RIJKSUNIVERSITEIT TE GRONINGEN

STUDIES IN DIALOGICAL LOGIC

ERIK C.W. KRABBE

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0. INTRODUCTION

This dissertation consists of papers on dialogical logic, grouped around three themes. The papers of Part 1 are concerned with the pragmatic and intuitive foundations of a dialogical approach in logic. They yield a number of exemplars of propositional dialectic systems (dialogue games) that form the subject matter for the metatheoretical studies of Part 2. In Part 2 one will also find the completeness theorems that connect (propositional) dialogical logic with other branches of logic, viz., the theory of deduction and the theory of models. The papers of Part 3 extend the dialogical approach so as to include modal operators. They contain material on the foundations and on the metatheory of modal dialectic systems, including their connections with other branches of modal logic.

The present introduction to these papers contains an attempt at terminological clarification (Section 0.1) and a number of remarks on the separate papers (Section 0.2). A brief note on the dialogical treatment of quantifiers is included in the appendix to this dissertation.

0.1. Logic, theory of argumentation, and formal dialectic

0.1.1. The garbs of logic

Contemporary dialogical logic owes its existence to Lorenzen's definition of a number of logical constants in terms of distinctively associated "attacks" and "defenses". (1) The dialogical approach to logic is characterized by this particular way of introducing logical constants and by a matching way of formulating a clarifying definition for the concept of 'logical validity'.

One may distinguish three main types of logic, each characterized by its own way of introducing logical constants and its own way of redefining 'validity'. In From Axiom to Dialogue the authors speak of the "modes of presentation", or "garbs", of logic: (2)

(1) The deduction-theoretic or derivational garb: logical constants are defined (implicitly) by the system of axioms and rules of inference to which they pertain. 'Validity' is reconstructed as derivability in such a system.

(2) The model-theoretic or semantic garb: logical constants are defined by means of semantic rules. 'Validity' is reconstructed as immunity from counterexample. What constitutes a counterexample is stipulated explicitly by a theory of structures/models.

(3) The dialogue-theoretic, dialogical or dialectical garb: logical constants are defined (implicitly) by the dialogue game (or, dialectic system) to which they pertain. 'Validity' is reconstructed as the availability of a winning strategy in debate.

The common purpose of the papers constituting this dissertation is to contribute to the development of logic in this third garb.

0.1.2. Theory of argumentation and logic

Dialogical logic may, of course, be studied for its own sake or for the purpose of constructing new, interesting validity concepts. This is one of the motivations for the present work, but at the same time, in Paper 1 and elsewhere, I have tried to contribute to the construction of verbal instruments for the resolution of conflicts of opinion. Thus the foundations of dialogical logic here presented are at the same time motivated from the perspective of a future comprehensive theory of argumentation.

It is hard to fix a serviceable boundary line between the (extensions of the) terms "theory of argumentation" and "logic". Both are concerned with argumentation, or reasoning, and both try to draw a distinction between what is sound or proper and what is not. Moreover, both "disciplines" are beset by the "normative-descriptive ambiguity". Either discipline is sometimes taken to include the other. [3]

I propose to use "logic" as a comprehensive term and to assign to the theory of argumentation those logical questions about argumentation that are, rightly or wrongly, neglected by most contemporary logicians and that can, presumably, be profitably studied in an interdisciplinary way. [4]

Since the social aspect of argumentation _is_ usually neglected by logicians, and since this aspect is particularly apt to become a common hunting ground for a number of disciplines, it should not surprise us that theorists of practical argumentation attend to it, if only by stressing the speaker-listener relationship. Consequently, of the three garbs of logic mentioned in Section 0.1.1, the dialogical one seems most akin to the aims of theory of argumentation.

The foundations of dialogical logic in Paper 1 (and in other parts of this dissertation) will, I hope, be incorporated in a comprehensive theory of argumentation formulated as a theory of debate. [5]

0.1.3. Formal, normative, descriptive

In *From Axiom to Dialogue* three senses of "form" and "formal" are kept apart by means of subscripts, "formal$_1$" standing for a sense related to Platonic forms, "formal$_2$" for a sense related to linguistic forms (modes of construction), and "formal$_3$" for a sense related to procedural rigor. [6] It is stressed that formal logic, today, is formal$_2$ and formal$_3$, not formal$_1$.

There is a fourth sense of formal (formal$_4$) according to which it means 'nonempirical' or even 'normative'. Thus, when Hamblin introduced the term "formal dialectic" he contrasted it with "descriptive dialectic", though stressing that "neither approach is of any importance on its own". [7]

As the term is used in this dissertation, "formal" in "formal dialectics" means 'formal$_3$' only. Consequently, any rigorous system of procedures or rules for the resolution of conflicts of avowed opinion by verbal means is called a "system of formal dialectics" or a "dialectic system" for short, whether its roots are primarily normative or primarily descriptive. [8] Logic in its dialectical garb shares in the "normative-descriptive ambiguity" familiar to students of applied logic and philosophy of logic.

As to the nonempirical aspect of logic, elsewhere I have argued that neither primarily normative not primarily descriptive logic is a "merely formal", in the sense of "nonempirical" science. [9] Thus, the primarily normative studies of Part 1 are in fact based upon *empirical intuitions* as to what rules are probably acceptable to most people. [10] Consequently, *the resulting theories about which dialectic rules are commendable are falsifiable*.

There is a fifth sense of formal (formal$_5$)! This sense occurs when "formal dialogue games" are contrasted with "material dialogue games". This sense is not intended by "formal" in "formal dialectics" either. I shall return to this matter in Section 0.1.5.

0.1.4. Dialogical logic, formal dialectics, dialogue games

I use the term "dialogical logic" for the dialogue-theoretical (or, dialogical or, dialectical) garb of logic (Section 0.1.1). The term "formal dialectics" was explained in the preceding section. One may, of course, study dialogical logic with aims other than that of constructing systems for the verbal resolution of conflicts of opinion. In such cases I would prefer the (rather neutral) term "dialogue game", instead of "formal dialectics" or "dialectic system", for the systems studied or constructed.

0.1.5. Material and nonmaterial systems

In the writings of Lorenzen and Lorenz the material dialogues clearly have priority over the nonmaterial (formal$_5$) ones. Not only are the material dialogues introduced before the formal$_5$ ones in most texts (if the latter are treated at all), [11] but they also constitute the locus where the logical constants are introduced. Systems of rules for formal$_5$ dialogues are then used to reconstruct logical notions, such as 'validity' or 'logical truth'.

For the latter purpose, however, one need not have recourse to formal$_5$ dialogues or dialogue games at all. For, equivalently, one may define the class of logical truths, for instance, as the class of sentences such that there is a <u>formal$_5$ winning strategy</u>, for the Proponent of each of them, in a <u>material</u> dialogue game. A formal$_5$ strategy, for a party N, is simply a strategy according to which N never makes certain kinds of moves. [12] The moves from which N abstains are the "material" ones, i.e., those that depend upon the content or meaning of some nonlogical constant. In practice this means no more than that some modes of ending the game are excluded, viz., those that depend upon the truth value of an atomic sentence ("material closure"). Since the expedient of first defining formal$_5$ <u>games</u> or <u>dialogues</u> is thus easily bypassed, their use by Lorenzen and Lorenz is clearly of secondary importance. [13]

In <u>From Axiom to Dialogue</u> and in this dissertation as well, the procedure is reversed. The first examples of complete dialectic systems reached are systems that do not include any material rules or moves (Paper 2). This order is more or less dictated by the desire to have an acceptable system of dialectics, whether or not any material agreement obtains.

For, it is clear that even if a certain company (seeking an instrument for the verbal resolution of conflicts) does not agree about the truth value of any atomic sentence -- nor upon any procedure for attaining such an agreement -- it may nevertheless be able to agree upon a set of nonmaterial rules for rigorous debate. In this situation systematic debate is still possible. In the reverse situation -- with agreement about some atomic sentences but lack of agreement about the nonmaterial rules -- debate is impossible. The nonmaterial dialectic systems are, therefore, indispensable and constitute the more fundamental case from which material systems can be derived.

This reversal of priority stems from a difference in aims. Lorenzen's purpose, in introducing the dialogues, is to fix a clear, teachable meaning for the logical constants, whereas in From Axiom to Dialogue and in this dissertation the dialectic systems are (models for) systems for the resolution of conflicts.

Note that both, the material and the nonmaterial ($formal_5$) dialectic systems, are systems of formal ($formal_3$) dialectics.

0.2. Foundations, metatheory, modality

0.2.1. Foundations

Part 1 of this dissertation is concerned with foundations of dialogical logic. These foundations are independent of other garbs of logic. The central aim of dialectic systems is taken to be the resolution of conflicts of avowed opinions by verbal means. Hence, Paper 1 starts with a definition of conflict of avowed opinions and of what constitutes a resolution of a conflict. If such conflicts are to be resolved by verbal means, the parties should ideally first try to agree upon a set of rules to provide the necessary regulation of their debates. That is, they should try to agree upon a system of "formal dialectics". The rules that constitute the proposed systems of formal dialectics are "founded" in certain primary norms, in the sense that they are implementations of these norms. They are, then, hierarchically ordered, in the sense that a rule is introduced as a (possible) implementation of another rule, which again implements some other rule, and so on, up to the primary (basic) norms. With the exception of one rule -- rule F_2D 1 in Section 1.16 -- the rules are language-invariant. Examples, however, pertain to propositional languages of the form $\mathfrak{J}_D$ (with implication only), or of the form $\mathfrak{P}_D$ (with implication, conjunction, veljunction and negation), or of the form $\mathfrak{P}_D^{\wedge}$ (with a falsum constant in addition). The subscript "D" indicates that the languages are dialectically augmented (with questions, etc.). (14)

Part of Paper 1 pertains to the level of concrete utterances or inscriptions of sentences, while part pertains to the more abstract level of sentences. Variables for sentences are "U", "V", "W" ...; utterances of them are indicated by "$\underline{U}$", "$\underline{V}$", "$\underline{W}$" .... A sentence may be a declarative sentence (U,(?)U), or a question (U?,?), or an exclamation (U!,!). Again, a declarative sentence may be either assertive (U) or hypothetical ((?)U). The term statement is reserved for utterances of declarative sentences in "use" (not "mention"). (15)

In Paper 2 a number of finished dialectic systems are described. These systems are best looked upon as simplified models of what a full-fledged and workable dialectic system will look like. The constructive and classical systems are equivalent to dialogue games as introduced by Lorenzen or Lorenz. In fact, the constructive dialectic systems are virtually identical to the constructive formal_5 games of Lorenzen. (16) These systems (and the minimal ones in $\mathfrak{P}^{\wedge}$) are, moreover, the most "natural" ones. For minimal systems in $\mathfrak{P}$, and for classical systems, some rather "unnatural" rules are needed.

In Paper 3 material procedures and moves are subjoined to the systems of Paper 2 (see Section 0.1.5 above). At the end of this paper another material system, MatDial, is independently formulated, mixed conflicts of complete opposition taking the place of the earlier simple conflicts.

Paper 4 gives an alternative, quite simple, motivation for some dialectic rules, restricted, however, to contexts of immanent criticism of verbalized systems of thought.

0.2.2. Metatheory

In Part 2 some facts about dialectic systems are proved, which are independent of other logical garbs. Among other things, in Paper 5 it is shown that the dialectic systems of Papers 2 and 3 fulfill the norms of Paper 1. Also in Paper 5, there is a "dialogical" proof of the equivalence of the systems with a *falsum*-constant and the corresponding systems without a *falsum*-constant. In the case of minimal logic, this is not trivial. In Paper 7 it is shown that all the systems of Paper 2 are *invertible*. [17]

Further, Part 2 contains completeness theorems that connect dialogical logic with other garbs. The results are well-known; [18] what is new is the manner of proof. I have aimed at simple, visually suggestive, proofs. Moreover, I have arranged this whole part of metatheory in one circle of theorems. The circle starts with winning strategies in dialectic systems, represented by closed dialogical tableaux. It then moves to closed deductive tableaux (Paper 6), thence to natural deduction, to axiomatics, to semantic validity, to closed semantic tableaux, and back again to winning strategies (Paper 7). [19] Other arrangements are quite possible (thus, one may go from closed deductive tableaux to closed dialogical tableaux without much trouble), but the "full circle" seems particularly attractive.

I should, perhaps, apologize for the use of König's lemma in some places, for in propositional logic it is not needed and therefore constitutes an unnecessary nonconstructive element in the proofs offered here. However, since propositional syntax serves only to exemplify the methods here presented, and since in predicate logic the lemma is needed in any case, whereas in propositional logic it leads to much simplification, I felt entitled to use it. My purpose was not constructivity but simplicity.

Paper 8 proves a general completeness theorem for material dialectic systems. It is more abstract than any of the other papers. Paper 9 repeats this proof for one special case: the system MatDial introduced in Paper 3.

0.2.3. Modality

The papers of Part 3 contain their own introductions. Therefore little remains to be said about them here. Paper 10 arose from an independent motivation, viz., the wish to find a dialectic system corresponding to a noncumulative logic. This investigation quite naturally led me to a first exploration of modal dialectics, an exploration that is continued in Paper 11. In these two papers one will find both "foundational" and "metatheoretic" sections, which extend the normative foundations and the metatheory of the preceding parts so as to incorporate modal operators.

PART 1

FOUNDATIONS

1. [Chapter III] Conflicts of Opinion and Methods for their Resolution[1]

(with E.M. Barth)

One modern way of sharpening the vague pre-scientific notion of 'logical validity' runs as follows:

> The step from a set, Π, of premises to a conclusion, Z, is *dialectically valid* (in a system σ) if and only if:
>
> there is (given the dialectic system σ) a winning strategy for a Proponent of Z, relative to Π as the set of concessions made by the Opposition (in a discussion carried out according to the rules of the system σ).

This definition of validity is due to Paul Lorenzen, who first formulated it in *Ein dialogisches Konstruktivitätskriterium.*[2] It is based on the concept of having a winning strategy, which is defined in Section 15 below. We shall formulate methods for deciding about validity in this sense in Chapter V, but first we shall discuss, in the present chapter, the general features of the dialectical "garb" of logic and formulate, in Chapter IV, a number of dialectic systems, to be used as values for the variable "σ" in the definition of dialectical validity.

The dialectical garb is the most recent one donned by modern logic. This is not to say that it is the most recent one in the history of logic. Indeed, its roots go back to Aristotle's *Topics* and *Sophistical Refutations*, which are, presumably, older than the *Prior Analytics*, wherein lie the roots of the other garbs.

In medieval times *dialectica*, as part of the *trivium*, was incorporated in the undergraduate university curriculum. The closest medieval equivalent to the dialectical garb of modern logic is found in the treatises on the Obligation Game.[3] The word *dialectica*, however, is not restricted to the dialectical garb of logic. In Stoic and medieval logic and in the 16th century, *dialectica* was simply the word for logic.[4]

In modern "formal" logic, the dialectical garb was inaugurated by P. Lorenzen (see Section I.1, *sub* (iii) 8). From him originate the modes of "attacking" and "defending" statements, of certain logical forms$_2$ given in our rule F_2D 1 (Section 16). His preference for a dialectical, or dialogical, logic of contest over the *pious solitaire* of the prevalent monological systems is already apparent in *Logik und Agon*. There he equates the system that is, misleadingly, known as

---

1 The text of this chapter is a slightly modified version of Sections 1 through 16 of our paper [FD1]. Sections 17, 18 and 19 of the paper are incorporated in the next chapter (Chapter IV), Section 20 is incorporated in Section XI.7.

2 Lorenzen [DKn], p. 195 (Lorenzen and Lorenz [DLg], p. 12).

3 See Hamblin [F11], pp. 126ff.: "One of the earliest – perhaps the earliest – of the treatises on Obligation is ascribed to William of Sherwood".

4 See Scholz [AGL], pp. 7, 8.

classical logic with a logic of cooperative debates (dialectics, in Plato's and Aristotle's sense), and intuitionistic (constructive) logic with a logic of competitive debates (eristics). In *Ein dialogisches Konstruktivitätskriterium* he proposes the use of the availability or want of dialogue rules that make a sentence suitable for critical debate (*dialogisch-definit*) as a criterion of constructivity in mathematics. In later publications his dialogical logic forms part of a radical reconstruction program for scientific and philosophical language.[5] Complete systems of dialectical logic were first devised by K. Lorenz in *Arithmetik und Logik als Spiele* and proved by him to be equivalent to certain sequent systems (see Section I.1 *sub* (iii) 5).

In this book we shall first study dialectics as an independent subject and later explore its connections with the other "garbs". We do not intend to offer a reconstruction program in which we would have to assume, initially, that nothing but the simplest everyday language is understood by the reader. On the contrary, we speculate that we and the readers already have a lot of logical rules in common, and we will continue to try to take advantage of this in our exposition.

The program of this chapter was briefly indicated at the end of Section I.3. We shall first define what we mean by a "conflict of avowed opinion" and what we consider a resolution of such a conflict to be. If such conflicts are to be resolved by *verbal* means, the parties should ideally first try to agree upon a set of rules to provide the necessary regulation of their debates. That is, they should try to agree upon a system of $formal_3$ dialectics[6]. The rules that constitute our proposed systems of $formal_3$ dialectics will be "founded" in certain primary *norms*, as implementations of these norms. They are, then, hierarchically ordered, in the sense that a rule is introduced as a (possible) implementation of another rule which again implements some other rule, and so on up to the primary (basic) norms. Except for the one rule F_2D 1 in Section 16, the rules are *language-invariant*. In Chapter IV, we arrive at a number of systems for debates in languages of the forms $\mathfrak{J}_D$, $\mathfrak{T}_D$ and $\mathfrak{T}_D^{\wedge}$. These dialectic systems are equivalent to those constructed by Lorenzen and Lorenz and hence to the systems of logic in the other "garbs".

We would like to stress that the present dialectic systems, though formally complete in the sense of modern logic, certainly do not yet constitute "complete" theories of argumentation in the wider sense of theories ready for use, but only ⌐ problems of interpretation, definition, and clarification, as treated by Arne Naess.[7] A future, fully-grown theory of argumentation will have to deal with

these problems so as to unite the approaches of Lorenzen (and others) with those of Naess (and others) in one system of dialectics. Cf. Section XI.7 for a list of tasks that remain.

---

5 See Kamlah and Lorenzen $[LPr]_2$, Lorenzen and Schwemmer $[KLE]_2$.

6 As noted at the end of Section I.3, we owe the expression "formal dialectics" to C. L. Hamblin ([F11], Chapter 8). Our use of subscripts after "formal" was also explained in Section I.3.

7 Naess [CAr].

⌐ possible frameworks for such theories. For one thing, we do not here discuss

1.1. [III.1] Conflicts of avowed opinions

Def. 1 A (full, mature, overt) *conflict of avowed opinions* is a quadruple ⟨*Con*, *T*, B, A⟩ where *T* is a statement, *Con* a finite set (possibly empty) of statements, A is a user or group of users of language, and B is a (another, or the same) user or group of users of language, and which satisfies the following conditions:
A has made the statement *T* (the *thesis*) that has been communicated to B, and has not withdrawn this statement;
B has made the statements in *Con* (the *concessions*), and has not withdrawn these statements, which have been communicated to A;
B has challenged A with respect to *T* relative to *Con*; that is to say, B has communicated to A non-acceptance of *T*, in the sense of having used some expression that is lexically classified as an expression of non-acceptance (disbelief, doubt, disagreement), and has not withdrawn this challenge;
The conflict shall be called *pure* or *simple* as long as A has not also challenged B with respect to one or more of the statements in *Con*; otherwise, it shall be called *mixed*.

Until further notice, we shall be concerned exclusively with pure conflicts of opinions, i.e., with conflicts involving exactly one initial thesis *T*.

We are not *directly* concerned with "what" the expressions express; only with the expressions themselves and with the assumption that these expressions have been observed by the users of language in the quadruple. Def. 1 should therefore contain reference throughout to a language, in the sense that the *definiendum* ought to be:

An overt conflict of avowed opinions *in a language* $\mathfrak{L}$,

and the *definiens*:

A quadruple ⟨*Con*, *T*, B, A⟩ where *T* is a statement *made in or translatable into* $\mathfrak{L}$ (etc.).

For clarity's sake, we have omitted these references in the formulation of Def. 1, but the definition should nevertheless be understood in this way.

Our concern is also exclusively with overt conflicts: someone (B) must have "challenged" someone (A) with respect to the latter's avowed opinion. No challenge, no conflict. Of course, there may yet be a (tacit) *difference* of avowed opinion:

A *difference of avowed opinion* is simply a triple ⟨*T*, B, A⟩ where *T* is a statement of a certain sentence made by A, and B is another (group of) user(s) of language who has not made any unretracted statement of the same sentence or of a translation of it into another language.

Whether there is a difference of avowed opinion or not, there may be an opposition of (avowed or tacit) opinion (belief):

An *opposition of* (avowed or tacit) *belief* is a triple ⟨T*, B, A⟩ where T* is a proposition, A and B are as before, and which satisfies the following conditions:

A believes that T*,
B doubts whether T* or believes that T* is not the case.

We shall not deal here with mere differences of avowed opinion, nor shall we be directly concerned with opposing beliefs. For, though undoubtedly they exist, they are to a high degree secondary or "derived": they owe their existence *in part* to past communication (in the widest sense) between users of languages, as well as to other kinds of information (in the widest sense), to patterns of nutrition and to genetic make-up (and some of these "categories" are again relatable to economic conditions, to schooling, etc.). They owe their existence *in part* to past discussion and to reports on past discussions.

1.2. [III.2] Deciding to discuss. Dialogue attitudes

The parties in a conflict of avowed opinions may decide to try to *resolve* the conflict, in the following sense of "resolve":

Def. 2 By *the conflict* ⟨*Con*, *T*, B, A⟩ *has been resolved*, we shall mean that one of the conditions in Def. 1 is no longer satisfied by this quadruple.

This is the case when one of the statements in *Con*, or *T* itself, has been withdrawn, and also when the challenge – the expression of doubt as to *T* – has been withdrawn; also when A or B is no longer a user of language (e.g., due to death). A decision to discuss *may* be: (1) *a decision to try to resolve* the conflict by verbal methods (which normally means, to strive toward the withdrawal of some statement or other expression, since people do not easily die as a result of a verbal feud). However, it may also be (2) a decision merely to exchange opinions, or (3) to annoy one another by verbal means in the presence of a third party.

We shall deal exclusively with discussions of the first kind, and so in this book from now on use of the word "discussion" will be restricted to discussions of this kind.

The decision to discuss or not to discuss the thesis in question may hinge on several things. One of these is whether a suitable instrument, acceptable to both parties, can be found for pursuing a discussion to this end: conflict resolution. Even before such an instrument is found, admittedly, we may sometimes ascribe certain *propositional attitudes*[1] to the parties involved: we may say that A is positively committed to *T*, B negatively committed to *T*, but positively committed to the elements of *Con*, in the following sense:

By *language user* X *is positively committed to the statement U*, we shall mean: X *intends* to defend *U* systematically against criticism of *U*, provided a suitable system of $formal_3$ dialectics, acceptable to both parties, can be found.

By *language user* X *is negatively* (or, *critically*) *committed to the statement U*, we shall mean that X *intends* to criticize *U* systematically, provided etc.

[1] For the earlier notion of 'propositional attitude', see especially K. J. J. Hintikka's [SPA] and several essays in his [II0].

However, intentions are affairs too "internal" to be of fundamental importance in a theory of argumentation. The notions and expressions we shall rather use are the following, which can be *applied* to the persons involved in a conflict of avowed opinions only after some system σ of formal$_3$ dialectics has been agreed upon:

Def. 3 By *language user* X *is in σ-pro-position to the statement U relative to language user* Y, we shall mean that X and Y have agreed to a discussion according to the rules of σ and have agreed that as soon as Y has offered criticism of *U* according to the rules of σ, X shall have an obligation to defend *U* against this criticism according to the rules of σ. This will be abbreviated to: X pro$_\sigma$ *U* (relative to Y).

Def. 4 By *language user* X *is in σ-contra-position to the statement U relative to language user* Y, we shall mean that X and Y have agreed to a discussion according to the rules of σ and have agreed that X shall have an *unconditional* right to criticize *U* in accordance with the rules of σ. This will be abbreviated to: X contra$_\sigma$ *U* (relative to Y).

Def. 5 By *language user* X *is in σ-neutral position* (*σ-zero-position*) *to the statement U relative to language user* Y, we shall mean that X and Y have agreed to a discussion according to the rules of σ and that X is neither in σ-pro-position nor in σ-contra-position to *U* relative to Y. This will be abbreviated to: X neutral$_\sigma$ *U* (relative to Y).

These relations are quaternary relations between persons (X, Y, where possibly X = Y), one statement (of a sentence), and a system of formal$_3$ dialectics. They obtain only when the language users have decided upon a system σ and have agreed on certain conditions. We shall call these quaternary relations *statemental* (rather than propositional) *dialogue attitudes*, i.e., *dialogue attitudes toward statements of sentences.*

Def. 3 implies that Y has granted to X a *conditional right* to defend *U*, whereas Def. 4 stipulates that Y has granted to X an *unconditional right* to criticize *U* (see further Section 10: Rights and obligations).

1.3. [III.3] Proponent and Opponent

We can now define two system-dependent dialectical roles, which are called *Proponent* and *Opponent* (of *T*, with respect to *Con*) in a discussion according to the rules of a system σ:

Def. 6 The role of *Proponent* in an σ-discussion issuing from a conflict ⟨*Con*, *T*, B, A⟩ is the role taken by a language user (or group of language users) X involved in the conflict (i.e., A or B), when at the start of the discussion:
X pro$_\sigma$ *T*, not X contra$_\sigma$ *U*, for any statement *U* in *Con*.

Def. 7 The role of *Opponent* in an σ-discussion issuing from a conflict ⟨*Con*, *T*, B, A⟩ is the role taken by a language user (or group of language users) X involved in the conflict (i.e., A or B), when at the start of the discussion:
X contra$_\sigma$ *T*, X pro$_\sigma$ *U*, for every statement *U* in *Con*.

Given our definition of a conflict (Def. 1), it will usually be A who takes the role of Proponent and B who takes the role of Opponent. If the goal is conflict resolution, then the dialectical roles *must* be distributed in this way.

We have defined these dialectical roles in terms of the three statemental dialogue attitudes, or dialogue attitudes towards stated sentences. Proponent and Opponent are relations involving four terms, viz., a person or group of persons X, a conflict, a system of $formal_3$ dialectics, and an σ-discussion issuing from the conflict.

Since A may be the same person as B, X in Def. 6 may be the same person as X in Def. 7. Even when this is the case, Proponent (P) and Opponent (O) are different dialectical *roles*. With this in mind, we can say: *At the outset of a discussion of the kinds to be considered here,* P *opposes none of the statements in the (pure) conflict, and* O *is neutral to none of them.* (Mixed conflicts will be mentioned again in Section 11.).

In the *formulation* of our rules for systems of $formal_3$ dialectics (from Section 5 onward), we shall not take into account the possibility that A = B. This is not to say that these rules cannot be applied in the self-critical case. Only that, in order to do so, the rules must be reformulated in such a way that all ascriptions of dialogue attitudes and of other rights and duties are relativized to roles. Thus, instead of saying "X has this or that dialogue attitude to statement *U*", we must say "X *when-playing-the-part-of-Proponent* (*Opponent*) has this or that dialogue attitude to statement *U*", etc.[1]

1.4. [III.4] Speech acts

Let *U* be any *assertive* or *hypothetical utterance* of a well-formed declarative sentence (of some language $\mathcal{L}_D$), or, for short, *statement* (in $\mathcal{L}_D$). The person who made the utterance *U*, in speech or in writing, will be called *The Speaker* (*relative to U*). Assume that this utterance is perceived by some hearer or reader who may (as far as others know) be critically inclined towards *U*; we shall call such a person *a Critical Listener* (*relative to U*). A Critical Listener may *express doubt* concerning *U* or, in other words, *criticize* this statement. We shall not yet go into the question of *how* a statement can be systematically criticized or attacked, but shall simply assume to start with that it can be done, and by verbal means. We shall call such an expression of doubt, or element of criticism, *aU*, for "verbal attack on *U*". (Note that *U* is always a statement, never a person or the properties of a person.)

The Speaker (relative to *U*) may now attempt to defend his/her statement against possible consequences of the "attack" *aU*. This can basically be done in two different manners:

(1) by launching a *counter-attack* on the general position of The Critic, in accordance with the following definition:

Def. 8 A *counter-attack*, or *counter-criticism*, is a verbal attack (by The Speaker) on a statement (by The Critic) *toward which The Critic has the attitude of pro-position*.

[1] We would like to thank F. H. H. Schaeffer for his constructive criticism on this section.

We shall say that The Speaker (relative to *U*) has chosen a *counteractive* or *indirect defense* of the criticized statement *U*, and symbolize this as follows: *ca*. There is a second defense type:

(2) by meeting the criticism of *U* "head-on", i.e., by giving a reply designed to remove The Critic's qualms on this special point, at least on condition that The Critic can be brought to accept the reply. A defense move of this kind will be called a *protective* or *direct defense*, and will be symbolized by: *pU*. Such a protective reply will usually be a function of the words used in the statement *U* and of its syntactical form, and also of the words used in the attack *aU*.

Since a defense remark *pU* may again be the object of criticism, even a protective defense will *usually* be merely conditional, the condition being that the defense itself can be defended if it is attacked, and so on (as discussed in more detail in Sections 6 and 7).

For a move in defense of *U* against the attack *aU* we shall sometimes write: *dU*.

Now let the statement *U* under consideration be a statement of a sentence U.

In the notion of so-called *rational discussions* the following principle, which we shall call *The Principle of (verbal) Externalization of Dialectics*, is often an important connotative component:

ExtDial
1. The *objects* of critical and defensive acts are to be *statements* (externalization of the objects of dialectics).
2. The relevant attitudes – relations of persons to objects – are to be *statemental dialogue attitudes* (externalization of propositional attitudes).
3. Whenever an attack or defense act is a verbal (speech) act, the range of words that can be used in an admissible verbal attack *aU* or an admissible verbal protective defense *pU*, as well as the range of syntactic forms one may resort to, depend functionally – and hence exclusively – on the words in, and syntax of, the sentence U and the sentences – if any – offered as definitions or clarifying reformulations of (parts of) U (externalization of dialectical *Wechselwirkung*).[1] "Whether a certain move is permissible shall depend on what has been said, and not on intentions, beliefs, etc."

Where this principle is accepted, we may to some extent shift our theoretical attention from *utterances* (statements, utterances of questions, utterances of exclamations) to *sentences*. Then we may speak of a criticism (or attack) aU of (or on) the sentence U. That is to say, an utterance of the sentence aU will constitute a verbal attack on an utterance of U, e.g., on *U*. Similarly we may speak of a protective defense pU of U against an attack aU.

When there is more than one kind of verbal attack possible on a sentence U, we shall speak of an attack or critical remark of the first kind, of the second kind, and so on:

$a_1U, a_2U, \ldots,$

---

[1] *Wechselwirkung* (interaction) is an old Kantian category, preceding the logical notion of functionality.

and when any of these kinds may be meant:

a_iU.

Different kinds of attack on U may require different kinds of protective moves in defense of U. A protective defense of U appropriate[2] to an attack a_1U will be called p_1U, and so on. If there is more than just one appropriate protective defense against a_1U, we shall have to distinguish between $p_{11}U$, $p_{12}U$, and so on; in general the j'th kind of protective defense against an attack a_iU will be called:

$p_{ij}U$.

The systems of $formal_3$ dialectics we are to discuss in this chapter are all based on the principle that counteractive as well as protective moves are permitted, and to both parties. So we can display our technical symbols in the following general schema:

Figure III.1

Sentence (declarative)	Possible critical moves (attacks)	d_iU, i.e., moves in defense of U against a_iU	
		Protective (direct)	Counteractive (indirect)
U	a_iU	$p_{ij}U$	ca

Example

For concrete examples of $forms_2$ of sentences and coordinate forms of attack and of protective defense, the reader is referred to Rule F_2D 1 in Section 16. A preview of this rule and of the explanation following it is essential, since we shall be using it in the examples and exercises (though not in our systematic exposition).

Notice that whereas Proponent and Opponent are dialectical roles pertaining to the discussion as a whole, the roles Speaker and Critical Listener pertain to only one locution at a time. If a language user A takes the role of Proponent and B that of Opponent, they keep these roles throughout the discussion. But when B makes a statement or poses a question, A will *ipso facto* be called Critical Listener with respect to this locution, and *vice versa*. Proponent and Opponent are defined in terms of their dialogue attitudes to the statements in the initial conflict.

We propose to make a verbal (and conceptual) distinction among "criticizing", "attacking", and "challenging": One *criticizes* (a proposition expressed by) a sentence, relative to a set of *concessions* (a set of statements of conceded declarative sentences), by *attacking* a *statement* of that sentence, thereby *challenging*

---

2 Or, which is generated by . . . . This concept, *appropriate* or *belonging to the set of moves that can be generated*, is a relative one: a move is appropriate given (i.e., relative to) some system σ of $formal_3$ dialectics.

the *debater* who made the attacked statement to *defend* that statement relative to the concessions. in order to *remove The Critic's doubts with respect to the said sentence.* One does not challenge a sentence or a statement, nor – except in a transferred, metaphorical sense – attack or even criticize the debater. We shall also say that one party *challenges the other party with respect to a statement.* But the objects both of criticism and of attack are *externalized.*

1.5. [III.5] The elementary rules of some plausible systems of $formal_3$ dialectics

The following assumptions will be part of this $formal_3$ dialectics (of which we shall later distinguish a number of variants):

FD E1 Some participant(s) in a discussion issuing from a conflict ⟨*Con*, *T*, B, A⟩ shall take the part of *Proponent* and some the part of *Opponent* of *T* relative to *Con*, as defined in Def. 6 and Def. 7.

FD E2a When no other rule prescribes a different attitude, then O shall assume contra-position toward *each* of P's statements (in the discussion) and pro-position toward *each* of its own.

FD E2b When no other rule prescribes a different attitude. then P shall assume the neutral position toward *each* of O's statements (in the discussion) and pro-position toward *each* of its own.

FD E2a and FD E2b constitute a fundamental asymmetry of the parties (cf. Section 11, on mixed conflicts).

The statemental dialogue attitudes that O and P have, either by virtue of Definitions 6 and 7 (Section 3) or by virtue of FD E2, are summarized in the following schema:

Figure III.2

N	N's statemental dialogue attitude toward:			
	Initial thesis *T*	Elements of *Con*	N's own statements	Other party's statements
Proponent	pro	neutral	pro	neutral
Opponent	contra	pro	pro	contra

P is not supposed to remain passive when its statements are attacked. P may, in fact must, defend the attacked statement (Def. 3; Def. 6) and:

FD E3 Any defense act may be protective or counteractive, but not otherwise.

We introduced this terminology already in Section 4. (Observe that there is no reason for formulating FD E3 so as to pertain only to the Proponent.)

Furthermore we suggest:

FD E4 (*The Principle of Externalization of Dialectics*; cf. Section 4).

And as the last of the elementary rules we suggest:

FD E5 When party N performs a speech act that is not among those permitted to N by the rules of this system of formal dialectics, or if it performs a non-permitted, nonverbal action that reduces the other party's chances of winning the discussion, then the other party, $\bar{N}$, may *if it so wishes* withdraw from the discussion without losing it.

This rule has several crucial consequences. It makes it risky to produce irrelevant remarks, such as shifting to another topic of discussion, offering arguments *ad hominem/virum/feminam*, or in other ways abusing one's adversary. It may be strengthened to:

FD E5super ... then N has lost all its rights in the discussion and N's behavior is to be called *irrational with respect to the present dialectical situation* by the company[1] that has adopted this system of $formal_3$ dialectics.

If this rule – FD E5super – is adopted, a debater who is insulted, ridiculed or otherwise abused (fired from his/her job, sent to an asylum or physically hurt) without having committed any non-permitted (relative to the other FD-rules) action in the course of the discussion, has won the discussion as a whole.

1.6. [III.6] Systematic dialectics

We suggest that the following rule be adopted, and that the norm it expresses be called *the fundamental norm of a systematic dialectics* of defense against verbal attacks on statements. It is independent of the five *elementary* rules (E-rules) of Section 5.

FD S1 A Proponent shall be given the opportunity to attempt to defend an attacked statement of its own by making *another statement*, provided it assumes the pro-position toward the latter.

This norm has to be translated into operational terms. We need some terminology:

Def. 9 A *stage* in a discussion is any bit of locution in which exactly one of the parties in the discussion is entitled to speak (functions as The Speaker) and that is not immediately preceded, nor immediately followed, by another bit of locution in which the same partly is entitled to speak.

Def. 10 A *chain of arguments* in a discussion issuing from a conflict ⟨*Con*, *T*, B, A⟩ consists of a number of stages in chronological order, such that the *first stage* consists of an attack on *T* by O (the language user(s) taking the part of Opponent).
An *appropriate*[1] *chain of arguments* is a chain of arguments such that each stage consists of a speech act permitted by the chosen system of formal dialectics in virtue of the stages that precede it in the chain (excluding the action of "retracing one's steps" and reacting to an earlier local conflict; cf. Section 13).

[1] The expression "company" is here used in the same sense as in Rupert Crawshay-Williams' [MCR].

[1] See Note 2 of Section 4.

Notice that these definitions are merely meant to stipulate a use of language. We could have said: "By a 'stage' we shall understand any bit of locution in which . . .", etc. Similar observations obtain for the definitions that follow.

Def. 11 A(n) *(appropriate) discussion* issuing from a conflict C consists of a number of stages in chronological order, such that:
(i) each stage, s, of the discussion is a stage in at least one (appropriate) chain of arguments issuing from C, and such that all stages preceding s in the chain are stages of this discussion, too. (Such a chain, up to and including s at least, is said to be an (appropriate) chain of arguments *in* the discussion.)
(ii) if two chronologically consecutive stages of a discussion do not belong to the same chain of arguments, then the transition from one chain of arguments to another is sanctioned (is an appropriate one) by the rules of the chosen system of formal$_3$ dialectics. (See Section 13.)
(iii) the same user(s) of language take(s) the part of Proponent (Opponent) in each chain of arguments in the discussion.

Consequences. Initial parts of (appropriate) chains of arguments (of discussions) issuing from a conflict C are themselves (appropriate) chains of arguments (discussions) issuing from C.
It is not excluded that a stage of a discussion belongs to several chains of arguments in the discussion.

We shall not introduce any rule explicitly requiring the Opponent to attack every one of the Proponent's statements. Consequently we cannot assume that every statement the Proponent makes will be attacked. It is therefore possible, and it will turn out to be clarifying, to introduce an expression to distinguish those of the Proponent's statements that are attacked by the Opponent in the course of the discussion from those that are not. We suggest the following terminology, which will be used here:

Def. 12 With the exception of the initial thesis T, which will count as T_1, any statement attacked by the Opponent in the course of the discussion, and no other statement, is to be called an *intermediary thesis*.
Every intermediary thesis, and also the initial thesis, is to be called a *local thesis*.

Def. 13 The set consisting of the Opponent's initial concessions together with all statements made by the Opponent, in a certain chain of arguments, *before* the Opponent attacked the nth local thesis, T_n, in that chain, will be called *the nth set of local concessions* or *the set of local concessions relative to* T_n in that chain of arguments. This set will be called Con_n (the chain of arguments being understood).

Consequence. In every chain of arguments, $Con_n \subseteq Con_{n+1}$, for every n such that there is a next local thesis, T_{n+1}, in the chain.

Def. 14 The quadruple ⟨Con_n, T_n, B, A⟩, where A and B are the same as in the initial conflict ⟨*Con*, *T*, B, A⟩, T_n is the nth local thesis and Con_n is the nth set of local concessions in a given chain of arguments, will be called *the nth local conflict in that chain of arguments*, and *a local conflict* of the discussion as a whole.

Def. 15 By *the nth local discussion within a given chain of arguments* or, *the local discussion (in that chain of arguments) issuing from the nth local conflict (in that chain of arguments)*, we shall understand that part of the chain that begins right *after* T_n and ends just before O's attack on T_{n+1}, if there is a T_{n+1}, or, if there is no T_{n+1}, that comprises the rest of the said chain.

Def. 16 The local thesis T_n will be called *the local thesis of* (but not *in*!) the nth local discussion, and Con_n will be called *the set of local concessions of* (but not *in*!) that local discussion. We shall say that T_n and Con_n *pertain* to the nth local discussion.

Consequence. The local thesis and the local concessions pertaining to a given local discussion are *constraints on* that local discussion, but T_n and the elements of Con_n are not made within that local discussion.

It will not be necessary to give explicit definitions of our use of the terms *initial concessions*, *intermediary concessions*, *initial conflict*, *intermediary conflict*, *initial discussion*, *intermediary discussion.*

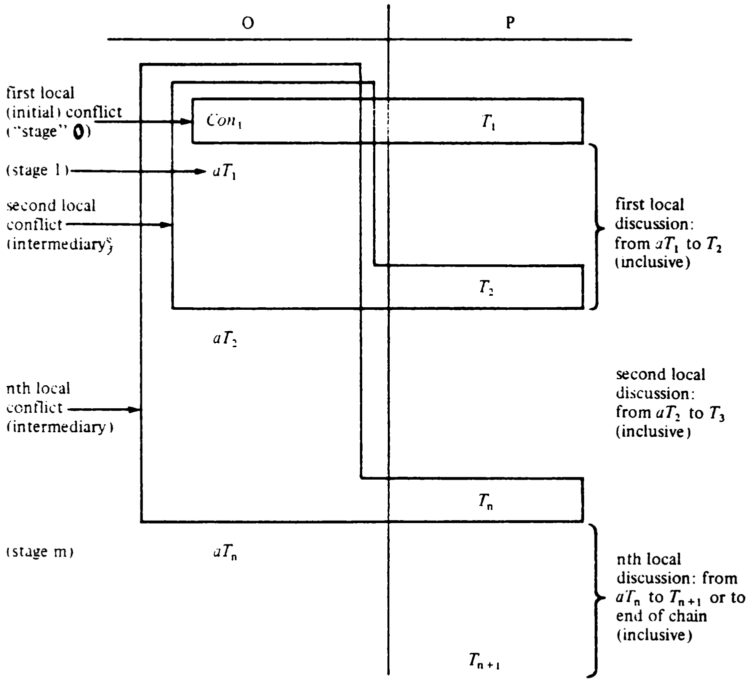
Now, remember that everything the Proponent says in a pure, or simple, conflict of opinions is said with the ultimate goal of removing the Opponent's doubts as to the initial thesis T_1. So if after the Opponent's first attack, P says something else that again is attacked, and that thereby becomes the next local thesis T_2, then P will have to defend T_2 in some way or other, *in order to* refute in this way the Opponent's attack on the initial thesis T_1, and so on, in agreement with the fundamental norm FD S1. So let us adopt this rule:

FD S2 Every intermediary thesis T_n belonging to a given chain of arguments is to be regarded as a *conditional defense* of the preceding local thesis T_{n-1}, in the sense that as soon as T_n has somehow been unconditionally defended in that chain of arguments, T_{n-1} is to be regarded as unconditionally defended in that chain of arguments, too.

(This rule is instrumental in satisfying FD S1.)

We shall depict chains of arguments by two coordinated columns. In the left column we shall write down O's utterances (beginning with the initial concessions) and in the right column we shall write down P's utterances (beginning with the initial thesis, *T*, i.e., T_1). We shall use a separate line for each stage. The chronological ordering of the stages will be rep resented by the vertical ordering of the inscriptions in the columns (earlier utterances appearing higher than later ones in the same chain of arguments). When these notational conventions are used, a representation of a chain of arguments will look like Figure III.3:

Figure III.3



Example

The following discussion can be generated by either of the systems of $formal_3$ dialectics to be developed in this chapter. The language used is J_1 (dialectically augmented). The statements of the original conflict are found above the dotted line. Jumps from one chain of arguments to another are indicated by dashes. The stages are numbered. The Pope wins.

Here there are *two* (appropriate) *chains of arguments*, one consisting of stages 1 through 6, and the other consisting of stages 1, 2, 3, 4, 7, 8, 9, 10. So stages 1 through 4 belong to *both* chains. The jump from the first to the second chain executed by Olga at stage 7 is allowed by the rules of Section 13 (cf. Def. 11 (ii) above). In the first chain of arguments we have three local discussions:

Con_1 = {(a), (b)}, T_1 = (c), aT_1 = stage 1,
$Con_2 = Con_1 \cup$ {stage 1} = {(a), (b), stage 1}, T_2 = stage 2, aT_2 = stage 3,
$Con_3 = Con_2$, T_3 = stage 4, aT_3 = stage 5.

In the second chain we have only two local discussions. Stage 4 is not attacked in this chain and, therefore, does not count as a local thesis in it. Note that all attacks by the Pope are (and must be) counteractive defenses.

Olga	Pope	Pragmatic character of dialectical move
(a) if there is matter then there is mind (b) if there is mind then God exists	(c) if there is matter then God exists	
. . . .	. . . .	
1. (?)there is matter		(attack on (c))
2.	God exists	(defense of (c) against 1)
3. God exists?		(attack on 2)
4.	(?)there is matter	(attack on (a))
5. there is matter?		(attack on 4)
6.	You said so yourself!	(unconditional defense of 4, see Section 7)
- - - - - - - - - -	- - - - - - - - - -	- - - - - - - - - -
7. there is mind		(defense of (a) against 4)
8.	(?)there is mind	(attack on (b))
9. God exists		(defense of (b) against 8)
10.	You said so yourself!	(unconditional defense of 2)

Exercise

Consider the following discussion in $\mathcal{T}_0$:

Olga	Pope	
(a) A v [B v (C & A)]	(b) (A & C) v B	
. . . .	. . . .	
1. ?		(attack on (b))
2.	A & C	(defense of (b) against 1)
3. L?		(attack on 2)
4.	?	(attack on (a))
5. A		(defense of (a) against 4)
6.	A	(defense of 2 against 3)
- - - - - - - - - -	- - - - - - - - - -	
7. B v (C & A)		(defense of (a) against 4)
8.	?	(attack on 7)
9. C & A		(defense of 7 against 8)
10.	R?	(attack on 9)
- - - - - - - - - -	- - - - - - - - - -	
11. B		(defense of 7 against 8)
- - - - - - - - - -	- - - - - - - - - -	
12.	B	(defense of (b) against 1)
13. B?		(attack on 12)
14.	?	(attack on (a))
15. B v (C & A)		(defense of (a) against 14)
16.	?	(attack on 15)
17. B		(defense of 15 against 16)
18.	You said so yourself!	(unconditional defense of 12)

The Pope won again! Could Olga have done better? Analyze this example into *chains of arguments* and *local discussions*. Indicate the local thesis and the local concessions of each local conflict.

1.7. [III.7.] Realistic dialectics: The possibility of unconditional defense

The next basic rule we suggest is *the fundamental norm of the possibility of unconditional defense*:

FD R1 The opportunity granted to the Proponent in FD S1 shall be realistic: in some cases it must be possible for an attacked statement to be defended unconditionally.

Again we need a definition:

Def. 17 By *an appropriate Ipse dixisti!-remark*, we shall understand an utterance of the words *Ipse dixisti!* or *You said so yourself!* by a debater who has incurred an obligation to defend a sentence that has already been stated, hypothetically or assertively, by the other party – provided the obligation has not in the meantime been lost or overruled by other obligations.

(Note that this definition is framed for either debater, i.e., for the Opponent as well as for the Proponent.)

The *Ipse dixisti!*-remarks constitute a type of protective defense move that is independent of the (specific) internal structure (the $form_2$) of the sentence to be defended as well as of the kind of attack involved. Such protective defense moves will be called "general", whereas the protective defenses that depend on the specific structure of the sentence to be defended, or on a specific kind of attack, will be called "structural". The schema in Section 4 is therefore to be expanded as follows:

Figure III.4

Sentence (declarative)	Possible critical moves (attacks)	d_iU, i.e., moves in defense of U against a_iU		
		protective (direct)		counter-active (indirect)
U	a_iU	$P_{ij}U$		ca
		general	structural	

In Section IV.1.3 we shall encounter yet another type of general protective defense. If we use the symbol

$[d_iU]$

to mean that the debater in whose column this symbol occurs has, at the stage corresponding to the level of the inscription in the column, incurred an obligation – and hence a right – to defend the sentence U against some attack a_iU, and if we use an exclamation sign as an abbreviation for *Ipse dixisti!*, then the following depicts an appropriate *Ipse dixisti!*-remark made by the Proponent:

Figure III.5

	O	P	
Con_n	.	.	
	.	.	
	.	T_n	
	a_iT_n	$[d_iT_n]$	
	.		
	.	.	
	.	.	nth local discussion
	.	.	
(Stage s)		!	

if and only if T_n, i.e., the *sentence* of which T_n is a statement, has been stated – hypothetically or assertively – by O in this chain of arguments *before stage s*. For it does not matter, of course, whether O stated T_n first and attacked it as uttered by P afterwards, or attacked T_n first and stated it afterwards, provided P at stage s has no other obligation with a higher priority than $[d_iT_n]$, i.e., as long as T_n still is the local thesis. So each of the following schemas shows an appropriate use of *Ipse dixisti!* by P (using sentence variables rather than statement variables, and writing "U!" instead of "!"):

Figure III.6

	O	P
	.	
	.	
	U	
	.	
	.	
Stage n	aU	[dU]
	.	
	.	
Stage n + k		[dU]
Stage n + k + 1		U!

	O	P
	.	
	.	
	aU	[dU]
	.	
	.	
Stage n	U	[dU]
	.	
	.	
Stage n + k		[dU]
Stage n + k + 1		U!

(One may require that k be zero, i.e., that the *Ipse dixisti!*-remark be made as soon as possible; but this is not necessary.)

Now we believe most people will agree to the following *closure rule*:

FD R2 At every stage in a chain of arguments the local thesis at that stage, in that chain of arguments, shall be said to have been *unconditionally defended* by the Proponent *if* it was defended by an *appropriate Ipse dixisti!-remark*.

(This rule is instrumental in achieving the goal expressed in FD R1.)

If FD R2 is adopted, it is natural to adopt also:

FD R3 If, at a certain stage in a certain chain of arguments, the local thesis at that stage, in that chain of arguments, has been unconditionally (or successfully) defended, then O *loses its rights* to pursue the discussion *in that chain of arguments.*

Example

In the example of Section 6, stages 6 and 10 are appropriate *Ipse dixisti!*-remarks. In 6, the local thesis 4 is unconditionally defended in the first chain of arguments, so the local theses 2 and (c) are also unconditionally defended in this chain (FD S2). According to FD R3, Olga has now lost her rights to pursue the discussion in the first chain of arguments. Olga can now either give in or make the transition to another chain of arguments, as is indeed done at stage 7. In 10, the local thesis 2 (and hence (c)) is unconditionally defended in the second chain of arguments. Olga has now lost her rights in the second chain as well.

Observe that we are talking about discussion that issues from conflicts containing only one initial thesis. Since O does not have any initial thesis, this party has no intermediary thesis either. FD R1 and FD R2 are therefore not applicable to O, and for the same reason there are no "natural" rules for O corresponding to FD R1 and FD R2. Similarly, there is no "natural" rule for P corresponding to FD R3; that is to say, P cannot lose its rights as a consequence of an appropriate *Ipse dixisti!*-remark by O.

There is indeed no *point* in granting O a right to make such remarks, since O has nothing to defend at the outset of the discussion.[1] This is characteristic of "pure" discussions issuing from conflicts in which there is only one thesis. We may therefore reformulate Def. 17 so as to be applicable only to P, and from now on we shall assume the reformulated definition:

Def. 17^P ... by a Proponent who has incurred, etc.

(In mixed conflicts – cf. Section 11 – one will need the original Def. 17.) Whichever definition we choose, Def. 17 or Def. 17^P, we now have some important consequences:

Consequence. The Proponent in a discussion with only one thesis cannot *lose* its rights but can only exhaust them (unless a rule is adopted saying that performing irrelevant speech acts involves loss of rights, see FD E5 super).

Consequence. (To the Opponent) Do not state the local thesis! Do not make a sentence you have stated into a new local thesis by attacking it!

Exercise

Using brackets to denote structural protective defense rights, we can rewrite the example of Section 6 thus:

[1] This recommendation runs counter to what was said on p. 88 in E. M. Barth and J. L. Martens [AHm].

Olga	Pope
(a) if there is matter then there is mind (b) if there is mind then God exists	(c) if there is matter then God exists
. . . .	. . . .
1. (?) there is matter	[God exists]
2.	God exists
3. God exists?	
4. [there is mind]	(?) there is matter

etc.

Complete this.
Do the same for the discussion in the exercise of Section 6.
(Stage 1 will read: 1. ? | [A & C, B].)

1.8. [III.8.] Winning and losing: Definitions and immediate effects

Why should the debaters enter into a discussion at all? There must be some possible – spiritual, if not material – immediate result, desired by the debater in question.[1] The following rules answer this question:

FD W1 If, in a certain chain of arguments, one party has (1) lost its rights in that chain of arguments or (2) exhausted[2] its rights in that chain of arguments, then this party shall express that the other party has *won* (with respect to) that chain of arguments by *rational* means.

FD W2 If and only if one party has won a certain chain of arguments by rational means, then this party *may* express that the other party has "lost (with respect to) that chain of arguments" provided it adds: "My adversary in this discussion has used *rational* arguments and so was rational with respect to *every* stage of the discussion (in this chain of arguments)".

It is, then, not irrational to use any combination of one's rights in a discussion, even if they express contrary propositional dialogue attitudes. *It is not irrational to lose a discussion*, provided the loss is not due to acts as described in FD E5. But it is – we suggest – irrational *not to admit* that one has lost:

FD W3 When a losing party violates FD W1, this party shall be called irrational by the company[3] that has adopted this system of formal$_3$ dialectics.

When both FD W1 and FD W2 are adopted, the discussions fall under the heading of "two-person zero-sum games". This is not to say that every chain of arguments will be won by one of the parties; that they will be, in fact, follows from the above for most human beings if, but only if, we also know that a party that does not suffer enforced loss of rights will eventually exhaust its rights, so that any chain of arguments is bound to come to a "logical" (non-enforced) end after a finite number of stages.

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1 See here the intriguing paper by Robin Giles, [LSB].
2 A party who has just completed a stage has never exhausted its rights; it has the duty and hence the right to remain silent during the next stage. Of course, further rules will have to determine when a stage is completed.
3 See Note 1 to Section 5.

Def. 18 A chain of arguments will be said to be *completed* (in a given discussion) if, and only if, it has been won by one of the parties.

A discussion may come to an end through loss or exhaustion of all rights. But an end to the debate may also be enforced upon the debaters, e.g., through lack of time. In either case we shall say that the discussion has been *closed*. It then seems natural to say:

FD W4a P has *defended T successfully* relative to *Con* and O in a certain discussion, D, if and only if the discussion D is *closed* and P has won the last completed chain of arguments in D (at least one chain being completed).

FD W4b O has *refuted T successfully* relative to *Con* and P in a certain discussion, D, if and only if D is *closed* and P has lost the last completed chain of arguments in D (and also if no chain has been completed).

If FD W1, W2 and W4 have already been adopted, it is natural also to adopt the following rule:

FD W5a When P had defended *T* successfully relative to *Con* and O in a discussion D, O shall express (privately or publicly, depending on arrangement) *that this is so* (e.g., by calling P "winner of this discussion by rational means").

FD W5b When O has refuted *T* successfully relative to *Con* and P in a discussion D, P shall express (privately or publicly, according to arrangement) *that this is so* (e.g., by calling O "winner of this discussion by rational means").
O *may*, *if it so wishes*, express instead withdrawal of doubt with respect to *T* (relative to *Con*; or, *if* O *so wishes*, absolutely, i.e., relative to any set of concessions whatsoever). And P *may*, instead, express withdrawal of belief.

FD W6 The winning party *may*, but is not required to, express that the other party has lost the discussion, with the same proviso as in FD W2.

FD W7 A closed discussion may be reopened, provided neither party has lost nor exhausted its rights.

We leave the task of formulating the "natural" consequences of reopening a closed discussion to the reader.

Example

In the example of Section 6, both chains of arguments are completed and won by the Pope. The discussion presumably has been closed. Since the Pope won the last chain of arguments, he has defended (c) successfully relative to {(a), (b)} in this discussion. The discussion may be reopened since Olga can still make a transition to another chain of arguments (with grim prospects, though).

Exercise

Who has won the discussion in the exercise of Section 6? Who do you think would have won had the discussion been closed immediately after stage 17? (See Section 13.)

1.9. [III.9.] Connection between expression of statemental dialogue attitudes and loss of rights

Now, observe carefully that the rules, FD R2–W6, laying down the conditions for winning and losing, are formulated *without mention of statemental dialogue attitudes*. We can, however, formulate a connection between these rules and attitudes.

When, at a certain stage, the Opponent has conceded or concedes the local thesis of that stage, the Opponent has made a statement of a sentence, a statement of which it has attacked. Since this latter statement is the local thesis, O has *expressed* contra-position toward it in the current local discussion; it was this expression of contra-position that caused this statement to be the local thesis. O's own statement of the same sentence, the concession in question, belongs to the local set of concessions, so O is in pro-position toward that statement. Moreover, O has *expressed* pro-position toward this statement in the context of the local discussion simply by stating it (we shall say). Conversely, when in a certain local discussion O is in pro-position toward one statement (its own) of a certain sentence and has expressed contra-position toward another statement (by P) of the same sentence, then the latter statement *is* the local thesis of the current local discussion at that stage, and O has conceded it. Remembering that P never takes the attitude of contra-position to any of O's statements in a discussion having only one thesis, we can therefore say, quite generally:

Consequence. When in a certain chain of arguments a party *has expressed* (cf. FD E4) pro-position toward one statement of a sentence and *has expressed* contra-position toward another statement of the same sentence *within the same local discussion*, then this party runs the risk of losing its rights to pursue this chain of arguments at the next stage.

Def. 19 Two dialogue attitudes toward statements will be called *contrary* if, and only if, one cannot *express* both attitudes toward (different statements of) the same sentence in the same chain of arguments without risking *immediate* loss of rights (in that chain of arguments).

Consequence. Within the confines of each local discussion, pro-position and contra-position are *contrary dialogue attitudes* (but not complementary, in the sense of the Excluded Middle; for there is also the third attitude of non-commitment).

So it is dangerous to *express* contrary dialogue attitudes toward statements of the same sentence. It is *not* irrational *to be in both statemental dialogue attitudes at the same time with respect to statements of the same sentence or of the same "meaning"* (which we do not need to characterize at all here). In fact, the Opponent will often be in this state, but must beware of *expressing* both attitudes *in the same local discussion*. What is more, this is something only the Opponent *can* do. The Proponent never *has* contrary attitudes toward the same sentence, because the Proponent never assumes the attitude of contra-position toward any statement in discussions with only one thesis.

Exercise

Return to the example and exercise of Section 6, and note the stages at the end of which Olga has expressed contrary dialogue attitudes.

1.10. [III.10.] Rights and obligations

It is time to explain how we use the words "right" and "obligation". We have said already that the symbol (written in the column of party N):[1]

[d_iU] shall mean that N is obliged to defend a statement of U against an attack a_iU, i.e., is obliged to make a defense move d_iU, i.e., that N has a general right to make a defense move d_iU. (This move may be a counteractive one.)

Using these brackets to express the totality of N's rights, we can also write:

[ca; p_iU] meaning that N can choose between the two more specific rights: counter-attack and protective defense.

If at the stage in question there are k possibilities for making a protective defense against the attack a_iU, then we may write:
[ca;$p_{i1}U, \ldots, p_{ij}U, \ldots, p_{ik}U$] for the totality of N's particular rights at this stage.

Clearly,

$$[d_iU] = [ca;p_iU] = [ca;p_{i1}U, \ldots, p_{ij}U, \ldots, p_{ik}U]$$

That N is obliged to defend U, i.e., to carry out *some* defense move d_iU, means that N is obliged to choose among the k + 1 particular rights. Remember that the *general* defense move, U! or !, is always included among the defense rights p_iU.

1.11. [III.11] Mixed conflicts

The conflicts we have defined and discussed so far will be called *simple* or *pure* conflicts. Often, in practice, all the participants in a discussion have theses to defend, and so each party will be the Proponent of at least one thesis and the Opponent of at least one. The idea is that such *complex*, or *mixed*, conflicts may usually be analyzed as sets of simple conflicts superimposed upon each other *without interfering with one another*. This may be an idealization in need of considerable refinement if we want to cover the whole ground of what we should like to call rational argumentation. We may have to add a "thermodynamical" component of some kind or other. However, to study superimposed simple conflicts that do interfere with each other, or whose resolution contains features of interference among the moves due to the various simple conflicts, will be work for the future; the theory of how to resolve pure conflicts of the above type is as far as we shall go in this book (but see Section IV.5.2, on material discussions).

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[1] We use "N" to refer to either party, O or P, and "$\bar{N}$" to refer to the adversary of N. Cf. FD E5 in Section 5.

What we can and do show in this book is that several well-known derivational and model-theoretic systems of elementary logic correspond to a dialectics for discussions issuing from simple (pure) conflicts (Chapter XI).

If the reader has qualms about the fundamental asymmetry in simple conflicts of opinions and feels that everything has to be symmetrical in order to be "just", then it is likely that he or she has in mind the – very common – situation that we call a mixed, or complex, conflict (with or without interference among the constituent simple conflicts).

1.12. [III.12.] "Natural" rules; "Consequences"

(i) *The "naturalness" of the rules suggested here*

The argument we, the authors, offer to bring our readers to accept and adopt the dialectical rules here propounded has the status of *persuasion*, not of "proof". Proofs can exist only with reference to a set of logical rules that have been accepted by the reader already. In this argument, the authors employ – or assume that the readers employ – some of the rules for which we have argued. If this brings the reader to accept a proposed rule, then the reader and the authors *de facto* already share some *rules of thought* – for better or worse. We "play" upon modes of thought we expect the readers already to follow, in order to bring about an explicit agreement to certain $formal_3$ as well as (later) to certain $formal_2$ dialectical rules, which then achieve the status of *logical rules* (of thought and of verbal behavior). $Formal_2$ dialectical rules concern the syntactical operations ("logical constants") of the chosen language.

In each case where we expect that the large majority of people can be brought to agree upon a certain rule as a part of $formal_3$ dialectics (provided they are explicitly confronted with it and made acquainted with the motivation given for it here or with a similar motivation), we shall say, for short, that we think it is a *natural rule.* This expression, then, will be used in a way that carries no implications about innateness or universality – though, on the other hand, nor do we explicitly exclude the possibility that some of these rules may be innate or universal.

As to the question of the relation between logic in its dialectical garb and the wider field of the Theory of Argumentation, our answer is: *The subject called "Logic" corresponds to that part of the Theory of Argumentation that studies systems of language-invariant $formal_3$ dialectical rules and language-dependent $formal_2$ dialectical rules based on ($formal_2$) syntactical rules.* In addition, the Theory of Argumentation will contain analyses of rules that should be followed in order to bring people who are in, or who may develop, a conflict of avowed opinions to agree to enter upon a rational discussion. Such *discussion-promoting rules* may be called *rational of the second order* – the $formal_2$ and $formal_3$ rules being *rational of the first order*. ("Do not abuse the other party!" and many other discussion-promoting rules are often said to be "rational" even in the absence of any system of $formal_3$ dialectics accepted by both parties.)

If second-order rational behavior is "regimented" and its rules systematized, we may speak of *$formal_3^2$ rules* (and perhaps even $formal_3^3$ rules, $formal_3^4$ rules, can be formulated).

No higher-order rules (rules for higher-order rational behavior) will be discussed here. Notice that the FD-rules could have been called F_3D-rules (except for the one F_2D-rule, or set of such rules, in Section 16).

(ii) *A note on the meaning of "consequence" in this book*

To say that such and such *is* a consequence of rules and definitions so far accepted is nonsense unless reference is made to some system of logic according to which it is a consequence – but to do so *at this moment* would be premature. Therefore, whether it shall be treated as a consequence or not must again be considered a matter for democratic decision-making in the company. In order not to confuse our readers with too many numbers, "consequences" will not be numbered, and what we have just said about "consequence" should be applied to any preceding or following statement called a consequence in this book.

1.13. [III.13.] Thoroughgoing dialectics

As *the fundamental norm of a thoroughgoing dialectics* we suggest:

FD T1 At every point in the discussion the Opponent shall have the opportunity to test, in all possible manners, the tenability (against criticism) of P's last statement up to that point; and the Proponent shall have the opportunity to attempt a defense of the local thesis in all possible manners.

We shall implement this norm by rules that – in principle – permit both parties to try out different (continuations of) chains of arguments (*lines of attack* in O's case, *lines of defense* in P's case) within one and the same discussion issuing from one and the same conflict. If a party has lost or exhausted its rights within a chain of arguments, it will – in principle – still be granted a right *to open up another chain*. (Indeed, there will be nothing else for such a party to do.) We shall also grant this right to any party that is willing to abandon a chain of arguments in which it still has rights remaining, provided it expresses that the other party has won the abandoned chain.

When a chain of arguments is abandoned, the discussion need not start from scratch: *different chains of arguments may share an initial sequence of stages.*

FD T2 If N is to be the speaker at the next stage, N may (and sometimes must) *retrace its steps*, i.e., abandon the current chain of arguments and supplant its speech act at some stage (at which it was the speaker) in this chain by some other speech act (one that is, by the rules of the system, permissible at this stage). In that case, and if no other rule intervenes, the parties shall take the dialogue attitudes, assume the obligations, and hence have the rights, that they would have had if the supplanted speech act and the speech acts at the following stages in the old chain had not been made.

FD T3 Whenever N abandons a chain of arguments, N shall be said to have lost it, and $\bar{N}$ to have won it; FD W1, W2 and W3 shall be applied accordingly.

In Section 15 we shall somewhat restrict this right to retrace one's steps (in FD D3). Instead of the expression "retracing one's steps", we could also have used the expression "returning to an earlier situation".

Exercise

Determine who has won (lost) each chain of arguments in the discussions in the example and in the exercise of Section 6.

Reconsider the exercise of Section 8.

1.14. [III.14.] Orderly dialectics

We suggest that the following norm be called *the fundamental norm of orderly dialectics*:

FD O1 At the completion of any stage in any chain of arguments, the dialogue attitudes and obligations (and thus the rights) of each party should be determinable from a study of the discussion up to and including that stage; this should also hold, for each party, for the relative priority (urgency) of its obligations, if there be more than one.

This requires that, at each stage of the discussion, we reach clear decisions about the dialogue attitudes of each participant to every statement uttered in the discussion at any stage. Rule FD E2 says that O shall take the contra-position to each of P's statements and that each party shall take the pro-position to each of its own statements. But this rule does not say whether, and for how long, these dialogue attitudes are to be retained. Adoption of the following rules will fill this gap:

FD O2a Whenever at some stage of some chain of arguments a party N *expresses* its contra-position to a statement *U*, N shall lose its contra-position to *U* in that chain of arguments at the completion of that stage.

(Note that N still retains the right to attack *U* counteractively.)

FD O2b Whenever at some stage of some chain of arguments a party N defends a statement *U* protectively against a certain attack on *U*, N shall, at the completion of that stage of the said chain of arguments, lose its obligation (and hence its rights) to defend *U* any further *against this attack.*

We do not propose that N should lose its pro-position to *U* in this chain of arguments after a protective defense of *U*, since $\bar{N}$ may want to launch a new attack on *U* on another occasion in the same chain of arguments; and according to the rules we have suggested so far, $\bar{N}$ obtains the right to attack *U* for counteractive purposes more than once. Such combinations of attacks are important for $\bar{N}$. It is therefore not to be expected that a company will agree to a rule that largely destroys this possibility, as would a rule to the effect that N should lose its pro-position to *U* in a chain of arguments after its first protective defense of *U* in that chain of arguments.

For the same reason, we do not propose to adopt a rule like FD O2b for *counteractive* defenses. The whole point of such defenses is that they may elicit responses from one's adversary that may – alone or in combination – provide material on which a realistic protective defense can be based. So a party should, we think, keep its obligation – and hence its rights – to defend a statement against a certain attack until it has provided a *protective* defense of it against this attack (unless this obligation is canceled in virtue of other rules).

For clarity's sake, we suggest an explicit. new rule:

FD O3 At the completion of each stage in each chain of arguments, both parties shall retain the dialogue attitudes and obligations they had at the beginning of that stage, unless a change is provided for by some other rule; the same shall hold for the relative priorities – if any – of the obligations.

The next rule in this section is intended to implement FD O1 by exploiting the organization of a chain of arguments into a chain of local discussions, as described in Section 6. We propose:

FD O4a Any local discussion shall be concerned with only one (local) thesis (viz., the last one, *the* local thesis pertaining to this local discussion).
FD O4b At any stage, the parties shall have rights and obligations with respect to the statements *in or pertaining to* the current local discussion only.
FD O4c At any stage, the parties shall be allowed to be in non-neutral dialogue attitudes toward the statements *in or pertaining to* the current local discussion only.

This rule we propose to implement as follows:

FD O5a At the completion of a stage that initiates a new local discussion (i.e., an attack by O, creating a new local thesis), both parties shall assume (or retain) the neutral position to all statements that do not belong to or pertain to this local discussion.
FD O5b At the completion of a stage that initiates a new local discussion (i.e., at the completion of an attack by O), *all* obligations incurred at earlier stages shall be canceled (and hence also all rights incurred at earlier stages).

As long as no new rules interfere, we shall have the following consequences:

Consequences. At the completion of any stage that is part of a local discussion, L, with thesis T, we have the following distribution of obligations and dialogue attitudes:

(i) O is in pro-position to *all* its own statements in the current chain of arguments.
O is in contra-position to all the statements P has made so far *within* L, and to these only.
Hence O is no longer in contra-position to T in this chain of arguments.
O is obliged to defend (protectively or counteractively) all those among its own statements in this chain of arguments that P has attacked *in the course of* L (compare FD O5b!) and which O has not yet defended protectively. There are no other obligations, and hence no other rights, for O, T having been attacked already at the onset of the present local discussion L.

(ii) P is in pro-position to all of its own statements *pertaining to* L (i.e., including T) and to no other statements. P is

obliged to defend T if and only if P has not yet defended T protectively.
P has no other unconditional defense obligations.
P is in contra-position to no statement whatsoever.
P has only one unconditional defense obligation at a time.

These consequences will be considerably strengthened (indeed, "superseded") in the next section.

An alternative to FD O4b, implemented by FD O5b, would be to let the parties keep the defense obligations they have incurred in earlier local discussions, but to have these obligations ordered in the sense that a more recently incurred obligation has a higher priority than one incurred at an earlier stage.

Exercises

1. Look carefuly at each of the consequences and see if you agree. Add supporting arguments and explanation where necessary, to convince yourself and those who *de facto* share (some of) your rules of thought. (In the future we shall simply ask you *to show that* something ***holds***; such an instruction should always be understood as short for the more elaborate formulation above; cf. Section 12.)
2. If no other rules interfere, what are the dialogue attitudes and defense obligations of each of the parties at the end of each stage in the discussions of the example of the exercise of Section 6?

1.15. [III.15.] Dynamic dialectics

In Section 12 we proclaimed that no higher-order rules would be discussed in this book. This is not to say that the set of first-order rules we formulate does not show any internal structure. On the contrary, we have already seen – and it will become even clearer in the present section – that some of the rules will be instrumental in satisfying a norm expressed in another, more basic, more fundamental, or "higher" rule. In this sense there is a hierarchy *within* the set of first-order rules to be formulated here.

An extremely fundamental rule of the highest practical and philosophical importance, which a company may or may not adopt, is the following, which we shall call *the fundamental norm of dynamic dialectics*:

FD D1 The system of FD-rules applied in a discussion shall be designed *to promote the revision and flux of opinions* in any company in which these rules are adopted.

(Of course, when one party revises its opinion, it does not follow that the other party revises its opinion, too – but neither does it follow that it does not, as some people seem to think.)

This norm should be translated into operational terms (i.e., to be "implemented"). If necessary, this should be done *via* other, more concrete rules that have to be satisfied by the rules which deal directly with verbal "operations" in discussions:

FD D2 The rules shall be such that *unavoidable* decisions as to the outcome of discussions will be reached *as soon as possible.*

(This rule is instrumental in bringing about the goal expressed in FD D1.) In implementing this rule, FD D2, we shall clearly have to *limit the rights of the* *parties* so as to avoid evasive and repetitive speech acts that do not contribute to the goal of reaching a clear decision.

(1) We first propose a rule that will often limit the *number of chains of arguments* within a discussion:

FD D3 At the start of any stage (in any chain of arguments), the party, N, who is to be the speaker at that stage, may open only one new chain of arguments branching off at that stage with the same dialectical move, i.e., with the *same sentence* used *in virtue of the same right.*

In other words, if a stage s_1 is already in some chain of arguments immediately followed by a stage s_2 in which N was the speaker, it is not permitted that N retrace its steps (in virtue of FD T2) in order to react once more to s_1 by the dialectical move on which s_2 was based.

FD D3 implements FD D2, and hence FD D1, without breaking the fundamental norm FD T1 of a thoroughgoing dialectics. It prevents the parties from repeating themselves by repeating a chain of arguments.

(2) Equally important is:

FD D4 The rules shall *aim at bringing it about* that each chain of arguments *ends*, with a clear result as to whether or not someone has won, and if so, who has won (*within a finite number of steps*).

(This decision is an instrument for satisfying FD D2 in the case of unavoidable results, and in any case for satisfying FD D1.)

As we hinted in Section 8, there is a *prima facie* possibility of "never-ending", "infinitely long", "indefinitely long" chains of arguments. FD D4, if adopted, is a decision to minimize the risk of running into such chains. A chain of arguments that, if continued, would lead to exhaustion or loss of rights on the part of one of the debaters may also be broken off for external reasons (e.g., lack of time) before completion. So it will not be possible to preclude inconclusive chains unless we arbitrarily decree that one of the parties is to be called the winner in all such cases. We can, however, do a lot to minimize the risk of no conclusion's being reached as to winning and losing.

In implementing FD D4 we shall clearly have to further *limit the rights of the parties* so as to avoid, *within a given chain of arguments*, those repetitions of attacks and defenses that are not instrumental in leading to a clear decision as to who has won that chain of arguments. The following rules, D5 through D8, are intended to deal with this. We claim that these rules provide a natural solution (in the sense of Section 12), but certainly not that they provide the only, or even the only "natural", solution. The limitations we propose pertain to the length of stages, to the length of local discussions, and to the length of whole chains of arguments.

(2a) As to the *length of stages*, we propose:

FD D5 Each stage shall contain one and only one utterance of one and only one sentence. (This sentence may be declarative or interrogative or an exclamation.)[1]

[1] In some cases it may be necessary to explain to the other party the dialectical status of a statement (i.e., in virtue of what right the statement is made). We do not count the statements used in such clarifications as stages.

(2b) The next rule concerns the *length of local discussions.* It is intended to help to bring it about that each local discussion contain not more than a *finite number of stages*:

FD D6 When party N has a certain defense obligation, and hence a general right to carry out a counteractive defense, this party may not attack a statement made by $\bar{N}$ twice in the same manner in virtue of this right (in the same chain of arguments).

Consequence. Since in a local discussion P has an obligation to defend the local thesis, and no other obligation, P may carry out an attack *of a certain kind* on a statement of O's only *once within a given local discussion*. P is, however, allowed to attack a statement of O's in two or more *different manners*, even within the same local discussion. In a new local discussion an attack of a certain kind may be carried out anew (as far as the rules go that we have proposed so far).

Example 1

	Olga	Pope	
	(a) A & B	(d) C	
	(b) A → C		
	(c) B → C		
	. . . . . .	. . . . . .	
1.	C?		(attack on (d))
2.	[A]	L?	(attack on (a))
3.	A		(defense of (a) against 2)
4.	[B]	R?	(attack on (a))
5.	B		(defense of (a) against 4)

The two attacks on (a), both counteractive defenses in virtue of the attack on (d), are allowed, since they are of different kinds. After stage 5 no more attacks on (a) are allowed within the current local discussion.

Example 2

	Olga	Pope	
	(a) [A v (A → B)] → B	(b) B	
	. . . .	. . . .	
1.	B?		(attack on (b))
2.	[B]	(?) A v (A → B)	(attack on (a))
3.	?	[A , A → B]	(attack on 2)
4.		A → B	(defense of 2 against 3)
5.	(?)A	[B]	(attack on 4)
6.	[B]	A v (A → B)	(attack on (a))

The repeated attack on (a) (of the same kind!) is allowed, since it occurs in another local discussion, and hence in virtue of a different right to execute counteractive defenses.

(2c) Finally, we need rules concerning the *length of chains of arguments*, which we have so far broken up into local discussions. We want rules that will prevent, as far as possible, and without interfering with other rules in an undesirable manner, a *chain of arguments* from consisting of an infinite *number of local discussions.*

We have not yet excluded the possibility that chains of arguments may run on indefinitely due to renewed attacks by O *in virtue of different defense obligations* FD O2a provides a safeguard against repeated attacks in virtue of contra-position, and FD D6 against repeated attacks made in virtue of one and the same defense obligation. But by a series of counteractive attacks made in virtue of different defense obligations, it is still conceivable that an Opponent attack the same statement *U* again and again, thereby conferring on this statement the status of local thesis in a (possibly indefinitely long) series of local discussions (each new local discussion being brought into being by a new attack on this statement, *U*). In order to preclude such series of local discussions with the same statement as local thesis in each, it suffices to adopt the following rule or an equivalent thereof:

FD D* O shall not use the local thesis as an object for counteractive defense.

The rule we shall in fact propose, however, is stronger; like FD D*, it excludes counteractive attacks on the local thesis, yet it has a still more dynamic effect than the D*-rule would, since it pertains not just to the Opponent but to both dialectical roles. Furthermore, it is formulated so as to implement FD O1 and thereby to clarify and simplify the analysis of what happens. So we expect it to be no less acceptable to our readers than the D*-rule. Here it is:

FD D7 Both parties shall assume the neutral position to a statement *U* uttered by P as soon *as* O *has had an opportunity to react to it*, i.e., as soon as O has completed the next stage, whether this latter stage constitutes an attack on *U* or not, i.e., whether a new local discussion has started or not.

Consequences. O is obliged to attack any statement of P's either at once or not at all, and hence once at the most.
O will be in contra-position to one statement at a time, at most.
O will have a defense obligation with respect to one statement at a time, at most.
O's right to carry out counteractive defense moves is nullified.

This rule, in contradistinction to most of the other rules, mentions the two dialectical roles by name. At first sight, the rule appears to be to O's disadvantage, since it sets a stricture on O's freedom of action, and since one might think that O could sometimes profit from a larger stock of locutions on both sides. So we may assume that the rule is acceptable at least to whoever is going to take the role of Proponent and to those who sympathize with him/her. In order to persuade you to accept this rule, we shall have to convince you that an Opponent who wins a discussion after having made delayed or repetitive attacks could actually have won it without making them, and that this rule is not really advantageous for P after all.[2]

Suppose we have completed our system of formal dialectics, in some way acceptable to a certain company of which you are a member. Let us call this system σ_r, where "r" refers to the reader and his/her company. And suppose that the system comprises the rules proposed here prior to FD D7, but not FD D7 itself. We now define the notion of a *dialogue situation*, to be used below, as follows.[3]

Def. 20 *a* By *a dialogue situation* (*given a system* σ_r *of formal dialectics*) *at the completion of a stage* (or at the very beginning of a chain of arguments), we mean the total constellation of obligations, rights and dialogue attitudes the two parties have at the completion of this stage, including the right to be the speaker at the next stage.

b Dialogue situation S is *dialectically equal* (*relative to* σ_r) *to* dialogue situation S' if and only if the following holds for each party N:

(i) According to the rules of σ_r, N is in pro-position to statements of exactly the same sentences in S as in S' and is in contra-position to statements of exactly the same sentences in S as in S';

(ii) According to the rules of σ_r, N has defense obligations for statements of exactly the same sentences and with respect to exactly the same kinds of attack in S as it has in S'; and furthermore, N has rights – and even the *same* rights – of counteractive defense with respect to statements of exactly the same sentences in S as it has in S';

(iii) According to the rules of σ_r, N is to be the speaker at the next stage in S if and only if, according to the rules of σ_r, N is to be the speaker of the next stage in S'.

In our attempt to persuade our readers that FD D7 does not favor Proponents at the cost of Opponents, we shall also need the concept of a winning strategy. It can be defined as follows:

Def. 21 By *party* N *has a winning strategy* for (or in) a dialogue situation S according to the system σ_r of formal dialectics, we shall mean that, whenever it is N's turn to speak in the ensuing discussion, there is a way in which N can make use of the rights it has on the strength of σ_r to make such moves that, whatever remarks $\bar{N}$ makes, each chain of arguments in the discussion ends after a finite number of steps and with the result that N has won it.

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2 In his dissertation W. Kindt already proved, for a large class of dialectic systems, that rules like our FD D7 are not really advantageous for the Proponent (in other words, that they are not "unjust" to the Opponent). Our present argument in favor of adoption of FD D7 does not, however, depend upon Kindt's theorem. For our purposes a much simpler argumentation suffices, since we need to reckon only with such dialectic systems as are plausible extensions of the system of FD-rules developed thus far. See Kindt [ATD], *Satz* 6.7 and *Korollar* 6.8 (1), p. 19.

3 We assume that the ***number*** of previous statements of any sentence U toward which N is in pro- or in contra-position is dialectically irrelevant according to the system.

Let S be the dialogue situation that obtains as soon as P (at some stage, s, of the discussion) has uttered U (i.e., at the beginning of the next stage). Suppose that in this situation, S, O has a certain winning strategy (on the strength of some completion σ_r, acceptable to the reader, of our formal dialectics, not including FD D7). This means that there is a manner in which O can make use of its rights such that each chain of arguments issuing from this new stage will end in loss – through exhaustion of rights – for P.

Suppose that O – employing this winning strategy – for some reason prefers *not* to attack U immediately, but attacks it at some later stage in a certain manner. We leave open whether this is the first attack on U or is in the middle of a whole series of attacks, which, in virtue of FD O5a, must *all* attack U, and whether in the latter case the attacks are of similar or different kinds. Let S'' be the dialogue situation that obtains immediately after this – delayed or repeated – attack was made. O had a certain winning strategy at an earlier stage, and we have assumed that O has since acted in accordance with this winning strategy; hence O, in situation S'', still has a winning strategy. Now let S' be the dialogue situation that would have obtained if O had attacked U in this same way immediately after P had uttered it.

What we want to convince you of is that O would have a winning strategy in this situation, S', as well. (This is the situation that will be enforced on O if the company adopts rule FD D7.)

Since attacks by O start new local discussions and the parties have non-neutral dialogue attitudes toward only those statements in or at least pertaining to the new local discussion, the only possible difference between S' and S'' is that in S'' the Opponent may be inpro-position to some additional concessions. Since O's winning strategy in situation S'' must hold good against any debater (by definition of "winning strategy"), it must *a fortiori* hold good against any "moderate" debater who, for some reason or other, does not attack these additional concessions. This implies that O can exhaust all the other rights of such a "moderate" debater. But these other rights are just those rights the Proponent would have in dialogue situation S' (i.e., if O attacked U in this manner right away). And this means that O has a winning strategy in situation S', too, since, as we saw, S'' differs from S' *only* in the presence of these additional concessions. In fact, O can therefore even *improve* its winning strategy for (or, in) S from a dynamic point of view by bringing about situation S', i.e., by attacking U at once, since this will shorten some of the possible chains of arguments. O's dynamically improved winning strategy does not admit of any delayed or repeated attacks on U (according to the same argument applied to each hypothetically delayed or repeated attack). And if O has no winning strategy in S at all, it runs the risk of losing the discussion in both cases, i.e., whether we adopt FD D7 or not. True, we have not said anything that can be used to weigh these risks against each other; however, we hope to have convinced the reader that if an Opponent loses against a certain debater, then this result was not enforced by the new rule, and that if an otherwise certain victory for the Proponent does not come about before the discussion is closed for external reasons (e.g., lack of time), this is not the fault of the new rule – on the contrary.

For these reasons, we expect most of our readers to accept rule FD D7.

(Remember that O can attack *U* in every possible manner, provided each attack is treated as a new line of attack, defining a new chain of arguments – cf. FD T2.)

Even if the company adopts FD D7, an "infinite" number of local discussions may still be generated if P repeats the sentence T in a local thesis *T* again and again; for O might conceivably react to every statement of this sentence in the same manner. We therefore propose:

FD D8 After an attack by O, P may not repeat the sentence T in the new local thesis *T* within the same chain of arguments as long as the set of local concessions has not been augmented with any statement of a new sentence.

This rule seems at first sight advantageous for O, since it restricts P's freedom of action.

As before, we shall try to convince our readers that a Proponent (above it was an Opponent) having a winning strategy on the strength of an acceptable system σ_r that does *not* contain this rule, FD D8, but may contain FD D7, also has a winning strategy if this rule is added to the system. Again we make use of the concept of a dialogue situation, as defined in Def. 20.

Suppose that, given the system σ_r approved by our reader, the Proponent has a winning strategy for a local conflict with local concessions *Con* and local thesis *T*; this clearly means that whatever remarks O makes, provided they are permitted by the system σ_r, there is a way in which P can make use of its rights such that each chain of arguments in the discussion is guaranteed to end with a winning remark (e.g., with an appropriate *Ipse dixisti!*-remark) by P. We think the reader is likely to agree that we have the following *Consequence of* σ_r:

Consequence. A winning strategy for P in a local discussion issuing from ⟨*Con*, *T*, B, A⟩ cannot require a repetition (by P) of the sentence T in *T* before the set *Con* of O's concessions has been augmented with a statement of a new sentence (unless new strictures are put on O's reactions).

For P, in making such a repetition, cannot count on O's reacting in another manner the second time unless new strictures are put on O's reactions; hence a description of a winning strategy for P cannot assume that this is the case (unless, etc.), and hence cannot require that P make the repetition. By repeating T, P runs the risk that O will attack the new statement of T in exactly the same way as before and so merely revive the old dialogue situation (as defined in Def. 20). And this risk is not inconsequential for P's position. Certainly, if this happens only a finite number of times and if there is still time to pursue the discussion, P may in some cases still be certain to win. But P would obviously also have won – and sooner – had it not made the repetition, since at any stage of the discussion the possibilities for winning – for each party – depend entirely[4] on the dialogue situation at that stage (and on the time left, or on other limits – if any – set on the number of remarks permitted to the discussants in one discussion). And if P goes on and on repeating T, then P even runs the risk of O's

[4] See preceding note.

stubbornly reacting to *every* statement of T in exactly the same manner, *ad infinitum*, which would prevent P from ever making a winning remark (such as *Ipse dixisti!*) in this chain of arguments. In other words, a Proponent will never need a right to make such a repetition while *Con* is constant, in order to win (at least as long as no new strictures are put on O's rights of reaction to these repetitions). So, as added to the rules we have proposed so far, FD D8 is not disadvantageous to P after all.

We have now precluded the possibility that chains of arguments may run on indefinitely due either to repetitive attacks by O on the same statement, or to repetitions of a cycle consisting of a statement of a certain sentence by P and a certain reaction by O, etc.

Example 3

In Example 2 the local thesis 2 is repeated at stage 6. This is allowed on account of the new concession made at stage 5. An earlier repetition of the local thesis 2 would have violated FD D8.

Exercises

1. Review the discussions of the example and of the exercise of Section 6 and also those of the examples of the present section. Assume that the attacks and defenses of F_2D 1, Section 16, are the only possible moves.
   a Is FD D3 observed?
   b Enumerate the stages that can serve to continue the discussion by some admissible transition to a new chain of arguments.
   c Are FD D7 and FD D8 observed?
   d Enumerate the stages that can serve to continue the discussion without a transition to a new chain of arguments.
2. Show that the consequences mentioned after FD D7 hold.
3. Show that the following consequence holds (given that all stages contain either an attack or a protective defense):
   Each utterance by O (apart from the attack on the initial thesis) will be a reaction to an utterance by P that immediately precedes it in the chain of arguments.

1.16. [III.16.] Dynamic dialectics (II)

Another instrument for achieving the goal expressed in FD D1 is the following:

FD D9 Only such structural operators (non-referring operators, "logical constants") shall be employed for which there are clear definitions of their meaning-in-use, informing potential debaters how sentences containing these operators can be attacked and defended.

In order to satisfy FD D9, which also is instrumental in bringing about the goal expressed in FD D1, we propose the following rule about definitions of meanings-in-use of structural operators:

FD D10 The definitions of the meanings-in-use of the structural operators shall, where possible, be such as to bring about a *decomposition* of the sentences involved in the conflict (and, therefore, in the process of resolving it).

This rule can be implemented by a $formal_2$ rule – the first so far – as follows:

F_2D 1 The meanings-in-use of some frequently used structural operators – as far as the $forms_2$ of attacks on, and of the structural protective defenses of, sentences containing them are concerned – shall be as given by the following *strip rules for logical constants.*[1]

Figure III. 7

	(Speaker:) U	(Critic:) aU	structural pU	
$Rule_{\rightarrow}$	$V \rightarrow W$	(?) V	W	
$Rule_{\sim}$	~V	(?) V	none Λ	in languages without any Λ in languages with Λ
$Rule_{v}$	V v W	?	V W	Speaker may choose any one
Γ& $Rule_{\wedge}$ ſ	V & W	L ? R ?	V W	if the attack was: L ? if the attack was: R ?
$Rule_{At}$	U (atomic)	U ? (provided The Critic is in contra-position to the statement *U* in question)	none	

Explanation

$Rule_{\rightarrow}$ Take a sentence U of the form $V \rightarrow W$, where V and W are themselves well-formed sentences. A Critical Listener may challenge (a statement of) such a sentence[2] by saying:

I am not convinced of W in the case that V.

or:

But suppose that V; can you defend your statement in that case? I am willing to defend V, for the purpose of this debate.

In Section II.4 we said that we would symbolize an attack of this kind as follows:

(?)V.

Remember that we use the question mark in various ways; it means one thing when put in front of a sentence and within parentheses, and

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[1] These "strips", as we may call them, were first formulated by P. Lorenzen. Cf. the introductory remarks to this chapter.

[2] Recall what was said in Section 4 about the possibility of shifting the focus of attention from statements to sentences, so that now *sentences* may be regarded as objects of criticism and defense (p. 17).

another when it is put after. In a statement, *(?)V*, of (?)V the declarative sentence V is *stated hypothetically*, or *stated (merely) exploratively*, or *granted*. *(?)V* is here a declarative statement, though not an assertive one.
In saying that this is *the* way to attack a conditional sentence V → W, we are saying that the operational meaning of the connective → is to make a claim that an assertive statement of W is defendable as soon as V is granted.
Note that a(V → W) = V, p(V → W) = W (using the notation introduced in Section 4, and restricting the use of "p" to structural defense moves).

$\text{Rule}_{\sim}$ Let U be of the form ~V. A Critical Listener may challenge a statement of U by simply granting V. If no Λ-sentence is available in the language, The Speaker can retort only by counteractive defense or *general* protective defense. Otherwise, there is the possibility of a *structural* protective defense using a statement of Λ. In the latter case, ~V is *dialectically equivalent* to V → Λ.
a(~V) = (?) V, p(~V) = Λ (if Λ is available).

Rule_{v} Let U be of the form V v W. The Speaker can now defend his/her statement of U protectively either by granting V or by granting W. (In the case of P, it would be more natural to say: by taking the burden of proof for V or for W.) One such protective defense is sufficient. Note that whoever utters V v W is not bound to know whether V or W is true. He/she is bound only to *defend* either V or W, or to react by counteractive defense or *general* protective defense, as soon as his/her statement is attacked.
a(V v W) = ?, p_1(V v W) = V, p_2(V v W) = W.

$\text{Rule}_{\&}$ Let U be of the form V & W. The Critic can now choose between two modes of attack. Given the mode of attack chosen by The Critic, The Speaker has no choice as to which *protective* defense is called for.
a_1(V & W) = L?, a_2(V & W) = R?, p_1(V & W) = V, p_2(V & W) = W.

Rule_{At} Let U be atomic. Since a further decomposition is not possible, there is no structural protective defense (satisfying FD D10). The attack can never be used to get more concessions from The Speaker. So we propose *not to allow counteractive defense moves of this type.* It seems natural, however, to allow attacks ensuing from contra-position to atomic statements, for how else could the discussion start if the thesis is atomic?
If no Λ is available, there are similar difficulties with the $\text{Rule}_{\sim}$ – see Section IV.2.2.

We expect the readers to agree with us on the following: Rules FD 9 – F_2D 1 are justified by (in the sense that they satisfy) – without following from – the norm described in FD D1. However, they may also, of course, be adopted, *in the absence of alternative preferred rules*, by a company whose members are not particularly interested in the goals described in FD D1.

The following alternative strip rules do not satisfy FD D10. This would have been alright had they still satisfied FD D2 in the same measure as does F_2D 1; but it seems highly improbable that they do:

Figure III.8

U	aU	pU
If V, then W	V and not W	
Not V	Not (notV)	
V or W	Neither V nor W	
U (atomic)	Not U	{ Not (not U) { U

If Not U may be an attack on U, then a structural protective defense, it seems, would have to be either a negation of that in turn – in which case the situation has become considerably more complex than the one from which we started – or simply a repetition of the first claim, U. In the latter case, since nothing has been achieved at all except postponement of a possible loss for one of the parties, such a rule does not satisfy FD D2. A discussion of the other (not to be recommended) rules in this schema is left to the readers.

F_2D 1 satisfies FD 10 because none of the strips in it involves any structural operator other than the one to be defined.

Exercise

Find (natural) decomposing dialectical rules for:
1) the biconditional: U *if and only if* V (U ≡ V);
2) Sheffer's stroke: ***not both*** U ***and*** V (U|V).
Which FD-rule has to be changed in order for the rule for Sheffer's stroke to be usable?

2. [Chapter IV] Variants of Formal$_3$ Dialectics

(with E.M. Barth)

In order for a company to complete the system of formal$_3$ dialectics set out in the last chapter, the following are sufficient:

(i) Fix a language (-form$_2$), to be used in the debates.
(ii) Expand the rule F_2D 1 in order to treat those structural operators in the language for which no strip rules have as yet been given.
(iii) Consider the possibility of making one or more changes in, or additions to, the rules suggested in the preceding chapter.
(iv) Decide that the final set of rules contains *all* the rules; i.e., *that no move is to be allowed if it is not generated by these rules* (for sanctions, see FD E5 or FD E5super). In other words, decide that any move that is not generated by the system is to be regarded as a *fallacy.*

In this chapter we shall assume that the language to be used is of one of the forms $\mathfrak{J}_D$, $\mathfrak{T}_D$ or $\mathfrak{T}_D^{\wedge}$. Since Rule F_2D 1 provides strip rules for all the structural operators we are interested in, we are not concerned with task (ii). The provisos, changes and additions will pertain to the rules in Section III.7 (Realistic dialectics) and in Section III.14 (Orderly dialectics).

2.1. [IV.1] Constructive dialectic systems

2.1.1. [IV.1.1] A dialectic system with constructive implication

Assume, first, that the participants have chosen a language of the form $\mathfrak{J}_D$, and that they have decided not to make use of any logical constant save →. The following rule, which will be called *the fundamental closure rule of formal$_3$ dialectics*, has the force of strengthening the "*if*" in the closure rule formulated in Section III.7 (FD R2):

FD R2≡ . . . if and only if it was defended by an appropriate *Ipse dixisti!*-remark.

And now a definition:

Def. 1 By *constructive*-IF *dialectics* (constructive dialectics for $\mathfrak{J}_D$-languages, CID) we shall understand the system of rules proposed in Chapter III, with FD R2 replaced by FD R2≡, and with F_2D 1 limited to Rule$_\rightarrow$ and Rule$_{At}$.

Example 1

The discussion in the example of Section III.6 is a CID-discussion.

2.1.2. [IV.1.2] A dialectic system with constructive implication, conjunction, veljunction and negation

Now assume that the company has chosen a language of the form $\mathfrak{T}_D$, i.e., a language containing →, ~, v, and &, but no decidedly false sentences. Assume, moreover, that the company has decided not to make use of any other logical constants. The defining strip for ~V, which we then assume in F_2D 1, is the following (see Figure III.7, Section III.16):

Figure IV.1

U	aU	structural pU
~X	(?)X	none

There are no alternatives in F_2D 1 pertaining to the other connectives. We shall henceforth distinguish between (constructive-, etc.) NOT *dialectics* and (constructive-, etc.) Λ *dialectics*, meaning in the first case a dialectic system for languages of the form $\mathfrak{T}_D$, and in the second case a dialectic system for languages of the form $\mathfrak{T}_D^{\Lambda}$.

Def. 2 By *constructive*-NOT *dialectics* (constructive dialectics for $\mathfrak{T}_D$-languages, CND) we shall understand the system of rules proposed for $\mathfrak{T}_D$-languages in Chapter III, with FD R2 replaced by FD R2≡.

Example 2

CND

Olga	Pope
	(a) if both there is mind and it is not the case that there is mind then God exists
. . . .	. . . .
1. (?) both there is mind and it is not the case that there is mind	[God exists]
2.	God exists
3. God exists?	[ ]
4. [there is mind]	L ?
5. there is mind	
6. [it is not the case that there is mind]	R ?
7. it is not the case that there is mind	
8. [ ]	(?) there is mind
9. there is mind?	[ ]
10.	You said so yourself!

(We use the empty brackets "[ ]" to indicate the absence of structural protective defense rights after an attack on a negation or atom.)

The Pope wins this chain of arguments. He has refuted Olga's *line of attack* (line of criticism; cf. Section III.13).

Example 3

CND

Olga	Pope
(a) either it is not the case that there is matter or there is mind	(b) if there is matter then there is mind
. . . .	. . . .
1. (?) there is matter	[there is mind]
2. [it is not the case that there is matter, there is mind]	?
3. it is not the case that there is matter	
4. [ ]	(?) there is matter
5. there is matter?	[ ]
6.	You said so yourself!

The Pope wins the chain of arguments.

Example 4

CND

Olga	Pope
(a) if God exists then there is mind	(b) if it is not the case that God exists then it is not the case that there is mind
. . . .	. . . .
1. it is not the case that God exists	[it is not the case that there is mind]
2.	it is not the case that there is mind
3. there is mind	[ ]
4. [there is mind]	God exists
5. God exists?	[ ]

Olga wins! The Pope did not transgress any of the rules, but has exhausted his rights in the chain of arguments (FD D8!).

Example 5

CND

Olga	Pope
	(a) either God exists, or it is not the case that God exists
. . . .	. . . .
1. ?	[God exists, it is not the case that God exists]
2.	God exists
3. God exists?	[ ]

The Pope has exhausted his rights in the chain of arguments.

2.1.3. [IV.1.3] A dialectic system with constructive Λ

Assume this time that the company has chosen a language with n "absurd" or "decidedly false" sentences Λ_i. This means that the defining strip for ~V in F_2D 1 will be:

Figure IV.2

U	aU	structural pU
~X (or: $X \rightarrow \Lambda_i$)	(?)X	Λ_i

For the sake of simplicity, we usually drop the index "i" in "Λ_i", i.e., we restrict our attention to languages of the form $\mathcal{J}_D^\Lambda$. This change of language form makes it possible for the company to contemplate different ways of treating a statement of an absurd sentence. Compare the discussion fragments below.

Figure IV.3

language of the form $\mathcal{J}_D$

O	P
.	
.	
.	
~X	
[ ]	(?)X
aX	

language of the form $\mathcal{J}_D^\Lambda$

O		P	
~X (or: $X \rightarrow \Lambda$)			
[Λ]		(?)X	
1 aX	2 Λ	1	2

When ~X is, or may be, defined as $X \rightarrow \Lambda$, the Opponent has two options, 1 and 2, in reacting to P's attack. Option 1 is the same as in the fragment on the left. The problem now arises of whether O's opportunity to state Λ (option 2) should influence P's chances of winning the discussion.

Clearly, if we want a Λ dialectics that is just as P-friendly as the constructive-NOT dialectics, then we shall have to introduce a second closure rule, based on the following terminological convention:

Def. 3 By *an appropriate Absurdum dixisti!-remark*, we shall understand an utterance of the words *Absurdum dixisti!* or, *You said something absurd!*, by debater N in a situation where $\bar{N}$ has expressed contra-position toward some statement or other, and also has accepted, by stating Λ, pro-position toward a statement of Λ.

The additional closure rule is this:

FD R2≡v . . . *if and only if* it was defended by an appropriate *Ipse dixisti!*-remark *or* by an appropriate *Absurdum dixisti!*-remark.

Now we can provide another definition:

Def. 4 By *constructive-Λ dialectics* (constructive dialectics for $\mathcal{J}^{\Lambda}_{D}$-languages, C$\Lambda$D) we shall understand the system of rules proposed in Chapter III for $\mathcal{J}^{\Lambda}_{D}$-languages, with R2 replaced by R2≡v.

Consequence. (advice to the Opponent) Do not state a *decidedly* false (absurd) sentence!

The same warning can be expressed quite generally, rather than being directed explicitly to one and only one of the parties (cf. Section III.9):

Consequence. Do not express pro-position toward an absurdity (decidely false sentence) if you have already expressed contra-position toward any statement whatsoever!

This is something the Proponent cannot do in a pure (simple) discussion, where the Proponent *is never* in contra-position to any statement whatsoever. Proponents *in simple discussions* can state absurdities, in the sense of decidedly false sentences, without running the risk of an *Absurdum-dixisti!*-remark from the other party. Constructive-Λ dialectics is recommended to all who agree

(i) that one need not take seriously a language user who professes to be critical, in the sense of being in contra-position to one or more statements, while at the same time professing pro-position toward a statement of a sentence to whose falsity or absurdity he/she has already agreed; whereas

(ii) since the Proponent in a discussion issuing from a simple, or pure (one-thesis), conflict does not take the oppositional or critical attitude of contra-position to any statement whatsoever in the whole discussion, this party should be allowed to "get away with" a statement of a Λ-sentence and to *win* the discussion (or a part of it) on certain conditions, notwithstanding its expression of pro-position toward a Λ-sentence; the condition being, of course, that the Opponent expresses either some such decidedly false statement or contrary attitudes toward statements of the same sentence.

It does not really matter whether we say:

(1) The parties (including the Proponent) have agreed (in advance) upon the falsity of Λ, but in the discussion P's attitude toward Λ is going to be the same (i.e., the neutral) attitude that P takes toward O's (positive) concessions,

or:

(2) Only the Opponent has expressed in advance that it is going to treat Λ as absurd or otherwise false.

Example 6

If we choose "mind is matter" as a Λ-sentence of $\mathfrak{T}_1^{\Lambda}$, then this is a CΛD-chain of arguments:

CΛD

Olga	Pope
	(a) if mind is matter then God exists
. . . .	. . . .
1. (?) mind is matter	[God exists]
2.	You said something absurd!

The Pope wins this chain of arguments.

Example 7

Return to Example 2. In order to transform this chain of arguments into one in CΛD, we merely have to rewrite stage 8:

8. [mind is matter]	(?) there is mind

The option of stating "mind is matter" offers no bright prospects to Olga – the Pope can retort with an *Absurdum dixisti!*-remark:

8. [mind is matter]	(?) there is mind
9. mind is matter	
10.	You said something absurd!

The Pope wins this chain of arguments.

In the exercises below we shall begin to make use of Beth's *sequents* (Chapter I.1 *sub* (iii), 6 and 7). A sequent Π/Γ is an ordered pair of sets of sentences Π and Γ. In the case where Γ contains just one sentence Z we shall write:

Π/Z

instead of

$\Pi/\{Z\}$.

Further, we shall write:

$\Pi, \Pi'/\Gamma$

instead of

$\Pi \cup \Pi'/\Gamma$

and

$\Pi, U \to V/\Gamma$

instead of

$\Pi \cup \{U \to V\}/\Gamma$

etc.

Also we shall suppress quotes, writing:

A → B / A

instead of

"A → B" / "A",

etc.

So A → B / A = {"A → B"}/{"A"}.

"∅" denotes the empty set.

By a discussion (chain of arguments) *for* a sequent Π/Z, we mean a discussion (chain of arguments) issuing from a conflict where the concessions are statements of sentences in Π and the thesis is a statement of the sentence Z.

Exercises

1. Write out one completed CID-chain of arguments for each of the following sequents:

a A → (B → C), A → B / A → C
b (A → B) → (B → C), B / C
c A → B, B / A
d ∅ / A → (B → A)
e A, B / C
f (A → B) → A / (A → B) → B

2. For each of the following sequents, write one completed CND-chain of arguments won by P, and one completed CND-chain of arguments won by O. (Recall FD D6 and FD D8.)

a A → ~B / B → ~A
b ~(A & B) / ~A v ~B
c ∅ / ~~(A v ~A)

3. Do Exercise 2 for C∧D.

2.2. [IV.2.] Minimal dialectic systems

The systems that we shall call *minimal* dialectic systems (for languages of the forms $\mathfrak{T}_D$ and $\mathfrak{T}_D^{\wedge}$) are *stricter for Proponents* than are the constructive systems; i.e., some options open to P in the constructive dialectic systems will no longer be so in the minimal ones.

2.2.1. [IV.2.1] A dialectic system with minimal implication

"Minimal-IF dialectics", or "MID", is just another name for constructive-IF dialectics (CID); we introduce it simply for symmetry's sake:

Def. 5 MID = CID.

2.2.2. [IV.2.2] A dialectic system with minimal Λ: A stricture on the P-win-conditions

Again, assume that the company has chosen a language of the form $\mathfrak{T}_D^\Lambda$ (and again, that they have decided not to make use of any logical constants save →, &, v, ~, Λ). And assume that an appropriate *Absurdum dixisti!*-remark is found to be insufficient as an unconditional defense of the local thesis.

Def. 6 By *minimal-Λ dialectics* (minimal dialectics for languages of the form $\mathfrak{T}_D^\Lambda$, MΛD) we shall understand the system of rules proposed for $\mathfrak{T}_D^\Lambda$-languages in Chapter III, but with FD R2≡ instead of FD R2.

Clearly, this dialectic system is less "P-friendly" than CΛD, and hence than CND (which will be shown to be equivalent to CΛD in Section V.5).

Example 1

All discussions in the examples and exercises of Chapter III are MΛD-discussions (using the dialectical extension either of $\mathfrak{T}_1^\Lambda$ or of $\mathfrak{T}_0^\Lambda$).

Example 2

If we again choose "mind is matter" as the Λ-sentence of $\mathfrak{T}_1^\Lambda$, then Example 2 of the preceding section can be transformed into an MΛD chain of arguments by rewriting stages 8 and 9 and omitting stage 10:

.
.
.

8. [mind is matter]	(?) there is mind
9. mind is matter	

Olga wins! (Although the Pope has not transgressed any of the rules, he has exhausted his rights in this chain of arguments.)

2.2.3. [IV.2.3] A dialectic system with minimal implication, conjunction, veljunction and negation

A system that will be shown to be equivalent to MΛD (in Section V.5) is obtained by choosing a language without any Λ and applying *restricted* constructive-NOT dialectics, or minimal-NOT dialectics, which contains a rule restricting the attack possibilities of the Proponent:

FD M-NOT The Proponent may attack a statement of a negative sentence ~U only if the local thesis itself is a statement of some negation ~V.

The reason for wishing to adopt such a rule is that attacks by P (i.e., counteractive defense moves by P) on a negation, according to Rule$_\sim$ in the version without any structural protective defense, are *prima facie* pointless. For the object of a counteractive defense is to bring it about that the other party makes further statements, in order for an appropriate protective defense to be carried out. Whenever a negation, *~U*, is attacked by P (who must, in that case, utter a statement *U*), there is no choice for O but to attack this latter statement *U* and thereby to start a new local discussion. Hence an attack by P on a negation can never serve the normal purpose of counteractive defense: such an attack can never elicit from O any statements of new sentences (without starting a new local discussion). There is, therefore, some point to rules that restrict P's right to carry through such attacks. However we go about this matter (for the company can, after all, decide *not* to restrict P's right to attack negations – see constructive-NOT dialectics in Section 1.2), we should, at the very least it seems, grant P a right to attack a negation in the case where the local thesis itself is some negation. For in that case there are *no protective defense possibilities* for P, and counteractive defense moves cannot in any case serve their "normal purpose". So the rule FD M-NOT, though by no means the only, or even the most reasonable, solution, is not arbitrary; it constitutes a somewhat awkward device that may come in handy if the disputants agree to employ a language containing sentences without an associated structural protective defense possibility.

We now give the following definition:

Def. 7 By *minimal*-NOT *dialectics* (minimal dialectics for languages of the form $\mathcal{T}_D$, MND) we shall understand the systems of rules proposed in Chapter III for $\mathcal{T}_D$-languages, but with FD R2≡ instead of FD R2, and with the addition of the restriction FD M-NOT on moves permissible to the Proponent.

Example 3

All discussions in the examples and exercises of Chapter III are MND-discussions (using either $\mathcal{T}_1$ or $\mathcal{T}_0$).
The chain of arguments in Example 2 of Section 1.2 is an MND-chain up to and including stage 7. Stage 8 is not permitted in MND. Indeed, in MND the Pope has already exhausted his rights after stage 7 has been completed.

Exercises

1. Why is the chain of Example 3 in Section 1.2 not an MND-chain of arguments?
2. Write out completed MND- and M∧D-chains for the sequent of Example 3 in Section 1.2.
3. For the same sequent, write out a C∧D-chain that is not an M∧D-chain, and which is won by the Pope.

2.3. [IV.3.] Classical dialectic systems

We may make things easier for Proponents by assuming, instead of FD O2b as applied to the Proponent, the following principle, which we shall call *the fundamental norm of non-constructive dialectics*:

FD K At each stage of each chain of arguments, the Proponent shall *retain* its *unused* protective defense rights (including the rights to make *Ipse dixisti!*-remarks) with respect to a local thesis even after the latter has been defended protectively in that chain of arguments. (The Proponent shall not retain its *used* rights.)

If the company decides to adopt FD K, then P retains the right to present protective defenses of a former local thesis, and may thus profit from concessions that O has stated in the meantime, in the contest about other local theses. We do not say that the Proponent should retain pro-position, since such a stipulation would grant the Opponent an opportunity to make a local thesis the object of an unlimited number of counteractive defense moves (something we prevented by FD D7). Notice further that, if we want to avoid that a debater is faced with conflicting rules, adoption of FD K *requires* that FD O2b be limited to the Opponent (and that FD O4b and O5b be adapted). No further modifications of the rules are required.

In checking our argument in favor of FD D7 and D8 (Section III.15), the reader may notice that some changes are called for in the case where FD K is included. As to FD D7, this is a simple affair: the possible differences between S' and S'' (p. 41) involve not only the set of O's concessions but also the set of P's defense rights. However, a "moderate" Proponent will no more use the additional rights than it chooses to attack the additional concessions. Hence our argument that O must have a winning strategy in S' if it has one in S'' stands.

The argument in favor of FD D8 fails since, even if the supply of concessions has not been augmented, the supply of defense rights may have changed in such a way that – upon P's repetition of a former local thesis – it is *not* possible for O to revive an old dialogue situation. Take, for instance, the following "chain of arguments":

Figure IV.4

Olga	Pope
(a) (A → B) → C	(b) A → B
. . . .	. . . .
1. (?) A	[B]
2.	B
3. B ?	[ ]
4. [C]	(?) A → B (violates FD D8)

Since at stage 4 the Pope has violated one of the rules of the system, viz. FD D8, Olga may now withdraw from the discussion without losing it. However, if FD K is included, there is no way in which Olga can revive an old dialogue situation – certainly not by asserting "C", but also not by attacking "A → B". For in the latter case the Pope will have both the right to defend by means of "B" and the conditional right to defend the atom "B" by means of *Ipse dixisti!* (against 3), and this combination of rights on the Pope's side has not yet occurred (the right to defend by means of "B" was used at stage 2).

There is no *prima facie* guarantee that P cannot profit by the repetition of a former local thesis in such cases. It is, however, possible to argue for FD D8 – in the case where FD K is included – along the following lines. As before, let σ_r be an acceptable system of formal$_3$ dialectics including FD D7 and FD K. We must show that the addition of FD D8 is acceptable to P.

First consider the system σ_r', similar to σ_r, but according to which *all* protective defense rights are retained (used or not used). This system is – in a sense – maximally P-friendly and will, therefore, be acceptable to P if σ_r is too. In σ_r' P can never profitably repeat a local thesis. For, if P were to do so, O could restore a previous dialogue situation: the one that obtained just before P repeated the thesis. It follows that FD D8 is not really harmful to P and may be added to σ'_r. Moreover, it may now be strengthened to: "P is never to repeat *any* local thesis". It remains to be seen that it will not bother P if only the unused defense rights are retained. But, by FD D7 and its consequences, it can be argued that all statements by P that constitute protective defenses are at the next stage attacked by O. Hence, if FD D8 (strengthened form) is included in the system, protective defense rights cannot be used twice by P so as to make two statements of the same sentence; for the first of these statements would have been attacked and hence would be a local thesis. Therefore, if σ_r is acceptable to P, so too is the combination of σ_r and the strengthened form of FD D8, and hence also the combination of σ_r and FD D8 as stated.

2.3.1. [IV.3.1.] A dialectic system with classical implication

Assume that the participants have chosen a language of the form $\mathfrak{J}_D$.

Def. 8 Our system of *classical*-IF *dialectics* (classical dialectics for $\mathfrak{J}_D$-languages, KID) is obtained from the system of constructive-IF dialectics by the addition of FD K, the limitation of FD O2b to the Opponent, and the adaptation of FD O4b and O5b so as to allow for the exception to these rules stated by FD K.

Example 1

Every MID-discussion is a KID-discussion.

Example 2

KID

	Olga	Pope
	(a) if if God exists then there is mind then God exists	(b) God exists
	. . . .	. . . .
1.	God exists?	[ ]
2.	[God exists]	(?) if God exists then there is mind
3.	(?) God exists	[there is mind]
4.		You said so yourself!

(The Pope wins by the retained right to make an *Ipse dixisti!*-remark.)

2.3.2. [IV.3.2.] A dialectic system with classical $\wedge$

Assume that the participants have chosen a language of the form $\mathfrak{J}_D^{\wedge}$.

Def. 9 Our system of *classical*-$\wedge$ *dialectics* (classical dialectics for $\mathfrak{J}_D^{\wedge}$-languages, K$\wedge$D) is obtained from the system of constructive $\wedge$-dialectics as KID is obtained from CID.

Example 3

Every C$\wedge$D-discussion is a K$\wedge$D-discussion.

Example 4

KΛD

Olga	Pope
∅	(a) either God exists or it is not the case that God exists
. . . .	. . . .
1. ?	[God exists, it is not the case that God exists]
2.	God exists
3. God exists?	[ ]
4.	it is not the case that God exists
5. (?) God exists	[mind is matter]
6.	You said so yourself!

(The Pope wins; cf. Example 5 in Section 1.)

2.3.3. [IV.3.3] A dialectic system with classical implication, conjunction, veljunction and negation

Assume that the participants have chosen a language of the form $\mathcal{T}_D$.

Def. 10 Our system of *classical*-NOT *dialectics* (classical dialectics for $\mathcal{T}_D$-languages, KND) is obtained from the system of constructive-NOT dialectics as KID is obtained from CID.

Example 5

Every CND-discussion is a KND-discussion.

Example 6

KND

Olga	Pope
(a) it is not the case that it is not the case that God exists	(b) God exists
. . . .	. . .
1. God exists?	[ ]
2. [ ]	(?) it is not the case that God exists
3. (?) God exists	[ ]
4.	You said so yourself!

(The Pope wins.)

As a possible, equivalent, alternative to FD K, we mention the following "classical structural protective defense moves", which could be added to F_2D 1 in order to obtain a classical dialectics (P being The Speaker and O The Critic – not the other way around):

Figure IV.5

F_2D K

U	aU	classical pU	
X → Y	(?) X	~~Y	
~X	(?) X	none (or: ~~Λ)	
X v Y	?	~~(X or Y)	∠ v
X & Y	l ? r ?	~~X ~~Y	
U (atomic)	?	~~U	

In fact it would be sufficient to add the classical protective defenses for the veljunctions and for the atomic sentences, but we shall not go further into that matter here.

The rule F_2D K conflicts with FD D10, but is in accord with FD D9. F_2D K is acceptable only if the other rules of the system of dialectics jointly guarantee a sufficient implementation of FD D1 and FD D2.

Exercises

1. Change Example 4 into a KND-chain won by P.
   Change Example 6 into a KΛD-chain won by P.
2. Write out a KΛD- (or KND-) chain won by O that is not a CΛD- (or CND-) chain.
3. Write out a KΛD- (or KND-) chain won by P that is not a CΛD- (CND-) chain for the sequent:
   ~A → A / A.

2.4. [IV.4.] Summary

Thus far we have defined eight different dialectic systems (for MID = CID), each with its own variant on dialectical validity. For example, with respect to languages of the form $\mathcal{T}_D$, we now have at our disposal *three* concepts of validity: viz., constructive dialectical validity, minimal dialectical validity, and classical dialectical validity.

All these systems share the *Elementary Rules* (Section III.5), the rules of *Systematic Dialectics* (III.6), the rules of *Realistic Dialectics* (III.7), the rules concerning the effects of *winning and losing* (III.8), the rules of *Thoroughgoing Dialectics* (III.13), and the rules of *Dynamic Dialectics* up to and including FD D10. They all take their *$formal_2$* rules from F_2D 1, but which parts of F_2D 1 apply depends on the language $form_2$ to which the dialectical rules pertain. The only further differences between the systems concern

(i) the way in which the rules of *Realistic Dialectics* are completed (FD R2≡ or FD R2≡v);

(ii) the presence or absence of FD K, with a corresponding choice of rules for *Orderly Dialectics.* If FD K is not included in the system, then the rules of *Orderly Dialectics* are those of Section III.14. If FD K *is* included, there are modifications to FD O2b, FD O4b and to FD O5b;

(iii) the presence or absence of FD M-NOT.

These differences are summarized below.

Figure IV.6

→ DEFENSIVE STRENGTH →

Rule \ Dialectic system	MID = CID	KID
FD K		•
FD R2≡	•	•

Rule \ Dialectic system	MND	CND	KND
FD K			•
FD R2≡	•	•	•
FD M-NOT	•		

Rule \ Dialectic system	MAD	CAD	KAD
FD K			•
FD R2≡v		•	•
FD R2≡	•		

We have put a dot where we want to say that a rule is included in a system.

In the following theorem, we state the obvious relationships between the systems defined thus far, pertaining to the existence of winning strategies for the Proponent:

Theorem 1 If P has a winning strategy for a dialogue situation in a minimal (constructive) system, then P has a winning strategy for that situation in the corresponding constructive (classical) system.

The reader may check this by inspecting the definitions, in order to see that a change from minimal to constructive dialectics, or from constructive to classical, always makes things easier for P.

3. [IV.5] Formal$_3$ material dialectic systems

In this section (and its subsections) we shall study how evaluations of (some) atomic statements as "true" or "false" can be brought to bear upon the course of a formal$_3$ discussion. We want to discuss the possibility of a Proponent's defending its local thesis unconditionally by pointing to the "truth" of the thesis or by pointing to the "falsity" of a concession. That is to say, we suggest that the company should contemplate the possibility of adding one or more rules to generate "material moves" – calling all moves that could so far be generated "formal$_2$ moves". Yet we want also to preserve our implementations of the norms in Chapter III. In particular, we should take care not to obstruct the basic norms of orderly and of dynamic dialectics and our present implementations of them.

Clearly, for each atom that may be subjected to "material" moves, the company should then come to an agreement about its "material" status; i.e., should decide whether it should be called "true" or "false" or should remain without such a predicate. For this status will determine which "material" remarks the company will allow The Speaker to make, and disagreement over this status will lead to gaps in the distribution of rights and duties in the discussion, and thus obstruct implementation of the fundamental norm of orderly dialectics (FD O1, Section III.14). But this is not to say that the truth value of each atomic sentence must be decided before the discussion starts. It is sufficient for the company to agree in advance to employ some definite procedure(s) by which, for each atomic sentence, a decision can be reached (within a reasonable time limit). Such a procedure may employ all kinds of *ostensive means* – including *experimentation* – *consultation* of authorative sources, and *computations* of various kinds.

In this section we shall assume that the company has adopted two such procedures. By the *material truth procedure*, it can decide, for any given atom X, whether X is to be accepted in the company or not. By the *material falsity procedure*, the company can, for any given atom X, decide whether X is to be rejected or not. In the case where the material truth procedure, if applied, would lead to acceptance of X, we say that a company that accepts the procedure *implicitly accepts* X. Notice that the company need not be aware of this. When this procedure has in fact been applied, with positive outcome, to what apparently was an implicitly accepted atom X, we say that the company now *explicitly accepts* X. If the outcome was negative, we say that the company now *explicitly does not accept* X, or that X is *explicitly not-accepted* by the company. Notice that not all atoms X that are not explicitly accepted need to be explicitly not-accepted: they may have no explicit status at all.

We employ a similar terminology in connection with the material falsity procedure. In the case where this procedure, if applied, would lead to a rejection of X, we say that a company that accepts the procedure *implicitly rejects* X; and after the procedure has been applied with positive outcome to what apparently was an implicitly rejected atom X, we say that the company now *explicitly rejects* X. If the outcome was negative, we say that the company now *explicitly does not reject* X.

In what follows, let $\mathbb{T}$ be the class of implicitly accepted atoms and let $\mathbb{F}$ be the class of implicitly rejected atoms. Let $\mathbb{T}_0$ ($\mathbb{F}_0$) be the class of explicitly accepted (rejected) atoms. Clearly $\mathbb{T}_0 \subseteq \mathbb{T}$ and $\mathbb{F}_0 \subseteq \mathbb{F}$. We shall assume $\mathbb{T} \cap \mathbb{F} = \emptyset$, and hence $\mathbb{T}_0 \cap \mathbb{F}_0 = \emptyset$, but we shall leave open the possibility that $\mathbb{T} \cup \mathbb{F}$ does not exhaust the class of all atoms. Atoms that are explicitly accepted (rejected) shall also be called explicitly not-rejected (not-accepted).

The classes $\mathbb{T}$ and $\mathbb{F}$ are fixed once a material truth procedure and a material falsity procedure are adopted. On the other hand, we get different values for "$\mathbb{T}_0$" and "$\mathbb{F}_0$" as more and more implicit acceptations (rejections) are turned into explicit acceptations (rejections).

3.1. [IV.5.1] Pure conflicts: Material procedures subjoined to our former dialectic systems

In order to subjoin material procedures for truth and falsity to our systems of formal$_3$ dialectics defined in the preceding sections, we shall define two *material moves* (cf. Def 17^P in Section III.7 and Def. 3 in Section 1.3):

Def. 11 By an *appropriate Verum dixi!-remark*, we shall understand an utterance of the words *Verum dixi!* or *I told the truth!* by a Proponent who has incurred an obligation to defend a sentence that is explicitly accepted by the company, provided this obligation has not in the meantime been lost or overruled by other obligations.

Def. 12 By an *appropriate Falsum dixisti!-remark*, we shall understand an utterance of the words *Falsum dixisti!* or *You uttered a falsehood!* by a debater N in a situation where $\bar{N}$ has expressed contra-position toward some statement or other and also has assumed pro-position toward a statement of some sentence that is explicitly rejected by the company.

Notation:

Figure IV.7

	Speaker: X	Critic: aX	general "material" pU
For $X \in \mathbb{T}_0$:	X	X ?	*Verum dixi!* or: X!!
For $X \in \mathbb{F}_0$:	X	*Falsum dixisti!* or: X??	none

From each of the systems defined in the preceding sections, a formal$_3$ material dialectics can now be obtained by (i) an agreement about material procedures for truth and falsity, and (ii) the following implementation of the fundamental norm of the possibility of unconditional defense (to replace FD R2, Section III.7):

FD R2M . . . if and only if it was defended by an appropriate *Ipse dixisti!*-remark [or by an appropriate *Absurdum dixisti!*-remark], or by an appropriate *Verum dixi!*-remark, or by an appropriate *Falsum dixisti!*-remark.

The reference to *Absurdum dixisti!*-remarks should be inserted only if such a clause was included in the original system to which the material procedures are subjoined.

Clearly only the Proponent will profit from the introduction of material procedures in our dialectic systems. Since we are still discussing *pure*, or *simple*, conflicts, this is quite acceptable. For in a dialectics devised for the resolution of pure conflicts, there is only one Proponent and the other party (role) – the Opponent – does not have a thesis to defend. Therefore a *Verum dixi!*-remark by the Opponent is just as pointless as is an *Ipse dixisti!*-remark (cf. Section III.7). On the other hand, appropriate *Falsum dixisti!*-remarks by the Opponent are precluded by the wording of Definition 12. And quite rightly so, as the Proponent should also be able to defend, at least relative to *some* sets of concessions, an explicitly rejected thesis (cf. Section 1.3, on *Absurdum dixisti!*-remarks).

We think, therefore, that FD R2M is a *natural* (Section III.12) way of incorporating material procedures into formal$_3$ dialectic systems devised for the resolution of *pure* conflicts.

Thus far we have disregarded those sentences accepted/rejected implicitly but not explicitly. The Proponent can exploit such sentences only if granted some rights to demand that a material procedure be applied (by, or on behalf of, the company). The Proponent may hope thereby to bring it about that some sentence moves from the class of merely implicitly to the class of explicitly accepted (rejected) sentences. In order not to obstruct the effect of the rules of dynamic dialectics, such rights should be limited to certain well-defined circumstances and atoms:

FD DM1 Let the local thesis, *T*, be atomic. If the sentence T is neither explicitly accepted nor explicitly not-accepted, then the Proponent has a right to interrupt the discussion and to demand that the material truth procedure be applied to T.

FD DM2 Let *U* be any atomic concession. If the sentence U is neither explicitly rejected nor explicitly not-rejected, then the Proponent has a right to interrupt the discussion and to demand that the material falsity procedure be applied to U.

The rules FD DM1 and 2 show how one can incorporate material procedures into a formal$_3$ dialectics while yet satisfying the norm of dynamic dialectics, even when the material procedures are applicable to a potential infinity of sentences: in each discussion, the number of applications of material procedures will be finite.

Systems of formal$_3$ dialectics to which material procedures and moves are subjoined are called (*formal$_3$*) *material* systems. All systems defined in the preceding sections were *non-material*, in the sense that no such procedures or moves were subjoined to them. Such systems are often called *formal* (*formal$_2$*, as we would say), because the meaning (content, matter) of the sentences does not figure in any of the (formal$_2$) moves generated by these systems. The sentences may consequently be regarded as mere sentence forms$_2$.

Systems containing one or more *absurd*, or *decidedly false*, sentence(s), Λ_1, Λ_2, . . . Λ_i, as well as a procedure for deciding which sentences are to be counted as absurd, form a borderline case. The reader may have noticed a striking similarity between absurd sentences and "materially" rejected sentences, particularly explicitly rejected sentences. It might seem as if there were no difference between the former, the "decidedly false" sentences, and the latter, the "explicitly rejected" sentences. There are, however, three important differences. These are:

(i) *Connection with negation.* The absurd sentences permit a structural protective defense for an attacked statement of a negation. There is no such connection between negation and the sentences in $\mathbb{F}_0$ or in $\mathbb{F}$.

(ii) *Different recommendations to the company.* With respect to absurdity, we would recommend that the company, in order not to complicate the application of the rule for negation, select some simple formal$_2$ procedure to check whether a statement is a statement of an absurd sentence or not. Preferably the Λ_i should simply be enumerated, and the enumeration regarded as a part of the definition of the dialectic system. (Or, the company may use one Λ-sentence as a "symbol" or as an abbreviation for what is, in our opinion, a veljunction: the veljunction of all the Λ_i.) With respect to falsity, in the material falsity procedure ostensive and other non-formal$_2$ means are tolerated. Clearly the absurd sentences are then to be compared with the *explicitly rejected* sentences rather than with the implicitly rejected ones. But they differ from the latter as well: if the company follows our recommendations, the absurd sentences will be fixed once and for all (by formal$_2$ criteria), whereas more and more sentences from $\mathbb{F}$ may be added to $\mathbb{F}_0$ as the discussions proceed.

(iii) *Difference in minimal dialectics.* In minimal dialectics it is not the case that P can win any chain of arguments in which a concession Λ appears. Yet, even in minimal dialectics, P can win any chain of arguments in which a concession $U \in \mathbb{F}$ appears.

As to our "borderline case", we take the following terminological decision: if the *only* procedure subjoined to a dialectic system is a simple formal$_2$ procedure to check whether a statement is a statement of an absurd sentence or not, we shall call the resulting system non-material.

As far as the study of winning strategies is concerned, the material systems for the resolution of simple conflicts "reduce" to the corresponding non-material ones (at least if the dialectics is constructive or classical). For, instead of granting to the Proponent the rights contained in FD R2M and FD DM1 and 2, we may imagine that all sentences in $\mathbb{T}$, as well as the negations of all sentences in $\mathbb{F}$, were stated by O at the onset of the discussion. Since O has conceded all the elements of $\mathbb{T}$, P can now make an appropriate *Ipse dixisti!*-remark wherever it could previsouly make an appropriate *Verum dixi!*-remark (Figure IV.8). And since O has also conceded the negations of all the elements of $\mathbb{F}$, P can replace an appropriate use of a *Falsum dixisti!*-remark – with respect to an atomic statement U by O – with an attack on $\sim U$. O can react only by an attack on P's statement of U or, in languages with an absurd sentence, by stating such an absurd sentence (the two options are depicted in Figure IV.9). In the first case P can win the chain of arguments by an appropriate *Ipse dixisti!*-remark, and in the second case, by an *Absurdum dixisti!*-remark.

Figure IV.8

O	P
. . . . .	. . . . . .
	U
U?	[ ]
	Verum dixi!

$\Rightarrow$

O	P
U	
. . . . .	. . . . . .
	U
U?	[ ]
	Ipse dixisti!

Figure IV.9

O	P
. . . . .	. . . . . .
U	
	Falsum dixisti!

$\Rightarrow$

O		P	
~U			
. . . .		. . . .	
U			
[Λ]		(?) U	
[1] U ?	[2] Λ	[1] [ ] *Ipse dixisti!*	[2] *Absurdum dixisti!*

(Two options for O. If there are no absurd sentences in the dialectic, there is only one "option".)

So the imagined additions to the class of initial concessions are at least as advantageous to P as are the rights contained in FD R2M and FD DM1 and 2. It can also be shown that the additions to the concessions are not more advantageous to P than is the introduction of the material procedures and rules.[1] We shall not study further the material dialectic systems for the resolution of pure conflicts, but instead turn at once to the problem of the resolution of *mixed* conflicts.

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[1] Atomic concessions can *only* be exploited by *Ipse dixisti!*-remarks. Hence each concession of an atom X may be replaced, without harm to P, by the addition of X to $\mathbb{T}$. P may then use *Verum dixi!* instead of *Ipse dixisti!*
After the theory of winning strategies for P has received further attention (Chapter V), it will be easy to show that the use P makes of concessions of the form ~X, X atomic, can be restricted, without harm to P, to attacks on ~X in a situation where X also is among the concessions. Hence each concession of the form ~X, X atomic, may be replaced, without harm to P, by the addition of X to $\mathbb{F}$. P may then instead make a *Falsum dixisti!*-remark.

3.2. [IV.5.2] Mixed conflicts under complete opposition: A formal_3 material dialectic system

As we said in Section III.11, we shall not attempt to analyze the discussions arising from mixed conflicts in terms of superimposed pure conflicts. We shall, instead, by way of example, define a natural material dialectic system. After, we shall scrutinize the FD-rules of Chapter III and determine the extent of their applicability to discussions arising from mixed conflicts. We shall then see if our system of material dialectics for mixed conflicts fulfills the basic norms.

The introduction of material procedures for acceptance and rejection of sentences and the *Verum dixi!*- and *Falsum dixisti!*-remarks connected with them give us the opportunity to define a rather simple system of dialectics for the resolution of one type of mixed conflicts: *mixed conflicts under complete opposition*.

Def. 13 A *mixed conflict* (of avowed opinions) *under complete opposition* is a quadruple $\langle St_B, St_A, B, A\rangle$, where St_A and St_B are sets of statements (not both empty), A and B are (groups of) users of language, and which satisfies the following conditions:

(i) A (B) has made the statements in St_A (St_B), which have been communicated to B (A), and has not withdrawn any of these statements;

(ii) A (B) has challenged B (A) with respect to *every* statement in the set St_B (St_A) (hence *complete* opposition).

We shall not consider mixed conflicts of any other types.

In a mixed conflict of this kind, there may – by definition – be more than one challenge. When both parties may have theses to defend, we cannot identify one of the dialectical roles as the role of Proponent and the other as Opponent. We shall, therefore, dub the roles White (W) and Black (B):

Def. 14 The role of *Black* in a discussion issuing from a conflict $\langle St_B, St_A$, B, A$\rangle$ is the role taken by A or B according to whether A or B makes the first utterance *in* the discussion. The other role is the role of *White*.

In order to make it equally possible for both parties to make *Verum dixi!*- and *Falsum dixisti!*-remarks, we must first revise Definitions 11 and 12:

Def. 15 *a* By an *appropriate Verum dixi!-remark*, we shall understand (in the present context) an utterance of the words *Verum dixi!* or *I told the truth!* by any party that has incurred an obligation to defend a sentence which has been explicitly accepted by the company, provided the obligation has not in the meantime been lost or overruled by other obligations.

b By an *appropriate Falsum dixisti!-remark*, we shall understand (in the present context) an utterance of the words *Falsum dixisti!* or *You uttered a falsehood!* by any party, in a situation where its adversary has assumed pro-position toward a statement of some sentence that has been explicitly rejected by the company.

We shall exclude moves consisting of an *Ipse dixisti!*-remark from the dialectic system which we shall presently define.

Each party may demand that a material procedure be applied and subsequently base a *Verum dixi!*- or a *Falsum dixisti!*-remark on the outcome of such an application:

FD M1 If party N has incurred an obligation to defend an atomic sentence T (i.e., if a statement of T made by N has been attacked by $\bar{N}$), N has a right to interrupt the discussion and first demand that the material truth procedure be applied to T, provided T has not yet been explicitly accepted or explicitly not-accepted.

FD M2 If party N has made a statement of an atom U that has not yet been explicitly rejected or explicitly not-rejected, $\bar{N}$ has a right to interrupt the discussion and to demand that the material falsity procedure be applied to U.

FD M3 Each party may make appropriate *Verum dixi!*- and *Falsum dixisti!*-remarks. After such a remark has been made, it is the other party's turn to move. Chains of arguments are lost (won) only through exhaustion of one's (the other party's) rights.

We assume that the language chosen is of the form $\mathcal{T}_D$. Hence no *Absurdum dixisti!*-remarks will occur in the discussions. We further assume that *all* the atomic sentences used are either in $\mathbb{T}$ or in $\mathbb{F}$.

Def. 16 By *material dialectics for the resolution of mixed conflicts under complete opposition* for languages of the form $\mathcal{T}_D$ (MatDial, for short), we shall understand the following system of rules:

Elementary Rules

(i) Some participant(s) shall take the part of White and some the part of Black. A party that has made no statements must take the part of Black.

(ii) *Each party shall assume contra-position toward each statement by the other party.* (I. e., the *complete opposition* in the conflict is to be retained throughout the discussion.) Hence it is not possible to distinguish counter-attacks from other attacks.

(iii) Each party shall assume pro-position toward each of its own statements.

(iv) FD E5 (Section III.5).

The only asymmetry in the rules:

(v) Each chain of arguments starts with B attacking a statement made by W (cf. Def. 14).

As to stages:

(vi) FD D5 (Section III.16).

(vii) A stage at which party N is The Speaker will consist either
 (a) of an attack (possibly a *Falsum dixisti!*-remark) on a statement by $\bar{N}$,
 or (b) of a protective defense (possibly a *Verum dixi!*-remark).

Formal$_2$ Rules and Material Rules

(viii) Attacks and defenses will be effected according to F_2D 1 (with the version of Rule$_\sim$ that does not contain a protective defense), or according to FD M3. The discussion may be interrupted according to FD M1 and FD M2.

Orderly dialectics

(ix) FD O2 and FD O3 (Section III.14).

A dynamic principle

(x) A statement may be attacked once only, and an attacked statement may be defended once only (in each chain of arguments).

The effects of winning and losing

(xi) The rules FD W of Section III.8, modified so as to pertain to the new material moves (the modifications may be left to the reader).

Thoroughgoing, yet dynamic, dialectics

(xii) FD T2 (Section III.13) and FD D3 (Section III.16).

Example

Let the language used be $\mathcal{T}_1$.
Let "God exists" ϵ 𝕋
"there is mind" ϵ 𝔽
"there is matter" ϵ 𝔽.

B	W
(a) if both God exists and there is mind then there is matter	(b) it is not the case that either God exists or there is matter
. . . .	. . . . . .
1. (?) either God exists or there is matter	[ ]
2. [God exists, there is matter]	?
3. God exists	
4. [there is matter]	(?) both God exists and there is mind
5. L ?	[God exists]
6. [God exists!!]	God exists?
7. God exists!!	
8.	God exists
9. God exists?	[God exists!!]
10.	God exists!!
11. there is matter	
12. [ ]	there is matter??

(W has exhausted its rights and loses the chain of arguments.)

Which of the basic FD-rules (norms) are implemented by our system of material dialectics? Let us first look at the elementary rules of Section III.5. We do not have the roles O and P, "*the* Opponent" and "*the* Proponent". Each party is both Opponent (of the other party's statements) and Proponent (of its own). Hence we cannot have the rules FD E1 or FD E2. A similar task is, however, performed by rules (i), (ii) and (iii). We do not have FD E3 either, since, by virtue of (ii), each counteractive defense move may also be described as an attack by virtue of contra-position. The spirit of FD E4 is preserved as long as the procedures for truth and falsity have been agreed in the company and all applications of these procedures can be somehow intersubjectively checked by the members of the company. FD E5 is itself included.

Since there is no unique Proponent, there is no point in preserving the systematic dialectics of Section III.6. The rules in that section were intended to give *the* Proponent systematic defense possibilities. The norm present in FD S1, when read as pertaining to both players, is nonetheless implemented. With two exceptions, viz., the case of attacked negations and the case of attacked atomic $\mathbb{F}_0$-elements, there is always the possibility of defending an attacked statement by making another statement. Instead of the *Ipse dixisti!*-remark and the *Absurdum dixisti!*-remark, we now have the possibility of carrying out successful protective defenses by the material rule FD M3. So the norm FD R1 (Section III.7) is implemented. The rules for winning and losing (Section III.8) are adopted, as well as the rule FD T2, implementing FD T1, of thoroughgoing dialectics (Section III.13).

As to orderly dialectics (Section III.14), we have implemented FD O1 by FD O2 and O3, and (vii), but FD O4 and O5 do not apply, since we miss the systematic build-up of chains of arguments given in Section III.6. The dynamic norms FD D1, D2, D4, D9 and D10 (Sections III.15 and 16) are implemented by the inclusion of FD D3, D5, F_2D 1, and (x). On the other hand, FD D7 and D8 are too closely connected with Section III.6 to be adopted here.

Summing up: The system here defined for the resolution of conflicts under complete opposition satisfies all the basic rules (norms) of Chapter III that pertain to the features of this system.

We shall return to this system in Section IX.2.

4. FORMAL DIALECTICS AS IMMANENT CRITICISM OF PHILOSOPHICAL SYSTEMS

4.1. [1] Introduction

In the following* I shall present a new argumentational interpretation of the formal dialogue-games which we owe to Professors Lorenzen and Lorenz. This interpretation differs from that given in [1] in that it is restricted to conflicts arising from a context of immanent criticism, whereas [1] dealt with pure conflicts in general. It will become apparent that the attacks and defenses in Professor Lorenzen's "strips" can be given *two different interpretations*, according to what happens to be the dialectical role of the utterer of the attacked statement. There are also a number of quite natural consequences pertaining to the structural rules.

My starting-point shall be a remark by Frank Van Dun ([3], p. 106)

> (..) *formal* dialogues - these being dialogues where one participant has all the facts and the other all the logic, so to speak.

Let us call the party with "all the facts":Black (Black, too, has logic of course), and the one with nothing but logic: White. You will presently understand why it is I don't call them Opponent and Proponent.

4.2. [2.] *The provocative thesis*

Imagine the following dialectical situation: Black is the "proponent" of a philosophical system, i.e., he/she/it knows or pretends to know what the world is like and what there is; this philosophy is laid down in a set of statements. White wants to attack this philosophy by immanent criticism, i.e., he/she/it wants to beat Black on its own ground. This is a situation where Black has all the facts, for the facts are determined by the system. White, on the other hand, has nothing but logic. Actually, White doesn't even have logic, at least not until the parties agree upon a system of formal dialectics...

I shall argue that Lorenzen's and Lorenz's formal dialogue-games constitute particularly suitable instruments for the resolution of conflicts in this kind of situation.

In the formal dialogue-games the party which has nothing but logic has some thesis to defend, and is called: Proponent. In the situation I depicted it is the party with al the facts (Black) which has a thesis (the philosophical system). Therefore, if I were to use the words "Proponent" and "Opponent", I would assign the name of "Proponent" to Black, and the name of "Opponent" to White. This, however, would be very confusing, as the dialectical role of Black will turn out to be equal to that of Lorenzen's Opponent and the role of White to that of the Proponent. I, therefore, stick to the names "Black" and "White".

One way in which White can start its immanent criticism is to put forward a statement which is known to be unacceptable to Black and to claim that it is part and parcel of Black's system. (Indeed, presumably all immanent criticism can be presented in this form.) For instance, in an attack on a theistic system, White can put forward the STONE-proposition: "God is able to make a stone He cannot Himself lift". Or, if White thinks that Black's system is inconsistent or otherwise absurd it may put forward a *falsum-statement* i.e., employ a sentence ($\wedge$) which, in the system of formal dialectics agreed upon, expresses an absurdity. I shall call the statement put forward by White at the start of the discussion: *provocative thesis*. It is unimportant whether or not this thesis is believed to be true by White. Not only that this is *dialectically* unimportant, for that is always the case if the dialectics is externalized (see The Principle of Externalization of Dialectics in **Paper 1,Section 4,p.17)** but it is also unimportant for our judgment of White's veracity. Whereas in most situations you are supposed to adhere to your thesis (at least until your defeat in debate), this is not the case here. By its *provocative thesis*, T, White is merely announcing that Black cannot reasonably doubt T[1], unless Black is willing to abandon or modify its system.

The provocative thesis can have any grammatical form. If a first order language is employed, the following forms seem particularly well suited to express a provocation: the disjunctive form (dilemma: "God is either not loving or not omnipotent"), the existential form (strange entities: "Some circles are square"), denial (of pieces of common sense knowledge: "There is no knowledge") and the atomic form ("You're nuts"). The STONE-proposition can be expressed in existential form: "there is some way in which God....", or "there is some possible world such that....".

What if Black accepts the provocative thesis without more ado? In that case, we can either say that there was no discussion at all or that an abortive discussion has taken place. If someone has to be called the winner of such an abortive discussion it must be White. However, even if White is called the winner, this party has failed in its attempt at immanent criticism through picking an unsuitable provocative thesis. The provocative thesis should be unpalatable to Black.

4.3. [3.] *Critical interpretation of the logical constants*

If Black rejects the provocative thesis the result will be a *simple conflict of avowed opinions* (Def. 1, p. 13):

$$\langle \mathit{SYST}, T, \text{Black}, \text{White} \rangle,$$

where "*SYST*" stands for the uttered or published part of Black's system (*SYST* is a set of statements), and "T" for the provocative thesis. The debaters may now pick a system of formal dialectics to resolve their conflict. In [1] an argument is developed in favor of several systems of formal dialectics (coinciding with the dialogue games of Lorenzen and Lorenz) as suitable instruments for conflict resolution, provided that the parties in the conflict want to implement various fundamental norms. Nothing is said about the origin of the conflict. I now want to show that, if the origin of the conflict is such as sketched above, the systems of formal dialectics in [1] become particularly appealing.

What forms can Black's rejection of the thesis take? The attacks described in Lorenzen's "strips" are, I think, quite acceptable. (They were recently accepted by the present company!) I would only like to add (as is done in [1]) the possibility of a rejection of an atomic thesis by means of a simple expression of doubt (indicated by "?"). Hence there are now three cases in which Black can reject the thesis by a simple "how come?", viz., when the thesis is atomic, when it is disjunctive, and when it is existential. Let us consider the other forms which provocation and rejection can take:

If $T = T_1 \;\&\; T_2$, I think we can understand White's provocation to imply that Black must admit both T_1 and T_2 (unless Black abandons or modifies its system). So it is presumably in order to have Black choose one of them for rejection. It makes little sense to have an initial provocative thesis of this form, since you may as well put forward either conjunct. However, conjunctive provocations may very well occur later on in the discussion.Similar considerations apply to $(\forall x)Ax$.

If $T = T_1 \rightarrow T_2$, we may take White's provocation to imply that, if Black enlarges its system by a statement T_1, it will become unreasonable for it to maintain doubt about a provocative thesis T_2. In order to reject this, Black should, *for the sake of argument*, become the advocate of an enlarged system $SYST \cup \{T_1\}$ (Black needn't *believe* T_1) and then reject T_2.

The case $T = \sim T_1$ may be treated similarly, equating $\sim T_1$ with $T_1 \rightarrow \wedge$. Here, White's provocation consists of a claim that $SYST \cup \{T_1\}$ is absurd.

In all these cases, where the rejection involves more than a simple "how come?", it is obvious that White should answer immediately by a protective defence move according to Lorenzen's strips. We may even merge attack and defence move, and go from a situation

$$SYST/_{BLACK} A \& B \qquad \text{(Black is to be the speaker)}$$

immediately to:

$$SYST/_{BLACK} A \text{(Again, Black is to be the speaker).}$$

On the other hand, it is convenient to stick to the principle that the parties take turns; we may then stipulate:

Special Rule 1 A rejection (by Black) of a conjunction, a conditional, a negation or an universal statement should be answered immediately by White by means of a protective defence, constituting a fresh provocation.

We cannot have this rule in the case of atomic statements, for in that case I wouldn't know of any protective defence, at least not within the context of an indoor game.[2] We cannot have it for disjunctive or existential statements either. White's provocation $\overline{I}_1 \vee \overline{I}_2$ doesn't imply either a provocation $\overline{I}_1$ or a provocation $\overline{I}_2$; hence, in this case it seems appropriate to grant White a right to postpone its choice of a protective defence. Similarly for $(\exists x) Ax$.

4.4. [4.] *Information-seeking interpretation of the logical constants*

Once Black has rejected a provocative thesis of a form to which the special rule doesn't apply, i.e., a thesis which is either disjunctive or existential or atomic, White may start to cross-examine Black, on account of its system, with the following ends in view:

(i) to make Black the advocate of a system which contains a statement of the very sentence used in the provocative thesis, or otherwise:

(ii) to make Black the advocate of a system stated in sufficient detail for White to choose a protective defence.

For the forms of these questions (by White) and answers (by Black) we should again consult Lorenzen's "strips". However, this time we interpret these strips in a *strikingly different way* not as rejections and provocations, or even as attacks and defences of some sort or other, but as (information-seeking) questions and answers. White doesn't doubt the system or reject any statement of it *within* the game of immanent criticism. *White asks questions to get more information* about the system.

Black has, so we assumed, all the facts (as they are determined by the system). It therefore seems reasonable to require of Black that it answer all questions without delay:

Special Rule 2 All questions put to Black by White should be answered by Black in its next move.

Note that this implies that Black must, on demand, specify one of the alternatives contained in a statement $S_1 \vee S_2$. When $(\exists x)$ Ax is the object of a question, Black must point out an example Aa. In strip-form:

Black	White	Black
$S_1 \vee S_2$	which?	S_1
		S_2
$(\exists x)Ax$	example?	Aa

(all choices to be made by Black)

When $S_1 \rightarrow S_2$ is a statement in Black's system, and Black is questioned on account of this statement, we may imagine White remarking:

> You say that in your system S_2 would be the case if S_1 were the case. Well, I would say that, from your point of view, you cannot reasonably doubt that S_1 is the case - So I'm willing to put forward a provocative thesis S_1, instead of the one we are presently discussing. On the other hand, if you don't reject S_1, you must incorporate a statement S_2 in your system (for that's the content of your remark $S_1 \rightarrow S_2$). I'm also willing to continue our discussion about the present thesis, if you explicitly make a statement S_2, i.e., if you submit such a statement to questioning or other dialectical use.

Again Black ought to answer without delay. Schematically we have:

Black	White	Black	White
$S_1 \to S_2$	Shall I put S_1 or will you put S_2 ?	Please put S_1	S_1 (fresh provocation)
		S_2 (added to the system) (Black chooses)	(next question, or protective defence)

Compare:

White	Black	White
$T_1 \to T_2$	T_1 (for the sake of argument)	T_2 (fresh provocation)

For $\sim S_1$ we may, similarly, imagine White remarking:

> Your system, would, you say, be absurd if S_1 were the case. Shall I put forward a provocative thesis S_1 or do you admit the absurdity of your system (by stating $\wedge$)?

The last option may or may not constitute a loss for Black (Cp. Sections **1.3 and 2.2 of Paper 2)**. Information-seeking questions on account of atomic statements are not admissible, as there is no more detailed information to be had.

How long may this questioning go on? White should be allowed to ask all relevant questions, i.e., all questions which are relevant to the uttered statements of the system. So White may ask both L? and R? on account of each conjunctive statement made by Black. In the course of the questioning more and more statements are added to the system and White should be allowed to put forward questions on account of these new statements as well. If no universal statement appears on Black's side the number of questions will always be finite. There are several techniques available in order to secure that only a finite number of questions can be asked, even when universal statements appear. One may require of White that it announces, and keeps within, a fixed limit before the discussion begins, or one may use techniques which employ ordinal numbers as in [2]. Anyhow, the discussion is naturally segmented into *local discussions*, as in Section 6 of **Paper 1**, each with its own local provocative thesis. Such a new thesis can either be the result of a protective defence move or the "fresh provocation" which results from a question aimed at a conditional or a negation; it marks the end of the previous local discussion and the beginning of the next one. Throughout the discussion Black employs the critical interpretation of the logical constants as to White's statements and White employs the information-seeking interpretation as to Black's statements.

White *wins* a local discussion if and only if

(1) the local provocative thesis is stated by Black, or

(2) (optional)$\wedge$ is stated by Black, or

(3) White wins the local discussion L' which orginates in L, (i.e. which starts with the rejection of the new provocation which ended L).

Blacks wins if White does not win, i.e., in the last local discussion: if White exhausts its rights of putting questions (without reaching situation (1) or (2)) and if, moreover, there is no possibility of protective defence. White/Black wins the discussion (or the chain of arguments, [1]) if and only if it wins the first local discussion. The effects of winning and losing can be determined as in **Paper 1, Section 8.** In some cases it seems reasonable that Black, having lost, should either announce a change in the system or abandon it or admit the provocative thesis into the system, but we cannot oblige Black to do so, unless it has been defeated in every possible way.

NOTES

(*) I would like to thank Prof. E.M. Barth and J. Vrieze for their advice while preparing my two contributions to this volume.

(1) I use roman capitals as sentence-variables and corresponding italic capitals as variables for statements (utterances) of sentences.

(2) What is wanted is a structural protective defence. For, a general protective defence is not a statement and hence would not constitute a fresh provocation. Moreover, it would obviously be unfair to White to require an immediate reaction of the latter type. Cp. Paper 1, Section 7.

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PART 2

METATHEORY

5. [Chapter V] Winning Strategies and Dialogical Strategy Tableaux

This chapter will be devoted to the study of *winning strategies* (cf. Def. 21 in Section III.15) and to the dialectical reconstructions of the notion of 'logical validity' that they provide.

In the first section we shall concentrate on those features of *dialogue situations* (Def. 20a, Section III.15) that are essential for the *existence of winning strategies* and, therefore, relevant to the problem of *dialectical validity*. These are largely the same features that make dialogue situations *equal* (Def. 20b, Section III.15). They will be used to define *dialogue sequents*, each of which represents some class of equal dialogue situations. In doing so we shall, for theoretical purposes, simplify our sequents by dropping the rules FD D6 and FD D8 of what will henceforth be called the "official" systems.

The second section will take up the notions of *strategy* and *strategy tree*. Some terminology for trees will be introduced and one lemma about trees in general will be proved. The dialogical (P-strategy) tableaux in Section 3 provide us with a convenient method with which to find and describe winning strategies for the Proponent. We shall in most places restrict our exposition to the non-material systems. (We shall return to material dialectics in Section IX.2.) In Section 4 we shall show that FD D2 is sufficiently implemented by our rules: each discussion ends after a finite number of stages. In Section 5 we shall establish the equivalence of the systems of Λ-dialectics with the corresponding systems of NOT-dialectics.

From now on we shall concentrate upon winning strategies for (completed) *chains of arguments*. The rules FD T2 and FD D3 (Sections III.13 and 15) jointly guarantee that whoever "has" a winning strategy for *all* (completed) chains of arguments that may in principle issue from a certain conflict "has" a winning strategy for all discussions as well (assuming, in the case of a P-winning strategy, that there is time enough for the debaters to complete at least one chain of arguments. See FD W4a, Section III.8).

5.1. [V.1] Dialogue sequents

What features of a dialogue situation are relevant to a party's chances of winning or losing a chain of arguments within which this situation occurs? The following list is intended to give a complete answer to this question (for non-material systems). It suffices to establish:

(i) whose turn it is to move (i.e., who is to be the speaker at the next stage);

(ii) what sentence was used in the local thesis of the current local discussion (unless the next stage will inevitably open a new local discussion); in classical dialectics former local theses are relevant as well;

(iii) what sentences, if any, were stated by O thus far in this chain of arguments (or as initial concession), and which of them cannot, on account of FD D6, be attacked (or cannot be attacked in certain ways) during the rest of the current local discussion;
(iv) what protective defense rights, if any, are available to the parties;
(v) what sentence, if any, was stated by P in the preceding stage (or, if there is no preceding stage, what is the initial thesis);
(vi) what sentences are excluded from being stated by P, in virtue of FD D8.

Def. 1 We shall codify the relevant features by means of *dialogue sequents*:

$$S = \langle \Pi, \Delta, T, N, \Phi, \Gamma \rangle$$

where

(a) N is either P or O (the party whose turn it is to move);
(b) T is a sentence or a set of sentences (the sentence used in the current local thesis, or, in classical dialectics, the set that consists of all the sentences used thus far in local theses in the chain of arguments);
(c) Π and Φ are sets of sentences (Π is the set of sentences stated by O thus far in the chain of arguments or as initial concessions, and Φ is either empty or of the form {Z}, where Z was stated by P in the preceding stage or, if there is no preceding stage, where Z is the sentence stated as initial thesis);
(d) Δ and Γ are sets of sentences (Δ is the set of sentences that may be stated by O, at a later stage, by virtue of a structural protective defense right, and similarly for Γ and P).

This sextuple will usually be written more compactly as:

$\Pi; \Delta / T /_N \Phi; \Gamma$ (Cf. p. 52.)

This will be called our *official notation*.

Notice that Δ is either empty or of the form {U} or of the form {U, V}.[1] (See the last but one of the consequences listed immediately below FD D7, Section III.15.) In minimal and constructive dialectics Γ will also have one of these forms; in classical dialectics Γ may be a larger set. In the case of Δ and Γ (but not in the case of Π and Φ), we shall use brackets instead of braces, writing "[U, V]" instead of "{U, V}", etc., in conformity with the use of brackets introduced in Section III.7 (cf. also Section III.10).

Our dialogue sequents do not mirror all of the information mentioned in (i)–(vi) above. In Def. 1 we disregarded the attacks already made by P during the current local discussion, i.e., we simply have a set Π of sentences but not of indexed sentences (where the indices would indicate whether a sentence can still be attacked and, if so, how many times and in what ways). Furthermore, we omitted the set of sentences called for by item (vi). *In the case of classical dialectics*, we shall also disregard the distinction between used and unused rights

[1] Since {U} = {U, U}, the first form is really a special case of the latter.

on P's side; thus Γ is simply the set of sentences that P *is or has ever been* permitted to state by virtue of structural protective defense rights in the current chain of arguments. These omissions together amount to a considerable simplification in the structure of the dialogue sequents, but we must, of course, show that our study of strategy by means of the simplified sequents will be relevant to the "official" dialectic systems defined in the preceding chapter. Our simplifications are tantamount to the following changes in the rules of $formal_3$ dialectics:

(1) drop FD D6 (Section III.15)
(2) drop FD D8 (Section III.15)
(3) (in classical dialectics:) the Proponent shall retain *all* of its protective defense rights, both used and unused.

We call our systems as originally defined the *official systems*. The modified systems will be called *P-liberalized systems*, since all the present modifications increase the number of options for P and, therefore, seem to give P an advantage over O. However:

Lemma 1 P has a winning strategy for a dialogue situation *S* according to ("on the strength of") a P-liberalized system if, and only if, P has a winning strategy for *S* according to the corresponding official system. (Cf. Def. 21 in Section III.15.)

Proof[2] A winning strategy for P according to the official system will of course be effective in the P-liberalized system as well. As to the converse: it is, we think, quite obvious that the repetitions within one and the same local discussion, which were ruled out by FD D6, can never be of any use for P. O can always react to two attacks of the same kind on the same statement in exactly the same way; so all P may hope to achieve by these repeated attacks is that several distinct statements of the same sentences are made by O, and that can never be profitable for P. In other words, if P has a winning strategy on the strength of the P-liberalized system it has one against "stubborn" opposition (opposition always reacting in the same way) and from this winning strategy against stubborn opposition the repeated attacks, by P, may be omitted (together with O's stubborn reactions on these attacks, etc.), which gives us a winning strategy for P according to the P-liberalized system *with* FD D6 *added*. For constructive and minimal dialectics the arguments adduced in Section III.15 in favor of the acceptability, for P, of FD D8 suffice to show that if there is a winning strategy for P according to the P-liberalized system augmented by FD D6, then there is also a winning strategy for P according to the same system *with both* FD D6 *and* FD D8 *added*. Since, for the minimal and constructive dialectics, the latter system *is* the official system, this settles the matter in these cases. To the classical dialectics we may, similarly, add the *strengthened form of* FD D8 (Section IV.3). Observe that the repetitive use of a protective defense right would always violate the strengthened form of FD D8.

[2] Our proofs are $informal_2$. We are simply trying to show that something holds to those who *de facto* share some of our rules of thought (cf. Section III.12 and Section III.14 Exercise 1).

Hence the restriction of retained rights to unused rights is not a new restriction at all, once FD D8 (strengthened form) is included in the system. We conclude that if there is a winning strategy for P according to a P-liberalized classical system augmented by FD D6, then there is also a winning strategy for P according to the corresponding official system with the strengthened form of FD D8. The latter winning strategy holds good for the official system●

In this book we are mainly concerned with the existence or non-existence of winning strategies *for P*. For it is the concept of a winning strategy for P which figures in the dialectical reconstruction of 'validity' (see introduction to this part of the book). So, by virtue of Lemma 1, we may safely substitute the P-liberalized systems for the official systems in the study of strategy.[3] *The P-liberalized systems are not advocated as substitutes for the official systems in any other context.* You may have realized that in the P-liberalized systems the dynamic norms, FD D1, D2, D4, are insufficiently implemented. We therefore recommend the official systems for all practical purposes.

In our notation we shall make use of all the simplifications introduced in Paper 2, Section 1.3, pp. 52,53, as well as of similar ones.

Elements of Γ and Δ must alway be indicated by brackets. Hence

$$\Pi, U/T/_O V$$

is short for

$$\Pi \cup \{U\}; \emptyset/T/_O\{V\}; \emptyset$$
$$= \langle \Pi \cup \{U\}, \emptyset, T, O, \{V\}, \emptyset \rangle$$

whereas

$$\Pi; [U]/T/_O[V]$$

is short for

$$\Pi; \{U\}/T/_O\ \emptyset; \{V\}$$
$$= \langle \Pi, \{U\}, T, O, \emptyset, \{V\} \rangle$$

The use of a specific Greek letter: "Π", "Δ", "Φ", "Γ" will generally indicate the position in the sequent. "Π" is used for the concessions, "Δ" for protective defense rights on O's side, "Φ" for the set containing P's last sentence, "Γ" for protective defense rights on P's side. "/∅/" is shortened to "/" and "$/\emptyset/_N$" to "$/_N$", etc. (see type OI below).

In concrete examples we omit quotes around the sentences or formulas of the languages from which these examples are taken.

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3 If handled with care our sequents may also be used to depict strategies for O (see next section). It can be shown that there is a winning strategy for O in the official system if and only if there is a no-loss strategy for O in the P-liberalized system. (A no-loss strategy for N guarantees that N will not lose any chain of arguments, if N employs this strategy, though it may not guarantee that N will win every chain of arguments.)

Examples

official notation	*shorthand notation*
$\Pi; \emptyset/T/_O \{Z\}; \Gamma \cup \{U, V\}$	$\Pi/T/_O Z; \Gamma, [U, V]$
$\Pi \cup \Pi' \cup \{U\}; \emptyset/\emptyset/_P \emptyset; \Gamma' \cup \{W\}$	$\Pi, \Pi', U/_P \Gamma', [W]$
$\{$"$A \rightarrow B$"$\}; \emptyset/\emptyset/_O \{$"$B \rightarrow A$"$\}; \emptyset$	$A \rightarrow B/_O B \rightarrow A$
$\{$"$A \rightarrow B$", "B"$\}; \emptyset/$"$B \rightarrow A$"$/_P \emptyset; \{$"A"$\}$	$A \rightarrow B, B/B \rightarrow A/_P [A]$

Every non-material chain of arguments starts with a situation of type:

$$\Pi/_O Z \qquad \text{(type OI)}$$

where the chain is a chain for Π/Z in the sense of Section IV.1.3. Since the only thing O can do is to attack Z, the next situation must be

$$\Pi, a_iZ/Z/_P[P_{i1}Z, \ldots, P_{in}Z] \quad \text{(for i = 1 or i = 2)}$$

– see Section III.10. Diverging slightly from the conventions introduced in that section we shall now write:

$$[d_iZ] = [P_{i1}Z, \ldots, P_{in}Z] \quad \text{(omitting ca)}$$

where the $P_{ij}Z$ are all the possible *structural* protective defenses. The situation may then be rendered as:

$$\Pi, a_iZ/Z/_P[d_iZ].$$

If no statement needs to be made in an attack of the i-th kind on a statement of Z, then a_iZ shall denote the empty set.

In virtue of F_2D 1 we have the following possibilities for a_iZ and $[d_iZ]$ (omitting the index where it is not reaily needed):

Figure V.1

$Z = V \rightarrow W$	$aZ = V$	$[dZ] = \{W\}$	
$Z = \sim V$	$aZ = V$	$\begin{cases}[dZ] = \emptyset \\ [dZ] = \{\Lambda\}\end{cases}$	in NOT-dialectics in Λ-dialectics
$Z = V \vee W$	$aZ = \emptyset$	$[dZ] = \{V, W\}$	
$Z = V \,\&\, W$	$\begin{cases}a_1Z = \emptyset \\ a_2Z = \emptyset\end{cases}$	$[d_1Z] = \{V\}$ $[d_2Z] = \{W\}$	
Z is atomic	$aZ = \emptyset$	$[dZ] = \emptyset$	

In each non-material chain the *second* dialogue situation is of the following general type:

$$\Pi/T/_P \Gamma \qquad \text{(type P)}$$

where $\Gamma = \{U, V\}$ or $\Gamma = \{U\}$ or $\Gamma = \emptyset$.

What further types of situations can arise? In a situation of type P the Proponent can either

(i) defend the local thesis unconditionally; in that case the chain is won by P (this situation will not be represented by a sequent); or
(ii) defend by means of a structural protective defense move; *in non-classical dialectics* this brings us back to a situation of type OI (the thesis T may be omitted in the sequents depicting this type of situation, since the next stage will inevitably open a new local discussion); or
(iii) attack a statement made by O. Then either it is the case that in this attack P does not make a statement of its own but challenges O in an interrogative way, in which case P brings about a situation of type

$\Pi; \Delta/T/_O \Gamma$ (type OII, $\Delta \neq \emptyset$)

or it is the case that P does make a statement of a sentence, Z; if O gets no protective defense rights as a consequence of this move, then we are back to OI (in non-classical dialectics), otherwise we have:

$\Pi; [U]/T/_O Z; \Gamma$ (type OIII)

(Notice that in non-classical dialectics Γ is restricted to the same forms in types OII and OIII as in type P.)

These are all the types there are (in non-classical dialectics). In a situation of type OII, O can do nothing but defend protectively (using a sentence in Δ). The result will be a situation of type P. In a situation of type OIII, O may choose between a protective defense move and an attack on Z. If O takes the first course of action, Z disappears from the sequent (FD D7!), and if O attacks Z, then [U] disappears from the sequent (for in that case a new local discussion starts – FD O5b!). So, whatever O's choice may be, the next situation will be of type P (cf. Exercise 3 of Section III.15).

The same analysis may be applied to classical dialectics, provided we take as our first type

$\Pi/T/_O Z; \Gamma$ (type OI, classical)

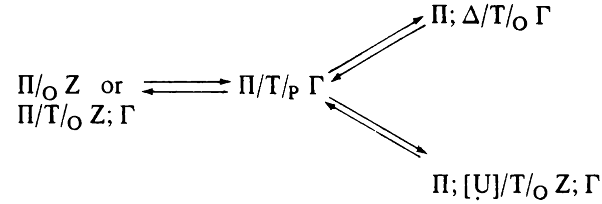
Thus we have established:

Lemma 2 In non-material chains of argument only situations of the types indicated below occur. They succeed each other in the way indicated by the arrows:

Figure V.2

A survey of the different types of dialogue situations followed by a schema showing the possible transitions between situations of these types (for non-material dialectics).

Type OI	is	$\Pi/_O Z$ or $\Pi/T/_O Z; \Gamma$	
Type P	is	$\Pi/T/_P \Gamma$	
Type OII	is	$\Pi; \Delta/T/_O \Gamma$	$(\Delta \neq \emptyset)$
Type OIII	is	$\Pi; [U]/T/_O Z; \Gamma$	



For the study of strategies in material dialectics we need different sequents. In the system MatDial, for instance, it is important to know what *unattacked* statements, (other than statements of the form U!!), each party has made. We are not interested in attacked statements (see rule (x) of Def. 16 in Section IV.5.2). Furthermore, the number of unattacked statements of one and the same sentence may be of importance. Similarly, we need to codify only the unused defense rights, but again the number of "similar" defense rights – i.e., defense rights involving the same sentences – is relevant. And, of course, it is of crucial importance which atomic sentences are "true", i.e., what $\mathbb{T}$ is.

Def. 2 A *material dialogue sequent* is a sextuple

$$S = \langle \vec{\Pi}, \vec{\Delta}, \mathbb{T}, N, \vec{\Phi}, \vec{\Gamma} \rangle$$

where

(i) $\mathbb{T}$ is a class of *atoms* – the "true" atoms;
(ii) N – this is the party whose turn it is to move – is either B or W (see Section IV.5.2);
(iii) $\vec{\Pi}$ and $\vec{\Phi}$ are *sequences* of sentences ($\vec{\Pi}$ contains exactly one sentence for each unattacked statement made by B, and similarly for $\vec{\Phi}$ and W);
(iv) $\vec{\Delta}$ and $\vec{\Gamma}$ are sequences of sets of sentences of the forms {U}, {U, V} and {U!!} ($\vec{\Delta}$ contains one set of sentences for each of B's rights to defend a statement, and similarly for $\vec{\Gamma}$ and W).

This sextuple will usually be written more compactly as:

$$\vec{\Pi}; \vec{\Delta}/\mathbb{T}/_N\, \vec{\Phi}; \vec{\Gamma}$$

This will be called our *official notation*.

Again we shall use a simplified notation:

official notation	*shorthand notation*
$\langle U, U \rightarrow V \rangle; \langle \{V\}, \{W, Z\} \rangle / \{W, U\} /_B\, \emptyset; \langle \{V\} \rangle$	$U, U \rightarrow V; [V], [W, Z]/W, U/_B\, [V]$
etc.	

Exercises

1. The following dialogue sequents are given in simplified notation: rewrite them according to the official notations of Definition 1 and Definition 2:
   a $\Pi, \Pi'/_O Z$
   b $\Pi; [U, V]/W/_O Z$
   c $\Pi, U \rightarrow V, U; [V]/T/_O U; [W], \Gamma$

d A → B, B → C, B/B → C/$_P$ [C]
e A → B; [B]/A/$_W$ B → A; [A, B], [C], [A!!]

2. Does Lemma 2 hold for the P-liberalized systems, for the official systems, or for both?
3. Show that in the non-material systems O's rights can never be exhausted in a chain of arguments.
4. Write down all the sequents that depict a situation that may (in CND) follow A → B, A v C, C/C/$_P$ [B, C].
5. Try to define a type of sequents for the material systems of Section IV.5.1.

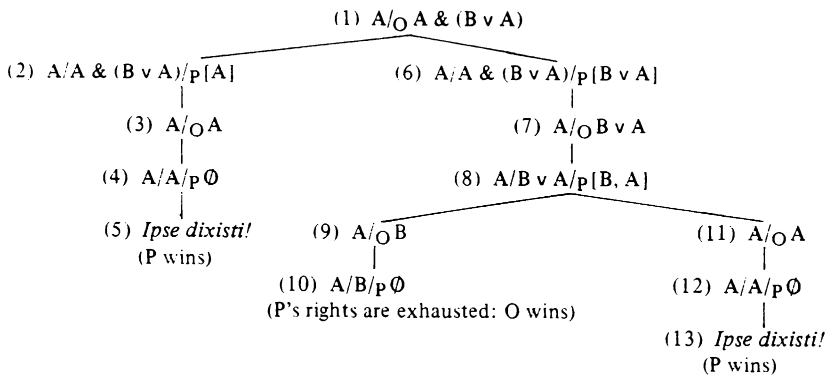
5.2. [V.2] Strategy diagrams

5.2.1. [V.2.1.] Tree diagrams

For any dialogue situation, and for any system of formal$_3$ dialectics, we may draw a diagram depicting all the possible chains of arguments that may issue from it (or, at least, initial segments of these chains of arguments).

Example 1

Let the dialectics be any of CAD, CND, MAD, MND. The following diagram depicts all the possible chains of arguments for A/A & (B v A):



Explanation: The initial sequent (1) is placed at the top.[1] Here O is to make a move and can choose between L? and R?. Sequents representing the *results* of these moves are shown on the next level ((2) and (6)); each of them is joined by a line to the sequent from which it originates. In both situations there is only one possible move for P; this gives us two sequents again on the next level ((3) and (7)), etc. Each level is either an O-level (with O-sequents, i.e., sequents subscripted "O") or a P-level (with P-sequents, i.e., sequents subscripted "P"). Each downward path indicated by lines depicts a possible chain of arguments. All chains of arguments are completed.

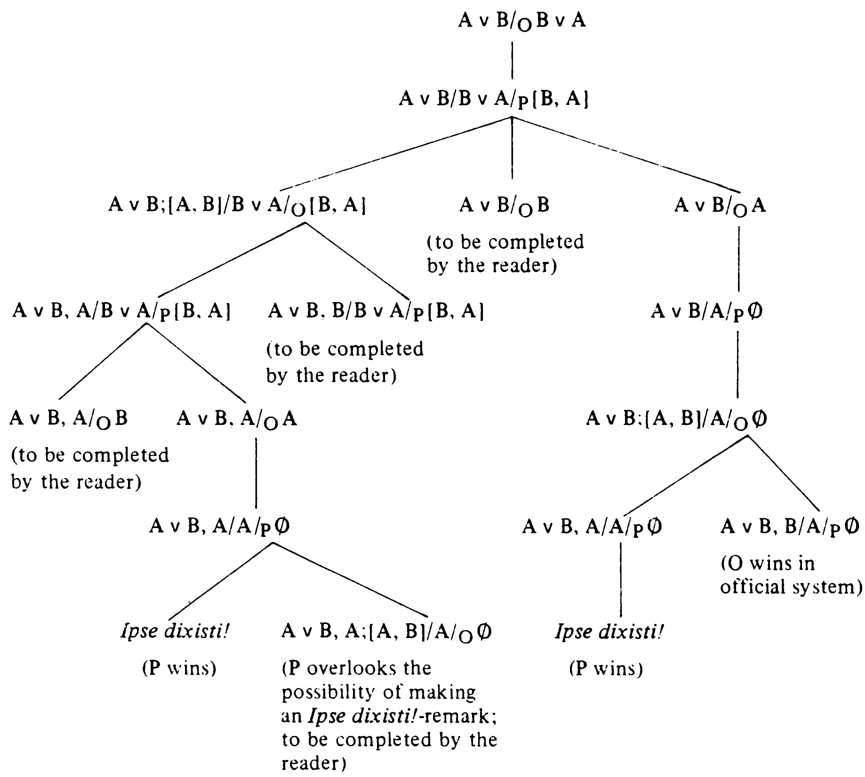
[1] Actually an *inscription of a name of* the initial sequent is placed at the top (cf. Section II.2). We shall avoid these cumbersome expressions (but cf. the beginning of Section 3).

In the diagram of Example 1 all the chains of arguments were completed. In general, however, we must envisage the possibility of infinite chains of arguments (though not in the official systems, as will be shown in Section 4). The following diagram shows only initial fragments of all possible chains of arguments issuing from a situation in P-liberalized dialectics.

Example 2

Let the dialectics be constructive or minimal. The following diagram of chains of arguments for A v B/B v A (to be completed by the reader) may be interpreted in two ways:

(1) if the dialectics is an official system, all possible chains of arguments are shown completely;
(2) if the dialectics is P-liberalized, only initial segments of all possible chains of arguments are shown, because possibilities for P that do not exist in the official systems are disregarded.



In the diagrams of Example 1 and Example 2 no sequent is joined to more than one sequent above it: they are *tree diagrams* in the sense of Def. 4 below. Though not all diagrams depicting possible chains of arguments need to be tree diagrams, one can always put them into that form. We now turn to the mathematical notion of a *tree*, which is of primary importance in the study of strategy and tableau methods.

Def. 3 ⟨R, A, r⟩ is a *tree* (or, R is a tree on A with *root* r) if and only if

(1) A is a set and R is a binary relation on A
(2) $r \in A$
(3) no $x \in A$ is such that xRr
(4) for every $x \in A$ such that $x \neq r$ there is exactly one element $y \in A$ such that yRx (y is called the R-*predecessor* of x)
(5) for every $x \in A$ there is a finite sequence $\langle x_1 \ldots x_n \rangle$ $(n \geqslant 1)$ such that
(a) $x_1 = r$
(b) $x_n = x$
(c) $x_i R x_{i+1}$, for all i such that $1 \leqslant i < n$.

The elements of A will be called *nodes* of the tree. A tree is called *finite* (*infinite*) if A is finite (infinite). If xRy, y is called an R-*successor* of x. A node without R-successors is called a *final* node. An infinite sequence $\langle x_1 \ldots \rangle$ (finite sequence $\langle x_1 \ldots x_n \rangle$) such that $x_{i-1} R x_i$ for all $i > 1$ (for all i such that $1 < i \leqslant n$) is called an infinite (finite) R-*path* from x_1 (to x_n). We say that x_1 *dominates* x_n, if there is an R-path from x_1 to x_n. Thus the root dominates all the nodes of the tree. An R-path from r to a final node is called a *finite branch* of the tree. An infinite R-path from r is called an *infinite branch* of the tree.

Def. 4 A *tree diagram* is a pair ⟨T, f⟩ such that T is a tree and f is a function defined for the nodes of T.

The function f associates a distinct entity with each node. The same entity may be associated with different nodes. In this book the associated entities will be sequents of different kinds. Examples 1 and 2 show how (finite) tree diagrams can be represented on paper. If the branches of a tree diagram depict (possibly uncompleted) chains of arguments for a sequent according to some system of dialectics, we say that it is a tree diagram *for* that sequent *in* that system.

We now turn to the notion of *strategy*. A strategy may be roughly described as a determinate way a party may make use of its rights in any situation that may arise in a discussion.[2] It will suffice to define *strategy diagrams*. By an N-*sequent* we mean a sequent $\Pi, \Delta / T /_N \Phi, \Gamma$ (or $\vec{\Pi}, \vec{\Delta} / \mathbb{T} /_N \vec{\Phi}, \vec{\Gamma}$).

Def. 5 A tree diagram in system σ is an N-*strategy diagram* in system σ if and only if

(i) each non-final node which has an associated N-sequent has exactly one successor;
(ii) each node which has an associated $\overline{N}$-sequent, S, has as many successors as there are sequents representing a situation that may result from a move by $\overline{N}$ in the situation represented by S, each of these sequents being associated with one of these successors;

[2] More accurately: a strategy for N is a function f which for each kind of dialogue situation (for each sequent) S in which N is to be the speaker of the next stage, and in which N's rights are not exhausted, determines a move for N, f(S) being the kind of situation (sequent) that results from this move.

(iii) all sequents associated with final nodes represent situations in which the speaker of the next stage has exhausted its rights or can make a winning remark (*Ipse dixisti!*, etc.). The winning remark is usually written underneath.

Example 3

Turn to the diagram of Example 1. If we omit (2), (3), (4) and (5) we obtain an O-strategy diagram. If we omit (9) and (10) (or (11), (12) and (13)) a P-strategy diagram is obtained.

Def. 6 An N-strategy diagram in system σ is an N-*winning strategy diagram* in σ if and only if

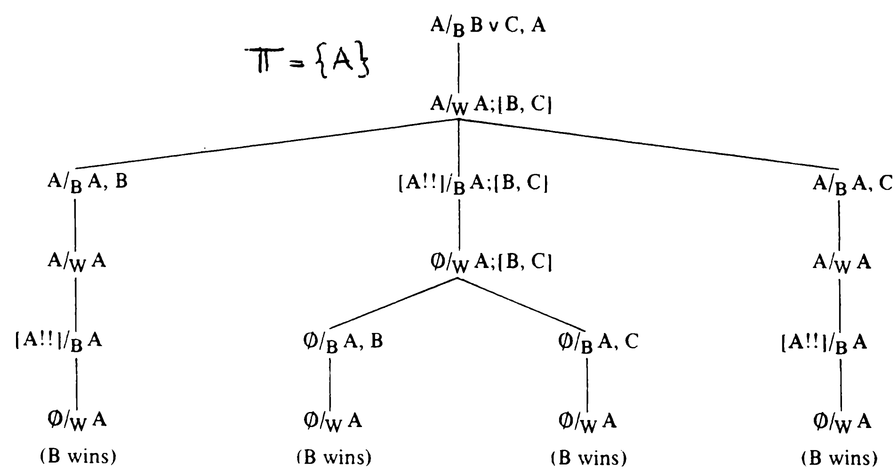
(i) all branches are finite;
(ii) all sequents associated with final nodes represent situations such that
  (a) if N = P: N is to be the speaker of the next stage and can make a winning remark.
  (b) if N ≠ P (i.e., N = O, or σ is MatDial and N = W or N = B): $\bar{N}$ is to be the speaker of the next stage and $\bar{N}$ has exhausted its rights.

Example 4

The P-strategy diagram obtained from the diagram in Example 1 by omitting (9) and (10) is a P-winning strategy diagram.

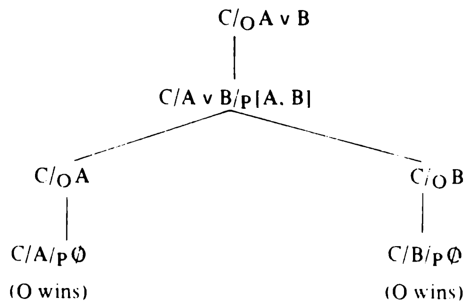
Example 5

The following tree diagram is a B-winning strategy diagram in material dialectics (since Π = {A} throughout the diagram, we may omit Π as a part of the sequents):



Example 6

An O-winning strategy diagram for C/A v B in MΛD, MND, CΛD, and CND:



The last diagram depicts an O-winning strategy even in the P-liberalized systems, but usually a diagram depicting an O-winning strategy in the official systems does not even constitute an O-strategy diagram in the P-liberalized systems ~~(cf. Exercise 5)~~.

5.2.2. [V.2.2] Rules for constructing P-winning strategy diagrams

We are primarily interested in P-winning strategy diagrams; therefore, we now explicitly list the rules for constructing such a diagram from the top down (in non-material P-liberalized systems). The rules are of three kinds: (1) rules that tell you exactly what sequents to place under O-sequents (representing *all* choices for O); these are here called *compulsory rules*; (2) rules that together determine what O-sequents you can place under a P-sequent, from which one is to be chosen in each case; these are here called *choice rules*; (3) rules that say what sequents may appear at final nodes: *closure rules*. The compulsory rules pertain to sequents representing situations of the types OI, OII, OIII, the choice rules and the closure rules to sequents representing situations of type P (see Figure V.2, Section 1). Rules are further distinguished according to the principal operator involved. The names of the rules reflect this classification.

We first list the rules for CΛD; afterwards we shall indicate how the rules for the other systems are obtained.

Compulsory rules in CΛD

OI→	under	Π/$_O$ U → V	you must write	Π, U/U → V/$_P$[V]
OI&	under	Π/$_O$ U & V	you must write both and	Π/U & V/$_P$[U] Π/U & V/$_P$[V]
OIv	under	Π/$_O$ U v V	you must write	Π/U v V/$_P$[U, V]
OI$^{\Lambda}_{\sim}$	under	Π/$_O$ ~U	you must write	Π, U/~U/$_P$[Λ]

OI_{At}	under	$\Pi/_O U$ (U atomic)	you must write	$\Pi/U/_P \emptyset$
OII^3	under	$\Pi; \Delta/T/_O \Gamma$ $(\Delta \neq \emptyset)$	you must write a sequent for every	$\Pi, U/T/_P \Gamma$ $U \in \Delta$
OIII→	under	$\Pi; [U]/T/_O V \rightarrow W; \Gamma$	you must write both and	$\Pi, U/T/_P \Gamma$ $\Pi, V/V \rightarrow W/_P [W]$
OIII&	under	$\Pi; [U]/T/_O V \& W; \Gamma$	you must write all of	$\Pi, U/T/_P \Gamma$ $\Pi/V \& W/_P [V]$ $\Pi/V \& W/_P [W]$
OIIIv	under	$\Pi; [U]/T/_O V \vee W; \Gamma$	you must write both and	$\Pi, U/T/_P \Gamma$ $\Pi/V \vee W/_P [V, W]$
$OIII^{\Lambda}_{\sim}$	under	$\Pi; [U]/T/_O \sim V; \Gamma$	you must write both and	$\Pi, U/T/_P \Gamma$ $\Pi, V/\sim V/_P [\Lambda]$
$OIII_{At}$	under	$\Pi; [U]/T/_O V; \Gamma$ (V atomic)	you must write both and	$\Pi, U/T/_P \Gamma$ $\Pi/V/_P \emptyset$

Choice rules in CAD

Pd	under	$\Pi/T/_P \Gamma$	you may write	$\Pi/_O Z$ for any $Z \in \Gamma$
P→	under	$\Pi, U \rightarrow V/T/_P \Gamma$	you may write	$\Pi, U \rightarrow V; [V]/T/_O U; \Gamma$
P&L	under	$\Pi, U \& V/T/_P \Gamma$	you may write	$\Pi, U \& V; [U]/T/_O \Gamma$
P&R	under	$\Pi, U \& V/T/_P \Gamma$	you may write	$\Pi, U \& V; [V]/T/_O \Gamma$
Pv	under	$\Pi, U \vee V/T/_P \Gamma$	you may write	$\Pi, U \vee V; [U, V]/T/_O \Gamma$
$P^{\Lambda}_{\sim}$	under	$\Pi, \sim U/T/_P \Gamma$	you may write	$\Pi, \sim U; [\Lambda]/T/_O U; \Gamma$

Closure rules in CAD

Pid At final nodes sequents of the form $\Pi, U/U/_P \Gamma$ may appear. (We usually write *Ipse dixisti*! underneath.)

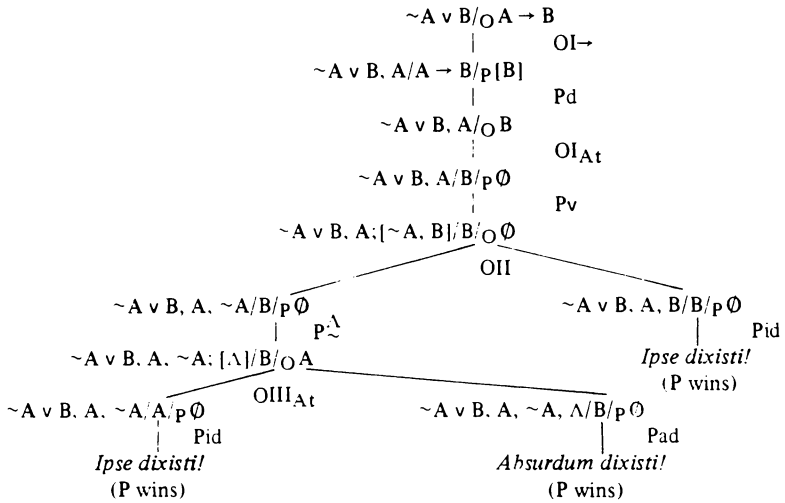
Pad At final nodes sequents of the form $\Pi, \Lambda/T/_P \Gamma$ may appear. (We usually write *Absurdum dixisti*! underneath; if T = Λ we may of course write *Ipse dixisti*! instead.)

---

[3] Where a "T" appears it is supposed that, for some i, $a_i \Gamma \in \Pi$ and $[d_i T] \subseteq \Gamma$.

Example 7

A P-winning strategy diagram in C∧D, marked with the names of the rules:



In order to obtain the rules for the other systems you must make the following changes:

CID(= MID): retain only OI→, OI$_{At}$, OIII→, OIII$_{At}$, Pd, P→, Pid.

M∧D: omit Pad.

MND: omit Pad; replace OI$\overset{\wedge}{\sim}$, OIII$\overset{\wedge}{\sim}$, P$\overset{\wedge}{\sim}$ by:

OI~	under $\Pi/_O$ ~U	you must write	Π, U/~U/$_P$ ∅
OIII~	under Π; [U]/T/$_O$ ~V; Γ	you must write both	Π, U/T/$_P$ Γ and Π, V/~V/$_P$ ∅
$\overset{min}{P\sim}$	under Π, ~U/~V/$_P$ Γ	you may write	Π, ~U/$_O$ U

(The ~V in $\overset{min}{P\sim}$ reflects restriction FD M-NOT, Section IV.2.3.)

CND: omit Pad; replace OI$\overset{\wedge}{\sim}$ and OIII$\overset{\wedge}{\sim}$ by OI~ and OIII~; replace P$\overset{\wedge}{\sim}$ by
P~ under Π, ~U/T/$_P$ Γ you may write Π, ~U/$_O$ U.

In order to obtain the rules for the classical systems KID, K∧D and KND from MID, C∧D and CND respectively, it suffices (i) to write "/T/$_O$" instead of "/$_O$", "/T, U → V/$_P$" instead of "/U → V/$_P$", etc., in the OI- and OIII-rules and in Pd, Pid and P~, and (ii) to add "Γ" on P's side of the sequent descriptions wherever it does not appear already. "T" stands for a class of local theses, and "Γ" for a set of used or unused structural protective defense rights. For instance:

OI$^K_{\rightarrow}$	under	Π/T/$_O$ U → V; Γ	you must write	Π, U/T, U → V/$_P$ [V], Γ
OI$^K_{At}$	under	Π/T/$_O$ U; Γ (U atomic)	you must write	Π/T, U/$_P$ Γ
Pd	under	Π/T/$_P$ Γ	you may write	Π/T/$_O$ Z; Γ for any Z ∈ Γ.

5.2.3. [V.2.3.] König's Lemma on trees

We conclude this section with a theorem on trees in general (known as the "Tree Theorem" or as "König's Lemma") which will be needed several times in the rest of this book.

Def. 7 A tree ⟨R, A, r⟩ is *finitely branching* if for any x ∈ A the set of R-successors of A is finite.

Lemma 3 (König's Lemma)

The following three properties of trees are incompatible:

(i) being finitely branching
(ii) having only finite branches
(iii) being infinite.

[- -]

5.3. [V.3.] Dialogical strategy tableaux

The tree diagrams used as examples in the preceding section were represented on paper in such a way that the reader could immediately grasp their structure. Yet the tree diagrams must not be confused with their representations on paper. There must be some set of conventions determining what counts as an inscription of a description of a tree diagram and how such inscriptions are to be interpreted. The conventions we actually used were, we trust, clear enough from the examples. For instance: there is to be a spot on paper for each node, labeled by an inscription of the name of the associated sequent; the root is represented at the top, etc. This is perhaps the clearest system for the description of tree diagrams, but in practice it leads to a lot of drudgery. As the reader will have noticed, one has to copy O's concessions and P's defense rights and the local thesis over and over again. The *dialogical tableaux* provide us with timesaving notational machinery for the description of P-winning strategy diagrams in non-material dialectics. The idea is: not to rewrite the elements that are retained, but to have conventions which tell you to look for the retained elements higher up in the figure drawn on paper. These conventions we shall now explain by means of an example in which we shall construct a dialogical tableau step by step. The notation used in the preceding section – henceforth to be called *tree form notation* – will be shown at the right. Our initial sequent will be A → B, B → C/A → C. ~~The winning strategy to be depicted is one possible solution of Exercise 7b of the preceding section.~~

First divide the paper into two columns. The left column will be O's column, the right one will be P's column. The columns are used in the same way as in the descriptions of chains of arguments. So we are going to have utterances and rights of O (P) represented on the left (right). The initial conflict is represented thus:

Figure V.3

MID

O	P
$A \to B$	$A \to C$
$B \to C$	

$A \to B, B \to C /_O A \to C$

The first move always consists of an attack by O on the thesis: we must apply the compulsory rule OI $\to$. We simply *add* inscriptions representing the new elements ensuing from this attack to the inscriptions already present:

Figure V.4

MID

	O	P
	$A \to B$	$A \to C$
	$B \to C$	
OI$\to$	A	[C]

$A \to B, B \to C /_O A \to C$

$A \to B, B \to C, A / A \to C /_P [C]$

Next apply a choice rule:

Figure V.5

MID

	O	P
	$A \to B$	
	$B \to C$	$A \to C$
OI$\to$	A	[C]
P$\to$	[B]	A

$A \to B, B \to C /_O A \to C$

$A \to B, B \to C, A / A \to C /_P [C]$

$A \to B, B \to C, A; [B] / A \to C /_O A; [C]$

We must now apply $OIII_{At}$. This gives us two new sequents. We therefore split both O's column and P's column into a first and a second column. The two first columns (marked "1") go together to describe the sequent $A \to B, B \to C, A/A/_P \emptyset$; the two second columns (marked "2") go together to describe $A \to B, B \to C, A, B/A \to C/_P [C]$.

Figure V.6

MID

	O (1)	O (2)	P (1)	P (2)
	$A \to B$			
	$B \to C$		$A \to C$	
OI$\to$	A		[C]	
P$\to$	[B]		A	
$OIII_{At}$	A?	B	[]	

$A \to B, B \to C /_O A \to C$

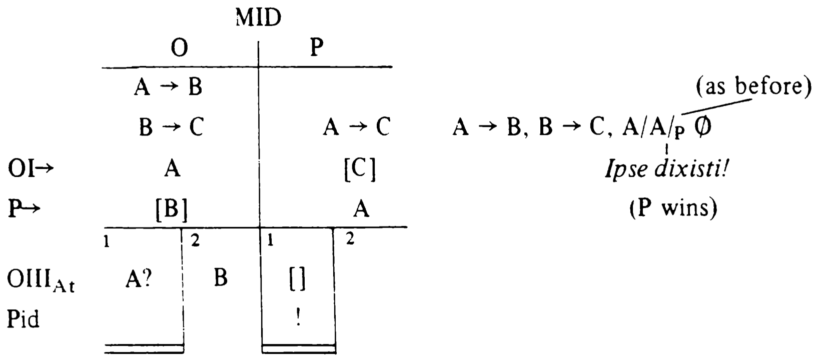
$A \to B, B \to C, A / A \to C /_P [C]$

$A \to B, B \to C, A; [B] / A \to C /_O A; [C]$

$A \to B, B \to C, A/A/_P \emptyset$ $\qquad$ $A \to B, B \to C, A\ B / A \to C /_P [C]$

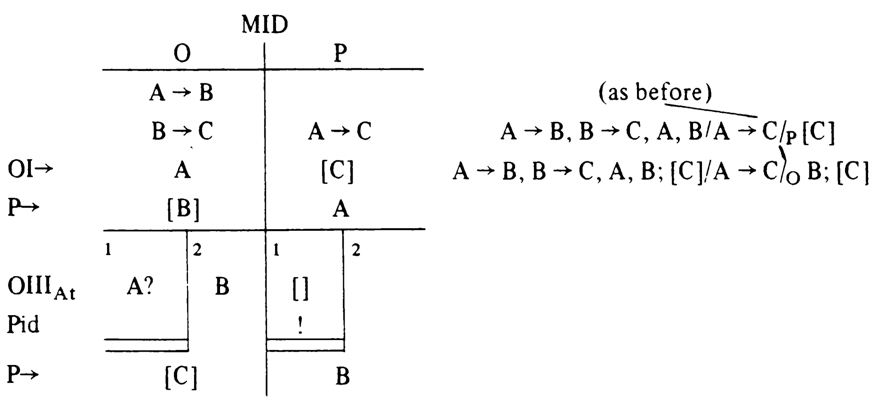
We have written the usual "[ ]" for the empty set of protective defense rights obtained by O's attack on the atom "A". This is important in non-classical dialectics, for it indicates that (in column 1) P's defense right is not given by "[C]". Each of the two pairs of columns, which are customarily called *subtableaux*, must from now on be developed *wholly independently* from the other. It is convenient to develop first some subtableau in which a closure rule may soon be applied. In fact we can immediately apply the closure rule Pid in subtableau 1. The winning remark is represented by "!", and the fact that the chain of arguments is completed and won by P is indicated by a double horizontal line. A (sub)tableau that has been completed in this way is said to be *closed*.

Figure V.7



Since subtableaux may themselves split into further subtableaux and so on, within a few steps the columns may get inconveniently narrow. In order to minimize this effect, we re-allot the vertical space set free by the closure of a subtableau to its immediate neighbor.[1] In our example all the vertical space is now allotted to subtableau 2. We apply a choice rule:

Figure V.8



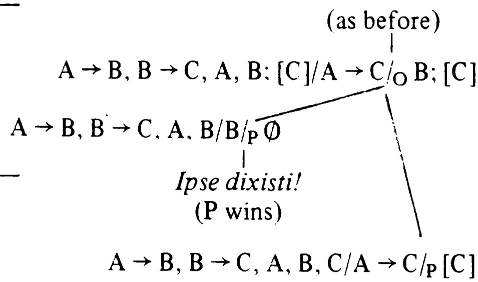
[1] This is a matter of convenience; if there is space enough it needn't be done.

At the next step we must again apply $OIII_{At}$. This splits subtableau 2 into subtableau 21 (pronounced two-one) and subtableau 22 (pronounced two-two). In subtableau 21 we can immediately obtain closure:

Figure V.9

MID

	O		P	
	$A \to B$			
	$B \to C$		$A \to C$	
$OI\to$	A		[C]	
$P\to$	[B]		A	
	1	2	1	2
$OIII_{At}$	A?	B	[]	
Pid			!	
$P\to$	[C]		B	
	21	22	21	22
$OIII_{At}$	B?	C	[]	
Pid			!	

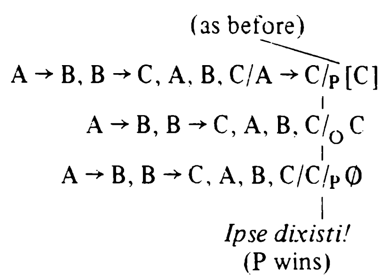


We finish the tableau by Pd, OI_{At}, and Pid:

Figure V.10

MID

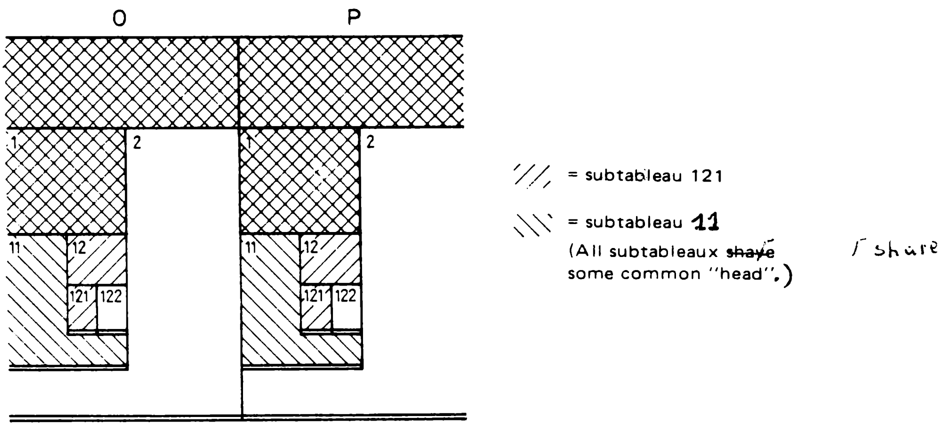
	O		P	
	$A \to B$			
	$B \to C$		$A \to$ ~~B~~ C	
$OI\to$	A		[C]	
$P\to$	[B]		A	
	1	2	1	2
$OIII_{At}$	A?	B	[]	
Pid			!	
$P\to$	[C]		B	
	21	22	21	22
$OIII_{At}$	B?	C	[]	
Pid			!	
Pd			C	
OI_{At}	C?		[]	
Pid			!	



The tableau method is sure to save a lot of time and work. The method is, moreover, completely equivalent to that of the previous section, i.e., a closed tableau (a tableau with only closed subtableaux) can always be rewritten in tree form, and *vice versa*. However, in some respects the tableaux are less clear than the diagrams in tree form, and error is more likely. So, please heed the following caveats:

CAVEAT 1 The structure of subtableaux on the left must exactly match the structure given on the right, e.g.:

Figure V.11



(All subtableaux share some common "head".)

CAVEAT 2 Never apply a rule to expressions in one subtableau, while putting some of the new expressions in another subtableau. This would be equivalent to an obviously nonsensical "jump" from one branch to another in a strategy diagram!

CAVEAT 3 A protective defense right on O's side can only be used if it is represented by the bottommost expression in O's column in the subtableau (and then it *must* be used in at least one subtableau).

CAVEAT 4 A sentence in P's column can be attacked only at the very next line in the subtableau (and there it *must* be attacked in at least one subtableau).

CAVEAT 5 In non-classical dialectics a structural protective defense right on P's side can only be used if it is represented by the bottommost bracketed expression in P's column (in the subtableau), and applications of Pid must pertain to the bottommost local thesis in P's column.

CAVEAT 6 A tableau that is not closed (i.e., one containing a subtableau that is not closed) does not represent a P-winning strategy diagram, but neither does it show that no such strategy diagram is obtainable.

Example 1

The winning strategy of Example 7 in Section 2 is to be represented in a tableau as follows:

CΛD

	O		P	
	~A v B		A → B	
OI→	A		[B]	
Pd			B	
OI_{At}	B?		[ ]	
Pv	[~A, B]		?	
	1	2	1	2
OII	~A	B		
		Pid		!
$P^{\Lambda}_{\sim}$	[Λ]			A
	11	12	11	12
$OIII_{At}$	A?	Λ	[ ]	
Pid		Pad	!	!

In a closed dialogical tableau each subtableau corresponds to a completed chain of arguments that may occur if P uses the strategy.

The construction rules of the preceding section can be rewritten in a way that suggests the tableau technique of representing P-winning strategy diagrams. By way of example we write out the rules for the construction of P-winning strategy diagrams in CID and KID in this way:

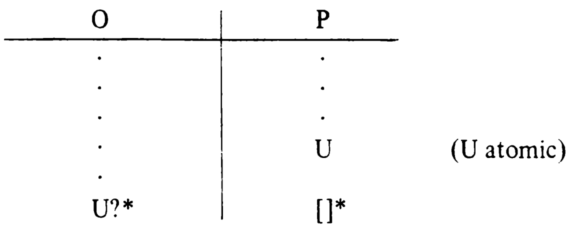
Figure V.12

Compulsory rules

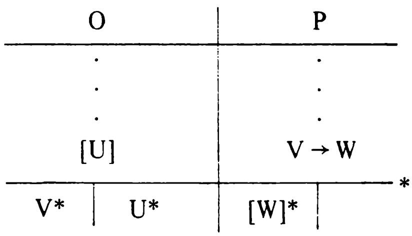
	O	P
	.	.
OI→	.	.
	.	.
		U → V
	.	
	U*	[V]*

(The asterisk indicates what is added to the tableau by virtue of the rule.)

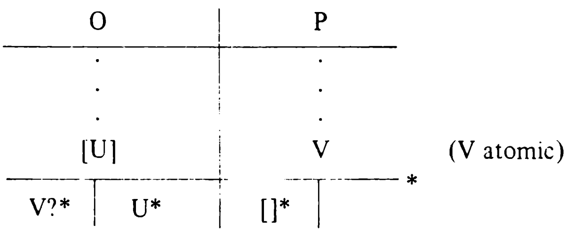
OI_{At}



OIII→

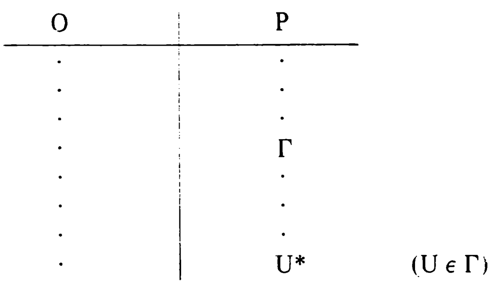


$OIII_{At}$



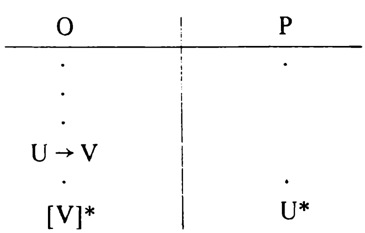
Choice rules

Pd



Condition: In non-classical dialectics U must be picked from a bracketed expression in P's column that has no other bracketed expression below it in the same column.

P→



Closure rules

	O	P
Pid	.	.
	.	.
	.	.
	U	.
	.	.
	.	!*
	.	.

Condition: In non-classical dialectics the *last* attack by O in the sub-tableau must be an attack on U; in classical dialectics it suffices that there be some attack on U by O in the sub-tableau.

Exercises

1. Rewrite the tree diagrams for the sequents in Exercises 7–11 of Section 2 as dialogical tableaux.
2. Construct closed dialogical tableaux for the following sequents (prefer a minimal to a constructive, and a constructive to a classical system):
   a (C → B) → A/B → A
   b Ø/~~~A → ~A
   c ~A & ~B/~(A v B)
   d ~(A v B)/~A & ~B
   e A v C, ~A v B, ~C v D/B v D
   f Ø/~~(A v ~A)

5.4. [V.4] Some simple properties of dialectic systems

The Tree Theorem (König's Lemma, Section 2) makes it possible to show in a simple way that our dynamic rules (Section III.15 and 16) successfully implement FD D2 and hence the fundamental norm of dynamic dialectics, FD D1. We first establish that FD D6 achieves its intended goal:

Lemma 4 Each local discussion in a chain of arguments according to an official system of (non-material) dialectics will end after a finite number of stages. (This will also hold if material procedures are attached as in Section IV.5.1.)

Proof In order to prove this we introduce a *measure* of *the complexity of dialogue situations.* The only features that matter are:

(i) the number and kind of concessions;
(ii) for each concession: whether it has been attacked in the local discussion; for conjunctions: whether they have been attacked by means of a question L? or by means of R?, or both;
(iii) which protective defense rights are available for O.

Determine the complexity of the situation by counting units in the following way:
For each occurrence of →, ~, v, in an unattacked (part of a)[1] concession, count *two units*.

For each occurrence of & in an unattacked (part of a) concession count *four units*. For a principal operator & in an unattacked concession count *four units*; if the concession has been attacked once, count *two units*. For O's unused simple ([U]) or complex ([U, V]) defense obligation count *one unit* (there is at most one such obligation); for the rest, let U and V contribute to the measure as if they were unattacked concessions.

At the start of a local discussion the dialogue situation has some finite complexity. One can verify that at each stage within the local discussion the complexity of the situation decreases by one or more units. For instance, if O is the speaker then O must use some protective defense right (for otherwise O would attack a statement of P's and start a new local discussion); if this is a simple right [U], the complexity goes down by one unit, and if the right is complex (of the form [U, V]), then the complexity may decrease even more. If P attacks, say, a negation ~V in a Λ-dialectic system, the two units contributed by the principal ~ are lost, as well as the contributions of all the operators in V. [Λ] contributes one unit, so the complexity goes down at least by one unit, etc.

If the local discussion does not come to an end earlier, we are bound to reach a situation of zero complexity sooner or later. Suppose it is O's turn to move in a situation of zero complexity. There cannot be a protective defense right for O, so O will attack (or switch to another chain of arguments). This ends the local discussion. Suppose next that it is P's turn to move in a situation of zero complexity. There cannot be any statement for P to attack, so P will offer a structural protective defense, unless P offers a general protective defense or has exhausted its rights. In the first case O is sure to start a new local discussion by an attack on the statement constituting P's defense. In the second case the chain of arguments is completed.

This completes our proof of the lemma●

As the reader may remember (if not, see Section IV.5.2), in debates based on MatDial there are no local discussions; and yet we can prove the following lemma, and in an entirely similar way at that:

Lemma 5 In MatDial each chain of arguments contains a finite number of stages.

The proof is left to the reader.

Thus we have shown that FD D6 – in MatDial, rule (x) – achieves its purpose. We must now turn to FD D7 and FD D8. Do these rules suffice to preclude chains of arguments consisting of an infinite number of local discussions? We

---

[1] We shall say, in this connection, that L? (R?) leaves the right (left) conjunct unattacked.

first observe that our systems of formal$_3$ dialectics all have the so-called *subformula property*:

Lemma 6 (*The subformula property*) In a discussion issuing from a conflict ⟨*Con*, *T*, B, A⟩ the only statements that can ever occur are

(i) statements of subsentences (subformulas) of the sentences (formulas) that are stated in $Con \cup \{T\}$;
(ii) statements of Λ;

Lemma 6 is a direct consequence of rule F_2D 1; the only rule (so far) according to which locutions (other than exclamations) are uttered in a discussion.

Since $Con \cup \{T\}$ is always finite, the number of sentences that may be stated in a discussion issuing from ⟨*Con*, *T*, B, A⟩ is finite. Hence at the sentence level there is only a finite number of possible local conflicts in such a discussion. Now FD D7 and FD D8 guarantee that within a chain of arguments no two local conflicts can occur that are the same at the sentence level. This gives us:

Lemma 7 In our official systems of dialectics – with or without material procedures attached to them – each chain of arguments contains a finite number of local discussions.

We have now reached a point where we can apply Lemma 3 in order to obtain the result that each discussion must terminate after a finite number of stages. We first define:

Def. 8 A system of formal dialectics is *locally finite* if and only if each conceivable discussion that proceeds according to the rules of the system contains a finite number of stages.[2]

Theorem 2 The system MatDial and each of our official systems of dialectics, with or without material procedures attached, is locally finite.

Proof We already know that each chain of arguments contains a finite number of stages: for MatDial this is the content of Lemma 5, and for the other systems it follows from Lemma 4 and Lemma 7.

The number of possible moves in each dialogue situation is also finite (if it is O's turn to move, there are at most three possibilities; if it is P's turn, the number of possibilities may be considerably greater, but will still be finite). It follows that a tree diagram depicting all the possible chains of arguments that may issue from a given conflict has the following two properties:

(i) it will be finitely branching
(ii) it will have only finitely long branches.

By Lemma 3 (Section 2) we have ascertained that the diagram will then contain only a finite number of nodes.

Each discussion consists of consecutive segments of chains of arguments depicted by the branches of the diagram (Def. 11, Section III.6, Section III.13). FD D3 guarantees that no segment of a branch will appear

---

2 This concept applies to all games, not just to dialectic systems. The French term is *localement fini*, the German term is *partienendlich*. See Berge [TGJ], p. 24.

twice in the discussion. Hence the discussion can contain no more than a finite number of stages●

Another fundamental property of our systems is the following:

Lemma 8 Any winning strategy diagram has a finite number of nodes.

Proof This may be seen to follow from Lemma 3. Alternatively, you may realize that a winning strategy diagram is part of a (total) diagram depicting all chains of arguments that can possibly issue from a given conflict, and that it – the part – must therefore be finite, since the total diagram is finite●

[- -]

Consider the following restriction on the moves permissible to P:

Rule R_{At} An *Ipse dixisti!*-remark may be made only if the local thesis is atomic.

Theorem 3 P has a winning strategy for a possible dialogue situation S on the strength of a P-liberalized system σ if, and only if, P has a winning strategy for S on the strength of σ with the rule R_{At} added to it.

Proof The "if" part is trivial: a winning strategy respecting the restriction is also a winning strategy if the restriction is removed from the system.

Suppose that a tree diagram τ is a P-winning strategy diagram for a certain dialogue sequent S_0. The final nodes of τ must be such that some closure rule, Pid or Pad, applies. Consider one such final node with associated sequent S. If Pad applies there is no conflict with R_{At}. Otherwise Pid applies and

(3) $S = \Pi, U, a_iU/U/_P[d_iU]$,[3]

or, in classical dialectics,

$(3)^K$ $S = \Pi, U, a_iU/T, U/_P[d_iU], \Gamma$.

From now on we shall assume (3) and leave it to the reader to adapt our proof to the classical case. It suffices to show that there is a P-winning strategy for S that respects R_{At}. For, if we show this in general, τ can clearly be extended by means of such diagrams in order to obtain a P-winning strategy diagram for S_0 that respects R_{At}.

We now use *structural induction*. We want to establish that:

(4) for each natural number n: if all sentences V with fewer than n occurrences of logical operators are such that there are P-winning diagrams that respect R_{At} for all dialogical situations

$S^i(\Pi, V) =_{Df} \Pi, V, a_iV/V/_P[d_iV]$

– i.e., for each class Π and for each mode of attack on V[4] – then the same holds for each sentence W with exactly n logical operators.

So suppose that W has exactly n logical operators, and that there is, for each Π' and j, a P-winning strategy diagram for $S^j(\Pi', V)$ respecting R_{At}, whenever V has fewer than n logical operators.

Case a: W is atomic. We cannot use the supposition about the V's, but a one-node diagram for $S^i(\Pi, W)$ is already a P-winning strategy diagram that respects R_{At}.

Case b: W is complex. We shall show that P can reduce the situation depicted by $S^i(\Pi, W)$ to situations depicted by sequents $S^j(\Pi', V)$ where V is a proper subsentence of W. Essentially, P *will have to copy* O's *moves*.

We show how to begin a dialogical tableau for $S^i(\Pi, W)$ in Figure 13, an explanation of which follows below.[5]

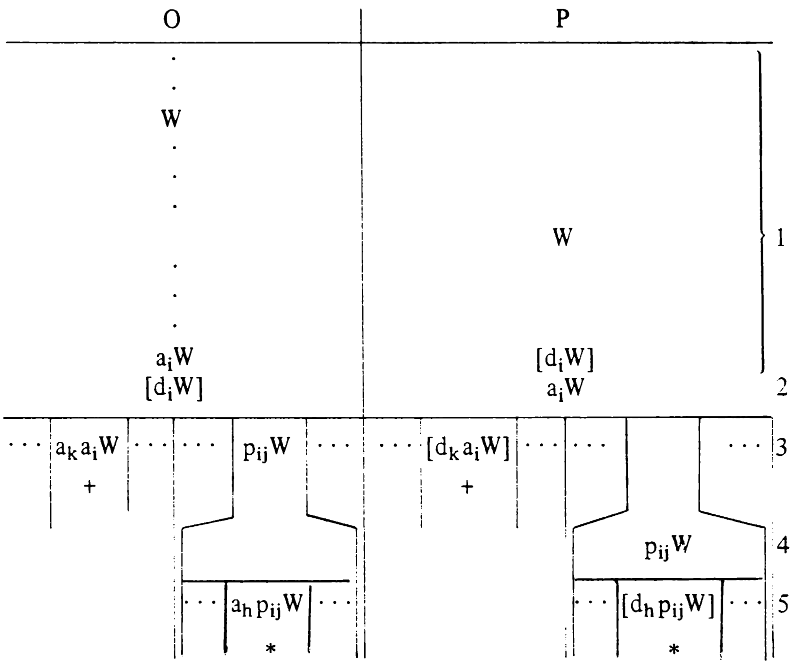
---

[3] We assume that we only deal with *possible* dialogue situations, i.e., situations that can occur in discussions. Since S_0 represents a possible situation, S too represents a possible situation. Therefore, if the local thesis is U, a_iU and $[d_iU]$ must occur (for some i) in the way indicated.

[4] If V is not a conjunction there is just one mode of attack and the index i may be omitted. If V is a conjunction we have $S^1(\Pi, V)$ and $S^2(\Pi, V)$.

[5] If conjunction is not present (in MID) the indices i, k and h can be omitted. If veljunction is not present (again in MID) the index j can be omitted. Try first to read the tableau for these simple cases.

Figure V.13



Comment:

1. $S^i(\Pi, W)$.
2. P, too, *attacks* W in the i-th manner.
3. O may either *attack* or *defend*. There is one subtableau for each mode of attack on a_iW and one subtableau for each structural protective defense in $[d_iW]$.
4. P chooses a similar structural protective *defense*.
5. O *attacks*: one subtableau for each mode of attack on $p_{ij}W$.

Subtableau + gives us a sequent

$S_+^k = \Pi, W, a_iW, a_k a_i W / a_i W /_P [d_k a_i W]$.

Subtableau * gives us a sequent

$S_*^{jh} = \Pi, W, a_iW, p_{ij}W, a_h p_{ij} W / p_{ij} W /_P [d_h p_{ij} W]$.

This looks worse than it is. Subtableaux of kind + (with $a_k a_i W$) appear only if $a_iW \neq \emptyset$ (i.e., if W is either a conditional or a negation). There is one such subtableau for each way in which a_iW can be attacked. S_+^k is of the form $S^k(\Pi', a_iW)$. Since a_iW has fewer than n logical operators we know, by our supposition, that there is a P-winning strategy diagram for S_+^k that satisfies R_{At}. So we are able to complete the subtableaux of kind + successfully.

Similarly, subtableaux in which O defends are only present if $[d_iW] \neq \emptyset$. In that case P can use exactly the same protective defense that was used by O, thus forcing O to attack $p_{ij}W$. For each mode of attack on $p_{ij}W$ we then get a subtableau (for the h-th mode, the h-th subtableau) with another sequent S_*^{jh}. But S_*^{jh} is of the form $S^h(\Pi', p_{ij}W)$. Again $p_{ij}W$ has fewer logical operators than W has, so we can successfully complete all these subtableaux as well. This completes the proof●

Exercises

1. Define a measure of complexity for situations in material chains of arguments and prove Lemma 5.
2. In the proof of Theorem 3, Case b is treated abstractly, covering at once all the logical forms that W may have. Another way to go about this is to distinguish cases according to the principal operator of W and to indicate how in each case a P-winning strategy diagram that respects R_{At} may be obtained. Elaborate some of these cases (in CND).
3. Adapt the proof of Theorem 3 to the classical systems.
4. Where does the proof of Theorem 3 go wrong if we consider the official systems instead of the P-liberalized systems? Does the theorem hold for the official systems?
5. Consider the following change in the non-material P-liberalized systems:

   1. Add R_{At};
   2. Let each situation of the form: Π, U/T/p[U], Γ be won by P.

   Show that there is a P-winning strategy in a changed system if and only if there is one in the corresponding original system.

5.5. [V.5] Equivalence of NOT-dialectics with $\wedge$-dialectics

In this section we shall establish the equivalence of

(i) M$\wedge$D with MND
(ii) C$\wedge$D with CND
(iii) K$\wedge$D with KND.

Without such equivalences there would be no point in calling different dialectic systems by the same name: "minimal", or "constructive", or "classical".

By *system σ is equivalent to system σ'* we mean the following: Let Π/Z be any sequent such that all sentences in Π and the sentence Z belong to a fragment of language to which both σ and σ' pertain; then P has a winning strategy for $\Pi/_O Z$ in σ if and only if P has one in σ'.

In each case we compare a NOT-dialectics and a $\wedge$-dialectics which pertain to languages $\mathfrak{L}_D$ and $\mathfrak{L}_D^{\wedge}$ (not specified here) of the forms $\mathfrak{F}_D$ and $\mathfrak{F}_D^{\wedge}$ respectively, where $\mathfrak{L}_D^{\wedge}$ is just the language obtained from $\mathfrak{L}_D$ by addition of $\wedge$. We are, of course, concerned only with sequents Π/Z in which $\wedge$ does not occur, i.e., such that $\wedge$ does not occur as a subsentence of Z or as a subsentence of a sentence in Π, and hence not as Z or as a sentence in Π.

The proof of the equivalences is, with one exception, fairly easy, especially now that we can visualize winning strategies in the form of dialogical strategy tableaux. *The exception concerns the transformation of a closed* M$\wedge$D-*tableau into a closed* MND-*tableau*. We shall, therefore, first present a proof of the equivalences in which one link is still missing. This proof will establish the equivalence of C$\wedge$D and CND and of K$\wedge$D and KND, and also shows how a closed MND-tableau can be transformed into a closed M$\wedge$D-tableau.

The missing link will be provided as Lemma 11. All other lemmas and definitions in this section will work up to it. You may want to defer study of the latter part of the section until you have seen some more proofs (e.g., those in Chapter VII).

Theorem 4 Let Π/Z be a sequent such that Λ does not occur in Z nor in any sentence of Π.
There is a closed dialogical tableau for Π/Z on the strength of a certain system of NOT-dialectics if and only if there is a closed dialogical tableau for Π/Z in the corresponding Λ-dialectics.[1] This holds whether the system is minimal, constructive, or classical.

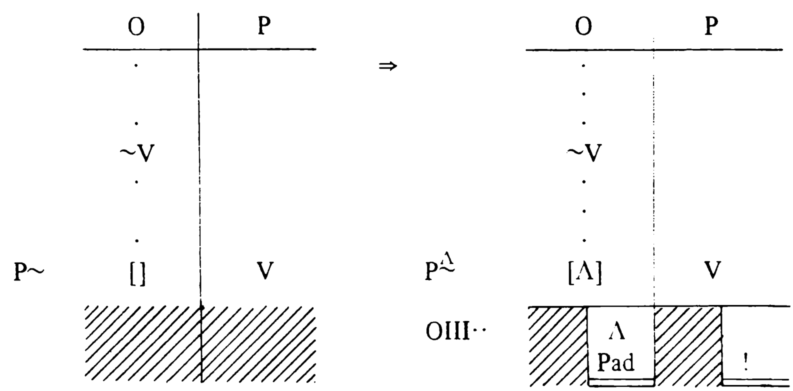
Proof (i) NOT-dialectics ⇒ Λ-dialectics.
Let a closed dialogical tableau for Π/Z in MND, CND or KND be given. Replace each application of OI~ by one of OI$\overset{\Lambda}{\sim}$:

Figure V.14

	O	P			O	P
		~U	⇒			~U
OI~	U	[ ]		OI$\overset{\Lambda}{\sim}$	U	[Λ]

In the same way, replace each application of OIII~ by one of OIII$\overset{\Lambda}{\sim}$.
If the negation dialectics is CND or KND, replace each application of P~ in the following way:

Figure V.15



Here ▨ indicates the subtableaux that contain attacks on V. (Compare Figure IV.3 in Section IV.1.3.)

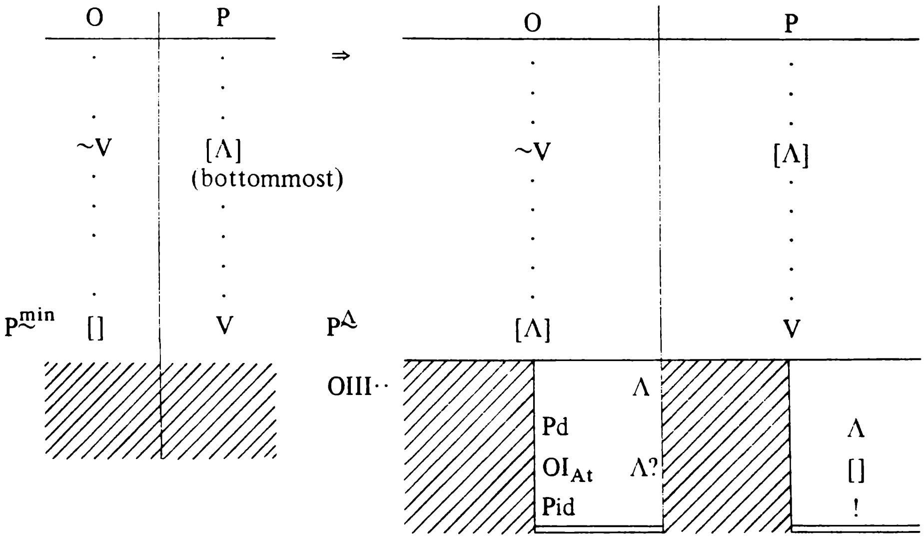
If the NOT-dialectics is MND, we don't have Pad in the corresponding Λ-dialectics (MAD). However, in this case we have to replace

---

[1] It does not matter if we say "there is a closed dialogical tableau for Π/Z" or "there is a P-winning strategy diagram for Π/Z" or "there is a winning strategy, for P, for Π/$_O$ Z". All these expressions are equivalent.

applications of $P^{min}_{\sim}$, not of P~. $P^{min}_{\sim}$ can be applied only if the local thesis is a negation ~U. The last attack, by O, in the subtableau must then have been an attack on ~U and the bottommost defense right on P's side of the subtableau must be the empty [ ]. This has already been changed into [Λ]; so we can carry out the following replacement:

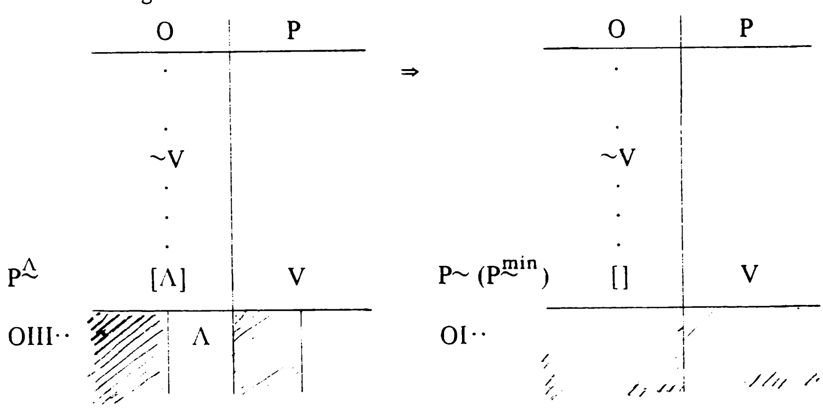
Figure V.16



By these changes a closed dialogical tableau in NOT-dialectics is transformed into a closed dialogical tableau in the corresponding Λ-dialectics.

(ii) Λ-dialectics ⇒ NOT-dialectics.
We first replace each application of $P^{\Lambda}_{\sim}$ by one of P~ (or $P^{min}_{\sim}$); we omit the subtableau in which O defends by Λ:

Figure V.17



If we are going from MΛD to MND we must replace applications of $P^{\Lambda}_{\sim}$ by applications of $P^{min}_{\sim}$; i.e., at each replacement the local thesis must be a negation. This will be the case if, and only if, the strategy in MΛD, as depicted by the given tableau, respects FD M-NOT (formulated in Section IV.2.3). Below we shall give a separate proof establishing that in MΛD each closed dialogical tableau *for a sequent in which no* Λ *occurs* can be transformed into one that respects FD M-NOT and that hence uses, instead of $P^{\Lambda}_{\sim}$, the following rule:

$P^{\Lambda min}_{\sim}$: under Π, ~U/~V/$_P$ Γ you may write Π, ~U; [Λ]/~V/$_P$ U; Γ.

⌈We shall now ~~Below we shall~~ simply assume that this transformation has been effected.

When all applications of $P^{\Lambda}_{\sim}$ have been replaced by applications of P~ (or of $P^{min}_{\sim}$), the resulting tableau will not contain any Λ or [Λ] on O's side. For Λ did not occur in the initial sequent and the system has the "subformula property" (Lemma 6), whence it follows that application of the rule $P^{\Lambda}_{\sim}$ is the only way to make Λ appear on O's side of the tableau.

We now replace each application of $OI^{\Lambda}_{\sim}$ by an application of OI~ (and similarly $OIII^{\Lambda}_{\sim}$ by OIII~):

Figure V.18

	O	P			O	P
		~U	⇒			~U
$OI^{\Lambda}_{\sim}$	U	[Λ]		OI~	U	[]

The result will, in general, not yet be a correct tableau in NOT-dialectics. However, there is only one possible defect: there may be alleged applications of Pd involving [Λ], i.e., the right to state Λ, where this right is no longer present. Such applications are always, of necessity, followed by OI_{At}, thus:

Figure V.19

	O	P	
	.		
	.	[]	
	.	.	
	.	.	
"Pd"	.	Λ	(incorrect)
$\bar{O}I_{At}$	Λ?	[]	

We claim that a correct tableau can be obtained by canceling these lines, wherever they occur. The only defect that could possibly ensue would consist of *Ipse dixisti!*-remarks by P based on the omitted OI_{At}, which made Λ the local thesis. But, since Λ does not appear on O's side of the tableau, there are no such remarks●

We now turn to the missing link in the proof: that each closed MΛD-tableau *for a sequent* S *in which* Λ *does not occur* can be transformed into a closed MΛD-tableau for S that employs $P^{\Lambda min}_{\sim}$ instead of $P^{\Lambda}_{\sim}$. Those applications of $P^{\Lambda}_{\sim}$ in which the local thesis is a negation are, equally, applications of $P^{\Lambda min}_{\sim}$. All the other applications of $P^{\Lambda}_{\sim}$ are not applications of $P^{\Lambda min}_{\sim}$.

Def. 9 The applications of P that are *not* applications of $P^{\Lambda min}_{\sim}$ we shall call *illicit applications* of $P^{\Lambda}_{\sim}$.
A tableau without illicit applications of $P^{\Lambda}_{\sim}$ will be called a *well-arranged* tableau.

Our goal is now to show the following:

> Each closed MΛD-tableau for a sequent Π/Z with no occurrence of Λ (i.e., each P-winning strategy diagram for such a Λ-free sequent) can be transformed into a *well-arranged* closed MΛD-tableau for the same sequent.

We shall prove this by establishing something more general, viz., Lemma 11 below, of which it is a special case. We generalize the problem in two respects:

(i) we consider dialogue sequents of all types, not only those of type OI,
(ii) we consider all sequents in which Λ does not occur *essentially* (see Def. 11 below), not only those in which Λ does not occur at all.

In preparation of the proof we now offer some preliminary definitions and lemmas.

Def. 10 A closed dialogical tableau will be said to have the *Λ-property* if each application of Pd involving the use of a protective defense right [Λ] is *immediately* followed by OI_{At} and *Ipse dixisti!*.

Lemma 9 Each closed dialogical tableau for a dialogue sequent S can be transformed into a closed dialogical tableau (for S) having the Λ-property. If the original tableau is well-arranged, so is the transformed tableau.

Proof First we remove all the applications of Pd involving [Λ] and the (compulsory) applications of OI_{At} that follow them. This can only invalidate some *Ipse dixisti!*-remarks by P that were based on the omitted applications of OI_{At}, for these made Λ into a local thesis. But since the Pd that used [Λ] is omitted, there is, on these occasions, still a bottommost defense right [Λ] in P's column. Hence we can re-insert an application of Pd using this [Λ] and an application of OI_{At} just above the *Ipse dixisti!*-remark. Clearly no applications of $P^{\Lambda}_{\sim}$ are made illicit by the procedure●

We shall prove that it is possible to eliminate the illicit applications of $P^{\Lambda}_{\sim}$ not only from tableaux for sequents in which Λ does not occur, but from all tableaux for sequents in which Λ does not occur *essentially*, in the following sense:

Def. 11 Λ occurs *essentially* in the sequent S if and only if one of the following cases applies:

(i) Λ occurs as a *proper* subsentence in a sentence contained in S;
(ii) S is of the form $\Pi; [U]/T/_O\, \Lambda; \Gamma$;
(iii) there is a defense right $[\Lambda]$ on P's side in S, but the local thesis, in S, is not a negation;
(iv) S is of the form $\Pi/_O\, \Lambda$, and $\Lambda \notin \Pi$;
(v) Λ is the local thesis in S, but Λ is not among O's concessions in S.

Inessential occurrrences of Λ in S include all occurrences of Λ as one of O's concessions, or in a defense right $[\Lambda]$ on O's side.

Lemma 10 *a* If Λ does not occur essentially in S, and if S′ is the result of an application of any rule (of MΛD), other than Pd, on S then Λ does not occur essentially in S′.

b If an MΛD-tableau has the Λ-property and Λ does not occur essentially in the initial sequent, then Λ does not occur essentially in any sequent in the tableau.

Proof *a* Suppose case (i) applies to S′; then case (i) also applies to S, since the rules have the "subformula property" (cf. Lemma 6 in Section 4).
Suppose case (ii) applies to S′: then the rule must be P$\overset{\Lambda}{\sim}$ or P$\rightarrow$. The concession attacked by P must be $\sim\Lambda$ or $\Lambda \rightarrow W$ (for some W); so case (i) applies to S.
Suppose case (iii) applies to S′. If S and S′ share both the local thesis and the defense right $[\Lambda]$ for P, case (iii) applies to S as well. This excludes many rules. Only the OI-rules, the corresponding parts of the OIII-rules, and Pd remain to be considered. Of these OI_{At}, the corresponding part of $OIII_{At}$, and Pd do not give a defense right to P. OI$\overset{\Lambda}{\sim}$, and the corresponding part of OIII$\overset{\Lambda}{\sim}$ do not give us an S′ to which case (iii) applies (the local thesis will be a negation). If any of the other rules was applied, case (i) applies to S.
Suppose case (iv) applies to S′. The rule must have been Pd. This is the exception we allowed.
Suppose case (v) applies to S′. Since no rule cancels concessions, Λ is not among O's concessions in S. If Λ is the local thesis in S, case (v) applies to S. Otherwise the rule must have been OI_{At} (or the corresponding part of $OIII_{At}$) and case (iv) (or case (ii)) applies to S.

b We saw that an essential occurrence of Λ can only be introduced by an application of Pd involving $[\Lambda]$, but if the tableau has the Λ-property, Λ will always be among O's concessions when Pd is thus applied ●

Lemma 11 A P-winning strategy diagram in MΛD for a sequent S in which Λ does not occur essentially can be transformed into a well-arranged P-winning strategy diagram in MΛD, for the same sequent.

Proof It is sufficient, in virtue of Lemma 9, to show this for diagrams having the Λ-property. We use induction on strategy trees, considering only those with the Λ-property.

So suppose that the lemma holds for all P-winning strategy diagrams in MΛD having the Λ-property and containing fewer than n nodes. Let τ be a P-winning strategy diagram for a dialogue sequent S (in which Λ does not occur essentially) containing exactly n nodes. Let τ have the Λ-property. We must show how to transform τ into a well-arranged P-winning strategy diagram for S.

If τ contains just one node, $P^{\Lambda}_{\sim}$ cannot have been used in τ, so τ need not be transformed. If τ contains more than one node, the root must have one or more successors with associated sequents $S^{(i)}$. By Lemma 10(b) none of the $S^{(i)}$ contains an essential occurrence of Λ. For each $S^{(i)}$ there is contained in τ a P-winning strategy diagram τ_i. Each τ_i may, on our supposition, be replaced by a well-arranged diagram τ_i^* for $S^{(i)}$. By Lemma 9, we may moreover assume that the τ_i^* have the Λ-property. By these replacements τ is transformed into a P-winning strategy diagram τ^* for S.

If τ^* is well-arranged, we are through. Further, if the local thesis in S is Λ, we are sure that Λ is among the concessions in S (otherwise Λ would occur essentially in S), and, if it is P's turn to move in S, S consitutes a one-node P-winning strategy diagram, which is of course well-arranged. The only remaining possibility is that τ^* starts with an illicit application of $P^{\Lambda}_{\sim}$ in the following way ($S = \Pi, \sim U/W/_P\ \Gamma$):

Figure V.20

MΛD

	O	P	
	.		
	.	W	1
	.		
	~U	Γ	2
$P^{\Lambda}_{\sim}$ illicit!	[Λ]	U	

…	a_iU	…	Λ	…	$[d_iU]$	…	3

Comment:

1. Local thesis W, W is not a negation, W ≠ Λ.
2. P's protective defense rights Γ.
3. O may *attack* or *defend.* There is one subtableau for each mode of attack on U and one subtableau for defense.

The subtableaux in which O attacks give us sequents
$S^{i}_{+} = \Pi, \sim U, a_iU/U/_P\ [d_iU]$.
The subtableau in which O defends gives us the sequent
$S^{+} = \Pi, \sim U, \Lambda/W/_P\ \Gamma$.

τ^* has the Λ-property. There are no essential occurrences of Λ in τ^*.
τ^* contains (i) a well-arranged P-winning strategy diagram τ^i_+ for each of the S^i_+;
(ii) a well-arranged P-winning strategy diagram τ^+ for S^+, which, of course, has the Λ-property and no essential occurrences of Λ.

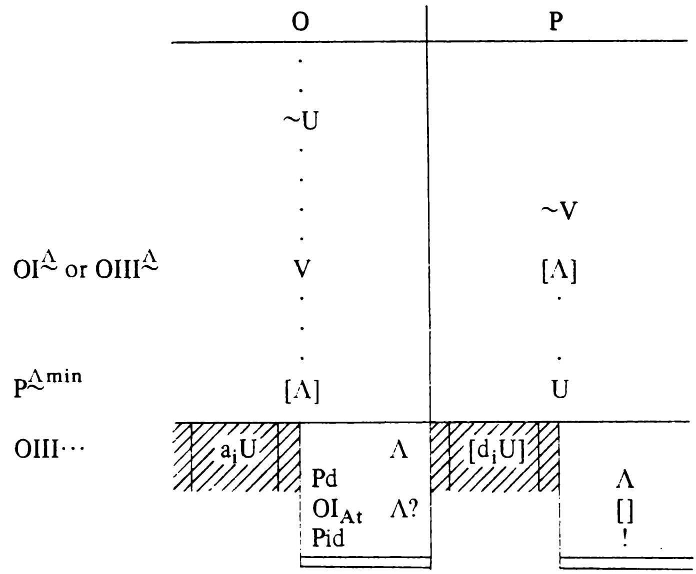
We are going to transform τ^+ into a well-arranged P-winning strategy diagram for S.

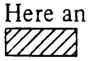
Imagine τ^+ in tableau form and omit the Λ that is among the initial concessions (unless $\Lambda \epsilon \Pi$), turning S^+ into S. In MΛD this can only invalidate some *Ipse dixisti!*-remarks made by P. $W \neq \Lambda$; so these *Ipse dixisti!*-remarks cannot occur before W is replaced by another local thesis. In what circumstances can Λ become the local thesis in τ^+? Only if Λ is first "stated" by P. Recall that Λ does not occur essentially in S. So $\Lambda \notin \Gamma$, for W is not a negation. Hence P's "statement" of Λ cannot derive from the initial defense rights Γ. Again, we can have no Λ on P's side of the tableau in virtue of P$\overset{\Lambda}{\sim}$ or P→, since this would constitute an essential occurrence of Λ. The only remaining possibility is that a statement of Λ derives from a defense right [Λ] that is obtained further down in the tableau. Since Λ occurs nowhere essentially, and since τ^+ has the Λ-property, the context of each of P's inappropriate *Ipse dixisti!*-remarks must be as follows:

Figure V.21

	O	P	
	.		
	.		
	.		
	~U		
	.		
	.	~V	
	.		
OI$\overset{\Lambda}{\sim}$ or OIII$\overset{\Lambda}{\sim}$	V	[Λ] (bottommost defense right)	
		.	
		.	
		.	
Pd		Λ	
$OI_{\Lambda t}$	Λ?	[]	
Pid		!	(Possibly incorrect, since the Λ on the left has been omitted)

We now replace these fragments of τ^+ by:



Here an application of $P^{\Lambda min}_{\sim}$ is inserted. The subtableaux indicated by  are closed in the same way as the well-arranged tableaux τ^i_+. There may now be more concessions than in S^i_+ but that doesn't matter. The result is a closed dialogical tableau in MΛD, for S, which is well-arranged ●

6. Transformation of dialogical Lorenzen-tableaux into deductive Beth-tableaux

[Chapter VII. Deductive Tableaux]

[- -]

6.1. [VII.1] Deductive tableaux with implication as the only logical constant

In the following we shall regard sequents Π/Z as condensed statements of *deduction problems*, i.e., problems of the form: can Z be deduced from Π (according to system σ)? In this context Z shall be referred to as the *concludendum* (the conclusion to be reached, i.e., the desired conclusion) in the problem and the sentences in Π will be called its *premises*. Instead of applying deduction rules to the premises, we apply *reduction rules* to the problem Π/Z as a whole, in order to reduce it to lesser problems. Finally, we hope to arrive at *trivial problems*; these are the deduction problems represented by sequents of the form Π, U/U, where the concludendum is identical with one of the premises.

We shall *state* the problem by drawing a so-called *deductive tableau*:

Figure VII.1

	Prem.	Concl.
Π	------	

	------	Z

and shall speak – as before – of the *left column* (containing the premises) and the *right column* (containing the desired conclusion) of the tableau.

The meaning of each logical constant can now be determined by two reduction rules, one which is applicable if this constant occurs as the principal operator of a premise (the *left rule*), and one rule which is applicable if this constant occurs as the principal operator of the concludendum (the *right rule*). Together these rules stipulate exactly how an occurrence of a logical operator as principal operator of a sentence can be exploited in deductions. (Sometimes we have two closely associated tableau rules instead of one – left or right – rule.) For the conditional there is the following *right rule*:

→r A problem $\Pi/U \rightarrow V$ reduces to the problem $\Pi, U/V$.

This rule can be "justified" from the point of view of systems for natural deduction. For, provided CP and TRIV are included in the systems, we can say:

Let us enter the antecedent of the conclusion, U, as an hypothesis in the hope that we can find a deduction of V from Π and U; if we can, then we may apply CP to that deduction, which gives us precisely a deduction of $U \rightarrow V$ from Π.[1] Since the problem $\Pi/U \rightarrow V$ can, in this way, be solved as soon as $\Pi, U/V$ is solved, we have now *reduced* the former problem to the latter.

Our *left rule* for the conditional is:

→l A problem $\Pi, U \rightarrow V/Z$ reduces to the two problems $\begin{cases} \Pi, U \rightarrow V/U \text{ and} \\ \Pi, U \rightarrow V, V/Z. \end{cases}$

Again, this rule can be "justified" from the point of view of systems for natural deduction, provided, this time, that MP is among the rules (or is imitable). For if *both* problems to the right can be solved, we can first derive U from the sentences in $\Pi \cup \{U \rightarrow V\}$ (first problem) and immediately apply MP to U and $U \rightarrow V$ so as to obtain V. Once V is obtained we solve the second problem ($\Pi, U \rightarrow V, V/Z$) and derive Z. Altogether this will constitute a solution to the original problem: how to derive Z from $\Pi \cup \{U \rightarrow V\}$. So $\Pi, U \rightarrow V/Z$ is reducible to the *two* problems described in the rule, both of which one must solve in order to solve the former.

By repeated applications of these rules we hope to arrive at a *set of trivial deduction problems*. These shall be called *closed*:

---

[1] Presence of TRIV in the system is sometimes necessary. For though a premise may freely be repeated in a deduction even if the rule TRIV were not included, the same does not hold for an hypothesis (see Def. 2 in Section VI.1.1).

Closure Rule:

c Every problem Π, U/U is *closed*.

The reductions give rise to a *tree of sequents*, as is shown by the following example:

Example 1

$A \to B, B \to C/A \to C$ ⇐ given problem

⇐ application of $\to$ r

$A \to B, B \to C, A/C$

⇐ application of $\to$ l to "$A \to B$"

$A \to B, B \to C, A/A$ (closed) — $A \to B, B \to C, A, B/C$

⇐ application of $\to$ l to "$B \to C$"

$A \to B, B \to C, A, B/B$ (closed) — $A \to B, B \to C, A, B, C/C$ (closed)

Def. 1 *a* A *deductive tableau for a sequent* Π/Z, *based on a system* σ *of reduction rules*, is a tree diagram[2] in which each node is associated with a sequent, and such that, for each non-final node, the associated sequent is reducible (by one application of one rule in σ) to the sequent(s) associated with its successor(s); Π/Z itself is associated with the root of the tree.

b A deductive tableau is *closed* if, and only if, (i) each branch is finite, and (ii) each final node is associated with a closed sequent.

Each reduction rule will reduce a problem either to *one* or to *two* problems. Hence we may invoke Lemma 3 from Section V.2 to show:

Lemma 1 A closed deductive tableau has a finite number of nodes.

(Cf. Lemma 8 of Section V.4.)

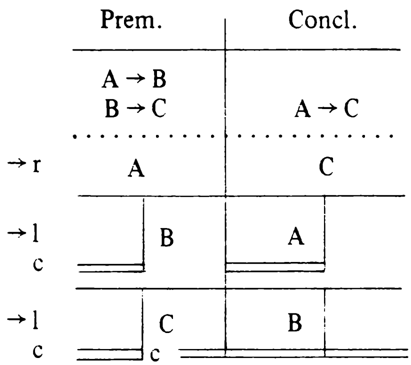
As in the case of P-winning strategy diagrams we can save a lot of rewriting of constituents of problems if we employ the tableau notation that was explained in Section V.3. In the left column, marked "Prem.", we write down the premises and intermediary conclusions, which function as premises at the next stage in the process of problem reduction. We get more and more "premises" as we proceed, since there is no reduction rule that drops a premise. In the right column, marked "Concl.", we write the *concludendum*, which may (but need not) be different at each stage. Note that it is always *the bottommost sentence on the right* (in a subtableau) that counts as the concludendum (of that subtableau), the other sentences on the right having been successively supplanted by the next. Consequently, *right rules can only be applied to this bottommost (occurrence of a) sentence.*

[2] See Section V.2.

The subtableaux correspond to different branches in the tree, just as was explained in Section V.3.

Example 2

The deductive tableau of Example 1 in tableau notation:



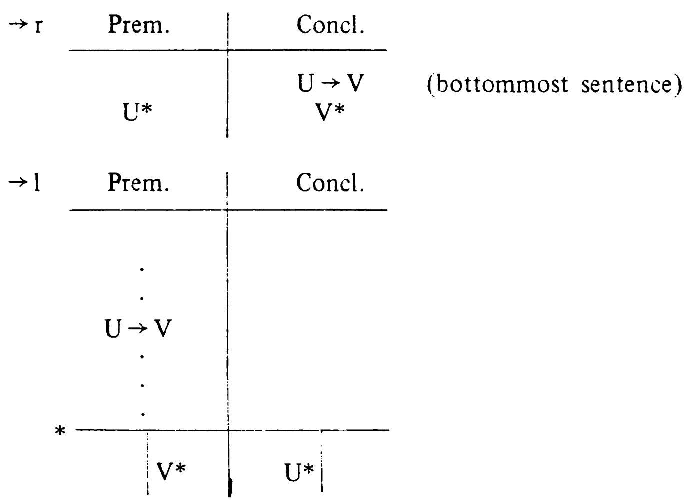
(The double horizontal lines indicate closure)

As a result of the application of → r the occurrence of "A → C" is supplanted by "C", and no rule can be applied to this occurrence of "A → C" any more.

We entreat the reader to re-read the caveats 1, 2, 5, 6 of Section V.3, all of which apply, in analogous manner, to deductive tableaux.

We shall now reformulate the rules → r and → l in tableau form. (For simplicity's sake we drop the current concludendum "Z" from → l.) The asterisks indicate what should be added by virtue of the rule.

Figure VII.2



	Prem.	Concl.	
c			
	U		
		U	(bottommost sentence)
*			

These three rules are all the rules needed for minimal-IF deductive tableaux (which coincide with constructive-IF deductive tableaux).

Def. 2 Our *system of rules for constructing minimal-* (and *constructive-*) IF *deductive tableaux* (MIdt, or CIdt) consists of the rules → l, → r and c.

[- -]

Classical Introduction of a Conditional:

→K A problem Π/Z reduces to the problem Π, Z → U/Z.

In tableau form:

Figure VII.9

	Prem.	Concl.
		Z
→K	Z → U*	Z*

The following schemata show, in a quite general way, how one can reinstate any supplanted concludendum, using no other logical constant than →.

Figure VII.10

Prem.	Concl.
	U
	.
	.
	.
	.
	.
	.
	V
	(U is supplanted by V)

⇒

	Prem.	Concl.
		U
→K	U → V	U . . . insert this line
		.
		.
		V
→l	V	U
c		

The second subtableau in the tableau on the right closes at once; in the first subtableau U is reinstated, i.e., appears as the concludendum of the problem stated last (in the open subtableau).

(It can also be done in other ways, by means of ~, or by means of → and ∧ – cf. Section 3 – and in still other manners.)

Def. 3 Our *system of rules for constructing classical*-IF *deductive tableaux* (KIdt) consists of the same rules as the system MIdt, with the rule → K added.

[- -]

6.2. [VII.2.] Tableau transformation in purely implicational languages

In this section we shall show how each closed *dialogical* tableaux constructed according to MID (KID) can be transformed into a closed *deductive* tableau constructed according to MIdt (KIdt). Thus the method of deductive tableaux will be shown to be *complete with respect to* the dialectical garb, and the dialectics to be *sound with respect to* the derivational garb. The converse will follow from the equivalence of all garbs, to be proved in Chapter XI.

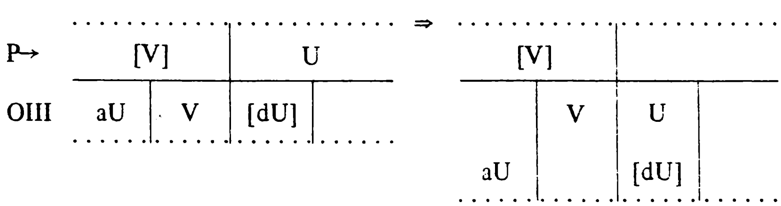
We shall describe a completely mechanical and quite practical method by means of which one can bring about the desired transformation. This method will work only if the given dialogical tableau satisfies the rule R_{At} of Section V.4 (restriction of *Ipse dixisti!* to atomic sentences), but in view of Theorem 3 (Section V.4) this restriction is quite harmless. (The proof of Theorem 3 in fact provides us with a mechanical method for transforming any closed dialogical tableau into one satisfying R_{At}.) First we divide the (closed, given) dialogical tableau into *units.* A unit consists of one move by P followed by all *possible* reactions on O's side. Now each tableau *starts* with an application of an OI-rule (OI→ or OI_{At}) representing O's attack on the initial thesis. This first application of an OI-rule shall count as *half a unit*. Afterwards, going down one particular subtableau, we encounter a number of whole units, each of which consists either of an *attack* by P followed by O's possible reactions (P→ followed by an OIII-rule) or of a *protective defense move* by P followed by an attack by O (Pd followed by an OI-rule). Each subtableau *ends* with an application of Pid, which again shall count as *half a unit*.

It is convenient to separate the units by dotted lines (see Example 1).

We now successively carry out the following changes in the (closed, given) dialogical tableau:

1. (*Ad* P→:) In each unit that consists of an application of P→ followed by an application of an OIII-rule, there is a division of our tableau into subtableaux (see Figure VII.12, tableau-fragment on the left). Let U be that sentence which appears in P's column on account of the application of P → (U is the antecedent of the attacked conditional U → V). This inscription of U should be transferred to the top of the first subtableau, where it should be placed on an inserted line, at the same level as V. The original inscriptions of the first subtableau should be pushed down one line (of text).

Figure VII.12

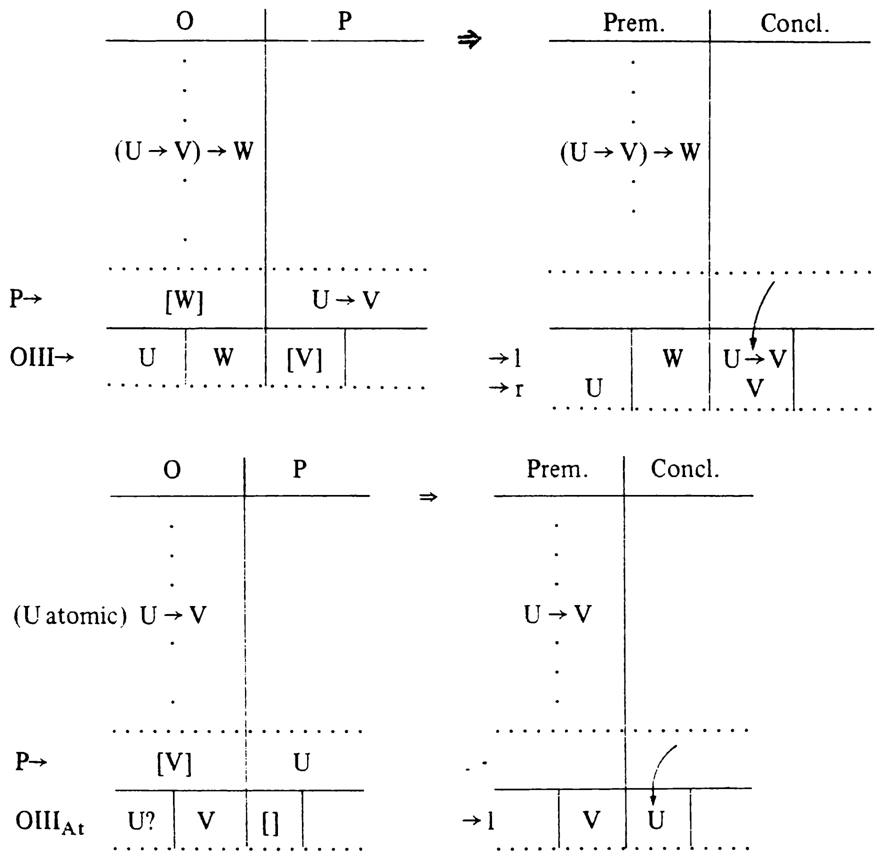


2. Each exclamation mark should be replaced by a repetition of the (atomic) local thesis.
3. All questions should be erased, as well as, in O's column, all inscriptions [dX].
4. For every sentence X, all inscriptions of [X] in P's column are to be replaced by inscriptions of X; inscriptions of the "empty brackets" [] should be erased.

If these instructions are followed, the units and half-units of the dialogical tableau are transformed either into applications of rules for the reduction of deduction problems, or else into "void" reductions, i.e., "reductions" of a sequent (problem) to itself, according to the following schemata:

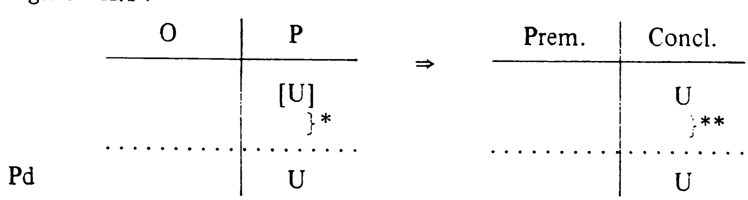
A. Units consisting of an attack by P followed by O's reactions:

Figure VII.13



B. Half-units consisting of an application of Pd

Figure VII.14



Pd is transformed into a repetition of a concludendum. If the *current* concludendum is repeated, i.e., if there is no inscription in the part of the deductive tableau indicated by **, this reduction is "void". Provided that the given dialogical tableau was constructed according to MID, this will indeed be the case. For, first, the bracketed expression [U] to which Pd is applied is then the bottommost *bracketed* expression in P's column and, second, all inscriptions of sentences in part * of the dialogical tableau are antecedents of conditionals that were attacked by P, and hence have been pushed down by us into other subtableaux, according to Instruction 1[1]. If, however, the dialogical tableau was constructed according to KID, this transformation may lead to a non-trivial reoccurrence of a former concludendum (see further Instruction 6, p. 128).

C. Half-units consisting of an attack by O

Figure VII.15

	O	P			Prem.	Concl.
			⇒			
		U → V				U → V
OI→	U	[V]		→ r	U	V

	O	P		Prem.	Concl.
			⇒		
		U (U atomic)			U
OI_{At}	U?	[]			

A half-unit of type B and a half-unit of type C combine to make a whole unit of the second type described above.

D. Half-units which lead to closure of subtableaux

Figure VII.16

	O	P			Prem.	Concl.
			⇒			
		U (U atomic)				U
	U?	[]				
		} *				} **
Pid		!		c		U
	(U occurs somewhere in O's column)				(U occurs somewhere in the *Prem*-column)	

[1] They cannot have been pushed into the very same subtableau we are now considering. For each inscription that, in accordance with Instruction 1, is pushed down into a sub-

Note that, if the local thesis U is first pushed down into some subtableau according to Instruction 1, then this subtableau must be just the one we are now considering. For Instruction 1 tells us to push certain inscriptions down into those subtableaux in which they are attacked (see above), and U *is* attacked in the subtableau we are now considering. So U will certainly appear in the *Concl*-column after we have worked through all the instructions. Instruction 2 tells us to replace the exclamation mark by an inscription of U. The diagrams in Figure VII.16, therefore, truly depict the effect of our transformations on a half-unit Pid. Hence each application of Pid will be transformed into a repetition of a concludendum together with an application of the closure rule c. In MID the repetition of the concludendum is trivial and constitutes a void reduction, since we can argue about the parts * and ** in the same way as under B.

In order to obtain a closed deductive tableau we need to add the following instructions:

5. Repetitions of the concludendum which constitute a "void" reduction should be removed.
6. In each case where a *supplanted* concludendum is repeated – we saw that this can only happen in classical logic, i.e., if we go from KID to KIdt – the repetition should be justified by an inserted application of $\rightarrow$ K (and $\rightarrow$ 1) as shown in Figure VII.10 (Section 1.) .

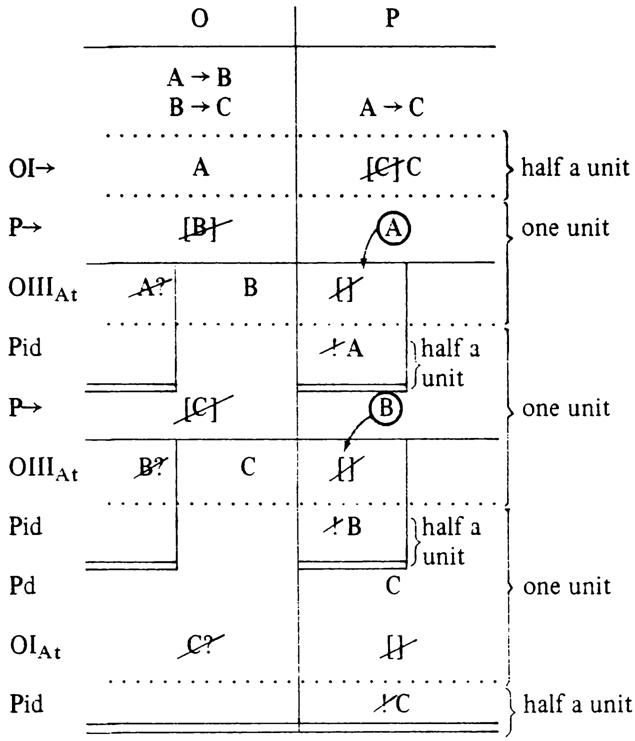
Thus we have established:

Lemma 5 A closed dialogical tableau for a sequent Π/Z which is constructed and closed according to the rules of MID (KID) can, by a completely mechanical procedure, be transformed into a deductive tableau for the same sequent, constructed and closed according to the rules of MIdt (KIdt).

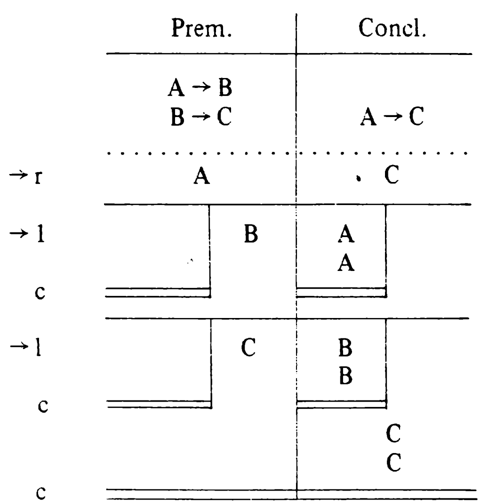
Example 1

Transformation of a closed MID-tableau for $A \rightarrow B, B \rightarrow C/A \rightarrow C$ into a closed MIdt-tableau. We devide the tableau into units and half-units and indicate how Instructions 1 through 4 are executed:

tableau is, in that subtableau, immediately followed by a bracketed expression. But in the subtableau we are now considering the [U] indicated is the *bottommost* bracketed expression.



After execution of these instructions we obtain the following tableau, which is a nearly correct deductive tableau.



After execution of Instruction 5 (see above) we obtain exactly the closed deductive tableau of Example 2, Section 1.

Exercises

1. Turn to the dialogical tableaux, closed in MID (KID), that you were asked to make in Chapter V (V.3, Exercises 1, 2a). If they do not satisfy R_{At} already, first construct tableaux that do satisfy R_{At} for the same sequents. Then show how these are to be transformed into closed deductive tableaux (cf. Example 1).
2. Prove the converse of Lemma 5 for MIdt and MID. HINT: Use induction on the tree structure of deductive tableaux (Section V.4). Hence assume:

   for each natural number n: each deductive tableau (constructed in MIdt) containing fewer than n nodes has the property that there exists (on the strength of MID) a P-winning strategy for its initial sequent,

   and try to show that the tableaux with n nodes have the same property. (Assume that you have a tableau with n nodes and distinguish cases according to the first rule applied.)

6.3. [VII.3.] Tableau transformation in full sentential languages

We shall now formulate reduction rules for problems stated by means of conjunction, veljunction and negation (in addition to implication). We shall also display most of these rules in tableau notation.

[- -]

$\& l_1$ A problem Π, U & V/Z reduces to the one problem Π, U & V, U/Z. Or:

Figure VII.17

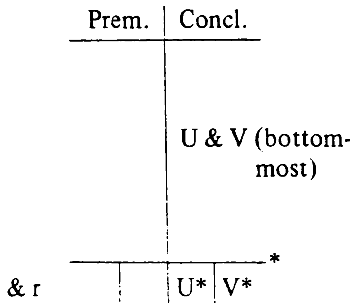
	Prem.	Concl.
	.	
	.	
	.	
	U & V	
	.	
	.	
$\& l_1$	U*	

$\& l_2$ A problem Π, U & V/Z reduces to the one problem Π, U & V, V/Z.

& r A problem Π/U & V reduces to the two problems $\begin{cases}\Pi/U \text{ and} \\ \Pi/V.\end{cases}$

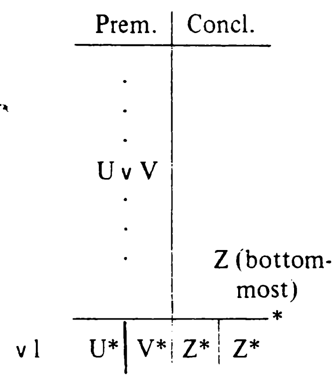
Solution of *both* Π/U and Π/V guarantees that there is a solution for Π/U & V. Or:

Figure VII.18



v l A problem Π, U v V/Z reduces to the two problems { Π, U v V, U/Z and Π, U v V, V/Z.
Again, a solution of *both* problems on the right guarantees that there is a solution for the problem on the left. Or:

Figure VII.19



$\vee r_1$ A problem $\Pi/U \vee V$ reduces to the one problem Π/U. Or:

Figure VII.20

	Prem.	Concl.
		U v V (bottom-most)
$\vee r_1$		U*

$\vee r_2$ A problem $\Pi/U \vee V$ reduces to the one problem Π/V.

$\sim l_\Lambda$ A problem $\Pi, \sim U/Z$ reduces to the two problems $\begin{cases} \Pi, \sim U/U \text{ and} \\ \Pi, \sim U, \Lambda/Z. \end{cases}$
Solution of *both* problems on the right guarantees the existence of a solution to the problem on the left.

$\sim r_\Lambda$ A problem $\Pi/\sim U$ reduces to the one problem $\Pi, U/\Lambda$.

The following rule is a rather unusual one; its application requires a negation in each column:

$\sim l_{min}$ A problem $\Pi, \sim U/\sim V$ reduces to the one problem $\Pi, \sim U/U$. Or:

Figure VII.21

	Prem.	Concl.
	.	
	.	
	.	
	~U	
	.	
	.	
	.	~V (bottom-most)
$\sim l_{min}$		U*

It is a special case of a rule that does not require a negation in the *Concl*-column.

$\sim l$ A problem $\Pi, \sim U/Z$ reduces to the one problem $\Pi, \sim U/U$. Or:

Figure VII.22

	Prem.	Concl.
	.	
	.	
	.	
	~U	
	.	
	.	
	.	Z (bottom-most)
~l		U*

~r A problem Π/~U reduces to the one problem Π, U/~U. Or:

Figure VII.23

	Prem.	Concl.
		~U (bottom-most)
~r	U*	~U*

This is another example of a rule which tells us to repeat the concludendum.

Λc Every problem Π, Λ/Z is *closed*. Or:

Figure VII.24

	Prem.	Concl.
	.	
	.	
	.	
	Λ	
	.	
	.	
	.	Z (bottom-most)
Λc	═══════	═══════*

The last two rules also tell us to repeat the concludendum:
→ΛK A problem Π/Z reduces to the one problem Π, Z → Λ/Z. Or:

Figure VII.25

	Prem.	Concl.
		Z (bottom-most)
→ΛK	Z → Λ*	Z*

~K A problem Π/Z reduces to the one problem Π, ~Z/Z. Or:

Figure VII.26

	Prem.	Concl.
		Z (bottom-most)
~K	~Z*	Z*

Def. 4 *a* Our *system of rules for constructing minimal-Λ deductive tableaux* (MΛdt) comprises the rules → l, → r, & l_1, & l_2, & r, v l, v r_1, v r_2, ~ l_Λ, ~ r_Λ and c.

b Our *system of rules for constructing minimal-*NOT *deductive tableaux* (MNdt) consists of the same rules, but with ~ l_{min} and ~ r replacing ~ l_Λ and ~ r_Λ.

c Our *system of rules for constructing constructive-Λ deductive tableaux* (CΛdt) consists of the rules of MΛdt with Λc added.

d Our *system of rules for constructing constructive-*NOT *deductive tableaux* (CNdt) consists of the rules of MNdt but with the full ~ l instead of ~ l_{min}.

e Our *system of rules for constructing classical-Λ deductive tableaux* (KΛdt) consists of the rules of CΛdt with → ΛK added.

f Our *system of rules for constructing classical-*NOT *deductive tableaux* (KNdt) consists of the rules of CNdt with ~ K added.

[- -]

The transformation procedure of Section 2 can be extended to the new systems for tableau construction:

Theorem 9 A closed dialogical tableau for a sequent Π/Z which is constructed and closed according to the rules of any of our systems for dialogical tableau construction can, by a completely mechanical procedure, be transformed into a deductive tableau for the same sequent, constructed and closed according to the rules of the corresponding system for deductive tableau construction.

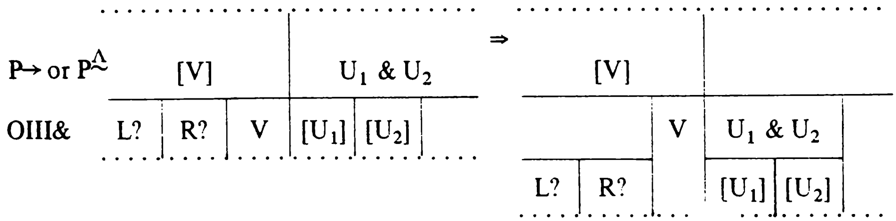
[- -]

Proof of Theorem 9:

First, the given closed dialogical tableau (which again is assumed to satisfy R_{At}) should be divided into units as in Section 2. An application of Pad of course counts as half a unit, just as an application of Pid. We adapt the instructions that are to be followed so that they can be applied to dialogical tableaux in which there appear other logical constants than just →.

1. As we know, in each unit consisting of an application of P→ or of $P^{\overset{\wedge}{\sim}}$ and an application of an OIII-rule, there is a division of the tableau into subtableaux. Let U be that sentence which appears in P's column on account of the application of P→ or $P^{\overset{\wedge}{\sim}}$. (U is then either the antecedent of an attacked conditional or else the negated sentence of an attacked negation.) When U is not a conjunction, the inscription of U we are concerned with should be transferred to the top of the first subtableau, where it should be placed at an inserted line of text (see Figure VII.12 in Section 2). If $U = U_1 \& U_2$ the unit will show a division into *three* subtableaux. In this case the following transformation should be carried out:

Figure VII.27



2. In each application of Pid the exclamation mark should be replaced by a repetition of the (atomic) local thesis.
3. All other exclamation marks, as well as all questions, should simply be removed, as well as, in O's column, all inscriptions [dU].
4. All inscriptions of [U] in P's column are to be replaced by inscriptions of U; however, inscriptions of the "void brackets" [] and of [U, V] should be removed.

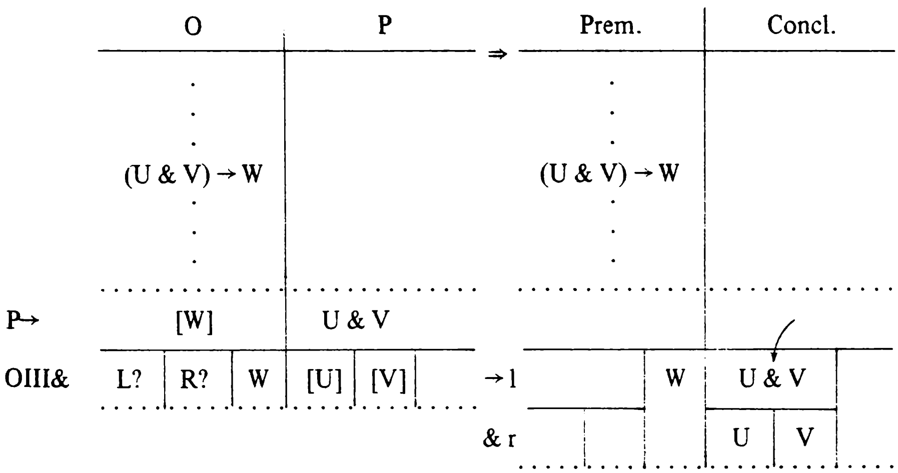
If the Instructions 1 through 4 are followed, the units of what was a dialogical tableau will end up looking exactly like applications of reduction rules (only without repetitions of the concludendum in v l and ~ r), or like "void" reductions, or like repetitions of a *supplanted* concludendum. The latter possibility arises only in classical tableaux. In the case of a classical tableau "applications" of v r_1 or of v r_2, on a supplanted concludendum, may appear as well. The number of *kinds of units* to be analyzed is quite large. We shall give a complete list of these and, by way of example, some diagrams. The other diagrams (for kinds of units) are left to the reader.

A. Units consisting of an attack by P followed by O's reactions

P → + OIII → ⇒ → l + → r }
P → + $OIII_{At}$ ⇒ → l } (See Section 2 for diagrams)
P → + OIII & ⇒ → l + & r

Diagrams illustrating this last type of transformation:

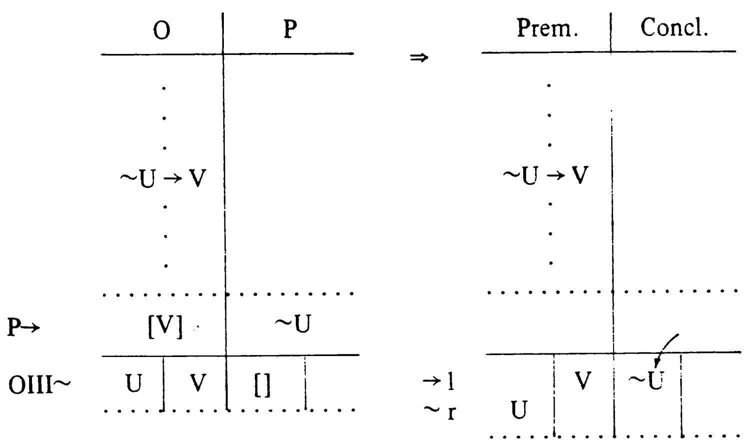
Figure VII.28



P→ + OIIIv ⇒ → l
P → + OIII^ ⇒ → l + ~r^
P → + OIII~ ⇒ → l + ~ r (a repetition of the concludendum is missing in the first subtableau)

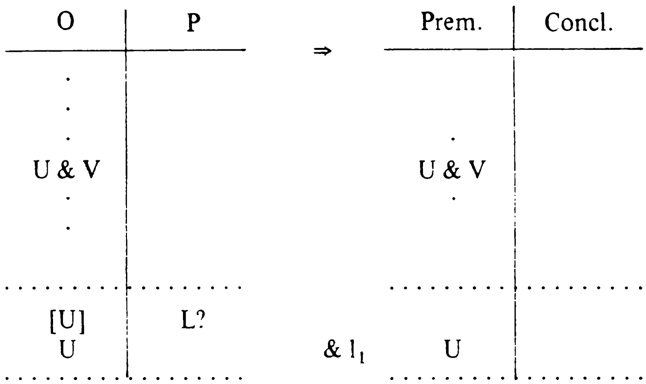
Diagrams for this last type of transformation:

Figure VII.29



P&L + OII ⇒ & l_1. Or:

Figure VII.30

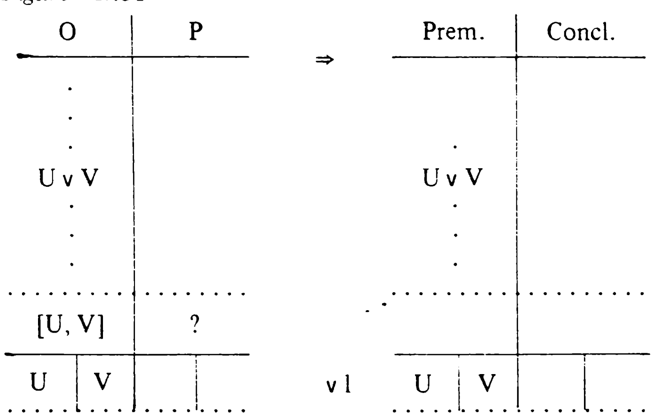


P&L
OII

P&R + OII ⇒ & l_2
Pv + OII ⇒ v l (repetitions of the concludendum in the sub-tableaux are missing)

Diagrams for this last type of transformation:

Figure VII.31



Pv
OII

$P^{\overset{\Lambda}{\sim}}$ + OIII →⇒ ~ l_Λ + → r
$P^{\overset{\Lambda}{\sim}}$ + $OIII_{At}$⇒ ~ l_Λ
$P^{\overset{\Lambda}{\sim}}$ + OIII& ⇒ ~ l_Λ + & r
$P^{\overset{\Lambda}{\sim}}$ + OIIIv ⇒ ~ l_Λ
$P^{\overset{\Lambda}{\sim}}$ + $OIII^{\overset{\Lambda}{\sim}}$ ⇒ ~ l_Λ + ~ r_Λ

} These are similar to the combinations with P →

For combinations with $P^{min}_{\sim}$ or with P~, see under E.

B. Half-units consisting of an application of Pd

Applications of Pd are transformed (by someone following our instructions) into "void" reductions, or into repetitions of a *supplanted* concludendum, or into applications of $\vee r_1$ or of $\vee r_2$:

Figure VII.32

	O	P		Prem.	Concl.
		U v V	⇒		U v V
OIv	?	[U, V]			
		} *			} **
Pd		U	$\vee r_1$		U

(See Section 2, Figure VII.14 for another set of diagrams.) As in Section 2 it may be argued that, if the system is non-classical, there are no sentences at all in part**. The [U, V] in the dialogical tableau was the bottommost bracketed expression in the subtableau under consideration. This excludes the possibility that applications of $P{\sim}$ or of $P_{\sim}^{min}$ occur in part *; for each such application would have been followed by an application of an OI-rule, and so a bracketed expression would have been placed in part *. So the only inscriptions of sentences in part * are those which derive from $P{\rightarrow}$ or from $P^{\Lambda}_{\sim}$, and these were already pushed down into subtableaux other than the one we consider here. In classical systems, however, we may get a repetition of a *supplanted* concludendum or an "application" of $\vee r_1$ or of $\vee r_2$ on a *supplanted* concludendum by following these instructions (see further Instruction 6, on p. 140).

C. Half-units consisting of an attack by O

$OI{\rightarrow}\ \Rightarrow\ \rightarrow r$ } (See Section 2 for diagrams)
$OI_{At}\ \Rightarrow$ void }
$OI\&\ \Rightarrow\ \& r$

Diagrams illustrating this last type of transformation:

Figure VII.33

	O		P			Prem.		Concl.	
			U & V		⇒			U & V	
OI&	L?	R?	[U]	[V]	& r			U	V

$OIv\ \Rightarrow$ void
$OI^{\Lambda}_{\sim}\ \Rightarrow\ \sim r_{\Lambda}$
$OI{\sim}\ \Rightarrow\ \sim r$ (a repetition of the concludendum is missing)

A half-unit B and a half-unit C combine to make a whole unit consisting of a protective defense move by P together with O's possible reactions.

D. Half-units which conclude subtableaux

Pid ⇒ c (See Figure VII.16 in Section 2)
Pad ⇒ Λc

In classical systems Pid may give rise to repetition of a *supplanted* concludendum.

E. Half-units consisting of an attack by P for which no structural protective defense is possible

$P\sim \Rightarrow \sim l$
$P_{\sim}^{min} \Rightarrow \sim l_{min}$

Diagrams for this last type of transformation:

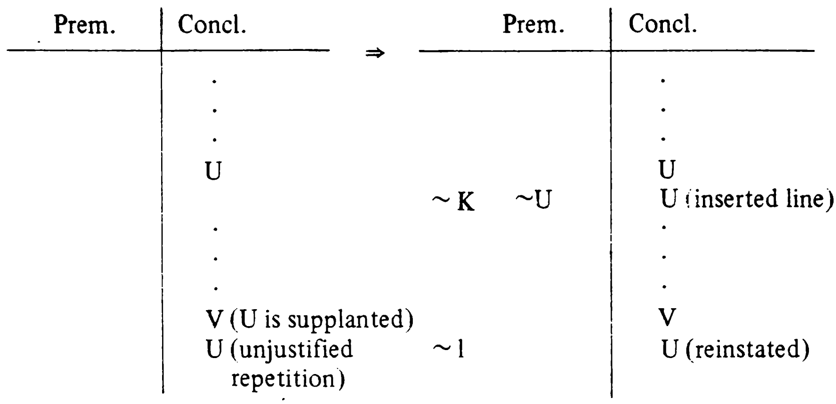
Figure VII.34

	O	P	⇒	Prem.	Concl.
		~V local thesis			~V
OI~	V	[]	~ r	V	}**
	~U	}*		~U	
	. . . .	. . . .		. . . .	. . . .
$P_{\sim}^{min}$	[]	U	$\sim l_{min}$		U

The ~U shown in O's column may also coincide with the V or appear higher up in that column (above the V). The rule $P_{\sim}^{min}$ is not included in classical systems. We can show that ~V is the concludendum at the moment $\sim l_{min}$ is applied, by the now familiar arguments about the parts * and **. A half-unit of type E can continue with a half-unit of type C such as to form a unit of type A.
Having carried out Instructions 1 through 4 we need to follow two additional instructions:

5. Repetitions of the concludendum should be added at the places we have indicated, whereas repetitions which constitute other "void" reductions should be removed.
6. For classical systems: whenever a *supplanted* concludendum is repeated (or a rule is "applied" to it) this should be put right by (repetition of the concludendum and) insertion of some applications of rules. At the end of Section 1 (Figure VII.10) we saw how this can be done by means of → K and → 1. It can, however, also be done by means of ~ K and → l:

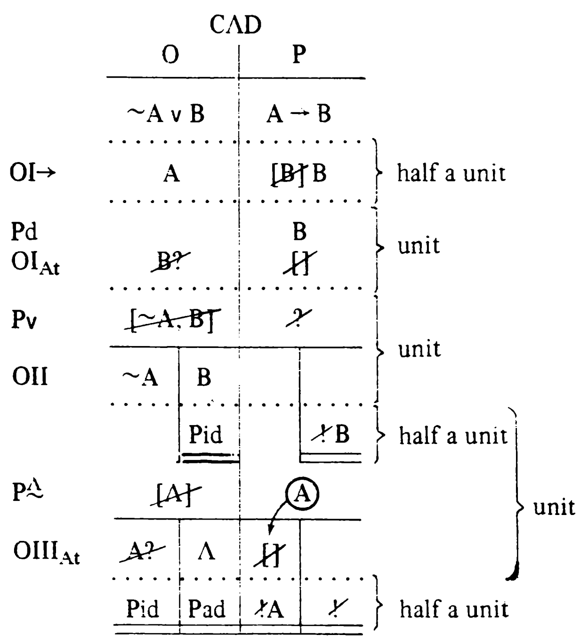
Figure VII.35



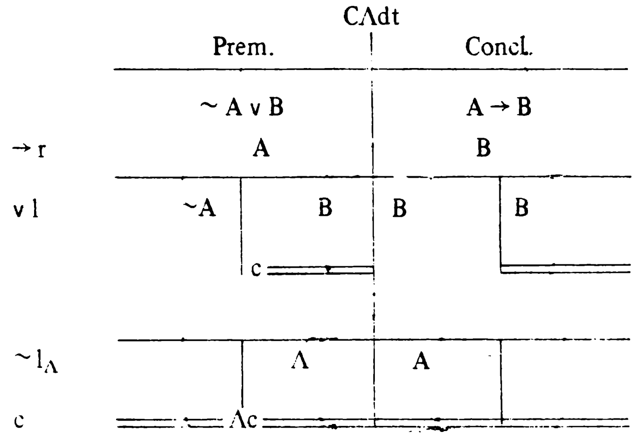
– and also by means of $\sim \Lambda K$, $\sim 1_{\Lambda}$ and Λc●

Example 1

We shall transform the closed CΛD-tableau in Example 1 of Section V.3 into a closed deductive tableau constructed according to CΛdt. We start with Instructions 1 through 4.



After making the corrections of Instruction 5 we have the following deductive tableau:



[- -]

Example 2

We do the same for a closed MND-tableau:

	O	P	
		~~(A v ~A)	
OI~	~(A v ~A)	~~~?~~	half a unit
$P^{min}_{\sim}$ OIv	~~?~~ ~~?~~	A v ~A ~~[A, ~A]~~	unit
Pd OI~	A	~A ~~?~~	unit
$P^{min}_{\sim}$ OIv	~~?~~ ~~?~~	A v ~A ~~[A, ~A]~~	unit
Pd OI_{At}	~~A?~~	A ~~?~~	unit
Pid		~~A~~ Γ	half a unit

⇒

	Prem.	Concl.
		~~ (A v ~A)
~ r	~(A v ~A)	~~ (A v ~A)
$\sim l_{min}$		A v ~A
$v\, r_2$		~A
~ r	A	~A
$\sim l_{min}$		A v ~A
$v\, r_1$		A
c		

Γ/A

[- -]

Example 3

And this time for a closed KND-tableau:

	O	P		⇒	Prem.	Concl.
		A v ~A				A v ~A
OIv	~~?~~	~~[A, ~A]~~	half a unit	~ K	~(A v ~A)	A v ~A
Pd		~A	unit	v r_2		~A
OI~	A	~~[]~~		~ r	A	~A
Pd		A	unit	~ l		A v ~A
OI_{At}	~~A?~~	~~[]~~				
Pid		~~!~~A	half a unit	v r_1		A

7. Transformation of semantic Beth-tableaux into dialogical Lorenzen-tableaux

[Chapter XI. The Unity of the Garbs – and What Next]

[- -]

7.0. Rules for the construction of semantic tableaux

(Excerpts from [AD1],X)

In the following we shall look upon all sequents Π/Γ – and hence also upon sequents Π/Z – as condensed statements of classical model-theoretic *evaluation problems* (*validity problems*), i.e., problems of the form: can a classical model for Π/Γ – or, a counter-example to Π/Z – be found? (Is the sequent classically *in*valid?)

In this context we shall speak of the sentences in Π as the "sentences to be given the value **T**" and of the sentences in Γ (or the one sentence Z) as "the sentences (sentence) to be given the value **F**". We apply *reduction rules* to the problem Π/Γ (or, to the problem Π/Z) in order to reduce it to lesser problems. Finally we hope to arrive at *trivial evaluation problems*. These are of two kinds. First, obviously, there is *no* model for a sequent of the form Π, U/Γ, U. Hence all such sequents are *trivially valid*. On the other hand there are sequents which, as one might say, "represent their own model".

The meaning of each logical constant can now be determined by two reduction rules, one that is applicable if this constant occurs as the principal operator of a sentence which is to be given the value **T** and one that is applicable if this constant occurs as the principal operator of a sentence which is to be given the value **F**. For the conditional we have the following *right rule*:

→R A sequent Π/Γ, U → V reduces to the one sequent Π, U/Γ, U → V, V.

[- -]

Our *left rule* for the conditional is:

→L A sequent Π, U → V/Γ reduces to the two sequents $\begin{cases} \Pi, U \to V/\Gamma, U \\ \Pi, U \to V, V/\Gamma. \end{cases}$

[- -]

We add a *Closure Rule* for sequents that are trivially valid:

C Every sequent Π, U/Γ, U is *closed* – or, more appropriately, leads to closure of the investigation (the semantic tableau).

[- -]

Just like dialogical and deductive tableaux, semantic tableaux, too, are tree diagrams, of a certain kind:

Def. 1 *a* A *classical semantic tableau for a sequent* Π/Γ *based on a system* σ *of reduction rules* is a tree diagram in which each node is associated with a sequent, and such that, for each non-final node, the associated sequent is reducible (by one application of one rule of σ) to the sequent(s) associated with its successor(s); Π/Γ itself is associated with the root of the tree.

b A classical semantic tableau is *closed* if, and only if,
(i) each branch is finite, and
(ii) each final node is associated with a closed sequent.

[- -]

Def. 2 Our *system of rules for constructing classical*-IF *semantic tableaux* (KIst) consists of the rules → L, → R, and C.

[- -]

In contradistinction to the systems for deductive tableaux, the systems of rules for constructing semantic tableaux have only one left rule for conjunction:

& L A sequent Π, U & V/Γ reduces to the one sequent Π, U & V, U, V/Γ.

[- -]

Our right rule for conjunction is:

& R A sequent Π/Γ, U & V reduces to the two sequents $\begin{cases} \Pi/\Gamma, U \,\&\, V, U \\ \Pi/\Gamma, U \,\&\, V, V. \end{cases}$

[- -]

A left rule for veljunction:

v L A sequent Π, U v V/Γ reduces to the two sequents $\begin{cases} \Pi, U \vee V, U/\Gamma \\ \Pi, U \vee V, V/\Gamma. \end{cases}$

Because we are studying sequents Π/Γ where Γ may contain more than one sentence, the right rule for veljunction is simpler than the corresponding rules for the construction of deductive tableaux:

v R A sequent Π/Γ, U v V reduces to the one sequent Π/Γ, U v V, U, V.

A left rule for negation:

~ L A sequent Π, ~U/Γ reduces to the one sequent Π, ~U/Γ, U.

A right rule for negation:

~ R A sequent Π/Γ, ~U reduces to the one sequent Π, U/Γ, ~U.

[- -]

We add a new closure rule, pertaining to an absurd sentence Λ:

CΛ Every sequent $\Pi, \Lambda/\Gamma$ is *closed*.

[- -]

Def. 7 *a* Our *system of rules for constructing classical*-NOT *semantic tableaux* (KNst) consists of the rules $\rightarrow$ L, $\rightarrow$ R, & L, & R, v L, v R, ~ L, ~ R, and C.

b Our *system of rules for constructing classical-*Λ *semantic tableaux* (KΛst) consists of the same rules as KNst with in addition the rule CΛ.

[- -]

In the study of classical semantic tableaux we found it profitable to consider "set-set" sequents Π/Γ, rather than just "set-sentence" sequents Π/Z. For even if a tableau *starts* with a "set-sentence" sequent Π/Z, sequents of the general type Π/Γ, with Γ containing more than one sentence, will have to be considered at later nodes.

In the theory of constructive semantic tableaux a second generalization is called for: even if a tableau starts with one sequent Π/Γ (or Π/Z), we shall soon be led to validity problems which involve more than one sequent.

[- -]

It will therefore be profitable to focus on *sets of sequents* (instead of just on sequents) right from the beginning.

[- -]

We shall denote sets of sequents by "Σ", "Σ'", etc. Furthermore we shall write[2]

"$\Sigma;(\Pi/\Gamma)$" instead of "$\Sigma \cup \{\Pi/\Gamma\}$"

and

"$\Sigma;(\Pi/\Gamma);(\Pi'/\Gamma')$" instead of "$\Sigma \cup \{\Pi/\Gamma; \Pi'/\Gamma'\}$"

etc., because the expressions on the left are easier to read.

Employing these notational conventions we now formulate our *right rule*[3] for the conditional:

$\rightarrow R^c$ = $\rightarrow R^m$ } A set of sequents $\Sigma;(\Pi/\Gamma, U \rightarrow V)$ reduces to the one set of sequents $\Sigma;(\Pi/\Gamma, U \rightarrow V);(\Pi, U/V)$.

---

2 We use semicolons rather than commas to separate (names of) sequents, because commas are already used to separate (the names of) the sentences within the sequents.

3 We shall give two names, one with a "c" and one with an "m", to each rule that figures both in constructive and in minimal systems.

[- -]

Our *left rule* for the conditional is:

$\rightarrow L^c$ / $= \rightarrow L^m$: A set of sequents $\Sigma; (\Pi, U \rightarrow V/\Gamma)$ reduces to the two sets of sequents $\begin{cases} \Sigma; (\Pi, U \rightarrow V/\Gamma, U), \text{ and} \\ \Sigma; (\Pi, U \rightarrow V, V/\Gamma). \end{cases}$

[- -]

Obviously, if there is no model for a certain sequent there can be no model for any set containing it. Therefore, since $\Pi, U/\Gamma, U$ is trivially valid, any set $\Sigma; (\Pi, U/\Gamma, U)$ is trivially valid. This means that *one* closed sequent in a set suffices to make the whole set closed. We add this as a *Closure Rule*:

C^c / $= C^m$: Every set of sequents $\Sigma; (\Pi, U/\Gamma, U)$ is *closed*.

The rules we have so far formulated constitute a system which we shall call MIst or CIst:

Def. 10 Our *system for constructing constructive*-IF *semantic tableaux* (CIst) consists of the rules $\rightarrow L^c$, $\rightarrow R^c$ and C^c.
Our system for constructing minimal-IF semantic tableaux (MIst) is exactly the same system as CIst.

Our definition of "semantic tableau" is entirely analogous to Def. 1:

Def. 11 *a* A *constructive (minimal) semantic tableau for a set of sequents* Σ, *based on a system* σ *of reduction rules*, is a tree diagram in which each node is associated with a set of sequents, and such that for each non-final node, the associated set of sequents is reducible, by one application of a rule of σ, to the set(s) of sequents associated with the successor(s) of this node. The set Σ as a whole is associated with the root.

b A *constructive (minimal) semantic tableau for a sequent* Π/Γ is a constructive (minimal) semantic tableau for the set $\{\Pi/\Gamma\}$.

c A constructive (minimal) semantic tableau for a set of sequents is *closed* if, and only if, each branch of the tableau is finite and each of its final nodes is associated with a closed set of sequents.

[- -]

A left rule for conjunction:

$\left.\begin{array}{r}\&\,L^c\\ =\&\,L^m\end{array}\right\}$ A set $\Sigma;(\Pi, U \& V/\Gamma)$ reduces to the one set $\Sigma;(\Pi, U \& V, U, V/\Gamma)$.

A right rule for conjunction:

$\left.\begin{array}{r}\&\,R^c\\ =\&\,R^m\end{array}\right\}$ A set $\Sigma;(\Pi/\Gamma, U \& V)$ reduces to the two sets $\left\{\begin{array}{l}\Sigma;(\Pi/\Gamma, U \& V, U)\\ \Sigma;(\Pi/\Gamma, U \& V, V).\end{array}\right.$

A left rule for veljunction:

$\left.\begin{array}{r}v\,L^c\\ =v\,L^m\end{array}\right\}$ A set $\Sigma;(\Pi, U \vee V/\Gamma)$ reduces to the two sets $\left\{\begin{array}{l}\Sigma;(\Pi, U \vee V, U/\Gamma)\\ \Sigma;(\Pi, U \vee V, V/\Gamma).\end{array}\right.$

A right rule for veljunction:

$\left.\begin{array}{r}v\,R^c\\ =v\,R^m\end{array}\right\}$ A set $\Sigma;(\Pi/\Gamma, U \vee V)$ reduces to the one set $\Sigma;(\Pi/\Gamma, U \vee V, U, V)$.

A left rule for negation:

$\sim L^c$ A set $\Sigma;(\Pi, \sim U/\Gamma)$ reduces to the one set $\Sigma;(\Pi, \sim U/\Gamma, U)$.

A right rule for negation:

$\sim R^c$ A set $\Sigma;(\Pi/\Gamma, \sim U)$ reduces to the one set $\Sigma;(\Pi/\Gamma, \sim U); (\Pi, U/\emptyset)$.

A closure rule concerning Λ:

$C\Lambda^c$ Every set of sequents: $\Sigma;(\Pi, \Lambda/\Gamma)$ is *closed*.

[- -]

Def. 12 *a* Our *system of rules for constructing constructive*-NOT *semantic tableaux* (CNst) consists of the rules $\rightarrow L^c$, $\rightarrow R^c$, $\& L^c$, $\& R^c$, $v L^c$, $v R^c$, $\sim L^c$, $\sim R^c$ and C^c.

b Our *system of rules for constructing constructive*-Λ *semantic tableaux* ($C\Lambda$st) consists of the same rules as CNst, and, moreover, the rule $C\Lambda^c$.

[- -]

$\sim L^m$ A set of sequents $\Sigma;(\Pi, \sim U/\Gamma)$
reduces to the two sets of sequents $\begin{cases} \Sigma;(\Pi, \sim U/\Gamma, U) \\ \Sigma;(\Pi, \sim U, \Lambda/\Gamma). \end{cases}$

$\sim R^m$ A set of sequents $\Sigma;(\Pi/\Gamma, \sim U)$
reduces to the one set of sequents $\Sigma;(\Pi/\Gamma, \sim U);(\Pi, U/\Lambda)$.

Def. 17 Our *system for constructing minimal-*Λ (*minimal*-NOT) *semantic tableaux*, MΛst (= MNst), consists of the rules $\rightarrow L^m$, $\rightarrow R^m$, & L^m, & R^m, vL^m, vR^m, $\sim L^m$, $\sim R^m$, and C^m.

[- -]

7.1. [XI.1] How to prove the missing link

Clearly, at this stage, there is only one link missing: the one between closed semantic tableaux and winning strategies for the Proponent. We shall, therefore, first concentrate upon the following theorem:

Theorem 28 If, in any of our systems for constructing semantic tableaux, a semantic tableau for Π/Z can be brought to a closure, then there is a P-winning strategy for $\Pi/_O Z$[1] on the strength of the corresponding system of formal_3 dialectics.

Our proof of this theorem will be preceded by *four lemmas*, two for classical and two for non-classical systems. Of two similar lemmas, we shall always state and prove the easier first. Before we proceed with the technicalities, let us explain the idea and plan of the proof.

The most conspicuous difference between semantic and dialogical tableaux is that, whereas the latter contain bracketed formulas, [U], the former do not. We shall have to *add* brackets to some of the formulas in the closed semantic tableau:

U ⇒ [U]

in order to be able to read it as a closed dialogical tableau. We have to show that these additions are defendable. This is the reverse of the situation in Chapter VII, where we showed how to transform dialogical into deductive tableaux by, among other things, erasure of the brackets in the former.

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[1] Or, amounting to the same, a P-winning strategy diagram (a closed dialogical tableau) for $\Pi/_O Z$ Cf. Paper 5, Section 2.1, pp. 92, 93

The reader will remember from Section V.1 that in an expression of the form

$$\Pi; \Delta/_N \Phi; \Gamma$$

the "Γ" stands for a class of rights or possible moves [. . .], i.e., "contains" brackets. What we want to accomplish can, therefore, be expressed roughly as a defendable change of sequents Π/Γ into sequents $\Pi/_P\Gamma$.[2]

In Lemmas 3 and 4 we show that for any closed semantic tableau – say, for the sequent Π/Γ – there is a winning strategy for the Proponent in discussions issuing from the fictitious – "theoretical" – situation $\Pi/_P\Gamma$. That is, the situation in which the Proponent has no thesis to defend but is given just the sentences in Γ as structural "defense" rights (for the defense of the "zero thesis", one might say). For a sequent Π/Z this means that if a semantic tableau for Π/Z has been – or can be – brought to a closure, then a dialogical tableau for $\Pi/_P[Z]$ can also be brought to a closure, i.e., then there is a winning strategy for the Proponent in a situation as depicted by $\Pi/_P[Z]$.

It remains to be shown that if the Proponent has a winning strategy for $\Pi/_P[Z]$, then it has one also for $\Pi/_O Z$. This last step is taken care of in Lemmas 1 and 2. These lemmas will also be used in our proofs of Lemmas 3 and 4. This is the reason why we prove the lemmas in the present order.

7.2. [XI.2.] Constructive and minimal systems. Equivalences concerning the existence of winning strategies

In the proofs, we shall need to take into consideration not only chains of arguments issuing from situations of type OI, or $\Pi/_O Z$, but also chains of arguments starting from a situation of type $\Pi/_P\Gamma$.[1] Situations of this latter type cannot ever occur in "normal" chains of arguments (i.e., those issuing from a situation of type OI), for in situations of type P, which do occur in such chains, there is always a current local thesis (or, in classical systems, a non-empty set consisting of the current local thesis and former local theses).[2] Nevertheless, our dialectical systems determine uniquely what chains of arguments could issue from such a fictitious situation $\Pi/_P\Gamma$, and they also determine what a winning strategy diagram for such a situation consists of.[3]

Whether the dialectical system studied is classical or not, we shall consider chains of arguments issuing from fictitious situations $\Pi/_P\Gamma$ where the class of structural defense rights, Γ, may be empty or contain more than two elements. In such chains there may also occur situations of the types $\Pi; \Delta/_O \Gamma$ and $\Pi; [U]/_O Z; \Gamma$ (without local thesis!) where, again, Γ is empty or contains more

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2 By the notational conventions in Section V.1, "$\Pi/_P\Gamma$" stands for "$\Pi; \emptyset/\emptyset/_P\emptyset; \Gamma$" (p. 86).

1 Cf. Note 2 to Section 1.

2 The fictitious situation $\Pi/_P\Gamma$ must be understood to include a general right for P to execute counteractive defense moves (in defense of a fictitious "zero thesis"). Hence, in a chain starting from such a situation $\Pi/_P\Gamma$, there will, in general, also be situations of the equally fictitious types $\Pi; \Delta/_O\Gamma$ and $\Pi; [U]/_O Z; \Gamma$.

3 The rights in Γ must be treated as derived from a fictitious attack by O on P's "zero thesis".

than two elements. But, if the dialectical system is non-classical, the possibility of such situations is *restricted to the first local discussion.* For, after the first attack by O (which starts the second local discussion)[4], all the rights in Γ will be canceled by virtue of FD O5b (Section III.14). If O attacks Z in a situation $\Pi; [U]/_O Z; \Gamma$, a situation $\Pi, aZ/Z/_P [dZ]$ will follow, and not $\Pi, aZ/Z/_P [dZ], \Gamma$. Similarly, if, in a situation $\Pi/_P \Gamma$, P uses a $Z \in \Gamma$ in a protective defense move, then a situation $\Pi/_O Z$ follows, and not $\Pi/_O Z; \Gamma$. For O's next step will inevitably open a new local discussion, so P can no longer use the rights in Γ.

Lemma 1 In each of the constructive or minimal dialectical systems the following conditions are equivalent:

(i) there exists a P-winning strategy for $\Pi/_P [Z]$;
(ii) there exists a P-winning strategy for each $\Pi, a_iZ/Z/_P [d_iZ]$;
(iii) there exists a P-winning strategy for $\Pi/_O Z$.

Proof From (ii) it immediately follows that (iii), since the situations $\Pi, a_iZ/Z/_P [d_iZ]$ are precisely the situations that O can call into existence in a situation $\Pi/_O Z$.

Given a P-winning strategy for $\Pi/_O Z$, P can win each chain of arguments issuing from $\Pi/_P [Z]$ simply by first using the right to state Z, which move creates a situation $\Pi/_O Z$, and then employing the given winning strategy. Hence (i) follows from (iii).

Finally, let us try to defend the step from (i) to (ii). We shall, in informal terms, describe how P can use a given winning strategy diagram for $\Pi/_P [Z]$ as a manual for winning each chain of arguments that issues from $\Pi\, a_iZ/Z/_P[d_iZ]$.

To begin with, P should simply carry out its moves in the first local discussion *as if* the initial situation were $\Pi/_P [Z]$ and in accordance with its winning strategy for that situation. Let the winning strategy for $\Pi/_P [Z]$ prescribe some attack on a statement of U ($U \in \Pi$), for instance as P's *first* move. Then P can execute the same attack in the situation $\Pi, a_iZ/Z/_P [d_iZ]$, etc.[5] The (possible) extra occurrence now of a_iZ and the right to make an *Ipse dixisti!*-remark on account of Z may be ignored, since they do not make things more difficult for P. Clearly, the only difficulty that may arise is that the given strategy prescribes a use of the right [Z]. *This can only happen at the end of the first local discussion*. For as soon as the second local discussion starts P loses the right [Z]. Consequently, if the last stage of the first local discussion

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[4] In the case where the chain is not "normal" it is convenient to deviate from Def. 15 of Section III.6, and to call the part of the chain up to O's first (actual) attack (or to the end of the chain if there is no attack by O at all!) the ***first*** local discussion, etc. The "local thesis" of this first local discussion is P's fictitious "zero thesis".

[5] This may be explained in terms of tree diagrams, as follows. In its winning strategy diagram for $\Pi/_P[Z]$, P should make the following changes, from the root downwards, in each path that constitutes a possible first local discussion.
For each sequent associated with a node in such a path, P should:
(i) replace the [Z] on the right by $[d_iZ]$;
(ii) add a_iZ to the concessions (this must be done *throughout* the tree);
(iii) add Z as "current local thesis".

does *not* consist of a use of [Z], i.e., if the second local discussion starts with an attack by O on a statement that P made by virtue of a *counter-active* defense move, all goes well.

Now consider the possibility that the given strategy (for $\Pi/_P[Z]$) prescribes the use of the right [Z] in a situation which occurs in the first local discussion. This situation must be of the form $\Pi'/_P[Z]$, where $\Pi \subseteq \Pi'$. The *actual* situation is now $\Pi', a_iZ/Z/_P[d_iZ]$, and the right [Z] is not available. However, if the given winning strategy prescribes the use of [Z] in a situation $\Pi'/_P[Z]$, there is clearly a P-winning strategy for $\Pi'/_O Z$ contained in it. In the situation $\Pi'/_O Z$, O may call into existence the situation $\Pi' a_iZ/Z/_P[d_iZ]$, hence there must be a P-winning strategy for that situation too●

7.3. [XI.3] Classical systems. Equivalences concerning the existence of winning strategies

For classical systems we have a similar lemma. It differs from the preceding one in a number of details:

Lemma 2 In each of the classical dialectical systems the following conditions are equivalent:

(i) there exists a P-winning strategy for $\Pi/_P[Z], \Gamma$;
(ii) there exists a P-winning strategy for each $\Pi, a_iZ/\{Z\}/_P[d_iZ], \Gamma$
(iii) there exists a P-winning strategy for $\Pi/_O Z; \Gamma$.

Proof Again it is easy to see that (iii) follows from (ii), and that (i) follows from (iii).

Let us concentrate on the step from (i) to (ii). As in the preceding lemma, P can use a winning strategy diagram for $\Pi/_P[Z], \Gamma$ as a manual also for winning each chain of arguments that issues from $\Pi, a_iZ/\{Z\}/_P[d_iZ], \Gamma$. So long as no use of the right [Z] is prescribed, P can simply carry out its moves as if the initial situation were $\Pi/_P[Z], \Gamma$. P should do so, not only in the first local discussion, but throughout the chain of arguments. Hence, the only difficulty that may arise is that the strategy could prescribe a use of [Z] in a situation $\Pi'/T/_P[Z], \Gamma'$, whereas the actual situation P has to deal with is $\Pi', a_iZ/T \cup \{Z\}/_P[d_iZ], \Gamma'$ ($\Pi \subseteq \Pi'$ and $\Gamma \subseteq \Gamma'$). However, in that case, there must be a P-winning strategy for $\Pi'/T/_O Z; \Gamma'$ (contained in the one given) and hence for $\Pi', a_iZ/T \cup \{Z\}/_P[d_iZ], \Gamma'$, since O may call that situation into existence●

7.4. [XI.4] Classical systems. From closed semantic tableau to winning strategy for the Proponent

In the proofs that now follow we shall write "W_P" to denote the set that consists of those dialogue sequents for which a P-winning strategy "exists" on the strength of the dialectic system under consideration. Thus we write "$\Pi/_P\Gamma \in W_P$" instead of "there is a P-winning strategy for $\Pi/_P\Gamma$", etc.

Lemma 3 For all classical systems:
if there is a closed semantic tableau for Π/Γ, then there is a P-winning strategy for $\Pi/_P\Gamma$[1] on the strength of the corresponding dialectical system.

Proof We focus on the degree of complexity of the given object, the closed semantic tableau, and use induction. The complexity of a tableau is, for our purpose, determined by the number of its nodes, so we shall concentrate on that number. Let us, therefore, assume:

(*) If there is a closed semantic tableau containing fewer than n nodes for a sequent Π/Γ, then $\Pi/_P\Gamma \in W_P$.

Now let a closed semantic tableau τ, containing exactly n nodes, be given for a sequent Π/Γ. We must show that $\Pi/_P\Gamma \in W_P$. We split the demonstration into cases according to which rule is applied first in τ.

case C The ordinary closure rule is applied immediately! Some sentence, say U, must occur on both sides of the tableau: $U \in \Pi \cap \Gamma$.
Hence, we can write "Π, U" for Π and "Γ, [U]" for Γ:[2]
$\Pi\,_P\Gamma = \Pi, U/_P\Gamma, [U]$.
Clearly, each $\Pi, U, a_iU/\{U\}/_P\Gamma, [U, d_iU] \in W_P$ (P can make an *Ipse dixisti!*-remark).
Hence $\Pi, U/_P\Gamma, [U] \in W_P$ (Lemma 2).

case CΛ The Λ-closure rule is applied immediately! That is, $\Lambda \in \Pi$.
So $\Pi/_P\Gamma(= \Pi, \Lambda/_P\Gamma) \in W_P$, since P can make an *Absurdum dixisti!*-remark. (Of course, this case occurs only if the dialectical system is KΛD and the system for semantic tableaux is KΛst.)

case → L $\Pi/\Gamma = \Pi, U \rightarrow V/\Gamma$ and is reduced to $\Pi, U \rightarrow V/U, \Gamma$ and $\Pi, U \rightarrow V, V/\Gamma$ by the semantic tableau rule → L. For these latter sequents, τ contains closed semantic tableaux with fewer than n nodes. Hence, by (*):

1. $\Pi, U \rightarrow V/_P\Gamma, [U] \in W_P$ and
2. $\Pi, U \rightarrow V, V/_P\Gamma \in W_P$.

From 1 it follows, by Lemma 2, that

3. each of the sequents $\Pi, U \rightarrow V, a_iU/\{U\}/_P\Gamma, [d_iU] \in W_P$.

From 2 and 3 we conclude that

4. the situation $\Pi, U \rightarrow V; [V]/_O U; \Gamma \in W_P$, for in this situation O can call into existence only the situations in 2 and 3. Hence $\Pi, U \rightarrow V/_P\Gamma \in W_P$, for P can bring about the situation in 4 by an attack on $U \rightarrow V$.

case → R $\Pi/\Gamma = \Pi/\Gamma, U \rightarrow V$ and is reduced to $\Pi, U/\Gamma, U \rightarrow V, V$ by the semantic tableau rule → R. For this latter sequent, τ contains a closed semantic tableau with n–1 nodes. Hence, by (*):

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1 Cf. Note 2 to Section 1.
2 By the notational conventions of Section V.1, p. 86, "Π, U" and "Γ, [U]" stand for "$\Pi \cup \{U\}$" and "$\Gamma \cup \{U\}$", respectively. Further, if $U \in \Pi$, then $\Pi \cup \{U\} = \Pi$, etc.

1. $\Pi, U/_P \Gamma, [U \to V, V] \in W_P$. Also
2. $\Pi, U/U \to V/_P \Gamma, [U \to V, V] \in W_P$ (P need not use the extra right to make an *Ipse dixisti!*-remark).
3. $\Pi/_O U \to V; \Gamma, [U \to V] \in W_P$ (O has no choice but to bring about the situation in 2).
4. $\Pi/_P \Gamma, [U \to V] \in W_P$ (P can state $U \to V$).

For KIst and KID this concludes the proof.

case & L $\Pi/\Gamma = \Pi, U \& V/\Gamma$ and is reduced to $\Pi, U \& V, U, V/\Gamma$. For this latter sequent, τ contains a closed semantic tableau with n-1 nodes. Hence, by (*):

1. $\Pi, U \& V, U, V/_P \Gamma \in W_P$. We can successively conclude:
2. $\Pi, U \& V, U; [V]/_O \Gamma \in W_P$ (O *must* state V).
3. $\Pi, U \& V, U/_P \Gamma \in W_P$ (P *can* ask: R? with respect to U & V).
4. $\Pi, U \& V; [U]/_O \Gamma \in W_P$ (O *must* state U).
5. $\Pi, U \& V/_P \Gamma \in W_P$ (P can ask: L? with respect to U & V).

case & R $\Pi/\Gamma = \Pi/\Gamma, U \& V$ and is reduced to $\Pi/\Gamma, U \& V, U$ and $\Pi/\Gamma, U \& V, V$. For these latter sequents τ contains closed semantic tableaux with fewer than n nodes.
Hence, by (*):

1. $\Pi/_P \Gamma, [U \& V, U] \in W_P$ and
2. $\Pi/_P \Gamma, [U \& V, V] \in W_P$. Also
3. $\Pi/U \& V/_P \Gamma, [U \& V, U] \in W_P$ and
4. $\Pi/U \& V/_P \Gamma, [U \& V, V] \in W_P$ (P need not use the extra right).
5. $\Pi/_O U \& V; \Gamma, [U \& V] \in W_P$ (O must attack U & V and, therefore, bring about either the situation in 3 or the situation in 4).
6. $\Pi/_P \Gamma, [U \& V] \in W_P$ (P can state U & V).

The cases v L, v R, ~ L, and ~ R must be left to the reader●

From Lemma 3 (together with Lemma 2) Theorem 28 can be proved for the classical systems. However, we shall first state and prove a similar lemma for the non-classical systems.

Exercise

Complete the proof of Lemma 3.

7.5. [XI.5] Constructive and minimal systems. From closed semantic tableau to winning strategy for the Proponent

The situation is somewhat more complicated here, for we now have to deal with *semantic tableaux for sets of sequents* (Section X.3).

Lemma 4 For all constructive and minimal systems, except the combination MNst (= MAst) - MND:
if there is a closed semantic tableau for a set Σ of sequents, then there is sequent $\Pi/\Gamma \in \Sigma$ such that there is a P-winning strategy for $\Pi/_P \Gamma$ on the strength of the corresponding dialectical system.

Proof We use induction, just as for Lemma 3. So let us assume:

(*) If there is a closed semantic tableau containing fewer than n nodes for a set of sequents Σ, then there is a $\Pi/\Gamma \in \Sigma$ such that $\Pi/_P \Gamma \in W_P$.

Let a closed semantic tableau τ, containing n nodes, be given for a set of sequents Σ. We must show that there is a sequent $\Pi/\Gamma \in \Sigma$ such that $\Pi/_P \Gamma \in W_P$. We split the demonstration into cases according to which rule is applied first in τ.

The cases C^c and $C\wedge^c$ are similar to the cases C and C \ in the preceding proof and are left to the reader. In the remaining cases Σ is reduced to one or two sets of sequents, Σ_1 (and Σ_2). τ contains a closed semantic tableau for Σ_1 (and a closed semantic tableau for Σ_2) with fewer than n nodes. By (*), Σ_1 (and Σ_2) must contain a sequent Π_1/Γ_1 (and a sequent Π_2/Γ_2) such that $\Pi_1/_P \Gamma_1 \in W_P$ (and $\Pi_2/_P \Gamma_2 \in W_P$). If Π_1/Γ_1 (or Π_2/Γ_2) occurs in Σ we are through; otherwise Π_1/Γ_1 (and Π_2/Γ_2) must be the very sequent(s) that result(s) from the application of the rule. We shall henceforth assume that they are.

case $\rightarrow L^c$ $\Sigma = \Sigma; (\Pi, U \rightarrow V/\Gamma)$[1] and reduces to $\Sigma; (\Pi, U \rightarrow V/\Gamma, U)$ and $\Sigma; (\Pi, U \rightarrow V, V/\Gamma)$.

By our assumptions:

1. $\Pi, U \rightarrow V/_P \Gamma, [U] \in W_P$
2. $\Pi, U \rightarrow V, V/_P \Gamma \in W_P$.

We shall describe how P may exploit the winning strategies for the situations in 1 and 2 in order to win each chain of arguments that issues from a situation $\Pi, U \rightarrow V/_P \Gamma$. P should, to begin with, carry out its moves in the first local discussion as if the situation were $\Pi, U \rightarrow V/_P \Gamma, [U]$ and in accordance with its winning strategy for that situation. If the first local discussion ends without the use of the right [U] being prescribed by the strategy, all goes well (cf. the proof of Lemma 1). Now consider the possibility that the given strategy (for $\Pi, U \rightarrow V/_P \Gamma, [U]$) prescribes the use of the right [U] in a situation $\Pi', U \rightarrow V/_P \Gamma, [U]$ ($\Pi \subseteq \Pi'$) that occurs in the first local discussion. The actual situation is now $\Pi', U \rightarrow V/_P \Gamma$ and possibly $U \notin \Gamma$. In this case:

3. $\Pi', U \rightarrow V/_O U \in W_P$ (by virtue of a winning strategy contained in the one for the situation in 1). Hence
4. each $\Pi', U \rightarrow V, a_iU/U/_P [d_iU] \in W_P$ (Lemma 1).[2]

Since $\Pi \subseteq \Pi'$, we conclude from 2:

5. $\Pi', U \rightarrow V, V/_P \Gamma \in W_P$. It follows that
6. $\Pi', U \rightarrow V; [V]/_O U; \Gamma \in W_P$ (for O can bring about only the situations in 4 and 5).

1 "Σ" denotes a set of sequents. $\Sigma; (\Pi/\Gamma); (\Pi'/\Gamma') = \Sigma \cup \{\Pi/\Gamma\} \cup \{\Pi'/\Gamma'\}$, etc.

2 This is an inessential application of Lemma 1. The step can also be taken immediately.

7. $\Pi', U \rightarrow V/_P \Gamma \in W_P$ (P can attack $U \rightarrow V$).[3]

case $\rightarrow R^c$ $\Sigma = \Sigma; (\Pi/\Gamma, U \rightarrow V)$ and reduces to $\Sigma; (\Pi/\Gamma, U \rightarrow V); (\Pi, U/V)$. By our assumptions:

1. $\Pi, U/_P [V] \in W_P$. Also
2. $\Pi, U/U \rightarrow V/_P [V] \in W_P$ (P need not use the extra right). Hence
3. $\Pi/_O U \rightarrow V \in W_P$ (O must attack $U \rightarrow V$).
4. $\Pi/_P [U \rightarrow V] \in W_P$ (P can state $U \rightarrow V$).
5. $\Pi/_P \Gamma, [U \rightarrow V] \in W_P$ (P need not use the extra rights).

For CIst and CID this concludes the proof.

case $\& R^c$ $\Sigma = \Sigma; (\Pi/\Gamma, U \& V)$ and reduces to $\Sigma; (\Pi/\Gamma, U \& V, U)$ and $\Sigma; (\Pi/\Gamma, U \& V, V)$. By our assumptions:

1. $\Pi/_P \Gamma, [U \& V, U] \in W_P$ and
2. $\Pi/_P \Gamma, [U \& V, V] \in W_P$.

We shall describe how P may exploit its winning strategies for the situations in 1 and 2 in order to win each chain of arguments that issues from a situation $\Pi/_P \Gamma, [U \& V]$. P should, to begin with, carry out its moves as if the situation were $\Pi/_P \Gamma, [U \& V, U]$ and in accordance with its winning strategy for that situation. If the first local discussion ends without the use of the right [U] being prescribed by the strategy, all goes well. The only difficulty that may arise is that the strategy could prescribe the use of the right [U] in a situation $\Pi'/_P \Gamma, [U \& V, U]$ $(\Pi \subseteq \Pi')$ that occurs in the first local discussion. The actual situation is now $\Pi'/_P \Gamma, [U \& V]$, and possibly $U \notin \Gamma$. In that case

3. $\Pi'/_O U \in W_P$ (by virtue of a winning strategy contained in the one for the situation in 1).

It would not be a good policy for P to state $U \& V$, for O may very well react by R? instead of L? Rather, P should shift its ground and from now on employ the winning strategy for the situation in 2, which, since $\Pi \subseteq \Pi'$, may also be applied to $\Pi'/_P \Gamma, [U \& V, V]$ (ignoring the extra concessions). The only difficulty that may now arise is that the strategy could prescribe the use of the right [V] in a situation $\Pi''/_P \Gamma, [U \& V, V]$ $(\Pi' \subseteq \Pi'')$ before the end of the first local discussion. The actual situation is now $\Pi''/_P \Gamma, [U \& V]$, and possibly $V \notin \Gamma$. In that case

4. $\Pi''/_O V \in W_P$ (by virtue of a winning strategy contained in the one for the situation in 2). From 3 and 4 we conclude:
5. $\Pi''/U \& V/_P [U] \in W_P$ $(\Pi' \subseteq \Pi''$, P can state U), and
6. $\Pi''/U \& V/_P [V] \in W_P$ (P can state V). Hence
7. $\Pi''/_O U \& V \in W_P$ (O can bring about only the situations in 5 and 6).

---

3 This method of treating the case $\rightarrow L^c$ is also employed by G. Haas in his [HKS], p. 143. However, our manuscript was already in the press before we became acquainted with this paper.

8. $\Pi''/_P[U \& V] \in W_P$ (P can state U & V).
9. $\Pi''/_P\Gamma, [U \& V] \in W_P$ (P need not use the extra rights).

case v R^c $\Sigma = \Sigma; (\Pi/\Gamma, U \vee V)$ and reduces to $\Sigma; (\Pi/\Gamma, U \vee V, U, V)$. By our assumptions:

1. $\Pi/_P\Gamma, [U \vee V, U, V] \in W_P$.

Let P use its winning strategy for this situation also for $\Pi/_P\Gamma, [U \vee V]$. The only difficulty that may arise is that the strategy could prescribe the use of the right [U] or of the right [V] before the end of the first local discussion. Note that the strategy cannot prescribe the use of both these rights in the first local discussion, unless U = V and both rights coincide. For, once P has stated U or V, the first local discussion will end. Let us treat the case where the strategy prescribes the use of the right [U] in a situation $\Pi'/_P\Gamma, [U \vee V, U, V]$ which occurs in the first local discussion (the case for [V] is similar). The actual situation is now $\Pi'/_P\Gamma, [U \vee V]$, and possibly $U \in \Gamma$. In that case

2. $\Pi'/_O U \in W_P$. Hence
3. $\Pi'/U \vee V/_P[U, V] \in W_P$ (P can state U).
4. $\Pi'/_O U \vee V \in W_P$ (O must attack $U \vee V$).
5. $\Pi'/_P[U \vee V] \in W_P$ (P can state $U \vee V$).
6. $\Pi'/_P\Gamma, [U \vee V] \in W_P$ (P need not use the extra rights).

The other cases must be left to the reader●

Exercise

Complete the proof of Lemma 4.

7.6 [XI.6] Full circle

By combining the lemmas we now have a

Proof of Theorem 28

Let there be a closed semantic tableau for Π/Z.

(i) For classical systems:
By Lemma 3: $\Pi/_P[Z] \in W_P$.
By Lemma 2: $\Pi/_O Z \in W_P$.

(ii) For minimal and constructive systems, excepting the combination MNst–MND:
A closed tableau for Π/Z is a closed tableau for the set $\{\Pi/Z\}$. By Lemma 4 we can now defend that $\Pi/_P[Z] \in W_P$ (there being no other sequent in $\{\Pi/Z\}$ but Π/Z). By Lemma 1, $\Pi/_O Z \in W_P$.

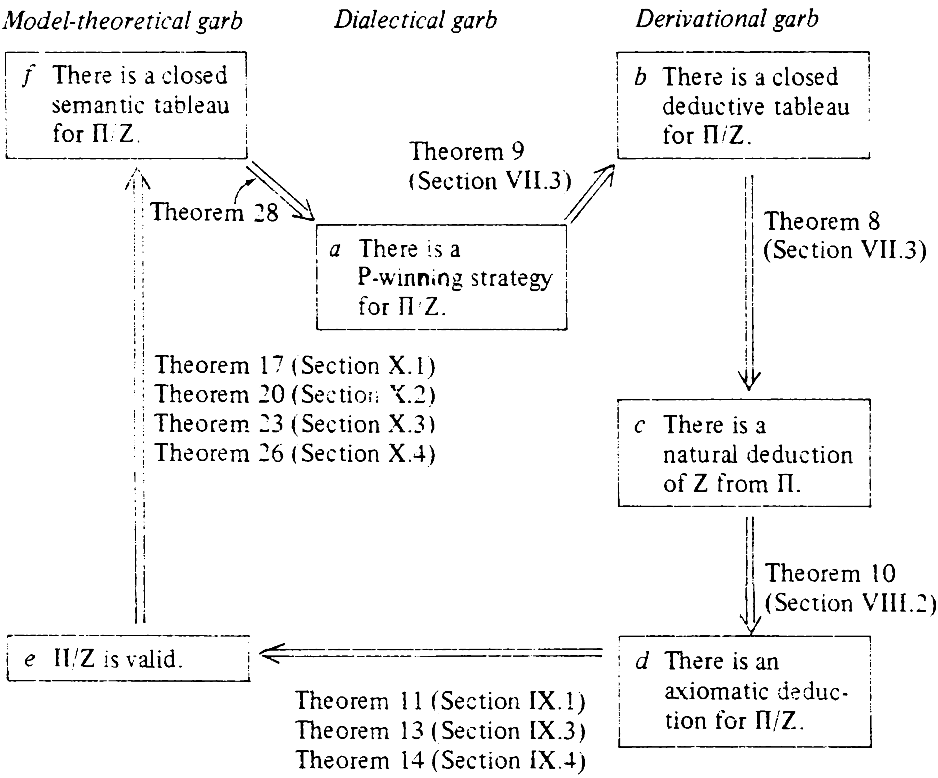
(iii) For MNst (= MΛst) and MND;
Since Theorem 28 has already been shown to hold for MΛst - MΛD, there is a P-winning strategy for $\Pi/_O Z$ in MΛD. By the equivalence of NOT-dialectics with Λ-dialectics (Theorem 4, Section V.5) there is a P-winning strategy for $\Pi/_O Z$ in MND as well●

Thus we have completed and closed a circle of theorems which together establish the unity of the garbs and methods of modern logic (as far as treated in this book):

Theorem 29 Let $\mathfrak{L}$ be a language of one of the forms $\mathfrak{J}$, $\mathfrak{T}$ or $\mathfrak{T}^{\Lambda}$. Let Π/Z be a sequent containing only sentences of $\mathfrak{L}$. The following conditions are equivalent, provided they refer to corresponding systems (i.e., systems whose abbreviated names start with the same two upper-case initials "MI", "CN", etc.) and that these systems pertain to $\mathfrak{L}$ (or $\mathfrak{L}_D$):

a There is a P-winning strategy for $\Pi/_O Z$.[1]
b There is a closed deductive tableau for Π/Z.
c There is a natural deduction of Z from Π.
d There is an axiomatic deduction for Π/Z.
e Π/Z is minimally/constructively/classically valid in $\mathfrak{L}$.[2]
f There is a closed semantic tableau for Π/Z.

Figure XI.1



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[1] See Note 1 to Section 1.
[2] Read "minimally" if the names of the systems begin with "M", "constructively" if they begin with "C" and "classically" if they begin with "K".

Proof See Figure XI.1 The arrows correspond to the implications among the conditions *a* through *f* that we have established. Each theorem mentioned is a (universal) conditional. In each case the arrow points from the antecedent of this conditional to its consequent●

Now that we have completed a full circle, the well-known theorem called by the names *Gentzen's Hauptsatz* and *(Cut-) Elimination Theorem* (as applied to tableau systems) follows at once:

Consequence (*Gentzen's Hauptsatz*)
Let $\mathfrak{L}$ be as in Theorem 29. Let U and Z be sentences of $\mathfrak{L}$ and let Π/Γ be a sequent containing only sentences of $\mathfrak{L}$. The following assertions hold, relative to the tableau systems or (non-material) dialectic systems pertaining to $\mathfrak{L}_D$ that were defined in this book:

a If $\Pi, U/_O Z \in W_P$ and $\Pi/_O U \in W_P$ then $\Pi/_O Z \in W_P$.
b If there are closed deductive tableaux for both Π, U/Z and Π/U, then there is one for Π/Z.
c If there are closed semantic tableaux for both Π, U/Γ and Π/U, Γ, then there is one for Π/Γ.

Proof It is easy to check that, if both Π, U/Γ and Π/U, Γ are (minimally, or constructively, or classically) valid, Π/Γ, too, is valid. Whence the assertions *a*, *b* and *c* may be derived by Theorem 29●

8. THE ADEQUACY OF MATERIAL DIALOGUE-GAMES

The concept of a *material dialogue-game** is explained by P. Lorenzen, by K. Lorenz, and, from a somewhat different point of view, by K. J. J. Hintikka.[1] Whereas in *formal* dialogues the formulas uttered are meaningless schemata, *material* dialogues are carried through in an interpreted language: their sentences—at least the elementary ones—may have truth-values, and these truth-values have their bearing on the possibilities of winning or losing. Each of the three authors mentioned has asserted, at least implicitly that his game is *adequate* in the following sense: there exists a winning strategy for the proponent of a thesis, iff this thesis is true according to classical semantical theory.[2] K. Lorenz's proof of *Hauptsatz 1* can be reinterpreted to establish the adequacy of his *reine* (*faktische*) *Dialogspiele*.[3]

In this paper I will present a rather general definition of "*material dialogue-game*", though one limited to games in which all the elementary sentences are either true or false. This definition makes it possible to state and prove a theorem asserting the *adequacy* with respect to any two-valued model theory $\mathfrak{M}$ of all material dialogue-games that have three properties to be explained shortly: *local finiteness*, *regularity*, and *accordance in logical rules with the particular model theory under consideration.* These, to my opinion, are properties a reasonable material dialogue-game should have. The proof of the theorem is straightforward, once its key-concept—that of a *P-favorable position* in a game—has been defined. The adequacy of most known material dialogue-games follows as special cases of the theorem.

8.1. *A definition of 'material dialogue-game'* Material dialogues must be held in a *language*. In the following, let $\mathfrak{L}$ be some fixed language, with sentences, *A*, *B*, *C*, . . ., some of them elementary. It is not required that

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$\mathfrak{L}$ be a language for sentential logic or for quantifier logic. Further, there must be *players*. We shall only consider games with two disputants, White(W) and Black(B). I will use 'P' as a variable over $\{W, B\}$, and denote P's adversary as $\overline{P}$.

A dialogue-game in $\mathfrak{L}$ shall be determined by its *positions*, and its *permitted moves*. These will now be treated in succession. A *position* x shall be a seven-tuple consisting of

(1): the *player*, P_x, whose turn it is at x.
(2): a *valuation*, v_x, of all the elementary sentences of $\mathfrak{L}$: For A elementary $v_x(A) = \mathbf{T}$ or $v_x(A) = \mathbf{F}$.
(3), (4): sets $\mathbf{A}(W)_x$ and $\mathbf{A}(B)_x$ of *assertions* already made by W and B before the current x was reached.
(5), (6): sets $\mathbf{D}(W)_x$ and $\mathbf{D}(B)_x$ of *defense sets*[4] of W and B at x.
(7): a *structural-rule function*, f_x, assigning natural numbers to assertions in $\mathbf{A}(W)_x$ and $\mathbf{A}(B)_x$, to defense sets in $\mathbf{D}(W)_x$ and $\mathbf{D}(B)_x$, and to elements of these latter sets (f_x may be empty or only partially defined).

By an *assertion* I mean a labelled sentence $\langle A, n\rangle$—where n is a natural number—; thus it makes sense to say that P has asserted A twice. Assertions $\langle A, n\rangle$ and $\langle B, m\rangle$ are *equiform* iff $A = B$. P's assertions represent possibilities of attack for $\overline{P}$. An attack usually provides the disputant whose assertion is attacked with some possible retorts; these constitute what I have called *defense sets*. A *defense set*, therefore, is defined to be a set of assertions. It is not necessary to introduce challenges, like "?" and "?n", as special components of the positions of the games, since their influence upon the situation is determined completely by the defense sets they introduce.[5] The *structural-rule functions* serve in formulating structural rules; more explanation will follow the definition of "material dialogue-game".

A *move* is an ordered pair $\langle x, y\rangle$, where x and y are positions and $P_x \neq P_y$. Whereas the set of positions is the same for all dialogue-games in $\mathfrak{L}$, the set of *permitted moves*, $\mathbf{R}$, also called the *game relation*, may be different for different games. Each game has its rules, and its moves should conform to them. The rules of a material dialogue-game, which determine its permitted moves, are of two kinds: the *logical rules* (*allgemeine Spielregel*), which determine the kinds of attack and the relevant retorts that may occur in the game, and the *structural rules* (*spezielle Spielregel*), which determine when and how often these kinds of attack and these retorts may be used in a particular *tournament*[6] of the game.[7]

Without loss of generality we may suppose that for each natural number i and each complex sentence A of $\mathfrak{L}$ it makes sense to speak of an attack of the i-th kind on A. If there are only finitely many(k)kinds of attack possible (as in the case of languages for sentential logic), we can let the attack of the i-th kind coincide with the attack of the k-th kind, for $i > k$. Universal sentences make good examples of sentences that may be attacked in infinitely many ways. Let A be a complex sentence of $\mathfrak{L}$, then we shall denote by $\alpha_i(A)$ the set of sentences that must be simultaneously asserted in an attack on A of the i-th kind. In most known games $\alpha_i(A)$ is empty or contains at most one sentence, e.g., $\alpha_1(A \rightarrow B) = \{A\}$, $\alpha_1(A \vee B) = \emptyset$, $\alpha_1((\forall x)A(x)) = \emptyset$; an attack on a "Shefferstroke-sentence" $A|B$ involves two sentences: $\alpha_1(A|B) = \{A, B\}$. By $\delta_i(A)$ we shall denote the set of relevant retorts, from which a player may pick one if he is attacked by an attack of the i-th kind on A, e.g., $\delta_1(A \rightarrow B) = \{B\}$, $\delta_1(A \vee B) = \{A, B\}$, $\delta_i((\forall x)A(x)) = \{A(a_i)\}$ (a_i the i-th individual constant), $\delta_1(A|B) = \emptyset$. A *logical rule* may now be defined as a set $\{\langle\alpha_i, \delta_i\rangle\}_{i\epsilon\omega}$ of pairs of functions, such that for each complex sentence A of $\mathfrak{L}$ and for each natural number i $\alpha_i(A)$ and $\delta_i(A)$ are (possibly empty) sets of sentences of $\mathfrak{L}$.

Let $\mathbf{L} = \{\langle\alpha_i, \delta_i\rangle\}_{i\epsilon\omega}$ be a logical rule; we shall then say that the move $\langle x, y\rangle$ *conforms to* $\mathbf{L}$ *as an attack of the i-th kind on* the complex sentence A, iff for some $n\langle A, n\rangle \in \mathbf{A}(\overline{P}_x)_x$ and the only differences between x and y—except that, by definition of move, $P_y \neq P_x$—concern:

(1) the set of assertions of P_x; here assertions (each with a label not occurring in x) corresponding to all the sentences in $\alpha_i(A)$—if any—are added to $\mathbf{A}(P_x)_x$ in order to obtain $\mathbf{A}(P_x)_y$.
(2) the set of defense sets of $\overline{P}_x$; here exactly one defense set containing assertions (each with a label not occurring in x) corresponding to all the sentences—if any—in $\delta_i(A)$ is added to obtain $\mathbf{D}(\overline{P}_x)_y$.
(3) the structural rule function.

In addition to attacks on complex sentences conforming to the logical rule, we may have *attacks on elementary sentences* and *defense moves*. A move $\langle x, y\rangle$ is said to constitute an *attack on the elementary sentence A*, iff A is elementary, and for some $n\langle A, n\rangle \in \mathbf{A}(\overline{P}_x)_x$, and the only differences between x and y—except that $P_y \neq P_x$—concern:

(1): the set of defense sets of $\overline{P}_x$; here a defense set $\{\langle A, m\rangle\}$ (where m does not occur in x) is added to obtain $\mathbf{D}(\overline{P}_x)_y$.
(2): the structural function.

A *defense move* $\langle x, y\rangle$ consists in adding exactly one assertion (with a label not occurring in x) equiform to an element of a defense set in $\mathbf{D}(P_x)_x$ to $\mathbf{A}(P_x)_x$ in order to form $\mathbf{A}(P_x)_y$; further, the structural rule function may undergo some changes in this case as well. It is not excluded that a move belongs to several of these types at once.

Using the vocabulary explained above we define a *material dialogue-game* as an ordered pair

$$\mathbf{G} = \langle \mathbf{L_G}, \mathbf{R_G}\rangle, \text{ such that}$$

(1) $\mathbf{L_G}$ is a logical rule.
(2) $\mathbf{R_G}$ is a set of moves.
(3) if $\langle x, y\rangle \in \mathbf{R_G}$, then either

(i) $\langle x, y\rangle$ is an attack on a complex sentence comforming to $\mathbf{L}_{\mathbf{G}}$,

or

(ii) $\langle x, y\rangle$ is an attack on an elementary sentence,

or

(iii) $\langle x, y\rangle$ is a defense move.

Positions not in the domain of $\mathbf{R}_{\mathbf{G}}$ will be called *end positions* of **G**. If x is an end position of **G** and a tournament of **G** ends at x, then P_x will be said to have *lost* the tournament and $\overline{P}_x$ will be said to have *won* the tournament. We do not admit draws. There is no further loss of generality: all games we are interested in can be brought into this form, if necessary by introducing some dummy moves.

Of course, given a dialogue-game **G**, a move may fall under one of (3), (i)-(iii) and yet fail to be a permitted move of **G**; any further restrictions put on $\mathbf{R}_{\mathbf{G}}$ may be said to belong to the structural rule. Such restrictions can be formulated in terms of the numbers assigned to the assertions and defense sets by the structural rule functions. For instance, if you want to allow three attacks on each assertion, and no more, the number $f_x(\langle A, n\rangle)$ may indicate how many attacks are still allowed; this number should be three at the introduction of $\langle A, n\rangle$ in the tournament and go down by one each time an attack is made on $\langle A, n\rangle$; $f_x(\langle A, n\rangle) = 0$ may indicate that the assertion is "dead", that is, that it can no longer be used. Or again, if you want the game to be over as soon as B asserts a true elementary sentence, all you have to do is this: consider the moves $\langle x, y\rangle$ that consist of the positing of a true elementary sentence by B, and permit only those that have a structural rule function f_y such that $f_y(Q) = 0$ for all assertions and defense sets Q in y, where 0 indicates that Q is dead. It should be noticed that the valuation of the elementary sentences of $\mathfrak{L}$ remains fixed during each tournament in **G**: words should not change their meaning in the course of a discussion.

8.2. *Conditions a reasonable material dialogue-game should fulfill* The definition of material dialogue-games given above is rather wide, and it is not to be expected that all games conforming to it be adequate with respect to a certain given model theory, or even that they be intuitively acceptable in any sense whatever. I will now discuss the conditions that dialogue-games must fulfill in order to be called reasonable.

First, the structural rules should not be too liberal. It seems reasonable not to allow any disputant to let the discussion drag on without an ending; therefore, stipulations to prevent this should be part of the structural rule. If all the tournaments of a game end after a finite number of moves, the game is called *locally finite.*[8] Thus, the first condition a reasonable dialogue-game must fulfill is that it be locally finite.

Second, the structural rules should not be too stringent. All problems statable in the language should be discussable. The structural rule should not prevent a dialogue on a certain problem to get started at all. By a *problem* I here mean an ordered quadruple $\langle L, R, \mathsf{v}, P\rangle$, where L stands for a set of sentences to be defended by B and R for a set of sentences to be defended by W, and where v is a valuation of the elementary sentences and P is the player making the first move. Let a position x be, by definition, a *starting position* of a material dialogue-game **G**, iff

(1) $\mathbf{D}(W)_x = \mathbf{D}(B)_x = \emptyset$
(2) if $\langle A, n\rangle \in \mathbf{A}(\overline{P}_x)_x$, and it is not the case both that A is elementary and that $\mathsf{v}_x(A) = \mathbf{T}$, then the structural rule of **G** permits P_x to attack A in the next move. If A is complex, attacks of any kind provided by $\mathbf{L_G}$ are permitted.

All reasonable material dialogue-games should provide starting positions for all problems, i.e., they must fulfill the following condition:

(1) For any problem $\langle L, R, \mathsf{v}, P\rangle$ there exists a starting position x of **G**, such that

(a) $A \in L (A \in R)$ iff there is a n such that $\langle A, n\rangle \in \mathbf{A}(B)_x (\langle A, n\rangle \in \mathbf{A}(W)_x)$
(b) $\mathsf{v}_x = \mathsf{v}$
(c) $P_x = P$

Such a position x will be called a *starting position in* **G** *for* $\langle L, R, \mathsf{v}, P\rangle$.

There is another respect in which the structural rule must not be too stringent: disputants should have a right of immediate response. If one of P's assertions has been attacked, P should be allowed to produce a relevant retort in the next move, and if $\overline{P}$ makes an assertion, P should be allowed to attack that assertion in the next move. However in some situations this right of immediate response should be cancelled in order not to clash with the condition of local finiteness. Hence, we put the following two conditions on a reasonable game **G**.

(2) If $\langle x, y\rangle \in \mathbf{R_G}$ and this move introduces a new defense set containing an assertion $\langle A, n\rangle$ into the set of defense sets of P_y, and it is not the case both that A is elementary and that $\mathsf{v}_x(A) = \mathbf{F}$, then P_y may defend himself in the next move by an assertion $\langle A, m\rangle$ (where m is a lable not in y).
(3) If $\langle x, y\rangle \in \mathbf{R_G}$ and this move introduces a new assertion $\langle A, n\rangle$ into the set of assertions of P_x, and it is not the case both that A is elementary and that $\mathsf{v}_x(A) = \mathbf{T}$, then P_y may attack A in the next move; in case A is complex, P_y may use any kind of attack provided by the logical rule $\mathbf{L_G}$.

A game in which the structural rule is not too stringent, that is, a game fulfilling conditions (1), (2), and (3), will be called *regular*.[9]

Thirdly, the *logical rule* should be in accord with a choice of logical constants in $\mathfrak{L}$ and with the meanings of these logical constants.

These constants and their meanings are given by a *model theory* $\mathfrak{M}$. There may be several such model theories for $\mathfrak{L}$. Each model theory provides models M, N, . . . based on interpretations of the non-logical constants of $\mathfrak{L}$. The internal structures of model theories and of models do not concern us here, but it will be assumed that we are dealing with *two-valued model theories*, i.e., that with each model M of a theory $\mathfrak{M}$ there is associated a valuation v_M of *all* the sentences of $\mathfrak{L}$, assigning truth or falsity to them:

$$v_M(A) = \mathbf{T} \text{ or } v_M(A) = \mathbf{F}.$$

A reasonable logical rule should, for each false sentence, provide a mode of attack that is both honest and ruthless, i.e., such that only true assertions need to be made in the attack and such that all permitted retorts are false; for a true sentence the logical rule should not provide such a mode of attack. A logical rule meeting this condition will be said to be *in accord with* the model theory concerned. More precisely, a logical rule $\mathbf{L} = \{\langle \alpha_i, \delta_i \rangle\}_{i \in \omega}$ is said to be *in accord with* a model theory $\mathfrak{M}$, iff for every model M of $\mathfrak{M}$ and for every complex sentence A of $\mathfrak{L}$:

(1) if $v_M(A) = \mathbf{F}$, then there is a natural number i and a pair $\langle \alpha_i, \delta_i \rangle \in \mathbf{L}$ such that

(a) for all $B \in \alpha_i(A)$: $v_M(B) = \mathbf{T}$
(b) for all $B \in \delta_i(A)$: $v_M(B) = \mathbf{F}$

(2) if $v_M(A) = \mathbf{T}$ and i is a natural number and $\langle \alpha_i, \delta_i \rangle \in \mathbf{L}$, then either there is a $B \in \alpha_i(A)$ such that $v_M(B) = \mathbf{F}$, or there is a $B \in \delta_i(A)$ such that $v_M(B) = \mathbf{T}$.

This concludes my discussion of the properties a reasonable material dialogue-game should have; the adequacy theorem can now be formulated.

8.3 *The adequacy theorem and its proof*

Adequacy theorem *Let* **G** *be a material dialogue-game (in a language* $\mathfrak{L}$*) that is both locally finite and regular. Let* $\mathfrak{M}$ *be a two-valued model theory for* $\mathfrak{L}$*, such that* $\mathbf{L}_G$ *is in accord with* $\mathfrak{M}$*. If* M *is a model of* $\mathfrak{M}$ *and x is a starting position of* **G** *for* $\langle L, R, v, P\rangle$*, such that* v *and* v_M *agree on elementary sentences of* $\mathfrak{L}$*, then*

(a) *if for all assertions* $\langle A, n\rangle \in \mathbf{A}(P)_x$ *it holds that* $v_M(A) = \mathbf{T}$*, and if for some assertion* $\langle A, m\rangle \in \mathbf{A}(\overline{P})_x$ *it holds that* $v_M(A) = \mathbf{F}$*, then there is a winning strategy for P in x.*
(b) *if for all assertions* $\langle A, n\rangle \in \mathbf{A}(\overline{P})_x$ *it holds that* $v_M(A) = \mathbf{T}$*, then there is a winning strategy for* $\overline{P}$ *in x.*
(c) *if* $L = \emptyset$*, and* $R = \{A\}$*, and* $P =$ B *(in this case* B *may be called opponent and* W *proponent of the thesis A under the empty set of assumptions), then there is a winning strategy for* W *in x iff* $v_M(A) = \mathbf{T}$ *(otherwise, if* $v_M(A) = \mathbf{F}$*, there is a winning strategy for* B *in x).*

Proof: Part (c) of the theorem expresses what may be called "simple adequacy", and follows from (a) and (b) and the fact that not both W and B can have a winning strategy in the same position. To prove (a) and (b) we need to define the concept of a *P-favorable position.* It will be obvious from this definition (see below) that the positions described under (a) and (b) are P- and $\overline{P}$-favorable (with respect to M) respectively: condition (1) of the definition is fulfilled by hypothesis, and condition (3) is trivial, since $\bigcup(\mathbf{D}(P)_x) = \bigcup(\mathbf{D}(\overline{P})_x) = \emptyset$, as is condition (4) in the case of (b), since $P_x \neq \overline{P}$; condition (2) is fulfilled in virtue of the text under (a) and (b) above, condition (4)—in the case of (a)—is fulfilled because it is given that for some assertion $\langle A, m\rangle \in A(\overline{P})_x$ it holds that $v_M(A) = \mathbf{F}$ and because attacks on this assertion are permitted, since x is a starting position. The theorem follows then from the lemma, stated below, that a player P has a winning strategy for any P-favorable-position. Q.E.D.

Definition of P-favorable (with respect to M and **G**): A position x will be said to be *P-favorable with respect to a two-valued model* M *and a material dialogue-game* **G**, iff

(1) v_M and v_x agree on elementary sentences.
(2) if $\langle A, n\rangle \in \mathbf{A}(P)_x$, then $v_M(A) = \mathbf{T}$.
(3) if $\langle A, n\rangle \in \bigcup(\mathbf{D}(\overline{P})_x)$, then $v_M(A) = \mathbf{F}$.
(4) if $P_x = P$, then either there is an $\langle A, n\rangle \in \mathbf{A}(\overline{P})_x$ such that $v_M(A) = \mathbf{F}$ and attacks of all kinds on A are permitted by the structural rule of **G** as P's next move, or there is an $\langle A, n\rangle \in \bigcup(\mathbf{D}(P)_x)$, such that $v_M(A) = \mathbf{T}$ and defense by means of an assertion $\langle A, m\rangle$ is permitted by the structural rule of **G** as P's next move.

Lemma *Let* **G**, $\mathfrak{M}$, *and* M *fulfill the conditions of the Theorem. If x is P-favorable with respect to* M *and* **G**, *then there is a winning strategy for P in x.*

Proof: The set of P-favorable positions with respect to M constitutes a *pseudo-cycle* for P,[10] that is: once a tournament has moved into a P-favorable position (with respect to M), P can keep it that way and $\overline{P}$ cannot make the situation not P-favorable. The proof of this proceeds by cases:

A: $P_x = P$

A1: There is an assertion $\langle A, n\rangle \in \mathbf{A}(\overline{P})_x$, such that $v_M(A) = \mathbf{F}$ and attacks by P on A are permitted.

A1.1: A is elementary: If P attacks A in a move $\langle x, y\rangle$, then y will be P-favorable, since only a set $\{\langle A, m\rangle\}$ will have been added to $\mathbf{D}(\overline{P}_x)_x$ in order to form $\mathbf{D}(\overline{P}_x)_y$; condition (3) of the definition of "P-favorable" will be satisfied, for $v_M(A) = \mathbf{F}$; condition (4) will be satisfied trivially.

A1.2: A is complex. Since $\mathbf{L_G} = \{\langle \alpha_i, \delta_i\rangle\}_{i\in\omega}$ is in accord with $\mathfrak{M}$, and M is a model of $\mathfrak{M}$, there is a permitted attack move $\langle x, y\rangle$ and a natural number i, such that $\langle x, y\rangle$ conforms to $\mathbf{L_G}$ as an attack of the i-th kind on A, and such that for all $B \in \alpha_i(A)$: $v_M(B) = \mathbf{T}$ and for all $B \in \delta_i(A)$: $v_M(B) = \mathbf{F}$. Such a y is P-favorable again.

A2: There is no such assertion. Then, by condition (4), there must be an $\langle A, n\rangle \in \bigcup(\mathbf{D}(P_x)$ such that $v_M(A) = \mathbf{T}$ and a defense move $\langle x, y\rangle$ is permitted consisting of adding an assertion $\langle A, m\rangle$ to $\mathbf{A}(P_x)_x$. Obviously, y is P-favorable again.

B: $P_x = \overline{P}$

B1: $\overline{P}$ cannot move and loses the tournament.

B2: $\overline{P}$ can move. Say he moves by $\langle x, y \rangle$. Then $\langle x, y \rangle$ must be an attack on a true sentence or a defense move using a false sentence. It is trivial that y fulfills the conditions (1) through (3) of the definition of "P-favorable".

B2.1: $\langle x, y \rangle$ is an attack on an elementary sentence A. $v_M(A) = \mathbf{T}$. This will give P a defense set $\{\langle A, m \rangle\}$; since **G** is *regular* he may use this defense set in his next move, hence condition (4) is fulfilled.

B2.2: $\langle x, y \rangle$ is an attack on a complex sentence A. $v_M(A) = \mathbf{T}$. Since this attack must be conforming to the logical rule, it will consist of adding assertions corresponding to elements of a set $\alpha_i(A)$ to $\mathbf{A}(P_x)_x$ and adding a defense set containing assertions corresponding to the elements of a set $\delta_i(A)$ to $\mathbf{D}(\overline{P}_x)_x$. $\mathbf{L_G}$ is in accord with $\mathfrak{M}$, hence either there is a $B \in \alpha_i(A)$ such that $v_M(B) = \mathbf{F}$, and P may (by regularity) attack the corresponding assertion in the next move, or there is a $B \in \delta_i(A)$ such that $v_M(B) = \mathbf{T}$, and P may (by regularity) use B in a defense move. In either case condition (4) has been fulfilled by y.

B2.3: $\langle x, y \rangle$ is a defense move using a false sentence. Then $\overline{P}$ has to add a false assertion to $\mathbf{A}(\overline{P})_x$ in order to obtain $\mathbf{A}(\overline{P})_y$. By regularity, P may attack that assertion in the next move, hence y fulfills condition (4) in this case as well.

Thus the set of P-favorable positions with respect to M constitutes a pseudo-cycle for P. Moreover, in virtue of condition (4), this set does not contain any end positions z such that $P_z = P$, hence no end positions with loss for P. Since **G** is *locally finite* it follows that there is a winning strategy for P in every position of the set. (Indeed this strategy has been described, implicity, in this proof and boils down to attacking falsehoods and telling the truth). Q.E.D.

8.4 *Final remarks* The material games in [3] and [5], referred to in footnote 1, are special cases to which the adequacy theorem applies. The same holds for the material games in [4], if they are modified as follows:

(1) For the opponent a rule for winning the game should be instituted that is exactly analogous to that already present for the proponent.[11]
(2) In all games (except the *streng-konstruktive* one) *Wiederholungs-~~schranke~~*, present in the first edition of [4], should be restored. Schranken

Since *regularity* is a rather weak condition to be set upon the structural rules, we may conclude that the particular form of these rules is largely irrelevant for the existence of winning strategies in material dialogue-games—this in contradistinction to the situation in formal dialogue-games. There seems to be no smooth connection between the material and the formal games.[12]

It remains an open problem if, and how, the adequacy theorem can be extended to cover many-valued models.[13]

NOTES

1. P. Lorenzen in Kamlah and Lorenzen [4], Ch. VII, esp. p. 221 (Ch. VII is Lorenzen's); K. Lorenz in [5] (*faktische Dialogspiele*, p. X); K. J. J. Hintikka in [3].

2. P. Lorenzen in [4], p. 219: "Bei Beschräkung auf Junktoren und auf wahrheitsdefinite Primaussagen gilt darüber hinaus, dasz jede tautologisch-wahre Aussage stets konstruktiv dialogisch verteidigbar ist". K. Lorenz in [5], p. 44 (quoted in note (3)), also in [6], p. 92. K. J. J. Hintikka in [3], pp. 68, 69: "The following observation has struck me as being especially suggestive: There is a very close connection between the concept of a truth-value of a sentence and the game-theoretical concept of the value of the correlated game. If I have a winning strategy the value of the game is the payoff of winning, i.e., the "value" of winning the game. This is also precisely the case in which the sentence is true. Hence the payoff of winning as a value of the game can be identified with the truth-value "true" of the sentence, and correspondingly for falsity".

3. Lorenz in [5], p. 40 ff. On p. 44 Lorenz remarks: "Der Halbformalismus Ω_r ist unter dem Namen semantischer Halbformalismus bereits bekannt. Er definiert nichts anderes als die übliche klassische Zuordnung der beide Wahrheitswerte w_r und f_r zu logisch zusammengesetzten Aussagen C auf Grund ihrer Zuordnung zu den direkten Teilaussagen von C". *Hauptsatz 1* says that there is a winning strategy for the proponent (opponent) of a sentence A in a *reines Dialogspiel mit entscheidbarer Basis* if $\prec A$ ($A \prec$) is provable in Ω_r. Thus the existence of winning strategies is connected with provability in Ω_r, and Ω_r is connected with semantics.

4. It is possible to define dialogue-games with either assertions or sets only and to consider an assertion as a special kind of set or the other way around. This has been done by Drieschner in [2].

5. This has been remarked by F. van Dun in [8], p. 107.

6. Stegmüller, [7], p. 84: "We distinguish between a *game* as a type and a 'concrete performance' of the game or a *tournament*". My use of the words "game" and "tournament" agrees with Stegmüller's.

7. For structural vs. logical rules cf. Lorenz [5], p. 15, 20, and Stegmüller [7] p. 85 ff.

8. Locally finite = *localement fini* = *partienendlich*. *Cf.* Berge [1] p. 24.

9. My use of the word "regular" partially conforms to that of K. Lorenz in [5], p. 20.

10. *Cf.* Berge [1], p. 20.

11. Lorenzen in [4], p. 213.

12. Lorenz [5], p. XI: "Die reine Logik ist leer". That is: Lorenz's simplest type of material dialogue-games (*reine Dialogspiele*) does not lead in any straightforward way to equally simple formal games.

13. (*Added 1977*); Connections between dialogue-theory and many-valued logics have been studied by R. Giles, e.g. in [9], esp. p. 411.

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9. [IX.2.] The adequacy of material dialectics

In this section we shall link the semantics in the preceding section with the material dialectics treated in Section IV.5.2 (MatDial).[1] The crucial concept needed to span this conceptual bridge is that of an N-*favorable* dialogue situation. We shall first define this concept and then show that there is a winning strategy for N (N is either *White* or *Black*) for all dialogue situations that are N-favorable. In the following we shall assume that the company has chosen a language of the form $\mathfrak{I}$ as well as truth values **T** and **F**. For each choice of material procedures the company may make, we shall now assume that $\mathbb{T}$ (the class of implicitly accepted atomic sentences) and $\mathbb{F}$ (the class of implicitly rejected atomic sentences) together *exhaust* the class of atomic sentences (cf. Section IV.5.2). An agreement about material procedures, and hence about a class $\mathbb{T}$ (and $\mathbb{F}$) will then, in a natural way, lead to an interpretation I and hence a model $M = \langle \mathbf{T}, \mathbf{F}, I \rangle$:

---

[1] A general treatment of the connection between kinds of material dialectics and model-theoretic semantics is found in Krabbe [AMD]. We here restrict ourselves to the formal$_3$ material dialectic system MatDial of Paper 3, Section 2.

Def. 4 Suppose that the company has chosen some specific material procedures, and that $\mathbb{T}$ is the class of implicitly accepted atomic sentences. Let M be the classical model such that for each atom U:

(i) if $U \in \mathbb{T}$, then $I_M(U) = \mathbf{T}$
(ii) if $U \in \mathbb{F}$ (i.e., $U \notin \mathbb{T}$), then $I_M(U) = \mathbf{F}$.

A dialogue situation[2] of MatDial, with $\mathbb{T}$ as its class of implicitly accepted sentences, is N-*favorable* if and only if:

(1) each unattacked statement made by N is a statement of a sentence U such that $v_M(U) = \mathbf{T}$;
(2) each sentence that $\bar{N}$ can use in a structural protective defense of an attacked statement is a sentence U such that $v_M(U) = \mathbf{F}$. Moreover, there is no opportunity for $\bar{N}$ to make a *Verum dixi!*-remark;
(3) if N is to be the speaker at the next stage: either there is an unattacked statement, made by $\bar{N}$, of a sentence U such that $v_M(U) = \mathbf{F}$; or there is a sentence U that can be used by N to exercise a protective defense right (possibly of the form [U!!]) such that $v_M(U) = \mathbf{T}$.

Note that, if N is not to be the speaker at the next stage, (1) and (2) suffice to make a situation N-favorable.

In N-favorable situations N can win simply by attacking falsehoods and telling the truth:

Lemma 2 There is a winning strategy for N for each N-favorable dialogue situation.

Proof If it is N's turn to move, the clause under (3) guarantees that there is something N can do. So N cannot be the loser as long as the situation is N-favorable. We shall show that N can make sure that from an N-favorable situation only N-favorable situations will issue, and that $\bar{N}$ cannot prevent this. Since each chain of arguments is bound to end after a finite number of stages (Section V.4, Lemma 5), N can in this way make sure of winning the chain of arguments.
It therefore suffices to establish the following:
For each N-favorable situation:

(i) if it is N's turn to move, then there is always an opportunity for N to make a move such that afterwards the situation is again N-favorable;
(ii) if it is $\bar{N}$'s turn to move, then there is no opportunity for $\bar{N}$ to make a move such that afterwards the situation is no longer N-favorable.

---

[2] A dialogue situation is the total constellation of obligations and rights *within* a chain of arguments (Def. 20*a* of Section III.15), as represented by a sequent:

$$\vec{\Pi}; \vec{\Delta}/\mathbb{T}/_N \vec{\Phi}; \vec{\Gamma}$$

(end of Section V.1, Def. 2). For simplicity, we usually omit reference to chains of arguments in this section. For instance, we say "unattacked statement made by N" instead of "statement made by N in the current chain of arguments and not yet attacked in that chain".

Consider any N-favorable situation. Let $\mathbb{T}$ and M be as in Def. 4.

ad (i) Since it is N's turn to move, (3) applies. So there is either a false statement[3] for N to attack or a true statement with which N is entitled to defend one of its former statements (or N is in a position to make a *Verum dixi!*-remark). We must check, if (a) N attacks a false statement in the right way or (b) defends by means of a true one, that the situation will remain N-favorable, i.e., will afterwards satisfy (1) and (2) (it being then $\bar{N}$'s turn to move).

(a) N attacks a false statement.

(a→) Let this be a statement of $U \to V$. $v_M(U \to V) = \mathbf{F}$, so by *Sem* → (Lemma 1): $v_M(U) = \mathbf{T}$ and $v_M(V) = \mathbf{F}$. In its move, N adds a true statement (of U) to its other true statements, and adds to other such rights on $\bar{N}$'s side a right to defend by means of a false sentence (V). So the situation after N's move is again N-favorable.

(a&) Let N attack a statement of U & V. $v_M(U \,\&\, V) = \mathbf{F}$, so, by *Sem* &, either $v_M(U) = \mathbf{F}$ or $v_M(V) = \mathbf{F}$. In the first case N should attack by L?, in the second, by R? (if both U and V are false, it doesn't matter). Clearly such an attack merely adds to other such rights on $\bar{N}$'s side a right to defend by means of a false sentence. So, if N attacks the statement of U & V in the right way, the situation after N's move is again N-favorable.

Consideration of the other cases under (a) is left to the reader.

(b) N defends by means of a true sentence. Since only a true statement is added to N's other true statements, the situation will remain N-favorable. (The same holds if N makes a *Verum dixi!*-remark.)

ad (ii) If $\bar{N}$ can move at all, it can only (c) attack a true statement on N's side or (d) defend by means of a false sentence. In neither case will the supply of N's statements or of $\bar{N}$'s defense rights be augmented; so conditions (1) and (2) of Def. 4 will certainly continue to be fulfilled. Consider condition (3).

(c) $\bar{N}$ attacks.

(c→) $\bar{N}$ attacks a statement of $U \to V$. $v_M(U \to V) = \mathbf{T}$, hence, by *Sem* →, $v_M(U) = \mathbf{F}$ or $v_M(V) = \mathbf{T}$. In the first case $\bar{N}$ makes a false statement that can be attacked by N in the next move. In the second case $\bar{N}$ grants to N a right to defend itself by means of a true sentence. In both cases condition (3) is fulfilled.

[3] By a "false statement", we mean, of course, a statement of a sentence U such that $v_M(U) = \mathbf{F}$, etc.

Consideration of the other cases under (c) is left to the reader.

(d) $\bar{N}$ defends.
In that case $\bar{N}$ will make a false statement, and this statement can be attacked by N in the next move●

The following theorem, which states the "adequacy" of MatDial relative to classical semantics, is an immediate consequence of Lemma 2. We shall use the shorthand notation introduced at the end of Section V.1 and hence write $\Pi/\mathbb{T}/_B Z$ for $\langle\bar{\Pi}, \emptyset, \mathbb{T}, B, \langle Z\rangle, \emptyset\rangle$, i.e., the situation at the start of a material discussion in which Black (B) has made statements of the sentences in Π and White (W) has stated Z only.

Theorem 12 (*Adequacy Theorem*)

a White has a winning strategy for $\emptyset/\mathbb{T}/_B Z$ in the formal_3 material dialectic system MatDial if and only if Z is true in the classical model M in which all of the implicitly accepted atomic sentences (sentences in $\mathbb{T}$) are true, and all of the implicitly rejected atomic sentences (sentences in $\mathbb{F}$) are false.

b The following conditions are equivalent:

1 For all $\mathbb{T}$: White has a winning strategy for $\Pi/\mathbb{T}/_B Z$ in MatDial.

2 Π/Z is classically valid.

Proof a If $v_M(Z) = \mathbf{T}$, the situation $\emptyset/\mathbb{T}/_B Z$ is W-favorable. So W has a winning strategy by Lemma 2. If $v_M(Z) \neq \mathbf{T}$ then $v_M(Z) = \mathbf{F}$ and the situation $\emptyset/\mathbb{T}/_B Z$ is B-favorable. So, again by Lemma 2, there is a winning strategy for B. In that case, of course, there cannot also be a winning strategy for W.

b Suppose that Π/Z is not classically valid. Let M be a classical counter-example to Π/Z:

$v_M(U) = \mathbf{T}$ for all $U \in \Pi$
$v_M(Z) = \mathbf{F}$.

Let $\mathbb{T} =_{Df} \{U | U$ is an atom and $I_M(U) = \mathbf{T}\}$
Then $\Pi/\mathbb{T}/_B Z$ is B-favorable, so there is, by Lemma 2, a winning strategy for B and hence no winning strategy for W. Conversely, suppose that Π/Z is classically valid. Let $\mathbb{T}$ be any set of atoms. Let M be the classical model with, for all atoms U:

$I_M(U) = \mathbf{T}$ if and only if $U \in \mathbb{T}$.

If $v_M(Z) = \mathbf{T}$ then $\Pi/\mathbb{T}/_B Z$ is W-favorable, so there is a winning strategy for W by Lemma 2.
If $v_M(Z) = \mathbf{F}$, there must be some $U \in \Pi$ such that $v_M(U) = \mathbf{F}$, for otherwise M would be a counter-example to Π/Z. $\Pi/\mathbb{T}/_B Z$ is not itself W-favorable, but the situation issuing from B's first move (an attack on Z) will be W-favorable (since the false statement of Z disappears from W's supply of unattacked statements, and W may attack a false statement [of U] in the next move). So, by Lemma 2, W has a winning strategy for all situations that may follow $\Pi/\mathbb{T}/_B Z$ and hence for $\Pi/\mathbb{T}/_B Z$●

Exercises

1. Complete the proof of Lemma 2.
2. With the help of Theorem 12, try to determine for which sequents *a* through *i* in Exercise 2 of Section IV.5 and Exercise 6 of Section V.2 there exists a W- (B-) winning strategy in material dialectics.
3. *a* An analogue of the adequacy theorem holds for the $formal_3$ dialectic systems, with material procedures subjoined to them, of Section IV.5.1 (except for MND). Assume that $\mathbb{T}$ and M are as in Definition 4 and that $\Lambda \in \mathbb{F}$. A situation will be called P-*favorable* if and only if
   (i) when it is O's turn to move:
      a P's last statement is true (in M)
      b each statement O may make by virtue of a structural protective defense right is false;
   (ii) when it is P's turn to move:
      at least one of O's statements is false, or else P can exercise a structural protective defense right by stating some true sentence, or else P can make a winning remark.
   Show that the analogue of Lemma 2 holds (unless the system concerned is MND with material procedures subjoined to it).

   b A situation will be called O-*favorable* if and only if:
   (i) all O's statements in the chain of arguments are true (in M);
   (ii) the local thesis is false;
   (iii) each sentence that P may use in a structural protective defense of the local thesis is false;
   (iv) when it is O's turn to move, then either O may attack a false statement made by P, or O may make a true statement by virtue of some structural protective defense right.
      Again show that the analogue of Lemma 2 holds.

   c Formulate and prove an adequacy theorem for the systems of Section IV.5.1 (except MND . . .). Assume that $\mathbb{T}$ and $\mathbb{F}$ exhaust the set of atoms.

PART 3

MODALITY

10. Noncumulative Dialectical Models and Formal Dialectics

10.0. Introduction *)

In [AD1] Barth and Krabbe advance a new philosophical motivation (intuitive interpretation) for Kripke's semantics for constructive (intuitionistic) and minimal logic.(1) Kripke intends the nodes (elements) **H**, **H**' of a set **K** (of a model structure) to represent (possible) evidential situations of ours.(2) Further, he suggests that we read **H**$\mathcal{R}$**H**' as follows: in situation **H** we may, as far as we know, advance to situation **H**'.(3) In [AD1], on the other hand, the elements d, d' of the set D (corresponding to Kripke's **K**) in a dialectical structure represent possible dialectical situations of a specifiable dialectical subject,(4) and dRd' is read accordingly. An evidential situation is characterized, at least partly, by the set of sentences verified or verifiable by us in that situation. Hence the "values" assigned to sentences (by a model) represent the predicates "verified" and "not verified".(5) A dialectical situation is characterized, at least partly, by the set of sentences upon which positive agreement has been reached by a dialectical subject in that situation. Therefore the "values" assigned to sentences (by an interpretation/valuation) are these:

A, for Agreement

N, for Non-agreement(6)

There is no reason why there should be just one acceptable intuitive interpretation for any particular structure of semantics. Adoption of the dialectical interpretation does not commit one to a rejection of Kripke's own "monological" interpretation: the plausibility of both interpretations can be independently criticized.

The following (preservative/cumulative) property of models is debatable under both intuitive interpretations:

If v_M (U,d) = **A** and if dRd', then also v_M(U,d') = **A**.(7)

Under Kripke's interpretation, this expresses the Principle of Preservation/Cumulation of Information or Knowledge: "*we* don't forget".[8] Under the dialectical interpretation, we confront an isomorphous principle, the Principle of Preservation/Cumulation of Agreement. Both principles are strong idealizations for, of course, often we do forget, and often we come to disagree anew on some previously settled issue. The Principle of Cumulation of Information is plausible only under special circumstances, for instance, if *we* are an intuitionist mathematician proving more and more theorems.[9] In [AD1] it is argued that the Principle of Cumulation of Agreement is a realistic assumption *with respect to some companies* (dialectical subjects) and *for a limited period of time*.

What happens if we drop this principle? That is the first question I want to answer in this paper. So I am looking for a theory of dialectical models to encompass the behavior of companies that may relapse into disagreement on formerly agreed issues, i.e., a theory of noncumulative dialectical models.[10] Constructive and minimal variants of noncumulative dialectical models will be found in Section 1.

The next question I wish to pursue is whether, and how, these semantic theories can be connected with an acceptable formal dialectics. This leads to an exploration of *modal dialectics* in Sections 2 and 3. The central notions here are (i) a distinction between "contingent" and "strict" statements in terms of debaters' rights; and (ii) the Opponent's right to withdraw contingent concessions under appropriate circumstances. I shall continue the study of modal dialectics in another paper.[11]

In the fourth section I shall briefly consider noncumulative deductions and show that a "full-circle theorem" holds for noncumulative logics, i.e., that the semantic, the ~~formal dialectic~~, and the deduction--theoretic approaches yield the same logics.

dialogical

10.1. Noncumulative semantics and semantic tableaux

In [AD1] a normal dialectical structure is, by definition, an ordered quadruple $\mathfrak{S} = < \mathbf{A}, \mathbf{N}, D, R >$, where $\mathbf{A} \neq \mathbf{N}$, $D \neq \emptyset$, and $R \subseteq D \times D$, and such that R is both reflexive and transitive on D.[1] There is no need to change this definition, since nothing in it reflects a presumption of the Principle of Cumulation. A minimal dialectical structure is, according to the definition given in [AD1], a quintuple $\mathfrak{S} = < \mathbf{A}, \mathbf{N}, D, Abs, R >$ such that $< \mathbf{A}, \mathbf{N}, D, R >$ is a normal structure, $Abs \subseteq D$, and such that for all $d, d' \in D$: if dRd' and $d \in Abs$, then $d' \in Abs$ (Abs is called "the set of absurd situations").[2] The last condition expresses a cumulative principle. For it is tantamount to a Principle of Preservation of Agreement on Absurdity. Therefore I shall henceforth drop this condition from the definition of "minimal dialectical structure". Those structures that do satisfy the condition may be called "cumulative minimal structures". On cumulative minimal structures a logic may be based that respects the Principle of Preservation, at least for the special case where the dialectical subject agrees on $\wedge$ (an absurd sentence) or on a contradiction.

What about models? I can take over the relevant definitions from [AD1] almost verbatim.[3] So an interpretation on a normal or minimal structure $< \mathbf{A}, \mathbf{N}, D, (Abs,) R >$ will be a function I defined for all pairs $< U, d >$, where U is an atom of the language (but $U \neq \wedge$)[4] and $d \in D$, with values in $\{\mathbf{A}, \mathbf{N}\}$. A constructive dialectical model is a pair $M = < \mathfrak{S}, I >$ such that $\mathfrak{S}$ is a normal structure and I is an interpretation on $\mathfrak{S}$. Similarly for a minimal dialectical model. Clearly a constructive model is the same thing, structurally, as an S4-model.

I have now cleared the notions of 'structure' and 'model' of cumulative features. The semantic rules remain to be examined. These determine the values of complex sentences (in a situation) in terms of the values of their immediate constituents (in the same or in other situations). Together such rules are to define, for each model M = < **A**, **N**, D, (Abs,) R, I >, a (constructive or minimal) valuation v_M such that $I \subseteq v_M$ and such that v_M is a function defined for all pairs < U,d >, where U is a sentence of the language and d∈D, with values in $\{$**A**, **N**$\}$. It will be convenient to repeat briefly the semantic rules for cumulative models: (5)

<u>Sem</u>^{c}& v_M(U&V,d) = **A** iff v_M(U,d) = v_M(V,d) = **A**.

<u>Sem</u>^{c}v v_M(UvV,d) = **N** iff v_M(U,d) = v_M(V,d) = **N**.

<u>Sem</u>^{c}→ v_M(U→V,d) = **N** iff for some d' such that dRd' = v_M(U,d') = **A** and v_M(V,d') = **N**.

For constructive models:

<u>Sem</u>^{c}~ v_M(~U,d) = **N** iff for some d' such that dRd': v_M(U,d') = **A**.

<u>Sem</u>^{c}**Λ** v_M(**Λ**,d) = **N**.

For minimal models:

<u>Sem</u>^{m}~ v_M(~U,d) = **N** iff for some d'∉Abs such that dRd': v_M(U,d') = **A**.

Sem^m**Λ** v_M(**Λ**,d) = **A** iff d∈Abs.

Must these semantic rules be modified? As far as I can see, there are three options:

(1) The semantic rules for ~ and → express cumulative principles, viz., that agreement on sentences of the forms U→V and ~U is cumulative. What makes these rules cumulative is the reference to situations d' with dRd'. One must therefore reformulate these rules "in terms of d only":

<u>Sem→</u> v_M(U→V,d) = **N** iff v_M(U,d) = **A** and v_M(V,d) = **N**.

<u>Sem~</u> v_M(~U,d) = **N** iff v_M(U,d) = **A**. (6)

(2) Even if one does not assume the Principle of Cumulation when applied to atoms (and their compounds by means of v and &), he may still allow the principle to hold for statements of the forms ~U and U→V. No change in the semantic rules is needed.

(3) One may change the semantic rules in some more radical way. For instance, one could introduce the third value **R** for "rejection by the company".

The first option makes the relation R superfluous. If one drops R, he adheres to the Principle of Incoherence, i.e., he assumes that the dialectical subject may shift from one dialectical situation into any other. The set of sentences agreed upon in a situation puts no constraints then on the sets of agreed sentences in the possible developments of that situation. "Any situation may develop into any other situation". But this assumption does not lead to any interesting theory of models. If, furthermore, the new semantic rules for → and ~ are those formulated above, then for any d∈D, the rules are equivalent to the classical ones. We may conclude that the (most plausible) logic of Incoherent Noncumulative Models is classical logic.

In this paper I shall take the second option (leaving the third for further study). Thus there will be no change in the semantic rules. Agreement on U→V in a dialectical situation d is the same as agreement to agree on V as soon as agreement upon U has been reached. Agreement on U→V puts a constraint on which developments of the dialectical situation are still possible. And similarly for ~U. In other words, sentences of the forms U→V and ~U express a dialectical subject's irreversible decisions, by which the subject regulates its own future behavior. I shall call these sentences strict sentences.(7) Even for strict sentences, cumulation of agreement is plausible only "with respect to some companies and for a limited period of time". Indeed, it seems that if we want to reject all cumulation, we are stuck with option 1. Yet we have advanced: cumulation of just strict sentences is plausible for more companies and for longer stretches of time than is cumulation throughout.

I shall speak of constructive* and minimal* validity (in some language $\mathcal{R}$) if validity with respect to the present dialectical semantics is meant.[8] Examples of minimally* (and hence constructively*) valid sentences are all sentences of the forms: $\sim(U\&\sim U)$, $(U\rightarrow V)\rightarrow\{(V\rightarrow W)\rightarrow(U\rightarrow W)\}$, $(U\rightarrow W)\rightarrow\{(V\rightarrow W)\rightarrow[(U\vee V)\rightarrow W]\}$, $[U\rightarrow(V\rightarrow W)]\rightarrow[(U\&V)\rightarrow W]$, $\sim U\rightarrow[(V\rightarrow U)\rightarrow\sim V]$, $(V\rightarrow U)\rightarrow(\sim U\rightarrow\sim V)$. Examples of constructively* (and hence minimally*) invalid sentences are: $A\rightarrow(B\rightarrow A)$, $A\rightarrow[B\rightarrow(A\&B)]$, $[(A\&B)\rightarrow C]\rightarrow[A\rightarrow(B\rightarrow C)]$, ~~$\sim A\rightarrow(A\rightarrow\sim B)$~~ $A\rightarrow(\sim A\rightarrow\sim B)$.

Complete and sound systems of semantic tableaux are easily devised. One may adapt the rules given in the literature for constructing S4--tableaux,[9] treating $U\rightarrow V$ as $\Box(U\rightarrow V)$ and $\sim U$ as $\Box\sim U$. Here I shall follow the treatment in [AD1][10] and indicate the modifications that need to be made.

Let $\Pi^* =_{Df} \{U \mid U\in\Pi$ and U is a strict sentence$\}$. Π^* is called the <u>strict part</u> of Π. Instead of the rule $\rightarrow R^c$ ($= \rightarrow R^m$), we now employ the following <u>right rule</u> for the conditional:

$\rightarrow R^*$ A set of sequents $\Sigma;(\Pi/\Gamma,U\rightarrow V)$ reduces to the set of sequents $\Sigma;(\Pi/\Gamma,U\rightarrow V);(\Pi^*,U/V)$.[11]

Instead of the constructive and the minimal right rules for negation, we have:

$\sim R^{c*}$ $\Sigma;(\Pi/\Gamma,\sim U)$ reduces to $\Sigma;(\Pi/\Gamma,\sim U);(\Pi^*,U/\emptyset)$.

$\sim R^{m*}$ $\Sigma;(\Pi/\Gamma,\sim U)$ reduces to $\Sigma;(\Pi/\Gamma,\sim U);(\Pi^*,U/\Lambda)$.

The proofs of soundness and completeness are standard.[12]

10.2 Noncumulative formal dialectics: withdrawal of contingent concessions

Is it possible to devise plausible systems of formal dialectics that correspond to noncumulative dialectical semantics? This section will contain an attempt to construct such systems, but the attempt will only partly succeed. For, as we shall see, the systems we end up with do not fully correspond to noncumulative semantics; though the two approaches fully agree on sequents of the form $\emptyset$/Z, this does not hold generally for sequents Π/Z.

Clearly, if a system of formal dialectics is to correspond to noncumulative dialectical semantics at all, it must give a special status to statements of the forms $\sim$U and U$\rightarrow$V. Since from a semantic point of view such statements express irreversible decisions, i.e., constraints upon the possible developments of a dialectical situation, their acceptance by the company is of greater moment than the acceptance of statements of other forms. Therefore, so it seems, it should be harder for a Proponent (P) to defend statements of the forms $\sim$U and U$\rightarrow$V. This means that, in local discussions about such statements, either the Proponent's rights must be restricted or the Opponent's (O's) rights expanded.

A distinction between a number of kinds of statements - each with its own proper means of defense - is of course nothing uncommon. Thus most of us distinguish observation statements, mathematical statements, (alleged) empirical laws, moral and nonmoral value judgements, definitions, (alleged) laws of logic, metaphysical statements, etc. Much of philosophy is concerned with classifications of statements, with problems of demarcation, and with the proper way to defend or to establish statements of each particular type. What we are interested in right now is a distinction (<u>any</u> distinction) between two types of statements such that we can speak of <u>strict</u> statements on the one hand and <u>contingent</u> statements on the other, and such that strict statements may be used to defend either strict or contingent statements, but contingent statements may be used to defend contingent statements only. I give some traditional examples:(1)

(1) analytic statements vs. synthetic statements or synthetic *a priori* statements;

(2) statements of logic vs. mathematical statements;

(3) mathematical statements vs. statements of science;

(4) statements of *prima facie* duty vs. statements of actual duty;

(5) metaphysical statements vs. physical statements;

(6) synthetic *a priori* statements vs. synthetic statements.

On each line the strict statements are on the left and the correlated contingent statements are those on the right that are not strict, i.e., that are not also included among the statements on the left.

Suppose that a company wants to acknowledge some such distinction of strict and contingent statements. The traditional examples show that such distinctions meet cultural wants, and therefore that this is not an unrealistic assumption. We may then say that the company acknowledges the following principle, which I call the *fundamental norm of two-leveled dialectics*.(2)

FD $L_2$1 A strict (local) thesis is to be defended, ultimately,(3) on the basis of strict concessions.

I shall, moreover, suppose that the company acknowledges the elementary rules and fundamental norms of dialectic systems, as described in [AD1], Ch. III.(4) I shall try to construct plausible systems of formal dialectics for such a company. So my task consists of finding acceptable implementations of FD $L_2$1 to be adjoined to the FD-rules of [AD1], Ch. III, IV. At the same time, I am to end up with systems that agree with the validity notions taken from noncumulative dialectical semantics - this determines my choice of language forms and choice of class of strict statements. Clearly there is no guarantee that the two demands can be met by any one system.

Two implementations of FD $L_2 1$ seem equally suitable:

FD $L_2 2$ As soon as O has completed a stage in which O attacks a strict statement, O shall have a right to assume the neutral position to any or all contingent concessions to which it had the attitude of pro-position before the attack.

FD L_2* As soon as P has chosen and carried out a structural protective defense for a strict local thesis, O shall have a right to assume the neutral position to any or all contingent concessions that appear at that moment in the chain of arguments.

If FD $L_2 2$ is adopted, and if the Opponent takes full advantage of its rights, P cannot use any of O's contingent concessions in a defense of a strict (local) thesis, with the exception of statement(s) made by O in its attack on the (local) thesis. If FD L_2* is adopted, and if the Opponent again takes full advantage of its rights, the use of contingent concessions in a defense of a particular strict local thesis is restricted to the one local discussion having this as its local thesis. In this local discussion, P may try to get additional strict concessions. The fundamental norm FD $L_2 1$ and its two implementations are independent of any particular choice of strict versus contingent statements.

For the present, we want to identify the strict statements with those of the forms ~U and U→V, and the contingent statements with all other statements. But if we do so, neither FD $L_2 2$ nor FD L_2* seems satisfactory, for the following reasons. (1) FD $L_2 2$ makes it impossible to defend "A→B" successfully upon the basis of a concession "(A→B)&C". The concession is not an implication, nor a negation, and is therefore contingent. Hence O may withdraw this concession immediately after its attack on the thesis.

(Unless, of course, O prefers to attack some statement, and start a fresh local discussion, before P has carried out the structural protective defense move in question.)

The presence of the concession therefore offers no advantage to P. If P had a winning strategy for $(A \rightarrow B)\&C/_0 A \rightarrow B$ it would have one also for $\emptyset/_0 A \rightarrow B$. Since U&V/U is valid and $\emptyset/A \rightarrow B$ is invalid according to all noncumulative semantic systems, this shows that FD $L_2$2 cannot be incorporated in a formal dialectics corresponding to any of these semantic systems.

(2) Thus we turn to FD L_2*, a rule that seems to overcome the difficulties of the foregoing type. However, FD L_2* makes it impossible to defend $A \rightarrow A$. For assume that 0 withdraws each concession as soon as permitted. Then only one chain of arguments is possible, there being no further choices to be made by either party:

	O	P
1		$A \rightarrow A$
2	A	[A]
3		A
4	?A	[ ]

At the end of stage 4 the Proponent's rights are exhausted. For at stage 3 O has withdrawn the concession A, made at stage 2. Hence, a defense by _Ipse dixisti!_ is precluded, and P's only structural protective defense right has already been used at stage 3. So there is an O-winning strategy against $A \rightarrow A$! But of course $A \rightarrow A$ is valid according to noncumulative semantics. This shows that FD L_2* is unacceptable as well: it is _too_ P-_restrictive_.

One way to amend FD L_2* would presumably be to forbid the withdrawal of a concession made _in_ the attack on a strict local thesis, i.e., to read

FD L_2** ..., O shall have a right to assume the neutral position to any or all contingent concessions that appear at that moment in the chain of arguments, with the exception of a statement made by O _in_ its attack on the strict local thesis.

However, FD L_2* - and _a fortiori_ FD L_2** - is _also too_ P-_liberal_, if the formal dialectics is to correspond with a noncumulative semantics. These rules allow the following P-winning strategy for ~(A&B),B$/_0$A→C, whereas this sequent is invalid in noncumulative semantics.[8]

	O		P	
	~(A&B)			
	B		A→C	
1	A		[C]	
2	[ ]		A&B	
3	L?	R?	[A]	[B]
4			A	B
5	A?	B?	[ ]	[ ]
6			!	!

Nowhere is there a withdrawal of concessions; for the right [C] is never used by P, and hence the conditions of FD L_2* (or, FD L_2**) never apply. The following diagram depicts a constructive* counterexample to ~(A&B),B/A→C.[9]

It may be thought that some other division of statements into strict and contingent would dispose of these difficulties. I will have to await a proposal for this. In any case, the following two divisions certainly do not eliminate these difficulties:

(1) Let the strict statements be precisely those that contain a → or a ~.

(2) Let the strict statements be those that are "fully modalized", i.e., such that each elementary substatement occurs within the scope of a → or a ~.

If we adopt (1), it would indeed be possible to defend A→B on the basis of a concession (A→B)&C; but it would also be possible to defend A→B on the basis of a concession (A→C)&B (assuming either implementation of FD $L_2$1), whereas (A→C)&B/A→B is invalid in all noncumulative semantic systems. The proposal is too P-liberal. If we adopt (2), the very objections stated before, under the assumption that the strict statements are precisely those of either the form U→V or ~U, still obtain.

Henceforth I shall adopt FD $L_2$2. It can then be shown that the limited kind of correspondence, mentioned at the beginning of this section, holds.

Def. 1 For each nonmaterial constructive or minimal dialectic system $\mathfrak{S}$ (as defined in [AD1], Ch. IV), $\mathfrak{S}$* shall be the system obtained from $\mathfrak{S}$ by the inclusion of FD $L_2$1 and FD $L_2$2 in the rules, and by strengthening FD D8 to FD D8* (see below).

Thus we obtain the five systems: MID*, MΛD*, MND*, CΛD* and CND*.

In [AD1], Section III.15 (Dynamic dialectics) the authors tried to persuade the reader to accept the rules FD D7 and FD D8. The reader was asked to imagine a completed system of formal dialectics (in which the other FD-rules were to be included), and it was then argued that the dynamic rules FD D7 and FD D8 could be added without either party losing an opportunity to act according to a winning strategy.

In that context, withdrawal of concessions was not considered. If withdrawal of concessions, on the strength of FD $L_2$2, is admitted, the argument in favor of FD D7 holds as before,(10) but not that in favor of FD D8. However, a slightly adapted argument can be given in favor of FD D8, or in favor of the following expanded version of FD D8:(11)

FD D8* (a) After an attack by O, P may not repeat the sentence T in the new local thesis <u>T</u> within the same chain of arguments as long as the set of local concessions has not been augmented by any statement (not as yet withdrawn) of a new sentence, moreover:

(b) if <u>T</u> is itself a strict statement, some further strict statement must have been added to the concessions after O's attack on <u>T</u>.

10.3. Some properties of noncumulative formal dialectics

All properties of (nonmaterial) minimal and constructive dialectics discussed and proven in [AD1] Ch. IV and V can be shown, in similar ways, to extend to the noncumulative systems. Thus we have the obvious theorem concerning relative logical strength:

Theorem 1* If P has a winning strategy for a dialogue situation in a minimal* system, then P has a winning strategy for that situation in the corresponding constructive* system.(1)

Further, we can repeat the argument that the FD-rules together successfully implement the fundamental norm of dynamic dialectics:

Theorem 2* Each of the noncumulative systems of formal dialectics is locally finite.(2)

We now turn to the study of strategy. For this purpose it is again convenient to introduce P-liberalized systems. These systems are no longer locally finite but of equal logical strength as the original official systems.(3)

Def. 2 The P-liberalized system corresponding to any particular noncumulative system of formal dialectics is obtained by cancellation of the rules FD D6 and FD D8*.

Lemma 1* P has a winning strategy for a dialogue situation S according to a P-liberalized noncumulative system iff P has a winning strategy for S according to the corresponding official system.(4)

As before, one can describe the P-winning strategies (on the strength of either the official or the P-liberalized systems) by means of dialogical strategy diagrams or tableaux. In these tableaux we need only reckon with such Opponents as actually withdraw any concession they are allowed to withdraw (maximally severe Opponents). Let us then denote a withdrawal of all contingent concessions with a horizontal dashed line (withdrawal line), e.g.:

O	P
	A→B
- - - - - -	- - - - - -
A	[B]

The withdrawal line indicates that all contingent concessions above it in the same subtableau are withdrawn.

I here list those rules for the construction of dialogical P-winning strategy diagrams or tableaux that differ from the corresponding rules in [AD1].[5]

$OI^{*}_{\rightarrow}$ Under $\Pi/_{O}U\rightarrow V$ you must write $\Pi^{*},U/U\rightarrow V/_{P}[V]$.[6]

$OI^{\wedge *}_{\sim}$ Under $\Pi/_{O}\sim U$ you must write $\Pi^{*},U/\sim U/_{P}[\wedge]$.

$OI^{*}_{\sim}$ Under $\Pi/_{O}\sim U$ you must write $\Pi^{*},U/\sim U/_{P}\emptyset$.

$OIII^{*}_{\rightarrow}$ Under $\Pi;[U]/T/_{O}V\rightarrow W;\Gamma$ you must write both $\Pi,U/T/_{P}\Gamma$ and $\Pi^{*},V/V\rightarrow W/_{P}[W]$.

Similar substitutions of Π^{*} for Π are to be made in the formulations of the rules $OIII^{\wedge *}_{\sim}$ and $OIII^{*}_{\sim}$.

Preparatory to proving the soundness of the method of dialogical strategy tableaux with respect to the method of deductive tableaux (Section 4), we need again to establish:

<u>Theorem</u> 3* P has a winning strategy for a dialogue situation <u>S</u> on the strength of a (P-liberalized) noncumulative system $\mathfrak{S}$ iff P has a winning strategy for <u>S</u> on the strength of $\mathfrak{S}$ with the following rule (R_{At}) added to it: an <u>Ipse dixisti</u>!-remark may be made only if the local thesis is atomic.

The proof proceeds in the usual way by a structural induction.[7]

Another matter again is the equivalence of NOT-dialectics with $\wedge$-dialectics:

<u>Theorem</u> 4* Let Π/Z be a sequent such that $\wedge$ does not occur in Z nor in any sentence of Π.

There is a closed dialogical tableau for Π/Z on the strength of MND* (CND*) iff there is a closed dialogical tableau for Π/Z on the strength of MΛD* (CΛD*).

The proof for the equivalence of CND* and CΛD* is in no way different from that for the equivalence of CND and CΛD.(8) For the minimal systems we may even have a simpler proof than before.(9)

Finally, we turn to the soundness of the method of semantic tableaux relative to the dialectics, or - amounting to the same - to the completeness of the method of dialogical tableaux relative to the semantics.(10) Ideally we would like to obtain:

(*?) If, in any of the systems for constructing noncumulative semantic tableaux, a semantic tableau for Π/Z can be brought to a closure, then there is a P-winning strategy for $\Pi/_0 Z$ on the strength of the corresponding system of noncumulative formal dialectics.(11)

But in view of what was said in the preceding section, (*?) does not hold. Sequents like (A→B)&C/A→B and A,A→(B→C)/B→C are minimally* valid (so closed semantic tableaux can be constructed for them), whereas there are no P-winning strategies for $(A\to B)\&C/_0 A\to B$ or $A,A\to(B\to C)/_0 B\to C$. The following weaker correspondence theorem will have to do:

<u>Theorem</u> 5* If, in any of the systems for constructing noncumulative semantic tableaux, a semantic tableau for Π/Z can be brought to a closure, then there is a P-winning strategy for $\Pi/_P [Z]$ on the strength of the corresponding system of noncumulative formal dialectics.

There is nothing new to the proof of this.(12) There must, as a consequence, be P-winning strategies for $(A\to B)\&C/_P [A\to B]$ and $A,A\to(B\to C)/_P [B\to C]$, as may be independently checked by dialogical tableaux.

Let us call a dialectic system σ <u>invertible</u> (13) if the following holds:

(**) There is a P-winning strategy for $\Pi/_O Z$ on the strength of σ iff there is a P-winning strategy for $\Pi/_P [Z]$ on the strength of σ.

Clearly, since Theorem 5* holds but (*?) does not, the noncumulative systems of formal dialectics are not invertible.(14) But as long as $\Pi = \emptyset$, (**) will hold, simply because the situation depicted by $\emptyset/_P [Z]$ can, in dialogues, only be followed by a situation depicted by $\emptyset/_O Z$. Hence (*?) too holds as long as $\Pi = \emptyset$. It may further be shown that (**), and hence (*?), hold as long as $\Pi = \Pi^*$, i.e., as long as Π is identical with its strict part. ($\Pi = \emptyset$ is of course a special case of $\Pi = \Pi^*$.) Similarly if Z is contingent.(15) Thus, for these special cases, the dialectics is complete with respect to the semantics.

One may of course enforce a general completeness result (*?) by suitable changes in the formal dialectics. It would suffice to stipulate that discussions start from situations $\Pi/_P [Z]$ (instead of $\Pi/_O Z$). This, however, runs counter to the foundations of dialectics in [FD1], where the Proponent is to assume the neutral position towards each of O's concessions and has no unconditional right to attack these concessions.(16)

That discussions should start from situations $\Pi/_P [Z]$ rather than $\Pi/_O Z$ is, however, quite acceptable if the context of the discussion is one of "immanent" criticism of Π by means of a provocative thesis Z. But in that context the names "Opponent" and "Proponent" are misleading, for it is P that opposes the propositions put forward by O.(17)

10.4. Noncumulative deductions. The full circle theorem

It still remains to be shown that the dialectics is sound relative to the semantics or, amounting to the same thing, that the semantics is complete relative to the dialectics. This can easily be done without going through all the deductive methods expounded below. Yet I think it would be a pity to ignore the deductive systems that correspond to noncumulative semantics and dialectics. For one thing, there is not a little pleasure to be derived from studying these systems for their own sake. As we shall see, some of them are quite elegant, others rather weird. Deductive tableaux of the kind here described are almost completely unknown, but the systems of natural deduction and the axiom systems will lead us to comparisons with known systems. And this is another motivation for studying them.

Once it has been decided to devote a section to noncumulative deduction, I may as well skip the direct proof of the soundness of dialectics and unite this result, together with other soundness/completeness results, into one agreeable Full Circle Theorem.

My program is therefore as follows. I shall formulate first a system for the construction of deductive tableaux, then a system of natural deduction, and lastly an axiom system for each of the noncumulative logics. I shall not repeat such rules as are taken over unchanged from [AD1], except that a complete list of postulates for the axiom systems will be given in order to facilitate comparison with systems defined elswhere. I shall then establish the fundamental unity of the semantic, dialectic, and deductive methods (Full Circle Theorem). In the proofs I shall again restrict myself to those parts that differ from the corresponding parts in [AD1]. Let us proceed.

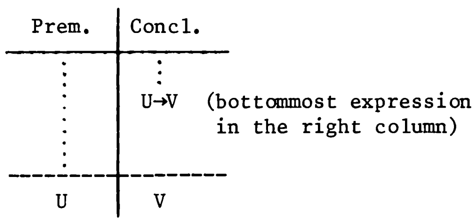
When constructing a deductive tableau according to a noncumulative system, one is to make use of cancellations lines.(1) These are horizontally drawn, dashed lines in a subtableau, which indicate a cancellation of all the contingent premises higher up in the same subtableau. They correspond to the withdrawal lines in dialogical tableaux. One should draw a cancellation line if, and only if, applying one of the following three rules:

→r* Π/U→V reduces to Π*,U/V.

~r* Π/~U reduces to Π*,U/~U.

$\sim r^*_\wedge$ Π/~U reduces to Π*,U/∧.

The following diagram shows how to draw a cancellation line when applying →r*:



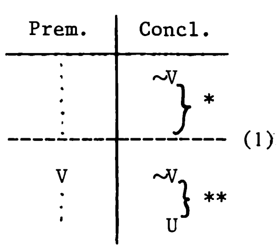
→r*

Each application of a rule (reduction rule or closure rule) must refer only to such premises as are still operative (i.e., not yet canceled).[2]

Except for the three rules just mentioned and the rule $\sim 1^*_{min}$, the rules for noncumulative deductive tableau construction are identical to the corresponding rules in [AD1].[3] A noncumulative rule $\sim 1^*_{min}$,[4] is somewhat hard to formulate. Note that the original rule

$\sim 1_{min}$ Π,~U/~V reduces to Π,~U/U

will not do. For instance, A,~A/A is minimally* valid, but A,~A/~B not. Without additional machinery, it seems not possible to formulate a suitable rule in terms of a simple sequent reduction. However, it is ~~easy enough to do~~ readily done by means of a diagram with provisos:



$\sim 1^*_{min}$

<u>Proviso</u> 1 ~U must appear somewhere among the Premises.

<u>Proviso</u> 2 The cancellation-line (1) (apparently introduced by ~r*) must be the bottommost cancellation line (in the subtableaux).

<u>Proviso</u> 3 There must be no expressions in the parts indicated by * and **, except for repetitions of ~V by virtue of an application of v1.

The systems of natural deduction, which now follow, are of the type introduced by Fitch.(5) Besides the ordinary subordinate deductions (6) with ordinary scope indicators, these systems feature <u>strict subordinate deductions</u> marked off by <u>strict scope indicators</u> (labeled "□"). As a (fully equivalent) alternative to a Fitch-style reiteration rule (i.e., a restriction on trivial deductions), the following restriction on the application of rules within a strict subordinate deduction is put into force.

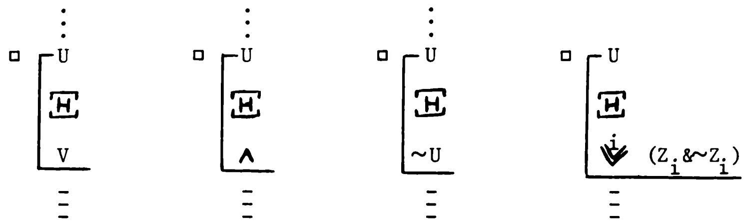
RESTRICTION: The sentences and subordinate deductions that are used as premises in an application of a rule, in order to obtain a new sentence occurrence within a strict subordinate deduction, must either

(i) themselves be strict, or

(ii) occur within the same strict subordinate deduction.

It is this restriction that constitutes the difference between strict and ordinary subordinate deductions.

I can take over all the rules for minimal and constructive systems of natural deduction postulated in [AD1],(7) except for the <u>Rule of Conditional Proof</u> (CP), the <u>Reductio ad ∧ Rule</u> (R∧), the <u>Minimal Reduction rule</u> (MR), and the Rule <u>Ex Contradictione Sequitur Quaelibet Negatio</u> (ECQN). These rules are to be replaced by the following:



∴ U→V CP* ∴ ~U R∧* ∴ ~U MR* ~U MR**

In the formulation of the rule MR**, the expression " $\bigvee^{i}(Z_i \& {\sim} Z_i)$" stands for any finite disjunction of explicit contradictions (the case of just one contradiction included). The rule MR** is to replace the rule ECQN when the system MNnd is changed to MNnd*. Clearly, in the latter system, MR* is redundant. For, if we can derive ~U strictly from U, we can also obtain the explicit contradiction U&~U.

A full list of postulates for the noncumulative axiom systems now follows.

Rules

$$\frac{U \quad U\to V}{\therefore\ V}\ \text{MP} \qquad \frac{U \quad V}{\therefore\ U\&V}\ \text{CONJ}$$

Axiom-schemata:

<u>Axs</u> 1* U→(V→U), if U is strict, i.e., if U is either of the form W→Z or of the form ~W.

<u>Axs</u> 2 [U→(V→W)]→[(U→V)→(U→W)]

<u>Axs</u> 3* U→U

<u>Axs</u> &1 (U&V)→U

<u>Axs</u> &2 (U&V)→V

<u>Axs</u> &3* (U→V)→{(U→W)→[U→(V&W)]}

<u>Axs</u> v1* (UvU)→U

<u>Axs</u> v2 U→(UvV)

<u>Axs</u> v3* (UvV)→(VvU)

<u>Axs</u> v4* {[(U→V)vW]&(UvW)}→(VvW)

<u>Axs</u> v5* [(UvW)&(VvW)]→[(U&V)vW]

<u>Axs</u> Λ_1 $\sim U \rightarrow (U \rightarrow \Lambda)$

<u>Axs</u> Λ_2 $(U \rightarrow \Lambda) \rightarrow \sim U$

<u>Axs</u> Λ_3 $\Lambda \rightarrow U$

<u>Axs</u> $\sim_1$ $\sim U \rightarrow (U \rightarrow V)$

<u>Axs</u> $\sim_2$ $(U \rightarrow \sim U) \rightarrow \sim U$

<u>Axs</u> $\sim^*_{min}$ $[U \rightarrow \bigvee^{i} (Z_i \& \sim Z_i)] \rightarrow \sim U$

The unstarred postulates are taken over unchanged from [AD1].[8]

Def. 3 (a) For each of the constructive or minimal systems, defined in [AD1] for the construction of deductive tableaux or for natural deduction, a corresponding noncumulative system is found by effecting the modifications indicated above. The names of these systems are: MIdt*, MΛdt*, CΛdt*, MNdt*, CNdt*, MInd*, MΛnd*, CΛnd*, MNnd*, CNnd*.

(b) The system MIax* consists of <u>Axs</u> 1*, 2, 3* and MP.

(c) Each of the following four systems consists of MP, CONJ, <u>Axs</u> 1*, 2, 3*, &1, &2, v1*, v2, v3*, v4*, v5*, and, in addition, certain axiom-schemata for ~ and for Λ, as follows:

Γ&3*,

MΛax*: <u>Axs</u> Λ_1, Λ_2

CΛax*: <u>Axs</u> Λ_1, Λ_2, Λ_3

MNax*: <u>Axs</u> $\sim^*_{min}$

CNax*: <u>Axs</u> $\sim_1$, $\sim_2$

It is clear from the semantics, given in Section 1, that MIax* is S4→, i.e., the strict implication fragment of S4.[9] CNax* (CΛax*) is the fragment of S4 with the following logical constants: (i) strict implication and negation, (ii) ordinary conjunction and disjunction, and (iii) in the case of CΛ*ax, an absurd sentence Λ.[10] The minimal systems are weaker than the corresponding constructive systems. The Λ-systems are equivalent to the corresponding N-systems as long as only Λ-free sentences

ΓNOT

are involved (Theorem 4*). Finally MIax* gives us exactly the implicative fragment of the other systems. All these remarks are corollaries to the Full Circle Theorem:[11]

Theorem 6* The following conditions are equivalent, provided that they refer to corresponding noncumulative systems (i.e., systems whose abbreviated names start with the same two uppercase initials) and that the sequent Π/Z consists of sentences of some language $\mathcal{L}$ to which these systems pertain:

a. There is a P-winning strategy for $\Pi/_P[Z]$.[12]

b. There is a closed deductive tableau for Π/Z.

c. There is a natural deduction of Z from Π.

d. There is an axiomatic deduction for Π/Z.

e. Π/Z is minimally*/constructively* valid in $\mathcal{L}$.[13]

f. There is a closed semantic tableau for Π/Z.

Proof

From *a* to *b*: A closed dialogical tableau can always be transformed into a closed deductive tableau. The very same instructions that were formulated for this purpose in [AD1] [14] will lead to the desired result here as well. These instructions work for tableaux starting with $\Pi/_P[Z]$ [15] and yield a closed deductive tableau for Π/Z. Of course withdrawal lines in the dialogical tableau will become cancellation lines in the deductive tableau. Each illicit "application" of a left rule in the deductive tableau (i.e., an application on a canceled premise) cannot but originate from an equally illicit application of a choice rule in the dialogical tableau. Therefore, if the dialogical tableau is correct so will be the deductive tableau into which it is transformed.

From _b_ to _c_: The technique of the "tape theorem" (16) can be applied to the noncumulative deductive tableaux. It can be checked that illicit "applications" of rules of natural deduction (i.e., such applications as do not respect the restriction formulated above) could only derive from illicit applications of tableau rules. Applications of the rule $\sim l^*_{min}$ need additional treatment, since the rule ECQN is not included in MNnd*. An application of $\sim l^*_{min}$ is permitted only where the concludendum is some negation ~V and where the last right rule that was applied in the subtableau is ~r* on ~V. So V appears uncancelled on the left. Consider any application of ~r* on a particular concludendum ~V. It may be followed, either immediately or after applications of left rules only, in at least one subtableau by an application of $\sim l^*_{min}$. (17) Let $Z_1, \ldots, Z_{n-1}$ be the formulas that appear on the right by virtue of such applications of $\sim l^*_{min}$. (18) Let Z_n be V. Now take any occurrence on the right of ~V that is either the repetition of the concludendum called for by ~r*, or a repetition of the concludendum that derives from the first repetition by a number of applications of ∨l.

Suppose first that this occurrence is followed, either immediately or after applications of left rules only, in at least one subtableau by an application of $\sim l^*_{min}$. Replace ~V by the disjunction $\bigvee\limits_{1 \leqslant i \leqslant n} (Z_i \& \sim Z_i)$ (with association to the left).

If the supposition does not hold, keep the occurrence of ~V and insert $\bigvee\limits_{1 \leqslant i \leqslant n} (Z_i \& \sim Z_i)$ above it.

Example Let U = "(A&~A)v[(B&~B)v{[Av(BvC)]&~[Av(BvC)]}]"

	Prem.		MNdt* Concl.	
	C→~[Av(BvC)]		~[Av(BvC)]	
	~A			
	~B			
~r*	Av(BvC)		~[Av(BvC)]	
v1	A	BvC	~[Av(BvC)]	~[Av(BvC)]
$\sim 1^*_{min}$			A	
c				
v1	B	C	~[Av(BvC)]	~[Av(BvC)]
$\sim 1^*_{min}$			B	
c				
→1		~[Av(BvC)]	C	
c		c		

This tableau is transformed into:

Prem.		Concl.	
C→~[Av(BvC)]		~[Av(BvC)]	
~A			
~B			
Av(BvC)		U	
A	BvC	U	U
		A	
B	C	U	U
		B	~[Av(BvC)]
	~[Av(BvC)]	C	

Note that this operation leaves the applications of v1 intact. All applications of ~r* are to be thus treated with respect to all repetitions of the concludendum of the kind indicated.

Now let us "stretch the tape". Each of the bottommost new occurrences of $\bigvee_{1\leq i\leq n} (Z_i \& \sim Z_i)$ can be accounted for by means of the Conjunction Rule (CONJ) and the Addition Rules (AD_1, AD_2). So after the insertion of a few lines we obtain an MNnd*-deduction. The applications of ~r* in the tableau are transformed into applications of MR**.

From c to d: Note that for none of the noncumulative axiom systems does the deduction theorem hold. E.g. A,B⊢A, but not A⊢B→A. Natural deductions can nevertheless be transformed into axiomatic deductions by a process of "conditionalization". I shall now describe this process.

Phase 1 Eliminate all applications of Separation Rules, of Addition Rules, of Ex Falso Rules, (19) and of Reductio Rules (20) in favor of the axioms &1, &2, v2, v3*, Λ_1, Λ_3, $\sim_1$, $\sim_2$, ~min*, Λ_2, and the rules MP and CP*. At the end of phase 1 we obtain a hybrid deduction which contains, besides axioms, applications of no other rules but: MP, CONJ, CD (Rule of Case Distinction, or Constructive Dilemma) CP* and TRIV (the Trivial Deduction Rule). (21)

Phase 2 Eliminate, successively, all applications of CP* and CD. At each step, eliminate an application that uses only innermost subordinate deductions, i.e, such subordinate deductions as do not contain an other subordinate deduction. Subordinate deductions that are not used by CP* or CD can simply be omitted. So this phase gets rid of all subordinate deductions. The result is an axiomatic deduction.

<u>For CP*</u> This can be done in the usual way. (22) Note that we need to make the insertion

		X
X	⇒	X→(U→X)
		U→X

(where X is operative in, but is itself outside, the subordinate deduction) only if X is strict, for a contingent sentence is not operative in a strict subordinate deduction, if it does not occur in that deduction. It is also important that all axioms are strict.

<u>For CD</u> Let the veljunctive premise be UvV, let the first subordinate deduction contain n lines on which are consecutively written U_1, ..., U_n (U_1=U), and let the second subordinate deduction contain m lines on which are consecutively written V_1, ..., V_m (V_1=V). Let the conclusion of the application of CD that we are going to eliminate be Z (= U_n = V_m).

(1) For each line (containing, say, W) operative in, but not itself within, the first subordinate deduction, the following insertion is to be made just above the first subordinate deduction:

W	TRIV
W→(WvV)	<u>Axs</u> v2
WvV	MP

, utilizing <u>Axs</u> 1*, <u>Axs</u> 2, <u>Axs</u> 3*, <u>Axs</u>&*, and MP.

(2) Remove the scope indicator of the first subordinate deduction. Replace U_1 (= U) at the top of the (former) first subordinate deduction by UvV (this can be justified by TRIV).

(3) In the (former) first subordinate deduction, working downwards, replace each U_i (on the i-th line of the subordinate deduction) by U_ivV ($1 \leqslant i \leqslant n$), inserting axioms as stated below. Suppose we have arrived at the i-th line of the subordinate deduction:

(i) If U_i was derived by TRIV, we may use TRIV to derive U_ivV.

(ii) If U_i was obtained from W and W→U_i by MP, we now have WvV and (W→U_i)vV at our disposal and we can obtain U_ivV by CONJ, Axs v4* and MP.

(iii) If U_i was obtained by CONJ, we use v5* instead.

Now turn to the second horn of the dilemma:

(4) For each line (containing, say, W) operative in, but not itself within, the second subordinate deduction, make the following insertion just above the second subordinate deduction:

W	TRIV
W→(WvZ)	Axs v2
WvZ	MP

(5) Remove the scope indicator of the second subordinate deduction. Replace V_1 (= V) at the top of the (former) second subordinate deduction by VvZ. Insert an axiom (ZvV)→(VvZ) above it. Since ZvV is on the last line of the (former) first subordinate deduction, we may use MP for justification.

(6) In the second subordinate deduction, make the replacements $V_i \Rightarrow V_i \vee Z$ ($1 \leq i \leq m$) and insert axioms as under (3).

(7) The last line of the (former) second subordinate deduction is now $Z \vee Z$. From this Z may be obtained by Axs $\vee 1^*$ and MP.

From _d_ to _e_: Validity or soundness of the axiom systems can be shown in the standard way by a deductive induction.

From _e_ to _f_: See the end of Section 1.

From _f_ to _a_: Theorem 5* in Section 3 ●

11. MODAL DIALECTICS

11.0. Introduction

Systems of formal dialectics are instruments for conflict resolution.[1] The language forms incorporated in these systems should be rich enough to make the systems attractive to potential debaters. Therefore, the supply of logical constants functioning in the languages of these forms cannot remain limited to the propositional connectives. If one looks for new logical constants that may serve to enrich the propositional languages, quantifiers and modalities are obvious candidates.[2] Quantifiers have been dealt with extensively by P. Lorenzen and K. Lorenz.[3] The incorporation of modalities in systems of formal dialectics is the subject of the present paper.[4]

In Section 1 I shall discuss contemporary contributions to my subject. Section 2 contains my proposal for a modal dialectics and for its normative foundation. Here I shall continue along the lines of an earlier paper.[5] The central notions are (i) pragmatic distinctions between classes of statements according to a relation of (dialectical) strictness defined wholly in terms of debaters' rights, and (ii) the Opponent's right to withdraw concessions of a certain degree of strictness under appropriate conditions.

Section 3 establishes a "Full Circle Theorem"[6] for the proposed modal dialectic systems. It appears that the corresponding derivational and semantic systems are of a multiply S4-type with a constructive (intuitionistic) or minimal basis. In Section 4 I add some conclusions and perspectives.

11.1. Towards a modal dialectics

I shall in turn consider (i) the constructive and dialectical foundations of modal logic as proposed by P. Lorenzen,[1] (ii) the incorporation of modality in the material language-games of K.J.J. Hintikka,[2] (iii) the theory of "levels of discourse" of M. Marčinko,[3] and (iv) my own noncumulative dialectics.[4] Each of these approaches will be discussed solely with respect to its merits as a contribution to a future modal dialectics, without implying that it was intended for just that purpose or denying any other merits it may have.

11.1.1. Dialogical logic and modal operators

In the course of a systematic and critical reconstruction of scientific language, Lorenzen repeatedly discusses modal notions in connection with dialogues.[5] In [NLE] he proposes the following simple rule for □:[6]

Figure 1

	(Speaker:)U	(Critic:)aU	structural pU
Rule$_{\Box}$	□V	?	V

This rule taken alone would give us a "void" sentential operator like "it is the case that - ", but Lorenzen adds another rule:[7]

> □-defense rule: If the proponent defends a □-formula he may attack only the □-formulae (the beginning □ deleted) put by the opponent beforehand.

Thus formulated, this □-defense rule may easily be misinterpreted. Followed to the letter, the rule makes it impossible for the Proponent (P) successfully to defend a thesis "□A" on the basis of a concession "B&□A" granted by the Opponent (O). The latter formula is simply not a □-formula, and therefore P can no longer use it for a counteractive defense.[8] This weird consequence (9) was certainly not intended by Lorenzen, who must, therefore, have had a different interpretation in mind. Fortunately, it is not hard to reformulate the □-defense rule so as to preclude unintended interpretations of the foregoing type. The following formulation by J.P. Murphy probably agrees with Lorenzen's original intentions. (10)

> If the proponent makes this defense [viz., the structural protective defense of a □-formula] and $U_1, \ldots, U_n$ are the only statements on lines (1) - (m) on the opponent's side of the mat [i.e., put by the opponent <u>before the protective defense</u> was made], then, for each i $(1 \leqslant i \leqslant n)$,
>
> (1) if $U_i = \Box V$, the opponent deletes the initial □ when this defense is made
>
> and
>
> (2) if $U_i \neq \Box V$, the opponent deletes U_i when this defense is made.

According to this formulation of the □-defense rule, the restriction on attacks by P (i.e., the limitation of the supply of concessions) becomes operative only <u>after</u> the protective defense move according to $\text{Rule}_\Box$ is made.

Both $\text{Rule}_\Box$ and (the Murphy-formulation of) the □-defense rule are <u>prima facie</u> suitable instruments for the resolution of conflicts that involve modal statements. They do not for their application presuppose any notion of possible worlds or of an alternative relation between them. Nor do they presuppose that the debaters have at their disposal any means for establishing the truth values of elementary statements. (11) But it should be noted that they do not exclude the use of such means ("material procedures") either. Both rules will be incorporated in the system of modal dialectics to be developed in Section 2.

Lorenzen, apparently, did not set great store by these modal dialogue rules, for they do not reappear in his later treatments of modality. (12)

Yet in these later works, too, modal notions are reconstructed critically in terms of dialogical procedures. One would, therefore, expect some reformulation of the modal dialogue rules to be part of this reconstruction. However, the dialogues on the basis of which Lorenzen now introduces modal notions are, in fact, not themselves modal dialogues but material dialogues in a metalanguage. On the one hand, Lorenzen does reconstruct the monadic concept of 'necessity' as 'relative necessity', but this turns out to be only a synonym for "logical implication" (where this latter term has its dialogical meaning) and no theory of modal dialogues is needed to elucidate that concept. [13] On the other hand, there is the notion of a modal implication (modallogische Implikation), which is explained in terms of "generally" applicable winning strategies in certain metadialogues (dialogues on logical implications), viz., such winning strategies as are independent of the (fixed) class of premises (or, concessions) W that appears in the different statements about logical implication uttered in the dialogue. [14] To characterize the class of correct modal implications one may indeed introduce a system of modal dialectics, but it can also be done in other ways, e.g., by rules for constructing deductive tableaux. This explains why the modal dialogues of [NLE] could gradually disappear from the scene. In [NLE] there are still two dialogue rules (quoted above). In $[LPr]_2$ these are replaced by a $\Box$-rule without an indication of how this rule is to be applied in dialogues (i.e., of who has a right to, or is obliged to, perform an act of the kind indicated by the rule). In $[KLE]_2$ the $\Box$-rule (now called $\Box$-Schritt) is no longer presented as a dialogue rule of an ~~indepently~~ independently formulated modal dialectics, but as a reduction rule (Entwicklungsschritt) in a system for the construction of modal (deductive) tableaux -- the tableau system, of course, being designed to characterize exactly the correct "modal implications".

Clearly then, for our purpose of finding a suitable system of modal dialectics, the earlier approach by Lorenzen, in [NLE], is the more interesting one.

11.1.2. Game-theoretical semantics and modal operators

Game-theoretical semantics is a special branch of semantics. It competes with other approaches in semantics, notably with (Tarski-Carnap--Montague-style) model-theoretic semantics. (15) As such, it is not intended as a theory of argumentation. Its systems are made neither for purposes of human communication not for the purpose of conflict resolution, i.e., they are not dialectic systems. (16) Considerations pertaining to norms for human communication or for critical debates are absent from the papers in the field of game-theoretical semantics. (17) Nevertheless, the principal theoretical structures of game-theoretical semantics, viz., the contests between "Myself" and "Nature", are easily reinterpreted as regimented debates between two humans or two groups of humans. It is, therefore, reasonable to expect clues to a future modal dialectics from the writings on modal game--theoretical semantics. In Hintikka [QLQ] we find the following typical rule, "G-knows", in which principles of game-theoretical semantics are combined with concepts belonging to possible-world semantics: (18)

(G-knows) If the game has reached a world ω' and a sentence of the form

a knows that X,

where 'a' is a proper name, Nature may choose an epistemic a-alternative ω" to ω'. The game is continued with respect to ω" and X', where X' results from X by replacing all pronominal cross--references to the initial 'a' by 'a'.

This rule is formulated so as to apply to (a fragment of) English. Below

I formulate an abstract version of this rule, suitable for any language - whether natural or artificial - with a necessity operator (to be denoted by "□"). This necessity operator need not to be the epistemic one (a knows that ...) and therefore I shall not mention "a" and replacements involving "a". It will be clear at once that the use of such a rule for □ by (groups of) human beings presupposes two things:

(i) that there is a supply Ω of "possible worlds" and a binary relation R (the alternative relation) on Ω available (in some strong, pragmatic sense) (19) to the debaters;

(ii) that each statement is made relative to some $\omega\in\Omega$.

The rule is eligible as a supplement to rule F_2D 1 in [AD1]. (20)

Figure 2 (21)

	(Speaker:)U	(Critic:)aU	structural pU
$MatRule_{\Box}$	□V-at-ω	ω'?	V-at-ω'
		(provided ωRω')	

According to this rule a Critic of a statement of the form □V-at-ω can attack this statement by choosing some $\omega'\in\Omega$ such that ωRω'. So R constitutes a restriction on the choices open to the Critic. The only structural protective defense available to the "first Speaker" is to make a statement of the form V-at-ω'. Clearly, the supply Ω of "possible worlds" must be available at least to the extent that debaters can choose and name (or otherwise indicate) members of it. Each statement contains exactly one indication of an $\omega\in\Omega$.

In game-theoretical semantics each play (tournament) of a game ends with some atom (possibly some negated atom) U-at-ω. The winner and the loser of the play are determined by the truth value of U at this possible world ω according to the underlying model $<\Omega,R,I>$. (In some of the games it is also relevant whether or not the roles of the players are in the end interchanged - on account of an odd number of negated sentences.) If these games are to be adapted for use by (groups of) humans it is required

(iii) that there be a method by which to determine the truth value of each atomic statement relative to each $\omega\in\Omega$.

Such a method I shall call a complete modal material procedure (complete modal m.p.).

The use of underlying models $<\Omega,R,I>$, unobjectionable though it is in game-theoretical semantics, seems questionable in a context of human debates. In particular, each of the suppositions (i)-(iii) then seems unrealistic. How does a company of debaters get hold of a structure $<\Omega,R>$ of possible worlds, in the sense that they are able to choose and name members of Ω and make their statement relative to them? And how to determine whether to accept or reject a certain atom U relative to a certain $\omega\in\Omega$? The problem can be described as that of finding externalizations, for purposes of communication and discussion, of the bare notions of a possible world, of alternativeness, and of truth of an atom relative to a possible world. (22)

If one, nevertheless, assumes (i)-(iii), the modal games of game--theoretical semantics can be reformulated as modal systems of material dialectics. "Myself" and "Nature" are then to be replaced by dialectical roles to be taken by (groups of) humans. (23)

In my [AMD], in which material dialectic systems are treated abstractly, modal operators were not considered. However, the results in that paper can easily be extended so as to include modal systems of all sorts. Instead of a simple underlying valuation, there is now an underlying model $<\Omega,R,I>$, and instead of the assumption that each atom is either true or false simpliciter, there is now assumption (iii) above. Furthermore each assertion in a dialogue must now be made relative to some specified $\omega\in\Omega$. The following is a generalized formulation of the Adequacy Theorem that, in [AMD], was formulated and proved for nonmodal material dialectics: (24)

Modal Adequacy Theorem Let $\mathbf{G}$ be a modal material dialogue game (in a language $\mathcal{L}$) that is both locally finite (25) and regular. (26) Let $\mathfrak{M}$ be a modal model theory for $\mathcal{L}$ such that the logical rule of $\mathbf{G}$ is in accord with $\mathfrak{M}$. (27) Let M be a model according to $\mathfrak{M}$ and let us consider any starting position (28) of $\mathbf{G}$ with M as its underlying model.

Then:

(a) if all initial assertions made by the first Speaker (29) are true in M whereas at least one initial assertion made by the second Speaker (30) is false in M, then there is a winning strategy for the first Speaker;

(b) if all initial assertions made by the second Speaker are true in M, then there is a winning strategy for the second Speaker;

(c) if there are no initial assertions made by the first Speaker ("Black"), whereas there is exactly one initial assertion of some sentence A, made by the second Speaker ("White") relative to a possible world ω, then there is a winning strategy for White iff A is true at ω in the model M according to the theory $\mathfrak{M}$ (otherwise, if A is false at ω in M according to $\mathfrak{M}$, there is a winning strategy for Black).

Proof See [AMD]. The present theorem can be proved in the same way as its nonmodal cognate● (31)

Rules such as "G-knows" and "$\text{MatRule}_{\Box}$" are indeed in accord with current modal model theories. For, if $\Box U$ is false at ω (in the underlying model $M = \langle \Omega, R, I \rangle$), the Critic of a statement U-at-ω may choose an $\omega' \in \Omega$ such that $\omega R \omega'$ and such that U is false at ω'. This constitutes an attack that is both honest and ruthless. (32) If $\Box U$ is true at ω, no such attack is possible.

Let me summarize the merits and drawbacks of the modal systems of material dialectics that were suggested by modal game-theoretical semantics. First of all the assumptions (i)-(iii) seem pragmatically unrealistic -- and much more so than the nonmodal assumption of two-valuedness, analogous to (iii), which underlies the nonmodal system MatDial (33) and other material dialectic systems. On the other hand, once these assumptions are granted, the Modal Adequacy Theorem is there to tell us that the dialectics agrees with pre-existing notions of modality (of our verbal and cognitive culture) to exactly the same extent as the corresponding model theory does. And, although such an agreement with pre-existing notions or habits of speech is not al all decisive, it will probably help to make a dialectic system more attractive (to people sharing in our verbal and cognitive culture) as an instrument of conflict resolution. So this must count as a merit, assuming, of course, that the corresponding model theory does, to some degree, accurately depict these pre-existing notions of modality.

The Adequacy Theorem, furthermore, shows us that we can widely vary the structural rules of the dialogue games without interfering with the established connection between semantics and dialectics. Thus the structural rules of the systems of modal dialectics that were suggested by game-theoretical semantics may be chosen in such a way that all the more fundamental norms of formal dialectics (34) are implemented.

11.1.3. The theory of levels of discourse

In 1978 R. Inhetveen reported upon a dialogical approach to modal logic by M. Marčinko that can roughly be described as intermediate between that of Lorenzen and that of Hintikka. (35) Marčinko's central notion is that of a level of discourse (Dialogebene). At each move in a discussion the "level of discourse" at which that move takes place must be explicitly indicated. A level is identified with a finite index sequence $L = < l_1,\ldots,l_n >$. (36) So at each move the party that makes the move should mention such an index sequence. Usually the level of a move is to be the same as that of the move by the other party to which it reacts. A shift to a new level of discourse occurs only in connection with an attack on a □-formula according to the following rule: (37)

Figure 3

	(Speaker:)U	(Critic:)aU	structural pU
$\text{Rule}^{M}_{\Box}$	L:□V	L':? (provided LRL')	L':V

Once more we meet with a relation R. (Cf. Figure 2.) This time R relates not possible worlds but levels of discourse, i.e., index sequences. The extension of R is to be fixed by the rules of the dialectic system. Inhetveen mentions several simple specifications such as: (38)

LR_2L' iff L = L' or L' is a continuation by one index of L;

LR_3L' iff L = L' or L' is a continuation of L, etc.,

but he does not discuss any reasons for preferring one over the other.

This whole technique of index sequence manipulation may be viewed as an attempt at "externalization for purposes of communication and discussion" of the bare notions of 'possible world' and of 'statement relative to a possible world'. (39) The relation R, again, may be regarded as an externalization of the notion of an 'alternative relation between possible worlds'. (40)

There is, however, nothing in the Marčinko-systems that corresponds to a complete modal m.p. The games, as described by Inhetveen, are wholly "formal" in the sense that no material procedures or material closure rules are applied in them. The only situation in which P wins a "chain of arguments" (41) is the situation where O has both stated and attacked a statement of some one sentence _at the same "level of discourse"_. (42)

The present framework allows for the introduction of innumerable dialectic systems. One obtains different systems not only on account of different determinations of R, but also on account of different choices of structural rules (e.g. "classical" of "constructive" ones). In contrast with the systems of Section 1.2, the Marčinko-systems are highly sensitive to changes in the structural rules. For several choices of R - together with "classical" structural rules - a dialectic system is obtained yielding a logic (43) that is demonstrably equipollent to some well-known axiomatically characterized modal logic, e.g., T, B, S4 or S5. (44)

There remains the pragmatic question of why a company of potential debaters interested in methods for conflict resolution should employ sequences of indices at all. Do these sequences stand for anything tangible? Or, can they be made to? What is needed is at least one of two things. Either we must be given more intuitive background, i.e., some indication of traditional (dia)logical practices that are critically reconstructed by means of index sequence manipulation, more or less in the way in which the traditional practices of generalization are reconstructed by means of (purely syntactic) variable manipulation. Or else, if no such intuitive background can be given, it needs to be explained how index sequence manipulation implements some norm(s) of verbal dialectics and furthers the goal of verbal conflict resolution. (45)

One possible intuitive background for the index sequences is to be found in the description of generalized Lorenzen-dialogues - which are n-person games - by F. van Dun. (46) Following his suggestions concerning modalities and combining these with Marčinko's, one might interpret each index sequence as (the proper name of) a debater and read LRL' as: L' is a co-player of L. A statement by player L of □A then binds all co-players of L to the defense of A. L's adversary may, in the context of an attack on □A, select the co-player of L that is to take on the defense. Once more, there are many different possible rulings as to the co-player relation R (Van Dun speaks also of "different kinds of partnership" characterized by "distributions of responsibility") (47) and, therefore, many different possible systems of generalized Lorenzen-dialogues.

In Section 2 I shall adopt the idea of distinct levels of discourse. I shall, however, not make use of index sequences. Instead the level at which each statement is made will be apparent from its grammatical form, or more precisely, from its principal modal operator. Further, I want to incorporate the use of modalities in systems for two-party debates. For, in my opinion, the notion of a two-party debate is more fundamental than the notion of a generalized Lorenzen-dialogue. The latter can presumably be defined in terms of complexes of two-party debates, together with a higher order ruling as to who should (or may) start a debate with whom and about what. Further, the use of modal operators need not to be restricted to debates within the context of a generalized Lorenzen-dialogue. Traditional verbal practice suggests rather that they have a role to play within the constituent two-party debates.

11.1.4. Noncumulative dialectics as a two-leveled modal dialectics

In [NDM], Section 2, I attempted to construct a plausible system of formal dialectics meeting the following requirements:

(1) agreement with the elementary rules and fundamental norms of dialectic systems set forth in [AD1], Ch. III; (48)

(2) implementation of the fundamental norm of two-leveled dialectics: (49)

FD $L_2$1 A strict (local) thesis is to be defended, ultimately, on the basis of strict concessions;

(3) agreement with the validity notions taken from noncumulative dialectical semantics. (50)

The fundamental norm of two-leveled dialectics is the norm that is acknowledged by any company of debaters that wants - for some reason or other - to discriminate between two kinds of statements, in [NDM] called strict statements and contingent statements, in such a way that strict statements may be used in the defense of statements of either kind, but contingent statements may be used in the defense of contingent statements only. The terms "strict" and "contingent" merely serve to mark the two classes of statements, without further metaphysical or epistemological implications. A company's reasons for introducing the distinction at all may very well be metaphysical or epistemological or whatever, but these reasons are of no concern for the logical task at hand, viz., the implementation of the norm once acknowledged.

In [NDM] I gave several traditional examples to serve as an intuitive background for the distinction between strict and contingent statements. (51) For instance, the strict statements could be identified with analytic statements (and the contingent statements with synthetic ones), or the strict statements could be taken to be statements of prima facie duty (and the contingent statements those of actual duty), etc. The same intuitive background serves to introduce one to the task of constructing a (two-leveled) modal dialectics. In fact, the two tasks, of constructing a noncumulative dialectics and of constructing a modal dialectics, are so closely related that in [NDM] I called the search for noncumulative dialectics "an exploration of modal dialectics". (52) The task of constructing a (two-leveled) modal dialectics can be described simply as that of finding a (family of) dialectic system(s) that meets the first two requirements: (1) and (2). The third requirement is of no concern here.

There is a lesson to be learned from the search for a noncumulative dialectics in [NMD]. That search was not completely successful in that the proposed systems complied with requirement (3) only in a weak sense. This partial failure was seen to be a consequence of the fact that these systems are not "invertible". (53) I was unable to find an invertible system (fully complying with all three requirements) because of difficulties in implementing the fundamental norm FD $L_2 1$. To implement this norm straightforwardly, O is to be granted a right to withdraw its contingent concessions in certain circumstances. But in what circumstances exactly? The very same question turns up if the purpose is to construct a modal dialectic system, independent of requirement (3).

The root of the trouble met with in [NDM] seems to lie in the portmanteau character of the logical constants → and ~. In noncumulative dialectic systems these syntactic operators are simultaneously charged with <u>two</u> <u>distinct</u> dialectical jobs:

(i) U→V and ~U are <u>strict</u> sentences, i.e., when the (local) thesis is of one of these forms this is a signal that O has certain rights to withdraw its contingent concessions (either at once or in the near future, or ...).

(ii) U→V and ~U are to be attacked and protectively defended according to the usual Lorenzen rules ("strip rules") for conditional sentences and negations. So, in an attack the Critic should state U, etc. (54)

These two jobs can be separated by the introduction of a new dialectical constant, □, for the first of them. We can then write □(U→V) and □~U instead of the old U→V and ~U. It will further be possible to retain the Lorenzen rules for → and ~ and at the same time to give a more satisfactory implementation of FD $L_2$1 by means of a separate rule for □. (55)

Thus it appears that the introduction of a syntactic operator of necessity can be defended as a means to a pragmatic end, viz., that of formulating rules of formal dialectics intended for the implementation of the fundamental norm FD $L_2$1, or a related norm. The norm FD $L_2$1, again, expresses a company's decision to discriminate - for one reason or another - between a class of strict statements and a class of contingent statements. Thus the ultimate ground for the introduction, or toleration, of modal operators in a language lies in the (theoretical, practical or poetical) reasons for having a distinction between classes of statements.

11.2. A theory of modal dialectics as many-leveled dialectics

At the end of the preceding section we met with a motivation for the introduction of a modal operator "□". In the first part of the present section I shall proceed with the forementioned implementation of the fundamental norm FD $L_2 1$, taking advantage of "□". First, however, I shall present a generalization of that fundamental norm so as to encompass the discrimination between not just two, but any number of classes of statements. The generalized norm, FD L1 below, will then be implemented within the framework of a dialectic system as given in [AD1], III.

The second part of this section contains rules for the construction of modal dialogical tableaux. Also, I shall point out some simple properties, both of the modal dialectic systems and of the corresponding dialogical tableaux systems.

11.2.1. The fundamental norm of many-leveled dialectics and its implementation

From a dialectical point of view there is no reason to stick to just one □-operator. For, obviously, a company may have many reasons for wanting to discriminate between not just two, but more levels of (dialectical) strictness. For instance, in order of increasing strictness; synthetic *a posteriori* statements/synthetic *a priori* statements/analytic statements. Another sequence is: statements of separate (alleged) facts/statements of empirical generalizations/statements of empirical (theoretical) laws/ mathematical statements/logical statements. As I observed elsewhere,(1) such examples are tied to certain philosophical schools or positions. More examples can be drawn from Lorenzen's schematic classification of statements.(2) This schema, moreover, suggests that it would be unduly restrictive to suppose that the ordering of classes of statements as to strictness is in all cases linear (or, simple). It will suffice to suppose that the ordering is a strict partial ordering, i.e., transitive and asymmetric. In the following I shall, therefore, assume that a system of formal dialectics is to be constructed for a company that, for some reason or other, has adopted an (exhaustive and exclusive) classification ③ of statements. Further, I shall assume that on the set of admitted (kinds or) classes of statements there is defined some strict partial ordering relation, "(dialectically) stricter than". This relation induces a second strict partial ordering, viz., an ordering of the statements themselves. This second ordering will also be denoted by the words "(dialectically) stricter than":

> U is dialectically stricter than V iff $U \in K$ and $V \in L$ for some classes of statements K and L such that K is dialectically stricter than L. ④

Let us not forget what these assignments of relative strictness are supposed to be about. As with the absolute distinction between strict and contingent statements in [NDM], ⑤ making assignments of relative strictness expresses the company's intention to discriminate between different kinds of statements with respect to the way in which these are to be defended in a critical debate. Let me formulate this intention as a norm - analogous to FD $L_2$1 ⑥ - to be called the fundamental norm of many-leveled dialectics:

FD L1 A (local) thesis is to be defended, ultimately, on the basis of concessions that are as strict as or stricter than this thesis.

FD L1 comprises, as special cases, norms FD L_n1 for each n, where n indicates the number of classes of statements. Taking n = 2 gives us FD $L_2$1. If we are to implement FD L1 (and, therefore, each FD L_n1) some rights of withdrawing concessions other than the ones mentioned in the norm must be granted to O. But, as we know from the study of two-leveled dialectics,[7] it is not acceptable that O be allowed to execute a withdrawal of concessions <u>immediately</u> after its attack on the thesis. First P must get an opportunity to elicit new concessions of the appropriate strictness by means of counterattacks. To implement this, let there be a "sign of degree of strictness", or "sign of necessity", for each type of statement distinguished by the company.[8] This I propose to execute as follows. Let I be a set of indices such that there is exactly one index $i \in I$ for each kind (K_i) of statement. Let us define:

$i \prec j$ iff K_j is stricter than K_i;

$i \preceq j$ iff $i \prec j$ or $i = j$.

Let us, further, associate one syntactic (unary and propositional) operator $\boxed{i}$ with each $i \in I$. The language $\mathcal{L}$, originally used by the company, is correspondingly extended to a language $\mathcal{L}_I$. In $\mathcal{L}_I$ the degree of strictness of each statement is explicitly indicated by means of its principal operator.[9] Hence the degree of strictness of a statement is determined by the sentence this statement expresses, two statements of the same sentence always having the same degree of strictness. For each necessity operator, $\boxed{i}$, let us adopt the following rule (formulated in terms of sentences!):

<u>Figure 4</u>

	(Speaker:) U	(Critic:) aU	structural pU
Rule $_{\boxed{i}}$	$\boxed{i}$ V	?	V

Clearly, this is in the present context the analogue of Lorenzen's rule (Figure 1). The Speaker may be either O or P. If O is the Speaker, and P the Critic, what the rule amounts to is simply that as long as $\boxed{i}\,\underline{V}$ is a concession, P may make use of V in its defense of the local thesis.

This is acceptable, since a concession $\boxed{i}\,V$ is simply a concession V with a symbol added to indicate the level of strictness. If, on the other hand, P is the Speaker, and O the Critic, then the rule allows us to separate the moment of O's attack "aU" from that of its withdrawal of concessions, for the latter operation can be suspended until after the execution of P's structural protective defense move. The following rule grants such withdrawal rights to O, and therefore suitably implements FD L1:

FD L2 If P has answered an attack on a local thesis of the form $\boxed{i}\,V$ by carrying out the structural protective defense move (V) assigned by Rule$_{\boxed{i}}$, O shall have a right to assume the neutral position to any or all concessions that are not of the form $\boxed{j}\,W$, where $i \lesssim j$, immediately before O attacks V. (10)

Immediate consequence: If the original thesis is of the form $\boxed{i}\,V$, P may - in each chain of arguments - in the first local discussion exercise all kinds of countercriticism on all of O's original concessions (as well as on concessions made by O in the course of that first local discussion), but ultimately P will state V (unless, of course, the first local discussion ends in some other way) (11) and must then defend its statement of V on the basis of those concessions of O's that are of the form $\boxed{j}\,U$, where $i \lesssim j$.

Let us call a withdrawal of concessions as described in FD L2 an i-withdrawal. It will be indicated by a numbered dashed line in O's column (i-withdrawal line).

Example 1 There are two levels of strictness: 0 and 1. Statements of level 1 are distinguished by the adverb "necessarily".

	Olga	Pope	Explanation
	a) if there is mind then necessarily God exists b) there is matter	(c) necessarily God exists	Original conflict of avowed opinions: (a) and (b) are concessions, (c) is the thesis.
1.	?	[God exists]	Olga attacks (c). The Pope obtains a right to use "God Exists" in a protective defense.
2.	[necessarily God exists]	(?) there is mind	The Pope defends counteractively, by means of an attack on (a).
3.	necessarily God exists		Olga defends protectively against the Pope's attack.
4.		God exists	The Pope defends protectively against Olga's attack.
(1)	------------------------		
	?	[ ]	Olga attacks the Pope's last statement. According to FD L2 this attack is preceded by a 1-withdrawal. Only 3 is not withdrawn.
5.	[God exists]	?	The Pope attacks 3.
6.	God exists		Olga defends protectively.
7.		You said so yourself!	The Pope makes an appropriate <u>Ipse dixisti</u>!-remark and wins the chain of arguments.

Note that an i-withdrawal is not to count as one of O's moves, in the sense that it would be followed by a move by P. On the contrary, an i--withdrawal is always followed immediately by an attack by O!

The rules FD L1, FD L2 and Rule $\boxed{i}$ are to be adjoined to the other rules of formal dialectics in [AD1] up to and including FD D6, and also including F_2D 1, in order to form a system of (many-leveled) modal dialectics for languages with both ordinary propositional connectives (→, &, ∨, ~) and modalities ($\boxed{i}$). As in [NDM] (12) we must reconsider the original argumentation in favor of adoption of the rules FD D7 and FD D8. For FD D7 this does not present any difficulty, (13) whereas FD D8 can even be strengthened: (14)

FD $D8^L$ (a) After an attack by O, P may not repeat the sentence T in the new local thesis T within the same chain of arguments, as long as the set of local concessions has not been augmented by any statement (not as yet withdrawn) of a new sentence;

(b) If P has executed the structural protective defense right of Rule $\boxed{i}$, by making a statement of V, P may not within the same chain of arguments execute a structural protective defense right according to the same rule and involving a statement of the same sentence V, unless some fresh concession of the form $\boxed{j}$ W has appeared (and has not again been withdrawn), where $i \not\prec j$.

We can now define five kinds of modal dialectic systems MID^L, etc., corresponding to the minimal and constructive systems MID, etc., of [AD1]. (Classical modal systems will be considered briefly in Section 4.)

Def. 1 Let L = < I, $\prec$ > be some partially ordered set of indices. For each constructive or minimal dialectic system σ (as defined in [AD1], IV), σ^L shall be the system obtained from σ by the inclusion of FD L1, FD L2 and $\text{Rule}_{\boxed{i}}$ (for each $i \in I$) among the rules, and by strengthening FD D8 to FD $D8^L$.

Thus we obtain the nonmaterial systems MID^L, etc., and also MID^L, etc., with material procedures and moves subjoined to them.[15]

In view of our intuitive background for the discrimination between classes of statements, the material procedures, too, should be relativized to an index $i \in I$. Thus instead of one class $\mathbb{T}$ of implicitly accepted atoms we get a class $\mathbb{T}_i$ for each level $i \in I$. And, similarly, for each $i \in I$, a class $\mathbb{F}_i$. For instance, $\mathbb{T}_i$ may stand for the observationally verifiable atoms, $\mathbb{T}_2$ for atoms verifiable by algorithmic calculation, etc. The assumption that $\mathbb{T}_j \subseteq \mathbb{T}_i$ if $i \prec j$, i.e., that the class of admitted procedures and hence that of implicitly accepted atoms narrow down as statements become stricter, is then plausible. Further, together with the right to assume the neutral position to concessions not of the form $\boxed{j}\,W$, where $i \precsim j$, mentioned in FD L2, there should be a right to suspend, for the rest of the chain of arguments, all material procedures that are not indexed by a j such that $i \precsim j$. Thus an i-withdrawal may include not only the withdrawal of concessions but also the suspension of material procedures.[16] In this way one can obtain dialectic systems that on the one hand do not depend on the notions of 'possible worlds', 'truth-at-a-possible-world', etc., nor on complete modal m.p.'s (Section 1.2), but which on the other hand do not restrict modal discussion to purely formal (nonmaterial) debates (Sections 1.1 and 1.3).

11.2.2. Modal dialogical tableaux. Some simple properties

In this section I shall list the modal analogues of some simple theorems about dialectic systems and dialogical tableaux that were shown to hold in [AD1].[17] I shall indicate where changes in the original proofs are called for.

Theorem 1^L If P has a winning strategy for a dialogue situation in a minimal modal system, then P has a winning strategy for that situation in the corresponding constructive modal system. (18)

Theorem 2^L Each of the modal systems of formal dialectics is locally finite. (19)

Def. 2 The P-liberalized system corresponding to any particular modal system of formal dialectics is obtained by cancellation of the rules FD D6 and FD $D8^L$. Furthermore, in these systems the i--withdrawals are always taken to comprise all the concessions eligible for withdrawal.

The last characteristic in Def. 2 can hardly said to be "P-liberalizing", but it would be confusing to choose a name for these systems here other than that used in [AD1] and [NDM]. (20) As before, P-liberalized systems are introduced only for the study of strategy. So we need to establish:

Lemma 1^L P has a winning strategy for a dialogue situation S according to a P-liberalized modal system iff P has a winning strategy for S according to the corresponding official system.

Proof If P has a winning strategy according to the official system it must have one that holds against an Opponent who always withdraws the maximal number of concessions. This latter strategy is a winning strategy according to the P-liberalized system. Conversely, if P has a winning strategy according to the P-liberalized system, this strategy equally holds good against Opponents that, perhaps, do not make a maximal use of their rights of withdrawal. It remains to be checked that the winning strategy may be so reformed as to comply with FD D6 [21] and FD $D8^L$. [22]

There are four new rules for the construction of modal P-winning strategy diagrams, or modal dialogical tableaux (for P-liberalized systems). To state these rules, let

$$\Pi^i =_{Df.} \Pi \cap \{\boxed{j}\, V \mid i \lessdot j\}$$

Π^i is called the i-<u>kernel</u> of Π.

<u>New compulsory rules</u>

O I $\boxed{i}$ Under $\Pi/_0\ \boxed{i}\, U$ you must write $\Pi/\boxed{i}\, U/_P\ [U]$.

O III $\boxed{i}$ Under $\Pi;[U]/T/_0\ \boxed{i}\, V;\Gamma$ you must write both $\Pi,U/T/_P\Gamma$ and $\Pi/\boxed{i}\, V/_P\ [V]$.

<u>New choice rules</u>

Pd (reformulated) If $T \neq \boxed{i}\, V$ for any $i \in I$:

Under $\Pi/T/_P\Gamma$ you may write $\Pi/_0\ Z$ for any $Z \in \Gamma$.

Pd^i Under $\Pi/\boxed{i}\, V/_P\Gamma$ you may write $\Pi^i/_0\ Z$ for any $Z \in \Gamma$. [22a]

P $\boxed{i}$ Under $\Pi,\boxed{i}\, U/T/_P\Gamma$ you may write $\Pi,\boxed{i}\, U;[U]/T/_0\Gamma$.

According to the rule Pd, the moment of withdrawal is attached to the moment of P's protective defense move by virtue of Rule $\boxed{i}$, rather than to the moment of O's attack, which immediately follows. In view of FD D7 this makes no difference. [23] In the tableau notation, i-withdrawals are denoted by i-withdrawal lines.

<u>Example 2</u> Let $L = \langle I, \lessdot \rangle, I = \{1,2\}, \lessdot = \{\langle 1,2 \rangle\}$. The following tableau for $B \rightarrow \boxed{2}\,\boxed{1}\, A, \boxed{1}\, B/\boxed{1}\,\boxed{2}\, A$ closes in MID^L:

	O	MID^L	P
	$B \rightarrow \boxed{2}\boxed{1}A$		
	$\boxed{1}B$		$\boxed{1}\boxed{2}A$
	....		....
$OI\boxed{1}$	?		$[\boxed{2}A]$
$P\boxed{1}$	[B]		?
OII	B		
$P\rightarrow$	$[\boxed{2}\boxed{1}A]$		B
$OIII_{At}$	1: ?	2: $\boxed{2}\boxed{1}A$		1: [ ]	2:
Pid			1: !
Pd	1 - - - - - - - -		$\boxed{2}A$
$OI\boxed{2}$	?		[A]
Pd	2 - - - - - - - -		A
OI_{At}	?		[ ]
$P\boxed{2}$	$[\boxed{1}A]$		?
OII	$\boxed{1}A$		
$P\boxed{1}$	[A]		?
OII	A		
Pid			!

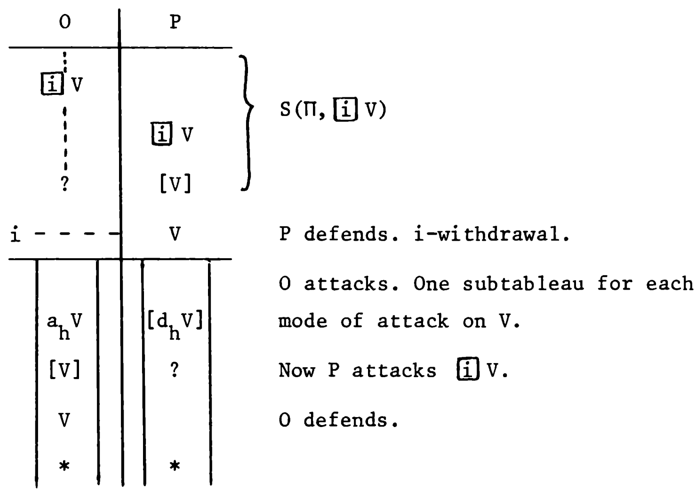
Note the 1-withdrawal line and the 2-withdrawal line that go with the applications of Pd.

<u>Theorem 3^L</u> P has a winning strategy for a dialogue situation <u>S</u> on the strength of a P-liberalized modal system σ iff P has a winning strategy for <u>S</u> on the strength of σ with the following rule (R_{At}) added to it: an <u>Ipse</u> <u>dixisti!</u>-remark may be made only if the local thesis is atomic.

<u>Proof</u>

The proof for this theorem is largely similar to that of Theorem 3 in [AD1].[24] But the diagram used in that proof does not suit the case $W = \boxed{i}V$. For, unless it happens itself to be of the appropriate strictness, the pW (= V)[25] on the left may be withdrawn as P defends by means of the same pW. Instead of the diagram in [AD1] we may use the following one in which P <u>first</u> defends:

<u>Figure 5</u>



Subtableau * gives us a sequent $\Pi^i, \boxed{i}V, a_hV, V/V/_P [d_hV]$ and this sequent is of the form $S^h(\Pi,V)$. Since V has one logical operator less than $\boxed{i}V$, we may apply the induction hypothesis, etc.●

<u>Theorem 4^L</u>

Let Π/Z be a sequent such that $\wedge$ does not occur in Z nor in any sentence of Π. There is a closed dialogical tableau for Π/Z on the strength of MND^L (CND^L) iff there is a closed dialogical tableau for Π/Z on the strength of $M\wedge D^L$ ($C\wedge D^L$).

There are some minor changes needed in the proof, given in [AD1], that closed M$\wedge$D-tableaux can be transformed into closed MND-tableaux. (26)

Finally, there is now the following theorem, which, as we know, fails for the noncumulative systems: (27)

Theorem 5^L (Invertibility Theorem) There is a P-winning strategy for $\Pi/_O Z$ on the strength of a modal dialectic system σ iff there is a P-winning strategy for $\Pi/_P [Z]$ on the strength of the same system.

This can be proved as in [AD1]. (28)

11.3. Semantic and derivational systems corresponding to modal dialectic systems

In the preceding section modal systems, and as a consequence modal logics, [1] were established on a purely dialectic basis, without any intermingling considerations of a deduction-theoretic or model-theoretic kind. I shall now briefly present systems for the construction of deductive tableaux, for natural deduction, for axiomatic deduction, for model-theoretic semantics, and for the construction of semantic tableaux corresponding to the modal dialectic systems. At the end of this section I shall establish the fundamental unity of the different approaches to modal logic. This unity is what is formulated in the Full Circle Theorem below. [2]

The nomenclature for the systems is the same as that in [AD1], [3] but there is a superscript "L" added to each name. "L" stands for some partially ordered set of indices $< I, \prec >$. To define a system completely one should fix a value for "L" and also a language $\mathcal{L}_I$ to which the system pertains. What follows will, however, be independent of the choice of I.

11.3.1. A survey of systems

For the construction of modal deductive tableaux, i-cancellation lines will take the place of i-withdrawal lines. [4] In each apllication of the following reduction rule one should draw an i-cancellation line:

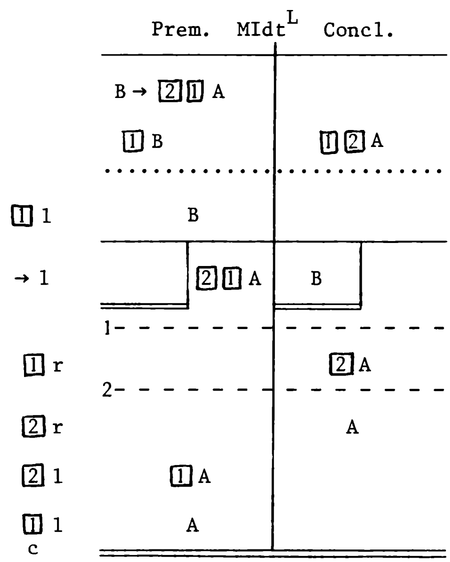
$\boxed{i}$r $\quad \Pi/\boxed{i}U$ reduces to Π^i/U.

An i-cancellation line indicates a cancellation of all premises not of the form $\boxed{j}V$ $(i \prec j)$ above it in the same subtableau. How such a line is to be drawn is shown in Example 3, below. There is one other new type of reduction rule:

$\boxed{i}$l $\quad \Pi, \boxed{i}U/Z$ reduces to $\Pi, \boxed{i}U, U/Z$.

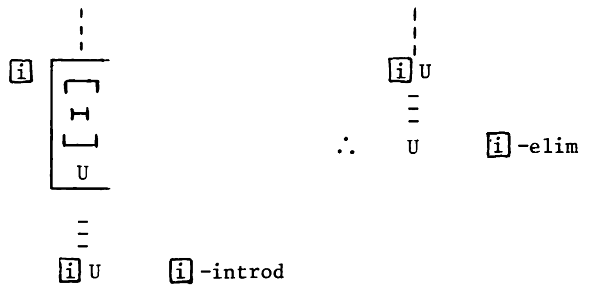
In a system ($MIdt^L$, etc.) based on an index structure $L = < I, \prec >$, there must of course be included a rule [i] r and a rule [i] l for each i∈I. The other rules of these systems are identical with those of the corresponding systems in [AD1], (5) but no rule should ever be applied to a canceled premise!

<u>Example 3</u> Let L be as in Example 2. The following is a closed deductive tableau for the same sequent as used in that example.



The systems of natural deduction that link up with these deductive tableaux are those of the type introduced by Fitch. (6) Since there is a necessity operator for each i∈I, we need i-<u>strict</u> <u>subordinate</u> deductions, marked off by i-<u>strict</u> <u>scope</u> <u>indicators</u>, for each i∈I. The i-strict scope indicators will be labeled "[i]". For each degree of strictness. We must have an introduction rule and an elimination rule for necessity:

Figure 6 [7]



These rules, and also the (constructive or minimal) rules for ordinary propositional connectives to be taken from [AD1], VI, are in their application subjected to a restriction. Let us say that an i-strict scope indicator which starts - but is not retracted - between the conclusion of an application of a rule and one of its premises, [8] i-separates the conclusion from this premise.

RESTRICTION: No application of a rule may be such that the conclusion is i-separated from one of its premises, unless this premise is of the form $\boxed{j}$ V or a j-strict subordinate deduction, where $i \measuredangle j$.

In fact, this is the way in which i-strict subordinate deductions differ from ordinary ones!

As to axiom systems, these are the modal postulates to be adjoined to the (constructive or minimal) postulates in [AD1], VIII:

Axs $\boxed{i}$	$\boxed{i}(U \to V) \to (\boxed{i}U \to \boxed{i}V)$	(For each $i \in I$.)
Axs $j \measuredangle i$	$\boxed{i}U \to \boxed{j}U$	(For all $i, j \in I$ such that $j \measuredangle i$.)
Axs T_i	$\boxed{i}U \to U$	(For each $i \in I$.)
Axs 4_i	$\boxed{i}U \to \boxed{i}\boxed{i}U$	(For each $i \in I$.)
$\boxed{i}$-Nec	$\dfrac{U}{\therefore\ \boxed{i}U}$	(Provided no use of premises is made in the deduction of U.)

Thus we obtain all kinds of propositional multiply modal systems of an S4-type and on a constructive (intuitionistic) or minimal basis. For $L = < \{1\}, \emptyset >$, i.e., if there is only one necessity operator, these systems, or close variants thereof, have been propounded by H.B. Curry, (9) whereas $CNax^L$ was put forward tentatively by R.A. Bull as a plausible intuitionist logic of necessity. (10)

It is now easy to formulate a (dialectical) semantic theory, along the lines of Kripke, (11) for each of the present modal logics. Let an L-normal dialectical structure ($L = < I, \prec >$) be, by definition, an ordered quadruple $\mathfrak{J} = < L, \mathbf{A}, \mathbf{N}, D, R >$ such that $\mathbf{A} \neq \mathbf{N}$, $D \neq \emptyset$. R is a function defined on $I^* = I \cup \{0\}$, such that $R_i \subseteq D \times D$ and R_i is reflexive and transitive for each $i \in I^*$, whereas for all $i, j \in I^*$: $R_i \subseteq R_j$ if $i \preceq j$, and $R_0 \subseteq R_i$ for all $i \in I$.

An L-minimal dialectical structure is a quintuple $\mathfrak{J} = < L, \mathbf{A}, \mathbf{N}, D, Abs, R >$, such that $< L, \mathbf{A}, \mathbf{N}, D, R >$ is an L-normal dialectical structure and such that $Abs \subseteq D$. An (R_0-)cumulative interpretation of $\mathcal{L}_I$ on an L-normal or L-minimal dialectical structure is a function I defined for all pairs $< U, d >$, where U is an atomic sentence of the language, $\mathcal{L}_I$, (but $U \neq \Lambda$) and $d \in D$, with values in $\{\mathbf{A}, \mathbf{N}\}$ and such that if $I(U,d) = \mathbf{A}$ and dR_0d' then $I(U,d') = \mathbf{A}$. An L-constructive (L-minimal) dialectical model is simply a pair $M = < \mathfrak{J}, I >$, where $\mathfrak{J}$ is an L-constructive (L-minimal) dialectical structure and I is a cumulative interpretation on $\mathfrak{J}$. For each model M there is a valuation function, v_M, defined for all pairs $< U, d >$ such that U is a sentence of $\mathcal{L}_I$ and $d \in D$, which is an extension of I ($I \subseteq v_M$), and takes its values from $\{\mathbf{A}, \mathbf{N}\}$. The values of v_M for the complex sentences are given by the semantic rules. These are the constructive (or minimal) rules in [AD1], (12) but with R_0 instead of R, and (for all $i \in I$):

$Sem^c \boxed{i}$ (= $Sem^m \boxed{i}$) $v_M(\boxed{i}\, U, d) = \mathbf{A}$ iff for all d' such that dR_id': $v_M(U, d') = \mathbf{A}$.

The notions of an L-<u>constructively</u> (L-<u>minimally</u>) <u>valid</u> sequent are to be defined in the usual way.[13]

Finally, a matching system for the construction of semantic tableaux comprises the following rules, besides those formulated in [AD1]:[14]

$\boxed{i}$ R   A set of sequents $\Sigma;(\Pi/\Gamma,\boxed{i}\,U)$ reduces to the set of sequents $\Sigma;(\Pi/\Gamma,\boxed{i}\,U);(\Pi^i/U)$.

$\boxed{i}$ L   A set of sequents $\Sigma;(\Pi,\boxed{i}\,U/\Gamma)$ reduces to the set of sequents $\Sigma;(\Pi,\boxed{i}\,U,U/\Gamma)$.

11.3.2. Full cicle

<u>Theorem 6^L</u>   The following conditions are equivalent, provided that they refer to corresponding modal systems and that the sequent Π/Z consists of sentences of some language $\mathcal{L}_I$ to which these systems pertain:

<u>a</u> There is a P-winning strategy for Π/Z.

<u>b</u> There is a closed deductive tableau for Π/Z.

<u>c</u> There is a natural deduction of Z from Π.

<u>d</u> There is an axiomatic deduction for Π/Z.

<u>e</u> Π/Z is L-minimally/L-constructively valid in $\mathcal{L}_I$.

<u>f</u> There is a closed semantic tableau for Π/Z.

<u>Proof</u>   It is no problem to extend the proofs contained in [AD1] so as to cover modal systems of the present kind. Below I shall only indicate additions and modifications peculiar to the modal case.

From <u>a</u> to <u>b</u>: Of the instructions for the transformation of closed dialogical tableaux into closed deductive tableaux,[15] the fourth should be amended to read:

4a. Inscriptions [ ] and [U,V] should be removed. All inscriptions (in P's column) of [U] that were entered on the strength of OI$\boxed{i}$ or OIII$\boxed{i}$ should be removed. All other inscriptions of [U] are to be replaced by inscriptions of U.

4b. Each occurence of an i-withdrawal line in O's column opposite an inscription of U in P's column should be replaced by an i-cancellation line that runs on over the inscription of U:

Figure 7

```
Pd    i-----|U   ⇒        i-----|-----
                   [i] r         U
```

This single modification suffices. There are of course some additional types of units that need to be considered, viz.:

P→ + OIII$\boxed{i}$	⇒	→1
$P^{\wedge}_{\sim}$ + OIII$\boxed{i}$	⇒	$\sim 1_{\wedge}$
P$\boxed{i}$ + OII	⇒	$\boxed{i}$ 1
Pd	⇒	either a void reduction, or vr_1, or vr_2, or $\boxed{i}$ r
OI$\boxed{i}$	⇒	void.

From b to c: The technique of the "tape theorem" [16] can be applied to modal deductive tableaux. It can be checked that illicit "applications" of rules of natural deduction (i.e., such applications as do not respect the restriction formulated above) could only derive from illicit "applications" of tableau rules (i.e., applications that make use of a canceled premise). [17] Applications of $\boxed{i}$ r ($\boxed{i}$ 1) are transformed into applications of $\boxed{i}$-introd ($\boxed{i}$-elim).

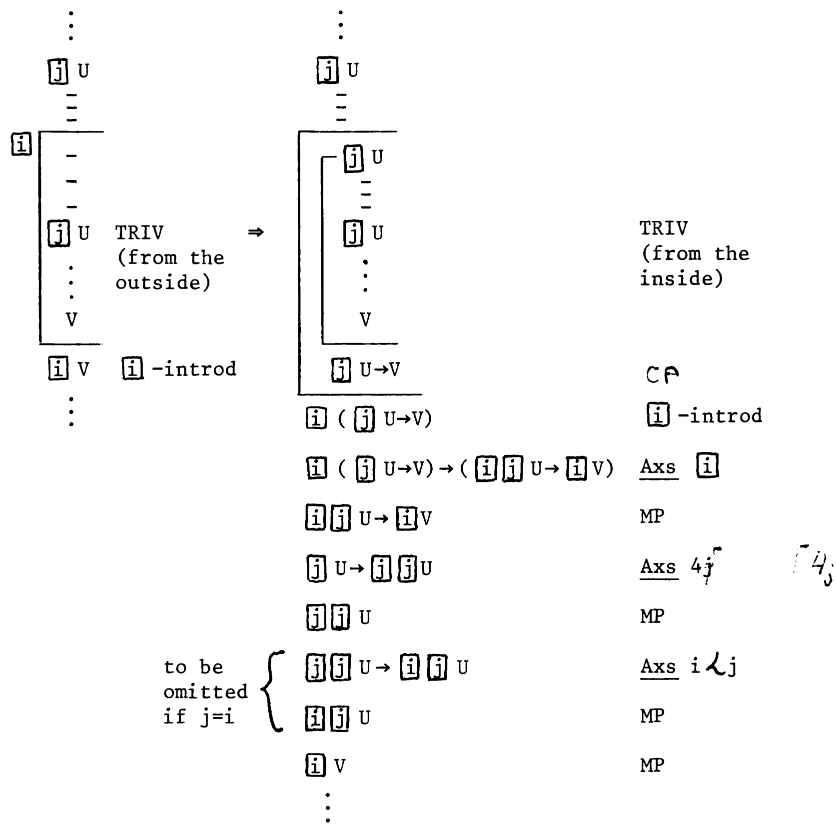
From c to d: Without loss of generality we may assume that the part

Ξ in [i]-introd is empty and that TRIV is the only rule such that premise and conclusion are sometimes i-separated. (18) Modal natural deductions can then be transformed into modal axiomatic deductions as follows: (19)

Phase 1 Eliminate all applications of rules other than MP, CP, TRIV, and [i]-introd in the usual way. (20) For [i]-elim use Axs T_i and MP. Thus one obtains a hybrid deduction which contains, besides axioms, applications of no rules other than MP, CP, TRIV, and [i]-introd.

Phase 2 Eliminate, successively, all strict subordinate deductions as follows. Leave out those that are not used in any application of [i]-introd. Then take any innermost strict subordinate deduction, i.e., one that does not contain another strict subordinate deduction. Let this deduction be i-strict. In it, only nonmodal rules are applied. The only sentences "from outside" used in it are of the form [j] U ($i \not\leq j$). The only rule that is applied for this purpose is TRIV. Figure 8 shows how one application of TRIV that brings in a sentence [j] U from the outside can be eliminated. Repeating this procedure again and again, we can get rid of all of them. Note that the occurrence of [j] U introduced into the i-strict deduction by TRIV is moved to the top of the i-strict deduction where it becomes an hypothesis for the application of CP.

Figure 8



In the end our i-strict subordinate deduction is transformed into one that makes no use of material from the outside. Let the application of [i] -introduction that immediately follows this i-strict deduction introduce [i] W (at the n-th line of the entire deduction). W is, within the i-strict deduction, derived on the strength of MP, CP and TRIV only. By the usual technique for eliminating CP, [21] we may obtain an axiomatic proof for W, and, by adding an application of [i] -Nec, for

$\boxed{i}$W. This proof may be placed on top of the original deduction and the i-strict subordinate deduction can be omitted. [22] The only problem that remains is to justify $\boxed{i}$W at line n (of the original deduction) in some way other than by $\boxed{i}$-introd.

This can be done by TRIV unless the $\boxed{i}$W occurrence that is the conclusion of the axiomatic proof is j-separated from line n, where it is not the case that $j \not\leq i$.

Let j_1 ..... j_m be the degrees of <u>all</u> separating scope indicators (in order from above to below). Then it suffices to extend the axiomatic proof of $\boxed{i}$W to one of $\boxed{j_1}$ ..... $\boxed{j_m}$ $\boxed{i}$W by $\boxed{j_m}$-Nec, ..., $\boxed{j_m}$-Nec, for $\boxed{i}$W can then be obtained by m applications of TRIV.

Now treat some other innermost subordinate deduction, etc. When they are all eliminated we have obtained a hybrid deduction that consists of three parts: (i) an axiomatic proof, (ii) the premises of the original deduction, (iii) a deduction which employs, besides axioms only MP, CP, and TRIV.

<u>Phase 3</u> Eliminate CP from part (iii) and the result is an axiomatic deduction. Note that the proviso that goes with $\boxed{i}$-Nec is satisfied.

From <u>d</u> to <u>e</u>: Validity or soundness of the axiom systems can be shown in the standard way by a deductive induction.

From <u>e</u> to <u>f</u>: The proof of this step, too, is standard. [23]

From <u>f</u> to <u>a</u>: The proof in [AD1] for this step holds good for modal systems. [24] We only need to add two cases in the proof of Lemma XI.4:

case $\boxed{i}$L $\Sigma = \Sigma;(\Pi, \boxed{i}U/\Gamma)$ and reduces to $\Sigma;(\Pi,\boxed{i}U,U/\Gamma)$.

By the assumptions made earlier in the proof of the lemma:

1. $\Pi, \boxed{i}U, U/_P\Gamma \in W_P$. It follows that
2. $\Pi, \boxed{i}U; [U]/_0\Gamma \in W_P$ (0 must state U).
3. $\Pi, \boxed{i}U/_P\Gamma \in W_P$ (P can attack $\boxed{i}U$).

case $\boxed{i}R$ $\Sigma = \Sigma;(\Pi/\Gamma, \boxed{i}U)$ and reduces to $\Sigma;(\Pi/\Gamma, \boxed{i}U);(\Pi^i/U)$. By the assumptions:

1. $\Pi^i/_P[U] \in W_P$.

From 1 it follows, by Theorem 5^L (p.232), that

2. $\Pi^i/_0U \in W_P$. Hence
3. $\Pi/\boxed{i}U/_P[U] \in W_P$ (P can state U).
4. $\Pi/_0\boxed{i}U \in W_P$ (0 must attack $\boxed{i}U$).
5. $\Pi/_P[\boxed{i}U] \in W_P$ (P can state $\boxed{i}U$).
6. $\Pi/_P\Gamma, [\boxed{i}U]$ (P need not use the extra rights).

This conludes the circle ●

11.4. Conclusions and perspectives

In Sections 1.4 and 2.1 it was shown how one can argue for the introduction of modalities from the point of view of the construction of systems of formal dialectics conceived as instruments for the resolution of conflicts. The systems given in Def. 1 implement the idea of levels of discourse by means of Rule [i] and FD L2. Moreover they admit the possibility of subjoined material procedures. Thus they share in some of what is best in the theories of Lorenzen, Murphy, Marčinko and Hintikka.

Many vexing problems of contemporary modal logic were not encountered -- sometimes simply because they are outside the scope of this paper. For instance, I said nothing about modal predicate logic, deontic logic, or counterfactuals. (1) These subjects, too, should be scrutinized from the point of view of conflict resolution. (2) Other familiar problems I have not dealt with because, though they would fall within the scope of this paper had we encountered them, they simply did not arise: Once modalities are understood as devices that make certain forms of debate possible, there is - in that context - no problem left about the "real meaning" of necessity, or about the reality of "possible worlds". Again, I did not encounter any problem in connection with sentences containing iterated or nested modalities. The rules of modal dialectics, though not made deliberately to handle such sentences, are quite capable of telling us how to deal with them in a debate.

Another matter that was left out is the problem of classical modal dialectics. This problem can be understood in two ways.

(i) What happens if the rules FD L1, FD L2 and Rule [i] are subjoined to the classical dialectic systems? (3)

(ii) Can one find some "more or less" plausible dialectic system that exactly yields a certain multiply modal classical logic of S4-type?

The answer to the first question is that, while of course it can be done, the resulting dialectic system has nothing to recommend it. For instance, there would be a P-winning strategy for $\Box A \vee \Box{\sim}A$ ($\Box = \boxed{1}$):

Figure 9

KND^L?!

		O	P	
			$\Box A \vee \Box{\sim}A$	
OI$\vee$		?	$[\Box A, \Box{\sim}A]$	
Pd			$\Box A$	
OI$\Box$		?	~~$[\Box A]$~~ ⌈	⌈$[A]$
Pd	1	----------	A	
OI$_{At}$		?	[ ]	
Pd			$\Box{\sim}A$	(P has retained this defense right)
OI$\Box$		?	$[{\sim}A]$	
Pd	1	----------	${\sim}A$	
OI$\sim$		A	[ ]	
Pid			!	(P has retained this <u>general</u> protective defense right)

This is highly counterintuitive. Undoubtedly such results can be avoided if we adapt the dialectical rules for the purpose of avoiding them. But it seems hardly feasible to argue for such adaptions straightforwardly from the point of view of conflict resolution. The problem lies with "the fundamental norm of non-constructive dialectics", FD K, (4) which in [AD1] is not separately motivated but is introduced as a norm a company might decide to adopt and which leads to classical dialectics, i.e., dialectic sytems that yield classical logic. So here the logic one wants to end up with motivates the choice of a dialectical rule. This brings us to question (ii).

The answer to question (ii) is certainly affirmative. In fact it suffices to stipulate that with each i-withdrawal P loses all its protective defense rights (structural and general). FD K must, of course, be reformulated so as to allow for this. The metatheory for the ensuing systems of formal dialectics (KID^L, etc.) can be developed in a manner parallel to that of their constructive and minimal cognates. The move Pid in the tableau of Figure 9 is not permitted in KND^L thus defined!

This can all be done. In this paper, however, I have wanted to argue more straightforwardly in favor of certain modal dialectic systems and not on the basis of some known logic one wants the systems to yield.

As to S5 and other modal systems one can ask questions analogous to question (ii); but I have not taken up these matters either, and for the same reason.[5] I do not deny that such questions and the formulation of answers to them have an heuristic value. Indeed, the present paper is based largely upon insights gained in studying noncumulative logics and asking precisely such a question.[6] In [AD1] the classical dialectic systems and MND are also introduced from the point of view of pre-existing logics.[7] In this paper I have treated MND^L alongside the other systems, because it took little trouble to do so, not because I believe there is much to say for MND or MND^L as a dialectic system. MID^L on the other hand is merely a fragment of the other systems.

The most attractive modal propositional dialectic systems I know of, therefore, are at present CND^L, CΛD^L and MΛD^L.

APPENDIX

12. Essentials of the dialogical treatment of quantifiers

Quantifiers have been dealt with extensively by P. Lorenzen and K. Lorenz.[1] My purpose in the present paper is to show how the results of the foundational studies in Part 1, and of the metatheoretical studies in Part 2, can be adapted so as to be applicable to dialectical languages that contain quantifiers. For, indeed, the results of Parts 1 and 2 can be applied to predicate logic, albeit not without some modifications and additions. I shall briefly indicate the difficulties involved in such an application and show how these are overcome. Lorenzen and Lorenz, as well as other authors on the dialogical treatment of quantifiers, have dealt with these difficulties, and solved them in one way or another. Here, I shall attempt a self-contained survey of these matters that can be read continuously with Parts 1 and 2 of this dissertation.

Section 1 deals with a foundational problem: *how to keep discussions finite*, i.e., how to strengthen the rules of dynamic dialectics appropriately. This section applies equally to classical, constructive and minimal logic. From Section 2 onward I shall concentrate mainly on constructive predicate logic, without a *falsum* constant ($\wedge$). But what is said can easily be seen to hold for the other logics that were treated in Part 2.

Section 2 deals with the problem of finding *finite representations for infinite winning strategy diagrams* - a problem that does not arise in propositional (modal or nonmodal) logic, because in that context all winning strategy diagrams are finite (contain a finite number of nodes). I shall attach the name "closed dialogical tableau" to the finite representations. Consequently, a "closed dialogical tableau" will be something different from a P-winning-strategy diagram, whereas earlier in this dissertation these terms stood for the same "structures", though they were associated with different notations for these structures.

Section 3 extends the transformation techniques of Papers 6 (from dialogical tableaux to deductive tableaux) and 7 (from semantic tableaux to dialogical tableaux) to predicate logic. It will be seen that closed tableaux constructed on the strength of (variants of) Beth's original system for quantificational constructive semantic tableaux admit of a smooth transformation into closed dialogical tableaux (by the methods of Paper 7). These systems diverge considerably from Kripke's tableau system for intuitionistic logic.[2] The latter system and its close variants I call *Kripke-type systems*; the former ones, *Beth-type systems*. (Tableaux of both types are sometimes referred to as Beth tableaux for intuitionistic/constructive logic.) Kripke-type systems directly link up with Kripke model theory for constructive predicate logic. Hence, they admit of a straightforward completeness proof. Beth-type systems, too, are known to be complete relative to Kripke model theory. (3) They yield closed tableaux that are smoothly convertible into closed dialogical tableaux. It therefore seems desirable that a "full circle" should contain both types of system, i.e., the metatheorems for semantic tableaux should take us from "validity" to a "closed Kripke-type semantic tableau", and thence to a "closed Beth--type semantic tableau", and finally to a "closed dialogical tableau". The step from Kripke-type tableaux to Beth-type tableaux wil be taken care of in Paper 13, which constitutes the second part of this appendix.

12.1. How to keep discussions finite

Let us assume that we deal with an uninterpreted (4) first-order language having as logical operators: implication, conjunction, veljunction, negation, and universal and existential quantifiers. Let this be a language containing an infinite number of individual variables and an infinite number of individual parameters. As metalogical variables I shall use "x", for individual variables, and "a", "b", and "c", for individual parameters. "U(x)" and "V(x)" stand for a sentence form with just x free. "U(a)" stands for the result of substituting a for the free occurrences of x in U(x), etc.

The following formal$_2$ dialectical rules originate with Lorenzen: (5)

F_2D 2

	(Speaker:)	(Critic:) aU	structural pU	
$Rule_\forall$	$\forall xU(x)$	a?	U(a)	(Critic may choose any parameter a)
$Rule_\exists$	$\exists xU(x)$	?	U(a)	(Speaker may choose any parameter a)

It is easily seen that the FD-rules of dynamic dialectics in Section 15 of Paper 1 do not suffice to guarantee the local finiteness (Definition 8, Section 4 of Paper 5) of dialectic systems containing such rules. For one thing the proof of Lemma 4 in Section 4 of Paper 5 breaks down: there is an infinite number of parameters for the Critic to choose in $Rule_\forall$. Now suppose that $\forall xU(x)$ is a concession, made by the Opponent (O), i.e., that the Proponent (P) is the Critic. P may then go on indefinitely attacking

∀xU(x) in various ways, each time with the choice of another parameter. Because each attack is different, FD D6 (Section 15 of Paper 1) does not rule this out. There is no reason why a local discussion could not go on indefinitely.

Several ways exist of reinstating local finiteness. The simplest, though perhaps not the most natural, method I know of is given by the following FD-rule: (6)

FD D11 At the start of the discussion P is to choose an upper limit for the number of stages that may occur in any chain of arguments. Each chain of arguments that reaches this maximum without being completed (by virtue of other rules) shall be broken off and counted as lost by P. P is to be the next speaker (if there remains anything at all for P to say).

Clearly, it is in P's interest to select a sufficiently large number m for this maximum. But, assuming that P wants to start a debate at all, it is also in P's interest not to exaggerate. Otherwise, no one would be willing to take the Opponent's part! The smaller the number m, the easier an Opponent will be found, and the bolder P's claim. The number $\frac{1}{m}$, therefore, expresses a "degree of pretension" on the part of P. According to FD D11 P should, in the interest of an implementation of the fundamental norm of dynamic dialectics (Section 15 of Paper 1), announce this "degree of pretension". (7)

FD D11 suffices to guarantee finite chains of arguments. It is not yet sufficient to guarantee finite discussions, i.e., local finiteness, for the new rules also open the possibility that an indefinite number of chains of arguments branch off at one and the same stage in the discussion. For instance, party N may time after time give up a chain of arguments and try another attack on ∀xU(x) with another choice of parameter, thus starting a new chain each time. Since all such attacks are really different, FD D3 (Section 15, Paper 1) does not rule this out. To prevent any such events, I propose to adopt:

FD D12 (k) Each party is allowed to retrace its steps (8) k times at most.

The number k is to be fixed by the company or by the disputants together before they engage in a discussion. What value for k would be acceptable, for all parties involved, depends upon a number of factors, such as the complexity of the statements in the conflict, the time available and the extent to which "changing one's mind" is tolerated.

Together, FD D11 and FD D12 are sufficient to guarantee local finiteness: Theorem 5 in Section 4 of Paper 5 holds.

12.2. Finite descriptions of infinite winning strategies

Consider any dialogue sequent of the form $\Pi/_0 \forall xU(x);\Gamma$ (or, $\Pi,[V]/T/_0$ $\forall xU(x);\Gamma$). Since, in a situation depicted by such a sequent, there is an infinite number of parameters that may be used in an attack on $\forall xU(x)$ on the strength of $Rule_\forall$, there is an infinite number of options for O. Consequently, an infinite branching occurs in any P-strategy diagram that contains a sequent of this type. The same holds if one of the attacked concessions is of the form $\exists xU(x)$, since there is an infinite number of ways of defending such a concession protectively on the strength of $Rule_\exists$. (Of course, FD D11 and FD D12 (k) jointly guarantee that only a finite number of these possibilities is realized in any one discussion.) It is now no longer possible to conclude, by König's lemma, that all P-winning strategy diagrams are finite, and Lemma 8 in Section 4 of Paper 5 actually fails: the number of nodes in some P-winning strategy diagrams is undoubtedly infinite.

In order to be able to apply the methods of Papers 6 and 7, I shall first show that, if there is any P-winning strategy at all (for some particular dialogue situation), one may represent at least one P-winning strategy (for that situation) by a finite tree diagram. The finite tree diagrams that are used for this purpose, and which are smoothly rendered in tableau notation, will henceforth be called closed dialogical tableaux.

Let us call an O troublesome if, on each occasion when this party selects a parameter, it selects a "fresh" parameter, i.e., a parameter that does not occur in the dialogue situation at the moment the choice is made. Such an O is appropriately called "troublesome", because its behavior minimizes the chances of equiform statements being uttered by both parties, and hence the opportunities for P to make an appropriate Ipse dixisti!-remark (Section 7 of Paper 1). Even a troublesome O may not care which "fresh" parameter is used. Let us call O finicky if it insists on selecting one of the fresh parameters itself, not-finicky if it is willing to leave that choice to P. The rules, formulated below, for the construction of dialogical tableaux in (constructive) predicate logic yield "winning strategies" that hold good, prima facie, if and only if O is troublesome-but-not-finicky.

If CND (Section 1.2 of Paper 2) is taken as the propositional basis, the additional rules for predicate logic are the following: [11]

Compulsory rules

OI∀ under $\Pi/_0\forall xU(x)$ you must write ~~$\Pi/_P[U(a)]$~~ $\Pi/\forall xU(x)/_P[U(a)]$ (a "fresh")

OI∃ under $\Pi/_0\exists xU(x)$ you must write ~~$\Pi/_P[U(x)]$~~ [12] $\Pi/\exists xU(x)/_P[U(x)]$

OII^x under $\Pi;[U(x)]/T/_0\Gamma$ you must write $\Pi,U(a)/T/_P\Gamma$ (a "fresh")

OIII∀ under $\Pi;[V]/T/_0\forall xU(x);\Gamma$ you must write $\Pi,V/T/_P\Gamma$
and $\Pi/\forall xU(x)/_P[U(a)]$ (a "fresh")

OIII∃ under $\Pi;[V]/T/_0\exists xU(x);\Gamma$ you must write $\Pi,V/T/_P\Gamma$
and $\Pi/\exists xU(x)/_P[U(x)]$

Choice rules

Pd^x under $\Pi/T/_P[U(x)]$ you may write $\Pi/$~~T~~$/_0U(a)$ (for any a)

P∀ under $\Pi,\forall xU(x)/T/_P\Gamma$ you may write $\Pi,\forall xU(x);[U(a)]/T/_0\Gamma$ (for any a)

P∃ under $\Pi,\exists xU(x)/T/_P\Gamma$ you may write $\Pi,\exists xU(x);[U(x)]/T/_0\Gamma$.

There are no new closure rules.

In order to see how a dialogical tableau, constructed and closed on the strength of these rules, can be used to get the better of any Opponent, not only the troublesome-but-not-finicky ones, we need two lemmas that are analogous to similar lemmas proved by Gentzen for sequent systems. [13] Here we shall just state them:

Lemma 1 Except for possible transgressions of the freshness condition, a correct dialogical tableau remains correct if some parameter b is everywhere substituted for some other parameter a. Moreover, the freshness condition will be met as well, provided that, for each parameter c that was ever introduced as "fresh" into the tableau, either (i) $c \neq a$ and $c \neq b$, or
(ii) $c = a$ but b meets the freshness condition (in the relevant places) as well as does c.

Lemma 2 Let $\mathcal{T}$ be a closed dialogical tableau for a dialogue sequent S. Let $a_1,\ldots,a_n$ be distinct parameters used as "fresh" parameters in $\mathcal{T}$. Let $b_1,\ldots,b_n$ be distinct parameters that do not occur in

$\mathcal{T}$. Then $\mathcal{T}$ can, by successive parameter substitutions in parts of the tableau, be transformed in a closed dialogical tableau $\mathcal{T}'$, for S, in which $b_1,\dots,b_n$ are used as fresh parameters, instead of $a_1,\dots,a_n$.

(Lemma 2 is proved with the help of Lemma 1.)

Suppose that P has at its disposal a closed dialogical tableau for a dialogue situation S̲, represented by a dialogue sequent S. Let O be any Opponent. There is only one problem for P: that O may on some occasion select a parameter, fresh or not, different from the fresh one that appears in the tableau. For instance, let a occur in the tableau as a fresh parameter, while O selects the individual parameter b. In that case, what P should do is substitute b for a in the part of the tableaux that is still needed. Trouble can arise only if for some other fresh parameter, c, neither (i) nor (ii) of Lemma 1 holds. P should, therefore, first replace such fresh parameters by other ones distinct from both a and b (Lemma 2). Consequently, condition (i) of Lemma 1 will always be met. In that way P will, by substituting b for a, obtain a closed dialogical tableau that is adapted to O's choice of parameter. Since a was fresh in the original tableau, the new tableau wil start from exactly that dialogue situation that arose as a consequence of O's choice, b. These considerations, which may be repeated for classical and minimal systems, suffice to establish the following theorem (where by a "P-m-winning strategy diagram" is meant a P-winning strategy diagram that has m stages at most in each branch):

Theorem 1 — There is a closed constructive (minimal, classical) quantificational dialogical tableau for a sequent $\Pi/_0 Z$ iff, for some choice of m according to FD D11, there is a P-m-winning strategy diagram (14) for $\Pi/_0 Z$, pertaining to the corresponding system of formal dialectics.

Corollary — If there is a P-winning strategy for $\Pi/_0 Z$ in a system that does not include FD D11, then there is also one in the corresponding system that does include FD D11 (i.e., for some m there is a P-m-winning strategy diagram).

12.3. Transformations

In transforming dialogical tableaux into deductive tableaux expressions of the form [U(x)] should simply be removed. The proof of Theorem 9 in

Section 3 of Paper 6 then applies. The new rules for quantificational deductive tableaux are: (15)

∀r A problem Π/∀xU(x) reduces to the problem Π/U(a) (a "fresh", i.e., not occurring in the given problem).

∀l A problem Π,∀xU(x)/Z reduces to the problem Π,∀xU(x),U(a)/Z (for any parameter a).

∃r A problem Π/∃xU(x) reduces to the problem Π/U(a) (for any parameter a).

∃l A problem Π,∃xU(x)/Z reduces to the problem Π,∃xU(x),U(a)/Z (a "fresh").

The transformations of parts of the dialogical tableau can be grouped as follows, so far as quantifiers are concerned: (16)

A. Units consisting of an attack by P followed by O's reactions

P→ + OIII∀ ⇒ →l + ∀r

P→ + OIII∃ ⇒ →l ~~+ ∃r~~ l

P∀ + OII ⇒ ∀l

P∃ + OII^x ⇒ ∃l

B. Half-units consisting of an application of Pd^x

These are transformed into applications of ∃r in the same way as applications of Pd involving expressions [U,V] are transformed into applications of $\vee r_1$ or $\vee r_2$.

C. Half-units consisting of an attack by O

OI∀ ⇒ ∀r

OI∃ ⇒ void

Thus Theorem 9 in Section 3 of Paper 6 can be extended to constructive quantificational systems, and similarly to minimal and classical quantificational systems:

Theorem 2 A closed constructive (minimal, classical) quantificational dialogical tableau for a sequent $\Pi/_0 Z$ can, by a completely mechanical procedure, be tranformed into a deductive tableau for Π/Z, constructed and closed according to the rules of the corresponding system for deductive tableau construction.

From here the circle of transformations can be continued in the usual way: going from closed deductive tableaux to natural deduction, and from natural deduction to axiomatic deduction, and thence to validity according to (some variant of) Kripke model theory. These steps are quite unproblematic.

As to (constructive or minimal, quantificational) semantic tableaux systems, we have a choice between the two types of systems mentioned in the introduction, Kripke-type systems and Beth-type systems. I shall now opt for a Beth-type system, whereas in Paper 13 it will be shown how to insert a Kripke-type system into the circle.

The quantificational rules for (constructive and minimal) semantic tableau construction (Beth-type) are the following: (17)

∀R A set of sequents $\Sigma;(\Pi/\Gamma,\forall xU(x))$ reduces to the set of sequents $\Sigma;(\Pi/\Gamma,\forall xU(x));(\Pi/U(a))$ (a "fresh").

∀L A set of sequents $\Sigma;(\Pi,\forall xU(x)/\Gamma)$ reduces to the set of sequents $\Sigma;(\Pi,\forall xU(x),U(a)/\Gamma)$ (for any parameter a).

∃R A set of sequents $\Sigma;(\Pi/\Gamma,\exists xU(x))$ reduces to the set of sequents $\Sigma;(\Pi/\Gamma,\exists xU(x),U(a))$ (for any parameter a).

∃L A set of sequents $\Sigma;(\Pi,\exists xU(x)/\Gamma)$ reduces to the set of sequents $\Sigma;(\Pi,\exists xU(x),U(a)/\Gamma)$ (a "fresh").

(The quantificational rules for classical systems are even easier to formulate.) For the quantificational Beth-type systems (and the quantificational extensions of classical semantic tableau systems), Theorem 28 of Paper 7 can be proved by the method used there:

<u>Theorem 3</u> If, in any of the constructive or minimal quantificational systems (Beth-type) for constructing semantic tableaux, a semantic tableau for Π/Z can be brought to a closure, then there is, for some choice of m according to FD D11, a P-m--winning strategy diagram for $\Pi/_0 Z$ on the strength of the corresponding system of formal dialectics. The same holds for classical quantificational systems.

Recall that in Paper 7 the terms "P-winning strategy (diagram)" and "closed dialogical tableau" were synonymous, whereas in the present context they stand for distinct, but equivalent (Theorem 1!), notions. When adapting

Lemmas 1 and 2 from Paper 7 to the quantificational case, one should, preferably, stick to winning strategies, whereas in the context of Lemmas 3 and 4 the notion of a closed tableau is to be used instead.

13. Permutation of reductions in constructive Kripke-type semantic tableaux

In this paper I shall show how a Kripke-type system for the construction of semantic tableaux [1] can be inserted into the metatheoretical circle. I shall first briefly describe a Kripke-type system (Section 1), largely skip its completeness proof since it is standard (Section 2), and then show how closed tableaux constructed according to this system can be transformed, by a permutation of the applications of reduction rules, into closed tableaux according to a Beth-type system. [2]

13.1. A Kripke-type system

Reduction rules in Kripke-type systems operate on structured sets of sequents, each sequent being associated with a "world" in a so-called "world tree". The worlds (which are simply to become the nodes of a world tree) shall be finite sequences of positive integers. Let us use n, m, ... for positive integers, and α, β, ... for finite sequences of positive integers, i.e., worlds. If $\alpha = < m_1,\ldots,m_p >$ and $\beta = < n_1,\ldots,n_q >$ then "αβ" stands for $< m_1,\ldots,m_p,n_1,\ldots,n_q >$, "αn" stands for $< m_1,\ldots,m_p,n >$, etc. αn is called an immediate successor of α. If β = αγ, for some γ,β is called a continuation of α ~~and we write α≤β. If, moreover, γ ≠ ∅ α<β~~.

Def. 1 [3] A world tree $\underline{T}$ is a set of sequences of positive integers such that 1) $\emptyset \in \underline{T}$
2) if $\alpha n \in \underline{T}$ then $\alpha \in \underline{T}$
3) if $\alpha(n+1) \in \underline{T}$ then $\alpha n \in \underline{T}$

Def. 2 A structured set of sequents, Σ, is a function defined on a world tree $\underline{T}_\Sigma$ with sequents as values. (Notation: Σ_α for the value of Σ at the world α.)

The reduction rules, known from [4] Beth-type systems, can now be reformulated. For the noncreative rules this can be done straightforwardly. For instance:

&L If $\Sigma_\alpha = \Pi, U\&V/\Gamma$, then Σ reduces to Σ', given by:

1) $\underline{T}_{\Sigma'} = \underline{T}_\Sigma$;

2) If $\beta \neq \alpha$: $\Sigma'_\beta = \Sigma_\beta$.

Further, we write $\alpha \leq \beta$ iff either β is a continuation of α or there are $\gamma, \alpha_1, \beta_1, m, n$, such that (1) $\alpha = \gamma m \alpha_1$, (2) $\beta = \gamma n \beta_1$, and (3) $m < n$. If, moreover, $\alpha \neq \beta$ we write $\alpha < \beta$

3) $\Sigma'_\alpha = \Pi, U\&V, U, V/\Gamma$.

The creative rules extend the world tree in a standard way. For instance:

$\rightarrow$R If $\Sigma_\alpha = \Pi/\Gamma, U \rightarrow V$ then Σ reduces to Σ', given by:

1) $\underline{T}_{\Sigma'} = \underline{T}_\Sigma \cup \{\alpha n\}$. Here, either Σ is not defined for any immediate successor of α and n = 1, or n-1 is the largest integer m such that Σ is defined for αm.
2) If $\beta \in \underline{T}_\Sigma$, then $\Sigma'_\beta = \Sigma_\beta$.
3) $\Sigma_{\alpha n} = \Pi, U/V$. (5)

In addition there is a Propagation Rule for formulas on the left. It serves to copy formulas that appear "on the left" in a world α so that they appear also "on the left" in an immediate successor of α. By repeated application, formulas can be propagated from α to any β such that ~~$\alpha < \beta$.~~ β is a continuation of α ($\alpha \neq \beta$).

PR Let β be an immediate successor of α, (6) and let Σ be defined for β (and hence for α). Further let $\Sigma_\alpha = \Pi_\alpha/\Gamma_\alpha$, $\Sigma_\beta = \Pi_\beta/\Gamma_\beta$, $U \in \Pi_\alpha$. Then Σ reduces to Σ', given by:

1) $\underline{T}_{\Sigma'} = \underline{T}_\Sigma$
2) If $\gamma \neq \beta$; $\Sigma'_\gamma = \Sigma_\gamma$
3) $\Sigma'_\beta = \Pi_\beta, U/\Gamma_\beta$.

The Propagation Rule is the only one to which the method of Lemma 4 in Section 5 of Paper 7 does not apply. So our task reduces to showing how applications of this rule can be eliminated from closed tableaux. But let me first complete the description of the tableau system. Only one further remark is in order.

With each α for which Σ is defined, we associate a set of parameters $P_\Sigma(\alpha)$. This set is always to contain a certain fixed parameter, say "a_o", and also precisely those parameters that appear in any of the sequents Σ_β such that ~~$\beta < \alpha$~~ $\angle$. Applications of the rules $\forall$L and $\exists$R (on Σ at α) may then be restricted to parameters in $P_\Sigma(\alpha)$. Further, "a_o" is never to count as a "fresh" parameter, and can, therefore, not be used in $\forall$R or $\exists$L.

13.2. Completeness

The system just described is readily seen to be complete: Let Σ be a structured set of sequents such that there is no closed tableau for Σ. Apply the

$\angle$ α is a continuation of β

reduction rules in a systematic way, regularly giving attention to universal (existential) formulas on the left (right) in the light of newly introduced parameters, and regularly executing all possible applications of the Propagation Rule. In this way one will inevitably construct a finite or infinite structured Hintikka set of sequents Σ' ~~such that Σ ∈ Σ'~~ that "extends" Σ. We need not go into the details here. (7)

13.3. Transformation of closed Kripke-type tableaux into closed Beth-type tableaux. Preliminary sketch

We are concerned with the question of how a closed semantic tableau constructed according to the Kripke-type system of Section 1 can be transformed into a closed semantic tableau constructed according to the Beth-type system of Paper 12, Section 3. Clearly, it suffices to eliminate the Propagation Rule. For whenever this rule is not used in a closed Kripke-type tableau, we can simply omit the whole rigmarole of world trees in order to obtain a closed Beth-type tableau!

To show how applications of the Propagation Rule may be eliminated it suffices, again, to show that the order of applications of rules in a closed semantic tableau can be permuted in such a way that worlds that have an immediate successor will never tolerate any further additions to their sequents. For in that case all the propagations of formulas on the left can be taken care of by the creative rules; the formulas are "placed into the new world" simultaneously with the introduction of that world. (8) The proof proceeds by first introducing some terminology. Then one, rather unwieldy lemma is stated and proved. From this lemma the desired (meta)theorem easily follows.

13.4. The Permutation Lemma

First, some new technical terms:

Def. 3 The degree of an application of a rule is to be the world (i.e., the sequence of integers) where, according to the rule, a formula is introduced (or, in the case of a closure rule, where the closed sequent in question is located).

Degrees are ordered by $\preceq$ and $\prec$.

Def. 4 An application of a rule in a construction of a semantic tableau is to be called _wrongly timed_ iff some rule-application of lower degree takes place further down, in such a way that both applications of rules lie on the same branch in the tableau.

Def. 5 A construction of a tableau is to be called _regular_ if no application of a rule is wrongly timed. The resulting tableau is also called _regular_.

Def. 6 The _degree of a (construction of a) tableau_ is the degree of the first application of a rule in the tableau. (9)

Lemma 1 (The Permutation Lemma) Let $\mathcal{T}$ be a closed tableau for Σ of degree α. Let r be the first rule application in $\mathcal{T}$, and let r, and r only, be wrongly timed. Then $\mathcal{T}$ can be transformed into a regular closed tableau $\mathcal{T}'$ for Σ of degree β, where β is the degree of the application of some rule that in $\mathcal{T}$ immediately follows upon r.

Proof By a course-of-values induction on the number of applications of rules in $\mathcal{T}$.

Assume that the lemma holds for any $\mathcal{T}'$ with less than n applications of rules (regardless of Σ and α). Let $\mathcal{T}$ have exactly n applications of rules, and let Σ,α and r be as stated. $n \geqslant 2$, otherwise r could not be wrongly timed. The application of the rule that immediately follows r (or, if r splits the tableau, at least one of the two rule-applications that follow r) must be of a degree lower than α. Otherwise, this second rule-application (or, if r splits the tableau, at least one of the immediately succeeding rule-applications) would be wrongly timed as well! If r does not split the tableau, the rule-application that immediately follows r shall be called r_1, and its degree β. If r splits the tableau we select as r_1 a rule-application that immediately follows r in one branch and such that its degree, β, is lower than or equal to the degree of r's immediate successor in the other branch. Now, whether r splits the tableau or not, we have $\beta \angle \alpha$.

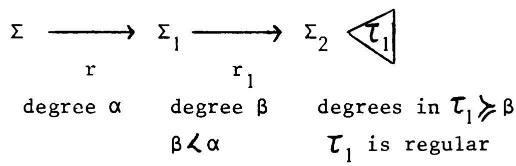
I shall show that $\mathcal{T}$ can be transformed into a closed tableau $\mathcal{T}'$ for Σ of degree β, where the first rule-application is analogous to r_1.

If r_1 is an application of a closure rule, the tableau closes by virtue of some other world than that in which formulas are introduced by r, i.e., the closure rule is immediately applicable to Σ. Thus Σ constitutes a one-node tableau with all the required properties. Henceforth I shall assume that r_1 is not an application of a closure rule.

<u>case 1</u> Neither r nor r_1 splits the tableau.

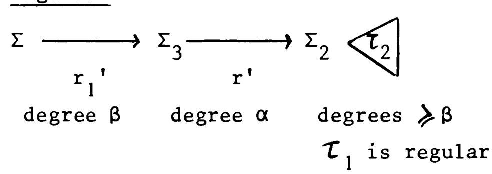
Let r carry Σ into Σ_1, whereas r_1 carries Σ_1 into Σ_2. Let $\mathcal{T}_1$ be the closed tableau for Σ_2 contained in $\mathcal{T}$. We may then picture the situation as in Figure 1.

<u>Figure 1</u>



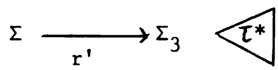
Since $\beta \not< \alpha$, a rule-application analogous to r_1 on Σ is possible: the formula occurrences from which r_1 starts cannot have been introduced by r. (There are no problems with the freshness condition in the case where r is an application of ∀R or ∃L, since application of ∀L and ∃R at β are restricted to parameters in $P_{\Sigma}(\beta)$ and hence cannot involve the "fresh" parameter introduced at α by r.) So let us first carry Σ into Σ_3 by a rule-application r_1', analogous to r_1, and then Σ_3 into Σ_2 by a rule-application r', analogous to r. The tableau can then be completed by adjoining $\mathcal{T}_1$ (Figure 2).

<u>Figure 2</u>



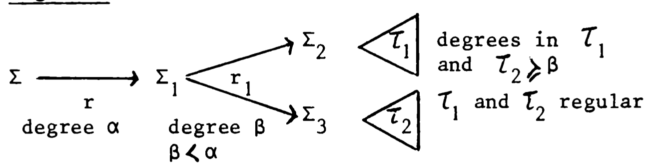
If the tableau Figure 2 is regular we are through; if not, only r' can be wrongly timed. The tableau as from Σ_3 contains n-1 applications of rules, and r' is the only wrongly timed application of a rule in it. Hence, by the induction hypothesis, this tableau can be transformed into a regular closed tableau $\mathcal{T}^*$ for Σ_3 of some degree $\not< \beta$. All the applications of rules in $\mathcal{T}^*$ will hence have a degree $\not< \beta$. Thus we find a regular tableau of degree β for Σ (Figure 3).

Figure 3



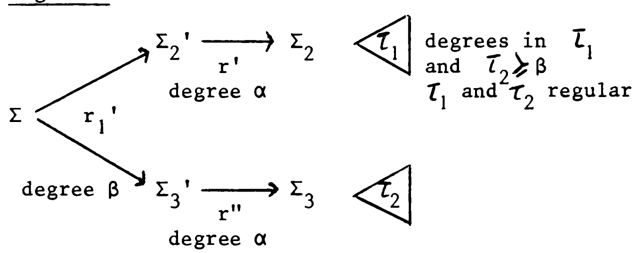
case 2 r does not split the tableau, but r_1 does. The given tableau is now structured as shown in Figure 4.

Figure 4



Again we can first carry Σ into Σ_2' and Σ_3' by a rule-application r_1', analogous to r_1, and then we can carry Σ_2' into Σ_2, and Σ_3' into Σ_3 by rule-applications r' and r" analogous to r (Figure 5).

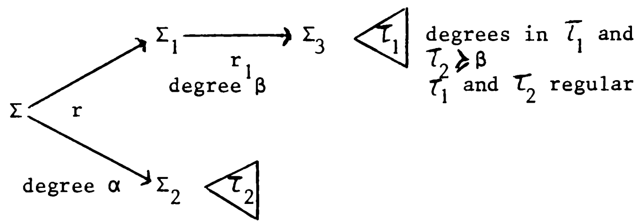
Figure 5



If the tableau in Figure 5 is regular we are through; otherwise we must apply the induction hypothesis, either to the subtableau for Σ_2' or to the subtableau for Σ_3', or both.

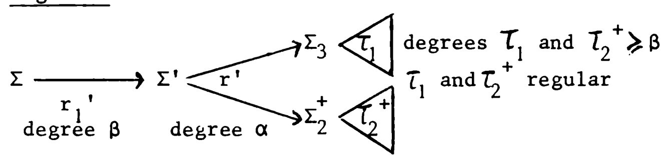
<u>case 3</u> r does split the tableau, but r_1 does not.
The given tableau is now structured as shown in Figure 6.

<u>Figure 6</u>



This can be transformed, in the usual way, into the tableau shown in Figure 7.

<u>Figure 7</u>

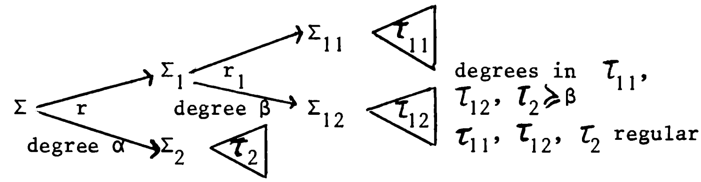


r_1' is analogous to r_1, r' to r. If r_1 introduces a fresh variable it may be necessary, in order to preserve the correctness of the tableau, first to change some of the fresh variables used in part $\mathcal{T}_2$ (Lemma 2 in Section 2 of Paper 12 applies to semantic tableaux as well). Further, $\mathcal{T}_2^+$ may differ from $\mathcal{T}_2$ in that some rule-applications in $\mathcal{T}_2^+$ are of a degree higher than the corresponding applications in $\mathcal{T}_2$. This is to be expected if r_1 is an application of a creative rule. Moreover, $\mathcal{T}_2^+$ and Σ_2^+ may differ from $\mathcal{T}_2$ and Σ_2 by the presence of some additional formula(s), introduced by r_1'. Notwithstanding these differences, $\mathcal{T}_2^+$ will be regular and of degree $\geqslant \beta$. So we may (if necessary) apply the induction hypothesis, as usual, to the subtableau for Σ'.

<u>case 4</u> Both r and r_1 split the tableau.
There is nothing new to this case except for its structural complexity. So I shall just show the figures. The structure of the given tableau is shown in

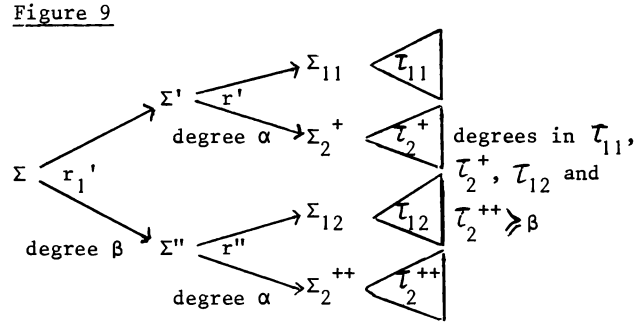
Figure 8.

Figure 8



By "permuting" r_1 and r we get the tableau shown in Figure 9.

Figure 9



Again r_1' is analogous to r_1, r' and r" to r. Σ_2^+, Σ_2^{++}, τ_2^+ and τ_2^{++} differ from the "unplussed" Σ_2 and τ_2 only in that there may be some extra formulas, introduced by r_1'. Again, we can (if necessary) apply the induction hypothesis to the subtableaux for Σ' Σ" ●

13.5. Immediate consequences of the Permutation Lemma

The following, simpler, lemma is an immediate consequence of Lemma 1:

Lemma 2 Any closed tableau for a structured set of sequents Σ in which, at most, the first application of a rule is wrongly timed can be transformed into a regular closed tableau for Σ.

Going one step further we obtain:

Lemma 3 Any closed semantic tableau for a structured set of sequents Σ can be transformed into a regular closed semantic tableau for Σ.

Proof By a straightforward course-of-values induction on the number of applications of rules in the given tableau, and using Lemma 2.

From a regular semantic tableau the applications of the Propagation Rule can easily be eliminated. For let α be any world and let β be an immediate successor of α. As soon as β is introduced (from α), no new formulas will be added to α any further. So all the formulas propagated from α to β are present already at the moment of β's introduction. Hence all applications of the Propagation Rule can be shifted to a cluster that follows immediately upon β's introduction. Finally, if the creative rules are reformulated so as to allow for the propagation of formulas (on the left) present at the moment of application, we may drop the Propagation Rule from the system. Thus we get to the desired result:

Theorem A constructive (or minimal) quantificational semantic tableau, for Π/Z, constructed and closed on the strength of the rules of a Kripke-type system can, by a completely mechanical procedure, be transformed into a closed tableau, for Π/Z, on the strength of the corresponding Beth-type system.

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NOTES

Notes to 0

1. Lorenzen [LAg] and [DKn]. Following the appearance of Aristotle's Topics, dialogue or debate has been a subject of interest to logicians at intervals over the centuries. However, in the age of "modern" logic (beginning with Frege, 1879), there has been little such interest on the part of logicians until Lorenzen.

2. Barth and Krabbe [AD1] (henceforth to be referred to simply as "[AD1]"), I.1. Cf. Figure I.1, loc. cit., for subdivisions, and for divisions cutting across that into "garbs". For instance, classical and constructive (intuitionistic) logic can appear in any of the three garbs.

3. Cf. the definition of theory of argumentation in Van Eemeren, et al. $[Arg]_2$, p. 49, and also the comparison with logic, op. cit., p. 130. Matters of demarcation are not what is really interesting: one can have it either way. Cf. Krabbe [TAD].

4. This definition is of course stipulative, but, I presume, lexically largely correct. Cf. the emphasis on an interdisciplinary approach in Van Eemeren, et al. $[Arg]_2$, p. 115. In Section 12 of Paper 1 below ([AD1], III.12), "theory of argumentation" is still taken to be the more comprehensive term. But, as I said in the preceding note, one can have it either way. My present choice is mainly motivated by Van Benthem's essay on the relationship between logic and theory of argumentation, [LAr].

5. Cf. [AD1], XI.7.

6. [AD1], I.3.

7. Hamblin [F11], p. 256. The authors of [AD1] owe the term "formal dialectic(s)" to Hamblin. The term does not appear in the works of Lorenzen and Lorenz.

8. For a further analysis of these roots, see [AD1], I.4 (or, Barth [Ev1], Section 4) on problem-solving validity and (semi-)conventional validity.

9. [AD1], II.1; this section was written mainly by the present author.

10. For "empirical intuitions", see loc. cit. Cf. Paper 1 Section 12 ([AD1], III.12) on "natural" rules.

11. E.g., Kamlah and Lorenzen $[LPr]_2$, Lorenzen and Schwemmer $[KLE]_2$, Lorenz [ALS], [DSG].

12. With one exception: the strategy may prescribe that one copy one's adversary's move, even if it is of a forbidden kind. Cf. Lorenz [DRE], Note 21: "... dass formale Endgewinnstellungen Gewinnstellungen (für P) bleiben, wenn die Primdialoge wieder in den Dialog miteinbezogen werden...".

13. It is all the more surprising that Hintikka characterized the games of Lorenzen and Lorenz as "indoor games". Hintikka [LGQ], p. 71 ([LLG], p. 81).

14. Cf. [AD1], II.4.

15. Cf. [AD1], II.4, esp. Figure II.3.

16. Kamlah and Lorenzen $[LPr]_2$, p. 221.

17. The term "invertible" is defined in Paper 10, pp. 192, 193.

18. The first completeness proofs for dialogue games were given by Lorenz in [ALS] and in [DSG]. Lorenzen is not very explicit on the matter of completeness proofs, sometimes he refers to Lorenz. Kindt, in [ATD], draws attention to some problems in Lorenz's proofs and treats the matter on a highly abstract level. G. Haas, [HKS], independently, found methods of proof akin to mine.

19. Papers 6 and 7 omit certain steps that can be found in [AD1], Ch. VI-X.

Notes to 4

See p. 79.

Notes to 8

See p. 168.

Notes to 10.0

*) I would like to thank Prof. E.M. Barth and C.A.M. Roy for their advice while preparing this paper.

1. Kripke [SAI.I]. Semantics for minimal logic (with a falsum-operator) is briefly treated by M. Fitting in [ILM], p. 40. The new motivation for these semantic theories and the corresponding translation of epistemic/ information-theoretic terms into dialectical terms originated with E.M. Barth. In Barth and Krabbe [AD1] (henceforth to be referred to simply as "[AD1]") only propositional logics are studied, but among these are constructive and minimal logics both with and without a falsum-constant, $\wedge$.

2. Kripke [SAI.I], p. 98: "We intend the nodes **H** to represent points in time (or 'evidential situations'), at which we may have various pieces of information." To represent points in time is, of course, not the same as to represent evidential situations. The latter intuitive interpretation is to be taken more seriously. For one thing, interpreting the nodes as points in time makes Kripke's story inconsistent; for this see again p.98: "Now given a point in time **G** ... For all we know, we may remain 'stuck' at **G** for an arbitrarily long time." How can one remain stuck at a point in time, even for a short time? As to the we who are to have the various pieces of information, I surmise that a knowing or thinking subject is meant (not necessarily identical with Kripke and his readers), a monological counterpart of the dialectical subject.

3. Op. cit., p. 99: " ... we say **H** $\mathcal{R}$ **H**' if, as far as we know, at the time **H**, we may later get enough information to advance to **H**'."

4. [AD1] IX.3: "By a dialectical subject we shall understand any company

of users of language that are or that have been or that may become engaged in critical discussions, with themselves or with one another." This is the company in R. Crawshay-Williams's sense, see his [MCR].

5. "If, at a particular point **H** in time, we have enough information to prove a proposition A, we say that ϕ(A,**H**) = **T** ...", Kripke [SAI.I], p. 98. This seems to imply that "= **T**" stands for "is provable" or "is verifiable". Elswhere, however, Kripke interprets "= **T**" as "is verified" or "is proved". Unless it is assumed that we prove everything provable, or verify everything verifiable, the two readings do not coincide. The "provability" or "verifiability"-reading seems to be the more realistic one, i.e., seems to agree better with real thinking subjects (cf. Note 2), for, whatever the evidential situation, an infinite number of sentences will get the value **T**.

We do not have to restrict the means of verification to proof. The alternative monological reading of "= **T**" as "can be known" and "= **F**" as "cannot yet be known" would do just as well: constructive logic can thus be interpreted as an epistemic logic.

6. "Agreement" must here be understood as either explicit or implicit agreement. For, whatever the dialectical situation, an infinite number of sentences will get the value **A**. Cf. the preceding note.

7. [AD1], IX.3, Lemma 4; Kripke [SAI.I], p. 94:
"for any **H**,**H**'$\in$**K** such that **H**$\mathcal{R}$**H**'... if ϕ(**A**,**H**) = **T**, then ϕ(**A**,**H**') = **T**"
Henceforth I shall use the symbolism of [AD1].

8. Kripke [SAI.I], p. 99. My italics.

9. This is only an example. The Principle of Cumulation of Information holds for any thinking subject that sticks to Normal Science (in the

sense of T.S. Kuhn).

10. The idea of simply dropping the Principle of Cumulation from the assumptions underlying Kripke-style semantics was proposed by E.M. Barth. Some of the resulting logics can be found in Hacking [WSI], but Hacking does not mention noncumulation. Cf. Notes 9 and 10 to Section 4 below. At present there seems to be a growing interest in matters of cumulation versus noncumulation. Cf. Woods and Walton [ACF], Mackenzie [QBN]. The "quantum logics" developed by P. Mittelstaedt and E.-W. Stachow, though they are motivated in a completely different manner, resemble the noncumulative logics in this paper and are literally noncumulative. None of the quantum logics, however, includes, or is included in, any of the noncumulative logics here presented. For the quantum logical law $A \rightarrow [(A \rightarrow B) \rightarrow A]$ is not valid in the present logics, whereas, conversely, the latter do (but the quantum logics do not) have the normal properties of distributivity for 'and' and 'or'. Cf. Mittelstaedt [QLg].

11. Paper 11.

Notes to 10.1

1. [ADl], IX.3, Def. 5.

2. [ADl], IX.4, Def. 10.

3. [ADl], IX.3, Def. 6, Def. 7, IX.4, Def. 11, Def. 12.

4. It is supposed that we deal with some language for propositional logic with an infinite number of atoms A, B, ... A', ..., and with the connectives $\rightarrow$, &, $\vee$, $\sim$ (language of the form $\mathfrak{P}$) and possibly the propositional constant $\wedge$ (language of the form $\mathfrak{P}^{\wedge}$). Cf. [ADl], II.3.

5. [AD1], IX.3, Def. 8, IX.4, Def. 13.

6. More precisely, a semantic rule for an n-ary connective C is certainly cumulative if it can be given in the form $v_M(C(U_1,\ldots,U_n),d)$ = **A** iff for all d' such that dRd': Φ(d'). The (cumulative) rules for → and ~ can be written in this form. But could there not be plausible rules for ~ and → that are not cumulative and yet that are different from the rules "in terms of d only" given above? If we start looking for some very complicated rule, we are taking option (3). If the rules are simple there is probably no plausible alternative. Let me explain what I mean by a simple rule. Take a unary operator ⊕. The rule must be of the form "v_M(⊕U,d) = **A** iff ....". On the right-hand side there must appear some (expression equivalent to a) disjunction. Each disjunct is to describe some condition of the set {d'/dRd'} sufficient to have v_M(⊕U,d) = **A**. The conditions must be mutually exclusive. Each condition must be given by a (noncontradictory) conjunction of the statements v_M(U,d) = **A**, (∃d')(dRd' and v_M(U,d') = **A**), and (∃d')(dRd' and v_M(U,d') = **N**), where each of these three may be replaced by its negation. There are (in view of the reflexivity of R) four consistent conjunctions of this type, equivalent to

(1) v_M(U,d) = **A** and (∃d')(dRd' and v_M(U,d') = **N**)

(2) v_M(U,d) = **N** and (∃d')(dRd' and v_M(U,d') = **A**)

(3) (d')(if dRd' then v_M(U,d') = **A**)

(4) (d')(if dRd' then v_M(U,d') = **N**)

There are therefore sixteen possible simple rules for a unary connective (including the empty disjunction on the right, i.e., the rule v_M(⊕U,d) = **N**). Rules with a disjunct (1) or (3) are in no way acceptable as a rule for ~. So we are left with (2), (4), and the disjunction of (2) and (4) to fill out the right-hand side of the semantic rule. The first of these rules seems to render: " not yet agreed, <u>but not excluded</u>", rather than "not". The second is the rule $\underline{Sem}^c\sim$. The third is equivalent to the proposed rule "in terms of d only" (for '(2) or (4)' is equivalent to '$v_M(U,d) = \mathbf{N}$'). For binary connectives, the conjunctions must be built up from $v_M(U,d) = \mathbf{A}$, $v_M(V,d) = \mathbf{A}$, $(\exists d')(dRd'$ and $v_M(U,d) = v_M(U,d') = \mathbf{A})$, etc., and their negations. This gives us 32 consistent conjunctions and 2^{32} eligible disjunctions ... Up to now I have not found an attractive alternative among them.

7. This terminology corresponds to that of Hacking in [WSI].

8. These validity concepts may be applied to languages with or without a <u>falsum</u>-constant $\mathbf{\Lambda}$.

9. Kripke [SAM.I]; Hintikka [KB1]; Schütte [VSM]; Smyllyan [GIM]; Fitting [ILM], [TMP]~~; Barth and Krabbe [AD1]~~. Constructive*-validity coincides with S4-validity, if we read "→" as strict implication and "∼" as strict negation.

10. Sections X.3 and X.4. Thus we obtain four "starred" systems for the construction of semantic tableaux: MIst*, MΛst*, CΛst*, CNst*. Since MΛst = MNst, MΛst* = MNst*.

11. [AD1], X.3. Notation: Π, Γ ... denote sets of sentences. Ordered pairs of sets of sentences Π/Γ are called <u>sequents</u>. We write $\Pi/\Gamma, U\rightarrow V$ instead of $\Pi/\Gamma \cup \{U\rightarrow V\}$, etc. Σ, Σ' ... denote sets of sequents. We write $\Sigma;(\Pi/\Gamma)$ instead of $\Sigma \cup \{\Pi/\Gamma\}$, etc.

12. [AD1], X. The proofs must of course be adapted to the present situation. The most important modification is that, if Σ is a Hintikka set of sequents, then a model for Σ can be found, if we define the relation R

as follows:

$\Pi/\Gamma R \Pi'/\Gamma'$ iff $\Pi^* \subseteq \Pi'$

(instead of $\Pi/\Gamma R \Pi'/\Gamma'$ iff $\Pi \subseteq \Pi'$). See X.3, Lemma 12, X.4, Lemma 17.

Notes to 10.2

1. Such examples are, of course, tied to certain philosophical schools or positions. More examples may be found in Lorenzen's "System of Truths", [NLE], p. 61; Lorenzen and Schwemmer $[KLE]_2$, p. 236. This system (and also our Kantian examples (1) and (6) as well as (2) and (3), which are incorporated in Lorenzen's system) suggests a stricter than relation, which is a strict partial ordering (i.e., transitive and asymmetrical) of these kinds of statements. This will be pursued in Paper 11. Cf. Curry [FML], Section 8 A1, pp. 359, 360. What I am interested in is not field--dependency in Toulmin's sense: strict and contingent statements may very well be distinguished within one and the same field.

2. The notion of a dialectical or dialogical level (Dialogebene) I owe to M. Marčinko, about whose as yet unpublished work I was informed by R. Inhetveen, [KWL]. In Marčinko's dialectic there is a level (a finite index sequence $L = 1_1, \ldots, 1_n$) associated with each statement in a discussion. The levels are chosen by the disputants, according to fixed rules. In this paper, however, there are just two levels (strict and contingent) and they are fixed by the grammatical form of the statements. See Section 1.3, Paper 11.

3. The word "ultimately" gives this norm its due degree of vagueness. I still leave open the possibility that contingent concessions may be used to elicit additional strict concessions (implementation FD L_2* below).

4. There the reader will also find definitions or explanations of all the dialectical notions not explained in the present paper, e.g., Opponent (O), Proponent (P), thesis, concession, stage, neutral position, structural protective defense, local discussion, local thesis, chain of arguments, etc. ([AD1], III = Paper 1 and [AD1], IV = Papers 2 and 3 of this dissertation.)

5. In such a dialogue sequent, concessions are written on the left and P's last statement on the right, whereas the index indicates who is to make the next move.

6. If we are dealing with a purely implicational language, we must adduce another example, e.g., $A, A\to(B\to C) /_0 B\to C$.

7. As usual, I write O's statements and rights in the left column and P's statements and rights in the right column. Structural protective defense rights are indicated by square brackets.

8. This example holds good for constructive noncumulative semantics, without $\wedge$. An example that holds for a purely implicational language, and hence for minimal noncumulative semantics as well, is:
$A\to[B\to(C\to D)], B /_0 A\to(C\to D)$

9. If an atom does not show at a dialectical situation this means that it is to be assigned the value **N** (otherwise **A**). [AD1], IX.3.

10. See Paper 1, p. 41. U must be either contingent or strict. Notice that between the stage at which P utters U and the (deferred or repeated) attack by O on U that gives rise to situation S", there cannot (in the same chain of arguments) occur any attacks by O on other statements. For in that case P would have lost pro-position to U by FD O5a. Therefore, even if withdrawal of concessions on the strength of FD $L_2 2$ is

admitted, it still holds that the only difference between *S'* and *S''* is that in *S''* the Opponent may be in pro-position to some additional concessions. A withdrawal of concessions may indeed have taken place in the meantime, but only on account of an attack on *U*, and hence only if *U* is strict. Such a withdrawal of concessions therefore does not affect those concessions present in situation *S'* (with the possible exception of *aU*, if this statement is contingent - but a statement *aU* is included in *S''* anyway).

11. Part of the argument in Paper 1, pp. 42, 43, is no longer convincing in the present situation. Repetition of the sentence from a former local thesis before the set of concessions is augmented will no longer guarantee O the opportunity to revive an old dialogue situation. For O may in the meantime have lost pro-position to a number of contingent concessions. However, if that is the case, the situation after the second attack by O on a statement of T can only be less favorable to P than was the first attack. Let the dialogue situation after the first attack be *S*, and the one after the second attack, *S'*. The only difference between *S* and *S'* is that, possibly, some contingent concessions available in *S* are not available in *S'*. Let P have a winning strategy for *S*. And assume that P uses this winning strategy. Then the winning strategy for *S* must contain one for *S'*. This latter winning strategy may also be applied to *S*. For P may ignore the additional concessions, i.e., those present in *S* but not in *S'*. Hence P may improve on its winning strategy for *S* by using the one for *S'* instead. Therefore it cannot really be advantageous for P to have a right to repeat a thesis before any fresh concession is available.

The second part of FD D8* may be argued for in the same way as was done for FD D8 in Paper 1, pp. 42, 43. Repetition of a strict thesis before a fresh strict concession has appeared gives O the opportunity to revive an old dialogue situation.

Notes to 10.3.

1. Cf. Theorem 1 of Paper 2 (p. 60). ([AD1], IV = Papers 2 and 3 of this dissertation; most of [AD1], V is included in Paper 5.)

2. Cf. Theorem 2 of Paper 5 (p. 106).

3. Cf. the explanation on p. 85.

4. Cf. Lemma 1 of Paper 5 (p. 85). For the argument as to FD D8*, see Note 11 to Section 2.

5. Paper 5, Section 2.2.

6. The definition of Π^* was given at the end of Section 1.

7. Paper 5, pp. 107 ff. Some modifications are called for. (1) If a_iW is strict, then $S^k_+ = (\Pi,W)^*, a_iW, a_ka_iW/a_iW/_P\, [d_ka_iW]$. For, the set of concessions is $\Pi^* \cup \{W\}^* \cup \{a_iW, a_ka_iW\}$ instead of $\Pi \cup \{W, a_iW, a_ka_iW\}$. This is so because an attack on a_iW allows O to withdraw its contingent concessions. Of course a_iW, being strict, cannot be withdrawn. Hence we can use the induction hypothesis as before. (2) If $p_{ij}W$ is strict, then $S^{jh}_* = (\Pi,W,a_iW)^*, p_{ij}W, a_hp_{ij}W/p_{ij}W/_P\, [d_hp_{ij}W]$, because an attack on $p_{ij}W$ allows O to withdraw its contingent concessions. Again $p_{ij}W$, being strict, cannot be withdrawn. So we can once more apply the induction hypothesis as usual.

8. Paper 5, Section 5.

9. <u>Loc</u>. <u>cit</u>. There are two changes to be made. (1) In the proof of Lemma 10 (a), where it was supposed that case (v) aplies to S', we read: "Since no rule cancels concessions ...". This, of course, is no longer

true. Fortunately, nothing turns on this general property of the "cumulative" systems. A mere rewording will suffice. Read: "Since no rule but $OI\overset{\wedge}{\sim}^*$, $OIII\overset{\wedge}{\sim}^*$, $OI\overset{*}{\rightarrow}$, $OIII\overset{*}{\rightarrow}$ cancels concessions, and since S' cannot be the result of any of these rules (at least not in that subtableau in which concessions are withdrawn) (for $\wedge$ is the local thesis!), we may exclude the possibility that $\wedge$ is among O's concessions in S."

(2) In the proof of Lemma 11 there seems to be a more serious difficulty. Withdrawals of contingent concessions may prevent the shaded subtableaux in Figure V.21 from being brought to closure in the same way as the tableaux $\mathcal{T}^i_+$. But actually a much simpler proof, not involving Figure V.21, is possible where $M\wedge D^*$ is concerned. In order to transform the tableau $\mathcal{T}^*$, which starts with an illicit application of $P\overset{\wedge}{\sim}^*$ (= $P\overset{\wedge}{\sim}$), into a well-arranged tableau for S, it suffices to take

$\Gamma+$ tableau $\mathcal{T}^{\Gamma}_{+}$ and omit the $\wedge$ on the left (unless $\wedge \in \Pi$). This turns S^+ into S and can only invalidate some Ipse dixisti!-remarks made by P. However, if $\mathcal{T}^+$ is constructed on the strength of $M\wedge D^*$ there are no such invalidated remarks. First $W \neq \wedge$ and second $\wedge \notin \Gamma$; hence $\wedge$ cannot appear on P's side without a previous attack by O on a negation. But such an attack would lead to a withdrawal of the $\wedge$ on the left and, consequently, deprive P of its right to make the aforementioned Ipse dixisti!-remark.

10. Cf. Paper 7.

11. Cf. Theorem 28 of Paper 7 (p. 149).

12. Cf. (proof of) Lemma 4 (pp. 154 ff.). Read Π^* (or Π'^*) for Π (or Π') where necessary. Reference to Lemma 1 from [AD1], XI.2 in this proof is inessential. Remember that there is no system for semantic tableaux MNst different from $M\wedge st$ in [AD1], and hence that $MNst^* = M\wedge st^*$ (as applied to sequents without $\wedge$) in the present paper. So from a closed semantic

MNst*-tableau we first obtain a P-winning strategy in M∧D*. Theorem 4* (which may be generalized so as to include sequents $\Pi/_P [Z]$) tells us that there is also a P-winning strategy in MND*.

13. The name invertible was suggested by the relationship between the present notion and the direct inversion of inferences in sequent systems. Curry [FML], Section 5D1.

14. That is, the analogue of Lemma 1 of Paper 7 (p. 151) does not hold.

15. For these special cases of (**), the proof of Lemma 1 of Paper 7 can be adapted. The crucial point is that in these cases, but not in general, we can conclude that there is a P-winning strategy for $\Pi/_O Z$ if a P-winning strategy for each $\Pi, a_i Z/Z/_P [d_i Z]$ is given.

16. Cf. Paper 1, Sections 1-5.

17. Cf. Paper 4. In that paper discussions are still supposed to start with a rejection of the provocative thesis, i.e., they start from a situation $\Pi/_B Z$ (B corresponds to O).

Notes to 10.4.

1. Cancellation lines occur in the modal semantic tableaux of Beth and Nieland [SCL], and in the modal deductive tableaux of Barth [NDM]. In these publications they were not separately discussed and not called by any special name. Extensive use of cancellation lines, both in modal deductive and modal semantic tableaux, was made in a compendium by Barth and Krabbe [AD1.III] ("strikte strepen").

2. Of course a right rule is to be applied always to the bottommost expression in the right column of the subtableau, the concludendum. The

concludendum is always operative.

3. See Sections 1 and 3 of Paper 6.

4. For use in the system MNdt*.

5. Fitch [SLg].

6. Fitch's term is "subordinate proof". Ordinary subordinate deduction will remain in use in connection with CD.

7. [AD1], VI.1 and VI.2.

8. Cf. [AD1], VIII.1 and VIII.2, where some well-known axiom systems are formulated.

9. See Hacking [WSI], where the postulates of MIax* occur. Cf. Diaz [DCC].

10. In these cases a comparison with Hacking's systems is more involved. For one thing Hacking's systems are incomplete with respect to the Kripke semantics for S4, if deductions from premises are taken into account. For instance, a deduction for A,B/A&B cannot be constructed on the strength of these systems. Proof: it can be shown by induction that if one of Hacking's systems admits a deduction for U_1 .... U_n/V, the corresponding "possibility" sequent $\Diamond U_1, \ldots, \Diamond U_n / \Diamond V$ is valid in S5. Of course $\Diamond A, \Diamond B / \Diamond (A\&B)$ is not valid in S5.

However, if we restrict ourselves to deductions from strict premises, CNax* can be shown to be equivalent to the Lewy calculus with postulates 16 and 17 ([WSI], p. 52). The same holds for C$\wedge$ax* and the Lewy calculus with postulate 19, provided we subjoin the postulates $\wedge_1$ and $\wedge_2$ to the latter system.

11. Cf. Theorem 29 of Paper 7 (p. 158).

12. Recall that the dialectic systems are not invertible.

13. Read "minimally*" if the names of the systems start with "M", and "constructively*" if they start with "C".

14. See Sections 2 and 3 of Paper 6.

15. The given dialogical tableau is supposed to satisfy the rule R_{At}. This can always be brought about according to Theorem 3*.

16. [AD1], VII.1 and VII.3, Theorem 8.

17. Cf. diagram with provisos on pp. 195, 196.

18. When Z_i is entered in the right column by virtue of $\sim 1^*_{min}$, $\sim Z_i$ must figure as a premise. We admit the case n = 1, but this is not important.

19. Ex Contradictione Sequitur ∧ (EC∧), Ex ∧ Sequitur Quodlibet (E∧Q), and Ex Contradictione Sequitur Quodlibet (ECQ).

20. R∧*, RM* and RM**.

21. These eliminations can be effected in the usual way, [AD1], VIII.2. For AD_2 use v2, v3* and MP.

22. [AD1], VIII.1.

Notes to 11.0

1. For the concept of a formal ($formal_3$) dialectics as an instrument for the resolution of conflicts of avowed opinion, see Barth and Krabbe [AD1] (henceforth to be referred to simply as "[AD1]"), Ch. III (= Paper 1 of this dissertation).

2. Remarks to the same effect were made in [AD1], Section XI.7 (What next in the Theory of Argumentation).

3. Lorenzen and Lorenz [DLg]. The formal ($formal_2$) attacks and structural protective defense moves relevant to universally or existentially quantified sentences were first stated explicitly by P. Lorenzen in [DKn] (p. 196; [DLg] p. 12). If one were to include these new possibilities in the formal dialectic systems of [AD1] without taking any precautions, the systems thus obtained would clash with the norms of a dynamic dialectics. For, in general, there would be an infinite number of ways to attack a universally quantified statement (and also an infinite number of ways to defend an existentially quantified sentence). By this feature the local finiteness of the systems would be destroyed. Consequently, Theorem 2 of Paper 5, Section 4, would no longer hold. There is a number of ways to strengthen the rules of a dynamic dialectics (Paper 1, Section 15) so as to restore local finiteness. Cf. K. Lorenz's theorizing about the role of, and the justification of, Angriffsschranken in [DSG]; [DRE] and [CCR]. The simplest method I know of stipulates that the Proponent selects some natural number n at the start of the discussion and that, subsequently, the discussion shall be closed after, at most, the n-th stage. Cf. Paper 1, Section 8, for the rules for winning or losing a discussion. Cf. Kindt [ATD], p. 26. See Paper 12 below.

4. Modality, too, is given considerable attention by P. Lorenzen. See

Section 1.1 below.

5. Paper 10, Section 2.

6. I.e., a theorem concerning the equivalence of dialectic, derivational and semantic systems (soundness and completeness relations) analogous to Theorem 29 in [AD1] (Paper 7, Section 6, in this dissertation).

<u>Notes to 11.1</u>

1. Lorenzen [NLE], [KDS], [PTM], Kamlah and Lorenzen $[LPr]_2$, Lorenzen and Schwemmer $[KLE]_2$.

2. See several essays in Saarinen (ed.) [GTS], notably Hintikka [QLQ] and Saarinen [BLO].

3. See Inhetveen [KWL].

4. Paper 10.

5. See Note 1.

6. Lorenzen [NLE], p. 63. The column titles, the choice of notation and the way the rule is named are taken from [AD1]. "aU" stands for "verbal attack on U"; "pU" stands for "protective defense of U" (in general, there is also the possibility of counteractive defense or countercriticism). A pU is called "structural" if its form depends on the grammatical structure of U or aU. Cf. Paper 1, Section 4 and 7.

7. <u>Loc</u>. <u>cit</u>. I have written "□" instead of Lorenzen's "Δ", and will continue to do so throughout this section.

8. A similar problem is discussed in Paper 10, Section 2, in connection with the rule FD $L_2 2$. Cf. Section 1.4 below.

9. It is a weird consequence, for one would expect the modal dialogue rules to be conservative with respect to ordinary (nonmodal) implications, like U&V/V.

10. Murphy [MLg], p. 1. I have adapted the notation and added the explanations in brackets.

11. Cf. Section 1.2, pp. 211, 212, below.

12. Cf. Lorenzen [KDS], [PTM], Kamlah and Lorenzen $[LPr]_2$, and Lorenzen and Schwemmer $[KLE]_2$.

13. Cf. Lorenzen and Schwemmer $[KLE]_2$, p. 114: $\Box_W A_t \leftrightharpoons W \angle A_t$. Here W stands for a set of sentences expressing our present knowledge (Wissen) and $\ulcorner A_t$ A_t stands for a sentence about the future ($t > 0$, i.e., t later than now). Cf. op. cit. p. 113. Lorenzen admits two kinds of modalities as meaningful: "mellontic" ones (i.e., those concerned with statements about the future) and deontic ones. I am not concerned here with the latter. Nor is the fact that Lorenzen restricts meaningful "ontic" or "logical" necessity to the "mellontic" case of interest in the present context. The discussion about a modal sentence $\Box_W A_t$, then, is a discussion about whether or not $W \angle A_t$ holds, and this is a material discussion in a metalanguage about an atomic thesis, $W \angle A_t$, of that metalanguage. In that sense, the necessity operator is introduced by Lorenzen on the basis of material dialogues in a metalanguage.

14. Consider a metalanguage in which the atoms are of the form $\Box_W A$. Let the logical constants of this metalanguage be some dialogically defined connectives and quantifiers. Now let $\Pi/_0 Z$ be a dialogue sequent (Paper 5, Section 1) built from sentences of that language with the same subindex W in all atoms. This dialogue sequent depicts a dialogue situation and that situation may figure as the initial situation of (ma-

terial) dialogues in the metalanguage. It appears that for some such $\Pi/_{0}Z$ there is a describable P-winning strategy that holds *irrespective of the choice of* W. If this is indeed the case for $\Pi/_{0}Z$ one may suppress the index W (and, if one desires, substitute distinct propositional letters/predicate letters for "object language" sentences/sentence forms). The result is a (correct) modal implication. The class of correct modal implications can be characterized by a tableau system that is essentially based on what Lorenzen calls the "Rule of Aristotle", which rule again can be shown to hold on account of a version of Gentzen's *Hauptsatz* that applies to the dialogue theory of the object language. See Lorenzen and Schwemmer $[KLE]_2$, pp. 114-116.

15. "... the present collection is intended for those scholars interested in or actively working on semantics who wish to compare the results achieved in game-theoretical semantics, and this approach generally, with rival theories (such as Montague grammar)" Saarinen (ed.) [GTS], Introduction, p. vii. The word "semantics" I use in the narrow sense of a theory about the relations between linguistic phenomena and extra-linguistic entities, not in the broader sense of any theory of "meaning" whatsoever, including the pragmatically based ones.

16. Conversely, the pragmatically based dialectic systems of Lorenzen and Lorenz and in [AD1] are not *semantic* systems, except in the broader sense of that term (see preceding note). It is in this wider sense, I think, that one should take the word "semantisch" in the title of Lorenz [DSG].

17. The merits of game-theoretical semantics lay elsewhere. In my opinion they are most evident in Saarinen [BLO], where the superiority of the game-theoretical approach over some earlier, model-theoretic, ones (by ~~H.~~ J.A.W. Kamp, D. Gabbay, etc.) is shown.

18. "It is easy to extend game-theoretical semantics to epistemic, modal, and doxastic concepts, at least in some of their typical uses. The basic idea is a straightforward combination of possible-world semantics for these notions with our game-theoretical principles", Hintikka [QLQ], in Saarinen (ed.) [GTS], p. 41; the rule "G-knows" is quoted from p. 42.

19. Of course, Ω and R must be decidable, for in each dialogue situation it must be determinable what the rights and duties of each party are. Cf. the norm FD 01 in Paper 1, Section 14. One may even demand more, viz., that Ω and R be decidable by rather simple means, if the dialectic system based on them is to be a practicable one. Here the notions of 'simplicity' and 'practicability' are dependent upon a cultural parameter, involving, say, the state of computer technology.

20. Paper 1, Section 16.

21. Cf. Figure 1 above, and also Note 6.

22. On externalization cf. [AD1], I.7 _sub_ (xi) ff. and The Principle of Externalization of Dialectics, _op_. _cit_., III.4 (= Paper 1, Section 4).

23. If, and only if, the dialectic system is intended as an instrument for the resolution of _pure_ conflict of opinion are these roles those of Proponent (P) and Opponent (O). Cf. Paper 1, Section 3. But the modal games of game-theoretical semantics are most easily reinterpreted as modal analogues of MatDial (Paper 5, Section 2) and hence as dialectic systems designed for the resolution of _mixed conflicts under complete opposition_ (_loc_. _cit_.). So the roles will usually be designated as White (W) and Black (B).

24. Paper 8, Section 3 (pp. 165 ff.).

25. A game is called _locally finite_ iff each of its plays ends after a finite number of moves.

26. A dialogue game is called <u>regular</u> iff (1) for each pair of classes of initial assertions to be assigned to the parties, and for each underlying model, there is a <u>starting position</u>, i.e., a position in which The Speaker is allowed, in order to start the debate, to attack any one of its adversary's assertions (with the possible exception of true atomic assertions), (2) rights of attack and of protective defense, which are obtained by virtue of the logical rule, may aways be executed as the <u>next</u> move, i.e., the structural rule of the game allows one to react immediately (with possible exceptions for attacks on true atomic assertions and defense by means of false ones). Cf. Paper 8, p. 164.

27. I.e., for each model $M = \langle \Omega, R, I \rangle$ according to $\mathfrak{M}$, and for each complex sentence U of $\mathcal{L}$: (1) if U is <u>false</u> at ω in M (according to $\mathfrak{M}$, $\omega \in \Omega$), then the logical rule of G provides a means to attack a statement of U relative to ω in a way that is both honest and ruthless, i.e., in such a way that, if a sentence V has to be stated relative to a world $\omega' \in \Omega$ in the execution of the attack, V is true at ω' in M, and further, if the attacks allows a structural protective defense by means of a sentence W relative to ω'', then W is false at ω'' in M; (2) if U is <u>true</u> at ω in M, then the logical rule of G does <u>not</u> provide a means to attack a statement of U relative to ω with the features described above. Cf. Paper 8, p. 165.

28. See Note 26.

29. I.e., the first Speaker in the discussion. In the case of a pure conflict: O. Cf. Note 23.

30. I.e., the adversary of the first Speaker. In the case of a pure conflict: P. Cf. Note 23.

31. Alternatively, the Modal Adequacy Theorem may be derived from the nonmodal one, if we are willing to envisage a language that provides a name for each possible world.

32. Cf. Note 27.

33. Paper 3, Section 2; Paper 8, p. 165.

34. Cf. Paper 1, sections on systematic, thoroughgoing, orderly and dynamic dialectics.

35. Inhetveen [KWL]. Unfortunately, I have up till now been unable to obtain the (uncompleted?) dissertation by M. Marčinko (Erlangen) to which Inhetveen refers.

36. Here $l_1,\ldots,l_n$ are probably intended to stand for positive integers or members of some other well-determined set of indices. That levels are identified with index *sequences* instead of mere indices appears to be a matter of technical convenience.

37. The name of the rule and its schematic formulation are mine. Cf. Inhetveen [KWL] . There is no change of level of discourse. involved in any other type of move, unless modalities other than □ are taken as primitive.

38. *Op. cit.* I have rendered Inhetveen's formulas in words.

39. Cf. Section 1.2 above, p. 212.

40. In an *orderly* dialectics the relation R should be decidable. In a *dynamic* dialectics it should be *quickly* decidable. Cf. Paper 1, Section 14 (FD O1) and 15 (FD D2).

41. For the term "chain of arguments", see Paper 1, Section 6. Inhetveen uses the term *Zweig*.

42. This is the situation where an appropriate *Ipse dixisti!*-remark can be

made by P. Cf. Paper 1, Section 7. The requirement that both statements are made at the same level of discourse is new.

43. By the "logic yielded by a dialectic system" I mean the set of sequents Π/Z such that there is a P-winning strategy - on the strength of the system - for the dialogue situation given by $\Pi/_0 Z$ (i.e., such that Z is the thesis, sentences of Π are stated as concessions, and 0 is to move first).

44. According to Inhetveen this is proved by Marčinko. One can also show such connections to hold by means of (some adaptation of) methods set forth here. The ~~earliest~~ easiest thing to do is to compare dialogical tableaux that express P-winning strategies in Marčinko-systems with semantic tableaux that employ index sequences to name possible worlds. To show that for each closed dialogical tableau there is a corresponding closed semantic tableau, one can adapt the method in Paper 6, there used to transform closed dialogical tableaux into closed deductive tableaux. To show the converse one, can adapt the method used for classical systems in Paper 7, Sections 3 and 4.

45. As stated in the introduction to this section, my criticism pertains only to Marčinko's systems viewed as a contribution to modal dialectics, i.e., as systems for conflict resolution. Whether or not Marčinko had such applications in mind, I do not know.

46. Van Dun [MOF], part II, pp. 121-135.

47. Op. cit., p. 132.

48. = Paper 1 of the present thesis. There the reader will also find definitions or explanations of all the dialectical notions not explained in the present paper, e.g., Opponent (O), Proponent (P), thesis, concession, stage, neutral position, structural protective defense (cf.

Note 6), local discussion, local thesis, chain of arguments (cf. Note 41), Ipse dixisti!-remark (cf. Note 42), etc. A Dutch translation of a former version of [AD1], III and part of Ch. IV is Barth and Krabbe [FD1].

49. Paper 10, p. 184.

50. Paper 10, Section 1.

51. Paper 10, Section 2. Cf. my remark above on the lack of an intuitive background for index sequences (Section 1.3).

52. Paper 10, p. 178.

53. Paper 10, pp. 192, 193. A system is called invertible if there is a P-winning strategy for $\Pi/_{0}$ Z iff there is one for $\Pi/_{P}$ [Z]. In the non-cumulative dialectic systems the implication from right to left fails.

54. Cf. $\text{Rule}_{\rightarrow}$ and $\text{Rule}_{\sim}$ in F_2D 1, Paper 1, Section 16.

55. I shall pursue this - in a generalized form - in the next section. Essentially I shall revert to rule FD L_2^* which I rejected in Paper 10, pp. 185, 186.

Notes to 11.2

1. Paper 10, Section 2, Note 1.

2. Lorenzen [NLE], p. 61. Lorenzen and Schwemmer $[\text{KLE}]_2$, p. 236.

3. I.e., a partition on the basis of the equivalence relation "equally strict". The classification may, without loss of generality, be supposed to be exhaustive since the nonassigned statements can be assigned to one additional class. It may also be supposed to be exclusive; if it happens not to be so, then a more refined classification can be found that is exclusive while expressing essentially the same distinctions.

4. I use "U", "V", etc., as linguistic variables for declarative sentences, and "U̲", "V̲", etc., as variables for statements (utterances of declarative sentences). Cf. [AD1], II, Conventions 1 and 3.

5. Paper 10, Section 2.

6. *Loc. cit.*(The norm is quoted on p. 218 above.)

7. *Loc. cit.*

8. If there is a first element, K_0, in the strictness ordering, i.e., a class of ("totally contingent") statements such that any other class K is stricter than K_0, then an operator $\boxed{0}$ for K_0 is redundant.

9. For statements of the form $\boxed{i}$ U this degree is i, and for statements not of such form it is, say, zero. Note that in $\mathcal{L}_I$ it is possible to make statements of zero strictness even when originally no first element K_0 was admitted (cf. preceding note). Thus the recommendation of the systems of formal dialectics to be defined below implies a recommendation to distinguish a class of "totally contingent" statements. Without loss of generality we may use "0" as an index for this "lowest level" and assume that $0 \notin I$, i.e., that I contains indices for higher levels only.

10. This rule is a variant of FD L_2* in Paper 10, Section 2. I prefer to stipulate that the withdrawal of concessions take place *immediately before* O's attack on V̲, instead of *immediately after* P's statement V̲, because it would be unfair if O withdrew the concessions and then went on to discuss something else instead of V̲ (and thence something else instead of $\boxed{i}$ V). As soon as FD D7 is included in the system the two formulations become equivalent.

11. Either by an attack by O on some (hypothetical) statement made by P in the context of a counterattack, or by some winning remark by P, or

because one of the parties abandons the chain of arguments.

12. Paper 10, Section 2, Notes 10 and 11.

13. Paper 1, Section 15: if the right to execute an i-withdrawal is connected with attacks on U, it still holds that the only difference between situations S' and S" is that in S" O may be in pro-position to some additional concessions. For, if we assume that O makes the maximal use of its rights of withdrawal, the concessions in S' are those of the concessions in S that are of the form $\boxed{j}\,V$, where $i \leq j$, and these concessions are also present in S". Note that no withdrawals but i-withdrawals can intervene, since between S and S" all attacks by O must be attacks on U (FD 05a, Paper 1, Section 14).

14. ~~The first part of FD D8~~ may be defended as in Paper 10, Section 2, Note 11. The second part may be defended as in Paper 1, Section 15: repetition of this defense move before a fresh concession of appropriate strictness has appeared gives O the opportunity to revive an old dialogue situation.

15. Paper 3, particularly Section 1.

16. At the end of Paper 3, Section 1, it was pointed out that (constructive) material systems in a sense reduce to the corresponding nonmaterial ones. A similar reduction is possible in the present context. For P it makes no difference (as far as the existence of winning strategies is concerned) whether $\boxed{i}\,U$ is conceded or $U \in \mathbb{T}_i$. Nor does it make any difference for P whether $\boxed{i}\sim U$ is conceded or $U \in \mathbb{F}_i$. This can be shown by simple strategic considerations.

17. Paper 2, Section 4; Paper 5; Paper 7, Section 2. Cf. Paper 10, Section 3.

18. Cf. Theorem 1 of Paper 2 (p. 60) and Theorem 1* of Paper 10 (p. 190).

19. Cf. Theorem 2 of Paper 5 (p. 106) and its proof. In Lemma 4 count two units for each occurrence of [i]. Cf. also Theorem 2* of Paper 10 (p. 190).

20. Paper 5, Section 1; Paper 10, Section 3, Def. 2.

21. Paper 5, Section 1, proof of Lemma 1 (p. 119).

22. Cf. Note 14 above. The same arguments that were used for the adoption of FD $D8^L$ can now be used again to show that Lemma 1^L holds.

22a. The rul Pd (reformulated) and the rules Pd^i, for each i, ar henceforth considered to constitute special cases of

23. Cf. Note 10 above.

24. Paper 5, Section 4 (pp. 107-110).

25. Since for a formula [i] V there is only one mode of attack and only one protective defense, one can omit the subscripts in the expression $P_{ij}W$.

26. Paper 5, Section 5. Lemma 9 does not quite hold in general (the proof fails for the case where [Λ] derives from a thesis [i]Λ), but it holds if the original sequent does not contain an essential occurrence of Λ. Again, in the proof of Lemma 10(a), case (v), we cannot say that "no rule cancels concessions", since Pd does, but Pd cannot give rise to a sequent to which case (v) applies - after Pd there is <u>no</u> local thesis in the sequent! Finally, in Figure V.21 we should realize that if Λ is not withdrawn, neither is ~U.

27. Paper 10, Section 3. The present Theorem 5^L is, therefore, not an analogue of Theorem 5*, <u>loc</u>. <u>cit</u>., but of the property there denoted as (**).

28. Paper 7, Section 2, Lemma 1 (pp. ~~299, 300~~). [151, 152

<u>Notes to 11.3</u>

1. Cf. Section 1, Note 43.

2. Cf. Theorem 29 of Paper 7 (p. 158) and Theorem 6* of Paper 10 (p. 199).

3. The first component of each name is either "C" for "constructive" or "M" for minimal (we are not, now, concerned with classical systems). The second component is "I" for "implication(al)", or "N" for "negation", or "$\wedge$" for systems with both $\wedge$ and negation. The third component is "D" for "dialectic", or "dt" for "deductive tableau", or "nd" for "natural deduction", or "ax" for "axiom system", or "st" for "semantic tableau". Recall that "MI = CI".

4. Cf. Paper 10, Section 4, Note 1.

5. See Sections 1 and 3 of Paper 6.

6. Fitch [SLg], in particular Ch. 3. Cf. Paper 10, Section 4 (pp. 196, 197).

7. The meaning of the dots and dashes and of $\boxed{\vdash\!\dashv}$ is explained in [AD1], VI.1.1.

8. I.e., the i-strict subordinate deduction indicated by the scope indicator starts after the premise but is not finished before the conclusion. A (local) premise of an application of a rule must not be confused with a (global) premise of the entire deduction. A local premise may be either the occurrence of a sentence (e.g., the premise of an application of $\boxed{i}$ -elim) or a subordinate deduction (e.g., the premise of an application of $\boxed{i}$ -introd).

9. Curry [FML], Ch. 8, in particular 8c. Curry's distinction of an inner and an outer system is one way in which a company may distinguish strict from contingent statements. It is, therefore, not surprising that the same logics result from his analysis and mine. Curry also presents sequent systems and natural deduction systems (Gentzen-type) for these modal logics, *loc. cit.* On the other hand *classical* multiply modal systems analogous to the present ones are treated in Fitting [LSM],

Goble [GMd], Rennie [MMM], Rescher and Manor [MEP].

10. Bull [SMC], p. 6: "In my opinion the most plausible of these systems for the role of intuitionist logic of necessity is that obtained by adding to *IC* the Godel [*sic*] rule and axioms for S4".

11. Kripke [SAM.I]. I shall use the "dialectical" terminology of [AD1], IX.3 and 4.

12. [AD1], IX.3, Def. 8 and IX.4, Def. 13.

13. *Op. cit.* IX.3, Def. 9 and IX.4, Def. 14.

14. [AD1], X.3 and 4. See Paper 7, Section 0.

15. Paper 6, proof of Theorem 9 (pp. ~~203-209~~). 135-141

16. [AD1], VII.1.1, Lemma 2.

17. For instance, consider Figure VII.6 in [AD1]. If the conclusion of the application of MP, V, is i-separated from its first premise, U→V, there must be an i-strict scope indicator that starts somewhere in the fragment denoted by $\equiv$. The initial part of this scope indicator appears - as the left part of an i-cancellation line - somewhere on the tape between the U→V and the U. If it occurs where the tape runs through subtableaux other than the ones displayed in the schema, the corresponding right part of the i-cancellation line must be passed through by the same tape fragment. Consequently, the i-strict subordinate deduction would lie wholly within the fragment $\equiv$. If the initial part of the scope indicator occurs somewhere between the U→V and the horizontal line in the tableau fragment shown, then U→V is canceled and the application of →1 is illicit.

18. Emptiness of $\equiv$ implies that no i-strict scope indicator is started in $\equiv$. The desired result can be reached by inserting additional applica-

tions of TRIV.

19. Cf. Paper 10, Section 4, proof of Theorem 6*, From c to d (pp. 202-205).

20. [AD1], VIII.2, Theorem 10.

21. E.g., [AD1], VIII.1, Lemma 1.

22. The reason for this is that I do not want to have to handle $\boxed{i}$-Nec when eliminating the other strict subordinate deductions.

23. E.g., [AD1], X.3 and 4. Some obvious modifications and additions must be made. In Definition 13 (and 18) there should be clauses pertaining to $\boxed{i}U$. In Σ, $\boxed{i}U$ has had sufficient attention by the left rules, concerning the sequent Π/Γ, iff, if $\boxed{i}U\in\Pi$ then $U\in\Pi$. In Σ, $\boxed{i}U$ has had sufficient attention by the right rules, concerning the sequent Π/Γ, iff, if $\boxed{i}U\in\Gamma$ then there is in Σ a sequent Π'/Γ' such that $\Pi^i\subseteq\Pi'$ and $U\in\Gamma'$. In Lemma 12 (and 17), for each $i\in I\cup\{0\}$, R_i should be defined by: $\Pi/\Gamma R_i \Pi'/\Gamma'$ iff $\Pi^i\subseteq\Pi'$. Finally, $\boxed{i}R$ must be counted among the creative rules. (In this note, let $\Pi^c = \Pi$.)

24. Paper 7, Theorem 28. This theorem is, for minimal and constructive systems, based on Lemma 1 in Paper 7, Section 2 and Lemma 4 in Paper 7, Section 5 (= Lemma XI.4 of [AD1]). Lemma 1 corresponds to Theorem 5^L, above. Recall that for the case of minimal negation we also need Theorem 4^L.

Notes to 11.4

1. D. Lewis remarks that one can define a kind of counterfactual connective in a language $\mathcal{L}_I$ with $I = \{1, \ldots, n\}$:
$U \Box\!\!\rightarrow V =_{Df.} (\sim\boxed{1}\sim U \& \boxed{1}(U\rightarrow V)) \vee \ldots \vee (\sim\boxed{n}\sim U \& \boxed{n}(U\rightarrow V)) \vee \boxed{n}\sim U.$
(assuming $1 \prec 2 \prec \ldots \prec n$). This seems to offer an interesting approach to a dialectical introduction of the counterfactual. Lewis [Cnt], p. 44.

2. Cf. the program outlined in [AD1], XI.7.

3. The classical dialectic systems are defined in Paper 2, Section 3.

4. P. 55.

5. For classical S5 (with one necessity operator) one may use the following FD-rule: Suppose that at some definite moment $U_1, \ldots, U_n$ are among O's concessions and P has Ipse dixisti!-rights with respect to $V_1, \ldots, V_m$. Then P obtains a right for the rest of the chain of arguments to reinstate simultaneously the concessions $U_1, \ldots, U_n$ and to add $[V_1], \ldots, [V_m]$ to its protective defense rights, provided (i) P agrees to let a withdrawal of concessions and protective defense rights precede this reinstatement, (ii) the reinstatement does not lead to a dialogue situation that has already existed in the chain of arguments, (iii) the reinstatement does not count as a move by P.

6. Paper 10, Section 2.

7. For MND see the principle FD M-NOT in Paper 2, Section 2.3 (p. 54).

Notes to 12

1. Lorenzen and Lorenz [DLg].
2. Beth [SCI], [FMt], pp. 449 ff.; Kripke [SAI.I], Section 2.
3. Fitting [ILM], Ch. 5.
4. That is to say, only nonmaterial systems of formal dialectics will be studied here.
5. Lorenzen [LAg], [DKn]. The rule is stated here for ease of comparison with F_2D 1 in Section 16 of Paper 1 (p. 44).
6. Cf. Note 3 to the introduction of Paper 11. Lorenz's solution in [DRE], using ordinal numbers, is less arbitrary and therefore perhaps more natural.
7. That $\frac{1}{m}$ expresses a "degree of pretension" was suggested to me by

E.M. Barth. Discussion-promoting rules are needed in order to restrain people from obstructing a discussion at the start by making their claims too weak (selecting too great a number m). Cf. Section 12 of Paper 1. This matter will not be pursued here.

8. For the notion of 'retracing one's steps', see Section 13 of Paper 1 on thoroughgoing dialectics. Notice that nothing similar to retracing one's steps occurs in the dialogues of Lorenzen and Lorenz, i.e., discussions on the basis of their systems do not contain more than a single chain of arguments. Consequently, they have no need for anything like the present rule.

9. Once more we meet with a need for discussion-promoting rules.

10. The present reduction to finite tree diagrams has been used by me in teaching courses since the mid-seventies. The same subject is now treated by W. Felscher in his [ITD], whose skeletons correspond to closed dialogical tableaux. A similar point of view is also present in Kindt [ATD], Sections 2 and 10 on Quasifinitheit.

11. The propositional rules were formulated in Paper 5 (pp. 94-96).

12. "[U(x)]" stands for the right to defend protectively by uttering any statement of the form U(a).

13. "Umbenennung von freien Gegenstandsvariabelen", Gentzen [ULS], pp. 198, 199. In Szabo's volume: pp. 90, 91.

14. With a finite or infinite number of nodes, but with m stages at most in each branch.

15. The propositional rules are listed in Sections 1 and 3 of Paper 6.

16. Cf. Section 3 of Paper 6.

17. The propositional rules are listed in Section 0 of Paper 7. The "Σ" is redundant, and could have been omitted, now that the completeness proof for the system (which is not repeated here) is to be taken from Fitting [ILM], Ch. 5, instead of [AD1], X.3. I retain the "Σ" in order to preserve continuity with Paper 7 and [AD1], X. The completeness proof of [AD1], X.3 fails, not only because a sensible tableau construction may be infinite - this hurdle can be taken as in classical predicate logic - but because even an infinite construction does not necessarily yield a Hintikka set of sequents. For, because of the presence of quantifiers, it is not generally possible to restrict the application of creative rules to internally completed sequents (loc. cit. Def. 16).

Notes to 13

1. Kripke-type systems are those similar to Kripke's system in [SAI.I], Section 2. See below, Section 1.
2. Beth-type systems are those similar to Beth's system in [SCI] and [FMa], p. 449. Cf. Fitting [ILM], Ch. 5. Cf. Section 3 of Paper 12.
3. Cf. Schütte [VSM], p. 22: "Indexbaum".
4. The creative rules are $\rightarrow$R, $\sim$R, and $\forall$R.
5. In view of the Propagation Rule, formulated below, "$\sqcap$" may be omitted, if one wishes.
6. One may also have the rule for any β such that $\alpha \prec \beta$, if one wishes.
7. If the tableau does not yield a "finitary" Hintikka set, an infinite branch will be generated containing an infinite Hintikka set, or a finite Hintikka set that contains infinitely many formulas in its sequents. It is obvious enough how structured Hintikka sets should be defined.
8. On the other hand, for the smooth completeness proof of Section 2 the Propagation Rule is essential.
9. Henceforth, we shall assume that the tableaux are provided with an "analysis" showing how they were constructed.

S U M M A R Y

This dissertation consists of a number of papers on dialogical logic. The dialogical approach in logic is characterized by the way logical constants are defined: not by means of semantic rules, rules of inference, or axioms, but by means of rules that determine when and how sentences with a given logical constant as their principal operator are attacked and defended. A further, allied, characteristic concerns the way one goes about refining the concept of 'logical validity'. In the case of the dialogical approach, this is done in terms of the availability of a winning strategy for the party that wants to uphold some thesis in discussion. The hope is that some of the dialectic systems (i.e., systems of rules for conducting a critical discussion) created in this branch of logic, including those set forth in this dissertation, can in the future be extended to a comprehensive theory of argumentation, of which they will form the "logical skeleton".

In Part 1 of this dissertation, the pragmatic and intuitive foundations are laid for a number of dialectic systems. These foundations are independent from what goes on in other branches of logic, such as the theory of models and proof theory.

Paper 1 (written jointly with E.M.Barth) starts with a definition of the fundamental notion of a 'conflict of avowed opinions'. The paper focuses on 'simple' conflicts, i.e., those conflicts in which exactly one thesis is at issue. The thesis is upheld by a "Proponent" (P) and opposed by an "Opponent" (O). Thereupon, the foundations are laid for (formal) dialectic systems that serve as instruments for the resolution of conflicts by verbal means. These dialectic systems consist of norms and rules for conducting an orderly debate between two parties, P and O, which generally ends with one party winning the discussion. The norms and rules are hierarchically ordered, starting with a number of primary, or fundamental, norms that (the authors assume) most if not all potential debaters will consent to and accept, provided they are explicitly confronted with them. In this sense, these fundamental norms and rules may be called "natural". Two examples of such norms are: <u>the fundamental norm of a systematic dialectics</u> (FD S1; Section 6) and <u>the fundamental norm of dynamic dialectics</u> (FD D1; Section 15). The first of these stipulates that P should be given the opportunity to attempt to defend one

of its own statements that has been attacked, by making another statement (provided that P is willing to uphold this new statement). The second norm requires that a dialectic system be designed in such a way as to promote the revision and flux of opinions.

In order to implement these primary norms, secondary norms or rules are proposed, and so on, up to the rules that regulate the actual courses of discussions. Thus the above-mentioned fundamental norm FD S1 leads to an analysis of discussions into chains of arguments, local discussions, and stages, whereas the norm FD D1 leads to various measures intended to prevent discussions from running on indefinitely. With the exception of Lorenzen's "strip rules", included as F_2D 1 (Section 16), all the rules are language-invariant. (The examples, however, always employ some specific propositional language.)

In Paper 2 (also written jointly with E.M.Barth) eight definite propositional dialectic systems are defined, in which the norms and rules of of the preceding paper are included. Anticipating the results of Part 2, these systems are called "minimal", "constructive" or "classical". The constructive (i.e., intuitionistic) and classical systems are equivalent to the dialogue games known from the works of P.Lorenzen and K.Lorenz. The constructive systems are, in fact, virtually identical with Lorenzen's dialogue games that go by the same name. Clearly then, the system of norms and rules of Paper 1 provides a foundation for these traditional dialogue games.

Paper 3 treats of the way in which the dialectic systems, defined earlier, can be enriched by the introduction of "material moves". A move is to be called "material" if the truth value of an elementary sentence is at issue, it being assumed that the parties agree upon the use of one or more "material procedures" by which to establish the truth or falsity of such a sentence. The paper is concerned with the rules that regulate the way one can, in the course of a discussion, invoke material procedures, and the effects this has on the rights and duties of the parties. The particular rules of the material procedures lie themselves outside the scope of this dissertation. The second part of the paper describes a material dialectic (formal) system (MatDial; Section 5.2) that is independent of the nonmaterial systems of the preceding paper, and that is suitable for the resolution of one type of "mixed" conflict, viz., conflicts in which both parties explicitly reject each other's statements.

The foundations for dialectic systems given in the first paper are not unique. In the short Paper 4, another motivation is offered for what are in practice the same dialectic rules. On the one hand this motivation is simpler than that given in Paper 1, but on the other, it applies only to one type of (simple) conflict, viz., those involving a situation in which a system of opinions is immanently criticized. The proponent of that system is assigned a role in the discussion equal to the role of Opponent defined in Section 3 of Paper 1, and vice versa: the opponent of the system of opinions is, within the discussion, the Proponent (of a provocative thesis).

In Part 2 the dialectic systems, already defined and justified, become the objects of further study, and are related to logical systems of other types.

Paper 5 starts by introducing dialogue sequents in which the crucial elements of dialogue situations are codified. For each dialectic system, an equipollent variant is defined in which the rules of conduct pertaining to P are more lenient. This facilitates the description and study of (winning) strategies. The various situations that can occur in a dialogue are grouped according to type (Section 1). Further, the rules for constructing P-winning strategy diagrams (Lorenzen's dialogical tableaux) are reformulated and classified. Both tree notation and tableau notation are used (Sections 2 and 3). In Section 4 it is shown that the norm FD D1 is implemented to the extent that discussions are guaranteed finite (Theorem 2). Section 5 demonstrates -- within the confines of dialogue thery -- that the dialectic systems incorporating a propositional falsum-constant are equivalent to the corresponding systems without this constant. The larger part is devoted to the proof that this holds for minimal logic.

In Paper 6 it is shown, by graphic description, how closed dialogical (Lorenzen-)tableaux can be transformed into closed deductive (Beth-) tableaux.

Paper 7 is devoted to the step that takes one from closed semantic (also Beth-)tableaux to closed dialogical (Lorenzen-)tableaux. For that purpose, it is first shown that each system is invertible (Lemmas 1 and 2). A dialectic system is called invertible if the existence of a P-winning strategy does not depend on whether the first move consists, as usual, of an attack by O on the thesis, or rather of some move by P, P having the right to pronounce the thesis.

Papers 6 and 7, together with some other steps that are not included in this dissertation, make it clear how dialogical tableaux can be inserted into a "circle of metatheorems" in which, besides the two kinds of Beth-tableaux already mentioned, natural deduction, axiomatics, and (model-theoretic) semantics find their proper place.

Paper 8 deals abstractly with material dialectic systems. According to the Adequacy Theorem there proved, whosoever is right (from a semantic point of view) will be able to carry his point, provided that the dialectic system satisfies three quite plausible conditions.

In Paper 9 this proof is repeated with respect to a concrete example: the adequacy of the system MatDial (Paper 3).

In Part 3 the exploration is extended to modal operators. Both the foundations of modal dialectic systems and metatheoretic questions are discussed.

Paper 10 originates in the following questions, that were put to the author: What would a noncumulative logic be like (i.e., a logic based on Kripke models for intuitionistic logic, but without the Principle of Cumulation)? and: Can a plausible dialectic system be constructed that corresponds to this noncumulative logic? It was through these questions that modal dialectic systems were reached. In Section 1 various ways of refining the concept of a 'noncumulative logic' are discussed. Section 2 lays the foundations for dialectic systems with two levels (of strictness) at which statements can be made. The following fundamental norm is proposed: a strict thesis is to be defended, ultimately, on the basis of strict concessions. Several ways of implementing this norm by means of further rules are investigated. While none of these implementations is fully satisfactory, one particular set of systems is singled out as preferred and is allotted further metatheoretic study. The noncumulative systems turn out not to be invertible (Section 3). It is nevertheless possible to establish a circle of metatheorems (Section 4).

Paper 11 discusses modal dialogue theory (based upon constructive propositional logic), starting with a review of the current literature. Notably, the contributions of Lorenzen, Murphy (Section 1.1), Hintikka (Section 1.2), Marčinko, and Van Dun (Section 1.3) are discussed. In Section 1.2, moreover, a Modal Adequacy Theorem is formulated that is a corollary to the General Adequacy Theorem of Paper 8. In Section 1.4 the relationship Paper 11 bears to Paper 10 is expounded. The introduction of a necessity operator is motivated by the way this operator serves to eliminate certain logical operators with a portmanteau character, viz., implication and negation in the way they occur in Paper 10. From now on the necessity operator shall indicate the degree of strictness, whereas implication and negation retain their ordinary dialectical meaning. The normative foundations for modal dialectic systems are then laid. The fundamental norm of Paper 10 returns in a generalized form (Section 2.1): any number of levels shall from now on be a possible choice. Modal dialogical tableaux are dealt with in Section 2.2. The modal dialectic systems are invertible (Theorem 5^L). After, a circle of metatheorems is, once more, established (Section 3). The last section contains, among other things, some remarks about classical modal systems. However, the modal dialectic systems that stand out as the most attractive ones are those that are based on a constructive or minimal propositional logic.

The Appendix contains two papers on dialogical predicate logic. The first (Paper 12) shows how the foundational reflections of Part 1 and the metatheory of Part 2 can be adapted so as to be applicable to predicate logic. In Section 1 there is a brief discussion on how debates can be kept within bounds. Section 2 demonstrates how "infinite" winning strategies (something unheard of in the context of propositional logic) can be depicted by (finite) dialogical tableaux. In Section 3 it is shown that the establishment of a circle of metatheorems in predicate logic is unproblematic. Kripke's semantic tableaux for nonclassical logic, however, are not yet included in the circle.

Paper 13 serves to insert these last-mentioned tableaux into the circle of metatheorems. To that end it is shown that the applications of rules in a closed semantic (Kripke-)tableau can be permuted so as to yield a standard form suitable for transformation into a closed semantic (Beth-) tableau, and thus into a tableau that is more akin to a dialogical or a deductive tableau.

SAMENVATTING

Dit proefschrift bestaat uit een aantal opstellen over dialogische logica. Kenmerkend voor de dialogische aanpak is dat logische constanten niet middels semantische regels, deductieregels of axioma's worden gedefinieerd, maar door regels die vastleggen hoe en wanneer zinnen met een bepaalde logische constante als hoofdoperator worden aangevallen en verdedigd. Een tweede, hiermee samenhangend, kenmerk is dat het begrip 'logisch geldig' hier verscherpt wordt in termen van de beschikbaarheid van een winstrategie in discussies voor de verdediger van een these. Sommige van de discussiesystemen die door deze tak van logica zijn voortgebracht, waaronder die welke in dit proefschrift zijn beschreven, zullen hopelijk kunnen worden uitgebouwd tot een meer omvattende argumentatietheorie, waarvan ze dan het "logisch geraamte" zullen vormen.

In Deel 1 worden de pragmatische en intuïtieve grondslagen gelegd voor een aantal discussiesystemen. Deze grondslagen zijn onafhankelijk van wat er in andere deelgebieden van de logica, zoals modeltheorie en bewijstheorie, gebeurt.

Opstel 1 (geschreven samen met E.M. Barth) begint met een definitie van het centrale begrip 'geschil over geuite meningen'. Het opstel concentreert zich op "eenvoudige" conflicten, d.w.z. conflicten waarbij precies één these centraal staat. De these wordt door een "Proponent" (P) verdedigd en door een "Opponent" (O) bestreden. De discussiesystemen waarvan vervolgens de grondslagen worden gelegd (ook "(formeel) dialectische systemen" geheten), dienen als instrumenten voor het oplossen van zulke geschillen met verbale middelen. Zij bestaan uit normen en regels voor het voeren van een geordend strijdgesprek tussen twee partijen, P en O, aan het eind waarvan doorgaans één van de partijen de discussie gewonnen heeft. Deze normen en regels zijn hiërarchisch geordend, te beginnen met een aantal primaire, of fundamentele, normen, die -- naar de schrijvers verwachten -- de instemming kunnen krijgen van verreweg de meeste potentiële discussianten, mits zij er expliciet mee worden geconfronteerd. In die zin kunnen deze fundamentele normen en regels "natuurlijk" worden genoemd. Als voorbeelden noem ik de fundamentele norm van een systematische dialectiek (FD S1; Par. 6) en de fundamentele norm van een dynamische dialectiek (FD D1; Par. 15). De eerste norm houdt in dat P de gelegenheid moet krijgen om te proberen een eigen uitspraak die is aangevallen, te verdedigen door een andere uitspraak te doen (waar P dan achter moet staan). De tweede houdt in dat een discussiesysteem zo moet worden ontworpen dat de herziening en doorstroming van meningen erdoor wordt bevorderd.

Om deze primaire normen te (helpen) verwezenlijken worden secundaire normen ~~als~~ of regels voorgesteld, enz., tot aan de regels die, in de praktijk, de gang van de discussie regelen. Zo geeft de bovengenoemde fundamentele norm FD S1 aanleiding tot een geleding van discussies in ketens van argumenten, lokale discussies en spreekbeurten, terwijl de norm FD D1 aanleiding geeft tot allerlei maatregelen om te verhinderen dat discussies eindeloos voortkabbelen. Met uitzondering van Lorenzen's "strip-regels" die zijn opgenomen als F_2D 1 (Par. 16), zijn alle regels taal-onafhankelijk. (De voorbeelden echter maken steeds gebruik van propositie-logische talen.)

In Opstel 2 (eveneens geschreven samen met E.M. Barth) worden acht concrete propositie-logische dialectische systemen gedefinieerd waarin de normen en regels uit het vorige opstel zijn opgenomen. Deze systemen worden, vooruitlopend op Deel 2, vast "minimaal", "constructief", of "klassiek" genoemd. De constructieve (= intuïtionistische) en klassieke systemen zijn equivalent met dialoogspelen bekend uit het werk van P. Lorenzen en K. Lorenz. De constructieve systemen zijn zelfs (vrijwel) identiek met de formele constructieve dialoogspelen van Lorenzen. Deze vanouds bekende dialoogspelen blijken dus door het stelsel van normen en regels uit Opstel 1 te kunnen worden onderbouwd.

Opstel 3 handelt over de wijze waarop de eerder beschreven systemen met "materiële zetten" kunnen worden verrijkt. (Par. 5.1) Onder materiële zetten versta ik zulke waarbij het vaststellen van de waarheidswaarde van een elementaire zin aan de orde komt. Dit veronderstelt dat er tussen de partijen overeenstemming bestaat over één of meer "materiële procedures" waarmee deze waarheidswaarden kunnen worden toegekend. Het gaat in dit opstel om de regels volgens welke materiële procedures worden aangeroepen tijdens een discussie en om de gevolgen daarvan voor de rechten en plichten van de partijen. De procedureregels zelf vallen buiten het bestek van dit proefschrift. Voorts heb ik een materieel dialectisch formeel systeem (MatDial) beschreven (Par 5.2) dat onafhankelijk is van de niet-materiële systemen uit het vorige opstel, en dat geschikt is voor het oplossen van één soort "gemengde" conflicten (namelijk conflicten waarbij de partijen elkaars beweringen expliciet verwerpen).

De fundering van dialoogsystemen in het eerste opstel is niet uniek. In het korte Opstel 4 geef ik een andere motivering voor wat in de praktijk dezelfde discussieregels zijn. Deze motivering is weliswaar eenvoudiger dan die in het eerste opstel, maar zij is alleen van toepassing op een bepaald soort van (eenvoudige) conflicten, nl. die waarin er sprake is van immanente kritiek op een systeem van opvattingen. De "proponent" van laatstgenoemd systeem krijgt in die discussie een rol toegewezen die gelijk is aan de rol van Opponent zoals die in Par. 3 van Opstel 1 is gedefinieerd, en vice versa: de "opponent" van het systeem is binnen de discussie de Proponent (van een provocatieve these).

In Deel 2 worden de eerder gefundeerde en gedefinieerde dialectische systemen verder bestudeerd, terwijl tevens de verbanden met andersoortige logische systemen worden gelegd.

Opstel 5 begint met het invoeren van dialoog sequenten die dienen om de belangrijkste elementen in een dialoogsituatie te coderen. Tevens wordt naast ieder dialectisch systeem een equipollente variant gedefinieerd waarin de gedragsregels voor de Proponent versoepeld zijn. Aldus wordt het beschrijven en bestuderen van (win)strategieën vergemakkelijkt. De verschillende situaties die in een dialoog kunnen voorkomen, heb ik in types ingedeeld (Par. 1). Voorts heb ik de regels voor het construeren van P-winstrategie--diagrammen (= Lorenzen's dialoogtableaus) opnieuw geformuleerd en ingedeeld. Zowel de boom-notatie als de tableau-notatie worden gebruikt (Par. 2 en 3). In Par. 4 wordt aangetoond dat de norm FD D1 in zoverre verwezenlijkt is dat discussies altijd eindig zijn (Stelling 2). In Par. 5 toon ik - zonder het terrein van de dialoogtheorie te verlaten - aan dat de systemen met een propositionele falsum-constante equivalent zijn met de overeenkomstige systemen zonder deze constante. De meeste arbeid gaat daarbij zitten in het bewijs dat dit ook geldt voor de minimale logica.

In Opstel 6 wordt op aanschouwelijke wijze aangetoond hoe gesloten (Lorenzen) dialoogtableaus kunnen worden omgezet in gesloten deductieve (Beth) tableaus.

Opstel 7 is gewijd aan de stap van gesloten semantische (eveneens Beth) tableaus naar gesloten (Lorenzen) dialoogtableaus. Daartoe wordt eerst van alle systemen aangetoond dat ze invertibel zijn (Lemma's 1 en 2). Een dialectisch systeem is invertibel indien het voor het bestaan van een P-winstrategie niet uitmaakt of de eerste zet, zoals gebruikelijk, bestaat uit een aanval van O op de these, of dat P het eerst aan de beurt is, met het recht om de these te poneren.

De Opstellen 6 en 7, tezamen met een aantal andere stappen die hier niet zijn opgenomen, laten zien hoe de dialoogtableaus kunnen worden ingevoegd in een "cirkel van metastellingen" waarvan, behalve de al genoemde twee soorten Beth-tableaus, ook natuurlijke deductie, axiomatiek en (modeltheoretische) semantiek deel uitmaken.

Opstel 8 geeft een abstracte behandeling van materiële dialoogsystemen. De daar bewezen adequaatheidsstelling houdt in dat, mits het systeem aan een drietal plausibele voorwaarden voldoet, wie (semantisch gezien) gelijk heeft dat ook (met dialectische middelen) kan krijgen.

In Opstel 9 herhaal ik dit bewijs voor een concreet geval: de adequaatheid van het systeem MatDial (Opstel 3).

In Deel 3 worden modale operatoren bij het onderzoek betrokken. Zowel kwesties betreffende de fundering van modale discussiesystemen als metatheoretische kwesties komen hier aan de orde.

Opstel 10 kwam voort uit de volgende aan mij gestelde vragen: hoe ziet een niet-cumulatieve logica eruit (d.w.z. een logica gebaseerd op Kripke--modellen voor intuïtionistische logica maar zonder cumulatie-principe), en: is er een plausibel dialectisch systeem te construeren dat aan deze niet--cumulatieve logica beantwoordt? Via deze vraagstelling kwam ik terecht bij modale discussiesystemen. In Par. 1 worden verschillende verscherpingen van het begrip 'non-cumulatieve logica' besproken. In Par. 2 leg ik de grondslagen voor dialectische systemen met twee niveaus van striktheid waarop uitspraken gedaan kunnen worden. Als fundamentele norm stel ik voor dat een strikte these uiteindelijk verdedigd moet worden op grond van strikte concessies. Verschillende mogelijkheden om de norm middels verdere regels te verwezenlijken worden onderzocht. Hoewel geen van deze mogelijkheden in alle opzichten voldoet, heb ik toch een keus gemaakt voor een bepaalde groep systemen, waarvan de metatheorie dan in de rest van het opstel wordt behandeld. Het blijkt dat de niet-cumulatieve systemen niet invertibel zijn (Par. 10.3). Nochtans is het mogelijk een cirkel van metastellingen rond te krijgen (Par. 10.4).

Opstel 11, waarin de modale dialoogtheorie (op basis van constructieve propositie-logica) systematisch behandeld wordt, begint met een bespreking van de literatuur. Met name de bijdragen van Lorenzen, Murphy (Par. 1.1), Hintikka (Par. 1.2), Marčinko en Van Dun (Par. 1.3) komen aan de orde. In Par. 1.2 wordt tevens een modale adequaatheidsstelling geformuleerd, die volgt uit de algemene adequaatheidsstelling van Opstel 8. In Par. 1.4 wordt het verband met Opstel 10 uit de doeken gedaan. De motivering voor het invoeren van een noodzakelijkheidsoperator is, dat met behulp daarvan de dialectische dubbelrol van implicatie (en negatie), zoals die voorkomt in Opstel 10, kan worden voorkomen. De noodzakelijkheidsoperator dient voortaan om de striktheidsgraad aan te duiden, terwijl implicatie en negatie hun gewone (dialectische) betekenis houden. Vervolgens leg ik de normatieve grondslag voor modale discussiesystemen. De fundamentele norm uit Opstel 10 keert hier in meer algemene vorm terug (Par. 2.1): ieder aantal van niveaus behoort voortaan tot de mogelijkheden. Modale dialoogtableaus worden behandeld in Par. 2.2. De modale dialoogsystemen zijn invertibel (Th. 5^L). Daarna wordt weer een cirkel van metastellingen geconstrueerd (Par. 3). De laatste paragraaf bevat o.a. enkele opmerkingen over klassieke modale systemen. Als meest aantrekkelijke systemen komen echter naar voren die welke gebaseerd zijn op constructieve of minimale propositielogica.

In de Appendix zijn twee opstellen over dialogische predikatenlogica opgenomen. In het eerste daarvan (Opstel 12) laat ik zien met welke aanpassingen de fundamentele beschouwingen uit Deel 1 en de metatheorie uit Deel 2 kunnen worden overgeheveld naar de predikatenlogica. In Par. 1 wordt kort besproken hoe de discussies eindig gehouden kunnen worden. In Par. 2 laat ik zien hoe "oneindige" winstrategieën, waarvan in de propositielogica geen sprake was, toch door (eindige) dialoogtableaus kunnen worden uitgebeeld. In Par. 3 blijkt de cirkel van metastellingen zonder moeilijkheden ook in de predikatenlogica te kunnen worden aangelegd, waarbij echter Kripke's semantische tableaus voor niet-klassieke logica's vooralsnog niet in de cirkel zijn opgenomen.

Opstel 13 dient om laatstgenoemde tableaus in te schuiven in de cirkel van metastellingen. Hiertoe laat ik zien dat de volgorde van regeltoepassingen in gesloten semantische (Kripke) tableaus op een bepaalde manier veranderd kan worden, zodat een standaardvorm wordt bereikt die zich leent voor omzetting in een gesloten semantisch (Beth) tableau, en daarmee in een tableau dat dichter bij dialoogtableaus en deductieve tableaus staat.

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LIST OF ERRATA (SECOND EDITION)
to "Studies in Dialogical Logic" by Erik C.W. Krabbe
16 August 1985

A. CRUCIAL

1. p.12, main text, between lines 6 and 5 from the bottom: <u>Insert</u>
   possible frameworks for such theories. For one thing, we do not here discuss
2. p.182, line 7: <u>For</u> $\sim A \rightarrow (A \rightarrow \sim B)$ <u>read</u> $A \rightarrow (\sim A \rightarrow \sim B)$
3. p.197, between lines 6 and 5 from the bottom: <u>Insert</u>
   <u>Axs</u> &3* $(U \rightarrow V) \rightarrow \{(U \rightarrow W) \rightarrow [U \rightarrow (V \& W)]\}$
4. p.253, rule OI∀: <u>For</u> $\Pi/_p[U(a)]$ <u>read</u> $\Pi/\forall x U(x)/_p[U(a)]$
5. p.253, rule OI∃: <u>For</u> $\Pi/_p[U(x)]$ <u>read</u> $\Pi/\exists x U(x)/_p[U(x)]$
6. p.253, rule Pd^x: <u>For</u> $\Pi/T/_0 U(a)$ <u>read</u> $\Pi/_0 U(a)$
7. p.258, line 9 of Section 13.1: <u>For</u> and we write $\alpha \preccurlyeq \beta$. If, moreover, $\gamma \neq \emptyset$: $\alpha \prec \beta$.
   <u>read</u> . Further, we write $\alpha \preccurlyeq \beta$ iff either β is a continuation of α or there are γ, α_1, β_1, m, n, such that (1) $\alpha = \gamma m \alpha_1$, (2) $\beta = \gamma n \beta_1$, and (3) $m < n$. If, moreover, $\alpha \neq \beta$ we write $\alpha \prec \beta$.
8. p.259, line 12: <u>For</u> $\alpha \prec \beta$ <u>read</u> β is a continuation of α $(\alpha \neq \beta)$
9. p.259, line 6 from bottom: <u>For</u> $\beta \preccurlyeq \alpha$ <u>read</u> α is a continuation of β

B. CONFUSING

1. p. 44, Figure III.7, left column, line 5: <u>For</u> $Rule_{At}$ <u>read</u> $Rule_{\&}$
2. p. 44, Figure III.7, left column, line 6: <u>For</u> U(atomic) <u>read</u> $Rule_{At}$ U(atomic)
3. p. 68, bottommost line: <u>For</u> W <u>read</u> B
4. p.84, bottom of Def.1: <u>For</u> (Cf. <u>read</u> (Cf. p.52.)
5. p.89, line 3: <u>Delete</u> , other than statements of the form U!!,
   [Utterances of this form are not statements.]
6. p.100, Figure V.10, P.column: <u>For</u> $A \rightarrow B$ <u>read</u> $A \rightarrow C$
7. p.113, line 11: <u>For</u> Below we shall <u>read</u> We shall now
8. p.138, second line below Figure VII.30: <u>For</u> P∨ <u>read</u> P∨ + OII
9. p.142, Example 2, bottom left: <u>For</u> ~~!A~~ <u>read</u> /A
10. p.143, Example 3, left: <u>For</u> [A, ~ A] read ~~[A, ~ A]~~
11. p.185, line 11 from bottom: <u>After</u> local thesis. <u>insert</u> (Unless, of course, O prefers to attack some statement, and start a fresh local discussion, before P has carried out the structural protective defense move in question.)
12. p.198, line 13 from bottom: <u>After</u> &2 <u>insert</u> ,&3*
13. p.202, line 5: <u>For</u> the new occurrences <u>read</u> the bottommost new occurrences
14. p.203, line 5: <u>For</u> usual way. <u>read</u> usual way, utilizing <u>Axs</u> 1*, <u>Axs</u> 2, <u>Axs</u> 3*, <u>Axs</u> &3*, and MP.
15. p.227, line 9: <u>For</u> $\mathbb{T}_i$ <u>read</u> $\mathbb{T}_1$
16. p.229, New choice rules, at the end of rule Pd^i: <u>Add footnote</u>
   The rule Pd (reformulated) and the rules Pd^i, for each i, are henceforth

considered to constitute special cases of Pd.

17. p.235, RESTRICTION, line 3: For i-strict subordinate read j-strict subordinate

18. p.237, Theorem 6^L, line 5: For $\sqcap/_P Z$ read $\sqcap/_0 Z$

19. p.238, line 13 from bottom: For P[i] read P[i] + OII

20. p.244, Figure 9, P-column: For [□A] read [A]

21. p.255, line 13: Delete + Hr

22. p.260, line 5: For such that $\Sigma \subseteq \Sigma'$ read that "extends" Σ

23. p.271, under Inhetveen, R.: This entry should read:

[KWS] Ein konstruktiver Weg zur Semantik der "möglichen Welten". In: E.M. Barth and J.L. Martens (eds.) [AAT].

24. p.273, bottom: Add

Naess, A.

[CAr] Communication and Argument. Elements of Applied Semantics, Oslo (Universitetsforlaget) and London (Allen & Unwin), 1966. Translation of: En del elementaere logiske emner, Oslo [1947, etc.].

25. p.281, line 3: For $v_M(C,U, \ldots U_n, d)$ read $v_M(C(U_1, \ldots, U_n), d)$

26. p.287, line 14: For τ^* read τ^+

27. p.298, Note 44, line 3: For earliest read easiest

28. p.301, Note 14 should read:

This may be defended as in Paper 10, Section 2, Note 11.

29. p.305, Note 23, bottom: Add

(In this note let $\sqcap^0 = \sqcap$.)

C. MERELY ANNOYING

1. p.vii, line 4 from bottom: For let read led
2. p.3, line 2: After dissertation insert)
3. p.4, line 2 from bottom: For more of less read more or less
4. p.23, Figure III.3, left, line 7: After intermediary add)
5. p.59, Figure IV.5, right column, line 4: For ~ ~(X or Y) read ~ ~(X ∨ Y)
6. p.62, Figure IV.7: For Speaker: X read Speaker: U
7. p.62, Figure IV.7: For Critic: aX read Critic: aU
8. p.87, line 8: For (type 01) read (type OI)
9. p.94, last line before Section 5.2.2.: Delete (cf. Exercise 5)
10. p.97, lines 6 and 5 from the bottom: Delete The winning strategy to be depicted is one possible solution of Exercise 7b of the preceding section.
11. p.101, Figure V.11, right, line 3: For shave read share
12. p.108, main text, line 11 from bottom: For as P-winning read a P-winning
13. p.110, line 15 from bottom (illegible) should start:
    of language to which both σ and σ' pertain;
14. p.153, line 18 (illegible) should end: (P can make an
15. p.167, line 9 from bottom: For schranke read schranken

16.p.174: Delete the running headline at the top of the page.

17.p.178, line 2 from bottom: For formal dialectic read dialogical

18.p.184, bottommost line: For language-forms read language forms

19.p.186, line 15 (bottom of diagram): For ?A read A?

20.p.187, line 11 (fifth numbered line of the diagram) should read:
5 A? | B? | [ ] | [ ]

21.p.187, line 7 from bottom: For statement read statements

22.p.195, two bottommost lines of the main text: For easy enough to do read readily done

23.p.196, line 2 from bottom: For rule read Rule

24.p.197, line 13 from bottom should read:
∴ V MP ∴ U & V CONJ

25.p.198, bottommost line: For N-systems read NOT-systems

26.p.200, line 13 from bottom: For Z_1, ... Z_{n-1} read Z_1, . . . ,Z_{n-1}

27.p.201, topmost line should read:
Example Let $U = "(A \& \sim A) \vee [(B \& \sim B) \vee \{[A \vee (B \vee C)] \& \sim [A \vee (B \vee C)]\}]"$

28.p.202, line 9: For application read applications

29.p.206, title: For M O D A L D I A L E C T I C S read Modal dialectics

30.p.209, line 5: For infact read in fact

31.p.216, line 15: For "cassical" read "classical"

32.p.234, line 4 from bottom: For i∈I. We read i∈I, we

33.p.234, line 2 from bottom: For "$\boxed{i}$". For read "$\boxed{i}$", for

34.p.237, line 6: For $\Sigma;(\Pi/\Gamma, \boxed{i}U),(\Pi^i/U)$ read $\Sigma;(\Pi/\Gamma, \boxed{i}U);(\Pi^i/U)$

35.p.239, line 4 from bottom: For occurence read occurrence

36.p.240, Figure 8, rightmost column: On top of $\boxed{i}$-introd add CP

37.p.240, Figure 8, rightmost column: For Axs 4j read Axs 4_j

38.p.240, Figure 8, middle column: The text to be omitted if j = i pertains to the formulas $\boxed{j}\boxed{j}U \rightarrow \boxed{i}\boxed{j}U$ and $\boxed{i}\boxed{j}U$. This should be indicated by a brace.

39.p.244, line 3: For $\Box A \vee \Box \sim A$ read $\Box A \vee \Box \sim A$

40.p.244, line 6: Idem

41.p.244, line 7: For OIv read OIv

42.p.252, lines 8 and 7 from the bottom: For minimalizes read minimizes

43.p.266, line 10: For reformulated read formulated

44.p.278, Note 2, line 6: For inconsistent, for read inconsistent; for

45.p.278, Note 2, line 11: For knowing of thinking read knowing or thinking

46.p.282, Note 9, line 2: Delete ; Barth and Krabbe [AD1]

47.p.282, Note 10, line 1: For Section read Sections

48.p.283, Note 2, line 3: For [KWL] read [KWS]

49.p.285, Note 11, line 8: For was the first attack. read was the situation after the first attack.

50.p.289, Note 8, line 1: For [AD1] read Cf. [AD1]

51.p.292, Note 3: For [KWL] read [KWS]

52.p.293, Note 13, line 3: For At read A_t

53.p.294, bottommost line: For H. Kamp read J.A.W. Kamp

54.p.296, Note 27, end of line 2: Add ,

55.p.297, Note 35, line 1: For [KWL] read [KWS]

56.p.302, Note 28: For pp. 299, 300 read pp. 151, 152

57.p.304, Note 15: For pp. 203-209 read pp. 135-141

58.p.306, line 3: For P.35 read p.35

59.p.315, line 4: For normen als regels read normen of regels

60.p.316, line 16: For situaties die een dialoog read situaties die in een dialoog

61.p.319: After Fitting, M.C., delete 243, 274n,

62.p.319: For Kamp, H. read Kamp, J.A.W.

63.p.177, title: For Noncumulative Dialectical Models and Formal Dialectics read Noncumulative dialectical models and formal dialectics

64. p. 262, line 4: For Theorem 5 read Theorem 2

65. p. 7, line 6 from below: For viz.; read viz.,

66. p. 209, line 5 from below: For indepently read independently

S T E L L I N G E N

behorende bij: Erik C.W. Krabbe, *Studies in Dialogical Logic*, 1982

de analyse van de door Frede, Kneale en Mates behandelde stoïcijnse
logismen — volgens Frede's reconstructie van het stoïcijnse systeem
r zulke analyses — behoeft geen gebruik te worden gemaakt van Frede's
jectuur voor het zo geheten vierde "thema" van dit systeem.
M. Frede, Die stoische Logik, Göttingen (Vandenhoeck & Ruprecht), 1974.
W. Kneale en M. Kneale, The Development of Logic, Oxford, etc. (Clarendon/Oxford U.P.), 1962.
B. Mates, Stoic Logic, Berkeley en Los Angeles (Univ. of Calif. P.), 1953.

ilay's rol bij de wedergeboorte van de temporele logica in onze eeuw
i door Prior overschat en is niet die van "founding father". Bij
ilay zijn de temporele bepalingen namelijk predikaten van gebeurtenis-
, terwijl de temporele logica pas kon opbloeien door temporele bepa-
gen niet te behandelen als deel van een predikaat of subject maar als
zinsoperatoren. Deze grote stap is door Prior zelf genomen.
J.N. Findlay, 'Time: A Treatment of Some Puzzles', in: J.J.C. Smart (red.), Problems of Space and Time, Londen en New York (Macmillan), 1964, blz. 339-355. (Oorspronkelijk verschenen in de Australasian Journal of Psychology and Philosophy 19, 1941, blz. 216-235.) Zie vooral noot 16, blz. 355.
A.N. Prior, Past, Present and Future, Oxford, etc, (Clarendon/Oxford U.P.), 1967. Zie Ch. I, 'Precursors of Tense-Logic', met name blz. 1 en par. 4.
A.N. Prior, Time and Modality. Being the John Locke Lectures for 1955-6 Delivered in the University of Oxford, Oxford, etc. (Clarendon/Oxford U.P.), 1957. Zie Appendix A, met name blz. 105-108.

Segerbergs systeem LinDAS (voor z.g. reële tijd) zijn de postulaten
r pseudo-dichtheid ($GGp \rightarrow Gp$ en $HHp \rightarrow Hp$) overbodig. Een postulaat dat
fhankelijk is van de postulaten van LinDA (voor z.g. rationele tijd)
dat gebruikt kan worden om een systeem voor reële tijd te definiëren is:

$$\Box(Gp \rightarrow PGp) \rightarrow (Gp \rightarrow Hp)$$

voorkomt bij Rescher en Urquhart.
es: Gp: het zal altijd zo zijn dat p; Hp: het is altijd zo geweest dat p;
: het is ooit zo geweest dat p; $\Box$p: het was, is, en zal altijd zo zijn
p.)

- E.C.W. Krabbe, *Propositionele Tijdslogica* (Doctoraalscriptie, Universiteit van Amsterdam), 1972. Zie Lemma II.24 (blz. 38) en Stelling V.3 (blz. 105).
- N. Rescher en A. Urquhart, *Temporal Logic*, Wenen en New York (Springer), 1971. Zie (G8) op blz. 96.
- K. Segerberg, 'Modal Logics with Linear Alternative Relations', *Theoria* 36, blz. 301-322 (1970). In de postulaten S_0 en S_1 op blz. 315 is per ongeluk het symbool "□" voor het antecedent weggevallen.

ɪ illocutionaire handeling welke bestaat uit het invoeren van een onderɛelling met het doel een voorwaardelijke bewijsvoering (bijv. een *reductio l absurdum* of een gevalonderscheiding) in te zetten, is moeilijk in te ɜlen volgens de taxonomie van Searle. Hetzelfde geldt voor het doen van ɔncessies door de Opponent in een strijdgesprek. In tegenstelling tot ɱpirische hypothesen kunnen deze handelingen niet worden opgevat als: beɜerders (*assertives*) waarbij de graad van het geloof en van de gebondenɜid (*commitment*) het nulpunt nadert of bereikt.
- J.R. Searle, 'A Taxonomy of Illocutionary Acts', in: J.R. Searle, *Expression and Meaning. Studies in the Theory of Speech Acts*, Cambridge (Cambridge U.P.), 1979, blz. 1-29. (Oorspronkelijk verschenen in 1975.) Zie vooral blz. 13. Hierboven is gebruik gemaakt van de Nederlandse vertaling door R. Grootendorst: 'Een taxonomie van illocutionaire handelingen', in: F.H. van Eemeren en W.K.B. Koning (red.), *Studies over taalhandelingen*, Amsterdam en Meppel (Boom), 1981, blz. 145-174. Zie vooral blz. 157.

ɜ Nederlandse taal beschikt over een aantal fraaie inheemse technischɔgische termen, zoals "kettingrede", "ontkenning", "strijdig", waarvan ɜt gebruik dan ook kan worden aanbevolen. De even fraai klinkende uitrukking "bewijs uit het ongerijmde", waarmee een (klassieke) *reductio ad bsurdum* wordt aangeduid, is echter minder bruikbaar, daar verwarring met ewijsvoering van het type *ex falso sequitur quodlibet* voor de hand ligt.

olgens de definitie die Naess in zijn *Elementaire Argumentatieleer* geeft oor de term "precisering" -- welke definitie teruggrijpt op een eerder egeven definitie van de term "interpretatie" -- wordt de vraag of U (in e context K) een precisering (in K) van T (in K) is, onder meer beslist oor wat U en T in allerlei andere contexten dan K kunnen uitdrukken.

ie van precisering sluit niet aan bij veel van Naess' eigen en verdere beschouwingen. Aan de hand daarvan blijkt duidelijk en van belang is, juist omdat een formulering zelfs binnen een en ingscontext verschillende beweringen kan uitdrukken, en niet om-ulering in verschillende contexten verschillende beweringen en.

Elementaire argumentatieleer, Baarn (Ambo), 1978. Vertaling bbink van En del elementaere logiske emner, Oslo [1947, etc.].

eren dat tussen Nederlandse wijsgeren in de eenentwintigste kussionszusammenhang ontstaat, is het nodig dat de Centrale iten het op korte termijn eens worden over de inhoud -- in gro- van een landelijk algemeen verplicht te stellen gedeelte van ogramma wijsbegeerte ter grootte van ongeveer één studiejaar. ller, Hauptströmungen der Gegenwartsphilosophie. Eine kritische g [I], 4e druk, Stuttgart (Alfred Kröner), 1969. Zie blz. XLI.

én van deze stellingen zal worden bestreden.

én van deze stellingen is juist.

Amsterdam, 29 maart 1982