On Logical Nihilism

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Abstract

This thesis is concerned with a family of position which are labelled as *logical nihilism*. There are two main theses which are denoted by this term: First, Gillian Russell defends the thesis that there are no logical laws. Second, Aaron Cotnoir maintains that we cannot capture natural language consequence with our formal tools. The goal of this thesis is twofold as well: First, the arguments both authors provide for their respective position are presented and subsequently supplied with clarifications and further refinement. The second step of the thesis is to criticize these arguments in their strengthened form.

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Contents

Introduction: Three Varieties of Nihilism

This thesis is concerned with the recent emergence of "nihilism" in the philosophical literature on logic. In the last few years several articles by different authors have been published which have the proclaimed aim of defending nihilism with regards to logic. In this thesis I give the first lengthy in-print discussion of this fairly recent trend: What is this new, negative outlook on logic all about? In the following I will use the word "logic" or "logical system" in a fairly naive way: A language supplied with a consequence relation.

The first problem, however, which any account of logical nihilism faces is that the term is used by the authors in a divergent manner. Thus, one has to distinguish these three theses:

1. [**Logical Nihilism**] There are no logical laws.
2. [**Formalizability Nihilism**] Formal languages cannot capture the consequence relation of natural language.
3. [**Nihilism about Applied Logic**] Applied logic is a vain exercise and should be dropped in favour of pure logic.[1]

The first kind of nihilism is concerned with our concept of logical consequence. According to this position there is no argument which actually exemplifies a logical consequence relation. This version of nihilism is defended in several recent articles by Gillian Russell. It should be quite clear that the second form of nihilism, which I have labeled formalizability nihilism, is quite different than the first one: According to it, formal languages fail to model all aspects of natural language consequence. This thesis is defended by Aaron Cotnoir (2018). The third version of nihilism, on the other hand, operates on a different level. First, let us introduce the notion of *formal* or *pure* and *applied* logic (compare Priest 2006a, 195f.): If we are concerned with logic on its own and the properties which

[1] A fourth use of "logical nihilism" can be found in Estrada-Gonzalez (2012, p. 179): There he uses it to denote the thesis that it is possible that every sentence is false or, at least, that no sentence has a designated truth value. However, since he only mentions it in passing I will not go into this matter.

logical systems exhibit we typically talk about pure logic. Here, a logical system is a mathematical entity and we investigate its mathematical properties and relations to other systems without interpreting it in any other way. If, on the other hand, logic is used for some other purpose and, thus, an external question of correctness is applied we are typically talking about applied logic. Here, it is not enough to describe the mathematical entity on its own but one has to add some interpretation, i.e. some relation to extra-mathematical objects. Thus, the third version of nihilism says that applied logic is not a fruitful enterprise. In parts of Franks (2014) one can see such a thesis shining through. However, Franks does not seem to defend this position.[2] In short, these are the three varieties of nihilism which will play a role in this thesis in which the first two will be discussed in way more detail.

These three stances in the philosophy of logic, however, share more than just a common name: All three authors see themselves as replying to the recent debate between logical monists and pluralists. Thus, in their own way, each position is seen as a negative perspective for the outlook of this debate. This reference to the monism-pluralism debate is not arbitrary but there is a common story to be told that links it with the three versions of nihilism: The monism-pluralism debate is a dispute within applied logic, i.e. the participants of the debate argue about whether one or many logical consequence relations capture what they are after. The intended interpretation of these authors is something like natural language consequence, i.e. some entailment-relations which we are faced with in natural language. Further, a general presupposition in this debate seems to be that the logical system we are dealing with is *non-degenerate*: A system is degenerate if its consequence relation is either empty or trivial. Given this picture one can bring all three versions of nihilism together: They all maintain that no non-degenerate representation of natural language consequence is possible. First, Russell's nihilism argues that the correct logical system has to be degenerate. Cotnoir's formalizability nihilism amounts to the claim that there is no correct representation of natural language. Finally, given the other two versions of nihilism in the background, Frank's version of nihilism makes sense: The project of applied logic seems to be futile.[3]

This thesis is structured in three parts: In the first chapter, I will give a detailed

[2]Actually, it is not clear at all what Franks is aiming for in his paper. As far as I can see, the most charitable interpretation of his text is that he tries to make a plea for a clear division of labour: Questions of pure logic are interesting on their own and should not be subject to the criticism of philosophers which are concerned with applied logic. This seems to be a correct view to endorse. However, it is hard to see whether there is any valuable philosophical thesis to discuss here.

[3]One might take Russell (2018b) as an example of this connection: There she proposes a method of *lemma incorporation* as a reaction to her nihilism. While proper applied logic will only have a degenerate logical consequence relation we can still inquire into structures which have an (artificially) restricted scope, i.e. we can still make claims like "if we are only concerned with bivalent sentences then the law of excluded middle will be valid". This, however, looks quite similar to Frank's version of nihilism. After all, we are merely concerned with pure logic in order to make these statements. Thus, her method of lemma incorporation might be seen as giving up on applied logic proper.

discussion of Russell's argument for logical nihilism. This will contain some clarifying refinements of her presentation and an in depth assessment of the various steps in her argument. As I will show there is some vagueness in her argumentation which demands for a proper treatment. I will provide several ways to make her account precise. However, no matter which way one chooses, I will show in how far this clarification makes her position vulnerable to criticism: The counterexamples she puts forward against the validity of our most fundamental logical laws are not justified.

The second chapter will be concerned with Cotnoir's arguments for formalizability nihilism. Here, I will go through his discussion of pluralism, semantic paradoxes, universal quantification and vagueness. Thereby, I will cast serious doubt upon the success of his argumentation: I will show in how far his appeal to the debate of logical monists and pluralists is different from Russell's and that it does not provide a point in favour of his nihilism. Second, I argue that the troubles of semantic closure, universal quantification and vagueness are not limited to formal languages and, thus, cannot provide a firm ground for formalizability nihilism. Last, I concede that higher-order-vagueness might establish his intended thesis. However, in order justify this step he has to take a stance towards the project of formalization which makes the label "nihilism" futile.

The third chapter, on the other hand, will be different in tone: Since the third version of nihilism has no clear proponents I will rather concern myself with mapping the philosophical landscape on the topic of applied logic. This will give me the opportunity to place the first two versions of nihilism within these debates and, thereby, provide a further elucidation of their relation. Last, I draw a conclusion in which I sum up the results of these discussions and point towards further questions for future research.

Chapter 1

Logical Nihilism

In this chapter I will spell out the first version of logical nihilism: There are no laws of logic. In a couple of papers Gillian Russell introduced this idea in recent years. Her general reasoning towards this thesis is that since logic is supposed to be absolutely general we can always produce counterexamples to alleged logical laws (compare 2018, p. 308). However, in order to motivate this general line of attack her papers seem to exhibit a more complicated argument (especially Russell 2017a). As I understand her she puts forward the following four-stepped argument:

1. [**Logical Consequence**$_{sem}$] $\Gamma \models \varphi$ iff in all models M with $M \vDash \gamma$ for all $\gamma \in \Gamma$: $M \vDash \varphi$.
2. [**Universalism**] There is no constraint as to which models can be quantified over in the definition above.
3. [**Monsters**] If universalism is true then we can always find a model M such that $M \vDash \gamma$ for all $\gamma \in \Gamma$ but $M \nvDash \varphi$.
4. $\therefore$ [**Logical Nihilism**] It is never the case that $\Gamma \models \varphi$. *[from 1,2 and 3]*

So, if we accept a semantic take on the concept of logical consequence, i.e. define it in terms of truth-preservation over a set of models, but do not restrict the domain of quantification then we will include unforeseen models which will be counterexamples to our most basic logical principles (compare Payette and Wyatt 2019, 3f.). However – as presenting this argument to logicans has taught me – this argument has to be false: We can prove that the relation picked out by the semantic definition of logical consequence is reflexive and, thus, we can show that no matter what logic for all : $\varphi \models \varphi$. Hence, there has to be something wrong with this argument. The aim of this chapter is to point out where the mistake exactly lies and what it tells us about the prospects of logical nihilism.

In this chapter I want to take a closer look on this argument and account for every step in it: First, I am going to present the background of the premises of the argument. Hence, the first section of this chapter will introduce the philosophical debate on the concept of logical consequence. What is the motivation for understanding logical consequence in terms of models? In the second section, I will outline the motivation which Russell puts forward for her assumption of universalism: According to her, the recent trends in the philosophy of logic, i.e. the discussion of logical pluralism, has brought to light several arguments for universalism. Next, I will present Russell's own countermodels to the most basic logical laws, i.e. conjunction elimination and identity. In this section I will refine Russell's way of setting up these models and present it with some internal criticism. We will see that the argument which I stated above does not go through but has to be adapted in some important ways. Last, I present some external criticisms of her position, or rather some criticisms of the starting points of her argument.

1.1 The Semantic Concept of Logical Consequence

In this section I am going to present some background for the starting point of Russell's argumentation. Since most people would say that the concept of logical consequence is at the very heart of logic I will have to restrict my attention here to really basic points in order to keep this section short. In a certain sense this whole thesis is about the concept of logical consequence and here I am only interested in introducing some basic notions. Therefore, I will come back to some of the issues of this presentation in later parts of this work. This section will proceed as follows: First, I am going to point to some characterization which logical consequence has received. Second, I turn to the formal, mathematical way of spelling out this concept: Either one goes down a syntactic route or a semantic one. Last, I will present some different versions of the semantic approach since this has some important consequences for Russell's argumentation.

1.1.1 Logical Consequence? What's that?

In order to introduce the semantic definition of logical consequence I have to give a broad sketch of what this definition is supposed to capture. Following Beall et al. (2019), I will present some platitudes which are usually ascribed to this concept.

First, logical consequence is usually used to describe some form of relation between the premises and the conclusion of an argument. A argument is called valid iff the conclusion follows from the premises. There is a certain issue about what exactly the premises and conclusions are. A classical candidate for the relata of logical consequence are propositions. However, as work on indexicals and other context-sensitive expressions has shown, it might make sense to take uttered sentences as the relevant entities. To answer this question, however, would go beyond the scope of this thesis.[1] Since context

[1] As Russell (2008) notes, this metaphysical question about the relata of our consequence relation could

sensitivity plays an important role in the later parts of this chapter it makes sense to regard the relata of logical consequence as somehow context dependent. Hence, my discussion in 1.3.3. will be about how exactly context and consequence are related and not about whether they are related at all.

Second, not any argument which we would except as "correct" or "good" exhibits a logical consequence relation. In order to be ascribed this status to an argument the truth of the premises has to necessitate the truth of the conclusion: It is not possible that the conclusion turns out to be false, given that the premises are true. Hence, one usually distinguishes inductive and deductive arguments: While the truth of the premises of an inductive argument merely makes the truth of the conclusion more probable (in some sense), a deductive argument rests upon a necessary relation.

Third, this necessitation has to be due to the form of the argument, else it would be a merely material inference. Whatever the relata of logical consequence are we seem to have a *hylomorphistic* idea of how they are composed. This idea can be cashed out in different ways (compare MacFarlane 2000, 50f.): The formal part could be the one which exemplifies the necessary conditions for having a sentence in the first place. If these formation rules lie below every possible sentence there seems to be a clear connection how necessity comes about. Second, one could regard the formal part of a sentence as the structure which stays the same even though the object under consideration change. In modern terms: the form of a sentence is the set of its permutation-invariant components. Last, one could regard the form as the structure which stays if we abolish any relation of the language to the world, i.e. if we abstract from any semantic content. In these last two, it seems also quite clear how one could say that necessity is secured: No matter how the world turns out to be our sentence's form is unaffected.

Thus, our logical theory tries to give a systematic account of this pretheoretic conception. The relation which our logic describes has to account for our judgments of formality and necessity: If our formal apparatus declares certain inferences as valid which are not formal or necessary then it seems to be the wrong "logic".

1.1.2 Semantic and Syntactic Approaches to Logical Consequence

In the introduction I already mentioned the distinction of formal and applied logic. Here, we want to apply formal logic to our concept of logical consequence as I described it in the previous section: How can we use mathematical tools to formalize this concept? There are two main traditions of how such an attempt is supposed to be done: A syntactic, proof-theoretic one and a semantic, model-theoretic one.[2] In the most common

be seen as a case for logical pluralism. Thus, there might not even be a question to answer here.

[2] Beall et al. (2019) mention a third, hybrid analysis of logical consequence which combines the above mentioned approaches. However, the aim of the section is to clarify what the semantic conception of logical consequence is since this is the only requirement we need in order to understand Russell's

presentations of logical consequence, of course, both proof-systems and model-theoretic semantics are used. Usually we show that some system of proofs is correct and complete with regards to some model-theoretic construct. Here, however, we are concerned with a more fundamental, philosophical issue: Which definition captures the essence of our concept of logical consequence better, i.e. which one should seen as the primitive?

First, let us have a short look into the syntactic account of logical consequence. According to this tradition logical consequence is best spelled out in terms of proofs: An argument is valid iff there exists a proof which given the premises as an input will output the conclusion. But how, one might ask, does a proof guarantee that the truth of the premises necessitates the truth of the conclusion? Given the characterization of logical consequence I sketched above this seems to be a reasonable thing to ask in this context.[3] This worry is usually answered by some form of anti-realism concerning “truth”. So, for instance, “φ is true” could be reinterpreted as “there exists a proof of φ” (compare Schroeder-Heister 2018). Now we can understand how the necessary “truth-preservation” works: Given that there is a proof of every premise then a valid argument, i.e. a proof from the premise to the conclusion, guarantees that there is a proof of the conclusion.

In opposition to this anti-realistic understanding of logical consequence we can also go down a realistic way which is commonly associated with the semantic, model-theoretic formalization of logical consequence: Here, “truth” is understood along the lines of Tarski (1935) as satisfaction in a mathematical model. So, if all the premises are satisfied in a model, then the conclusion has to be satisfied as well. In order to account for the necessity of logical consequence Tarski (1936) defines logical consequence along the following lines: In all models in which the premises are satisfied, so is the conclusion. Hence, we use the mathematical entity, i.e. the *model*, to represent some instance of an argument, i.e. some sort of *case*. Thus, logical consequence is inherently linked to the idea of a counterexample. As S. Shapiro (1998, p. 141) puts it: ”The realm of [formal] models may be thought of as representing the source of counter-models in natural language reasoning.” There are different interpretations of both these terms: Of course there are different precise mathematical definitions of what a model is. For now, we can use the term as standing for some of these case-representations. On the other hand, there are different interpretations of what these cases are which the model-theoretic machinery is trying to capture. This is basically the topic of the next section. For the current purpose it is enough to note that

argument. Thus, it makes sense to restrict the presentation to these two traditions in order to make out a clear contrast. Similarly I will leave aside considerations like S. Shapiro (2007) who conjectures that these two formal approaches pick out two different intuitive conceptions and, thus, are a way to disambiguate two notions that are commonly conflated.

[3] However, people who regard this approach to logical consequence as fundamental might not agree with the characterization of the basic tenants of logical consequence which I have given above. The fundamental properties logical consequence are quite different in their view. My presentation of this issue in 3.2. will be rather along these lines, i.e. not on the level of two mathematical descriptions of one common concept but on the level of a disagreement about this concept itself.

the model-theoretic, semantic approach to logical consequence is understood as truth-preservation in all cases. This is supposed to guarantee necessity. However, there is a notorious difficulty to pin down how exactly this definition is supposed to account for the formality of logical consequence.

As I have stated above Russell starts her discussion of logical nihilism from the point of a semantic account of logical consequence. If one adopts a syntactic, proof-theoretic account of logical consequence it is harder to see how an argument for logical nihilism could work. This is due to the fact that the proof-theoretic account of logical consequence is not really related to the idea of counterexamples. Hence, a route to proof-theoretic nihilism would have to look significantly different. Of course, the syntactic approach, just like the semantic approach faces a certain problem of restricting the admissible instances of the definition. In fact, this was Tarski's main motivation to introduce his alternative: If we define logical consequence by proofs of a deductive system then it seems to depend on the chosen proof system at hand and, thereby, appears to be quite arbitrary. Thus, given certain principles of restriction of admissible proof-systems it might turn out that there are admissible systems with an empty consequence relation. However, in order to end up in nihilism one would have to argue that any non-empty consequence relation contradicts some principle of being an admissible instance which seems to be a very long shot. Thus, I will conjecture that there is no clear path to proof-theoretic nihilism which would be parallel to Russell's ideas. If she would want to argue for nihilism within the syntactic approach she would have to take quite different philosophical grounds as her starting point.

1.1.3 Some Varieties of Semantic Definitions

In this section I am going to present some versions of semantic conceptions of logical consequence. Even though this field of debate is not directly related to the discussion that follows I think these varieties of understanding the concept of logical consequence are a nice way to spell out one of the central mistakes of Russell's argument which I will diagnose in 1.3.3. As Russell (2018c, pp. 332-340) realized one can broadly distinguish four different ways to spell out such a notion.[4]

The first two accounts of logical consequence are labelled as *representational* and as *interpretational* (compare Etchemendy 1990). According to the representational conception logical consequence is understood as truth-preservation across all possible ways the world could be. Thus, in this version of the semantic definition models are taken to represent different configuration of the world. The interpretational account, on the other

[4]In order to avoid confusion it is useful to point out that Russell takes "case" to denote one of the specific varieties while I take it as a name that is independent of these varieties: The semantic definition quantifies over models which are formal representations of cases. What exactly cases are is left upon and has to be interpreted further, for instance, along one of the four lines which Russell presents.

hand, takes models to represent different assignments of meaning to the (non-logical) vocabulary. Hence, logical consequence amounts to truth-preservation under all variations of meaning.[5]

A third variety of the semantic concept of logical consequence is the so called *substitutional* account. According to this account we do not reinterpret our language but simply substitute every (non-logical) expression with a grammatically correct counterpart. Such a theory was endorsed by Quine (1970, 49ff.) but it was already opposed by Tarski (1936, 7ff.): If we just substitute expression of our language for each other then our concept of logical consequence will depend heavily on the expressiveness of our language (compare Etchemendy 1990, 30ff.). However, there are some advantages to this approach to logical consequence, as Halbach (2018) has pointed out in his recent paper: The substitutional approach does not necessarily need a *model-theoretic* formalization of the concept of logical consequence. According to Halbach, this is an advantage since the reliance on model-theory is problematic. Thus, we should not be to quick in disregarding substitutionalism in favor of interpretationalism.[6]

The last and youngest attempt to understand the semantic conception of logical consequence is the so called *universalist* approach which was put forward by Williamson (2013; 2017).[7] Instead of reinterpreting the non-logical vocabulary we substitute suitable variables for these expressions and then use a universal quantifier to bind them. Thereby we get the *universal closure* of the *shell* of a sentence. If a sentence is a logical truth then the universal closure will turn out to be true. This idea can be extended to logical consequence: We have to check whether any assignment that makes the premises true also makes the conclusion true. For example, if we want to check weather some argument $Pa \wedge Qb \models Pa$ is correct we will first create the universal closure of the shells of the respective sentences, i.e. $\forall X \forall Y \forall x \forall y (Xx \wedge Yy)$ and $\forall X \forall x (Xx)$. Second, we check whether $\forall X \forall Y \forall x \forall y ((Xx \wedge Yy) \rightarrow Xx)$ is true. If ones takes the "$\rightarrow$" here to denote a material conditional then this should be a second-order language instance of the definition of logical consequence which I have given in the previous section: In order for this sentence to be true, all assignments that make the premises true have to make the conclusion true. This

[5] S. Shapiro (1998, p. 148) proposes, on the other hand, that both these notions are in play when we interpret our model-theoretic description of logical consequence: A case is any interpretational variation of our language with regards to any representational possibility. Thus, these two varieties are not exclusive.

[6] I will leave the details of these problems aside for this thesis. They should not matter for the questions we are concerned with. However, *prima facie* one might wonder whether this abandoning of model-theory does block Russell's argument which I have stated above. After all, there we have made reference to models. However, all Russell needs in order to get her argument of the ground is that there is a substitution instance for "I am walking down the street. Hence, I am walking down the street" such that the premises are true while the conclusion becomes false. Whether we use model-theory or some other formal tool to pick out such (alleged) substitution instances is not important. Hence, as far as I can see, the problems which Halbach is trying to tackle do not have any impact for the current argument.

[7] Note that this use of "universalism" is different and completely independent of the one above. It is a unlucky ambiguity. However, down the line it should be made clear by the context about which thesis I will be talking.

approach to logical consequence is special since it does not formulate logical consequence in a meta-language but in the object language of second-order logic. Thereby, it differs quite a lot from the other three varieties.

For the purpose of this chapter it does not really matter which of these conceptions we follow.[8] In order to give a presentation of Russell's argument for nihilism it is enough to accept some version of the semantic definition of logical consequence since they provide a nice elucidation of what the opaque term "case" is supposed to stand for. As Russell (2018c) notes, however, these different characterizations may lead to different results if we try to resist nihilism: She argues that universalism has a better case than the other two. But, as I already mentioned in the beginning of this section, I will not go down this route. Rather, I use these different ways to specify cases in order to spell out the basic idea of the semantic definition. This should be conducive to make one of the flaws in Russell's argument clearer.

1.2 From Pluralism to Universalism

Let us assume that we accept some version of the semantic approach to the concept of logical consequence. Hence, we subscribe to the view that logical consequence is best understood as truth-preservation in *all* models, i.e. all representations of possible cases which are specified in some of the ways I presented above. But why should we adopt universalism, i.e. why should we also buy the thesis that the *all* in the definition of logical consequence quantifies over representations of any case whatsoever. Russell's motivation for this further claim starts from the debate of logical pluralism which has taken up a big spot in the philosophy of logic of the last two decades. Her line of thought is roughly the following: Logical pluralism is faced with some serious challenges by the defenders of monism. The point of resistance for pluralism is to show that these arguments lead to extremely weak logics, or even to nihilism.[9] This is meant as a *reductio* of the monistic challenges and, thus, used as a defensive tool by pluralists. However, Russell objects that this step of the argument is fallacious: It is only a *reductio* if we have ruled out nihilism

[8]The formal apparatus I describe in 1.3. is a first-order logic posed in the ordinary model-theoretic setting since Russell poses her argument in these terms. This apparatus, however, is only closely tied to the first two characterization of logical consequence. The other two are usually formalized in a different way: universalism is supposed to be stated in second-order logic and substitutionalism is supposed to be independent of model-theory at all. However, as I already indicated, I maintain that the problems I will highlight in 1.3. are independent of this issue and should generalize to the other two notions.

[9]She expresses this line of reasoning in Russell (2018b, p. 308), (2018, pp. 344f.) as well as (2019). However, this line of thought can already be found in Beall and Restall (2006, 92f.) and Bueno and Shalkowski (2009, p. 300) which Steinberger (2019, p. 16) has comprised as the "Threat of Logical Nihilism". It is important not to conflate this kind of reasoning with the "argument from diversity" which Cotnoir (2018, pp. 304-309) proposes and which will be the subject of section 2.1. There the argument is a dilemma: Either logic is plural or not. It can be neither. So their is no logic. Even though I will take some of Cotnoir's points up in the following discussion, I maintain that there is a general difference between his argument and the one we are faced with right now.

as a possible position. Thus, this whole line of thought should be seen as an argument for nihilism rather than a *reductio* of monism.

The subsections of this section will, again, follow Russell's reasoning. First, I am going to present the idea of logical pluralism and sketch the theories Russell discusses. Second, I look into the authors which defend logical monism and give an overview of their most effective arguments against logical pluralism. Last, I will evaluate whether the presented dynamic of the discussion in the philosophy of logic really leads to a motivation for universalism: It seems clear that the dialectic situation is not a decisive argument in favour of universalism. Rather, it can be seen as raising the basic question whether there is something which restricts our domain of models, i.e. whether we can set boundaries to the set of cases which are governed by our logic.

1.2.1 Logical Pluralism

Logical pluralists maintain that there is more than one correct relation of logical consequence. The basic idea of logical pluralism can already be found (implicitly) in Tarski (1936, 10f.) since he accepts that his take on logical consequence is underdetermined and open for several equally good refinements. Another example is Carnap (1937, 51f.) who argues for a tolerant view on the plurality of different possible logics. An explicit discussion of "logical pluralism" starts with the work of Haack (1978), but it received a new renaissance with Beall and Restall's (2000; 2006) take on the matter. The entry in the *Stanford Encyclopedia of Philosophy* distinguishes several versions of pluralism of which some can be omitted in this chapter (Russell 2019).[10] Thus, I will rather follow Cook (2010, 493ff.) and his way of introducing pluralism. Even though his discussion is too broad for the purpose of this chapter, I will take certain aspects of his presentation on board: According to him, a formal logic $\langle \mathcal{L}, \Rightarrow \rangle$ is understood as a pair of a language $\mathcal{L}$ and a consequence relation $\rightarrow$. $\mathcal{L}$ provides a certain set of statements and $\Rightarrow$ relates a subset of these statements with a singular one.[11] Now, logical pluralism amounts to the fact that there are different $\Rightarrow$ and a general classification of pluralisms can be given as follows:

1. **Carnapian Tolerance Pluralism**: There exists a plurality of consequence relations, but each consequence relation is paired up with a different language. Thus, there will be several correct pairs of the form $\langle \mathcal{L}_1, \Rightarrow_1 \rangle$ and. $\langle \mathcal{L}_2, \Rightarrow_2 \rangle$.[12]

[10]For an even broader characterization see Caret (2019, pp. 3-7). Note, that the "pluralism about modelling" which both discuss seems to be concerned with the topic of the second chapter. This group of theories could even be labelled *formalizability pluralism.* Russell even notes that there seems to be a clear connection between this line of pluralism and Cotnoir's nihilism.

[11]We could also treat $\Rightarrow$ as a relation between sets of statements in order to account for systems which allow for multiple conclusions. However, as far as I can see, the discussion in this thesis should be independent of this issue.

[12]As Stei (2017, p. 2) notes, one could think that the change from $\Rightarrow_1$ to $\Rightarrow_2$ follows trivially from

2. **Substantive Logical Pluralism**: There exists a plurality of consequence reltaions within one language. Hence, there will be several correct pairs of the form$\langle \mathcal{L}, \Rightarrow_1 \rangle$ and $\langle \mathcal{L}, \Rightarrow_2 \rangle$.

So, first, one could have a plurality of languages and a plurality of correspondence relations, or, second, one could have several correspondence relations within one and the same language. However, since this chapter is concerned with the descent from pluralism to universalism which Russell describes I will restrict my attention to substantive logical pluralism. This is due to the fact that the monist replies which I will present in the next section are first and foremost directed at a substantive version of pluralism. This is a historical rather than a systematic point: Carnapian tolerance pluralism may lead down a similar way to universalism. However, this form of pluralism is usually already paired with certain other stances in the philosophy of logic like instrumentalism (compare Restall 2002).

Is it possible that there are several relations of logical consequence within one and the same language? The answer of substantive logical pluralists is "Yes". But how does this plurality come about? In the case of Carnapian Tolerance Pluralism the answer was – more or less – straightforward: Due to the differences in our language we should adopt a different kind of logic. Substantive Logical Pluralists, on the other hand, have to give a more demanding answer: There is some feature of the logical consequence relation that cannot be accounted for in a monistic theory, i.e. a theory with only one pair $\langle \mathcal{L}, \Rightarrow \rangle$. This description would be underdetermined and, thus, has to be relativized to a certain factor.[13] Now, there are several theories in the debate which differ with regards to what exactly this relativizing feature is supposed to be: Beall and Restall (2006) assume that the general semantic definition of logical consequence is underdetermined and, thus, one has to relativize logic to the different instantiations which it might take. Field (2009), on the other hand, rejects the semantic conception of logical consequence and opts for a plurality of epistemic norms which logic can take.[14] A different account again is presented by Bueno and Shalkowski (2009) who take it that the modality of logic can take different

our switch from $\mathcal{L}_1$ to $\mathcal{L}_2$: $\Rightarrow$ is a relation of sets and elements of a certain language. Thus, if we look at different languages, then a $\Rightarrow$ can never be co-extensive with another $\Rightarrow$ of a different language. But there are two major things this argument does not take into account: First, it may be possible to speak of an overlap of languages, i.e. elements which two languages share. There is nothing in the formal structure we have defined so far that would prohibit such a scenario. Second, one could have a different idea of what the identity of consequence relation amounts to. Maybe proper co-extensionality is not the criterion we should use but, rather, some meta-theoretic properties of the relation.

[13] Cook (2010, p. 492) maintains that one has to keep apart pluralism and relativism, i.e. the thesis that the correctness of a logic is relative to some factor. But, even though both theses could come apart, some relativistic property seems to be the only satisfactory explanation for pluralism.

[14] I think it will be clear from my discussion of Beall and Restall's view that they are also committed to a certain relativization to different norms (compare also Commandeur 2018). The fact that Restall (2014) proposes a proof-theoretic version of pluralism which starts from the same motivations as Beall and Restall (2006) might speak in favour of this claim. After all, proofs might be seen as naturally linked to our epistemic capacities. Thus, a plurality of correct proof procedures might correspond to a plurality of epistemic norms.

forms. Zardini (2018), on the other hand, assumes that there are certain domains to which logic has to be relativized. The most influential version of substantive logical pluralism is Beall and Restall's. Thus, I will spend the rest of this section to elaborate their account.

As I already mentioned, Beall and Restall's case-based approach takes it that the classical semantic conception of logical consequence is underdetermined. If one describes logical consequence in this tarskian tradition it amounts to:

(TT) $\Gamma \models \varphi$ iff in every model in which every $\gamma \in \Gamma$ is true φ is true as well.

In the definition of logical consequence for classical logic models are supposed to represent possible worlds, i.e. cases which are complete and consistent. Hence, the informal version of Tarki's thesis is that in every possible worlds in which the premises are true, so is the conclusion. But, according to Beall and Restall (2006, p. 29), this informal thesis has to be generalized to the following:

(GTT) An argument is valid$_x$ iff in every case$_x$ in which the premises are true so is the conclusion.

This sentence is polysemous: There are different specifications that case$_x$ can take and, thus, GTT can define different conceptions of validity.

Furthermore, Beall and Restall (2006, pp. 14-23) identify three basic features which a logic must have. Thereby, they give further criteria for what kind of system counts as an admissable instance of GTT. First, they take it for granted that logical validity is a *necessary* relation between the premises and the conclusion of an argument: If the premises are true then it follows by necessity that the conclusion holds as well. Second, logic is in some way *normative* for thought. If we do not obey the laws of logic then we are doing something wrong. Last, logic is concerned with the *formal* structure of our thought. There is extensive literature on all three of these aspects of logic. For the moment it is enough to note that Beall and Restall use certain features which they take to characterise the logical consequence relation to restrict GTT: Only relations which exemplify all three ideas can be called "logic". Three possible admissible instances of GTT which Beall and Restall consider are classical, relevance and intuitionistic logic which correspond to the following restrictions of cases:

Definition. *Logical Consequence*$_{Clas}$ $\Gamma \models_{Clas} \varphi$ iff in all *possible worlds* w with $w \models \gamma$ for all $\gamma \in \Gamma$: $w \models \varphi$.

Definition. *Logical Consequence*$_{Rel}$ $\Gamma \models_{Rel} \varphi$ iff in all *situations* s with $s \models \gamma$ for all $\gamma \in \Gamma$: $s \models \varphi$.

Definition. *Logical Consequence*$_{Int}$ $\Gamma \models_{Int} \varphi$ iff in all *constructions* c with $c \models \gamma$ for all $\gamma \in \Gamma$: $c \models \varphi$.

Thus, there are three special subsets of the set of cases which are picked out by one of the instances of logical consequence, i.e. possible worlds, situations and constructions. All three relations, they say, fulfill the necessity, formality and normativity constrain and, hence, have equal right to be called “correct”.

1.2.2 ... and the Monistic Replies

There are a lot of arguments in the literature which are directed against logical pluralism. But I will only be able to discuss some of these in the current section. Others will be omitted because they are either not really effective, or they do not establish a conclusion that would make the threat of universalim apparent. Since the aim of this section is to clarify Russell’s argument for universalism I will restrict myself to the following four lines of attack: An argument of generality, an argument of necessity, an argument of formality and an argument of normativity. As one can easily see, these are the three components of Beall and Restall’s view: GTT plus three restrictions thereof. Each of the arguments tries to show that pluralism fails due to the fact that the restriction of GTT is in tension with the other tenets.

The Argument of Generality

This line of attack goes back to Priest (2006a, 202ff.) and it is already echoed in Beall and Restall (2006, p. 92) (compare Russell 2019). It is a straightforward attack on the restriction of the domain of the quantifiers in the formulation of GTT and, thereby, the most clear sign of universalism in the debate. The argument goes as follows: The idea of a semantic conception of logical consequence as introduced by Tarski was to quantify over *all* models because this would create a consequence relation that was invariant with respect to the different ways which the world or our language could be. Thus, restricting our definition to a subset of all models seems to be in contradiction with this basic idea. As Beall and Restall correctly point out, this argument is clearly not an argument for universalism but starts from it and points towards the incompatibility of universalism and pluralism. The other three arguments below, on the other hand, can be seen as a justification for the points this reasoning here takes for granted.

The Argument of Necessity

The argument of necessity runs quite parallel to the argument of generality above. In their article, Bueno and Shalkowski (2009, 299ff.) point out that Beall and Restall’s version of pluralism cannot account for their own demand of admissible instances of GTT (compare Cotnoir 2018, p. 306.): Only those classes of cases are admissible instances which pick out a necessary relation. However, if we cash out the concept of necessity in the standard way, i.e. as a quantification over possible worlds then we run into a certain problem:

The laws of logic are supposed to be necessary. Hence, they hold in every possible world or in every case. Since we are talking about logical laws (and not physical laws) this quantification is supposed to be unrestricted. But we assumed in our formulation of GTT that the jurisdiction of logical laws is restricted to a subset of cases. Thus, we can conclude that all admissible instances which Beall and Restall put forward are not necessary consequence relations, but merely contingent ones. Further, we can see how this reasoning makes the demand for universalism more apparent: If we want our consequence relation to be a necessary, we have to drop the restrictions on the quantification in our definition of logical consequence.

The Argument of Normativity

The most discussed objection to pluralism is known as the "collapse" problem, but one can also label it as an argument of normativity (compare Russell 2019). Let us assume that the normativity of logic amounts to the fact that if one believes in the premises of a valid argument then one ought to believe in the conclusion of the argument. However, if we also assume that logical pluralism is correct and that there are two relations of logical consequence $\Rightarrow_1$ and $\Rightarrow_2$ which differ in their extensions, then the following situation can arise: We believe in a set of sentences Γ which is the set of premises for some valid inference of $\Rightarrow_1$ with the conclusion φ. In $\Rightarrow_2$, on the other hand, φ is not a consequence of Γ. So, given that logical pluralism is correct what are we obliged to believe in this situation?[15] Either we follow the demand of the stronger logic, or we only comply to the weaker one. Either way there is a collapse into monism at the horizon: If we allways opt for the strongest logic at hand, then the existence of weaker relations of logical consequence seems to be in vain. The strongest logic trumps the weaker one when it comes to its normative implications and, thus, monism is back in place. If we go down the other route and only listen to the laws of the weaker logic pluralism looses its bite in a similar fashion: If we are allowed to ignore the demands of the stronger logic, then the only "correct" logic seems to be a minimal one. Hence, pluralism has collapsed into monism.[16]
Even though this argument is the most discussed objection to logical pluralism it is not as straightforwardly connected to a universalist stance as the others: In the first place it is just a defence of monism. However, if we opt for the second option which I mentioned above and become monists that deem the weakest possible logic as correct there might

[15] This kind of argument is usually traced back to Williamson (1988, p. 112) who applied a similar line of thought in a different context.

[16] For early presentations of this argument see Priest (2006a, p. 203) and Read (2006, 194f.) and further defence is provided in Keefe (2014) and Stei (2017). There are several lines of response in the literature. First, one could argue that logic is not normative in the relevant sense (compare Russell 2017b and Blake-Turner and Russell 2018). Second, one could argue that there is still a case for pluralism even if we restrict ourselfes to groups of logics which have the same normative consequences (Barrio et al. 2018). Third, one could argue for some sort of plurality of epistemic norms (see Field 2009, Caret 2017 and Commandeur 2018).

be a direct worry that some sort of minimalism and, subsequently, nihilism would come about.

The Argument of Formality

That Beall and Restall's pluralism may run into problems with the formality constraint is already pointed out by Paseau (2007, 392f.). However, his concerns are rooted in the semantic conception of logical consequence which is expressed by GTT. The argument I want to present here, on the other hand, is concerned with a different kind of pluralism: Domain-based logical pluralism. Such a version of pluralism is presented in Lynch (2008, 133f.) and further developed and defended by Pedersen (2014), Yu (2017) and Commandeur (2018). The basic idea of domain-based logical pluralism is to attribute the restricting or relativizing force (which, for example, determines which logics are suitable instances of GTT) to "domains". Domains are here taken to be constituted by the subject matter at hand.[17] Thus, there might be a mathematical, a physical or an ethical domain. The authors I mentioned usually introduce domain-based pluralism in order to create a bridge between alethic and logical pluralism. However, in our context the motivation for a domain-based pluralism can also be seen in its force to resist some of the other three arguments I mentioned above. For instance, if every domain has its own unique relation of logical consequence then the argument from normativity seems to be blocked: The question of which inferences shall we accept is *prima facie* answered by the subject matter at hand (compare Steinberger 2019, 13f.).

Now that I have introduced domain-based logical pluralism I can present one central argument against it.[18] This argument from formality was presented by Cotnoir (2018, 307f.): If the subject matter under consideration constitutes the domain we have to consider and, thereby, determines which logic is at play, then the laws of logic seem to depend on the subject matter. However, we have said that logic should be *formal*. Hence, if formality is cashed out in terms of subject matter invariance we have a problem: The concept of logical consequence in a domain-based logical pluralism turns out to be a material consequence relation. Thus, the hylomorphism which is common to most accounts of logical consequence (and is most definitly accepted by Beall and Restall) prohibits a relativization of logical consequence to domains. Hence, the monist can maintain that proper, formal logical consequence is the relation which stays the same accross all domains (compare 1.1.1.). Thereby, we end up in a similar predicament as in the arguments above: Logic is in some sense universal to all the precisifications the pluralist puts forward.

[17]This relativization can be seen as a proper restiction of the scope of logical laws. Hence, domain-based pluralism is not a *global* version of pluralism as the GTT-style pluralism claims to be (compare Haack 1978, p. 223).

[18]It is not the only argument, though. Priest (2006a, p. 198) for instance argues against a domain-based pluralism along the lines of the argument from generality and Steinberger (2019) raises the worry of cross-domain discourse.

1.2.3 A Slippery Slope to Universalism?

What are we supposed to make of this line of thought? In order to evaluate this claim one should distinguish two different interpretations of the thesis I have labelled as "Universalism" since conflating these two might lead to flawed objections.

- **Universalism$_1$**: The quantifier in the definition of logical consequence has to range over the whole set of cases.
- **Universalism$_2$**: The set of cases itself is – in some sense – unrestricted.

The first version of universalism is basically the denial of the idea that is captured by GTT, i.e. there is a set of cases and we can define logical consequence for a subset of these. Instead, we have to take the whole set of cases into consideration. Universalism$_2$, however, is concerned with the membership of the set of cases itself and is the denial of the idea that there are strict constraints on what counts as a case. I leave it deliberately vague how strong this claim has to be. Now, all the arguments which arise out of the discussion of pluralism and which I have presented above are concerned with universalism$_1$: They show that a pluralist position runs into troubles since it does not quantify over the whole sets of cases. Here, I take it that they make some good points: The restrictions of the GTT-kind seem to lead into trouble. If we take something to be a case then our quantifier in the definition of logical consequence better range over it.

However, if we take universalism to amount only to universalism$_1$ then we have to be cautious of the following: Usually any monist position is a universalist one! It would be quite unusual to believe, for instance, that classical logic is correct but to maintain that there are incomplete or inconsistent cases.[19] There might be incomplete and inconsistent models but they do not pick out any entity that is important to our concept of logical consequence. Hence, universalism$_1$ plays only a secondary role with regards to the topic of this work: If you recall the outline of the argument that I take Russell to give then we have to take universalism to be a thesis that provides sufficient justification to add cases which are picked out by the monstrous model to the set of cases. So if the classical logician is correct these monsters, as we will see in the next section, are definitely not in the set of cases. To conclude: universalism$_1$ is not sufficent to justify Russell's argument.

I hope this remarks have made it clear that we need something stronger: Universalism has to include a claim along the lines of universalism$_2$, i.e. the set of cases itself has to be broad enough to incorporate the Russell's monsters.[20] This, however, is not established

[19] This position is called *logical particularism* and is defended by Payette and Wyatt (2018; 2019). They take a similar starting point as Russell's nihilism but then drop the demand that a law of logic has to be general. Thus, *generalism* in their terminology corresponds to universalism$_1$ in the present one.

[20] Russell does not accept an unrestricted version of universalism in which any mathematical construct whatsoever picks out a case. As will become clear in the next section, Russell is trying to keep her models quite conservative. For instance, she is trying to avoid a change of meaning of the logical constants. Thus, there seem to be natural limits even to this kind of universalism.

by the discussion above. At most, the arguments have shown that if we think that our set of cases contains situations as well as constructions – like Beall and Restall – then we have to opt for a logical consequence relation that is weak enough to capture both of these. Thus, we have established a conditional claim which depends on our acceptance of some cases. But, universalism of the kind we need is not justified by this discussion alone: The question whether Russell's models pick out some members of the set of cases has to be evaluated on its own!

I conclude this chapter on a positive note, though: The debate of pluarlists and monists has shown that there is still an open question how exactly we are supposed to restrict our set of cases. A direct line towards the premise that Russell needs does not exist here. However, one could say that the existence of non-classical logic and the acceptance of more and more cases put the question of universalism on the table: What are the boundaries of the set of cases? If it does contain more than classical possible worlds than what else are we supposed to accept? Thus, one could say that there is a *slippery slope* towards universalism: We have accepted more and more cases in the past, so why not the ones Russell is describing with her monstrous models?

1.3 Monstrous Models

Can we really create such bizarre models such that even the most basic laws of our common logics fail? According to Gillian Russell (2017a) this is in fact possible. In this section I will discuss these kinds of models. The idea behind them can be semi-formally presented in the following fashion: Let us assume that we are working in a setting with a context-sensitive language which includes special predicates like *Conj-white* and *Prem-green*. *Conj-white* is true of any object iff the object is white and the predicate is used in a conjunction. Thus, a simple, singular sentence like *Snow is Conj-white* is always false. Similarly, *Prem-green* is truly predicated of an object iff the object is in fact green and the predicate is used in a sentence which is part of the premises of an argument. If it is used in the conclusion it is always false. Now, we can create a counterexample to conjunction elimination. Consider, for example, the following derivation:

$$\frac{\textit{Snow is Conj-white} \wedge \textit{The wall is Conj-white}}{\textit{Snow is Conj-white}}$$

Let us assume that the premises are true. This is perfectly possible because the predicate *Conj-white* is used in the context of a conjunction. However, the conclusion is false. Thus, conjunction elimination fails. A counterexample to identity can be created in a parallel way:

$$\frac{\textit{The grass is Prem-green}}{\textit{The grass is Prem-green}}$$

Again, let us assume that *The grass is Prem-green* is true as it occurs in the premises. But the same sentence has to false when *Prem-green* occurs as the conclusion. Hence, the rule of identity has to fail as well. This is especially surprising since – as I already mentioned – one can prove this rule simply by means of the ordinary definition of logical consequence.[21] Hence, the need for elaboration should be apparent. Further, the question which was left open at the end of the last section was whether Russell's monsters can be said to pick out a case. Thus, the exact properties of these models are at the center of a correct assessment of her views.

This section is structured as follows: First, I will give a presentation of Russell's idea of a countermodel to conjunction elimination which I will subsequently refine. Second, I will do the same with her idea of a countermodel to identity. A general result of both these precisifications of Russell's monstruous models will be that her refutation of these basic laws of logic is not as straightforward as she thinks: The concept of logical consequence which she presupposes diverges in some severe fashion from a standard one. Last, I will present some remaining worries which one could have with these models.

1.3.1 Countermodels to Conjunction Elimination

The monstrous models which Russell (2017a, 130ff.) has in mind are supposed to be a variation of classical Tarski-models for first order logic. According to Russell we start from a normal first-order language consisting of constants, variables, n-ary predicates and the logical connectives. The models M which she takes under consideration are a pair $\langle D, I\rangle$ consisting of a domain D and an interpretation function I. As in the classical case, D is a non-empty set of objects. I, on the other hand, is special: Instead of being a simple function from our language L to D it is a two-placed function which takes an expression of our language and a *sentence-position value* and maps these into D. The sentence-position values are either *solo*, $left$ or $right$ and depend on the position which the expression under evaluation takes within a bigger syntactic construct.[22] From there she just continues to define a one-placed valuation-function in a recursive manner:

1. $V(\Pi t_1 ... t_n) = 1$ iff $I(\langle t_1, ..., t_n\rangle) \in I(\Pi, X)$. ($X$ is a variable which can take either $right$, $left$ or *solo*.)

2. $V(\neg\varphi) = 1$ iff $V(\varphi) = 0$.

3. $V(\varphi \wedge \psi) = 1$ iff $V(\varphi) = 1$ and $V(\psi) = 1$.

4. $V(\varphi \to \psi) = 1$ iff $V(\varphi) = 0$ or $V(\psi) = 1$.

[21] Rumfitt (2015, 42f.) turns this point around: For him any relation that could play the role of logical consequence has to be reflexive.

[22] Note that Russell does not try to give a formal model to predicates like *Conj-white*. Here idea here is a bit broader: She wants to represent predicates which are relative to the position they obtain in the sentence they were used in. Thus, it is not about being in the scope of a conjunction or not.

5. $V(\varphi \vee \psi) = 1$ iff $V(\varphi) = 1$ or $V(\psi) = 1$.

6. $V(\exists x\varphi) = 1$ iff there is a $\varphi[\alpha/x]$ such that $V(\varphi[\alpha/x]) = 1$.

7. $V(\forall x\varphi) = 1$ iff for all $\varphi[\alpha/x]$: $V(\varphi[\alpha/x]) = 1$.

However, there are a few problems which arise from this rough picture. This has two reasons: First, Russell does not give us a definition of logical consequence. She just puts forward a model in which $V(\varphi \wedge \psi) = 1$ but $V(\varphi) = 0$ and concludes that she has created a counterexample to conjunction elimination. However, in order to build such a counterexample it is of utmost importance to give a clear account of logical consequence. The second shortcoming of her presentation is that she does not give us a formal definition of sentence-position values: They just appear in the definition of truth for atomic sentences without any prior formal introduction. But, as will be made clear clear in the discussion below, this step is far from trivial. Thus, her account of possible counterexamples is not precise and in need for further refinement: We have to add a formal definition of logical consequence and a formal representation of sentence-position values. Else Russell has not made a sufficient step beyond the informal examples I have presented in the introduction to this section.

In the following I will present two such representations of sentence-position values: a semantic one which sees them as semantic entities and a syntactic one which places them in our language. It is easy to see that both ways are incompatible with Russell's original claim that we can just keep standard models and combine them with a standard language: The semantic refinement will abandon the view that it is a simple Tarski-model which we are considering and the syntactic refinement introduces a more complicated language. I do not claim that these are the only, or even the best ways to capture Russell's thoughts. However, they seem to be very natural translations which preserve as much of Russell's models as possible.

Semantic Refinement

First, we start with a standard language $\mathcal{L}$ which consists of

1. a set of individual constants $a, b, c...$,

2. a set of individual variables $x, y, z...$,

3. a set of n-ary predicates $P, Q, R...$,

4. the logical connectives $\wedge, \vee, \neg, \rightarrow$ and quantifiers $\forall$ and $\exists$,

5. the auxiliary symbols (and),

A *term* of $\mathcal{L}$ is either an individual constant or a variable. Now we can recursively define the set of *formulas* of $\mathcal{L}$:

1. If P is a n-ary predicate and $t_1, ...t_n$ are terms then $Pt_1, ...t_n$ is a formula.
2. If φ is a formula then $(\neg\varphi)$ is a formula.
3. If φ and ψ are formulas then $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$ and $(\varphi \to \psi)$ are formulas.
4. If φ is a formula then $\forall(\varphi)$ and $\exists(\varphi)$ are formulas.

This is all a standard way set out a language for first-order logic.

However, while the language we use here is quite standard, the semantics is not: A *sentence-positioned model* is triple $\langle D, I, Sp\rangle$ with the domain D being a set of objects, the set of sentence-position values $Sp = \{R, L, S\}$ and the interpretation-function I being a two-placed function from our language and Sp to the domain: If X is some sentence position in Sp and t a term of $\mathcal{L}$ then $I(t, X) \in D$ and if Π is a n-ary predicate of $\mathcal{L}$ then $I(\Pi, X) \in D^n$. Now we can extend the interpretation function to a valuation function V from the formulas of $\mathcal{L}$ and some sentence-position value X to the truth-values $\{0, 1\}$:

1. $V(\Pi t_1...t_n, X) = 1$ iff $\langle I(t_1, X), ..., I(t_n, X)\rangle \in I(\Pi, X)$.
2. $V(\neg\varphi, X) = 1$ iff $V(\varphi, X) = 0$.
3. $V(\varphi \wedge \psi, X) = 1$ iff $V(\varphi, L) = 1$ and $V(\psi, R) = 1$.
4. $V(\varphi \to \psi, X) = 1$ iff $V(\varphi, L) = 0$ or $V(\psi, R) = 1$.
5. $V(\varphi \vee \psi, X) = 1$ iff $V(\varphi, L) = 1$ or $V(\psi, R) = 1$.
6. $V(\exists x\varphi, X) = 1$ iff there is a $\varphi[\alpha/x]$ such that $V(\varphi[\alpha/x], X) = 1$.
7. $V(\forall x\varphi, X) = 1$ iff for all $\varphi[\alpha/x]$: $V(\varphi[\alpha/x], X) = 1$.

As one can see the truth-conditions of the connectives are quite different than in the semantics of classical logic: They behave as modal-operators which shift the sentence-position value according to which the subformulas have to be evaluated. Thereby, I am diverging from Russell's original definition of the truth-clauses. However, this formal mechanism seems to be necessary to represent the shift-from one sentence-position value to another while maintaining a compositional semantics. Now, Russell's thought seems to be the following: If we consider logical consequence relations between formulas we always treat the premises and the conclusions as single sentences on their own. Thus, the definition of logical consequence has to look like this:

Definition. *Logical Consequence* $\Gamma \models \varphi$ iff in all models M with $V(\gamma, S) = 1$ for all $\gamma \in \Gamma$ it has to be the case that $V(\varphi, S) = 1$.

This definition is in line with the usual way logical consequence is defined, i.e. it is a quantification over models and in all the structures in which the premises are true the conclusions are true as well. Hence, in this case the peculiarity of Russell's system does not stem from a dismissal of the common way to define logical consequence but from the truth-clauses of the connectives. In the face of the other refinements I will present below, this is an important feature of the current system.

If we assume the usual introduction and elimination rules for the connectives it is easy to see that the rule of conjunction elimination is prone to failure. We can demonstrate this by considering a simple countermodel: Let us take a model M with $D = \{o_1, o_2\}$ and the following interpretation function: $I(t_1, X) = o_1$,[23] $I(t_2, X) = o_2$ and $I(P, R) = \{o_2\}$, $I(P, L) = \{o_1\}$, $I(P, S) = \varnothing$. Now we can consider the following sentences: We know that $V(Pt_1 \wedge Pt_2, S) = 1$ because $V(Pt_1, L) = 1$ and $V(Pt_2, R) = 1$. However, $V(Pt_1, S) = 0$ and $V(Pt_2, S) = 0$. Thus, we have a counterexample to conjunction elimination: The conjunction is true even though both conjuncts taken by themselves are false.

To conclude, we are able to produce a counterexample to conjunction elimination because we add some further structures to our models. Thereby, we shift from standard Tarski-models to a special version of Kripke-models and redefine logical consequence in a way that is standard for modal logic.[24] Then we redefine the logical connectives as modal-operators: Instead of quantifying over the set of accessible possible worlds like $\Diamond$ and $\Box$, though, they just shift the world of evaluation of the subformulas. Thereby, we have changed the truth-clause of conjunction in a rather substantive way.

Syntactic Refinement

There are several problems one has to face if one wants to add sentence-positions into the language. The biggest one is to avoid a clash with the usual recursive approach towards creating formal languages: If we want to keep building our language out of some basic particles we have to deal with the problem that the position of a sub-formula depends on the complex formula it is part of. However, I think one can resolve some of these worries in the following way.

A *sentence-positioned language* $\mathcal{L}_{SP}$ is an extension of the language L which I presented above: The language's vocabulary contains everything that $\mathcal{L}$ contains plus three sentence-position values L, R, S. Again, a *term* of $\mathcal{L}_{SP}$ is either an individual constant or a variable and an *atomic formula* of $\mathcal{L}_{SP}$ is a n-ary predicate followed by n terms. For reasons that will become clear later, the set of formulas of $\mathcal{L}_{SP}$ is also the same as in $\mathcal{L}$. At this point we will leave the picture above and introduce some further structures: A *building block*

[23] Russell's plan is to represent special predicates which are sensitive to the sentence position. Thus, the interpretation of terms can be regarded as rigid, i.e. they are interpreted in the same fashion in every context.

[24] The models $M = \langle D, I, Sp \rangle$ are just first-order Kripke-Models with three worlds, an empty accessibility-relation and an invariable domain.

of $\mathcal{L}_{SP}$ is recursively defined as follows:

1. A pair $\langle \varphi, X\rangle$ of an atomic formula φ and a sentence-position value X is a building block.

2. If φ is a building block then $\neg\varphi$ is a building block.

3. If φ is a building block then $\exists x\varphi$ is a building block.

4. If φ is a building block then $\forall x\varphi$ is a building block.

Now we can give a recursive definition of a *positioned compound* of $\mathcal{L}_{SP}$:

1. If φ is a sequence of two building blocks and one two-placed connective of the form $(\chi\Box\psi)$ where the sentence-position value in χ is L and the sentence-position value in ψ is R, then φ is a positioned compound.

2. If φ is a positioned compound then $\neg\varphi$ is a positioned compound.

3. If φ and ψ are positioned compounds then $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$ and $(\varphi \rightarrow \psi)$ are positioned compounds.

4. If φ is a positioned compound then $\exists x\varphi$ and $\forall x\varphi$ are positioned compounds.

Now the set of *positioned formulas* of $\mathcal{L}_{SP}$ is just (non-recursively) defined as:

1. If φ is a building block with the sentence-value S then it is a positioned formula.

2. If φ is a positioned compound then it is a positioned formula.

In this way we can capture Russell's proposal without changing some basic properties of the language.

Now we can go over to the semantics and define the interpretation function. As I pointed out in the beginning of this section, Russell takes the interpretation function to take two arguments into consideration: If a is an individual constant and X any sentence-position value then $I(a, X) \in D$ and if Π is an n-ary predicate and X any sentence-position value then $I(\Pi, X) \in D^n$. Then we extend this definition of the interpretation function to a valuation function for the formulas of our language. However, as I pointed out so far, our new language does contain sentence-position values. Thus, we have to change the base case of the recursive definition in order to create a valuation function for the positioned formulas of our $\mathcal{L}_{SP}$:

1. $V(\langle \Pi t_1...t_n, X\rangle) = 1$ iff $\langle I(t_1, X), ..., I(t_n, X)\rangle \in I(\Pi, X)$.

2. $V(\neg\varphi) = 1$ iff $V(\varphi) = 0$.

3. $V(\varphi \wedge \psi) = 1$ iff $V(\varphi) = 1$ and $V(\psi) = 1$.

4. $V(\varphi \to \psi) = 1$ iff $V(\varphi) = 0$ or $V(\psi) = 1$.

5. $V(\varphi \vee \psi) = 1$ iff $V(\varphi) = 1$ or $V(\psi) = 1$.

6. $V(\exists x\varphi) = 1$ iff there is a $\varphi[\alpha/x]$ such that $V(\varphi[\alpha/x]) = 1$.

7. $V(\forall x\varphi) = 1$ iff for all $\varphi[\alpha/x]$: $V(\varphi[\alpha/x]) = 1$.

Thus, the recursive clauses for the connectives stay classical. Now we can go on to define logical consequence. In order to adapt the logical consequence relation to our more fine-grained language we have to make it more variable. If we want to account for Russell's intuition we have to define logical consequence in a way that is not sensitive to the sentence-positions in question: The shift of a sentence position is not supposed to make a change. Now we can make use of the fact that we did not abandon the concept of a formula of the language: We can associate a positioned formula $\varphi*$ to each formula φ by adding the sentence-position values to the atomic formulas. For each formula the following $*$-function will put out a unique sentence-positioned formula. The recursive definition of $*$ goes as follows:

1. If φ and ψ are atomic formulas, then $\varphi* = \langle\varphi, S\rangle$, $(\neg\varphi)* = \neg\langle\varphi, S\rangle$, $(\varphi\Box\psi)* = \langle\varphi, L\rangle\Box\langle\psi, R\rangle$, $(\exists\varphi)* = \exists\langle\varphi, S\rangle$ and $(\forall\varphi)* = \forall\langle\varphi, S\rangle$.

2. If φ and ψ are not atomic formulas, then $(\neg\varphi)* = \neg\varphi*$, $(\varphi \wedge \psi)* = \varphi * \wedge\psi*$, $(\exists\varphi)* = \exists\varphi*$ and $(\forall\varphi)* = \forall\varphi*$.

By using this device we can build the insensitivity into the definition of logical consequence.

Definition. *Logical Consequence* Let Γ be a set of formulas and φ be a formula. Then $\Gamma \models \varphi$ iff all models with $V(\gamma*) = 1$ for all $\gamma \in \Gamma$ are such that $V(\varphi*) = 1$.

As usual, we take logical consequence to be a relation between a set of formulas and a single formula. These formulas, however, are not the items which evaluated with regards to their truth. Instead, we add a biconditional clause that links the consequence relation of formulas to truth-preservation facts between the corresponding positioned formulas. Hence, if we take up this refinement of Russell's model we have to redefine logical consequence in a very non-standard fashion.[25]

In order to create a counterexample to conjunction elimination we can pretty much use the same example we used in the case of the semantic refinement. But instead of a triple we only have a model $M = \langle D, I\rangle$ and treat the sentence position value as a

[25] Alternatively, we could have changed the inference rules, i.e. we could have defined conjunction elimination in a sentence-position invariant fashion while keeping the concept of logical consequence classical. However, adapting logical consequence instead of every single inference rule seems to be a simpler refinement.

component which is given by our syntax. Hence, in order to evaluate the sentences Pt_1, Pt_2 and $Pt_1 \wedge Pt_2$ we have to use the *-function to find the corresponding positioned formulas Pt_1*, Pt_2* and $(Pt_1 \wedge Pt_2)*$. Now $(Pt_1 \wedge Pt_2)* = \langle Pt_1, L\rangle \wedge \langle Pt_2, R\rangle$ and $V(\langle Pt_1, L\rangle \wedge Pt_2, R\rangle) = 1$, but $Pt_1* = \langle Pt_1, S\rangle$, $Pt_2* = \langle Pt_2, S\rangle$ and $V(\langle Pt_1, S\rangle) = 0$ as well as $V(\langle Pt_2, S\rangle) = 0$. So, we have again a counterexample to conjunction elimination.

1.3.2 Countermodels to Identity

In the case of Russell's presentation of counterexamples to identity the situation is even worse then with the couterexamples to conjunction-elimination. Here Russell (2017a, p. 132) just points towards the fact that one could adapt her idea of sentence-position sensitive predicates to account for argument-position sensitive ones. So, just as her account of conjunction elimination failure was in need of a formal refinement so is her account of identity failure: We have to give a formal representation of argument-positions. Here – again – we have two options: Either we regard them as semantic entities and, hence, opt for a semantic refinement, or we treat them on syntactic grounds and, thereby, go for a syntactic refinement.

Semantic Refinement

The semantic refinement of counterexamples to identity can actually borrow a lot from the above refinement of counterexamples to conjunction elimination. We can take the same language $\mathcal{L}$ and immediately start to give it a semantics: Again, an argument-positioned model is a triple $\langle D, I, A\rangle$ with the domain D, a set of argument-position values $A = \{P, C\}$ and a two-placed interpretation-function I from expressions of $\mathcal{L}$ and some argument-position value $X \in A$ to D. The interpretation-function works just as above and the valuation function is defined as follows:

1. $V(\Pi t_1...t_n, X) = 1$ iff $\langle I(t_1, X), ..., I(t_n, X)\rangle \in I(\Pi, X)$.
2. $V(\neg\varphi, X) = 1$ iff $V(\varphi, X) = 0$.
3. $V(\varphi \wedge \psi, X) = 1$ iff $V(\varphi, X) = 1$ and $V(\psi, X) = 1$.
4. $V(\varphi \to \psi, X) = 1$ iff $V(\varphi, X) = 0$ or $V(\psi, X) = 1$.
5. $V(\varphi \vee \psi, X) = 1$ iff $V(\varphi, X) = 1$ or $V(\psi, X) = 1$.
6. $V(\exists x\varphi, X) = 1$ iff there is a $\varphi[\alpha/x]$ such that $V(\varphi[\alpha/x], X) = 1$.
7. $V(\forall x\varphi, X) = 1$ iff for all $\varphi[\alpha/x]$: $V(\varphi[\alpha/x], X) = 1$.

Thus, we use the classical truth conditions and do not treat the connectives as modal operators. Instead, the work has to be done by a rather drastic change in the definition of logical consequence. Since we are concerned with the law of identity which does not make use of any connectives this should not come as a suprise.

Definition. *Logical Consequence* $\Gamma \models \varphi$ iff in all models M with $V(\gamma, P) = 1$ for all $\gamma \in \Gamma$ it has to be the case that $V(\varphi, C) = 1$.

If we keep in mind that the argument-position values are like possible worlds in modal logic this new definition of logical consequence amounts to the fact that the premises and conclusions of arguments are evaluated in different worlds. This definition should look quite strange: The context of evaluation of the premises is completely independent of the context of evaluation of the conclusion. However, one has to keep in mind that Russell is only trying to accommodate special predicates which are sensitive to the argument-position. Thus, the interpretation function for normal, argument-position insensitive predicates will be the same in P as it is in C.

A counterexample to the law of identity can be easily created: Just take some model $M = \langle D, I, A \rangle$ such that some formula Fa has $V(Fa, P) = 1$ but $V(Fa, C) = 0$. Thus, we have a counterexample to $\varphi \models \varphi$. If we have – for instance – a sentence which contains one of Russell's *Prem*-predicates, we know that the evaluation in P and C can diverge from each other.

Syntactic Refinement

There are two possible ways to approach a syntactic refinement, i.e. two ways to add argument position values to the language: First, one could strengthen the language in such a way that it can express arguments and then give sentences in the premises some value and conclusion a different one.[26] A second and – as I think – easier strategy would be to just add some values to the language in a rather *ad hoc* way and then, subsequently, change the definition of logical consequence to make it clear that these values do represent argument-positions. Since the second option is more conservative Russell should prefer it over the first. Hence, I will opt for the second option and neglect the first one.

An *argument-positioned language* $\mathcal{L}_{AP}$ consists of the same elements as $\mathcal{L}$ plus a set of argument-position values $A = \{P, C\}$. The set of formulas of $\mathcal{L}_{AP}$ is the equivalent to the set of formulas of $\mathcal{L}$. But – just like in the case of $\mathcal{L}_{SP}$ – we have to add some further structure. Since we do not have to worry about the same worries of building a formal language in a recursive fashion here – as it was the case with $\mathcal{L}_{SP}$ – we can immediately go on to define the set of *argument-positioned formulas* of $\mathcal{L}_{AP}$:

1. If φ is an atomic formula and X an argument-position value then $\langle \varphi, X \rangle$ is an argument-positioned formula.

[26] Of course, this option was also possible in the case of the semantic refinement. This, however, would have created a clear shift from first-order logic towards second-order logic. Thus, in order to keep our refinements conservative, we do not need to take such a formalization under consideration.

2. If φ is an argument-positioned formula then $\neg\varphi$ is an argument-positioned formula.

3. If φ and ψ are argument-positioned formulas then $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$ and $(\varphi \to \psi)$ are argument-positioned formulas.

4. If φ is an argument-positioned formula then $(\exists\varphi)$ and $(\forall\varphi)$ are argument-positioned formulas.

So, just as above, we have created a new set of special formulas by decorating all the atomic formulas with an additional value.

As in the case of the syntactic refinement of countermodels to conjunction elimination the semantics stays classical in many ways. But, again, we have to bear in mind that instead of evaluating formulas we are evaluating the argument-positioned formulas. Thus, the recursive definition of the valuation function looks alike: The only divergence from the usual truth-clauses comes in form of a new base clause. Another parallel to the cases of conjunction elimination failure comes in form of a redefinition of the logical consequence relation by use of a formulas and argument-positioned formulas. But this time we will use two functions ($*$ and $+$). The $*$-function looks like this:

1. If φ is an atomic formula then $\varphi* = \langle \varphi, P \rangle$.

2. If φ and ψ are formulas then $(\neg\varphi)* = \neg\varphi*$, $(\varphi \wedge \psi)* = \varphi * \wedge \psi*$, $(\exists\varphi)* = \exists\varphi*$ and $(\forall\varphi)* = \forall\varphi*$.

Here, we are associating a formula with an argument-positioned formula in which all the argument-position values are P. The $+$-function, on the other hand, goes as follows:

1. If φ is an atomic formula then $\varphi+ = \langle \varphi, C \rangle$.

2. If φ and ψ are formulas then $(\neg\varphi)+ = \neg\varphi+$, $(\varphi \wedge \psi)+ = \varphi + \wedge \psi+$, $(\exists\varphi)+ = \exists\varphi+$ and $(\forall\varphi)+ = \forall\varphi+$.

In this case we have a function which gives us argument-positioned formulas who only bear the value C. So, intuitively speaking, the *-function relates formulas to their use as premises and the $+$-function relates them to their use as conclusions. Now we can make a familiar move in defining logical consequence as follows:

Definition. *Logical Consequence* Let Γ be a set of formulas and φ be a formula. Then $\Gamma \models \varphi$ iff all models with $V(\gamma*) = 1$ for all $\gamma \in \Gamma$ are such that $V(\varphi+) = 1$.

Here, logical consequence is treated as a relation between a set of formulas and a formula which depends on facts about truth-preservation between $*$-variations of the premises and $+$-variations of the conclusion.

As we already saw in the case of conjunction elimination, we can use almost the same counterexample for the syntatic refinement as we used in the semantic one. Let us take some model $M = \langle D, I \rangle$ and a sentence Fa. We know that $Fa* = \langle Fa, P \rangle$ and $Fa+ = \langle Fa, C \rangle$. Now we assume that $V(\langle Fa, P \rangle) = 1$ but $V(\langle Fa, C \rangle) = 0$. Hence, by our definition of logical consequence the law of identity fails.

1.3.3 Some Internal Criticisms

Let me start with a short summary of what I have shown so far: If we want to give a precisification of Russell's monstrous models we run into a certain dilemma: Either we have to change the truth-clauses of the connectives (as was the case in the semantic refinement of the countermodels to conjunction elimination) or we have to change the definition of logical consequence from straightforward truth-preservation in all models to something different (as was the case in the other three refinements). Since Russell wants to present counterexamples to conjunction elimination as well as the law of identity it is a reasonable assumption that she wants to keep the classical truth-clauses for conjunction and the common definition of logical consequence. This, however, seems to be impossible. Thus, we have to check whether her divergences are justified.

Truth-Clauses and the Meaning of Connectives

In the semantic refinement of the counterexamples to conjunction elimination we have seen that it is possible to create counterexamples to conjunction elimination simply by changing the truth-clauses for the connective $\wedge$. Of course this does not come as a surprise. For instance, the following simple change of the truth clauses would have settled the issue: $\varphi \wedge \psi$ is true iff φ is true or ψ is true. Hence, it is possible that the conjunction is true while one of the conjuncts is false. But, of course, this change in the truth-clauses of our connective brings with it a clear change of meaning: $\wedge$ does not represent a conjunction but a disjunction. Hence, we have to ask the question whether the truth-clauses we used to characterize $\wedge$ still pick out a conjunction or not.

As a short reminder, the truth-clause looked as follows: $V(\varphi \wedge \psi, X) = 1$ iff $V(\varphi, L) = 1$ and $V(\psi, R) = 1$. I already noted that the connectives are interpreted as modal operators which shift the context of evaluation of the subformulas: The conjunction is true (in whatever place it may appear) iff the left conjunct is true according to L and the right conjunct is true according to R. The intuitive interpretations of L and R were "being left of a connective" and "being right of a connective". If this representation is correct then I do not think that we changed the meaning of the conjunction: In order for the conjunction to be true it has to be the case that both conjunctions are true with respect to their position in the sentence. If something has gone wrong in this system it has to be in the very idea of a divergence of L and R: If the interpretation function would yield the same results no matter whether it takes L or R as a second argument, conjunction elimination would still be intact. Thus, it is rather the idea that the context of evaluation is shifted when we consider the truth of a conjunct as it appears in the conjunction in the premise and the truth of a conjunct as it appears alone in the conclusion which is to blame. This brings us to the second issue.

Logical Consequence and Context Shifts

In the last subsection I showed that even the first of the four refinements which keeps the definition of logical consequence more or less untouched has to tell a story about context shifts, not from the premises to the conclusion per se, but – in the case of conjunction elimination – a shift from the context of the subformulas of the premise to context of the conclusion. For the other three refinements, syntactic or semantic, it seems clear that such a story has to be told anyway.[27] So, the peculiar property of Russell's monsters is that they allow for some kind of intra-argument context shifts. Hence, the central premise of Russell's reasoning, i.e. the premise which was supposed to be justified by universalism, is that these formal models pick out cases and are, thereby, relevant for our evaluation of logical laws. However, this premise is highly contested (e.g. Rumfitt 2015, p. 33) and is in need for a clear defence.

Since Russell wants to capture special contexts in her logic it does make sense to have a look into logics of context-sensitivity like Kaplan's logic of demonstratives LD. In his logic Kaplan (1979, p. 92) defines logical truth as truth in all context of all models.[28] As Russell (2012) notes this definition of logical truth can be generalized to a definition of logical consequence which is quite similar to the one usually used in modal logics:

Definition. *Logical Consequence*$_{LD}$ $\Gamma \models_{LD} \varphi$ iff in all contexts c of all models M if $M, c \models_{LD} \gamma$ for every $\gamma \in \Gamma$ then $M, c \models_{LD} \varphi$.

This is how logical consequence is defined in ordinary context-sensitive logics and, I think, it is quite easy – under the semantic understanding of logical consequence – to find a justification of this modification of the usual way to define logical consequence: In the standard (non-modal) way of defining logical consequence we are quantifying over models

[27]Ordinary context sensitive logics are similar to my semantic refinements. *Prima facie* the syntactic refinements, however, work differently and manifest an even greater departure from the classical way to define logical consequence: The relation of logical consequence is defined by truth-preservation between syntactic entities which are different from the premises and conclusions. But this trick of mapping premises and conclusions into different syntactic constructs which are evaluated in one and the same model does effectively the same job as introducing different points of evaluation. Thus, I will regard both refinements as different formal models of the same phenomenon which Russell is trying to capture and treat the syntactic refinements on a pair with the semantic one.

[28]In order to understand this definition, it is important to note that one has to distinguish between the *context* and the *circumstance* of the context (compare MacFarlane 2014, 76ff.). As Kaplan (1989, 548ff.) puts it, the context (of utterance) determines the circumstance of evaluation. Thus, one can discern classical logic from logics like LD: For classical logic a valid sentence is true in all circumstances. For LD a sentence is valid if it is true in all contexts. However, there are circumstances which will never be determined by a context(, for instance, those in which no individuals exist). Similarly, in our current case there might be circumstances in which there is no language and, thus, no linguistic context to look for.

because the models are supposed to pick out the cases in question. Since, we are concerned with truth-preservation it is reasonable to assume that the cases are supposed to fix the truth and falsity in question.[29] Hence, a model can only pick out a case if it fixes the truth-conditions as well. However, if we switch to modal systems the truth does not only depend on the model in question but usually on the point of evaluation, i.e. a world or, here, a context. Thus, in order to pick out a case we need a pair $\langle M, c\rangle$. Now, if we want to keep our definition of logical consequence as quantification over case-representations we have to quantify over all pairs $\langle M, c\rangle$ and, thereby, we end up with a definition like the one of LD. Thus, a natural by-product of this reasoning is that just as the models do not shift from the premises to the conclusions in regular logics so the context cannot shift from the premises to the conclusions either.[30]

However, in the case of linguistic contexts like arguments and conjunctions Russell (2018b, 315f.) claims that the context is allowed to shift if we go from the premises to the conclusions: The premise is simply a different linguistic context then the conclusion. But why does this simple appeal to a context-shift from the premises to the conclusions not generalize to other parameters which may shift from premises to conclusions? A natural example are parameters like the speaker, the time and the place: The inference from "I am in the train to Cologne" to "I am in the train to Cologne" leads from a true premise to a false conclusion if we are changing the context in which we utter the premise and the conclusion. This should not come as a surprise for anybody who learned about indexicality. Thus, it is not clear what is supposed to be special about linguistic contexts: Russell just assumes that it is less intuitive to keep a linguistic context fixed in the course of an argument than it is to keep others parameters fixed. Here it may depend on what one means with "logic". But, as far as I can see, in an ordinary understanding of what logic is there is no tight connection between it and linguistic contexts and it seems perfectly fine to abstract from those: If we want to know whether a sentence is true for the purpose of finding out facts about logical consequence we neither want to include facts about the context of utterance, nor do we want to include facts about the linguistic context. However, one might wonder whether there are contexts from which logic can not abstract and, thus, whether there could be context shifts between premises and conclusion after all.[31]

[29] In any of the four interpretations of "case" I have presented in 1.1.3. this seems to be a basic assumption: On the representational account it should be clear that the possible constellations to be represented are the things that fix the truth. On the interpretational and substitutional account it should also be clear that a possible reinterpreation or substitution of the non-logical vocabulary, fixes the truth-value of the sentence in question. Last, a possible assignment of a shell in the universalist account is, again, nothing else but an instance which fixes a truth-value.

[30] In a similar vain one could argue that the main means that contexts are supposed to provide is to fix the meaning of a sentence. However, changing the meaning of expressions within an argument can hardly be seen as providing counterexamples to logical laws. In fact, we usually regard these as paradigmatic instances of mistakes.

[31] An interesting example would be the semantic context. Russell's *con-white* is only sensitive to

One might object that there are three different examples where logicians are giving up on a fixed context within the argument. The first example is *Dynamic Predicate Logic* DPL which allows for a shift within the argument (compare Groenendijk and Stockhof 1991). This is motivated by the fact that meaning which is – as I pointed out above – supposed to be fixed by the context has an inherently dynamic component to it. Thus, by shifting to a dynamic notion of logical consequence we are able to capture natural language phenomena like anaphorical bindings. Thus, in order to systematize all facets of natural language *discourses* in our formal models we have to allow for such a shift within the argument. This application of logic, however, seems to be far off from the motivation Russell has in mind. In the case of discourse representation the argument I have put forward above is obviously mistaken: The discourse of DPL is not the argument of the standard logicians. Here it would in fact be weird to abstract from context-shifts since we try to capture a dynamic phenomenon which is essentially governed by these shifts. So, there is a way to introduce counterexamples to conjunction-elimination and identity, but the argument would look quite different than Russell's: One would have to argue that the relation of logical consequence is not static but inherently dynamic. Hence, Groenendijk and Stockhof (1991, p. 26) provide a clear disambiguation of these two notions: They highlight that the notion of entailment with which they are concerned is a truly dynamic one and that a translation of the usual, static definition into DPL fails to capture our inferences in discourses.

The two other exceptions of this orthodoxy are Zardini (2014) and Radulescu (2015) who both argue that the fixing of the context from premises to conclusions is an unjustified step. Zardini argues that our standard logics undergenerate since there are inferences which make an essential use of context shifts: From the truth of "It will be sunny in two days" you can infer two days later "It is sunny". If we want to capture these inter-contextual inferences – *Fregean entailments* in Zardini's words – we have to drop the requirement of a fixed context. In a similar vein Radulescu appeals to Frege's idea that the thought expressed by "It will be sunny today" is in fact the same thought as expressed by "It is sunny" uttered two days later. However, I think the basic motivation of these authors is similar to the one of the dynamic logicians above and, thus, quite different from Russell's project: If our notion of logical consequence is supposed to capture a dynamic phenomenon then it cannot keep the context fixed. However, Russell's approach seems to be routed in a debate about static entailment. Thus, the examples given by Zardini and Radulescu are quite different than her motivation.

syntactic features and her *prem-white* is only sensitive to some syntactic features of the metalanguage. Since we assumed an semantic conception of logical consequence it seems possible to keep these context fixed. However, we could also have things like *false-white* (which applies to an object which is white and the sentence in which it is used is false) or *invalid-green* (which applies to an object which is green and the sentence is part of an invalid inference). However, these would not lead to nihilism and,as is commonly known and further discussed in the second chapter of this thesis, if a logic is strong enough to express such things about its own semantics then it is prone to paradoxes anyway.

To Conclude...

I have tried to point out that the refinements I have presented in the previous section fare quite well: They do not lead to problems with the meaning of connectives. Second, I have shown where the turning point of Russell's models is to be found: A new definition of logical consequence which allows for a context shift from the premises to the conclusion. This, however, is not in the spirit of the ordinary context-sensitive logician's take on the matter. Further, I have tried to make clear that her motivation of the peculiarity of linguistic context for the definition of logical consequence is misguided. But I do not think that this shows that the overall argument which I described at the beginning of this chapter is thereby blocked *per se*. Rather, it shows how monstrous these models really are and puts a lot of pressure on the premise of universalism: If she wants to incorporate her counterexamples in the set of cases Russell has to assume that universalism allows her to make a small change of the definition of logical consequence which she needs, i.e. she has to apply a broader notion of logical consequence. This, however, is not the usual way she spells out universalism: It is not the unrestricted quantification over special models which does the job but it is her rejection of the implicit restriction to fixed contexts. Hence, if the presentation of Russell's argument that I have given in the beginning of this chapter is in fact correct then the argument is fallicious. Rather, we have to update it in the following fashion:

1. [**Logical Consequence**$_{sem}$] $\Gamma \models \varphi$ iff in all models M: If $M, w \vDash \gamma$ for all $\gamma \in \Gamma$ at some point of evaluation w then $M, w' \vDash \varphi$ for some point of evaluation w'.[32]

2. [**Universalism**] There is no constraint as to which pairs of models and points of evaluation can be quantified over in the definition above, i.e. there is no guarantee that w and w' are the same points of evaluation.

3. [**Monsters**] If universalism is true then we can always find two pairs $\langle M, w\rangle$ and $\langle M, w'\rangle$ such that $M, w \vDash \gamma$ for all $\gamma \in \Gamma$ but $M, w' \nvDash \varphi$.

4. $\therefore$ [**Logical Nihilism**] It is never the case that $\Gamma \models \varphi$. *[from 1,2 and 3]*

So, we have to generalize the definition of logical consequence in order to make it clear were this stronger version of universalism can attack.[33] In analogy to Beall and Restall's GTT one could maybe label it as a *Universalized Kripke Thesis* since it lifts one constraint from the usual definition of logical consequence in modal logics. The discussion in this chapter has shown that this argument does in fact go through: Given that we accept this

[32] Again, we could make a syntactic analogy to this fine-grained refinement of the logical consequence relation: $\Gamma \models \varphi$ iff in all models M with $M \vDash \gamma*$ for all $\gamma \in \Gamma$: $M \vdash \varphi+$ where $*$ and $+$ are functions wich put out some truth-evaluable formula.

[33] It was pointed out to me that one could see this version of the argument as Russell's plan all along. Thus, one may also read it as a mere explication of her reasoning.

generalized notion of logical consequence and a sufficiently strong universalism we should accept logical nihilism.

1.4 Concluding Remarks

I want to conclude this chapter with some external remarks on Russell's position. So far, I have only given an internal criticism of the way she presents her argument and the motivations she points to in order to justify her premises. As my analysis has shown the crucial step in her reasoning is to allow for her monstrous models to pick out cases. So far I have drawn some doubt upon her motivation of this claim.

Why should we accept a thesis like universalism$_2$ in the first place? Even if we are not concerned with Russell's monsters we might still wonder what sense universalism taken as a thesis about the set of cases itself makes. After all, the overarching debate in which Russell engages is not merely concerned with mathematics but is trying to capture something else. Thus, people usually point out that there are natural bounds to the set of cases: No matter what exactly a "case" is supposed to be, not everything goes.[34] This seems to be a generally accepted idea. (The effort Russell takes to create counterexamples which do not affect the truth-conditions of the connectives is a clear sign that even she accepts this.) Hence, just pointing towards universalism is not a justification to accept any model as a case-representation. One has to give different reasons for it. In the case of Russell's monsters, however, I think I have given sufficient grounds to disregard them as proper case-representations.

But this was only one case-study. A more general question remains: What is the criterion by which we can decide weather a given mathematical model does in fact represent a case? If the semantic definition would be the only thing determining the range of cases we would still end up with quite a diverse set. As Russell notes, even if her purported counterexamples do not work we might still find ourselves in an unsatisfying position: Nihilism might be false, but we do want to call more rules a "logical law" than just the very basic ones we discussed earlier. This is an issue which I will pick up in the third chapter. For now, let us look into Cotnoir's nihilism.

[34] One could take the possibilism debate as a point of resistance towards this point (compare Mortensen 1989; Estrada-Gonzalez 2011). However, the formal details of this position are quite different from Russell's proposal (see Estrada-Gonzalez 2012).

Chapter 2

Formalizability Nihilism

Cotnoir's argumentation for his version of nihilism is not as straightforward as Russell's: He does not propose one line towards nihilism but launches a number of attacks against the idea that *natural language consequence* could be captured with formal means.[1] First, natural language consequence is, just as the concept of consequence I laid down in the last chapter, a relation between a set of sentences of natural language and a single sentence of natural language. Since, Cotnoir's nihilism is about the formalization of this relation he can take this relation as a given fact. We can – for the purpose of this chapter – just assume that we have an intuitive grasp of what follows from what in natural language. A second important note which is essential for the understanding of Cotnoir's position is that "formal" is here not taken in the sense of the formality-property of logical consequence we have encountered in the first chapter. Now we are concerned with "formal" in the sense of "artificial" or "mathematical". A formal consequence relation, in this sense, is defined on a formal language and, thus, a relation between a set of formal sentences and a singular one. Now, Cotnoir is trying to show that the formal consequence relation is in principle incapable of representing the relation which we encounter in natural language. However, he leaves it quite unclear what exactly "representing" or "capturing" is supposed to consist in. A first, reasonable step in order to make this more explicit would be to say that a consequence relation $\models$ of a formal language $\mathcal{L}$ represents natural language consequence iff for all sets of sentences of natural language Γ and single sentences of natural language φ such that $\langle\Gamma, \varphi\rangle$ is part of natural language consequence: a representation $R(\Gamma) \in \mathcal{L}$ of Γ and a representation $R(\varphi) \in \mathcal{L}$ of φ are such that $R(\Gamma) \models R(\varphi)$. Now we have converted the issue of consequence-representation into an issue about sentence representation. However, not any sentence representation would work: We do not want to allow, for instance, that our "language" $\mathcal{L}$ is just the set of natural numbers N. Then an argument for formalizability nihilism seems to be quite hard to come by. Thus, Cotnoir

[1] Cotnoir actually uses the term "natural language inference". However, philosophers usually take "inference" to denote the act of drawing a conclusion. Since, this does not seem to be what Cotnoir is after in his paper I will speak of a natural language consequence relation in order to avoid misunderstandings.

has to set some demands on mathematical tools in place as to when exactly they count as a representation of natural language. Reasonable demands would be defining a syntax and a semantics that, again, represent natural language syntax and semantics.[2] For the following discussion it is important to highlight the fact that Cotnoir takes "formalization" to be a project of correct representation as opposed to some process of optimization.

His attacks on this idea can be classified into two different classes: First, arguments from diversity and, second, arguments from expressive limitations of formal languages. Neither is supposed to make a decisive point for Cotnoir's nihilism. Rather, he takes them to be abductive grounds which motivate formalizibility nihilism as a unified theory which can deal with a diversity of problems in the philosophy of language. Thus, I will start this chapter with a quick view on Cotnoir's argument from diversity which I regard as utterly misguided. Further, I will try to give some pointers towards an argument from diversity that would make more sense. Then I will turn in more detail to the arguments from expressive limitations. The structure of these arguments are quite straightforward: There are some sentences which natural languages, but no formal language can express. Since, natural language consequence is a relation between sentences of natural language and formal consequence a relation between sentences of formal languages we can conclude that no formal consequence relation can ever completely capture natural language consequence. Hence, the second section of this chapter deals with semantic closure of natural and formal languages. In the third section I discuss the problem of universal quantification and in the fourth one the problem of vagueness. Each of these three topics has a vast and complex literature. Thus, in order to stay within the limits of this thesis, I have to restrict the presentation of the problems of formalization to a broad outline of the debates. However, as will become clear in the subsequent discussion, this should not have any bearing on my criticism of the three arguments form expressive limitations. Last, I close this chapter with some concluding remarks on Cotnoir's position.

2.1 An Argument from Diversity?

Cotnoir starts his defence of formalizability nihilism with a discussion of logical pluralism, a topic which I already introduced in the first chapter. According to Cotnoir this debate can be seen as an argument for his version of nihilism, rather than Russell's version. I will start this chapter with an evaluation of this argument. Second, I will introduce a different argument from diversity which stems from a different kind of pluralism, i.e. a pluralism of formalizations of natural language consequence.

[2] At this point I will omit a further definition of what a "natural" or a "formal language" is. Since the arguments I will present below do not hinge on any specific definition of these concepts I will leave them in this way. For a more detailed account of these concepts see – for example – Novaes (2012, pp. 52-64).

2.1.1 Not Many. Not One. So None.

So, just like Russell, Cotnoir starts from the discussion of logical pluralism. However, he presents quite a different argument which can be put forward in the form of a dilemma:

1. [**Problem of Monism**] There cannot be a *single* consequence relation which captures natural language consequence.

2. [**Plurality Problems**] Logical pluralism cannot be correct, i.e. there cannot be a *plurality* of consequence relations which capture natural language consequence.

3. ∴ [**Formalizability Nihilism**] There is no logic that captures natural language consequence.

The problems of monism show that they cannot provide the consequence relation of natural language. The shortcomings of pluralism show that they cannot provide the consequence relation of natural language either. These are our only options. Hence, formalizability nihilism has to be true.

In his paper, Cotnoir seems to take it for granted that there is some motivation for logical pluralism. Thus, he is mainly concerned with pointing out the troubles of such a position which we have already seen in the first chapter. However, one should make the motivation for logical pluralism clear: Why should we become logical pluralists in the first place? In the last chapter we presented a case for *underdetermination* pluralism: Since logical consequence is underdeterminate there are several relation we are allowed to call "correct". This is a conceptual motivation of logical pluralism. In the case of *alethic* pluralism, for example, one usually puts forward an extensional motivation which is called the *scope problem*: For any truth-property we put forward there are counterexamples. Hence, we should opt for a plurality of truth-properties (compare Lynch 2009, p. 4). Thus, it is important to bear in mind that logical pluralism is not motivated by some extensional failure of some logical consequence relations but rather by the success of several relations.

I think we have everything in place now to see why Cotnoir's argument is not sufficient to justify its conclusion: The motivation of pluralism is that the concept of logic is *prima facie* underdeterminate, i.e. the principle governing it leaves different admissible instances open. However, as the arguments from the last chapter have shown, there is a clash between these principles and the idea that there is more than one logic. Hence, there can only be one admissible instance. So, there is, in fact, some clash between these two positions. However, Cotnoir seems to draw the following conclusion: There is no admissible instance. This is to quick. In order for this conclusion to be justified, it has to be the case that the above tension shows that nothing can play the role of logical consequence, i.e. that there is some inconsistency in the concept itself which prohibits any possible instance. But, as my presentation of logical pluralism has shown, this inference seems to be wrong: it does not say that if there is only one instance, then there is none. It is a thesis about underdetermination: There are several correct instances. This is why the arguments I have presented in 1.2.2. are seen as *collapse* arguments: Logical pluralism is supposed to collapse into monism. As far as I can see, nobody in this debate draws such a radical conclusion as Cotnoir. In the worst case, people conclude that the situation merely shows that the concept of logical consequence is quite dark but not that it is inconsistent: There are several good candidates of logical consequence, but we know that only one can be correct. (This seems to be the predicament of many logical monists.) In order to pin down which one is, in fact, correct we need further elucidation and arguments. As Russell points out, the arguments against logical pluralism usually undermine the motivation of logical pluralism and, thus, we end up in monism again. However, this direct "collapse" is why she sees them as arguments for weaker logics and, subsequently, for her version of nihilism. So, if she is correct we already have the only admissible instance at hand. Hence, I want to conclude that Cotnoir's argument from diversity seems to make an unjustified leap.

In order to prove some sort of nihilism along these lines, he *prima facie* needs to show a stronger thesis: The principles which govern the concept of logical consequence are inconsistent, i.e. nothing can play the role logical consequence is supposed to have.[3] However, even this stronger thesis cannot really establish formalizability nihilism since it would rather establish that there is no consequence relation in natural language after all, i.e. the argument would not have established any shortcomings of formalization. Thus, such an argument would establish some sort of nihilism but not the one Cotnoir is defending in his paper.

Something similar has to be said about the other arguments from diversity which Cotnoir mentions to close off his first line of attack, as well as the argument from normativity which he presents at the end of his paper. First, he points towards Varzi (2002) who generalizes Tarski's (1936) line of thought that there is a plurality of logical consequence relations due to the fact that there are different ways to draw the line between logical and non-logical vocabulary. Then he points towards a proposal by Sagi (2014): Instead of determining the correct consequence relation by referring to the set of logical vocabulary one could as well give *semantic constraints*, i.e. a general statement such that a possible interpretation of the language is only admissible if it satisfies this statement. But Cotnoir follows Glanzberg (2015) in arguing that this would yield at most *analytic entailment* and not what we usually call logical consequence since the requirement of formality which I mentioned in the previous chapter is not satisfied. However, this reasoning on its own does not establish formalizability nihilism. At most, it points towards the general philo-

[3] At this point, I do not want to claim that there is no way from Cotnoir's premises to his intended conclusion. But he definitly has to show that the dialectic in the debate of logical pluralism is misguided. This is why I think that his argument is inherently different than Russell's: In order for his argument to work, he has to make a clear case against any sort of monism.

sophical problem to discern analytic entailment from logical consequence which is *prima facie* independent of any issue of formalization. Hence, again, Cotnoir's argument rather establishes that there is no proper relation of logical consequence (since the formality constraint is not met) than providing a reason to believe in formalizability nihilism.[4] The second argument from diversity which Cotnoir adds to his reasoning starts from Russell's (2008) idea of a possible pluralism. This pluralism is of a different kind than the one of Beall and Restall which we have encountered above. According to her, the extension of our logical consequence relation depends on what we take as its relata. Hence, it is a kind of pluralism that stems from metaphysical grounds. However, this very starting point of her pluralism should make it clear that Cotnoir's formalizability nihilism is misplaced: Why should any metaphysical issue like this pose a serious problem to our attempts of formalizing natural language consequence? Cotnoir goes on to argue that neither a consequence relation between propositions nor between sentences can be seen as correct and, thereby, tries to establish his nihilism. But, again, he has rather established that there is no logical consequence relation at all than that formalizability nihilism is true. Last, I will make a short detour to the end of Cotnoir's paper because the argument he presents there shows the same mistake: The argument from normativity basically says that the concept of logical consequence is inconsistent because the concept of an objective normative relation is inconsistent. This, again, is no argument for or against formalizability nihilism but, rather, an argument against the fact that there is a natural language consequence relation at all, i.e. it has nothing to do with formalizations.

I want to close this section with some remarks on what Cotnoir is trying to argue for at this point: His proclaimed aim is to show that there can be no formal system which represents natural language consequence. In the case of his current argumentation this seems to be what I have called the *concept* of logical consequence and described in the first

[4]Glanzberg (2015, 71f.) explicitly says that his argumentation is about the very existence of a relation of logical consequence in natural language, i.e. whether natural language semantics determines something we would call a relation of logical consequence. Indeed, this thesis has some similarities to Cotnoir's formalizability nihilism. However, it should be clear from the very formulation of the aim Glanzberg's paper that they argue for different things: Cotnoir says that we cannot capture natural language consequence with formal means. Glanzberg, however, maintains that we cannot legitimately talk about a relation of natural language consequence. Thus, as I pointed out above, Glanzberg can legitimatly refer to the philosophical distinction between analytic entailments and proper, formal logical consequence within his argumentation. This step, however, looks very weird when we translate it into Cotnoir's project: If natural language semantics does not determine a consequence relation but only lexical entailments what does formalizability nihilism amount to? There are two different things Cotnoir could have in mind here: Either we keep the definition of formalizability nihilism. Then, however, Glanzberg's reasoning seems to establish that nihilim is false: If there is no relation of natural language consequence out there than it would be false to ascribe some failure to our formal means to capture this very relation. On the other hand, if we change the definition of formalizability nihilism and understand Cotnoir's "natural language inference" as the lexical entailments of natural language then Glanzberg's ideas do not help him to establish nihilism, i.e. Glanzberg does not argue that there are no formal means to capture lexical entailment. Thus, I would like to conclude that Cotnoir's reference to Glanzberg's paper is ill advised. However, Glanzberg's no-logic-in-natural-language thesis is very interesting on its own, since it could be seen as a version of nihilism on its own. I will come back to it in the conclusion of this thesis.

chapter of this thesis: It is, for instance, a necessary and formal relation between sentences. Thus, there are certain properties this relation exemplifies. Now, his argument from diversity is trying to establish the claim that no consequence relation in formal languages can play this role, i.e. can represent a relation with these properties. I have shown that his argument for this view is misguided. The arguments from expressive limitations, on the other hand, try to establish formalizability nihilism on a completely different route: They are not concerned with whatever properties one take the consequence relation to instantiate but, rather, with the extensional adequacy of the representation (compare Cotnoir 2018, p. 313). This is a different topic and it makes quite a difference: After all, a relation might be said to exemplify whatever property we ascribe to "logical consequence" but still be extensionally inadequate to capture the natural language consequence relation. An illustrative example would be a system of logical nihilism which we encountered in the last chapter, i.e. a system with an empty consequence relation. One might maintain, that all the (non-exsistent) rules of this logic are necessary, formal and maybe even normative. However, it would make no sense in Cotnoir's project to assume that such a system is a correct representation of natural language consequence. Remember that he seems to assume that we have an intuitive grasp of which pairs of sets of natural language sentences and single sentences are in the relation of logical consequence.

2.1.2 The Diversity of Formalizations

A proper argument from diversity would have to start from a different kind of pluralism: It cannot be a pluralism that emerges from some underdetermination in the principles governing our concept of logical consequence. Rather, it has to be some kind of pluralism with regards to the formalization of natural language inference patterns. As Cotnoir (2018, 320f.) notes at the very end of his paper, there is such a version of pluralism in the literature: The view that there are several correct formalizations of natural language, i.e. the group of theories which I labelled as formalizability pluralists in the first chapter. However, Cotnoir merely presents some points of agreement and disagreement which his nihilism has with these views instead of producing a proper argument from diversity. In this section I will try to sketch such an argument which is in line with the things he says about formalizability pluralism. I will keep the basic structure of Cotnoir's original argument from diversity:

1. [**Impossibility of Formalizability Monism**] Our formal means of modelling are incapable of producing a uniquely correct representation of natural language consequence.

2. [**Plurality Standards**] The standards of formalizability pluralism are too low. They do not talk about *complete correctness* for formal systems.

3. ∴ [**Formalizability Nihilism**] There is no logic that *completely* captures natural language consequence.

So, again, the argument starts from the motivation of some kind of pluralism. But, this time, the second premise is quite different: Rather than arguing that this form of pluralism is impossible it takes pluralism on board but points out that all their "correct" formal systems fail some higher standard of correctness, i.e. capturing natural language consequence *completely*.

However, in the context of Cotnoir's discussion it does not really make a lot of sense to dive into such an argument. This is due to the fact that the motivation of formalizability nihilism is either uninteresting for Cotnoir, or already part of the other argument of Cotnoir. The first, uninteresting version of formalizability pluralism is presented by Terres Villalonga (2017). In her version of pluralism classical and relevant logics are both correct since they model two different aspects of natural language: classical logic is only concerned with truth-preservation while relevant logic takes some gricean maxims under consideration. However, this form of pluralism is rather a disambiguation about the objects which are modelled than a substantial version of formalizability pluralism. The second family of formalizability pluralists take the phenomenon of vagueness as the starting point of their discussion.[5] Since natural language is vague and mathematical models are precise there might be several models which can be said to be a correct representation of natural language. However, as we will see in the fourth section of this chapter, Cotnoir takes this very phenomenon to be an argument for logical nihilism. Hence, my new version of the argument from diversity would not add anything new to his discussion. It merely points towards the disagreement between formalizability pluralism and nihilism.

2.2 Semantic Closure

As I pointed out in the introduction to this chapter, the arguments from expressive limitations follow a very clear path. In the case of semantic closure it looks like this (compare Cotnoir 2018, p. 310):

1. [**Shortcoming of Formal Languages**] Formal languages cannot express their own semantics and maintain consistency/non-triviality.

2. [**Power of Natural Languages**] Natural languages can (and do) express their own semantics while maintaining consistency/non-triviality.

3. ∴ [**Formalizability Nihilism**] There is no logic that captures natural language consequence.[6]

[5]Compare Cook (2002), (2010, 500f.) and S. Shapiro (2006, 49f.).

[6]I take this argument to be a correct explication of Cotnoir's reasoning. In his own summary of the

Formal languages, so to speak, are faced with a certain dilemma: either they are incomplete or inconsistent/trivial. Natural language on the other hand is complete, but not trivial. Thus, formal languages cannot represent the semantic closure of natural language. Further, if there is a whole part of natural language that cannot be captured in formal systems it seems to be correct that no logic can represent all instances of the natural language consequence relation there are.

This section will proceed along the following lines: First, I introduce the problems which arise in semantically closed formal languages. The discussion about these semantic paradoxes is quite big and philosophers have come up with different answers to the challenge. Thus, my presentation will mainly be restricted to the discussions Cotnoir refers to. Second, I want to discuss how natural languages fare with these paradoxes. Yes, languages like English seem to be complete with regards to their own semantics. However, it is far from clear whether they are as innocent as Cotnoir claims, i.e. they might be as inconsistent/trivial as formal languages.

2.2.1 A Dilemma of Incompleteness and Inconsistency

In the following I will present the problem of semantically closed languages as a dilemma of incompleteness and inconsistency/triviality: Either we accept that our language has expressive limitations and is, thus, incomplete or we run into semantic paradoxes and, thereby, accept that the theories formulated in this rich language turn out to be inconsistent or even trivial. First, I am going to present the problem and then I go into some solutions that are discussed in the literature.

The Basic Problem(s)

Cotnoir (2018, pp. 310-313) presents three different arguments that are supposed to establish the impossibility of semantically closed formal languages. Each of these arguments is based on a semantic paradox: First, the classic paradox of the liar. Second, Field's take on Gödel's incompleteness results. And, third, the Curry paradoxes. In the following I will give a short sketch of each of these results.

In order to create the liar paradox we need the following ingredients (compare Beall et al. 2017). If our language is semantically closed it is reasonable that it contains a truth predicate Tr. Further, it should be possible to name our sentences, i.e. sentences can occur as subjects of predicates. Hence, if A is a sentence, then $\ulcorner A \urcorner$ is the name of A. If we want to express that a sentence A is true we can use $Tr(\ulcorner A \urcorner)$. Further, it is reasonable that this truth-predicate follows the so called T(arski)-schema: $Tr(\ulcorner A \urcorner) \leftrightarrow A$ (Tarski 1935). Now

argument he is just referring to the fact that natural language is semantically closed while no formal language is semantically closed. This more detailed way of putting it, however, should make it more clear where the problems of semantic closure are to be located.

we just have to be able to create the following self-referential sentence: $L \dashv\vdash \neg Tr(\ulcorner L \urcorner)$. This is usually done by a method called *diagonalization*: The diagnoalization lemma states that for any sufficiently strong theory F: $F \vdash D \leftrightarrow A(\ulcorner D \urcorner)$, i.e. we can proof that a sentence D is equivalent to a sentence which ascribes D some property $A(...)$ and is, in this sense, self-referential (compare Raatikainen 2018).[7] If we take $A(...)$ to be $\neg Tr(...)$ we get the sentence L which is the basic version of the liar-sentence. If we assume classical logic, it is easy to show that we have created a paradox. We only need the law of excluded middle ($\models \varphi \vee \neg\varphi$), basic adjunction ($\varphi, \psi \models \varphi \wedge \psi$) and proof by dilemma (if $\Gamma \models \varphi \vee \psi$, $\Gamma, \varphi \models \chi$ and $\Gamma, \psi \models \chi$ then $\Gamma \models \chi$):

1. $Tr(\ulcorner L \urcorner) \vee \neg Tr(\ulcorner L \urcorner)$ (by law of excluded middle)
2. $Tr(\ulcorner L \urcorner)$ (Assumption)
3. L (from 2 by T-schema)
4. $\neg Tr(\ulcorner L \urcorner)$ (from 3 by definition of L)
5. $Tr(\ulcorner L \urcorner) \wedge \neg Tr(\ulcorner L \urcorner)$ (by 2 and 4)
6. $\neg Tr(\ulcorner L \urcorner)$ (Assumption)
7. L (from 6 by definition of L)
8. $Tr(\ulcorner L \urcorner)$ (from 7 by T-schema)
9. $Tr(\ulcorner L \urcorner) \wedge \neg Tr(\ulcorner L \urcorner)$ (by 6 and 8)
10. $Tr(\ulcorner L \urcorner) \wedge \neg Tr(\ulcorner L \urcorner)$ (by dilemma on 1, 5 and 9)

Due to the law of excluded middle L is either true of false. If it is true then the negation of L is true. In classical logic, however, this amounts to the fact that L is false. If L is false, on the other hand, it has to be the case that $\neg L$ is true. But this is just what L says. Thus, L is true. So, either way the conjunction of the liar and its negation follows and thus we have proven a contradiction as a theorem. Further, the law of explosion ($\varphi \wedge \neg\varphi \models \psi$) is valid in classical logic. Thus, any sentence of our language has to be true and, thereby, any theory stated in the language is trivialized.

A second, related problem which Cotnoir points to are Gödel's incompleteness results (compare Raatikainen 2018): First, any sufficiently strong system S, i.e. some axiomatized proof-system in which one can proof a certain amount of arithmetical facts, will be incomplete, i.e. there will be arithmetical truths which are not provable in S. Second, any such system S will not be able to prove its own consistency. This result, however, looks troubling if we consider languages which are semantically closed (compare Field

[7] I will leave the subtleties of diagnoalization aside for this thesis.

2008, 200ff.). By semantically closing our language we add some sort of validity- and truth-predicate to our language and should be able to derive that the relation is in fact truth-preserving. This, however, conflicts with Gödels second incompleteness theorem: If we can show that our consequence relation is truth-preserving than we have shown that our system is consistent.

The third problem which Cotnoir cites can be produced if we have a language with both a truth predicate and a validity predicate or consequence connective. Given these two one can create a version of the so called Curry paradox (compare Beall and Murzi 2013 and Beall and L. Shapiro 2018): Just like we have seen in the case of the liar our semantically closed languages should be able to talk about their own sentences. Thus, a validity predicate should take the following form: $Val(\ulcorner\varphi\urcorner,\ulcorner\psi\urcorner)$. Now we can use the diagonalization lemma which I stated above to show that for any sentence φ of the language there exists a curry-sentence κ such that $\kappa \leftrightarrow Val(\ulcorner\kappa\urcorner,\ulcorner\varphi\urcorner)$. We simply take $A(...)$ to be $Val(...,\ulcorner\varphi\urcorner)$. Thus, the Curry-sentence is equivalent to the fact that it entails φ. Now, given that our validity predicate or connective follows a modus-ponens like rule ($Val(\ulcorner\varphi\urcorner,\ulcorner\psi\urcorner) \wedge \varphi \models \psi$) and allows for contraction (if $\varphi \models Val(\ulcorner\varphi\urcorner,\ulcorner\psi\urcorner)$ then $\models Val(\ulcorner\varphi\urcorner,\ulcorner\psi\urcorner)$) we can derive any φ, as well as the law of identity ($\varphi \models \varphi$) is in place we get the following:

1. $\kappa \models \kappa$ (by identity)
2. $\kappa \models Val(\ulcorner\kappa\urcorner,\ulcorner\varphi\urcorner)$ (from 1 by the definition of κ)
3. $\models Val(\ulcorner\kappa\urcorner,\ulcorner\varphi\urcorner)$ (from 2 by contraction)
4. $\models \kappa$ (from 3 by the definition of κ)
5. $\models \varphi$ (from 3 and 4 by modus-ponens rule)

Thus, we have proved that any φ is a theorem, i.e. we have trivialized any theory stated in our language. Note that we were able to do this without the detour of contradictions and explosion.

...and Some Solutions

Cotnoir's discussion of the possible solutions to these problems is contained to the liar paradox. Here he basically argues that the two most common replies to it do not work: First, the introduction of truth-value gaps and, second, the introduction of truth-value gluts. Both answers are prone to so called revenge liars and, thus, do not escape the dilemma of incompleteness and inconsistency.[8] As he notes, this is not a decisive point

[8] Thereby he omits a whole bunch of responses to the liar (compare Field 2008 and Beall et al. 2017). Some solutions like Tarski's hierarchical structure can be omitted here because they give up on complete semantic closure anyway. Others routes, however, seem to be open still.

against the possibility of accounting for the semantic closure in natural language. It is just supposed to create some motivation for the first premise above by a pessimistic form of meta-induction, i.e. so far many attempts to solve the semantic paradoxes have failed and, thus, there is reason to believe that they are not solvable.[9]

The first strategy to handle the liar is to regard the problematic sentence as a truth-value gap, i.e. it is a sentence which is neither true nor false. Thus, the law of excluded middle fails in full generality. A famous example of such an approach is Kripke's (1975) theory of truth. His system allows for partially defined predicates, i.e. the extension and anti-extension of a predicate may not be exhaustive. Now, if Tr is one of those predicates we can block the reasoning that created the liar: L is neither true nor false but undefined. Thus, if we switch to a paracomplete system which allows for gaps we can avoid the original liar. However, this line of response falls into the so called revenge liar. One way to create this revenge problem is to consider determinacy: Since our language is semantically complete we want to be able to express the fact that some sentence is determinatly true or false, i.e. that it is just true or just false. So, lets $Tr_D(\ulcorner\varphi\urcorner)$ be true if φ is true and false if is false or neither true nor false. But if we can express such a predicate we can again use a diagnoalization to create the following sentence: $L_R \leftrightarrow \neg Tr_D(\ulcorner L_R\urcorner)$, i.e. we take $A(...)$ to be $\neg Tr_D(\ulcorner ...\urcorner)$. Now, the only thing which is needed in order to create a paradox within the paracomplete account is to assume that the law of excluded middle is valid for determinate-truth ascriptions ($\models_D (\ulcorner\varphi\urcorner) \vee \neg(\ulcorner\varphi\urcorner)$). However, this seems to be a direct consequence of the definition of Tr_D. Now, we can use a parallel derivation as in the case of the standard liar to prove that $L_R \wedge \neg L_R$. Hence, if we want to be able to express determinate truth the paracomplete approach does not yield a way out of the paradoxes. Further, Cotnoir argues that this revenge is just an instance of a general scheme to create revenge problems for any theory that tries to account for semantic closure. This general scheme consists of three principles:

> [**Characterization**] There is a property Q such that only *rogue* sentences of a language $\mathcal{L}$ have Q.
>
> [**Semantic Closure**] We can express Q in $\mathcal{L}$ and there exists a name $\ulcorner A\urcorner$ for every sentence A of $\mathcal{L}$.
>
> [**Revenge Immunity**] No sentence which ascribes Q to a *rogue* sentence is itself Q.

The general route towards a revenge paradox looks like this: we take a sentence R such that $\ulcorner R\urcorner \dashv\vdash Q\ulcorner R\urcorner$. Hence, R is *rogue* and, by characterization, $Q\ulcorner R\urcorner$ is true. Further,

[9] Cotnoir does not discuss the solutions to the other two problems at length. Thus, I will omit these points here. However, as should become clear in due course, the Curry paradox will play a role in his discussion of the liar. Since he does not mention it at that point, I will try to make it explicit.

this amounts to the fact that R is true. However, by revenge immunity, we can also conclude that R is not *rogue*. Hence, R is both an element of the set of *rogue* sentences and not. Thus, Cotnoir concludes that any system which exhibits these three principles runs into the problems of some sort of revenge liar.

The second strategy, which Cotnoir presents as an answer to the troubles of the first one, is to maintain semantic closure on the pain of inconsistency, i.e. by introducing truth-value gluts: One has to accept that there can be inconsistent sentences. This, however, does not render our system useless since inconsistencies do not necessarily render our theories trivial. This line of reply to the liar is prominently defended by Priest (1984): If we use a paraconsistent logic, i.e. a logic which rejects the principle of explosion, we can keep our language semantically closed. Even though contradictions may arise in our theories they are not automatically trivialized by that fact. However, it is questionable whether the paraconsisent account is not liable to a revenge of triviality as well. Here, Cotnoir follows Beall (2015) to argue that even in Priest's paraconsistent theory we can find a route from semantic completeness to post-inconsistency or triviality. In order to do this, Beall is referring to the *trivializer*-paradox which is a variation of the Curry-paradox for validity predicates I presented above. Let us use $\perp$ to denote triviality, i.e. all sentences of a language are true. Then we can call a sentence φ a trivializer, i.e. $Triv(\ulcorner\varphi\urcorner)$, iff $\varphi \vdash \perp$. So, it is reasonable to assume that $\varphi \wedge Triv(\ulcorner\varphi\urcorner) \vdash \perp$. Now, just as in the case of the Curry-paradox diagnoalization should yield us with the following sentence: $T \dashv\vdash Triv(\ulcorner T\urcorner)$. If we insert this formula in the above principle we get $T \wedge Triv(\ulcorner T\urcorner) \vdash \perp$ which is just $T \wedge T \vdash \perp$. This, again, should be equivalent to $T \vdash \perp$ and, further, to T itself. But if we have proven T we can just use modus ponens to derive $\perp$. So, even though on the face of it Priest rejected explosion in order to accept contradictions he still ends up in a trivial system. Hence, Beall concludes, there is a direct route from semantic closure to triviality.[10] Cotnoir takes this quite different kind of revenge as a sign that the paraconsistent strategy faces as severe obstacles as the paracomplete one.

2.2.2 Semantic Paradoxes in Natural Language

How does natural language handle these paradoxes? As Cotnoir notes, we usually seem to accept that natural languages is semantically closed. E.g. we can express facts about the semantics of English without switching to a different language: "The things he said turned out to be true." seems to be a fine English sentence. Hence, it seems to be quite uncontested that natural languages are semantically closed. A point of resistance towards this conclusion is expressed by Kripke (1975, p. 714): Most of our semantic notions are products of philosophical reflection on the semantics of natural language but

[10] In his reply Priest (2016b) points towards the fact that substructural logics are able to deal with the Curry paradox and, thereby, with Beall's trivializer-paradox.

not necessarily part of natural language itself. Even though “truth” and “false” might be part of natural language, notions like “truth-value gap” or ”determinedly true” might not be. Thus, he concludes that natural language might not be as universal as some philosophers hoped. However, in order for Kripke’s reasoning to establish a clear point of defence against Cotnoir’s nihilism we have to exclude the resources needed in order to create the Curry paradox from natural language. Here one might wonder whether “validity” and closely related terms are really restricted to philosophers discourse about semantics or whether they are part of natural language. Thus, I will accept that natural language is sufficiently universal, i.e. any formal representation will be object to Cotnoir’s attack. But in order for his argument to work we need more: if we want to show that formal languages are not suited to represent natural language in all its parts we have to show that the above dilemma does not apply to natural languages, i.e. that natural language can be complete without producing inconsistencies/trivialities in theories stated in natural language. Else the troubling properties of formal languages would not be problems for a formalization of natural language. In this section I want to argue against this point: Can natural language express their own semantics? Yes, it appears so. Does natural language produce inconsistencies? Yes, by the same rights.

What is the evidence we have for the semantic closure of natural language? Well, we seem to be able to express the semantics of English in English itself. But what about problematic sentences like the liar or the revenge liar we have seen above? They seem to be expressible as well: “This sentence is false” seems to be a perfectly fine English sentence. Does this sentence create inconsistencies? Yes, as we know from antiquity the liar sentence seems to have inconsistent properties. We can more or less use the same deduction as we did above to create a liar in natural language:

> “This sentence is false” is either true of false. Let us assume first that “This sentence is false” is true. Then, however, we can conclude by its very meaning that it is false. Hence, we have to say that “This sentence is false” is true and false. If, on the other hand, we assume that “This sentence is false” is false, we can see that the conditions for its being the case are met. Hence, it has to be true. So, we can again conclude that “This sentence is false” is true and false. Thus, either way, we end up in a contradiction.

So, even if we are not faced with formal languages but only concerned with semantic closure for natural languages we run into similar problems.[11] This result suggests that the problem of a dilemma of incompleteness and inconsistency is not a problem of formalization but its roots run deeper.[12]

[11] Thus, Chihara (1979) and Eklund (2002) both argue that natural language is inconsistent.

[12] This point can also be made the other way around: If philosophers which are not fond of inconsistencies try to exclude the liar from the set of truth-evaluable entities they have to give up on the full semantic closedness of our language.

However, as the discussion of Priest's theory made clear, it does make sense to take Cotnoir to argue for a more severe dilemma, i.e. a dilemma of triviality and semantic incompleteness. Even though it does not seem like a big concession to argue that natural language is, in fact, inconsistent, the fact that natural language contains trivializing sentences might be harder to swallow. Is there any reason to believe that natural language does not contain sentences which are trivializers? Well, by the semantic completeness of natural language which we have accepted above we know that Beall's trivializer sentence will be expressible in English: "It follows from this sentence that every sentence of English is true" sounds like a fine English sentence. So, just as in the case of inconsistency there is *prima facie* good reason to allow for triviality in natural language. Thus, I conclude that if a semantically complete formal language has to contain trivializers then so does natural language.[13]

To conclude, I do not think that Cotnoir's argument works: In order to make a point in favour of formalizability nihilism he has to show that the problems of formal languages are peculiar to formal languages and do not occur in natural ones. However, he does not provide any reason to believe that. In fact, Cotnoir (2018, p. 319) seems to think that formalizability nihilists are *free* to disagree with inconsistency theories of natural language. As my above reasoning shows this idea is misguided: In order for the argument from expressive limitations to work formalizability nihilists *have to* disagree with inconsistency theorists. Further, as the discussion of this section made sufficiently clear this dispute is from being an *isosthenia*: The reasons we have to believe in the semantic closedness of natural language are just as good as the reasons we have into believing that it is inconsistent or even trivial. Hence, Cotnoir's argument is blocked.

2.3 Universal Quantification

The second argument from expressive limitations runs in a parallel fashion (compare Cotnoir 2018, p. 314):

1. [**Shortcoming of Formal Languages**] Formal languages cannot express universal quantification.

2. [**Power of Natural Languages**] Natural languages can express universal quantification.

3. ∴ [**Formalizability Nihilism**] There is no logic that captures natural language consequence.

[13] The thesis that natural language is, in fact, trivial is defended, for instance, by Azzouni (2003, 2007, 2013). Bueno (2007), on the other hand, sees the existence of paraconsistent logics as a clear point of resistance towards tirivalism. However, both sides of this debate seem to agree on the conditional thesis which I propose: If there is no formal way out of triviality, there will be no way out for natural language as well.

Again, there is a part of natural language that is in principle not captured by any formal language. Here, however, we are talking about the subset of natural languages which makes use of universal quantification.

Just like in the case of semantic closure, I start this section with a presentation of the philosophical debate of universal quantification. Again, this debate could easily fill a Master thesis on its own. Hence, the following sketch should rather be seen as an introduction into the debate which follows Cotnoir's own presentation. Then, I turn towards the question how universal quantification is supposed to work in natural languages.

2.3.1 Formal Systems and Universal Quantification

Again, I will follow the structure of the first argument from expressive limitations. First, I present the problem that universal quantification faces when it is added to formal languages.

The Basic Problem

The problem of universal quantification in formal languages can be derived from the following two principles.[14] The first principle can be formulated like this (compare Cartwright 1994, p. 7):

Definition. *All-in-one principle* The objects in the domain of quantification make up a set or a set-like entity.

Formal representations of quantification make essential use of sets: Classical first order logics involve a domain of objects over which we quantify. In order to check whether a quantified sentence is true we use some formal machinery to go through the objects of this set. Hence, it seems to be a necessary requirement for our quantified sentences in order to have a determinate sense that they range over a determinate set (compare Priest 1995, 138ff.). The second principle goes as follows:

Definition. *No universe* There is no universal set.

[14] For a detailed presentation of this problem see Rayo and Uzquiano (2006) and Florio (2014). I have omitted some other arguments one could put forward against universal quantification because these arguments do not fit the current point Cotnoir is trying to make, i.e. they are clearly independent of any issue of formalization and are concerned with other aspects of generality absolutism. Studd (2019, pp. 4-10), for instance, presents some objections which Cotnoir does not consider: First, one could argue that there are always sortal restrictions at work when we use quantifiers. Second, one could argue that using a universal quantifier in metaphysics would commit us to metaphysical realism. A related issue is mentioned by Rayo and Uzquiano (2006): One could simply maintain that the range of our quantifiers is determined by the conceptual framework which we use. All of these issues apply to the semantics of natural language quantification and, thus, would not be solved by Cotnoir's formalizability nihilism. Further, they draw attention to the tension between generality absolutism and the *Löwenheim-Skolem* theorem: We know that any theory which is satisfied by an infinite model can be satisfied by a finite one. Hence, there will never occur an epistemic reason to talk about an infinite domain. However, this point is an epistemic one and does not establish the impossibility of universal quantification in any language.

This second premise is based on the set-theoretic paradoxes which can be created when we allow for a universal set. First, if we work in a set theory which accepts the *axiom of separation* we can deduce the existence of the Russell-set from the existence of a universal set: The axiom of separation ($\forall A \exists M \forall X(x \in M \leftrightarrow (x \in A \wedge P(x)))$) says that if we have some set A we can use some of its elements which satisfy any condition P to create a new set M. If we have a universal set, however, we know that our quantifier in the axiom ranges over any set. Now we just introduce the following condition: $\neg(x \in x)$. Thereby, we have created the Russell-set, i.e. the set which contains all and only those sets which are not members of themselves. From this set, however, it is easy to create Russell's paradox: We can prove that the Russell set itself is a member of itself and that it is not. Second, the universal set itself should run into Cantor's second antinomy, or some sort of Burali-Forti paradox: Let us take a universal, all encompassing set U. But then we know by the definition of U that even the powerset $P(U)$ has to be part of U. However, by Cantor's theorem we know that the cardinality of $P(U)$ is greater than the one of U and, thus, it cannot be a member of U. So, given that there is no all encompassing set-like entity and given that universal quantification needs an all encompassing set-like entity we might conclude that universal quantification is impossible in formal languages: Any model-theoretic semantics is in principle incapable of representing a universal quantifier.

There are some parallels to the first argument from expressive limitations: Instead of creating an outright inconsistency/triviality in the language itself the problems above show that the theory which states the semantics of our language, i.e. set theory, is inconsistent/trivial if we have to account for a universal quantifier.[15] Thus, there is a similar dilemma lurking in the background of the problems of universal quantification: Either our object-language can only express a restricted quantifier, or our semantic theory of this language is inconsistent/trivial.

... and Some Solutions

If we want to avoid relativism with respect to our quantification we have to drop one of the premises of the above argument. Hence, there are broadly two lines of absolutist responses to the problem of universal quantification: First, one could drop the All-in-one principle and opt for a different kind of quantification. Or, second, one could accept the existence of a universal set-like entity and defend one's system against the paradoxes. In the following I will give a short glimpse on how such responses in the literature look and how Cotnoir criticizes them.[16] Again, these are not proofs of the impossibility of a

[15] If we assume that our language has to be semantically closed, then the problems of universal quantification are, in fact, inconsistencies in the language itself.

[16] Just as in the case of the semantic paradoxes a big part of the literature is concerned with accepting that our quantification is not unrestricted and, thus, deals with the correct way of how to correctly restrict it. I will not go into replies that have to give up on complete unrestrictedness and, thereby, concedes to the attack of formalizability nihilism. For example, one could also talk about a hierarchy of quantifiers and

formal system which uses an universal quantifier. Cotnoir is just trying to motivate his first premise by highlighting some defects in the most promising replies to the problems I outlined above.

A famous way of dropping the All-in-one principle is to refer to *plural* quantification. In classical formal languages we usually do not talk about plurals but analyze any plural term away, e.g. as a conjunction of claims about some individuals. However, it is also possible to add names for plurals and plural variables to our language and, thus, define a notion of plural quantification. These systems were introduced in order to give first-order logic the power to express set-theory without thereby committing to the existence of sets (compare Linnebo 2017). However, Cotnoir maintains by referring back to worries of Linnebo (2003) and other authors that even plural quantification remains committed to some set-like entity and, thus, runs into the same paradoxes when applied unrestrictedly. Thus, he remains skeptical about the outlook of unrestricted plural quantification. Another line of attack, which one could employ against this strategy, is Williamson's (2003, 425f.) argument from logical consequence: One way of including the concept of logical consequence is by generalizing over all interpretations of the non-logical vocabulary (compare 1.1.3.). However, if our quantifier is unrestricted, interpretations should themselves be objects which are quantified over. Given these assumptions we can recreate Russell's paradox without referring to the notion of a set-like entity. All we need is the following:

1. For all o, $I(F)$ is an interpretation under which P applies to o iff F applies to o.
2. Define R as: For every o, R applies to o iff o is not an interpretation under which P applies to o.
3. For all o, $I(R)$ is an interpretation under which P applies to o iff o is not an interpretation under which P applies to o.
4. $I(R)$ is an interpretation under which P applies to $I(R)$ iff it is not an interpretation under which P applies to $I(R)$.

Thus, given some reasonable assumptions about the semantics of our language we can derive an inconsistent entity, i.e. the Russell-interpretation. Hence, using an All-in-one free account of universal quantification still comes with significant costs.

If we, on the other hand, reject No-universe we have to accommodate the fact that certain set-theoretic paradoxes might arise in our system. Thus, we have to change our

domains. But just like Tarski's proposal of a hierarchies of truth-predicates above this account would have some expressive limitations that make its success for the current purpose questionable: We will never be able to express universal quantification but we will be able give to characterize the semantics of a logic of n-th order in a logic of order $n+1$ (compare Linnebo 2006 and Linnebo and Rayo 2012). Cotnoir launches a similar attack on approaches which try to fix universal quantification on proof-theoretic grounds: If we try to discern restriced and unrestricted quantifiers by writing in some *open-endedness* in the inference rules of the latter – as, for instance, Williamson (2006) – then we will not make a step beyond the hierarchical approach.

system in some way. Here, Cotnoir discusses several options and presents some problems of each. First, one could try to switch from set theory to mereology, since there one can have a universal sum. Thus, we do not regard the domain of quantification as a set but as a mereological sum. However, Cotnoir maintains that mereology can only yield models of the size of certain limit cadinals which is a restriction after all. Thus, he claims that this system would not have unrestricted quantification. Second, one could opt for some alternative set theories like Priest's (2006) paraconsistent variation: This system can have a universal set since the paradoxes which it produces do not trivialize our theory. Here, Cotnoir claims that there are certain inferences which are at the core of our conception of restricted quantification. However, if we combine these three principles we can show that some version of explosion is valid.[17] Thus, paraconsistent logic would be too weak to capture our restricted quantifier and, thereby, should not be seen as a satisfying account of quantification. A third option would be do adopt a different kind of set-theory like Quine's New Foundations since there a universal set exists. According to Cotnoir, however, this system lacks intuitive appeal.

2.3.2 Universal Quantification in Natural Language

In this section, again, I will have a critical look into Cotnoir's motivation of the second premise of his argument. However, the case of universal quantification is a bit darker than the one of semantic closedness. This is due to the fact that it is not even that clear whether natural language has a universal quantifier. The main sources Cotnoir (2018, p. 314) cites as a motivation of this are philosophers talking about quantifiers in metaphysics: If a physicalist utters "Platonic forms do not exist", she does not intend this as a remark on some restricted domain. The point is not that platonic forms do not exist *here*, but that they do not exist *at all*. Further, as Williamson (2003, p. 427) puts it, the denial of the assumption that we are capable of universal quantification leads to an inconsistency: Let us call this position generality relativism. According to generality relativism our quantifiers are always restricted and, thus, our claims of generality are always relative to some domain. However, as generality relativists we want to state our own position and, hence, argue against generality absolutism: There is no all-encompassing domain. However, this thesis seems to be meant to have a claim towards absolute generality and, thereby, should contain an unrestriced quantifier. Hence, if the generality relativist gets it right she has no expressive tools to properly state her own position. Further, Williamson points out that the problems of generality relativism are not restricted to this metaphysical perspective. Rather, it is in conflict with certain semantic principles and, thereby, endangers the selfunderstanding of our language and thought. Thus, this shortcoming of

[17] These are *modus ponens* (All As are Bs. x is an A. Hence, x is a B.), *weakening* (Everything is a B. Hence, all As are Bs.) and *contraposition* (All As are Bs. Hence, everything non-B is a non-A.). Given these principles we can proof that explosion is valid.

relativism can be seen as a motivation for generality absolutism.[18] Hence, the case for generality relativism seems quite bad and a point of resistance towards Cotnoir's nihilism should come from a different angle. Let us accept that natural language has a universal quantifier, i.e. generality absolutism is correct. Now, in order for Cotnoir's argument to work, we have to show that universal quantification in natural languages does not produce similar problems as in formal languages: Else, there would be no genuine motivation for formalizability nihilism. If both natural and formal universal quantification are plagued with paradoxes why should there be a general difference between the two? Thus, our approach will be similar to the one in the case of semantic paradoxes: We accept that Cotnoir gets something right but one of the steps in his argument is not motivated at all. Thus, there is no wedge between formal and natural language and, thereby, no reason to become a formalizability nihilist.

The central question of this section has to be the following: Can we create inconsistencies using universal quantification in natural language which are similar to the problems we run into in the formal case? Our argument for the existence of semantic paradoxes in natural language was quite easy: we just had to point out that the liar seems to be an English sentence which seems to be inconsistent. Here, however, the case is not that clear: As I already pointed out at the end of the last section, the problem of universal quantification is not a straightforward inconsistency in the object language but rather a problem which arises out of our attempts to give a semantic theory of the universal quantifier. Thus, we have to investigate whether there is a problem of the semantics of natural language generality absolutism which mirrors the ones we have discussed above.

As far as I can see, it makes sense to set up this discussion as a dilemma: Either the semantics of the natural-language quantifier is governed by an All-in-one principle or not, i.e. either we presuppose the existence of an all-encompassing set-like entity when using this quantifier or not. Thus, rather than arguing decisively for either of these options I will show that both lead to paradoxes. However, it makes sense to point out that any motivation we had for using the All-in-one principle in formal systems seems to apply for natural language as well.[19] The first horn of the dilemma can be dealt with in quite straightforward fashion: If natural language semantics does, in fact, rely on a set-like entity then there is no difference to the standard quantifier of formal languages: The

[18] These seem to be mainly philosophical points. However, from a purely linguistic point of view it seems to be quite hard to discern a proper universal quantifier from one which is in fact restricted but has a really big domain. Peters and Westerstrahl (2006, 47ff.) make one linguistic point in favour of unrestricted quantification: They conjecture that due to the fact that we can learn "every" independent of any restriction of it (e.g. "every sentence", "every day", "everybody"...) natural language might be capable to express an unrestricted quantifier. But even they regard this question as a mainly philosophical or logical problem and not as a linguistically fruitful subject.

[19] For instance, as Cotnoir notes, our best account of quantifiers in natural language is *generalized quantifier theory* which makes use of the All-in-one principle. However, it might turn out that the universal quantifier of natural language behaves differently after all. So I will not regard these abductive grounds as a decisive points against Cotnoir.

semantic theory of natural language will have the same problems as the semantic theory of formal language, i.e. the semantic theory of natural language will introduce the same paradoxes that arise from a universal set-like entity. Hence, if the semantics of formal languages cannot deal with these paradoxes then a semantic theory of natural language has no prospects to deal with them either.

Thus, let us look into the second horn of the dilemma: the only route which is still open for the formalizability nihilist is to question All-in-one for the universal quantifier of natural language. However, we have already encountered All-in-one-independent problems of generality absolutism above. Studd (2019, pp. 10-15), for instance, argues that it is not the assumption that our domain of quantification is itself set-like that produces the problems. Rather, it is due to the fact that there are *indefinite extensible* concepts which is an idea he takes over from Dummett's work: A concept F is indefinite extensible iff we can show that any domain which tries to comprise all Fs misses at least one of the Fs. Let us take, for instance, the concept *collection*. Let D be the domain of our alleged universal quantifier. Again, we do not have to make any assumption about what properties D has, i.e. it might not behave set-like. Now we can show, however, that there is a collection, call it the *Russell collection* R, which is not in D: R is the collection of all the non-self-membered collections which we quantify over, i.e. all the non-self-membered collections in D. R cannot be a member of itself, since R does only contain non-self-membered collections. Now suppose for *reductio* that R is a member of D, i.e. we are quantifying over R. But then R is a non-self-membered collection in D and, thereby, R is a member of itself. Thus, by *reductio*, R cannot be a collection in D. Hence, D does not comprise all collections and, further, our domain of quantification is not absolutely general. We have shown that there is something (the collection R) which we do not quantify over. Thus, all we need for the argument to work is the existence of indefinite extensible concepts and not the All-in-one principle.[20] Again, just as in the case of the first horn, there is little reason to believe that natural language semantics would fair any better with universal quantification, i.e. the problems of generality absolutism seem to be quite independent of our formal machinery.

Even though the situation is not as straightforward as in the case of the semantic paradoxes, I still conclude that Cotnoir's argument is blocked: There are no good reasons to believe that universal quantification in natural language is innocent. If it is governed by an All-in-one principle then we have to run into the same problems. If it is not, however, we have sufficient reason to believe that generality absolutism on its own will have the

[20] Williamson's argument which I mentioned above is another instance of a problem created by an indefinite extensible concept and is, thereby, independent of All-in-one. Similarly the argument which Grim (1991, pp. 116-122) puts forward against our ability to talk about the set of all truths or all propositions rests on indefinite extensible concepts. Just as the other two authors, Grim highlights that the problems are not solved by dropping some All-in-one principle and to appeal to "raw quantification", i.e. quantification without any formal semantic theory.

same effect on the semantics of natural language universal quantification as it has on its formal counterpart. Thus, it seems rather to be the case that generality absolutism is at the bottom of these paradoxes and it is not a peculiarity of formal language universal quantification that it leads to an inconsistent semantic theory. Thereby, we have again shown that the alleged problems of formalization seem to be independent of formalization and, further, we have no reason to believe that formal languages could not capture the subset of natural languages which uses universal quantification. I think this result is in line with the general outlines of the debate on generality absolutism and relativism: They are theories of a problematic issue in semantics, no matter whether we are concerned with natural or formal language semantics. To conclude, Cotnoir's second argument from expressive limitations is blocked as well.

2.4 Vagueness

Cotnoir (2018, 317f.) concludes his list of arguments of expressive limitations with a short look into the topic of vagueness. This topic, as he correctly notes, is one of the properties which are classically used in order to characterize the distinction between formal and natural languages: While formal languages are clear and precise, natural languages are vague and unclear. Even though Cotnoir does not formulate his argument precisely it seems one can put it into a similar scheme as the first two:

1. [**Shortcoming of Formal Languages**] Formal languages cannot account for vagueness.

2. [**Power of Natural Languages**] Natural languages are inherently vague.

3. ∴ [**Formalizability Nihilism**] There is no logic that captures natural language consequence.

So, just as before, there is a whole set of sentences of natural language, i.e. the ones containing vague predicates, which are not expressible in formal languages. Thus, no formal system can capture natural language consequence completely. Even though on this general level the discussion of vagueness is quite parallel to the other two arguments the details look quite different: Due to the fact that his presentation is really short there are a lot of unclear points what exactly this argument is supposed to be.

Instead of going into the problems of formal languages first and then into the question of vagueness in natural language I will proceed differently in this section. Since Cotnoir presents two very different arguments in order to motivate the first premise of the above argument I will structure this section along these lines. Hence, I am going to start with the question whether vagueness induces inconsistency in formal languages and discuss whether this establishes Cotnoir's point. Second, I will look into higher-order vagueness and whether this is a good motivation for the first premise. As will be made clear by the following presentation these two arguments and their evaluation will be quite different.

2.4.1 A Dilemma of Precision and Inconsistency

The first argument which Cotnoir presents in order to show that formal languages cannot account for vagueness can be put into the following form:[21]

1. [***Sorites*** **Paradox**] If we add precise representations of vague terms to our logic we can create a paradox and, thus, end up in contradiction.

2. [**Inherent Vagueness**] Natural languages as inherently vague.

3. $\therefore$ Formalizability nihilism is correct.

This line of attack is parallel to the other two arguments above: If we want to have a formal language that can express vague terms then we end up in inconsistencies. Thus, we have a dilemma of some limitation, i.e. being precise, or inconsistency. Similarly, my line of resistance against this argument will be on the level of natural language inconsistencies. Thus, my presentation of the Sorites paradox will be quite short. I will omit a presentation of possible solutions here.

The *Sorites* Paradox

The Sorites paradox can be formulated if the following three conditions are given (compare Hyde and Raffman 2018): First, there has to be a some *Sorites series*, i.e. an ordered line of instances $i_1, ..., i_n$ of a predicate. Second, any neighboring instances have to be indiscriminable. So, if the predicate applies in the one instance i_x it should apply in the next i_{x+1} as well. Last, there are predicates such that they apply to the first element in the line, i.e. $P(i_1)$, but not to the last one, i.e. $\neg P(i_n)$. When we try to represent vague predicates in our formal systems this seems to be exactly what we have to deal with.

[21] Actually, Cotnoir's formulation would rather suggest the following argument:

1. [***Sorites*** **Paradox**] Vaguenss induces inconsistencies into languages.
2. [**Natural Vagueness**] Natural language is vague.
3. $\therefore$ Natural language is inconsistent.
4. $\therefore$ There can be no coherent logic of natural language.

This argument is puzzling to say the least. It seems to run in a completely different direction as the other two arguments. There we assumed that natural language is in fact consistent and, thus, tried to establish formalizability nihilism by pointing out that any attempt of a complete formal representation of natural language would be inconsistent. Now, however, Cotnoir starts from the inconsistency of natural language and concludes from there that a formalization of it is impossible. I suppose that this is due to the limited space this argument is given and that the following version of the argument is, in fact, the intended one. Else, I could just conclude that the argument seems to be quite false: Why should formal languages not be capable of being inconsistent?

Given these three elements we can create a paradox in the following way: We know that $P(i_1)$ and that $\neg P(i_n)$. We know that $P(i_x) \rightarrow P(i_{x+1})$. Hence, we can use modus ponens to derive $P(i_2)$ and by a second application $P(i_3)$ etc. Thus, after $n-1$ applications we will derive $P(i_n)$. So, if our formal system allows for this operation we will have to conclude that the last element in the Sorites series does in fact have the property in question.

Natural Language Vagueness

Given my discussion of the last two chapters it should be quite clear what will happen here: Cotnoir's argument presupposes that vague natural languages are not plagued by the Sorites paradox while formal languages which try to capture vagueness are. However, this idea is, again, quite problematic: Why should the Sorites paradox be a problem of formalization? Just as in the case of the semantic paradoxes the case is quite clear: The Sorites does arise in natural language just as it does in formal one. The semi-formal presentation of the paradox which I gave in the last section could be easily stated using natural language alone. Thus, the paradox should not be the product of our formalization but, rather, be at the heart of vagueness as such. Again, one could say that the Sorites is older than any of our modern formal languages. Thus, if we say that formal systems are lead into inconsistency due to the paradox then, by the same rights, natural languages are inconsistent as well (compare Eklund 2002). Thus, if we want to state the problem of vagueness in the form of a dilemma we can conclude that it applies to formal as well as natural languages.

2.4.2 Higher-Order Vagueness

The most promising argument for formalizability nihilism can in fact be found in the few remarks Cotnoir makes about higher-order vagueness. This second argument from vagueness runs along a different line and it seems to be way more convincing than the first one since it departs in its structure from the problem of semantic closure and universal quantification. His argument goes as follows:

1. [**Higher-Order Vagueness**] Vague languages require a vague meta-language.
2. [**Precision**] Formal systems are precise, i.e. a formal representation of vagueness has to have a precise meta-language.
3. $\therefore$ Formal systems cannot represent vagueness.

So, in order to give a precise, formal system which represents some vague parts of natural language consequence we have to rely on a vague metalanguage on our own. Hence, if our aim was to give a complete and precise account of natural language we seem to have a problem. This is an interesting argument because it forces us to think about the criteria we have for a successful complete formalizations: Cotnoir's argument seems to rest on the assumption that a formal system has to be precise. Hence, if a formal system is supposed to be a complete representation of natural language, vagueness cannot pop up at any point.

Vagueness Strikes Back

As Wright (2010, p. 527) notes one can distinguish three varieties of higher-order vagueness:

1. **Borders of Borderlines**: If we characterize the semantics of vague expressions as a triple of extension, to which the expression definitly applies, anti-extension, to which the expression definitly not applies, and a borderline-set which contains the cases of indeterminacy, we are faced with the following coutnerintuitive consequence: There is a determinate borderline between the extension and the borderline-set and between the borderline-set and the anti-extension. Thus, our formal approach has created a cut-of point where the truth-value is supposed to switch. However, intuitively there is no such point: Just as there is no unique shade that is clearly "red" while the one next to it in the spectrum is "not red", there is no unique shade that is "red" while the next one is "neither red nor not red". Thus, vagueness is not contained to the first level but demands for vagueness on the second level. However, if we use the same account of vagueness on the second level we will just reiterate the issue: If we treat vagueness of any order n as such a triple we will have to account for vagueness on the $n + 1$-th level.

2. **Vagueness of "vague"**: The term "vague" may be vague as well, i.e. there might be predicates which are definitely vague, predicates which are definitely not vague and some borderline cases of predicates which are neither.

3. **Iteration of Definite-Operator**: In classical accounts of vagueness a definite-operator Δ which takes an expression and maps it into the truth-values true and false, i.e. a sentence $\Delta(\varphi)$ is true if φ is true, or false and it is false if φ is neither. However, for similar reasons as in the case of borderlines above one might think that sentences like $\Delta(\varphi)$ might themselves come out as neither true nor false. Thus, for instance, $\Delta(\varphi)$ does not have to be equivalent to $\Delta(\Delta(\varphi))$. Hence, our ascriptions of "definiteness" may turn out to be indefinite.

For our current purpose we only have to be concerned with the first version of higher-order vagueness. This problem should be familiar to anyone who ever took a course that tackled the issue of vagueness from a formal standpoint: The precision of the mathematical models seems to be in tension with the vague phenomenon that they are supposed to model. So in order to prevent a reduction of vagueness to precision we usually keep the metalanguage of our logic vague as well (compare Sainsbury 1997, 253ff.). Thus, the best we can do in order to represent vague predicates in our formal systems is to have an infinite hierarchy of precise representations. However, just as in the case of semantic closure and universal quantification, one might wonder whether such a hierarchy is a proper, complete model of vagueness. After all, we will not be able to express a term that is really vague on any level of the hierarchy. [22] Hence, one can, in fact conclude, that there is no precise representation of vagueness, i.e. a model for vagueness has to be vague at some point. This result is different from the ones we have looked in so far: We are not faced with some dilemma, but with a definite shortcoming of formal systems.

As far as I can see, this general point is not contested in the literature on higher-order vagueness. There the debate is rather concerned with the possibility of higher-order Sorites paradoxes and how to account for them. But my reasoning here is not motivated by these paradoxes, but by the general limitations of formal systems. So, even authors who oppose the idea that there is some special problem of higher-order Sorites paradoxes follow along in these points: For example Hyde (1994), Bobzien (2010) and Wright (2010) which are all critical towards higher-order vagueness do not oppose my current line of thought: They merely suggest that the "problems" of higher-order vagueness do only arise under a false conception of what first-order vagueness does, in fact, consist in. One might even say that they all argue against the very attempt of a completely precise representation of vagueness in the first place.

Precision and Formalization

Given that we have established the first premise it is easy to show how formalizability nihilism can be established: Natural language is vague. Thus, given higher-order vagueness there can be no completely precise representation of it. Formal systems are precise. Hence, no formal system can capture natural language consequence. This argument seems to be sound. However, we have not looked closer into the claim that formal systems have to be precise or, rather, that they have to be precise all the way down. Thus, in this section, I am giving a short motivation for this premise.

Why did philosophers or scientists in general set out to create formal languages? As Frege (1879, p. v) puts it formal systems are tools for a specific purpose (compare Frege 1882): While natural language is good for everyday purposes a formal language aims to be good only with regards to the expansion of our knowledge by scientific inquiries. Just as our limps and eyes are well suited to handle ordinary tasks so is our natural language. And just as artificial instruments like microscopes etc. might replace these

[22] This point might not be a decisive point against the possibility to account for vagueness. But – just like Cotnoir's negative meta-induction in the other cases – I will stop my investigation on this negative outlook instead of providing a proper conclusion on this matter.

when conducting more fine grained research so does a formal language replace the natural one. Now, there are different advantages which philosophers through time have ascribed to formal languages (compare Maat 1999): They are supposed to get rid of ambiguities, are less lively and may capture the essence of things better than ordinary language. But the aim of formalization which is important for the issue of the current section is of course precisification: The vagueness of natural language is supposed to stand in the way of our scientific inquiry. Hence, we introduce artificial, formal languages which are precisely defined. So, there is some motivation behind this premise of Cotnoir's reasoning.

This idea, however, should not be seen as a decisive point in favour of Cotnoir. One could still disagree whether formal languages have to be that precise. Tye (1994), for instance, presents a formal account of vagueness that makes use of vague formal entities. So for him formalization does not require complete precision. So all I have shown is that given we accept total precision as a central aim of the project of formal languages then the problem of higher-order vagueness is a strong point for formalizability nihilism.

2.5 Concluding Remarks

The results of this chapter can be summarized quite easily: We have put formalizability nihilism in its place by pointing out the positives and negatives of Cotnoir's argumentation. Just like in the case of Russell's nihilism our critical evaluation has provided several severe internal criticisms of Cotnoir's paper. The argument from diversity seems to be far off and both of his main arguments of expressive limitations suffer from a severe jump or an unjustified assumption. The problems of semantic paradoxes, universal quantification and paradoxes of vagueness do not arise from formalization. They are part of natural language as well and, thus, cannot drive a wedge between natural language consequence and our attempts to capture it in a system of formal logic. The only positive prospect for formalizability nihilism seems to consist in the very motivation people had to start the project of formalization: Formal languages are inherently precise while natural language allows for vagueness. The problem of higher-order vagueness highlights the fact that this difference remains even after the development of further formal means to capture vague terms.

What are the upshots of this discussion? I think just as the results of my evaluation are twofold so should be the consequences. First, let me spell out the consequences of the negative outcomes, i.e. the failure of Cotnoir's arguments from paradox.[23] Most paradoxes that philosophers care about have their roots in natural language consequence and are not created by an attempt to capture this consequence relation. Hence, using formal methods to spell out the structures of these paradoxes seems to be a legit enterprise. JC Beall

[23] I will cast aside the failure of the argument from diversity since I do not see any consequences arising from its (severe) failures.

(2003, p. 1) puts it nicely:

> Philosophers work at the limits of language; that is where standard principles are challenged; that is where paradoxes are found. The task, at least with respect to natural language, is to learn from such limiting cases; the task is to figure out what the paradoxes teach us about the very language we speak.

Thus, the picture which Cotnoir presents gets it – in a sense – wrong: The paradoxes which we have discussed in this chapter are not some kind of downside of formal systems but rather one of their most useful applications![24] By clearing up the principles which might govern our natural language consequence we are able to make progress in understanding these paradoxes. Else we might have to leave them in the dark.

As the last section has shown however, there is some right to formalizability nihilism: In the case of higher-order vagueness we are faced with a phenomenon that emerges out of our attempts of formalization or, rather, precisification. Natural language is vague and it seems that formal languages cannot capture this vagueness without losing their precision. Here Cotnoir seems to be on the right track. But his formalizability nihilism is far from being a unified view in the philosophy of language which solves a lot of our problems – as he claims – and, thereby, abductively justified. Rather, it is a general view on the relationship of natural and formal languages which provides an answer to one particular debate, i.e. vagueness.

I want to conclude this chapter with a dilemma that Cotnoir's stance faces: Either the thesis which he labels "logical nihilism" is not motivated, or his label "logical nihilism" is misplaced: Let us assume that the reasoning I presented above is correct and Cotnoir's position has some justification. Then, however, we have to make use of the fact that formal systems are inherently precise! However, it is quite hard to motivate this demand on formal systems if we just set them out as a model of natural language: If our aim is to capture natural language in a one-to-one representation why should we even think that this representation will turn out to be precise? Thus, in order to motivate Cotnoir's position the aim of precisification has to be build in somewhere into the conception of a formal language. However, if we understand our formal language as a precisification of natural language consequence it should not come as a surprise that the thesis which Cotnoir calls "logical nihilism" is true: A one-to-one correspondence is not possible since natural language is vague and – given our aim of precisification – formal systems have to be precise. Thus, if Cotnoir's stance is motivated the label "logical nihilism" does not really fit it: One-to-one correspondence is not the aim of logic, so we should not measure

[24] Cotnoir agrees that despite of nihilism logic remains useful for the purpose of (incomplete) formalization. However, it would be a weird aim of our logical systems to account for paradoxes if these just arise out of our formalization. If we aim to represent aspects of natural language consequence and, according to Cotnoir, these paradoxes are not part of natural language consequence then our logic should cast them aside and restrict itself to the parts of language it can consistently model.

it with this standard. The other horn of the dilemma can be created with an analogous reasoning: If we take the term "logical nihilism" to be a good depiction of the thesis which Cotnoir is trying to defend, then we should understand the aim of formalization to consist in a complete capturing of natural language consequence. However, given this aim, it is quite hard to motivate the premise that formal systems have to be precise. Thus, the justification of his thesis ceases.

Chapter 3

Nihilism and the Science of Logic(s)

So far, I have discussed the two challenges posed by the first two versions of nihilism. As I outlined in the introduction the third version of nihlism works on a different level: It is concerned with the very project of applied logic. Applied logic, so the third nihilist says, is useless and we should constrict ourselves to formal, mathematical logic. Since there is no clear argument for this position proposed by its defendant I will not present a detailed discussion of this stance in this chapter like I did in the previous two. Rather, I will use this chapter to tackle the issue of applied logic in general. The aim of the chapter will be to supply the discussion of the first two versions of nihilism with a discussion of the presuppositions of the very project they were posed in and, thereby, place them within a broader framework.[1] Thus, just as with the summary of certain debates in the first two chapters I have to make a disclaiming note: The distinctions and discussions I am drawing upon in this chapter are far from exhaustive. Again, a comprehensive answer to the question "What is applied logic?" would probably exceed the boundaries of a thesis on its own. Hence, I will try to delve upon these debates only in so far as it is useful for an elucidation and assessment of logical and formalizability nihilism.

I divide this chapter into three sections each trying to draw some distinctions with regards to a particular sub-question of "What is applied logic?". First, there is the very general question which philosophers have asked and answered differently: Is there any peculiar domain of application for logic? Or should we rather regard logic as a tool? Second, given that we answered the first question with "yes", we may ask what this special application of logic is? What is this thing according to which logic may be correct or incorrect? Last, one could ask how we could come to know this, i.e. ask for the epistemology of logic. As will become clear in due course Cotnoir and Russell seem to agree on the answer to the first question, but may differ in their answer to the second

[1] Alternatively, I could have started this work with some parts of this chapter. However, I would like to draw some conclusions from the discussion of nihilism in the previous chapters. Thus, it made sense to end the thesis with these general remarks. This, on the other hand, leads to minor overlap when it comes to the remarks I have made in 1.1.1.

one. Further, even though the previous chapters have highlighted some clear points of resistance against both versions of nihilism, one may take the reasoning of both authors to motivate a weaker thesis as an answer to the third question, i.e. some version of skepticism towards logic respectively.

3.1 Logic as a Mere Tool and Logic as a Science

Philosophers of logic usually agree upon the fact that there are multiple fruitful applications of logic: One can use logic, for instance, in other sciences like formal semantics in linguistic or computational science. Further, one can use logic for very concrete, practical things like paraconsistent logics for databases or fuzzy logis for washing machines. This plurality of applications seems to be uncontested. However, the dispute starts if we wonder whether there is some peculiar thing that logic is trying to capture or get right, i.e. even though we may be able to use pure logic for any of the applications I listed above, there is something further like logic *proper*. Is there a canonical interpretation of the purely mathematical systems which pure logic is concerned with or is there anything which the formal language and its consequence relation are supposed to represent? One can roughly draw a distinction between two different answers in the debate: First, one could accept that there is a proper domain of application of formal logic. I will call this position *substantialism*. Second, one could deny this thesis, i.e. there is no logic proper. I will call this position *instrumentalism*.[2] For the instrumentalist the correctness of a logic amounts to its usefulness to some purpose (e.g. to model some phenomenon). The substantialist, on the other hand, maintains that there is something wich logic is trying to capture.[3] According to them, there is something like a pretheoretic notion of logical consequence which is the object of our inquiry, i.e. the peculiar aim of applied logic is to capture the proper consequence relation. The question what exactly this relation is supposed to be will be tackled in the next section. For now, it is enough to state: Substantialists claim that there is a canonical application of the mathematical systems of logic, while the instrumentalist denies this.

The notion of instrumentalism with regards to logic goes back to Haack (1978, p. 221). According to her instrumentalism is characterized by a negative answer to the question whether there is an issue of correctness in logic. Rather, systems of logic are seen as more or less fruitful. This idea, however, was famously preceded by Carnap's (1937, p. 52) pragmatic view on excepting logics and languages:

[2]One could also labe this thesis *deflationism* about the aim of logic: While substantialism has an inflated concept of what logic is supposed to establish, the instrumentalist has a sparse view of its purpose.

[3]Substantialists may not deny the fact that there are other useful applications of logic. Thus, one can view the substantialist thesis as a claim over and above instrumentalism: Seeing logic as a tool has its proper place. However, there is something more to it than just usefulness.

> *In logic there are no morals.* Everybody is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he whishes to discuss it, he must state his methodology clearly, and give syntactical rules instead of philosophical argument.

We already encountered Carnap's pluralism in 1.2.1. Here, however, we are rather concerned with his pragmatic view on logic that is in the background of this pluralism. A more contemporary instance of instrumentalism is, for instance, Kouri (2016). She argues that one should understand logic as a tool which enhances our deductive capacities. In a similar vain, Allo (2017) places logic within the bigger framework of constructionism: Logic is a technological artifact that is used in our processing of information.

I think it is quite easy to see that the first version of nihilism does only make sense within a substantialist framework: Russell's logical nihilism is surely concerned with the question what the correct logic is and not which logics are useful for what purpose. She would not deny that there are formal systems which are usefull tools, but the nihilism which she proposes is directed towards a different issue. Further, it is hard to determine any useful or fruitful application of a logic with an empty consequence relation. Other non-degenerate logics might have their place and justification but they do not represent *the* laws of logic. The case of Cotnoir's formalizability nihilism is a bit different: As I stated in the conclusion of the second chapter, Cotnoir does in fact make some good points with regards to the question whether logic is a good tool to create a complete one-to-one model of natural language consequence. However, this is only one application of logic and, thus, only a very limited negative result which can hardly be labelled nihilism. Hence, I conclude, in order to get Cotnoir's intentions right one should assess him within the substantalist stance on logic: He thinks that the canonical application of logic is to capture "natural language inference". Since, he deems this impossible he takes a nihilistic stance on logic. To conclude, one could also turn these two points around: instrumentalist stances on logic seem to have quite some resources against either version of nihilism, i.e. they might accept that the nihilist get something right, but their overall picture of logic is misguided. Carnap, for instance, might have agreed that there is no non-degenerate logic which captures the proper notion of logical consequence. However, he would disregard such a question as an external one (compare Carnap 1950). The questions which are actually interesting and important are the internal one: Given one language and some aim what is the best consequence relation to adopt.

3.2 What is the Object of Logic?

As we have seen, both versions of nihilism only makes sense if they are formulated within a substantialist stance on logic. Thus, we will now turn to the different views one could have within the substantialist framework. One may wonder, after all, what this peculiar substance, which logic is supposed to capture, consist in? Here, there are several options one could look into and the following ideas are far from exhaustive. Rather, they highlight two important presuppositions and differences of the two versions of nihilism: First, as we have seen in the beginning of the first chapter, Russell has to rely upon a certain version of logical consequence. Second, Cotnoir has to assign an important role to natural language.

3.2.1 Law-first vs. Truth-first

One distinction which we have already encountered in the outline of Russell's nihilism is the distinction between a law-first and a truth-first view on logic. There I looked into the formal tools of proof-theory and model-theory and provided some explanations what these formal methods might be about. Here, however, I am going to talk directly about the issue of what logical consequence is taken to be.[4] As the name already indicates the disagreement between these two views is about the primacy of their respective concept: Both parties agree that whatever logic is about should tell us something about the laws we ought to reason with and it should tell us about the form-induced, necessary preservation of truth. I will start this section with a short sketch of both views and then add some observations on their relation to the two versions of nihilism.

According to the law-first approach the aim of logic is to unveil the laws of correct reasoning, or correct methods of proof. Rather than being about truth and necessity, the primary object of logic is the notion of a correct inference and, thereby, the notion of proof.- Truth-first approaches, on the other hand, maintain that logic describes the laws of truth. Frege (1918, p. 58) puts it as follows:

> As the word "beautiful" in aesthetics and "good" in ethics, so does "truth" point logic the way. Indeed, all sciences have truth as an aim; logic, however, deals with it in a very different manner. It [= logic] relates to truth just about as physics relates to mass or to heat. Discovering truths is the aim of all sciences: Logic is assigned to recognize the laws of being true. (*my translation*)

Thus, logic is concerned with entailment relations between truth-apt objects. Thereby, the laws of correct reasoning are only of secondary interest: The correct way of reasoning is the one which captures the logical truth-preservation relation between premises and conclusions. Hence, there is a direct route from the idea that logical consequence is necessary form-induced truth-preservation to the idea that it provides the laws of correct

[4] As I also mentioned in 1.1.1. one might argue that we should just disambiguate two notions of logical consequence, one along the line of laws and one along the line of truth. However, for the purpose of clarifying nihilism this should not matter.

reasoning. However, there are similar moves that one can make within a law-first approach to capture the link to the notion of truth which was so significant for Frege. For instance, one might argue that the meaning of the logical connectives are given by their inferential use: Their meaning consists in the inference rules which govern their use in proofs. Thus, given such an *inferentialist* picture of their meaning one might argue that a valid inference cannot lead from truth to falsity since it only explicates the meanings at hand (compare Brandom 2000).

Where do the approaches of Russell and Cotnoir fit in with regards to this distinction? Let me start off with Cotnoir's formalizability nihilism: the outlines of his position seem to suggest a quite persuasive stance on the current matter. All that is needed for his nihilism to work is that natural language consequence is the medium between our theory of logic and whatever we are trying to capture.[5] Thus, the outline of formalizability nihilism is independent of this debate about the object of logic. Russell's logical nihilism, on the other hand, seems to be rooted within the truth-first tradition: As I already mentioned in the introductory sections to the first chapter, she presupposes that the semantic definition of logical consequence gets it right since she makes essential use of counterexamples. Law-first approaches, however, do not operate with counterexamples: In their view, it does not make sense to say that conjunction-elimination fails in some contexts. Rather, if the connective under investigation is not governed by the rules on conjunction-introduction and -elimination it is not a conjunction after all.[6]

3.2.2 Language and Thought

A second, classical question with regards to logic is whether it describes our language or some non-lingual structure. This debate is even more convoluted than the last one. Thus, the following disclaimer is in place: I will restrict my remarks on some observations of Russell's and Cotnoir's nihilisms.

Cotnoir sees the aim of logic to consist in a correct representation of natural language consequence. His arguments for nihilism take it as central that natural language is the object of our logical inquiries. Thus, his position is strictly tied to the idea that logic is concerned with language. The situation with Russell's nihilism, on the other hand, is a bit different. Her position, i.e. the thesis that there are no laws of logic, could be seen as being about language as well as non-lingual structures. However, her argument for nihilism, as we have seen in the first chapter, does presuppose a certain stance on the relata of logical consequence: logical consequence is not a relation between propositions,

[5] This idea seems to be the default of most contributors to this debate. But one can also find explicit statements of this thesis: Priest (2014a), for example, calls natural language the canonical application of logic and Payette and Wyatt (2018) maintain that the whole debate of logical pluralism makes only sense if we take it to be about natural language.

[6] Note that Russell maintains that she does not change the meaning of the connectives since the truth-clauses are (supposed) to stay classical.

but between context-sensitive sentences. Thus, on can still make out a clear connection between logic and language in her position. Although this is not as tight as in Cotnoir's case.

Thus, we can draw a similar conclusion as in the case of the instrumentalism and substantialism debate: Both versions of nihilism presuppose a view on the object of logic which some philosophers might object to: If we regard the goal of logic as unveiling the relations between propositions there are some points of resistance against both versions of nihilism. First, Russell's argument looses one of its most central premises. Second, Cotnoir's nihilism seems to be off topic, i.e. it would not be a nihilism about "logic" anymore.

3.3 And How can we know of it?

Now, given that we have fixed what the aim of our applied logic should be we can still ask how we can know whether our logic is, in fact, correct. This is an issue in the epistemology and methodology of logic. However, the success of a stance on this issue might depend on the answer one has given to the last question. After all, if we change the object under investigation the corresponding epistemology might have to change as well. I will start this section by pointing out how one could draw skeptical conclusions from the preceding discussions of logical and formalizability nihilism. Second, I sketch the landscape of positive replies to this skepticism.

3.3.1 From Nihilism to Skepticism

So far we have pointed out that Russell and Cotnoir have slightly different presuppositions when it comes to what they take logic to be. But what about the epistemology of logic? Here, *prima facie* both authors do not have to take clear stances. After all, logical as well as formalizability nihilism are concerned with the very possibility of logic itself and not with our knowledge of it. However, in the following, I will draw some skeptical ideas from the discussion of both versions of nihilism above: Even though the arguments of both Cotnoir and Russell did not establish their respective version of nihilism we might have to accept some skeptical lessons from their ideas.

We concluded the section on Russell's nihilism with an open issue: I have argued against Russell's motivation for universalism and, especially, her alleged counterexamples to our most basic laws. However, it is not clear at all where the boundary between models that pick out cases and models that do not lays. So, even if the possibility of logical nihilism by means of Russell's monstrous models is blocked there is still an issue of *logical skepticism*, i.e. if we are not able to determine the set of cases then we cannot know the extension of our logical consequence relation. The only thing we might have established is that conjunction elimination and the law of identity should be covered by this relation. Thus, Russell's argument for nihilism can be rather seen as an expression of this form of skepticism: If we want to have more laws of logic then we have to find some criterion by which we can determine the set of cases.[7] One could take some of the arguments presented by Etchemendy (1990) as a predecessor of this form of skepticism: the semantic definition of logical consequence does not tell us by itself what the laws of logic are since, as Tarski already realized, it does not fix the set of cases. Thus, the semantic definition does not provide us with firm grounds to decide disputes about which logical laws are valid in full generality. To conclude, I would like to point out that this general lesson is in line with other works by Russell (2014; 2015; 2018) which are, rather, concerned with the epistemology of logic. This open end of our discussion of nihilism can be seen as one of the central question of her writings in this area.

Cotnoir poses his formalizability nihilism as an answer to certain troubles of the philosophy of language. As he puts it, there are a lot of troubles which we run into when trying to give a formal account of natural language consequence. Since these problems are so overwhelming there are abductive grounds to accept formalizability nihilism as a unified solution to all of these problems. I think the last chapter has shown in how far this idea is misguided: Cotnoir's diagnosis that it is our attempt of formalization that produces these problems gets it wrong. However, just as in the case of Russell's nihilism one can easily make a transition towards seeing Cotnoir's original motivation as a basis for skepticism with regards to logic: the vast mass of problems which we run into in our logical inquiries makes a positive outlook quite implausible. My argumentation of the second chapter has merely provided grounds to believe that this is not an issue of formalization. Cotnoir's negative meta-induction, however, stands as it is: If we share his negative outlook on the debates on the semantic paradoxes, universal quantification or vagueness then we should also accepts his pessimism. Thus, one could say that even though Cotnoir's and Russell's nihilisms are quite different, they are still in alignment with respect to the skeptical upshots which I have made out in their position: Our attempts to give an account of logical consequence have failed since no satisfying answer to the different paradoxes is at hand.

Last, it makes sense to point towards the difference between these two skeptical lines of thought: Russell points towards the general inconclusive reasons we have when we are only considering the semantic definition of logical consequence. Cotnoir, on the other hand, takes other reasons into consideration: The "correct logic" is the one that does not fall into the numerous paradoxes and problems which he draws attention to. These consideration, however, lead to a negative result: There does not seem to be a satisfying answer to these issues. Further, one could bring the two versions of skepticism into a

[7] One could take this as the general lesson which Russell intends with her "nihilism". After all, she says that even if her monstrous models are too outrageous to be accepted some minimal logic that would survive the collapse arguments against logical pluralism would not be satisfying on its own.

common story: If we try to give further arguments to determine which logic is the correct one, i.e. answering the russellian skepticism, we will probably draw attention to the paradoxes and problems to discern different logics from each other. Then, however, we should at some point be facing cotnoirian skepticism since no account presents a satisfying answer to the paradoxes.

3.3.2 A Pathway of Science?

These skeptical ideas, however, are commonly rejected by philosophers of logic in favor of a more positive stance on the prospects of logic. In the following I want to distinguish two broad approaches to the epistemology of logic which might differ in respect to the answers they give to the skeptical challenges presented in the last paragraph: An *exceptionalist* and an *anti-exceptionalist* epistemology. According to the exceptionalist the science of logic is quite different from our usual scientific conducts, according to the anti-exceptionalist there is no such categorical distinction.[8] Again, the following characterization of the landscape of the debate in the epistemology of logic does not claim to be exhaustive. Rather, I want to sketch two significantly different positions and show how they handle the two lines of skepticism I have extracted from Russell's and Cotnoir's nihilisms respectively.

Let me start with the classical, exceptionalist picture of logic and, thus, point out how their anti-skeptical resources look. A central claim of the exceptionalist epistemology of logic is that logic, just like mathematics, is supposed to be *apriori*: While other sciences rely on empirical methods and information gathered by experience the grounds to believe in a law of logic are quite different.[9] There is – of course – a vast plurality of different views on what aprioricity consists in. However, for the purpose of this thesis there are only a view disputed properties which are interesting to us: fallibility and transparency. The classical stance on these issue seems to be what Wright (2018, p. 428) has labeled *logical euclideanism*: what correct reasoning consists in is obvious to us and our justification for believing in the correctness of logical laws amounts to certainty. However, this might not be the only view that an exceptionalist about logic can have on this issue: One might say that, just as in philosophy, we are concerned with a domain which is only knowable apriori, but which is still in need of close scrutiny and intellectual work in order to make the object under investigation apparent. Thus, our theories of logic might be fallible.

Anti-exceptionalism finds its routes in Quine (1953) who famously argued for the continuity of natural science and philosophy or logic. However, this general stance has

[8]The terms exceptionalist and anti-exceptionalist stem from Williamson (2007) who labels stances on philosophy along these lines. Note that there are different ways to be an exceptionalist about logic than just epistemological concerns: One can ask just as well whether the object of logic is completely different than the one in natural sciences: E.g. logic is supposed to be normative.

[9]I have called this view "classical" because it resembles the traditional view that philosophers had throughout history on our grasp of logic. Explicit endorsements and defences of this thesis can be found in Peacocke (1987) and Boghossian (2000).

received a renaissance in recent years.[10] Even though the actual practice of natural science and logic might be quite different, our considerations on which logic is the correct one should be sensitive to the results of natural science. As a consequence, theories of logic are not apriori and – just like theories in natural science – subject to change and revision. Thus, there cannot be an anti-exceptionalism along the euclidean lines I mentioned above: The grounds we have for deciding what logic is correct are not immediately transparent to us.

How do the two varieties of logical skepticism fit into this picture? The two versions were the following: First, a skepticism towards our criteria for determining the scope of logical consequence and, second, a pessimistic stance on the outlook of solving problems and paradoxes which are naturally linked to logic. I think it is a general observation that the existence of skepticism in either form speaks against the idea which I have labelled above as euclideanism: If the extent of logical consequence is transparent why should we ever become skeptical about the possibility of a criterion to determine the set of cases? Similarly, if euclideanism is true how are we to account for the troubles and paradoxes which lead Cotnoir to his negative meta-induction? I think it is quite easy to see that euclideanism is a bad account of how we are actually investigating logical consequence and the possibility of skepticism makes this divergence very apparent: If we would have such a criterion there should not be any skeptics of either variety around.[11] Thus, in order to make sense of skepticism in either form we should either follow a non-euclidean exceptionalism or some version of anti-exceptionalism: logical consequence is not as transparent as philosophers in the past thought it to be.

If euclideanism is out of the picture, however, I think the two positive epistemologies at hand give quite similar answers to the two versions of skepticism we encountered: they do not share Cotnoir's pessimistic metainduction about the prospects of the philosophy of language in general and, thereby, try to provide some criterion as to which logic is the correct one. As I mentioned earlier, the natural connection between the two skeptical ideas is that we take russellian skepticism as the product of trying to determine the set of cases by means of the semantic definition of logical consequence alone, while cotnoirian skepticism is the result of trying to come up with other grounds to determine the extension of logical consequence. Hence, if both positive epistemologies present a view on the prospect of this extended set of grounds, which differs from Cotnoir's pessimistic stance, we have answered to both versions of skepticism.[12]

[10]More recent defenders of this position are Maddy (2002), Priest (2014; 2016), Russell (2014; 2015), Hjortland (2017) and Williamson (2017). As Hjortland notes though there are some important differences between these positions. However, I will not go into this issue here.

[11]This observation can be reasonably extended to most discussions in the philosophy of logic: If it would be completely transparent to us what correct reasoning consists in there should not be any substantial dispute about which logic is the correct one.

[12]Of course, anti-exceptionalism will take reasons into account which will go beyond the scope of apriori reasoning and, thereby, differ from any version of exceptionalism. Whether this provides a better prospect

3.4 Concluding Remarks

As I mentioned in the introduction to this chapter this part of the thesis differs from the other two in a quite significant way: Instead of arguing in depth whether the arguments which Russell and Cotnoir propose are sound, I restricted myself to stating some observations on the general relationship between the two versions of nihilism and their relation to other issues in the philosophy of logic. These observations are the following: First, nihilism should not be confused with instrumentalism. In fact, instrumentalism could be seen as an anti-dote to the nihilist tendencies. Second, when it comes to the object of our logical investigations there are some slight differences between Russell and Cotnoir. Third, even though the arguments for nihilism might not go through, the reasons which Russell and Cotnoir put forward in favour of them might be seen as motivating some form of skepticism respectively.

in answering to the skeptical challenge can be left as an open question.

Conclusion

Let me sum up the results of this discussion: First, I have shown in how far Russell's argument for logical nihilism is misleading. I think the two main points one can take away from this discussion is that (1) universalism is not motivated by the failure of pluralism on its own. Rather, its main motivation stems from the fact that we accept more and more kinds of cases. (2) Russell's monstrous models are quite different from other non-classical models. Their peculiar property of shifting the context of evaluation while assessing the extension of logical consequence makes them incapable of picking out cases. Second, the discussion of Cotnoir's family of attacks has highlighted several results: (1) The plurality of formal systems which Cotonir appeals to in his argumentation is no ground for his formalizability nihilism. (2) Semantic paradoxes, paradoxes of universal quantification and the Sorites paradox should not be seen as products of our formal machinery: They are independent of our attempts to formalize natural language consequence. (3) The problem of higher-order vagueness which Cotnoir appeals to does only establish his nihilstic thesis if we conflate two ends that are usually ascribed to formal systems: First, as aiming to capture natural language consequence and, second, as precisifying natural language. Finally, the last chapter conjectured that, even though the arguments for either type of nihilism are blocked, one may see them as an expression of skepticism towards the philosopher's appeal to logic.

The preceding discussion had to leave many questions unanswered and many important topics were reduced to mere side-notes: The success of logical pluralism in the first chapter, the answers to the paradoxes in the second chapter, as well as the complete landscape of philosophical approaches to logic in the third chapter are still very much open issues and this thesis has done little towards a proper answer to these tasks. However, I want to highlight some topics which were quite peculiar for the development of this thesis and which I see as concrete question for future research: First, in my criticism of Russell's argument for nihilism I drew upon a distinction which I called static and dynamic concepts of logical consequence. I pointed out that people working on these topics usually have one intended conception in mind and, thus, one should keep them apart. The ill advised step in Russell's reasoning is just to conflate the two. However, there has been only little research conducted on how these two notions exactly relate and what the philosophical bases for such a distinction could be. Second, in my discussion of formalizability nihilism

I mentioned Glanzberg's thesis that there is no relation of logical consequence determined by natural language semantics. As I said in the second chapter, this idea is different from Cotnoir's nihilism, but may be legitimatly called a version of "nihilism" as well. At least, Glanzberg points out an interesting negative result about the limits of natural language semantics. Thus, even though I was not able to tackle his reasoning in the scope of this thesis it would be a valuable addition to have an in-depth discussion of his paper that mirrors the discussions I have provided in the first two chapters.

Bibliography

Allo, Patrick (2017). "A Constructionist Philosophy of Logic". In: *Minds & Machines* 27, pp. 545–564.

Azzouni, Jody (2003). "The Strenghtened Liar, the Expressive Strength of Natural Language, and Regimentation". In: *The Philosophical Forum* 334, pp. 329–350.

— (2007). "The Inconsistency of Natural Languages: How we live with it". In: *Inquiry* 50, pp. 590–605.

— (2013). "Inconsistency in Natural Language". In: *Synthese* 190, pp. 3175–3184.

Barrio, Eduardo et al. (2018). "Substructural Logics, Pluralism and Collapse". In: *Synthese*. DOI: 10.1007/s11229-018-01963-3.

Beall, JC (2003). "Introduction". In: *Liars and Heaps. New Essays on Paradox*. Ed. by JC Beall. Oxford/New York: Oxford University Press, pp. 1–5.

— (2015). "Trivializing Sentences and the Promise of Semantic Completeness". In: *Analysis* 75, pp. 573–584.

Beall, JC et al. (2017). "Liar Paradox". In: *Stanford Encyclopedia of Philosophy* (Fall2017 Edition). Ed. by Ed Zalta. URL: https://plato.stanford.edu/archives/fall2017/entries/liar-paradox/.

— (2019). "Logical Consequence". In: *Stanford Encyclopedia of Philosophy* (Spring2019 Edition). Ed. by Ed Zalta. URL: https://plato.stanford.edu/archives/win201i/entries/logical-consequence/.

Beall, JC and Julien Murzi (2013). "Two Flavours of Curry Paradox". In: *Journal of Philosophy* 110, pp. 143–165.

Beall, JC and Greg Restall (2000). "Logical Pluralism". In: *Australasian Journal of Philosophy* 78, pp. 475–493.

— (2006). *Logical Pluralism*. Oxford: Clarendon Press.

Beall, JC and Lionel Shapiro (2018). "Curry's Paradox". In: *Stanford Encyclopedia of Philosophy* (Summer2018 Edition). Ed. by Ed Zalta. URL: https://plato.stanford.edu/archives/sum2018/entries/curry-paradox/.

Blake-Turner, Christopher and Gillian Russell (2018). "Logical Pluralism without the Normativity". In: *Synthese*. DOI: 10.1007/s11229-018-01939-3.

Bobzien, Susanne (2010). "Higher-order Vagueness, Radical Unclarity, and Absolute Agnosticism". In: *Philosophers' Imprint* 10.10, pp. 1–30.

Boghossian, Paul (2000). “Knowledge of Logic”. In: *New Essays on the Apriori*. Ed. by Paul Boghossian. Ed. by Christopher Peacocke. Oxford/New York: Oxford University Press, pp. 229–254.

Brandom, Robert (2000). *Articulating Reasons. An Introduction to Inferentialism*. Cambridge(Mass.)/London: Harvard University Press.

Bueno, Otavio (2007). “Troubles with Trivialism”. In: *Inquiry* 50, pp. 655–667.

Bueno, Otavio and Scott Shalkowski (2009). “Modalism and Logical Pluralism”. In: *Mind* 118, pp. 295–321.

Caret, Colin (2017). “The Collapse of Logical Pluralism has been Greatly Exaggerated”. In: *Erkenntnis* 82, pp. 739–760.

— (2019). “Why Logical Pluralism?” In: *Synthese*. DOI: 10.1007/s11229-019-02132-w.

Carnap, Rudolf (1937). *The Logical Syntax of Language*. London: Kegan Paul.

— (1950). “Empiricism, Semantics, and Ontology”. In: *Revue Internationale de Philosophie* 4, pp. 20–40.

Cartwright, Richard (1994). “Speaking of Everything”. In: *Nous* 28, pp. 1–20.

Chihara, Charles (1979). “The Semantic Paradoxes: A Diagnostic Investigation”. In: *The Philosophical Review* 88, pp. 590–618.

Commandeur, Leon (2018). “Logical Functionalism”. Master Thesis. Universiteit van Amsterdam.

Cook, Roy (2002). “Vagueness and Mathematical Precision”. In: *Mind* 111, pp. 225–247.

— (2010). “Let a Thousand Flowers Bloom: A Tour of Logical Pluralism”. In: *Philosophy Compass* 5, pp. 492–504.

Cotnoir, Aaron (2018). “Logical Nihilism”. In: *Pluralisms in Truth and Logic*. Ed. by Jeremy Wyatt (et al.) Basingstoke: Palgrave Macmillan, pp. 301–329.

Eklund, Matti (2002). “Inconsistent Languages”. In: *Philosophy and Phenomenological Research* 64, pp. 251–275.

Estrada-Gonzalez, Luis (2011). “On the Meaning of Connectives (Apropos of a Non-Necessitarianist Challenge)”. In: *Logica Universalis* 5, pp. 114–126.

— (2012). “Models of Possibilism and Trivialism”. In: *Logic and Logical Philosophy* 21, pp. 175–205.

Etchemendy, John (1990). *The Concept of Logical Consequence*. Cambridge(Mass.): Harvard University Press.

Field, Hartry (2008). *Saving Truth from Paradox*. Oxford/New York: Oxford University Press.

— (2009). “Pluralism in Logic”. In: *The Review of Symbolic Logic* 2, pp. 342–359.

Florio, Salvatore (2014). “Unrestricted Quantification”. In: *Philosophy Compass* 9, pp. 441–454.

Franks, Curtis (2014). “Logical Nihilism”. In: *The Metaphysics of Logic*. Ed. by Penelope Rush. Cambridge: Cambridge University Press, pp. 109–127.

Frege, Gottlob (1879). *Begriffsschrift, eine der arithmetischen nachgebildete Formalsprache des reinen Denkens*. Halle an der Saale: Louis Nebert.

— (1882). "Über die wissenschaftliche Berechtigung einer Begriffsschrift". In: *Zeitschrift für Philosophie und philosophische Kritik* 81, pp. 48–56.

— (1918). "Der Gedanke. Eine logische Untersuchung". In: *Beiträge zur Philosophie des deutschen Idealismus* 2, pp. 58–77.

Glanzberg, Michael (2015). "Logical Consequence and Natural Language". In: *Foundations of Logical Consequence*. Ed. by Colin Caret. Ed. by Ole Hjortland. Oxford/New York: Oxford University Press, pp. 71–120.

Grim, Patrick (1991). *The Incomplete Universe. Totality, Knowledge, and Truth*. Cambridge(Mass.)/London: The MIT Press.

Groenendijk, J.A.G. and M.J.B. Stockhof (1991). "Dynamic Predicate Logic". In: *Linguistics and Philosophy* 14.1, pp. 39–100.

Haack, Susan (1978). *Philosophy of Logics*. Cambridge: Cambridge University Press.

Halbach, Volker (2018). "The Substitutional Analysis of Logical Consequence". In: *Nous*. DOI: `doi:10.1111/nous.12256`.

Hjortland, Ole (2017). "Anti-exceptionalism about Logic". In: *Philosophical Studies* 174, pp. 631–658.

Hyde, Dominic (1994). "Why Higher-Order Vagueness is a Pseudo-Problem". In: *Mind* 103, pp. 35–41.

Hyde, Dominic and Diana Raffman (2018). "Sorites Paradox". In: *Stanford Encyclopedia of Philosophy* (Summer2018 Edition). Ed. by Ed Zalta. URL: `https://plato.stanford.edu/archives/sum2018/entries/sorites-paradox/`.

Kaplan, David (1979). "On the Logic of Demonstratives". In: *Journal of Philosophical Logic* 8, pp. 81–98.

— (1989). "Demonstratives: An Essay on the Semantics, Logic, Metaphysics, and Epistemology of Demonstratives and Other Indexicals". In: *Themes from Kaplan*. Ed. by Joseph Almog (et al.) Oxford/New York: Oxford University Press, pp. 481–566.

Keefe, Rosanna (2014). "What Logical Pluralism cannot be". In: *Synthese* 191, pp. 375–390.

Kouri, Teresa (2016). "Logical Instrumentalism". Dissertation. Ohio State University.

Kripke, Saul (1975). "Outline of a Theory of Truth". In: *The Journal of Philosophy* 72.19, pp. 690–716.

Linnebo, Øystein (2003). "Plural Quantification Exposed". In: *Nous* 37.1, pp. 71–92.

— (2006). "Sets, Properties, and Unrestricted Quantification". In: *Absolute Generality*. Ed. by Augustin Rayo. Ed. by Gabriel Uzquiano. Oxford/New York: Oxford University Press, pp. 149–178.

Linnebo, Øystein (2017). “Plural Quantification”. In: *Stanford Encyclopedia of Philosophy* (Summer2017 Edition). Ed. by Ed Zalta. URL: `https://plato.stanford.edu/archives/sum2017/entries/plural-quant/`.

Linnebo, Øystein and Augustin Rayo (2012). “Hierarchies Ontological and Ideological”. In: *Mind* 121, pp. 269–308.

Lynch, Michael (2008). “Alethic Pluralism, Logical Consequence and the Universality of Reason”. In: *Midwest Studies in Philosophy* 32, pp. 122–140.

— (2009). *Truth as One and as Many.* Oxford/New York: Oxford University Press.

Maat, Jaap (1999). “Philosophical Languages in the Seventeenth Century: Dalgarno, Wilkins, Leibniz”. Dissertation. Universiteit van Amsterdam.

MacFarlane, John (2000). “What Does it Mean to Say that Logic is Formal?” Dissertation. University of Pittsburgh.

— (2014). *Assessment Sensitivity: Relative Truth and its Applications.* Oxford/New York: Oxford University Press.

Maddy, Penelope (2002). “A Naturalistic Look at Logic”. In: *Proceedings and Addresses of the American Philosophical Association* 76.2, pp. 61–90.

Mortensen, Chris (1989). “Anything is Possible”. In: *Erkenntnis* 30, pp. 319–337.

Novaes, Catarina Dutilh (2012). *Formal Languages in Logic. A Philosophical and Cognitive Analysis.* Cambridge: Cambridge University Press.

Paseau, Alexander (2007). “Review of *Logical Pluralism*”. In: *Mind* 116, pp. 391–396.

Payette, Gillman and Nicole Wyatt (2018). “Logical Particularism”. In: *Pluralisms in Truth and Logic.* Ed. by Jeremy Wyatt (et al.) Basingstoke: Palgrave Macmillan, pp. 277–299.

— (2019). “Against Logical Generalism”. In: *Synthese.* DOI: `10.1007/s11229-018-02073-w`.

Peacocke, Christopher (1987). “Understanding the Logical Constants: A Realist’s Account”. In: *Proceedings of the British Academy* 63, pp. 153–200.

Pedersen, Nikolaj Jang Lee Linding (2014). “Pluralism x 3: Truth, Logic, Metaphysics”. In: *Erkenntnis* 79, pp. 259–277.

Peters, Stanley and Dag Westerstrahl (2006). *Quantifiers in Language and Logic.* Oxford/New York: Oxford University Press.

Priest, Graham (1984). “Semantic Closure”. In: *Studia Logica: An International Journal in Symbolic Logic* 43, pp. 117–129.

— (1995). *Beyond the Limits of Thought.* Cambridge: Cambridge University Press.

— (2006a). *Doubt Truth to be a Liar.* Oxford/New York: Oxford University Press.

— (2006b). *In Contradiction: A Study of the Transconsistent.* 2nd ed. Oxford/New York: Oxford University Press.

— (2014a). “Logical Pluralism: Another Application for Chunk and Permeate”. In: *Erkenntnis* 79, pp. 331–338.

Priest, Graham (2014b). “Revising Logic”. In: *The Metaphysics of Logic*. Ed. by Penelope Rush. Cambridge: Cambridge University Press, pp. 211–223.

— (2016a). “Logical Disputes and the Apriori”. In: *Logicue & Analyse* 236, pp. 347–366.

— (2016b). “Old Wine in (Somewhat Leaky) New Bottles: Some Comments on Beall”. In: *Australasian Journal of Logic* 13.5, pp. 89–95.

Quine, W.v.O. (1953). “Two Dogmas of Empiricism”. In: *From a Logical Point of View*. Cambridge(Mass.): Harvard University Press, pp. 20–46.

— (1970). *Philosophy of Logic*. Cambridge(Mass.)/London: Harvard University Press.

Raatikainen, Panu (2018). “Gödel’s Incompleteness Theorems”. In: *Stanford Encyclopedia of Philosophy* (Fall2018 Edition). Ed. by Ed Zalta. URL: `https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/`.

Radulescu, Alexandru (2015). “The Logic of Indexicals”. In: *Synthese* 192, pp. 1839–1860.

Rayo, Augustin and Gabriel Uzquiano (2006). “Introduction”. In: *Absolute Generality*. Ed. by Augustin Rayo. Ed. by Gabriel Uzquiano. Oxford/New York: Oxford University Press, pp. 1–19.

Read, Stephen (2006). “Monism: The One True Logic”. In: *A Logical Approach to Philosophy. Essays in Honor of Graham Solomon*. Ed. by David DeVidi. Ed. by Tim Kenyon. Dordrecht: Springer, pp. 193–209.

Restall, Greg (2002). “Carnap’s Tolerance, Meaning, and Logical Pluralism”. In: *The Journal of Philosophy* 99, pp. 426–443.

— (2014). “Pluralism and Proofs”. In: *Erkenntnis* 79, pp. 279–291.

Rumfitt, Ian (2015). *The Boundary Stones of Thought. An Essay in the Philosophy of Logic*. Oxford/New York: Oxford University Press.

Russell, Gillian (2008). “One True Logic?” In: *Journal of Philosophical Logic* 37, pp. 593–611.

— (2012). “Lessons from the Logic of Demonstratives: What Indexicality Teaches Us about Logic, and Vice Versa”. In: *New Waves in Philosophical Logic*. Ed. by Greg Restall. Ed. by Gillian Russell. Basingstoke: Palgrave Macmillan, pp. 190–216.

— (2014). “Metaphysical Analyticity and the Epistemology of Logic”. In: *Philosophical Studies* 171, pp. 161–175.

— (2015). “The Justification of the Basic Laws of Logic”. In: *Journal of Philosophical Logic* 44, pp. 793–803.

— (2017a). “Introduction to Logical Nihilism”. In: *Logic, Methodology and Philosophy of Science – Proceedings of the 15th international Congress*. Ed. by Hannes Leitgeb (et al.) London: College Publications, pp. 125–135.

— (2017b). “Logic isn’t Normative”. In: *Inquiry*. DOI: `10.1080/0020174X.2017.1372305`.

— (2018a). “Deviance and Vice: Strength as a Theoretical Virtue in the Epistemology of Logic”. In: *Philosophical and Phenomenological Research*. DOI: `10.1111/phpr.12498`.

Russell, Gillian (2018b). "Logical Nihilism: Could there be no Logic?" In: *Philosophical Issues* 28, pp. 308–324.

— (2018c). "Varieties of Logical Consequence by their Resistance to Logical Nihilism". In: *Pluralisms in Truth and Logic*. Ed. by Jeremy Wyatt (et al.) Basingstoke: Palgrave Macmillan, pp. 331–361.

— (2019). "Logical Pluralism". In: *Stanford Encyclopedia of Philosophy* (Spring 2019 Edition). Ed. by Ed Zalta. URL: `https://plato.stanford.edu/archives/spr2019/entries/logical-pluralism/`.

Sagi, Gil (2014). "Formality in Logic: From Logical Terms to Semantic Constraints". In: *Logique & Analyse* 227, pp. 259–276.

Sainsbury, RM (1997). "Concepts Without Boundaries". In: *Vagueness: A Reader*. Ed. by Rosanna Keefe. Ed. by Peter Smith. Cambridge(Mass.)/London: The MIT Press, pp. 251–264.

Schroeder-Heister, Peter (2018). "Proof-Theoretic Semantics". In: *Stanford Encyclopedia of Philosophy* (Spring 2018 Edition). Ed. by Ed Zalta. URL: `https://plato.stanford.edu/archives/spr2018/entries/proof-theoretic-semantics/`.

Shapiro, Stewart (1998). "Logical Consequence: Models and Modality". In: *The Philosophy of Mathematics Today*. Ed. by Matthias Schirn. Oxford/New York: Oxford University Press, pp. 131–156.

— (2006). *Vagueness in Context*. Oxford/New York: Oxford University Press.

— (2007). "Logical Consequence, Proof Theory, and Model Theory". In: *Oxford Handbook of Philosophy of Mathematics and Logic*. Ed. by Stewart Shapiro. Oxford/New York: Oxford University Press, pp. 651–670.

Stei, Erik (2017). "Rivalry, Normativity , and the Collapse of Logical Pluralism". In: *Inquiry*. DOI: `10.1080/0020174X.2017.1327370`.

Steinberger, Florian (2019). "Logical Pluralism and Logical Normativity". In: *Philosophers' Imprint* 19.12, pp. 1–19.

Studd, James (2019). *Everything More or Less. A Defence of Generality Relativism*. Oxford/New York: Oxford University Press.

Tarski, Alfred (1935). "Der Wahrheitsbegriff in den formalisierten Sprachen". In: *Studia Philosophica* 1, pp. 261–405.

— (1936). "Über den Begriff der logischen Folgerung". In: *Actualités scientifiques et industrielles* 394, pp. 1–11.

Terres Villalonga, Pilar (2017). "From Natural to Formal Language: A Case for Logical Pluralism". In: *Topoi*. DOI: `10.1007/s11245-017-9490-8`.

Tye, Michael (1994). "Sorites Paradoxes and the Semantics of Vagueness". In: *Philosophical Perspectives* 8, pp. 189–206.

Varzi, Achille (2002). "On Logical Relativity". In: *Philosophical Issues* 12, pp. 197–219.

Williamson, Timothy (1988). “Equivocation and Existence”. In: *Proceedings of the Aristotelian Society* 88, pp. 109–127.

— (2003). “Everything”. In: *Philosophical Perspectives* 17, pp. 415–465.

— (2006). “Absolute Identity and Absolute Generality”. In: *Absolute Generality.* Ed. by Augustin Rayo. Ed. by Gabriel Uzquiano. Oxford/New York: Oxford University Press, pp. 369–389.

— (2007). *The Philosophy of Philosophy.* Oxford: Blackwell.

— (2013). *Modal Logic as Metaphysics.* Oxford/New York: Oxford University Press.

— (2017). “Semantic Paradoxes and Abductive Methodology”. In: *Reflections on the Liar*. Ed. by Bradley Armour-Garp. Oxford/New York: Oxford University Press, pp. 325–346.

Wright, Crispin (2010). “The Illusion of Higher-Order Vagueness”. In: *Cuts and Clouds. Vagueness, Its Nature, and Its Logic*. Ed. by Richard Dietz. Ed. by Sebastiano Moruzzi. Oxford/New York: Oxford University Press, pp. 523–549.

— (2018). “Logical Non-Cognitivism”. In: *Philosophical Issues* 28, pp. 425–450.

Yu, Andy Demfree (2017). “Normativity for Alethic-Logical Pluralists”. In: *Inquiry.* DOI: 10.1080/0020174X.2017.1392893.

Zardini, Elia (2014). “Context and Consequence. An Intercontextual Substructural Logic”. In: *Synthese* 191, pp. 3473–3500.

— (2018). “Generalised Tarski’s Thesis Hits Substructure”. In: *Pluralisms in Truth and Logic*. Ed. by Jeremy Wyatt (et al.) Basingstoke: Palgrave Macmillan, pp. 237–276.