IMPLICIT AND EXPLICIT STANCES IN LOGIC

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Abstract We identify a pervasive contrast in modeling styles in logic between what may be called more 'implicit' and more 'explicit' approaches. The former change the meaning of logical constants and consequence to accommodate new topics entering the field, while explicit approaches extend classical logical systems with new vocabulary. We discuss the contrast in intuitionistic vs. epistemic logic, default reasoning, logics of questions, information dynamics and games, and then define the stances more sharply. New technical issues arise concerning dualities and merges between the two styles of logical analysis. Finally, we discuss what the contrast means for an understanding of logic as a repertoire of natural stances.

1 Two faces of logic

Logic has two faces that co-exist in how practitioners, or textbook authors, present the field. One way of taking logical languages is as a medium for *description of the world*, focused on truth and how things really are. Logical operators like “not”, “and”, or “some”, and complex formulas then express complex assertions about the world, or maybe better: assertions whose construction history is complex (a syntactically complex formula may well express a simple fact). This descriptive truth-oriented view of logic goes well with the traditional semantic notion of valid consequence: in any model where the premises are true, so is the conclusion. Thus, logical consequences support inferences that are safe under all circumstances.

But here is another view of logic, which has equally distinguished roots in the history of the subject. Logic is about argumentation, refutation, agreement, disagreement, and learning, and logical operators are expressions of ‘control’ structuring such attitudes and activities. This *functional view of logic* ties it to information flow, communication, and action in general. While it may be less well represented in logic textbooks, it is one of the driving forces behind the game paradigm for evaluation and proof (Lorenzen 1955), procedural semantics (Suppes 198x) as well as modern epistemological and dynamic views of logic (see van Benthem 2011). Logical constants now have to do with rights and duties of agents in central logical tasks.

2 Explicit and implicit approaches to logical analysis

In this paper, we take both views as equally fundamental. And in particular, we observe that much innovative research in logic over the last decades has consisted in bringing 'functional themes' to the classical descriptive conception of logical systems. But when that happens, a second choice point appears. If we want to take information flow and interaction seriously, while keeping them grounded in descriptive correctness where appropriate, how should we do this? It is not just a matter of reinterpreting classical systems – or is it?

Take the special case of information and *knowledge*, crucial notions on the functional view of logic. Here is one way of taking them seriously: we add modal operators for knowledge on top of our existing languages, and study the resulting richer *epistemic logics* that sit on top of classical propositional or predicate logic. This approach aims for conservative extension of existing logic, so to speak, and it creates widely used formalisms in various disciplines, though they are sometimes considered (very unreasonably) peripheral extensions of the pure core of logic.

But here a second way of taking knowledge seriously, that may be called 'radical chic'. We penetrate the old order, and *redefine* the meanings of the existing logical operators, and perhaps also that of the notion of consequence, to include what we deem crucial aspects of knowledge. A typical instance of this alternative approach is *intuitionistic logic* that does not add knowledge operators, but encodes behavior of knowledge precisely in its deviations from the laws of classical consequence. This is much more mysterious, especially, since the behavior of knowledge will often not show in the presence of new laws, but in the *absence*, or modification of old laws. That mystery is why (again, unreasonably) implicit logics often have a much better press than explicit ones for possessing philosophical depth and significance.

In this paper, we will take both explicit and implicit approaches seriously, and we will see that their co-existence and interplay raises a number of interesting points for our understanding of logic. We will do a number of case studies, noticing some general issues that arise, and then proceed to a general discussion of what all this means.

3 A pilot study: epistemic logic

Epistemic logic (Hintikka 1962) was originally proposed as an analysis of the notion of knowledge based on the intuitive conception of information as a current range of possible candidates for the actual situation (or call them 'worlds' or 'states': what's in a name). This range may be large, in which case we know little, or it may be small (perhaps in the course of successive updates eliminating possibilities) and then we know a lot. More precisely, an agent *knows that* φ at a current world s if φ is true in all worlds in the current range of s, the 'epistemically accessible' worlds from s, given by some binary relation $s \sim t$. To the syntax of, say, propositional logic as our elementary base language, we now add a clause for constructing formulas $K\varphi$ – that can be subscripted to $K_i\varphi$ in case we want to distinguish between the knowledge of different agents. Then the truth definition reads as follows:

$\boldsymbol{M}, s \models p$	iff	$s \in V(p)$
$\boldsymbol{M}, s \models \neg\varphi$	iff	*not* $\boldsymbol{M}, s \models \varphi$
$\boldsymbol{M}, s \models \varphi \wedge \psi$	iff	$\boldsymbol{M}, s \models \varphi$ *and* $\boldsymbol{M}, s \models \psi$
$\boldsymbol{M}, s \models K\varphi$	iff	$\boldsymbol{M}, t \models \varphi$ *for all* t *with* $s \sim t$.

This is clearly propositional logic extended. It is often assumed that epistemic accessibility is an equivalence relation (but see below). The resulting modal logics are of the well-known *S5* type, and we will not explain epistemic logic any further in this paper (cf. van Benthem 2010).

It seems fair to say that few people believe any more in epistemic logic as it stands as a viable vehicle for analyzing the philosophical notion of knowledge. But what it is good at is describing another fundamental notion, the above *semantic information* that an agent has at her disposal. What is also true is that the simple perspicuous syntax of epistemic logic is still in wide use as a lingua franca for framing epistemological debates, for instance, about the validity of

'omniscience' $K(\varphi \rightarrow \psi) \rightarrow (K\varphi \rightarrow K\psi)$, or 'introspection' $K\varphi \rightarrow KK\varphi$. [1]

Epistemic logics are conservative extensions of classical logics, the debates are about their further axioms and the precise deductive strength one wants the logics to have.

[1] This lingering of old languages will come as no surprise to students of political debates.

Another noticeable line in the development since the 1960s has been increases in expressive strength, adding operators of new kinds, such as *common knowledge* $C_G\varphi$, one natural case of what might be called 'group information'. Knowledge comes natural also with the notion of *belief*, but we will discuss that later.

This is a shamelessly brief exposition of epistemic logic: see a classical text like Fagin, Halpern, Moses & Vardi 1995 for much more.

4 Intuitionistic logic

Intuitionistic logic had its origins in the analysis of constructive mathematical proof, with logical constants acquiring their meanings in natural deduction rules via the Brouwer-Heyting-Kolmogorov interpretation ('BHK'). In the 1950s, Beth and Kripke proposed models over trees of finite or infinite sequences, and in line with the proof idea, intuitionistic formulas are true at a node of such a tree when 'verified' there in some strong intuitive sense. A simple version uses *partially ordered* models $\boldsymbol{M} = (W, \leq, V)$ with a valuation V, setting:

$\boldsymbol{M}, s \models p$	iff	$s \in V(p)$
$\boldsymbol{M}, s \models \varphi\wedge\psi$	iff	$\boldsymbol{M}, s \models \varphi$ *and* $\boldsymbol{M}, s \models \psi$
$\boldsymbol{M}, s \models \varphi\vee\psi$	iff	$\boldsymbol{M}, s \models \varphi$ *or* $\boldsymbol{M}, s \models \psi$
$\boldsymbol{M}, s \models \varphi\to\psi$	iff	*for all* $t \geq s$, *if* $\boldsymbol{M}, t \models \varphi$, *then* $\boldsymbol{M}, t \models \psi$
$\boldsymbol{M}, s \models \neg\varphi$	iff	*for no* $t \geq s$, $\boldsymbol{M}, t \models \varphi$ [2]

Note what happens here. There is no separate new operator for knowledge or information, but the classical logical constants are *reinterpreted*, making negation and implication sensitive to the informational structure of the current model. In particular, an intuitionistic negation $\neg\varphi$ says that the formula φ is not just 'not true', but that it will never become true at any further stage. [3]

[2] In line with the idea of accumulating certainty, the valuation is *persistent*, i.e., "if $\boldsymbol{M}, s \models p$, and $s \leq t$, then also $\boldsymbol{M}, t \models p$." The truth definition lifts the persistence to all formulas.

[3] This is a thumbnail sketch. The intuitive interpretation of intuitionistic models as records of inquiry poses many problems that we ignore here. For instance, the original intuition of proof steps is not represented in our stages, making for a rather loose connection with the original constructive *BHK* interpretation.

This *meaning loading* makes the laws of reasoning for negation and implication different from those for classical propositional logic. Famously, this makes Excluded Middle $p \vee \neg p$ invalid, as this fails at states where p is not yet verified, though it will later become so. These deviations from classical logic are informative in that they implicitly tell us about properties of knowledge of agents. The failure of Excluded Middle tells us that they cannot 'decide' everything a priori. Thus meaning loading makes both the remaining validities informative (since they now say something new), and it packs information in the absence of certain classical laws.

This is a shamelessly brief exposition: see Troelstra & van Dalen 1988 for details.

5 The contrast: epistemic logic versus intuitionistic logic

So, we have two major research areas in logic, both meant to take information and knowledge seriously – but doing so in very different ways. Let us highlight the major differences that showed in the above:

epistemic logic	explicit, conservative language extension of classical logic
intuitionistic logic	implicit, meaning change old language, non-classical logic

Summarizing the difference once more, consider the obvious fact that we do not know the answer to every question right now (and maybe never will). This shows as follows in intuitionistic logic. The Excluded Middle formula $\varphi \vee_{int} \neg_{int} \varphi$ is not valid (where the indices highlight the fact that this non-validity occurs on a particular understanding of negation and disjunction), though some special cases are, under special circumstances (see the literature on intuitionistic logic). In contrast with this, Excluded Middle is unrestrictedly valid in epistemic logic, but it should not be confused with the invalid formula $K\varphi \vee_{class} K\neg_{class} \varphi$.

Much more can obviously be said about these two paradigms. But for the purposes of the present paper, we will just stipulate that both are interesting sets of intuitions, both have in fact generated a rich mathematical theory, and both seem worthy instances of a 'logical' modus operandi. Moreover, even with just this one example, we assume that the reader has grasped the point – and we will now feel free to use the terminology 'implicit' versus 'explicit' in other cases.

6 Comparisons and connections

There is more to the above situation than mere co-existence. Already in the 1930s, Gödel gave a translation ***t*** from intuitionistic logic into the modal logic *S4* that connects an implicit logic to an explicit counterpart in the following precise sense:

$$IL \vdash \varphi \text{ iff } S4 \vdash \boldsymbol{t}(\varphi), \quad \text{for all propositional formulas } \varphi.$$

This is often taken to mean that intuitionistic logic has now been 'embedded' in a richer epistemic logic that, in particular, also supports reasoning about non-persistent formulas which can become false at later stages. But there is much more to it.

For a start, notice that the logic translated into is not the earlier epistemic *S5*, but *S4*. And this difference is not just a matter of dropping one axiom, but it stands for a whole different style of thought (van Benthem 2010). [4] The reflexive transitive accessibility relation of *S4* does not just represent an epistemic range, it also has an essential *temporal* aspect of 'progressive inquiry'. Intuitively, like in the above models for intuitionistic logic, each such model describes an *informational process* where an agent learns progressively about the state of the actual world, encoded in a propositional valuation. At endpoints of the tree, all information is in, and the agent knows the actual world. The generalization is that we also allow atomic facts to become false along the way, making *S4*-models also suitable for modeling informational 'retractions', as well as real world change.

We will return to this interpretation below, including the role of time and informational events in the 'static' epistemic logic that we presented earlier. At this stage, however, we can see a caveat already. Mere translations between implicit and explicit stances, while mathematically quite interesting, and while perhaps also useful as manuals for relating talk in different logical frameworks and communities, are definitely not the last word in understanding their relations.

Of other reactions in the literature to this choice of approaches, we mention two.

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[4] The discussion that follows here demonstrates that the usual modal game of varying sets of axioms between, say, *S5, S4, T* or *K*, has little content unless we have reasons for the switch. Moving from one modal logic to another is not just a matter of fiddling with proof strength, it may involve wholly different worlds of thought. We will return to this point later.

One could construe the emergence of the two as resulting from a difference in *perspective* on the same phenomena of information flow and knowledge. Epistemic logic would give what philosophers call a 'third person perspective', describing the knowledge, or semantic information, that agents really have from the standpoint of an external modeler. By contrast, intuitionistic logic might represent the 'first person perspective' of someone engaged in an informational situation, and just talking about what the world is like, with the usual linguistic conventions tacitly built in about how it is all about the speaker's and hearer's information. I find this a serious proposal for harmony, but I do not think it fits all cases of implicit versus explicit – and I am not even sure that it fits the spirit of intuitionistic logic particularly well.

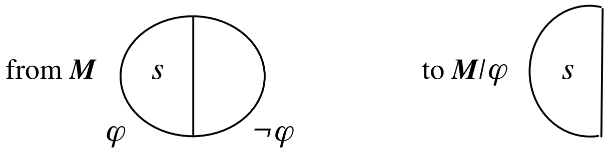
Another response has been more mathematical, and perhaps more opportunistic. Given that both epistemic logic and intuitionistic logic make sense, a weaker technical test of compatibility than translation is *merging* these systems consistently. And indeed, quite a few mixed systems exist of 'intuitionistic model' or 'intuitionistic epistemic' logic. There may be good reasons for going this way in concrete settings, but in general, I do not see what the juxtaposition solves. Even the idea of combining virtues can be questioned: some matches combine the worst features of the partners, and they are mainly an effective way of having the problems of both.

7 Dynamic logic of information change

Let us now move a bit further in time. Knowledge and information of agents are one crucial feature of what we have called the functional aspect of logic, but there is one more, namely, action, or in a social setting, *interaction* between different agents.

In recent years, work in the tradition of 'logical dynamics' (van Benthem 1996) has made informational actions a focal point of logical theory. Crucial informational actions that drive rational behavior of agents then come in three broad kinds that occur together in natural scenarios. Acts of *inference* are important ('drawing conclusions'), but so are acts of *observation*, and of *communication*. Van Benthem 2011 shows how all these actions, or the events that embody them, can be dealt with in current 'dynamic-epistemic logics', by adding an explicit vocabulary for them to existing logical systems, and analyzing their major laws.

Model update To make this a bit more concrete, here is a typical pilot system that makes the dynamic actions behind basic epistemic logic explicit. The key idea in dealing with these is now that informational action is model change. The simplest case is a *public announcement* $!\varphi$ of hard information: learning with total reliability that φ is the case eliminates all current worlds with *P* false:



As we said earlier, getting information by shrinking a current semantic range of options is a common idea in many fields, that works for information flow by being told, or through observation. We call this *hard information* because of its irrevocable character: in the update step, all counter-examples are eliminated. [5]

Public announcement logic Public announcements are treated as first-class citizens of logical theory in a *public announcement logic* (*PAL*) extending epistemic logic with a dynamic modality for public announcements, interpreted as follows: [6]

$$\boldsymbol{M}, s \models [!\varphi]\psi \quad \text{iff} \quad \text{if } \boldsymbol{M}, s \models \varphi, \text{ then } \boldsymbol{M}|\varphi, s \models \psi$$

The dynamic modality has a complete logic that records delicate phenomena of truth value change and order dependence, whose precise nature need not concern us here. We just display the crucial *recursion law* for knowledge after update, the basic 'recursion equation' of the *PAL* dynamics, that is axiomatized completely by

> the laws of epistemic logic, plus some obvious axioms for Boolean compounds after update, as well as the following crucial equivalence:
>
> $[!\varphi]K_i\psi \quad \leftrightarrow \quad \varphi \to K_i(\varphi \to [!\varphi]\psi)$ [7]

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[5] It may be thought that this is simple, with actions $!\varphi$ leading to knowledge $K\varphi$. But things are more subtle. Dynamics typically involves truth value change for complex formulas. While an atom p stays true after update (the physical facts do not change), complex epistemic assertions may change their truth values: before the update $!p$, I did not know that p, but afterwards I do. This results in *order dependence*: a sequence $!\neg Kp; !p$ makes sense, but the permuted $!p\ ;\ !\neg Kp$ is contradictory. By contrast, *factual* propositional statements containing no epistemic operators have their truth value invariant under public announcements, and they are invariant under a wide variety of update actions.

[6] The antecedent "*if* $\boldsymbol{M}, s \models \varphi$" reflects the assumption that the announcement is truthful.

There is a general method behind this example. Where possible, one 'dynamifies' a given static logic, making its underlying actions explicit and defining them as suitable model transformations. The heart of the dynamic logic is then a compositional analysis of post-conditions for the key actions via recursion laws. This leads to conservative extensions of the base logic, though its expressive requirements sometimes force some redesign of the base language.

Many further notions can be treated in this style: such as dynamic changes in agents' *beliefs*, *inferences*, current agenda *issues*, or even *preferences* – where model change will now often consist in changing the order structure of a given model, rather than eliminating worlds. Moreover, more systems like the above can also deal with public and private events in multi-agent scenarios like games (van Benthem 2013).

Remark Let us now return to an earlier point showing how our dynamic approach, even though not changing classical meanings, can be radical and non-conservative in other ways. While adding a dynamic component looks like conservative enrichment, it may still have a more radical thrust for received views in the static world. Consider the well-known variety of modal logics such as *K*, *S4*, or *S5* over models with different conditions on their accessibility relations. From a dynamic viewpoint, one might ask *why* these special conditions came to hold: perhaps, as a result of some epistemic action. For instance, one might think of transitive relations as arising from a model transformation ***tc*** of 'reflection' or 'exploration' replacing a current accessibility relation by its transitive closure. Then, the '*K4*-modality' is really an ordinary *K*-modality over models that have undergone this transformation, that is:

K is really ***[tc]*** □.

In this way, modal *S4* over preorders is really a sublogic of some propositional dynamic logic over arbitrary models. In other words, variety of modal logics may

[7] The reader may also want to check other interesting laws like $[!\varphi][!\psi]\chi \leftrightarrow [!(\varphi \wedge [!\beta]\psi)]\chi$. The total system can be used to reduce formulas with dynamic operators to pure epistemic base formulas – though this is not an essential feature: such a reduction no longer works in recent generalized 'protocol models' for *PAL*: see below.

dissolve into one modal logic over different operators for model transformation, explaining instead of just postulating the special properties that seemed sacrosanct.

8 Implicit dynamics in intuitionistic logic

We have now extended epistemic logic, an explicit approach to knowledge, with explicit informational actions. One of the things that make sense at this stage, given an awareness of co-existing implicit approaches, is ask what the same move would mean in the latter realm. Does intuitionistic logic support information dynamics?

One way of investigating this is by adding *PAL*-type operations to intuitionistic logic, as has been done in Palmigiano et al. 2010. But maybe more interesting is seeing whether intuitionistic logic already has an implicit theory of informational action. Recall that intuitionistic models as above intuitively stood for pre-ordered stages of some process of inquiry, where endpoints stand for final stages where we know the valuation for all proposition letters.

What are the steps in such a process? Looking at some simple examples, we see that moves from one state to a successor come in two kinds.

Example The hidden dynamics of intuitionistic models.
Consider two models $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, where the first refutes the classical double negation law $\neg\neg p \rightarrow p$, and the second the weak excluded middle $\neg p \vee \neg\neg p$:

Here is what the annotations say. The two branches of $\boldsymbol{M}_2$ may be viewed as public announcements of which endpoint, viewed as a classical valuation, we can get to. This is like *PAL*-style learning by elimination of worlds. But in other steps, like the one in $\boldsymbol{M}_1$, there is no such elimination of endpoints, and we merely get more proposition letters true at the next stage. Van Benthem 2010 explains the latter as a new type of informational action, namely, 'awareness raising' $\#\varphi$ that some fact φ is the case, involving syntactic in addition to semantic information.

But, there is still one more basic dynamic feature of interest in these models. Through their branching structure, intuitionistic models register *two* notions of semantic information, that have also been proposed for *PAL* itself (Hoshi 2009):

(a) *factual information* about how the world is;

but also, on a par with this:

(b) *procedural information* about our current investigative process.

How we can get our knowledge matters. While endpoints record eventual factual information states, the branching tree structure of intuitionistic models themselves, with available and missing intermediate points, encodes further non-trivial information: agents' knowledge of the latter procedural kind.

Once more, this changes our earlier translational view of how intuitionistic logic and epistemic logic connect. The obvious epistemic logic seems *S5*, and the fact that we had to translate into *S4* reflects the tree-like nature of intuitionistic models, standing for arbitrary temporal procedures of inquiry. Thus, the proper explicit counterpart to intuitionistic logic might be some temporal version of dynamic epistemic logics (cf. van Benthem 2011). In fact, temporal *protocol models* with a designated set of admissible histories have also been proposed in the latter realm for the purpose of modeling procedural information beyond local dynamic steps (cf. van Benthem, Gerbrandy, Hoshi & Pacuit 2009 on systematic connections between *PAL* and epistemic-temporal logics over branching time).

Thus, implicit and explicit stances can learn from each other, as their line of view highlights different aspects of the phenomenon that forms their shared interest.

9 Dynamic semantics, meaning as information change potential

However, the currently more relevant 'implicit' counterpart to dynamic epistemic logics like *PAL* is another one, that we shall discuss now.

Here is a basic idea from the area of *dynamic semantics* of natural language (Groenendijk & Stokhof 1991, Veltman 1996). The appealing and influential guiding intuition here is one of 'information change potential':

> the meaning of an expression is its potential for changing information states of someone who accepts the information conveyed.

This sounds much like a plea for taking informational actions seriously, but this time, they are treated, not by adding new operators, but by loading the meanings of classical logical constants with dynamic features.

These ideas can be implemented in many different ways, witness the references. Here, we will use 'update semantics' for propositional logic. Given a universe of information states, viewed as sets of propositional valuations, considerations, each formula φ induces a state transformation $[[\varphi]]$ by the following recursion:

$$[[p]](S) = S \cap [[p]],$$
$$[[\varphi \vee \psi]](S) = [[\varphi]](S) \cup [[\psi]](S)$$
$$[[\varphi \wedge \psi]](S) = [[\psi]][[\varphi]](S),$$
$$[[\Box\varphi]](S) = S \text{ if } [[\varphi]](S) \neq \emptyset, = \emptyset \text{ otherwise}.$$

Note how conjunction now stands for a dynamic sequential composition of actions, while an existential modality becomes a 'test' on the current information state. As with intuitionistic logic, these new meanings result in deviations from classical logic. For instance, conjunction is no longer commutative, because of the typical order dependence of dynamic procedures. [8]

Indeed, information about the informational process is encoded in the logic of this system, especially when we define an appropriate notion of *dynamic consequence* saying that after processing the information in the successive premises, the conclusion has no further effect:

> $\varphi_1, \ldots, \varphi_n \mid= \psi$ iff for every information state X in any model, $\varphi_n(\ldots(\varphi_1(X))$ is a fixed point for $[[\psi]]$: this set stays the same under an update $[[\psi]]$.

This notion of consequence behaves differently from classical logical consequence, and its deviations encode crucial features of the update process, such as its sensitivity to ordering of premises, or to repetitions of the same premise. [9]

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[8] However, also as in intuitionistic logic, such classical laws may still hold in special cases. For instance, $p \wedge q$ has the same update effects as $q \wedge p$, since testing facts has no side-effects.

[9] As for relevant technical results, van Benthem 1996 proves a complete axiomatization for the structural rules that hold for dynamic consequence. Rothschild & Yalcin 2012 prove that any family of update operators satisfying both Permutation and Contraction must consist of classical world eliminations for underlying standard static propositions.

There are much more sophisticated systems of dynamic semantics for other classes of expressions, with different notions of state change and dynamic consequence – but the above is a fair description of their basic mechanics.

10 The contrast returns: dynamic semantics vs dynamic logic of information

The reader will long have seen where we are heading. Dynamic epistemic logics like *PAL,* but also update semantics for propositional logic both take information change very seriously, with strong analogies in scenarios and intuitions. And both systems have an account for the dynamics of informational actions. But one does so explicitly, and the other implicitly:

> *Dynamic semantics* keeps the actions implicit, while giving the old language of propositional logic richer 'dynamic meanings' supporting a new notion of consequence, with a mathematical theory that differs from standard propositional logic,
>
> *Dynamic epistemic logic* makes the actions explicit, provides them with explicit recursion laws, extends the old base language while retaining the old meanings for it, and in all this, it still works with standard consequence. [10]

This contrast raises many further issues. Methodologically, for instance, one can wonder to which extent our distinctions in this case are a matter of description level, with dynamic semantics closer to semantic views of natural language, and dynamic epistemic logic closer to pragmatic views of speech acts. Also relevant is our earlier distinction between first-person and third-person perspectives. Perhaps natural language syntax does come with many dimensions of meaning for speakers, whereas a theorist would want to disentangle these. We will return to these issues later on.

Translations Technically, one can ask whether the two systems are intertranslatable in ways similar to what we had for epistemic and intuitionistic logic. There are some

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[10] It has to be said that things may not always be this easy to classify. Some systems of dynamic semantics also retain a classical base logic in the form of *fragments*, like with atomic facts in update semantics. Also, some dynamic semantics in the recent literature on epistemic modals employ ideas much closer to dynamic epistemic logic. And finally, it has also been claimed that the move from dynamic semantics to, say, *PAL* may be one of *degree* to which one wants to formalize the relevant informational scenarios.

technical problems here, [11] but indeed van Benthem 1989 already provided a faithful translation from update semantics for propositional logic into the modal logic *S5*. And precise translations from dynamic semantics in the assignment-changing style of Groenendijk & Stokhof 1991 into propositional dynamic logic *PDL* were already given in van Eijck & de Vries 1991. Finally, van Benthem 1996 axiomatizes the structure theory of dynamic consequence by translating into a simple extension of basic modal logic. More sophisticated results must surely be possible.

The case of questions Once we see the above contrast, we can predict that it will arise in many further areas of dynamic semantics, whether or not with a pre-existing dynamic logic variant. A case in point are current systems of *inquisitive semantics* (Groenendijk 200x), where propo-sitional formulas now get richer compositionally computed 'inquisitive meanings' having to do with their role in directing discourse or inquiry. The resulting logic of the propositional connectives is now a non-classical system of 'intermediate logic', related to Medvedev's 'logic of problems' from the constructive tradition, whose behavior encodes subtle features of inquiry.

One would expect the existence of an explicit counterpart then, and one has been found in recent dynamic epistemic logics of inquiry based on explicit acts of 'issue management', where questions and related actions modify current 'issue structures' on top of epistemic models (van Benthem & Minica 2012). [12]

We end with an issue of local interest to this author: is the Amsterdam trademark paradigm of dynamic semantics perhaps the true inheritor of Dutch intuitionism?

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[11] One striking technical difference is that update semantics recurses on the structure of the formulas viewed as actions, whereas *PAL* does *not* recurse on the formulas inside announ-cements $!\varphi$, but rather on the postconditions for their modalities $[!\varphi]$. The closest counter-part in *PAL* to the central dynamic semantics recursion would be to add a repertoire of model changing *conversational programs* built from atomic actions $!\varphi$ through constructions like sequential composition, guarded choice and iteration. (Note, for instance, that such a logic would distinguish at least the following three conjunctions of assertions: (a) $!\varphi ; !\psi$, (b) $!\psi ; !\varphi$, and $!(\varphi \wedge \psi)$.) It is known that logics of such programs over *PAL* can be highly complex: Miller & Moss 2004 – so one should enter this realm of expressive power with care.

[12] The *Synthese* issue Hamami & Roelofsen 2013 has logics of questions in both styles.

11 Dynamics of soft information

Our discussion so far has centered on knowledge. However, the points that we made are not confined to this special cognitive attitude. In particular, the implicit/explicit distinction applies just as well to logics of *belief*. These also show interesting features of their own, so we develop this theme a bit further.

Belief and conditional belief Epistemic-doxastic models for belief order epistemic ranges by a relation of *relative plausibility* $\leq_i xy$ between worlds x, y. These models interpret operators of absolute and conditional belief:

$\boldsymbol{M}, s \models B_i\varphi$ iff $\boldsymbol{M}, t \models \varphi$ for all $t \sim_i s$ *maximal* in the order $\leq_i$ on $\{u \mid u \sim_i s\}$

$\boldsymbol{M}, s \models B_i^{\psi}\varphi$ iff $\boldsymbol{M}, t \models \varphi$ for all $\leq_i$-maximal t in $\{u \mid s \sim_i u$ and $\boldsymbol{M}, u \models \psi\}$

On binary world-independent orderings, a good addition is 'safe belief', a standard modality intermediate in strength between knowledge and belief as defined above:

$\boldsymbol{M}, s \models [\leq_i]\varphi$ iff $\boldsymbol{M}, t \models \varphi$ for all t with $s \leq_i t$ [13]

Logics for these notions are the conditional logics of Lewis 1973, Burgess 1981.

Belief change Beliefs support useful default inferences going beyond what we know. But ty-pically, beliefs may be refuted by new information, leading to acts of belief change, learning, and *correction*. A first type of action triggering matching model changes are the earlier public announcements. Here is the basic dynamic recursion law for belief change in this setting:

$$[!\varphi]B_i\psi \quad \leftrightarrow \quad \varphi \rightarrow B_i^{\varphi}\,[!\varphi]\psi$$

A similar principle for updating conditional beliefs then axiomatizes the system completely. There is also a recursion law for safe belief under public announcement, which is even simpler:

the recursion law $[!\varphi][\leq_i]\psi \leftrightarrow (\varphi \rightarrow [\leq_i][!\varphi]\psi)$ holds on plausibility models.

But our richer belief models also support new transformations. In addition to hard informa-tion, there is *soft information*, when we take a signal as serious, but not infallible. Its mechanism is not eliminating worlds, but *changing plausibility order*. A widely used soft update is a *radical upgrade* $\Uparrow\varphi$ changing a current model M to

[13] Safe belief can define absolute belief $B_i\,\varphi$ and conditional belief $B_i^{\psi}\varphi$ (Boutilier 1994).

$M⇑\varphi$, where all φ-worlds become better than all $\neg\varphi$-worlds; within these zones, the old order remains. The corresponding modality satisfies $M, s \models [⇑\varphi]\psi$ iff $M⇑\varphi, s \models \psi$ – and its logic can again be axiomatized completely.

Powerful recursion laws have been found that capture belief changes under a wide variety of soft events (van Benthem & Liu 2007, Baltag & Smets 2007, Girard, Liu & Seligman 2012). An area where this variety makes sense is Learning Theory (Gierasimczuk 2010), as different update rules for transforming models locally have different effects in converging to true beliefs about the actual world. [14]

Logics of belief change in the preceding style are explicit companions of dynamic epistemic logic. We refer to van Benthem 2011 for details, and for connections with more postulational approaches to belief revision theory.

12 Non-monotonic logic and deviant consequence relations

Where there is explicit logic, there is implicit logic. How can one give an account of belief revision in implicit style? One line is dynamic semantics as before, using ordering changes to provide new meanings for natural language expressions like conditionals. We refer to Veltman 1996, Yalcin 2012, and Rodenhauser 2012 for some state of the art samples of this style of working. Many of our earlier points will be found to apply, but we will not pursue these here.

Instead, in this paper, we prefer to introduce a new line of comparison. One way of treating plausibility changes implicitly has been through non-standard notions of *consequence*. Around 1980, the study of common sense problem solving practice in AI modeled this as *non-standard inference*. In particular, *circumscription* (McCarthy 1980, Shoham 1986) says that

> a conclusion need not be true in all models of the premises,
> but only in the most preferred, or most plausible models.

The result is a 'non-monotonic' style of reasoning that deviates from classical logic. In particular, a conclusion C may follow from a premise P in the sense, but not from the extended set of premises $P, Q,$ because the maximal models for the conjunction $P \wedge Q$ need not be maximal among the models of P.

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[14] Again there is a connection with long-term procedures, a theme that we have seen before.

Non-standard consequence as implicit logic Over the last decades, complete sets of structural rules have been found for circumscription and many other forms of defeasible inference. Their subtle differences with the rules of classical logic (usually they support weaker variants) encode facts about the underlying mechanism of inspecting less or more plausible models. More generally, we see the vast area of non-standard consequence relations under study today as an implicit approach to the underlying forms of dynamics of inference and information.

Making it explicit Can we also extract this same information explicitly? The answer is positive for many reasoning styles. For instance, a circumscriptive consequence

$$\varphi_1, \ldots, \varphi_n \Rightarrow \psi$$

is easily translated into our earlier dynamic logic for belief change, using the formula

$$[!\varphi_1]\ldots[!\varphi_n]B\psi$$

This juxtaposition of perspectives raises some very interesting issues of redescription. Again we see a trade-off, similar to the one we have discussed earlier for implicit and explicit logics. Non-standard consequence differs from classical consequence, all on the original language. By contrast, dynamic logics of belief change enrich the original language, but they can now work with a perfectly classical consequence relation, explaining the 'deviant' features of non-standard consequence through the recursion laws for the new dynamic operators. [15]

Discussion One can have preferences for one approach over the other here. For instance, our explicit approach is an 'emancipation of informational events' that may be of interest per se. But van Benthem 2009 proposes a two-way perspective that does not take sides. Given a non-standard notion of consequence, one can tease out the informational or other events motivating it intuitively (these are often easy to find), and write their explicit dynamic logic. This dissolves 'non-standard deviation' from classical logic into 'dynamic extension' of classical logic. [16]

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[15] For instance, our earlier logic of belief change now *explains* the non-monotonicity precisely, in showing how $[!\varphi]B\psi$ does not imply $[!\varphi][!\varphi']B\psi$.

[16] Van Benthem 2009 relates this case study to generating implicit dynamics via a systematic process of abstraction, and of finding explicit dynamics via general representation theorems.

But also vice versa, given a dynamic logic of informational events, one can package some of its basic structure in the form of new consequence relations, and study those per se. The latter perspective will even add to the existing fund of styles of reasoning. In particular, our logics of belief change predict the existence of new styles of default inference based on different informational events and attitudes, such as

'soft-weak circumscription' $[\Uparrow\varphi_1]\ldots[\Uparrow\varphi_n]B\psi$

'soft-strong circumscription' $[\Uparrow\varphi_1]\ldots[\Uparrow\varphi_n]K\psi$,

where the premises are just taken as soft upgrades, not as public announcements. [17] These all have different structural rules reflecting the differences in the underlying process of drawing a conclusion.

Thus, if we think of comparing consequence relations only, implicit and explicit approaches may well live in harmony, and we can sometimes perform a Gestalt Switch one way or the other.

13 A final case: logic and games

Our final instance of the implicit/explicit contrast, with again some special flavors of its own, occurs at the currently thriving interfaces of logic and games.

Logic of games One conspicuous direction here in recent decades has been logic *of games:* the study of the logical structure of games in terms of players' actions, information, and preferences. This is what drives 'epistemic game theory', as well as many other logical systems that analyze reasoning with notions of rationality, the structure of players' strategies, the existence of game-theoretic equilibria, and many other key features of interactive social behavior.

There is a flourishing literature here involving standard combined modal logics of action, knowledge, belief, and preference (cf. van der Hoek & Pauly 2006, Perea 2012), as well as richer formalisms. We can view such logics as explicit ways of identifying major structures in games, giving them matching notions in a formalism that extends classical logical languages, and then identifying the laws of reasoning for such combined systems. An appreciable body of theory has accumulated in this manner, that we cannot survey here (cf. van Benthem 2013).

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[17] In this same format, the earlier 'update-to-test' consequence relation of update semantics would be expressed by a *PAL* validity making the conclusion *knowledge*: $[!P_1]\ldots[!P_k]\ K\varphi$.

Logic as games Logic of games may be the classical thrust of foundational research in the logical spirit. But there is also another connection between the two fields, perhaps an even more intimate one, that has been called logic *as games*. Starting from the 1950s, basic logical notions such as truth, valid consequence, or invariance between models have been cast as two-player games by a variety of authors (Ehrenfeucht 1953, Lorenzen 1955, Hintikka 1973). For instance, in many areas of logic today, truth of a formula φ in a model $\boldsymbol{M}$ is analyzed as existence of a *winning strategy* for the 'Verifier' player in a natural evaluation game for φ played over $\boldsymbol{M}$.

While such games are often viewed as didactic devices, they have become essential in recent semantics for computational logics, such as fixed-point languages, or 'game semantics' for linear logic. Indeed, in such semantics, the meaning of a formula is a generic game that can be played over arbitrary models. And so, basic logic becomes basic game algebra for the relevant games, a system that may well deviate in its validities from ordinary logics. For instance, van Benthem 2004 shows how the basic game algebra underlying first-order predicate logic is decidable, and hence considerably different from predicate logic itself. And thus, we see the typical features of an implicit approach once more. The classical truth-value based meanings of predicate logic are replaced by game-based meanings. And for in-stance, strategic powers of players in games are reflected not in explicit logics describing these powers, [18] but in the game-theoretic laws of predicate logic newly understood.

'Logic of games' versus 'logic as games' seems a major and still ill-understood duality in how to approach the contact between two realms that show natural affinities. The contrast between these perspectives, and their sometimes complex interactions are the grand theme of van Benthem 2013, even though no definitive view is developed of their coexistence and possible technical reductions. What is claimed there, however, is that the creative tension between the two directions of thought seems a powerful driving force in the formation of a new field.

14 Two stances: implicit versus explicit

No definition Having worked through all these examples, the reader might now feel entitled to a precise definition of the 'implicit/explicit' (*IE*) contrast. But perhaps

[18] Strategic powers are definable in modal logics of *forcing*, a species of 'logics of games'.

disappointingly, we do not have such a tight description to offer – and the best we can do is define by example. Still, we did give general features that should be helpful. Implicit approaches enrich old meanings, and locate subtle information in deviant notions of consequence – explicit approaches identify new items, introduce new vocabulary for them, and conservatively extend classical logic. In many concrete cases, that seems enough to attach the two labels to existing logics in a significant way.

As a consequence of this rather loose description, there need not be unique implicit or explicit stances on a phenomenon. Perhaps we should admit a sliding scale, from less to more explicit.

Concrete consequences But then, there are other ways of showing that a proposed distinction is real than providing an iron-clad definition. In particular, it is easy to show that the contrast pointed out in this paper has interesting *consequences*. Here are just a few concrete issues that arise once you see it:

If we *see one stance* on a topic, we can usually *find the complementary one*, whether or not this existed already. In this way, the *IE* contrast becomes a force for logical system building. We saw this with dynamic semantics of questions, which then suggested an explicit companion logic of issue modifying events, now studied as such. Likewise, explicit logics of belief change suggested new deviant notions of consequence in the area of nonmonotonic logic. [19]

Seeing two systems as representing different stances on the same thing may lead to creative *borrowing of ideas between stances*. One example is epistemic logic, that has developed a rich analysis of varieties of multi-agent knowledge and group knowledge. Intuitionistic logic could conceivably profit from the same ideas, creating accounts of mathematics that would be closer to the idea of even mathematical research as a (highly successful) social activity. [20] But also vice versa, it has become

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[19] Even so, I am definitely not saying that this system building will always be an automatic switch between stances. For instance, one outstanding challenge has been the area of non-classical substructural 'resource logics' (Gabbay & Schroeder-Heister 199x, Priest 200x). What would be an explicit dynamic counterpart, with appropriate notions of resources and (presumably) proof dynamics?

[20] Some multi-agent aspects have been introduced into *justification logics* (Artemov 200x) that derive from merging implicit and explicit aspects of proof-based constructive logics.

clear that explicit logics for analyzing games benefit from introdu-cing game terms inspired by game semantics for first-order logic (Parikh 1985).

Finally, our contrast also suggests similarities, and one may try to *borrow ideas inside the same stance* on different topics. Here is one example that has long exercised me, though it is still no more than a suggestion. One feature of assigning meanings that makes intuitionistic logic so attractive is the proof-theoretic *BHK interpretation* of the basic logical constants. This makes intuitionistic logic an intriguing meeting place of proof theory and model theory. But then, could the same sort of analysis be applied to dynamic semantics, another implicit paradigm for highlighting information dynamics? Is there an as yet unknown proof-theoretic, or at least combinatorial side to dynamic semantics as a view of language use that would mirror the power and elegance of the proof-theoretic interpretation?

Mathematics to the rescue? But could not we define our contrast purely technically, through a notion of translation or reduction between logical systems? I am convinced that more can be said along these lines, especially, on the theme of relative inter-pretations between non-classical logical systems and classical ones with extended vocabulary. But at the current state of ignorance concerning general translation methods or representation results, even in the specific case studies that were discussed in this paper, I am not prepared to say more.

Co-existence: mixing and merging If we cannot translate stances into each other in an impec-cable manner, a telling perspective on their relationship is that they can often be *merged*. For instance, many joint systems exist in the literature with features of both: intuitionistic modal logics (Dosen 198x), merges of logic games and game logics (van Benthem 2005, Agotnes & van Ditmarsch 2011), joint 'dynamic-epistemic inquisitive logics (Mascarenhas & Roelofsen 2012), joint systems of linear game logic and temporal logic (van Benthem 2013), and so on.

Degrees and shifts Finally, the existence of merges also calls to mind our earlier point that the *IE* contrast may be one of degree. Indeed, some moves in the literature may be classified as shifts toward 'externalizing' or 'internalizing' a given system. At one extreme, we will have to package all new structure into very rich meanings for a fixed base language – while at another, our strongest explicit theory might come to look suspiciously like a fully *formalized meta-theory* of the implicit semantics.

15 Conclusion, and broader repercussions

This paper may be a bit hard to classify. We have drawn attention to a significant contrast that we see running through much of the logical literature, between (more) *implicit* and (more) *explicit* approaches. Seeing that contrast reveals patterns in the research landscape, while also suggesting new questions that may not have been asked systematically before. For lack of a better word, one might call this ‘philosophy of logic’, maybe even with more justice than more abstruse studies of issues that seem to have no resonance in actual things that logicians do.

But I am sure that this bland stance will not satisfy many philosophers. [21] So, what if we try to tighten the screws? Should we ‘fight it out’: claiming that one stance is better, or at least more powerful than another? Indeed, there some such true claim may be lurking in the background. I myself am confident that there must be some sort of Universal Defining Power result for classical logic, in the sense that any reasonable semantics for any deviant logic can always be formalized in an explicit classical meta-theory. But even if true, how much of a victory is this?

In any case, my own philosophical inclinations tend toward plurality and harmony. What I would find a more interesting question would be a deeper *explanation* of the naturalness and utility of having the two stances, since I tend to see fundamentally different yet complementary ways of viewing things as an asset for an intellectual field – and in particular, for a methodological discipline like logic. The earlier-mentioned connection to first-person versus third-person perspectives is one attempt in this direction, the two stances would then be indexical perspective shifts – but I have also expressed my doubts whether this is really a satisfying way of viewing all the instances of the contrast that I have brought up in this paper.

But we need not locate the contrast in terms of pure logic, mathematics, or philosophy only. Another way of seeing its point might be in *applications* of logic.

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[21] It does not even satisfy all fellow logicians. After an invited lecture on this subject at *Advances in Modal Logic*, Copenhagen 2012, where the two stances widely resonated with the audience, I was attacked for not coming out in favor of explicit approaches (since that was where most of my own work lies), and asked to stop being diplomatic all the time.

The two stances make eminent sense in applications of logic to epistemology or computation, and perhaps it would be revealing to investigate more systematically what sort of light they throw on specific phenomena in these areas. As a special case of application, one might also think of current interfaces between logic and *cognition*, with natural language as a key example. Indeed it has been claimed that, as a result of evolutionary development, natural language is a messy practice of meaning-overloaded syntax in a more implicit multi-dimensional style, whereas explicit logics of information and agency would be our theorists' attempts at creating analytical order in this chaos. I am not sure that I subscribe to this view of the facts, since the observable basis of language carrying meanings is so much richer than syntax. But I agree that searching for analogous contrasts in empirical reality might help us better understand the thrust of this paper.

16 References (to be extended)

Johan van Benthem, 1991, 'Implicit versus Explicit Views of Knowledge', invited lecture, Proceedings TARK conference. – 1996, *Exploring Logical Dynamics*, CSLI Stanford. – 2009, 'The Information in Intuitionistic Logic', *Synthese*. – 2011, *Logical Dynamics of Information and Interaction*, Cambridge Univ. Press. – 2012, *Logic in Games*, The MIT Press. And all the others…