Hintikka's Thesis Revisited

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Abstract

We discuss Hintikka's Thesis [Hintikka 1973] that there exist natural language sentences which require non–linear quantification to express their logical form. Our basic assumption is that the criterion for adequacy of logical form is its compatibility with sentence truth conditions. It can be established by observing linguistic behaviour of language users. Our empirical research shows that there is a statistically significant preference to interpret Hintikka–like sentences with the *most* quantifier as having some linear (we call them conjunctional) logical forms and that there are differences between understanding Hintikka–like sentences with the *most* quantifier and these with proportional quantifiers. The former are more often understood by people as having conjunctional reading, when the latter are usually treated as branching sentences. Our conclusion is that some of the Hintikka–like sentences have logical form expressible in elementary logic, despite what Hintikka stated.

1 Introduction

1.1 Hintikka's Thesis

Jaakko Hintikka claims in [Hintikka 1973] that sentences like:

(1) Some relative of each villager and some relative of each townsman hate each other.

essentially require non–linear quantification for expressing their logical form. In particular, the logical form of sentence 1 should be written down – using branching quantifiers[1] – as follows:

(2) $$\begin{matrix} \forall x \exists y \\ \forall z \exists w \end{matrix} ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))),$$

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[1] For introduction into the theory of branching (Henkin) quantifiers see e. g. [Krynicki, M. Mostowski 1995] or [Westerståhl 1987].

where unary predicates V an T denote the set of villagers and the set of townsmen, respectively. The binary predicate symbol $R(x, y)$ denotes the relation "x and y are relatives" and $H(x, y)$ the relation "x and y hate each other". The above formula is equivalent to the following second–order sentence:

(3) $\exists f \exists g \forall x \forall z((V(x) \wedge T(z)) \Rightarrow R(x, f(x)) \wedge R(z, g(z)) \wedge H(f(x), g(z))))$

which is not equivalent to any first–order sentence (see the Barwise–Kunen Theorem in [Barwise 1979]).

Hintikka's reading of sentence 1 — given by the formula 2 — is called "the strong reading". However, sentence 1 can be assigned "weak readings", i.e. linear logical forms which are expressible in elementary logic. Let us consider the following candidates:

(4) $\forall x \exists y \forall z \exists w((V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))) \wedge$
$\wedge \forall z \exists w \forall x \exists y(V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w)))$.

(5) $\forall x \exists y \forall z \exists w((V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w)))$.

(6) $\forall x \forall z \exists y \exists w((V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w)))$.

Notice that all weak readings are implied by the strong reading, 2. Therefore, we can say that Hintikka's reading is stronger than any of the first–order readings. Formulae 4 – 6 are also ordered according to the inference relation which occurs between them. Obviously, formula 4 implies formula 5, which implies formula 6. By Hintikka Thesis we mean the following statement:

Hintikka's Thesis

Sentences like 1 have essentially non–linear logical forms. For example, sentence 1 should be interpreted by formula 2 and not by any of the formulae 4 – 6.

Because of its many philosophical and linguistic consequences Hintikka's claim has sparked lively controversy (see: [Barwise 1979], [Gabbay, Moravcsik 1974], [Guenthner, Hoepelman 1976], [Hintikka 1976], [M. Mostowski 1994], [M. Mostowski, Wojtyniak 2004], [Stenius 1976], and [Szymanik 2004]). In this article, some arguments presented in the discussion are analysed and criticized once again. Especially, the proposition to identify the logical form of sentence 1 with the first–order formula 4 is considered for the first time. Moreover, we propose to look at the Hintikka's Thesis referring to the data gained from empirical research.

Our conclusion is that sentences like 1 have logical form expressible in elementary logic, despite what Hintikka stated. In particular we claim that adequate reading of Hintikka's sentence is formula 4 and not 2. We have assumed that the criterion for adequacy of logical form is its compatibility with sentence truth conditions. It can be established by observing linguistic behaviour of language users. Therefore, we claim that truth conditions of Hintikka's sentence 1 are expressed rather by first–order formula 4 than by formula with branching quantification 2. The reading given by formula 4 will be called "the conjunctional reading" of sentence 1.

1.2 Other examples of Hintikka–like sentences

Other examples of Hintikka–like sentences were given by Jon Barwise [Barwise 1979]. These sentences seem to be more natural for our communication than Hintikka's sentence[2], but their logical form is still controversial.

(7) Most villagers and most townsmen hate each other.

(8) Exactly half of all villagers and exactly half of all townsmen hate each other.

(9) One third of villagers and half of townsmen hate each other.

1.3 Hintikka-like sentences are symmetric

Let us briefly remind Hintikka's arguments from the work [Hintikka 1973]. Hintikka observed that we have strong linguistic intuition that sentence 1 is equivalent to the sentence:

(10) Some relative of each townsman and some relative of each villager hate each other.

However, when we agree that formula 5 is an adequate logical form of sentence 1, then we have to agree analogously that adequate reading of sentence 10 is represented by the formula:

(11) $\forall z \exists w \forall x \exists y ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w)))$.

But 5 and 11 are not logically equivalent, therefore it would be counterintuitive to treat them as correct logical forms of sentences 1 or 10.

Notice that a similar argument works when we consider sentences 7–9 [Barwise 1979]. It is enough to observe that sentence 7 is intuitively equivalent to the sentence:

(12) Most townsmen and most villagers hate each other.

However, the possible linear reading of 7:

(13) $\mathsf{MOST}\ x\ (V(x), \mathsf{MOST}\ y\ (T(y), H(x,y)))$.

is not equivalent to an analogous reading of 12. Hence, the logical form of sentences like 7 is not given by formula 13.

However, we can not conclude Hintikka's Thesis so easily, because we should first consider the remaining first–order candidates, formulae 4 and 6. Hintikka does not consider these formulae, at all. And other authors focus only on formula 6.

[2] Notice that to formulate these examples we use quantifiers not definable in elementary logic: *Most*, *exactly half*, *one third*.

Exactly the same possibility occurs when thinking about sentences 7–9. Instead of branching logical form 14 (equivalent to formula 15) we can consider conjunctional logical form, formula 16. This is an alternative for formula 13.

(14) $$\begin{matrix} \mathsf{MOST}\ x : V(x) \\ \mathsf{MOST}\ y : T(y) \end{matrix}\ H(x, y).$$

(15) $$\exists A \exists B[\mathsf{MOST} x(V(x), A(x)) \wedge \mathsf{MOST} y(T(y), B(y)) \wedge \\ \wedge \forall x \forall y(A(x) \wedge B(y) \Rightarrow H(x, y))].$$

(16) $$\mathsf{MOST}\ x\ (V(x), \mathsf{MOST}\ y\ (T(y), H(x, y))) \wedge \\ \wedge\ \mathsf{MOST}\ y\ (T(y), \mathsf{MOST}\ x\ (V(x), H(y, x))).$$

2 Referential meaning

2.1 Barwise's test

In his paper on Hintikka's Thesis Barwise gives two arguments for the statement that the proper logical form of sentence 1 is the elementary formula 6 [Barwise 1979]. One of his arguments refers to the referential meaning of the sentence, i.e. the method of establishing its logical–value in a given situation[3]. Barwise's idea on how to choose between proposed readings is to ask native English speakers about their interpretations of sentence 1 in given situations. Barwise shows a picture (see 1) and asks whether some dot in each hut and some star in each house are not connected by a line. In spite of making no systematic research, Barwise claims that:

> In our experience, there is almost universal agreement that some dot in each hut and some star in each house are not connected by a line. (...) The reader who agrees with this is rejecting Hintikka's claim for a branching reading of 1 [Barwise 1979].

2.2 Our research

2.2.1 Method

Our research was inspired by Barwise's claim. The research was supposed to show whether people tend to understand Hintikka–like sentences in a strong or conjunctional way. For example, whether they understand sentence 7 as 14 or 16. In order to do this, a short test was constructed. It consists of five tasks. Each of these is meant to check how people understand one concrete sentence. In every task subjects were shown one sentence together with two pictures. These pictures represented appropriate models. Subjects were asked to decide whether the sentence is

[3]The second argument explores the notion of negation normality, proposed by Barwise [Barwise 1979]. For short a discussion of this argument and its philosophical assumptions see also [M. Mostowski, Szymanik 2005]. For us it is important to notice that the argument from negation normality is put forward by Barwise for the thesis that sentences 1, 7 – 9 have logical forms expressible in elementary logic.

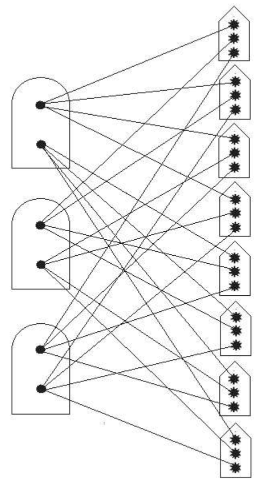

Figure 1: Barwise's picture

true or false in each of the given pictures. In one of them ($Picture_{br}$), both — the branching and conjunctional readings were true, and in the other ($Picture_{con}$) only the conjunctional reading was true. Let us present one of these tasks. (The whole test can be found in Appendix Part A). In this example, the left picture is $Picture_{con}$ and the right — $Picture_{br}$.

(17) Most circles and most squares are connected by a line.

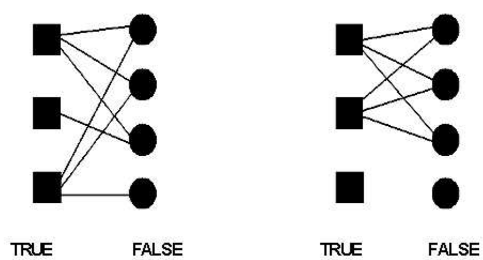

Figure 2: $Picture_{con}$ and $Picture_{br}$

We will write $Picture_{br} = 1$ or $Picture_{con} = 1$ to express that a subject decided that the sentence is true in the picture presenting branching or conjunctional reading, respectively. Analogously $Picture_{br} = 0$ or $Picture_{con} = 0$ signifies the decision that the sentence is false in branching or conjunctional pictures.

There were four possible answers to every task.

If the decision was $Picture_{con} = 1$ and $Picture_{br} = 1$, then the subject was interpreted as assigning conjunctional reading to a given sentence.

If the decision was $Picture_{con} = 0$ and $Picture_{br} = 1$, then we accept that the subject preferred branching reading.

The other two cases are more complicated. It happened that a subject answered according to the scheme: $Picture_{con} = 1$ and $Picture_{br} = 0$. We can try to think of it as we think of the situation $Picture_{con} = 0$ and $Picture_{br} = 1$ and say that this situation refers to the conjunctional reading. However, these two situations are not similar. The answer $Picture_{con} = 1$ suggests that the subject interprets the sentence in the conjunctional way, but if he did it consequently he should answer also $Picture_{br} = 1$, because conjunctional reading is also true in $Picture_{br}$. This is why the answer $Picture_{con} = 1$ and $Picture_{br} = 0$ has no "logical" sense. But it can be explained in a reasonable way. First of all, people may not recognize the inference relation between the branching and conjunctional readings. The conjunctional reading is simply easy enough to be applied. Secondly, people answering positively to only one of the questions (easier one), could assume that the other must be false. That can follow from the usual research scheme: if one is true the other is false. Such answers were interpreted as "preferring–conjunctional reading".

The last possibility is that the answers are: $Picture_{br} = 0$ and $Picture_{con} = 0$. What do these answers mean? One possible meaning is that the subject understood a given sentence in a third mysterious way, which was not predicted by the theoretical part of the research. For example, they could think that sentence 17 is true in the picture in which there are lines connecting circles with circles and squares with squares. There were very few such answers.

2.2.2 Subjects

We asked 43 subjects, 19 of them were first–year students taking the elementary course of logic. Remaining group were older students engaged in advanced logic course. Before the test, they were given instructions and some examples to explain the idea of "the sentence being true (false) in the picture (in the model)". There were no time limits, and it practically took the subjects about 20 minutes to solve the test.

2.2.3 Results

The subjects were divided into two groups according to their answers. The first group was the group of subjects who prefer branching reading, i.e. those who in most tasks decided $Picture_{con} = 0$ and $Picture_{br} = 1$. The second was the group of subjects who prefer conjunctional reading, that is in most of tasks chose $Picture_{con} = 1$ and $Picture_{br} = 1$ or $Picture_{con} = 1$ and $Picture_{br} = 0$. There were 11 people with branching preference and 15 with conjunctional one. The rest of the subjects performed an equal number of branching and conjunctional answers in the test (this group is called "balanced"). The difference between conjunctional and branching group is not statistically significant ($p < 0,50, df = 1, \chi^2 = 0,61$). Therefore, our research does not show that there is a general preference to interpret Hintikka-like sentences as having conjunctional logical forms (see Table 1 and Appendix Part C).

	branching	conjunctional	balanced
number of subjects of given preference	11	15	17

Table 1: Preference results

The question arises whether or not assigning the reading to each sentence depends on quantifiers used to formulate it. There are such differences which are an important result of the research. Sentences with *most* quantifier, according to most subjects, have conjunctional logical form. These sentences are:

(18) Most circles and most squares are connected by a line. (task 1)

(19) Most circles and most circuits are connected by a line. (task 3)

(20) Most squares and most circles are connected by a line. (task 6)

For the first task there were: 22 conjunctional and 4 preferring–conjunctional answers and 10 branching answers ($p < 0,05, df = 1, \chi^2 = 7,11$). For the third task there were: 27 conjunctional and 3 preferring–conjunctional answers and 6 branching answers($p < 0,05, df = 1, \chi^2 = 30$). For the sixth task there were: 22 conjunctional and 5 preferring–conjunctional and 10 branching answers ($p < 0,05, df = 1, \chi^2 = 7,81$).

Sentences with proportional quantifiers (*two third* or *half*) were often understood by subjects in the branching way. These sentences were:

(21) Two thirds of circles and half of the circuits are connected by a line. (task 2)

(22) Half of the circles and three fourths of the squares are connected by a line. (task 4)

(23) Two thirds of the squares and three fourths of the circles are connected by a line. (task 5)

For the second task there were: 29 branching answers and 7 conjunctional ($p < 0,05, df = 1, \chi^2 = 13,44$). For the fourth task there were: 28 branching answers and 8 conjunctional answers ($p < 0,05, df = 1, \chi^2 = 11,11$). For the fifth task there were: 25 branching answers and 11 conjunctional and 1 preferring conjunctional answer ($p < 0,05, df = 1, \chi^2 = 4,56$).

The table of results for each task is presented below (see also Appendix Part B):

These results allow us to state that there are statistically significant differences between Hintikka–like sentences with *most* quantifier and these with proportional quantifiers. The former are more often understand by people as having conjunctional reading, while the latter are usually treated as branching sentences.

	task 1	task 2	task 3	task 4	task 5	task 6
branching	10	29	6	28	25	10
linear and preferring-linear	26	7	30	8	12	27

Table 2: Results for tasks

2.3 Discussion

Marcin Mostowski and Dominika Wojtyniak claim that native speakers' inclination toward first–order reading of Hintikka's sentence cannot be accepted as an argument against Hintikka's Thesis [M. Mostowski, Wojtyniak 2004]. The authors prove that the problem of recognizing the truth–value of formula 2 in finite models is an $NPTIME$-complete problem (see: Theorem 2.1 in [M. Mostowski, Wojtyniak 2004]). Assuming the Church Thesis in its psychological version, Edmond's Thesis [Edmonds 1965], and $P \neq NP$, they claim that our minds are not equipped with any mechanisms of recognizing NP-complete problems[4]. In other words, we cannot perform an algorithm for checking the truth–value of formula 2 in the presented diagrams by means of our "hardware" in a single step. The authors conclude that this observation refutes arguments from native speakers' intuition.

It can be shown that formula 14 also defines an NP–complete class of finite models [Sevenster, manuscript]. Moreover, model–checking for all branching proportional formulae, of the form 24, is an NP–complete problem [M.Mostowski, Szymanik, manuscript].

(24) $$\begin{matrix} p \text{ of all } x : V(x) \\ q \text{ of all } y : T(y) \end{matrix} \; H(x, y), \text{ where } 0 < p, q < 1 \text{ are rational.}$$

Therefore, Mostowski and Wojtyniak should consequently assert that also native speakers' inclination toward first–order reading of the sentences 7 says nothing about the logical form of this sentence.

We cannot agree with such a conclusion. The presented arguments play in favour of thesis (under stated assumptions) that our minds are not equipped in any mechanism, allowing us to recognise the truth–value of the strong readings in finite models. If it was true that model-checking for the strong reading of 1 and 7–9 is beyond human mind ability, then it would mean that these sentences must be understood and interpreted by speakers as first–order sentences. Although, as our results show sentences like 8–9 are mostly understand as branching sentences. And facts about the computational complexity of these sentences seem do not explain linguistic behaviour of our subjects, because they interpret some of the NP–complete sentences in linear way and other in a branching way.

[4]Influence of the computational complexity of some natural language construction on its understanding was partially proved using neuro–imaging studies in the work [McMillan, Clark et al. 2005]. See [Szymanik 2005] for a discussion.

3 Inferential meaning

We can gain knowledge about the logical interpretation of sentences not only by analysing referential meanings which are assigned to these sentences by native speakers. The other way is to analyse the inferential dependencies of these sentences. The problem of the adequate reading of sentences 1 or 7 – 9 can be reduced to the problem of determining the logical form of some "easier" sentences which are implied by sentences 1, 7 – 9.

For example, if we assume that sentence 25 is true, then we agree that sentence 1 implies sentence 26:

(25) Mark is a villager.

(26) Some relative of Mark and some relative of each townsman hate each other.

If we interpret Hintikka's sentence as formula 6, then we must agree that sentence 26 is true in the picture.

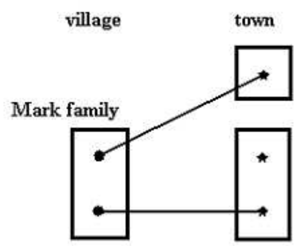

Figure 3: Illustration for inference argument.

Mostowski observed that this is a dubious consequence of assigning interpretation 6 to sentence 1 [M. Mostowski 1994]. Mostowski claims that sentence 26 is false in the above model[5]. Therefore, sentence 26 is not implied by sentence 1 understood as 6. However, it is implied by the strong reading because formula 2 is also false in the model. Therefore, Mostowski concludes that Hintikka's sentence must have branching form. Mostowski does not consider the conjunctional reading of Hintikka's sentence. And if we interpret sentence 1 by formula 4 the problem observed by Mostowski disappears, because formula 4, just as formula 2, is false in the above model.

[5]Mostowski implicitly interprets sentence 26 as $\exists x[R(Mark, x) \wedge \forall y(T(y) \Longrightarrow \exists z(R(y,z) \wedge H(x,z)))]$. It was observed in [Szymanik 2004] that sentence 26 can be also interpreted as $\forall y[T(y) \Longrightarrow \exists x(R(Mark, x) \wedge \exists z(R(y,z) \wedge H(x,z)))]$. In this case the problem does not arise.

4 Conclusions

We argue that some of Hintikka–like sentences have logical form expressible by first–order formulae, despite what Hintikka claims. The reasons for treating such natural language sentences as first–order expressible are as follows. For Hintikka's sentence we should focus on four possibilities: 2, 4–6. Hintikka's argument — given in the first section — allows us to reject formula 5 which is not symmetrical in contrast with Hintikka's sentence. A similar argument gives reasons to reject linear readings of sentences 7–9. Mostowski's inferential argument suggests that formula 6 is also not the appropriate reading of sentence 1. For sentences 7–9 an analogous formula does not exist.

Therefore, we have to choose between the conjunctional and branching reading. The following arguments support the conjunctional reading. First of all its simplicity. Secondly, our empirical results indicate that people prefer such readings for Hintikka–like sentences with *most* quantifiers. Moreover, the aforementioned Barwise's arguments agree with our empirical results. And, last but not least, there are the arguments from computational complexity. Branching readings — as NP–complete — can be too difficult for language users. The conjunctional reading is much easier in this sense.

Another option is that Hintikka–like sentences are just ambiguous. Observed differences in the understanding of such sentences with *most* quantifiers (usually interpreted in the conjunctional way) and with proportional quantifiers (usually percived as branching sentences) play in favour of such a claim. The question appears: what are the reasons for such diffrence?

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A Test

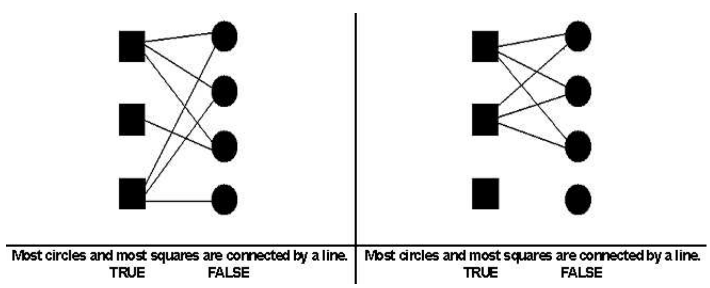

Figure 4: Task 1

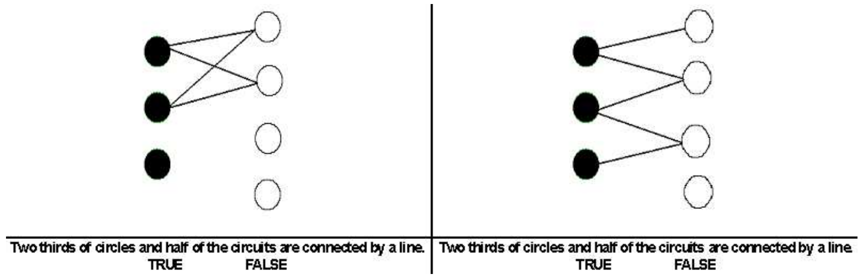

Figure 5: Task 2

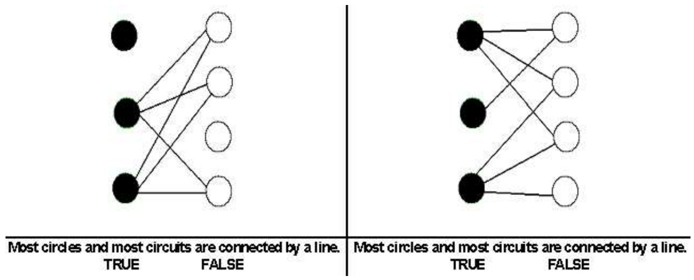

Figure 6: Task 3

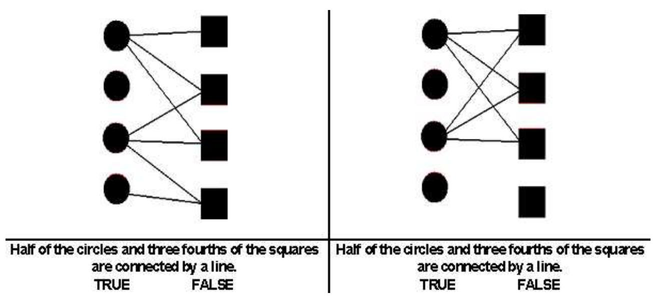

Figure 7: Task 4

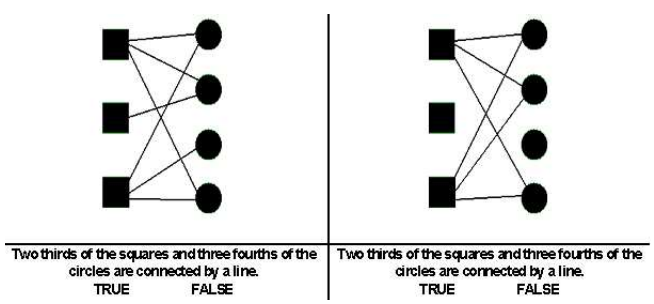

Figure 8: Task 5

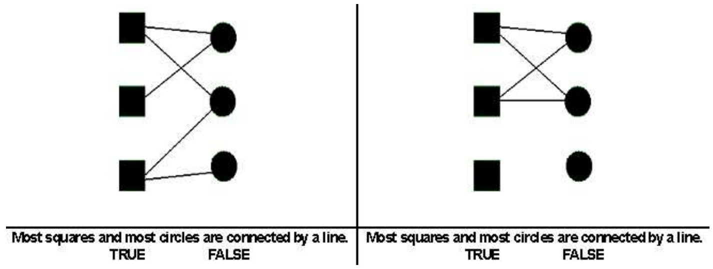

Figure 9: Task 6

B Tasks

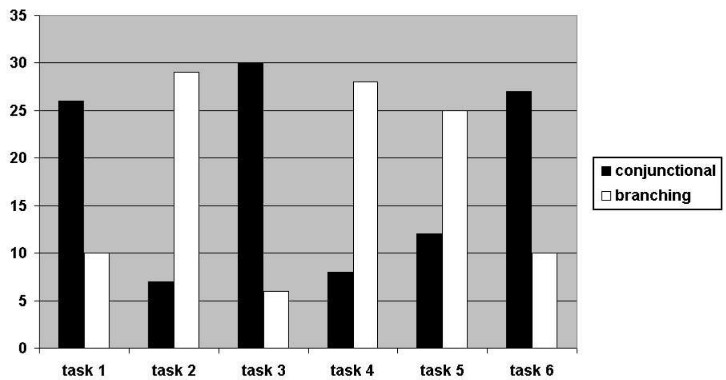

Figure 10: Number of aswers of two types in each task.