Probability in Dynamic Epistemic Logic

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1 Introduction

Many of those who are currently working in the field of dynamic epistemic logic have an interest in game theory. However, probability has not been incorporated into this logic yet, although it is an important aspect of game theory. In this article I wish to make a first step in combining the two. My main inspiration comes from the work in probabilistic epistemic logic in [HT93] and the work in dynamic epistemic logic in [BMS98, Ger99].

2 Probabilistic Epistemic Kripke Models

The standard model in the semantics of epistemic logic is a multimodal Kripke model. Probability can be added to these models in a similar fashion as Halpern et al. added probability to their models for multi-agent systems. The basic probabilistic notion they use is that of a probability space, which can be used to model an agent's information about probabilities. Analogous to the role possible worlds play in the accessibility relations, the sample spaces of these probability spaces are sets of possible worlds.

I provide a general definition of probabilistic epistemic Kripke models with which we can interpret the language of probabilistic epistemic logic. This contains formulas like $(\mathrm{Pr}_a(\phi) \leq \alpha)$, which can be read as 'the probability a assigns to ϕ is less than or equal to α.' However, given these models, it is still not immediately clear how one should model a specific situation (if, for example, an agent cannot distinguish between two different probability distributions). I provide an approach on how to construct a probabilistic epistemic Kripke model from two Kripke models: one for the *nonprobabilistic* information (i.e. propositional and epistemic information) and another for the *probabilistic* information. It is often easier to think about these domains of information separately. These models can be multiplied, yielding a *probabilistic epistemic* Kripke model.

3 Updates

Then I turn my attention to performing updates these models. The updates I consider are restricted to tests, because other updates cannot be modeled quite

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as elegantly, though new developments in this area are forthcoming. The idea for well-founded semantics for subgroup updates with tests is by Baltag [BMS98] (he calls these updates announcements) and I add probability to it. The only addition to the definition of the accessibilty relation of the update program that is needed is saying what happens to the probability spaces. The short answer is that exactly the same happens to them as what happens to the accessibility relations. Those worlds where ϕ does not hold are no longer considered. So the new sample spaces can only contain worlds where ϕ holds. The new probability assigned to an event is its old conditional probability given that it is in the new sample space.

4 The Monty Hall Dilemma

Finally I will discuss the famous puzzle called the Monty Hall Dilemma. Suppose you have won a quiz and the quiz master offers you a choice of three doors. Behind one of the doors is a car, behind the other two there are goats. You pick a door, say door number one, and the quiz master who knows where the car is has to open a door containing a goat. Say he opens door number three and then asks you whether you would like to switch to door number two. The question is whether it is to your advantage to switch. Well, in this case the probability the car is behind door number two is two thirds and therefore you should switch. In this puzzle probability, knowledge and information change all play a role; the probabilistic answer to the question whether it is to your advantage to switch, the knowledge the quiz master has about the location of the car, and the information change that is caused by the quiz master opening a door. These three aspects can be analysed with probabilistic dynamic epistemic logic.

5 Questions for Further Research

There are many interesting issues that still have to be dealt with. For example, what is the best way to model ignorance about probabilities? How can you model an update with a sentence that has probabiliy zero, but is not logically inconsistent? I discuss these and other issues.

References

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