Gatekeepers in Social Networks: Logics for Communicative Actions

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written by

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Abstract

Information control has been subject of investigation in the fields of information theory and social network analysis. Gatekeepers play a central role, as they are the agents that can manipulate the information flow between groups of agents. In this thesis, we present a formal model, which captures the essential properties of gatekeepers in a network. Firstly, we use graph theory to explore the structural dimension of gatekeepers. Secondly, we use notions from Dynamic Epistemic Logic and Coalition Logic to model the dynamism of their informational capabilities. We find that gatekeepers can be categorized in three distinct subclasses, which differ in terms of their structural properties, or type of information control. This approach contributes both to the field of information theory and communication science, by providing insights on which agents can exert information control and of what type, and to the 'logic in the community' agenda, by investigating an intersection between logic and the social sciences.

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Contents

1 Introduction

In early 2018, the Facebook-Cambridge Analytica scandal erupted. It came out that personal data of almost 90 million Facebook users have been harvested, largely without permission [1], and strategically used to design several political campaigns around the world. These included Donald Trump's campaign for the presidential elections of 2016 [2] and the Brexit referendum [3]. Facebook controlled the harvesting of information, and its strategic use was allowed by Cambridge Analytica, which constructed psychographic profiles of Facebook users out of it [4]. Alexander Nix, head of Cambridge Analytica, stated that with such profiles it is possible to achieve microtargeted persuasion and manipulate voters decision making [5][4].

This is a case of unregulated information control, where access to personal data of millions of people were used to sway voters and direct their choices to a desired outcome. Before the revelation of data misuse, Facebook users around the world were hardly aware that their private information could be so deployed, and what consequences the control of such a huge amount of information could have. And as important as knowing *that* information can be strategically controlled in such negative ways, is to know exactly *who* can do so, i.e. who controls the gates where this information flows. Only knowing who these agents are we can limit their power and protect our personal data from being illicitly used.

In the literature, agents that have the power to control the information flow between two groups of agents are called *gatekeepers* [6][7][8], and in this thesis we focus on them. The notion was crafted in the mid-twentieth century by the psychologist Kurt Lewin, who conceptualized gatekeeping as "including all form of information control" [9]. Since then, the concept has been applied to very distinct fields, ranging from chemistry and biology [10] to political science [11], sociology [12], or management and technology theories [13]. However, Barzilai-Nahon [6] shows that the systematic study of such concept remains compartmentalized, namely the "discourse on the topic of gatekeeping is conducted within each discipline, in relative isolation". This results in the concept of gatekeeper lacking a full theoretical status [6]. Consequently, she worked towards a generalization of the gatekeeping theory, aiming to achieve a discipline-neutral framework that encompasses all the different applications of the notion [7]. In order to achieve this generalization, she proposes to take the perspective of networks and information societies.

In this thesis, we aim at contributing to Barzilai-Nahon's project, by providing a formalization of the gatekeeper phenomenon in social networks. While Barzilai-Nahon concentrates on the role of the gated, we will focus on network gatekeepers themselves and on the analysis of their informational capabilities. We will propose a distinction internal to the notion of gatekeeper, namely the distinction between its positive power to enable the information flow and its negative power to block it. We will show that the positive and negative capabilities can be described in terms of graph theoretical properties of social networks, and that they capture the necessary or sufficient conditions to either enable or block the information flow between two groups. By combining or generalizing these properties, we will construct distinct versions of network gatekeepers. Notably, the distinction between positive and negative capabilities, of necessary and sufficient conditions to control the flow, as well as of kinds of gatekeepers, are captured for the first time in this work. We believe that this provides insightful theoretical specifications, that will contribute to the understanding of which agents in the network can exert information control, and to what extent.

To this structural and graph theoretical analysis of gatekeepers, the logical and informational ones will follow. Through suitably defined logics, we will represent the informational capabilities that gatekeepers have, and thus the correspondence between their structural and informational properties. We will use modal languages to characterize the network structure and the structural notions that we will introduce. In particular, we will use an hybrid version of propositional dynamic logic (PDL) to express some basic network properties, such as the existence of paths from a particular node to another. The logic we will use for this part resembles a non-epistemic form of Facebook logic, as proposed by Seligman et al. [14], and it is based on Social Network Models, as in Smets et al. [15]. We will then move to describe the informational capabilities that each kind of gatekeeper possesses, by means of a mixture of different logics. We will use notions from dynamic epistemic logic [16] to model the dynamics of posting actions. The framework that will result similar to the public broadcast network as proposed by Roelofsen [17], in that every agent can only publicly address the rest of her friends as a whole. This kind of communicative action will be used as a model transformer, introducing dynamism to network logic. In addition, we will use notions from coalition logic [18] [19], or STIT logic [20], to express that gatekeepers can force an outcome, namely enabling or blocking the information flow, if working together as a coalition.

Note that the logic part of this thesis (chapter 5 and 6) intends to investigate the relationships between the informational dynamics, and the social structures in which these take place, from a logic perspective. Then, while on the one hand this work advances the understanding of the gatekeeping dynamics, on the other it also contributes to van Benthem [21] or to Seligman et al. [22] agenda of exploring the intersections between logic and the social sciences.

To conclude the introductory part, let us see the structure of this thesis. In the second chapter, we will firstly present some of the literature on gatekeeping theory, and secondly some notions of graph theory that will be taken for granted in what will follow. In the third chapter, we will introduce the structural notions that will then be used to construct the three versions of gatekeepers in social networks. The latter will compose the fourth chapter. In the fifth chapter, we will introduce network logic and provide the characterization of the structural notions in this logic. In the sixth chapter, we will introduce the coalition logic for posting action, and then use it to capture the informational capabilities of gatekeepers. We will conclude with the seventh chapter, by suggesting possible future directions of the present work.

2 Preliminaries

2.1 Gatekeepers in the literature

The concept of gatekeeper has spawned an extensive and diverse literature. In all such literature, the notion is used as a metaphor, referring to objects or agents that control the passage of some other objects, of some other agents, or simply of some information, through a gate. In this brief overview, we will focus on the branches of this literature that apply the metaphor to settings that are both social and informational, i.e. to agents and their interactions, for this is the topic of the present thesis. We will see how (1) communication theory, (2) sociology or anthropology, and (3) economics or business theory use the notion, and what aspects of gatekeeping they focus on. Note that the perspective we will take in this thesis will differ from the ones taken there, as we will not study the concept as *applied* to some particular field or phenomena. Rather, we will take a more abstract perspective, even though still social and informational, and study gatekeepers as (unspecified) agents entertaining determinate central positions in a social network. As said in the introduction, Barzilai-Nahon [6] already paved the way for this kind of study. Then, after having briefly presented the three branches mentioned above, we will also see the core elements of Barzilai-Nahon's network gatekeeping theory. Note that we mainly consider Barzilai-Nahon's work, as it is the most systematic analysis of gatekeepers in social networks to date. However, the concept has been studied from the same social network perspective also by other scholars, and so we will mention also the definition of gatekeeper that has been put forward by them.

(1) *Communication Theory:* In this field, the gatekeeping metaphor is mainly applied to editors and journalist, namely to those agents that decide what information is closed off from media attention and what information can instead pass through the gate. This literature analyses the process through which the information is selected and shaped, underlying "how even single, seemingly trivial gatekeeping decisions can come together to shape an audiences view of the world" [8]. Hence, the primary focus here is on the selection process that the gatekeeping activity involves, in particular on the biases that it introduces in the information system.

(2) *Sociology and Anthropology:* In these fields, the focus is rather on the fact that gatekeepers have access to qualitative information that is unavailable to some other agents. The gatekeeping metaphor is used, for example, to refer to key informants, who are essential parts of ethnographic fieldwork. In this kind of investigation, the researcher lives in a community or a tribe and documents her findings. A key informant is a member of the community that has extensive and specialized knowledge about it, and is willing to share it with the researcher [12]. Hence, the primary focus here is on the access to reliable information that gatekeepers can make available to the researcher.

(3) *Economics or Business Theory:* These fields use the gatekeeping metaphor in yet another sense. They apply it to traders or brokers, namely to agents who connect mutually disconnected parts of the community and thereby have access to the heterogeneous information residing there [12]. This access is described as an informational advantage, for the information they have access to is used to come up with new ideas and thus to obtain new

gains. Hence, the primary focus here is on the capability of gatekeepers to combine the diversified information they have access to, and the gains they derive from them.

These three examples briefly illustrate how the notion is used in the literature. Let us now see the main elements of Barzilai-Nahon's *Network Gatekeeping Theory*, which relates more closely to the present work. This theory is divided in two parts: *network gatekeeping identification*, and *network gatekeeping salience*. The identification part is a descriptive theory and it systematizes the answer to the question: "who are network gatekeepers and what constitutes network gatekeeping and its mechanisms?". The salience part is instead a normative theory, aimed at explaining to whom and to what gatekeepers should pay attention. Since this thesis will concentrate on the description and representation of the notion of gatekeeper in networks, we will consider only the first and not the second part of the theory.

The descriptive theory introduces the basic definitions of network gatekeepers and of their capabilities. Its primitive constructs are the following:

- *Network gatekeeper*, defined as "an entity (people, organizations, or governments) that has the discretion to exercise gatekeeping through a gatekeeping mechanism in networks, and can choose the extent to which to exercise it contingent upon the gated standing";

- *Gatekeeping*, defined as "the process of controlling information as it moves through a gate";

- *Gatekeeping mechanism*, defined as the "tool, technology or methodology" through which these activities are performed;

- *Gated* are the agents that are subject to the gatekeeping;

- *Gate* is the channel through which the controlled information passes.

In this thesis, we will mainly focus on the first two primitive constructs. By taking "control" to mean both enabling and blocking the information flow, we will contribute to Barzilai-Nahon's study and specify what network gatekeepers are from a purely structural or graph theoretical perspective.

Note that Barzilai-Nahon's definition of network gatekeeper does not explicitly mention the structural properties that these agents must have. However, by equating gatekeepers to agents that can perform a set of specific actions *in* *networks*, she must be implicitly assuming that for an agent to be a gatekeeper, he must occupy a particular position in the network, i.e. he must satisfy some particular structural properties. In fact, other authors represent gatekeepers as agents that have specific properties in the network. For example, D. Easley and J. Keinberg represent gatekeepers in terms of structural properties [25]. They define gatekeepers as those agents that lie in every path between other two distinct agents. We will see below that this is their global version of the gatekeeper, but they also distinguish a local version of it.

2.2 Graph Theory for Social Networks

As mentioned above, social network structures can be represented through graph theoretical means. In this section, we will introduce the basic notions from graph theory that will be used in this work.

A graph is defined as a pair composed by a set of nodes and a set of edges [26].

Definition 2.1. (*Graph*). We say that a *graph* N is a pair $N = (\mathcal{A}, R)$, where $\mathcal{A}$ is a non-empty and finite set of nodes and R is a relation between them.

We define a social network as a graph where nodes are interpreted as agents and edges as relations between them. The graph is meant to represent a snapshot of a social network in time, so to capture its configuration in a given moment. Then, the set of agents is finite and the relations between them fixed. The relations represent social relations as friendships or acquaintanceships. These are two-directional relations, in the sense that the agents can exchange information in both directions. Since we are interested in modelling the interaction between agents, we consider only relationships between distinct agents, and exclude reflexive relationships as well as solitary agents (agents not related with any other agent). Such properties are represented by symmetric relationships, with no self-loops and where every agent has at least a friend or acquaintance in the network.

Definition 2.2. (*Network*). We say that a *network* N is a pair $N = (\mathcal{A}, R)$, where $\mathcal{A}$ is a non-empty and finite set of agents and R is a relation such that for every $a \in \mathcal{A}$

$$(a, a) \notin R \qquad \text{(Irreflexivity);}$$

for every $b \in \mathcal{A}$, if $a \neq b$, then $(a, b) \in R$ iff $(b, a) \in R$ (Symmetry);

there exists at least a $b \in \mathcal{A}$ such that $(a, b) \in R$. (Seriality);

Given a network, we introduce the notion of path, taken from the literature on graph theory [26], which will be used extensively in this thesis.

Definition 2.3. (*Path*). Let $N = (\mathcal{A}, R)$ be a social network and consider some $a, b \in \mathcal{A}$. We say that $P \subseteq N$ is a *path* iff it is a sequence of agents that links a to b, i.e. $P := (aRx_1R \ldots Rx_nRb)$, with $x_i \in \mathcal{A}$.

To express that an agent a lies on a path P, we will say that $a \in P$. This is a little abuse of notation, as paths are sets of ordered pairs, but we are confident that the reader will not be mislead by it. Similarly, we will say that a set of agents $A \subseteq \mathcal{A}$ *is* a path, when actually it is the case that the agents in A *form* a path.

The next definitions capture the notion of non-redundant and minimal paths belonging to a social network $N = (\mathcal{A}, R)$.

Definition 2.4. (*Non-Redundant Path*). Let a, b be two agents and $P := (aRx_1R \ldots Rx_nRb)$ be a path linking them. We say that P is a *non-redundant* path between a, b iff for every $x_i, x_j \in P$ we have $x_i \neq x_j$.

The non-redundancy of a path implies that no agent belonging to it appears more than one time, thereby avoiding cycles. Also minimal paths avoid circles, but for another reason. Let us see their definition.

Definition 2.5. (*Minimal Path*). Let a, b be two agents and P be a path linking them. We say that P is a *minimal* path between a, b iff for every $P' \subset P$, P' is not a path linking a, b.

Minimal paths are minimal connections between two points, so they can contain no cycle. Note that minimality does not coincide with non-redundancy. The following example shows it.

Example 2.6. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 1. We claim that there exists a minimal and non-redundant path between two agents, and a non-redundant but not minimal path in it, thus the two notions do not coincide.

Let $P := (aRbRcRdReRf)$ and $P' := (aRbReRf)$. Note that in P' there exists no subset that is a path linking a, f, thus P' is a minimal path between the two agents. Moreover, note that $P' \subset P$, so P is not a minimal path connecting

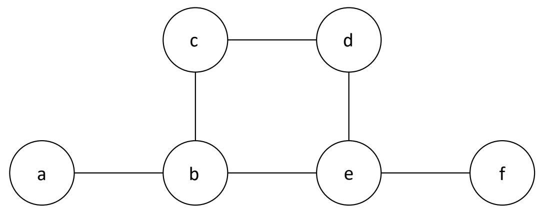

Figure 1

a, f. However, P is a non-redundant path between them, as there is no agent that appears two times in the path. Hence, we can conclude that there exists paths that are non-redundant but not minimal and that the two notions do not coincide.

However, the minimality of a path implies its non-redundancy, as the next proposition shows.

Proposition 2.7. *For any path P linking two agents a, b in a social network $N = (\mathcal{A}, R)$, if P is a minimal path between a, b then P is a non-redundant path between a, b.*

Proof. Let $N = (\mathcal{A}, R)$ be a social network. We proceed by contraposition. Consider some $a, b \in \mathcal{A}$ and let P be a redundant path between a, b. Since R is not reflexive, then the redundancy of P implies that at least two agents are repeated in the path, i.e. $P := (aR \dots Rx_iRx_j \dots Rx_iRx_j \dots Rb)$. Consider $P' \subset P$ such that $P' := (aR \dots Rx_iRx_j \dots Rb)$, i.e. P' does not contain the repetition of x_i, x_j. The sub-path P' still links a, b. By definition this means that P is not minimal. We can conclude that for any path P linking two agents a, b, if P is a minimal path between a, b then P is a non-redundant path between a, b. □

3 Structural Notions

In this chapter, we introduce the structural definitions that will be used later for the construction of gatekeepers in social networks. We distinguish five notions that capture the sufficient conditions for sets of agents to either enable or block the information flow between groups in the network. Not many of these notions will have the property of being necessary to enable or block the information flow. This property will instead belong to gatekeepers, which will result from the interplay between notions that we will introduce here.

The chapter is structured as follows. For each notion, we firstly provide the definition, an example illustrating it and its property of being a set of agents that is sufficient to enable or block the information flow. Subsequently, we uncover some other properties of the notion, in particular those that will be relevant for our subsequent discussion. We start with the definition of group of agents, disconnected groups and connector between them. Then, we propose the notions of bridge and bridging set between them. All these notions will be shown sufficient to enable the information flow. Lastly, we introduce the definitions of blocking set and its minimal version, which will be shown sufficient to block the information flow between the groups.

3.1 Groups

Social networks were formally discussed in the preliminary chapter. In the same section we introduced the notion of connected network, namely a network where for every two agents there exists at least a path that links them. If that happens, then the agents in the network are *closed under communication flow*. This means that the information can spread among the agents, as there exists at least one channel through which the information can circulate. The notion of group we propose is meant to capture the notion of being closed under communication flow.

Note that in what follows, we will use the convention that lower case letters represent agents, while upper case letters represent sets or groups of agents.

Definition 3.1. (*Group*). Let G be any non-empty $G \subseteq \mathcal{A}$ of a network $N = (\mathcal{A}, R)$, and consider some distinct $a, b \in G$. We call G a *group* iff there exists a path $P := (a = a_0 R a_1 R \dots R a_n = b)$, such that for every $a_i \in P$, we have $a_i \in G$.

In other words, a group is a set of connected agents. Importantly, the existence of a path between two agents is a *sufficient* condition for the information to flow among those agents. In every definition we will introduce below, remember that when the agents form a group, then they satisfy this sufficiency condition.

Note that not every set of agents forms a group. By definition, if a set of agents G is not a group, then for every two agents g, g' in G there is no path connecting them and the information flow between them is blocked. Generalized for two distinct groups G, G' instead of single agents, this amounts to say that there is no agent $g \in G$ and no agent $g' \in G'$ such that $(g, g') \in R$. In other words, the union of G, G' does not form a group.

Definition 3.2. (*Disconnected Groups*). Let G, G' be two distinct groups. We say that G, G' are *disconnected groups* iff $G \cup G'$ is not a group.

The notion of disconnected groups is central in our discussion. In this thesis, we focus on the power that gatekeepers have to control the information flow between two groups of agents. Then, the two groups must not be directly connected to each other, as otherwise they would be capable of direct communication and no control of the flow by external agents would be possible. Clearly, this also implies that another important notion is the one defining the agents that connect two disconnected groups, which we call the *connector*.

Definition 3.3. (*Connector*). Let G, G' be two disconnected groups. We say that $B \subset \mathcal{A}$ is a *connector* between G, G' iff $G \cup G' \cup B$ is a group.

This notion will be particularly relevant in the definitions that follow. Essentially, it is the set of agents that allows two disconnected groups to be closed under communication flows, i.e. that connects them. Since it allows the set to form a group, then a connector is sufficient to enable the information flow between them. However, connectors might contain agents that are not sufficient nor necessary to connect the groups, and thus to enable the flow. The example below will clarify this, as well as the distinctions between groups, connectors and disconnected groups.

Example 3.4. Let G, G', G'' be the three distinct sets of agents represented in Figure 2. We claim that (i) G is a group, while G', G'' are not; (ii) for any two of these groups, they are disconnected; (iii) $\{d, b\}$ is a connector between $\{a\}$,$\{c\}$, and $\{l\}$ is a connector between $\{m\}, \{i\}$.

Proof. (i) Consider $G = \{a, b, c, d\}$. Since $G \neq \emptyset$ and for every $x, y \in G$ there exists a path $P := (x = x_0 R x_1 R \ldots R x_n = y)$, such that for every $x_i \in P$, we have $x_i \in G$, then by definition of group G is a group.

Consider $G' = \{e, f, g, h\}$. There exists an agent, namely e, such that for any $x \in G'$ such that $x \neq e$ there is no path P such that $P := (x = x_0 R x_1 R \ldots R x_n = e)$. So G' does not satisfy the definition of group.

Consider $G'' = \{l, m, n\}$. In G'' there exists an agent, namely n, such that for any path $P := (x = x_0 R x_1 R \ldots R x_k = n)$, P contains an agent not in G'',

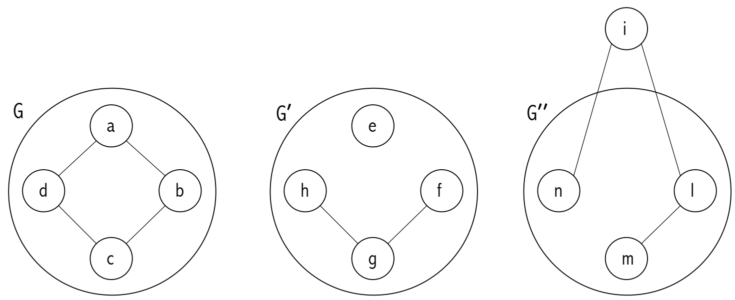

Figure 2

namely i. Then, G'' does not satisfy the definition of group.

(ii) Consider G, G'. There exists no $g \in G$ and $g' \in G'$ such that $(g, g') \in R$. Thus, G, G' are disconnected. Analogous reasoning holds for G, G'' and G', G''.

(iii) Consider $\{d, b\}$. Note that $\{a\}$ and $\{c\}$ are trivially groups and that $\{a\} \cup \{c\}$ is not a group, i.e. $\{a\}$ and $\{c\}$ are disconnected. We showed above that G is a group, then $\{d, b\}$ is a connector between $\{a\}$ and $\{c\}$.

Consider $\{l\}$. Note that $\{i\}$ and $\{m\}$ are trivially groups and that $\{i\} \cup \{m\}$ is not a group, i.e. they are disconnected. Since $\{l\} \cup \{i\} \cup \{m\}$ is a group, then $\{l\}$ is a connector between $\{i\}$ and $\{m\}$. □

Part (iii) of this example helps us appreciating the distinction between the necessary and sufficient sets of agents to enable the information flow. The set of agents $\{d, b\}$ is necessary and sufficient to enable the information flow between a and c. Necessary, because without it the information would not flow; Sufficient, because with it the information can flow. If we go at the level of agents, every agent in the set is sufficient, but not necessary for the information to flow, as the reader can easily verify. Instead, the set and the agent in the set $\{l\}$ are both necessary and sufficient to enable the flow between i and m.

Properties of groups, disconnected groups and connectors

The first property we show is about existence and (non) uniqueness of groups. Clearly, the notion of group is not unique, as there might be distinct connected sets in a network. Moreover, recall that in the definition of social networks, we excluded the existence of solitary agents. Then, there always exists at least a set of two connected agents, i.e. for every network there always exists at least one group.

The second property we show is about disconnected groups, and it is almost immediate from their definition. It says that if two groups of agents are disconnected then they can have no agents in common. This is because otherwise the groups will be connected through those common agents.

Proposition 3.5. *For any two distinct groups G, G', if they are disconnected, then $G \cap G' = \emptyset$.*

Proof. Let G, G' be two distinct and disconnected groups. Suppose towards contradiction that $G \cap G' = \{g_0, \ldots, g_n\}$. Since G, G' are distinct, then there exist a $g_i \in G$ and a distinct $g_j \in G'$ such that $(g_i, g_j) \in R$, by definition of group. This contradicts the assumption that G, G' are disconnected. Hence, $G \cap G' = \emptyset$. □

We now turn to the existence and non-emptyness of connectors between two groups. The non-emptyness depends on the fact that we assume to be working in a connected component of the network.

Proposition 3.6. *For any two disconnected groups G, G' in the same connected component N of the network, there exists a connector C between them.*

Proof. Let G, G' be two disconnected groups in the same connected component N of the network. Consider $N' = N \setminus (G \cup G')$. Since $G \cup G'$ is not a group, but $(G \cup G') \subset N$ and N is connected, i.e. a group, then $G \cup G' \cup N'$ must be a group. This means that N' is a connector between G, G'. □

Proposition 3.7. *For any two disconnected groups G, G' in the same connected component N of the network, and for every connector C between them, we have $C \neq \emptyset$.*

Proof. Let G, G' be two disconnected groups in the same connected component N of the network and let C be an arbitrary connector between them. Suppose towards contradiction that C is empty. Then $G \cup G' \cup \emptyset = G \cup G'$ is a group, which contradicts the assumption that G, G' are disconnected. Thus, $C \neq \emptyset$. Since C and G, G' were chosen arbitrarily, this holds for every two disconnected groups and for every connector between them. □

Note that connectors are not unique, as it is clearly possible to define two sets of agents B, B' that connect the disconnected groups.

3.2 Bridge

A bridge is the minimal set of agents that connects two disconnected groups. It is minimal, as each member of the bridge has the property of being necessary for the two groups to be connected through the bridge itself. However, we show that a bridge is not necessarily the only connection between two disconnected groups. Multiple bridges can connect two groups at the same time, i.e. the notion of bridge is not unique. Moreover, we show that a bridge is a linear path and a minimal connector between two disconnected groups, and that its agents form a group. The realistic exemplification of this notion will come after the analysis of local gatekeepers.

Definition 3.8. (*Bridge*). Let G, G' be two disconnected groups and consider some $B \subset \mathcal{A}$. We say that B is a *bridge* between G, G' iff

(B+) B is a connector between G, G';

(B-) for every $b \in B$, $B \setminus \{b\}$ is not a connector between G, G'.

Recall that by definition of connector between two disconnected groups, clause (B+) means that $G \cup G' \cup B$ is a group. Then bridges are set of agents that are sufficient to enable the information flow between the groups. Moreover, by clause (B-), every agent in a bridge is necessary for the connection of the two groups through the bridge itself.

Note that bridges might contain just one agent. When this will be the case, we will call the bridge *single-agent* bridge. We will apply the same terminological rule for every notion we will introduce from now on.

Let us see an example of what a bridge is and what it is not.

Example 3.9. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 3, and let G, G' be the two disconnected groups in it. We claim that $B = \{a, b\}$ is a bridge between G, G', but $B' = \{a, b, c\}$ is not.

Proof. Consider $B = \{a, b\}$. All the agents in B are connected by a path, i.e. B is a group. Note that a is connected to group G and b is connected to group G'. Then, the set $G \cup G' \cup B$ is a union of groups connected to each others,

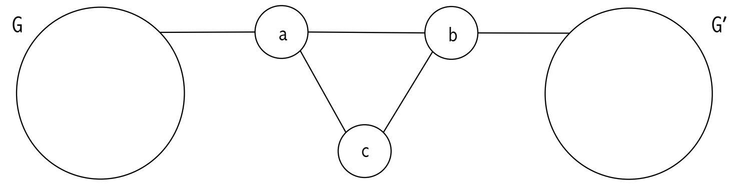

Figure 3

i.e. it is a group itself. Then, B is a connector between G, G' and satisfies (B+). Moreover, $G \cup G' \cup (B \setminus \{a\})$ is not a group, as G is disconnected from $(B \setminus \{a\}) \cup G'$. Analogously for b. This amounts to say that for every agent $x \in B$, $G \cup G' \cup (B \setminus \{x\})$ is not a group, i.e., B satisfies (B-) as well. Thus, B is a bridge between G, G'.

Consider $B' = \{a, b, c\}$. The set $\{a, b, c\}$ connects the two otherwise disconnected groups G, G'. Then, $G \cup G' \cup \{c\}$ is a group, i.e. B' is a connector between G, G' and it satisfies (B+). But $G \cup G' \cup (B' \setminus \{c\}) = G \cup G' \cup B$ and we saw that B is a connector between G, G'. This means that B' does not satisfy (B-). Thus, B' is not a bridge between G, G'. □

Properties of Bridges

The first two properties we introduce are existence and non-uniqueness of bridges between two groups.

Proposition 3.10. *For every connected component N of a social network, for every two disconnected groups G, G' in N, and for every connector C between G, G', there exists a bridge $B \subset C$ between G, G' such that $B \neq \emptyset$.*

Proof. Let G, G' be two disconnected groups in the same connected network $N = (\mathcal{A}, R)$. Consider a connector $C \subset \mathcal{A}$ between them. Since N is connected, by Proposition 3.6, there exists at least one connector C such that $C \neq \emptyset$. If C is a bridge, then we are done. If not, then it means that there exists some $b \in C$ such that $G \cup G' \cup (C \setminus \{b\})$ is still a group. So consider $C' = C \setminus \{b\}$. We know that $G \cup G' \cup C'$ is a group. If C' is a bridge, then we are done. If not, then it means that there exists some $b' \in C'$ such that $G \cup G' \cup (C' \setminus \{b'\})$ is still a group. Repeat the procedure till you get to some C^n such that $G \cup G' \cup (C^n \setminus \{b^n\})$ is not a group anymore. Then C^n is a bridge between G, G'. Since G, G' and C were chosen arbitrarily, we can conclude that for any $G, G' \subseteq \mathcal{A}$ that are two disconnected groups in the same connected component N of the network, and for any connector C between them, there exists a bridge C between G, G'. □

To ensure existence of the notions that we are going to introduce from now on, let us assume that the network where they lie is connected.

Example 3.11. (*Non-uniqueness of bridges*). Let $N = (\mathcal{A}, R)$ be the network represented in Figure 4, and let G, G' be the two disconnected groups represented in it. We claim that B and B' are bridges between G, G'.

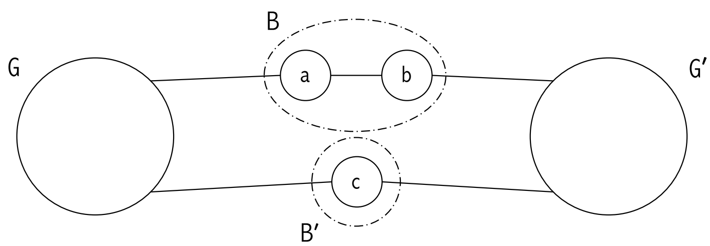

Figure 4

Proof. Consider $B = \{a, b\}$. All the agents in B are connected by a path, i.e. B is a group. Note that a is connected to group G and b is connected to group G'. Then, the set $G \cup G' \cup B$ is a union of groups connected to each others, i.e. it is a group itself. Then, B is a connector between G, G' and satisfies (B+). Moreover, $G \cup G' \cup (B \setminus \{a\})$ is not a group, as G is disconnected from $(B \setminus \{a\}) \cup G'$. Analogously for b. This amounts to say that for every agent $x \in B$, $G \cup G' \cup (B \setminus \{x\})$ is not a group, i.e., B satisfies (B-) as well. Thus, B is a bridge between G, G'.

Consider $B' = \{c\}$. The set $\{c\}$ connects the two otherwise disconnected groups G, G'. Then, $G \cup G' \cup \{c\}$ is a group, i.e. B' is a connector between G, G' and it satisfies (B+). Moreover, $G \cup G' \cup (B \setminus \{c\}) = G \cup G'$. Since $G \cup G'$ are disconnected, then $G \cup G'$ is not a group. This means that B satisfies (B-) as well. Thus, B' is a bridge between G, G'.

We can conclude that between the same two groups G, G' there can be more than one bridge. Hence, bridges are not unique. □

We now move to show a minimality property of bridges. Because of clause (B-), bridges are minimal connectors between the groups.

Proposition 3.12. *For any two disconnected groups G, G' and for any connector $B \subset \mathcal{A}$ between them, the following are equivalent:*

(1) B satisfies (B-);

(2) B satisfies (B-)': for every $B' \subset B$, B' is not a connector between G, G'.

Proof. Let $B \subset \mathcal{A}$ be a connector between two disconnected groups G, G' .

(1) $\Rightarrow$ (2). Let B satisfy (B-). By definition, B is then a bridge between G, G'. By Proposition 3.10 we know that $B \neq \emptyset$, so let $B = \{b_1, \ldots, b_n\}$ for some arbitrary n. We proceed by induction on the cardinality of $B' \subset B$, with $B' \neq \emptyset$. We show that for every m, there does not exist any B', with $|B'| = m$ and $0 < m < n$, such that $G \cup G' \cup (B \setminus B')$ is a group.

- Base Case: Let $m = 1$. Then $|B'| = 1$ and $B' = \{b_1\}$ for some $b_1 \in B$. By (B-) definition of bridge, $G \cup G' \cup (B \setminus \{b_1\})$ is not a group. Hence, when $m = 1$, $G \cup G' \cup (B \setminus B')$ is not a group.

- Induction step: Assume that the induction hypothesis holds for m, i.e. there exists no $B' \subseteq B$, with $|B'| = m$, such that $G \cup G' \cup (B \setminus B')$ is a group. For simplicity, let us call B_m the sets with m elements, and B_{m+1} the sets with $m + 1$ elements. Now we want to show that for B_{m+1} too it is the case that for every $B_{m+1} \subseteq B$, we have $G \cup G' \cup (B \setminus B_{m+1})$ is not a group. Suppose towards contradiction that this is not the case. Then, there exists at least one $B_{m+1} \subseteq B$, such that $G \cup G' \cup (B \setminus B_{m+1})$ is a group. Consider the B_m such that $B_m \subset B_{m+1}$. Then $(B \setminus B_m) = (B \setminus B_{m+1}) \cup \{b_i\}$ for some $b_i \in B$. By induction hypothesis we know that $G \cup G' \cup (B \setminus B_{m+1}) \cup \{b_i\}$ is not a group, and by assumption we know that $G \cup G' \cup (B \setminus B_{m+1})$ is a group. Now suppose that b_i is connected to either G, G' or $(B \setminus B_{m+1})$. Then $G \cup G' \cup (B \setminus B_{m+1}) \cup \{b_i\}$ would be a group, which is not. So b_i must be disconnected from $G \cup G' \cup (B \setminus B_{m+1})$. But $b_i \in B$, and B is a bridge. By (1) above we know that B is a group, so for no $b_i \in B$ it can be the case that b_i is disconnected from some other $b_j \in B$. We know that at least one $b_j \in B$ such that $b_j \neq b_i$ exists, as $B_m \subset B_{m+1} \subseteq B$ and each of them is non-empty, which means that $|B| \geq 2$. Contradiction.

Thus, for every $B_{m+1} \subseteq B$, we have $G \cup G' \cup (B \setminus B_{m+1})$ is not a group.

We can conclude that for every m, there does not exist any B', with $|B'| = m$ and $0 < m < n$, such that $G \cup G' \cup (B \setminus B')$ is a group. Since n was chosen arbitrarily, this holds for every n such that $B = \{b_1, \ldots, b_n\}$. Moreover, let $B'' = (B \setminus B')$. Since for every B' we have that $B' \subset B$, then our result is equivalent to saying that for every B'', we have that $B'' \subset B$ and $G \cup G' \cup B''$ is not a group, i.e. B satisfies (B-)′.

(2) ⇒ (1). Assume that (B-)′ holds for B, i.e. assume that for every $B' \subset B$, $G \cup G' \cup B'$ is not a group. Consider an arbitrary $b \in B$ and let $B' = (B \setminus \{b\})$. Since $B' \subset B$ then $G \cup G' \cup B'$ is not a group. By definition, B satisfies (B-).

We can conclude that for any two disconnected groups G, G' and for any connector $B \subset \mathcal{A}$ between them, B satisfies (B-) iff B satisfies (B-)′. □

The above proposition shows that clause (B) of definition of bridge is equivalent to clause (B′). The latter clause says that bridges are minimal connectors between the two groups, and that they can be equivalently defined in terms of minimal connectors. We now show another way in which bridges can be equivalently defined, namely as minimal paths between the groups. This result also follows from the proposition just proved.

Proposition 3.13. *For any two disconnected groups G, G' and for any $B \subset \mathcal{A}$, the following are equivalent.*

(1) B is a bridge between G, G';

(2) B is a minimal path between some $g \in G$ and some $g' \in G'$.

Proof. Let G, G' be two disconnected groups and consider some $B \subset \mathcal{A}$. B is a minimal path between some $g \in G$ and $g' \in G'$ iff there exists no $B' \subset B$ such that $G \cup G' \cup B'$ is a group iff B is a minimal connector between G, G' iff B is a bridge between G, G', by Proposition 3.12.

We can conclude that B is a bridge between G, G' iff B is a minimal path between some $g \in G$ and some $g' \in G'$. □

By Proposition 2.7, if a path between two agents is minimal, then it is also non-redundant. Therefore, this proposition implies that bridges are also non-redundant paths between two agents in the groups.

Proposition 3.14. *For any $B \subset \mathcal{A}$, if B is a bridge between two disconnected groups G, G', then (1) $B \neq \emptyset$; (2) $(G \cup G') \cap B = \emptyset$; (3) B is a group.*

Proof. Let $B \subset \mathcal{A}$ be a bridge between two disconnected groups G, G'.

(1) By definition of bridge, B is a connector between G, G'. Then, by Proposition 3.7, $B \neq \emptyset$.

(2) Suppose towards contradiction that $(G \cup G') \cap B = B'$ and let $B'' = B \setminus B'$. Clearly, $B'' \subset B$ and $G \cup G' \cup B''$ is a group, which contradicts the minimality of bridges. Hence $(G \cup G') \cap B = \emptyset$.

(3) By Proposition 3.10 we know that $B \neq \emptyset$. So let $B = \{b_1, \ldots, b_n\}$ for some arbitrary n. We proceed by induction on the cardinality of B. We prove that for every n, B is a group.

- Base Case: If $B = \{b_1\}$ then B is trivially a group.

- Induction step: Assume that the induction hypothesis holds for n, i.e. that $B' = \{b_1, \ldots, b_n\}$ is a group. Now we want to show that $B = B' \cup \{b_{n+1}\}$ is a group too.
  Suppose towards contradiction that B is not a group. Since by induction hypothesis B' is a group, the only agent in B disconnected from the other members of B must be b_{n+1}. Since B is a bridge between G, G', then by (B+) we know that $G \cup G' \cup B$ is a group. Then, three cases: (i) there exists some $g \in G$ and some $g' \in G'$ such that $(g, b_{n+1}) \in R$ and $(b_{n+1}, g') \in R$; (ii) there exists some $g \in G$ such that $(g, b_{n+1}) \in R$; (iii) there exists some $g' \in G$ such that $(b_{n+1}, g') \in R$. In each of the three cases, $G \cup G' \cup (B \setminus \{b_{n+1}\})$ is a group, which contradicts (B-) of bridge definition. Hence, $B = \{b_1, \ldots, b_n, b_{n+1}\}$ is a group.

We can conclude that for every n, $B = \{b_1, \ldots, b_n\}$ is a group.

□

Linearity of the Bridge

In this subsection, we show that bridges are linear sets of agents connecting two groups, i.e., paths that do not branch. To explain this, we introduce the definition of linearity. Recall that social networks are based on a relation R that is defined as irreflexive and symmetric. For such a relation, linearity amounts to saying that every agent a, if a is in a bridge B, then a has at most two other friends in B.

Definition 3.15. (Linear Order.) For every $B \subseteq \mathcal{A}$, B is *linearly ordered* iff for every $a \in B$ there exist at most two other $b, c \in B$ such that $(a, c) \in R$ and $(a, b) \in R$.

Proposition 3.16. *For any $B \subset \mathcal{A}$, if B is a bridge between some disconnected groups G, G', then B is linearly ordered.*

Proof. Let $B \subset \mathcal{A}$ be a bridge between some disconnected groups G, G'. Suppose towards contradiction that B is not linearly ordered, i.e. there exists some $a \in B$ such that for three distinct agents $b, c, d \in B$, (distinct both from a and between each others), we have $(a, b) \in R$, $(a, c) \in R$, $(a, d) \in R$. By (B+) and Proposition 3.14, B is a group that connects G to G', and $G \cup G'$ are not connected. But a, b, c, d are distinct agents, so that in B there exist at least two distinct paths P_1, P_2, such that $G \cup G' \cup P_1$ is a group and $G \cup G' \cup P_2$ is a group. We consider two cases: (i) $|P_1| > |P_2|$ or $|P_1| < |P_2|$ but not both; (ii) $|P_2| = |P_1|$. Suppose (i), then there exists some $P \subset B$, namely P_1 or P_2, but not both, such that $G \cup G' \cup P$ is a group. This contradicts the minimality of bridges. Now consider (ii). Since $P_2 \neq P_1$ then $|B| > |P_2| = |P_1|$. But then there exists some $P \subset B$, namely P_1 or P_2 such that $G \cup G' \cup P$ is a group, which contradicts the minimality of bridges.

Since there are no other options, we can conclude that for every $B \subset \mathcal{A}$, if B is a bridge between two disconnected groups G, G', then B is linearly ordered. □

To clearly see what we mean with linearity, we propose the following example.

Example 3.17. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 5, and let G, G' be the two disconnected groups represented in it. We claim that (i) the set $\{a, b\}$ is a bridge between G, G', thus also linearly ordered; (ii) the set $\{a, b, c\}$ is linearly ordered, but not a bridge between G, G'; (iii) the set $\{a, b, c, d\}$ is not linearly ordered, thus not a bridge between G, G'.

Proof. Consider the set $\{a, b\}$ and call it B. In Example 3.9, we showed that B is a bridge between G, G'. Then by Proposition 3.16, B must be linearly ordered. In fact, note that the only agent in B to which a is connected is b, and the only

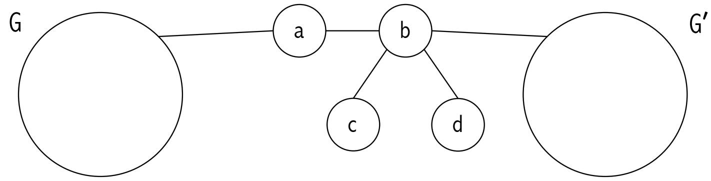

Figure 5

agent in B to which b is connected is a. Since a, b are the only elements in B, then B is linearly ordered.

Consider the set $\{a, b, c\}$ and call it B'. In B', every agent is connected to at most two other agents. So B' is linearly ordered. But B' is not a bridge, as $G \cup G \cup (B' \setminus \{c\})$ is a group, which contradicts clause (B-) of definition of bridge.

Consider the set $\{a, b, c, d\}$ and call it B''. In B'', there exists an agent who is connected to three other agents, namely b. So B'' is not linearly ordered. By Proposition 3.16, B'' is not a bridge between G, G'. In fact, $G \cup G' \cup (B \setminus \{d\})$ is a group, which contradicts clause (B-) of definition of bridge. Analogously if we use c instead of d. □

3.3 C-Local Gatekeeper

In this section, we introduce a notion to represent the agents belonging to bridges. We call every such agent a *C-local gatekeeper*. It is a $C-$local gatekeeper and not simply a local gatekeeper, because we define it relative to the connector C to which the bridge belongs. In this way, it will later be simpler to characterize the notion in the logic.

Definition 3.18. (*C-Local Gatekeeper*). Let G, G' be two disconnected groups and let $C \subset \mathcal{A}$ be a connector between G, G'. We say that an agent $c \in C$ is a *C-local gatekeeper* between G, G' iff there exists a bridge $B \subseteq C$ with $c \in B$.

This notion takes inspiration from the definition of local gatekeeper given by Easley and Kleinberg in [25].

> [F]or X to be a *local gatekeeper*, there should be two nodes Y and Z such that Y and Z each have edges to X, but not to each other. (our italics).

This kind of gatekeeper, call it NCM local gatekeeper, differs from the C-local gatekeeper in two ways. Firstly, it is a gatekeeper between two agents, whereas the C-local gatekeeper is between two groups. Secondly, local gatekeepers have direct relations with the two agents they locally gatekeep, whereas for C-local gatekeepers this is not necessarily the case. This, because the latter belong to bridges, which might not be single-agent bridges. Indeed, the next proposition shows that the definitions of C-local gatekeeper and NCM local gatekeeper express the same notion, when the C-local gatekeeper belongs to a single-agent bridge.

Proposition 3.19. *For every $C \subset \mathcal{A}$ that is a connector between some disconnected groups G, G', b is a C-local gatekeeper between G, G' iff there exists some $g, g' \in \mathcal{A}$, such that b is an NCM local gatekeeper between g, g'.*

Proof. Let $C \subset \mathcal{A}$ be a connector between some disconnected groups G, G' and consider some $b \in C$.

($\Rightarrow$) Assume that b is a C-local gatekeeper between G, G' and suppose towards contradiction that there exists no $g, g' \in \mathcal{A}$, such that b is an NCM local gatekeeper between them. This means that for every $g, g' \in \mathcal{A}$ such that $(g, g') \notin R$ we have $(g, b) \notin R$ and $(b, g') \notin R$. But b is a C-local gatekeeper between G, G', so there exists a bridge $B \subseteq C$ between G, G' such that $b \in B$. By (B+) of definition of bridge, B is a connector between G, G' so $G \cup G' \cup B$ is a group. Since G, G' are disconnected by definition, then B must be connected with both of them. Then we distinguish two cases: (1) $B = \{b\}$; (2) $B = \{b_1, \ldots, b_n, b\}$. If (1) then there are some $g \in G$ and $g' \in G'$ such that $(g, b) \in R$ and $(b, g') \in R$, otherwise B would not be a connector between G, G'. But we assumed that for every $g, g' \in \mathcal{A}$ such that $(g, g') \notin R$ we have $(g, b) \notin R$ and $(b, g') \notin R$. Contradiction. So consider (2). Since by Proposition 3.14, B is a group, then either there exists some distinct b_i, b_j such that $(b_i, b) \in R$ and $(b, b_j) \in R$, or there exists some $g \in G$ or $g' \in G'$, such that $(g, b) \in R$ or $(g', b) \in R$. But we assumed that for every $g, g' \in \mathcal{A}$ such that $(g, g') \notin R$ we have $(g, b) \notin R$ and $(b, g') \notin R$. Contradiction. We can conclude that if b is a C-local gatekeeper between G, G', then that there exists some $g, g' \in \mathcal{A}$, such that b is an NCM local gatekeeper between them.

($\Leftarrow$) Assume that there exists some $g, g' \in \mathcal{A}$, such that b is an NCM local gatekeeper between them. To prove that b is a C-local gatekeeper we need to prove that there exists a bridge $B \subseteq C$ between G, G' such that $b \in B$. Since b is an NCM local gatekeeper between g, g', then $\{g\} \cup \{g'\} \cup \{b\}$ is a group and $(g, g') \notin R$, by definition. Then $\{b\}$ satisfies both clauses of definition of bridge, i.e. it is a bridge between $\{g\}, \{g'\}$. Since $g \in G$ and $g' \in G'$, $\{b\}$ is a bridge between G, G'. Thus, b belongs to a bridge between G, G'.

Hence, if B is a bridge between some disconnected groups G, G', then for every $b \in B$, b is a C-local gatekeeper between G, G' iff b is an NCM local gatekeepers. □

We conclude the sections about bridges and C-local gatekeepers with a realistic example that illustrates the two notions.

Example 3.20. (*Citizen-Mayor Bridges*). Imagine the situation in which Ann, a citizen of a metropolis, wants to communicate an information to her mayor. Suppose that Ann is neither a friend nor an acquaintance with her mayor and that there are only two ways in which Ann can achieve her communicative goal. One of them is by sending an email to Bob, the mayor's secretary, who will then pass it on to the mayor. Bob constitutes a bridge between Ann and the mayor, as he connects them (thus {Bob} satisfies (B+)), and if he decides to block the information, the mayor does not receive it (thus {Bob} satisfies (B-)). Then, Bob is a local gatekeeper between Ann and the mayor, as he belongs to a bridge between them. Note that if Bob decides not to communicate Ann's information to the mayor, then it is not the case that the mayor cannot receive the information at all, but rather that she will not receive it from Bob. This, because Ann can try to achieve her goal following the other available way. This is by sending the information through a public organization that deals with matters of the same nature as the information Ann is interested in communicating. Ann can contact one of the members of the organization, Chen, who is in contact with Margit, who is the mayor's counselor regarding those matters. Margit often shares dinners with the mayor, and they discuss the same kind of issues that Ann wants to communicate. Chen and Margit form a bridge between Ann and the mayor. They together have the capability to communicate Ann's information to the mayor. However, if each of them decides to block it, then the information does not go through.

In this example, Bob on the one hand, Chen and Margit on the other, form two bridges between Ann and the mayor, who represent the two disconnected single-agent groups.

3.4 Bridging Set

Bridging sets are connectors containing only C-local gatekeepers between two otherwise disconnected groups of agents. Recall that a C-local gatekeeper is an agent lying on a bridge. We will see that this implies that bridging sets are formed only by bridges, which is why, stretching the English language a little bit, we call them *bridging* sets. The notion of bridging set is an important notion, because each kind of gatekeeper we will introduce below is a particular case of bridging set. We will show that bridging sets always exist, but they are not unique.

Definition 3.21. (*Bridging Set*). Let G, G' be two disconnected groups and consider some $C \subset \mathcal{A}$ such that C is a connector between G, G'. We say that C is a *bridging set* between G, G' iff for every $c \in C$, c is a C-local gatekeeper between G, G'.

Since bridges are connectors between two disconnected groups, then these sets are sufficient to enable the information flow between the groups.

To give a clearer picture of the notion, we immediately show that bridging sets are composed only by bridges, i.e. a bridging set is a union of *some* bridges between those groups.

Proposition 3.22. *For any two disconnected groups G, G', and for any $C \subset \mathcal{A}$ the following are equivalent:*

(1) C is a bridging set between G, G';

(2) C is the union of some bridges $B_1, \ldots, B_n$ between G, G', i.e. $C = \bigcup_{i=1}^{n} B_i$, with $1 \leq i \leq n$.

Proof. Let G, G' be two disconnected groups and consider some $C \subset \mathcal{A}$. The set C is a bridging set between G, G' iff every $c \in C$ is a C-local gatekeeper between the groups, by definition of bridging set. This is the case iff for every $c \in C$, there exists a bridge $B \subseteq C$ between G, G' and $c \in B$, by definition of C-local gatekeeper. Let us call $B_1, \ldots, B_n$, with $n \geq 1$, the bridges between G, G', such that for every $c \in C$, $c \in B_i$ with $1 \leq i \leq n$. Then by definition of union we have that $C = \bigcup_{i=1}^{n} B_i$. We can conclude that A is a bridging set between G, G' iff $C = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are some bridges between G, G'. □

We now move to illustrate the notion with an example. The example also shows the differences and similarities between bridging sets and connectors. They both are sets of agents that connect the two groups and they both can contain several bridges. However, bridging sets have the additional property to be formed just by bridges, whereas connectors are sets of agents that simply to connect two other disconnected groups.

Example 3.23. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 6, and let G, G' be the two disconnected groups represented in it. We claim that (i) C is a connector between G, G' and a bridging set between G, G'; (ii) C' is a connector between G, G' but not a bridging set between them.

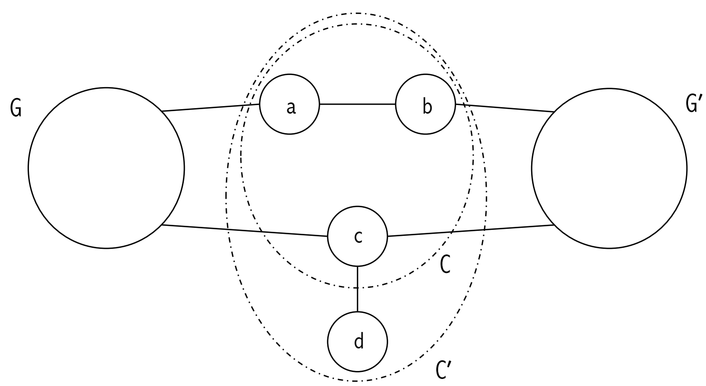

Figure 6

Proof. Consider $C = \{a, b, c\}$. Since for some agent $g \in G$ and some $g' \in G'$, we have $(g, a) \in R$, $(b, g') \in G'$, then $G \cup G' \cup \{a, b\}$ is a group. By the same reasoning, $G \cup G' \cup \{c\}$ is a group. Note that G, G' are disconnected, then (B+) holds for $\{a, b\}$ and for $\{c\}$. Then $G \cup G' \cup \{a, b\} \cup \{c\}$ is a group and since $C = \{a, b\} \cup \{c\}$, then C is a connector between G, G'. Moreover, $G \cup G' \cup (\{a, b\} \setminus \{a\})$ is not a group, and thus $(\{a, b\} \setminus \{a\})$ is not a connector between G, G'. The same holds for $\{b\}$. Since $G \cup G' \cup (\{c\} \setminus \{c\}) = G \cup G'$ and G, G' are disconnected, then $G \cup G' \cup (\{c\} \setminus \{c\})$ is not a group, i.e. $(\{c\} \setminus \{c\})$ is not a connector between G, G'. Thus, (B-) holds for $\{a, b\}$ and for $\{c\}$, i.e. they are two bridges between G, G'. Since $C = \{a, b\} \cup \{c\}$, then C is also a bridging set between G, G'.

Now consider $C' = \{a, b, c, d\}$. Since $C' = C \cup \{d\}$, and $(d, c) \in R$ then $G \cup G' \cup C'$ is a group and C' is a connector between G, G'. But $G \cup G' \cup (C' \setminus \{d\})$ is a group, and thus $(C' \setminus \{d\})$ is a connector between G, G'. This means that $\{d\}$ does not lie on any bridge between G, G'. Thus, C' is not a bridging set between G, G'.

We can conclude that C is a connector between G, G' and a bridging set between G, G', while C' is a connector between G, G', but not a bridging set between them. □

Note that the example also shows that bridging sets are not necessarily groups. This is because they are formed by bridges that do not always intersect.

Properties of Bridging Sets

The first property we show is that bridging sets between disconnected groups always exist when the network is connected. This is immediately derivable from the existence of bridges, of which bridging sets are composed.

Proposition 3.24. *For any two disconnected groups G, G', there exists a bridging set B between G, G'.*

Proof. Let G, G' be two disconnected groups. By Proposition 3.10 we know that there always exists a bridge B between G, G'. By Proposition 3.22, a bridging set is a union of bridges, i.e. B is a bridging set. We can conclude that for any two disconnected groups G, G' in N, there exists a bridging set B between G, G'. □

Recall that bridges between two given disconnected groups are not unique. This implies that bridging sets between two groups are not unique too.

Example 3.25. (*Non-uniqueness of Bridging Sets*). To show the non-uniqueness of bridging sets, we recall Figure 3 above. There, B and B' are two bridges between two otherwise disconnected groups G, G'. Since bridges are connectors and then each of them contains only C-local gatekeepers. This means that both B and B' satisfy the definition of bridging sets, i.e. the example in Figure 3 shows the existence of two bridging sets. We can conclude the non-uniqueness of the notion.

We conclude the section with a realistic example of bridging sets that exemplifies the properties that they have.

Example 3.26. (*Citizen-Mayor Bridging sets*). Recall Example 3.20 above. In that example we considered a citizen, Ann, who wants to communicate an information to her mayor. We assumed that there are just two paths that Ann can use to achieve her communicative goal: Bob forms the first, and Chen and Margit the second. We showed that they are both bridges, which means that each of these agents belongs to some bridge between Ann and the mayor. Hence, each of them is a C-local gatekeeper and forms a bridging set between Ann and the mayor. By the same reasoning, also the set comprising Bob, Chen and Margit together forms a bridging set.

3.5 Blocking Set

Blocking sets are sets of agents that are capable to block the information flow between two otherwise disconnected groups of agents. They have such capacity, because every connection between the two groups passes through them. Note that this notion is of fundamental interest to this thesis, as we ultimately aim at modeling the capacity that some agents have to control the information flow,where blocking is the negative side of this capacity. We show that blocking sets between two agents in a connected component of the network always exists, but they are not unique. Moreover, their agents relate with the definition of global gatekeeper given by the social network literature [25].

Definition 3.27. (*Blocking Set*). Let G, G' be two disconnected groups and consider some $A \subsetneq \mathcal{A}$ such that $A \cap (G \cup G') = \emptyset$. We say that A is a *blocking set* between G, G' iff every connector C between G, G' contains an element of A, i.e. for all $C \subset \mathcal{A}$, if $G \cup G' \cup C$ is a group, then $C \cap A \neq \emptyset$.

Thus, blocking sets are sufficient to block the information flow between two groups, as they contain at least one agent in every connector between them, which is sufficient to block every connection. However, blocking sets might not contain entire connections between the groups, as entire connectors, bridges or bridging sets. Then, they are not always sufficient to enable the information flow.

To better see what this definition amounts to, let us consider the following example.

Example 3.28. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 7, and let G, G' be the two disconnected groups represented in it. We claim that A and A' are blocking sets between G, G'.

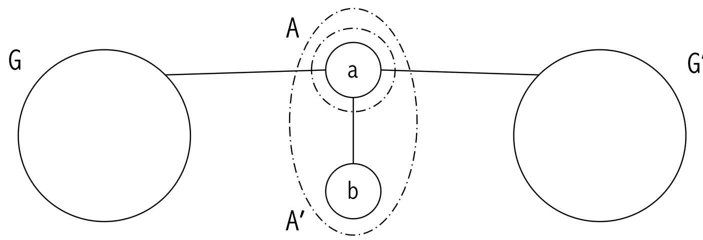

Figure 7

Proof. For a set of agents to be a blocking one between two disconnected groups G, G', the set must intersect every connectors between G, G' and be disjoint from the groups. The only connectors are A and A'.

Consider $A = \{a\}$. Clearly, $A \cap (G \cup G') = \emptyset$, and $A \cap A \neq \emptyset$. Moreover, $A \subset A'$, then $A' \cap A \neq \emptyset$. Thus, A is a blocking set between G, G'.

Consider $A' = \{a, b\}$. Clearly, $A' \cap (G \cup G') = \emptyset$ and since $A' \cap A' \neq \emptyset$. Moreover, $A' \cap A \neq \emptyset$. Thus, A' is a blocking set between G, G'.

Since $A' \neq A$, we can conclude that there exist two blocking sets between G, G' and that blocking sets are not unique. □

Properties of Blocking Sets

Blocking sets between two groups are defined on the assumptions that the groups are disconnected and lie in a connected component of the network. These assumptions imply that a blocking set between them always exists and it is never empty.

Proposition 3.29. *For any two disconnected groups G, G', there exists a blocking set B between them.*

Proof. Let G, G' be two disconnected groups in the same connected component $N = (\mathcal{A}, R)$ of the network and consider $C = \mathcal{A} \setminus (G \cup G')$. Since N is connected, then C is a connector between G, G' and $C \neq \emptyset$, as otherwise $G \cup G'$ would be a group, which contradicts their disconnectedness. Moreover, for every other connector C' between G, G', we have $C' \cap C \neq \emptyset$. Thus, C is a non-empty blocking set between G, G'. □

Example 3.30. (*Non-uniqueness of Blocking Sets*). Recall Example 3.28 above. Let $N = (\mathcal{A}, R)$ be the network in Figure 7, and let G, G' be the two disconnected groups represented in it. In that example, we showed that A and A' are blocking sets between G, G'. Therefore, we proved the existence of two distinct blocking sets between the same groups, i.e. the non-uniqueness of the notion.

Recall that in this thesis we are interested in modelling the agents that are necessary and sufficient to enable and block the information flow between two disconnected groups. For the blocking part, these agents are represented by the ones which, if removed from the network, would cut the connections between the groups. Then, Figure 7 shows that blocking sets do not capture all and only the agents that are necessary and sufficient to cut such connections. For example, agent b is neither necessary nor sufficient to block the flow between the groups. To illustrate, compare it with agent a. If we remove a from the network, there is no connection left between G, G', so that then G, G' become two disconnected components of the network. This means that a is sufficient to cut the connections between G, G' and therefore to block the information flow. Moreover, a is also necessary to that goal, as if a is not removed, then the connection between the groups remains uncut. However, this does not hold for agent b. If we remove agent b from the network, the two groups remain connected through agent a. This implies that b is neither necessary nor sufficient to block the flow. Despite that, in the example above we showed that $A = \{a\}$ and $A' = \{a, b\}$ are both blocking sets. This means that the notion of blocking set also captures agents that are not relevant with respect to blocking (or enabling) the information flow between two disconnected groups.

The next proposition is about the relations between blocking and bridging sets. It shows that every bridging set between two otherwise disconnected groups must have some elements in common with every blocking set between the two groups. This result will be useful later, as we will build one of the definitions of gatekeepers on the notions of blocking sets and bridging sets.

Proposition 3.31. *For any two disconnected groups G, G', and for any $A \subset \mathcal{A}$ such that $A \cap (G \cup G') = \emptyset$, the following are equivalent*

(1) A is a blocking set between G, G';

(2) For every bridging set B between G, G', we have $B \cap A \neq \emptyset$.

Proof. Let G, G' be two disconnected groups and consider some arbitrary $A \subset \mathcal{A}$ such that $A \neq \emptyset$ and $A \cap (G \cup G') = \emptyset$.

(1) ⇒ (2) Assume that A is a blocking set between G, G' and consider an arbitrary bridging set B between G, G'. Suppose towards contradiction that $B \cap A = \emptyset$. Since by assumption $A \cap (G \cup G') = \emptyset$, then $A \cap (G \cup G' \cup B) = \emptyset$. By definition of bridging set, $G \cup G' \cup B$ is a group. Since $(G \cup G') \subset (G \cup G' \cup B)$, then, by definition of blocking set, $(G \cup G' \cup B) \cap A \neq \emptyset$. So we have $A \cap (G \cup G' \cup B) = \emptyset$ and $(G \cup G' \cup B) \cap A \neq \emptyset$. Contradiction. Then $B \cap A \neq \emptyset$. Since B was chosen arbitrarily, this holds for every bridging set B between G, G'.

(2) ⇒ (1) Assume that for every bridging set B between G, G', we have $B \cap A \neq \emptyset$. Suppose towards contradiction that A is not a blocking set, i.e. there exists a connector C between G, G' such that $C \cap A = \emptyset$. By Proposition 3.10, we know that there exists a bridge $C' \subset C$ between G, G'. By definition of bridging set, we know that C' is a bridging set between G, G'. Then, $C' \cap A \neq \emptyset$. But we assumed that for every bridging set B between G, G', we have $B \cap A \neq \emptyset$. Contradiction. We can conclude that A is a blocking set between G,G′.

We can conclude that for any two disconnected groups G, G', and for any $A \subset \mathcal{A}$ such that $A \cap (G \cup G') = \emptyset$, we have (1) iff (2). □

Let us now see an interesting relation of the notion of blocking set with another notion in the literature on social networks.

Blocking sets in the literature

The interesting connection is between the notion of blocking sets and the definition of global gatekeeper that *Network, Crowds and Markets* (NCM) [25] provides. The two notions are equivalent, when the blocking set is a single agent.

This, because D.Easley and J.Kleinberg define global gatekeepers as the following:

> We say that a node X is a [global] *gatekeeper* if, for some other two nodes Y and Z, every path from Y to Z passes through X.

They define the global gatekeeper as the single agent that belongs to every path connecting other two agents in the network. In that sense, the notion coincides to our notion of blocking set, as by definition also the blocking set belongs to every path that connects the two groups. Note that global gatekeepers are agents with properties that are distinct from local gatekeepers that we discussed above. This notion has a global flavor, as it refers to every path between two agents, while the local version of it just refers to the paths between two agents passing through a given third agent. Then, global gatekeepers are also local gatekeepers, but not vice versa. In more formal terms, the definition of global gatekeeper is the following.

Definition 3.32. (*Global Gatekeeper*). Given two disconnected agents $a, b \in \mathcal{A}$, such that there exists at least a path $P := (a = x_1Rx_2R, \dots, Rx_n = b)$, we say that a node $c \in \mathcal{A}$ is the *global gatekeeper* between a, b iff for every such path P, we have $c \in P$.

Note that in this definition we added some conditions that were not present in the original NCM definition. One such condition is the existence of a path connecting a, b. If no such path exists, then a, b are not connected and thus no gatekeeper c can exist between them. Moreover, we added the condition that the agents a, b are disconnected. Recall that this means that $\{a\} \cup \{b\}$ is not a group. If it were a group, then it would be the case that $(a, b) \in R$, so that again there could not exist any gatekeeper belonging to every path between them.

We now prove that in some cases, global gatekeepers are equivalent to blocking sets. Since global gatekeepers are single agents, then, if any equivalence between the two is possible, it must be one in which blocking sets are singletons. The following proposition shows that in that case the equivalence holds.

Proposition 3.33. *For any two disconnected G, G' and any $a \in \mathcal{A}$ such that $\{a\} \cap (G \cup G') = \emptyset$, the following are equivalent:*

(1) a is a global gatekeeper between every $g \in G$ and every $g \in G'$;

(2) $A = \{a\}$ *is a blocking set between* G, G'.

Proof. Let G, G' be two disconnected groups and consider some $a \in \mathcal{A}$ such that $\{a\} \cap (G \cup G') = \emptyset$. Call $A = \{a\}$.

(1)⇒ (2) Assume that a is a global gatekeeper between some arbitrary $g, \in G$ and $g' \in G'$. By definition of global gatekeeper, for every path $P := (g = x_1Rx_2R, \ldots, Rx_n = g')$, we have $a \in P$. By definition of group, the agents in every such path form a group. Then, every set $G'' = (G \cup G' \cup P)$ is a group, as it is a union of connected groups. Clearly, for every such G'', we have that $(G \cup G') \subset G''$. Since for every P, $a \in P$, then $\{a\} \cap G'' \neq \emptyset$. Thus, $\{a\}$ is a blocking set between G, G'.

(2)⇒ (1) Assume that A is a blocking set between G, G'. Then, for every G'' such that $(G \cup G') \subset G''$, we have that $A \cap G'' \neq \emptyset$. Let g, g' be two arbitrary agents in G, G' respectively. By definition of group, we have that for every path $P := (g = x_1Rx_2R, \ldots, Rx_n = g')$, we have $A \cap P \neq \emptyset$. Since $A = \{a\}$, this means that $a \in P$ for every such path. Thus, a is a global gatekeeper between g, g', by definition. Since g, g' were chosen arbitrarily, this holds for every $g \in G$ and $g' \in G'$.

We can conclude that a is a global gatekeeper between some $g \in G$ and some $g' \in G'$ iff $A = \{a\}$ is a blocking set between G, G'. □

This is a rather interesting equivalence, as it reveals a fundamental distinction between our and NCM's understanding of the notion of gatekeeper. For them, a gatekeeper is necessary and sufficient to block the information flow between two groups, but only necessary to enable it. The global gatekeeper might not be sufficient for the enabling. This, because by definition it is an agent that belongs to every path between the groups, but this does not mean that there is no other agent that is necessary as well to enable the information flow. The perspective under which gatekeepers are agents that only have the capacity to block the information flow is different from the perspective we take in this thesis. We understand gatekeepers as agents that have the power both to block *and* to enable the information flow between two groups, i.e. gatekeepers control the information flow between them. As we saw in the preliminaries, this perspective aligns with the one of other scholars, in particular Barzilai-Nahon (2008) [27].

Moreover, note that Proposition 3.33 provides further insights about the fact that global gatekeepers differ from the local ones (C-local gatekeepers). One of them is that a C-local gatekeeper is not always a blocking set. To see this, recall Example 3.11. There, we showed that a bridge is not necessarily the unique connection between two disconnected groups. Therefore, there exist connections (groups) that a C-local gatekeeper does not intersect or block. This amounts to say that it is not always a blocking set.

Example 3.34. (*Citizen-Mayor Blocking Sets*). Recall Example 3.20 above. We assumed that there are just two bridges between Ann and the mayor: Bob, and Chen and Margit. Now, a blocking set is at least composed by one agent for every bridge, as it must block every connection. Then, Bob must be in every blocking set between Ann and the mayor. Otherwise he alone would form a connection between them, i.e. a group that in this case would not intersect the blocking set. Moreover, either Chen or Margit (or both) must belong to the blocking set too. If none of them belongs to it, then they would form an unblocked connection. So the blocking set must contain Bob and either Chen or Margit. Indeed, if they all decide to block the information, then Ann does not have any other way to communicate with the mayor, and the information flow between them is blocked.

4 The Gatekeepers

In this chapter, we construct the definitions of gatekeepers in social networks. Informally, we define gatekeepers as agents that have the power to enable or block the information flow between two disconnected groups. We represent them formally through structural properties of the network in which they are embedded. In particular, by combining or generalizing the notions introduced thus far, we can define gatekeepers as sets of agents that are necessary and sufficient to enable and block the information flow between the groups.

The chapter is structured similarly to the previous one. This means that for each of the distinct kind of gatekeeper we introduce, we propose its formal definition together with a picture to exemplify it (for the first and the third notions, the definition will be followed by a proposition that gives a clear intuition of what the notions amounts to) and the illustration of why it is sufficient and necessary to enable or block the information flow. Then, we show which properties the

notion has, and conclude its analysis by providing a realistic example.
We start with the introduction of the gatekeeping set. This kind of gatekeeper combines the definition of blocking set with the definition of bridging set. Then, we will introduce the gatekeeping bridge, which is a special case of the definition of bridge and of gatekeeping set. Lastly, the notion of grand gatekeeper, which is an extension of the notion of bridging set. We will conclude the section with a diagram that summarizes all the notions introduced in chapter 3 and 4, and highlights the relationships they entertain with each other.

4.1 Gatekeeping Set

The notion of gatekeeping set is grounded on the notion of bridging set and blocking set. This implies that it can enable *and* block the information flow between two given groups. We show that it is a union of bridges, but that the agents it contains do not necessarily form a group. The gatekeeping set always exists, but it is not unique. In the later sections, we will show that every other kind of gatekeepers are special cases of this one.

Definition 4.1. (*Gatekeeping Set*). Let G, G' be two disconnected groups. We say that $A \subset \mathcal{A}$ is a *gatekeeping set* between G, G' iff

(GS+) A is a bridging set between G and G';

(GS-) A is a blocking set between G and G'.

Recall that bridging sets between two groups are sufficient to enable the information flow between the groups, whereas blocking sets are sufficient to block that flow. Since gatekeeping sets are defined as bridging and blocking sets, then they are sufficient to enable and block the information flow. Moreover, each of them is also necessary to enable and block the information flow between the groups. This comes from the fact that they are a special kind of bridging set, namely such that they are also blocking sets (or a special kind of blocking set, i.e. they are such that they are also bridging sets). Then, if they do not block the information flow, their bridging capacity is sufficient to allow the flow between the groups; vice versa if they do not enable the information to flow, their blocking capacity is sufficient to block the flow between the groups.

Before proceeding with an illustrative example, we show that gatekeeping sets are unions of some bridges. This is immediate from their definition, in particular from the fact that they are bridging sets between G, G'.

Proposition 4.2. *For any two disconnected groups G, G', if $A \subset \mathcal{A}$ is a gatekeeping set between G, G', then A is a union of some bridges $B_1, \ldots, B_n$ between G, G', i.e. $A = \bigcup_{i=1}^{n} B_i$, with $1 \leq i \leq n$.*

Proof. Let G, G' be two disconnected groups and let $A \subset \mathcal{A}$ be a gatekeeping set between G, G'. By definition, A is a bridging set between G, G'. By Proposition 3.22, $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are some bridges between G, G'. □

Note that the above proposition does not show an equivalence between gatekeeping sets and union of bridges. This is because not every union of bridges is also a blocking set. The following example illustrates this and the notion of gatekeeping set itself.

Example 4.3. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 8. We claim that A is a gatekeeping set between G, G', but $A' = \{a, d\}$ is not.

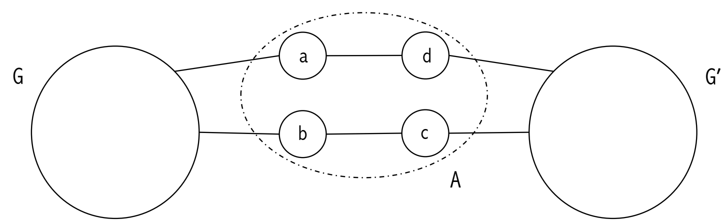

Figure 8

Proof. Consider $A = \{a, b, c, d\}$. It is a bridging set between G, G', because every agent in it lies on a bridge between G, G', i.e. every agent is an A-local gatekeeper between G, G'. Moreover, for every bridge B between G, G', we have $B \subset A$. Since it is clearly the case that $A \cap A \neq \emptyset$, then by definition of blocking set, A is a blocking set between G, G'. It follows that A is a gatekeeping set between G, G'.

Now consider $A' = \{a, d\}$. is a bridge between G, G', so it contains only A'-local gatekeepers. By definition of bridging set, we know that A' is a bridging set between G, G'. However, A' is not a blocking set between G, G', as it does not intersect $A'' = \{b, c\}$, which is a bridge between G, G', thus also a bridging set between them. This means that A' is not a gatekeeping set between G, G'. □

Note that the agents in the gatekeeping set represented in the example above do not form a group, but that it can also be otherwise. Moreover, this examples makes clear that the agents in a gatekeeping set are neither necessary nor sufficient to enable or block the information flow between the groups. This is because in such gatekeeper there might exist several disjoint and disconnected paths between the groups.

Properties of Gatekeeping Set

Gatekeeping sets always exist and they are not unique.

Proposition 4.4. *For any two disconnected groups G, G', there exists a gatekeeping set $A \subset \mathcal{A}$ between them.*

Proof. Let G, G' be two disconnected groups. By Proposition 3.24, there exists a bridging set $B \subset \mathcal{A}$ between G, G'. Consider all the bridging sets $B_1, \ldots, B_n$ between G, G'. Call A the union of all of them, i.e. $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G'. By definition, for every bridging set B_i between G, G', B_i is a connector, i.e. $G \cup G' \cup B_i$ is a group. This implies that $G \cup G' \cup \bigcup_{i=1}^{n} B_i$ is a group too, and that A is a connector between G, G'. Since A is a union of bridging sets, then by definition of bridging set, for every $a \in A$ there exists a bridge $B \subseteq A$ between G, G' such that $a \in B$. By definition of C-local gatekeeper, this means that for every $a \in A$, a is an A-local gatekeeper between G, G'. By definition of bridging sets, this amounts to say that A is a bridging set between G, G'. Then, we can use Proposition **??** to get that $(G \cup G') \cap A = \emptyset$. Moreover, we clearly have that for every bridging set B_i between G, G', $A \cap B_{\neq} \emptyset$, as $A = \bigcup_{i=1}^{n} B_i \neq \emptyset$. So we can use Proposition 3.31 to conclude that A is a blocking set between G, G'. Since at least one bridging set exists, then $A \neq \emptyset$. □

Example 4.5. (*Non-Uniqueness Gatekeeping set*). Let $N = (\mathcal{A}, R)$ be the network represented in Figure 9. We claim that both A, A' are gatekeeping sets between G, G', and thus that the notion of gatekeeping set is not unique.

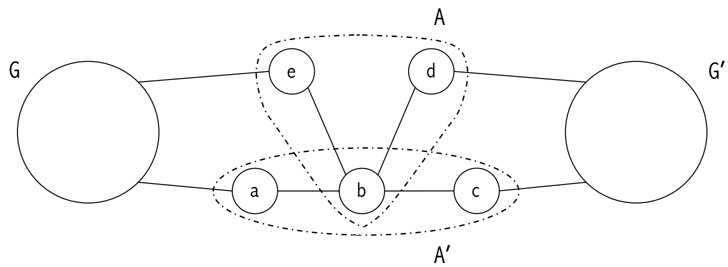

Figure 9

Proof. Consider $A = \{e, b, d\}$. Since $G \cup G' \cup A$ is a group, then A is a connector between G, G'. This means that A satisfies clause (B+) of definition of bridge. Moreover, $G \cup G' \cup (A \setminus \{e\})$ is not a group anymore, i.e. $(A \setminus \{e\})$ is not a connector between G, G'. The same holds if we substitute b or d to e. Then, for every agent x in A it is the case that $(A \setminus \{x\})$ is not a connector between G, G'. This means that A satisfies also clause (B-) of definition of bridge, i.e. A is a bridge between G, G'. Then, every agent in A is an A-local connector between G, G', i.e. A is a bridging set. Now, there are two connectors between G, G', either A or A'. Since $A \cap A = A$ and $A \cap A' = \{b\}$, then for every connector C between G, G' we have that $A \cap C \neq \emptyset$. By definition of blocking set, this means that A is a blocking set between G, G'. Since $A \neq \emptyset$, we can conclude that A is a gatekeeping set between G, G', by definition of gatekeeping set.

Now consider $A' = \{a, b, c\}$. Since $G \cup G' \cup A'$ is a group, then A' is a connector between G, G', i.e. A' satisfies clause (B+) of definition of bridge. Moreover, for every $x \in A'$, $(A' \setminus \{x\})$ is not a connector between G, G', as $G \cup G' \cup (A' \setminus \{x\})$ is not a group anymore. It follows that A' is a bridge between G, G'. Since $A' \cap A' = A'$ and $A' \cap A = \{b\}$, then for every connector C between G, G' we have that $A' \cap C \neq \emptyset$. Thus A' is a blocking set between G, G'. Since $A' \neq \emptyset$, we can conclude that A' is a gatekeeping set between G, G'.

Since $A \neq A'$, we can conclude that the notion of gatekeeping set is not unique.

□

In the example above, note that $A \cup A'$ is a gatekeeping set too. This is because $A \cup A'$ is a union of bridges, thus a bridging set by Proposition 3.22. Moreover, it clearly intersects every connector between G, G', as the only connectors are A and A'. It follows that A is a gatekeeping set, by definition 4.1. This shows another property of gatekeeping sets, namely that they might contain other gatekeeping sets in them.

The last property of gatekeeping set we show is also another consequence of their being defined as bridging sets: they have no agents in common with the groups that they connect.

Proposition 4.6. *For any $A \subset \mathcal{A}$, if A is the gatekeeping set between two distinct groups G, G', then $A \cap (G \cup G') = \emptyset$.*

Proof. Let $A \subset \mathcal{A}$ be the gatekeeping set between some distinct groups G, G'. By clause (GS-) of definition of gatekeeping set, A is a blocking set between G, G'. By definition of blocking set, $A \cap (G \cup G') = \emptyset$. □

Before concluding the section about gatekeeping sets, we propose a realistic example of the notion, taken from Italian politics.

Example 4.7. (*Berlusconi and his Gatekeeping set*). Imagine the following situation. We are in Italy, year 1994. Berlusconi just became the prime minister. In that position, he gained control over the three public television channels, and since he already owned the other three major channels, plus several newspapers, the Italian public information and media space were in his hands. As it was later established [28][29], at that time he was entertaining close relationships with members of "Cosa Nostra", one of the Italian mafia clans. Now imagine that Marco, an independent journalist, has gotten in possession of some information about these relationships and wanted to inform the population about it through the public television channels. He contacts Carlo and Diana, two workers for one of the public televisions. Carlo is thrilled by the news and wants to spread it as soon as possible. Diana instead is not, and suggests to ignore it. In fact, Diana is a member of Cosa Nostra, so spreading that information is not in her interests. She immediately gets in touch with Berlusconi, who indeed decides to block it. Carlo is powerless. He cannot choose to spread it, as such decision would anyway have to pass through Berlusconi, who now controls the media space. Thanks to the power he has, Berlusconi can decide to block that information and spread its opposite, namely that he is instead trying to fight mafia. So he instructs his most loyal employees Diana and Elmo to spread it, and the result is that a big portion of population receives that false information.

The situation just described is represented in Figure 9, where $G' = \{Marco\}$, $G = \{\text{the public}\}$, Diana is d, Carlo is c, Berlusconi b and Elmo e. Berlusconi, Diana ed Elmo can both block the information about the relationships between Berlusconi and members of the mafia (indeed, Berlusconi alone can achieve that), and they can also enable the flow of false information. This means that they are a blocking set and a bridging set between the public and the mafia, i.e. they form a gatekeeping set.

4.2 Gatekeeping Bridge

Gatekeeping bridges are a special case of gatekeeping sets. They are the minimal connectors and blocking sets between two groups, and they are unique, but not always existing. We show that they are gatekeeping sets and bridges, but that vice versa does not hold. In addition, we show that we can draw an interesting relationship between them and the notion of global gatekeeper in the literature.

Definition 4.8. (*Gatekeeping Bridge*). Let G, G' be two disconnected groups. We say that $A \subset \mathcal{A}$ is the *gatekeeping bridge* between G, G' iff

(B+) A is a connector between G, G';

(GB-) for all $A' \subset \mathcal{A}$, if A' is a connector between G, G', then $A \subseteq A'$.

By this definition, a gatekeeping bridge is a connector. We saw in Chapter 3 that connectors are sufficient to enable the information flow between the groups they connect. Then, gatekeeping bridges are sufficient to enable it too. Moreover, they are also necessary for that, as we will see below that they are the unique sets of agents that connect the groups. Then, if they do not enable it nobody else can. By the same reasoning, they are necessary and sufficient to block the information flow.

Example 4.9. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 10. We claim that B is the unique connector between G, G' which is also a gatekeeping bridge between G, G'.

Proof. Consider $B = \{a, b, c\}$. We first show that B is a gatekeeping bridge between G, G' and then show that for any other connector between G, G', this is not a gatekeeping bridge. Clearly, B is a connector between G, G', as $G \cup G' \cup B$

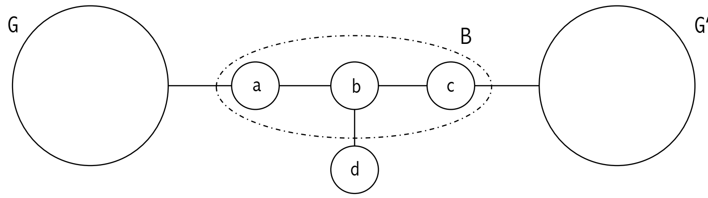

Figure 10

is a group. So A satisfies clause (B+) of definition of gatekeeping bridge. Note that the only connector $B' \subset \mathcal{A}$ between G, G' such that $B' \neq B$ is $B' = \{a, b, c, d\}$. Since $A \subset A'$, then A satisfies also clause (GB-) of definition of gatekeeping bridge. Thus A is a gatekeeping bridge between G, G'.

Now note that the only other connector B' is not a gatekeeping bridge between G, G'. This is because there exists a connector, namely B, such that $B' \not\subseteq B$, i.e. B' does not satisfy clause (GB-) and is not a gatekeeping bridge between G, G'. □

Properties of Gatekeeping Bridge

The first property of gatekeeping bridges we show is that they do not always exist. To show this we use one of the examples we introduced above.

Example 4.10. (*Non-Existence of Gatekeeping Bridge*). Consider the example proving the non-uniqueness of gatekeeping sets, namely Example 4.5. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 9. We claim that neither A nor A' are a gatekeeping bridge between G, G', and thus that a gatekeeping bridge between two disconnected groups G, G' does not always exist.

Proof. In Example 4.5 we have that $G \cup G' \cup A$ and $G \cup G' \cup A'$ are both groups. This means that A and A' are two connectors between G, G', and that they satisfy (B+) of definition of gatekeeping bridge. But neither of them satisfy clause (GB-). Consider A. For A this is because there exists a connector A' between G, G' such that $A \not\subseteq A'$. Analogously for A'. Thus, neither A not A' are a gatekeeping bridge between G, G'. We can conclude that a gatekeeping bridge between two disconnected groups G, G' does not always exist. □

So gatekeeping bridges between two disconnected groups do not always exist.

However, when they do they are unique.

Proposition 4.11. *For any $A \subset \mathcal{A}$, if A is the gatekeeping bridge between G, G', then A is unique.*

Proof. Consider some $A \subset \mathcal{A}$ such that A is the gatekeeping bridge between some G, G'. We want to prove that for any $A' \subset \mathcal{A}$, if $A \neq A'$ then A' is not the gatekeeping bridge. Consider some arbitrary $A' \subset \mathcal{A}$ such that $A' \neq A$. We have two cases, either A' is not a connector between G, G', or it is. In the first case, A' does not satisfy clause (B+), so A' is not a gatekeeping bridge between G, G'. So consider the second case. Note that A is the gatekeeping bridge, so by clause (GB-) of its definition we get $A \subseteq A'$. Since by assumption $A \neq A'$ then $A \subset A'$. So we have that $G \cup G' \cup A$ is a group and $A' \nsubseteq A$, which means that clause (GB-) of definition of gatekeeping bridge does not hold for A'. Thus A' is not the gatekeeping bridge. Since A' was chosen arbitrarily, we can conclude that for any $A' \subset \mathcal{A}$, if $A \neq A'$ then A' is not the gatekeeping bridge. Hence, A is unique. □

The next proposition shows that gatekeeping bridges between two disconnected groups are in fact bridges between the two groups.

Proposition 4.12. *For any $A \subset \mathcal{A}$, if A is the gatekeeping bridge between two disconnected groups G, G', then A is a bridge between G, G'.*

Proof. Let $A \subset \mathcal{A}$ be the gatekeeping bridge between G, G'. By definition of gatekeeping bridge, we know that $A \neq \emptyset$, and A is a connector between G, G'. Now consider some arbitrary $a \in A$. Let $A' = A \setminus \{a\}$. Clearly $A \nsubseteq A'$, so by the contrapositive of (GB-) we know that A' is not a connector between G, G', i.e., $(A \setminus \{a\})$ is not a connector between G, G'. Since a was chosen arbitrarily, then the result holds for every $a \in A$. Thus, A satisfies (B+) and (B-). We can conclude that if A is a gatekeeping bridge between some disconnected G, G', then A is a bridge between G, G'. □

Since they are (unique) bridges, gatekeeping bridges have all the properties that (unique) bridges have.

Proposition 4.13. *For any $A \subset \mathcal{A}$, if A is the gatekeeping bridge between some disconnected groups G, G', then (1) $(G \cup G') \cap A = \emptyset$; (2) A is a group; (3) A is linearly ordered; (4) A is the minimal set of agents such that A is a connector between G, G'.*

Proof. Let $A \subset \mathcal{A}$ be the gatekeeping bridge between G, G'.

(1) By Proposition 4.12, we know that A is a bridge between G, G'. By Proposition 3.14, we know that $(G \cup G') \cap A = \emptyset$.

(2) By Proposition 4.12, we know that A is a bridge between G, G'. By Proposition 3.14, we know that bridges are groups. Thus A is a group too.

(3) By Proposition 4.12, we know that A is a bridge between G, G'. By Proposition 3.16, we know that bridges are linearly ordered. Thus A is linearly ordered too.

(4) Consider an arbitrary $A' \subset \mathcal{A}$ and suppose that $A' \subset A$. By Proposition 4.12, A is a bridge between G, G', so we can use Proposition 3.12 to get that A' is not a connector between G, G'. Since A' was chosen arbitrarily, then for every $A' \subset \mathcal{A}$ such that $A' \subset A$ we have that A' is not a connector between G, G'. This amounts to say that A is the minimal set of agents such that A is a connector between G, G'. □

Gatekeeping bridges are not only a special case of bridges. They are also a special case of gatekeeping sets.

Proposition 4.14. *For any $A \subset \mathcal{A}$, if A is a gatekeeping bridge between some disconnected G, G', then A is a gatekeeping set between G, G'.*

Proof. Let $A \subset \mathcal{A}$ be the gatekeeping bridge between G, G'. By definition of gatekeeping bridge, $A \neq \emptyset$ and A is a connector between G, G'. By (GB-), for every $A' \subseteq \mathcal{A}$, if A' is a connector between G, G', then $A \subseteq A'$. This implies that for every connector A' between G, G', we have that $A \cap A' \neq \emptyset$. By definition 3.27, A is a blocking set between G, G', i.e. A satisfies (GS-) of definition of gatekeeping set. Moreover, by Proposition 4.12, A is itself a bridge. This means that every $a \in A$ is an A-local connector between G, G'. By definition of bridging set, we have that A is a bridging set between G, G'. This amounts to say that A satisfies (GS+) of definition of gatekeeping set and thus also that A is a gatekeeping set between G, G'. We can conclude that if A is a gatekeeping bridge between some disconnected G, G', then A is a gatekeeping set between G, G'. □

Notably, the two propositions above show that for every set A, if A is a gatekeeping bridge between two disconnected groups G, G', then A is both a bridge and a gatekeeping set between the groups. However, the other way around does not hold, i.e. if A is a gatekeeping set and a bridge between two disconnected groups G, G', then A is not always a gatekeeping bridge. To illustrate, consider the following example.

Example 4.15. (*Gatekeeping Set + Bridge ≠ Gatekeeping Bridge*). Consider again the example we used to prove the non-uniqueness of gatekeeping sets, i.e. Example 4.5. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 9. We claim that A and A' are both gatekeeping sets and bridges between G, G', but not gatekeeping bridges between G, G'.

Proof. In Example 4.5, we showed that A and A' are both bridges and gatekeeping sets between G, G'. Instead, in Example 4.10 we showed that A and A' are not gatekeeping bridges between G, G'. Thus, we can conclude that A and A' are both gatekeeping sets and bridges between G, G', but not gatekeeping bridges between G, G'. □

Gatekeeping Bridge and the Literature

In this subsection, we discuss a relationship between the proposed definition of gatekeeper and the one of global gatekeeper, as proposed by Easley and Kleibnerg [25]. Recall that given two agents, a global gatekeeper is an agent that lies in every path connecting them. Since a gatekeeping bridge is the unique set of agent that connects two disconnected groups, then it must be the case that it lies in every path connecting the two groups. To show this, we generalize the definition of global gatekeeper, lifting it from being defined between two agents to being defined between two groups.

Definition 4.16. (*Global Gatekeeper between two Groups*). Given two disconnected groups G, G', such that for every agent $g \in G$ and $g' \in G'$ there exists a path $P := (g = x_1 R x_2 R, \ldots, R x_n = g')$, we say that an agent c is a *global gatekeeper* between G, G' iff for every such path P, we have $c \in P$.

Given this notion, we can now show that a gatekeeping bridge between two groups G, G' is equivalent to the set of global gatekeepers between G, G'.

Proposition 4.17. *For any two disconnected groups G, G', and for any $A \subset \mathcal{A}$, the following are equivalent:*

(1) A is the gatekeeping bridge between G, G';

(2) *A is the set of all global gatekeepers between two groups G and G'.*

Proof. Let G, G' be two disconnected groups and consider some $A \subset \mathcal{A}$.

(1) $\Rightarrow$ (2). Assume that A is the gatekeeping bridge between G, G'. By definition of gatekeeping bridge, for every $A' \subset \mathcal{A}$, we have that $A \subseteq A'$. It follows that for every path P between any $g \in G$ and $g' \in G'$ we have that $A \subseteq P$. This means that for every agent $a \in A$, $a \in P$, and that every agent in A is a global gatekeeper between G, G'. Now suppose towards contradiction that there exists a global gatekeeper a between G, G' such that $a \notin A$. By definition of global gatekeeper between groups, this means that a belongs to every path between G, G'. But by Proposition 4.11 the only connector between G, G' is A. Since $a \notin A$ then a does not belong to every path between G, G'. Contradiction. We can conclude that A is the set of all global gatekeepers between G, G'.

(2) $\Rightarrow$ (1). Assume that A is the set of all global gatekeepers between G, G'. Suppose towards contradiction that A is not the gatekeeping bridge between G, G'. This means that for some $A \not\subset A'$, A' is a connector between G, G'. Call $A'' = A \setminus A'$. Clearly $A'' \subseteq A$ and $A'' \cap A' = \emptyset$. Since A' is a connector between G, G' then $G \cup G' \cup A'$ is a group, i.e. there exists a path connecting G, G' passing for the agents in A'. Then, for at least some $a \in A''$, there exists some path P between G, G' such that $a \notin P$. Since $a \in A''$ then $a \in A$. Since for all $a \in A$, a is a global gatekeeper between G, G' then for every path P between G, G' $a \in P$. Contradiction. We can conclude that A is the gatekeeping bridge between G, G'.

Since G, G' and A were chosen arbitrarily, we can conclude that for any two disconnected groups G, G', and for any $A \subset \mathcal{A}$,A is the gatekeeping bridge between G, G' iff A is the set of all global gatekeepers between two groups G and G'. □

Proposition 4.17 shows that gatekeeping bridges generalize the notion of global gatekeeper between groups. If the global gatekeeper between two groups was itself a generalization of the global gatekeeper between agents, then the gatekeeping set is yet another kind of generalization of the same notion. This, because gatekeeping bridges are sets of agents lying on every path between groups, and not only a single agent, as in the definition of global gatekeeper.

Moreover, note that the fact that a gatekeeping bridge is composed only by global gatekeepers means that each of them is sufficient to block the information flow between the groups, as each of them lies in every path that exists between the groups. However, it is not necessary, as every other member of the gatekeeping bridge can block it as well. On the other hand, global gatekeepers are necessary to enable the information flow between the groups, but not sufficient to that, as the reader can immediately verify.

To conclude the discussion of gatekeeping bridges, let us now consider a realistic example of them.

Example 4.18. (*Reporting to the General Secretary*). Imagine to be Aikilah, a young and talented social researcher working for the Dutch Minister for Social Affairs and Employment. Among Aikilah's tasks, there is the one to find the newest and most interesting reports about relevant social research, and inform the General Secretary about it. The Secretary does not have much time to read the very long reports that social researchers write, so Aikilah needs to make a very condensed summary of them. To check that the report that she will write complies with the standards, it will have to go through several steps. In each of these steps, a different employee will check whether the requirements of brevity and clarity are satisfied, and pass it on to the next. Only if all of the checking agents approve it, the report will arrive to the General Secretary; otherwise it will be blocked and sent back to Aikilah.

The situation can be represented by Figure 10. There, we have $G = \{\text{Aikilah}\}$, $G' = \{\text{The General Secretary}\}$, and a, b, c are three agents that control Aikilah's report and eventually block it and send it back. These agents form a gatekeeper between Aikilah and the General Secretary, as any other path that will lead her report to its goal, has to pass through them.

4.3 Grand Gatekeeper

The notion of grand gatekeeper extends the definition of bridging set between two disconnected groups of agents, making it maximal. This is one of the reasons why we call it *grand* gatekeeper, since it contains all the bridges and bridging sets between two groups, thus also all the agents that, together, can control or gatekeep the information flow. Moreover, it coincides with the maximal gatekeeping set. We show that the grand gatekeeper is particularly well-behaved, as it always exists (recall that we assume to be working in a connected network) and it is unique. In addition, we show that it is a blocking and a gatekeeping sets, but that vice versa does not always hold. Lastly, we prove that every gatekeeping bridge is a grand gatekeeper, and that grand gatekeepers contain all local gatekeepers between two groups.

Definition 4.19. (*Grand Gatekeeper*). Let G, G' be two disconnected groups. We say that $A \subset \mathcal{A}$ is the *grand gatekeeper* between G, G' iff A is a maximal bridging set, i.e.

(i) A is a bridging set between G, G';

(ii) for every $A' \subset \mathcal{A}$ such that $A \subset A'$, A' is not a bridging set between G, G'.

Grand gatekeepers are thus defined as bridging sets. We saw above that bridging sets are sufficient to enable the information flow between the groups they connect. Then, grand gatekeepers are sufficient to enable it too. In addition, the fact that they are unique implies that they are also necessary for that (we will see the proof of their uniqueness below). This, because if they do not enable the information to flow then nobody else can. This is the same reasoning we used for gatekeeping bridges, and can be used also to show that they are necessary and sufficient to block the information flow.

In the previous chapter we introduced bridging sets between two groups, and we showed that they are the union of *some* bridges between them. The following proposition proves that the grand gatekeeper between two groups is the union of *all* the bridges between them.

Proposition 4.20. *For any G, G' that are two disconnected groups in the same connected component of the network, and for any $A \subset \mathcal{A}$ the following are equivalent:*

(1) A is the grand gatekeeper between G, G';

(2) A is the union of all the bridges $B_1, \ldots, B_n$ between G, G', i.e. $A = \bigcup_{i=1}^{n} B_i$, with $1 \leq i \leq n$.

Proof. Let G, G' be two disconnected groups in the same connected component of the network N and consider some $A \subset \mathcal{A}$.

Then $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G' iff for every $a \in A$, we have $a \in A' \subseteq A$ such that A' is a bridge between G, G', i.e., clause (i) of Definition 4.19 holds.

Moreover, $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G' iff for all sets $B_j \subset \mathcal{A}$ such that $B_j \notin A$, B_j is not a bridge between G, G' iff $B_j \cup A$ is not a bridging set between G, G' iff clause (ii) of Definition 4.19 holds.

Therefore we have $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G' iff clause (i) and (ii) of Definition 4.19 hold, i.e., A is the grand gatekeeper between G, G'. □

Example 4.21. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 10. We claim that the set $A = \{a, b, c, d\}$ is the grand gatekeeper between G, G'.

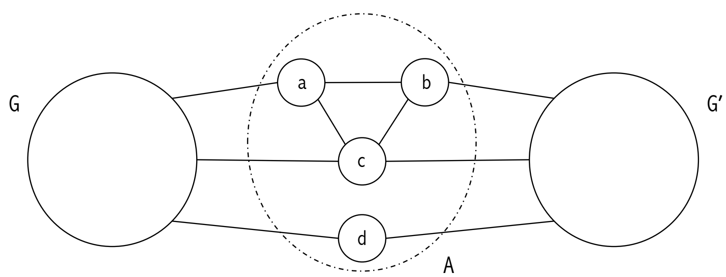

Figure 11

Proof. In order for A to be a grand gatekeeper between G, G', it should be a bridging set, and contain all the bridges between G, G'. Clearly, $B = \{d\}$ is a bridge between G, G', as $(B \setminus \{d\})$ is not a connector between G, G', while B is. Also the set $B' = \{a, b\}$ is a bridge, as $(B' \setminus \{a\})$ or $(B' \setminus \{b\})$ are not connectors between G, G', while B' is. Moreover, the set $B'' = \{c\}$ is a bridge too, as $(B'' \setminus \{c\})$ is not a connector between G, G', while B'' is. Therefore, all the agents in A lie on bridges between G, G', i.e. they are A-local gatekeepers between G, G'. Then, A is a bridging set between G, G'. Note that there exist no other bridge between G, G'. Thus, A contains all the bridges between G, G', i.e. it is the union of all of them. By Proposition 4.20, this amounts to say that A is the grand gatekeeper between them. □

In this example, we can appreciate the fact that the agents in a grand gatekeeper are neither necessary nor sufficient to enable or block the information flow between the groups. As for the gatekeeping set, this depends on the fact that several disjoint paths connecting the groups can coexist in a grand gatekeeper.

Properties of Grand Gatekeeper

The first property we show is that the grand gatekeeper between two disconnected groups always exists.

Proposition 4.22. *For any two disconnected groups G, G', there exists a grand gatekeeper $A \subset \mathcal{A}$ between G, G'.*

Proof. Let G, G' be two disconnected groups. By Proposition 4.20, we know that for any grand gatekeeper A between two disconnected groups, we have $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G'. By Proposition 3.10 we know that there always exists at least one bridge between G, G'. Hence, there always exists a grand gatekepeer between G, G'. □

Proposition 4.23. *For any two disconnected groups G, G', the grand gatekeeper $A \subset \mathcal{A}$ between G, G' is unique.*

Proof. Let G, G' be two disconnected groups. By Proposition 4.22 we know there exists a grand gatekeeper B between them. Suppose towards contradiction that there exists another grand gatekeeper A' between them. By Proposition 4.20, $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G'. Moreover, $A' = \bigcup_{i=1}^{n} B_i$ too. Therefore, $A = A'$, which contradicts that A and A' are distinct sets. We can conclude that for any two disconnected groups G, G', the grand gatekeeper A between G, G' is unique. □

For what concerns the relationships between grand gatekeepers and the other notions we introduced so far, we now show that a grand gatekeeper is a blocking set and thus also a gatekeeping set between the groups. However, vice versa does not hold, namely a gatekeeping set is not always a grand gatekeeper.

Proposition 4.24. *For any $A \subset \mathcal{A}$, if A is the grand gatekeeper between some disconnected groups G, G', then A is a blocking set between G, G'.*

Proof. Let G, G' be two disconnected groups and suppose that $A \subset \mathcal{A}$ is the grand gatekeeper between them. By Proposition 4.20, $A = \bigcup_{i=1}^{n} B_i$, where $B_1, \ldots, B_n$ are all the bridges between G, G'. By definition, A is a bridging set, thus a connector between G, G'. Then, for all the A-local gatekeepers a between G, G', we have that $a \in A$. Since bridging sets are formed by local gatekeepers, it follows that for every bridging sets B' between G, G', we have $A \cap B \neq \emptyset$. Then we can use Proposition 3.31 to get that A is a blocking set between G, G'. □

Proposition 4.25. *For any $A \subset \mathcal{A}$, if A is the grand gatekeeper between some disconnected groups G, G', then A is a gatekeeping set between G, G'.*

Proof. Let G, G' be two disconnected groups and suppose that $A \subset \mathcal{A}$ is the grand gatekeeper between them. By definition, a grand gatekeeper is a bridging set. By Proposition 4.24, A is a blocking set between G, G'. Thus, A satisfies both clauses of definition of gatekeeping set, and since by definition of gatekeeping set we have that $A \neq \emptyset$, we can conclude that A is a gatekeeping set between G, G'. □

Example 4.26. (*Gatekeeping Sets ≠ Grand Gatekeepers*). Consider again the example we used to prove the non-uniqueness of gatekeeping sets, i.e. Example 4.5. Let $N = (\mathcal{A}, R)$ be the network represented in Figure 9. We claim that both A and A' are gatekeeping sets between G, G', but neither is a grand gatekeeper between G, G'.

Proof. In Example 4.5, we showed that A and A' are both gatekeeping sets and bridges between G, G'. By Proposition 4.20, a grand gatekeeper is the union of all bridges between G, G'. But neither A nor A' contain the other, so neither of them is the grand gatekeeper between G, G'. □

Even if the notion of gatekeeping set does not coincide with the one of grand gatekeepers, the following proposition shows that if we consider the maximal gatekeeping set between two groups, then the two notions coincide.

Proposition 4.27. *For any two disconnected groups G, G', and for any $A \subset \mathcal{A}$, the following are equivalent:*

(1) A is the grand gatekeeper between G, G'.

(2) *A is a maximal gatekeeping set between G, G', i.e. A is a gatekeeping set and for any $A \subset A'$, A' is not a gatekeeping set between G, G'.*

Proof. Let G, G' be two disconnected groups and consider some $A \subset \mathcal{A}$.

(1) ⇒ (2). Let A be the grand gatekeeper between G, G'. By Proposition 4.27, we know that A is a gatekeeping set between G, G'. Consider an arbitrary $A \subset A'$. Since A is the grand gatekeeper between G, G', then by Proposition 4.20, A contains all the bridges $B_1, \ldots, B_n$ between G, G', i.e. $A = \bigcup_{i=1}^{n} B_i$. This means that A' contains some agents a such that there exists no bridge B' between G, G' with $a \in B'$. Then, A' is not a bridging set between G, G', by definition of bridging set. This amounts to say that A' is not a gatekeeping set between G, G', by definition of gatekeeping set. Since A' was chosen arbitrarily, we can conclude that for every A' such that $A \subset A'$, A' is not a gatekeeping set between G, G'. We can conclude that if A is the grand gatekeeper between G, G', then A is the maximal gatekeeping set between G, G.

(2) ⇒ (1). Let A be the maximal gatekeeping set between G, G'. By Proposition 4.22, we know that the grand gatekeeper A' between G, G' exists. By the first part of this proof, we know that it is the maximal gatekeeping set, i.e. $A = A'$. We can conclude that if A is the maximal gatekeeping set between G, G', then A is the grand gatekeeper between G, G'.

We can conclude that for any two disconnected groups G, G', and for any $A \subset \mathcal{A}$, A is the grand gatekeeper between G, G' iff A is the maximal gatekeeping set between G, G'. □

Another relation between grand gatekeepers and the notions we introduced so far is with the gatekeeping bridge.

Proposition 4.28. *For any $A \subset \mathcal{A}$, if A is the gatekeeping bridge between some disconnected groups G, G', then A is the grand gatekeeper between G, G'.*

Proof. Let G, G' be two disconnected groups and suppose that $A \subset \mathcal{A}$ is the gatekeeping bridge between them. By definition of gatekeeping bridge, A is a connector between the groups. By Proposition 4.11, A is the unique gatekeeping bridge between G, G', and by Proposition 4.12, A is a bridge between G, G'. It follows that A is the unique bridge between G, G'. Then, for every $A' \subset \mathcal{A}$ such that $A \subset A'$, A' is not a bridging set between G, G'. For suppose not. Then there is some A'-local gatekeeper a between G, G' such that $a \in A'$ but $a \notin A'$. This implies that there exists a bridge $B \subset A'$ between G, G' such that $B \neq A$, which contradicts the uniqueness of A. Hence, we can conclude that A is the maximal bridging set between G, G', i.e. A is the grand gatekeeper between them. □

Grand Gatekeeper and the Literature

In this subsection, we draw a connection between grand gatekeepers and the notion of C-local gatekeeper. Recall that this notion is inspired by Easley and Kleinberg [25] and it represents the agents belonging to bridges between two groups, in a given connector between the groups. We now show that the grand gatekeeper between two disconnected groups is the set that contains all the local gatekeepers between the groups. We achieve this result by taking the connector on which the local gatekeepers lie as the whole social network. It is possible to see the network as a connector because we assume to be working in a connected network. A connected network is in fact a set of agents such that there exists at least a path connecting all of them, so also the ones belonging to any two disconnected groups. Then, the set of all agents in the network is a connector between the groups.

Proposition 4.29. *For any two disconnected groups G, G' in a connected network $N = (\mathcal{A}, R)$, and for any $A \subset \mathcal{A}$, the following are equivalent:*

(1) A is the grand gatekeeper between G, G';

(2) A is the set of all the $\mathcal{A}$-local gatekeepers between G, G'.

Proof. Let G, G' be two disconnected groups and consider some $A \subset \mathcal{A}$.

(1) ⇒ (2). Assume that A is the grand gatekeeper between G, G' and suppose towards contradiction that there exists an $\mathcal{A}$-local gatekeeper a between G, G' such that $a \notin A$. This is equivalent to say that there exists a bridge $B \subset \mathcal{A}$ between G, G' such that for some $a \in B$, $a \notin A$, i.e. $B \nsubseteq A$. But by Proposition 4.20, $A = \bigcup_{i=1}^{n} B_i$, for all the bridges $B_1, \ldots, B_n$ between G, G'. So for every B_i between G, G', we have $B_i \subseteq A$. Contradiction. We can conclude that A is the set of all the $\mathcal{A}$-local gatekeepers between G, G'.

(2) ⇒ (1). Assume that A is the set of all $\mathcal{A}$-local gatekeepers between G, G'. This means that for every $b \in \mathcal{A}$ such that for any bridge B between G, G', it is the case that $b \in B$, we have $b \in A$. It follows that for every bridge $B_1, \ldots, B_n$ between G, G', we have $B_i \subseteq A$, i.e. $A = \bigcup_{i=1}^{n} B_i$. By Proposition 4.20, we can conclude that A is the grand gatekeeper between G, G'.

Since G, G' and A were chosen arbitrarily from a connected network N, we can conclude that for any two disconnected groups G, G' in a connected network $N = (\mathcal{A}, R)$, and for any $A \subset \mathcal{A}$, A is the grand gatekeeper between G, G' iff A is the set of all the N-local gatekeepers between G, G'. □

With this proposition we conclude the analysis of the grand gatekeepers' properties. As it is by now become usual, to conclude the whole section about grand gatekeepers, we propose an example illustrating how this notion is instantiated in the real world.

Example 4.30. (*The Grand Gatekeeper between Citizen-Mayor*). In one of the examples above, we have already seen a realistic example of grand gatekeeper. Consider again Example 3.26. There we said that the set comprising Bob, Chen and Margit forms a bridging set between Ann and the mayor. Since this set contains all the minimal connections (bridges) existing between the two, then it is the grand gatekeeper between them.

4.4 Diagram of the structural notions

So far we have introduced many structural notions. To provide the reader with a clearer picture of them and of the relations they entertain with each other, we propose the diagram in Figure 12 below. It is to be read as a Venn diagram, as the relations between the mentioned notions are set theoretical relations. To give an example of how to read it, the gatekeeping bridge is represented as a subset of bridge, blocking set, grand gatekeeper, gatekeeping set, bridging set, and connector.

Since the relationships between notions in the graph are set theoretical ones, then, if a notion is a subset of another one, it shares all the properties of its superset. This means that through this diagram we can also represent the

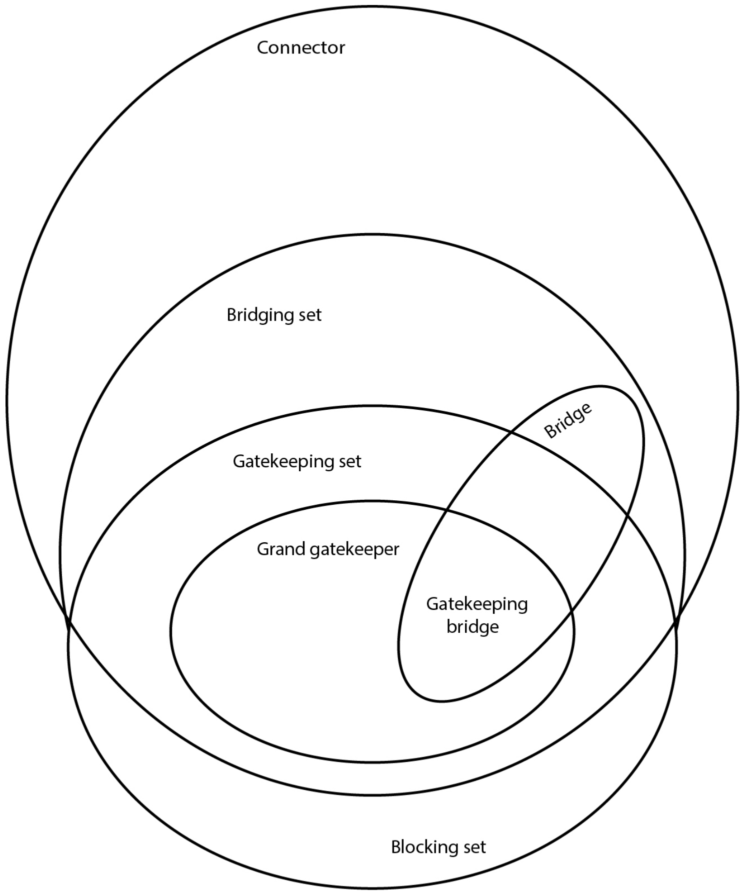

Figure 12

relationships between the notions and the properties of being necessary or sufficient to enable or block the information flow. To illustrate, recall that a connector has the property of being sufficient to enable the information flow between the groups. Then, every notion represented as a subset of connectors has this property too, e.g. bridges and bridging sets are sufficient to enable the flow between the groups they bridge. Moreover, recall that blocking sets are sufficient to block the flow. Then, since gatekeeping set, grand gatekeeper, and gatekeeping bridge lie at the intersection between blocking sets and connectors, this means they are sufficient to enable and block the information flow between the groups. For what concerns the necessity property, recall that connectors, bridging sets, bridges and blocking sets are not always necessary to enable or block the information flow between the groups. However, we showed that their interplay in the notion of gatekeeping set makes the latter necessary both to enable and to block the information flow between the groups. Then, grand gatekeeper and gatekeeping bridge are necessary to enable and block it too, for they are subsets of the gatekeeping set.

Importantly then, the three kinds of gatekeepers coincide on the property of being necessary and sufficient to block and enable the information flow between the groups, i.e. on the fact that they can control the information flow between them. Recall that Barzilai-Nahon defined gatekeepers as agents that control the information flow as it moves through the gate [7]. Then, by taking the gated as the groups that the gatekeeper controls, and the gate the relationships between the gated and the gatekeeper itself, we obtain that the three kinds of gatekeeper all satisfy the unique definition that Barzilai-Nahon provides.

5 Network Logic

Network logic is a hybrid version of propositional dynamic logic (PDL) that we will use to model social networks. As we said in the introduction, similar network logics have already been introduced. However, each of these logics contain elements that we do not need. For example, Smets et al. [15] or Christoff et al. [30] propose a static social network logic, but they introduce it together with propositional variables to express agents' personal features. Also Seligman et al. [14] propose a network logic, but an epistemic version of it. We need neither feature variables, as we do not need to qualify the properties of agents' in the network, nor epistemic operators, as we do not discuss knowledge or belief. For the scope of this thesis, all we need is a logic that represents the network structure and the notions we introduced so far. Then, the network logic we propose is simpler than the two above, and essentially amounts to the one proposed by Smets et al. or Christoff et al., but without the propositional variables expressing agents' features. We will see that this simple logic is sufficient to represent almost all the notions we introduced. This logic will be based on social network models, which is a setting already introduced and used by Smets et al. [15].

The structure of this chapter is quite simple. After having briefly presented PDL and hybrid logic, we will introduced the syntax and semantics of network logic. Then, we will characterize the structural notions in this logic.

5.1 Syntax and Semantics of Network Logic

The network logic we are going to introduce is an hybrid version of PDL, which is a branch of modal logic [31]. The peculiarity of PDL is that it allows the construction of modalities from basic programs π. For example, $[\pi]$ is a modality constructed from program π, and $[\pi]\phi$ is to be read as "for every execution of program π we will reach a state where ϕ is true". In addition, complex programs can be constructed from basic programs, by combining the latter with each other through some operation. The operations we will use below are choice (if π_1 and π_2 are programs, then so is $\pi_1 \cup \pi_2$), composition (if π_1 and π_2 are programs, then so is $\pi_1; \pi_2$), iteration (if π is a program, then so is π^*), test (if ϕ is a formula, then $?\phi$ is a program). When providing the definition of the network language satisfaction, we will also see what these programs mean.

In this basic version of PDL, one cannot call single agents or states in the network. Considering an hybrid version of PDL then means adding expressive power to PDL to allow it. We add expressing power by introducing a new set of propositional variables, the nominals $Nom = \{g, i, j, \dots\}$ and by expanding the valuation function in the models so to bind these new variables with fixed agents. Note that we add only the nominals for *some* of the agents in the network, and not all of them. This is because networks might be composed by millions of agents, but not all such agents are relevant for the purposes of analyzing network gatekeepers.
In addition to nominals that call single agents, we introduce also a second sort of propositional variables $G, I, J, \dots$ that denote groups. This is a novelty of this thesis. We call $Nom^G = \{g, i, j, \dots, G, I, J, \dots\}$ the new set of propositional variables for both types of nominals. Moreover, we also introduce to the language a set $\mathcal{D}$ of propositional variables $\mathcal{D} = \{d, d', \dots\}$, which represents data-bits, namely information that agents exchange.

Definition 5.1. (*Static Network Language*). Let $Nom^G = \{g, i, j, \dots, G, I, J, \dots\}$ be a non-empty set of propositional variables for nominals, and let $\mathcal{D} = \{d, d', \dots\}$ be a set of propositional variables for data-bits, disjoint from Nom^G. We define the *static network language* over $Nom^G \cup \mathcal{D}$ as follows:

$$\phi ::= g \mid G \mid d \mid \neg\phi \mid \phi \vee \psi \mid \exists\phi \mid [\pi]\phi.$$

$$\pi ::= R \mid ?\phi \mid \pi_1 \cup \pi_2 \mid \pi_1; \pi_2 \mid \pi^*.$$

with $\{g, G\} \subseteq Nom^G, d \in \mathcal{D}$.

We define the $\neg, \vee$ as usual. The intuitive meaning of the existential modality $\exists\phi$ is "there exists an agent that satisfies ϕ". We have seen above the intuitive meaning of $[\pi]\phi$.

Definition 5.2. (*Static Network Model*). A *static network model* for social networks is a tuple, $M = (\mathcal{A}, R, V)$, where $(\mathcal{A}, R)$ is a social network as in Definition 2.2, and $V : Nom^G \cup \mathcal{D} \to \mathcal{P}(\mathcal{A})$ is a valuation function. The valuation function V satisfies the following conditions:

- for all propositional variables for data-bits $d \in \mathcal{D}$, $V(d) = B \subseteq \mathcal{A}$, such that for every $b \in B$, $M, b \models d$;
- for all nominal variables for single agents $g \in Nom^G$, $V(g) = B \subseteq \mathcal{A}$, where B is a singleton subset of $\mathcal{A}$, e.g., $B = \{b\}$, such that $M, b \models g$;
- for all nominal variables for groups $G \in Nom^G$, $V(G) = B \subseteq \mathcal{A}$, where B is a group, such that for every connected $b \in B$, $M, b \models G$;

Definition 5.3. (*Static Network Language Satisfaction*). Let $M = (\mathcal{A}, R, V)$ be a static network model and consider some agent $a \in \mathcal{A}$. Then we define:

$$
\begin{array}{lll}
M, a \models d & \text{iff} & a \in V(d), \text{ where } d \in \mathcal{D}; \\
M, a \models g & \text{iff} & V(g) = \{a\}, \text{ where } g \in Nom^G; \\
M, a \models G & \text{iff} & a \in V(G), \text{ where } G \in Nom^G; \\
M, a \models \neg\phi & \text{iff} & M, a \not\models \phi; \\
M, a \models \phi \vee \psi & \text{iff} & M, a \models \phi \text{ or } M, a \models \psi; \\
M, a \models \exists\phi & \text{iff} & \text{there exists an } a' \in \mathcal{A} \text{ such that } M, a' \models \phi\ ; \\
M, a \models [\pi]\phi & \text{iff} & \text{for every } b \in \mathcal{A}, \text{ if } aR_\pi b \text{ then } M, b \models \phi.
\end{array}
$$

Where R_π is defined inductively as follows:

$$
\begin{array}{lll}
R_R & = & \{(x, y) | (x, y) \in R\} \\
R_{?\phi} & = & \{(x, y) \,|\, x = y \text{ and } y \models \phi\} \\
R_{\pi_1 \cup \pi_2} & = & R_{\pi_1} \cup R_{\pi_2} = \{(x, y) \,|\, xR_{\pi_1} y \text{ or } xR_{\pi_2} y\} \\
R_{\pi_1;\pi_2} & = & R_{\pi_1} \circ R_{\pi_2} = \{(x, y) \,|\, \text{ there exists } z \in \mathcal{A} \text{ such that } (x, z) \in R_{\pi_1} \text{ and } (z, y) \in R_{\pi_2}\} \\
R_{\pi^*} & = & (R_\pi)^* = \text{reflexive and transitive closure of } R_\pi.
\end{array}
$$

Abbreviations. We use the following abbreviations:

$[R]\phi$	iff	$[R_R]\phi$;
$[R]^*\phi$	iff	$[R_{\pi^*}]\phi$;
$\forall\phi$	iff	$\neg\exists\neg\phi$;
$\langle\pi\rangle\phi$	iff	$\neg[\pi]\neg\phi$;
$\Box\phi$	iff	$[R]\phi$;
$\Diamond\phi$	iff	$\langle R\rangle\phi$.

We define validity in a model ($M \vDash \phi$) and truth at a state in a model ($M, a \vDash \phi$) as usual.

5.2 Logical Characterizations of Structural Notions

We now use the static network language to give a characterization of connectors, bridges, local gatekeepers, bridging sets and blocking sets, as well as a characterization of the first two notions of gatekeepers we presented in chapter 4.

A preliminary note. In the following, we will use indexes from $\mathbb{N}$ to merely distinguish the agents involved, being their particular order irrelevant.

5.2.1 Connector

Recall that a connector is a set that connects two disconnected groups. It allows the existence of a path between the two groups, therefore also the communication between them.

Definition 5.4. (*Abbreviation for Connector*). Let $Connector(G_1, b_1, \ldots, b_n, G_2)$ be the abbreviation for the following formula:

$$(G_1 \wedge [?G_1; R]^* \neg G_2) \rightarrow \langle ?(G_1 \vee \bigvee_{i=1}^{n} b_i); R\rangle^* G_2)$$

Meaning: To explain the meaning, we split the formula in antecedent and consequent.

- Antecedent: $(G_1 \wedge [?G_1; R]^* \neg G_2)$. From any agent in G_1, for every path of agents all belonging to G_1, this does not lead to an agent in G_2.

- Consequent: $\langle ?(G_1 \vee \bigvee_{i=1}^{n} b_i); R\rangle^* G_2$). Assuming the antecedent true, from any agent in G_1, call it x, there exists a path of agents belonging to G_1 or to the connector, that links x with an agent in G_2.

We now prove that the truth of the formula in a model corresponds to the existence of a connector between two disconnected groups in the model.

Proposition 5.5 (Characterization of Connector)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model, let $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. Then $\{V(b_1), \ldots, V(b_n)\}$ is a connector between the disconnected groups $V(G_1)$ and $V(G_2)$ iff $M \vDash Connector(G_1, b_1, \ldots, b_n, G_2)$.*

Proof. Let $M = (\mathcal{A}, R, V)$ be a static network model and consider an arbitrary $x \in \mathcal{A}$. We want to prove that $M, x \vDash (G_1 \wedge [?G_1; R]^* \neg G_2) \rightarrow \langle ?(G_1 \vee \bigvee_{i=1}^{n} b_i); R\rangle^* G_2$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a connector between $V(G_1)$ and $V(G_2)$, which are two disconnected groups.

Suppose that $M, x \vDash (G_1 \wedge [?G_1; R]^* \neg G_2)$. This is the case iff $M, x \vDash G_1$ and $M, x \vDash [?G_1; R]^* \neg G_2$. By $M, x \vDash [?G_1; R]^* \neg G_2$ we know that for every $y \in \mathcal{A}$ such that $xR_{(G_1;R)^*}y$, we have $M, y \nvDash G_2$. Since by definition $R_{(G_1;R)^*} = (R_{(G_1;R)})^*$, then this is the case iff for every finite path $P := (xRz_0R \ldots Rz_n = y)$, such that for every z_k with $0 \leq k < n$, we have $M, z_k \vDash G_1$, it is the case that $M, y \nvDash G_2$. This holds iff for every z_k, with $0 \leq k < n$, in every such path P, we have $z_k \in V(G_1)$ and $z_n \notin V(G_2)$, i.e. $V(G_1) \cup V(G_2)$ is not a group. By definition, this means that $V(G_1), V(G_2)$ are disconnected groups.

We now show that $M, x \vDash \langle ?(G_1 \vee \bigvee_{i=1}^{n} b_i); R\rangle^* G_2$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a connector between the disconnected $V(G_1)$ and $V(G_2)$.

For simplicity, let us call $\pi = (?(G_1 \vee \bigvee_{i=1}^{n} b_i); R)$. Then, $M, x \vDash \langle \pi\rangle^* G_2$ iff there exists some $y \in \mathcal{A}$ such that $xR_{\pi^*}y$ and $M, y \vDash G_2$. Since $R_{\pi^*} = (R_\pi)^*$, then this is the case iff there exists a finite path $P := (x = z_0 R_\pi z_1 R_\pi \ldots R_\pi z_n = y)$ with all $z_i \in \mathcal{A}$ and $M, y \vDash G_2$. Since $\pi = (?(G_1 \vee \bigvee_{i=1}^{n} b_i); R)$ then the finite path P is such that for all z_k, with $0 \leq k < n$, we have $M, z_k \vDash (G_1 \vee \bigvee_{i=1}^{n} b_i)$ and $z_k R z_{k+1}$ and $M, y \vDash G_2$. This means that P is such that for all $z_k \in P$, either we have $z_k \in V(G_1)$, or $z_k \in \{V(b_1), \dots, V(b_n)\}$ (but not both) and $z_n = y \in V(G_2)$. Therefore, there exists a path such that for all its elements they belong to $V(G_1) \cup \{V(b_1), \dots, V(b_n)\} \cup V(G_2)$, i.e. $V(G_1) \cup \{V(b_1), \dots, V(b_n)\} \cup V(G_2)$ is a group. Since we proved that $V(G_1), V(G_2)$ are disconnected groups, this amounts to say that $\{V(b_1), \dots, V(b_n)\}$ is a connector between $V(G_1), V(G_2)$. □

5.2.2 Bridge

Recall that a bridge is a minimal connector between two disconnected groups, by Proposition 3.12. Let us now see their characterization in network logic.

Definition 5.6. (*Abbreviation for Bridge*). Let $Bridge(G_1, b_1, \dots, b_n, G_2)$ be the abbreviation for the following formula:

$$Connector(G_1, b_1, \dots, b_n, G_2) \wedge (G_1 \to \bigwedge_{i=1}^{n} [?(\bigvee_{j=1, j \neq i}^{n} b_j); R]^* \neg G_2)$$

Meaning: The two conjuncts represent the clauses composing the definition of bridges: the first represents (B+), the second (B-). Since the first was explained above, we focus on the second one.

(B-): $(G_1 \to \bigwedge_{i=1}^{n} [?(\bigvee_{j=1, j \neq i}^{n} b_j); R]^* \neg G_2)$. From any agent in G_1 there exists no path of agents that passes from all the agents $b_1, \dots, b_n$ in the bridge *except one* and reaches G_2.

Proposition 5.7 (Characterization of Bridge)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model, let $b_1, \dots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. Then $\{V(b_1), \dots, V(b_n)\}$ is a bridge between $V(G_1)$ and $V(G_2)$ iff $M \vDash Bridge(G_1, b_1, \dots, b_n, G_2)$.*

Proof. Let $M = (\mathcal{A}, R, V)$ be a static network model. In what follows we will prove the equivalence for each of the two conjuncts separately.

(1) We want to prove that $M \vDash Connector(G_1, b_1, \dots, b_n, G_2)$ iff $\{V(b_1), \dots, V(b_n)\}$ satisfies (B+) of definition of bridges. That clause exactly says that $\{V(b_1), \dots, V(b_n)\}$ is a connector. We proved in Proposition 5.5 that the equivalence holds.

(2) We want to prove that $M \vDash (G_1 \rightarrow \bigwedge_{i=1}^{n}[?(G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j); R]^* \neg G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ satisfies (B-) of definition of bridges.

Consider an arbitrary $x \in \mathcal{A}$ and suppose that $M, x \vDash G_1$. Take an arbitrary $1 \leq i \leq n$ and define $\pi_i = (?(G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j); R)$. Then $M, x \vDash [\pi_i]^* \neg G_2$ iff for every $y \in \mathcal{A}$ such that $x R_{\pi_i^*} y$ we have $M, y \nvDash G_2$. Since $R_{\pi_i^*} = (R_{\pi_i})^*$, then this is the case iff for every finite path $P := (x = z_0 R_{\pi_i} z_1 R_{\pi_i} \ldots R_{\pi_i} z_m = y)$ with all $z_i \in \mathcal{A}$ we have $M, y \nvDash G_2$. By definition of π_i, this means that for every path P such that for all z_k, with $0 \leq k < m$, it is the case that $M, z_k \vDash G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j$, for some arbitrary $1 \leq i \leq n$, we have $M, z_m \nvDash G_2$. This is the case iff every P is such that for all $z_k \in P$, either we have $z_k \in V(G_1)$, or $z_k \in (\{V(b_1), \ldots, V(b_n)\} \setminus V(b_i))$ (but not both) and $z_m \notin V(G_2)$, which is equivalent to say that every path such that all its agents belong to $V(G_1) \cup (\{V(b_1), \ldots, V(b_n)\} \setminus V(b_i))$ does not reach $V(G_2)$, i.e. $V(G_1) \cup (\{V(b_1), \ldots, V(b_n)\} \setminus V(b_i)) \cup V(G_2)$ is not a group. Since i was chosen arbitrarily, this holds for all $1 \leq i \leq n$, i.e. for every $V(b_i) \in \{V(b_1), \ldots, V(b_n)\}$, we have that $V(G_1) \cup (\{V(b_1), \ldots, V(b_n)\} \setminus V(b_i)) \cup V(G_2)$ is not a group. This means that (B-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1)$,$V(G_2)$.

Hence, $M, x \vDash G_1 \rightarrow [\pi_i]^* \neg G_2$ for some arbitrary $1 \leq i \leq n$, iff (B-) holds for $\{V(b_1), \ldots, V(b_n)\}$. Since $\pi_i = (?(G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j); R)$, and since i was chosen arbitrarily, then this holds for all $1 \leq i \leq n$ i.e. $M, x \vDash G_1 \rightarrow \bigwedge_{i=1}^{n}[?(G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j); R]^* \neg G_2)$ iff (B-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1)$,$V(G_2)$.

Since also x was chosen arbitrarily this holds for all agents in $\mathcal{A}$, i.e., $M \vDash (G_1 \rightarrow \bigwedge_{i=1}^{n}[?(G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j); R]^* \neg G_2)$ iff (B-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1)$,$V(G_2)$.

By putting the two conjuncts together we obtain $M \vDash Connector(G_1, b_1, \ldots, b_n, G_2) \wedge (G_1 \rightarrow \bigwedge_{i=1}^{n}[?(G_1 \vee \bigvee_{j=1, j\neq i}^{n} b_j); R]^* \neg G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ satisfies clauses (B+) and

(B-) of definition of bridge, i.e. $\{V(b_1), \ldots, V(b_n)\}$ is a bridge between $V(G_1)$ and $V(G_2)$. $\square$

5.2.3 Local Gatekeeper

Local gatekeepers are agents that belong to bridges. They are defined using the notion of connector, namely the bridge they belong to is part of a connector between two disconnected groups.

Definition 5.8. (*Abbreviation for C-Local Gatekeeper*). Given a connector $C = \{V(c_1), \ldots, V(c_n)\}$, with $n \geq 0$ between $V(G_1), V(G_2)$, let $C - LocalGtkpr(G_1, b, G_2)$, be the abbreviation for the following formula:

$\bigvee\{Bridge(G_1, c_1, \ldots, c_n, G_2) \mid n \geq 0$ with $c_1, \ldots, c_n$ all distinct and such that $c_i = b$ for some $1 \leq i \leq n\}$

Meaning: This is a disjunction of formulas representing bridges between the groups G_1, G_2. The bridges are formed by subsets of agents in the connector C, and the C-local gatekeeper is then one of the agents belonging to one of the bridges.

Proposition 5.9 (Characterization of C-Local Gatekeeper)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model, let $C = \{V(c_1), \ldots, V(c_n)\}$ be a connector between $V(G_1)$ and $V(G_2)$, and let $c_1, \ldots, c_n, b$ be nominals for single agents, while G_1, G_2 be group nominals. Then, $\{V(b)\}$ is a C-local gatekeeper between $V(G_1)$ and $V(G_2)$ iff $M \vDash LocalGtkpr(G_1, b, G_2)$.*

Proof. Let $M = (\mathcal{A}, R, V)$ be a static network model and consider an arbitrary $x \in \mathcal{A}$. Then we have $M, x \vDash LocalGtkpr(G_1, b, G_2)$ iff $M, x \vDash Bridge(G_1, c_1, \ldots, c_n, G_2)$ for some distinct $c_1, \ldots, c_n$ and such that $c_i = b$ for some $1 \leq i \leq n$. By Proposition 5.7, this is the case iff there exists a bridge $\{V^M(c_1), \ldots, V^M(c_n)\}$ between $V^M(G_1)$ and $V^M(G_2)$, for some distinct $c_1, \ldots, c_n$ and such that $c_i = b$ for some $1 \leq i \leq n$. This amounts to say that there exists a bridge $\{V^M(c_1), \ldots, V^M(c_n)\}$ between $V^M(G_1)$ and $V^M(G_2)$, for some distinct $c_1, \ldots, c_n$, such that for some $V^M(c_i)$, with $1 \leq i \leq n$, we have $V^M(c_i) = V^M(b)$. Since $C = \{V(c_1), \ldots, V(c_n)\}$ is a connector between $V(G_1)$ and $V(G_2)$, by definition of C-local gatekeeper, this means that $V^M(b)$ is a C-local gatekeeper between $V(G_1)$ and $V(G_2)$.

We can conclude that $M \models LocalGtkpr(G_1, b, G_2)$ iff given a connector $C = \{V(c_1), \ldots, V(c_n)\}$ between $V(G_1)$ and $V(G_2)$, $\{V(b)\}$ is a C-local gatekeeper between $V(G_1)$ and $V(G_2)$. □

5.2.4 Bridging Set

A bridging set between two disconnected groups is a set containing only bridges between these groups.

Definition 5.10. (*Abbreviation for Bridging Set*). Let $BridgSet(G_1, b_1, \ldots, b_n, G_2)$, where $n \geq 1$ be the abbreviation for the following formula:

$$\bigwedge_{i=1}^{n} (b_i \to C - LocalGtkpr(G_1, b_i, G_2)$$

Meaning: Every agent b_i in the bridging set is a C-local gatekeeper. Recall that this means that there exists a bridge in the connector C

Proposition 5.11 (Characterization of Bridging Set)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model and let $b_1, \ldots, b_n$ be nominals for single agents and G_1, G_2 be group nominals. Then $\{V(b_1), \ldots, V(b_n)\}$ is a bridging set between $V(G_1)$ and $V(G_2)$ iff $M \models BridgSet(G_1, b_1, \ldots, b_n, G_2)$*

Proof. Let $M = (\mathcal{A}, R, V)$ be a static network model and consider an arbitrary $x \in \mathcal{A}$. Suppose that $M, x \models b_i$ for some arbitrary $1 \leq i \leq n$. Then $M, x \models C - LocalGtkpr(G_1, b_i, G_2)$ iff $V(b_i)$ is a C-local gatekeeper between G_1, G_2, by Proposition 5.9. Since i was chosen arbitrarily, this means that for all $1 \leq i \leq n$ we have that $V(b_i)$ is a C-local gatekeeper between G_1, G_2. By definition of bridging set, $\{V(b_1), \ldots, V(b_n)\}$ is a bridging set between $V(G_1)$ and $V(G_2)$. □

5.2.5 Blocking Set

A blocking set between two disconnected groups is a set that intersects every connector between the two groups.

Definition 5.12. (*Abbreviation for Blocking Set*). Let $BlockSet(G_1, b_1, \ldots, b_n, G_2)$, where $n \geq 1$ be the abbreviation for the following formula:

$$((G_1 \wedge [?G_1 : R]^* \neg G_2) \rightarrow [R; ?\bigwedge_{i=1}^{n} \neg b_i]^* \neg G_2)$$

Meaning: The antecedent has the same meaning as the antecedent of the definition of connector. Then, assuming the antecedent true, the consequent says from any agent in G_1, every path that does not pass through any agent $V(b_i)$ in the blocking set, does not reach G_2.

Proposition 5.13 (Characterization of Blocking Set)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model, let $b_1, \ldots, b_n$ be nominals for single agents and G_1, G_2 be group nominals. Then $\{V(b_1), \ldots, V(b_n)\}$, with $\{V(b_1), \ldots, V(b_n)\} \cap (V(G_1) \cup V(G_2)) = \emptyset$, is a blocking set between $V(G_1)$ and $V(G_2)$ iff $M \vDash BlockSet(G_1, b_1, \ldots, b_n, G_2)$.*

Proof. Let $M = (\mathcal{A}, R, V)$ be an static network model. Consider an arbitrary $x \in \mathcal{A}$ and suppose that $M, x \vDash G_1 \wedge [?G_1; R]^* \neg G_2$. In the proof of Proposition 5.5, we showed that $M, x \vDash [?G_1; R]^* \neg G_2$ iff $V(G_1)$ and $V(G_2)$ are disconnected groups. So we can conclude this being the case here too, and move to show that $M \vDash [R; ?\bigwedge_{i=1}^{n} \neg b_i]^* \neg G_2$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a blocking set between $V(G_1), V(G_2)$.

For simplicity, define $\pi = (R; ?\bigwedge_{i=1}^{n} \neg b_i)$. Then $M, x \vDash [\pi]^* \neg G_2$ iff for every $y \in \mathcal{A}$ such that $x R_{\pi^*} y$ we have $M \nvDash G_2$. Since $R_{\pi^*} = (R_\pi)^*$, then this is the case iff for every finite path $P := (x = z_0 R_\pi z_1 R_\pi \ldots R_\pi z_m = y)$ with all $z_k \in \mathcal{A}$ we have $M, y \nvDash G_2$. By definition of π, this means that for every path P such that for all z_k, with $0 \leq k < m$, it is the case that $M, z_k \vDash \bigwedge_{i=1}^{n} \neg b_i$, we have $M, z_m \nvDash G_2$. For each P, the agents in it form a distinct group, which we call P'. Then the above is equivalent to say that for every group P' such that $(P' \cap V(G_1)) \neq \emptyset$, if $(P' \cap \{V(b_1), \ldots, V(b_n)\}) = \emptyset$ then $P' \cap V(G_2) = \emptyset$, thus also $V(G_2) \not\subset P'$. This is equivalent to its contrapositive, namely for any group P' such that $(P' \cap V(G_1)) \neq \emptyset$, if $V(G_2) \subset P'$ then $(P' \cap \{V(b_1), \ldots, V(b_n)\}) \neq \emptyset$. Now consider any group G_3 such that $P' \subseteq G_3$ and $V(G_1) \subset G_3$. It is the case that if $V(G_2) \subset P'$ then $(P' \cap \{V(b_1), \ldots, V(b_n)\}) \neq \emptyset$ iff if $V(G_2) \subset P'$ then $(V(G_3) \cap \{V(b_1), \ldots, V(b_n)\}) \neq \emptyset$. It follows that for any group $V(G_3)$ such that $(V(G_1) \cup V(G_2)) \subseteq V(G_3)$ we have $(V(G_3) \cap \{V(b_1), \ldots, V(b_n)\}) \neq \emptyset$. By definition of blocking set this means that $\{V(b_1), \ldots, V(b_n)\}$ is a blocking set between $V(G_1)$

and $V(G_2)$.

Hence, $M, x \vDash ((G_1 \wedge [?G_1; R]^* \neg G_2) \rightarrow [\pi]^* \neg G_2)$, iff $\{V(b_1), \ldots, V(b_n)\}$ is a blocking set between $V(G_1)$ and $V(G_2)$. Since $\pi = (R; ?\bigwedge_{i=1}^{n} \neg b_i)$, then $M, x \vDash ((G_1 \wedge [?G_1; R]^* \neg G_2) \rightarrow [R; ?\bigwedge_{i=1}^{n} \neg b_i]^* \neg G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a blocking set between $V(G_1)$ and $V(G_2)$.

Since x is chosen arbitrarily, this holds for all agents in $\mathcal{A}$, i.e. $M \vDash ((G_1 \wedge [?G_1; R]^* \neg G_2) \rightarrow [R; ?\bigwedge_{i=1}^{n} \neg b_i]^* \neg G_2)$ iff the set $\{V(b_1), \ldots, V(b_n)\}$ is a blocking set between $V(G_1)$ and $V(G_2)$. □

5.2.6 Gatekeeping Set

A gatekeeping set between two groups is defined as being both a bridging set and a blocking set. Then, it can be expressed as a conjunction of the formulas characterizing the bridging and blocking sets.

Definition 5.14. (*Abbreviation for Gatekeeping Set*). Let $GtkpSet(G_1, b_1, \ldots, b_n, G_2)$, where $n \geq 1$ be the abbreviation for the following formula:

$$BlockSet(G_1, b_1, \ldots, b_n, G_2) \wedge BridgSet(G_1, b_1, \ldots, b_n, G_2)$$

The meaning of the formula is quite clear, so we skip the explanation of it and move to the proof of equivalence.

Proposition 5.15 (Characterization of Gatekeeping Set)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model and let $b_1, \ldots, b_n$ be nominals for single agents and G_1, G_2 be group nominals. Then $\{V(b_1), \ldots, V(b_n)\}$ is a gatekeeping set between $V(G_1)$ and $V(G_2)$ iff $M \vDash GtkpSet(G_1, b_1, \ldots, b_n, G_2)$.*

Proof. Let $M = (\mathcal{A}, R, V)$ be a static network model. By Proposition 5.13 we know that $M \vDash BlockSet(G_1, b_1, \ldots, b_n, G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a blocking set between $V(G_1)$ and $V(G_2)$. By Proposition 5.11 know that $M \vDash BridgSet(G_1, b_1, \ldots, b_n, G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a bridging set between $V(G_1)$ and $V(G_2)$. By definition of gatekeeping set, $\{V(b_1), \ldots, V(b_n)\}$ is a gatekeeping set between $V(G_1)$

and $V(G_2)$ iff it is a bridging set between $V(G_1)$ and $V(G_2)$ and it is a blocking set between $V(G_1)$ and $V(G_2)$. Hence, $M \vDash BlockSet(G_1, b_1, \ldots, b_n, G_2) \wedge BridgSet(G_1, b_1, \ldots, b_n, G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a gatekeeping set between $V(G_1)$ and $V(G_2)$. □

5.2.7 Gatekeeping Bridge

As in the other cases, in the following characterization each conjunct of the formula represents the clauses defining gatekeeping bridges: the first represents (B+) and the second (GB-).

Definition 5.16. (*Abbreviation for Gatekeeping Bridge*). Let $GtkpBridge(G_1, b_1, \ldots, b_n, G_2)$ be the abbreviation for the following formula:

$$Connector(G_1, b_1, \ldots, b_n, G_2) \wedge (G_1 \to (\bigwedge_{i=1}^{n} [?(\neg b_i); R]^* \neg G_2))$$

Meaning: The formula is a conjunction, where the meaning of the first conjunct is clear. The second conjunct means that every path starting from any agent that satisfies G_1, and not passing from any agent b_i in the gatekeeping bridge, does not arrive to G_2.

Proposition 5.17 (Characterization of Gatekeeping Bridges)**.** *Let $M = (\mathcal{A}, R, V)$ be a static network model and let $b_1, \ldots, b_n$ be nominals for single agents and G_1, G_2 be group nominals. Then $\{V(b_1), \ldots, V(b_n)\}$ is a gatekeeping bridge between $V(G_1)$ and $V(G_2)$ iff $M \vDash GtkpBridge(G_1, b_1, \ldots, b_n, G_2)$.*

Proof. Let $M = (\mathcal{A}, R, V)$ be a static network model. As we did in the previous proof, in what follows we will prove the equivalence for each of the three conjuncts separately.

(1) We want to prove that $M \vDash Connector(G_1, b_1, \ldots, b_n, G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ satisfies (B+) of definition of gatekeeping bridge. This is the case iff $\{V(b_1), \ldots, V(b_n)\}$ is a connector between $V(G_1)$ and $V(G_2)$. By Proposition 5.5, we know that $M \vDash Connector(G_1, b_1, \ldots, b_n, G_2)$ iff $\{V(b_1), \ldots, V(b_n)\}$ is a connector between $V(G_1)$ and $V(G_2)$.

(2) We want to prove that $M \vDash (G_1 \to (\bigwedge_{i=1}^{n}[?(\neg b_i);R]^*\neg G_2))$ iff $\{V(b_1), \ldots, V(b_n)\}$ satisfies (GB-) of definition of gatekeeping bridge. Consider an arbitrary $x \in \mathcal{A}$ and suppose that $M, x \vDash G_1$. For simplicity, define $\pi_i = (?(\neg b_i);R)$. Then $M, x \vDash \bigwedge_{i=1}^{n}[\pi_i]^*\neg G_2$ iff for every $1 \leq i \leq n$ we have $M, x \vDash [\pi_i]^*\neg G_2$. Consider an arbitrary $1 \leq i \leq n$. Then the above is the case iff for every $y \in \mathcal{A}$ such that $xR_{\pi_i^*}y$ we have that $M, x \nvDash G_2$. Since $R_{\pi_i^*} = (R_{\pi_i})^*$, then this is the case iff for every finite path $P := (x = z_0R_{\pi_i}z_1R_{\pi_i}, \ldots, R_{\pi_i}z_m = y)$ with all $z_k \in \mathcal{A}$, we have $M, y \nvDash G_2$. By definition of π_i, this means that for every path P such that for all z_k, with $0 \leq k < m$, it is the case that $M, z_k \vDash \neg b_i$, we have $M, z_m \nvDash G_2$. For each such P, the agents in it form a group, which we call P'. Consider an arbitrary such P'. The above is then equivalent to say that if for every $z_k \in P'$, we have $M, z_k \vDash \neg b_i$, then for no $z_k \in P'$, we have $z_k \vDash G_2$, i.e. then P' is not a connector between G_1, G_2. Note that for every $z_k \in P'$ it is the case that $M, z_k \nvDash b_i$ iff $\{V(b_1), \ldots, V(b_n)\} \not\subset P'$. Then again the above is equivalent to say that if $\{V(b_1), \ldots, V(b_n)\} \not\subset P'$, then P' is not a connector between G_1, G_2. By taking its contrapositive, this amounts to say that if P' is a connector between G_1, G_2 then $\{V(b_1), \ldots, V(b_n)\} \subset P'$. Since P' was chosen arbitrarily, this holds for every set of agents. This means that (GB-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1), V(G_2)$.

Hence, $M, x \vDash G_1 \to [\pi_i]^*\neg G_2$, for some arbitrary $1 \leq i \leq n$ iff (GB-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1), V(G_2)$. Since $\pi_i = (?(\neg b_i);R)$, and since i was chosen arbitrarily, then this holds for all $1 \leq i \leq n$, i.e. $M, x \vDash G_1 \to (\bigwedge_{i=1}^{n}[?(\neg b_i);R]^*\neg G_2)$ iff (GB-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1), V(G_2)$.

Since x was chosen arbitrarily this holds for all agents, i.e., $M \vDash G_1 \to (\bigwedge_{i=1}^{n}[?(\neg b_i);R]^*\neg G_2)$ iff (GB-) holds for $\{V(b_1), \ldots, V(b_n)\}$ and $V(G_1), V(G_2)$.

Then, by putting the two conjuncts together, we have $M \vDash \mathit{Connector}(G_1, b_1, \ldots, b_n, G_2) \wedge (G_1 \to (\bigwedge_{i=1}^{n}[?(\neg b_i);R]^*\neg G_2))$ iff $\{V(b_1), \ldots, V(b_n)\}$ satisfies (B+) and (GB-) of definition of gatekeeping bridge iff $\{V(b_1), \ldots, V(b_n)\}$ is a gatekeeping bridge between $V(G_1)$ and $V(G_2)$. □

This proposition concludes the proofs about the characterizations of structural notions in the logic. We have characterized the notion of connectors, bridges, bridging sets, gatekeeping bridge, and gatekeeping sets. As the reader will have noticed, the characterization of one of the notions is missing.

Open Question: characterization of the grand gatekeeper between two disconnected sets.

We conjecture that this characterization can only be given by introducing names for all the agents in the network, so only by switching from a network model to what Seligman et al. call a *named agent model* [22]. This would allow to express the maximality condition that characterizes the grand gatekeeper.

6 Logic for Communicative Actions

In the previous chapters, we introduced several structural notions, which we used them to construct the definitions of gatekeepers. Then, we showed that almost all the notions can be characterized using network logic, which is a static logic. Yet, we have not discussed the other fundamental part of the gatekeeper phenomenon, namely its informational dimension and the capability of gatekeepers to control the information flow. Only representing the dynamics of the information flow of between the groups we can fully capture this phenomenon.

In this chapter we do just that, i.e. we represent the power gatekeepers have to block or enable the flow. We will base the representation on network models, in which we will define the actions that agents in the network can perform. To represent the gatekeeper phenomenon, it will be sufficient to represent the situation in which agents have the binary choice of posting the information they have gathered from other agents or not to post it. Here, posting information means that agents make it available to all the friends they have. This kind of communication has then a public dimension, meaning that once an agent posted some information, all of her friends receive it. The posting actions are relatable to those allowed in the virtual social network Facebook, or Twitter, where agents can post information on their profile and all of their friends can read it. Relatable, but not precisely the same kind of actions, and in the next sections we will see why. More generally, posting actions are comparable to all the communicative actions which reach all the agents related to the communicator, at the same time. In this sense, they can be compared to public broadcasting models, as proposed by Roelofsen [17]. This, because also in those models agents can only publicly address the a set of agents as a whole.

The presence of gatekeepers in the network can impact the effectiveness of the communication, as they can control it. By 'control' we mean the capability of agents to force a particular event to happen. Gatekeepers then control the information flow in the sense that they have the capability to force the groups they gatekeep to receive or not some information. However, recall that in this thesis we assume that gatekeepers can be composed by more than one agent, and that in some cases one single agent might be sufficient to block or enable the information flow. Then, to see to it that an event will occur, the members of a gatekeeper sometimes need to join their forces and cooperate towards that outcome.

To provide a suitable representation for this kind of communicative actions and for the gatekeepers' capabilities, we will combine some notions from different logics of actions [32]. For example, we will use the notion of dynamic model update from Dynamic Epistemic Logic (DEL) [16], and the notion of coalition and of forcing an outcome from Coalition logic [18] and STIT logic [20].

The structure of this chapter is the following. In section 6.1, after having briefly introduced DEL, we will see the definitions required to model the dynamics of posting actions in network models. In section 6.2, again we will briefly discuss Coalition logic and STIT logic, to then we introduce the syntax and semantics of coalition logic for posting actions. In section 6.3, we will use the new logic to show the informational capabilities of gatekeepers.

6.1 Communication via Posting Actions

In this section, we represent the dynamics of information flow induced by the posting action. To achieve the representation, we introduce three distinct notions. First, we introduce a *global action function*, which maps every agent to a set of data, representing the action that each agent chooses to do, i.e. the information that each agent chooses to share. It has a global dimension, because it is defined for every agent in the model. Then, we introduce the notion of *executability of global actions at a model*. It provides the conditions under which an agent can share some information. The condition for an agent to post data d is that at least one of the agent's friends has previously posted d herself. This implicitly requires that, if some agent posts an information, then she makes it available to all of her friends. Then, the definition of executability of an action implicitly represents not only the conditions under which agents can post data, but also the public dimension of the action. The executable actions are used as model updates. Every such action induces a distinct update of the model. The definition of *model update through a global action* represents this, and it thus encodes the communicative dynamics.

The notion of model update, or information update, is taken from DEL. This is a branch of modal logic, which studies the dynamics occurring when an epistemic action takes place in a given model. To represent such dynamics, DEL adds to the modal language a new set of modalities describing the model-transforming actions [16]. For example, suppose that A is an epistemic action, namely such that it transforms the informational state of some agents. Then we can add to the language the model-transforming modality $[A]$. To illustrate how the model-transformer works, also in this thesis, consider a network model M. We have that $[A]\phi$ is true for an agent a in M iff in the updated model $(M)^A$, we have ϕ true at a, i.e. $M, a \vDash [A]\phi$ iff $(M)^A, a \vDash \phi$. Note that not every action A can be executed. For example, if an agent does not have an information d, she cannot communicate d to other agents. Then, for every agent to be able to communicate d, she must satisfy some requirements. In DEL these are usually called *the preconditions* of an action. Only if the preconditions are satisfied, then an action is executable and the model can be updated through that action.
For what concerns DEL, these basic notions should be sufficient for the reader to understand what follows.

The definitions we introduce in this section are based on network models, as in Definition 5.2. However, we now keep track, in the valuation function, of the model to which it belongs, i.e. instead of V, we write V^M. This is because we will encode the dynamics of the model in the update of the valuation function. It is this function that keeps track, for each model, of which agent receives which new information. Then, the valuation of data may vary from model to model.

Definition 6.1. (*Global Action Function*) Given a network model M, A global action $\alpha : \mathcal{A} \to \mathcal{P}(\mathcal{D})$ is a function that maps each agent to a set of data.

Definition 6.2. *(Executability of Global Actions at a Model)*. Given a network model M, we call a global function *α executable at M* iff for all $a \in \mathcal{A}$, we have

$$\alpha(a) \subseteq \{d \in \mathcal{D} \mid \text{ there exists } b \text{ such that } aRb \text{ and } b \in V^M(d)\}$$

Informally, this notion says that for every agent a, a can perform an action post(d) at model M if and only if at M agent a is informed of d, i.e. some of a's friends posted it (preconditions of an action). We assume that every agent who has a friend that posted some data d automatically receives it and is thus automatically informed of it. Note that 'being informed of d' is not here intended as 'knowing that d' or as 'believing that d. Instead, we take it as simply meaning that the agent possesses data d.

For convenience, we introduce also the notion of *global action set $GAct^M$*. This contains all the actions executable at M.

Definition 6.3. *(Global Actions Set)* We call $GAct^M$ the set containing every global action function that is executable at model M, i.e.,

$$GAct^M = \{\alpha \mid \text{ for all } a \in \mathcal{A}, \alpha(a) \text{ is executable at } M\}$$

Now, given a static network model M, every $\alpha \in GAct^M$ induces a distinct model update $(M)^\alpha$, as the following definition states.

Definition 6.4. (*Model Update through Executable Actions*). The update of the network model $M = (\mathcal{A}, R, V^M)$ through the global action α executable at M, is the network model $(M)^\alpha = (\mathcal{A}, R, V^{(M)^\alpha})$, where $V^{(M)^\alpha}$ is given by:

$$V^{(M)^\alpha}(d) = \{a \in \mathcal{A} \mid d \in \alpha(a)\} \cup V^M(d) \qquad \text{for } d \in \mathcal{D};$$

$$V^{(M)^\alpha}(G) = V^M(G) \qquad \text{for } G \in Nom^G.$$

The model update defines how a model is transformed, according to a global action α. If in model M agent a chooses to perform some executable global action α, such that $d \in \alpha(a)$, then in the updated model $(M)^\alpha$ the agent displays information d in her website. By definition of executable actions, it follows that she has made d available to all her friends, that can now spread the information themselves. This clarifies why our notion of posting action relates, but does not coincide to the Facebook or Twitter one. Since the relations between them will illustrate some specificities of our posting action, let us briefly consider them. On the one hand, it does align, as we also define it as having a public dimension. However, it does not completely align to it, because (i) the agents are allowed to post information just in case their friends have already posted it. This means that the action shares information that someone else already included in the system; (ii) we do not allow agents to explicitly avoid receiving posted information, which in Facebook is always possible; (iii) we do not allow agents to un-post information, as the set of data agents has posted cannot decrease, but only increase. In Facebook instead, one can always eliminate information from its own wall.

Note that for each action α, a distinct model update takes place. This represents the intuition that each executable global action α leads to a different outcome. By interpreting models as moments in time, we can read each executable action α as giving rise to a different history. Then, a branching time structure unfolds, where every branch represent the output of a global action, i.e. of the sum of individual choices to post or not to post data.

The two definitions that follow, will be useful in the next section, when we will use Coalition or STIT logics to define that a set of agents can force some outcome.

Definition 6.5. (*Global Action Equivalence*). Given a network model M, two global actions α, β in $GAct^M$, and a $A \subseteq \mathcal{A}$, we say that $\alpha \sim_A \beta$ if and only if for all $a \in A, \alpha(a) = \beta(a)$.

This notion states that the global actions α, β agree on some local information, namely there exists some set of agents $A \subseteq \mathcal{A}$ for which α, β coincide. Then, this definition does not concern the other agents in the network. They instead can decide to take any action they like.

Definition 6.6. (*Communication Sequence*). Let $M = (\mathcal{A}, R, V^M)$ be a network model. The communication sequence SQ_M is the sequence of network models

$$\langle M_0 = M, M_1 = (M_0)^{\alpha^{M_0}}, M_2 = (M_1)^{\alpha^{M_1}}, \ldots, M_{n+1} = (M_n)^{\alpha^{M_n}}, \ldots \rangle$$

such that for any $n \in \mathbb{N}$, $M_n = (\mathcal{A}, R, V^{M_n})$, where V^{M_n} is given by $V^{M_n}(d)$:

$$V^{M_0}(d) = V^M(d) \text{ and } V^{M_{n+1}}(d) = V^{(M_n)^{\alpha^{M_n}}}(d).$$

This definition just represents what a sequence of repeated updates amounts to.

We now move to illustrate the above definitions through two examples. For simplicity, we will assume that the agents are each time sharing only one single data bit d. In the figures, the gray color represents the valuation of d.

Example 6.7. (*Citizen-Mayor via Facebook*). Consider again the situation in which Ann wants to communicate information d to her mayor. Recall that Ann is a citizen of a metropolis, and she is not in direct contact with the mayor. Contrarily to the strategy she used in the examples above, this time Ann tries to communicate information d in another way. Since the information is about an issue that many of her fellow citizens feel as needing urgent addressing, she decides to spread it among her Facebook friends. She is sure that they will spread it as well and that ultimately also the mayor will receive and share it herself with all of her contacts.
Let $M = (\mathcal{A}, R, V^M)$ be the network model represented in Figure 13 on the left, and let a represent Ann. Node a is gray, which means that Ann has posted data d.

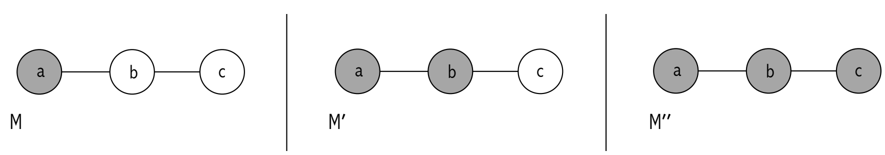

Figure 13

By posting information d, Ann has made it available to her friends, i.e. to Beatrix b. Since it is the case that $a \in R[b]$, and $a \in V^M(d)$, then by definition of executable action, we have $\alpha \in GAct^M$, with $d \in \alpha(b)$. This means that Beatrix can now repost d herself, which she actually does. Then, in the updated model $M' = (M)^\alpha$ we have $b \in V^{(M)^\alpha}(d)$. Now it is Beatrix who has made information d available to all of her friends, namely to the mayor c. At model $(M)^\alpha$ then, the mayor receives d and Ann realizes her communicative goal. Moreover, the mayor too posts d on her wall, and in model $M'' = (M)^{\alpha'}$ we have $b \in V^{(M)^{\alpha'}}(d)$.

Example 6.8. (*Multiple Citizen-Mayor via Facebook*). Now consider the following adaptation of Example 6.7. There are two citizens, Ann a and Eliver e, who do not know each other, but who share the aim of spreading information d via Facebook. They hope to make more people aware of it and to ultimately reach the mayor, who will eventually take action and share the information herself. Let $M = (\mathcal{A}, R, V^M)$ be the network model represented in Figure 14 on the left.

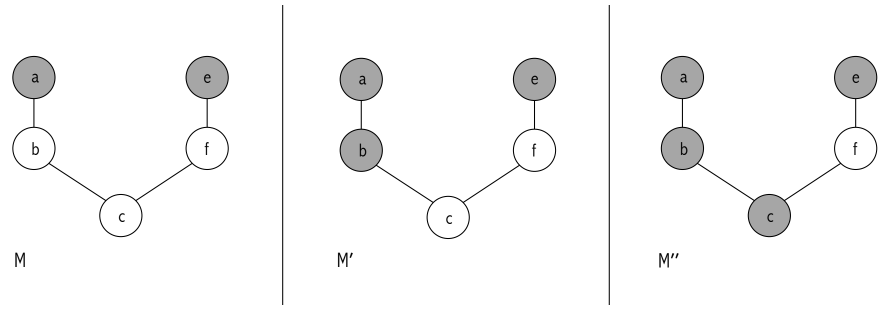

Figure 14

At M both Ann and Eliver posted d. Now Beatrix b and Fiona f, who are friends of Ann and Eliver respectively, received the information, so we have $\alpha \in GAct^M$, with $d \in \alpha(b)$ and $\alpha \in GAct^M$, with $d \in \alpha(e)$. This means that Beatrix and Fiona can repost the information themselves. However, Fiona is rather agnostic about that sort of issues, so she does not report d in her Facebook wall. Notice that by definition of executability of an action at a model, every agent at every model can perform the null-action, i.e. not spread any information with her friends. It follows that there exists some $\beta \in GAct^M$ such that $\beta(f) = \emptyset$, and $d \in \beta(b)$. At model $M' = (M)^\beta$ we have $f \notin V^{(M)^\beta}(d)$ and $b \in V^{(M)^\beta}(d)$. Then, at model $(M)^\beta$, the mayor receives d even if Fiona did not post it, as we have $b \in R[c]$, namely Beatrix is one of the mayor's friends. So at model M', we have $\beta' \in GAct^{M'}$ such that $d \in \beta'(c)$. This means that the mayor has the possibility to post the information herself, which again she actually does. Then, at model $M'' = (M)^{\beta'}$, we have $c \in V^{(M)^{\beta'}}(d)$.

6.2 Coalition logic for Posting Actions: Syntax and Semantics

In this section, we introduce coalition logic for posting actions. The language is an extension of the static network language of Definition 5.1, to which we add the coalition modality $\langle\langle A \rangle\rangle$, for some $A \subseteq \mathcal{A}$. This modality is used to express that if the agents in A perform the same action, they can force an outcome. When agents perform the same action, we say that they form a *coalition*.

Coalition logic is another branch of modal logic, firstly introduced by Pauly to express what a set of agents can achieve if working as a coalition [18]. There exist many variations of Coalition logic, e.g. [33][34]. Broersen et al. showed that this logic can be translated into a discrete-time version of STIT logic [35], which is another kind of action logic to express that agents can See To It That something will happen.
Coalition logic adds a coalition modality $[C]$ to the basic modal language. Then $[C]\phi$ is to be read "coalition C can achieve outcome ϕ", or "coalition C can force outcome ϕ". However, in Pauly version this is combined with games structures, which we do not need. Rather, we will combine a coalition modality with the equivalence relation for global actions we defined in the previous section. This strategy is inspired by STIT logic [20],[36], where a set of agents sees to it that ϕ iff for every choice the agents can make, ϕ will be the case.[1]

Note that the notion of coalition we adopt is a rather broad one. We assume that a coalition is not necessarily formed by an explicit agreement between the actors. It might happen by chance that they act in the same way and, if by doing so they enforce some outcome, we will still say that they form a coalition.

Definition 6.9. (*Coalition Language*). Let $Nom^G \cup \mathcal{D}$ be the set of atoms. We define the coalition language over $Nom^G \cup \mathcal{D}$ as follows:

$$\phi ::= g \mid G \mid d \mid \neg\phi \mid \phi \vee \psi \mid \exists\phi \mid [\pi]\phi \mid \langle\langle A\rangle\rangle\phi.$$

$$\pi ::= R \mid ?\phi \mid \pi \cup \pi \mid \pi;\pi \mid \pi^*$$

for some $\{g, G\} \subseteq Nom^G, d \in \mathcal{D}$ and $A \subseteq \mathcal{A}$.

We read $\langle\langle A\rangle\rangle\phi$ as "the agents in A force ϕ to happen". The other symbols are to be read as in definition 5.1. However, the intuitive interpretation of the satisfaction of data-bits d is now a more specific one. To illustrate, given a static network model M and an agent a, the intuitive interpretation of $M, a \models d$ is "agent a has made d available to all her friends". This represents the intuition that agent a posted data d on her website. Recall that having some information is not intended as believing or knowing that data, but just possessing such data.

The satisfaction definition of the new coalition modality $\langle\langle A\rangle\rangle$ is based on the notion of equivalence $\sim_A$ between actions in $GAct^M$ performed by some

[1] The semantics is more interesting and complex that this. The interested reader can find a comprehensive analysis in [20].

$A \subseteq \mathcal{A}$.

Definition 6.10. (*Coalition Language Satisfaction*). Let $M = (\mathcal{A}, R, V)$ be a network model and consider some agent $a \in \mathcal{A}$. Then the satisfaction definition is the same as 5.3 with the following clause added:

$M \models \langle\langle A \rangle\rangle \phi$ iff there exists a sequence of actions $\alpha_1, \ldots, \alpha_n$ in $GAct^{M_1}, \ldots, GAct^{M_n}$ such that for all $\beta_1, \ldots, \beta_n$ in $GAct^{M_1}, \ldots, GAct^{M_n}$, if $\alpha_1 \sim_A \beta_1, \ldots, \alpha_n \sim_A \beta_n$ then $(M)^{\beta_1, \ldots, \beta_n} \models \phi$.

Example 6.11. Consider Example 6.7 and the related Figure 13. Let $M = (\mathcal{A}, R, V^M)$ be the model represented there, and let the letters in the nodes be both the nominals associated with the agents, and the agents in $\mathcal{A}$. We claim that $M \models \langle\langle A \rangle\rangle \exists(c \wedge d)$, with $A = \{a, b\}$.

Proof. In Example 6.7, we showed that since $M, a \models d$ and $a \in R[b]$, then we have $\alpha \in GAct^M$, with $d \in \alpha(b)$, by definition of executability of α. Then, call $(M)^\alpha = M'$. By definition of model update through α, at M' we have $b \in V^{M'}(d)$ and since $b \in R[c]$, then we also have $\alpha' \in GAct^{M'}$, with $d \in \alpha'(c)$, by definition of executability of α' at M'. Now, call $M'' = (M')^{\alpha'}$. Then we obtain $M'', c \models d$. Since $V^M(c) = c$ then $M'', c \models c \wedge d$. By semantics of $\exists$, this amounts to $M'', c \models \exists(c \wedge d)$. It follows that for any other sequence of actions β, β' in $GAct^M$ and $GAct^{(M)^\beta}$, such that $\alpha \sim_{\{a,b\}} \beta, \alpha' \sim_{\{a,b\}} \beta'$ we have $(M)^{\beta,\beta'}, c \models \exists(c \wedge d)$. By semantics of coalition modality, this means that $M \models \langle\langle A \rangle\rangle \exists(c \wedge d)$, with $A = \{a, b\}$. □

Example 6.12. Now consider the other example above, namely Example 6.8, and its related Figure 14. Let $M = (\mathcal{A}, R, V^M)$ be the model represented there, and let the letters in the nodes be both the nominals associated with the agents and the agents in $\mathcal{A}$. We claim that $M \models \langle\langle A \rangle\rangle \exists(c \wedge \neg d)$, with $A = \{b, f\}$.

Proof. As we already mentioned above, every agent in every model has the possibility to remain silent and perform the null-action. This is because by definition of executability of global action at a model, there always exists some $\alpha \in GAct^M$, such that $\alpha(b) = \emptyset = \alpha(f)$. Then, call $(M)^\alpha = M'$. By definition of model update through α, at M' we have $b, f \notin V^{M'}(d)$ and thus also $c \notin V^{M'}(d)$. This means that $M, c \models c \wedge \neg d$. By semantics of $\exists$, $M'', c \models \forall(c \wedge \neg d)$. Since b, f are the only agents related to c and from which then c can get to know d, this means that for every other action β in $GAct^M$, such that $\alpha \sim_{\{b,f\}} \beta$ we have $(M)^{\beta}, c \models \exists(c \wedge \neg d)$. By semantics of coalition modality, this means that $M \models \langle\langle A \rangle\rangle \exists(c \wedge \neg d)$, with $A = \{b, f\}$. $\square$

6.3 Capturing the Informational Capabilities of Gatekeepers

In this section, we use coalition logic for posting actions to prove the capabilities of the structural notions to enable or block the information flow between the disconnected groups. We start by proving the capability of bridges, connectors, and bridging sets to enable the flow between the groups. Then, we prove that blocking sets can block it. Lastly, we prove that gatekeepers can control the information flow between the groups, i.e. both enable and block it.

Bridges

Proposition 6.13. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridge between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \models (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists(G_2 \wedge \Diamond d)$.

Proof. Let M be a network model. Assume that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridge between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$. By definition of bridge, $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a connector between $V^M(G_1)$ and $V^M(G_2)$. By definition of connector, this means that $V^M(G_1) \cup V^M(G_2) \cup \{V^M(b_1), \ldots, V^M(b_n)\}$ is a group. Since $V^M(G_1)$ and $V^M(G_2)$ are otherwise disconnected, there must exist some $V^M(b_i) \in \{V^M(b_1), \ldots, V^M(b_n)\}$ such that for some agent $g_1 \in V^M(G_1)$, we have $(g_1, V^M(b_i)) \in R$. Assume that $M \models (G_1 \to d)$. Since $g_1 \in V^M(G_1)$, then $M, g_1 \models G_1$. By our assumption this implies $M, g_1 \models d$, i.e., $g_1 \in V^M(d)$. Then, by $(g_1, V^M(b_i)) \in R$ and definition of executable action, we know there exists a global function $\alpha_1 \in GAct^M$ such that $d \in \alpha_1(V^M(b_i))$.

Now we have two cases: either (i) $V^M(b_i)$ is the only agent in $\{V^M(b_1), \ldots, V^M(b_n)\}$ or (ii) it is not.

(i) $V^M(b_i)$ is the only agent in $\{V^M(b_1), \ldots, V^M(b_n)\}$. By definition of bridge, there exists some $g_2 \in \mathcal{A}$ such that $M, g_2 \models G_2$ and $(V^M(b_i), g_2) \in R$. By $d \in \alpha_1(V^M(b_i))$ we get $M^{\alpha_1}, V^M(b_i) \models d$. Then $M^{\alpha_1}, g_2 \models G_2 \wedge \Diamond d$. By semantics of $\exists$, this implies that $M^{\alpha_1} \models \exists(G_2 \wedge \Diamond d)$. Then, there exists a sequence of actions α_1 in $GAct^M$ such that if for any other action $\beta \in GAct^M$, we have that $\alpha_1 \sim_{V^M(b_i)} \beta$, then $M^\beta \models \exists(G_2 \wedge \Diamond d)$. By semantics of coalition modality $\langle\langle b_i \rangle\rangle$, this amounts to $M \models \langle\langle b_i \rangle\rangle(\exists(G_2 \wedge \Diamond d))$. Therefore we can conclude that $M \models (G_1 \to d) \to \langle\langle b_i \rangle\rangle(\exists(G_2 \wedge \Diamond d))$.

(ii) $V^M(b_i)$ is not the only agent in $\{V^M(b_1), \ldots, V^M(b_n)\}$. Say the other agents are $b_1, \ldots, b_m$.

Claim: since $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridge between $V^M(G_1)$, $V^M(G_2)$, and we know that $(g_1, V^M(b_i)) \in R$, then there exists at most one other agent b_k among $b_1, \ldots, b_m$ such that $b_k \in \{V^M(b_1), \ldots, V^M(b_n)\}$ and $(b_k, V^M(b_i)) \in R$.

Suppose not. Then there exists at least one more b' with $b' \neq b_k$, such that $b' \in \{V^M(b_1), \ldots, V^M(b_n)\}$ and $(b', V^M(b_i)) \in R$. By definition of bridge, $V^M(G_1) \cup V^M(G_2) \cup \{V^M(b_1), \ldots, V^M(b_n)\}$ is a group, and we have $(g_1, V^M(b_i)) \in R$, $(b_k, V^M(b_i)) \in R$, and $(b', V^M(b_i)) \in R$. It follows that there exist at least two paths P_1, P_2, with $b_k \in P_1$ and $b' \in P_2$, such that $V^M(G_1) \cup V^M(G_2) \cup P_1$ is a group and $V^M(G_1) \cup V^M(G_2) \cup P_2$ is a group. This means that there exist some $B \subset \{V^M(b_1), \ldots, V^M(b_n)\}$ such that $V^M(G_1) \cup V^M(G_2) \cup \{V^M(b_1), \ldots, V^M(b_n)\} \setminus B$ is a group, which contradicts the minimality of bridges. We can conclude that there exists at most one agent b_k such that $b_k \in \{V^M(b_1), \ldots, V^M(b_n)\}$ and $(b_k, V^M(b_i)) \in R$.

Recall that there exists an $\alpha_1 \in GAct^M$ such that $d \in \alpha_1(V^M(b_i))$. Then, by $(b_k, V^M(b_i)) \in R$ and definition of executable action, we know there exists a global function $\alpha_2 \in GAct^{M^{\alpha_1}}$ such that $d \in \alpha_2(b_k)$. Since $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridge, then by Proposition 3.16 and $(V^M(b_i), b_k) \in R$, we know that there exists at most another agent b_j among $b_1, \ldots, b_m$ with $b_k \neq b_j$ such that $(b_k, b_j) \in R$. Then by $d \in \alpha_2(b_k)$ and definition of executable action, we know there exists an $\alpha_3 \in GAct^{M^{\alpha_3}}$ such that $d \in \alpha_3(b_j)$. By iterating analogous reasoning for all the agents in $\{V^M(b_1), \ldots, V^M(b_n)\}$, we get that there exists a sequence of actions $\alpha_1, \ldots, \alpha_n$ in $GAct^{M^{\alpha_1}}, \ldots, GAct^{M^{\alpha_n}}$ such that $\{V^M(b_1), \ldots, V^M(b_n)\} \subset V^{M^{\alpha_n}}(d)$. By definition of bridge there exists some $g_2 \in \mathcal{A}$ such that $M, g_2 \models G_2$ and some $b \in \{V^M(b_1), \ldots, V^M(b_n)\}$ such that $(b, g_2) \in R$. Since $b \in V^{M^{\alpha_n}}(d)$ then $M^{\alpha_n}, g_2 \models G_2 \wedge \Diamond d$. By semantics of $\exists$, this implies that $M^{\alpha_n} \models \exists(G_2 \wedge \Diamond d)$. Then, there exists a sequence of actions $\alpha_1, \ldots, \alpha_n$ in $GAct^{M^{\alpha_1}}, \ldots, GAct^{M^{\alpha_n}}$ such that if for any other action $\beta_1, \ldots, \beta_n$ in $GAct^{M^{\beta_1}}, \ldots, GAct^{M^{\beta_n}}$, we have that $\alpha_1 \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta_1, \ldots, \alpha_n \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta_n$, then $M^{\beta_1,\ldots,\beta_n} \models \exists(G_2 \wedge \Diamond d)$.

By semantics of coalition modality $\langle\langle b_1, \ldots, b_n \rangle\rangle$, this amounts to $M \vDash \langle\langle b_1, \ldots, b_n \rangle\rangle(\exists(G_2 \wedge \Diamond d))$. Therefore we can conclude that $M \vDash (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle(\exists(G_2 \wedge \Diamond d))$. □

Connector

Proposition 6.14. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a connector between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \vDash (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists(G_2 \wedge \Diamond d)$.

Proof. Let M be a network model. Suppose that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a connector between two disconnected groups $V^M(G_1), V^M(G_2)$. By Proposition 3.10, we know that in every connector there exists a bridge. Then we can apply Proposition 6.13 to get that $M \vDash (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists(G_2 \wedge \Diamond d)$.

Hence, if $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a connector between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then $M \vDash (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists(G_2 \wedge \Diamond d)$. □

Bridging Sets

In Proposition 3.22, we showed that bridging sets between two disconnected groups are union of some bridges between them. Then, they can enable the information flow between them and to show it we will use the result about capabilities of bridges above. Since they are not the union of *all* bridges (as the grand gatekeeper is), they cannot also block the information flow.

Proposition 6.15. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be two group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridging set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \vDash (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists(G_2 \wedge \Diamond d)$.

Proof. Let $M = (\mathcal{A}, R, V^M)$ be a network model. Assume that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridging set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$. By Proposition 3.22, we know that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a union of bridges between $V^M(G_1)$ and $V^M(G_2)$. Since by definition of bridging set $\{V^M(b_1), \ldots, V^M(b_n)\} \neq$

$\emptyset$, then there exists at least one such bridge in $\{V^M(b_1), \ldots, V^M(b_n)\}$. So we can apply Proposition 6.13, to get that $M \models (G_1 \to d) \to \langle\langle b_1, \ldots, b_n\rangle\rangle^* \exists(G_2 \wedge \Diamond d)$.

Hence, if $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridging set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then $M \models (G_1 \to d) \to \langle\langle b_1, \ldots, b_n\rangle\rangle^* \exists(G_2 \wedge \Diamond d)$. $\square$

Blocking sets

Proposition 6.16. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a blocking set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \models (d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d)$.

Proof. Let M be a network model. Suppose that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a blocking set between two disconnected groups $V^M(G_1), V^M(G_2)$ and assume that $M \models (d \to G_1)$. Consider an arbitrary $b \in \mathcal{A}$ such that $b \in V^M(G_2)$, i.e. $M, b \models G_2$. Since $V^M(G_1)$ and $V^M(G_2)$ are disconnected groups, then for all $a \in \mathcal{A}$, if $(a, b) \in R$, then we have $a \notin V^M(G_1)$, i.e. $M, a \models \neg G_1$. By our initial assumption, we get $M, a \models \neg d$. By semantics of $\Box$, this implies $M, b \models \Box\neg d$, which is equivalent to $M, b \models \neg\Diamond d$. So for every $b \in \mathcal{A}$ it holds that if $M, b \models G_2$ then $M, b \models \neg\Diamond d$. Thus $M \models G_2 \to \neg\Diamond d$.

By definition of global action $\alpha : \mathcal{A} \to \mathcal{P}(\mathcal{D})$ and executable action, there exists an action $\alpha \in GAct^M$ such that for every $V^M(b_i) \in \{V^M(b_1), \ldots, V^M(b_n)\}$ we have $d \notin \alpha(V^M(b_i))$, namely $\alpha(V^M(b_i)) = \emptyset$. Consider an arbitrary other action $\beta \in GAct^M$ and suppose that $\alpha \sim_{\{V^M(b_1), \ldots, V^M(b_n)\}} \beta$.

Claim: Since $M \models G_2 \to \neg\Diamond d$, then also $M^\beta \models G_2 \to \neg\Diamond d$.

For suppose not, i.e. $M^\beta \not\models G_2 \to \neg\Diamond d$. This means that there exists some $a \in \mathcal{A}$, such that $M^\beta, a \models G_2 \wedge \Diamond d$. Then $a \in V^M(G_2)$ and there exists some $b \in \mathcal{A}$ such that $(a, b) \in R$ and $b \in V^{M^\beta}(d)$. By definition of model update, this is the case iff $d \in \beta(b)$. By assumption $\beta \in GAct^M$, so β is executable at M. By definition of executable functions, $d \in \beta(b)$ iff there exists some $c \in \mathcal{A}$ such that $(b, c) \in R$ and $c \in V^M(d)$. Since in M we had $M \models (d \to G_1)$, then $c \in V^M(G_1)$. So we have $(a, b) \in R$, $(b, c) \in R$ and $c \in V^M(G_1)$, $a \in V^M(G_2)$ and $b \in V^{M^\beta}(d)$.
Since $V(G_1)$ and $V(G_2)$ are disconnected groups, this means that $\{b\}$ is a connector between them. Then, by definition of blocking set, we must have

$\{V^M(b_1), \ldots, V^M(b_n)\} \cap \{b\} \neq \emptyset$. This means that $\{V^M(b_1), \ldots, V^M(b_n)\} \cap \{b\} = \{b\}$ and $\{b\} \subseteq \{V^M(b_1), \ldots, V^M(b_n)\}$. Recall that we had $\alpha(V^M(b_i)) = \emptyset$ for every $V^M(b_i) \in \{V^M(b_1), \ldots, V^M(b_n)\}$, and that $\alpha \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta$. By definition of $\sim$, it follows that $\beta(V^M(b_i)) = \emptyset$ for every $V^M(b_i) \in \{V^M(b_1), \ldots, V^M(b_n)\}$. Then, $\beta(b) = \emptyset$, i.e. for every $d \in \mathcal{D}$, we have $b \notin V^{M^\beta}$. But we had $b \in V^{M^\beta}$. Contradiction. Hence, if $M \models G_2 \to \neg\Diamond d$, then also $M^\beta \models G_2 \to \neg\Diamond d$.

Therefore, there exists an action α such that if for every other action β we have $\alpha \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta$ then $M^\beta \models G_2 \to \neg\Diamond d$. By semantics of coalition modality $\langle\langle b_1, \ldots, b_n\rangle\rangle$, this amounts to $M \models \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d)$. So we can conclude that $M \models (d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d)$.

Hence, if $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a blocking set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then $M \models (d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d)$. □

Gatekeeping Bridge

Proposition 6.17. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a gatekeeping bridge between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \models ((G_1 \to d) \to \langle\langle b_1, \ldots, b_n\rangle\rangle\exists(G_2 \wedge \Diamond d)) \wedge ((d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d))$.

Proof. Let M be an network model. Assume that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is the gatekeeping bridge between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$. By Proposition 4.12, we know that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a bridge. So we can use Proposition 6.13, to get that the positive part of gatekeeping bridge's capability, i.e., $M \models (G_1 \to d) \to \langle\langle b_1, \ldots, b_n\rangle\rangle^*\exists(G_2 \wedge \Diamond d)$.

Now assume that $M \models (d \to G_1)$. By the fact that $V^M(G_1)$ and $V^M(G_2)$ are disconnected groups, and by applying exactly the reasoning we used for in the proof of Proposition 6.16, we obtain $M \models G_2 \to \neg\Diamond d$.

By definition of global action $\alpha : \mathcal{A} \to \mathcal{P}(\mathcal{D})$ and executable action, there exists an action $\alpha \in GAct^M$ such that for every $V^M(b_i) \in \{V^M(b_1), \ldots, V^M(b_n)\}$ we have $d \notin \alpha(V^M(b_i))$, namely $\alpha(V^M(b_i)) = \emptyset$. Consider an arbitrary other action $\beta \in GAct^M$ and suppose that $\alpha \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta$.

Claim: Since $M \models G_2 \to \neg\Diamond d$, then also $M^\beta \models G_2 \to \neg\Diamond d$.

For suppose not, i.e. $M^\beta \not\models G_2 \to \neg\Diamond d$. This means that there exists some $a \in \mathcal{A}$, such that $M^\beta, a \models G_2 \land \Diamond d$. Then $a \in V^M(G_2)$ and there exists some $b \in \mathcal{A}$ such that $(a, b) \in R$ and $b \in V^{M^\beta}(d)$. By definition of model update, this is the case iff $d \in \beta(b)$. By definition of executable function β, this is the case iff there exists some $c \in \mathcal{A}$ such that $(b, c) \in R$ and $c \in V^M(d)$. Since in M we had $M \models (d \to G_1)$, then $c \in V^M(G_1)$. So we have $(a, b) \in R$, $(b, c) \in R$ and $c \in V^M(G_1)$, $a \in V^M(G_2)$ and $b \in V^{M^\beta}(d)$.
Since $V(G_1)$ and $V(G_2)$ are disconnected groups, this means that $\{b\}$ is a connector between them. Then, we must have $\{V^M(b_1), \ldots, V^M(b_n)\} \subseteq \{b\}$, by definition of gatekeeping bridge. Since by definition, a gatekeeping bridge is not empty, then $\{V^M(b_1), \ldots, V^M(b_n)\} = \{b\}$. Now recall that $\alpha(V^M(b_i)) = \emptyset$ for every $V^M(b_i) \in \{V^M(b_1), \ldots, V^M(b_n)\}$ and that $\alpha \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta$. Then, $\beta(b) = \emptyset$, i.e. for every $d \in \mathcal{D}$, we have $b \notin V^{M^\beta}(d)$. But we had that $b \in V^{M^\beta}(d)$. Contradiction. Hence, if $M \models G_2 \to \neg\Diamond d$, then also $M^\beta \models G_2 \to \neg\Diamond d$.

Therefore, there exists an action α such that if for every other action β we have $\alpha \sim_{\{V^M(b_1),\ldots,V^M(b_n)\}} \beta$ then $M^\beta \models G_2 \to \neg\Diamond d$. By semantics of coalition modality $\langle\langle b_1, \ldots, b_n\rangle\rangle$, this amounts to $M \models \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d)$. So we can conclude that $M \models (d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d)$.

Hence, by putting together the two conjuncts we obtain that if $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a gatekeeping bridge between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then $M \models ((G_1 \to d) \to \langle\langle b_1, \ldots, b_n\rangle\rangle\exists(G_2 \land \Diamond d)) \land ((d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d))$. □

Gatekeeping Sets

Proposition 6.18. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a gatekeeping set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \models ((G_1 \to d) \to \langle\langle b_1, \ldots, b_n\rangle\rangle\exists(G_2 \land \Diamond d)) \land ((d \to G_1) \to \langle\langle b_1, \ldots, b_n\rangle\rangle(G_2 \to \neg\Diamond d))$.

Proof. Let $M = (\mathcal{A}, R, V^M)$ be a network model. Assume that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a gatekeeping set between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$. By definition of gatekeeping set, A is a bridging set between $V^M(G_1)$ and $V^M(G_2)$, and by Proposition 3.22 we know that A is a union of bridges between $V^M(G_1)$

and $V^M(G_2)$. Since by definition of gatekeeping set $\{V^M(b_1), \ldots, V^M(b_n)\} \neq \emptyset$, then $\{V^M(b_1), \ldots, V^M(b_n)\}$ is composed by at least one such bridge. So we can apply Proposition 6.13, to get that $M \models (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists (G_2 \wedge \Diamond d)$.

Moreover, by definition, a gatekeeping set between two groups is a blocking set. So we can apply Proposition 6.16, and derive that $M \models ((d \to G_1) \to \langle\langle b_1, \ldots, b_n \rangle\rangle (G_2 \to \neg \Diamond d))$.

Hence, by combining the two conjuncts we obtain that if $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a gatekeeping set between two disconnected groups $V^M(G_1)$, $V^M(G_2)$ then $M \models ((G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists (G_2 \wedge \Diamond d)) \wedge ((d \to G_1) \to \langle\langle b_1, \ldots, b_n \rangle\rangle (G_2 \to \neg \Diamond d))$. □

Grand Gatekeeper

Proposition 6.19. *Let $M = (\mathcal{A}, R, V^M)$ be a network model, $b_1, \ldots, b_n$ be nominals for distinct single agents and G_1, G_2 be distinct group nominals. If $\{V^M(b_1), \ldots, V^M(b_n)\}$ is the grand gatekeeper between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$ then*

$M \models ((G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists (G_2 \wedge \Diamond d)) \wedge ((d \to G_1) \to \langle\langle b_1, \ldots, b_n \rangle\rangle (G_2 \to \neg \Diamond d))$.

Proof. Let $M = (\mathcal{A}, R, V^M)$ be a network model. Assume that $\{V^M(b_1), \ldots, V^M(b_n)\}$ is the grand gatekeeper between two disconnected groups $V^M(G_1)$ and $V^M(G_2)$. By Proposition 4.20, A is the union of all the bridges between $V^M(G_1)$ and $V^M(G_2)$. By Proposition 3.10, there exists at least a bridge between them and it is non-empty. So we can apply Proposition 6.13, to get that $M \models (G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists (G_2 \wedge \Diamond d)$.

Moreover, by Proposition 4.24, a grand gatekeeper between two groups is a blocking set between them. So we can apply Proposition 6.16, and derive that $M \models ((d \to G_1) \to \langle\langle b_1, \ldots, b_n \rangle\rangle (G_2 \to \neg \Diamond d))$.

Hence, by combining the two conjuncts we obtain that if $\{V^M(b_1), \ldots, V^M(b_n)\}$ is a gatekeeping set between two disconnected groups $V^M(G_1)$, $V^M(G_2)$ then $M \models ((G_1 \to d) \to \langle\langle b_1, \ldots, b_n \rangle\rangle \exists (G_2 \wedge \Diamond d)) \wedge ((d \to G_1) \to \langle\langle b_1, \ldots, b_n \rangle\rangle (G_2 \to \neg \Diamond d))$. □

The following interesting fact is to be noted. When proving that a gatekeeper can both enable and block the information flow, we are implicitly also proving that a gatekeeper can decide to allow just a subset of the information to flow, while blocking its complement. Then, the propositions above show that the control the gatekeeper can exercise also comes in the form of shaping the information the groups receive. In gatekeeping theory, this is taken to mean that gatekeepers, as editors of newspapers or journals, do not merely block or enable the information to flow. By doing so, they actually *construct* the social reality.

> (..) the gatekeeping process is also thought of as consisting more than just selection. In fact, gatekeeping in mass communication can be seen as the overall process through which social reality transmitted by the news media is constructed, and is not just a series of in and out decisions. (Barzilai-Nahon, [6])

7 Conclusion and Future Works

In this section, we summarize what this thesis has achieved, and then discuss in which directions this work can be further developed.

We provided a formalization of the gatekeeper phenomenon in social networks. In chapter 1 and 2, we introduced the topic and the preliminaries that were necessary to motivate and introduce the rest of the work. In chapter 3, we provided a graph-theoretical representation of some structural notions that were later used to construct the gatekeepers. Important notions, were the connector, bridging set and blocking set. Chapter 4 contained the graph-theoretical representation of gatekeepers, each constructed out of some of the three notions above mentioned. In chapter 5, we used an hybrid version of PDL to characterize the notions of gatekeeper. We left as open question whether it was possible to characterize the grand gatekeeper with that logic. We think it is not, as to achieve that, one should add to the language an infinite amount of nominals, which we did not. In chapter 6, we expanded the network logic, making it dynamic. We defined a set of actions that we used as dynamic updates of the network models. This was encoded in the language through a coalition modality, which also expressed the capability that some agents have to force some outcomes.

From here, there are many paths that open up. We propose two of them.

Concerning the graph-theoretical part, it would be interesting to generalize the structural notions. All of them are now defined for two disconnected groups G, G', but they could be easily extended to n groups. Such generalizations could represent notions as the star network that Bruggeman describes [12]. To give an example of what this could look like, we provided a generalization of the gatekeeping set.

Definition 7.1. (*Gatekeeping Hub*). Let $G_1, G_2, \dots, G_n$ be disconnected groups and consider some $A \subset \mathcal{A}$ such that $A \neq \emptyset$. We say that A is the gatekeeping hub between disconnected $G_1, G_2, \dots, G_n$ iff

(GH-) for every $A' \subset \mathcal{A}$, if $\bigcup_{i=1}^{n} G_i \cup A'$ is a group, then $A \cap A' \neq \emptyset$;

(GS+) for every G_i, G_j, with $1 \leq i < j \leq n$, there exists some $A' \subseteq A$ that is a bridging set between G_i, G_j.

This notion has similar properties to the ones we showed for the other notions, but generalized to n groups. For example, if A is the gatekeeping hub between some disconnected groups $G_1, G_2, \dots, G_n$, then: (1) $\bigcup_{i=1}^{n} G_i \cup A$ is a group; (2) $G_j \cap G_k = \emptyset$; (3) $A \neq G_j$ and $A \neq G_k$; (4) $A \not\subseteq G_j \cup G_k$; (5) $A \not\subseteq \bigcup_{i=1}^{n} G_i$.

The other interesting direction would be to introduce more features to network models and represent more kinds of communication. For example, the posting action does not capture the directionality of exchanges of information, as it directs the posted message to every friend one has. One possibility would then be to define the *sending action*, a directed version of the posting action. This action could direct the message to a set of agents, namely a subset of one's friends. Then, also in this case, the sending action could be used as a model transformer and the update be again encoded in the valuation function V^M.

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