

RANDOM SEQUENCES

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GEBOREN TE KRIMPEN AAN DE IJSSEL

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*Tout a déjà été éprouvé et expérimenté à mille reprises,
mais souvent sans avoir été dit, ou sans que les paroles
qui le disent subsistent, ou si elles le font, nous soient
intelligibles et nous émeuvent encore. Comme les
nuages dans le ciel vide, nous nous formons et nous
dissipons sur ce fond d'oubli.*

Marguerite Yourcenar
Archives du Nord

Preface

The research on the foundations of probability reported here was part of project 22 – 110 of the Dutch Foundation for Scientific Research ZWO, whose support is gratefully acknowledged. Some of the central concerns of this essay took shape in a long afternoon stroll in Salzburg with Roger Cooke, who also intensely supervised their subsequent development. In spite of the heat generated by our debates, they have resulted in a state of low entropy in which few differences of opinion remain.

I owe a special debt to Johan van Benthem, who steered me toward mathematical logic long ago. The example of his technical acumen and systematic philosophizing has inspired me ever since. His already overburdened schedule notwithstanding, he took over part of my teaching duties during the last phase of the writing.

I am grateful to Joop Doorman, whose insistence on a philosophical justification of what was originally a collection of rather technical results, finally produced Chapter 2.

Lastly, I thank Mike Keane and Guus Balkema for mathematical assistance.

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Contents

1 Introduction	1
2 Roots of randomness: von Mises' definition of random sequences	6
2.1 Introduction	6
2.2 The frequency interpretation of probability	8
2.2.1 Methodological considerations	8
2.2.2 Kollektivs (informal exposition)	10
2.2.3 Strict frequentism: "Erst das Kollektiv, dann die Wahrscheinlichkeit"	11
2.2.4 Structure and task of probability theory	15
2.3 Axiomatising Kollektivs	16
2.3.1 The axioms	16
2.3.2 Some consequences of the axioms	19
2.3.3 Do Kollektivs exist?	21
2.4 The use of Kollektivs	24
2.4.1 The fundamental operations: definition and application	25
2.4.2 Necessity of Kollektivs	29
2.4.3 Strong limit laws	31
2.5 Making Kollektivs respectable: 1919 – 1940	33
2.5.1 Lawlike selections	33
2.5.2 The contextual solution	38
2.6 The Geneva conference: Fréchet's objections	41
2.6.1 Fréchet's philosophical position	41
2.6.2 Formal objections	43
2.6.2.1 Inconsistency	43
2.6.2.2 Ville's construction	43
2.7 Conclusions	51
Notes to Chapter 2	53
3 A new start: Martin-Löf's definition	55
3.1 Introduction	55
3.2 The definitions of Martin-Löf and Schnorr	57
3.2.1 Randomness via probabilistic laws	57
3.2.2 Recursive sequential tests	60
3.2.3 Total recursive sequential tests	61
3.2.4 An appraisal and some generalisations	65
3.3 Probabilistic laws	67
3.4 Martingales	70
3.5 Randomness via statistical tests	78
3.5.1 Types of statistical tests	78
3.5.2 Effective statistical tests	82
3.5.3 Discussion	83
3.6 Conclusion	85
Notes to Chapter 3	86
4 Place selections revisited	88

4.1 Introduction	88
4.2 Place selections from a modern perspective	89
4.3 Preliminaries	90
4.4 Effective Fubini theorems	92
4.5 Proof of the principle of homogeneity	99
4.6 New proof of a theorem of Ville	102
4.7 Digression: the difference between randomness and 2–randomness	113
Note to Chapter 4	114
5 Kolmogorov–complexity	115
5.1 Complexity of finite strings	116
5.1.1 Kolmogorov–complexity	116
5.1.2 Chaitin's modification	118
5.1.3 Conditional complexity	122
5.1.4 Information, coding, relative frequency	124
5.1.5 Discussion	127
5.1.6 Digression: resource–bounded complexity	128
5.2 Kolmogorov's program	128
5.3 Metamathematical considerations on randomness	131
5.3.1 Complexity and incompleteness	132
5.3.2 Discussion	135
5.4 Infinite sequences: randomness and oscillations	137
5.4.1 Randomness and complexity	138
5.4.2 Downward oscillations	139
5.4.3 Upward oscillations	142
5.4.4 Monotone complexity	144
5.5 Complexity and entropy	145
5.5.1 Dynamical systems	146
5.5.2 Metric entropy	146
5.5.3 Topological entropy	151
5.5.4 Kamae–entropy	156
5.6 Admissible place selections	157
5.6.1 Deterministic sequences	157
5.6.2 Admissibility and complexity	159
Notes to Chapter 5	160
6 Appendix: notation and definitions	162
6.1 Notation for sequences	162
6.2 Topology on 2^ω	162
6.3 Measures on 2^ω	162
6.4 Computability	163
6.5 Ergodic theory	164
References	166
Samenvatting (Dutch summary)	172

