Logical Functionalism

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Abstract

This thesis aims to develop a position called *logical functionalism*, as an improvement on previous logical pluralist positions. The main idea of logical functionalism is to take logical consequence to be a functional concept. From a pre-theoretic notion of logical consequence, I distil the core features of logical consequence, namely *formality*, *normativity* and *necessity*. Together these core features make up the logical consequence-role. I then show that this role can be performed in multiple different ways, resulting in different manifestations of the logical consequence-role and hence different logical systems. As such, I take logical functionalism as I have presented it here as an enhancement of previous logical pluralist positions. The main problem with the pluralist positions currently on the market I take to be a problematic relation between plurality and generality. This makes that these positions tend to collapse into either the strongest available logical system, or the weakest available system. Some generality therefore appears to be unavoidable, and I argue that logical pluralism needs to provide a satisfactory account of such generality, while at the same time retain the pluralist spirit of a multiplicity of equally correct logical systems. Logical functionalism, I maintain, provides such a satisfactory account. A non-problematic account of generality, that does not allow for a collapse into monism, can be found in the logical consequence-role, while the plurality can be found in the multiple different manifestations of this role.

Acknowledgements

In philosophy one feels forced to look at a concept in a certain way. What I do is suggest, or even invent, other ways of looking at it. I suggest possibilities of which you had not previously thought. You thought that there was one possibility, or only two at most. But I made you think of others. Furthermore, I made you see that it was absurd to expect the concept to conform to those narrow possibilities. Thus your mental cramp is relieved, and you are free to look around the field of use of the expression and to describe the different kinds of uses of it.

Ludwig Wittgenstein (Malcolm 1984, p. 43)

Contents

Introduction

In recent decades there have been many debates and subsequently new developments on logical pluralism. This is, arguably, most prominently due to a series of papers by JC Beall and Greg Restall, which culminated into their 2006 book *Logical Pluralism*. But Beall and Restall are not alone in their idea that there are several different, equally correct, logical systems. The historical roots of logical pluralism could be traced back to at least Rudolf Carnap (1937). Some notable logical pluralists these days are - besides Beall and Restall - Colin Caret (2017), Stewart Shapiro (2014), Hartry Field (2009) and Gillian Russell (2008). All have developed different positions that challenge logical monism, which on its turn clearly has an historical advantage. The most notable contemporary logical monists that take up the defence line against logical pluralism are - among others - Timothy Williamson (2013), Graham Priest (2006) and Stephan Read (2006). Historically it makes little sense to sum up some of the most prominent logical monists - name any historical logician and chances are most likely he or she can be placed among the ranks of the logical monists.

But this thesis is not primarily a historical overview of logical pluralism, nor an overview of the debate between the logical pluralists and the logical monists as such. There is a rich body of literature that focuses on several of such sub-fields of the discussion. For example, there are interesting analyses of the structure of the debate between the logical pluralist and the logical monist, and solutions aimed at resolving (or dissolving) such disputes - see for example Priest (2015) and Hjortland (2017). Others focus on the question of exactly how many logical systems logical pluralism allows for - such meta-theoretical considerations can be found, for example, in Sereni and Fogliani (2017). Yet others have connected logical pluralism with alethic pluralism and ontological pluralism - see for example Turner (2012), Cotnoir and Edwards (2015), Yu (2017), Pedersen (2014) and Lynch (2008). All these show interesting facets of the debate on logical pluralism, but will not be the focus of this thesis.

This thesis aims to put forth a new way of thinking about logical consequence, namely one that takes logical consequence to be a *functional* concept. As such, this thesis has been written as a defence of logical pluralism, for I believe that

such a conceptual framework avoids problems that versions of logical pluralism developed so far have been running into. In fact, I argue that they all run into one *family* of problems that lay bare a problematic relation between the pluralist spirit on the one hand, and some kind of generality on the other hand giving rise to this kind of so-called collapse problems. In this thesis, I argue that logical functionalism is a natural way to account for both the plurality and the generality of logical consequence. In developing logical functionalism, then, I claim to have enhanced logical pluralism as such and made the claim against logical monism more convincing. My suggestion is that if you want to be a logical pluralist - which I believe you should - then you have to be a logical functionalist. I take the other direction to be plausible as well: if you find logical functionalism to be convincing, you must endorse logical pluralism as well.

This thesis is structured as follows. In chapter 1, I will present logical pluralism. I will first pinpoint the crucial point of controversy between the pluralist and the monist, namely the existence of the *One True Logic*, before I will present the most prominent varieties of logical pluralism, categorised in the two categories *global logical pluralism* and *local logical pluralism*. In chapter 2, I will present the problems facing logical pluralism, and I will show that both global logical pluralism and local logical pluralism are challenged by a problematic relation between plurality and generality. In other words: for both kinds of logical pluralism, some kind of monism appears to be unavoidable, that threatens the pluralist spirit. In chapter 3, I will present the desiderata and a methodology of a solution to the overall problem facing logical pluralism presented in chapter 2. I will do so by formulating two desiderata that the solution must meet - *Plurality* and *Generality* - and I will argue that a functionalist methodology, on which I will briefly elaborate, might be suitable for developing a solution that meets these two desiderata. From chapter 3 it should be clear what I will do in the subsequent chapter: the idea is to identify a core *function*, or *role* the concept of logical consequence performs, which I will do by first identifying a pre-theoretic notion of the concept in play, and from that distil the core features that together define the role of the concept in play. In other words: what I am after is defining the *logical consequence-role*. In chapter 4, I will work this out, in order to develop logical functionalism, by formulating what I take to be the core features of logical consequence - *formality*, *normativity* and *necessity* - that together make up the logical consequence-role. I will provide a definition of the logical consequence-role and show that this role can be manifested in multiple different ways. A crucial feature appears to be the sense in which I take logical consequence to be normative, namely relative to some specific epistemic goal. In chapter 5 I will evaluate logical functionalism as presented in chapter 4, by considering how it fares against the problems facing logical pluralism presented in chapter 2, and by considering some resemblances and differences between logical functionalism and the logical pluralist positions presented in chapter 1. The aim

of this chapter is to consider the pay-off of logical functionalism, and indicate the place it takes up in the debate on logical pluralism. In chapter 6, I will discuss some open ends to my research by replying on some (potential) questions and objections, and by providing a brief outlook on future work. In chapter 7, I will conclude by summarising the main findings and arguments put forth in this thesis.

Chapter 1

Logical Pluralism

Logical pluralism holds that there are multiple different logical systems that are all equally correct, and thus that there is not *one* logical system with a privileged status over the others. In this chapter I will first discuss what is essentially at stake in the debate between the logical pluralist and the logical monist. I will argue that there is a non-controversial plurality over which both the pluralist and the monist agree. What is essentially at stake is whether or not there is a *One True Logic*, which I will briefly elaborate on. Then, I will present different versions of logical pluralism, namely Beall and Restall-pluralism, domain-relative pluralism and contextualism. These versions will be categorised into two kinds: global logical pluralism and local logical pluralism. The aim of this overview is to see, first, what the key claim of logical pluralism is, and second, how this claim has been worked out in versions of logical pluralism currently on the market.

1.1   A Non-Controversial Plurality

At first sight, logical pluralism might appear to be rather trivial. After all, anyone who is familiar with logic knows that there are different logical systems on the market. Besides classical logic, one can think of intuitionistic logic, relevant logic or paraconsistent logic. All these systems have their differences. For example, while classical logic allows for Double Negation Elimination (DNE), intuitionistic logic does not allow for this. Yet, both classical logic and intuitionistic logic are *explosive*: in both logical systems, everything follows from a contradiction. This is not the case in paraconsistent logic. In relevant logic the variable sharing principle is necessary, which holds that in a formula of the form $A \rightarrow B$, A and B must have at least one propositional variable in common, and an inference can only be valid if the premises and the conclusion have at least one propositional variable in common. Such a principle plays no role in the other logical systems. These are of course just some of the different logical systems on the market, and just some of the differences between them.

The plurality of logical systems in this sense I will call the *non-controversial plurality*. It is non-controversial because both logical monists and logical pluralists can acknowledge and accept it. Priest (2006), as a logical monist, for example takes this non-controversial plurality as a plurality of what he calls *pure logics*, where a pure logic is "a well-defined mathematical structure with a proof-theory, model theory, etc." (p. 195). Classical logic, intuitionistic logic, paraconsistent logic and relevant logic - to name a few - are all pure logics in this sense. Priest (2006) goes on to argue that "[t]here is no question of rivalry between [logical systems] at this level" (p. 195). I will follow Priest in this, and maintain that one is free to devise any such well-defined mathematical structure and call it a pure logical system. As long as all these systems are such structures, there is no rivalry between them and they can perfectly well co-exists next to each other.

What is at stake in the debate between the logical monist and the logical pluralist, then, is something more significant: the logical pluralist holds - while the logical monist denies - that within this non-controversial plurality *at least two* different logical systems are both equally correct, and that there is *no* one logical system with some kind of overall privileged status over the others. Another way to put this is that the logical monist holds - while the logical pluralist denies - that there is a *One True Logic*. To be a logical pluralist, you then need to hold that there is more than *One True Logic*. In the following, I will elaborate on what a *One True Logic* is, in what sense it enjoys a privileged status, and in what sense the logical pluralist take multiple logical systems to be equally correct.

1.2   One True Logic?

In its essence, this is what the debate between the logical monist and the logical pluralist is about: the existence of a *One True Logic*. The focus of this thesis is on logical pluralism, but in the following section I will briefly consider its counterpart, logical monism. What is a *One True Logic*? Or: what could it be? And what makes one believe that there is only one?

Goddu (2002) takes the core of logical monism to be the idea that "[t]here is one and only one correct answer as to whether a given argument is deductively valid" (p. 222). This is another way of phrasing what Beall and Restall (2000: p. 481) call 'Real Validity': the idea that there is only one *real* notion of validity. Given that a (deductively) valid argument is one where the conclusion is a logical consequence of the premises, this amounts to the idea that there is only one *real* logical consequence-relation. And all this can be taken to be different ways of expressing what the notion *One True Logic* entails: namely, there being only one notion of logical consequence and ultimately one way for an argument to be deductively valid.

In what sense is this *One True Logic* considered privileged over other logical systems? There are different ways to account for this. Recall that Priest (2006) acknowledges a plurality of pure logics. He writes that "[p]lurality is an issue of substance only if one is asking about *applied logics*" (p. 194). The idea is that one might be free to devise a pure logic, but pure logics are to be used for some specific purposes or applications. "Plurality is, then," according to Priest (2006), "an interesting issue only when we have one particular application in mind" (p. 194). Given a specific application, one logic will be more apt than another - this is something both the monist and the pluralist could agree on. So, the issue goes even deeper. According to Priest, it is about a *canonical application.* Thus, the *One True Logic* would be the correct logical system for the canonical application of logic. The challenge is to say which logic should be used for the canonical application of logic. A monist position would hold that for the core application of logic, only one logic is *correct*, and as such this logic has a privileged status of the other logic. According to Priest (2006), the canonical application of logic is the application of a logic in the analysis of reasoning of which the central purpose is "to determine what follows from what - what premises support what conclusions - and why. An argument where this is, in fact, the case is *valid*" (p. 194). Priest's take on what the canonical application of logic is to me seems reasonable: logic, in its most simple task-description, is to determine what follows from what. For now, we can allow ourselves here to stay neutral on the details of this issue, since the aim was to get an idea of what the *One True Logic* could be, and what we've seen is that it can be seen as the correct logic for the canonical application of logic. The monist takes there to be only one correct logical system suited for this application, while the pluralist holds that there could be different logical systems suited for this application.

How are we to decide which logic is the *correct* logic? There are several takes on this. Historically, logic has taken to be a special discipline of which the truths are *a priori*.[1] Call this *apriorism.* The idea is that logical truths and principles are independent of any empirical evidence, they are certain, and they are unrevisable. As such, one could take logical principles to be part of the *a priori* bedrock of knowledge. On the other hand, there is also the idea, which goes back to Quine (1957), that logical truths are attained in a similar way as (natural) science attains its truths, namely by abductive methods.[2] Call this *abductivism.* Somewhat loosely, abductivism can be seen as inference to the best explanation and what speaks in favour of a theory is a set of criteria, such as adequacy to the data, simplicity, elegance, fertility, fruitfulness, explanatory power, con-

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[1]See for example Frege (1893), Carnap (1937) and Gödel (1947).
[2]See also Hjortland (2017) for current research on this idea.

sistency, unifying power and avoidance of *ad hoc* elements.[3] For Priest (2015) these considerations yield a rational theory-choice whereby a *rationality index* as the weighted sum of its performance on each criterion decides which theory we should adopt - the theory with the highest rationality index, i.e. the theory that scores best on the rationality index. Which method of deciding the correct logic is the right one is open to debate, but I will not get into this issue here.

My aim was to make clear the idea of the *One True Logic*. Let me summarise the considerations I have presented in the above as follows. The logical monist holds that there is only *one* correct logic for (some) canonical purpose of logic. This logic is the *One True Logic*. Let me add to this that this *One True Logic* can be classical logic (as Timothy Williamson (2013) argues), as well as intuitionistic logic (as Michael Dummett (1991) argues), relevant logic (as Stephan Read (2006) argues) or paraconsistent logic (as Graham Priest (2006) argues). There are different ideas as to how one can decide which logic is the correct logic, but the logical monists - whatever logical system they take to be the *One True Logic* - all hold that only one system is correct.

1.3 Varieties of Logical Pluralism

Now that we have gained some insight in the ideas of logical monism, let me focus on logical pluralism. The logical pluralist holds - as we have seen - that there are several equally correct logical systems. Logical pluralism is an attractive position for several reasons. For one, there is this non-controversial plurality of logical systems of which none seems to obviously stand out, for which the logical pluralist seems to account for quite naturally. Furthermore, the logical pluralist promises to dissolve logical disputes between (monist) proponents of different logical systems. As Steinberger (2017) writes, "if [the logical pluralist] is right, the pluralist is a heroic character who delivers us from proclivity for fruitless squabbles" (p. 4). And regardless of the pay-off, it could be argued that given our definition of logic and logical consequence, this inherently gives us a genuine plurality of logical systems - the nature of logic *is* simply pluralistic. In later chapters of this thesis, I will argue for the latter.

There are different versions of logical pluralism - different ways of accounting for this possibility of plurality with respect to equally correct logical systems. In the next section, I will make a distinction between *Global Logical Pluralism* and *Local Logical Pluralism*[4]: while global logical pluralism holds that there are at

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[3]This list is not meant to be exhaustive and some criteria might be interchangeable, e.g. simplicity and elegance. For some, like the aforementioned, it is not always even clear what they mean. The details of this issue are irrelevant here.

[4]The terms Global [Logical] Pluralism and Local [Logical] Pluralism are due to Haack (1978). See also Hjortland (2013) and Stei (2017).

least two *all-purpose logics* - defined as a logic applicable to reasoning in all discourse domains[5] - local logical pluralism dispenses with the idea that there is *any* all-purpose logic, but maintains that there are only logical systems applicable for specific purposes.

1.3.1 Global Logical Pluralism

Global logical pluralism holds that there are at least two *all-purpose logics* which are both equally correct.[6] The global logical pluralist argues that there are several (at least two) different all-purpose logics, while the monist argues that there is only one all-purpose logic.

In the following, I will elaborate more on arguably the most influential version of global logical pluralism, put forth by Beall and Restall (2006).

Beall and Restall Pluralism

The version of global logical pluralism put forth by Beall and Restall (2006) - henceforth; Beall and Restall-pluralism - is arguably the most influential version of logical pluralism on the market, and it comes down to pluralism about logical consequence: the idea being that there are different ways of specifying the pre-theoretic notion of (deductive) logical consequence. Since the logical systems they allow for are not tied down to specific applications, as we will see in the next section on local logical pluralism, this version can be seen as a kind of global logical pluralism.

Beall and Restall (2000, 2006) take logical consequence to be the heart of logic, and their first step is to provide a general definition of this concept. That is no easy task. As we know from Tarski (1956), the concept of logical consequence is not *sharp*, and any *one* definition that satisfies all accounts of logical consequence appears to be impossible.[7] If this is in fact the case, then this might already hint at logical pluralism. After all, if there is not *one* definition that satisfies all accounts of logical consequence, then there might be *multiple* specifications of logical consequence. However, we might still want to identify at least some *core* of this notion. I will try to do exactly this in later chapters, in the form of the logical consequence-role. Beall and Restall, however, settle for what they call the *Generalized Tarski Thesis* (GTT):

> **(GTT)**: An argument is valid$_x$ iff in every case$_x$ in which the premises are true, the conclusion is true.

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[5]See Hjortland (2013). Hjortland writes that an all-purpose logic "is more or less what Priest [2006a: 196] calls the canonical application of logic" (p. 357).

[6]See for example Hjortland (2013).

[7]For more on this, see my reply on one potential objection in section 6.1.

The pluralism in logic arises from the fact that in this definition, the notion of *case* can be interpreted in different ways. The notion itself is *unsettled* and needs a specification. By specifying different interpretations of a case, one could arrive at different notions of validity. For example, Beall and Restall (2006) present three possible interpretations of a case: cases that are complete and consistent (i.e. possible worlds), cases that may be inconsistent and complete (i.e. situations), and cases that could be incomplete but consistent (i.e. constructions). The first specification is said to yield classical logic, the second relevant logic and the latter intuitionistic logic. However, the question remains what a case exactly is and how to specify one. Beall and Restall (2006) write on this:

> Cases, whatever else they are, are 'things' in which claims may be true. By specifying 'truth conditions' for claims, you thereby specify cases. [...] We do not pretend to have given precise individuation conditions for cases. We are not sure that such conditions can be given. However, we have said enough to indicate that we are *moderately* liberal with respect to what counts as a case: for any set of sentences, there is a case in which those sentences are true. (p. 89)

Let us consider how this works in practice. For example, if we take cases to be possible worlds, then the clauses for truth in a case - to be read now as truth in a possible world - already gives us an account of logical consequence. As Beall and Restall (2000) write, "with no further analysis of what a world w might *be*, or how *many* there might be, a story of consequence can be told" (p. 4). In fact, this is a *classical* story of consequence, that yields classical logic. On the other hand, you could also take cases to be situations rather then possible worlds, and provide truth conditions that take into account a notion of relevance.[8] This could yield relevant logic. Finally, if you take cases as constructions, you can provide truth conditions that take into account a condition of proof (or construction), which would yield intuitionistic logic. However liberal Beall and Restall (2006) are with respect to what counts as a case, there are some constraints: an instance of (GTT) is admissible only if it satisfies the settled role of logical consequence, and (hence) if its judgements about consequence are *necessary*, *normative* and *formal* (p. 35).

This version of logical pluralism is a kind of global logical pluralism, because the different logical systems that result from the different specifications of cases apply, at least potentially, unrestrictedly. That is, in Beall and Restall-pluralism, the logical system results from a specific specification of a case, but a case is not restricted to some specific parameter, such as for example a specific domain of inquiry.[9] Some specification of a case, for example one that takes a case to be

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[8]For example, see Beall and Restall (2000: p. 9).

[9]As is the case in domain-relative logical pluralism. See section 1.3.2.

a maximally possible world, might - at least potentially - be applied on either mathematics or ethics, or perhaps even both.

1.3.2 Local Logical Pluralism

Global logical pluralism holds on to the idea that there *is* a so-called 'all-purpose logic', being a logic that (at least potentially) applies unrestrictedly, although there is no *one* such logic. Local logical pluralism dispenses with this idea: there is no all-purpose logic. Rather, every logical system is *relativised* to some kind of parameter, such as a domain of inquiry in the case of domain-relative pluralism, or specific contexts in the case of contextualism.[10] But there are different versions of local logical pluralism, that all relativise logical systems to different parameters.

Domain-Relative Logical Pluralism

Domain-relative logical pluralism[11], argues that logical systems are relative to domains of inquiry. The position has been suggested, yet not worked out explicitly, by Michael Lynch (2008, 2009). For Lynch, domain-relative logical pluralism can function as an extension of his alethic pluralism. Validity can be defined in terms of truth: an argument is valid only when if the premises are true the conclusion is (or must be) true. If there are several different truth properties, as the alethic pluralist holds, then this might entail that there are several ways for an argument to be valid. As logical consequence is the concept that renders an argument valid or not, there might then be multiple logical consequence-relations connected to each domain-specific truth property. Therefore, to understand domain-relative pluralism it makes sense to briefly say something about alethic pluralism.

Alethic pluralism, as Edwards (2008) puts it, is "the thesis that sentences or propositions can be true in different ways" (p. 143). Yu (2017) defines alethic pluralism as the position which holds that "[t]here are distinct domain-specific truth properties, where domains are associated with subject matter" (p. 277). That is to say that for different subject matters there are different ways of assigning truth to a proposition about that subject matter. For example, we can think of a subject matter that falls within the domain of natural sciences, like particle physics. Let us assume that the correspondence theory of truth corresponds to this domain. That is, for some proposition about particle physics to be true, it has to correspond to the hard facts of reality on particle physics.[12] But,

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[10]Stei (2017) provides reasons that might compel us to 're-label' local logical pluralism to a kind of logical relativism. Although the matter is essentially terminological, the term 'relativism' might be too specific and hence misleading. *A kind of* relativism seems right, but 'local logical pluralism' seems to be more apt. Yet, nothing hinges on this terminological issue.

[11]Or: domain-specific logical pluralism.

[12]Other versions of the correspondence theory of truth might use isomorphism or congruence, rather than correspondence. Also, some might don't use propositions, but rather sentences. I

for another domain, such a correspondence theory of truth does not seem to fit. Think for example about the ethical domain. Arguably, there is no hard fact of reality that corresponds to the ethical proposition "Murder is wrong", so for the ethical domain the epistemically constrained notion of superassertibility[13] might be more fitting. A proposition is superassertible if this proposition is warranted by some state of information and warranted by all possible extensions of this state of information. Here, then, we take supperassertibilty as a truth property, which is to say that a proposition is true if and only if it is superassertible. Another example could be the mathematical domain, where truth can be assigned by internal coherence.[14] These examples would entail that there are three kinds of truth properties: the correspondence truth property, the superassertible truth property and the coherence truth property. Of course, there is still room for more truth-properties, but we have that there are *at least two* truth-properties. Which truth property applies to a sentence or proposition depends on the domain or subject matter that the sentence or proposition belongs to. This, in a nutshell, is alethic pluralism.

Alethic pluralism might be extended to domain-relative logical pluralism. The idea is, first, that inquiry is compartmentalised into different distinct domains, and second, that different domains of inquiry call for different logics, given that the domains have different truth-properties that we need to preserve in reasoning. For example, let us again take two different domains: the domain of natural sciences and the domain of ethics. Given that, let us assume, the domain of natural sciences has the correspondence truth-property - that for some proposition about, say, particle physics to be true, it has to correspond to the hard facts of reality on particle physics - this might lead the domain-relative pluralist to believe that classical logic governs this domain. On the other hand, if we take the ethical domain to have the supperassertible truth-property, this might lead the domain-relative pluralist to take it that the constructivist intuitionistic logic governs this domain. This, in a nutshell, is domain-relative logical pluralism.

Contextualism

Another version of local logical pluralism is contextualism, which is most prominently articulated by Caret (2017).[15] Contextualism holds that logical pluralism

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[13]The notion of superassertability is due to Crispin Wright (1992) and it corresponds with the notion of superwarrant that is being used often. See Pedersen (2014) for a brief discussion between the connection between these two notions.

[14]Although if mathematics is governed by intuitionistic logic, then supperassertability might be the right theory of truth. I have used the coherence theory of truth here for illustrative purposes, which appears to be justified as at least intuitively it seems like a plausible candidate. But again, don't take this as an implicit endorsement.

[15]But see also Hjortland (2013) and Shapiro (2014).

use correspondence and propositions here not as an implicit endorsement that this is the right way to think of it, but rather for illustrative purposes.

arises from the fact that the validity predicate is sensitive to the context it is being used in (Caret 2017: p. 750). In other words, the meaning of 'validity' differs among different contexts. Specific cases (such as maximally possible worlds, incomplete cases, inconsistent cases, etc. - similar to what Beall and Restall understand by a case) come to the fore in different contexts. Caret (2017) summarises the view as follows:

> [C]ontextualism about validity is the view that 'valid' and its cognates include a parameter for a *deductive standard*. In simple terms, a deductive standard is an admissible class of cases that function as logically salient alternatives. Each context selects for a deductive standard and this, in turn, gives content to validity attributions in that context. Validity always requires a connection between premises and conclusion that rules out counterexamples, but the salient range of cases may vary from one context to another. (p. 752)

The idea is simple: different contexts require different logics. However, the question then is: how does a context determine what kind of logic is connected to it? Caret (2017) takes the most plausible contextual factors that determine this to be 'structural' assumptions, "such as assumptions of consistency or determinacy" (p. 753). Thus, if a certain context assumes determinacy, then classical logic should be preferred over intuitionistic logic, since propositions in the latter logical systems might be undetermined. On the other hand, we could also imagine a context that assumes some kind of provability that allows for a deeper understanding of the phenomena in play. In such a context, a constructive logic, like intuitionistic logic, should be preferred over classical logic, because the former demands a proof while the latter does not. So, depending on the assumptions and the requirements that come along with a specific context, the deductive standards shift: strict contexts mandate weaker logics in which fewer inferences are valid, while less strict contexts mandate stronger logics in which more inferences are valid. This, in a nutshell, is contextualism.

1.4   Conclusion

In this chapter I have presented logical pluralism. We have seen what is at stake in the debate between the logical pluralist and its antagonist, the logical monist, namely the existence and status of the *One True Logic*. While the monist holds that there is only *One True Logic*, the logical pluralist holds that there are several different, equally correct logics, and hence denies the existence of one logical system with some kind of privileged status over the others. I then have presented some varieties of logical pluralism: Beall and Restall-pluralism, domain-relative logical pluralism and contextualism. The first is a kind of global logical pluralism, which holds that there are multiple all-purpose logics. The latter two are a kind of local logical pluralism, which holds that there is *no*

all-purpose logic, but rather a multiplicity of compartmentalised, or relativised, logics - relativised either to domain of inquiry or context respectively.

Chapter 2

Problems Facing Logical Pluralism

Logical pluralism has not gone undisputed. In the following, I will present problems for logical pluralism, both global logical pluralism and local logical pluralism. I argue that these problems can be seen as a *family* of problems that indicate a problematic relation between logical pluralism and some account of generality. Logical monism seems to lurk at every corner, and in the remainder of this thesis I will argue that the best way to tackle it is to provide an account of generality that does not harm the pluralist spirit. In other words: the logical pluralist will have to face logical monism, but how? First, let me consider the problems.

2.1   Problems Facing Global Logical Pluralism

Global logical pluralism faces problems. For one, there is the problem that is known as the collapse problem, which according to Caret (2017), is "an assault on the general framework of logical pluralism" (p. 742). Stei (2017) argues that any kind of global logical pluralism[16] is susceptible to collapse arguments. The problem starts out on the reasonable and widely accepted assumption that logic is normative for reasoning. That is, logic is supposed to provide us norms of how we ought to reason; it should give us guidance. The collapse argument is supposed to show that global logical pluralism does not provide us such normative guidance, unless it disposes with the pluralist spirit by accepting some form of logical monism. Let me show how the argument goes.

There are different ways of presenting the collapse argument. Stei (2017) presents three of them. First, he shows that Williamson already recognised a collapse ar-

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[16]Stei uses the term Global Consequence Pluralism, but it comes down to what have I labelled Global Logical Pluralism.

gument before Beall and Restall started working on their position. Williamson (1988: p. 122) formulates the problem roughly as follows: if we assume that there are different, but equally correct 'deducibility' relations and then discover that our beliefs have a certain consequence in the one sense, but not in the other sense, should we then accept that consequence or not? Priest (2006) is more specific, as he formulates the problem as follows:

> Let s be some situation about which we are reasoning; suppose that s is in different classes of situations, say, $K1$ and $K2$. Should one use the notion of validity appropriate for $K1$ or for $K2$? We cannot give the answer 'both' here. Take some inference that is valid in $K1$ but not $K2$, $\alpha \vdash \beta$, and suppose that we know (or assume) α holds in s; are we, or are we not entitled to accept that β does? Either we are or we are not: there can be no pluralism about this. (p. 203)

Finally, as Read (2006) presents the problem the collapse becomes explicit. He writes:

> [S]uppose there really are two equally good accounts of deductive validity, V_1 and V_2, that β follows from α according to V_1 but not V_2, and we know that α is true. [...] It follows V_1-ly that β is true, but not V_2-ly. Should we, or should we not conclude that β is true? The answer seems clear: V_1 trumps V_2. After all, V_2 does not tell us that β is false; it simply fails to tell us whether it is true. [...] V_1 and V_2 are not equally good. V_1 answers a crucial question which V_2 does not. (pp. 194-195) [17] [18]

So, given that we take the two logical systems and their accounts of deductive validity V_1 and V_2 to be equally correct, and furthermore that V_1 is able to (logically) confirm a claim while V_2 remain 'silent' on this, Read proposes to endorse V_1, because it answers a question - is β true or not? - which V_2 doesn't answer. In this sense V_1 trumps V_2, and that is why Read believes one should endorse V_1. But if V_1 answers the question if β is true or not, and V_2 does not, then V_1 and V_2 might not be equally correct after all. Logical pluralism collapses into logical monism: assuming that different logical systems are equally correct, the stronger system is to be preferred, because it is able to provide us an answer to the crucial question whether β is true or not. In other words: the logical system with the strongest available consequence relation appears to have some kind of

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[17]In the original passage Read uses K_1 and K_2 and $K2$ to label what I've labelled V_1 and V_2. I've done this to avoid confusion with the foregoing quote by Priest.

[18]Note that there is a difference in the problems as presented by Williamson and Priest, and the problem as presented by Read. Williamson and Priest argue that there is no pluralism whatever our answer to the challenge is, whereby they don't imply that there can be a clear answer to whether β follows or not. On the other hand, Read argues that there *is* a clear answer, as he writes that V_1 trumps V_2, but this answer causes a collapse into monism and thereby undercuts pluralism.

privileged status compared to the systems with a weaker consequence relation. This privileged system appears to be an apt candidate for the *One True Logic* logical pluralism tries to avoid, or denies its existence.

One remark about the collapse problem is in place. It appears to be that all the arguments presented in the above assume that there is some epistemic utility related to logical consequence. For example, when Read argues that one should endorse the logical system that *does* allow for some specific inference, where another logical system doesn't, there appear to be no other reason to do so but some kind of epistemic force that the former system has. After all, in the example presented above, V_1 trumps V_2 because the former provides us knowledge, namely that β is true, while the latter does not. As it is presented it is 'couched' in terms of normativity, but it appears to be that some epistemic significance plays a role in deciding what logical consequence-relation to endorse. I will get back to the issue of normativity and epistemic utility in chapter 4.

Bremer (2014) formulated seven problems for Beall and Restall-pluralism. I will mention two of them here, namely the common core problem and the formal common core problem.[19] The common core problem and the closely related formal common core problem both present a similar kind of worry as the collapse problem, although approached from a different angle: despite all the alleged pluralism in Beall and Restall-pluralism, a common core is (implicitly) assumed which could undermine the pluralist spirit. The underlying problem is that it is unclear in what sense we should take the arguments for logical pluralism to be 'valid, correct and convincing'. These arguments have to be 'valid, correct and convincing' for any audience whatsoever - so for proponents of all different logical systems. In order for these arguments to be so, we need a common core of accepted argumentative standards. There might be different logical preferences - one might, for some reason, prefer classical logic while another might prefer intuitionistic logic, for example - but we need to share at least some widely shared accepted standards in order for the arguments to be 'valid, correct and convincing' for all of us. Such standards might be found in the *intersection* of different (applied) systems of reasoning. However, this would be some kind of common core logic. In other words: above all the different logical systems Beall and Restall-pluralism allows for, there must be some substantial *common core* which allows for all these different systems to *be* a logical system in the first place. I

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[19] The other problems are that "[i]n [Beall and Restall's] version of logical pluralism standard logic is only *restricted* by the other contestants. The three candidates stand in sub/super-logic relations to each other" (p. 295), "Beall and Restall's choice what can be dealt with pluralistically in logic is idiosyncratic and not principled" (p. 296), that "Beall and Restall's comparison of logical pluralism with the Church-Turing-Thesis works to their disadvantage" (p. 297), that "Beall and Restall are not explicit about their methodology" (p. 298), and that "Beall and Restall's logical pluralism seems to stand in conflict with a well-established tradition of treating logical form in linguistics and cognitive science" (p. 299).

take it that this common core provides each particular logical system with some kind of *universal* deductive standard. As Bremer (2014) puts it, "[i]n presenting a general argument for pluralism Beall and Restall seem to presuppose the very universal core of logic the existence of which they reject" (p. 294).

In the way Beall and Restall formulate their position, such a universal core of logic threatens to undermine the pluralist spirit. I take this to be also because of the way they present the *Generalized Tarski Thesis* (GTT) as the general definition of logical consequence. Recall that (GTT) goes as follows:

> **(GTT)**: An argument is valid$_x$ iff in every case$_x$ in which the premises are true, the conclusion is true.

The pluralism in logic arises from the fact that in this definition, the notion of *case* can be interpreted in different ways. However, if we would quantify over *all* (possible interpretations of) cases, we then seem to get some generality after all. The point is that the intersection of all logical systems given by (GTT) might give us a *One True Logic* after all.

Beall and Restall (2006) address this objection (p. 92). They argue that if we would take the intersection of all logical systems, we would be left with an unbearably weak logical system, as the only inference left in this intersection might be the identity inference, that allows us to infer A from A. To take that to be the only really valid argument, they consider implausible and an unmotivated conclusion. They have a response to this objection that retains the pluralist spirit and retains a more plausible account of validity. For an argument Δ to be invalid in logic$_x$ but valid according to logic$_y$, there has to be a case$_x$ in which the premises of Δ are true but the conclusion false, and that there is no case$_y$ in which this is the case. To then render the argument invalid seems a too drastic move, say Beall and Restall, and they argue there is no principled reason to do so (p. 92). In fact, if we take logic$_y$ to bear all the marks essential to a logical consequence-relation - necessity, formality, normativity - there do seems to be a reason to opt for logical pluralism and render the objection not warranted. Beall and Restall's conclude that it would require some hostility toward logical pluralism to hold this objection against Beall and Restall-pluralism.

However, I believe that the above does point out a weakness in Beall and Restall's presentation. The (GTT) is supposed to be the general definition of logical consequence. We have seen that if we quantify over all the different possible specifications of cases, then we get a logic that is severely impoverished. However, Beall and Restall argue, if we were only a bit more sympathetic to logical pluralism, we would see that as logic$_y$ bears all the marks essential to a logical consequence-relation, we would have to accept it has a genuine logical consequence-relation. But I see no principled reason to disregard the possibility

of quantifying over all cases and accepting that we are then left with an unbearably weak logic. After all, Beall and Restall do not present their pluralism in a way that prioritises the admissibility constraints, i.e. the core features of logical consequence. They do not argue that as long as some relation adheres to these constraints, or has these features, *then* it is a genuine consequence relation. I see that the logic at the intersection of all logic is weak, and that it is implausible to take this notion of validity to be the one *real* notion of validity. But what means do Beall and Restall have to disregard the quantification over all cases? Some notion of epistemic utility could have been of use here, as a too weak logical system is of no epistemic use, but there is no mention of this in their presentation. The admissibility constraints are simply not in the business of disregarding this problematic universal quantification. All these constraints do is simply tell us which cases are admissible, but I don't see how a universal case could *not* be admissible. In fact, this is also not what Beall and Restall argue for. All they say is that we shouldn't disregard logic$_y$ in our example. Now, I grant that there is a tension between raising the objection of the universal quantification over cases *and* not disregarding logic$_y$. That is, one might raise the objection of the universal quantification over cases, but it seems that we *should* then disregard logic$_y$. However, while Beall and Restall write that the objection can only have force if the presumption against pluralism has already been made, I also take it to be that their reply only has force if the presumption in favour of pluralism has already been made. In other words, we have gained nothing. At the same time, I take this to show that Beall and Restall in their presentation of logical pluralism are not able to avoid a problematic relation with generality.

Thus, the main problem global pluralism needs to account for is how to maintain the pluralist spirit in the face of some generality. Taking the collapse problem and the problems put forth by Bremer (2014) to be genuine problems, some monistic, general features of logic appear to be unavoidable. So the challenge for the global pluralist will be to give an account of generality that retains the plurality of logical systems, if of course it wishes to maintain the pluralist position.

2.2 Problems Facing Local Logical Pluralism

Local logical pluralism also faces problems. A problem that especially faces domain-relative pluralism is the problem of mixed compounds. This problem originated in the discussion of truth-pluralism[20], but can be extended to the discussion of logical pluralism[21]. A mixed compound here generally is taken to be the result of mixing different domains of inquiry, in for example a conjunction or inference more specifically. In other words: a mixed conjunction or mixed inference can be seen as a special case of a mixed compound. Now, let us first briefly

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[20]See Tappolet (1997) and Tappolet (2000).
[21]See for example Wrenn (2015) and Steinberger (2017).

present the problem as it was presented in the discussion on truth-pluralism, and then see how it might extend to the discussion on logical pluralism.

Recall that alethic pluralism, as Edwards (2008) puts it, is "the thesis that sentences or propositions can be true in different ways" (p. 143), and that the ways of propositions being true is often relativised to a specific domain of inquiry. For example, some proposition in domain D_x may be true in virtue of truth property X, while another proposition in domain D_y may be true in virtue of truth property Y. Now, the problem of mixed compounds, which in this case is that of a mixed conjunction, asks in virtue of which truth property the conjunction of a proposition from D_x and D_y - $D_x \wedge D_y$ - is true (assuming that this conjunction is in fact true). For example, take domain D_x to be the ethical domain, which we take to be governed by the superwarrant truth-property, and take the proposition "Murder is wrong" to be a proposition from this domain. Then, take domain D_y to be the domain of natural sciences, which we take to be governed by the correspondence truth-property, and take the proposition "A gluon acts as the exchange particle for the strong force between quarks" to be a proposition from this domain. Our mixed conjunction then becomes as follows: "Murder is wrong and a gluon acts as the exchange particle for the strong force between quarks". Clearly, this mixed conjunction is not true in virtue of the correspondence truth-property, because the proposition doesn't seem to express a fact that has a clear referent in the actual physical world. On the other hand, the superwarrant truth-property seems to be ruled out as well. This leads Tappolet (2000) to argue that:

> [M]ixed conjunctions needs to be true in a further way. [...] But then each conjunct has to be true in the same way. This is what follows from the truism that a conjunction is true if and only if its conjunctions are true. Hence the question arises again why this further way of being true is not the only one we need. (p. 385)

The problem then seems to be that if the conjunction can not be true in virtue of one of the truth-properties of the conjuncts, then there must be some other way of being true. However, then *this* 'way of being true' appears to be a more generic notion of truth, which is something the alethic pluralist aims to avoid - it goes against the pluralist spirit that holds that there is not *one* generic truth-property.

The problem can be extended to logical pluralism.[22] The relevant question then is what we are to make of logically complex propositions and inferences which contain atomic parts that pertain to domains governed by distinct logical systems. Consider two domains: that of mathematics (DM) and that of macrophysics (DP). Assume for now that the domain of mathematics is associated with

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[22]See for example Steinberger (2017) and Wrenn (2015), taking on cues from Lynch (2008, 2009).

intuitionistic logic and the domain of macro-physics with classical logic. Then, let A be a mathematical proposition and B a macro-physical proposition, both true within their respective domains. Steinberger (2017) then asks:

> What are we to make of A $\star$ B, where $\star$ is a logical connective? There are three possibilities:
> (i) Treat $A \star B$ as if it belonged to DM;
> (ii) Treat $A \star B$ as if it belonged to DP;
> (iii) Treat $A \star B$ as belonging to $DM \bullet DP$,
> where $DM \bullet DP$ is a status that functionally depends on DM and DP while being distinct from both. (p. 16)

In this example, option (i) would be to choose for the weakest logic available, while option (ii) would be to choose for the strongest logic available. Option (iii) is more complex, and appears to hint at another level of generality in which an argument can be valid.

Lynch (2008, 2009) opts for option (i), and he resorts to a principle called *Logical Modesty*:

> Where a compound proposition or inference contains propositions from distinct domains, the default governing logic is that comprised by the intersection of the domain-specific logics in play. (Lynch 2008: p. 139, Lynch 2009: p. 102)

The idea here is that when we are confronted with a mixed compound, we should draw only on principles sanctioned by all logical systems in play. This naturally results in opting for the weakest logical system given that, for example, every theorem in intuitionistic logic is also one in classical logic, but not otherwise. However, this is bad news for the logical pluralist, as this is what Steinberger (2017) refers to as a *downward collapse* into monism. After all, if we take the weakest logical system available on the market, this would seem an apt candidate for the *One True Logic*, because *Logical Modesty* tells us that we should draw *only* on principles sanctioned by all logical systems in play.

There is also option (ii), to go for the strongest logic available. However, there are problems with this solution. For example, say that for some $A \in DM$ we have proved $\neg\neg A$, and take any B such that $B \in DP$. As the logical connective, let us consider $\vee$. We then get a disjunction - $\neg\neg A \vee B$ - of which one conjunct is governed by intuitionistic logic and the other by classical logic. Now, if we opt for the strongest logic available here, then our disjunction is governed by classical logic. But in classical logic our disjunction - $\neg\neg A \vee B$ - is equivalent to $A \vee B$. If we take B to be some known falsehood in DP, then we should infer A. However, A is part of a domain governed by intuitionistic logic, which does not allow for double negation elimination, and we had only proved $\neg\neg A$, and we

had *not* proved *A*.

Finally, there is option (iii). In the literature on logical pluralism, I have not found authors who would endorse this option. In the case of alethic pluralism, this option is strongly reminiscent to that of Edwards (2008). Edwards aims to solve the problem for the alethic pluralist, by taking a mixed conjunction to be true in virtue of both its conjuncts to be true in their own respect, thus making the truth of the conjunction to functionally depend on the truth of both conjuncts. For example, if one conjunct is true in the domain of mathematics and the other conjunct is true in the domain of macro-physics, then the conjunction is true in virtue of both conjuncts being true in the domain of mathematics and the domain of macro-physics respectively. However, Yu (2017) points out that what Edwards' solution leaves us with eventually is a conjunction that is 'logically' true, as he writes:

> Given that "Murder is wrong" and "Charity is good" are ethically true, the answer predicts that the conjunction "Murder is wrong and charity is good" is not ethically true, but rather 'logically' true - just like "There are no true contradictions". (p. 283)

This kind of 'logical truth', then, appears to be the most general truth-property, of which the alethic pluralist rejects its existence. So, this solution faces problems for the alethic pluralist. For the logical pluralist, this solution also doesn't seem to work: if a mixed compound is governed by another more general logic, would this be the *One True Logic*?

Thus, the main problem for local logical pluralism appears to be similar to the main problem for global logical pluralism: how to maintain the pluralist spirit in the face of some generality? It appears to be that some kind of generality is unavoidable, as the problem of mixed compounds shows.

2.3   A Family of Collapse Problems?

It is worth emphasising the similarity between the main problem for global logical pluralism and that for local logical pluralism. What both the collapse problem and the problem of mixed compounds show is a problematic relation between plurality and some kind of generality. The proposal I will develop in the following will explicitly address this problem. The problem is that, despite a pluralist spirit which holds that there are multiple different, equally correct logical systems and therefore no one privileged system, some form of monism appears to be unavoidable. Pluralism threatens to collapse into monism, either upward into the strongest logical system or downward into the weakest logical system. Either way, the system into which it collapses has some kind of privileged status over the over systems.

It is important to emphasise this similarity, for it can point into a direction in which we can enhance logical pluralism. One way of advancing could be to give an account of logic - and hence, logical consequence - that does *not* have such a problematic relation with generality. That is to say that our challenge is to come up with a pluralist theory of logic that does not collapse into monism. However, on the other hand, if some form of monism is *unavoidable*, we will have to provide an account of generality that respects the pluralist spirit.

2.4   Conclusion

In this chapter I have presented the problems facing logical pluralism - both global logical pluralism and local logical pluralism. I have argued that there is a 'family' of collapse problems facing these positions, that indicates a problematic relation between plurality and generality. The problem is that every pluralist position threatens to collapse into monism - either into the strongest logic available or the weakest logic available. Such a collapse appears to be *unavoidable*, at least in the way logical pluralism as been presented thus far. Recall that, as Steinberger (2017) points out, the logical pluralist can be "a heroic character who delivers us from proclivity for fruitless squabbles" (p. 4). However, if both the global logical pluralist and the local logical pluralist are untenable, because they tend to collapse into monism and hence undermine the pluralist spirit of a multiplicity of equally correct logical systems, we need to come up with a solution. In the next chapter, I will present the desiderata and methodology for such a solution.

Chapter 3

Desiderata and Methodology for a Solution

Given the problems for both global and local pluralism presented in the previous chapter, the challenge we are facing is the following. Taking the main problems facing global logical pluralism and local logical pluralism seriously, we need to account for some kind of generality. There appears to be a sense in which logic is 'monistic' in nature, but in order to retain the pluralist spirit, it also has to be 'pluralistic' in a sense. To put it differently: we want to allow for a genuine plurality of logical systems in the sense that there are multiple, equally correct logical systems, but we also want an account of what ties all these systems together as being a logical system in such a way that there is no problematic collapse into monism. So, we need to combine these two seemingly opposing tendencies. These tendencies can be formulated in two desiderata a solution to these problems should meet. In a sense, to paraphrase Lynch (2009) and given that the fundamental concept of logic is logical consequence, we want logical consequence to be both *One* and *Many*.

3.1   Desideratum: *Generality*

The first desideratum is that any account of logical consequence should be able to capture some kind of generality of the concept. In other words, we need an account of the general features of logical consequence. We have seen in the section on the problems facing logical pluralism that some kind of generality - or monism - is unavoidable. So, let me be a bit more precise. The kind of generality I have in mind for the solution to the problems facing logical pluralism, is one that does not entail a problematic relation with the pluralist spirit, and hence with the first desideratum. One way of making this more concrete is to guarantee that the combination of any set of different logical systems does not have a privileged status over these systems separately. Furthermore, every set

of general properties that runs across all different logical systems, and hence all different logical consequence-relations, should not form a threat to the pluralist spirit. That is, it should not allow for a collapse into monism. At the same time, we need an account of what ties all these different logical systems together.

3.2 Desideratum: *Plurality*

The second desideratum is that any account of logical consequence should be able to maintain the pluralist spirit. That is to say that it should be able to retain a multiplicity of (at least two) equally correct logical systems. This desideratum makes it explicit that I aim for a pluralist solution. The kind of pluralism I aim for, then, is one that does not collapse into monism. Thus, the aim is to allow for a set of equally correct logical systems, but also that any combination of each distinct logical system in this set does not yield a more general logical system. At the same time, in order to *be* a genuine logical system, we want each of these distinct logical systems to share some general properties. For more on this, see the *Generality*-desideratum, but it is important to emphasise this here as well. We want a genuine plurality, in the sense that there are several equally correct logical systems that do not collapse into monism, but at the same time we want this plurality to allow for some kind of monism, or at least that allows for an account of what ties all these different systems together. The challenge is to come up with such an account that is satisfying and effective.

3.3 Methodology

Having set the two desiderata I believe the solution to the problem facing logical pluralism should meet, let me elaborate on how I will be going to set up my solution - logical functionalism - and present the main motivations to take my method of choice.

I take the functionalist tradition to provide a well-suited method that allows us to combine the two seemingly opposing tendencies of generality and plurality. For example, functionalism in the philosophy of mind allows for a plurality of different manifestations of mental states, while retaining generality by taking the core function of these mental states to remain stable across the different manifestations. Furthermore, Lynch (2009) has interestingly adopted the functionalist tradition to account for the idea that 'truth' is a functional concept, again taking it to be *One* in the sense that there is a core function of the concept that remains constant, but at the same time taking it to be *Many* in the sense that there are different ways this core function can be performed, or manifested, by different truth-properties. My method will be similar as that of Lynch: in a sense, what Lynch did for truth, I hope to do for logical consequence.

If we take logical consequence to be a functional concept, then what I will have to do is to provide an account of what the *core function* of logical consequence is. I will do this by first providing an adequate pre-theoretic notion of logical consequence, which will comprise what I will call the *core features of logical consequence*, which can be seen as the most fundamental and historically (relatively) undisputed features of logical consequence. The core features will be selected on the basis of canonical prominence on the one hand, but also my own critical assessment on the other hand. By canonical prominence I mean the prominence (in terms of 'weight' or 'occurrence') some feature have been given in canonical discussions on logical consequence. From these core features I will define what can be called the *logical consequence-role.* The idea is that if some relation between the premises and the conclusion of an argument has the property of playing the logical consequence-role, then it can be regarded as a logical consequence-relation.[23]

3.4 Functionalism

Functionalism originated in the philosophy of mind. Roughly, the idea there is that what constitutes a mental state is not its internal make-up, but rather the *function* it has - or the role it plays - within the system of which it is a part. An (oversimplified) example could be the following. A mental state like 'pain' manifests itself in humans by certain neural properties, but by different neural properties in other animals. But, in both organisms it performs the same function, such as indicating something is wrong with the body. So, it is not the internal make-up that constitutes 'pain', as this differs between human and other animals. Yet, in both organisms it has the same function. For any mental state, if it has this function - or: if it performs this role - then it is 'pain'.

One key motivation for functionalism in general is that it allows for multiple realizability. The idea in the philosophy of mind is that a mental state like 'pain' can occur in a wide variety of different entities. Although, for example, humans and other animals are distinctively different, we might still want to say that they can both experience a mental state like 'pain'. Functionalism is able to account for this unity in the concept itself by formulating a common core - the function - while still retaining the plurality in manifestations by taking this function to be able to be realised in multiple different ways. In other words: both humans and other animals can experience 'pain', even though both are distinctively different and even though both manifestations of 'pain' are not identical.

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[23]These steps mimic the method Lynch (2009) puts forth.

3.4.1 Varieties of Functionalism

There are different versions of functionalism in the philosophy of mind. In the following I will make two distinctions between the varieties of theories in functionalism. First, there is a distinction between theories about what constitutes the core function - or: role - of some mental state. Roughly, these different theories can be classified into three categories: machine-state functionalism, psycho-functionalism and analytic functionalism.[24] Second, there is a distinction between theories about the kind of property that constitutes a (particular) manifestation of this role. Roughly, these different theories can be classified into two categories: role-functionalism and realizer-functionalism.[25]

What constitutes the core function?

There are roughly three kinds of theories about what constitutes the role of some mental state. Machine-state functionalism takes the mind to be essentially a Turing machine that computationally produces, on the basis of some input and a fully specified set of rules, some mental state.[26] So, for some entity to possess a mind - and hence, mental states - it should be something like a Turing machine. Psycho-functionalism, on the other hand, takes the best scientific explanation of the mind available, and from that derives the properties that any entity possessing a mind should have. To put it differently, what makes some entity a mental state is that it possess those properties which are specified by the best scientific theory available. Thus, in more general terms not limited to the philosophy of mind, this kind of functionalism can be labelled theoretical functionalism. Finally, rather than the best scientific theory available in the case of theoretical functionalism, analytic functionalism takes certain analytic or conceptual truths of mental states to determine whether some entity is a mental state or not.

I take analytic functionalism to be relevant kind of functionalism for my purposes, so let me now briefly elaborate on this version. As we've seen, analytic functionalism derives the definition of a concept from a theory that consists out of certain analytic or conceptual truths. In the philosophy of mind, analytic functionalism aims to derive the definition of a certain mental state - like 'pain' - from analytic of conceptual truths "about relations of the mental states to inputs, outputs and other mental states" (Shoemaker 1981: p. 104). Thus, we can take the analytic functionalist to abstract away from the specific content of the concept in play, in order to first devise a more universal (in a sense: topic-neutral) set of conceptual truths, under which different particular manifestations could fall. The need for sufficiently *general* conceptual truths then becomes apparent; if we are too restrictive in our choice of which conceptual truths we want to take

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[24]See for example Block (1980) and Schwarz (2013).

[25]See for example McLaughlin (2006) and Hiddleston (2011).

[26]See for example Putnam (1967).

into account, then the explanatory force erodes. On the other hand, if we are too permissive, then we will include manifestations of our concept in play that we don't want to include. The analytic functionalist's methodological solution for this dilemma is to take an ordinary, or *folk*, conception of the concept in play as a starting point.

Other versions of functionalism seem less useful for my purposes here. For example, the application of machine-state functionalism seems to be too much limited to the philosophy of mind. As we've seen, this version of functionalism puts the emphasis on a computational process like that of a Turing-machine to determine whether some entity has a mind or not. Now, of course logic could also be seen as a computational process[27], but we would like to say more than that. I don't see how machine-state functionalism could help us any further.

I maintain that theoretical functionalism is also not the relevant kind of functionalism. First of all, it is unclear to me which scientific theory we should take to decide over our logical concept of logical consequence. Recall that psycho-functionalism (as theoretical functionalism in the philosophy of mind) takes the best scientific explanation of the mind available, and from that derives the properties that any entity possessing a mind should have. Outside of the philosophy of mind, we could say that theoretical functionalism takes the best scientific theory available, of a specific scientific field, to determine the properties of a concept that is somehow connected to this specific scientific field. In the philosophy of mind, this scientific field might be cognitive psychology. For logic, however, I am not sure which scientific field we should look at to determine the properties of a logical concept such as logical consequence. Interestingly, Kouri Kissel and Shapiro (2017) might be seen as endorsing some form of theoretical functionalism, taking mathematics to be the relevant scientific field that is to have to final verdict on logical concepts.[28] They take it to be that our best mathematical theories should tell us which logical systems we ought to adopt. However, I maintain that sticking to logic by trying to find analytic and conceptual truths within this field of science to me seems more natural to do than taking another scientific discipline to decide on our logical theories. It is unclear to me why specifically mathematics - in the case of Kouri Kissel and Shapiro - and not some other scientific discipline should be the theory to decide on our logical theories, and I don't know how are we even to decide which scientific theory other than logic we should ascribe this role.

Second, even if we would choose to adopt theoretical functionalism and take the relevant scientific theory to be the metatheory of logic, then what we would then get are studies of concepts in a *pure* logic, rather than an *applied* logic.

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[27] See for example my discussion on the formal features of logical consequence in section 4.1.1.
[28] See for example p. 20. See also Shapiro (2014), p. 7.

What I am interested in is not merely the properties of a logical system as a well-defined mathematical structure with a proof-theory and model-theory, but rather in the function logical consequence in relation also with certain epistemic goals. Thus, the version of functionalism relevant for my purposes here should allow for such epistemic goals, but there is no place for these in the metatheory of logic. Thus, I maintain that not only theoretical functionalism as taking an extrinsic scientific theory to decide on logical concepts seems implausible, but that the same goes for theoretical functionalism as taking an intrinsic scientific theory to take up this role.

What kind of property constitutes a manifestation?

There are roughly two theories about what kind of property it is that constitutes a manifestation of the role in play. For example, if we're discussing the pain-role, what kind of property is the property of performing the pain-role? On the one hand, realizer-functionalism holds that a functional theory about pain merely provides definite descriptions that allow us to locate the manifestations of the pain-role in lower-level properties. Thus, according to this version, it is for example C-fiber stimulation - the lower-order property - that performs the pain-role in humans. On the other hand, role-functionalism holds that it is the higher-order property of having a property that performs the pain-role that counts as a manifestation of the pain-role.[29] Thus, this view considers pain to be a higher-order property.

I take it to be that role-functionalism seems better capable of preserving unity among different manifestations, for it allows the higher-order property in play to be a rigid designator whose reference remains constant in all possible manifestations. In other words, it seems more capable of allowing us to take one manifestation to be of the same kind as another manifestation. The main problem for logical pluralism that I have identified is exactly that there is a problematic account of plurality, such that it seems to conflict with some kind of generality. I've argued that in order to improve logical pluralism, we have to give a satisfying account of generality, such that there is no problematic relation between plurality and generality. Role-functionalism appears to be capable of providing such an account.

3.5   Conclusion

In this chapter I have presented the desiderata and methodology for a solution to the problems facing logical pluralism, as presented in the previous chapter. The solution, I maintain, should meet the desiderata *Generality* and *Plurality*. A solution that would meet these desiderata could allow logical consequence to be

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[29]See for example McLaughlin (2006), p. 45.

both *One* and *Many*, which might effectively solve the problems facing positions in logical pluralism currently on the market. As a methodology to develop such a position, I have argued to adopt functionalism. More specifically, I take analytic functionalism and role-functionalism to be the most useful kinds of functionalism for my purposes here.

Chapter 4

Logical Functionalism

In the foregoing I have presented logical pluralism and its problems, and the desiderata and methodology of a functionalist solution to these problems. Having laid out the groundwork, in the following I will set up logical functionalism. My main task is to identify and define the logical consequence-role. I aim to do so by extracting from a pre-theoretic notion the core features of logical consequence. These core features can then be taken to constitute the logical consequence-role.

4.1 Core Features of Logical Consequence

First, I will aim to identify the *core features* of logical consequence, which I will distil from the pre-theoretic notion of logical consequence. Functionalist theories often resort to a *folk* conception of the concept in play.[30] However, in the case of such a technical concept as logical consequence, it is unlikely that there is a conception of this concept that is widely shared among - and intuitive for - people in general. Therefore, it might be better to identify the pre-theoretic notion of logical consequence that logicians, mathematicians and philosophers have developed and discussed over time. That means that our pre-theoretic account will consist out of the most important, core features that have been ascribed to logical consequence over the history of philosophy and logic. Such features I take to have *canonical prominence*: canonically prominent features are those that are the most prominent in theories of logical consequence that have been developed over the course of time. They may not be shared by *literally* everyone, but at least they are shared by a considerable amount of respected thinkers. Yet, even in the case of canonical prominent features, there are debates as to what they exactly amount to, so there is often the need to make choices between competing (details of) features. Therefore, the (details of the) features presented here as being the core features of logical consequence are chosen not *merely* on the basis of canonical prominence, but also on my

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[30]See for example Lynch (2009).

own critical assessment and evaluation of these features. In the following, I will take the core features of logical consequence to be *formality*, *normativity* and *necessity*.

4.1.1 Formality

Logical form is a feature that has historically been recognised to play an important role in the concept of logical consequence.[31] Dutilh Noveas (2011) draws a helpful distinction between the various notions of formality that have been ascribed to logic over the course of history. Roughly, following Dutilh Novaes (2011), there are two kinds of formality: formal as pertaining to *form* and formal as pertaining to *rules*. The main difference, according to Dutilh Novaes, is

> a normative import and an emphasis on doings and actions. As pertaining to forms, the formal is essentially related to what something *is*; as pertaining to rules, the formal is essentially related to what something (or someone) *does*, and is generally characterised by a strict adherence to (usually explicitly formulated) rules, norms or laws. (p. 321)

I take both kinds of formality to be relevant to logical consequence. Let me explain.

First, let me consider formality as pertaining to forms. From this 'family' of variations of formality I take the main feature to be that being formal corresponds to abstraction from content. That is to say that logic is not concerned with a *particular* case or argument, but rather with a more *universal* framework that incorporates all the particular cases or arguments. Dutilh Novaes (2011) writes that "what is distinctive about logic is that it provides patterns of correct reasoning valid across different disciplines, that is, its topic-neutrality" (p. 314). A different way to articulate this idea comes from Boolos (1975), who compares logic with the natural sciences (p. 517): where sciences such as astronomy have a specific subject-matter - in the case of astronomy being celestial objects and phenomena related to these - logic has no such specific subject matter. Rather than putting the emphasis on a particular subject or phenomena, logic puts the emphasis on the general form or structure of an argument, whatever the subject-matter is.[32]

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[31] Hanson (1997) ascribes the emphasis on logical form to Tarski, Church and Quine (p. 375). Furthermore, Beall and Restall (2006) write that "[f]ormality is one feature of deductive logic that is almost invariably taken as distinctive" (p. 18).

[32] This way, one could argue that the subject-matter is in fact this form or structure of an argument and the properties related to these. I consider this to be essentially a terminological matter. As long as it is agreed that *if* logic has a subject-matter, *then* it is this kind of subject-matter, we have resolved that issue.

Let me briefly elaborate on the related notion of topic-neutrality. Formality and topic-neutrality are, as we can see in the above, intimately connected. For example, Glanzberg (2015) writes that

> [i]t is a common idea that to satisfy formality, the logical constants have to meet certain standards. The main standard is being 'topic-neutral', or not specifically about the particular things in the world, but only its general or 'formal' structure. (p. 96)

In other words, in order for logic to be formal, it (at least) needs to abstract away from the specific content of an argument. The idea is that logic is not about this specific content, but rather about the structure of the argument. Such a structure can be seen as a formal feature of the argument - it is the form of the argument - and this structure becomes explicit once we abstract away from content and do not consider any specific topic. Therefore, I maintain that by taking logical consequence to be formal in the sense of abstraction from content, I have also taken logical consequence to essentially topic-neutral.[33]

Now, consider formality as pertaining to rules. For this 'family' of variations on the notion of the formal, Dutilh Novaes (2011) distinguishes three versions: the formal as computable, the formal as pertaining to regulative rules and the formal as pertaining to constitutive rules. I maintain that in order to give logic its normative force, we should at least take two of these versions as relevant to logical consequence, namely the formal as computable and as pertaining to constitutive rules. First, one way of interpreting the formal is to interpret it as computability in the sense that one could define a formal system as a recursively axiomatized theory, or in the sense that one could define a formal language as a language with strict formation rules such that its syntax is absolutely exact. This way of defining the formal comes closest to the idea of a strict adherence to a fixed set of rules and I take this to be crucial in order to establish the other version of the formal as pertaining to rules, namely that which takes the formal to be pertaining to constitutive rules. The idea here is that such constitutive rules are taken to provide *conditions of possibility* for the existence of some kind of phenomenon. These constitutive rules allow us to distinguish right from wrong - if something doesn't adhere to these rules, then it is an improper kind of the phenomena in play, or not such a phenomena at all.

Now, if we combine this with the idea of formal as computable, then we get

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[33] As Dutilh Novaes (2011) writes, Kant rejects the characterisation of logic as topic-neutral. He puts emphasis on the idea that logic is completely *a priori*. I will not engage in an exegesis of Kant's idea, but rest content with the observation of Dutilh Novaes (2011) that "this particular aspect of his characterisation did not prevail. Indeed, later authors such as Frege will reject Kant's dismissal of logic as a tool to produce knowledge, and the general idea that topic-neutrality as a key feature of logic is to be found in several influential authors of the twentieth century" (p. 315).

a clearer picture of the kind of formality as pertaining to rules that I have in mind: I take logical consequence to be formal as computable, and that the rules that come along with this interpretation of the formal are essentially constitutive rules in the sense that if an argument does not adhere to a strict set of rules, then it is not a proper, or correct, logical argument, but perhaps an *informal* or incorrect argument.[34] In that sense, these strict set of rules have what I call *constitutive force*: they are capable of distinguishing a proper formal, logical argument, from an improper or incorrect one.

To summarise: I take logical consequence to be formal in the sense that it abstracts from content *and* that it is formal in the sense of pertaining to strict, regulative rules that have constitutive force.

4.1.2 Normativity

The idea that logic is normative is deeply ingrained in the tradition of what we take logic to be. To put it boldly: there is a widely shared intuition that logic prescribes the norms for how we ought to think or reason. Arguably the most common intuition these days is to argue that logic is normative for reasoning.[35] That is, logic is taken to show the *correct* ways of reasoning. But what kind of normativity is in play here? And what does this kind of normativity entail?

The relevant notion of normativity for logical consequence I take to be one which cannot be considered irrespective of epistemic goals. Perhaps this is a deviation from the canonical way of taking logical consequence to be normative, but I believe there are good arguments to do so. The worry with merely stating that logical consequence has to be normative is that one could stipulate any set of norms one likes. This way, every consequence relation can trivially be taken to be prescribing some set of norms. Yet, we don't want to prescribe just *any* set of norms, but the appropriate norms that apply to logical consequence. The norms that a consequence relation should prescribe are not about how one ought to dance, or how one ought to ride a bike. Rather, they are about how one ought to *reason*. But reasoning is essentially an epistemic activity. Thus, it would make sense to maintain that the norms logical consequence prescribes are essentially epistemic norms. This view locates the normative element not merely in the logical consequence-relation, but rather in the epistemic goals of deductive reasoning that the logical consequence-relation should contribute to. Such epistemic goals of deductive reasoning I take to be related to attaining

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[34]In a sense, I replace the Kantian laws of thought with a kind of computability. I believe this resonates with Dutilh Novaes (2011) writing that "if one is sympathetic to the Kantian idea of constitutive rules of thought, the exact *content* of such rules still remains to be determined, and it is by no means uncontroversial that they should coincide with the rules of logic as traditionally construed" (p. 329).

[35]See for example MacFarlane (2004).

some kind of informative truth from a set of premises, potentially accompanied by further constraints such as the ability to provide a proof of how this truth has been attained[36] or a constraint that demands some kind of relevance in the form of the variable sharing principle.[37] These constraints are used to specify the epistemic goals: you want to obtain a specific kind of knowledge, like one that allows you to really understand why something is the case, and the provability constraint specifies this and allows you to achieve this specific epistemic goal.

There is a way to make these considerations presented above more precise. In order to spell out the connection between principles of logic and the norms of reasoning, one could formulate so-called *bridge principles*.[38] Steinberger (forthcoming) argues that the following bridge principle (∗) is the correct bridge principle:

> If S believes that $A, ..., A_n \models C$, and S considers C or has subjective reasons to consider C, then S has reasons to (believe C, if S believes all of the A_i)

I take the epistemic goals of deductive reasoning to be (one of) those subjective reasons that gives S reasons to believe C, if S believes all of the A_i. They are subjective, because there can be different goals in different contexts. They provide S reasons to believe C, if S believes all of the A_i, because the epistemic goal enforces a specific logical consequence-relation, and if S has this specific epistemic goal then this is a compelling reason to endorse this logical consequence-relation, for it can contribute to achieving this epistemic goal.

Now, (∗) is perhaps a bit weaker than my claim: saying that S *has reasons* to believe C, if S believes all of the A_i might be something weaker than saying that respective to certain subjective reasons (such as epistemic goals) S *ought* to believe C, if S believes all of the A_i. Steinberger (forthcoming) seems to opt for the 'weaker' version for it accommodates "exceptional Preface-like situations in which our standard logical reasons are overridden by competing epistemic considerations" (p. 19).[39] In other words, allowing for a bit more flexibility enables us to also deal with these exceptional Preface-like situations, which seems to

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[36]Which would be the case in a constructive logic such as intuitionistic logic.

[37]Which would be the case in relevant logic.

[38]See MacFarlane (2004).

[39]Referring to the Preface-paradox, whereby one has good reasons to belief that a large set of non-trivial propositions $p_1...p_n$ is true, while at the same time one has good reasons to consider the conjunction of $p_1...p_n$ to be false. For example, I have meticulously thought-through every proposition in this thesis and lets assume I only write down these propositions when I consider them (for good reasons) to be true. At the same time, I am very much aware of my own fallibility, and for this reason I am also sure that I've made a mistake somewhere. Therefore, although I am confident in putting forth each separate proposition, I am not so confident about the conjunctions of all these propositions. This is paradoxical for the conjunction of $p_1...p_n$ is an obvious logical consequence of $p_1...p_n$.

be an advantage over a more 'strict' version. I am not sure if there really is a tension between my stricter version of Steinberger's weaker version. In fact, we might take it that I am pointing out to *the most compelling reason* to believe C, if S believes all of the A_i. Thus, I maintain that my considerations on the relation between the normative status of logical consequence and its presumable epistemic nature fit well with the bridge principle put forth by Steinberger. In fact, it seems to me that it is precisely such considerations that lead him to endorse ($*$) as the most appropriate bridge principle. For example, he writes:

> [L]ogic, on the whole, does not require us to have any particular beliefs (except towards obvious logical truths, perhaps). Rather, it prohibits certain constellations of doxastic attitudes. When we reason we must negotiate these global constraints on our system of beliefs with other epistemic norms - local norms that guide us in determining whether a given proposition is to be believed. This element of negotiation is well captured by the wide-scope reading. (p. 18)

And further on he writes, in arguing how ($*$) fares against the Preface-paradox:

> [M]y (logic-based) reasons for believing the conjunction [in the preface paradox] can be trumped by the weightier demand for epistemic humility: I am bound to have made a mistake somewhere along the way. Thus, the *reasons* operator allows our qualitative principle to accommodate exceptional Preface-like situations in which our standard logical reasons are overridden by competing epistemic considerations. (p. 19)

In other words, logical consequence does not have normative force irrespective of certain related epistemic norms. For example, if agent S's epistemic goal is to get a deeper understanding of why something is the case, then S has reasons to consider a conclusion that is valid according to a constructive logic such as intuitionistic logic, but not one that is valid classically. So, the epistemic goal in play determines what logical system is appropriate, namely the system that fits this particular epistemic goal.

First, consider a case where S has the epistemic goal of gaining deeper understanding, by which I mean that the goal is to get a clear understanding of how something is the case for example by providing a proof. Furthermore, say that S has proven $\neg\neg p$. What keeps S from making the (classically valid) inference to p? The answer is the epistemic goal. As long as S is not able to provide a proof for p, S has not gained a deeper understanding of how p is attained. Thus, as long as S has this specific epistemic goal, it will need to endorse intuitionistic logic.

Now, consider a case where the epistemic goal is to only make inferences that

'stay on topic'. Say than that we have proven p. What keeps S from making the (classically valid) inference to $(q \vee \neg q)$? The answer again is the epistemic goal. As the argument shifts the topic, and our epistemic goal was exactly to 'stay on topic', such an inference is not valid.

In all these situations, S then has reasons to believe a conclusion C, where C is valid according to the logic enforced by the specific epistemic goal, if S believes all of A_i. I maintain that given an epistemic goal, the *one* appropriate logical consequence-relation manifests itself. Then, if the agent agent S has an epistemic goal for which intuitionistic logic is most appropriate in obtaining this goal, then if S does *not* reason according to intuitionistic logic, S goes wrong and thus makes a mistake. The crucial insight here is that we cannot consider a logical consequence-relation without taking an epistemic goal in mind. The way these epistemic goals operate all naturally fit the $(*)$ bridge principle.

Let me elaborate more on what I mean when I say that the epistemic goal 'enforces' the appropriate logical consequence-relation, and - equivalently - that given an epistemic goal, the *one* appropriate logical consequence-relation manifests itself. The idea is that every *pure* logical system - in Priest's (2006) definition "a well-defined mathematical structure with a proof-theory, model theory, etc." (p. 195) - has certain features. Some of these features might be perfectly suited for obtaining a specific epistemic goal. For example, the feature of intuitionistic logic (as a pure logic) that demands a constructive proof is perfectly suited for the epistemic goal of obtaining a deeper understanding of *why* or *how* something is the case. After all, if you have provided a constructive proof, then you get a clearer insight in *why* or *how* some truth has been attained. So, this feature of intuitionistic logic is perfectly suited for this specific epistemic goal. Similarly, the variable sharing principle in relevant logic (as a pure logic) appears to be perfectly suited for the epistemic goal of 'staying on topic', or gaining relevant knowledge. Now, not every pure logic might have feature that make it relevant for some specific epistemic goal. After all, one is free to devise a well-defined mathematical structure with a proof-theory and model theory, without taking into account any epistemic goal. When a feature (or certain features) of this pure logic are suited to contribute to achieving some epistemic goal, then the pure logic becomes an applied logic - applied in connection to this specific epistemic goal. So, what I have been considering in the above - and will consider in the following - are *applied logics*. This is reasonable simply because this is where the rivalry between different logical systems arises. If we say that some relation between the premises and the conclusion of an argument is classically valid while intuitionistically invalid, irrespective of any epistemic goal, then we are talking about pure logics. But, at least for my purposes here, this is not the interesting issue. After all, "[p]lurality is an issue of substance only if one is asking about *applied logics*" (Priest 2006: p. 195).

Now, to summarise this section: I take logical consequence to be normative in the sense that it sets forth norms that contribute to achieving at least *some* epistemic goals of deductive reasoning and that can be formulated in bridge principle (∗).

4.1.3 Necessity

Necessity has been recognised to be a core feature of logical consequence for over two millenia, from Aristotle, to Leibniz, to Tarski.[40] The idea behind this is that we want logic - and hence, logical consequence - to necessitate that *if* the premises of an argument are true, then, in any case, so is the conclusion. Beall and Restall (2006) write that necessity is one of the oldest features determining logical consequence (p. 14). In their words, "[t]he truth of the premises of a valid argument necessitates the truth of the conclusion of that argument" (p. 14). But, what kind of necessity is the relevant one here? There are different ideas on this.[41]

One prominent interpretation of the notion of necessity takes it to be *truth in all possible worlds*. In fact, taking necessity to be truth in all possible world appears to be quite common ground, so it has considerable canonical prominence.[42] However, this seems to imply merely *one* kind of necessity, namely one where under any conditions, in any possible world or situation, *if* some premises of a valid argument are true, then so is the conclusion. In my discussion of normativity, however, I have argued that we need to consider logical consequence respective to an epistemic goal. Different epistemic goals, I have argued, give us different logical consequence-relations and hence different ways in which an argument can be valid. In other words: in a context with one epistemic goal, a conclusion might *not* necessarily follow from the premises, while in another context with a different epistemic goal, it might be. For example, for an epistemic goal that enforces classical logic, p necessarily follows from $\neg\neg p$, while for an epistemic goal that enforces intuitionistic logic this is *not* the case. Thus, there appears to be different kinds of necessity in play here.

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[40]See Hanson (1997).

[41]Beall and Restall (2006) write that "it is notoriously difficult to characterise the notion of necessity at work here. This difficulty is not made any easier by the fact that many people have attempted to reduce necessity to a notion defined in terms of logic such as inconsistency" (p. 15). Hanson (1997) writes that "[m]any philosophers think that a special kind of logical necessity (sometimes called 'narrow logical necessity') is required, and the literature abounds with distinctions between this kind of necessity and metaphysical, causal, and broadly logical necessities" (p. 380).

[42]See Beall and Restall (2006), who write that "[t]he characterisation of the necessary as that which is true in all possible worlds strikes many as a natural way to begin to put some flesh on these bones" (p. 15).

To accommodate this plurality, I will endorse a restricted notion of necessity, restricted by the epistemic goal in play. That means that I maintain to endorse a notion of necessity which says in all possible worlds restricted to the relevant epistemic goal, *if* some premises of a valid argument are true, so is the conclusion. Following Beall and Restall (2006), we can then interpret 'possible world' in different ways. For example, in a context with an epistemic goal in play that corresponds to intuitionistic logic, we can take possible worlds to be stages of construction. In a context with an epistemic goal in play that corresponds to relevant logic, we can take possible worlds to be situations. For a context with an epistemic goal that to corresponds to classical logic, we can take possible worlds to be just that.

This notion of necessity is reminiscent of the way Beall and Restall (2006) formulate it. Recall that they take the *Generalized Tarski Thesis* to be the general account of logical consequence, whereby the unsettled notion of a *case* can be interpreted in different ways: cases as possible worlds, cases as constructions and cases as situations, to name a few. For Beall and Restall, then, necessity comes down to truth in either all *possible worlds*, all *constructions* or all *situations*, depending on how a case is specified. Necessity, thus, is truth in all cases, under a certain specification of *case*. I follow Beall and Restall in this, although my formulation might be slightly different: necessity, in my case, is truth in all contexts selected by a specific epistemic goal.

To summarise: I take one core feature to be that logical consequence *necessitates* the conclusion of an argument such that in all contexts with some specific epistemic goal, if the premises of an argument are true, then so is the conclusion.

4.2   The Logical Consequence-Role

The core features of logical consequence that I have presented in the above are taken to indicate the features that a relation between the premises and the conclusion of an argument must have in order to have the property of being a logical consequence-relation. To have the property of being a logical consequence-relation, the relation must perform what I call the *logical consequence-role*. We can then define the logical consequence-role as follows:

> For every relation R between the premises and the conclusion of an argument Δ, R plays the logical consequence-role if and only if:
>
> (i) R is formal in the sense that it abstracts from matter and in the sense of pertaining to strict, regulative rules that have constitutive force.
>
> (ii) R is normative in the sense that it prescribes norms that con-

tribute to achieving an epistemic goal of deductive reasoning and that can be formulated in bridge principle (∗).

(iii) R necessitates the conclusion of an argument such that in all possible worlds restricted by the relevant epistemic goal, if the premises of an argument are true, then so is the conclusion.

This definition of the logical consequence-role is not to be equated with necessary and sufficient conditions of logical consequence, but there is a distinction to be made between the conditions formulated in the definition. (i) and (iii) can be considered to be the result of a conceptual analysis: they specify necessary conditions for some relation to be logical consequence-relation. These conditions need to be combined with (ii), In other words: what (ii) does is that it expresses the idea that given an epistemic goal, one ought to use the logical consequence-relation this epistemic goal enforces. What logical consequence then *does* is that it prescribes norms of how one ought to reason, or which arguments one ought to consider to valid respective to this epistemic goal.

These considerations also show that we take logical consequence to be a higher-order property on the relation between premises and the conclusion of an argument, hence putting logical functionalism at the side of role-functionalism. For any argument, there is a relation between the premises and the conclusion of this argument. The property of being a logical consequence I then take to be a property of a given consequence relation, if and only if this relation plays the logical consequence role. The property of being a logical consequence-relation can then be taken to rigidly designate a single property which can be defined as follows:

> The property of being a logical consequence-relation $=_{df}$ the property of performing the logical consequence-role.

To put it differently:

> For all F, F is the higher-order property of being a logical consequence-relation, if and only if, for all R, R has F if and only if R performs the logical consequence-role.

Let me consider an example. Say that we have an argument Δ with premises $P_1...P_n$ and a conclusion C. In any case, we could say that there is *some* relation R between the premises and the conclusion. I am interested, however, specifically in the logical consequence-relation. When can we say that this relation R is a logical consequence-relation? The answer is: as long as this relation plays the logical consequence-role, i.e. that it has the property of performing the logical consequence-role. By the definition of the logical consequence-role, that means that (i) this relation should be formal in the sense specified in the set of core features of logical consequence, (ii) given some epistemic goal this relation should

be such that reasoning in accord with the relevant logical system is the best way of moving us towards this goal, thereby prescribing the norms of how this goal can be achieved, and (iii) this relation should necessitate the conclusion based upon the premises such that in all possible worlds restricted by the relevant epistemic goal, if the premises of an argument are true, then so is the conclusion. In the next section, I will consider some specific arguments that illustrate how this works, thereby also showing the multiple realizability of the logical consequence-role.

4.3   Multiple Realizability

In the above I have defined the logical consequence-role, but I have not yet shown on how this supports logical pluralism. That is, I still have to argue for the logical consequence-role to be *multiple realizable*: the logical consequence-role can be *manifested*[43] in various different, equally correct ways.

Functionalism is perfectly suited to allow for a plurality of manifestations. With my definition of the logical consequence-role in hand I can show that there are multiple different ways of performing this role. I will consider three possible manifestations: the classical logical consequence-relation, the intuitionistic logical consequence-relation and the relevant logical consequence-relation. What I will need to show is that in all these logical systems (or at least two), the consequence relation is necessary, formal and normative in the senses specified in the definition of the logical consequence-role. That is, I will need to show that the relation between the premises and the conclusion in all these cases has the property of *being a logical consequence-relation.*

The examples I will provide in the following illustrate how to determine if some relation between the premises and the conclusion of an argument is a genuine logical consequence-relation. Let me first briefly elaborate, in general terms, how this process goes. First of all, I will assume a plethora of *pure logics*; in Priest's terms these are well-defined mathematical structures with a things like a proof-theory and a model theory. In the following examples, I will take the established systems of classical logic, intuitionistic logic and relevant logic, of which it appears to me obvious that we can take all of these to be formal, in the sense that they abstract from matter and adhere to strict, regulative rules, and hence that they satisfy condition (i) of the logical consequence-role. Thus, I will focus merely on whether they also adhere to conditions (ii) and (iii) - normativity and necessity. I will do so by considering three epistemic goals. First, I will consider 'simple' truth-preservation. Second, I will consider truth-preservation with the additional constraint of provability. Third, I will consider truth-preservation

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[43]'Manifested' (or: 'manifestation') and 'realized' (or: 'realisation') can here be used interchangeably.

with the additional constraint of relevance.

In general, this is how I think it should work. We have, as I have assumed, a plethora of *pure logics*. A pure logic satisfies the formality feature as defined in the logical consequence-role, but not necessarily the normativity and necessity feature, given that to satisfy these features one should consider an epistemic goal in relation to which we evaluate whether the relation between the premises and the conclusion of the argument in play contributes to achieving this specific epistemic goal. The kind of necessity in play follows from the epistemic goal. I've argued that we can not consider the normativity of logical consequence irrespective of an epistemic goal. The manifestations that I will present in the following are the manifestations on the level of *applied logics*. As I have stated earlier, this is the interesting level given that, pace Priest, plurality is an issue of substance only on this level. Thus, in the following examples, what I do is I assess a specific argument within a specific context. A context W is defined as a combination of a specific agent S, at a specific time t, at a specific location l and - most importantly for my purposes - with a specific epistemic goal EG. In a definition: $W = < S, t, l, EG >$. For my purposes, S, t and l don't do any work; only EG is relevant, because it is EG that enforces the appropriate logical consequence-relation. The aim then is to show that there are different kinds of logical consequence-relations respective to different epistemic goals. What ties these different manifestations together is that they all perform the logical consequence-role. That is, and I will show that this is so, the relation between the premises and the conclusion of the arguments in the following example all have the property of being a logical consequence-relation.

4.3.1 Classical consequence

First of all, consider the simple argument Δ from p to $q \vee \neg q$. Furthermore, let us assume that our epistemic goal is truth-preservation, without any additional constraints. Now, we need to establish whether the relation R between the premises and the conclusion of Δ is a logical consequence-relation. In order to do so, we need to see whether R plays the logical consequence-role, and thus whether it satisfies the conditions specified in the definition of the logical consequence-role.

First, let me consider (ii): R is normative in the sense that it sets forth norms that contribute to achieving at least *some* epistemic goal of deductive reasoning. In this example, we have assumed that our epistemic goal is truth-preservation without any further additional constraints. Given that this is our epistemic goal, we ought to accept the conclusion of Δ if we are to believe the premise. Why? Simply because our aim was to preserve truth without any additional constraints, which is exactly what the relation R in Δ does. If we dismiss the conclusion, then we must have gotten a different aim and hence we must have been working

in a different context.[44]

Second, let me consider (iii): R should necessitate the conclusion of an argument such that the conclusion is true in all possible worlds restricted to the relevant epistemic goal. In other words, it should be impossible for the premises to be true and the conclusion to be false in all possible worlds restricted to the relevant epistemic goal. Thus, if we assume that p, then it should be impossible for $q \vee \neg q$ to be false. In fact, since we are dealing with truth-preservation without any additional constraints, there is no possible world in which $q \vee \neg q$ is false - in other words: it's true. Thus, there is no way that the premise p is true, but the conclusion $q \vee \neg q$ is false. Hence, R necessitates the conclusion and satisfies (iii).

This is enough to have established one manifestation of the logical consequence-role. In the following, we will see if there could be manifestations different manifestations than the one sketched above.

4.3.2 Intuitionistic consequence

Second, consider again the argument Δ from p to $q \vee \neg q$, but let us now assume that our epistemic goal is to gain a deeper understanding of how some truth has been attained. We could say that we are then after truth-preservation with a further additional constraint, namely that of provability. In the previous, we have already established that R is *a* logical consequence-relation. However, this does not need to say that we can employ it for every epistemic goal. In this example, our epistemic goal is more strict - we don't only want to know what is the case, but also *how* this is the case by providing a proof. Therefore, we can't employ R here. Let me first consider why this is the case, before I will present another manifestation.

First of all, with our provability-constraint on truth-preservation, $q \vee \neg q$ does not hold in every possible world restricted to our epistemic goal. For this example, we can interpret possible worlds now as stages of construction. That is, $q \vee \neg q$ will *only* be true if at some stage we have constructed a proof that either q or $\neg q$ is the case. We could easily imagine a stage of construction where we did not (yet) come up with such a proof, and therefore in these stages $q \vee \neg q$ does not hold. Thus, R in this case does *not* necessitate the conclusion of Δ, for we can imagine a stage without a proof of the conclusion. This already is enough to disregard R in this case - as it does not satisfy (iii) - but we could simply add that R does not contribute to our epistemic goal, for it doesn't require a proof

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[44]Assuming, of course, that one aims to achieve the epistemic goal, which I take to be essential to rationality and deductive reasoning. So, we are - now and in the following - assuming *rational* agents.

of the conclusion and I have assumed that when the epistemic goal is to gain a deeper understanding of why or how something is the case, then one (perhaps, the most obvious) way to do this is by providing a proof. Again, this is to show that one logical consequence-relation can be a genuine manifestation of the logical consequence-role with respect to one epistemic goal (in one context), but not with respect to another epistemic goal. This is *not* to invalidate R as a logical consequence-manifestation, for we have already established in the previous subsection that it is.

Now, let me consider another manifestation of the logical consequence-role, namely one that suits our epistemic goal of gaining a deeper understanding of why something is the case - that is, our epistemic goal that contained the requirement of provability. Consider the argument Δ', from $p \to \neg\neg q$ and p to $\neg\neg q$. Now, we need to establish whether the relation R' between the premises and the conclusion of Δ' is a logical consequence-relation.

First, let me consider (ii): R' is normative in the sense that it sets forth norms that contribute to achieving at least *some* epistemic goal of deductive reasoning. Our epistemic goal in this case was to gain a deeper understanding of how some truth has been attained. If this is the epistemic goal, one can merely assert the conclusion if a proof has been provided. In the case of Δ', we can only conclude $\neg\neg q$ if we have a proof of this. Given that we assumed that we have a proof of $p \to \neg\neg q$ and p, we then have a proof of $\neg\neg q$. To be clear, we *don't* have a proof of q. Therefore, we *can't* conclude q. If we would conclude q in this context, then we would go wrong, because we would violate the provability constraint.

Second, let me consider (iii): R' should necessitate the conclusion of an argument such that the conclusion is true in all possible worlds restricted to the relevant epistemic goal. Again, we interpret possible worlds now as stages of construction. If we have a proof of $p \to \neg\neg q$ and a proof of p, then for every further stage after having proven this, we have that $\neg\neg q$. There is no way for there being a proof of the premises, but not of the conclusion. Thus, we have that (iii) holds for R'.

This is enough to have established another manifestation of the logical consequence-role. Let me present one more.

4.3.3 Relevant consequence

Third and finally, consider once more the argument Δ from p to $q \vee \neg q$, but let us now assume that our epistemic goal is to 'stay on topic'. That is, we are dealing with truth-preservation with the further additional constraint of relevance (or that of the variable sharing principle). Although we have already established

that R is a logical consequence-relation, we ought *not* to employ it here, since our epistemic goal is not only to know what is the case, but we also want to 'stay on topic'. Let me again first consider why we then ought not to employ R, before I will present another manifestation.

In this case, we can't conclude $q \vee \neg q$ from p because the conclusion is unrelated to the premises. In other words: the conclusion does not share any variable with the premises. In that sense, we can say that the conclusion is not relevant and that we have not 'stayed on topic', but rather diverted the topic. If we now interpret possible worlds as situations, then we can say that the conclusion is *not* true in every situation in which the premise is true, as from a situation in which the premise p is true we simply can't say anything about the conclusion $q \vee \neg q$, because it is not relevant. Hence, we have violated (i). Furthermore, R simply did not contribute to achieving our epistemic goal, hence it also violated (ii).

Now, consider the argument Δ'' from $p \rightarrow \neg\neg q$ and p to q. We need to establish whether the relation R'' between the premises and the conclusion of Δ'' is a logical consequence-relation, and more specifically, one that suits our epistemic goal - truth-preservation with the further constraint of relevance.

First, let me consider (ii): R'' is normative in the sense that it sets forth norms that contribute to achieving at least *some* epistemic goal of deductive reasoning. In this example, we have assumed that our epistemic goal is truth-preservation with the further additional constraint of relevance. By the variable sharing principle, R'' contributes to this epistemic goal, and hence it satisfied (ii).

Second, let me consider (iii): R'' should necessitate the conclusion of the argument such that the conclusion is true in all possible worlds restricted to the relevant epistemic goal, where we can now interpret possible worlds as situations. In fact, it does in a similar way as we have seen in the classical case, given that we can take any relevantly valid argument to be classically valid. Situations are, just like possible world, complete and consistent, hence truth-preservation over all situations guarantees truth-preservation over all possible worlds.

This is enough to have established another manifestation of the logical consequence-role. I have now presented three different manifestations of the logical consequence-role. Thereby, I have shown that the logical consequence-role can be realised in multiple different ways. In effect, I have shown that logical functionalism as I have presented it here implies logical pluralism, as I have been able to present different logical consequence-relations by making use of the logical consequence-role.

4.4 Conclusion

In this chapter I have developed logical functionalism. I have done so by first defining the core features of logical consequence, that I have distilled from the pre-theoretic notion. These core features I take to be *necessity*, *formality* and *normativity*. The first two features can be seen as the result of a conceptual analysis, the third feature - normativity - I have taken to be a essentially functional one. A special emphasis has been put on this notion of normativity: I have argued for the normativity of logical consequence to be relative to an epistemic goal. The core features together define the logical consequence-role and I have then showed multiple realizability by considering three consequence-relations: classical, intuitionistic and relevant.

In showing that the logical consequence-role can be multiple realizable, I have then showed that if logical functionalism is indeed the right account of logical consequence, then the case for logical pluralism has been strengthened. In fact, if logical functionalism is true, then effectively logical pluralism is true as well.

Essentially, the appropriate logical consequence-relation is determined by the relevant epistemic goal, but for any relation between the premises and the conclusion of an argument to be a genuine logical consequence-relation, it needs to have the property of being a logical consequence-relation, which is defined as the property performing the logical consequence-role.

In the next chapter I will consider how logical functionalism fares against our desiderata and the challenges I've presented for logical pluralism, and I will consider resemblances and differences with other positions. In other words: I will try to situate logical functionalism in the debate on logical pluralism.

Chapter 5

Logical Functionalism at work

In the above I have developed logical functionalism. Logical functionalism takes logical consequence to be a functional concept. For a relation to be a logical consequence-relation, it needs to have the higher-order property of being a logical consequence-relation, which is defined as the property of having a property that plays the logical consequence-role. This logical consequence-role, is defined by the core features of logical consequence that I have taken from the pre-theoretic notion of this concept. The pre-theoretic notion I have taken to be an informal account that includes the most important features that are, and historically have been, taken to be part of logical consequence. Furthermore, I have argued that there multiple realisations of this logical consequence-role, and hence that logical functionalism as I have presented it here entails logical pluralism.

Recall that my main motivation for developing this position was to enhance previous versions of logical pluralism. Let me now explicitly consider some enhancements of logical functionalism, that sets it apart from previous position. First, I will elaborate on some intrinsic enhancements. That is, I will show how logical functionalism is a preferable solution to the problems facing logical pluralism, based upon some of the features I have presented in the above. Second, I will elaborate more on some resemblances of logical functionalism with other positions discussed in this thesis, but also on the differences that sets logical functionalism apart from the rest.

5.1 Logical Functionalism as the Solution

I will first elaborate on how logical functionalism is a solution to the problems facing logical pluralism presented in chapter 2. Recall that the main problem was a problematic relation between plurality and generality, such that every pluralist theory unavoidably collapses into some kind of monism. In the methodology section, I have presented two desiderata - labelled plurality and generality - that a solution to this problem will have to meet. How does logical functionalism as

I have presented it here meets those desiderata?

Plurality

In what sense does logical functionalism meet the plurality desideratum? In other words: is logical functionalism capable of retaining a multiplicity of (at least two) equally correct logical systems? The answer is *yes*. This would be because of the fact that there are different ways of performing the logical consequence-role. That is, my definition of the logical consequence-role entails a multiplicity of different manifestations. Furthermore, these different manifestations are equally correct with respect to being a manifestation of the logical consequence-role. That is, they are equally worthy of the name 'logical consequence'. However, different manifestations - that is, some proper logical consequence-relations - are paired to different epistemic goals. In *this* sense, they can be considered to be *not* equally correct, although I would prefer to take them to be not equally *useful* for every epistemic goal.

Generality

In what sense does logical functionalism meet the generality desideratum? In other words: does logical functionalism's account of generality *not* entail a problematic relation with the pluralist spirit, and hence with the plurality desideratum? The answer is *no*, and thus that it meets the generality desideratum. This would be because of the fact that a combination - either the intersection or the conjunction - of all logical systems available does not generate a more universal logical system. For example, say that we have two relations - I, for intuitionistic logic, and C for classical logic - of which we have established that they both have the property of being a logical consequence-relation. That is, they both have the property of performing the logical consequence-role, which means that they are necessary, formal and normative as defined in the logical consequence-role. There is no way for I to collapse into C. All there is 'above' or 'across' all logical systems is the higher-order property of being a logical consequence-relation. This is what ties different logical systems together, this is what makes them similar and this is the only property they need to share. Thus, the reason why logical functionalism satisfies the generality desideratum can be put as follows: there is nothing in the set of core features and hence the definition of the logical consequence-role that could cause a problematic collapse. Their collapse is merely into the logical consequence-role, which is our general account of logical consequence. Thus, the logical consequence-role is our account of generality, but such an account of generality leaves open multiple different manifestations.

Blocking the Collapse

By meeting the two desiderata sketched above, I claim to have effectively solved the general problem facing both global logical pluralism and local logical pluralism. I've taken there to be a family of collapse problems, all indicating a

problematic relation between plurality and generality. The worry was that pluralism unavoidable collapses into monism, either into the strongest logical system available or the weakest logical system available. The challenge was to block this collapse. Logical functionalism does so. Let me explain how it does so.

The crux lies in the fact that, as I've stated in the above, logical functionalism collapses merely into the logical consequence-role. Let me now get back to the general collapse problem, as formulated by Williamson, Priest and Read.[45] When we are confronted with a plurality of logical systems, where one system *does* allow for some specific inference, while another logical system doesn't, logical functionalism tells us that we only need to endorse the logical consequence-relation relevant to the epistemic goal in play. Every logical consequence-relation is relative to an epistemic goal and the epistemic goal determines the appropriate logic. Different epistemic goals, different logical systems. Essentially, this is how plurality is maintained. The generality, in the form of the logical consequence-role, is non-problematic given that, first, there appears to be no epistemic goal for the logical consequence-role as such, and second, we could say that the 'common core' logical functionalism assumes is not a logical system at all. The latter is a direct response to the common core problem as presented by Bremer (2014).[46] The objection to Beall and Restall-pluralism there was that in the intersection of different (applied) systems of reasoning we would find something of a common core logic. The logical functionalist grants some kind of common core, presented in the logical consequence-role, but this merely defines the *role* logical consequence performs and is itself *not* a logical consequence-relation, and hence it enforces no one particular logical system.

The (GTT) in Beall and Restall-pluralism is not a logical consequence-relation as well, but we have seen that the (GTT) is vulnerable to a collapse because of a universal quantification over all possible cases, which could result in a *One True Logic*. Such a universal quantification is not possible in the case of the logical consequence-role. We might want to quantify over all possible relations, but this wouldn't work because of the normativity constraint, which tell us to consider every logical consequence-relation respective to an epistemic goal. However, I see no epistemic goal for the set of all possible relations combined, and hence we can't consider the universal quantification over relations to be competing with the manifestations of the logical consequence-role that *do* connect with an epistemic goal. Hence, there is no collapse.

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[45]See section 2.1 of this thesis.
[46]See section 2.1 of this thesis.

5.2 Resemblances and Differences

Logical functionalism has been put forth here as a novel position that solves problems that previous logical pluralist positions were not able to solve. However, there appear to be clear resemblances with other positions. In the following, I will focus on some resemblances between logical functionalism and the pluralist positions presented in this thesis. At the same time, I will point out the differences, in order to give logical functionalism a unique position in the debate on logical pluralism.

Resemblances and differences with Beall and Restall-pluralism
First of all, I will consider some resemblances with Beall and Restall-pluralism. At times, it appears to be that Beall and Restall (2006) hint at some form of logical functionalism. For example, they write:

> We cannot simply stipulate that relation R is a relation of logical consequence. The degree of success of any account of logical consequence will be the degree to which it works, which is measured in terms of the [core] concepts [of logical consequence; necessity, normativity and formality]. (pp. 28-29)

The idea expressed in this quote shows resemblance with logical functionalism: for something to count as a logical consequence-relation, it has to satisfy certain core concepts. One obtains pluralism if one can show that there are at least two different ways of satisfying these criteria, which is what Beall and Restall do. Now, Beall and Restall take the same core concepts logical consequence that I have presented as the core features of logical consequence, but even if they would have chosen different concepts the main idea could still come across as a functionalist one, resonating with analytic functionalism.

There is an even more explicit remark Beall and Restall (1999) make, responding to criticism by Graham Priest, that makes the resemblance between their position and logical functionalism ever more apparent. Recall that earlier I have paraphrased Lynch (2009) - who held that truth was both *One* and *Many* - in stating that logical consequence can taken to be both *One* and *Many*. In fact, Beall and Restall (1999) already phrased their position like this. They write:

> Graham Priest poses the question: "Logic: One or Many? Our answer is "both".
>
> > *One*: there is precisely one core notion of logical consequence, and that notion is captured in schema (V).
> > *Many*: There are many true instances of (V), each of which specifies a different consequence relation governing our language.

This one-many answer is what we call 'pluralism'. (p. 17)

The resemblance is obvious, and raises the question: in what sense is logical functionalism different than Beall and Restall-pluralism?

The most general answer is that logical functionalism does not entail a problematic relation between generality and plurality, but I will need to say more than that. The crux lies in the difference between taking the (GTT) as starting point as Beall and Restall-pluralism does, and taking the logical consequence-role as starting point as logical functionalism does. Recall the objection of the universal quantification over cases, presented in section 2.1.: by taking the (GTT) to be the general characterisation of logical consequence, there is the problem of a universal quantification over all possible cases, which could result in a unbearably weak, but general, logic. I've argued that while Beall and Restall write that this objection can only have force if the presumption against pluralism has already been made, I've also taken it to be that their reply only has force if the presumption in favour of pluralism has already been made. This points to a weakness. On this aspect, logical functionalism differs from Beall and Restall-pluralism by taking the logical consequence-role to be general characterisation of logical consequence. This is an enhancement because the objection of universal quantification over all cases cannot rise.

Resemblances and differences with contextualism
Another comparison can be drawn between logical functionalism and the (similar) contextualist positions endorsed by Caret (2017) and Hjortland (2013). Again, let me first point out to the resemblances.

One obvious resemblance is that both logical functionalism as I have presented it here and contextualism argue that different situations or contexts require different logics. I grant that me taking logical consequence to be relative to an epistemic goal fares quite close to contextualism taking logical consequence to be relative to a context, where structural assumptions determine what kind of logic governs this context. What these structural assumptions do for Caret (2017), the epistemic goals do for me. Thus, I grant that this element of my theory is 'contextualistic'.

Furthermore, when Caret (2017) discusses the objection to contextualism that by dropping the topic-neutrality of logic it throws out the baby with the bath water, he writes in response:

> My preferred solution is to adopt the view that logical terms have a *minimal meaning* that is invariant between difference specifications of cases. (p. 757)

The resemblance I find in the idea of retaining some minimal meaning that

remains constant over different specifications of cases, different domains or different contexts. This is quite similar to the idea in logical functionalism to take the logical consequence-role to remain invariant in a similar sense. The logical consequence-role might then be seen as having a kind of minimal meaning Caret proposes. Perhaps Hjortland (2013), to which Caret (2017) refers, presents an idea that resembles logical functionalism even more explicit, as he writes:

> The official line is that 'deductive consequence', or 'follows from', has multiple senses. But why not say that although the extension of the expressions vary, there is only one sense or intension? Crudely put, the meaning of the expression is a function from a type of cases (say, classical models) to an extension, i.e. a class of valid arguments (the classically valid arguments). (p. 361)

The resemblance of this idea with logical functionalism appears to be striking: there is only one *sense* of deductive (logical) consequence, but there is a plurality of extensions. Again, one might be tempted to take the logical consequence-role to be the one sense Hjortland puts forth.

Therefore, I take logical functionalism as I have presented it here to be more closely related to local logical pluralist positions, such as contextualism, than it is to Beall and Restall-pluralism. The reason for this is that by taking the normative force of logical consequence to be inherently related to certain epistemic goals of deductive reasoning, I have implemented some kind of contextualism in logical functionalism. After all, I have taken the epistemic goals to determine the context. This is in fact not much different than taking logical consequence to be relative to some context - one might in fact take these epistemic goals to be one kind of structural assumptions Caret takes to determine what kind of logic is connected to some context. But all this makes the question what then the real differences are between logical functionalism and contextualism (or local logical pluralism more in general) even more pressing.

The account of generality logical functionalism offers is more satisfying than what contextualism has to offer. In other words: logical functionalism, I argue, has a better story of what ties all the different logical systems together. What ties them together, namely, is that they all have the higher-order property of being a logical consequence-relation. If contextualists or local logical pluralists alike intended to provide exactly such an account of generality, then the difference between logical functionalism and such position might indeed be minimal. However, as far as I know, such an account of generality hasn't been worked out in much detail. Logical functionalism is an enhancement of contextualism in this respect.

5.3 Conclusion

In this chapter I have situated logical functionalism among other logical pluralist positions presented in this thesis. I have done so by first considering its intrinsic features that make that logical functionalism able to (dis)solve the problems presented in chapter 2. Furthermore, I have considered resemblances and differences with other positions, namely Beall and Restall-pluralism and contextualism. The crux, both in its intrinsic features and in its relation to other positions, I take to be that logical functionalism provides a much more satisfying account of the generality of logical systems than do other positions generally, and the pluralist positions specifically. Hence, I believe its not only plausible that logical consequence is a functional concept, but also that the logical consequence-role as I have defined it here is capable of avoiding a problematic relation between generality and plurality.

Chapter 6

Discussion and Future Work

In the previous chapters I have presented logical functionalism and tried to show its advantages. I have put forth logical functionalism as a plausible and effective account of logical consequence and tried to do so as comprehensively and detailed as possible. However, there remain some open ends, which I will discuss in this chapter. First, I will discuss some questions and objections, in an attempt to resolve all ambiguity or uncertainty that the reader might be left with. In doing so, I hope to clarify my case. Second, I will sketch possible future work on logical functionalism. I argue that every every philosopher of logic, both pluralists and monists, ought endorse the functionalist conceptual framework I have presented and defended in this thesis. I will discuss what such future work might consist out of.

6.1 Questions, Objections and Replies

First, I will discuss and reply to some (potential) questions and objections.[47]

Question: Are there any formal features of logical consequence implied by the logical consequence-role as presented here? For example, is the account of logical consequence presented here transitive or non-transitive? Or does it at least allow for non-transitivity?

Reply: Canonically, logical consequence has taken to be transitive. For some specific phenomena, this causes problems, such as in the case of vagueness: if $A \models B$, $B \models C$ and by transitivity then also $A \models C$, then we get the (in)famous Sorites Paradox. Alternative logical systems have been developed in order to

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[47]These questions and objections are a combination of questions and objections I have received from the audience of an early presentation of this thesis and from my supervisor in different stages of writing this thesis, as well as potential question and objections I have identified myself.

solve this problem, which take logical consequence to be non-transitive.[48]The question then is: does logical functionalism as I have presented here allow for such alternative logical systems?

The answer is that as long as a logical consequence-relation adheres to the logical consequence-role, then logical functionalism as presented here allows for such a logical consequence-relation. That is: as long as the consequence relation is necessary, formal and normative as defined in the definition of the logical consequence-role, then it is a proper logical consequence-relation and hence logical functionalism would then allow for it. In the case of an alternative logical system, one then only needs to show that it does in order to be worthy of the name 'logical system'. At this point, I don't see a principled reason as to why a logical system which takes the consequence relation to be non-transitive would, *a priori*, not be included. Yet, I haven't included it in the set of core features, as I take it to be that logical systems which take the consequence relation to be transitive could be similarly worthy of the name 'logical system'. Some manifestations of the logical consequence-role might be transitive, others might be not. That is why I haven't included this formal requirement in the set of core features - it is too strict. In setting up the core function of logical consequence, we should not have in mind any particular application of it. When we talk about specific applications of the consequence relation, we are talking on the level of the manifestations. But the manifestations should follow from the role, and not the other way around.

This reply to the question of transitivity extends to other formal features. For every account of logical consequence, as long as the relation between the premises and the conclusion of an argument has the property of being a logical consequence-relation, then logical functionalism allows for it. That means that there is a considerable amount of flexibility in logical functionalism as to what counts as a logical system. I take this to be a further advantage of the position.

Objection: The set of core features and the definition of the logical consequence-role entailed by these core features is vague and imprecise.

Reply: What I aim to achieve is *not* a precise (formal) definition of logical consequence. Rather, I have tried to identify the *core features* of logical consequence, distilled from a pre-theoretic notion of this concept. According to Shapiro (1998), detailed answers to the question of what this pre-theoretic notion of logical consequence amounts to are rare. This might be due to difficulties Tarski (1956) pointed out with respect to defining logical consequence. Tarski writes:

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[48]See for example Van Rooij (*manuscript*).

> With respect to the clarity of its content the common concept of consequence is in no way superior to other concepts of everyday language. Its extension is not sharply bounded and its usage fluctuates. Any attempt to bring into harmony all possible vague, sometimes contradictory, tendencies which are connected with the use of this concept, is certainly doomed to failure. We must reconcile ourselves from the start to the fact that every precise definition of this concept will show arbitrary features to a greater or less degree. (p. 409)

In other words, according to Tarski it is impossible to capture the entire concept of logical consequence in one definition, probably because there appears to *be* no clear demarcation, or at least there is no way of capturing such a demarcation in a definition, as the usage fluctuates drastically. However, this is of course not to say that there is *no* common ground as to what logical consequence essentially amounts to - there is *some* common ground, and I have tried to capture this in the set of core features. As such, my account of logical consequence is rather abstract, but that is exactly what should serve my aim best. As Lynch (2009) writes about his project on truth:

> The overall picture of truth given here is abstract. But of course, that is part of the point. This book has been about truth itself and truth itself, the functionalist theory claims, is a rather abstract property. (p. 192)

My project is, in this respect, not much different then Lynch's project. Logical consequence, I maintain, is an abstract concept with a wide range of different usages. My aim for the logical consequence-role is to provide some kind of generality that runs across all these different usages. Therefore, I can't be *too* strict on what I take into account as being part of the core function of logical consequence. On the other hand, of course, I also can't say too little on what the core features are - this is the fine line I've had to walk in defining the logical consequence-role. I believe I have maintained a proper balance and I hope to have succeeded in defining the core features of logical consequence, and hence in defining the logical consequence-role. I believe that what I have done is all one could expect if one wants to define the logical consequence-role.

Objection: Logical functionalism is vulnerable to the threat of logical nihilism: the logical consequence-role gives us an extremely shallow and unworkable definition of logical consequence.

Reply: I hold that the logical consequence-role is all we can say about logical consequence *per se*. In my view, logical consequence is an abstract property that can't be made more precise than my definition of the logical consequence-role. But that is not to give in to logical nihilism, for the following reason: logicians are not primarily working with the abstract property of logical consequence, but

rather with *manifestations of* this abstract property. We should evaluate these manifestations on their 'workability', for example by considering the epistemic utility. We should *not* evaluate the logical consequence-role on its 'workability', or its effective application on certain phenomena.

Objection: In section 5.3 different manifestations of the logical consequence-role are presented, whereby the classical logical consequence-relation is taken to be one that holds with respect to truth-preservation *without* any additional constraints, while the other two examples are presented as truth-preservation *with* constraints, namely that of provability and relevance. That seems to imply that the classical manifestation of the logical consequence-role is the null-manifestation, or unrestricted manifestation. That sounds like the classical manifestation is actually the most general one, and that might be the *One True Logic* after all. Hence, there is no genuine pluralism in logical functionalism as it is presented here. The collapse problems strikes back.

Reply: There is a difference in logical systems, and hence logical consequence-relations. They are not equally *good* with respect to any epistemic goal. However, they are equally correct in the sense that they all are genuine manifestations of the logical consequence-role. We should look for generality not in the manifestations, but in the role.

Furthermore, the epistemic goals I take to be essentially different. They cannot be reduced to each other and essentially stand in no subset relation to each other. For example, take an epistemic goal such as gaining a deep understanding of why something is the case, for which one might need to be able to provide a proof of how some kind of truth has been attained, which could demand a constructive logic such as intuitionistic logic. This is a fundamentally different epistemic goal then just knowing whether something is the case or not, which might correspond to classical logic. I maintain that the latter goal can not be reduced to the former, nor the other way around. Again, in the end they are fundamentally different. If I want to go to Rome via Berlin, then this is something different than wanting to go to Rome directly.

6.2   Future Work

In this thesis I've tried to put forth logical functionalism as a novel position in the philosophy of logic, that proposes a natural and effective account of logical consequence. I have presented one way of being a logical functionalist, but more work could be done. What might future work within the logical functionalist framework consist of?

Core features
An essential element in my display of logical functionalism is the set of core features of logical consequence. This set is intended to be exhaustive and of course as adequate as possible. However, future work might consist in revising or enhancing this list. This doesn't need to threaten logical functionalism as such. The main point of the thesis is to ignite a new way of looking at the concept of logical consequence by taking it to be functional. I have tried to do that by showing how it could work by a definition of the logical consequence-role that to me seems adequate. But it is not impossible that my thesis that logical consequence is a functional concept is correct, but that the set of core features from which I distil the logical consequence-role is not. This might be argued by showing that logical systems we deem reasonable do not meet the demands that my version of the logical consequence-role sets. I haven't been able to find these systems, but they might be there.

Logical Functionalism without Pluralism
Future work on the core features, as described above, might have consequences for the pluralistic agenda. That is: it might be possible that a refined set of core features results in logical monism, if one would argue that among the core features of logical consequence are certain principles that rule out certain logical consequence-relations, while vindicating only one such relation. I haven't been able to find such core features, but they might be there. For the set of core features presented in this thesis, and hence my definition of the logical consequence-role, such pluralism *is* implied, but different core features might result in a different definition of the logical consequence-role which might result in there only being *one* manifestation of this role.

Epistemic goals and the connection with the appropriate logic
In section 4.1.2. I have stated that the epistemic goal 'enforces' the appropriate logical consequence-relation, and - equivalently - that given an epistemic goal, the *one* appropriate logical consequence-relation manifests itself. The idea is that a pure logic has certain features that are perfectly suited for obtaining a specific epistemic goal. For example, the feature of intuitionistic logic (as a pure logic) that demands a constructive proof is perfectly suited for the epistemic goal of obtaining a deeper understanding of *why* or *how* something is the case. However, this is merely a first pass on the relation between an epistemic goal and its appropriate logical consequence-relation and more work might be done in order to get this relation more precise. For example, is there a systematic method for determining the appropriate logical consequence-relation of a specific epistemic goal? Could it be that one logical system has features that contradict each other? And also, could we demarcate epistemic goals better? Is there a systematic way to rule out absurd epistemic goals, perhaps?

Chapter 7

Conclusion

In this thesis I have presented logical functionalism as an improvement of logical pluralism. I have argued that the main problem facing logical pluralism - both global logical pluralism and local logical pluralism - is a problematic relation between generality and plurality. The problem is that every logical pluralist position currently on the market tends to collapse into some kind of monism, either into the strongest logical system available or the weakest logical system available. Such a collapse is unavoidable and is at odds with the pluralist spirit, which holds that there is a plurality of different equally correct logical systems.

The main challenge, I have argued, is to provide an account that allows for both this pluralist spirit and a non-problematic account of generality. By generality I mean an account of the general features of logical consequence such that this account does not allow for a collapse into monism. In order to achieve such an account, I have adopted a functionalist framework and applied this t the fundamental concept in logic, namely logical consequence. The idea then is to take logical consequence to be a functional concept. The main challenge for this approach is to capture the logical consequence-role, which can be seen as the role logical consequence performs. For any relation between the premises and the conclusion of an argument to have the property of being a logical consequence-relation, the relation must perform the logical consequence-role. The logical consequence-role is defined by the core features of logical consequence: formality, normativity and necessity. The core features I have taken from the pre-theoretic notion of logical consequence, which consists out of the most important, core features that have been ascribed to logical consequence over the history of philosophy and logic. Such features I take to have *canonical prominence*: canonically prominent features are those that are the most prominent in theories of logical consequence that have been developed over the course of time. I have made my selection of the core features of logical consequence on the basis of canonical prominence combined with my own critical assessment and evaluation of these features. After having defined the logical consequence-role, I have shown that this logical

consequence-role is multiple realizable and as such have shown that logical functionalism is a pluralist position.

There are two main developments in this thesis, that can be considered separately. First of all, logical functionalism meets the challenge of providing an account of logical consequence (and hence, logic) that allows for both plurality as well as a non-problematic generality. The plurality is achieved by the multiple realizability of the logical consequence-role, while the non-problematic generality is achieved by this logical consequence-role. This account of generality is non-problematic because all there is 'above' or 'across' all logical systems is the higher order property of being a logical consequence-relation. The logical consequence-role itself is no logical system and it allows for a genuine plurality of equally correct logical systems in the sense of the multiple realizations of the logical consequence-role.

Another development in this thesis is crucial to my presentation of logical functionalism, but could be considered a development in itself. It is that I have argued that we cannot consider any normative force of logical consequence without taking into account an epistemic goal of deductive reasoning. The idea is that if logic is seen as setting forth norms about how one ought to reason, then since reasoning is essentially an epistemic activity, it would make sense to maintain that the norms logical consequence sets forth are essentially epistemic norms.

To conclude: I maintain that taking logical consequence to be functional concept is a natural way to account for both the plurality and the generality of logical consequence. In fact, I argue that this is the right account of logical consequence. As such, logical functionalism is not only an enhancement of previous logical pluralist positions, but also an enhancement of previous accounts of logical consequence in general.

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