# CRISP：a semantics for focus－sensitive particles in questions 

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#### Abstract

Focus particles like only, too, even, etc. are well studied expressions in formal semantics. They received a lot of attention from different view points, e.g. presupposition theory and the study of scalar implicatures. However, these particles did not receive as much attention when occurring in questions (with the exception from focus intervention effects). Concentrating on too we present interesting data points on too in alternative questions, plain polar questions, and who-questions, showing that too is infelicitous in some questions, but not all. We restrict ourselves thereby to questions in matrix form. The explanation of these data would be a first step towards a general account of the distribution of too in questions.

In order to explain the data points, the thesis will develop a compositional inquisitive semantics with focus and presuppositions: CRISP. This is motivated both conceptually and technically, since the few accessible frameworks for such a study are either technically restricted, or conceptually ill-suited. We will show that CRISP can account for the data points.


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## Chapter 1

## Introduction

Many languages of the world comprise a class of words commonly referred to as focus particles. In English, this class consists among other expressions of the words only, also, just, either, and too. In German, we find expressions such as auch, nur, and sogar. ${ }^{1}$

The common semantic core of these words is that their meaning is focus sensitive:
(1) John only talked to Mary F $^{\text {. }}$
(2) John only talked ${ }_{F}$ to Mary.
(3) John talked to Mary ${ }_{F}$ too.
(4) John talked $F_{F}$ to Mary too.

Similar cases can be constructed for other members of this class in English. What we observe is that the meaning of these sentences depends on which constituent is focused. Example (1) means that John talked to Mary and no one else, but (2) means something completely different. It means that John talked to Mary but did nothing else. He may have talked to others, but all he did with respect to Mary was talking. A similar contrast holds for examples (3) and (4). The sentence in (3) means that John talked to Mary in addition to some other person, whereas the sentence in (4) means that John talked to Mary in addition to some other action he performed with respect to Mary.

A subclass of the focus particles is the class of additive particles. In English, this class comprises the prototypical examples also, either, too. The common semantic core of these particles is an inference they give rise to:
(5) $\mathrm{John}_{F}$ smokes too.
(6) John also smokes ${ }_{F}$.
(7) $\mathrm{John}_{F}$ doesn't smoke either.

This inference we will call the additive inference. ${ }^{2}$
Semantics research on additive particles in English has focused nearly exclusively on occurrences of such expressions in declaratives (cf. Rullmann (2003), Kripke (1991/2009), Heim (1992) among others). This is

[^0]an ongoing trend (cf. Ahn (2015)).
However, additive particles are not restricted in distribution to declaratives, they also occur in questions:
(8) Mary smokes. Does she $\operatorname{drink}_{F}$ too?
(9) Mary smokes. Does John ${ }_{F}$ smoke too?
(10) Mary or John smokes. Does Bill $_{F}$ smoke too?
(11) Does Mary $F_{F}$ dance or sing too? disjunctive polar reading
(12) Does Mary (only) dance ${ }_{F}$, or $\operatorname{sing}_{F}$ too?
(13) Does Mary dance $F_{F}$ too, or (only) $\operatorname{sing}_{F}$ ?
(14) Everybody smokes. Who drinks ${ }_{F}$ too?
(15) I want ice cream. Who ${ }_{F}$ wants ice cream too?
summoning question
(16) \#Does Mary dance $F_{F}$ or $\operatorname{sing}_{F}$ too? alternative reading
(17) \#Does Mary dance $F_{F}$ too, or $\operatorname{sing}_{F}$ too?
(18) Mary smokes. \#Who ${ }_{F}$ smokes too?
(19) Mary or John smokes. \#Who ${ }_{F}$ smokes too?

The data shows that too is in general unproblematic in polar questions as indicated by examples (8)-(11). Moreover, too is also felicitous in alternative questions when it occurs within exactly one of the disjuncts (examples (12) and (13)). In case of who-questions, we see that too is felicitous when associating with the verb (example (14)), but also when associating with the $w h$-phrase itself (given our data set).

But too is not felicitous in all questions, and so focus particles are not felicitous in all questions. Example (16) shows that too leads to a deviant question when it does not occur within one of the disjuncts but adjoins at the disjunction. Similarly, in example (17) we see that too cannot occur in both disjuncts. Example (18) offers an interesting contrast to example (15). Unlike in (15), associating too with the wh-phrase results in a deviant question. The same contrast is offered by example (19). Example (10) suggests in connection with item (19) that the latter is not deviant due to the disjunction preceding it. There must be something about associating too with the wh-phrase that makes the question deviant, but is absent in (15). The question now is why the data is that way, and this thesis provides an answer to it.

As a matter of fact, there are many semantic approaches to questions, for instance: alternative semantics, partition semantics, structured meanings, and inquisitive semantics, among others. But there are only a few semantic frameworks readily suitable for the study of additive particles in questions. This is, on the one hand, due to the research focus of the last decades. On the other hand, another reason can be seen in the involved semantics of additive particles. Additive particles are not only focus sensitive, they are also standardly seen as presuppositional. This is highly suggested by the inference test. Using an example with too we see:
(20) $\mathrm{John}_{F}$ smokes too.
$\leadsto$ Someone else smokes.
John $_{F}$ smokes.
$\leadsto$ Someone else smokes
It is not the case that $\operatorname{John}_{F}$ smokes too.
$\leadsto$ Someone else smokes
(23) $\# \mathrm{John}_{F}$ smokes too, but in fact no one else smokes.
(20) shows that there is an inference that someone besides John smokes. (21) shows that this inference arises from too. (22) highly suggests that this inference is not an entailment, but a presupposition. ${ }^{3}$ (23) suggests that the inference is not an implicature.

This makes clear that a semantic framework for the study of additive particles in questions needs to be capable of representing presuppositions and focus. A few such frameworks exist, notably the account of Aloni, Beaver, Clark, and van Rooij (2007), Beck (2006) which is the foundation for other accounts, such as Beck and Kim (2006), Kotek (2016), a.o., and Balogh (2009).

The framework of Aloni et al. (2007) is motivated by earlier work in dynamic semantics of questions and as a result it is very technical, as it models the dynamics of domains and information at the same time and in relation to each other at a subsentential level. It models focus as a presupposition trigger in the sense of Roberts (1996) and relies on a version of the presupposition operator $\partial$ of D . Beaver (2001) for modeling presuppositions.

The approach has to face some issues though which are technical in nature. One problem is that it is a first order semantics. It cannot deal with higher-order quantification. For the same reason, it can also only deal with focus on expressions of type $e$, such as proper names, John say. Focus on verbs or other constituents, which is empirically attested, can simply not be captured by the framework. Of course, this is a challenge that can be solved. The more severe problem lies in the fact that the framework is non-compositional (also a consequence of being first-order) and that its treatment of focus operators is non-compositional too. Again, these issues can be solved, but are cumbersome.

Balogh (2009) provides a first framework for focus particles (only) in inquisitive semantics. Her account relies on J. Groenendijk (2008) and J. Groenendijk and Roelofsen (2009) which is an older version of inquisitive semantics. The account is more pragmatically oriented and models focus in terms of the theme/rhemedistinction, which is very similar to the treatment of focus in Aloni et al. (2007). Also similar to Aloni et al. (2007), Balogh's account is a first-order semantics. This has the same consequences as above: it is non-compositional, focus is restricted to type $e$ expressions. Moreover, the computation of the theme is non-compositional, and so the focus semantics is non-compositional.

Beck (2006) is a compositional framework which makes use of alternative semantics for questions, uses a Rooth-style focus semantics (which is compositional), and relies on presuppositions à la Heim and Kratzer (1998). As such the framework is better off than the two previously mentioned. Nevertheless, the framework has to face some major challenges, not only technical, but also conceptual in nature. Recent work in inquisitive semantics has provided arguments against the architecture of alternative semantics (cf. Ciardelli, Roelofsen, and Theiler (2017)). The compositional machinery of alternative semantics relies heavily on pointwise function application, instead of the standard function application. This has severe drawbacks for the treatment of functors in natural language such as negation. The issue is that these expressions need access to the denotation of their argument at once. Such cannot be done when relying on point-wise function application. Consequently, these expressions must be dealt with syncategorematically within alternative semantics when compositionallity wants to be achieved. From a theoretical perspective, this is undesirable for the resulting grammar would need special rules for these functors (cf. Ciardelli et al. (2017): 4-5). Another issue lies with predicate abstraction. There is simply no straightforward way of defining such an operation satisfactorily (cf. Ciardelli et al. (2017): 5). Consequently, even though alternative semantics is compositional, the compositional machinery provides problems which cannot be resolved easily or not at all without moving to another framework. Other drawbacks of alternative semantics for questions are discussed in Ciardelli et

[^1]al. (2017) and Ciardelli and Roelofsen (2017).

Since the mentioned frameworks have major shortcomings, we propose to develop a new framework for the study of additive particles in questions in particular, and for focus phenomena (notably focus particles) in general. The system will be a compositional inquisitive semantics with presuppositions and focus and will allow for the study of too in questions, thereby allowing for explaining the presented data.

The major goals of the thesis are thus:

Goal 1: Providing a semantic explanation of the presented data on too in questions.

Goal 2: Providing an inquisitive semantics for the study of focus phenomena, especially focus particles.

In the remainder of this introduction we want to clarify the notion of focus to be used in the thesis, to introduce the underlying idea for the construction of the system, and to provide an overview of the content of the chapters.

### 1.1 Overview on the formal system CRISP

As described the major goals of the thesis are to provide an explanation for the data presented earlier by using an appropriate inquisitive semantics. We also saw that such a semantics needs to be developed in the first place. The first goal of the thesis will therefore be to develop such a formal system. The formal system will be called CRISP, which stands for Compositional Roothian Inquisitive Semantics with Presuppositions.

The system will make use of Rooth-style focus semantics, in particular in the spirit of Rooth (1992). Unlike other implementations of Rooth (1992), we will make the focus semantics part of the object language which will allow us to encode many aspects of the meaning of additive particles directly in their lexical entry. The other component comes from recent developments in inquisitive semantics. The system will rely on the compositional presuppositional inquisitive semantics of Champollion et al. (2017). This semantics, which is still under construction, draws from the compositional semantics of Theiler (2014) as well as Ciardelli et al. (2017). The system as such, however, will not simply be a fusion of these different components. As we will see some extensions and changes are in need in order to get these components together.

The resulting system will satisfy all the earlier stated requirements for the study of additive particles in questions, and thus too in questions. However, we can readily see that the system is not restricted to this class of expressions. It can in fact be used for the study of other focus particles, such as only, and other focus phenomena. It can be used in the study of presuppositions in questions (the initial intent of Champollion et al. (2017)). Therefore, the resulting system offers various applications for different phenomena in the study of questions, but also declaratives due to its being an inquisitive semantics.

### 1.2 Focus

A discussion of the notion of focus at work is necessitated by the very nature of the notion:
"The basic notions of Information Structure (IS), such as Focus, Topic and Givenness, are not simple observational terms. As scientific notions, they are rooted in theory, in this case, in theories
of how communication works."
(Krifka (2008): 243)
Krifka points out that there is not simply a notion or concept of focus that linguists make use of, or that we can find out in the wild. Focus is a theoretical term used by linguists in theorizing. As the quote makes clear as well, the notion of focus applicable for the linguist depends on the theory of the linguist. In our case, we said that we will make use of Roothian focus semantics, and so it makes sense to simply make use of its underlying notion of focus:
"Focus indicates the presence of alternatives that are relevant for the interpretation of linguistic expressions."
(Krifka (2008): 247)
We will see in the course of the thesis what this is meant to be.

### 1.3 Overview of the thesis

The thesis consists of two parts. The first part is devoted to the development of CRISP and comprises the chapters 2 and 3 . The second part is devoted to the application of CRISP in explaining the data and consists of chapters 4,5 , and 6 .

In chapter 2 we will introduce the materials used in the construction of CRISP in chapter 3. Starting with Roothian focus semantics, we will first introduce the basic ingredients of Rooth (1985). Particularly, we will introduce the idea of Rooth (1985) of how focus is semantically interpreted (signaling of alternatives) and how he captures this idea formally. In Rooth (1985) this idea is formalized by means of a compositional two-dimensional semantics. In the next step we will turn to the adjustments to Rooth (1985) presented by Rooth himself in Rooth (1985). The result will be a presuppositional focus semantics.

Once the essentials of Rooth (1985) and Rooth (1992) used in our framework are presented and clarified, we will turn to the compositional presuppositional inquisitive semantics of Champollion et al. (2017). We will provide the principal idea underlying this inquisitive semantics and provide its formal details. A small fragment will be provided at the end in order to show its working and familiarizing the reader with the notation.

In chapter 3 we will develop the formal semantics CRISP. The construction will proceed in two steps. We will first show that we cannot simply combine Rooth (1985), Rooth (1992) and Champollion et al. (2017). The problem we will encounter is that the novel operator introduced in Rooth (1992) cannot appropriately be implemented in Champollion et al. (2017). It would not be a presuppositional operator in the sense of the formal system of Champollion et al. (2017). We present then a few strategies that can resolve this issue. We argue for one of the solutions and construct CRISP along the lines of it. This will form the second step of the construction. Many of the technical details (the formal language, its semantics) will be provided in Appendix A to keep the discussion smooth and accessible.

Chapter 4 provides a first fragment of English. We will discuss the semantics of plain polar questions (e.g. Does John smoke?), disjunctive polar questions (e.g. Does Mary dance or sing?), and alternative questions
(e.g. Does Mary dance $F_{F}$ or $\operatorname{sing}_{F}$ ?). We will restrict ourselves here to questions in matrix form. The chapter will utilize earlier work in inquisitive semantics in stating a semantics for these constructions, in particular the semantics of lists as developed in Roelofsen and Farkas (2015).

Chapter 5 deals with who-questions which will be the only $w h$-questions we will consider in this thesis. Here, we will provide our own account, deviating from the fragment provided in Champollion et al. (2017) and other inquisitive semantics accounts of who. The change is motivated empirically and conceptually. The account provided there allows for an easy focus semantics of who and an easier ordinary semantics of who when compared to Champollion et al. (2017). It also allows for easier compositions of multiple wh-questions compared to Champollion et al. (2017) while having all the advantages of it with respect to mention-some readings. For mention-all readings we will follow the proposal by Dayal (1996) and assume that $D_{e}$ is closed under Link's (1983) individual sums. We will further utilize an exhaustivity operator $X$. This will be similar to ideas found in Klinedinst and Rothschild (2011) and Uegaki (2015).

Chapter 6 deals with the too in questions. It is divided into two parts. Starting from what we think to be the core data on $t o o$, its focus-sensitivity and the additive inference associated with it, we will discuss different ways of characterizing the latter. In this way we also provide a small overview on different semantics for too. In the next step we will consider a few syntactic and prosodic properties of too. Last, we provide an account of too in CRISP. The account will be motivated by empirical, conceptual and technical considerations. The second part is devoted to the explanation of the data.

Chapter 7 concludes.

Last we provide Appendix A which will contain most of the formal details on the type theory we are using, as well as the semantics of CRISP.

## Chapter 2

## Preliminaries: Roothian Focus Semantics and Presuppositional Inquisitive Semantics

This chapter provides the theoretical and formal background for the semantics developed in chapter 3 . We will consider the focus semantics of Rooth (1985), Rooth (1992) and the compositional inquisitive semantics with presuppositions of Champollion et al. (2017). The chapter is organized into two parts: we will first consider the focus semantics of Rooth (1985) and Rooth (1992). Considering them is motivated by two points: (i) it accords straightforwardly with our notion of focus, for we can take it to be the very source of that notion, and (ii) Rooth's semantics is particularly simple. We start with a discussion of the basic idea of Rooth (1985) and how this idea is formalized. We will see that Rooth (1985) provides us with the basic ingredients needed for modeling the semantics of focus-sensitive particles. This we will illustrate with the focus-sensitive particle only. The same particle motivates some changes though. We will see that the semantics for only we can assign in Rooth (1985) is too restrictive. This is why we will move from Rooth (1985) to Rooth (1992). Rooth (1992) is an extension of Rooth (1985). It resolves the issues surrounding only by means of a presupposition operator, ' $\sim$ '. The resulting focus semantics will be used in our formal semantics to be developed in chapter 3 .

The second part of the chapter is devoted to the compositional inquisitive semantics with presuppositions of Champollion et al. (2017). As the name suggests, this semantics provides us with a compositional procedure for the derivation of questions and assertions. Moreover, it includes a treatment of presuppositions. These aspects are crucial for our own endeavor as we are interested in additive particles in questions. The former, as we saw in chapter 1 are focus-sensitive (motivating the focus semantics part) and are presupposition triggers (motivating the presupposition part). The semantics of Champollion et al. (2017) provides us with the necessary tools to account for the presuppositional part of the semantics of additive particles (and other focus-sensitive operators, e.g. only). However, the framework of Champollion et al. (2017) is alone insufficient, as it lacks focus semantics. For this reason we will combine both frameworks into a single framework. This is the subject matter of chapter 3 .

### 2.1 Roothian Focus Semantics

We saw that particles like too and only are sensitive to focus marking. In particular we saw that either their truth-conditional meaning depended on this focus marking (only), or their presupposition did so (too). When we want to describe a formal semantics for such particles and additive particles in particular, we need to somehow model this influence of the focus marking. The question is how. Rooth (1985) provides a very easy but powerful answer to this. We will now discuss his idea of what focus does semantically and how we can formalize this. We will see that his basic idea can be spelled out in a two-dimensional semantics, which defines for each expression two semantic values, $\llbracket \cdot \rrbracket^{o}$ and $\llbracket \cdot \rrbracket^{f}$. The former is called the ordinary semantic value, the latter focus semantic value. We will discuss these in detail. We then show how Rooth (1985) can account for the meaning of only. The same word, we will show, motivates to extend Rooth (1985) to Rooth (1992). This extension is achieved by adding an operator ' $\sim$ '. We will discuss this in detail as well, showing how it accounts for the issues encountered earlier.

### 2.1.1 The Basic Idea of Rooth (1985) and its formal interpretation

In Rooth (1985) it is assumed that focus is an abstract syntactic feature $F$ marked on syntactic phrases in S-Structure. Given the generative view (cf. Rooth (1985): 10), the feature can find interpretation at LF (logical form) and realization on PR (phonological realization). Two questions thus arise:

1. How is the focus feature $F$ semantically interpreted?
2. How is the focus feature $F$ phonologically realized?

Rooth's (1985) answer to 1 . is that focus is semantically interpreted as signaling alternatives (cf. Rooth (1985): 10, 13). His answer to 2 . is that (in English at least) the feature is phonologically realized by intonational prominence (cf. Rooth (1985): 1). Rooth (1985) is interested in the semantics of focus and so are we. Nevertheless, the reader should keep in mind how focus is said to be phonologically realized. This is of importance for later stages of this thesis.

The signaling of alternatives is formally captured in the definition of the so-called p-sets:
Definition 2.1.1. ( $p$-sets, cf. Rooth (1985): 14)
Let $T(A)$ be the translation of the natural language expression $A$ in some formal language (e.g. TY ${ }_{2}$ or Montague's $I L$ ). Let $\llbracket T(A) \rrbracket^{o}$ be its ordinary value ${ }^{1}$. We define the focus value of $A, \llbracket T(A) \rrbracket^{f}$, recursively:
(i) If $T(A)$ bears an $F$-feature, then $\llbracket T(A) \rrbracket^{f}=\left\{\llbracket X \rrbracket^{o}: X\right.$ is of the same type as $\left.T(A)\right\}$
(ii) If $T(A)$ is not $F$-marked and $A$ is a non-complex phrase, then $\llbracket T(A) \rrbracket^{f}=\left\{\llbracket T(A) \rrbracket^{o}\right\}$
(iii) If $T(A)$ is not $F$-marked and $A$ is a complex phrase $[\beta \gamma]$, then

$$
\llbracket T(A) \rrbracket^{f}= \begin{cases}\left\{f(d): f \in \llbracket T(\beta) \rrbracket^{o} \wedge d \in \llbracket T(\gamma) \rrbracket^{o}\right\} & \text { if } T(\beta) \text { is of type }(\sigma, \tau) \text { and } T(\gamma) \text { is of type } \sigma \\ \left\{f(d): f \in \llbracket T(\gamma) \rrbracket^{o} \wedge d \in \llbracket T(\beta) \rrbracket^{o}\right\} & \text { if } T(\gamma) \text { is of type }(\sigma, \tau) \text { and } T(\beta) \text { is of type } \sigma\end{cases}
$$

Hence, Rooth provides a two-dimensional semantics for natural language. Each logical form phrase $A$ for some natural language fragment is assigned two different denotations. There is the focus insensitive denotation, formally denoted by $\llbracket \cdot \rrbracket^{o}$, usually referred to as ordinary semantic value, and there is the focus sensitive

[^2]denotation, formally denoted by $\llbracket \cdot \rrbracket^{f}$, usually referred to as focus semantics value. The focus semantic value of an expression $\alpha$ is a p-set. Later in this thesis, we will reserve the symbols $\llbracket \cdot \rrbracket^{o}$ and $\llbracket \cdot \rrbracket^{f}$ for the system CRISP.

When putting these definitions into practice, we may have something along the following lines: consider the little fragment of English below which comprises a proper name Mary and an intransitive verb laugh. In the table we use $\dot{X}$ for expressions of the object language and we use $X$ for the denotation of the object language symbol $\dot{X}$. This is because Rooth translates natural language expressions into a formal language (IL) and assigns this language a model-theoretic semantics. The notation helps us to not get confused.

| CAT | Item $\alpha$ | $\operatorname{Tr}(\alpha): \tau$ | $\llbracket \operatorname{Tr}(\alpha) \rrbracket^{o}$ | $\llbracket \operatorname{Tr}(\alpha) \rrbracket^{f}$ |
| :---: | :---: | :---: | :---: | :---: |
| PN | Mary | $\dot{m}: e$ | $m$ | $D_{e}$ or $\left\{\llbracket \dot{m} \rrbracket^{o}\right\}$ |
| IV | laugh | $\dot{L}:\langle e, t\rangle$ | $L$ | $D_{\langle e, t\rangle}$ or $\left\{\llbracket \dot{L} \rrbracket^{o}\right\}$ |
| S | Mary laughs | $\dot{L}(\dot{m}): t$ | 1 or 0 | $D_{t}$ or $\left\{\llbracket \dot{L}(\dot{m}) \rrbracket^{o}\right\}$ |

We assign these items very simple translations as displayed in the table: ${ }^{2}$ names are translated into constant symbols $\dot{c}$ of type $e$ and intransitive verbs are translated into constants $\dot{P}$ of type $\langle e, t\rangle$. The fragment allows us to form the sentence Mary laughs which is represented by the formula $\dot{L}(\dot{m})$ and results from standard function application. The ordinary semantic values are then the standard denotations of these expressions, i.e. for $\llbracket \dot{m} \rrbracket^{o}$ we have some individual in our domain of discourse $(m)$, for $\llbracket \dot{L} \rrbracket^{o}$ we get a set of individuals (the laughing individuals of the domain of discourse, L) and the sentence Mary laughs simply denotes a truth-value, 1 or 0 . Now, the focus semantic value of $M a r y_{F}$ is then simply the set of objects in $D_{e}$ which is $D_{e}$. Similarly for $l a u g h_{F}$, i.e. $\llbracket \dot{L} \rrbracket^{f}=D_{\langle e, t\rangle}=\left\{x: x \in D_{\langle e, t\rangle}\right\}$. So among the focus alternatives of laugh are properties such as laugh, die, but also $d o g$ and gun (which seems counter-intuitive, but that's what we get here). Now, for Mary ${ }_{F}$ laughs we get $\llbracket \dot{L}(\dot{m}) \rrbracket^{f}=\left\{L(x): x \in D_{e}\right\}$. Hence, alternatives to Mary are Billy, Ann, etc. So, here we get that among the alternatives to Mary laughs are Billy laughs, Anna laughs, etc. And similarly in case laughs is F-marked, $\llbracket \dot{L}(\dot{m}) \rrbracket^{f}=\left\{P(m): P \in D_{\langle e, t\rangle}\right\}$. Given these examples, the idea and its formal realization in Rooth (1985) should be clear.

Now we can provide a reasonable meaning rule for only: ${ }^{3}$
Definition 2.1.2. (meaning rule for only) ${ }^{4}$
$\llbracket \dot{\operatorname{onl}}(\dot{p}) \rrbracket^{o}=\forall q[(q \in C \wedge q) \rightarrow p=q]$, where $\dot{p}$ is the translation of $\mathrm{V}^{\prime}$
$\llbracket \operatorname{only}(\dot{p}) \rrbracket^{f}=\left\{\llbracket \operatorname{only}(\dot{p}) \rrbracket^{o}\right\}$
Here, $C$ is the domain of quantification for only and is identified with the focus-value of $\dot{p}$ (cf. Rooth (1992): 77). This provides a more or less adequate meaning rule for only. However, as we will see in the next subsection, it is too strong. With these points we will close this subsection.

[^3]
### 2.1.2 The Adjustments of Rooth (1992)

In Rooth (1992) the following considerations are put forward: take the sentence Mary only read ${ }_{F}$ The Recognitions. For the following, let us translate read simply by $\lambda y \cdot \lambda x \cdot R(x, y)$, and The Recognitions by $r$. We can then provide the following translation for the sentence: $\forall q[(q \in C \wedge q(m)) \rightarrow q=\lambda x . R(x, r)] .{ }^{5}$ Now, this is true if and only if, Mary did indeed nothing besides reading The Recognitions. But, under any reasonable circumstances, Mary did more with it. She at least looked at it! Since look at is a focus-alternative of read, the statement is judged false given the above. This shows that the above is too strong.

Rooth (1992) found a reasonable and easy solution to this issue. Instead of identifying the domain of quantification $C$ with the focus-alternatives of the $F$-marked phrase, we should take it to only restrict the value of $C$ leaving the actual value of $C$ to pragmatics (cf. Rooth (1992): 77-79). Indeed, this solves the issue. Assume it is indeed true that Mary read The Recognitions, but she did not understand it. Then, if $C$ consist only of the alternatives read The Recognitions and understood The Recognitions, then Mary only read $_{F}$ the Recognitions comes out true.

On the basis of the above observations as well as similar observations in other cases which we did not discuss here (cf. Rooth (1992): 79-82, 82-82, 84-85), Rooth proposes the Focus Interpretation Principle:

Definition 2.1.3. (Focus Interpretation Principle, Rooth (1992))
When interpreting focus at the level of a phrase $A$ add one of the following constraints:
(i) set case: $\llbracket A \rrbracket^{o} \in \Gamma \wedge \exists \gamma\left[\left(\llbracket A \rrbracket^{o} \neq \gamma\right) \wedge(\gamma \in \Gamma) \wedge\left(\Gamma \subseteq \llbracket A \rrbracket^{f}\right)\right]$
(ii) individual case ${ }^{6}: \gamma \in \llbracket A \rrbracket^{f} \wedge \gamma \neq \llbracket A \rrbracket^{o}$

The additional constraints which were not made explicit above are empirically motivated (cf. Rooth (1992): 90). The variable $\Gamma$ in (i) is introduced by the focus interpretation. Similarly for $\gamma$ in (ii). The constraints expressed by (i) and (ii) are thus constraints on the value of the variables $\Gamma$ and $\gamma$ respectively.

Note that the Focus Interpretation Principle as stated above can be simplified significantly:
Definition 2.1.4. (Focus Interpretation Principle, simplified)
When interpreting focus at the level of a phrase $A$ add one of the following constraints:
(i) set case: $\llbracket A \rrbracket^{o} \in \Gamma \wedge \Gamma \subseteq \llbracket A \rrbracket^{f} \wedge|\Gamma| \geq 2$.
(ii) individual case: $\gamma \in \llbracket A \rrbracket^{f} \wedge \gamma \neq \llbracket A \rrbracket^{o}$

It would be nice to subsume the individual case under the set case. Unfortunately, this is only possible, when we assume that always $\llbracket A \rrbracket^{o} \in \Gamma$ for $A F$-marked, for the individual case only requires that $\gamma \in \llbracket A \rrbracket^{f}$, but not that $\llbracket A \rrbracket^{o} \in \Gamma$.

The main problem for Rooth consists in making this part of his formal theory. He chooses to implement the Focus Interpretation Principle by means of a presupposition operator ' $\sim$ '. His reason is that the constraints expressed in the Focus Interpretation Principle do not show up in the content of a sentence (cf. Rooth (1992): 91-92). They stay backgrounded. This is a characteristic property of presuppositions. The Focus Interpretation Principle in terms of $\sim$ reads:

[^4]Set case: $\phi \sim \Gamma$ presupposes that $\Gamma$ is a subset of the focus value for $\phi$ and contains both the ordinary value of $\phi$ as well as an element distinct from it.

Individual case: $\phi \sim \gamma$ presupposes that $\gamma$ is an element of the focus value for $\phi$ and distinct from $\phi$ 's ordinary value.

Some clarifications about the syntax and the semantics of $\sim$ are in need. Following Rooth, we will treat $\sim$ as an operator that comes in on the level of logical form (cf. Rooth (1992): 94). It attaches with a free variable to a node at LF. This variable is introduced by the focus interpretation. Further, the variable is identified with a constituent, the antecedent, at LF of the appropriate type. The type is determined by the pragmatic or semantic construction which makes use of focus and triggers the focus interpretation. ${ }^{7}$. ~ is only a presuppositional operator, meaning that $\llbracket A \sim C \rrbracket^{o}=\llbracket A \rrbracket^{\circ}$. And last, $\sim$ blocks any kind of 'focus projection': $\llbracket A \sim C \rrbracket^{f}=\left\{\llbracket A \rrbracket^{o}\right\}$. This is because at this point the focus interpretation already happened (cf. Rooth (1992): 94-95).

Again, we can simplify this significantly by simplifying the set case: $\phi \sim \Gamma$ presupposes $\llbracket \phi \rrbracket^{o} \in \Gamma \wedge \Gamma \subseteq$ $\llbracket \phi \rrbracket^{f} \wedge|\Gamma| \geq 2$. The cardinality constraint can be dropped for the constructions we are interested in. This is because additive particles have such constraints as part of their meaning. We thus do not need to encode them as part of the meaning of $\sim$. For now this doesn't matter and we stick to the above.

Finally, let us see how this solves the issue with only. We have: ${ }^{8}$


Thus, focus is interpreted at the level of $\mathrm{V}^{\prime}$. This means: $\llbracket \lambda x \cdot R(x, r) \rrbracket^{o} \in C \wedge C \subseteq \llbracket \lambda x . R(x, r) \rrbracket^{f} \wedge|C| \geq 2$ is presupposed. Given the considerations above, $C$ may thus happen to be the set consisting only of read The Recognitions and understood The Recognitions. In that case, Mary indeed only read The Recognitions. The new semantics thus solves our initial problem.

[^5]
### 2.2 Compositional Inquisitive Semantics with Presuppositions

In this section we will introduce the compositional inquisitive semantics with presuppositions of Champollion et al. (2017). It will serve as basis for our semantics in Chapter 3. Similar to the preceding section, we will first provide the basic idea, followed by its formal interpretation. We will also provide a few facts about the presupposition projection of the connectives, quantifiers and operators used in this semantics as well as a few examples.

### 2.2.1 The Basic Idea of Champollion et al. (2017)

In inquisitive semantics the meaning of a sentence is taken to be the set of classical propositions (states) that contain enough information to resolve the issue expressed by it. Champollion et al. (2017) add presuppositions into this picture. The basic idea of Champollion et al. (2017) is to conceive of states (which they call possibilities) not simply as sets of worlds, but as updates. These are specified by an effect and its preconditions. The former is the at-issue information of the state, and the latter is the presupposition of the state. Intuitively, if an effect has preconditions, then it cannot come about in a world in which its preconditions are not satisfied. We may thus see the at-issue information as a set of worlds and its preconditions as another set of worlds. Further, the at-issue information must be included in its presuppositions. We thus arrive on a picture of states which formally sees them as pairs of sets of worlds.

### 2.2.2 Compositional Inquisitive Semantics with Presuppositions

The new perspective on states results formally in a switch from $T Y_{2}$ to $T T_{2} . T T_{2}$ is a relational type theory (cf. Appendix A for details), thus allowing for types such as $\alpha \times \beta$. The relational perspective comes with some technical advantages. State denoting expressions play a different role in the formal system from other expressions. They are supposed to provide two kinds of information, namely the presupposition, and the at-issue information. Now, when treating them directly as tuples, we have immediate access to both kinds of information. This will become clear in due course. We will deal with the technical aspects of $T T_{2}$ in Appendix A and leave type theory matters aside (if possible). The reader will note that there is no huge difference between $T Y_{2}$ formulas and the formulas at work in Champollion et al. (2017). The only essential difference is that we have expressions of relational types $\sigma \times \tau$.

We will formally introduce the basic notions of Champollion et al. (2017):
Definition 2.2.1. (State $s$ ) Let $W$ be the logical space (set of worlds). A tuple $s=\langle X, Y\rangle$ with $Y \subseteq X \subseteq W$ is a state. Here, $X$ is the presupposition and $Y$ the at-issue information.

States can be ordered. Assume we have state $s=\langle X, Y\rangle$ and $t=\langle U, V\rangle$. When is one of them an enhancement of the other? An enhancement should satisfy two things in our setting. First, as we would expect, it should be at least as informative as the state it enhances. So, when we find that $s$ enhances $t$ we suspect $s$ to provide at least the information $t$ provides. So, we should find that $Y \subseteq V$. Secondly, an enhancement should still satisfy the presuppositions of the state it enhances. So, we also should have that $X \subseteq U$. This provides us with the following formal definition:

Definition 2.2.2. ( $s \sqsubseteq t$, substate) Let $s=\langle X, Y\rangle$ and $t=\langle U, V\rangle$. We have that $s$ is a sub-state of $t$ (or that $s$ enhances $t$ ), in symbols $s \subseteq t$ if and only if $X \subseteq U$ and $Y \subseteq V$.

Clearly, $\sqsubseteq$ is a partial order on states.

Given the notion of state and the ordering on them we can define propositions.
Definition 2.2.3. (Proposition) A proposition $P$ is a non-empty, downward closed set of states, i.e. $P \neq \varnothing$ and for all states $s, t$, if $s \in P$ and $t \sqsubseteq s$, then $t \in P$.

Similar to states, also propositions can be ordered. In particular, they are ordered under the subset relation $\subseteq$, as can be seen easily. Moreover, the subset relation constitutes the entailment relation. In inquisitive semantics, we define entailment in terms of support.

Definition 2.2.4. (Support) Let $s$ be a state and $P$ a proposition. We say that $s$ supports $P$ iff $s \in P$.
Definition 2.2.5. (Entailment)

Let $P, Q$ be propositions. We have that $P$ entails $Q$, in symbols $P \vDash Q$, iff $\forall s(s \in P \rightarrow s \in Q)$.

In general: let $\Gamma=\left\{\phi_{1}, \ldots, \phi_{n}\right\}, \Delta=\left\{\psi_{1}, \ldots, \psi_{m}\right\}, m, n \in \mathbb{N}$, be sets of propositions. We have $\Gamma \vDash \Delta$ iff $\cap \Gamma \subseteq \cup \Delta$.

That entailment boils down to the subset-relation is obvious from the above.

The ontology of Champollion et al. (2017) gives rise to rather complex types. As stated earlier, we need to assign expressions denoting states type $\langle s, t\rangle \times\langle s, t\rangle$. And since propositions are non-empty downward closed sets of states, they are denoted by expressions of type $T:=\langle\langle s, t\rangle \times\langle s, t\rangle, t\rangle$. Given this, we can obtain the types of other linguistically relevant kinds of expressions easily.

The following abbreviations in the object language are used:
Definition 2.2.6. (Abbreviations in the object language)

| $\pi_{s}:=p_{1}(s)$ | the presupposition of state $s$ |
| :---: | :---: |
| $\alpha_{s}:=p_{2}(s)$ | the at-issue information of state $s$ |
| $s^{\top}=\left\langle\pi_{s}, \pi_{s}\right\rangle$ | maximizing the at-issue information of state $s$ |
| $s^{\perp}=\left\langle\pi_{s}, \varnothing\right\rangle$ | minimizing the at-issue information of state $s$ |
| $s \sqsubseteq t:=\alpha_{s} \subseteq \alpha_{t} \wedge \pi_{s} \subseteq \pi_{t}$ | $s$ is a substate of $t$ |
| $\operatorname{true}(P, w):=P(\langle\{w\},\{w\}\rangle)$ | $P$ is true at $w$ |
| $\operatorname{presup}(P):=\lambda s . P\left(s^{\perp}\right)$ | the presupposition of $P$ |
| $\|P\|:=\lambda w \cdot \operatorname{true}(P, w)$ | the truth set of $P$ |
| $s[P]:=\left\langle\pi_{s} \cap\right\| P\left\|, \alpha_{s} \cap\right\| P\| \rangle$ | updating state $s$ with proposition $P$ |

Here, $p_{1}, p_{2}$ are projection functions. Given these abbreviations we will write states $s$ as $\left\langle\pi_{s}, \alpha_{s}\right\rangle$ which is simply for convenience.

Next come the definitions of the inquisitive connectives and transplication:
Definition 2.2.7. (Inquisitive Connectives and Transplication)

| $\mathrm{T}:=\lambda s .(\lambda t . t)=(\lambda t . t)$ | verum (or top) |
| :---: | :---: |
| $\perp:=\lambda s .\left(\alpha_{s}=\varnothing\right)$ | falsum (or bottom) |
| $P \wedge Q:=\lambda s . P(s) \wedge Q(s[P])$ | conjunction |
| $P \rightarrow Q:=\lambda s .\left(P\left(s^{\perp}\right) \wedge \forall t \subseteq s .(P(t) \rightarrow Q(t[P]))\right.$ | inquisitive implication |
| $\rightarrow P:=P \rightarrow \perp$ | inquisitive negation |
| $P \mathbb{W}:=\lambda s . P(s) \vee Q(s)$ | inquisitive disjunction |
| $P_{Q}:=\lambda s . P(s) \wedge Q\left(s^{\top}\right)$ | transplication |

Here we added T . We use bold face $\mathrm{T}, \perp$ in order to sufficiently delineate these from the symbols $\mathrm{T}, \perp$ used in the abbreviations $s^{\top}$ and $s^{\perp}$. Last we will define inquisitive quantifiers and operators:

Definition 2.2.8. (Inquisitive existential and universal quantifier, projection operators and entailment)

| $? P:=P \nVdash \curvearrowleft P$ | inquisitive projection |
| :---: | :---: |
| $\sqcup S:=\left\langle\cup_{s \in S} \pi_{s}, \cup_{s \in S} \alpha_{s}\right\rangle$ | supremum |
| $!P:=\lambda s . \exists S .(\sqcup S=s \wedge \forall t \in S: P(t))$ | informative projection |
| $\langle ?\rangle:=\lambda P . \lambda s . P(s) \vee((P=!P) \wedge \rightharpoondown P(s))$ | conditional inquisitive projection |
| $\nVdash x . P:=\lambda s . \forall x . P(s)$ | inquisitive universal quantifier |
| $\exists x . P:=\lambda s . \exists x . P(s)$ | inquisitive existential quantifier |
| $P \vDash Q:=\forall s . P(s) \rightarrow Q(s)$ | entailment |

### 2.2.3 Projection Behavior of inquisitive connectives, quantifiers, and operators

In this section we will consider the presupposition projection behavior of the inquisitive connectives, quantifiers, and operators. We will supply two examples at the end of the subsection for illustration.

Fact 1. (Projection of $\mathbb{x}, \rightarrow, \pi) ~ x, \rightarrow, \rightarrow$ encode the projection behavior of conjunction, implication, and negation stated in Karttunen (1973). In particular:

$$
\begin{aligned}
\wedge & : \operatorname{presup}(P \wedge Q)=\lambda s \cdot P\left(s^{\perp}\right) \wedge Q\left(s[P]^{\perp}\right)=\lambda s \cdot P\left(\left\langle\pi_{s}, \varnothing\right\rangle\right) \wedge Q\left(\left\langle\pi_{s \cap|P|}, \varnothing\right\rangle\right) \\
\rightarrow & : \operatorname{presup}(P \rightarrow Q)= \\
& \lambda s \cdot P\left(s^{\perp}\right) \wedge \forall t \sqsubseteq s^{\perp} \cdot(P(t) \rightarrow Q(t[P]))=\lambda s \cdot P\left(\left\langle\pi_{s}, \varnothing\right\rangle\right) \wedge \forall t \sqsubseteq\left\langle\pi_{s}, \varnothing\right\rangle .\left(P\left(\left\langle\pi_{t}, \varnothing\right) \rightarrow Q\left(\left\langle\pi_{t \cap|P|}, \varnothing\right\rangle\right)\right)\right. \\
\rightarrow & : \operatorname{presup}(\neg P)=\lambda s \cdot P\left(s^{\perp}\right) \wedge \forall t \sqsubseteq s^{\perp}(P(t) \rightarrow \perp(t[P]))=\lambda s \cdot P\left(s^{\perp}\right)
\end{aligned}
$$

Fact 2. (Projection of $P_{Q}$ ) Transplication $P_{Q}$ presupposes $Q$ and all presuppositions of $P$ :
$P_{Q}: \operatorname{presup}\left(P_{Q}\right)=\lambda t . P_{Q}\left(t^{\perp}\right)=\lambda t .\left(\lambda s .\left(P(s) \wedge Q\left(s^{\top}\right)\right)\left(t^{\perp}\right)\right)=\lambda t . P\left(t^{\perp}\right) \wedge Q\left(t^{\top}\right)$
Fact 3. (Projection of $w$ ) Inquisitive disjunction presupposes the disjunction of the presuppositions of $P$ and $Q$ :

$$
\mathbb{W}: \operatorname{presup}(P W Q)=\lambda t \cdot P\left(t^{\perp}\right) \vee Q\left(t^{\perp}\right)
$$

Fact 4. (Projection of $\exists$ and $\nVdash$ ) The inquisitive existential quantifier presupposes the existential quantification over the presupposition of its scope. The inquisitive universal quantifier presupposes the universal quantification over the presuppositions of its scope. In particular:

$$
\begin{aligned}
& \exists: \operatorname{presup}(\nexists x . P)=\lambda t . \exists x P\left(t^{\perp}\right) \\
& \nVdash: \operatorname{presup}(\nVdash x . P)=\lambda t . \forall x P\left(t^{\perp}\right)
\end{aligned}
$$

Fact 5. (Projection of ? and !) The inquisitive projection? presupposes the presupposition of its argument, and the informative projection presupposes the existence of a maximal state which satisfies the presuppositions of its argument. In particular:

$$
\begin{aligned}
& ?: \operatorname{presup}(? P)=\lambda t . P\left(t^{\perp}\right) \vee(\curvearrowleft P)\left(t^{\perp}\right)=\lambda t . P\left(t^{\perp}\right)=\operatorname{presup}(P) \\
& !: \operatorname{presup}(!P)=\lambda t \cdot!P\left(t^{\perp}\right)=\lambda t .(\lambda s . \exists T(\sqcup T=s \wedge \forall u \in T: P(u)))\left(t^{\perp}\right)=\lambda t \cdot \exists T\left(\sqcup T=t^{\perp} \wedge \forall u \in T: P(u)\right)= \\
& \quad!p r e s u p(P)
\end{aligned}
$$

We will now consider a few easy examples. In the following let $P:=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)$ and $Q:=\lambda t . \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w . q(w)$. Here, $P$ and $Q$ denote atomic, non-presuppositional propositions.

Example 1. (Transplication $P_{Q}$ ) We have $P_{Q}=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)_{\lambda t . \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)}$. We have then for $\operatorname{presup}\left(P_{Q}\right)$ :

$$
\begin{aligned}
\operatorname{presup}\left(P_{Q}\right)= & \lambda u \cdot\left[\lambda s \cdot \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)_{\lambda t \cdot \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)}\right]\left(u^{\perp}\right) \\
& =\lambda u \cdot\left[\lambda v \cdot\left[\lambda s \cdot \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)(v) \wedge \lambda t \cdot \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)\left(v^{\top}\right)\right]\right]\left(u^{\perp}\right) \\
& =\lambda u \cdot\left[\lambda s \cdot \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)\left(u^{\perp}\right) \wedge \lambda t \cdot \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)\left(u^{\top}\right)\right] \\
& =\lambda u \cdot\left[\alpha_{u^{\perp}} \subseteq \pi_{u^{\perp}} \wedge \alpha_{u^{\perp}} \subseteq \lambda w \cdot p(w) \wedge \alpha_{u^{\top} \subseteq \pi_{u^{\top}} \wedge \alpha_{\left.u^{\top} \subseteq \lambda w \cdot q(w)\right]}}=\lambda u \cdot\left[\varnothing \subseteq \pi_{u} \wedge \varnothing \subseteq \lambda w \cdot p(w) \wedge \pi_{u} \subseteq \pi_{u} \wedge \pi_{u} \subseteq \lambda w \cdot q(w)\right]\right. \\
& =\lambda u \cdot\left[\pi_{u} \subseteq \lambda w \cdot q(w)\right] \\
& =\lambda u \cdot \pi_{u} \subseteq \lambda w \cdot q(w)
\end{aligned}
$$

The reader should note that we will make heavy use of transplication in studying presuppositional constructions in natural language, as it serves as our primary tool for encoding presuppositions. The above is an abstract illustration of this.

Next we will consider an example with conjunction:

Example 2. (filtering, $)$ Let us consider $Q \wedge P_{Q}$. We have for $\operatorname{presup}\left(Q \wedge P_{Q}\right)$

$$
\begin{aligned}
& \operatorname{presup}\left(Q \wedge P_{Q}\right)=\operatorname{presup}\left(Q \wedge\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)_{\lambda t \cdot \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w . q(w)}\right]\right) \\
& =\operatorname{presup}\left(Q \wedge\left[\lambda v \cdot\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)(v) \wedge \lambda t . \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)\left(v^{\top}\right)\right]\right]\right) \\
& =\operatorname{presup}\left(\lambda u \cdot \left[\lambda t . \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)(u)\right.\right. \\
& \left.\left.\wedge\left[\lambda v .\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot p(w)(v) \wedge \lambda t . \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)\right]\left(v^{\top}\right)\right](u[Q])\right]\right) \\
& =\operatorname{presup}\left(\lambda u \cdot \left[\alpha_{u} \subseteq \pi_{u} \wedge \alpha_{u} \subseteq \lambda w \cdot q(w)\right.\right. \\
& \left.\left.\wedge\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w . p(w)(u[Q]) \wedge \lambda t . \alpha_{t} \subseteq \pi_{t} \wedge \alpha_{t} \subseteq \lambda w \cdot q(w)\left(u[Q]^{\top}\right)\right]\right]\right) \\
& =\operatorname{presup}\left(\lambda u \cdot \left[\alpha_{u} \subseteq \pi_{u} \wedge \alpha_{u} \subseteq \lambda w \cdot q(w)\right.\right. \\
& \left.\left.\wedge \alpha_{u[Q]} \subseteq \pi_{u[Q]} \wedge \alpha_{u[Q]} \subseteq \lambda w \cdot p(w) \wedge \alpha_{u[Q]^{\top} \subseteq} \pi_{u[Q]^{\top}} \wedge \alpha_{u[Q]^{\top}} \subseteq \lambda w \cdot q(w)\right]\right) \\
& =\operatorname{presup}\left(\lambda u \cdot \left[\alpha_{u} \subseteq \pi_{u} \wedge \alpha_{u} \subseteq \lambda w \cdot q(w)\right.\right. \\
& \left.\left.\wedge \alpha_{u[Q]} \subseteq \pi_{u[Q]} \wedge \alpha_{u[Q]} \subseteq \lambda w \cdot p(w) \wedge \pi_{u[Q]} \subseteq \pi_{u[Q]} \wedge \pi_{u[Q]} \subseteq \lambda w \cdot q(w)\right]\right) \\
& =\operatorname{presup}\left(\lambda u \cdot\left[\alpha_{u} \subseteq \pi_{u} \wedge \alpha_{u} \subseteq \lambda w \cdot q(w) \wedge \alpha_{u[Q]} \subseteq \pi_{u[Q]} \wedge \alpha_{u[Q]} \subseteq \lambda w \cdot p(w)\right]\right) \\
& =\lambda s \cdot\left[\lambda u \cdot\left[\alpha_{u} \subseteq \pi_{u} \wedge \alpha_{u} \subseteq \lambda w \cdot q(w) \wedge \alpha_{u[Q]} \subseteq \pi_{u[Q]} \wedge \alpha_{u[Q]} \subseteq \lambda w \cdot p(w)\right]\right]\left(s^{\perp}\right) \\
& =\lambda s .\left[\alpha_{s^{\perp}} \subseteq \pi_{s^{\perp}} \wedge \alpha_{s^{\perp}} \subseteq \lambda w . q(w) \wedge \alpha_{s^{\perp}[Q]} \subseteq \pi_{s^{\perp}[Q]} \wedge \alpha_{s^{\perp}[Q]} \subseteq \lambda w \cdot p(w)\right] \\
& =\lambda s \cdot\left[\varnothing \subseteq \pi_{s} \wedge \varnothing \subseteq \lambda w \cdot q(w) \wedge \varnothing \subseteq \pi_{s[Q]} \wedge \varnothing \subseteq \lambda w \cdot p(w)\right] \\
& \equiv \text { T }
\end{aligned}
$$

This finishes our presentation of Champollion et al. (2017).

### 2.3 Summary

In this chapter we considered the basics of Rooth (1985) and Rooth (1992), as well as Champollion et al. (2017). We saw that in Rooth (1985) focus is a syntactic feature which is semantically interpreted as signaling alternatives to the usual denotation of the focused phrase. We saw that this idea is formally realized by utilizing a two interpretation functions ( $\llbracket \cdot \rrbracket^{o}$ vs. $\llbracket \cdot \rrbracket^{f}$ ). We then considered the syntax and semantics of the operator ~ introduced in Rooth (1992). In Champollion et al. (2017) we paid attention merely to the formal apparatus. We introduced the basic idea (states as updates which have preconditions and effects) and its formal interpretation. We further provided some facts about the projection behavior of the connectives, operators, and quantifiers used in Champollion et al. (2017).

In the next chapter we will develop a formal semantic system that makes use of the discussed components.

## Chapter 3

## CRISP: compositional Roothian inquisitive semantics with presuppositions

### 3.1 Introduction

In this chapter we will develop a compositional inquisitive semantics with focus and presuppositions. The semantics will result from a non-trivial combination of Rooth style focus semantics and Champollion et al. (2017) inquisitive semantics with presuppositions. The resulting semantics will be called CRISP: Compositional Roothian Inquisitive Semantics with Presuppositions. The development of this system was earlier motivated by minimal pairs like:
(a) Does Bill $_{F}$ smoke too?
(b) Does Bill smoke $F_{F}$ too?
(a) and (b) clearly mean different things, and this difference is due to the difference in focus structure. The semantic framework we will develop here is capable of capturing this difference.

Chapter 3 is organized as follows: we will start with the basics of CRISP. Here, we combine Rooth (1985, 1992) with Champollion et al. (2017). The idea is to take Champollion et al.'s semantics as the language in which we translate natural language expressions in the two-dimensional way of Rooth (1985). We will see that the object language of Champollion et al. is not rich enough for a proper implementation of Rooth's (1992) squiggle operator $\sim$. There are a number of solutions to this problem which will be discussed. Any solution though results in a substantial change of the semantics of Champollion et al.

### 3.2 First Attempt

In this section we will introduce CRISP. It makes use of different components of the frameworks introduced in Chapter 2. However, CRISP cannot be described by the "equation" CRISP = Rooth + Champollion et al. Unlike in Rooth's focus semantics, we will not recursively define new semantic values, i.e. model-theoretic objects, on top of the ordinary semantic values, i.e. the usual model-theoretic objects. Essentially, Rooth's
focus semantics assigns different denotations to a phrase $A$ on LF depending on whether $A$ is $F$-marked. Moreover, the constraints expressed by the semantics of $\sim$ come in on the level of model-theoretic semantics. In CRISP, we will move all this into the object language. The reason is that CRISP's type theory is rich enough to express all of this immediately in the object language. The formal semantics of its type theory takes care of the rest for us. This means, though, that we cannot simply do with Champollion et al.'s (2017) system. We will see that Champollion et al.'s object language lacks expressive power. In particular, it cannot talk about assignments. As a consequence CRISP will make use of a yet different type theory $\left(T T_{3}\right.$, see Appendix A).

The section is organized into different parts. We will first discuss how to implement Rooth's (1985) focus semantics as described in Chapter 2 in Champollion et al.'s semantics. This is an easy task. Next, we will consider how to do the same with ~. It will become clear that this is not possible without adding more expressive power to Champollion et al.'s semantics. Possible solutions to this issue will then be discussed in the next section (§3.3).

### 3.2.1 Implementing Rooth (1985) in Champollion et al. (2017)

In Rooth (1985) we have two different model-theoretic evaluation functions, $\llbracket \cdot \rrbracket^{o}$ and $\llbracket \cdot \rrbracket^{f}$. In general, for any LF phrase $A, \llbracket A \rrbracket^{f}$ denotes a set of objects $\delta$ such that $\llbracket A \rrbracket^{o} \in D_{\tau}$ implies $\delta \in D_{\tau}$. Put differently, $\llbracket A \rrbracket^{f}$ is the domain of interpretation of $\llbracket A \rrbracket^{o}$. We can perceive of $\llbracket A \rrbracket^{f}$ then simply as the characteristic function of the domain of interpretation of $\llbracket A \rrbracket^{o}, D_{\tau}$ for $A: \tau$. Thus, $\chi_{D_{\tau}}(x)=1$ iff $x \in D_{\tau}$. Given this functional perspective, we should be able to represent this immediately in a type theory. Indeed, for any type $\tau$ and any domain of interpretation $D_{\tau}$, we can characterize $D_{\tau}$ by the expression $\lambda x_{\tau}$. T.

We will then propose the following: instead of treating $\llbracket A \rrbracket^{o}$ and $\llbracket A \rrbracket^{f}$ as two different model-theoretic evaluations, we treat them as two different translations of $A$. Here, as in Rooth's original case, $\llbracket \cdot \rrbracket^{\circ}$ is primary to $\llbracket \cdot \rrbracket^{f}$ since the latter relies on values of the former. These translations are defined recursively and make use of function application, in the former case we use standard function application, and in the latter case we use point-wise function application.

Formally, we will define $\llbracket \cdot \rrbracket^{o}$ as follows:
Definition 3.2.1. ( $\llbracket \cdot \rrbracket^{o}$, ordinary semantic translation)
Let $A$ be a natural language expression for some fragment. Then $\llbracket A \rrbracket^{o}$ is determined by one of the following two items:
(i) if $A$ is a basic expression of the lexicon, then $\llbracket A \rrbracket^{\circ}$ is simply the translation of $A$ given in the lexicon.
(ii) if $A$ is a non-basic expression $[B C]$, then $\llbracket A \rrbracket^{o}=\llbracket B \rrbracket^{o}\left(\llbracket C \rrbracket^{o}\right)$, if $\llbracket B \rrbracket^{o}:\langle\sigma, \tau\rangle$ and $\llbracket C \rrbracket^{o}: \sigma$, or $\llbracket A \rrbracket^{o}=$ $\llbracket C \rrbracket^{o}\left(\llbracket B \rrbracket^{o}\right)$, if $\llbracket C \rrbracket^{o}:\langle\sigma, \tau\rangle$ and $\llbracket B \rrbracket^{o}: \sigma$.

Here $\cdot(\cdot)$ denotes standard function application.
Clause (i) simply says that in case $A$ is a basic expression of the lexicon, then its ordinary semantic translation is given by the lexicon. Clause (ii) says that for complex expressions $A$ the ordinary semantic translation is compositionally derived from the translations of the constituents of the expression $A$ using standard function application.

Next we need to define $\llbracket \cdot \rrbracket^{f}$. We will follow closely the definition given in Rooth (1985). The task consist in transforming the original definition into an appropriate translation. The first clause of Rooth's definition stated that for $A F$-marked, $\llbracket A \rrbracket^{f}$ is the set of objects in the model that are members of the same domain as $\llbracket A \rrbracket^{o}$. We may thus propose to read this instead as saying, if $A$ is $F$-marked, then $\llbracket A \rrbracket^{f}$ is the set of formulas of the same type as $\llbracket A \rrbracket^{\circ}$. We will state this formally below. The second and third clause of Rooth tell us what happens if $A$ is not $F$-marked. The second clause considers the case where $A$ additionally is a basic expression. Here we have that $\llbracket A \rrbracket^{f}$ is the singleton set containing the denotation of $\llbracket A \rrbracket^{o}$. We may propose to interpret this as saying that $\llbracket A \rrbracket^{f}$ is simply the set of formulas that are identical to $\llbracket A \rrbracket^{o}$ (modulo $\alpha$-conversion). The third clause provides us with the rule of what to do if $A$ is in addition a complex expression $[B C]$. It tells us that we compute $\llbracket A \rrbracket^{f}$ recursively from $\llbracket B \rrbracket^{f}, \llbracket C \rrbracket^{f}$, and point-wise function application:

Definition 3.2.2. (Pointwise-function application, $\odot)$ If $A \subseteq D_{\langle\sigma, \tau\rangle}, B \subseteq D_{\sigma}$, then $A \odot B=\{f(d): f \in$ $A$ and $d \in B\} \subseteq D_{\tau}$.

This we can carry over into our object language:
Definition 3.2.3. (Point-wise function application in the object language, $\odot$ )
Let $M, N$ be arbitrary terms, such that $M:\langle\langle\sigma, \tau\rangle, t\rangle$ and $N:\langle\sigma, t\rangle$. Then: $M \odot N=\lambda x_{\tau} \cdot \exists \delta \cdot \exists \epsilon \cdot M(\delta) \wedge N(\epsilon) \wedge$ $x=\delta(\epsilon)$.

This definition says that for any two terms (or forumlas; its really the same in type theory) $M, N$ such that $M:\langle\langle\sigma, \tau\rangle, t\rangle$ and $N:\langle\sigma, t\rangle, M \odot N$ is the set of objects that are obtained from applying all elements $\delta$ in $M$ to all elements $\epsilon$ in $N$. Further, we introduce the following abbreviation:

Definition 3.2.4. (1, classical verum $)^{1}$
$1:=(\lambda t . t)=(\lambda t . t)$
Given this, we can define $[\cdot]^{f}$ as follows:
Definition 3.2.5. ( $[\cdot]^{f}$, focus semantic translation)
Let $A$ be a natural language expression of some fragment. We have:
(i) if $A$ is $F$-marked and $\llbracket A \rrbracket^{o}: \tau$, then $\llbracket A \rrbracket^{f}=\lambda x_{\tau} . \mathbf{1}:\langle\tau, t\rangle$
(ii) if $A$ is not $F$-marked and $\llbracket A \rrbracket^{o}: \tau$, then $\llbracket A \rrbracket^{f}=\lambda x_{\tau} \cdot x=\llbracket A \rrbracket^{o}:\langle\tau, t\rangle$
(iii) if $A=[B C]$ is not $F$-marked, then $\llbracket A \rrbracket^{f}=\llbracket B \rrbracket^{f} \odot \llbracket C \rrbracket^{f}$ or $\llbracket A \rrbracket^{f}=\llbracket C \rrbracket^{f} \odot \llbracket B \rrbracket^{f}$, depending on the type of $\llbracket B \rrbracket^{f}$ and $\llbracket C \rrbracket^{f}$.

Clause (i) says that if $A$ is a $F$-marked natural language expression and its ordinary semantic translation $\llbracket A \rrbracket^{o}$ is of type $\tau$, then its focus semantic translation $\llbracket A \rrbracket^{f}$ is the characteristic function from expressions of type $\tau$ onto 1, i.e., the set of formulas of type $\tau$. Since this formula is interpreted as an element of $D_{\tau}$, and every element of $D_{\tau}$ can be referred to by such a formula, we can think of $\llbracket A \rrbracket^{f}$ as the set $D_{\tau}$. This captures then the Roothian story.

[^6]Clause (ii) says that if $A$ is a basic expression that is not $F$-marked and $\llbracket A \rrbracket^{o}: \tau$, then $\llbracket A \rrbracket^{f}$ is the characteristic function from expressions of type $\tau$ to the expressions that are $\llbracket A \rrbracket^{o}$. In other words, $\llbracket A \rrbracket^{f}$ denotes the singleton set containing $\llbracket A \rrbracket^{\circ}$.

Last, clause (iii) says that if $A$ is a complex expression $[B C]$ and is not $F$-marked, then $\llbracket A \rrbracket^{f}$ is the result of point-wise function application on $\llbracket B \rrbracket^{f}$ and $\llbracket C \rrbracket^{f}$. This also captures the Roothian original.

In this way we managed to implement Rooth's (1985) focus semantics into Champollion et al. (2017). The new framework looks similar to two-dimensional systems such as Karttunen and Peters (1979) twodimensional semantics for presuppositions. Unlike them we do not translate expressions into tuples, and let these tuples represent the meaning of an expression. We will simply have both translations around separately. We also do not want to make the claim that the meaning of a natural language expression $A$ is a tuple of its ordinary translation and its focus translations. We will see the focus feature $F$ itself simply to have a semantic effect on its carrier, and not as a part of its carriers meaning ${ }^{2}$.

### 3.2.2 Lack of expressive power

Above we provided definitions for $\llbracket \cdot \rrbracket^{o}$ and $\llbracket \cdot \rrbracket^{f}$. We can now ask how to implement Rooth's (1992) operator ' $\sim$ ' into our two-dimensional version of Champollion et al. (2017). Reconsidering the semantics of $\sim$ in Rooth (1992), we need to account for two clauses:
(i) $\llbracket A \sim C \rrbracket^{o}=\llbracket A \rrbracket^{o}+$ Presupposition: $C \subseteq \llbracket A \rrbracket^{f} \wedge|C| \geq 2$
(ii) $\llbracket A \sim C \rrbracket^{f}=\left\{\llbracket A \rrbracket^{o}\right\}$

Now, clause (ii) is really no problem at all to account for. But how to account for clause (i) in our semantics? The obvious thing to do is to use transplication:

Definition 3.2.6. $\left(\llbracket A \sim C \rrbracket^{o}\right.$, naive version)
(i) $\llbracket A \sim C \rrbracket^{o}=\llbracket A \rrbracket_{\lambda s . C \subseteq \llbracket A \rrbracket^{f} \wedge}^{o}|C| \geq 2$
(ii) $\llbracket A \sim C \rrbracket^{f}=\left\{\llbracket A \rrbracket^{o}\right\}$

This definition, however, is problematic. In Champollion et al. (2017) presuppositions are seen as preconditions on updates. $\sim$ is supposed to impose the condition $C \subseteq \llbracket A \rrbracket^{f} \wedge|C| \geq 2$ on $\pi_{s}$. But, this formula is insensitive to states and so cannot impose conditions on the presupposition $\pi_{s}$ of the state $s$. Hence, it cannot function as a presupposition. We can resolve this issue by making $\pi_{s}$ sensitive to the values of variables. This can be achieved in several ways, two of which we will discuss now.

### 3.3 CRISP

We just saw that implementing Rooth (1985) into Champollion et al. (2017) can be done, but that we cannot implement ~ into Champollion et al. (2017).

[^7]In the following we will discuss two different solution strategies to our problem encountered with implementing ~ into Champollion et al. (2017). One strategy consists in interpreting the variable $C$ with respect to each world in the state. The other strategy consists in interpreting the variable simply with respect to the state as such. Both strategies involve the use of assignment functions $g$. On the first strategy we make use of world-assignment pairs, and on the second we make use of state-assignment pairs.

### 3.3.1 The world-assignment pair strategy

By a world-assignment pair we understand a tuple $\langle w, g\rangle$ where $w$ is a possible world, and $g$ is an assignment function. In particular, an assignment function $g$ is a function mapping any variable $v$ of type $\tau$ on a unique element in $D_{\tau}$. So, assignments here are not functions $g: \mathbb{N} \rightarrow D_{e}$ as in e.g. Heim and Kratzer (1998), but functions $g: V A R_{\tau} \rightarrow D_{\tau}$ for each type $\tau$. Following dynamic semantics tradition we will call world-assignment pairs $\langle w, g\rangle$ possibilities and denote them by $i, j, \ldots$.

On this strategy we will implement ~ by the following clauses:
Definition 3.3.1. (semantics of ~, world-assignment pair account)
(i) $\llbracket A \sim C \rrbracket^{o}=\llbracket A \rrbracket_{\lambda s . \forall i \in \pi_{s} .\left[\llbracket A \rrbracket^{o} \in\left(g_{i}(C) \subseteq\left\lceil A \rrbracket^{f}\right) \wedge\left|g_{i}(C)\right| \geq 2\right]\right.}^{o}$, where $i=\left\langle w_{i}, g_{i}\right\rangle$ a possibility, with $w_{i}$ the world of $i$, and $g_{i}$ the assignment of $i$.
(ii) $\llbracket A \sim C \rrbracket^{f}=\lambda x_{\tau} \cdot x=\llbracket A \rrbracket^{o}$

On this account states are tuples consisting of sets of world-assignment pairs. We will define them as follows:

Definition 3.3.2. (state, world-assignment pair account) A state $s$ is a tuple $\left\langle\pi_{s}, \alpha_{s}\right\rangle$, such that $\alpha_{s} \subseteq \pi_{s}$ and $\pi_{s}$ a set of world-assignment pairs.

The ordering on states can be given by $\subseteq$ again:
Definition 3.3.3. (ordering on states, ᄃ) Let $s, t$ be states. We say that $s$ is a substate of $t$ (or an enhancement), in symbols $s \sqsubseteq t$, iff $\pi_{s} \subseteq \pi_{t} \wedge \alpha_{s} \subseteq \alpha_{t}$.

Clearly, $\sqsubseteq$ is an ordering on states. Propositions are then defined in the familiar way.

The account allows for further structuring and thus expressivity, considerations which we will not pursue here, but see the last chapter.

### 3.3.2 The state-assignment pair strategy

By a state-assignment pair we mean a triplet $s=\left\langle\pi_{s}, \alpha_{s}, g_{s}\right\rangle$ where $\pi_{s}$ and $\alpha_{s}$ are as before, and $g_{s}$ is the assignment function of the state $s$. On this account we refer to state-assignment pairs simply as states. Here, assignment functions are the same objects as on the first strategy.

On this strategy we give $\sim$ the following meaning rule:
Definition 3.3.4. (semantics of $\sim$, state-assignment pair account)
(i) $\llbracket A \sim C \rrbracket^{o}=\llbracket A \rrbracket_{\lambda s .}^{o}\left(C\left(g_{s}\right) \subseteq \llbracket A \rrbracket^{f}\right) \wedge\left|C\left(g_{s}\right)\right| \geq 2$
(ii) $\llbracket A \sim C \rrbracket^{f}=\lambda x_{\tau} \cdot x=\llbracket A \rrbracket^{o}$

This strategy, too, forces us to change the ontology of Champollion et al. (2017). States are now treated as triplets $\left\langle\pi_{s}, \alpha_{s}, g_{s}\right\rangle$ where $g_{s}$ is the assignment of the states $s$ and $\alpha_{s} \subseteq \pi_{s}$ are as before. As on the other account we can define an ordering on states. Before defining an ordering on states, we will define an ordering on assignments:

Definition 3.3.5. (Ordering on assignments, $\leq$ )
For $g, g^{\prime}$ assignments. We have $g \leq g^{\prime}$ iff $\operatorname{dom}(g) \subseteq \operatorname{dom}\left(g^{\prime}\right)$ and $\forall i \in \operatorname{dom}(g) . g(i)=g^{\prime}(i)$.
We define the following ordering on states:
Definition 3.3.6. (Ordering on states, $\sqsubseteq) ~$
Let $s=\left\langle\pi_{s}, \alpha_{s}, g_{s}\right\rangle$ and $t=\left\langle\pi_{t}, \alpha_{t}, g_{t}\right\rangle$ be states. We say that $s$ is a sub-state of $t$ (or that $s$ enhances $t$ ), in symbols $s \sqsubseteq t$, iff $\pi_{s} \subseteq \pi_{t}, \alpha_{s} \subseteq \alpha_{t}, g_{s} \geq g_{t}$.

Propositions, again, can be defined in the familiar way as non-empty downsets of states.

### 3.3.3 Comparing the strategies

Which strategy should we employ? Given the initial task, we should employ the second strategy. This strategy provides us with the minimum needed to solve the issue, whereas the first strategy provides us with more fine-grainedness than needed. As such the second strategy is preferable.

There are, however, some reasons to not pursue strategy 2. As we will see below, on strategy 2 more work needs to be done in the end. When we want to have assignments as total functions, we are forced to adjust the basic concepts of inquisitive semantics. On the other hand, we can maintain the inquisitive semantics framework as it is when taking assignments to be partial functions. But this means that we need to extend the simply typed $\lambda$-calculus in order to allow for partial functions. And, more severe, we will have to face serious issues when adjusting the operator ! of Champollion et al. (2017). These complications do not arise when we utilize the first strategy and assume that assignments are total functions. This we will make clear next.

First, let us consider whether the second strategy fares well with assignments as total functions. Here, we may initially note that the ordering on assignments defined above is more natural when taking assignments to be partial functions. Nevertheless it does not exclude the possibility of letting assignments to be total functions. But: when letting assignments be total functions, the ordering $\leq$ boils down to $=$ which is of course still an ordering, but not a particularly good one as we will see. Consider a sentence such as John walks.

Cleary, John walks is non-inquisitive meaning that it denotes a downset with a unique maximal state. On the second strategy, such a representation cannot be achieved. This is because the set of total assignment functions has no infimum and supremum. When sticking to the standard assumption that our language has infinitely many variables of type $\tau$ for each type $\tau$, this means that the proposition expressed by John walks would have infinitely many alternatives, i.e. maximal elements. On the other hand, if we assumed that our language only has finitely many variables of type $\tau$ for each type $\tau$, we would find that the proposition has a maximum only if the cardinality of the set of variables of type $\tau$ is 1 for each type $\tau$, for in any other case we will have more than one assignment function which will necessarily differ. This account, therefore, forces
us to change basic notions of inquisitive semantics - which is possible, but undesirable.

A more serious threat consists in the fact that on this account the definition of ! is extremely hard. In Champollion et al. (2017) the definition of ! involves $\sqcup$. We have $\sqcup S=\left\langle\bigcup_{s \in S} \pi_{s}, \bigcup_{s \in S} \alpha_{s}\right\rangle$. When moving from states to state-assignment pairs, we run into trouble. $\sqcup$ is supposed to provide us with the supremum of a set of states. When we now have states as triplets $\left\langle\pi_{s}, \alpha_{s}, g_{s}\right\rangle$ no such supremum exists, which is because the set of total functions has no such elements.

Some of these complications do not arise when we instead take assignments to be partial. This is because the set of partial functions has an infimum: the empty assignment $\varnothing$. Given the ordering on state-assignment pairs we defined, we avoid the problem with non-inquisitive propositions. However, partial functions impose a more complicated type theory (cf. Carpenter (1997): 45). Furthermore, they seem not to solve the issues surrounding the definition of $\sqcup$. For instance, we cannot simply define $\sqcup S:=\left\langle\bigcup_{s \in S} \pi_{s}, \bigcup_{s \in S} \alpha_{s}, \bigcup_{s \in S} g_{s}\right\rangle$, because the last component is not said to be a function in all cases. One may stipulate that $\sqcup S:=\left\langle\bigcup_{s \in S} \pi_{s}, \bigcup_{s \in S} \alpha_{s}, \varnothing\right\rangle$, but on which grounds? And is this empirically adequate? Whatever a solution to this problem may be (if such exists), it is non-trivial.

We take this to be sufficient motivation to utilize the first strategy. We will further make use of total assignments instead of partial assignments. The reason is that we can simply stick to the simple typed $\lambda$-calculus and only need to add a basic type $a$ for assignments. When we want to make use of partial assignments, we would need to extend the simply typed $\lambda$-calculus to allow for such expressions (cf. Carpenter (1997): 45).

### 3.3.4 Adjusting Champollion et al. (2017)

The solution we endorse has repercussions for the semantics we wanted to adopt. The semantics of Champollion et al. (2017) needs to be adjusted to the new ontology, meaning we move from $T T_{2}$ to $T T_{3}$, the differences being that we add a new basic type $a$ for assignment functions to $T T_{2}$. Further we let all variables of any type be of functional types, e.g. when a variable $x$ denoting an individual was of type $e$, then it's now of type $\langle a, e\rangle=\mathbf{e}$. Variables thus take assignments as arguments and output expressions of their usual types. We will do the same for constants of type $e$ : they now are of type e. More in Appendix A.

We also need to adjust the abbreviations in the object language, etc. We will list the abbreviations, connectives, quantifiers and operators below.

Definition 3.3.7. (Abbreviations in the object language) ${ }^{3}$

| $\pi_{s}:=p_{1}(s)$ | the presupposition of state $s$ |
| :---: | :---: |
| $\alpha_{S}:=p_{2}(s)$ | the at-issue information of state $s$ |
| $s^{\top}=\left\langle\pi_{s}, \pi_{s}\right\rangle$ | trivializing the at-issue information of state $s$ |
| $s^{\perp}=\left\langle\pi_{s}, \varnothing\right\rangle$ | contradicting the at-issue information of state $s$ |

[^8]| $s \sqsubseteq t:=\alpha_{s} \subseteq \alpha_{t} \wedge \pi_{s} \subseteq \pi_{t}$ | $s$ is a substate of $t$ |
| :---: | :---: |
| $(\mathbf{N e w}!) \operatorname{true}(P, i):=P(\langle\{i\},\{i\}\rangle)$ | $P$ is true for the possibility $i$ |
| presup $(P):=\lambda s . P\left(s^{\perp}\right)$ | the presupposition of $P$ |
| $(\mathbf{N e w}!)\|P\|:=\lambda i . \operatorname{true}(P, i)$ | the truth set of $P ;$ the at-issue information of $P$ |
| $s[P]:=\left\langle\pi_{s} \cap\right\| P\left\|, \alpha_{s} \cap\right\| P\| \rangle$ | updating state $s$ with the at-issue information of $P$ |
| $\mathbf{1}:=(\lambda t . t)=(\lambda t . t)$ | classical verum |

The definitions of the connectives and transplication stay symbol-wise the same. They are repeated for the convenience of the reader:

Definition 3.3.8. (Inquisitive Connectives and Transplication)

| $\mathrm{T}:=\lambda s . \mathbf{1}$ | verum (or top) |
| :---: | :---: |
| $\perp:=\lambda s .\left(\alpha_{s}=\varnothing\right)$ | falsum (or bottom) |
| $P \wedge Q:=\lambda s . P(s) \wedge Q(s[P])$ | inquisitive conjunction |
| $P \rightarrow Q:=\lambda s .\left(P\left(s^{\perp}\right) \wedge \forall t \sqsubseteq s .(P(t) \rightarrow Q(t[P]))\right.$ | inquisitive implication |
| $\sqcap P:=P \rightarrow \perp$ | inquisitive negation |
| $P \mathbb{W}:=\lambda s . P(s) \vee Q(s)$ | inquisitive disjunction |
| $P_{Q}:=\lambda s . P(s) \wedge Q\left(s^{\top}\right)$ | transplication |

Last, the definitions of the inquisitive operations, quantifiers, and entailment:
Definition 3.3.9. (Inquisitive existential and universal quantifier, projection operators and entailment)

| $? P:=P \nVdash \curvearrowleft P$ | inquisitive projection |
| :---: | :---: |
| $\sqcup S:=\left\langle\cup_{s \in S} \pi_{s}, \cup_{s \in S} \alpha_{s}\right\rangle$ | supremum of $S$ |
| $!P:=\lambda s . \exists S .(\sqcup S=s \wedge \forall t \in S: P(t))$ | informative projection |
| $\langle ?\rangle:=\lambda P . \lambda s . P(s) \vee((P=!P) \wedge \neg P(s))$ | conditional inquisitive projection |
| $($ New! $) \nVdash x . P:=\lambda s . \forall s^{\prime}\left[s\langle x\rangle s^{\prime} \rightarrow P\left(s^{\prime}\right)\right]$ | inquisitive universal quantifier |
| $($ New $!) \exists x . P:=\lambda s . \exists s^{\prime}\left[s[x] s^{\prime} \wedge P\left(s^{\prime}\right)\right]$ | inquisitive existential quantifier |
| $P \vDash Q:=\forall s[P(s) \rightarrow Q(s)]$ | entailment |

We omitted the definition of " $s\langle x\rangle s^{\prime \prime}$ " and " $s[x] s^{\prime}$ " intentionally in the above table. Instead we provide them now:

Definition 3.3.10. $\left(s\left\langle x_{1}, \ldots, x_{n}\right\rangle s^{\prime}\right.$, non-inquisitive assignment update)
Let $s, s^{\prime}$ be states and let $x_{1}, \ldots, x_{n}$ be variables of some fixed but arbitrary functional type $\alpha$. We define $s\left\langle x_{1}, \ldots, x_{n}\right\rangle s^{\prime}$ by the following claues:
(i) $\forall\langle w, g\rangle \in \pi_{s} \cdot \exists\left\langle w^{\prime}, g^{\prime}\right\rangle \in \pi_{s^{\prime}} . w=w^{\prime} \wedge \forall v . v \neq x_{1} \wedge \ldots \wedge v \neq x_{n} \rightarrow v(g)=v\left(g^{\prime}\right)$
(ii) $\forall\left\langle w^{\prime}, g^{\prime}\right\rangle \in \pi_{s^{\prime}} . \exists\langle w, g\rangle \in \pi_{s} \cdot w=w^{\prime} \wedge \forall v . v \neq x_{1} \wedge \ldots \wedge v \neq x_{n} \rightarrow v(g)=v\left(g^{\prime}\right)$
(iii) $\forall\langle w, g\rangle \in \alpha_{s} \cdot \exists\left\langle w^{\prime}, g^{\prime}\right\rangle \in \alpha_{s^{\prime}} . w=w^{\prime} \wedge \forall v . v \neq x_{1} \wedge \ldots \wedge v \neq x_{n} \rightarrow v(g)=v\left(g^{\prime}\right)$
(iv) $\forall\left\langle w^{\prime}, g^{\prime}\right\rangle \in \alpha_{s^{\prime}} \cdot \exists\langle w, g\rangle \in \alpha_{s} . w=w^{\prime} \wedge \forall v . v \neq x_{1} \wedge \ldots \wedge v \neq x_{n} \rightarrow v(g)=v\left(g^{\prime}\right)$

Let us take a closer look at what this definition says. Clause (i) says that for any possibility $\langle w, g\rangle \in \pi_{s}$ there is a possibility $\left\langle w^{\prime}, g^{\prime}\right\rangle \in \pi_{s^{\prime}}$ such that they differ at most in their second component (i.e. $w=w^{\prime}$ ), and further it holds that for any variable $v$ which is not among $x_{1}, \ldots, x_{n}, g, g^{\prime}$ agree on the values of $v$ (i.e. $\left.v(g)=v\left(g^{\prime}\right)\right)$. Thus, this clause says that for any possibility $\langle w, g\rangle$ in $\pi_{s}$ we must find a possibility $\left\langle w^{\prime}, g^{\prime}\right\rangle$ in $\pi_{s^{\prime}}$ such that the latter possibility differs from the former at most with respect to its assignment $g^{\prime}$ and that $g$ and $g^{\prime}$ differ only in the value they yield when $x$ is applied to them; they do not differ in the value when being the argument for $v \neq x$. The effect of this clause is to ensure that we do not remove any indices and do not lose any information about the values of the variables different from $x_{1}, \ldots, x_{n}$, when moving from state $s$ to $s^{\prime}$.

Clause (ii) says that for all possibilities $\left\langle w^{\prime}, g^{\prime}\right\rangle \in \pi_{s^{\prime}}$ there is some possibility $\langle w, g\rangle \in \pi_{s}$ such that they differ at most with respect to their second component (i.e. $w^{\prime}=w$ ), and for all variables $v$ which are not among $x_{1}, \ldots, x_{n}$ we have that $g^{\prime}$ and $g$ agree on their values (i.e. $\left.v\left(g^{\prime}\right)=v(g)\right)$. Thus, this say that any possibility in $\pi_{s^{\prime}}$ is a possibility obtained from a possibility $\langle w, g\rangle \in \pi_{s}$ by changing the second component $g$ of $\langle w, g\rangle$ to some $g^{\prime}$ where $g^{\prime}$ may only differ in the value it yields when $x$ gets applied to it; for any other variable $v$ it must behave exactly like $g$. The effect of this clause is to ensure that we do not add any indices and do not add arbitrary values of the variables different from $x_{1}, \ldots, x_{n}$, when moving from state $s$ to $s^{\prime}$.

Clause (iii) is like clause (i) but for $\alpha_{s}$ instead of $\pi_{s}$. Similarly, clause (iv) is like clause (ii) but for $\alpha_{s^{\prime}}$ instead of $\pi_{s^{\prime}}$.

Let us consider an example for $\nVdash$.
Example 3. Let us consider $\nVdash x[P(x)]$, where we assume that $P$ is a non-presuppositional atomic property (for instance "smoke"). Let $\mathcal{L}=\left\{\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{a}, g_{b}\right\rangle,\left\langle w_{b}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle,\left\langle w_{\varnothing}, g_{a},\right\rangle,\left\langle w_{\varnothing}, g_{b}\right\rangle\right\}$ be the logical space. Here we take $w_{i}$ to mean that $P$ holds true of $i$ in $w$. And we take $g_{i}$ to mean that $x g=i$. Let $s=\left\langle\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\},\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\}\right\rangle$. We want to know whether $\nVdash x[P(x)](s)$ is true. We have:

$$
\forall x[P(x)]=\lambda s . \forall s^{\prime}\left[s\langle x\rangle s^{\prime} \rightarrow P(x)\left(s^{\prime}\right)\right]
$$

Now, observe: we find that $s\langle x\rangle s^{\prime}$ for $s^{\prime}$ being: add the other states
(i) $s_{1}=\left\langle\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\},\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\}\right\rangle=s$
(ii) $s_{2}=\left\langle\left\{\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle\right\},\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\}\right\rangle$
(iii) $s_{3}=\left\langle\left\{\left\langle w_{a . b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle\right\},\left\{\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle\right\}\right\rangle$

We now need to check whether $P(x)\left(s^{\prime}\right)$. Observe that $P(x)\left(s^{\prime}\right)$ for $s^{\prime} \in\left\{s_{1}, s_{2}, s_{3}\right\}$ as $\alpha_{s^{\prime}} \subseteq P(x)$. Hence, $\nVdash x[P(x)](s)$ is true. Observe that $s_{2}$ and $s_{3}$ also satisfy the formula.

Graphically, we can represent the denotation of $\nVdash x[P(x)]$ for $\mathcal{L}$ as follows:

| $\left\langle w_{\varnothing}, g_{a}\right\rangle$ | $\left\langle w_{a, b}, g_{a}\right\rangle$ | $\left\langle w_{a}, g_{a}\right\rangle$ | $\left\langle w_{b}, g_{a}\right\rangle$ |
| :---: | :---: | :---: | :---: |
| $\left\langle w_{\varnothing}, g_{b}\right\rangle$ | $\left\langle w_{a, b}, g_{b}\right\rangle$ | $\left\langle w_{a}, g_{b}\right\rangle$ | $\left\langle w_{b}, g_{b}\right\rangle$ |

Figure 3.1: The denotation of $\nVdash x[P(x)]$ for $\mathcal{L}$. The purple area represents the at-issue content, the area enclosed by the dashed line represents the presupposition. Together, these areas represent the maximal element of the proposition denoted by $\nVdash x . P(x)$. We see that universal statements come about as noninquisitive and informative.

Definition 3.3.11. $\left(s\left[x_{1}, \ldots, x_{n}\right] s^{\prime}\right.$, inquisitive assignment update)
Let $s, s^{\prime}$ be states and let $x_{1}, \ldots, x_{n}$ be variables of some fixed but arbitrary functional type $\alpha$. We define $s\left[x_{1}, \ldots, x_{n}\right] s^{\prime}$ by the following clauses:
(i) $\pi_{s}=\pi_{s^{\prime}}$
(ii) $\alpha_{s}=\alpha_{s^{\prime}}$
(iii) $\forall\langle w, g\rangle \in \alpha_{s^{\prime}} \forall\left\langle w^{\prime \prime}, g^{\prime \prime}\right\rangle \in \alpha_{s^{\prime}} \forall v \cdot v=x_{1} \vee \ldots v=x_{n} \rightarrow v g^{\prime}=v g^{\prime \prime}$

Let us take closer look at what this definition says. Clause (i) and (ii) together say that we only consider the state $s$ itself. We will see soon why. Clause (iii) says that possibilities in $\alpha_{s^{\prime}}$ (i.e. $\alpha_{s}$ ) must yield the same value for $x_{1}, \ldots, x_{n}$, though possibly different once for the $x_{i}, 1 \leq i \leq n$. This ensures inquisitiveness. This is because the effect of clause (iii) is similar to a partitioning of the logical space itself, where we partition the logical space by considering the assignment functions of the possibilities. Practically, we keep apart parts of the logical space which yield different values for the $x_{1}, \ldots x_{n}$. This generates alternatives, and is thus the source of inquisitiveness. Clause (i) and (ii) alone are insufficient for this.

The role of clause (i) and (ii) is motivated by the role $s\left[x_{1}, \ldots, x_{n}\right] s^{\prime}$ plays in existential quantification. We want to find within the cells of the partition the states that support the existential. In this way we can check for each assignment whether it makes the existential statement true or not - loosely speaking.

Let us consider an example.
Example 4. Again let $\mathcal{L}=\left\{\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{a}, g_{b}\right\rangle,\left\langle w_{b}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle,\left\langle w_{\varnothing}, g_{a},\right\rangle,\left\langle w_{\varnothing}, g_{b}\right\rangle\right\}$ be the logical space. Let us consider the sentence $\exists x[P(x)]$, where $P$, again, is a non-presuppositional atomic property (we can think again of "smoke" here). Let us consider the state $s=\left\langle\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\},\left\{\left\langle w_{a, b}, g_{a}\right\rangle\right\}\right\rangle$. We want to know whether $\exists x[P(x)](s)$ is true. We have that $s[x] s$. We only need to check whether $\alpha_{s} \subseteq P(x)(s)$. This is the case since $x g=a$.

Next, consider the state $t=\left\langle\left\{\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{b}, g_{a}\right\rangle\right\},\left\{\left\langle w_{b}, g_{a}\right\rangle\right\}\right\rangle$. We find that $t[x] t$ but we do not have that $P(x)(t)$ under the assignment $g$. The reason is that $x g=a$ for $\alpha_{t}=\left\langle w_{b}, g_{a}\right\rangle$. So, $\alpha_{t} \nsubseteq P(x)$.

Graphically, we can represent the denotation of $\exists x[P(x)]$ for $\mathcal{L}$ as follows:
Later we will introduce further abbreviations from Champollion et al. (2017). These abbreviations concern specific aspects of question semantics.

| $\left\langle w_{\varnothing}, g_{a}\right\rangle$ | $\left\langle w_{a, b}, g_{a}\right\rangle$ | $\left\langle w_{a}, g_{a}\right\rangle$ | $\left\langle w_{b}, g_{a}\right\rangle$ |
| :---: | :---: | :---: | :---: |
| $\left\langle w_{\varnothing}, g_{b}\right\rangle$ | $\left\langle w_{a, b}, g_{b}\right\rangle$ | $\left\langle w_{b}, g_{b}\right\rangle$ | $\left\langle w_{a}, g_{b}\right\rangle$ |

Figure 3.2: The denotation of $\exists x[P(x)]$.

### 3.3.5 Two kinds of presuppositions

Given the adjustments, we can now show that the supposed presupposition of $\sim$ comes about as such in our formal semantics. For simplicity we assume that $\alpha$ is non-presuppositional and a sentence:

$$
\begin{aligned}
\operatorname{presup}\left(\llbracket A \sim C \rrbracket^{o}\right) & =\lambda t .\left(\lambda s . \llbracket A \rrbracket^{o}(s) \wedge \lambda s .\left(\forall i \in \pi_{s}: C\left(g_{i}\right) \subseteq \llbracket A \rrbracket^{f} \wedge\left|C\left(g_{i}\right)\right| \geq 2\right)(s)\right)\left(t^{\perp}\right) \\
& =\lambda t .\left(\llbracket A \rrbracket^{o}\left(t^{\perp}\right) \wedge \forall i \in \pi_{t^{\top}}: C\left(g_{i}\right) \subseteq \llbracket A \rrbracket^{f} \wedge\left|C\left(g_{i}\right)\right| \geq 2\right) \\
& =\lambda t . \forall i \in \pi_{t^{\top}}: C\left(g_{i}\right) \subseteq \llbracket A \rrbracket^{f} \wedge\left|C\left(g_{i}\right)\right| \geq 2 \text { (given our assumption) } \\
& =\lambda t . \forall i \in \pi_{t}: C\left(g_{i}\right) \subseteq \llbracket A \rrbracket^{f} \wedge\left|C\left(g_{i}\right)\right| \geq 2
\end{aligned}
$$

In general: $\llbracket A \sim C \rrbracket^{o}$ presupposes all presuppositions of $\llbracket A \rrbracket^{o}$ and the presupposition of $\sim$ as the reader can see easily from the second line.

This result is good, but also shows that we have different kinds of presuppositions at work in our new semantics. We will discuss this issue now.

In Champollion et al. (2017) presuppositions $\pi_{s}$ are modeled as sets of worlds, and in our amended system as sets of possibilities $i$. As such, presuppositions $\pi_{s}$ encode factual information, i.e. how the world looks like. The information required by $\sim$ is more specific than that. In fact, the presupposition imposed by $\sim$ is about the availability of specific entities. We thus have two kinds of presuppositions:
(i) presuppositions about factual information
(ii) presuppositions about the presence/ availability of entities

If these kind of conditions are not met by a state, the state's at-issue information will not update the common ground. And neither will the complement of the at-issue information with respect to the presupposition $\left(\pi_{s} \backslash \alpha_{s}\right)$ be able to update the common ground.

### 3.3.6 Presupposition projection of connectives, quantifiers and inquisitive operators

Before closing this section, let us observe that the earlier stated facts about the presupposition behavior of connectives and operators are preserved in the new system. We thus have:

Fact 6. (Projection of $\mathbb{x}, \rightarrow, \rightarrow) \mathbb{x}, \rightarrow, \rightarrow$ encode the projection behavior of conjunction, implication, and negation stated in Karttunen (1973). In particular:

$$
\begin{aligned}
\wedge & : \operatorname{presup}(P \wedge Q)=\lambda s \cdot P\left(s^{\perp}\right) \wedge Q\left(s[P]^{\perp}\right)=\lambda s \cdot P\left(\left\langle\pi_{s}, \varnothing\right\rangle\right) \wedge Q\left(\left\langle\pi_{s \cap|P|}, \varnothing\right\rangle\right) \\
\rightarrow & : \operatorname{presup}(P \rightarrow Q)= \\
& \lambda s \cdot P\left(s^{\perp}\right) \wedge \forall t \sqsubseteq s^{\perp} \cdot(P(t) \rightarrow Q(t[P]))=\lambda s \cdot P\left(\left\langle\pi_{s}, \varnothing\right\rangle\right) \wedge \forall t \sqsubseteq\left\langle\pi_{s}, \varnothing\right\rangle .\left(P\left(\left\langle\pi_{t}, \varnothing\right) \rightarrow Q\left(\left\langle\pi_{t \cap|P|}, \varnothing\right\rangle\right)\right)\right.
\end{aligned}
$$

$$
\backsim: \operatorname{presup}(\curvearrowleft P)=\lambda s \cdot P\left(s^{\perp}\right) \wedge \forall t \sqsubseteq s^{\perp}(P(t) \rightarrow \perp(t[P]))=\lambda s \cdot P\left(s^{\perp}\right)
$$

Fact 7. (Projection of $P_{Q}$ ) Transplication $P_{Q}$ presupposes $Q$ and all presuppositions of $P$ :

$$
P_{Q}: \operatorname{presup}\left(P_{Q}\right)=\lambda t . P_{Q}\left(t^{\perp}\right)=\lambda t .\left(\lambda s .\left(P(s) \wedge Q\left(s^{\top}\right)\right)\left(t^{\perp}\right)\right)=\lambda t . P\left(t^{\perp}\right) \wedge Q\left(t^{\top}\right)
$$

Fact 8. (Projection of $w$ ) Inquisitive disjunction presupposes the disjunction of the presuppositions of $P$ and $Q$ :

```
W:presup}(PWQ)=\lambdat.P(\mp@subsup{t}{}{\perp})\veeQ(\mp@subsup{t}{}{\perp}
```

Fact 9. (Projection of ? and !) The inquisitive projection ? presupposes the presupposition of its argument, and the informative projection! presupposes every proper substate of the supremum of $T$ satisfies the presuppositions of its argument. In particular:

```
? : \(\operatorname{presup}(? P)=\lambda t . P\left(t^{\perp}\right) \vee(\neg P)\left(t^{\perp}\right)=\lambda t . P\left(t^{\perp}\right)=\operatorname{presup}(P)\)
\(!: \operatorname{presup}(!P)=\lambda t .!P\left(t^{\perp}\right)=\lambda t \cdot(\lambda s . \exists T(\sqcup T=s \wedge \forall u \in T: P(u)))\left(t^{\perp}\right)=\lambda t \cdot \exists T\left(\sqcup T=t^{\perp} \wedge \forall u \in T: P(u)\right)=\)
    !presup( \(P\) )
```

For the quantifiers $\exists$ and $\nVdash$ we observe the following:
Fact 10. (Projection of $\exists$ and $\not W$ )
$\operatorname{presup}(\exists x . P)=\lambda t .\left(\exists s^{\prime}\left[\left\langle p_{t}, \varnothing\right\rangle=\left\langle\pi_{s^{\prime}}, \varnothing\right\rangle \wedge \forall i, i^{\prime} \in \pi_{s^{\prime}}\left[x g_{i}=x g_{i^{\prime}}\right] \wedge P\left(\left\langle\pi_{s^{\prime}}, \varnothing\right\rangle\right)\right]\right)$
$\operatorname{presup}(\nVdash x . P)=\lambda t .\left(\forall s^{\prime}\left[\left\langle\pi_{t}, \varnothing\right\rangle\langle x\rangle\left\langle\pi_{s^{\prime}}, \varnothing\right\rangle \rightarrow P\left(\left\langle\pi_{s^{\prime}}, \varnothing\right\rangle\right)\right]\right)$
Note that these boil down to existential and universal presuppositions. In case of $\exists x . P(x)$ we find that it is presupposed that under some assignment the presuppositions of $P$ hold, whereas in case of $\nVdash x . P(x)$ that it is presupposed that under all assignments the presuppositions of $P$ must hold.

Let us consider an example for the inquisitive existential $\exists$.
Example 5. Let us consider the sentence $\exists x . P_{Q}$, where $P, Q$ are both non-presuppositional and atomic. We take again $\mathcal{L}=\left\{\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{a}, g_{b}\right\rangle,\left\langle w_{b}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle,\left\langle w_{\varnothing}, g_{a},\right\rangle,\left\langle w_{\varnothing}, g_{b}\right\rangle\right\}$ to be the logical space. Observe:

$$
\operatorname{presup}\left(\exists x \cdot P_{Q}\right)=\lambda t .\left(\exists s^{\prime}\left[\left\langle\pi_{t}, \varnothing\right\rangle=\left\langle\pi_{s^{\prime}}, \varnothing\right\rangle \wedge \forall i, i^{\prime} \in \pi_{s^{\prime}}\left[x g_{i}=x g_{i^{\prime}}\right] \wedge Q\left(\left\langle\pi_{s^{\prime}}, \pi_{s^{\prime}}\right\rangle\right)\right]\right)
$$

Consequently, a state $s$ that supports the meaning expressed by $\exists x . P_{Q}$ must be such that its presupposition supports the meaning of $Q$, thus it must hold that $\pi_{s} \subseteq Q$. Pictorially:


Figure 3.3: We represent the denotation of $P$ by the light orange area, whereas the pruple area represents the denotation of $Q$. The area encapsulated by the black dashed lines displays again the alternatives making up the sentence denotation. We see that it contains only areas which are in support of $Q$ and $P$. In particular, the presupposition of a state must support $Q$ and its at-issue component must support $P$.

### 3.4 Summary

In this chapter we introduced CRISP. It resulted from a fusion of the semantics provided in Champollion et al. (2017) and the focus semantics of Rooth (1992). The fusion made necessary to change the former in a crucial way in order to make sense of the latter. We achieved this by transforming states - pairs of sets of worlds - into pairs of sets of world-assignment pairs (possibilities). This also made necessary changes of the way quantification works.

In the following chapters we will apply CRISP to model linguistic phenomena. Hereby we will encounter technical limitations of CRISP which we will address but leave for the future. The following chapters can thus be seen as a test of what CRISP can do so far and where we need to improve.

## Chapter 4

## Polar Questions and Alternative Questions

In the introduction we noted differences in felicity for polar questions and alternative questions with too. The pair below illustrates this point again:
(24) Does Mary ${ }_{F}$ dance or sing too?
(25) \#Does Mary dance ${ }_{F}$ or $\operatorname{sing}_{F}$ too?
disjunctive polar reading
alternative reading

We see that on the polar question reading the question is felicitous, but on the alternative question reading the question isn't. If we want to understand this, we first need to know what the meaning of alternative questions and disjunctive polar questions are. This holds for the other cases as well. The goal of this chapter is then to provide an account of the meaning of plain polar questions (e.g. Does Mary ${ }_{F}$ dance too?), disjunctive polar questions, and alternative questions in matrix form.

The chapter is organized as follows. We will first provide the basic assumptions of Champollion et al. (2017) on questions together with the semantics of lists as developed in Roelofsen and Farkas (2015) (§4.1). They provide a semantic framework for non-wh questions which uniformly accounts for the meaning of plain and disjunctive polar questions, as well as alternative questions. Further, the framework also accounts for the ambiguity of the question below:

Does Mary dance or sing?
a. Is it the case that Mary either dances or sings?
b. Which of the two is Mary doing: dancing or singing?
disjunctive polar reading
alternative reading

It is argued that the question is disambiguated by means of prosodic cues (cf. Pruitt and Roelofsen (2013)). The framework of Roelofsen and Farkas (2015) accounts for this. This we will consider in $\S 4.2$. We then show the working of the introduced framework by considering step by step the three different question types we are interested in, starting with plain and disjunctive polar questions (§4.3), followed by alternative questions (§4.4). It follows a short section on Hurford disjunctions in questions. Prima facie these provide a challenge for the analysis given in $\S 4.4$, but this challenge can be met by that analysis as we will show. We conclude with a summary (§4.6).

### 4.1 Basic Assumptions on Questions and the semantics of lists

Following Baker (1970), Champollion et al. (2017) assume that all questions (embedded or not) are headed with a silent question morpheme $Q$. $Q$ projects an interrogative nucleus. The complement of $Q$ is an abstract, schematically:
interrogative nucleus


Later, when discussing wh-questions, we will say more on the semantics of abstracts and Q. For now it suffices to say the following:

Definition 4.1.1. (Semantics of Q, Champollion et al. (2017): 13) ${ }^{1}$
$\llbracket \mathrm{Q}^{n} \rrbracket_{g}:=\lambda P_{\left\langle e^{n}, T\right\rangle} \cdot(\langle ?\rangle \nexists \vec{x} \cdot P(\vec{x}))_{!(?) \nexists \vec{x} \cdot P(\vec{x})}$
We will adjust this to our type theory:
Definition 4.1.2. (Semantics of Q in CRISP)
$\llbracket \mathrm{Q}^{n} \rrbracket^{o}:=\lambda P_{\left\langle\langle a, e\rangle^{n}, T\right\rangle} \cdot(\langle ?\rangle \nexists \vec{x} \cdot P(\vec{x}))_{!\langle ?\rangle} \exists_{\vec{x} \cdot P(\vec{x})}$
Since we assume that $Q$ is silent and we take the phonological realization of $F$ to be intonational prominence (Rooth (1985)), it doesn't make sense to assume that Q can be focused (i.e. $F$-marked), for intonational prominent parts of an utterance are not silent. We thus didn't provide a clause for that case.

Further, the reader can see that we take $Q$ to be a presupposition trigger. In particular, we take it to presuppose that there is an answer to the question (cf. Champollion et al. (2017): 13). This seems natural. After all, when we ask a question we take it to have an answer, and in cases where this presupposition fails the question seems odd. We will follow Champollion et al. (2017) in this without further ado.

In Roelofsen and Farkas (2015) it is assumed that polar questions and alternative questions are lists (Roelofsen and Farkas (2015): 371-372). Lists have a couple of features and a particular syntax: ${ }^{2}$


As the above schematic tree for a list indicates, these are either declarative (DECL) or interrogative (INT), or they are either closed (Closed), or open (OPEN). These features are commonly referred to as list classifiers (Roelofsen and Farkas (2015): 373). The items are (in the cases of interests) connected via disjunction

[^9](w). Moreover, negation can occur in lists. This part of a list is called body (Roelofsen and Farkas (2015): 373). We will discuss their semantics now.

Going from top to bottom, Decl and Int take over different jobs in lists. The job of Decl is to make the list purely informative (cf. Roelofsen and Farkas (2015)). We can capture this role easily in CRISP by utilizing '!'. Further, Decl is treated as a propositional modifier. Consequently, we will translate Decl by $\lambda P_{T}!P$ (cf. Roelofsen and Farkas (2015): 374).

Definition 4.1.3. (Translation of DECL)
$[\mathrm{DECL}]^{o}=\lambda P_{T} .!P$
Like Decl, Int is treated as a propositional modifier. Unlike Decl, it is assigned two roles in Roelofsen and Farkas (2015). First, it is supposed to ensure inquisitiveness. This means that whenever its argument is inquisitive it leaves it untouched, and if it is non-inquisitive, Int makes it inquisitive. This we can capture in CRISP by means of $\langle ?\rangle$, the conditional inquisitive projection operator (cf. Roelofsen and Farkas (2015) $: 374){ }^{3}$

The second role of InT consists in ensuring non-informativity. This is achieved by introducing a presupposition that the actual world be contained in at least one of the alternatives of the propositional argument of InT (cf. Roelofsen and Farkas (2015): 374). Consequently, we can translate Int as $\lambda P_{T} .\langle ?\rangle P_{!}$(?) $P$, which is the translation of $Q^{0}$. We will then treat Int simply as $Q^{0}$.

Definition 4.1.4. (Translation of Int)
$\llbracket \mathrm{INT} \rrbracket^{o}=\llbracket \mathrm{Q}^{0} \rrbracket^{o}$
Next let us consider Open and Closed. Both are treated as functions which take two arguments, a proposition and a propositional modifier, and return a proposition. Thus they are of type $\langle T,\langle\langle T, T\rangle, T\rangle\rangle$ (cf. Roelofsen and Farkas (2015) : 374).

Turning to Open, its role consists in leaving open whether a list item is true or not (cf. Roelofsen and Farkas (2015): 374). This can be captured in CRISP by applying ? to the propositional argument of Open. As the authors note, some caution is needed. Open is followed by Decl or Int. If Open allows any of the operators to apply, we face one of two problems. In case Int follows up on Open, Int would be redundant. This is because its argument is already inquisitive and the presupposition it adds to its argument is trivial for its argument. In case of DECL, we would derive a tautology, a meaning that is redundant in any context. Farkas and Roelofsen do not want any of these cases to occur(cf. Roelofsen and Farkas (2015)). The authors therefore assign Open a second role: blocking the working of the propositional modifiers. We agree on (at least) the second point. Consequently, we will translate Open by the term $\lambda P_{T} \cdot \lambda F_{\langle T, T\rangle} . ? P$. This term ensures that the propositional modifiers will not do their job while leaving it open whether or not the propositional argument holds.

Definition 4.1.5. (Translation of OPEN)
$\llbracket \mathrm{OPEN} \rrbracket^{o}=\lambda P_{T} \cdot \lambda F_{\langle T, T\rangle} . ? P$
Turning to Closed the authors assign it the role of implying that exactly one of the list items holds. In a declarative this implication is taken to be part of the at-issue content, whereas in interrogatives it is seen as a presupposition. The semantics of ClOSED is captured by treating it as an exclusive strengthening

[^10]operator (cf. Roelofsen and Farkas (2015): 375). Since the exact semantics of this operator is orthogonal to the interests of the authors in Roelofsen and Farkas (2015), they do not explicate it any further. We will later make use of an exhaustivity operator exh to capture the semantics of ClOSED. For this reason, deviating from Roelofsen and Farkas (2015), we will translate Closed by the term $\lambda P . \lambda F_{\langle T, T\rangle} . F(\operatorname{exh}(C, P))$. We will introduce exh in (§4.4).

Definition 4.1.6. (Translation of ClOSEd)
$\llbracket \mathrm{Closed}]^{o}=\lambda P_{T} \cdot \lambda F_{\langle T, T\rangle} \cdot F(\operatorname{exh}(C, P))$
So far we discussed the list classifiers. Let us turn to the body next. As is clear from the above, the body provides the propositional argument for the list classifiers. Roelofsen and Farkas (2015) assume that items translate into expressions of propositional types, and that disjunction (and negation) translate into $w$ and $\rightarrow$ respectively (cf. Roelofsen and Farkas (2015): 373). Moreover, they assume that to each translated item ! applies. The idea is that each item is supposed to provide one alternative to the list (cf. Roelofsen and Farkas (2015): 373). We will see below when considering the three question types individually that the prosody of a sentence factors into the translation process. We will provide the relevant details there.

### 4.2 Some facts on the prosody-meaning mapping in English question constructions

Why considering prosody? The reason is simple. Recent experimental studies (e.g. Pruitt (2008a), Pruitt (2008b), and Pruitt and Roelofsen (2013)) argue for that prosody disambiguates between the disjunctive polar reading and the alternative question reading of our earlier example repeated below:
(27) Does Mary dance or sing?

The final tone contour is supposed to be essential to this end. The studies found:
(28) A final falling tone is perceived as essential for the alternative question reading (cf. Pruitt and Roelofsen (2013): 644-645)

In contrast to this we will take the default final tone on polar questions (plain and disjunctive) to be rising (cf. Biezma and Rawlins (2012): 363). This leaves open the possibility of a falling tone (cf. Pruitt and Roelofsen (2013): 644).

In how far the prosodic features to be mentioned below play into the disambiguation is still debated (cf. Biezma and Rawlins (2012): 87; Pruitt and Roelofsen (2013): 647).

Of course, if prosody has a disambiguating role in English disjunctive questions, we assume it to have a semantic effect. In Roelofsen and Farkas (2015) the final tone contour of English disjunctive questions are assigned semantic effects. The final falling tone is said to signal that exactly one of the list items holds, whereas the rising intonation perceivable on disjunctive polar questions is said to not have any such effect (cf. Biezma and Rawlins (2012)). In particular, final falling tone signals the presence of Closed and final rising intonation signals the presence of Open (cf. Roelofsen and Farkas (2015): 372, 375).

When turning to the disjunctive phrase in alternative questions and disjunctive polar questions we also find prosodic differences. The disjunctive phrase of an alternative question is standardly pronounced with a
prosodic phrase break (approximately, there are breaks in the pronunciation of this part of the sentence) and a final rise on the left disjunct (cf. Roelofsen and Farkas (2015): 372). These are absent in case of disjunctive polar questions. This prosodic difference is utilized in Roelofsen and Farkas (2015). The prosodic phrase break plus rise on the left disjunct in alternative questions is taken to signal that disjunction separates two items, whereas the prosody of the disjunctive phrase in disjunctive polar question is seen as signaling that the disjunction is inside the item (cf. Roelofsen and Farkas (2015): 372).

Last we want to note that in Pruitt and Roelofsen (2013) it is argued that in alternative questions, the disjuncts are focused (cf. p. 646-647 therein). We will take this over as an assumption.

### 4.3 Plain polar and disjunctive questions

In this section we will be concerned with sentences such as in the example below:
(29) Does Mary dance?

Particularly, we are interested in its logical form. Clearly, the body does not contain a disjunction. In fact, the body is a single item. By assumption, we have final rising intonation, and so we have an open list. Since we have interrogative syntax, we clearly have InT at work. This gives us as a logical form:


What is missing is an analysis of item $_{1}$. We know that item ${ }_{1}$ includes application of ! to 【Mary dances】 ${ }^{o}$. The latter we need to determine now. We assume the following:

Definition 4.3.1. (Fragment)

| CAT | Item | $\llbracket \cdot \rrbracket^{o}: \tau$ | $\llbracket \cdot \rrbracket^{f}: \tau$ |
| :---: | :---: | :---: | :---: |
| PN | Mary | $m: \mathbf{e}$ | $\lambda x_{\mathbf{e}} \cdot \mathbf{1}:\langle\mathbf{e}, t\rangle$ |
| IV | dance | $\lambda x_{\mathbf{e}} \cdot \lambda s \cdot \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . D(x)(i)$ | $\lambda x_{\langle\mathbf{e}, t\rangle} \cdot \mathbf{1}:\langle\langle\mathbf{e}, t\rangle, t\rangle$ |

In general, we will translate proper names in the same manner as Mary, and intransitive verbs in the same manner as dance. Furthermore, we will introduce some notation since the translations of predicates is lengthy:
Notation 1. (hat notation, $\hat{P}$ )
For an atomic, non-presuppositional predicate $\lambda \vec{x} . \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . P(\vec{x})(i)$ we write $\hat{P}$.
For example, we write $\hat{D}$ for $\lambda x . \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . D(x)(i)$.

Given this, we have for item ${ }_{1}$ the following analysis:


This makes for the meaning of the whole construction simply ?! $\hat{D}(m)$ due to Open. This can be simplified to ? $\hat{D}(m)$.

The result fits our intuitions about the meaning of Does Mary dance?. Intuitively, the question asks whether Mary either dances, or not. The formalization in CRISP says that the question is resolved either in a state in which Mary dances, or in a state in which Mary doesn't dance.

We can make this visible by means of a picture:


Figure 4.1: Derivation of a plain polar question. The black dashed line represents $\pi_{s}$, the filled purple area represents $\alpha_{s}$, for $s$ an alternative (i.e. a maximal state supporting the proposition that Mary dances.) The orange area represents the complement of the purple area. It is added by Open. Again, we only represent maximal states. The orange area represents thus the proposition that Mary does not dance. Obviously, a state supporting the question supports either that Mary dances or that she does not dance.

Let us turn to our example under its disjunctive polar reading:
(30) Does Mary dance or sing?

We said that by assumption, on its disjunctive polar reading the question comes with a final rising tone. Thus, we have an open interrogative list (taking again interrogative syntax to mean that Int is at work). Furthermore, the disjunctive phrase is pronounced without a prosodic phrase break and rise on the left disjunct. Thus, the disjunction is treated as being inside the item. Consequently we have only one item. ${ }^{4}$ Therefore, disjunctive polar questions are but simply a special case of plain polar questions. The difference lies simply in a more complex body. We will address the form of disjunction below when discussing alternative

[^11]questions.


Figure 4.2: Derivation of a disjunctive polar question. The derivation of the disjunction is omitted.
We will shortly come back to polar questions with final fall when discussing exhaustivity (see §4.4.2).

### 4.4 Alternative questions

In this section we will be concerned with the logical form of alternative questions. As stated earlier, alternative questions like Does Mary dance or sing are pronounced with a final falling tone. This is taken to trigger Closed at the logical form. The effect of this operator is to exhaustify the meaning of the propositional argument (the meaning of the body) so that we arrive at the presupposition that exactly one of the items making up the body holds. We thus need to discuss exhaustification in order to properly understand the meaning of logical forms of alternative questions.

The section is organized as follows. We will first provide the logical form of our running example repeated below:
(31) Does Mary dance or sing?

This can be done (more or less) without discussing exhaustification.
We then turn to the question what exhaustification needs to do in order to achieve the proposed effect of Closed. We will find, in accordance with Roelofsen and Farkas (2015) that the operator involved in Closed, in our case exh, needs to get rid of the possibility that Mary does both things, dancing and singing. Only then will we derive the presupposition that exactly one of the items holds.

### 4.4.1 The logical form of alternative questions

As already stated, the alternative question in (31) has final falling intonation. As such, we have as classifiers Int and Closed at work on the logical form of the question. Moreover, we said that alternative questions come with a prosodic phrase break on the disjunctive phrase and rising intonation on the left disjunct of it. This is taken to signal that disjunction separates two items. Importantly, since items are taken to express propositions, i.e. $[i t e m]^{o}=X_{T}$, we are forced to assign (31) a logical form where disjunction takes two propositional arguments. These arguments are the informative propositions that Mary dances and that Mary sings. Thus we arrive at the following logical form:


We left the items unanalyzed as we think their form is clear. Importantly, we cannot have a logical form on which we first derive a narrow scope disjunction as below, for this always results in an item internal disjunction:


This logical form is inappropriate for an alternative question and indeed corresponds to that of a disjunctive polar question. ${ }^{5}$ This brings us back to disjunction in disjunctive questions. As far as matters stand, the following can be said: alternative questions need to be analyzed with disjunction taking propositional arguments. This is not the case for disjunctive polar questions. We may thus treat disjunction differently in these questions. The narrow scope analysis available in disjunctive polar questions may also correspond to the prosody of such questions, one may speculate. On the other hand, we may for simplicity assume that we have in both cases the same treatment of disjunction. From a logical point we can account with a wide scope analysis for both disjunctive polar questions and alternative questions. The semantics of lists, as is clear, alone does not settle this issue. What is needed is an empirical argument for or against a wide scope analysis of disjunction in disjunctive polar questions. In this thesis we do not want to go into such an argument, instead we assume that we have wide scope disjunction in both disjunctive polar and alternative questions. Again, from a logical perspective it does not matter, and a narrow scope analysis is available if needed. We leave a more sophisticated analysis for the future.

We want to comment on the wide scope analysis for alternative questions (and polar questions). We will

[^12]leave aside the fact that on such an account we need to make use of ellipsis or similar mechanisms when accounting for the surface structure of the question. Obviously, the questions are not pronounced with two full-fleshed disjuncts. We will in this thesis simply assume that an appropriate amount of the second disjunct stays silent which we indicate by bracketing "( )" on logical form. Again, we leave a more sophisticated analysis for the future. ${ }^{6}$

Returning to our example, (31) asks which of the two mentioned alternatives (items) holds, presupposing that exatcly one alternative holds. Our formal representation needs to account for this. In particular, it is the job of Closed to strengthen the presupposition of the question from at least one alternative is the case to exactly one alternative is the case. This can be done in different ways, but their is an easy one.

Observe that the disjunction as such leaves open the possibility that Mary both sings and dances. This holds for both our intuition and inquisitive semantics. Without the strengthening of Closed the presupposition of the question would then simply be that Mary dances, or sings, or both. What we then want to exclude is the possibility that Mary can do both. The effect of excluding this possibility consists in that we strengthen the presupposition of the question from at least one alternative holds to exactly one alternative holds, because in that case the alternatives become mutually exclusive. If Mary cannot both dance and sing, and we know she dances, then Mary does not sing and vice versa. The same point is made by Roelofsen and Farkas (2015) on page 375.

We will in the next section introduce the operator exh. It follows the tradition originated by J. A. Groenendijk and Stokhof (1984) and Szabolcsi (1983). We will draw heavily from Spector (2016).

### 4.4.2 Exhaustivity

We will now turn to the definition of exh. We will proceed as follows. We will first provide the version of the operator presented in Spector (2016). It will be our blue print for exh. In fact, the latter is simply the inquisitive version of the former.

However, we will treat exh as a covert focus operator which makes use of $\sim$. This means we will have to adjust the logical form of alternative questions slightly. This will be the second part of this subsection.

Before getting started, the reader should note that Spector (2016) is not using an inquisitive semantics framework. In his paper, propositions are sets of worlds (cf. Spector (2016): 6). Further, worlds are taken to be models throughout his paper (cf. Spector (2016): 6).

The operator of interest is $e x h_{m w}$ as defined in Spector (2016). The operator dates from work by J. A. Groenendijk and Stokhof (1984) and Szabolcsi (1983) and much other work in formal semantics and pragmatics among which are van Rooij and Schulz (2004), Schulz and van Rooij (2006), Spector (2003), Spector (2006) . The operator takes two arguments, a classical proposition ${ }^{7} \phi$ and a set of alternatives $\mathcal{A L \mathcal { L }}$, which is a set of classical propositions (cf. Spector (2016)). The idea is that exh $h_{m w}$ keeps the set of $\phi$-worlds that make as few as possible of the propositions in $\mathcal{A L \mathcal { L }}$ true. This set minimizes the set of true alternatives (cf. Spector (2016): 3).

In order to define $e x h_{m w}$ we need to define some auxiliary notions (cf. Spector (2016): 7). In the following

[^13]$\mathcal{A L T}$ is a set of propositions in the sense of Spector (2016), thus a set of sets of worlds.
Definition 4.4.1. (preorder on $\mathcal{A L T}, \leq_{\mathcal{A L T}}$ )

For a set of alternatives $\mathcal{A L T}$, we define $\leq_{\mathcal{A L T}}$ as: $u \leq_{\mathcal{A L T}} v$ iff $\{a \in \mathcal{A \mathcal { L } T}: a(u)=1\} \subseteq\{a \in \mathcal{A L T}: a(v)=1\}$
Here $u, v$ are worlds. By $a(u)=1$ it is understood that the proposition $a$ holds true of the world $u$ (or equivalently, that $a$ is true in model $u$ ). Similarly for $a(v)$. The definition of preorder on $\mathcal{A L \mathcal { L }}$, then, says that $u \leq \mathcal{A L T}$ means that $v$ makes at least the alternatives true that are true in $u$. Or, $v$ makes potentially more alternatives $a \in \mathcal{A} \mathcal{L T}$ true than $u$ does. Next we define the strict preorder on $\mathcal{A L \mathcal { T }},<_{\mathcal{A L} \mathcal{T}}$ (cf. Spector (2016): 7):

Definition 4.4.2. (strict preorder on $\mathcal{A L T},<\mathcal{A L T}$ )

Let $\mathcal{A L \mathcal { T }}$ be a set of alternatives. We define: $u<_{\mathcal{A L} \mathcal{T}} v$ iff $u \leq_{\mathcal{A L T}} v \wedge \neg\left(v \leq_{\mathcal{A} \mathcal{T}} u\right)$
Given these definitions, we can now define $e x h_{m w}$ (cf. Spector (2016): 7):
Definition 4.4.3. $\left(e x h_{m w}\right)$
Let $\mathcal{A L T}$ be a set of alternatives and $\phi$ a proposition. We define:
$\operatorname{exh}_{m w}(\mathcal{A L T}, \phi)=\left\{u: \phi(u)=1 \wedge \neg \exists v\left(\phi(v)=1 \wedge v<_{\mathcal{A L T}} u\right)\right\}$.
Equivalently: $\operatorname{exh}_{m w}(\mathcal{A L \mathcal { L }}, \phi)=\phi \cap\{u: \neg \exists v(\phi(v)=1 \wedge v<\mathcal{A L T} u)\}$
$e x h_{m w}$ works then as follows. Considering the first definition, exh $h_{m w}$ takes an alternative set $\mathcal{A L \mathcal { L }}$ and a proposition $\phi$ as argument. Next, it collects all those worlds that make $\phi$ true, i.e. the $\phi$-worlds. Further, it keeps only the $\phi$-worlds that are minimal with respect to $<\mathcal{A L \mathcal { L }}$. This means, that we end up with only those $\phi$-worlds that make the least alternatives $a \in \mathcal{A} \mathcal{L} \mathcal{T}$ true. Equivalently, we take the intersection of $\phi$ and the set of worlds $u$ which do not have any $\phi$-world predecessors with respect to $<\mathcal{A L \mathcal { L }}$ (note, $u$ may not satisfy $\phi$ in this case).

Obviously, we cannot simply take this over into CRISP. We need to adjust these definitions and put them into an appropriate type-theoretic costume. This can be done easily step by step. First $\mathcal{A L} \mathcal{T}$ is still a set of propositions, but in the inquisitive sense. For clarity, we denote this set by $C$. We then define the preorder $\leq_{\mathcal{A} \mathcal{L} \mathcal{T}}$ in CRISP as follows:

Definition 4.4.4. ( $\leq_{C}$, preorder on the set of alternatives $C$ )
Let $C$ be a set of alternatives, and $s, t$ states. We define $\leq_{C}$ as: $s \leq_{C} t$ iff $\{P \in C: P(s)\} \subseteq\{P \in C: P(t)\}$.
This mirrors the definition in Spector (2016) sufficiently. We could mirror the definition in Spector (2016) exactly, when we were to define $\leq_{C}$ in terms of truth too. This would mean we would consider only states $s=\langle\{i\},\{i\}\rangle$. Our definition is thus more general.

Next, we will define $<_{\mathcal{A L} \mathcal{T}}$ in CRISP. We will denote the strict preorder by $<_{C}$ :
Definition 4.4.5. $\left(<_{C}\right.$, strict preorder on $\left.C\right)$
Let $C$ be a set of alternatives. Let $s, t$ be states. We define: $s<_{C} t$ iff $s \leq_{C} t \wedge \neg\left(t \leq_{C} s\right)$
With these definitions at hand, we can define $e x h_{m w}$ in CRISP. We will denote it simply by exh.

Definition 4.4.6. (exh)
$\operatorname{exh}(C, \phi)=\lambda s . \exists s^{\prime}\left[s \sqsubseteq s^{\prime} \wedge \phi\left(s^{\prime}\right) \wedge \neg \exists u\left[\phi(u) \wedge u<_{C} s^{\prime}\right]\right]$
The operator works as follows. It takes a state $s$ as an argument and checks for whether there is a state $s^{\prime}$ such that $s$ is a substate of $s^{\prime}$ and $s^{\prime}$ supports $\phi$ (remember, we are now in an inquisitive framework, so support is the primary notion, not truth). It further checks whether there is no state $u$ such that $u$ supports $\phi$ but $u$ supports strictly less alternatives in $C$ than $s^{\prime}$. If these conditions are met, $s$ will be part of the new proposition (the condition $\exists s^{\prime} . s \sqsubseteq s^{\prime} \wedge \phi\left(s^{\prime}\right)$ ensures downward closure).

Before considering whether exh does what we promised we want to address $C$. Particularly, we will assume in this thesis that $C$ is a free variable. Moreover, the value of this variable is restricted by $\sim$. The assumption in sum: we assume in this thesis that exh is a focus operator. It, like too and only (see §6), comes accompanied with a free variable $C$ which it uses as a domain of quantification. This variable is restricted by focus interpretation that is by $\sim$. Consequently, $C$ is then a set of focus alternatives. ${ }^{8}$ Thus, we will assign alternative questions a slightly different logical form as initially done:


Given the logical form, we can show that exh does it's job. We have the following relevant values:

1. $\llbracket \mathrm{item}_{1} \rrbracket^{o}=!\hat{D}(m)=\hat{D}(m)$
2. $\llbracket$ item $_{2} \rrbracket^{o}=!\hat{S}(m)=\hat{S}(m)$

[^14]3. $\llbracket$ item $_{1} \rrbracket^{f}=\left\{\mathcal{P}(m): \mathcal{P} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
4. $\llbracket$ item $_{2} \rrbracket^{f}=\left\{\mathcal{Q}(m): \mathcal{Q} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
5. $\llbracket(1) \rrbracket^{o}=\hat{D}(m) w \hat{S}(m)$
6. $\left[(1) \rrbracket^{f}=\left\{\mathcal{P}(m) w \mathcal{Q}(m): \mathcal{P}, \mathcal{Q} \in D_{\langle\mathbf{e}, T\rangle}\right\}\right.$
7. $\llbracket(2) \rrbracket^{o}=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge\left(\alpha_{s} \subseteq D(m) \vee \alpha_{s} \subseteq S(m)\right)$

The effect of Closed is now the following. Let us assume that the presupposition triggered by $\sim C$ is satisfied and that $C$ is the set of Sauerland alternatives (cf. Sauerland (2004)) without conjunction, that is:

$$
C=\{\hat{D}(m) w \hat{S}(m)), \hat{D}(m), \hat{S}(m)\}
$$

$e x h$ will now systematically expel any state $s$ from the propositional argument of ClOSED that supports both alternatives $\hat{D}(m)$ and $\hat{S}(m)$, because such state would not be minimal. Hence, we rule out the possibility that Mary does both, dancing and singing. Note however that nothing more happens, since any state satisfying the prejacent $\hat{D}(m) w \hat{S}(m)$ necessarily satisfies one of the two disjunct. We will thus end up with a proposition in which the alternatives are mutually exclusive. This leads then to the presupposition that exactly one of the alternatives holds. The situation is displayed below by means of a diagram.


Figure 4.3: Schematic computation of an alternative question meaning. What we see is that Closed only affects the at-issue content of the disjunction (i.e. the body). But, we see that this leads to the strong presupposition introduced by Int. We assume here disjuncts without presuppositions. The computation is slightly more complicated in case the disjuncts have presuppositions themselves.

Let us note that we can also represent the meanings of plain polar questions and disjunctive polar questions in case they come with a final falling tone. First, observe that this means that the item is handed over to Closed. Second, we need to assume that there is focus marking. A reasonable position for this is the item as such. The alternatives are then propositions. The reasonable alternatives are the proposition expressed by the item and its negation. In that case exhaustification by exh is vacuous, for we will not affect the proposition expressed by the item. Application of the resulting meaning for Closed to Int results then in
a polar question meaning with the presupposition that one of the alternatives holds. This presupposition is trivial for polar questions. We present the following result:

Proposition 1. Let $P$ be an arbitrary proposition. We have ?! $P_{!?!P} \equiv ?!P$.
Proof. ( $\Rightarrow$ ). Trivial.
$(\Leftarrow)$. Assume $s \vDash ?!P$, for $s$ an arbitrary state. We need to show: $s^{\top} \vDash!?!P$. We have:

$$
!?!P\left(s^{\top}\right)=\exists T\left(\bigsqcup T=s^{\top} \wedge \forall t \in T: ?!P(t)\right)
$$

We need to find $T$ such that:
(i) $\sqcup T=s^{\top}$
(ii) $\forall t \in T: ?!P(t)$

Such $T$ can be given by $T_{0} \cup T_{1}$, where

$$
\begin{aligned}
& T_{0}=\left\{t: \alpha_{t} \subseteq \pi_{t} \subseteq \pi_{s} \cap|\sqcap!P|\right\} \\
& T_{1}=\left\{t: \alpha_{t} \subseteq \pi_{t} \subseteq \pi_{s} \cap|!P|\right\}
\end{aligned}
$$

We have $\sqcup T \subseteq s^{\top}$ by construction. We need to show $\bigsqcup T \sqsupseteq s^{\top}$. Observe that $\sqcup T \sqsupset s^{\top}$ if and only if $\pi_{s} \subseteq \bigcup_{t \in T} \alpha_{t}$. Let $i \in \pi_{s}$ be an arbitrary possibility. By $i \in \pi_{s}$ and $s \vDash ?!P$ we have $i \in \pi_{s} \cap|!P|$ or $i \in \pi_{s} \cap|\rightarrow!P|$. If the former, we have $\langle\{i\},\{i\}\rangle \in T_{1}$ and thus $i \in \bigcup_{t \in T} \alpha_{t}$. In the latter case, we have $\langle\{i\},\{i\}\rangle \in T_{0}$ and thus we have $i \in \bigcup_{t \in T} \alpha_{t}$. As $i \in \pi_{s}$ was arbitrary, we have for all $i \in \pi_{s}$ that $i \in \bigcup_{t \in T} \alpha_{t}$, and so $\pi_{s} \subseteq \bigcup_{t \in T} \alpha_{t}$. This shows that our choice of $T$ is a good one.

We now only need to show that $\forall t \in T: ?!P(t)$ holds. Observe: if $t \in T$, then $t \in T_{0}$ or $t \in T_{1}$. We have:

$$
\begin{aligned}
& t \in T_{0} \Rightarrow \alpha_{t} \subseteq \pi_{t} \subseteq \pi_{s} \cap|\neg!P| \\
& t \in T_{1} \Rightarrow \alpha_{t} \subseteq \pi_{t} \subseteq \pi_{s} \cap|!P|
\end{aligned}
$$

Thus ?! $P(t)$. Since $t$ was arbitrary, we have indeed for all $t \in T: ?!P(t)$. This finishes our proof.

### 4.5 Hurford Disjunctions

In this section we will address a data point that relates to exhaustification in disjunctive questions and therefore to the logical form of alternative questions. We assigned alternative questions a logical form which has exhaustfication hardwired into it. Particularly, since Closed applies to the body, which is where disjunction is located, exhaustificatio will always be applied on top of a disjunction in alternative questions. The data points to be presented in this section challenge ths position and make clear that exhaustification must locally apply in some but not all cases. Consequently, we need to allow for local exhaustfication in disjunctive questions. ${ }^{9}$

[^15]The analysis of plain/ disjunctive polar questions and alternative questions provided here accounts for cases like (26) and (29). However, the analysis cannot account for the following cases:
a. *Was John born in France or in Paris?
b. *Was John born in Paris or in France?
a. Did Mary read most or all of the books?
b. Did Mary read all or most of the books?

The cases displayed in (32) are problematic for our current analysis because (i) it cannot account for the deviance of these cases, and (ii) it assigns them inappropriate meanings. This is true on both the polar and the alternative readings of (32-a) and (32-b) and is obviously bad. In case of (33) the situation is slightly better.Unlike before, these questons are non-deviant. But, as before, we do not derive the correct meaning for these examples. What we derive as meanings for all these cases, i.e. the cases in (32) and (33), are plain polar questions. The reason is that the disjunctions on any reading reduce to one of the disjuncts. This can be seen easily from the fact that in (32) the meaning of the disjunct "John was born in Paris" is a subset of the meaning of the disjunct "John was born in France". In (33) we find the same situation. Given the meaning of disjunction in inquisitive semantics, the set of states that resolve at least one of the disjuncts, and the above fact, the disjunctions in (32) and (33) are always equivalent to one of its disjuncts.

The above data points are not new (cf. Ciardelli and Roelofsen (2017) for instance). These are instances of Hurford disjunctions (cf. Hurford (1974)) in questions. In the remainder of this section we want to show that our account can account for these data when subscribing to a few extra assumptions. In particular, we will make clear that the proposal provided in Katzir and Singh (2013) is available to us (as is also done in Ciardelli and Roelofsen (2017)).

As was noted in Ciardelli and Roelofsen (2017), the architecture of inquisitive semantics gives rise to the above facts and these in turn allow for adopting the explanation of the data points in (32) and (33) provided in Katzir and Singh (2013). We will adopt this explanation here. It makes use of a redundancy principle and local exhaustification by means of an exhaustifier in the sense of $\S 4.4$.

The redundancy principle reads as follows:
Definition 4.5.1. (Local redundancy check, Katzir and Singh (2013) $)^{10}$
$\mathbf{S}$ is deviant if S contains $\gamma$ and $\llbracket \gamma \rrbracket^{o}=\llbracket O(\alpha, \beta) \rrbracket^{o} \equiv_{c} \llbracket \zeta \rrbracket^{o}, \zeta \in\{\alpha, \beta\}$.
(Katzir and Singh (2013): 210)
Here, S is a sentence and $\gamma$ is a constituent. Furthermore $O$ is an operator. The principle then says that if a sentence $S$ is deviant if it has a constituent which is interpreted as a two-place operator and the meaning of that constituent is contextually equivalent $\left(\equiv_{c}\right)$ to the meaning of one of the arguments of the operator.

Clearly, the local redundancy check predicts Hurford disjunctions to be deviant. In particular, intuitively it should predict that the sentences in (32) and (33) are deviant, for as we saw earlier these sentences do not pass the local redundancy check. The sentences in (33) are felicitous though.

An explanation as to why the sentences in (33) are non-deviant can be provided when assuming that these sentences can receive a different parsing which involves an exhaustifier. Crucially, exhaustification

[^16]must happen locally, not above the level of disjunction - for then it is too late. Schematically, lending from the grammatical view of scalar implicatures (cf. Fox (2007), Chierchia, Fox, and Spector (2012) for instance), we can parse the disjunctions in the sentences in (32) and (33) as:
\[

$$
\begin{equation*}
\operatorname{exh}(A) w B \tag{34}
\end{equation*}
$$

\]

the rest of the derivation being the same as in the cases considered earlier so far. ${ }^{11}$
Here, we take $A$ to be the weaker disjunct and $e x h$ is the operator from $\S 4.4$. The effect of exhaustifying the weaker disjunct lies in breaking the contextual equivalence of the disjunction with this disjunct (if the context allows for it, cf. Katzir and Singh (2013) for cases where this strategy does not work out). Of course, in case of (32-a) and (32-b), exhaustifcation will not safe the structures. Focusing on (32-a) the weaker disjunct is "John was born in France". The effect of exhaustifying it lies in strengthening its meaning to something like "John was born in France, but not in Germany, etc.". Now, given our world knowledge, we know that the non-weaker disjunct "John was born in Paris" entails not only that John was born in France (the weaker disjunct), but also that John was not born in Germany, etc. Consequently, we still find that the disjunction as such is equivalent to its weaker disjunct "John was born in France, but not in Germany, etc.". Exhaustification does not safe the structure from deviance.

Exhaustification safes the sentences (33-a) and (33-b) though, for the weaker disjunction is now no longer equivalent to the disjunction (given the context does not force the equivalence). This can be seen from the fact that exhaustifying the weaker disjunct "Mary read most of the books" gives us something along the lines of "Mary read most of the books, but not all of them", and clearly this is no longer logically entailed by the other disjunct. Given an appropriate context, the new parse passes the local redundancy check.

We need to note the following: when we want to use exh from $\S 4.4$ we need to assume that the weaker disjunct is $F$-marked. Alternatively, we may assume that there is another exhaustifier involved in such cases. Yet another option is available: it may be that exhaustifiers like the one in $\S 4.4$ are informed about the relevant alternatives not only by focus interpretation as assumed for $e x h$. It may be that some other process can do the job as well, or that in fact exhaustification does not involve focus interpretation at all. We think these considerations are worth investigating (especially when we assume that scalar alternatives are focus alternatives), but leave this for the future. ${ }^{12}$

In sum: our account is in fact able to account for Hurford disjunctions by adopting the proposal presented in Katzir and Singh (2013). Consequently, disjunctive questions have in some but not all cases a more complex logical form than said earlier. Nevertheless, the overall picture stays more or less the same. We do not have to revise our list semantics in a crucial way, all we need to do is to allow for more complex bodies. Hence, our treatment of alternative questions with a hardwired exhaustifier on top of disjunction is not threatend by Hurford disjunctions as was initially suggested by the data points. The same goes for disjunctive polar questions. This is a point in favor of our analysis.

[^17]
### 4.6 Summary

In this chapter we considered three different question types: plain and disjunctive polar questions (§4.3) and alternative questions (§4.4). We used the list semantics of Roelofsen and Farkas (2015) to provide logical forms for these question types. This was necessary for we can only tell why too is deviant in some but not all questions when we know what their meaning is. We furthermore considered Hurford disjunctions in questions. This was motivated by the use of exhaustifcation in alternative questions. We saw that our treatment of alternative questions can account for Hurford disjuncions by making use of the account provided in Katzir and Singh (2013). This shows then that the place of exhaustfication assumed in list semantics is unproblematic (from this perspective).

## Chapter 5

## Who-Questions

In this chapter we will discuss who-questions in matrix form. We will zoom in on mention-some and strongly exhaustive readings, leaving weakly-exhaustive readings for the future. As in the case of polar and alternative questions, this is necessary for explaining the data on too.

The chapter is organized as follows. We will first discuss the account of Champollion et al. (2017) (§5.1). Here we will mostly be concerned with the derivation of mention-some readings. We will argue that some of the assumptions made are unwarranted and provide an alternative account which even simplifies the derivation of mention-some readings in Champollion et al. (2017) (§5.2). We then turn to the derivation of mention-all readings (§5.3). We will see initially a limitation of our present account: it cannot derive mention-all readings. We will overcome this limitation by following the proposal in Dayal (1996) and close $D_{e}$ under Link's (1983) individual sum. This allows for the missing alternatives. We then, in the second step, define an appropriate exhaustivity operator $X$ (see Klinedinst and Rothschild (2011) and Uegaki (2015) for similar ideas). We conclude with a summary (§5.4).

### 5.1 Champollion et al. (2017) on who

Let us start by repeating the basics. Following Baker (1970), Champollion et al. (2017) assume that all questions (embedded or not) are headed with a silent question morpheme Q. Q projects an interrogative nucleus. The complement of $Q$ is an abstract, schematically:
interrogative nucleus


The meaning of the question morpheme Q is:
Definition 5.1.1. (Semantics of Q, Champollion et al. (2017): 13)
$\llbracket \mathrm{Q}^{n} \rrbracket_{g}:=\lambda P_{\left\langle e^{n}, T\right\rangle} \cdot(\langle ?\rangle \nexists \vec{x} \cdot P(\vec{x}))_{!(?\rangle \nexists \vec{x} \cdot P(\vec{x})}$
Champollion et al. (2017), again following Baker (1970) propose the following translation for who: ${ }^{1}$
Definition 5.1.2. ([ $\left[\mathrm{who}_{i}\right]_{g}$, Champollion et al. (2017))
$\llbracket w h o_{i} \rrbracket_{g}=\lambda P_{\langle e, T\rangle}!P(g(i))$

[^18]The assumptions leading to this translation are (cf. Champollion et al. (2017): 5):
(i) who carries an index $i$ with it
(ii) $Q$ either binds this index or it triggers $\lambda$-abstraction over it

The example derivation below makes this visible:

Who smokes?


Below we provided an example denotation for "Who smokes?" in Champollion et al. (2017)


Figure 5.1: Denotation of "Who smokes?" given the logical space $\mathcal{L}=\left\{w_{a, b}, w_{a}, w_{b}, w_{\varnothing}\right\}$

The role of who in Champollion et al. (2017) thus consist in (i) providing an index which can be $\lambda$ abstracted and then function as an argument for the meaning of Q, (ii) making the abstract informative. who achieves this by taking a property as an argument, mapping it into the scope of !, and applying its index to the property. The semantics of who is thus reduced to a minimum. Instead, the burden lies on Q in the derivation. The question morpheme is responsible for providing a question meaning and to quantify over instances of a property. This can be seen best when comparing it to other accounts of who. Let us consider Ciardelli et al. (2017) to this end.

They provide the following meaning rule for who:
Definition 5.1.3. (who in Ciardelli et al. (2017): 20)
[who] $=\lambda P_{\langle e, T\rangle} \cdot \cup_{x \in D_{e}} P x$
This can be paraphrased in Champollion et al. (2017) by $\lambda P . \exists x . P x$. So, Ciardelli et al. (2017) assume that who is a generalized quantifier. Particularly, it is synonymous to someone on their account. What we see then is that Champollion et al. (2017) rip off the quantifier identity of who and degrade it to supply an index. Instead, the quantificational aspect of who is moved into the meaning of the question morpheme as is visible from the formulas.

Moreover, we see that no! is present in the meaning rule of who on the account of Ciardelli et al. (2017), and no index $g(i)$ either. This is because of the generalized quantifier treatment.

In sum, Champollion et al. (2017) split up the meaning as it is given in Ciardelli et al. (2017) and put most of it into the question morpheme, making who instead into an expression which allows for abstract formation.

The position of Champollion et al. (2017) is motivated by the following facts:
Given the basic assumptions on questions:
(a) accounts treating who as inquisitive existentials face the challenge of island constraints (cf. Champollion et al. (2017): 5)
(b) simply assuming that wh-phrases denote identity functions is naive. Such an approach runs into troubles with inquisitive abstracts as illustrated by (23) and (25) of Champollion et al. (2017) (cf. p. 6 therein)
(c) in non-wh-questions (e.g. alternative questions) often no! seems to be applied to the argument of the meaning of Q (Champollion et al. (2017): 6)

Item (a) addresses the account of Ciardelli et al. (2017) and Theiler (2014). Such accounts are less suitable given Champollion et al.'s basic assumption on questions. Item (b) motivates presence of !. We will address this point below. Item (c) is supposed to motivate that! is part of the meaning of wh-phrases. More on this below too.

The translation proposed by Champollion et al. (2017) circumvents these issues, as we can interpret whphrases in situ thereby avoiding island constraints, and we can deal well with inquisitive abstracts, due to the presence of ! in the translation of who. The translation is thus suitable for Champollion et al. (2017). The examples below illustrate this fact:


The relevant values are (in the semantics of Champollion et al. (2017)):
(i) $\llbracket$ walk $\rrbracket_{g}=\lambda x . \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w . W(x)(w)$
(ii) $\llbracket \mathrm{who}_{2} \rrbracket_{g}=\lambda P .!P(g(2))$
(iii) $\llbracket(1) \rrbracket_{g}=!\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w . W(g(2))(w)$
(iv) $\left[\right.$ that $\rrbracket_{g}=\lambda P_{T} \cdot P \quad$ (no translation provided by Champollion et al. (2017), but it does what it should)
(v) $\llbracket(2) \rrbracket_{g}=\llbracket(1) \rrbracket_{g}$
(vi) [think $\rrbracket_{g}=\lambda P_{T} \cdot \lambda x \cdot \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha \subseteq \lambda w \cdot T(P, x)(w) \quad$ (no translation provided by Champollion et al. (2017), but it does what it should)
(vii) $\llbracket(3) \rrbracket_{g}=\lambda x \cdot \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha \subseteq \lambda w \cdot T\left(\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot W(g(2))(w)\right], x\right)(w)$
(viii) $\left[\right.$ who $_{1} \rrbracket_{g}=\lambda P .!P(g(1))$
(ix) 【(4) $\rrbracket_{g}=!\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha \subseteq \lambda w \cdot T\left(\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot W(g(2))(w)\right], g(1)\right)(w)$
(x) [(5) $]_{g}=\lambda x_{1} \cdot \lambda x_{2} .!\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha \subseteq \lambda w \cdot T\left(\left[\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot W\left(x_{2}\right)(w)\right], x_{1}\right)(w)$
(xi) $\left[\mathrm{Q}^{n} \rrbracket_{g}:=\lambda P_{\left\langle e^{n}, T\right\rangle} .(\langle ?\rangle \exists \vec{x} \cdot P(\vec{x}))_{!(?)} \nexists \vec{x} \cdot P(\vec{x})\right.$
(xii) $\llbracket \mathrm{S} \rrbracket_{g}=\langle ?\rangle \exists x, y\left[!\lambda s \cdot \alpha_{s} \subseteq \pi_{s} \wedge \alpha \subseteq \lambda w \cdot T\left(\left[\lambda s \cdot \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w \cdot W(y)(w)\right], x\right)(w)\right] \quad$ (we dropped the presupposition)
(37) shows that the account can deal with multiple-wh questions easily. We do not encounter problems with island constraints. The wh-phrases stay in situ.


The relevant values are:

1. $\llbracket \operatorname{talk} \rrbracket_{g}=\lambda x . \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w . T(x)(w)$
2. $\llbracket$ walk $\rrbracket_{g}=\lambda x . \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda w . W(x)(w)$
3. $\llbracket$ or $\rrbracket_{g}=\lambda Q_{\langle e, T\rangle} \cdot \lambda P_{\langle e, T\rangle} \cdot \lambda x \cdot P(x) w ~ Q(x)$
4. $\left[\mathrm{who}_{1}\right]_{g}=\lambda P!!P(g(1))$
(38) shows that we can deal with inquisitive properties which would lead to an inquisitive abstract by utilizing !. We simply block the possibility of having inquisitive abstracts.

Now, the translation of Champollion et al. (2017) indeed solves the major issues (a) and (b), but we find that (c) is unwarranted. Moreover, the example (38) used by Champollion et al. (2017) could in principle be derived with a simpler meaning postulate for who when employing the prosody-meaning mappings of lists.

We agree with Champollion et al. that in the example (an alternative question) no such operator is scoping over the disjunctive phrase. And as such, we agree, Q can indeed not be the source for !. But this does not motivate the claim that it is the $w h$-phrase. It could also be due to intonation as we will argue on our second point.

So, consider the example of Champollion et al. (2017):

> Who walks or talks?
[(23) in Champollion et al. (2017)]

On the most natural reading we have that the disjunctive phrase is pronounced without a prosodic phrase break (thus as in a polar question). We could thus assume that the disjunctive phrase is to be translated exactly like the body of a polar question. In particular, this could be worked out as follows. Assume that [who】 ${ }^{o}=x_{\langle a, e\rangle}$ is simply a variable. So we treat who like a pronoun. Then, using the semantics of lists, we derive the following meaning for the body: ! $\left(\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . W(x)(i) w \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . T(x)(i)\right)$. The rest is then similar to Champollion et al. (2017): we lambda abstract over $x$ yielding $\lambda x!\left(\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq\right.$ $\left.\lambda i . W(x g)(i) w \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . T(x g)(i)\right)$. Last, we can apply the meaning of Q to this, giving us exactly the same as Champollion et al. (2017) derive modulo the differences in type theory: ${ }^{2}$

$$
\begin{equation*}
\exists x .!(\hat{W}(x) w \hat{T}(x)) \tag{40}
\end{equation*}
$$

So, even here it seems not necessary that who is the source of !. We may take intonation as a principal factor. In conclusion, there is no motivation beyond (b) for assuming that wh-phrases are the locus of !. And the issue in (b) may be solved in a different fashion. In particular, we may assume that wh-phrase in subject position together with a final falling intonation in root wh-questions triggers the presence of ! between the abstract and the body at LF, here displayed for the case of one $w h$-phrase:
interrogative nucleus


We have to admit though that this strategy seems not to fare so well in case the question is pronounced with a prosodic phrase break on the disjunctive phrase, that is similar to an alternative question. For, when utilizing the list semantics for the derivation of the disjunctive phrase meaning, we essentially end up with
(41) $\quad \lambda x \cdot \hat{W}(x) w \hat{T}(x)$

[^19]which corresponds to the disjunction criticized by Champollion et al. (2017). Moreover, when we now took the intonation to trigger Closed, we would end up with an exhaustified disjunction. We are not sure whether this is in any way appropriate as such an intonation is unnatural. We leave this point for now and return to it later (§5.2). Nevertheless, the point we made earlier stays untouched by this, as the relevant reading (we assume) can be derived without the assumption that the meaning of who utilizes !.

### 5.2 A very simple view on who

We will now provide a very simple view on who. The section is split into two subsections. We start by providing a logical form of who-questions in matrix form. The compositional derivation of it is easier, yet yields superficially the same results as the account of Champollion et al. (2017) (modulo the differences in type theory). Afterwards, we consider the denotation of who-questions. We will see that, as is the case in Champollion et al. (2017), we derive mention-some-readings in this way. Mention-all-readings are the subject of $\S 5.3$.

### 5.2.1 The logical form of who-questions

Before turning to who let us repeat the meaning of Q in CRISP:
Definition 5.2.1. (Semantics of Q in CRISP)
$\llbracket \mathrm{Q} \rrbracket^{o}:=\lambda P_{\left\langle\langle a, e\rangle^{n}, T\right\rangle} \cdot(\langle ?\rangle \nexists \vec{x} \cdot P(\vec{x}))_{!(?\rangle} \nexists \vec{x} \cdot P(\vec{x})$
In this thesis we will assume a very simple semantics for who. Unlike Champollion et al. (2017) the meaning rule for who will not include !. We will treat this as an aspect of wh-questions triggered by intonation. Moreover, we will also not assign it a functional form. The proposal is different from the usual inquisitive semantics treatments, as in Theiler (2014) and Ciardelli et al. (2016) for that matter:

Definition 5.2.2. $\left(\llbracket \text { who } \rrbracket^{o} \text { and } \llbracket \text { who } \rrbracket^{f}\right)^{3}$
[who】 ${ }^{o}=x_{i}$ which is of type $\langle a, e\rangle$
If who bears focus:
[who】 ${ }^{f}=D_{\langle a, e\rangle}$
Otherwise:
$\llbracket$ who $\rrbracket^{f}=\left\{\llbracket\right.$ who $\left.\rrbracket^{o}\right\}$
We treat who basically like $h e_{n}$ in Montague's PTQ. And our derivations will somewhat look like indirect derivations by means of $\mathrm{S} 8 n$ as the tree below illustrates. In particular, we can observe the following differences and similarities to the treatment of root $w h$-questions in Champollion et al. (2017). First, as in Champollion et al. (2017) we start with generating wh-questions in situ. However, unlike in Champollion et al. (2017), we add an intermediary step between the in situ $w h$-phrase and the abstract $i$, namely we add the operator !:

Definition 5.2.3. $\left.(\llbracket!]^{o}\right)$
$\llbracket!\rrbracket^{o}=\lambda P_{T}!!P$
This reflects our earlier strategy. In particular, we will take it that ! is triggered by final falling intonation together with the presence of who in subject position. This means that we split the meaning postulate

[^20]of Champollion et al. (2017) in its two parts. We take who only to denote an index (or in our case a free variable which we index, as is the case in Montague's PTQ). The informative projection is realized afterwards by another operator, !. Note that the meaning of ! asks for a propositional argument. The rest of the derivation is then the same as in Champollion et al. (2017) as is clear from the tree below.


The simpler meaning postulate for who also allows for simpler compositional derivations of multiple whquestions with a wh-phrase in object position. We do not need to type-shift (cf. Champollion et al. (2017): 7). We demonstrate this by an example of Champollion et al. (2017) Who loves whom?:


Last, the reader should note that the problematic case mentioned earlier can be dealt with successfully. We said that when we use the semantics of lists for the compositional derivation of the meaning of a disjunctive phrase such as walks or talks in Who walks or talks?, we in principle may be forced to derive the disjunction $\lambda x$.[! $\hat{W}(x) w!\hat{T}(x)]$ which denotes an inquisitive meaning. However, two facts must be considered. First, unlike in the case of disjunctive questions, the final falling tone is not taken to trigger Closed. We assume that the presence of who in subject position stops this mechanism, instead leading in combination with the prosodic structure to the triggering of !. Second, ! turns anything of type $T$ within its scope into an expression denoting an non-inquisitive and informative proposition. This avoids the earlier mentioned issues, by simply not letting them arise.

The motivation for our position is twofold. One reason is the above stated argument. There seems no conclusive argument to assign who the meaning it receives in Champollion et al. (2017). Second, if the meaning of who is simply a free variable, we have an easier time for calculating the focus value of it.

Last, the reader should note that we do not face any disadvantages from this. In particular, we still have all the nice features of Champollion et al. We can generate $w h$-questions from wh-phrases in situ, thus avoiding island constraints. And, we get along with inquisitive abstracts due to the logical form we assigned such questions.

### 5.2.2 Denotation of who-questions in CRISP

So far we dealt with the logical form of matrix who-questions. Now we want to know what denotations these have. We want to address this question by means of an example. We will consider "Who smokes?", which has the logical form displayed in (44):


We use again a diagram to make clear what the denotation of the formula is assuming the logical space $\mathcal{L}=\left\{\left\langle w_{\varnothing}, g_{a}\right\rangle,\left\langle w_{\varnothing}, g_{b}\right\rangle,\left\langle w_{a, b}, g_{a}\right\rangle,\left\langle w_{a, b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{a}, g_{b}\right\rangle,\left\langle w_{b}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle\right\}:$


Figure 5.2: Denotation of "Who smokes?" given the logical space $\mathcal{L}$.

It is important to note that the above gives us only mention-some readings, i.e. readings on which it is sufficient to specify some witness for the existential. The reason for this is two-fold. One reason is that the meaning of the question morpheme $Q$ is based on the inquisitive existential quantifier. The other reason is that we cannot express answers such as "Everyone". This is related to the inquisitive existential. It is clause (iii) which will not allow for such an answer given our current system. The mentioned reply usually corresponds to the overlap of all the alternatives in the question denotation, as can be seen from the denotation displayed in 5.1. Clause (iii) leads to mutual exclusivity and so no overlapping alternatives will ever be part of our question denotation for who-questions.

Wh-questions, however, also come with another reading, the so-called mention-all reading. On that
reading we are obliged to mention all witnesses of the existential. This corresponds to the so-called weakly exhaustive reading of $w h$-questions. Yet, another reading is the so-called strongly exhaustive reading. On this reading we are not only obliged to mention all witnesses of the existential, but also to mention all non-witnesses. We will address the derivation of strongly-exhaustive readings in the next section. ${ }^{4}$

Last, we want to address the shape of the denotation. In particular, we want to address why the possibilities $\left\langle w_{b}, g_{a}\right\rangle$ and $\left\langle w_{a}, g_{b}\right\rangle$ do not make up part of the denotation. The reason for this lies with (i) the presupposition triggered by $Q$, namely that there is an answer to the question, and (ii) in the way assignments work in CRISP. The presupposition triggered by $Q$ in this case is

$$
\begin{aligned}
!\exists x \hat{S}(x)\left(s^{\top}\right) & =\exists T\left[\bigsqcup T=s^{\top} \wedge \forall t \in T: \exists x \cdot \hat{S}(x)(t)\right] \\
& =\exists T\left[\bigsqcup T=s^{\top} \wedge \forall t \in T: \exists s^{\prime}\left[t[x] s^{\prime} \wedge \hat{S}(x)(t)\right]\right. \\
& =\exists T\left[\bigsqcup T=s^{\top} \wedge \forall t \in T: \exists s^{\prime}\left[t=s^{\prime} \wedge \forall i, i^{\prime} \in s^{\prime}\left[x g_{i}=x g_{i^{\prime}}\right] \wedge \alpha_{s^{\prime}} \subseteq \pi_{s^{\prime}} \wedge \alpha_{s^{\prime}} \subseteq S(x)\right]\right.
\end{aligned}
$$

It is the condition $\alpha_{s^{\prime}} \subseteq S(x)$ which cannot be met by the possibilities under consideration, for these assign $x$ values which are not within the set of smokers. And for this reason these possibilities are not part of any supportive state of the question denotation.

### 5.3 Mention-all readings for who-questions

As we discussed in the last section, we are only in a position to derive mention-some readings for whoquestions. This, we said, is a limitation for we know that who-questions come with other readings as well. We explained that this is related to the definition of the inquisitive existential quantifier. We also noted that in its current state our account can only derive mention-some readings. An adjustment is called for!

We will provide an adjustment which allows us to derive mention-all readings, in particular stronglyexhaustive readings. We employ a two step strategy: (i) we close the domain of individuals $D_{e}$ under Link's (1983) individual sum operator, as originally suggested in Dayal (1996) for questions. This gives us a separate alternative corresponding to the everyone-answer. (ii) we then define an exhaustivity operator $X$ which strengthens mention-some reading denotations to strongly exhaustive reading denotations (see Klinedinst and Rothschild (2011) and Uegaki (2015) for similar ideas).

### 5.3.1 Closing $D_{e}$ under individual sum

From here on we will assume that $D_{e}$ is closed under Link's $(1983)^{5}$ individual sum. We introduce new symbols to the vocabulary of CRISP:

Definition 5.3.1. (Extending CRISP)

| Symbol | Intended Meaning |
| :---: | :---: |
| For any two constant symbols $a, b$ of type $e, a \oplus b$ of type $e$ | plural object of $a$ and $b$ |
| The operator * of type $\langle\langle\mathbf{e}, T\rangle,\langle\mathbf{e}, T\rangle\rangle$ | provides plural denotation |

[^21]For instance, if Ann (a) and Bill (b) are atoms, then $a \oplus b$ will denote the join of $a$ and $b$, which is the group or the sum consisting of Ann and Bill.

Importantly, by extending $D_{e}$, we make it possible for assignments to map a variable onto a plural individual. For this reason, we will consider the effect of our extension on our definitions of $\exists$ and $\nVdash$.

Let us consider the sentences $\exists x . \hat{S}(x)$ and $\nVdash x . \hat{S}(x)$ (where we may think of $\hat{S}$ simply as "smoke" which allows for plural individuals as members of its denotation). Starting with the existential formula, we observe that we have the denotation displayed below (given the logical space $\mathcal{L}$ ):


Figure 5.3: The denotation of $\exists x . \hat{S}(x)$ given the logical space $\mathcal{L}$.

For the universal formula we have:


Figure 5.4: The denotation of $\nVdash x . \hat{S}(x)$ given the logical space $\mathcal{L}$.

Back to $w h$-questions. We will assume that VPs that combine with who are *-marked even though they are morphologically singular. ${ }^{6}$ The effect of this can be observed from the figure below which displays the denotation of "Who smokes?" given some logical space $\mathcal{L}$.


Figure 5.5: The mention-some denotation for "Who smokes?" given the logical space $\mathcal{L}$.

As we can see, the alternative (displayed by means of purple, orange, and blue) have the same presupposition. Moreover, we now have an alternative corresponding to the reply "Everyone". This is crucial. Note that the denotation is very similar to the denotation proposed in Hamblin (1973): each alternative in our denotation corresponds to one proposition in the Hamblin denotation of the question.

The reading we want to derive, the strongly exhaustive reading, corresponds to the denotation displayed in the next figure:


Figure 5.6: The denotation of the strongly exhaustive reading of"Who smokes?" given the logical space $\mathcal{L}$.

The reason for this is that the alternatives correspond to the replies "Everyone (smokes)." (the purple alternative), "(Only) Ann (smokes)." (the orange alternative, where we take $a$ to represent "Ann"), and "(Only) Bill (smokes)" (the blue alternative, where we take $b$ to represent "Bill").

[^22]
### 5.3.2 Deriving Strong Exhaustivity

Strong exhaustivity is usually understood in terms of partitions (J. Groenendijk \& Stokhof, 1982). The strong exhaustive answers corresponds to cells in a partition of the space of possible worlds (or a subset thereof if the question comes with a presupposition). Therefore, the usual way to go from a Hamblin denotation to a strong exhaustive denotation in inquisitive semantics is simply to define an equivalence relation between worlds that satisfy the same Hamblin answers, and to define the cells of the partition as the equivalence classes for this relation. However, as can be seen in 5.5 , the alternatives in our basic denotation are already non-overlapping since they each come with a different assignment for the variable $x$, so defining an equivalence relation on possibilities will not help.

To take the problem differently, we need an operation that will take the denotation in 5.5 and return the denotation in 5.6, where each possibility expresses an exhaustive answer (i.e., "both Ann and Bill", "only Ann", or "only Bill"). Note that in the world where both Ann and Bill smoke, all three alternatives in 5.5 are in principle possible, but we only want to keep the blue one, because its assignment maps the variable $x$ onto the correct short answer $a \oplus b$. This gives us an intuition of the kind of operation that is needed to arrive at the strong exhaustive denotation: whenever a world may correspond to multiple alternatives, we want to keep only the possibilities which belong to the most restrictive alternative. ${ }^{7}$

In the rest of this section, we offer a technical solution to this problem and show that it does return the correct strong exhaustive denotation when applied to a question.

First, we define an order on states based on the world component of possibilities, ignoring assignments.
For $t, t^{\prime}$ states, $t \leq t^{\prime}$ if and only if:
a. $\forall i \in \pi_{t}, \exists i^{\prime} \in \pi_{t^{\prime}}: w_{i}=w_{i^{\prime}}$
b. $\quad \forall i \in \alpha_{t}, \exists i^{\prime} \in \alpha_{t^{\prime}}: w_{i}=w_{i^{\prime}}$

This order is less restrictive than the standard order $\subseteq$. In particular, if $t=\left\langle\pi_{t},\left\{\left\langle w_{a b}, g_{a b}\right\rangle\right\}\right\rangle$ is the blue alternative in 5.5 , and $t^{\prime}=\left\langle\pi_{t^{\prime}},\left\{\left\langle w_{a b}, g_{a}\right\rangle\left\langle w_{a b}, g_{a}\right\rangle\right\}\right\rangle$ the red one, we have $t \leq t^{\prime}$ but $t \notin t^{\prime}$. This explains why these two states are proper alternatives in our system, although in Champollion et al.'s system the state corresponding to $t$ would be nested in the state corresponding to $t^{\prime}$ (and therefore wouldn't be an alternative as it wouldn't be maximal).

Next, we define an operator $\chi$ to exhaustify the alternatives in an inquisitive proposition $P$ with respect to each other:

For $P$ an inquisitive proposition and $t \in A L T(P)$,

$$
\begin{equation*}
\chi(t, P)=\left\langle\pi_{t}, \alpha_{t} \cap\left\{i \mid \forall t^{\prime} \in A L T(P),\left[\exists g:\left\langle w_{i}, g\right\rangle \in \alpha_{t^{\prime}}\right] \rightarrow t \leq t^{\prime}\right\}\right\rangle \tag{46}
\end{equation*}
$$

This operator restricts an alternative so that in the world domain, it does not overlap with any more informative alternative. In particular, the possibility $\left\langle w_{a b}, g_{a}\right\rangle$ in the red alternative would be removed because $\left\langle w_{a b}, g_{a b}\right\rangle$ is in the blue alternative, which is more informative (and similarly for $\left\langle w_{a b}, g_{b}\right\rangle$ in the green alternative).

[^23]Finally, we can define an operator that maps an inquisitive proposition $P$ onto its strong exhaustive denotation by taking the inquisitive disjunction of these alternatives, after downward closure:

$$
\begin{equation*}
\text { Given } P \text { an inquisitive proposition: } X(P)=\lambda s . \exists t \in A L T(P): s \sqsubseteq \chi(t, P) \tag{47}
\end{equation*}
$$

Note that $X$ is vacuous if $P$ is non-inquisitive, as it should be, since $\chi$ is vacuous when $A L T(P)$ is a singleton. Note also that we defined this operator in a way that does not affect the presuppositions of the question.

Let us make the working of $X$ visible. We assume the denotation displayed in 5.5. We proceed in two steps to make the working of $X$ clear. Step one consist in checking the $\leq$ relation between the alternatives. Formally, we have the following alternatives:
$\begin{array}{ll}\text { a. } & s_{1}=\left\langle\left\{\left\langle w_{a \oplus b}, g_{a \oplus b}\right\rangle,\left\langle w_{a \oplus b}, g_{a}\right\rangle,\left\langle w_{a \oplus b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle\right\},\left\{\left\langle w_{a \oplus b}, g_{a \oplus b}\right\}\right\rangle \text { (the purple alternative) }\right. \\ \text { b. } & s_{2}=\left\langle\left\{\left\langle w_{a \oplus b}, g_{a \oplus b}\right\rangle,\left\langle w_{a \oplus b}, g_{a}\right\rangle,\left\langle w_{a \oplus b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle\right\},\left\{\left\langle w_{a \oplus b}, g_{a}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle\right\}\right\rangle \text { (the orange } \\ \text { alternative) } \\ \text { c. } & s_{3}=\left\langle\left\{\left\langle w_{a \oplus b}, g_{a \oplus b}\right\rangle,\left\langle w_{a \oplus b}, g_{a}\right\rangle,\left\langle w_{a \oplus b}, g_{b}\right\rangle,\left\langle w_{a}, g_{a}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle\right\},\left\{\left\langle w_{a \oplus b}, g_{b}\right\rangle,\left\langle w_{b}, g_{b}\right\rangle\right\}\right\rangle \\ & \text { alternative) }\end{array}$
The following holds:
a. $\quad s_{1} \leq s_{1}, s_{1} \leq s_{2}, s_{1} \leq s_{3}$
b. $\quad s_{2} \leq s_{2}, s_{2} \nsucceq s_{1}, s_{2} \nsubseteq s_{3}$
c. $\quad s_{3} \leq s_{3}, s_{3} \neq s_{1}, s_{3} \not \leq s_{2}$

Given this, we can consider the working of $\chi(t, P)$. In our case $P=\nexists x \cdot \hat{S}(x)_{!\exists x . \hat{S}(x)}$, and $t \in s_{1}, s_{2}, s_{3}$. We will consider only one case here, the others being essentially the same. Let us consider the case $t=s_{2}$. We have to determine $\chi\left(s_{2}, \exists x . \hat{S}(x)_{!\exists x . \hat{S}(x)}\right)$ :

$$
\begin{aligned}
\chi\left(s_{2}, \exists x \cdot \hat{S}(x)_{!\exists x \cdot \hat{S}(x)}\right) & =\left\langle\pi_{s_{2}}, \alpha_{s_{2}} \cap\left\{i: \forall t^{\prime} \in A L T\left(\exists x \cdot \hat{S}_{!\exists x \cdot \hat{S}(x)}\right)\left[\exists g\left[\left\langle w_{i}, g\right\rangle \in \alpha_{t^{\prime}}\right] \rightarrow t \leq t^{\prime}\right]\right\}\right\rangle \\
& =\left\langle\pi_{s_{2}}, \alpha_{s_{2}} \cap\left\{\left\langle w_{a}, g_{a}\right\rangle\right\}\right\rangle \\
& =\left\langle\pi_{s_{2}},\left\{\left\langle w_{a}, g_{a}\right\rangle\right\}\right\rangle
\end{aligned}
$$

The second line of the computation follows from the fact that $\left\langle w_{a \oplus b}, g_{a}\right\rangle \in \alpha_{s_{2}}$. Now, we also find possibilities with the same world component in $\alpha_{s_{1}}$ and $\alpha_{s_{2}}$, namely $\left\langle w_{a \oplus b}, g_{a \oplus b}\right\rangle \in \alpha_{s_{1}}$ and $\left\langle w_{a \oplus b}, g_{b}\right\rangle \in \alpha_{s_{3}}$, but the latter alternative, $s_{3}$, is not such that $s_{2} \leq s_{3}$ and so $\left\langle w_{a \oplus b}, g_{a}\right\rangle$ is out. This makes clear the working of $X$. This also makes clear that $X$ derives the strongly exhaustive reading for "Who smokes?" from the mention-some reading denotation of "Who smokes?".

### 5.4 Summary

In this section we considered the semantics of who. We started with a critical discussion of Champollion et al.'s (2017) account of who. We noted that their semantics for who cannot be motivated in the way they'd like to and set out another proposal which paid attention to the prosody of wh-questions. The resulting account maintained all the advantages of Champollion et al.'s original account while even improving on Champollion et al. (2017) with respect to mention-some readings as demonstrated by (43). The new account also provides
an easy focus semantics for who.
We then discussed how to derive mention-all readings. Here, we made a difference between weakly exhaustive and strongly exhaustive readings. We then showed how to derive the latter leaving the weakly exhaustive readings for the future. The derivation for the strongly-exhaustive readings was made possible by adding plurals to CRISP. We used Link (1983) plural semantics for this end. With the new denotations at hand, we were able to define an exhaustivity operator $X$ which strengthens the mention-some denotation of a who-question to a strongly-exhaustive question denotation.

## Chapter 6

## too in Questions

This chapter is concerned with two topics, one being the meaning of too and the other being too in questions. The chapter is thus naturally split into two parts. Starting with the meaning of too, we will discuss some aspects on what may be called standard accounts of too with respect to the semantics of too. Concretely, we will zoom in on the additive inference of too and discuss different characterizations of it. This provides some theoretical background on different positions on the semantics of too and allows to position and motivate the meaning rule given for too in CRISP. We will also provide a few points on the syntax of too.

The second part of the chapter provides our explanation of the data. We will argue that the deviant cases are all subject to either redundancy (which we assume to be bad) or presupposition failure. In order to make this precise we will provide a rather primitive discourse model, which essentially consists only of an update function. In the explanation of the wh-question data we will encounter a few technical issue which a related to either (i) the fact that CRISP is not a dynamic semantics, or (ii) the way $\nexists$ works. We will also encounter a conceptual issue which is related to discourse management. We will address these issues clearly. Consequently, our explanation will not be spotless, but the technical and conceptual issues aside it seems to be on the right track. We will start with polar questions, followed by alternative questions and concluding with $w h$-questions.

### 6.1 On the syntax and semantics of too

In this section we will be concerned with the meaning of too. The principal goal consist in providing a semantic account of too in CRISP. This will allow us to study too in questions (and in declaratives alike) which is the empirical main goal of this thesis.

The section is organized as follows: we will first discuss the core data on too. These are its focus-sensitivity and the additive inference standardly observed in sentences with too. The discussion will provide us with a minimal overview of positions available for the semantics of too, differing in their characterization of the additive inference. Second, we will provide a few facts on the syntax of too which facilitate the discussion in the remainder of the thesis. We will not provide a thorough syntactic analysis of too. We conclude with a meaning rule for too in CRISP.

### 6.1.1 The core data: focus-sensitivity and additive inference

In the introduction of this thesis we pointed out that standardly too is seen as focus-sensitive particle that triggers an additive presupposition. In this section we will take a closer look at the data that give rise to this view. Further, we will discuss several conceptual and empirical issues surrounding the characterization of the additive inference. We will start with the basics, and derive the standard assumptions on the meaning of too step by step. We will then address questions which are crucial for the development of an adequate account of the meaning of too. In this way we will also provide a minimal overview over different positions in the literature. This also allows us later to position our own account within the bulk of literature on too.

Consider the minimal pair below:

John $_{F}$ smokes too.
John smokes $_{F}$ too.
(48) and (49) clearly differ in meaning. We can infer from (48), but not from (49), that someone besides John smokes. Similarly, we can infer from (49), but not from (48), that John does something else besides smoking. The difference in meaning cannot be due to a difference in lexical material, for clearly, the lexical material is the same. Further, the sentences also do not differ in their syntactic structure. However, they differ with respect to the placement of the $F$-feature. This placement gives rise to a different LF, thus to a difference in meaning. Therefore, we find that the meaning contribution of too depends on the position of focus. ${ }^{1}$ This we can capture with our focus semantics.

Next, the question is what kind of inference we have at hands. Is is an entailment (cf. Ahn (2015)), a presupposition (cf. Rullmann (2003), and many others), or an implicature? First, we can see that it is not cancellable, suggesting that it is not an implicature:

Implicature cancellation:
a. \#John ${ }_{F}$ smokes too, but in fact no one else does so.

We can then apply the usual projection tests for presuppositions (D. I. Beaver (1997)) and see that the additive inference projects from the scope of negation, modals, or from the antecedent of conditionals (we will come back to questions of course).
(51) Embedding:
a. It is not the case that $\mathrm{John}_{F}$ smokes too. $\quad \sim$ Someone besides John smokes.
b. $\mathrm{John}_{F}$ might smoke too. $\sim$ Someone besides John smokes.
c. If $\mathrm{John}_{F}$ smokes too, Mary will be angry. $\sim$ Someone besides John smokes.

The various tests highly suggest that the inference is a presupposition: the inference projects from various environments and hence cannot be an entailment, and when denying the inference explicitly within the same sentence, the sentence becomes inconsistent, thus providing evidence that it is not a implicature. Moreover, the inference test together with the observation that too does not provide anything to the at-issue content of a sentence suggests that there is nothing more to the meaning of too than the presupposition it triggers.

[^24]Our aim is now to properly characterize this presupposition, i.e. to say what exactly is presupposed. First of all, note that too seems to require a true antecedent, where the antecedent must be a focus alternative:
(52) Mary doesn't smoke. \#John ${ }_{F}$ smokes too.
(53) Smoking is unhealthy. \#John smokes too.
(54) Mary smokes. $\mathrm{John}_{F}$ smokes too.

Second, we note that too requires a distinct focus alternative, for observe:
(55) John smokes. \#John ${ }_{F}$ smokes too.
(56) Bill smokes. John $_{F}$ smokes too.

This can be seen as a reflex of a general ban against redundancy, thus making the meaning of too nonredundant. The task is now to spell out what is meant by distinct. We may initially assume that distinct simply means non-identical (cf. Rullmann (2003), Ahn (2015)). This seems motivated by the above example. However, the examples below suggests otherwise:

John collects cars. \#He collects BMWs ${ }_{F}$ too.
In (57) we find that the antecedent and the prejacent are non-identical. Nevertheless the sentence containing too is deviant. We further find that the prejacent John collects BMWs entails John collects cars, the antecedent. Notice that when we leave out too, we have that the second sentence can be seen as specifying which cars John collects
(58) John collects cars. He collects BMWs.

In that case, the discourse is fine. The problem seems indeed to be caused by too. Intuitively, (57) says something like Besides cars, John collects BMWs which is of course a contradiction.
(57) together with (58) suggests then that the actual constraint is non-entailment of the antecedent by the prejacent. However, there seem to be cases that put doubt on this. As recognized by Theiler (2018), the following discourse is also deviant:
(59) Mary and John smoke. \#John ${ }_{F}$ smokes too.

The question is whether we can explain this in terms of redundancy (and maintain the above position), or whether we have to blame too for the degradedness of the discourse (thus arguing against the above). Theiler argues that it is too that is to blame (cf. Theiler (2018): 4-5). The argument runs as follows: we need to ensure that the discourse is non-redundant without too. If adding too leads then to degradedness, it is too that is responsible for this. She provides the following example:
(60) Alice and Mary called. \#Mary ${ }_{F}$ called too.
(61) Alice and Mary called. This means in particular that Mary $F_{F}$ called (\#too).

The idea is that the second sentence without too is non-degraded. Adding too makes it degraded. Now, since too is purely presuppositional, the degradedness of the sentence must be related to the meaning of too. Consequently, Theiler (2018) suggests that what is required by too is that the antecedent and the prejacent
are logically independent.
We want to comment on this. We should note that This means in particular that Mary called has not quite the same meaning as Mary called. The first claims that Mary called is a logical consequence of Alice and Mary called. Mary $y_{F}$ called is simply an assertion that Mary called. This explains why Mary called is bad given the preceding sentence, whereas This means in particular that Mary called is not. This also means that the sentence is indeed non-redundant as wanted by Theiler. Observe though that the focus alternatives of This means in particular that Mary ${ }_{F}$ called are not simply propositions such as that Mary called, that Alice called, that Peter called. On the contrary, the alternatives too considers in this scenario are of the form This means in particular that Peter called, This means in particular that Mary called, etc.

It may thus be that adding too is bad for the reason that there is no appropriate antecedent available. This is not excluded by Theiler. Indeed, if the relevant alternatives in this case were that $X$ called, then we would have an appropriate antecedent at hand, namely that Alice and Mary called, but this is not the case. So, the argumentation in Theiler (2018) seems not necessarily to go through.

An example which supports Theiler's claim is the following:
(62) John read the book. \#He opened ${ }_{F}$ it too.

Here we have that the antecedent entails the prejacent. This was claimed by Theiler to be bad. Since no conjunction is involved we do not have to care about the local redundancy check.

So, this data point seems to support the claim by Theiler: the prejacent and the antecedent must be logically independent.

Unfortunately, there are cases which do not accord with Theiler's proposal. The following example found by A. Cremers makes this clear:
(63) Context: Mary is a student.

Mary smokes. \#[Three of the students $]_{F}$ smoke too.
There is no entailment in any direction between these sentences, nevertheless the sentence containing too is bad. ${ }^{2}$

We then see that the notion of distinctness is crucial for the semantics of too, but no consensus on what this notion exactly means exists. Instead, analyses differ along this parameter.

Another question the above characterization raises is whether any old alternative that is true and distinct will do. Kripke $(1991 / 2009)$ argues that this is not the case. His example suggests that too's presupposition is anaphoric, thus requiring a true distinct focus alternative that is salient in the discourse:
(64) $\mathrm{Sam}_{F}$ is having dinner in New York tonight too.
(Kripke (1991/2009): 373)

It is argued that the sentence is bad when uttered out of the blue. But this goes against any account of too that treats its presupposition as simply existential. In that case the presupposition should be satisfiable by any person other than Sam who is having dinner tonight in New York; and we know there are quite a few of them. The presupposition seems not satisfiable even if it is common knowledge that many people are

[^25]having dinner in New York tonight. Last, it seems also impossible to accommodate the presupposition for this reason. Consequently, Kripke concludes that the antecedent must be salient in the context, and so he argues that the presupposition of too is anaphoric.

But, as so often, we find counter examples to this analysis. Brasoveanu and Szabolcsi (2013) provide the following case, indicating that too is not necessarily anaphoric:
a. A: I see that you have submitted abstracts to phonology conferences, semantics conferences, historical conferences ... Do they all pass muster?
b. B: Don't worry, I seek out expert advice.
c. A: Always talk to your advisor too.
(Brasoveanu and Szabolcsi (2013): 61)
The example is supposed to show that anaphoricity is not necessary. Consider the meaning of the reply given by $\mathbf{A}$. This sentence means something along the lines of Whenever you submit an abstract talk to your advisor in addition to whatever expert or experts you are consulting (cf. Brasoveanu and Szabolcsi (2013): 61). On an anaphoric account of too the last sentence would be paraphrased by Always talk to your advisor in addition to them. The problem consists in the fact that the pronoun them must be resolved to some salient individual. But, the discourse does not provide such an individual. In particular, $\mathbf{B}$ leaves the group of experts or the expert unspecified. No reference to some particular individual takes place, and so no individual becomes salient in the discourse. The paraphrase provided by the anaphoric view is thus not appropriate for restating A's reply.

In sum, we see that there are many different accounts of too. They mainly differ along two parameters: distinctness and saliency. ${ }^{3}$ What they all agree on is the focus-sensitivity of too as well as its requirement that the antecedent be true. With this being said, we will next consider some syntactic properties of too.

### 6.1.2 On the syntax and prosody of too

Like other focus particles, the position of too in a sentence depends to a certain degree on the focus structure of the sentence (cf. König (1991): 12). Speaking on the level of surface structure, too usually occurs to the right of its associate (cf. Rullmann (2003): 371, König (1991): 22-23). This means that it usually will associate only with material to its left, so that cases like the once below are rule out:
(66) \#John too smokes ${ }_{F}$.
(67) \#John smokes too and Mary $F_{F}$ smokes.

In the remainder of the thesis, inspired from Rullmann (2003), we will assume that too is a VP adjunct. This assumption has some consequences.

When treating too as VP-adjunct only, it should not be able to associate with the subject for instance. Such cases seem empirically attested though:
(68) $\mathrm{John}_{F}$, too, smokes.

[^26]Following Rullmann (2003), we will assume the VP-internal subject hypothesis. Thus, we assume that the subject is generated within the VP. This allows too to associate with the subject. The reason is that too can only associate with $F$-marked material c-commanded by it, a consequence of how association with focus works in a Roothian focus semantics.

In this thesis we will furthermore interpret the subject in its base position, thus VP internal. This is done for simplicity. We are fully aware of the fact that the subject moves out of its VP internal position and leaves a trace of type e. This is, in principle, no obstacle. We can assume that when the subject is focused, so is its trace (cf. Selkirk (1995)). For details see Rullmann (2003): 367-368, 378-383.

We also want to provide a few facts on the prosody of too. In particular we want to note that too usually occurs with a falling pitch accent (cf. Rullmann (2003): 371). Further, sentences containing too usually have the so-called $B$ - $A$-pitch contour which is characteristic for contratsive topic constructions (cf. Rullmann (2003): 371 and references therein). Rullmann (2003) argues that these prosodic facts determine the semantics of too, but we will leave these details aside for the rest of this thesis, as it is not totally clear whether these characterizations are generalizable to questions, mainly because questions are understudied with respect to too, and in questions falling pitch contours have specific effects, especially sentence final. It is not clear to us in how far the final tone on too, if present in a question (which seems to be the case), affects the question meaning. Could it be taken to trigger exhaustification, or something else? Does it leave the question untouched and is only involved in the contrastive topic marking?

### 6.1.3 too in CRISP

We will provide the following meaning rule for too in CRISP:
Definition 6.1.1. ( $[\text { too }]^{o}$ and $\llbracket \mathrm{too} \rrbracket^{f}$ )
$[\text { too }]^{o}=\lambda X_{T} \cdot \lambda s . \forall i \in \pi_{s}, \exists P\left[P \in C g_{i} \wedge(X \not \vDash P) \wedge \operatorname{true}(P, i)\right] \wedge X(s)$
$\llbracket$ too $\left.]^{f}=\{\llbracket \text { too }]^{o}\right\}$
The rule is motivated both conceptually and technically. Starting with the technical aspect, the best we can do is modeling too as having an existential presupposition, for we are non-dynamic and so anaphoricity cannot be captured. On the other hand, we saw that too cannot be seen as strictly anaphoric. So, we think what we get is fine and can after all be improved on by making CRISP dynamic in the future.

Conceptually, the meaning rule takes distinctness to mean non-entailment of the antecedent by the prejacent. We think this is sufficient for the data we are dealing with since no case with a conjunctive antecedent is part of our data.

### 6.2 The Data

In this chapter we will discuss the following data:
(69) Mary smokes. Does she drink ${ }_{F}$ too?
(70) Mary smokes. Does John $_{F}$ smoke too?
(71) Mary or John smokes. Does Bill $F_{F}$ smoke too?

Does Mary ${ }_{F}$ dance or sing too?
disjunctive polar reading
Does Mary (only) dance ${ }_{F}$, or $\operatorname{sing}_{F}$ too?
Does Mary dance $F_{F}$ too, or (only) $\operatorname{sing}_{F}$ ?
Everybody smokes. Who drinks ${ }_{F}$ too?
I want ice cream. Who ${ }_{F}$ wants ice cream too?
summoning question
\#Does Mary dance ${ }_{F}$ or $\operatorname{sing}_{F}$ too?
alternative reading
\#Does Mary dance $F_{F}$ too, or sing ${ }_{F}$ too?
Mary smokes. \#Who ${ }_{F}$ smokes too?
Mary or John smokes. \#Who ${ }_{F}$ smokes too?

The primary goal of our considerations in this chapter is to understand why some of these items are deviant, whereas others are felicitous. Understanding this is key for understanding the distribution of too in questions.

Let us take a closer look. (69) and (70) make clear that too can occur in plain polar questions. Further, they make clear that too can associate with both the subject and the verb in a plain polar question. (71) makes clear that a question containing too can be asked felicitously after a disjunction. This means that a disjunction must be able to provide an antecedent for too. (72) and (77) show a contrast between alternative questions and disjunctive polar questions. On its disjunctive polar reading, i.e. (72), the sentence is felicitous, whereas on its alternative question reading the sentence is infelicitous. Importantly, (72) shows that too is felicitous in disjunctive sentences, and so this cannot be the reason why (77) is bad. On the other hand (73) and (74) illustrate that too can actually occur in alternative questions. They suggest in combination with (77) that the felicity of an alternative question with too depends on the position of too relative to the disjunction. We will see that the crucial relation is that between exh and too. Item (78) on the other hand makes clear that too cannot occur on each disjunct at the same time. And finally, item (79) and (80) show that too is not always felicitous in $w h$-questions. Whether a wh-question, or who-question in our case, with too is felicitous seems to depend on whether too associates with the $w h$-phrase or not, as suggested by (75), (79), and (80). If it does, the question is deviant, if not, the question is felicitous. (76) shows that there are exceptions to this rule. Explaining this data would be a first step towards a general account of the distribution of too in questions.

### 6.3 A simple discourse semantics

We are interested in explaining the above pattern. We will claim that in case of wh-questions this has to do with redundancy of the questions in the utterance context. If we want to provide an explanation along these lines we will have to make use of a discourse model which allows us to precisely formulate the notion of redundancy we have in mind. But the use of a discourse model is not only motivated by that. We are dealing with pairs of sentences. And, the meaning of the first sentence in this pair evidently influences the acceptability of the second sentence. If we want to capture this we need at least a crude model of the dynamics between these sentences, that is in how far the meaning of one sentence influences the context in a way that another sentence may or may not be accepted by the context after the first sentence got integrated in the context. The data thus presents itself in a way suggesting that the facts are related to discourse properties (at least in some cases).

### 6.3.1 A simple discourse model

A context $\mathcal{C}$ can be seen as comprising different kinds of information: it comprises the information of what was said, what i factual information, and which issues are still open, and many other kinds of information (I can reference Krifka here). In the basic inquisitive system InqB, a context $\mathcal{C}$ is seen as a proposition (add reference to LN). This, together with an appropriate context update operation, provides a very simple model of discourse dynamics.

Unfortunately, this won't do for us. Let $\mathcal{C}$ be a context in the sense of InqB. Since we have two meaning components - presupposition vs at-issue information - we need a slightly more involved update function than in InqB (where we can get away with set-theoretic intersection $\cap$ ). Let us further assume that the update proceeds as follows: we first check whether the presuppositions of the proposition we wish to update our context $\mathcal{C}$ with are satisfied in the context. If so, we proceed with checking for the at-issue content, if not, no update takes place (presupposition failure).

Now consider the following discourse:
(81) a. Mary smokes.
b. Does $\mathrm{John}_{F}$ smoke too?

Given $\mathcal{C}$ we will first try to update with (81-a). (81-a) imposes no restrictions on the presuppositions on the states in $\mathcal{C}$ and so the presupposition check is trivially satisfied. We next check whether the at-issue information is satisfied in the context by which we may mean that we check for some states in $\mathcal{C}$ whether they satisfy the at-issue information. Technically, we may conceive of this as (i) checking whether for each $\pi_{s}$ for each $s \in \mathcal{C}$ we have that $\pi_{s}$ satisfies the presuppositions in question, and (ii) intersecting the at-issue information of $\mathcal{C}$ with the at-issue information of the proposition in question - so, the InqB update.

Imagine now we updated successfully. We will then proceed in the same manner for (81-b). But, we will not make it pass the presuposition check. The reason is that since (81-a) does not impose any restrictions on the presuppositions of the states in $\mathcal{C}$, we will in general not have that the presupposition of (81-b) is met in $\mathcal{C}$. The update procedure stops. What we need is a different model, in particular we need different contexts.

We will model a context $\mathcal{C}_{\mathcal{L}}$ as a non-empty downward closed set of sets of world-assignment pairs relative to the logical space $\mathcal{L}$. Furthermore, we will define the following update operation: ${ }^{4}$

Definition 6.3.1. (Context update operation, $\Pi$ )
Let $\mathcal{C}$ be a context, and $P$ a proposition.

$$
\mathcal{C}\rceil P:= \begin{cases}\mathcal{C} \cap\{\alpha \mid \exists \pi:\langle\pi, \alpha\rangle \in P\} & \text { if for all } \alpha \in \mathcal{C}: \exists s \in P, \pi_{s}=\alpha \\ \text { undefined } & \text { otherwise }\end{cases}
$$

So, we are checking first whether for all sets of possibilities $\alpha$ in $C$, whether there is some state $s$ in the proposition $P$ denoted by the sentence $S$ such that $\alpha$ is identical to the presupposition $\pi_{s}$ of the state $s$. This boils down to checking whether the presupposition of the proposition $P$ is satisfied in the context $\mathcal{C}$. Hence, a sentence can only update a context if the presupposition of the proposition denoted by the sentence holds

[^27]throughout the state. This restriction is important. If we would update without any reservations we would possibly end up in a context $\mathcal{C}^{\prime}$ that no longer contains the actual state.

The second step consists in updating the context. We update the context $\mathcal{C}$ by intersecting it with the set of all sets of possibilities $\alpha$, such that for some set of possibilities $\pi$ we have that $\langle\pi, \alpha\rangle$ is a state in $P$. This boils down to saying that we update with any at-issue content of a state for which we also find that the presupposition of it is satisfied.

In case the presupposition check fails, the update procedure halts and no update takes place. This is the "undefined" case.

This is motivated by our notion of state itself. The presupposition component is conceived of as the restriction under which the context is updated with the at-issue content (ref champollion et al.). The above makes this idea formally precise.

Unfortunately, the proposed update model comes with a build in problem. Imagine we are discourse initial. Nothing has been said yet, nothing settled, etc. The logical space has not yet been reduced in its entirety. In that case we may take $\mathcal{C}=\wp(\mathcal{L})$. When now asking a $w h$-question such as "Who smokes?" we will not update, for the presupposition of the question is not a superset of all subsets of $\mathcal{L}$. In particular, we have that the presupposition of the question is a proper subset of $\mathcal{L}$ as made clear in the figure below. Technically, this is unproblematic, but conceptually it is. It has been argued by many scholars that discourse is not simply a (linear) process that is supposed to narrow down our uncertainty about what the world is, but it underlines constraints about how to do so. In particular, it has been argued that questions organize discourse in discrete steps and that assertions need to address the questions in order to be felicitous in the discourse (to be a proper move) (cf. Katzir and Singh (2015) and the references therein) Being sympathetic with this picture, we do not want to make use of the above model.

Instead, we will propose and make use of the model below. The idea comes from the following picture: a proposition in both Champollion et al. (2017) and in CRISP can be thought of as a list of offerings. When uttering a sentence, we propose a list to the discourse participants from which they are supposed to choose an item. The items are - in a sense - classical propositions (which come with a presupposition and an assertion). Alternatively, they may reject the list as such (presupposition denial, etc.). Usually, when we offer others something from a list, the items on the list are supposed to be available to them. Bringing this idea down to CRISP, we can think of the items as the alternatives and their substates of the proposition. What we then want is that each alternative can update the context. Hence, what we need to check is whether for each alternative the context is such that we can update it with a substate of it. The latter part comes from the notion of meaning used in inquisitive semantics.

We are now in a position to define the notions of support and redundancy:
Definition 6.3.2. (support)
Let $\mathcal{C}$ be a context and let $P$ be a proposition. We say that $P$ is supported by context $\mathcal{C}$ if we have that $\mathcal{C} \sqcap P \neq \varnothing$. We write $\mathcal{C} \vdash P$ in that case.

Definition 6.3.3. (redundancy) Let $P$ be a proposition. We say:

1. $P$ is redundant in context $\mathcal{C}$ if $\mathcal{C} \sqcap P=\mathcal{C}$
2. for $P$ an inquisitive proposition, $P$ is redundant in context $\mathcal{C}$ if $\exists s\left[s \in \operatorname{Alt}(P) \wedge \forall \alpha \in \mathcal{C} \sqcap P\left[\exists s^{\prime} \sqsubseteq s\left[\alpha_{s^{\prime}}=\right.\right.\right.$ $\alpha]$ ]]

The case in 1. corresponds to the situation where $P$ was already issued, as for instance in "A: Who smokes? B: Mary. C: Who smokes?" The question issues by A' is redundant in that case.

The case in 2., on the other hand, corresponds to the situation where a question is issued but the context is such that it is already resolved. This is the case above: B's utterance resolves C's question already.

## 6.4 too in Polar and Alternative Questions

In this section we will be concerned with items (69) - (74), (77), and (78). Starting with items (69) - (71), we will see that too is unproblematic in plain polar questions. Moreover, (69) and (70) make clear that in questions too can associate with constituents of different types. (71) makes clear that disjunctive sentences can provide appropriate antecedent for questions with too. Next, we will show that disjunctive polar questions with too do behave the same as plain polar questions with too. The polar questions discussed in this thesis therefore in general contrast with alternative questions. Here, we will find that alternative questions are bad unless focus operator - in our case too and exh - are not stacked. This will be concluded from items (73), (74), (76), and (78).

### 6.4.1 too in plain polar questions

Let us restate the data:
(82) Mary smokes. Does Mary drink ${ }_{F}$ too?
(83) Mary smokes. Does John ${ }_{F}$ smoke too?
(84) Mary or John smokes. Does Bill $F_{F}$ smoke too?
(82) and (83) differ in focus structure, but not in the antecedent. In (82) the question has a focus feature on the verb drink. In (83) we have the focus feature on the subject John. Both cases are felicitous. Item (84) illustrates that disjunctions are capable of providing antecedents for too. This is of importance for item (80) as it makes clear that there is an appropriate antecedent available, even though the immediately preceding sentence is a disjunction.

Starting with (82) and (83), let us first consider the logical form of the antecedent Mary smokes. Using the list semantics of Roelofsen and Farkas (2015) we assign "Mary smokes" the following logical form.


This is in need of an explanation. First, note that we assume that "Mary smokes" comes with its standard intonation, i.e. we have nuclear stress at the end of the sentence and a final fall. Second, because we assume that final falling tone triggers Closed and the latter involves exh which needs to associate with focus, we need to assume that there is $F$-marking below $\sim$. Third, given the first assumption, the only plausible focus
marking seems to be wide focus marking. Observe that we cannot have narrow focus marking for essentially this would mean that we need to deviate from the standard intonation (cf. Wagner (2017)).

The relevant values are:

1. $\llbracket$ Mary $\rrbracket^{o}=m$
2. $\llbracket$ smoke $\rrbracket^{o}=\hat{S}$
3. $\llbracket \mathrm{VP} \rrbracket^{o}=\hat{S}(m)$
4. $\llbracket \mathrm{VP} \rrbracket^{f}=\left\{\mathcal{P}: \mathcal{P} \in D_{T}\right\}$
5. $\llbracket(1)]^{o}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge \llbracket \mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(s)$
6. $\llbracket(2) \rrbracket^{o}=\lambda F . F\left(e x h\left(C, \lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge \llbracket \mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(s)\right)\right)$
$=\lambda F . F\left(\lambda s . \exists s^{\prime}\left[s^{\prime} \subseteq s \wedge \forall i \in \pi_{s^{\prime}}\left[C g_{i} \subseteq \llbracket \mathrm{VP}\right]^{f} \wedge \llbracket \mathrm{VP}\right]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(m)\left(s^{\prime}\right) \wedge \neg \exists t[\forall i \in$ $\left.\left.\left.\left.\pi_{t}\left[C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge \llbracket \mathrm{VP}\right]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(t) \wedge t<_{C} s^{\prime}\right]\right]\right)$
7. $\llbracket \mathrm{S} \rrbracket^{o}=!\lambda s . \exists s^{\prime}\left[s^{\prime} \subseteq s \wedge \forall i \in \pi_{s^{\prime}}\left[C g_{i} \subseteq \llbracket \mathrm{VP}\right]^{f} \wedge \llbracket \mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(m)\left(s^{\prime}\right) \wedge \neg \exists t\left[\forall i \in \pi_{t}\left[C g_{i} \subseteq\right.\right.$ $\left.\left.\llbracket \mathrm{VP} \rrbracket^{f} \wedge\left[\mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(t) \wedge t<_{C} s^{\prime}\right]\right]$

Assuming that the relevant alternatives are $\hat{S}(m)$ and $\rightarrow \hat{S}(m)$, we find that $\llbracket \mathrm{S} \rrbracket^{o}$ is equivalent to 【(1) $\rrbracket^{o}$.

The logical form of Does Mary $\operatorname{drink}_{F}$ too? is:


The relevant values are:

1. $\llbracket \mathrm{VP} \rrbracket^{o}=\hat{D}(m)=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq D(m)$
2. $\left[\mathrm{VP} \rrbracket^{f}=\left\{\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \mathcal{P}(m): \mathcal{P} \in D_{\langle\mathbf{e}, T\rangle}\right\}\right.$
3. $[1) \rrbracket^{o}=\hat{D}(m)_{\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket \mathrm{VP}\right]^{f} \wedge\left[\mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right.}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge \llbracket \mathrm{VP} \rrbracket^{o} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{D}(m)(s)$
4. $[2]]^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq[\mathrm{VP}]^{f} \wedge[\mathrm{VP}]^{o} \wedge\left|C g_{i}\right| \geq 2 \wedge\left(\left[(1]^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{D}(m)(s)\right.$
5. [(3)] $]^{o}=\lambda F . \lambda s . \forall i \in \pi_{s} \exists P\left[P,[\mathrm{VP}]^{o} \in C g_{i} \subseteq[\mathrm{VP}]^{f} \wedge\left([\mathbb{1}]^{o} \nLeftarrow P\right) \wedge\left|C g_{i}\right| \geq 2 \wedge \operatorname{true}(P, i)\right] \wedge\left(\alpha_{s} \subseteq \pi_{s} \wedge\left(\alpha_{s} \subseteq\right.\right.$ $\left.\left.D(m) \vee \alpha_{s} \cap D(m)=\varnothing\right)\right)$
6. $\left[\mathrm{S} \rrbracket^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P,[\mathrm{VP}]^{o} \in C g_{i} \subseteq[\mathrm{VP}]^{f} \wedge\left([\mathbb{1}]^{o} \nRightarrow P\right) \wedge\left|C g_{i}\right| \geq 2 \wedge \operatorname{true}(P, i)\right] \wedge\left(\alpha_{s} \subseteq \pi_{s} \wedge\left(\alpha_{s} \subseteq\right.\right.\right.$ $\left.\left.D(m) \vee \alpha_{s} \cap D(m)=\varnothing\right)\right)$

We see that the question as such presupposes that there is a distinct focus alternative $P$ to $\left[V P \rrbracket^{o}\right.$ $\left([\mathbb{1}]^{o} \neq P\right)^{5}$ which is true of any possibility $i \in \pi_{s}$. We thus presuppose that there must be a distinct true focus alternative to "Mary drinks" in a state for supporting the question.

Let us now consider the effect of asking "Does Mary $\operatorname{drink}_{F}$ too?" after uttering "Mary smokes." We will do so in two steps. First, we consider the effect when we are discourse initial starting with $\mathcal{C}=\wp(\mathcal{L})$ for $\mathcal{L}$ a logical space. Second, the general case.

Let us assume that the logical space $\mathcal{L}$ is the one depicted below and that the denotation of "Mary smokes" is as depicted below. This is assuming that the denotation of "Mary smokes" and $\hat{S}(m)$ coincide for $\mathcal{L}$, which is to say that each $i \in \pi_{a}$, with $a$ the alternative of $\hat{S}(m)$, satisfies the presupposition triggered by $\sim C$. ${ }^{6}$

| $\left\langle w_{m \oplus n, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\left\langle w_{m \oplus n, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\left\langle w_{m \oplus n, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\left\langle w_{m \oplus n, \varnothing}, g_{\{\hat{S}(m), \rightarrow \text {, }}(m)\right\}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $\left\langle w_{m, m \oplus n}, g_{\{\hat{S}(m),-, \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m, \varnothing}, g_{\{\hat{S}(m),-\hat{S}(m)\}}\right\rangle$ |
| $\left\langle w_{n, m \oplus n}, g_{\{\hat{S}(m),-\hat{S}(m)\}}\right\rangle$ | $\left\langle w_{n, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{n, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{n, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ |
| $\left.\left\langle w_{\varnothing, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \rightarrow}{ }_{S}(m)\right\}\right\rangle$ | $\left\langle w_{\varnothing, m}, g_{\{\hat{S}(m), \rightarrow, \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{\varnothing, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{\varnothing, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ |

Figure 6.1: The logical space $\mathcal{L}$ and the denotation of "Mary smokes" represented by the possibilities within the purple-lined rectangle. The purple indices represent the truth-values of $\hat{S}$ at that world, similarly for $\hat{D}$ ("to drink"). E.g. when we have $w_{m, m \oplus n}$ we have that it is true that Mary smokes, and it is true that Mary and Nora $(n)$ drink. The index at $g$ represents the value for $C$ at the possibility $i$. As before, the black dashed line represents the presupposition component of the alternative under consideration, here of "Mary smokes."

Next, we want to know what $\mathcal{C}^{\prime} \sqcap\left[\text { Does Mary drink }{ }_{F} \text { too? }\right]^{o}$ is. We need to check:

$$
\forall s\left[s \in \operatorname{Alt}\left(\left[\text { Does Mary drink } \operatorname{dtoo}_{F} ?\right]^{o}\right) \rightarrow \exists s^{\prime}\left[s^{\prime} \sqsubseteq s \wedge \forall s^{\prime \prime} \sqsubseteq s^{\prime}\left[\exists \alpha \in \mathcal{C}^{\prime}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]
$$

Observe that the condition is met. First, we determine the alternatives of [Does Mary drink ${ }_{F}$ too? $]^{\circ}$. It holds that in our new context $\mathcal{C}^{\prime}$ each $\alpha \in \mathcal{C}^{\prime}$ is such that for all possibilities $i \in \alpha$ we find $P$ a proposition such that $\operatorname{true}(P, i), \lambda s . \forall i \in \pi_{s}\left[T g_{i} \subseteq[\mathrm{VP}]^{f} \wedge[\mathrm{VP}]^{o} \wedge\left|T g_{i}\right| \geq 2\right] \wedge \hat{D}(m)(s) \nRightarrow P$, and $P \in C T g_{i}$ (we use $T$ for the earlier $C$ here to avoid reusing the same variable). ${ }^{7}$ Such is $\hat{S}(m)$. Hence, the presupposition of [Does Mary drink ${ }_{F}$ too?] ${ }^{o}$ is satisfied in $\mathcal{C}^{\prime}$. The alternatives are then:

[^28]\[

$$
\begin{aligned}
" \hat{D}(m) "= & \left\langle\left\{\left\langle w_{m \oplus n, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\right.\right. \\
& \left\langle w_{m, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle \\
& \left.\left\langle w_{m, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\right\}, \\
& \left\{\left\langle w_{m \oplus n, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\right. \\
& \left.\left.\left\langle w_{m, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\right\}\right\rangle \\
" \sqcap \hat{D}(m) "= & \left\langle\left\{\left\langle w_{m \oplus n, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\right.\right. \\
& \left\langle w_{m, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle \\
& \left.\left\langle w_{m, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\right\}, \\
& \left\langle w_{m \oplus n, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m \oplus n, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle,\left\langle w_{m, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle, \\
& \left.\left.\left\langle w_{m, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle\right\}\right\rangle
\end{aligned}
$$
\]

The situation is once more depicted in the next figure.

| $\left\langle w_{m \oplus n, m \oplus n}, g_{\{\hat{S}(m), \sqcap \hat{S}(m)\}}\right\rangle\left\langle w_{m \oplus n, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m \oplus n, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m \oplus n, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\left\langle w_{m, m \oplus n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m, m}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m, n}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ | $\left\langle w_{m, \varnothing}, g_{\{\hat{S}(m), \rightarrow \hat{S}(m)\}}\right\rangle$ |

Figure 6.2: To the left of the blue line we find the at-issue information of the alternative " $\hat{D}(m)$ " and its complement we find to its right. Note that our contexts only memorize at-issue information, such that they eventually shrink with each update.

We observe that the question is non-deviant. The question is neither inconsistent (that much we can read off its translation), and it is non-redundant in the given context $\mathcal{C}^{\prime}$, for neither do we find that $\mathcal{C}^{\prime} \cap\left[\text { Does Mary } \operatorname{drink}_{F} \text { too? }\right]^{o}=\mathcal{C}^{\prime}$, nor do we find that $\exists s\left[s \in \operatorname{Alt}\left(\left[\text { Does Mary drink }{ }_{F} \text { too? }\right]^{o}\right) \wedge \forall \alpha \in\right.$ $\left.\mathcal{C} \sqcap\left[\text { Does Mary drink }{ }_{F} \text { too? }\right]^{\circ} \exists s^{\prime} \sqsubseteq s\left[\alpha_{s^{\prime}}=\alpha\right]\right]$.

Let us now consider the general case. Let us assume we have a context $\mathcal{C} \vdash \llbracket$ Mary smokes $\rrbracket^{o}$. As before, we need to check whether we can update $\mathcal{C}$ with the proposition expressed by "Does Mary drink ${ }_{F}$ too?". As before, we observe that for each possibility $i$ for each $\alpha \in \mathcal{C}$ we find that $\hat{S}(m)$ holds, which makes a good antecedent for the question. Assuming that the restrictor of "too" $T$ is reasonable (i.e. contains $\hat{S}(m)$ next to $\hat{D}(m)$ ) we will find that the presupposition of the question is met by the context. ${ }^{8}$ We thus find that the update condition is met. The result of the update then depends on what $\mathcal{C}$ is. Importantly, the felicity of the question now only depends on whether the context has it that the question was already asked $\left.\left(\mathcal{C} \sqcap \llbracket \text { Does Mary } \operatorname{drink}_{F} \text { too? }\right]^{o}=\mathcal{C}\right)$ or the context $\mathcal{C}^{\prime}=\mathcal{C} \sqcap\left[\text { Does Mary drink }{ }_{F} \text { too? }\right]^{o}$ is such that it provides an answer to the question, which means that the prior context already resolves the issue. Note that the question cannot be inconsistent unless we have that Mary neither drinks nor not drinks, which seems impossible. In general, we find that the question is felicitous in the imposed context. Its felicity is only subject to general constraints.

The exact same explanation will do in the case of item (77). The logical form of Does John ${ }_{F}$ smoke too? is:

[^29]

The relevant values are:

1. $\llbracket \mathrm{VP} \rrbracket^{o}=\hat{S}(j)=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq S(j)$
2. $\llbracket \mathrm{VP} \rrbracket^{f}=\left\{\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \hat{S}(x): x \in D_{\mathrm{e}}\right\}$
3. $\llbracket(1)]^{o}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge \llbracket \mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}(j)(s)$
4. $\llbracket(2) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge \llbracket \mathrm{VP} \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2 \wedge\left(\llbracket(1) \rrbracket^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{S}(j)(s)$
5. $\left.\llbracket(3) \rrbracket^{o}=\lambda F . \lambda s . \forall i \in \pi_{s} \exists P[P, \llbracket \mathrm{VP}]^{o} \in C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge(\llbracket(1) \nRightarrow P) \wedge \operatorname{true}(P, i)\right] \wedge\left(\alpha_{s} \subseteq \pi_{s} \wedge\left(\alpha_{s} \subseteq S(j) \vee \alpha_{s} \cap\right.\right.$ $S(j)=\varnothing)$ )
6. $\llbracket \mathrm{S} \rrbracket^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P, \llbracket \mathrm{VP} \rrbracket^{o} \in C g_{i} \subseteq \llbracket \mathrm{VP} \rrbracket^{f} \wedge(\llbracket(1) \not \vDash P) \wedge \operatorname{true}(P, i)\right] \wedge\left(\alpha_{s} \subseteq \pi_{s} \wedge\left(\alpha_{s} \subseteq S(j) \vee \alpha_{s} \cap S(j)=\right.\right.$ $\varnothing$ )

We find that the question presupposes that there must be a distinct focus alternative $P$ to $\llbracket \mathrm{VP} \rrbracket^{o}$. As can be seen, the reasoning is basically the same as in our earlier case. Consequently, we arrive at the same conclusion: the felicity of the question is subject to general constraints such as bans on redundancy. But, nothing in the question itself makes it deviant.

Let us turn to items (84). The meaning of Does Bill $l_{F}$ smoke too? is obviously identical to that of Does $J o h n_{F}$ smoke too? except for the constant $j$ which is replaced by $b$. The question now is to determine the context introduced by the disjunctive sentence that precedes it.

For the disjunction Mary or John smokes we have the following logical form (assuming that we have no prosodic phrase break on the disjunction and final falling intonation):


## Decl

(4)


Closed
(3)


The relevant values are:

1. $\mathbb{[ 1}=(\hat{S}(m) w \hat{S}(j))$
2. $[2] \rrbracket^{o}=!(\hat{S}(m) w \hat{S}(j))$
3. $[(2)]^{f}=\left\{\mathcal{P}: \mathcal{P} \in D_{T}\right\}$
4. [(4)] ${ }^{o}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq[(2)]^{f} \wedge[(2)]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{S}(m) w \hat{S}(j))(s)$
5. [55] $]^{o}=\lambda F . F\left(e x h\left(C, \lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq[(2)]^{f} \wedge[(2)]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{S}(m) w \hat{S}(j))(s)\right)\right)$
$=\lambda F . F\left(\lambda s . \exists s^{\prime}\left[s \subseteq s^{\prime} \wedge \forall i \in \pi_{s^{\prime}}\left[C g_{i} \subseteq[(2)]^{f} \wedge[2]\right]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{S}(m) w \hat{S}(j))\left(s^{\prime}\right) \wedge \neg \exists t[\forall i \epsilon\right.$ $\left.\left.\left.\left.\pi_{t}\left[C g_{i} \subseteq \llbracket(2)\right]^{f} \wedge[(2)]^{o} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{s}(m) w \hat{S}(j))(t) \wedge t<_{C} s^{\prime}\right]\right]\right)$
6. $[\mathrm{S}]^{o}=!\lambda s . \exists s^{\prime}\left[s \subseteq s^{\prime} \wedge \forall i \in \pi_{s^{\prime}}\left[C g_{i} \subseteq[(2)]^{f} \wedge[(2)]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{S}(m) w \hat{S}(j))\left(s^{\prime}\right) \wedge \neg \exists t[\forall i \in\right.$ $\left.\pi_{t}\left[C g_{i} \subseteq[(2)]^{f} \wedge\left[(2]^{o} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{s}(m) w \hat{S}(j))(t) \wedge t<_{C} s^{\prime}\right]\right]$
$=\lambda s . \exists s^{\prime}\left[s \subseteq s^{\prime} \wedge \forall i \in \pi_{s^{\prime}}\left[C g_{i} \subseteq[(2)]^{f} \wedge[(2)]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{S}(m) w \hat{S}(j))\left(s^{\prime}\right) \wedge \neg \exists t[\forall i \in\right.$ $\left.\left.\pi_{t}\left[C g_{i} \subseteq[(2)]^{f} \wedge[(2)]^{o} \wedge\left|C g_{i}\right| \geq 2\right] \wedge!(\hat{s}(m) w \widehat{S}(j))(t) \wedge t<_{C} s^{\prime}\right]\right]$

For the following we will assume that $C g_{i}=\{!(\hat{S}(m) w \hat{S}(j)), \rightarrow!(\hat{S}(m) w \hat{S}(j))\}$ for all $i \in \mathcal{L}$. In that case, we find that $\left[\mathrm{S} \rrbracket^{o}=!(\hat{S}(m) w \hat{S}(j))\right.$.Further, we denote the restrictor of "too" by the variable $T$ in the following, assuming that $T g_{i}=\{\hat{S}(m), \hat{S}(j), \hat{S}(b)\}$ for all $i \in \mathcal{L}$ as is reasonable.

All it takes for us is to show that the disjunction provides a proper antecedent for the question "Does Bill $_{F}$ smoke too?". As before, we need to check the value for $\mathcal{C} \sqcap\left[\text { Does Bill }{ }_{F} \text { smoke too? }\right]^{o}$. In case we find that the update condition is satisfied, we found that the disjunction provides a proper antecedent.

So, let us assume that we have a context $\mathcal{C}$ with $\mathcal{C} \vdash \llbracket$ Mary or John smokes $\rrbracket^{o}$. Observe that

$$
\forall s\left[s \in \operatorname{Alt}\left(\llbracket \text { Does } \operatorname{Bill}_{F} \text { smoke too } ? \rrbracket^{o}\right) \rightarrow \exists s^{\prime}\left[s^{\prime} \sqsubseteq s \wedge \forall s^{\prime \prime} \sqsubseteq s^{\prime}\left[\exists \alpha \in \mathcal{C}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]
$$

because, by $\mathcal{C} \vdash$ ["Mary or John smokes" $\rrbracket^{o}$ and our further assumption on $C$ we have:
(a) possibilities $i$ under consideration are such that! $(\hat{S}(m) w \hat{S}(j))$ is true at $i$.
(b) by this we must have that for each such $i$ we have $\operatorname{true}(\hat{S}(x))$ for $x \in\{m, j\}$.
(c) this means that there is $P \in T g_{i}$ such that $P$ is not entailed by the prejacent of "too" and that $P$ is true at each possibility in the presupposition component. Hence, the presupposition of "too" is met in the context.
(d) Consequently, we find for each alternative of "Does Bill $F_{F}$ smoke too?" that there is a substate such that all of its substates' presupposition componenent is an element of the context.
(e) Conclusion: the update condition is met and we showed all we need to show.

Having checked that the presupposition of the question is satisfied, we can now check that updating the context with the question has a non-trivial effect (i.e., the sentence is neither redundant nor inconsistent).

Inconsistency is easy to rule out: since the question isn't informative, as long as the original context is consistent, its update with the question is consistent as well. To see this, assume that $\alpha \in \mathcal{C}$ is non-empty. It follows that $\alpha$ has a non-empty intercept with either $S(b)$ or with its complement. By downward closure, $\alpha^{\prime}$ which results from the intersection of $\alpha$ with such an alternative will be a non-empty set in the updated context.

Redundancy depends on our assumptions on prior discourse. Formally the sentence is non-redundant as long as we can find $\alpha$ in $\mathcal{C}$ such that $\alpha$ overlaps with both $S(b)$ and its complement. Concretely, there are three ways in which the context could make the question redundant:

1. it entails that Bill smokes,
2. it entails that Bill doesn't smoke,
3. it already raises the issue of whether Bill smokes.

Assuming that the context was minimal before the assertion of the disjunction is sufficient to prevent all three issues. Indeed, the disjunction is independent from Bill smoking or not, so it does not settle the issue, and it is purely informative, so it does not raise any issue either. As a consequence, the question is not necessarily redundant, and if it is it wouldn't be because of the disjunction. Of course, previous elements in the discourse may still make it redundant (e.g., if the disjunction follows the questions "Who smokes?"). In sum, CRISP gets the data correct.

### 6.5 On too in alternative questions

We observed earlier that too in an alternative question results sometimes in a deviant question. In particular, we perceived the following contrast:
(85)
(86) Does Mary dance ${ }_{F}$ too, or $*$ (only) $\operatorname{sing}_{F}$ ?
(87) Does Mary * (only) dance ${ }_{F}$, or $\operatorname{sing}_{F}$ too?
(88) \#Does Mary dance ${ }_{F}$ too, or $\operatorname{sing}_{F}$ too?

We will show that the difference in acceptability is due to a difference in logical form. In particular, (85) and (88) are unacceptable because of a clash between too and $e x h$, which results from the sequential position of the two operators at logical form and leads to redundancy. Crucially, in both (85) and (88) exh scopes over too. On the other hand we will see that overt only in (86) and (87) provides to the felicity of the questions. The effect of only is to provide an antecedent for too due to its presupposition. The presupposition stays untouched by Closed which scopes over both focus operators and so the questions are consistent and their presuppositions satisfied.

### 6.5.1 The felicitous cases

Let us start by discussing the felicitous cases. Since (86) and (87) are symmetric, we will focus on (86).

We first give an entry for only based on our definition of exh. The only difference is that we take only to presuppose its prejacent, while exh asserts it.

$$
\begin{align*}
& \llbracket \text { only } \rrbracket^{o}=\lambda X_{T} \cdot \lambda s . \exists s^{\prime}: s \sqsubseteq s^{\prime} \wedge X\left(s^{\top \top}\right) \wedge \neg \exists t:\left[X(t) \wedge t<_{C} s^{\prime}\right]  \tag{89}\\
& \llbracket \text { only } \rrbracket^{f}=\left\{\llbracket \text { only } \rrbracket^{o}\right\}
\end{align*}
$$

The logical form of (86) is:


The relevant values for the meaning of each disjunct are:

1. $\llbracket A \rrbracket^{o}=\hat{D}(m)$
2. $\llbracket B \rrbracket^{o}=\hat{S}(m)$
3. $\llbracket A \rrbracket^{f}=\llbracket B \rrbracket^{f}=\left\{\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \mathcal{P}(m): \mathcal{P} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
4. $\llbracket(1) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s}\left[{ }^{1} C g_{i} \subseteq \llbracket A \rrbracket^{f} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge\left|C^{1} g_{i}\right| \geq 2\right] \wedge \hat{D}(m)(s)$
5. $\llbracket(3) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s}\left[C^{2} g_{i} \subseteq \llbracket B \rrbracket^{f} \wedge \llbracket B \rrbracket^{o} \in C^{2} g_{i} \wedge\left|C^{2} g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(s)$
6. $\llbracket(2) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C^{1} g_{i} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge C^{1} g_{i} \subseteq \llbracket A \rrbracket^{f} \wedge\left(\llbracket(1) \rrbracket^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{D}(m)(s)$
7. $\llbracket(4) \rrbracket^{o}=\lambda s . \exists s^{\prime}\left[s \sqsubseteq s^{\prime} \wedge \forall i \in \pi_{s^{\prime}}\left[C^{2} g_{i} \subseteq \llbracket B \rrbracket^{f} \wedge \llbracket B \rrbracket^{o} \in C^{2} g_{i} \wedge\left|C^{2} g_{i}\right| \geq 2\right] \wedge \hat{S}(m)\left(s^{\top}\right)\right] \wedge \neg \exists t\left[\forall i \in \pi_{t}\left[C^{2} g_{i} \subseteq\right.\right.$ $\left.\left.\llbracket B \rrbracket^{f} \wedge \llbracket B \rrbracket^{o} \in C^{2} g_{i} \wedge\left|C^{2} g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(t) \wedge t<_{C^{2}} s^{\prime}\right]$

For simplicity, let us first assume that $C^{1}$ and $C^{2}$ are uniformly assigned the same value, a set consisting of only two elements: $\hat{D}(m)$ and $\hat{S}(m)$. This means that the only possible antecedent for $P$ in the first disjunct is $\hat{S}(m)$, and the only effect of only in the second disjunct is to negate $\hat{D}(m)$. On top of that, only presupposes the truth of $\hat{S}(m)$. The net result is that both disjuncts presuppose that Mary sings, the first disjunct asserts that she (also) dances, and the second disjunct asserts that she doesn't dance. This disjunction is therefore equivalent to the polar question "Does Mary dance?", but with a stronger presupposition (namely, that she sings).

Now, let's see what other values $C^{1}$ and $C^{2}$ could take. Let us first note that any value other than $\hat{S}(m)$ in $C^{2}$ is negated by only. Assuming that the presupposition of the first disjunct projects, this means that resolving $P$ to anything other than $\hat{S}(m)$ would result in a contradiction when evaluating the second disjunct. Conversely, since the second disjunct presupposes $\hat{S}(m)$, which is not supported by the assertive content of the first disjunct, the disjunction as a whole will anyway presuppose that Mary sings, so the presupposition of the first disjunct is necessarily satisfied as long as $\hat{S}(m)$ is in $C^{1}$. In short, the disjunction as a whole will only ever presuppose that Mary sings, and adding more alternatives to $C^{1}$ and $C^{2}$ has no effect but to strengthen the meaning of the second disjunct.

The disjunction in place, we can now turn to the top of the tree. Because this is an alternative question and not a polar question, we first apply Closed. There's a problem however: both disjuncts contain a focus-operator, and given the definition of the ~-operator, no alternative projects (or to be precise, the focus value of the whole disjunction is just the singleton of its ordinary value). This means that attaching a new $C^{3}$ to the tree will result in a presupposition failure, and so we cannot give a new restrictor to the exhaustifier introduced by Closed.

We see two solutions to this problem. The first would be to assume that focus-sensitive operators (more precisely, the $\sim$-operator) do not reset the focus alternatives. That is to say, each disjunct would project its own alternatives, which would be combined by disjunction by point-wise function application as usual. The problem is, these alternatives would then combine with too and only, resulting in a focus value consisting of arbitrary disjunctions of the form " $A$ too or only $B$ " with possibly complex presuppositions.

Another solution is to recycle the focus value of the disjuncts, under the assumption that it is identical across disjuncts. While this may seem far-fetched at first, Abenina-Adar and Sharvit (2018) have already proposed a very similar constraint to account for the contrast in (90).
a. Did John even eat the cake $_{F}$ ?
b. \#Did John even eat the cake ${ }_{F}$, or the candy ${ }_{F}$ ?

They argue for a constraint of Domain Uniformity, which requires all disjuncts in an alternative question to share the same focus value. Without going into the details of their argument, we can adopt this constraint, which in the present case means that $C^{1}$ and $C^{2}$ must be assigned the same value. It is only natural then, to assume that this shared restrictor can be picked up by Closed.

We can then apply Closed to the disjunction, but this is almost vacuous. The only-disjunct is already exhaustive of course. The too-disjunct presupposes the truth of the prejacent of only, so this cannot be negated by Closed. The result is simply to exclude any third alternative from the first disjunct (to the effect that Mary may be dancing on top of singing, but nothing else).

Finally, we can apply Int. It only adds the presupposition triggered by $Q$ to the meaning of the argument as the argument is already inquisitive.

A final note is in order. We haven't discussed the fact that dropping only from the disjunct not carrying the additive particle seems to be slightly degraded (though not completely ungrammatical). There may be several reasons for this. One could be very superficial, e.g., a parallelism requirement between the two disjuncts. We may have a more principled explanation however. The application of CLOSED adds to the disjunct with the additive particle the negation of all alternatives except the one that is presupposed by too. From the other alternatives, it removes all alternatives, including the antecedent of too... unless the antecedent of too is the second disjunct itself. Because of presupposition projection, the only way for the second disjunct not to be contradictory (and therefore vacuous) is to be picked as the antecedent of too. The problem now may have
to do with the way presuppositions project from disjunction (a problem we have largely ignored so far): the presupposition of each disjunct may be evaluated against the negation of the other disjunct (alternatively, this may be done incrementally, with the presupposition of the first disjunct projecting as is). If this is the case, the presupposition of too would be contradicted by the negation of the second disjunct. Inserting only would transform the assertive part of the second disjunct into a presupposition, which happily projects out of the whole disjunction.

### 6.5.2 The deviant cases

In this subsection we will discuss (85) and (88), and why they are deviant. Note that a naive explanation that would have been possible before we discussed (86) and (87) would simply attribute the badness of these sentences to a clash between the different focus operators. We saw however that in alternative questions, several operators may share the same restrictor without any problem. Our explanation will therefore rely on showing that the problematic cases always result in a redundant question.

Following the discussion of (86) and (88), let us assume the following logical form for (85), in which Closed shares its restrictor with too:
(91)


The relevant values are:

1. $\llbracket A \rrbracket^{o}=\hat{D}(m)$
2. $\llbracket B \rrbracket^{o}=\hat{S}(m)$
3. $\llbracket A \rrbracket^{f}=\llbracket B \rrbracket^{f}=\left\{\mathcal{P}(m) \mid \mathcal{P} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
4. $[1]]^{o}=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge\left(\alpha_{s} \subseteq D(m) \vee \alpha_{s} \subseteq S(m)\right)$
5. $\llbracket(1) \rrbracket^{f}=\left\{\lambda s \cdot \mathcal{P}(m)(s) \vee \mathcal{Q}(m)(s) \mid \mathcal{P}, \mathcal{Q} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
6. $\llbracket(3) \rrbracket^{o}=\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \forall i \in \pi_{s}, \exists P\left[P \in C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge|C g i| \geq 2 \wedge\left(\llbracket(1) \rrbracket^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right]$

$$
\wedge\left(\alpha_{s} \subseteq D(m) \vee \alpha_{s} \subseteq S(m)\right)
$$

7. 〔(4) $\rrbracket^{o}=\lambda F \cdot F\left(\operatorname{exh}\left(C,\left[(3) \rrbracket^{o}\right)\right)\right.$

We observe the following: (3) presupposes that there is some alternative $P \in C g_{i}$ for all $i \in \pi_{s}$ that is not entailed by (1), the disjunction. We then exhaustively strengthen the meaning of (3). exh will now negate systematically as many elements of $C$ as possible. Which elements these are depends on what $C$ is. In any case, the only elements that will not be negated are (1) and its two disjuncts $A$ and $B$. So, the presupposition of too must be satisfied by either $A$ or $B$. Next, observe that which of these will be the antecedent for too depends on what the answer to the question is. For, assume that the answer to the question is Mary only dances in addition to something else (but not singing, etc.). In that case, the at-issue information of the answer rules out $B$ as a possible antecedent, and only leaves $A$ as an antecedent. That is, if the strengthened left disjunct is the answer, then we presuppose that $A$, which effectively rules out the second alternative of the question meaning. Similarly in the other case. Hence, the presupposition boils down to presupposing an answer to the question. Thus, the question is redundant, hence deviant.

Turning to (88) we observe that Domain Uniformity applies and forces $C^{1}=C^{2}$ in the following logical form:


We have the following relevant values:

1. $\llbracket A \rrbracket^{o}=\hat{D}(m)$
2. $\llbracket B \rrbracket^{o}=\hat{S}(m)$
3. $\llbracket A \rrbracket^{f}=\llbracket B \rrbracket^{f}=\left\{\lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \mathcal{P}(m): \mathcal{P} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
4. $[1]]^{o}=\lambda s . \forall i \in \pi_{s}\left[C^{1} g_{i} \subseteq \llbracket A \rrbracket^{f} \wedge \llbracket A \rrbracket^{o} \in C_{g}^{1} i \wedge\left|C^{1} g_{i}\right| \geq 2\right] \wedge \hat{D}(m)(s)$
$=\lambda s . \forall i \in \pi_{s}\left[C^{1} g_{i} \subseteq \llbracket A \rrbracket^{f} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge \llbracket B \rrbracket^{o} \in C^{1} g_{i}\right] \wedge \hat{D}(m)(s) \quad$ (by Domain Uniformity)
5. [(2)] $\left.]^{o}=\lambda s . \forall i \in \pi_{s}\left[C^{2} g_{i} \subseteq \llbracket B\right]^{f} \wedge \llbracket B \rrbracket^{o} \in C^{2} g_{i} \wedge\left|C^{2} g_{i}\right| \geq 2\right] \wedge \hat{S}(m)(i)$
$=\lambda s . \forall i \in \pi_{s}\left[C^{1} g_{i} \subseteq \llbracket A \rrbracket^{f} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge \llbracket B \rrbracket^{o} \in C^{1} g_{i}\right] \wedge \hat{S}(m)(s) \quad$ (by Domain Uniformity)
6. [(3) $]^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C^{1} g_{i} \wedge\left[A \rrbracket^{o} \in C^{1} g_{i} \wedge[B]^{o} \in C^{1} g_{i} \wedge C^{1} g_{i} \subseteq\left[A \rrbracket^{f} \wedge(\llbracket(1)]^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge\right.$ $\hat{D}(m)(s)$
7. [4) $]^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C^{1} g_{i} \wedge\left[A \rrbracket^{o} \in C^{1} g_{i} \wedge[B]^{o} \in C^{1} g_{i} \wedge C^{1} g_{i} \subseteq\left[A \rrbracket^{f} \wedge\left([(2)]^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge\right.\right.$ $\hat{S}(m)(s)$
8. $\left.[5]^{o}=[(3)]^{o} w[4)\right]^{o}$

$$
\begin{aligned}
= & \lambda s . \forall i \in \pi_{s} \exists P\left[P \in C^{1} g_{i} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge \llbracket B \rrbracket^{o} \in C^{1} g_{i} \wedge C^{1} g_{i} \subseteq \llbracket A \rrbracket^{f} \wedge \operatorname{true}(P, i)\right] \wedge \\
& \left(\left(([(1) \neq P) \wedge \hat{D}(m)(s)) \vee\left(\left([(2)]^{o} \neq P\right) \wedge \hat{S}(m)(s)\right)\right)\right.
\end{aligned}
$$

```
9.[[6] ]}=\lambdaF.exh(\mp@subsup{C}{}{1},[55]\mp@subsup{]}{}{o}
    = \lambdaF.\lambdas.\exists\mp@subsup{s}{}{\prime}[s\sqsubseteq\mp@subsup{s}{}{\prime}\wedge\foralli\in\mp@subsup{\pi}{\mp@subsup{s}{}{\prime}}{}\existsP[P\in\mp@subsup{C}{}{1}\mp@subsup{g}{i}{}\wedge\llbracketA]\mp@subsup{]}{}{o}\in\mp@subsup{C}{}{1}\mp@subsup{g}{i}{}\wedge[[B\mp@subsup{]}{}{o}\in\mp@subsup{C}{}{1}\mp@subsup{g}{i}{}\wedge\operatorname{true}(P,i)\wedge((([\mathbb{1}\mp@subsup{]}{}{o}#
P)\wedge\hat{D}(m)(\mp@subsup{s}{}{\prime}))\vee(([(2)\mp@subsup{]}{}{\circ}\not\existsP)\wedge\hat{S}(m)(\mp@subsup{s}{}{\prime})))]\wedge\neg\existst[\foralli\in\mp@subsup{\pi}{t}{}\existsP[P\in\mp@subsup{C}{}{1}\mp@subsup{g}{i}{}\wedge\llbracketA\mp@subsup{\rrbracket}{}{o}\in\mp@subsup{C}{}{1}\mp@subsup{g}{i}{}\wedge\llbracketB\mp@subsup{\rrbracket}{}{o}\in\mp@subsup{C}{}{1}\mp@subsup{g}{i}{}\wedge
true}(P,i)\wedge((([1)\not=P)\wedge\hat{D}(m)(t))\vee(([(2)\not=P)\wedge\hat{S}(m)(t)))]\wedget<\mp@subsup{<}{\mp@subsup{C}{}{1}}{}\mp@subsup{s}{}{\prime}
```

Let us inspect [5)] ${ }^{\circ}$ and [6] $]^{\circ}$ closer. We observe the following for [5] ${ }^{\circ}$. We have a presupposition to the effect that there must be a true focus alternative $P$ such that $P$ is a focus alternative to either $A$ or $B$ or both. We also observe that $P$ can be a true and distinct focus alternative to $A$ and $B$ only in case $\left|C^{1}\right| \geq 3$. Last, observe in case $\left|C^{1}\right|=2$, we find that the left disjunct presupposes $\hat{S}(m)$, whereas the right disjunct presupposes $\hat{D}(m)$. Graphically:

## $\hat{D}(m) \quad \hat{S}(m)$

Figure 6.3: Denotation of $[(5)]^{o}$ given $\left|C^{1}\right|=2$.

Sticking to the case $\left|C^{1}\right|=2$, let us observe the effect of Closed. The role of Closed consists in exhaustifying the input meaning, a process we said is crucial for the derivation of alternative question meanings. However, in our case at hands, exhaustification will not happen. In fact, as will become clear in due course, $e x h$ will output its argument again. To see this let us consider again [6) $]^{\circ}$. We have:

$$
\begin{aligned}
& \llbracket(6)]^{o}=\lambda F . \lambda s . \exists s^{\prime}\left[s \sqsubseteq s^{\prime} \wedge \forall i \in \pi_{s^{\prime}} \exists P\left[P \in C^{1} g_{i} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge \llbracket B\right]^{o} \in C^{1} g_{i} \wedge \operatorname{true}(P, i) \wedge\right. \\
& \left.\left.\left(\left(([1)]^{o} \neq P\right) \wedge \hat{D}(m)\left(s^{\prime}\right)\right) \vee\left(\left([\text { (2) }]^{o} \neq P\right) \wedge \hat{S}(m)\left(s^{\prime}\right)\right)\right)\right] \\
& \wedge \neg \exists t\left[\forall i \in \pi _ { t } \exists P \left[P \in C^{1} g_{i} \wedge \llbracket A \rrbracket^{o} \in C^{1} g_{i} \wedge \llbracket B \rrbracket^{o} \in C^{1} g_{i} \wedge \operatorname{true}(P, i)\right.\right. \\
& \left.\wedge(([\text { (1) } \neq P) \wedge \hat{D}(m)(t)) \vee(([\text { (2) } \neq P) \wedge \hat{S}(m)(t)))] \wedge t<_{C^{1}} s^{\prime}\right] \\
& =\lambda F . \lambda s . \exists s^{\prime}\left[s \sqsubseteq s ^ { \prime } \wedge \forall i \in \pi _ { s ^ { \prime } } \exists P \left[P \in\{\hat{D}(m), \hat{S}(m)\} \wedge\{\hat{D}(m), \hat{S}(m)\} \subseteq \llbracket A \rrbracket{ }^{f} \wedge \operatorname{true}(P, i) \wedge\right.\right. \\
& \left.\left(([1) \neq P) \wedge \hat{D}(m)\left(s^{\prime}\right)\right) \vee\left(\left([(2) \nRightarrow P) \wedge \hat{S}(m)\left(s^{\prime}\right)\right)\right)\right] \\
& \wedge \neg \exists t\left[\forall i \in \pi _ { t } \exists P \left[P \in\{\hat{D}(m), \hat{S}(m)\} \wedge\{\hat{D}(m), \hat{S}(m)\} \subseteq \llbracket A \rrbracket^{f} \wedge \operatorname{true}(P, i) \wedge\right.\right. \\
& \left.(([1) \neq P) \wedge \hat{D}(m)(t)) \vee((\llbracket(2 \not \# P) \wedge \hat{S}(m)(t)))] \wedge t<_{\{\hat{D}(m), \hat{S}(m)\}} s^{\prime}\right]
\end{aligned}
$$

Now, observe that there is no $s^{\prime}$ such that $\left.s^{\prime} \in[5)\right]^{o}$ and $\left(\alpha_{s^{\prime}} \cap \hat{D}(m)=\varnothing\right) \vee\left(\alpha_{s^{\prime}} \cap \hat{S}(m)=\varnothing\right)$, because for any $\left.s^{\prime} \in[5]\right]^{o}$ it must hold that $\forall i \in \pi_{s^{\prime}}[\operatorname{true}(P, i)]$, where $P \in\{\hat{D}(m), \hat{S}(m)\}$, and $([(1) \neq P) \wedge \hat{D}(m)(s)$ or $\left([(2) \neq P) \wedge \hat{S}(m)(s)\right.$ - loosely speaking $s$ must support in any case either $\lambda s \cdot \hat{S}(m)\left(s^{\top}\right) \wedge \hat{D}(m)(s)$ or $\lambda s . \hat{D}(m)\left(s^{\top}\right) \wedge \hat{S}(m)(s)$. Consequently, exh will not be able to remove the overlap and so we will also not be able to derive an alternative question meaning. But this is not the reason for the deviance of the question. When computing $[6]^{\circ}$ we will find that we presuppose an answer to the question, but we also find that the question as such presupposes that one of the disjuncts is already established - the effect of too on the two disjuncts. Hence, the question presupposes that an answer is already available in the common ground. This makes the question redundant, for when asking it it must have been already settled. This is the reason for its deviance.

Let us now turn to the case where $\left|C^{1}\right| \geq 3$. In that case, we will find that exh will negate the alternatives $\mathcal{P}(m) \notin\{\hat{D}(m), \hat{S}(m)\}$ such that the at-issue content of the question indeed gets strenghtend, but the presupposition will be again that one of its answers must already be established in the common ground.

## 6.6 too in who-questions

In this section we will consider the data on who-questions. Unlike before, we will start by considering the deviant cases, i.e. (79) and (80). In both these cases we have association of too with the wh-phrase who resulting in deviance. We follow up with an explanation as to why association with the verb in who-questions is in general unproblematic as indicated by example (75). The explanations as before, will be spelled out in terms of redundancy, but also in terms of presupposition failure. Unlike before, we will make extra assumptions in our explanation. The reason being that we encounter a few technical issues which are related to the fact that CRISP is not a dynamic semantics and the way $\exists$ works. The details will be provided below.

We will argue that (79) and (80) are deviant because they are either redundant (mention-some reading), or the presupposition triggered by "too" is not satisfiable (strongly-exhaustive reading). However, some qualifications about the utterance context are in need as we will see. No such case usually occurs in case of (75).

Last, we will address a new data point due to Nadine Theiler: summoning questions (cf. (76)). These are wh-questions in which an additive particle associates with the wh-phrase yet the question is felicitous. We will see why.

### 6.6.1 too associating with the wh-phrase

We will start by providing the logical form of the interrogative sentence.


The relevant values are:

1. $\llbracket(1) \rrbracket^{o}=\hat{S}\left(x_{1}\right)$
2. $\llbracket(1) \rrbracket^{f}=\left\{\hat{S}(y): y \in D_{\mathbf{e}}\right\}$
3. $\llbracket(2) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \hat{S}\left(x_{1}\right)(s)$
4. $\llbracket(3) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge\left(\llbracket(2) \rrbracket^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{S}\left(x_{1}\right)(s)$
5. [4) $\rrbracket^{o}=\lambda x . \lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge\left(\llbracket(2) \rrbracket^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{S}(x)(s)$
6. $\llbracket \mathrm{S} \rrbracket^{o}=\exists x . \lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge\left(\llbracket(2) \rrbracket^{o} \neq P\right) \wedge \operatorname{true}(P, i)\right]$
$\wedge \hat{S}(x)(s)_{!} \nexists x . \lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq[(1)]^{f} \wedge[(1)]^{o} \in C g_{i} \wedge\left([(2)]^{o} \notin P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{S}(x)(s)$
$=\lambda s . \exists s^{\prime}\left[s=s^{\prime} \wedge \forall i, j \in \pi_{s^{\prime}}\left[x g_{i}=x g_{j}\right] \wedge \forall i \in \pi_{s^{\prime}} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1)\right]^{f} \wedge \llbracket(1)\right]^{o} \in C g_{i} \wedge$

$$
\left(\llbracket\left[2 \rrbracket^{o} \not \vDash P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{S}(x)\left(s^{\prime}\right)_{!\nexists x . \lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq[(1)]^{f} \wedge[(1)]^{\circ} \in C g_{i} \wedge\left([(2)]^{\circ} \not{ }^{\circ} P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{S}(x)(s)}
$$

The question presupposes that in its resolving state $s$ we find a true proposition $\hat{S}(y)$ such that $\llbracket(2) \rrbracket^{o} \not \approx$ $\hat{S}(y)$, and so in particular such that $\hat{S}(x) \not \models \hat{S}(y)$. Note that this means that there must be two salient propositions in the context, one being $\hat{S}(y)$, the other being $\hat{S}(x)$. This means also that $y$ must not be an individual part of $x$, and so we must have two salient individuals in our context.

Above we assigned "Mary smokes" a logical form which has it that we focus mark the VP. The form is repeated here again:


We will see that in discussing the wh-question cases, focus marking becomes very important. In particular, in the contexts we are interested in - the ones making the wh-question deviant - the prior assertion seems to have a different prosody than assumed so far. In particular, we will argue below that "Mary smokes." must have narrow focus marking on the subject. Only in that case will the question "Who ${ }_{F}$ smokes to?" be deviant due to redundancy or presupposition failure. We will show this by first considering the case where we assign "Mary smokes." the logical form repeated above. We will go through the derivation then and find that the question is non-redundant and consistent. Consequently, the context provided by "Mary smokes" does not render the question deviant. We will then consider a slightly different scenario, which intuitively imposes a different focus structure on "Mary smokes." (narrow focus on the subject). We will find that in that case the question becomes redundant or is subject to presupposition failure. We want to note here already that we need to make a few extra assumptions to get that result, but these assumptions are well-shared and argued for in the literature. As before, we will first consider what happens when we are discourse initial, hence if $\mathcal{C}=\wp(\mathcal{L})$. We then consider the general case.

Getting started, let us assume that we are discourse initial, hence we assume that $\mathcal{C}=\wp(\mathcal{L})$. We need to determine the effect of uttering "Mary smokes." on our context $\mathcal{C}$, i.e. we need to compute $C \sqcap \llbracket$ Mary smokes $\rrbracket^{o}$.

We need to check:

$$
\forall s\left[s \in \operatorname{Alt}\left(\llbracket \text { Mary smokes } \rrbracket^{o}\right) \wedge \exists s^{\prime} \sqsubseteq s\left[s^{\prime} \neq\langle\varnothing, \varnothing\rangle \wedge \forall s^{\prime \prime} \sqsubseteq s^{\prime}\left[\exists \alpha \in \mathcal{C}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]
$$

We know that "Mary smokes" denotes a non-inquistive informative proposition. Let us denote its unique alterntaive by $a$. It is now easy to check the above. Observe that $\pi_{a}=\mathcal{L}$, and we are done.

Next, we need to compute $C \bigcap\left\{\alpha: \exists \pi\left[\langle\pi ; \alpha\rangle \in \llbracket\right.\right.$ Mary smokes $\left.\rrbracket^{o}\right\}$. Given our logical space $\mathcal{L}$ as displayed below, we get:

$$
\begin{aligned}
\mathcal{C}^{\prime} & =\mathcal{C} \bigcap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in[\text { Mary smokes }]^{o}\right\}\right. \\
& =\wp(\mathcal{L}) \bigcap \wp\left(\alpha_{a}\right) \\
& =\wp\left(\alpha_{a}\right)
\end{aligned}
$$

$$
\left.\begin{array}{c}
\left\langle w_{\varnothing}, g_{m \oplus b}\right\rangle\left\langle w_{m \oplus b}, g_{m \oplus b}\right\rangle\left\langle w_{m}, g_{m \oplus b}\right\rangle
\end{array}\right\rangle\left\langle w_{b}, g_{m \oplus b}\right\rangle
$$

Figure 6.4: Denotation of "Mary smokes" for the logical space $\mathcal{L}$.
Next, let us consider the effect of "Who ${ }_{F}$ smokes too?" on the new context $\mathcal{C}^{\prime}$. We will stick to our two individual case allowing us to ignore the ambiguity of "Who $F_{F}$ smokes too?". 9

Again, we need to check:

$$
\forall s\left[s \in \operatorname{Alt}\left(\left[\mathrm{Who}_{F} \text { smokes too } ?\right]^{o}\right) \wedge \exists s^{\prime} \sqsubseteq s\left[s^{\prime} \neq\langle\varnothing, \varnothing\rangle \wedge \forall s^{\prime \prime} \sqsubseteq s^{\prime}\left[\exists \alpha \in \mathcal{C}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]
$$

Observe that we have two alternatives at hand - as displayed in the figure below. We denote them for ease of exposition by $a$ and $b$, where

$$
\begin{aligned}
& a=\left\langle\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle\right\}\right\rangle \\
& b=\left\langle\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\}\right\rangle
\end{aligned}
$$



Figure 6.5: Denotation of "Who $F_{F}$ smokes too?" for the logical space $\mathcal{L}$.

It holds,

[^30]for $a$ : we have for $s^{\prime}=a$ that $s^{\prime} \neq\langle\varnothing, \varnothing\rangle$ and each $s^{\prime \prime} \subseteq a$ is such that $\pi_{s^{\prime \prime}}=\alpha$ for some $\alpha \in \mathcal{C}^{\prime}$, because $\pi_{a}=$ $\left\{\left\langle w_{m \oplus n} g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\}$ (the rest follows by downward closure of both $\mathcal{C}^{\prime}$ and [Who ${ }_{F}$ smokes too?] ${ }^{\circ}$.) for $b$ : similarly.

So, the condition is met and we have

$$
\begin{aligned}
\mathcal{C}^{\prime \prime} & =\mathcal{C}^{\prime} \bigcap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in\left[\mathrm{Who}_{F} \text { smokes too? }\right]^{0}\right]\right\} \\
& =\wp\left(\alpha_{a}\right) \cup \wp\left(\alpha_{b}\right)
\end{aligned}
$$

We now encounter a problem though. As we can see, the question "Who ${ }_{F}$ smokes too?" is non-redundant given the preceding context on which we have established that Mary smokes, for we do not have that $\mathcal{C}^{\prime \prime}=\mathcal{C}^{\prime}$. We also do not have $\exists s\left[s \in \operatorname{Alt}\left(\left[\mathrm{Who}_{F} \text { smokes too }\right]^{\circ}\right) \wedge \forall \alpha \in \mathcal{C}^{\prime} \sqcap\left[\mathrm{Who}_{F} \text { smokes too }\right]^{\circ} \exists s^{\prime} \sqsubseteq s\left[\alpha_{s^{\prime}}=\alpha\right]\right]$.

The situation is in general very similar. For assume we are given a context $\mathcal{C}$ such that $\mathcal{C} \vdash[\text { Mary smokes }]^{0}$. Hence, our context is such that for all $\alpha \in \mathcal{C}^{\prime}$ we have that $\forall i \in \alpha$ Mary smokes is the case. If Mary is the only smoker, we will find that the update with the question "Who smokes too?" is not licensed, for there is no distinct individual besides Mary who smokes. Otherwise, we will have that the question is licensed, but non-redundant for we have not yet picked a cell in the question denotation.

We may be tempted to conclude that CRISP is unable to accout for our intuitions. This we claim is an unwarranted conclusion. What CRISP predicts is that the question "Who ${ }_{F}$ smokes too?" is not always redundant, in particular not necessarily in a context induced by the assertion "Mary smokes" where we have wide focus on the entire proposition. Observe that this is borne out. Importantly, the context we considered is one where the assertion "Mary smokes." with wide focus marking can be understood as answering the question "What happened?". In that case, the question "Who smokes?" is not necessarily resolved. In particular not necessarily on its mention-all reading. This CRISP gets correct. Now, what we cannot capture is that the question "Who smokes?" is somewhat degraded in such a context which we think is due to the fact that it must be resolved in a state where Mary smokes which we already know. An answer to the question is thus not entirely informative. So, we may think that the question is degraded for it imposes a discourse development which is not very effective (for the question requires from us to check again whether Mary smokes or not, which we already know). Indeed, the question "Who else smokes?" seems way better in this context. Note that CRISP does not predict that the mention-some reading of "Who smokes?" is redundant in the considered context - which intuitively it is. This is because of the way inquisitive existential quantification works. We address this below again.

This reasoning indicates that the context where "Who ${ }_{F}$ smokes too?" is deviant is one where "Who smokes?" has been settled. We will now show that CRISP predicts exactly this.

Let us assume again that we are discourse initial. We need to determine the effect of the implicit question "Who smokes?" on the initial context $\mathcal{C}=\wp(\mathcal{L})$. We will start by considering the mention-some reading of the question, which is the basic denotation.

As before, we need to check whether

$$
\forall s\left[s \in \operatorname{Alt}\left([\text { Who smokes? }]^{o}\right) \exists s^{\prime} \subseteq s\left[s^{\prime} \neq\langle\varnothing, \varnothing\rangle \wedge \forall s^{\prime \prime}\left[\exists \alpha \in \mathcal{C}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]
$$

Observe that we have three alternatives, which we denote by $a, b, c$ :

$$
a=\left\langle\left\{\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle,\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle,\left\langle w_{m}, g_{m}\right\rangle,\left\langle w_{n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle\right\}\right\rangle
$$

$$
\begin{aligned}
& b=\left\langle\pi_{a},\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m}, g_{m}\right\rangle\right\}\right\rangle \\
& c=\left\langle\pi_{a},\left\{\left\langle w_{m \oplus n}, g_{n}\right\rangle,\left\langle w_{n}, g_{n}\right\rangle\right\}\right\rangle
\end{aligned}
$$

We have,
for $a$ : for $s^{\prime}=\alpha$ we find that $\forall s^{\prime \prime} \sqsubseteq a\left[\exists \alpha \in \mathcal{C}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]$
for $b$ : similarly.
for $c$ : similarly.
Next, we compute $\mathcal{C} \cap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in\left[\mathrm{Who}_{F} \text { smokes }\right]^{o}\right\}\right.$. We have:

$$
\begin{aligned}
\mathcal{C}^{\prime} & =\mathcal{C} \bigcap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in\left[\text { Who }_{F} \text { smokes }\right]^{o}\right\}\right. \\
& =\wp(\mathcal{L}) \bigcap\left(\wp\left(\alpha_{a}\right) \cap \wp\left(\alpha_{b}\right) \cap \wp\left(\alpha_{c}\right)\right) \\
& =\wp\left(\alpha_{a}\right) \cap \wp\left(\alpha_{b}\right) \cap \wp\left(\alpha_{c}\right), \text { for } \alpha_{i} \subset \mathcal{L}, \text { where } i \in\{a, b, c\}
\end{aligned}
$$

Under its mention-all reading (which we take to be its strongly exhaustive reading for technical reasons) the effect of "Who smokes?" is differently:

$$
\begin{aligned}
\mathcal{C}_{S E}^{\prime} & =\mathcal{C} \bigcap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in\left[\text { Who }_{F} \text { smokes }_{S E} \rrbracket^{o}\right\}\right.\right. \\
& =\wp(\mathcal{L}) \bigcap\left(\wp\left(\alpha_{a}\right) \cap \wp\left(\alpha_{b}\right) \cap \wp\left(\alpha_{c}\right)\right) \subset \mathcal{C}^{\prime}
\end{aligned}
$$

The situation is depicted by the figure on the next page.


Figure 6.6: Basic denotation of "Who smokes?" for the logical space $\mathcal{L}$.

Next, we need to determine the effect of "Mary smokes." which we now conceive not simply as an assertion, but as an answer to the question "Who smokes?". Here, a few complications arise. We earlier stated that in this scenario the logical form of "Mary smokes." is as given below. In that case, we find that the assertion gives rise to the implication that no one besides Mary smokes, which is due to the narrow focus marking (cf. Wagner (2017)) and its effect on the working of Closed.

We think that this is somewhat inappropriate as an answer for the mention-some case (the basic denotation of the question) for not all possible replies are exhaustive; essentially a term answer such as "Mary." should settle the issue, but possibly leave open that other individuals also smoke. This we cannot capture with CRISP yet. What is needed is a way to determine the value of the variable $x$ bound by $\exists$ in the question. This can be realized when transforming CRISP into a dynamic semantics for then we can restrict the value of $x$ outside of its syntactic scope. So, what we want as a possible denotation is the orange area displayed above as a whole, which we cannot yet realize in CRISP - in fact, we cannot do better than the above at
the moment. We will for the sake of exposition assume that we can have this. The same complication arises when we want to consider the denotation CRISP provides for "Mary smokes." as an answer. Again, what we want is that the reply fixes a value for $x$, thereby fixing a cell of the partition induced by the wh-question. Yet, again for the very same reason, we cannot have this, and this is reflected in CRISP, for the denotation of "Mary smokes." on its exhaustive reading is not such that it resolves the issue expressed by the question. So, here too, we will assume for the sake of exposition that CRISP gives us what we want. Figure 6.7 depicts the denotations we will work with. Note that when considering the case where "Mary smokes." has its exhaustive denotation we also consider the mention-all case "Who smokes?" gives rise to, for "Mary smokes." is a mention-all reply. Consequently, we can skip the mention-all scenario.


Figure 6.7: Denotations of "Mary smokes" for the logical space $\mathcal{L}$. The big alternative (with at-issue information the orange area) represents the assumed mention-some reply, whereas the small alternative (with at-issue information the purple encapsulated area) represents the exhaustive reply.

We will then assume the following: as an answer to the mention-some reading of "Who smokes?" we'll let "Mary ${ }_{F}$ smokes." denote the proposition with the unique alternative
$M S=\left\langle\left\{\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle,\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle,\left\langle w_{m}, g_{m}\right\rangle,\left\langle w_{n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m}, g_{m}\right\rangle\right\}\right\rangle$,
but we assume that "Mary ${ }_{F}$ smokes." as a reply to the strongly-exhaustive readings denotes the proposition with the unique alternative

$$
S E=\left\langle\left\{\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle,\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{m \oplus n}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle,\left\langle w_{m}, g_{m}\right\rangle,\left\langle w_{n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m}, g_{m}\right\rangle\right\}\right\rangle
$$

which licenses the inference that only Mary smokes.
Let us then now consider the effect of updating the corresponding contexts with the answers. Considering the mention-some reply to the question first, we observe that the condition for the context update is satisfied:

$$
\forall s\left[s \in \operatorname{Alt}\left(\left[\text { Mary }_{F} \text { smokes } \rrbracket^{o}\right) \wedge \exists s^{\prime} \sqsubseteq s\left[s^{\prime} \neq\langle\varnothing, \varnothing\rangle \wedge \forall s^{\prime \prime} \sqsubseteq s^{\prime}\left[\exists \alpha \in \mathcal{C}^{\prime}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]\right.
$$

And we have for $\mathcal{C}^{\prime} \cap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in \llbracket\right.\right.$ Mary $_{F}$ smokes $\left.\rrbracket^{o}\right\}$ :

$$
\begin{aligned}
\mathcal{C}^{\prime \prime} & =\mathcal{C}^{\prime} \cap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in \llbracket \text { Mary }_{F} \text { smokes } \rrbracket^{o}\right\}\right. \\
& =\left(\wp\left(\alpha_{a}\right) \cup \wp\left(\alpha_{b}\right) \cup \wp\left(\alpha_{c}\right)\right) \bigcap \wp\left(\alpha_{M S}\right) \\
& =\wp\left(\alpha_{M S}\right)
\end{aligned}
$$

Now, the effect of updating $\mathcal{C}^{\prime \prime}$ with the question "Who ${ }_{F}$ smokes too?" consists in reducing the context to a set of possibilities such that each of these resolves the newly uttered question:

$$
\forall s\left[s \in \operatorname{Alt}\left(\left[\mathrm{Who}_{F} \text { smokes too? }\right]^{o}\right) \wedge \exists s^{\prime} \sqsubseteq s\left[s^{\prime} \neq\langle\varnothing, \varnothing\rangle \wedge \forall s^{\prime \prime} \sqsubseteq s^{\prime}\left[\exists \alpha \in \mathcal{C}^{\prime \prime}\left[\pi_{s^{\prime \prime}}=\alpha\right]\right]\right]\right]
$$

is satisfied for the two offered alternatives of the question denotation, the alternatives being

$$
\left\langle\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle\right\}\right\rangle
$$

$$
\left\langle\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle,\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\},\left\{\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\}\right\rangle
$$

For $\mathcal{C}^{\prime \prime} \cap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in\left[\mathrm{Who}_{F} \text { smokes too? }\right]^{o}\right\}\right.$ we get:

$$
\begin{aligned}
\mathcal{C}^{\prime \prime \prime} & =\mathcal{C}^{\prime \prime} \cap\left\{\alpha: \exists \pi\left[\langle\pi, \alpha\rangle \in\left[\text { Who }_{F} \text { smokes too? }\right]^{o}\right\}\right. \\
& =\wp\left(\alpha_{M S}\right) \cap\left(\wp\left(\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle\right\}\right) \cup \wp\left(\left\{\left\langle w_{m \oplus n}, g_{n}\right\rangle\right\}\right)\right) \\
& =\wp\left(\left\{\left\langle w_{m \oplus n}, g_{m}\right\rangle\right\}\right)
\end{aligned}
$$

Last, observe that we have

$$
\exists s \in \operatorname{Alt}\left(\left[\mathrm{Who}_{F} \text { smokes too } \rrbracket^{o}\right)\left[\forall \alpha \in \mathcal{C}^{\prime \prime}\right\rceil\left[\mathrm{Who}_{F} \text { smokes too }\right]^{o} \exists s^{\prime} \subseteq s\left[\alpha_{s^{\prime}}=\alpha\right]\right]
$$

Hence, the context which result from asking the question already resolves the question. This renders the question redundant in the context and makes it deviant, as was claimed.

On the other hand, when we consider "Mary $F_{F}$ smokes." a strongly-exhaustive reply, we find that the question "Who ${ }_{F}$ smokes too?" is not licensed in the subsequent context for its presupposition cannot be met. This makes sense for if we have that only Mary smokes is true, no one else smokes is true too. Hence, in either case we have that the question is deviant as was claimed. Obviously, when we consider the strongly-exhaustive denotation of "Who smokes?" the question "Who $F_{F}$ smokes too?" cannot be licensed for its presupposition cannot be satisfied.

In general, the situation is as follows. Assume we have a context $\mathcal{C}$ such that $\mathcal{C} \vdash\left[\right.$ Who smokes? $\rrbracket^{o}$, where we consider the basic denotation of the question. So, for each alternative of the question we find a substate such that all of its substates are such that their presupposition component is an $\alpha \in \mathcal{C}$. Now, this allows for updating $\mathcal{C}$ with $\llbracket$ Mary smokes $\rrbracket^{o}$ for its denotation is a subset of the denotation of the question (on both the mention-some reply denotation and the exhaustive denotation). We thus fix a cell in the question partition on the backdrop of which we then ask "Who ${ }_{F}$ smokes too?". In both cases - the mention-some reading and the strongly-exhaustive reading - we find that the question is redundant for we satisfy the redundancy criterion for inquisitive propositions.

We need to comment on an important technical issue here. In the above we always implicitly assumed that the question translations make use of the same variable. In the case we considered this is (as far as we know) unproblematic, but this is not in general so. The issue relates to the definition of $\exists$. In general, if we assume that the translations differ with respect to the variable bound by $\exists$ the redundancy will become contingent, for the variables may differ with respect to the values they receive at a possibility. We hope to resolve this issue in the future by going dynamic. For now, we need to acknowledge this issue.

### 6.6.2 too associating with the verb

Last, let us consider (75), here repeated as (93)

$$
\begin{equation*}
\text { Everybody smokes. Who drinks } F_{F} \text { too? } \tag{93}
\end{equation*}
$$

We have the following logical form for Everybody smokes with the translations in the tree:


As in the other cases we assume that $C=\{\nVdash x[\hat{S}(x)], \neg \nVdash x[\hat{S}(x)]\}$. The relevant values are:

1. $\llbracket$ Everybody $\rrbracket^{o}=\lambda P_{\langle\mathbf{e}, T\rangle} \cdot \not \not \not W[P(x)]$
2. $\llbracket(1) \rrbracket^{o}=\nVdash x[\hat{S}(x)]$
3. $\left[(2) \rrbracket^{o}=!\nVdash x[\hat{S}(x)]=\nVdash x[\hat{S}(x)]\right.$
4. $\llbracket(2) \rrbracket^{f}=\left\{\mathcal{P}: \mathcal{P} \in D_{T}\right\}$
5. 〔(3) $\rrbracket^{o}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket(2) \rrbracket^{f} \wedge \llbracket(2) \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \nVdash x[\hat{S}(x)](s)$
6. $\llbracket(4)]^{o}=\lambda F \cdot F\left(\operatorname{exh}\left(C, \lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket(2) \rrbracket^{f} \wedge \llbracket(2) \rrbracket^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \nVdash x[\hat{S}(x)](s)\right)\right)$
$=\lambda F . F\left(\lambda s . \exists s^{\prime}\left[s \sqsubseteq s^{\prime} \wedge \forall i \in \pi_{s^{\prime}}\left[C g_{i} \subseteq \llbracket(2)\right]^{f} \wedge \llbracket(2)\right]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \nVdash x[\hat{S}(x)]\left(s^{\prime}\right) \wedge \neg \exists t[\forall i \in$ $\left.\left.\left.\left.\left.\left.\pi_{t}\left[C g_{i} \subseteq \llbracket(2)\right]^{f} \wedge \llbracket(2)\right]^{o} \in C g_{i} \wedge\left|C g_{i}\right| \geq 2\right] \wedge \uplus x[\hat{S}(x)](t) \wedge t<_{C} s^{\prime}\right]\right]\right)\right)$

The question Who drinks too? has the following logical form: $_{\text {? }}$


The relevant values are:

1. $[(1)]^{o}=\hat{D}\left(x_{1}\right)$
2. $[(1)]^{f}=\left\{\mathcal{P}(x): \mathcal{P} \in D_{\langle\mathbf{e}, T\rangle}\right\}$
3. $\llbracket(2) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s}\left[C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge \mid C g_{\mid} \geq 2\right] \wedge \hat{D}\left(x_{1}\right)(s)$
4. $\left[(3) \rrbracket^{o}=\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge([(2) \| \neq P) \wedge \operatorname{true}(P, i)] \wedge \hat{D}\left(x_{1}\right)(s)\right.\right.$
5. [(4) $]^{o}=\lambda x . \lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1) \rrbracket^{o} \in C g_{i} \wedge\left(\llbracket(2) \rrbracket^{o} \| \neq P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{D}(x)(s)$ (Note that we also abstracted over $x_{1}$ in (2))
6. $\llbracket \mathrm{S} \rrbracket^{o}=\exists x$. $\left[\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq \llbracket(1) \rrbracket^{f} \wedge \llbracket(1)\right]^{o} \in C g_{i} \wedge(\llbracket(2)]^{o} \| \neq P\right) \wedge$

$$
\operatorname{true}(P, i)] \wedge \hat{D}(x)(s)]_{\left.\left.!\exists x .\left[\lambda s . \forall i \in \pi_{s} \exists P\left[P \in C g_{i} \wedge C g_{i} \subseteq[1)\right]^{f} \wedge[1)\right]^{\circ} \in C g_{i} \wedge\left([(2)]^{\circ} \| P\right) \wedge \operatorname{true}(P, i)\right] \wedge \hat{D}(x)(s)\right]}
$$

Clearly, picking $P=\hat{S}$ will always satisfy the presupposition of too, and this does not create any conflict with the answer of the question, since the antecedent is independent from the issue raised.

Applying $X$ to this question will not affect its felicity. So, we can also have a strongly-exhaustive reading.

### 6.7 Summoning questions: a new data point

We'd like to introduce a new data point due to Nadine Theiler. Theiler (2018) found that unlike claimed earlier in the literature not all felcitous cases of $w h$-questions with also associating with the $w h$-phrase receive a showmaster interpretation. The example below demonstrates this fact:

I want to graduate. $\mathrm{Who}_{F}$ also wants to graduate?
Importantly, the question is to be understood as "Who besides me wants to graduate?". Furthermore, usually it is the witness referred to in the assertion that functions as an antecedent for the additive particle and is
removed from the $w h$-domain (cf. Theiler (2018): 2, 9-10). These factors are essential for Theiler's explanation of the felicity of summoning questions as opposed to run-of-the-mill wh-questions with also. Without going into the details of her explanation - she uses a QUD approach similar to D. Beaver and Clark (2008) the overall idea is that because the witness referred to in the assertion is removed from the quantification domain of the wh-phrase, it cannot resolve the issue expressed by the wh-question, allowing it to function as an antecedent for the additive particle in the question. She argues that such a domain restriction is not possible in cases like (79) and (80), and is restricted to divisions which are realized in the grammar, such as the speaker/audience contrast. In sum, the domain restriction allows to satisfy the additive presuppositon of the question triggered by "also" (cf. Theiler (2018): 9-10).

Now, summoning questions are not restricted to "also":
I want to graduate. $\mathrm{Who}_{F}$ wants to graduate too?
The explanation provided by Theiler (2018) should also account for these cases, but what about CRISP? Can CRISP account for summoning questions?

The answer is that in its current evolutionary state it cannot. We simply do not have the means to compositionally derive such question meanings. This requires a mechanism that manipulates the domain of quantification accordingly. What we can do though is describe the situation summoning questions give rise to. This we may then take as a blue-print for an alternative explanation of the felicity of summoning questions. ${ }^{10}$

Let us assume that the assertion "I want to graduate" addresses the implicit issues "Who wants to graduate?". This means, from the perspective of CRISP, that we choose the cell in the denotation where the bound variable is assigned as value the speaker. Now, we can make sense of domain restriction in CRISP's current state by using a new variable in the translation of "Who $F_{F}$ wants to graduate too?" where the variable must be such that its value is never the speaker in the "I want to gradute"-cell. In that case we partition this cell again into smaller ones (if there are other people who want to graduate too) and the question is non-redundant (given the scenario we considered). Moreover, we find that the speaker functions as a proper antecedent in that case which does not resolve the newly expressed issue.

What we then see is that, as argued in Theiler (2018), domain restriction is essential in the explanation. Due to our weaker semantics for additive particles, we do not have that this makes the additive presupposition satisfiable, but that it makes the question non-redundant.

### 6.8 Summary

In this chapter we discussed the meaning of too as well as the data points motivating this thesis. We started out with a discussion of the semantics of too. We eventually assumed a meaning rule which takes the additive presupposition of too to be existential and spelled out distinctness as non-entailment of the antecedent by the prejacent. Together with syntactic assumptions inspired by Rullmann (2003) and Selkirk (1995), and our account of the meanings of plain polar questions, alternative questions, and who-questions, we attempted to provide explanations for all the data points.

Starting with polar questions, CRISP made the correct predictions and we did not encounter any technical issues. Turning to alternative questions we found the same. In fact, we conclude that with CRISP we can provide two different explanations for the data. We prefer the second explanation as it provides better inside

[^31]in why we encounter instances in which too is felicitous in alternative questions and such where it is not. This explanation relied on the constraint of Domain Uniformity which has been argued for already for in Abenina-Adar and Sharvit (2018).

Last, we considered the wh-question data. We started by considering association of "too" with the wh-phrase in questions such as "Who $F_{F}$ smokes too?". We initially claimed that the question is deviant in a context where we already established that "Mary smokes.", we found though that in fact this is not necessarily the case. Importantly paying attention to prosody and related information structural properties, we found that the question is deviant in a context in which we already addressed the question of who smokes as opposed to what happend (narrow focus vs wide focus). Consequently, we made a few extra assumptions: "Mary smokes." is an answer to the implicit question "Who smokes?". In that case we have been able to derive the deviance of the question, though we had to go over a few technical issues, whose solution is left for the future.

We also considered the newly introduced data point presented by Nadine Theiler. We haven't been able to account for it in a compositional manner, but we provided a sketch of an explanation which mirrors the explanation provided in Theiler (2018). The shortcomings are again due to the technical limitations of CRISP in its current state.

## Chapter 7

## Conclusion

This thesis started out with the observation that focus particles can occur in questions, but not in all questions as witnessed by the particle too. We then set out to find an explanation for the behavior of too in questions. Noticing that (to the best of our knowledge) there is no readily available semantics for such an endeavor, our first task was to provide a formal semantics suitable for the study of additive particles in questions. Consequently, we came up with CRISP, which combines Rooth (1985) and Rooth (1992) with the compositional presuppositional inquisitive semantics of Champollion et al. (2017).

With an appropriate system at hand, we first discussed the semantics of plain polar and disjunctive polar questions, delineating the latter from alternative questions. Hereby we concentrated exclusively on questions in matrix form and adopted the list semantics of Roelofsen and Farkas (2015).

Next we considered who-questions in matrix form. We initially assessed Champollion et al.'s (2017) view finding that its motivation is not ideal. Consequently, we provided another account of who which treats it somewhat like a pronoun in Montague's PTQ. This together with assumptions on the prosody-semantics interface allowed us to have simple focus values for who, while having all the advantages of Champollion et al. (2017). Moreover, we even improved on the latter as our derivations for mention-some readings are simpler. Due to our semantic architecture and the treatment of existential quantification, we had to face a challenge though when deriving mention-all readings. Our solution was to introduce plural denotations and a new operator $X$ for the derivation of strongly-exhaustive readings.

We then turned to the semantics of too and the data on too in questions. Here we focused first on the characterization of the additive inference which is characteristic for additive particles. We figured out that the characterization can differ along two different parameters: distinctness and saliency. For our own meaning rule for too we decided to make use of a non-salient account of too which was forced upon us by our semantics. For distinctness we followed D. Beaver and Clark (2008) and treated it as non-entailment of the antecedent by the prejacent.

With this account of too at hand we discussed the data. Starting with polar questions we showed that too behaves like in declaratives and nothing strange happens (as long as we do not have exh present). We then turned to alternative questions. We provided two explanations, whereby we prefer the second. It provides insides into why too is infelicitous in some but not all alternative questions, whereas the first explanation
would simply rule out any such case.
In case of who-questions, we provided different explanations. For mention-some questions we found that usually if the $w h$-phrase is focused and too associates with it, then the question is redundant given an appropriate antecedent for too. In case of mention-all questions, we saw that we will always have a presupposition failure. We then presented a new data point by Theiler (2018) which shows that who-questions with too in which too associates with who can be felicitous, but they involve domain restriction. Particularly, the speaker must have been excluded from the domain of discourse so that there is an antecedent for too which is not an answer to the question. too reads than similar to else in such questions. We also had to face several technical issues in this part of the thesis which we will address in the future.

In the remainder of this section, we would like to point out some future work. Obviously, we need to solve the technical issues addressed several times so far. Prima facie, the issues related to dynamics can be tackled right away since we have all the required resources at our disposal already. We only need to make them work. The real troublemaker, as far as we can tell, is $\exists$ for in one way or another it always caused trouble so far. One idea may be to tear apart existential quantification and inquisitiveness (p.c. Jakub Dotlačil) using an extra operator imposing inquisitiveness.

Going dynamic presumably allows not only to account for our technical issues, but also for a bunch of data considered in the literature, for instance, anaphora in questions, the work by Aloni et al. (2007) on focus particles in answers (in particular only), and, a major topic in dynamic semantics, presuppositions. In fact, a lot more can be done on presuppositions in CRISP and a lot more should be said on this topic, a topic we have indeed been largely quite on.

Yet another interesting data point are embedded questions which have been subject to linguistic research in and outside of inquisitive semantics for some while already.

We want to mention another interesting data point though, which was not yet really touched by inquisitive semantics: focus intervention effects. Given CRISP these can now be studied from a inquisitive semantics perspective as well, which may lead to different explanations due to the different architecture of inquisitive semantics compared to alternative semantics - which seems the dominant framework in that area of study.

We conclude then with the fact that CRISP, even though still in its development, offers interesting perspectives for inquisitive semantics research in particular and for the study of focus and presuppositions in questions and assertions as such.

## Appendix A

The appendix provides the formal details of the type theory $T T_{3}$ as well as the formal semantics for CRISP, which only adds to the semantics of $T T_{3}$ what the interpretation of non-logical constants looks like.

## Type theory $T T_{3}$

We provide the syntax followed by the semantics. We follow very closely hereCarpenter (1997) and Gamut (1991).

## Syntax

Definition 1. (The set of types $\mathbf{T}$ )
The set fo types $\mathbf{T}$ is the smallest set such that:
(i) $e, t, s, a \in \mathbf{T}$
(ii) $\sigma, \tau \in \mathbf{T}$, then $\langle\sigma \rightarrow \tau\rangle \in \mathbf{T}$
(iii) $\sigma, \tau \in \mathbf{T}$, then $\langle\sigma \times \tau\rangle \in \mathbf{T}$

Here $a$ is the type of assignment functions. The other types are standard types and need no further explanations.

Definition 2. (Vocabulary)
(i) For each type $\sigma$ a possibly empty set of constants of type $\sigma C O N_{\sigma}^{\mathcal{L}}$
(ii) For each type $\sigma$ an infinite set of variables of type $\sigma V A R_{\sigma}$
(iii) The logical connectives $\wedge, \vee, \rightarrow, \neg$
(iv) The quantifiers $\forall, \exists$
(v) Identity $=$, and lambda $\lambda$
(vi) The brackets (, ), [, ], 〈, $\rangle$

Definition 3. (Syntax)
Next we recursively define the set of well-formed expressions of type $\sigma$ of the language $\mathcal{L} W E_{\sigma}^{\mathcal{L}}$ :
(i) $V A R_{\sigma} \subseteq W E_{\sigma}^{\mathcal{L}}$
(ii) $C O N_{\sigma}^{\mathcal{L}} \subseteq W E_{\sigma}^{\mathcal{L}}$
(iii) If $A \in W E_{\langle\tau \rightarrow \sigma\rangle}^{\mathcal{L}}, B \in W E_{\tau}^{\mathcal{L}}$, then $A(B) \in W E_{\sigma}^{\mathcal{L}}$
(iv) If $A \in W E_{\sigma}^{\mathcal{L}}, B \in W E_{\tau}^{\mathcal{L}}$, then $\langle A, B\rangle \in W E_{\langle\sigma \times \tau\rangle}^{\mathcal{L}}$
(v) If $A \in W E_{\sigma}^{\mathcal{L}}$ and $x \in V A R_{\tau}$, then $\lambda x . A \in W E_{\langle\tau \rightarrow \sigma\rangle}^{\mathcal{L}}$
(vi) If $A \in W E_{\langle\sigma \times \tau\rangle}^{\mathcal{L}}$, then $p_{1}(A) \in W E_{\sigma}^{\mathcal{L}}$
(vii) If $A \in W E_{\langle\sigma \times \tau\rangle}^{\mathcal{L}}$, then $p_{2}(A) \in W E_{\tau}^{\mathcal{L}}$
(viii) If $A \in W E_{t}^{\mathcal{L}}$, then $\neg A \in W E_{t}^{\mathcal{L}}$
(ix) If $A \in W E_{t}^{\mathcal{L}}, B \in W E_{t}^{\mathcal{L}}$, then $A \circ B \in W E_{t}^{\mathcal{L}}$, with $\circ \in\{\wedge, \vee, \rightarrow\}$
(x) If $A \in W E_{t}^{\mathcal{L}}$, and $x \in V A R_{\sigma}$, then $\mathcal{Q} x[A] \in W E_{t}^{\mathcal{L}}$, with $\mathcal{Q} \in\{\forall, \exists\}$
(xi) If $A \in W E_{\sigma}^{\mathcal{L}}, B \in W E_{\sigma}^{\mathcal{L}}$, then $(A=B) \in W E_{t}^{\mathcal{L}}$
(xii) Every element of $W E_{\sigma}^{\mathcal{L}}$ for any type $\sigma$ is constructed in finitely many steps using (i)-(xi) only.

## Semantics

In this section we will follow again Carpenter (1997) and Gamut (1991).
Definition 4. (Domains) The basic types are interpreted on the domains $D_{e}, D_{a}, D_{s}, D_{t}$ respectively, which are interpreted as usual. We further have $D_{\langle\sigma \rightarrow \tau\rangle}=D_{\tau}^{D_{\sigma}}$ and $D_{\langle\sigma \times \tau\rangle}=D_{\sigma} \times D_{\tau}=\left\{\langle a, b\rangle: a \in D_{\sigma}, b \in D_{\tau}\right\}$

Definition 5. (Frame)
A frame is a set $D=D_{e} \cup D_{t} \cup D_{s} \cup D_{a}$.
Definition 6. (interpretation function / •/)
An interpretation function is a function $/ \cdot /: C O N_{\sigma}^{\mathcal{L}} \rightarrow \operatorname{Dom}_{\sigma}$ for all $\sigma \in \mathbf{T}$
Definition 7. (assignment function $\theta$ )
An interpretation function is a function $\theta: V A R_{\sigma} \rightarrow D o m_{\sigma}$ for all $\sigma \in \mathbf{T}$
Definition 8. ( $x$-variant of $\theta$ )
Let $\theta$ be an assignment function. Then $\theta[x:=a]$ is the assignment for which $\theta[x:=a](x)=a$ and $\theta[x:=$ $a](y)=\theta(y)$, for $y \neq x$.

Definition 9. (Model $\mathcal{M}$ )
A model $\mathcal{M}=\langle D, / \cdot /\rangle$ is a tuple, with $D$ a frame and $/ \cdot /$ an interpretation function.
Definition 10. (denotation $/ A /_{\mathcal{M}}^{\theta}$ )

1. $|x|_{\mathcal{M}}^{\theta}=\theta(x)$, if $x \in V A R_{\sigma}$ for some $\sigma \in \mathbf{T}$
2. $|c|_{\mathcal{M}}^{\theta}=\mid c /$, if $c \in C O N_{\sigma}^{\mathcal{L}}$, for some $\sigma$
3. $|A(B)|_{\mathcal{M}}^{\theta}=|A|_{\mathcal{M}}^{\theta}\left(/\left.B\right|_{\mathcal{M}} ^{\theta}\right)$
4. $\left.|\langle A, B\rangle|_{\mathcal{M}}^{\theta}=\left.\langle | A\right|_{\mathcal{M}} ^{\theta},|B|_{\mathcal{M}}^{\theta}\right\rangle$
5. $\left|p_{1}(\langle A, B\rangle)\right|_{\mathcal{M}}^{\theta}=|A|_{\mathcal{M}}^{\theta}$
6. $\left|p_{2}(\langle A, B\rangle)\right|_{\mathcal{M}}^{\theta}=/\left.B\right|_{\mathcal{M}} ^{\theta}$
7. $|\lambda x \cdot A|_{\mathcal{M}}^{\theta}=f$ such that $f(a)=|A|_{\mathcal{M}}^{\theta[x:=a]}$
8. If $A \in W E_{t}^{\mathcal{L}}$, then $/ \neg A /_{\mathcal{M}}^{\theta}=1$ iff $/ A /_{\mathcal{M}}^{\theta}=0$
9. If $A \in W E_{t}^{\mathcal{L}}, B \in W E_{t}^{\mathcal{L}}$, then $/\left.(A \wedge B)\right|_{\mathcal{M}} ^{\theta}=1$ iff $/\left.A\right|_{\mathcal{M}, w, g} ^{\theta}=|\psi|_{\mathcal{M}, w, g}^{\theta}=1$
10. If $A \in W E_{t}^{\mathcal{L}}, B \in W E_{t}^{\mathcal{L}}$, then $/\left.(A \vee B)\right|_{\mathcal{M}} ^{\theta}=1$ iff $/\left.A\right|_{\mathcal{M}} ^{\theta}=1$ or $/\left.B\right|_{\mathcal{M}} ^{\theta}=1$
11. If $A \in W E_{t}^{\mathcal{L}}, B \in W E_{t}^{\mathcal{L}}$, then $/\left.(A \rightarrow B)\right|_{\mathcal{M}} ^{\theta}=1$ iff $/\left.A\right|_{\mathcal{M}} ^{\theta}=0$ or $/ B / \mathcal{M}=1$
12. If $A \in W E_{t}^{\mathcal{L}}$ and $v \in V A R_{\sigma}$ where $\sigma \neq s$, then $/ \forall v[A] /_{\mathcal{M}}^{\theta}=1$ iff for all $d \in D_{\sigma}: /\left.A\right|_{\mathcal{M}} ^{\theta[x:=d]}=1$
13. If $A \in W E_{t}^{\mathcal{L}}$ and $v \in V A R_{\sigma}$ where $\sigma \neq s$, then $/ \exists x[A] /_{\mathcal{M}}^{\theta}=1$ iff for some $d \in \mathbf{D}_{\sigma}: /\left.[A]\right|_{\mathcal{M}} ^{\theta[x:=d]}=1$

## Interpretation of constants in CRISP

At last we want to clarify how predicate symbols and names are interpreted. Consider the sentence John walks. walks is translated by $\lambda x . \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq W(x)(i)$. We want to concentrate on the subformula $W(x)(i)$. The symbol $W$ is of type $\langle\mathbf{e},\langle\langle s \times a\rangle, t\rangle\rangle$, hence will be interpreted as a function mapping a functions from assignments to individuals to a function from possibilities to truth-values. But, what is that function? Assuming that we have applied an argument of type e already, we will think of it as the function below, given in set-theoretic notation:

$$
\left\{\langle w, g\rangle: x g \in\lceil W\rceil^{w}\right\}
$$

Here, $\lceil W\rceil^{w}$ denotes the extension of $W$ at world $w$. Consequently, we the interpretation of constants will provide us with intensional relations, which are evaluated with respect to a world of a possibility. The same we will do for $n$-ary predicate symbols.

For atomic, non-presuppositional predicates we make use of the following notation:
Notation 2. (hat notation, $\hat{P}$ )
For an atomic, non-presuppositional predicate $\lambda \vec{x} \cdot \lambda s . \alpha_{s} \subseteq \pi_{s} \wedge \alpha_{s} \subseteq \lambda i . P(\vec{x})(i)$ we write $\hat{P}$
Last, names are treated as functions in CRISP. This is in order to treat predicates uniformly. If we treated names simply as expressions denoting type $e$, we would have to type shift predicates depending on whether we want to feed them a proper name or a variable. This is inconvenient. Instead, we treat names as expressions of type e as well. Properties such as rigidity can be enforced onto them by demanding that under any assignment the name denotes the same individual. This can be done by means of a non-logical axiom.

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[^0]:    ${ }^{1}$ For a broad cross-linguistic overview and study the reader is referred to König (1991).
    ${ }^{2}$ The inference is more commonly referred to as additive presupposition, cf. Szabolcsi (2017)

[^1]:    ${ }^{3}$ This view got challenged recently by Ahn (2015).

[^2]:    ${ }^{1}$ For instance, if $A$ is $J o h n$, and its translation into $\mathrm{TY}_{2}$ is the constant symbol $j$ of type $e$, then its ordinary value is simply an individual in $D_{e}$.

[^3]:    ${ }^{2}$ We will provide an extensional fragment here, for simplicity. Rooth (1985) uses Montagues' IL as an intermediary between the natural language fragment and the formal semantics. We simply translate expressions of our framework into TY ${ }_{2}$ formulas.
    ${ }^{3}$ cf. Rooth (1992): 77, fn. 2 (77) for the form of the meaning postulate. The version stated here is even simpler, but sufficient for our purposes.
    ${ }^{4}$ We ignore the presuppositional aspect of only here.

[^4]:    ${ }^{5}$ We neglect the presupposition here as it is really about the assertion.
    ${ }^{6}$ The individual case arises from a discussion of contrastive focus not discussed here (cf. Rooth (1992): 79-82, 90).

[^5]:    ${ }^{7}$ This point seems clearest in Rooth's (1992) discussion of only on page 89.
    ${ }^{8}$ cf. Rooth (1992): 89.

[^6]:    ${ }^{1}$ We depart from the traditional symbol for truth, $T$, as this symbol is already used in Champollion et al. (2017) in the definition of $s^{\top}$, and because we used $T$ earlier already. The latter does not denote a truth-value, but a proposition and is thus of type $T$.

[^7]:    ${ }^{2}$ So, in this sense we take an expression $A$ with $F$-marking not simply as an atomic expression, but rather as a modified or complex expression. We will not try to pursue or clarify this point here further; it simply serves as a picture for displaying the differences.

[^8]:    ${ }^{3} p_{1}$ and $p_{2}$ are projection functions.

[^9]:    ${ }^{1}$ Remember that in Champollion et al. (2017) we only have $\llbracket \cdot \rrbracket_{g}$.
    ${ }^{2}$ We follow here Roelofsen and Farkas (2015), which assign lists a simplified syntax in their paper (cf. Roelofsen and Farkas (2015): fn. 7 (p.372).). Where necessary, we will address this point.

[^10]:    ${ }^{3}$ The operation does not yield an inquisitive proposition when applied to a tautology (cf. Roelofsen and Farkas (2015): 374). In that case it simply returns the input meaning.

[^11]:    ${ }^{4}$ We cannot have more than one item here for this would require to have a disjunction outside of an item, which we don't have.

[^12]:    ${ }^{5}$ The reason is that the effect of Closed is vacuous in this case. $Q^{0}$ then makes the non-inquisitive argument into an inquisitive proposition. Its presupposition is trivial in this case.

[^13]:    ${ }^{6}$ This might be unsatisfactory for the reader but going into these details leads us too far away from our actual topic - even though these issues might factor into the explanation.
    ${ }^{7}$ That is a set of worlds.

[^14]:    ${ }^{8}$ A similar assumption is made in Fox and Katzir (2011). There, too, the alternatives for exhaustification are seen as focus alternatives, but not Roothian ones for reasons that are somewhat orthogonal to our own interests.

[^15]:    ${ }^{9}$ The point also applies to disjunctive polar questions, but is evidently more interesting to us from the perspective of alternative questions.

[^16]:    ${ }^{10}$ Adjusted to our notation.

[^17]:    ${ }^{11}$ Note that Closed will then simply further strenghten the meaning of is argument, but its argument - if it passed the local redundancy check - will already be such that it denotes a meaning with two mutually exclusive alternatives. We think that this is unproblematic. Closed is after all not vacuous.
    ${ }^{12}$ Other considerations we leave for the future are stated in Katzir and Singh (2013).

[^18]:    ${ }^{1}$ Note that in the original system of Champollion et al. we do not have $\llbracket \cdot \rrbracket^{o}$ and $\llbracket \cdot \rrbracket^{f}$, but simply $\llbracket \cdot \rrbracket_{g}$.

[^19]:    ${ }^{2}$ We leave aside the presupposition of Q which they consider only in sec. 7 onwards.

[^20]:    ${ }^{3}$ We use set-theoretic notation for convenience. Nothing hinges on this of course.

[^21]:    ${ }^{4}$ We leave weakly-exhaustive readings for the future.
    ${ }^{5}$ We assume familiarity with Link (1983).

[^22]:    ${ }^{6}$ Note that this is necessary for independent reasons because, unlike singular which-questions, who-questions such as "Who smokes?" can receive plural answers (e.g., "Ann and Bill") and give rise to QVE effects ("For the most part, Mary knows who smokes").

[^23]:    ${ }^{7}$ Usually, this corresponds to the possibilities which assign the maximal (plural) individual to the variable, but it may not necessarily be the case when the predicate that combines with who is non-distributive or non-monotonic.

[^24]:    ${ }^{1}$ Clearly, when we leave out too in these sentences the additive inference disappears. Hence, the inference comes with too and thus is most plausibly seen as too's contribution to the meaning of the sentence.

[^25]:    ${ }^{2}$ It is not entirely clear to us why this is. We leave this for the future.

[^26]:    ${ }^{3}$ We ignore here the fact that some non-standard accounts such as Ahn (2015) are around which take the additive inference to be part of the assertion and thus treat it as an entailment.

[^27]:    ${ }^{4}$ Unfortunately, the discourse pragmatics are not spelled out in the handout of Champollion et al. (2017) and so we cannot say whether they would agree on what we do here.

[^28]:    ${ }^{5} P$ cannot be a proposition that is (contextually) equivalent or entailed by $\llbracket \mathrm{VP} \rrbracket^{o}$, for $\llbracket(1) \rrbracket^{o} \vDash \llbracket \mathrm{VP} \rrbracket^{o}$.
    ${ }^{6}$ This is a simplification. In general, the presupposition may not be satisfied by each such possibility $i$ and so "Mary smokes" is usually a stronger statement than $\hat{S}(m)$.
    ${ }^{7}$ The new context is indicated by the purple rectangle above. It is the intersection of the old context with the at-issue information of "Mary smokes."

[^29]:    ${ }^{8}$ This we need to assume here, for in principle we can have models where $T$ is not "reasonable", but something completely "random" such as $\{\neg \hat{S}(m), \hat{D}(m), \sqcap \hat{D}(n)\}$ say. What we need is a principle describing the "good" models and discarding the "bad" ones. We do not have such a principle which according to Rooth's theory is a pragmatic one.

[^30]:    ${ }^{9}$ The ambiguity between mention-some and strongly-exhaustive only plays a role when we have more than two individuals.

[^31]:    ${ }^{10}$ It's an alternative explanation for it explains the data point in terms of redundancy not in terms of presupposition failure.

