) allows us to capture an intended set of distinctions can be formulated as a purely logical question, i.e. the constructability of a LoA is connected to definability and characterisation of the formal system. In the context of the schemata, the premises afford certain constructions, while they constrain others, which can be captured by explicating the LoA of the language containing the exemplars of the schemata. But, the schemata itself is still a tool that needs to be used to arrive at a particular conclusion. When we adopt a LoA perspective on logical and mathematical proofs, it becomes clear that one needs an active agent that generates a theory - or a proof, from a given LoA.

To address the tension outlined and account for the human part of logic, we return to a few more concepts introduced earlier. Menary [2013] and Fabry [2018] introduce the notion of a cognitive niche to talk about how humans prepare their environment to accommodate easier management of tasks we need to perform. Furthermore, we have seen that humans actively participate in the construction of the way they understand their environment [Floridi, 2011]. In this context, an agent and the environment are related through the space of affordances [Gibson, 1979]. Since, as I have argued in Section 2, this relates to epistemic practices in general, it also relates to the practices of logic and science. Let us relate this to a toy example.

Consider a world imagined by Smolin and Unger [2014], a physicist and a philosopher, respectively, where chess was never invented. Arguably, the sentence 'Bishop can move only diagonally.' or the sentence 'Chess players take turns.' would not make much sense. In other words, such sentences that refer to an inexistent practice would not be true nor false, since they would not have any meaning. Moreover, such sentences would not be distinguish-

able in any useful way from say ‘Bishop can move only horizontally and vertically.’. Contrary to this situation, for us the sentence ‘Bishop can move only diagonally.’ or the sentence ‘White plays first.’ is not only true, but it is necessarily so - in the virtue of stipulated rules, but those rules are the rules of chess and disobeying them simply means that one is not playing the game. But this goes even further as there are many truths that were not determined/stipulated by the inventors but are still necessary, for instance ‘A bishop that started on a black square, can never eliminate pawns on white squares.’. This development is what Smolin and Unger [2014] call an evoked rule - a rule that emerges from practices and not stipulations of agents or by the facts of nature. To connect this intuition pump to the topic of this thesis, the truths and rules of logic are evoked in a sense that they are constrained by the game - the practice of logic, but also by our ‘nature’ - the way we experience and interact with the environment. This is also something pointed out in [Rovelli, 2016]. But, as we have seen before, the acceptance of certain practices, say mathematical, influences the way we utilize our ‘nature’ through enculturation. That is why a simple nature-nurture division is not so clear when one adopts a constructionist understanding of epistemic practices. Moreover, as the method of LoA demonstrates, epistemic processes are constrained by both epistemic agents and the environment they interact with. So, ‘ $2+2=4$ ’ is always true, as well as ‘Water is H₂O’. But if there were no beings which can evaluate these statements with respect to some LoA, these sentences would not be true, because they simply wouldn’t make sense, i.e. they wouldn’t answer any meaningful question to ask. It would be like asking ‘What is the fundamental nature of the Universe?’ without ever stating with respect to which model we ask the question; a physicist and a logician can give different, but still correct answers. So, we can see that our logico-mathematical concepts are rigidly constructed, i.e. once we have constructed a logico-mathematical objects we are highly constrained when it comes to attributing properties to that object. This is a consequence of the LoA that generated that specific concept. To further illustrate the point, as Rovelli argued, we have developed arithmetic because we experience the world in a predominantly discrete fashion, i.e. we are surrounded by countable objects. The reasoning is that if there were not beings that perceive discrete things, ‘ $2+2=4$ ’ would be as nonsensical as saying something about the Bishop while chess does not exist.

To explain in the vocabulary of the method of LoA - we cannot speak of the rules of chess outside of the chess-LoA. To evaluate the truthfulness of a statement about chess, we need to evaluate the statement at the appropriate LoA. More generally, there cannot be any knowledge that is independent

of a LoA. This thesis is the central tenet of the philosophy of information and constructionist epistemology in particular. Knowledge is produced with conceptual resources and from some data. A LoA explicates the constraints and affordances of those resources and data. When we apply this reasoning to logico-mathematical practices, we cannot produce any knowledge of logic and mathematics independently of the constraints imposed by our conceptual machinery which is situated in a specific temporal and embodied context.

But then the only way to tackle the tension is to say that the atemporality of the truths of logic and mathematics is guaranteed by its applicability in the types of problems it was designed for. Since, for instance, arithmetic was designed for organizing our cognitive niche, it would actually be surprising if arithmetic wasn't accommodating the truths about that niche. That is why certain temporal changes, still do not affect the way we can utilize a tool. Consider a Babylonian tablet that has marks representing the number '4506'. From the contemporary perspective, we would say that the third digit represents the number '0'. But, for ancient Babylonians the number '0' did not exist, since, what we call a number '0', in their representation of arithmetic, that was a place-holder, i.e. a place in the expression to which we can put some actual number [Hoyrup, 1994]. Even though there is a conceptual difference between the two understandings, we come to understand their calculations with ease. The reason for this is that, even though we operate with a different concept, the niche that arithmetic organizes is the same - our concept of '0' affords anything their concept of a place-holder afforded. The difference between the two is that our understanding of '0' lets us do more - our formal system is more expressive. But, this is a point of a more fine-grained LoA of mathematical practice. In other words, a LoA at which we consider formal tools *qua* historical developments is different from a LoA at which we consider what the tool affords in the broadest possible sense.

Such considerations connect nicely to the problem of the indispensability of mathematics. The indispensability of mathematics is an abductive claim. It starts with noting that mathematics wide applicability to empirical sciences is an intriguing phenomena. Additionally, I would claim that this is the case for logic too. Moreover, the applicability of logic and mathematics to a wide range of situations and the rigidity and persuasiveness they afford are remarkable features. Moreover, to return to the sciences, a great number of discipline in science draw on diverse areas of mathematics, from the use of Hilbert spaces in quantum mechanics to the use of differential geometry in general relativity [Colyvan, 2018]. As another example, biology relies on differential equations and statistics for modelling stochastic behaviour. The examples of sociology and economics spring to mind too and while in some

cases these methods are used to formulate empirical predictions, in others it is used for their elegance. Nowadays, it is hard to imagine how theories such as quantum mechanics and general relativity could even be stated without employing a substantial amount of mathematics [Colyvan, 2018]. But, why is mathematics so successful?

Well, the way I spelled out the interaction between agents and the environment in Section 2, mathematics was seen as a tool for organizing the world around us. But then, it shouldn't be a surprise that mathematics describes nicely the way around us is organized - after all, we used those same concepts to organize it in the first place. Following the constructionist epistemology, we say that we always view the environment from a specific LoA. On the concept of negation I have shown how agents can conceptualize a logico-mathematical tool. So, for instance, arithmetic as a tool makes the discreteness of objects a salient feature of the environment. These mathematical ways of thinking about the environment can, of course, be abstracted even more - consider the Babylonian and contemporary/Indian conception of what '0' is. Once we realize what features we can make more salient, i.e. what we make an observable, we can reuse it in other LoAs. If this is the case, the use of mathematics in other sciences amounts simply to the use of a level of abstraction whose observables are mathematical objects to view a model in a natural science, e.g. biology. As this gives us a quantitative characterization of a biological model, it essentially tells us something about how things encompassed by the biological model behave from the point of view where we adopt mathematical observables. But, it does not tell us anything about what the phenomena is in and of itself; as I argued before in line with Russo and Floridi, such in and of itself questions are senseless. What LoAs with such observables afford is exhaustiveness in the sense of requiring an agent to account for every possible model that can be constructed from a specific LoA. This is what differentiates mathematical proofs from everyday arguments. In everyday arguments, we evaluate only the most probable models of things argued about, i.e. everyday reasoning is defeasible. But, instead of drawing the conclusion that this is why it seems that logic and mathematics are indispensable, I make an opposing one in line with Novaes [2012].

In an everyday setting, defeasible reasoning suffices. We are allowed to make assumptions, infer with generics and argue with nonmonotonic inferences in general. In other words, a defeasible notion of consequence only requires us to inspect the most plausible models of the premises we have [Novaes, 2012], and see what holds in them. But, in cases of high epistemic vigilance, either when we argue with someone that has little trust in us or if that person's job is not to agree with us, an indefeasible notion of conse-

quence is required, i.e. we need to look at all the possible conclusions of our premises, i.e. what are all the ways our interlocutor can twist our words or environment it applies to. So, the specialness of logic does not come from some abstract platonic world nor mysterious innate abilities. Instead, it comes from our ability to exclude any objections for a certain claim. The same dialogical account holds for any deductive practice, i.e. for mathematics too [Novaes, 2012].

Here, I motivated the need for the new epistemology of logic and mathematics which takes their concepts as tools for shaping their practices, instead of the objects of their study. I have also pointed towards possible directions of how such epistemology can be formulated.

4.2 Logico-Mathematical Niche

Now, I turn to the main concepts I have relied on throughout the thesis - affordances and information, to formulate an account of the epistemic aspects of logico-mathematical practices.

Given the epistemic character of logic and mathematics illustrated in 4.1, I claim that we can understand the practices of logicians and mathematicians - but also of scientists too, as practices of niche construction. In particular, as the interplay between agents and their environment is mediated through technologies - they enable interaction and understanding, I will call this techno-niche construction. Agents construct tools, everyday or scientific, as part of organizing their world. In the case of logic and mathematics, humans have acquired mathematical objects through abstracting from our interaction with the world (perceptual or otherwise) via their respective point of interest, i.e. on which LoA they operated. The process of abstraction is useful since we can use those same abstractions to again interact with the world. So, logic and mathematics are understood as a cognitive tool that we use to shape our techno-niche we use for interacting with the world. As Azzouni [1994] pointed out, it is in co-empiricalness of mathematics, e.g. verification of ' $2+2=4$ ' in the empirical world, that gives rise to this interactivity between us and the world through conceptual tools such as numbers.

So, what are the implications of the view of the actual scientific practice as the construction of the techno-niche? Since the scientific practice is a human endeavour to organize the environment with accordance to a specific set of LoAs, looking at the products of the scientific practice from a purely ahistorical view devoid of the technologies that were used in knowledge production seems as a wrong way to go. In a sense, the context of justification

becomes irrelevant, since the knowledge constructing process is fully embedded in the context of discovery. The sciences continually revise the terms and inferential relations through which we understand the world. So the space of epistemic possibilities is not just the area where justification takes place, but also the arena in which inferential practices determine our conceptual understanding as science develops. So, the scientific practices provide a conceptual understanding of our environment that enables the working knowledge of scientists, i.e. scientific concepts simply are tools for operating in the environment, instead of simply being used with the aim of representation.

Without the conceptual understanding we would not be able to inhabit nor organize the world that is of our own making. Note that, it is not the case that we should eliminate thinking in terms of the context of justification and focus on the aspects of the context of discovery, for the latter also fails to explain the transferability of technologies both synchronically and diachronically. In the former sense, it fails to explain how we can utilize technological advancements transdisciplinary. For instance, why a certain mathematical breakthrough can have immediate implications on the practice of physics. In the latter sense, it fails to explain why there is a continuity in our technological development. For example, why the arithmetic calculations of ancient Babylonians and the contemporary ones are intranlatable and incommensurable, even though we operated with different conceptions of what the sign ‘0’ stands for. The answer for the synchronic challenge lies in the ability to apply the same observables in LoAs aiming to generate models of different phenomena.

We can make the same features salient of different systems, thus fostering the applicability of one method in several fields. For example, the notion of logical consequence can be used across fields, as well as say the concept of a discrete object. In the diachronic case, we rely on the ability to view the same system from the LoAs of different degrees of abstractedness. The fact that our technologies improved our practices and we can generate different models of the same system, say genetic code, does not mean that these LoAs are talking about entirely different things. On the contrary, it means that we can generate different models of the same system enabling us to construct information about it in several ways. This transferability of technologies and intertranslatability of LoAs is what characterizes human techno-niche and also what blocks the incommensurability claim.

To see how this is done, consider an example of observational incommensurability put forth by Kuhn [1962] where, as he claims, the practitioners of science that operate in different paradigms live in different worlds. If one scientist has a flat and the other a curved conception of the matrix of space,

they can look at the same phenomena, but for the former it can contain constrained bodies that fall slowly, with the solution to observational data being a compound, while for the other it a set of pendulums that repeat their motions again and again, changing the solution for description of the phenomena. Then, the two groups of scientists see different things when they look from the same point in the same direction. But, I claim that in the same way how different logicians can look at what negation is based on different observables, they can still talk with one another about the correct or at least more appropriate conception of negation. So, the problem here is, if we have different LoAs, how can we compare our models that are generated by those LoAs. Here, the feature of sociality discussed in 2.3.2 helps. As I explained there, Massimi [2018] relies on intertranslatability of perspectives to save the realist picture of the scientific practice. By how can this intertranslatability be done? Well, as long as two models are based on LoAs that differ in abstractedness, but still share some observables, intertranslatability is safe. For, only because we look at, for instance, logic from an abstract point of view and from a more applied one, it does not follow that the two are incompatible. Moreover, they can reinforce one another. Consider the case of logic and reasoning again. The information with which we ordinarily operate can change and it is always incomplete. So, in everyday reasoning, we often need to draw probable conclusions, but that are not necessary and we might change the conclusions after receiving new information.

In classical logic, we can only draw the necessary conclusions. Hence, in some cases, classical logic and everyday logic will give us different solutions to the question: Is this a good argument? This is one of the main criticisms concerning the normativity of classical logic for everyday reasoning. But, does that mean that a classical logician can never understand what is going on in everyday reasoning? As we have seen in the dialogical account of deductivism, deductive reasoning is not that disconnected from everyday reasoning. It is the limit case of everyday reasoning, i.e. the one in which we consider every possible model of our premises. So, the model of deductive reasoning is generated from a more fine-grained LoA in which we look at all of the models, on a lower more coarse-grained LoA, we find the everyday notion of reasoning. Given that, agents can easily move alongside the coarse-fine-gradedness axis simply by generating different models by including or excluding observables concerning the constraints on what models we ought to consider in our judgement of arguments. This is why the conception of the scientific and logico-mathematical practices as an endeavour led by the adoption of LoAs is a relationist one.

An objection can be made that sometimes, two agents can implement

completely different observables in their respective LoAs, thus making them not being different degrees of abstractedness. But, how can such a thing happen? Considering the technological view of practices defended here, this would require agents operating with completely different technologies. Not only that, but they would have to operate with technologies that the other party cannot adopt. But, what kind of thing should it be that two disjoint subsets of humans can adopt each a specific set of technologies that the other group cannot? This is not a problem of technologies. Here, the problem lies in how to even demarcate between two such groups of human agents. Given that technologies are human artifacts and that humans actively participate in shaping their techno-niche, as long as there is a reason to group all humans under one setting (namely, that of being human), it does not seem possible that such radical incommensurability could take place.

I have argued that we need to understand the practices of logic and mathematics as instances of techno-niche construction. Then, I have used such understanding to mount responses towards the indispensability and incommensurability problems concerning logic and mathematics.

4.3 The Philosophy of Logico-Mathematical Practices

In this final part, I want to highlight the benefits of the approach endorsed in this thesis and outline potential future research lines.

In 4.2 I have highlighted the importance of developing the method of LoA for examining the construction of the techno-niche, i.e. looking into the practice of science. This opens up a method for determining the ontological and epistemic commitments of scientific acts (observation, experimentation) by explicating the observables involved [Floridi, 2011]. In that way, we can explicitly see in what ways we can transfer technologies across disciplines, as well as how to achieve translatability of theories within and across disciplinary divides. It is precisely this ability to transfer technologies that bring about the engineering value of certain technologies - as I pointed out in the epistemic values subsection. Such explication fosters a push towards greater transferability and reproducibility of information, but also towards easier interdisciplinary research, since different methods and fields can have a common language (LoAs) for transferring information. Finally, this has implications on the study of scientific practice, since technology is on par with the scientists in terms of knowledge production.

In this case, research into the practice of science and that into fostering of interdisciplinary collaboration needs to develop a method for, once

that the relevant technologies were identified with analysing respective LoAs, transferring technologies across disciplinary borders. In a sense, this would amount to negotiating the adequate LoA to be shared among the researchers. Of course, except accounting for the difficulty of transfer of conceptual resources across disciplines, there is also an issue of the epistemic dependencies within a research group [Wagenknecht, 2015]. So, to be able to do this, the method needs to acquire a firmer ground in an empirical investigation of the ways conceptual resources are shared in and among research groups.

Now, I want to outline potential topics and motivation for their study. These topics are: cross-disciplinary transferability, intradisciplinary reuse, and theory-choice in logic and mathematics.

Cross-disciplinary transferability denotes actual cases and possibilities for transfer of tools across disciplines. In the context of formal methods, one can investigate how formal constructs developed in, say computational biology and particle physics, can be employed in other fields that are dealing with similar problems or even what are the features of a specific tool that enables it being used for a variety of tasks. This question is of great importance as it is essentially a question of maximal use of a tool we have developed. To address it, we can use the method of LoA to explicate what certain tools afford, and then investigate how can this be transferred to other domains of epistemic inquiry.

Intradisciplinary reuse can be seen as tool recycling. In the history of mathematics, we have many examples in which novel tools completely failed to achieve the purpose they were designed for, but that then initiated a conceptual breakthrough. For example, Giovanni Girolamo Saccheri tried to prove the parallel postulate (fifth postulate) of Euclid in his work *Euclides ab Omni Naevo Vindicatus*, by proving that, if we assume the sum of the internal angles of a triangle is either more or less than 180 degrees, the parallel lines are not straight. Around 150 years later, his proofs became theorems of elliptic geometry, i.e. his constructs were recycled to formulate a completely new idea in mathematics [Fitzpatrick, 1964]. So, Saccheri's work already afforded the construction of elliptic geometry, what lacked was such interpretation of his work. Method of LoA can be used in the study of the history of science, logic, and mathematics, to investigate such cases and, potentially, rediscover tools that can be recycled and applied today.

Theory-choice in logic and mathematics is understood in this thesis as being driven by abduction. With analysing the tools of these disciplines with the method of LoA, we can see the following. Any logico-mathematical artifact has three components. Its building blocks contained in the type variables,

observables which specify the mode of interpretation and the predicates constraining the observables. The choices can be made on all three levels, each level having a particular consequence of the resulting theory. Type variables determine the theoretical commitments of the resulting theory, as they specify what is being measured and how. Observables we choose to interpret the type variables determine the explanation of the system that the theory provides. Finally, the behaviour of the LoA tells us what are the most basic predictions of a theory, i.e. what would it take for the observables to be falsified. Hence, I believe that the method of LoA should be tested to see what benefits we can really have in the actual practice of logic and mathematics - but also of science in more general.

I outlined the benefits and potential applications of the method of LoA in the epistemology of sciences. Much work needs to be done, but with the conceptual foundations developed in this thesis new lines of research into the epistemic aspects of tools for inferential practices can be made.

5 Conclusion

The central problem of the thesis was answering the question of the epistemic role of technologies in the production of knowledge. In Section 2, I have argued that the concept of technology has to be broadened to include all of the human artifacts that shape our practices. Consequently, concepts are construed as cognitive technologies that shape our cognitive lives and, in particular, logico-mathematical concepts are tools for shaping our inferential practices, i.e. the improvement of our inferential capabilities amount to improvement of our tools for inference.

I have motivated this view by collecting and reviewing diverse literature, ranging from architecture and archaeology to cognitive science and mathematics, which suggest that the tools we use are constitutive of the knowledge-producing processes we engage in as epistemic agents. In the context of logic and mathematics, this points us to a different understanding of the epistemology of these fields - as outlined in Section 4. To be able to formulate such epistemology, in Section 3, I have introduced the method of levels of abstraction (LoA), which enables us to specify the affording and constraining role of technologies in information processing. Given that, I argue that we ought to adopt a new, informational, framework to study the practice of logic and mathematics because it allows us to systematically express the features of such practices relevant for the epistemic production.

Moreover, as I repeatedly point out throughout the thesis, we can scale up this framework to the scientific practice in general, because, in informational terms, both involve the same kind of epistemic practices. This follows from the conception of technologies I developed. As I have shown in Section 3, there are much more commonalities between seemingly radically different tools, e.g. between the microscope and the logical negation, than one might think. The commonalities lie in the way both tools shape the epistemic production. Moreover, this points us towards the value of logic and mathematics for the scientific practice. Since such formal concepts shape our inferential practices, they also shape the ways scientific inferences are made, thus giving a great responsibility to logicians and mathematicians as, at least partially, sculptors of the scientific thought. In Section 4, I highlight the importance of the empirical study of such influences. Furthermore, I looked at how such a framework can help us make sense of the apparent indispensability of mathematics and a prioricity of logic and mathematics, which resulted in the understanding of logico-mathematical practices as techno-scientific niche construction; and I argue that this can be expanded to all of the scientific practice. This leads us to the understanding of the scientific knowledge

as having a constructionist and non-representational character.

If we accept the characterization of technologies as developed in Section 2, my claim is that the right way to proceed with the study of epistemic processes is by adopting an informational framework as developed in the philosophy of information. Hence, this thesis can be read as a motivation for adopting information-based epistemology in general, as the right way to do philosophy. The benefits of such a perspective, as I have shown in Section 4, are stressing epistemic continuity - as opposed to incommensurability, interdisciplinarity - understood as examining a system from a variety of LoAs, and the break of the individual-collective and internal-external distinctions with respect to epistemic agents - knowledge is distributed and constructed, as opposed to isolated and discovered.

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