Finiteness, Invariance, and Analogy: A Minimal Model for Adaptive Processes

MSc Thesis *(Afstudeerscriptie)*

written by

Pepijn P.J. Kroes
(born June 27th, 1990 in Arnhem, the Netherlands)

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Members of the Thesis Committee:
Dr. Katrin Schulz
Dr. Benno van den Berg
Prof. Dr. Martin Stokhof
Prof. Dr. Fenrong Liu

Institute for Logic, Language and Computation

Abstract

Adaptive processes are processes that adapt to the environment in which they are situated by abstracting input taken from the environment and applying those abstractions to act on the environment and to determine what next to take as input. They involve a continuous oscillation between an inductive aspect of generalization and a deductive aspect of application. This thesis proposes a minimal model for such processes in terms of three key notions: finiteness, invariance, and analogy. I take cognition to be a prototypical example of an adaptive process and examine logic from the perspective of this model. In three early traditions of logical thinking—in ancient China, ancient India, and ancient Greece—both aspects of adaptive processes can be seen to be present. Classical logic, on the other hand, focuses solely on the deductive aspect. Finally, the wider applicability and explanatory value of the model are considered, as well as other instances of adaptive processes.

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To R,
thank you, for the continuous discussions and your singular insight.

And to someone who once observed “the snow is getting like sand,”
thank you for being there in times of distress,
and that we may meet again someday.

Contents

Chapter 1

Introduction

1.1 Topic, Motivation, and Approach

What is an adaptive process? In this thesis, I consider a process situated in an environment to be *adaptive* simply if it takes input from the environment, abstracts it in some way, and applies those abstractions to act on the environment and determine what next to take as input. Such a process can take many forms, but for most of this thesis, I will consider cognition as an adaptive process with the aim of exploring its relation to logic.

I will attempt to establish five points:

1. Induction and deduction are best understood in terms of each other, as two aspects and opposing tendencies of a finite adaptive process dealing with a potentially infinite amount of information.

2. Analogy provides a foundational mechanism for this process, through the ability to identify and create invariances.

3. Both aspects of this process can be seen in the early development of logical thinking in ancient China, ancient India, and ancient Greece.

4. Classical logic, when put in the context of this process, can be seen as focused on the deductive aspect in isolation.

5. As a minimal model for adaptive processes, the main notions involved—finiteness, invariance, and a continuous interaction of induction and deduction based on analogy—are ubiquitous and have explanatory value beyond the study of logic.

The first and second point will be covered in Chapter 2. The third will be the topic of Chapters 3 to 5. The fourth and fifth point will be discussed in Chapter 6.

Induction and deduction are understood broadly in this thesis, as general opposing tendencies to generalize from particulars and apply generalizations back again to particulars. While the view of induction and deduction as opposing and interacting tendencies is well known, for example, in our understanding of the scientific method, and the making of analogies has been seen by many as a fundamental cognitive process, I want to make an explicit connection between the two based on two types of analogies identified in the literature. I also want to stress the role of invariance (the notion of things staying the same or keeping things the same, as I will explain in Chapter 2), as a fundamental notion underlying adaptive processes. Moreover, I want to consider the implications of such processes being finite.

Studies of early logical thinking in ancient traditions are often focused on the extent to which they had logical, deductive reasoning in the modern sense. I will instead look at these traditions from this broader double-aspect perspective: the interaction of induction and deduction based on analogy. With regard to what we now call classical logic, exemplified by first-order logic, I want to consider how it can be understood from the perspective of cognition as an adaptive process. And what, if anything, it has gained and lost in comparison to logical thinking in the ancient traditions.

Finally, I hope to show that as a simple model with broad applicability, it can help to keep a few essential aspects of our thinking at the forefront of our mind to avoid common confusions, cognitive biases, and rigidity and a false sense of certainty in our thinking.

Many of the ideas and concepts involved in the model are separately discussed in many more elaborate theories, distracting through complexity from their basic interaction, or are only implicitly relied upon or applied in practice. My aim is not to introduce some novel, elaborate technical system. Rather, I want to rely on a minimal selection of some of the most well-known concepts, to propose a model for adaptive processes that is as simple as possible while aiming for the most explanatory value.

1.2 A Word about Words

In line with the above aim, I will use simple, plain English words and sentences whenever possible. Except for the concepts that are central to the thesis, the concepts of other authors, and where essential for the arguments being made, I will avoid the use of technical terms. The words used most frequently in our everyday language are in many cases sufficient and most useful. Despite the ambiguities inherent in natural language, these words are in some sense an optimal trade-off between efficiency and accuracy, between generality and specificity. Otherwise they would not have survived the pressures of the natural evolution of language and become among the most frequently used. As their meaning is most widely shared and invariant

among people, they provide the most solid foundation for building an argument, much like the choice of concepts for the model.

1.3 Overview of the Thesis

In its simplest sense, the general structure of the thesis is the introduction of a model (Chapter 2), providing support for it by providing instances (three early logical traditions, Chapters 3–5), and applying it to a further instance (modern classical logic, Chapter 6) and considering its implications.

Chapter 2 introduces a minimal model for adaptive processes. Its three main components—finiteness, invariance, and a process involving induction, deduction, and analogy—are discussed in the three main sections of this chapter. With the aim of using the model towards the study of logic, invariance is considered at four distinct levels: categories, language, argumentation, and logic. Since the components of the model are based on some of the most familiar notions, recurring across traditions, it allows for a comparison of different logical traditions at the level of our collective humanity, instead of from the perspective of any particular tradition.

Chapters 3–5 serve to illustrate the components of the model, providing support for its validity and its applicability to concrete instances. They are also intended to highlight that in early logical traditions both the inductive and deductive aspects of adaptive processes were present. For each tradition, I will focus on specific texts, and highlight one characteristic of the model in particular. Chapter 3 discusses the Mohist Canons in ancient China, with a particular focus on categorization. Chapter 4 discusses the Nyāya Sūtras in ancient India, with a particular focus on invariance. Chapter 5 discusses the work of Plato and Aristotle in ancient Greece, with a particular focus on the use of analogies. These chapters share a similar structure. The introduction gives some context for the tradition and the specific texts considered. The first section discusses the type of argumentation used in each tradition, in the context of which their theorizing developed. The second section forms the main part of each chapter where the relevant parts of the texts are discussed, part of it focused on the subject highlighted in the section title. The third section discusses specific analogical argument forms identified in the texts. The conclusion summarizes the main points of the chapter and highlights the connections with the different parts of the model.

Chapter 6 further develops the ideas behind the model introduced in Chapter 2. It also considers classical logic in terms of the model and suggests its wider applicability beyond logic.

Chapter 7 wraps up the discussion with concluding remarks.

Chapter 2

Proposal of the Model

The minimal model for adaptive processes rests on three main ideas, set out in the three main sections of this chapter: finiteness, invariance, and a continuous interaction of induction and deduction—of constructing and applying invariances—for which analogy will be argued to provide a basic underlying mechanism. Invariance is the central notion of the model and is simply taken to mean anything that stays the same or that is kept the same. Analogical comparisons allow us to perceive and construct invariances. And finiteness provides an important limitation to the construction of invariances and gives rise to the need for incremental adaptation.

I take cognition to be the prototypical example of an adaptive process. One aim of this thesis is to explore and compare early logical traditions and modern classical logic from the perspective of cognition as an adaptive process. For this reason, I will in this chapter introduce a model for adaptive processes from the perspective of cognition and with an aim towards exploring logical thinking. I take finiteness to refer to our finite lives and mental capacities. I consider different types of invariances underlying categories, language, argumentation, and logic. And the continuous process involving induction, deduction, and analogy, I discuss from the perspective of generalizing from experience into a mental model and applying the model back again to experience. For most of the thesis, therefore, I will use 'the adaptive process' in the singular, meaning cognition as prototypical example of an adaptive process.

Another aim of the thesis, however, is to make a broader point: that what I consider to be adaptive processes go beyond cognition. The main components introduced in this chapter, then, are at the same time meant as a more general model for 'adaptive processes' in the plural. The components being finiteness, invariance, and a continuous process of constructing and applying invariances based on an analogical kind of mechanism. At the end of Chapter 6, I will consider other instances of adaptive processes.

2.1 Finiteness and the Predictive Mind

As a starting point and foundation for the model, I want to consider the common-sense recognition that the lives we live are finite, and by extension, that the experience we have of the world is finite. Few things are as obvious when we consider the reality of our own lives and the lives of others, but it has important implications if we take it seriously. Half of what I want to say here has been compendiously expressed by Ernst Mach (1895, p. 186):

> When the human mind, with its limited powers, attempts to mirror in itself the rich life of the world, of which it is itself only a small part, and which it can never hope to exhaust, it has every reason for proceeding economically. Hence that tendency, expressed in the philosophy of all times, to compass by a few organic thoughts the fundamental features of reality.

Our finiteness has been a recurring theme throughout the history of philosophy. The arguments of René Descartes in his *Meditations on First Philosophy* (1641/2008) rest on the contrast between the finite nature of human beings and the infinite nature of God. We could think of Descartes' notion of an all-encompassing God, the cause of everything, as another description of what we conceive of as the abstract notion of the whole of reality. Descartes noted that our understanding grows incrementally; our imperfect "knowledge is being increased and perfected by degrees" (p. 244). Nonetheless, we are never able to grasp the whole completely: "there may be in God an infinity of things that I cannot comprehend, nor perhaps even compass by thought in any way; for it is of the nature of the infinite that it should not be comprehended by the finite" (p. 244).

Even if we do not make the assumption that there is an infinite reality, the environment we live in far exceeds our ability to fully internalize, simply because we are only a part of it. The *mental model* that we have of reality is therefore necessarily finite, incomplete, and fallible. Nevertheless, we are often led to think we know more than we do. We confuse our limited mental representation for reality itself since it is all we know. Our limited mental capacities force us to compress and abstract even the impressions we have from the parts of reality that we do experience.

Finite, fallible model

The story has become more complicated with advances in modern cognitive science. Traditionally it was often assumed that we are passive observers, the view that our brain is passively constructing an internal representation, or *explanation*, of the world. Nowadays, it is a common and well-supported thesis that from an evolutionary perspective, as part of the nervous system and connected with the senses, the brain's original and primary function is not thinking (let alone rational and abstract thought), but to control and preserve the body (cf., for example, Jekely et al., 2015).

Explanation

There is an interesting parallel here with Descartes, who noted that the brain "causes the mind to experience, among all the sensations which it is capable of impressing upon it, that one which is the best fitted, and generally the most useful for the preservation of the human body" (1641/2008, p. 277). The view has shifted to the idea that the brain is not just passively observing but actively generating hypotheses about the world. It constantly makes *predictions*—both about internal processes of the body it needs to regulate and the external world—to identify potential threats to and benefits for the body. Through feedback from the nervous system and the senses, the brain tests these hypotheses and adjusts them if necessary, seeking to minimize error (Howhy, 2013). The evolutionary advantage of this should be obvious. There is thus a constant interaction between what we perceive (bottom-up) and what we *expect* to perceive (top-down). What is more, our mental models direct our attention and influence what and *how* we perceive things, to some extent even getting ahead of what we perceive, actively constructing our perception, and thus reality as we experience it (Howhy, 2013).

Prediction

Making predictions based on incomplete information, due to our finiteness, inherently comes with a risk of error. Descartes argued that *errors in our judgments* arise from the interaction of two distinct human faculties (1641/2008, p. 252). On the one hand, we have a faculty of understanding, i.e., our knowledge, which he identified as finite and only incrementally growing. On the other hand, we have a faculty of will, i.e., our ability to make free choices. As he could conceive of no limit to this faculty of will, and no other faculty as having a greater extension, he regarded it as infinite in nature. He then identified the cause of making errors from the interaction of these two different faculties: "They arise from this cause alone, that I do not restrain the will, which is of much wider range than the understanding, within the same limits, but extend it even to things I do not understand" (p. 254). In other words, errors in judgment arise from extrapolating, i.e., making predictions, beyond our direct experience and knowledge.

Errors in judgment

The explanations and predictions we make are based on invariances of various kinds. This role of invariance forms the second part of the model, which I will discuss next. The constant interaction between explaining what we have perceived, and predicting what we will perceive next, forms the third part of the model, to be discussed in section 2.3.

2.2 Invariance

Invariance has been a fundamental notion in the philosophy of science, widely used by many authors (Weinert, 2004, pp. 62–74). I want to argue that, as the basis for every kind of sense-making that we engage in, it has an even more fundamental and ubiquitous significance than usually considered. Invariances underlie the patterns and regularities that we perceive in the

world, the meaning we attribute to words in natural language, and our ability to make arguments and draw conclusions from logical inferences. I will understand invariance rather broadly: anything that does not vary, anything that stays the same. It can be the sharing of particular features or a common structure among instances of a certain type of thing, such as that all birds have wings or that all triangles have three sides. We can also believe in invariances that do not correspond to reality, such as that all birds can fly (most can, but not all). Furthermore, I consider invariance a gradational concept, i.e., there can be lesser or greater degrees of sameness or constancy, less or more perfect invariances, for example, the regularity of a train following a timetable, or the increasing constancy of meaning of a concept as we get a better grasp of it over time. There are concrete invariances across space and time, regularities we perceive in our experience of the world, such as David Hume's 'uniformities in nature' (Hume, 1748/2007). There are also abstract invariances, such as abstract operations in logic and mathematics. Alfred Tarski, for example, defined logical constants as those operations that are 'invariant under permutations' (Tarski, 1986), which has been shown to be a necessary, though not sufficient, condition for logicality. In a sense, all meaning in language, knowledge, argumentation, and inference is based on some things staying the same or constant.

Since for most of this thesis I want to consider cognition as a prototypical adaptive process with an aim towards studying logic, I want to make a distinction between different types of invariance specifically for this purpose. I distinguish four general 'levels' of invariance that build on each other, in terms of invariance: categories, languages, argumentation, and logic.

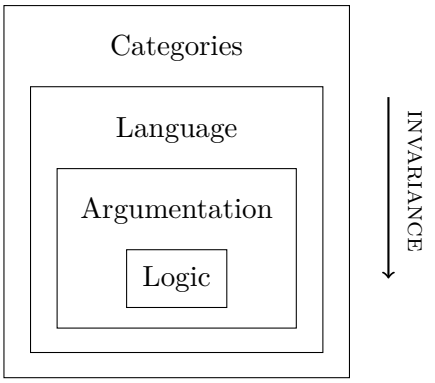


Figure 2.1: Levels of invariance

These four levels will serve as general reference points when discussing the ancient traditions in Chapters 3–5 to indicate the type of invariance involved. For the first level, I consider cognitive categories with a gradational structure (see section 2.2.1). We can see the invariance of language as based

on the invariance of such categories but in a more constant form. Likewise, we can see argumentation as a subset of language use that is generally more invariant than other types of language use. While logic is the most invariant, abstracting the invariances on which argumentation in natural language is based to an absolute degree in abstraction.[1] When we think of the interconnection of these levels, we can think, for example, of Boole (1847) when he wrote: "That which renders Logic possible, is the existence in our minds of general notions, our ability to conceive of a class, and to designate its individual members by a common name. The theory of Logic is thus intimately connected with that of Language." I will now briefly discuss each level and consider the type of invariance underlying it.

2.2.1 Categories

As the first level of invariance, I want to consider the mental categorizing we do of the things we encounter in experience; we constantly group things based on perceived similarities. While there is an extensive literature on categorization in cognitive science, I only want to mention a few key insights relevant to the current discussion. Specifically, from the empirical work of Eleanor Rosch, on what she calls "natural categories." Her work, starting in the 1970s, was focused on reconceptualizing the traditional, and then still dominant, conception of mental categories. The traditional, analytical conception of categories, going back to Aristotle, is based on discrete all-or-nothing membership; either an instance is a member, or it is not. They are categories with clear boundaries, and they have definite feature criteria for membership. Rosch aimed to show through experiments, with natural as well as artificial categories, that the way we categorize things mentally is far less exact and not definable in terms of precise membership criteria (Rosch, 1973, 1976). These insights contributed to a new wave of research on categorization (Rips et al., 2013) and align better with what we now know about the more continuous nature of the brain. I will highlight only a few of the main insights from Rosch's and related work.

Graded structure

First, natural categories have what Rosch called a *graded structure*. Unlike Aristotelian categories with definite boundaries, membership of natural categories is a matter of degree. Thus a natural category's internal structure is not discrete but continuous (see figure 2.2).

Prototype effect

Second, family resemblances determine the internal structure of natural categories, rather than being defined by a discrete set of feature criteria, i.e., membership is determined by whether an instance has enough characteristic features overlapping with members of the category (Rosch, 1975, p. 573). Moreover, natural categories exhibit a so-called *prototype effect*, the degree to which an instance is considered a member of a category is dependent on how much it deviates from the prototype for that category, which is like an abstracted 'ideal' member of the category that has the most characteristic

features. Membership of a category is efficiently determined by comparing new instances to the category's prototype.

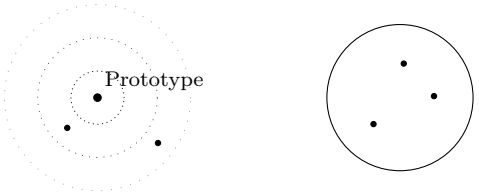


Figure 2.2: Graded vs. all-or-nothing category membership

Third, categories and the relations between them form a complex network (like the brain), which Mark Turner (1988) called *category structures*. We can think of this as a representation of the model we have of the world. On the one hand, some connections in the category structures are more basic and used more often; they are, what Turner called, more deeply *entrenched* than others. Such connections are stronger and more invariant (like connections in the neural networks of the brain become more established when used more often), and more other categories depend on them, making them have a "high degree of cognitive indispensability" (1988, p. 4). On the other hand, some connections are less entrenched or entirely new but have the potential to alter the category structures by proposing new connections to be established in them.[2] Section 2.3.3 further discusses these two types of connections in relation to two types of analogical comparison.

Category structures

As the first level of invariance, we can think of natural categories as intermediate between our immediate experience of the world and natural language. The invariances underlying natural categories are the perceived similarities among instances that are grouped together; we *perceive certain things staying the same* across instances and categorize on this basis. Rosch described the basis for our natural categorization as follows: "in the real world information-rich bundles of perceptual and functional attributes occur that form natural discontinuities […] basic cuts in categorization are made at these discontinuities" (1976, p. 385). Similarly, we can think of what James Gibson called "invariants in the stimulus flux" (Gibson, 1967, p. 162). He argued that besides the constant changes in stimuli across space and time, "certain higher-order variables of stimulus energy — ratios and proportions, for example — do *not* change." (Gibson, 1966, p. 3), regardless of perspective or stimulus intensity. He argued that these invariances "correspond to the permanent properties of the environment." In fact, (Rosch, 2008) has also considered natural categories in terms of Gibson's ecological theory of perception. Another connection can be made with what Hume called "uniformities in nature" (Hume, 1748/2007), certain law-like regularities or invariances in our experience of the world, with similar effects

Perceptual invariance

following similar causes. However, ideas about the invariances underlying our categorizing go back much further, as we will see in Chapters 3–5 when discussing the ancient traditions (e.g., what Plato called "cutting nature at the joints"). All of these point to the type of invariances on which natural categories are based. Some of these insights on natural categories, such as their graded structure and prototype effect, have also been shown to apply to artificial categories (Rosch, 1975), and even to ad hoc categories that "violate the correlational structure of the environment and are not well established in memory" (Barsalou, 1983) and goal-derived categories such as "things to pack in a small suitcase," some of which might become more entrenched with increased use (Barsalou, 1991). Since natural categories are mental constructs, they are fundamentally *private*; the unique encoding in our brain of how we categorize things differs per person. The next level of invariance to consider allows us to move beyond this private world of mental categorization.

2.2.2 Language

We come to the level of natural language when we give names to groups of things that we naturally categorize together, when we explicitly label implicitly formed categories for the purpose of communicating with other people or for introspection.

Whereas natural categories are private, language is *public*. Unlike the continuous nature of natural categories, language consists of a finite set of discrete units: words (which in turn are based on a finite set of discrete units: letters or characters, or vocal sounds in the case of spoken language). This added layer of abstraction and simplification allows us to communicate efficiently. We can take a natural category with a continuous structure, abstract it into the discrete unit of a word, and communicate the word to someone else, who can then consider their corresponding continuous natural category to grasp its meaning. When words refer to reasonably similar and constant underlying natural categories among people, it allows us to communicate thoughts, exchange experience, and adjust and extend our finite, fallible models of the world through comparison with those of others. A common source of confusion naturally arises from the differences in the underlying categories that words refer to for different people, formed on the basis of different experience. The confusion is compounded by the fact that we communicate in discrete words, static labels which make it seem like their meaning is constant and the same for different people, whereas the underlying categories are more fluid and gradational. There is therefore a continual struggle to synchronize the shared meaning of our words, i.e., to keep them more the same to be able to communicate.

The invariance on which words in natural language are based can thus be seen to come from two sources. On the one hand, the invariance of a

word is based on, and still close to, the invariance of the underlying category to which it refers. The meanings of words are not necessarily clearly defined and do not have clear boundaries. This accounts for one source of ambiguity in natural language (another source would be, for example, that words can have multiple meanings, referring to multiple different categories). On the other hand, the discreteness and consistent use of an explicit label given to a category make it more invariant. The boundaries of natural categories and the connections between them continually change with new experience and insights. We might encounter a new kind of bird, changing the category referred to by the word 'bird' and shifting perhaps what we would consider a prototypical example of it. However, those changes are, in the first place, private. The discrete and static word itself does not change, and more importantly, the public meaning of the word among people stays relatively more constant; it keeps referring to what we would collectively consider 'the same thing.' Hence, though arising from the invariances of natural categories, the invariances of words are also constructed between people: it is *interpersonal* and conventional. There are, of course, gradual as well as sudden changes in the meanings of words, but as public, shared constructs between people, and due to their discrete nature, words are relatively more constant than natural categories shaped by our continually changing experience.

Interpersonal invariance

Language also makes the complex network of interdependencies of the underlying categories it refers to more explicit.[3] Consider the fact that we make dictionaries for our languages, defining the words of a language only in terms of other words of that language. In contrast with explicitly defined terms, which I will discuss in the next section, the meanings of most natural language words are not deliberately constructed. The definitions in a dictionary merely document our actual use of language as it has naturally evolved. Hence such definitions are *descriptive*, an important contrast with the levels of argumentation and logic to be discussed next. On the other hand, one of the main similarities with argumentation and logic is that language is *sequential* in nature. Written or spoken language does not allow us to take in or express the richness of our experience at once. Instead, reading or hearing a sentence is much like sequentially following the steps of an argument or inference.

2.2.3 Argumentation

Language can be used for many purposes. We can exchange experiences, communicate our thoughts and coordinate our actions. We can evoke specific emotions and convey narratives, such as in literature. We can deliberately challenge the conventions and boundaries of words through metaphors and deviating from everyday language use, such as in poetry. However, when we use language explicitly to construct arguments, we get to the level of invariance of argumentation.

When we construct arguments, we do not (or rather, if we want to be effective, we would be better off trying not to) alter and evolve language. Instead, we rely on the common and shared understanding of words—the invariances on which words are based—to move from something shared that is known and familiar to both the sender and receiver, to something else that might as yet be unknown or unfamiliar to the receiver. We use language to actively construct invariances between things to try to lead someone to a new insight. We do not merely describe some experience or idea we had. We put forward reasons, to make a particular connection that we have in our category structures but that the other person might not have made before or had dismissed earlier. Philosophy, ancestor to natural philosophy (science), is a prototypical example of the use of language to make arguments.

Consistency

An essential part of argumentative language use is *consistency*, which is nothing other than a form of keeping certain things the same, invariant, over time. Through our language use, we make commitments to others about our views, which others can rely on as reasons for their arguments, for example, by pointing out new connections to consequences of those views that we might not have thought of ourselves. Consistency is also closely related to a more formal use of language. Therefore I want to consider as part of this level of invariance the use of formally defined terms. A *formal definition*, as expressed in natural language at this level, deliberately constructs a specific meaning for a word and limits its scope of use, either redefining an already existing word or inventing a new one. Unlike the descriptive definitions we might give for many natural language words, such as those documented in dictionaries, formal definitions are *normative* in nature. Such definitions make the meaning of a word more rigid, as the meaning is kept the same as captured in the definition, while the definitions in a dictionary might need to be updated as language evolves over time. The meaning of the words making up the definition can still change, but the constancy of the defined term itself as a label, and the increased effort of keeping its meaning the same between people, adds to its invariance. In this way, it is rather like the increased invariance of words as labels applied to natural categories but at a higher level of abstraction. This constructed meaning is common in philosophy in that many key words in philosophy mean something different, something more explicit and rigid, than their naturally evolved meaning in everyday language use.

Formal definition

As a level of invariance, argumentation in this sense is intermediate between natural language use as a whole and logical reasoning in a formal language. It is a subset of language use that is relatively more invariant than other uses of language through actively constructing invariances for other people to follow, and through increased consistency and reliance on more rigidly defined terms.

2.2.4 Logic

When we continue along the trajectory of making ever more general arguments based on ever stronger invariances, we end up at the level of logic. Seemingly cut loose from concrete experience and natural language, logic considers invariance purely in abstraction in a constructed *formal language*. Russell, Hume,[4] and others have noted that all *a priori* knowledge, derived from logical or mathematical reasoning, is *hypothetical* in character. We cannot derive anything about actual existence from purely logical or mathematical arguments (Russell, 1912/2001, p. 42). We can use it as a 'method of description,' as Wittgenstein called it (1956, I, §121), and as a model for prediction, but the complete certainty that we get from a logical argument stays in abstraction only. Thus logical arguments are confined to an abstract realm isolated from empirical reality and operate in a self-contained 'closed vessel' (Dilman, 1973, p. 150). I am thinking here particularly of classical logic which I will consider further in Chapter 6.

Formal language

The inferences in classical logic are based on a complete, *conceptual invariance* in abstraction. Tarski defined the logical constants as those that are "invariant under permutations," by which he meant that they stay constant regardless of any uniform transformations of the universe of discourse (Tarski, 1986). This has been shown to be a necessary, though not sufficient, condition for logical constants (Novaes, 2014). Logical inference rules, furthermore, are postulated, and specific instances of logical inferences using them are *true by definition*, by citing the inference rules involved. Likewise, logical terms are based on formal *logical definitions* which are entirely normative. Hence, in classical logic, invariance is taken to an absolute notion of keeping things the same in abstraction. At this level of invariance, there are no more gradations, only clear-cut discrete concepts. Particulars in the universe of discourse are not similar to each other to some degree but either equal or unequal. Furthermore, because of their complete equality, they are substitutable.

Conceptual invariance

Logical definition

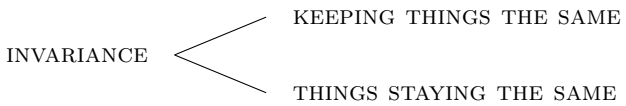
In effect, at the level of abstract classical logic, the notion of a natural category with graded structure, in which instances are similar to each other and membership is a matter of degree, gets flattened to the notion of an Aristotelian category, in which all instances are treated as *exactly the same* and membership is a matter of all-or-nothing (cf. figure 2.2), essentially making the characteristics of an implicit prototype explicit as definite criteria for category-membership. By making the invariance between instances complete, treating them as identical, they become interchangeable.

What we have seen then is a generally increasing invariance across the four levels discussed above. Natural categories have a gradational structure and are the most changeable, continually adapting based on new experience. Language is more invariant, as it applies constant labels to specific categories

(using a finite set of discrete letters or characters), which themselves are held relatively more the same between people for the purpose of communication. Argumentation is still more invariant, through its active construction of invariances by pointing out invariant relations between things, its reliance on consistency, and defining terms more explicitly with a specific scope of meaning and use. Logic then generalizes such argumentation in language into the most invariant form in a formal language in abstraction.

The characterization of these four levels of invariance and the clear differentiation between them as distinct levels is obviously a simplification. The boundaries between them are not as clear-cut, and there is a constant interaction between them. Naturally, categorizing is not the only thing we do. However, for this thesis, the idea of category structures based on natural categories, as presented in section 2.2.1, can serve as a useful (even if necessarily simplifying) representation for our mental model of the world. With regard to language, there are also syntactic and pragmatic aspects that are ignored here (which in their own ways rely on invariances, such as the regularities of the rules of grammar, and customs and conventions in language use as invariances over time). On the one hand, descriptive definitions in dictionaries, even if intended as documenting actual use, also become partly normative in explicitly limiting the scope of words as understood by a specific dictionary, consequently influencing the meaning and use of words for readers of that dictionary. On the other hand, what I have called formally defined terms at the level of argumentative language use are not always clearly defined and can change over the course of an argument, although they still tend to be generally more invariant. Nor is argumentation always more consistent than other uses of language, although it relies on consistency to be effective. Furthermore, formal terms are taken up in everyday language use and become more like other natural language words. Despite these and many other simplifications, I consider the distinct levels of invariance as presented here as at the right level of abstraction as themes of the early logical traditions that I want to focus on and for the study of invariances towards logical thinking in general.

Finally, I want to call attention to a certain change in characteristics along these levels of invariance. From private natural categories based on experience to public language. From the representational and descriptive nature of natural categories and language to the more normative nature of argumentation and logic. And from perceptual invariances to conceptual invariances. I want to make an important distinction between two general types of invariance:



On the one hand, we perceive certain things staying the same. We passively recognize existing invariances, such as perceptual invariances in our experience or well-established connections in our category structures. We can roughly call this a *descriptive* kind of invariance. On the other hand, we keep certain things the same. We actively construct invariances; we deliberately keep certain things constant and make new connections in our category structures. We can roughly call this a *normative* kind of invariance. We have seen natural categories as arising from perceiving things staying the same in experience. In language, this basis of things staying the same is combined with a layer of keeping things the same through words for communicating with other people. In argumentation, there is a further effort to keep things the same through consistent language use, actively creating new invariances, and formally defining terms. In classical logic, this keeping the same is taken to an absolute form in abstraction, such as in logical laws and inference rules. These two general types of invariance also correspond to the opposing tendencies of the process to be discussed in the next section.

Descriptive invariance

Normative invariance

2.3 The Adaptive Process

As the third part of the model, I want to consider a process of constructing and applying invariances in which we are continually engaged. I want to consider this process at the level of natural categories and how it allows us to move across the levels of invariance considered in the previous section. It is a process that allows us to learn, make sense of the new and unfamiliar, explain past experience, and predict future experience. As finite beings in an environment we cannot fully grasp, it allows us to adapt to the environment incrementally, which is why I refer to it as an *adaptive process*.

The process involves a continuous oscillation between two opposing tendencies, a *bottom-up* tendency to generalize from particular experience and a *top-down* tendency to apply generalizations back to experience. I will use the terms induction and deduction to refer to these general tendencies, to be explained below in sections 2.3.1 and 2.3.2. As abstract logical notions, induction and deduction are often studied and applied in isolation. I will argue that as general tendencies, they are better understood as two aspects of a single process and in constant interaction with each other, and that separating them, at the abstract level of logic as notions to be studied independently, is partly artificial. I will also argue that analogical comparison forms a basic mechanism underlying this adaptive process, allowing us to perceive and construct invariances, to be explained in section 2.3.3.

Bottom-up vs. top-down

The idea of an interplay between induction and deduction is clearly nothing new. For example, as both James and Russell pointed out, induction corresponds with an empiricist view of arriving at truth through empirical generalization from experience, while deduction corresponds with a ratio-

nalist view of arriving at truth through reasoning from abstract principles. James (1916) leaned more towards the empiricist side, Russell (1912/2001) more towards the rationalist side.[5] Yet both acknowledged that we need both tendencies for a well-rounded understanding of human cognition. It can be helpful to realize that many common distinctions line up with this fundamental dichotomy. Induction is concerned with the concrete, actual, and experimental, deduction with the abstract, hypothetical, and theoretical. Induction deals with the physical, causal, and descriptive, deduction with the mental, logical, and normative. Even such distinctions as Descartes' body versus mind, or the logical distinction between semantic and syntactic, between content and form, can be made in connection with this. The process considered here, in terms of invariance, treats such distinctions not as opposites that are completely separated but as different aspects of a single process involving their continuous interaction, and hence as not always easily separable. James (1916) made a distinction between percepts (that which we perceive in experience) and concepts (that which we conceive in thought) and similarly argued that they constantly interact and are not easily distinguishable. In section 2.1, we saw such an idea supported by the modern understanding of the brain in cognitive science: a constant interaction and blending of perception and prediction based on a cognitive model.

I will consider induction, deduction, and analogy in turn. The discussion of induction and deduction will be brief, as the main argument is in their interaction, in a process relying on analogy.

2.3.1 Induction

Particular to general

Induction has been described as an argument form going *from the particular to the general* (Russell, 1912/2001, p. 44; James, 1916, p. 35). It is our ability to generalize from individual instances to some general proposition or rule. It is the process by which we categorize. When we encounter a new and unfamiliar thing, we classify it, making sense of it by grouping it with similar things with which we are already familiar. In this sense, induction provides the basis for forming natural categories and enriching our category structures with new experience, incrementally increasing our finite empirical knowledge. As we encounter and incorporate new instances in our categories, the attributes considered typical for them can gradually change, shifting what we consider a prototypical example for them. Induction, understood in this way, describes the bottom-up tendency of the adaptive process, abstracting perceptual invariances in experience—perceived similarities between instances—into categories as a mental representation of general notions.

2.3.2 Deduction

Deduction has been described as an argument form going in the opposite direction, going *from the general to the particular* (Russell, 1912/2001, p. 44; James, 1916, p. 35). As a logical concept considered in isolation, it is usually understood as a logically valid argument form, providing complete certainty by definition. It allows us to infer the logical consequences of our explanatory and predictive model of the world. More importantly, however, as a characterization of the top-down tendency of the adaptive process, I will understand deduction as applying general notions to particular instances in experience. As we saw in section 2.2.4, deductive reasoning cannot be used to deduce the existence of something in experience since it is hypothetical in nature. To provide the certainty it promises, it has to stay in abstraction only. However, we naturally do apply general notions as encoded in categories to new experience, in the predictive application of our mental model. In Chapter 6, I will argue that the logical notion of deduction can be seen as arising from this more general process of applying generalizations.

General to particular

2.3.3 Analogy

An analogy makes a comparison between different things, pointing out similarities (invariances) despite the differences (variations) that exist between them. When we use analogies in arguments, it results in an argument form going *from particular to particular* (Alvargonzález, 2020) or *from general to general* (Juthe, 2015). We can thus represent induction, deduction, and analogy as in figure 2.3.

Particular to particular

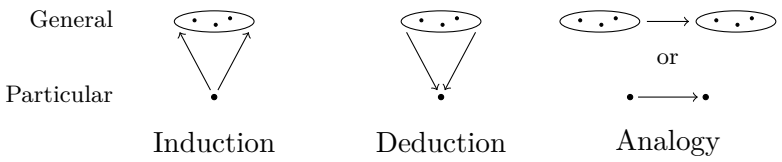


Figure 2.3: Three types of argument

Analogies have been argued to allow us to *perceive the possibility for a generalization* (Poincaré, 1914). We extrapolate something we know about one particular, for example, what category it belongs to, to another particular, or from one general statement to another, saying 'this is like that,' grouping them together and treating them as the same for some purpose. In this way, analogies provide the basis for perceiving and creating invariances, seeing or treating certain things as the same despite differences between them. We perceive invariances by noticing similarities between compared things; we create an additional invariance by treating them as the same for further purposes, such as by grouping them under a shared label.

Possibility for generalization

As an argument form going from particular to particular or from general to general, I want to argue that analogy provides a more elementary operation underlying both the bottom-up inductive and top-down deductive aspects of the adaptive process. I consider three reasons for this idea.

First, our ability to see similarities and make comparisons and connections between different things allows us to group them together, step-by-step from particular to particular. Hence, it provides the mechanism behind induction, generalizing from particular instances and forming natural categories. These generalizations encoded in categories—conceptual invariances created from perceptual ones—then provide the basis for deduction, making arguments back to particular instances. Hence, analogies provide the comparisons which form the basis for the bottom-up inductive tendency. Since this, in turn, provides the generalizations used by the top-down deductive tendency, they can be seen as underlying the whole process. A similar argument that both induction and deduction are reducible to comparisons between individual cases, i.e., analogies, was made by Wisdom (1991), I will consider this argument in Chapter 6.

Second, there is also another role for analogies. Turner (1988) has argued that all connections in our category structures arise from analogies: "Mentally, analogical and categorical connections between concepts are the same *kind* of connection. They differ in the *degree* to which they are entrenched in our general conceptual systems and in that part of our conceptual system I am calling our category structures" (p. 4). As we saw in section 2.2.1, Turner distinguished between two types of connections in the category structures. The more entrenched connections we no longer consider analogies because they have been frequently used. They do not seem novel to us anymore, but they once were: "Recognizing a connection between one zebra and another zebra, or between two red things, might have been, at one time, highly creative, but no longer" (p. 4). When creating a *new connection between categories* that were unrelated before, essentially noticing certain similarities between their prototypes, we consider it an analogy. Such comparisons, Turner argued, can be seen as proposing new connections to be established in the existing category structures. As a particular comparison between categories becomes more common and entrenched, it ceases to be an analogy to us. Consider the following example Whitehead (1929, p. 26) gives, as he imagines the discovery of the abstract notion of number by noticing the similarities between different groups (categories) of things:

Making new category connections

> We are merely thinking of those relationships between those two groups which are entirely independent of the individual essences of any of the members of either group. This is a very remarkable feat of abstraction; and it must have taken ages for the human race to rise to it. During a long period, groups of fishes will have been compared to each other in respect to their multiplicity, and

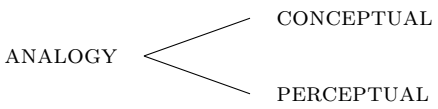
> groups of days to each other. But the first man who noticed the analogy between a group of seven fishes and a group of seven days made a notable advance in the history of thought. [...] the first man who entertained a concept belonging to the science of pure mathematics.

Starting as an analogy, a novel connection, it becomes more entrenched over time as a new category, after having encountered more groups of instances of particular sizes that we now start to notice and categorize this way, ceasing to be novel. As I mentioned in section 2.2.1, as a simplifying model, this idea of entrenchment accords well with our understanding of consolidating connections in neural networks in the brain through repeated use. Thus analogies allow us to make comparisons and new connections between different categories—from general to general—and construct new, more abstract categories out of them. These new categories then make us pay attention to different things. Since determining whether something is an instance of such a new category rests on somewhat different invariances than those on which the previous categories were based. Thus, the top-down deductive tendency, in turn, influences the bottom-up inductive tendency and determines what is compared next, what invariances in our experience are paid attention to, completing the constant interaction between them. Such an analogy suggesting a new connection can be seen as a new *hypothesis* to be tested and applied for prediction to new experience by the top-down deductive tendency. As we gather more evidence in support of the new connection through the bottom-up inductive tendency, the comparison is made more often and becomes more entrenched in the category structures. For natural, more entrenched comparisons between things that are already considered similar, there is a more passively perceiving them as staying the same, i.e., such comparisons reinforce a descriptive kind of invariance. For novel comparisons between things that seem more distant, we more deliberately have to keep them the same, i.e., such comparisons will establish a normative kind of invariance. Hence, these correspond to the two main types of invariance, as discussed in section 2.2, as well as to the two main types of analogies to be discussed next.[6]

Hypotheses

Third, another reason to think analogy might play this twin role, as underlying both sides of the adaptive process, is that there is a common distinction between two types of analogies and analogical arguments in the literature. Roughly, the distinction is between analogies that are predictive and analogies that are explanatory. Brown (1989) distinguished between predictive and proportional analogies, Govier (1989) between inductive analogy (empirical) and *a priori* analogy (normative). Similar distinctions were made by Guarini et al. (2009) and Walton (2014). Bartha (2010) distinguished between predictive and explanatory analogies, and Alvargonzález (2020) between exploratory and analytical. I will use a distinction roughly

corresponding to these, between what I will call perceptual and conceptual analogies, in line with the two sides of the adaptive process:



Perceptual analogies

Perceptual analogies compare a thing we already know or are more familiar with to another thing perceptually similar to it but not yet known or less familiar. For example, we might make an analogy between different species of cow. The invariance between the things compared is thus perceptual similarity, and the comparison is empirical, concerned with *actual experience*. Perceptual analogies often have a classificatory or predictive function. We perceive something new but similar to something we already know and predict something similar will happen as a result. We categorize new things we encounter in experience in terms of what we already know, i.e., based on our established category structures. In other words, this corresponds to the comparison underlying induction as described in the first reason above, and to common, categorical connections in our category structures used for incrementally extending already established categories.

Conceptual analogies

Conceptual analogies compare things we already know but that are considered quite different. For example, we might make an analogy between the concepts of knowing and seeing. The invariance between the things compared is conceptual rather than perceptual similarity. Conceptual analogies have a creative and often explanatory function, seeking new and creative connections between known but as yet unrelated things, explaining one thing in terms of another. As they suggest new explanations by recombining known things, they are a source of new *hypotheses* to consider for our predictive model of the world. One thing might be more familiar or simpler than the other, hence the explanatory value of the analogy, but the connection is made conceptually between already known things. In other words, they correspond to making new connections in our category structures as described in the second reason above. Thus they have the potential to create new categories by recombining already known ones.

As we will see, like perception and conception, the two types of analogy are not always easily separable, and often there is a mix of both involved. They correspond to a distinction made by Aristotle, as we will see in Chapter 5. I say 'rough correspondence' to the predictive vs. explanatory and empirical vs. normative distinctions in the literature since I am taking especially perceptual analogies in a broader sense. I want to emphasize the continuum Turner (1988) pointed out, between what are commonly considered 'analogies' on the one hand and regular 'comparisons' on the other, as not a difference in kind but in the degree of entrenchment in our category structures, as we saw above. For this reason, like Turner (1988), I take

all comparisons and connections in the category structures as analogical in nature, even if many are not regarded as novel anymore. Thus perceptual analogies also cover such well-established and common comparisons.

The two main types of analogy thus bring all of the above discussion together and correspond to the two sides of the adaptive process. Putting this all together, we can naturally represent the adaptive process (with our memory as the place of our category structures) as in figure 2.4.

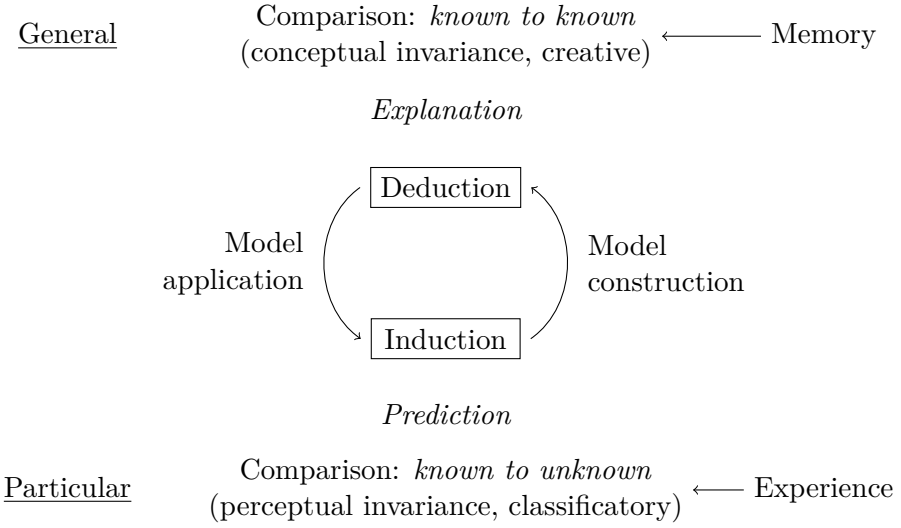


Figure 2.4: The adaptive process

We have seen the ways in which the two aspects of the adaptive process interact. The bottom-up inductive aspect, relying on perceptual analogies, generalizes from particulars in experience to general notions, encoded in the category structures. The established category structures can be seen as an explanatory model of the world. The top-down deductive aspect then applies those generalizations, i.e., common connections in the category structures, to new particulars in experience for prediction. In addition, conceptual analogies make new connections in the category structures, suggesting new categories by recombining already known ones in different ways. These new connections can be seen as hypotheses to be tested against new experience. As possible new categories predictively applied to new experience, they in turn direct attention and determine what next to consider as input for the bottom-up inductive aspect and how to categorize it. As more support is found in experience for a new connection, the connection is made more often and it becomes gradually more entrenched as a new category.

The overall structure of the adaptive process is nothing novel. It mirrors the well-known interaction of deductive theory and inductive experiment of the scientific method. This is not a coincidence, as science can, in fact, be

seen as a more rigorous and formalized, collaborative extension of our natural explorative behavior captured by this process (as we will see in section 6.3.2). Conversely, our familiarity with the scientific method provides a useful, familiar model by analogy to which we can understand some of the more complex cognitive processes that presumably gave rise to it. Beyond this, though, I want to argue that the logical notions of induction and deduction are better understood in this interconnected way as two aspects of a single process, rather than in isolation as they are often considered in logic. Moreover, that analogical comparison, of two main varieties, can be seen as an elementary operation underlying this process by allowing us to perceive and construct invariances. As analogies at the conceptual level create hypotheses to test through comparisons at the perceptual level, this process also accords well with the understanding of the brain as continually generating hypotheses about the world, as described in section 2.1.

As we will see in sections 6.3.1 and 6.3.2, analogical arguments have also been argued to play a mediating role between conservative and progressive social values in case law, as well as between conservative and progressive epistemic values in science. They mediate between things staying the same (invariant) and things changing (varying), between the familiar and the unfamiliar, between the known and the unknown, between the certain and the uncertain, between stasis and novelty. Since deduction is concerned with certainty and forms the basis for classical logic, which is static in nature, aiming to keep things the same, while induction is concerned with considering continually-changing experience, to find things staying the same, this provides another reason for thinking that analogies can have a more fundamental mediating role between the opposing tendencies of induction and deduction.

To summarize, I have considered three interconnected themes in this chapter:

1. Our finiteness explains the need for prediction based on incomplete information and consequently a source of error in our judgments. It also reminds us that any invariances we are familiar with, making up our explanatory and predictive model of the world, are based on a finite amount of experience.

2. The notion of invariance, of things staying the same or keeping things the same, underlies all of the sense-making in which we are engaged: from categorization to language to argumentation and ultimately logic.

3. An adaptive process, involving the opposing tendencies of induction and deduction, allows us to construct and apply an explanatory and predictive model of the world based on such invariances. Analogical comparison, allowing us to perceive and establish new invariances, can be seen as a mechanism underlying both sides of this process.

In the following three chapters, I will look at the early development of logical thinking in ancient China, ancient India, and ancient Greece through the lens of these concepts. Each of the main concepts introduced in this chapter can be identified in these three traditions, but for each, I want to highlight a specific aspect in particular. In Chapter 3, I will consider the Mohist Canons from the ancient Chinese tradition, with a particular focus on categorization. In Chapter 4, I will consider the Nyāya Sūtras from the ancient Indian tradition, with a particular focus on abstract invariance for inference. In Chapter 5, I will consider a few texts from Plato and Aristotle in the ancient Greek tradition, with a particular focus on the use of analogies in making arguments. Necessarily, the discussion of even single texts from these traditions will be limited given the scope of this thesis. The point is not to provide an exposition of these works but to focus only on the relevant parts from the perspective of the model introduced in this chapter. Finiteness will play a less prominent role in these chapters (although it will be implicit in various kinds of fallibility that will be discussed), but I will return to its importance in Chapter 6. In each tradition, categorization, language, and argumentation played a prominent role in their logical thinking. Most importantly, I will attempt to show that aspects from both sides of the adaptive process can be identified in each of these traditions.

Chapter 3

Mohist Canons in Ancient China

Analogical reasoning played a principal role in ancient Chinese philosophy. Many scholars have noted that analogical arguments seem to have been the characteristic way of arguing in ancient China (Xie, 2019). The most significant contributions to this were made by the Mohist school, founded by Master *Mòzǐ* (墨子, c. 400s–300s BCE) during the Hundred Schools of Thought (諸子百家) period. Fraser (2020c) notes that, like Socrates in the Greek tradition, they were the first in the Chinese tradition to look for objective moral standards and give well-reasoned arguments for their views, pioneering philosophical argumentation and debate. Their movement flourished during the Warring States era (479–221 BCE), the last period before the first unified Chinese empire, the Qin dynasty. "Mozi" also refers to the works written by members of the Mohist school. Six chapters of the Mozi are part of a later branch of the school, often referred to as the "later Mohist." Two of these form the *Canons* (*jīng* 經), a set of brief statements on many philosophical and other topics, along with two further chapters, the *Explanations* (*jīng shuō* 經說), providing commentary on each canon in a slightly longer form. They are followed by what are usually called the *Greater Selection* (*dà qǔ* 大取) and the *Lesser Selection* (*xiǎo qǔ* 小取), containing what appear to be fragments of essays (Fraser, 2020a). The writings of the later Mohists are considered among the "most important texts in the history of Chinese ethics, philosophy of language, epistemology, logic, and science" (Fraser, 2020a), yet they are also among the most obscure and difficult to interpret because of their terse and technical style and incomplete preservation.[7] I will focus mainly on these later Mohist writings, as they explicitly investigate analogical argumentation as an early form of making explicitly reasoned arguments. I will approach the subjects discussed from the levels of invariance and the two aspects of the adaptive process, starting with the argumentative context in which the Mohists developed their views.

3.1 Disputation

A major interest of pre-Qin philosophical thinkers in China was the knowledge and correct use of "names" (*míng* 名), to make words correspond to reality, or "rectifying names" (*zhèng míng* 正名) as Confucius called it (Fraser, 2020a). This was also an important theme for the later Mohists. Therefore they are sometimes considered part of the School of Names (*míng jiā* 名家), a broad label applied to thinkers from various schools of the Warring States era sharing an interest in language, disputation, and metaphysics. However, the later Mohists were critical of many other thinkers to whom this label is applied (Fraser, 2020d). Thinkers falling under this label were considered "disputers" because they disputed over the drawing of particular distinctions. According to Fraser, disputation (*biàn* 辯) was understood in various ways. In its broadest meaning, it is comparable to the Greek notion of dialectics (as we will see in Chapter 5), as "debate or reasoning aimed at knowledge" (Fraser, 2020a). In a more specific sense, it meant the evaluation of arguments over whether something is (*shì* 是) or is not (*fēi* 非) of a certain kind. As the later Mohists formulated it:

> As to disputation, by it we clarify the divisions between what is this and not and judge the guidelines of order and disorder; clarify places of sameness and difference and examine the patterns of names and objects; settle benefit and harm and resolve doubt. Only then can we lay out what is so of the myriad things and sort out parallels in groups of statements. (45.1a)[8]

The term *biàn* (辯) is therefore often translated as either "dialectics," "distinction drawing," "discriminating," or "disputation" (Fraser, 2020a). The Mohists identified four objects of knowledge: "Names, objects, matching, acting." (A80), which they respectively explained as: "That by which we say/assert is names. What we say/assert about is objects. Names and objects mating is matching. Proceeding with intent is acting." The notion of name refers to any type of word, all speech consists of names: "When sounds are uttered by speakers, they all consist of names" (A78). Objects or 'things' (*shí* 實) was understood broadly as well, including physical objects, events, and situations (Fraser, 2020a). Their aim was the proper matching (*hé* 合) of names with things. Here we see the applying of discrete labels (names) to things at the level of invariance of natural language. The later Mohists distinguished three types of names (A78): all-reaching names (*dá míng* 達名), kind names (*lèi míng* 類名), and personal names (*sī míng* 私名), also translated unrestricted, classifying, and private names, respectively (Liu and Zhang, 2010). The explanation corresponding to the canon mentions an example for each type of name:

> 'Thing' is an all-reaching name. Any object must take this name.

> Naming it 'horse' is a kind name. For what is similar to the object, one must use this name. Naming him 'John' is a personal name. This name stops in this object. (A78)

In other words, all-reaching names are generic and can refer to anything. Kind names apply a label to a group of similar things, what the Mohists called a kind (*lèi* 類). Personal names refer to one thing, closer to what we understand by "name" in English, like the name of an individual. It has been argued that these types of names point to a conception of quantification, in their extension (Liu and Zhang, 2010). All-reaching names correspond to universal quantification, as they apply to everything. Personal names correspond to particulars or existential quantification, applying to only a single thing. While kind names correspond to predicates, having as their extension multiple (but not all) things. The Mohists based most of their argumentation on kind names, on correct distinction drawing (*biàn* 辯), i.e., on categorization, which I will discuss next.

3.2 Categorization

The argumentation of the later Mohists was almost exclusively focused on kinds, on drawing correct distinctions between things and grouping similar things together under kind names. Their reasoning can therefore be called a kind-based reasoning (Liu and Zhang, 2010). To determine whether something fell within a particular kind, they compared it with a typical example of that kind, what they called a "standard" or "model" (*fa* 法).

The specific notion of a model that the later Mohists used originated with the early Mohists' focus on ethics. One of the main objectives of the early Mohists was to find universal and objective moral standards (Fraser, 2020c). They opposed the dominant Confucian view that focused on the importance of community-level rituals and tradition as determining what is right and what is wrong, as they observed that rituals and traditions of different communities could be incompatible with each other. The later Mohists extended this use of standards or models into all the other areas they investigated, including language, argumentation, and logic. These models could be very diverse things. Fraser (2020c) summarizes this as follows:

> The term *fa* denotes any guide, aid, or tool for following a norm or making a judgment. Explicit rules, laws, and definitions are all *fa*. But *fa* may also be role models, such as a virtuous parent, teacher, or ruler. They may be prototypes, exemplars, analogies, or pictures of some kind of thing, or they may be tools or measuring devices, such as a yardstick or a carpenter's compass and square. They may even be concepts, such as "the benefit (*lì*) of the world," which the Mohists take to be co-extensive with and

> thus a criterion for identifying what is *yi* (morally righteous). In short, any criterion or paradigm that helps us to make correct judgments or to act correctly can be a *fa*.

These models were taken to be exemplary instances of their kinds. They were compared to new instances, considering their similarities (*tóng* 同) and differences (yì 異), to determine whether they should be considered part of the kind. Something was categorized as being of a certain kind because some of its inherent characteristics were like the model for that kind. Their reasoning was primarily focused on this type of comparison and on "extending kinds" (*tuī lèi* 推類).

The Mohist notion of a model, to compare new instances to for membership of a kind, is in this way similar to the notion of a prototype in its role of determining membership of a natural category, as discussed in section 2.2.1. Moreover, their focus on matching names with things, and kind names in particular, is similar to applying explicit labels (words) to natural categories at the level of natural language, as discussed in section 2.2.2. Another similarity is that kinds, like natural categories, arise directly from experience with the world and are fixed by it to a certain extent. It has been argued that the flexibility and lack of restrictions on analogies used for extending kinds made the determination of kinds somewhat arbitrary, depending on the particular needs of an argument (Xie, 2019). Fraser (2020a), however, argues that kinds were not in any way arbitrary for the Mohists. They believed things in the world had a natural ordering, hence that there was a natural categorization of those things. According to the Mohists, kinds are fixed by nature to some extent, and our task is to carefully draw the correct distinctions and match the right words with their corresponding things. As we have seen in section 2.2.1, natural categories are not arbitrary either but based on perceptual invariances and natural 'cuts' in our sensory experience of reality. There is also a parallel here to Plato's use of models to determine kinds, as we will see in Chapter 5.

One of the ways in which kinds differ from natural categories is that they did not conceive of kinds as having degrees of membership. Mohist distinction drawing was based on binary choices, between 'this' (*shì* 是) or 'not' (*fēi* 非), 'so' or 'not so,' and so a certain thing was considered either an instance or not an instance of a particular kind. This all-or-nothing membership can be attributed to the fact that their concern was matching names with things, at the level of language with its discrete words, and for the purpose of argumentation. The important thing is that, unlike Aristotelian categories, they did not consider definite criteria for membership of a kind or provide an exhaustive list of features necessary for a match. Instead, membership was determined by similarity to a model, a prototypical instance of a kind.

More generally, we can identify here the bottom-up, inductive part of the adaptive process. The notion of extending kinds based on the process

of comparing models to new instances can be seen as the use of perceptual analogies from a known instance (the model) to an unknown, newly encountered instance to be categorized. But prediction comes with a risk of error because it involves new experience and goes beyond what is already known (what Descartes described as the 'will' going beyond the scope of the finite 'understanding,' as we saw in section 2.1). We see this too in the case of the Mohists. The later Mohists considered many ways in which extending kinds can go wrong, distinctions are drawn incorrectly, or arguments are invalid. Some of these have to do with the supposed scope of a kind name not matching the actual scope of the name. As we will see in the next chapter (section 4.2), there is a parallel here to the notion of 'false knowledge' in the Nyāya Sūtras in the Indian tradition. There are many other potential sources of error discussed in the Mohist Canons. They identified different sources of doubt, such as the unreliability of perception in certain circumstances, like in a fog mistaking a person for an ox (B10). A thing falling under a kind of a larger scope does not imply it falls under a kind of a smaller scope ("If a living thing, is it a bird?" B2). There has to be consistency in the use of kind names: with arbitrary usage the meaning becomes unclear (which they called "wild mentioning/citing," B66). And when one consistently uses a different kind name than what is typically used, it leads to confusion and contradiction (this they called "borrowing names," B8).

The upper part of the adaptive process can also be seen to be present in the Mohists' theorizing, in their notion of explanation (*shuō* 說). While the comparisons of models to new instances in many of their examples are based on visual similarities, i.e., perceptual invariances, the Mohists also identified abstract invariances as a basis for making arguments and acquiring knowledge. The later Mohists identified three sources of knowledge: "Knowing. By hearsay, by explanation, personal" (A80). There are parallels here with the means of acquiring knowledge in the Nyāya Sūtras in the Indian tradition, as we will see in the next chapter (section 4.2). When we receive information from others through language, we know it by hearsay (*wén* 聞); we might also call this testimony. When we observe something ourselves, we know it personally (*qīn* 親); this gives us the ability to extend kinds based on perceptual invariances, as we have seen. When we know something by explanation, we arrive at a conclusion without directly observing it ourselves and without learning about it by hearsay. Instead it is arrived at through reasoning or inference. The Mohists defined explanation as "that by which one clarifies/understands" (A72) and regarded it as a basis for arguments: "By explanations, present reasons" (45.1b). Fraser (2020b) notes that explanations are "the analogue of what we would call an argument, a justification, or a piece of reasoning." We see this in the structure of the Mohist Canons itself, with an "explanation" corresponding to each canon, providing reasons and further elaboration for what is stated in the canon. If we look at what these explanations are like, we see that

they often consist in giving concrete examples of the thing discussed in the canon, many even making an explicit analogy. Take, for example, canon A5: "Knowing is connecting." The corresponding explanation states: "Knowing is, by means of the knowing, passing something and being able to describe its features. Like seeing." This is part of a series of canons comparing knowledge to eyesight.[9] Here we can see the use of conceptual analogies, making new connections between already known things to explain one thing in terms of another, corresponding to the top part of the adaptive process. Such analogies were used to provide reasons, in this case, for the argument that knowing is connecting. The Mohist still considered this a form of applying a model to a new instance. In fact, as Fraser (2020c) notes, they considered all forms of reasoning to be a form of analogical reasoning, including deductive inference: "in their eyes the process of deducing a conclusion from a general rule or principle is in effect a special case of the more general cognitive process of comparing things to models and drawing distinctions."

In line with the above-discussed uses of models for extending kinds and explanations, Fraser (2020a) has noted that models served either an action-guiding role or a justificatory role for the Mohists. On the one hand, they served as "guidelines or decision criteria that direct action and practical reasoning" (Fraser, 2020a). On the other, they provided "fundamental standards of correctness by which to justify actions and judgments" (Fraser, 2020a). This twin role again corresponds to the predictive and explanatory parts of the adaptive process and the types of analogies involved: perceptual (action-guiding) analogies having a predictive function and conceptual (justificatory) analogies having an explanatory function.

Corresponding also to both sides of the adaptive process is the distinction the Mohists made between actual and hypothetical (or fictional) things. They noted the difference between what is so in the present or was so in the past but might not be so in the future (B16), the difference between things being absent because they once were but no longer are, and things being absent because they never were (B49), as well as the distinction we already saw between personal knowledge and knowledge by explanation. They recognized the need for both perceptual and abstract knowledge, and the role of memory: "If one could know only by means of the five routes [the senses], then after a duration knowledge would cease to fit the facts" (B46).

We have seen some of the ways in which categorization (kinds), language (names), and argumentation (disputation) played a role in the theorizing of the Mohists, corresponding to the levels of invariance discussed in section 2.2. What about logic? There are many logical notions to be found in the Mohist Canons. The Mohists frequently used the words "permissible" or "admissible" as opposed to the "impermissible," "perverse," "confused," or "contradictory," to indicate the validity or invalidity of arguments and to clarify logical paradoxes (Fraser, 2020a, p. 37). They used "this" and "that" to indicate contrasting things, showing implicit use of the law of

non-contradiction (Fraser, 2020b, p. 23). Even though they did not develop a symbolic language, they seemed to have used certain words as variables (Liu and Zhang, 2010; Xie, 2019). For example, some kind names such as "dog," "ox," and "horse," are used frequently as generic kind names. And the relations between "dog" and "hound" (same kind but different names) and "ox" and "horse" (different kinds) are taken as generic for different relations between kind names. They also applied negation (e.g., "non-ox," A74) and conjunction (B11), and gave conditional or counterfactual examples (45.1c; Fraser, 2020b, p. 78). As Fraser (2020b) notes, in multiple instances it is clear that they applied the law of excluded middle (A51, A74, B35). They also seem to have had a notion of quantification, as we already saw reflected in the different types of names in section 3.1, as well as in other cases (e.g., "Some is not all," 45.1c).

Most importantly, considering logic from the perspective of invariance, the Mohists had a notion of abstract, 'universal' invariance. The later Mohists used the terms "pervading" and "filling" to indicate the universal presence of some feature. For example, "Space is pervading different places" (A41), or "To fill/filled is nowhere not present" (A65), also: "All/-completely/covering is none are not so" (A43). Related to this is the recurring technical notion of "hard-white" (*jiān bái* 堅白), a more common theme among thinkers of the School of Names (Fraser, 2020d). It was used as a model for the notion of the complete correlation of two things or features of things, certain properties being mutually pervasive with each other. "Hard-white" referred to the "hardness and whiteness of a completely white stone" (Fraser, 2020d). We will see a similar notion in the Indian tradition in the next chapter. These notions were concerned with complete, or universal, invariances, a typical feature of logical or rationalistic thinking (section 2.2.4). We can see an example of this in the Mohist ethical doctrine of "all-inclusive care" (*jiān ài* 兼愛), also translated as "universal love," thought to be the world's earliest form of consequentialism (Fraser, 2020c). It advocated an inclusive concern for everyone. This did not mean equal care for everyone (they acknowledged the factor of the closeness of relations between people), but rather that everyone is deserving of some care, that no one should be excluded from moral consideration. All-inclusive care was an abstract, universal principle, as evident in the fact that it extended to everyone (including ourselves), not just to those people we know but even to unknown and hypothetical people. At that time, they did not know whether there was a finite number of people in the world, yet they believed that this posed no problem to the universality of their principle: "The limitless/infinite poses no obstacle to all-inclusiveness," (B73) and "Not knowing their number yet knowing they are all covered" (B74). Another aspect of this was that it applied to everyone equally, regardless of external circumstances or the length of one's life: "External circumstances cannot make our benefit to parents greater" (44.25) and "The sage acts for the sake

of the world...One long-lived, one short-lived, their benefit to the world is equal" (44.28). This type of complete and normative invariance, keeping things the same despite variations, and extending beyond the actual to any yet unknown and hypothetical people, is typical for logical thinking. I do not believe it is accidental that the school to first develop more explicit logical thinking in the ancient Chinese tradition was also the first to propose an explicit ethical theory based on universal principles, and as we saw, to search for normative, universal moral standards to overcome variations between the mutually inconsistent local rituals and traditions. And they did so through the use of models, by applying them through analogical extension to new cases, actual and hypothetical.

3.3 Argument Forms

As all of the theorizing of the Mohists took place in disputation with other schools of thought and situated in a culture of debate, I will briefly consider the argument forms they identified and used.

In the *Lesser Selection*, the later Mohists listed four general reason patterns they identified: illustrating (*pì* 辟), paralleling (*móu* 侔), pulling (*yuán* 援), and pushing (*tuī* 推). These were not unique to the Mohists; they were used by other thinkers in ancient China around that time. However, the Mohists explicitly documented and investigated them. They were also aware of potential weaknesses of these types of arguments:

> expressions in analogies, paralleling, pulling, and pushing become different as they proceed, become dangerous as they change direction, fail when taken too far, and leave their roots as they flow, and so one cannot fail to be cautious and cannot invariably use them. So statements have many methods, separate kinds have different reasons/causes, and so one cannot look at only one side. (45.1e)

All of these argument forms are instances of analogical argumentation based on the extension of kinds. They reflect a focus on "fair or unbiased analogical persuasion" (Fraser, 2020a): "Having it oneself, one doesn't condemn it in others. Lacking it oneself, one doesn't seek it in others" (45.1b). I will discuss each argument form briefly, together with the type of invariance underlying it and a corresponding weakness the Mohists identified.

Illustrating (*pì* 辟). The Mohists defined this argument form as "mentioning other things and using them to clarify it" (45.1d). This is the most typical argument by analogy. An analogy is drawn between a more familiar or easily understood thing and another thing, to clarify or establish a claim about the latter. Illustrating involves citing a model which is considered to have certain relevant similarities to the new thing being considered and

classifying it as part of the kind for which the model is used. The invariance on which this type of argument is based is the similarities between things (Fraser, 2020b). A weakness the Mohists identified was that different things are necessarily only partially similar and might be critically different: "Things have respects in which they are the same, yet it doesn't follow that they are completely the same" (45.1e).

Paralleling (*móu* 侔). The Mohists defined this as "placing expressions side by side and jointly proceeding" (45.1d). Fraser argues that this pattern is the most unfamiliar and vague, and there are no clear analogues to it in other logical traditions. A simple interpretation offered by Fraser (2020a) is that it "refers very generally to presenting a series of syntactically parallel utterances, one or more of which are used to argue by analogy that one or more others should be accepted." Hence the invariance on which this type of argument is based can be understood as the syntactic similarities between expressions (Fraser, 2020b).[10] According to Fraser, they were most cautious about this type of argument. A weakness the Mohists identified was that syntactically similar expressions do not always have a similar meaning: "Parallels between expressions are correct only up to a point" (45.1e).

Pulling (*yuán* 援). The Mohists defined this as "saying, 'You are so, how is it that I alone cannot be so?'" (45.1d). The disputant argues that the claim being argued for is of the same kind as a claim previously accepted by the opponent. The invariance on which this type of argument is based is the similarities between the commitments made by the participants in a disputation (Fraser, 2020b). A weakness the Mohists identified was that the grounds on which the claims are made might be critically different: "When things are so, there is that by which they are so. Their being so is the same, but that by which they are so isn't necessarily the same" (45.1e).

Pushing (*tuī* 推). The Mohists defined this as "on the basis that what they don't accept is the same as what they do accept, proposing it" (45.1d). The disputant argues that a claim not yet accepted by the opponent is of the same kind as a claim already accepted by the opponent, and therefore should also be accepted. The invariance on which this type of argument is based is the same as for "pulling." A weakness the Mohists identified was that the reasons for accepting the compared expressions might be critically different: "When people accept things, there is that by which they accept them. Their accepting them is the same, but that by which they accept them isn't necessarily the same" (45.1e).

Each of these argument forms can thus be seen as being based on a certain invariance (either perceptual, syntactical, or interpersonal) and establishing a new invariance between the things being compared, i.e., by extending a kind. Pulling and pushing furthermore make an explicit appeal to consistency among the disputants, an essential aspect at the level of invariance of argumentation, as we saw in section 2.2.3.

3.4 Conclusion

The later Mohists' theorizing on argumentation as far as covered in this chapter is neatly summarized in the following canon:

> By names, mention objects. By expressions, put across thoughts. By explanations, present reasons. Select and propose on the basis of kinds. Having it oneself, one doesn't condemn it in others. Lacking it oneself, one doesn't seek it in others. (45.1b)

On the level of categories, we have seen that the Mohist notion of extending kinds through comparison of new instances to models corresponds to the extending of natural categories through comparison of new instances to prototypes. We have seen that kinds are central to all the Mohists' reasoning. They arise in the first instance from experience, i.e., from what the Mohists called personal knowledge.

On the level of language, we have seen that the Mohists' concern was with correct naming, the matching of names with things, especially for kind names. This corresponds to the applying of explicit labels to natural categories. We have seen that the Mohists identified numerous sources of error in extending kinds and applying the correct names to them. Through communication, we also have the ability to acquire knowledge by hearsay.

On the level of argumentation, we have seen that the Mohists' arguing—disputation—amounted to the drawing of distinctions and the extending of kinds: determining whether something is or is not of a certain kind. We have also seen explanations as giving reasons, like an argument or inference, providing knowledge by explanation. Furthermore, we have seen that the four argument types the Mohists identified are based on various types of invariances, two of which directly appeal to consistency by the disputants, an important condition for effective argumentation (section 2.2.3).

On the level of logic, we have seen that the Mohists used many logical notions, all in relation to kinds. Most importantly, the notion of complete invariance in abstraction, as exemplified in the doctrine of all-inclusive care extending equally to all people, actual and hypothetical, as well as the early Mohists' search for objective moral standards to overcome the inconsistencies between varying local rituals and traditions.

As for the adaptive process, I identified the comparison to models for extending kinds as perceptual analogies and the bottom-up, inductive part of the process. While explanations, a form of giving reasons or making arguments, and often relying on concrete examples and analogies, I identified as corresponding to conceptual analogies at the upper part of the process. We have also seen the distinctions the Mohists made between actual and hypothetical, and the action-guiding and justificatory roles of models, both of which also correspond to both sides of the adaptive process. Finally,

Top	
Conceptual analogies	Explanations: giving reasons
Hypothetical	Knowledge by explanation, indirectly deduced
Bottom	
Perceptual analogies	Extending kinds
Actual	Personal knowledge, directly perceived
Interaction	
All reasoning analogical: "general cognitive process of comparing things to models and drawing distinctions" (Fraser, 2020c)	

Figure 3.1: Correspondences to the adaptive process.

we have seen that the Mohists considered all reasoning to be analogical, including deductive reasoning, and hence that they considered comparing things to models as fundamental.

Chapter 4

Nyāya Sūtras in Ancient India

Ancient Indian philosophy knows a rich tradition of philosophical debate. The main concern was epistemological, inquiring into the sources and justification of knowledge. The development of these debates over time has been described as a gradual arising of awareness and crystallization of the characteristics of sound reasoning (Matilal, 1998). The informal argumentation patterns used in debates gradually developed into theories of inference. Because of their origins in the context of debate, theories of inference were often intertwined with theories of argumentation and how to present an argument persuasively. Among the earliest written sources on these subjects were debate manuals from various schools of thought, used for instructing students in debate techniques. The Nyāya Sūtras ("aphorisms on logic"), attributed to Akṣapāda Gautama (c. 200 BCE), is one of the more systematic and influential of these manuals. It is the foundational text of the Nyāya school ("school of logic"), and much like the Mohist Canons of the Mohist school in ancient China, considered among the earliest texts in ancient India concerned with explicit theorizing on logical argumentation.[11] The main topic for this chapter will be the Nyāya Sūtras. I will occasionally refer to the broader context of Indian philosophical thought and developments beyond this particular text. However, since the rich Indian tradition of commentaries, and commentaries on commentaries, and so on, is incredibly complex and expansive, I will for the most part restrict focus to the original text. As we will see, the theorizing of the Nyāya was analogical in nature. Like with the Mohist Canons, I will approach the subjects discussed from the perspective of the model introduced in Chapter 2. I will start with the argumentative context of debate. The main part of the chapter, the section on invariance, will focus on the four means of acquiring knowledge identified by the Nyāya, and the invariance underlying one of them: inference, the discussion of which will be extended into the section on argument forms.

4.1 Debate

Debates were highly structured events in ancient India. Philosophers from different schools were pitched against one another in the presence of a jury evaluating the debate and a chairman who organized the event, and strict rules had to be obeyed (Matilal, 1998). Debate manuals offered theories of evidence and argumentation but also listed so-called "defeat situations," conditions under which one was considered defeated by an opponent, as well as fallacies and rhetorical tricks. The Nyāya Sūtras identifies three types of debate (I.II.1–3): honest debate or "discussion" (vāda), where the aim is establishing a truth, tricky debate or "wrangling" (jalpa), where debaters tried to win through any means necessary, including using rhetorical tricks, and destructive debate or "cavil" (vitaṇḍā), where the sole aim was to defeat an opponent's thesis, without putting forward a counter-thesis. They warned against participating in the latter two types of debate (IV.II.116). Discussion (vāda) is defined as: "adoption of one of two opposing sides. What is adopted is analysed in the form of five members, and defended by the aid of any of the means of right knowledge, while its opposite is assailed by confutation, without deviation from the established tenets" (I.II.1).[12] Like the Mohists' notion of disputation, discussion is thus concerned with a competition between opposing standpoints, establishing whether something is or is not the case. The "five members" is the Nyāya's formulation of the structure of a proper argument, which I will discuss in section 4.3.

Many debates in ancient India were concerned with the justification and sources of knowledge ("the means of right knowledge," pramāṇas). This led to a structured analysis of cognition and the logical connections between cognitive events or states (Bajaj, 2011). Bajaj notes that the Nyāya regarded a sentence in the natural language Sanskrit as the expression of a "thought" or cognitive state (jñāna). Every such cognition they considered to have a content-ness (viṣayatā), a logical structure usually not fully expressed by the Sanskrit sentence. They thought of inference in terms of such cognitive states expressed through Sanskrit sentences. Early Indian philosophers did not develop a symbolic language; like the Mohists, all their reasoning and theorizing were done in natural language. However, they developed more and more technical devices in their language use. This culminated in a rigorous technical language based on Sanskrit[13] in the work of Gāṇgeśa Upādhyāya (c. 1325 CE) of the Navya-Nyāya ("new logic") school, a later school of thought building on the foundations of the Nyāya school as well as the Vaiśeṣika school (Bajaj, 2011). In this way, Indian logic and theorizing were shaped by the study of language, allowing Indian philosophers to inherit to some degree the natural structure and interpretation of Sanskrit and its sensitivity to context, while revealing the implicit logical structure and avoiding inexactness and many of the ambiguities inherent in natural language. By building on the invariances underlying natural language, in-

stead of constructing a formal language from scratch, the Indian approach is "largely free from the philosophical and foundational problems inherent to the formal methodologies developed in the Western tradition" (Bajaj, 2011).

Since the primary concern of early Indian philosophers was epistemological, I now want to consider the sources of knowledge examined in the Nyāya Sūtras, as well as sources of errors in judgment, and the central notion of invariance on which inference (as a source of knowledge) rested.

4.2 Invariance

The Nyāya Sūtras identifies four means for acquiring 'right knowledge' (I.I.3): perception, inference, comparison, and verbal testimony. Other schools of thought identified less or more, from only perception (Cârvakas), to perception and inference (Vaiśesikas and Buddhas), to perception, inference, and verbal testimony (Sânkhyas), or more (Vidyābhuṣana, 1913).[14] I will consider each of the four types of knowledge the Nyāya identified in turn. Two of these will correspond to the two parts of the adaptive process, and a third will partly correspond to both.

Perception (pratyaksha) is defined in the Nyāya Sūtras as knowledge arising from the contact of the five senses ("eye, ear, nose, tongue and skin," III.I.58) with their corresponding objects ("colour, sound, smell (odour), taste (savour) and touch," III.I.58). This is comparable to the Mohists' notion of personal knowledge. Perception was considered the primary source of knowledge more generally among ancient Indian philosophers, prior to all other types of knowledge, hence also why it was considered a means of knowledge by all schools. The Nyāya considered perceptual knowledge to be always true (IV.II.105).

Inference (anumana) was regarded as going from a perceived thing to an unperceived one (II.I.108). It is of one of three kinds (I.I.5): *a priori* ("knowledge of effect derived from the perception of its cause"), *a posteriori* ("knowledge of cause derived from the perception of its effect"), and 'commonly seen' ("knowledge of one thing derived from the perception of another thing with which it is commonly seen"). These correspond to inferring something that *will be* (future), something that *has been* (past), and something that *is* (present), respectively. The Nyāya Sūtras considers an objection that inference is not a means of right knowledge because it can lead to errors in judgment in certain cases. For each of the three types of inference there might be another reason, another explanation, for the thing being perceived. For example, from observing ants carrying off their eggs, we might infer that it will rain, when in reality their nest is damaged. In response, the Nyāya Sūtras acknowledges that inference can lead to error, but that this does not make inference an invalid means of knowledge, noting that: "If in such cases any wrong inference is drawn, the fault is in the person, not

in the process" (II.I.99). In the case of a damaged nest, the ants will walk chaotically, whereas in anticipation of rain, they will walk orderly. Hence the specific inference is not applicable because the antecedent of the inference is not similar to the perceived thing. We can see that this corresponds to the top-down part of the adaptive process, applying a fallible explanatory model to something newly observed, with the inevitable possibility of making an error, of there being another explanation for the thing observed. We will see more about the invariance underlying inference below and its use in arguments during debates in section 4.3.

Comparison (upamana) is defined in the Nyāya Sūtras as "knowledge of a thing through its similarity to another thing previously well known" (I.I.6). An objection is considered that there is no precise standard for comparison, to which the Nyāya Sūtras replies that comparison is based on similarity in a high degree between well-known things (II.I.105–106). We might perhaps think of comparison and inference together as a rough analogue to the Mohists' notion of knowledge by explanation. Two differences with inference are noted. First, comparison is "knowledge of a *perceived* thing through its similarity to another thing also *perceived*," while inference is "knowledge of an *unperceived* thing through that of a thing *perceived*" (II.I.108). Second, comparison "is established through the compendious expression "so." 'As is a cow, *so* is a bos gavaeus'—this is an instance of comparison" (II.I.109, bos gavaeus is an Indian species of bison). So unlike inference, comparison is based on a notion of similarity. Hence comparison is based on perceptual similarity between perceived things. They also used the expression "not so" to indicate a lack of relevant similarity between compared things. There is a parallel here to the Mohists' distinction drawing, who also used the expressions "so" and "not-so" to indicate whether something was relevantly similar and fell under a certain kind. Another parallel is the recurring of certain words as examples or 'variables' standing in for any kind of category, such as "dog vs. hound" (Mohists) and "cow vs. bos gavaeus" (Nyāya) for similar categories with different names, and "ox vs. horse" (Mohists) and "cow vs. horse" (Nyāya) for different categories. The Nyāya Sūtras argues that comparison needs to be a distinct means of knowledge from perception for words to be able to signify general notions or genera: "That the name *bos gavaeus* signifies one and all members of the *bos gavaeus* class is not a result of perception but the consequence of a distinct knowledge called comparison" (I.I.6). We can thus see that comparison corresponds to the bottom part of the adaptive process, making perceptual analogies based on a high degree of similarity between perceived things, and a basis for the bottom-up, inductive tendency to derive general notions from experience.

Verbal testimony (śhabda) is defined as "instructive assertion of a reliable person" (I.I.7). This is similar to the Mohists' notion of knowledge by hearsay, but the Nyāya distinguished two kinds: words referring to matter which is seen, and words referring to matter which is not seen. The first

can be actually verified, while the second is not actually verifiable, but "we can somehow ascertain it by means of inference" (I.I.7). This distinction between seen and unseen directly relates to the two parts of the adaptive process, expressed at the level of natural language. The bottom, inductive part, concerned with experience, is verifiable, while the top, conceptual part, is concerned with inference in abstraction and not directly verifiable in experience. Even though verbal testimony and inference both reveal something not perceived, they are argued to be different. In verbal testimony, "we rely on unseen matter not simply because it is signified by words but because they are spoken by a reliable person" (II.I.113). They argue that while the "connection between a sign and the thing signified, which is the basis of inference, is obvious to perception" (this connection will be discussed in the second half of this section), the "connection between a word and the objects signified by it, which is the basis of verbal testimony, is not obvious to perception" (II.I.114). In fact, the Nyāya held that there was no natural connection between a word and an object, as they believed the meaning of words to be understood through convention (II.I.115–117). In addition, unlike the type of connection on which inference is based, there is no uniform connection between words and their meaning, as they can be used in different senses (II.I.118). Here we see a contrast between descriptive and argumentative language use. Unlike a reasoned argument or inference stating the inference steps involved to get to the conclusion, verbal testimony is merely giving a description of something. Therefore, the invariance on which it is based is the reliability of the speaker, i.e., on how often that person has been accurate on past occasions, a kind of interpersonal invariance over time.

Naturally, besides the means for acquiring right knowledge, the Nyāya Sūtras, like the Mohist Canons, also considered sources of doubt and error in judgments and knowledge. Doubt is defined as "conflicting judgment about the precise character of an object" (I.I.23). This could be due to similarities between different things, such as being unable to determine whether a tall object in the twilight is a man or a post. We saw a similar example in the case of the Mohists: mistaking a person for an ox in a fog (section 3.2). Doubt could also arise due to lack of similarity (such as being unable to determine whether sound is eternal because it is not comparable to either eternal or non-eternal things), from conflicting testimony, irregularities in perception, or non-perception. Similar to some of the difficulties of extending kinds considered by the Mohists, the Nyāya Sūtras also notes that comparison necessarily involves differences as well as similarities: "A cow possesses some characters in common with a *bos gavaeus* but there is no complete identity between them" (V.I.5). There is, in a way, no avoidance of doubt, indeed: "Examination should be made of each case where there is room for doubt" (II.I.68). When it comes to false knowledge, the Nyāya Sūtras makes a distinction between the essence and the appearance of an object. Our knowledge of something perceived is true knowledge; we can point to

something and say "that." However, if we describe it as something that it is not, by applying the wrong label to it in language (the word for a category or 'genus'), such as saying "man" when in reality it is a post (IV.II.105), then it becomes false knowledge. This is similar to the misapplication of a kind name in the Mohist tradition, as we saw in section 3.2.

For the remainder of this section, I want to consider inference in more detail and the invariance underlying it. Some context will help to better understand the Indian notion of inference. According to Matilal (1998), a basic ontological distinction underlying much of Indian logic is between *locations* (dharmin) and *properties* (dharma). Locations are understood as indicating both space and time, or they could be abstract entities. They are thought of as "possessing" certain properties. Matilal notes that the word "property" is an unfortunate translation of dharma since it has a much broader meaning (he prefers the words "locus" and "locatee," the latter meaning 'that which is being located'). For example, "the pot is blue" is understood as "the pot is blueness-possessing," but also, "there is a pot on the ground" is understood as "the ground is pot-possessing." So the same object could be considered a location in one cognition and a property of some other location in another cognition. Here we see one of the ways in which the Sanskrit language shaped Indian logic, as it is a feature of its grammar that allows for a conversion of location to property and vice-versa on a word level through the use of reciprocal substantive and possessive suffixes.[15] Matilal also provides a useful template for the type of inference that seems to be presupposed in all inference patterns of the Indian logicians (1998, p. 22). There are three components to it: the locus or *subject of the inference* (pakṣa), the *reason-property* (hetu), and the *to-be-inferred property* (sādhya). Matilal argues it is essentially a theory of substitution, where the reason-property (say "h") that is known to be a property of the subject (say "p") forces the to-be-inferred property (say "s") in its place by virtue of a logical relation ("pervaded-by") between the two properties:

$$p \text{ has } h\text{-pervaded-by-}s.$$

leads through substitution to:

$$p \text{ has } s.$$

The example Matilal gives is:

Sound has product-hood-pervaded-by-impermanence.

leads through substitution to:

Sound has impermanence.

That is to say, product-hood (the property of being produced through effort) is in some way associated with ("pervaded by") the property of impermanence. Hence, sound, because it is known to be a product, is inferred to be impermanent. The relation of *pervasion* (vyāpti) between two properties across locations—also called an "invariable," or "inference-warranting," or "concomitance" relation—is the core notion of invariance underlying inference in Indian logic. It is essentially a form of correlation, or a universal statement, saying that wherever the first property is present, the second property is present. This concept, what it is, how it is applied, and how we can know it, was an important topic of study in the Indian tradition. There is some parallel here to the Mohist notion of "pervasion" or "filling," of something being everywhere present.

As the notion of vyāpti is central to the study of inference in Indian logic, I want to briefly go beyond the Nyāya Sūtras to consider the problem of establishing such an invariance relation, as something similar will come back in the Greek tradition and Chapter 6. Dignāga (c. 480–540 CE), an influential Buddhist scholar considered to have laid the foundation for more deductive types of reasoning in the Indian tradition, considered the means of right knowledge to be of two kinds: perception and inference (corresponding to the lower and upper parts of the adaptive process). However, this led to a paradox about establishing the vyāpti on which inference is based (Gillon, 2021). The only kind of inference that he considered correct led to a conclusion about a particular instance: the subject of the inference (like that of the Nyāya). Hence there was no possibility to establish the vyāpti through inference. That meant that the only way to establish the vyāpti had to be through perception, i.e., by observing both the reason and to-be-inferred property in every single instance. However, this would include observing the to-be-inferred property in the subject of the inference, hence making inference redundant as a distinct means of knowledge. Īśvarasena, a student of Dignāga, seems to have been the first to realize this problem, and proposed as a solution that a vyāpti could simply be established on the basis of being unchallenged by any counter-examples (Gillon, 2021). Dharmakīrti (c. 600–660 CE), a student of Īśvarasena and another influential Buddhist scholar, did not believe this solution was satisfactory, as there is no guarantee that we will not encounter a counter-example in the future. Instead, he proposed that a vyāpti was either a causal relation or an identity relation, though this still did not fully resolve the problem (Gillon, 2021). The interesting thing to note here is that Dignāga's deductivist approach, allowing only deductive inference (based on a vyāpti) as a valid means of knowledge beyond perception, first gave rise to the paradox. Dignāga's student, Īśvarasena, offered an inductivist solution. Dharmakīrti, believing this solution to be insufficient, then offered his own by appealing to causal and identity relations, which correspond to both sides of the adaptive process (causality being concerned with experience and induction, and

identity being characteristic of abstract thought and deduction). The Nyāya, who also considered the more inductive type of knowledge from comparison (as well as verbal testimony) as valid means of knowledge, did not run into this problem.

Next, I want to look at the standard argument form put forth in the Nyāya Sūtras for use in debate, relying on a vyāpti but also showing its crucial combination with a concrete example.

4.3 Argument Forms

The standard argument form proposed in the Nyāya Sūtras, providing a standard for logical debate, consisted of five parts (or "limbs"): proposition, reason, example, application, and conclusion (I.I.32). The example given is:

PROPOSITION	This hill is fiery
REASON	Because it is smoky
EXAMPLE	Whatever is smoky is fiery, as a kitchen
APPLICATION	So is this hill (smoky)
CONCLUSION	Therefore this hill is fiery

In this example, "hill" is the subject location (pakṣa), "fiery" is the to-be-inferred property (sādhya), and "smoky" is the reason property (hetu). The *proposition* (pratijna) states what is to be established: that the subject location has the to-be-inferred property. The *reason* (hetu), as we have seen, is a property of the subject location that is known. The *example* (udaharana) is the most important part; here, the pervasion relation (vyāpti) is expressed, making explicit the invariant connection between the reason property and the to-be-inferred property. This relation is essentially a universal statement or rule that links the two properties across all locations and provides the invariance for the inference. In addition, an example instance of this rule is given. We will look at the role of this example next. Finally, the rule is applied to the subject location: the subject location has the reason property (*application*, upanaya), so according to the example, it also has to have the to-be-inferred property (*conclusion*, nigamana).

The third limb of the inference pattern, the example, as mentioned, is key to understanding this form of argument. The Nyāya Sūtras states that an example is a familiar instance, about which ordinary people and experts alike hold similar opinions (I.I.25). I would argue that we can see this as making sure that an argument is based on something evident and as invariant as possible across the understanding of different people, based on familiar, shared knowledge.[16] Depending on whether what is argued for is presence or absence of a property, examples are known to either possess or lack the to-be-inferred property, implying that the property is "invariably contained" or "invariable rejected" in the reason given (I.I.36–37). The inference relies

on a comparison of the example with the subject. However, as we have seen, the Nyāya Sūtras acknowledges that compared things are not identical, that there is a "difference between the subject and the example although the conclusion is drawn from a certain equality of their characters" (V.I.5), and that the "example happens to surpass the subject" (V.I.6) in certain ways. The Nyāya Sūtras considers certain types of counter-arguments that rely on pointing out additional similarities or differences to reach an opposite conclusion. However, such counter-arguments are futile because "no body can commit [these counter-arguments] if he bears in mind the equality of the subject and the example only in those characters which are warranted by the reason" (V.I.5). Here again, we see the idea of avoiding error in judgment by not going beyond what is known (cf. Descartes, section 2.1). Furthermore, given the self-evident status of the example, the characteristics that are relevant to the argument—the co-presence of the reason property and the to-be-inferred property—are considered to not stand in need of proof (V.I.6), and as unambiguously characterizing the vyāpti.

The comparison to an example as part of the inference pattern, based on similarities between a familiar, known thing and another thing, recalls the similar role of models by the Mohists in ancient China. Like the Mohist models, the way the Nyāya use examples is comparable to the role of prototypes of categories. However, in the vyāpti the Nyāya more explicitly express the specific similarities on which the comparison is based. Moreover, unlike an overall perceptual similarity, the vyāpti expresses a specific relation between two properties. Hence the comparison between the example and the subject corresponds to conceptual analogies. Since there is still a concrete example required, the Nyāya can in this regard be seen as somewhere between the Mohists in ancient China and Aristotle in ancient Greece: between the Mohists' more implicit standards or models as closer to natural categories and prototypes, and clearly delineated Aristotelian categories based on explicit necessary and sufficient conditions for membership and the use of universals in inference without reference to concrete instances.

Various reasons have been offered for why the Indian philosophers insisted on requiring concrete instances as examples. Gillon (2021) has argued, for example, that since they were concerned with persuasive arguments in debate and not with formal validity, requiring a concrete example distinct from the subject avoids allowing trivial circular arguments (using the subject as the example) from being considered 'good arguments.' Matilal (1998) has argued that it ensures that the to-be-inferred property is actually applicable to the subject, that it is a type of property that can be assigned to the kind of subject considered, by giving another, already known instance, similar to the subject, where both the reason and to-be-inferred properties are present.[17] Whatever the reasons, the fact that these arguments were based not only on an abstract, universal statement but also on the explicit inclusion of a concrete instance made them intrinsically analogical.

4.4 Conclusion

Some have argued that the development of Indian logic goes roughly from inductive towards more deductive argument forms (Oetke, 1996). This has been disputed by others, arguing instead that Indian logicians were aiming for more deductive argument forms from the start, that it was only a process of gradual refinement towards the ideal of making arguments as certain as possible (Taber, 2004). Taber reminds us that the arguments that have been written down are mostly abbreviated versions of the way they were presented in the context of an actual debate (p. 153):

> The early Indian logicians almost certainly knew that the examples they gave were not strictly conclusive; but they also almost certainly presented those particular examples because they are, on the whole, when taken in a natural way, strong arguments that approximate an ideal of reliability. The expectation that there should be some quite fixed, if not absolutely invariable, connection between *hetu* and *sādhya* seems to have been in place from the very start.

More importantly, inference in ancient Indian logic has been argued to be both inductive and deductive (Bajaj, 2011), and that Indian philosophers did not explicitly make the distinction, as Matilal (1998, p. 15) notes:

> The argument patterns studied were at best an unconscious mixture of the two processes. Yet it seemed that these mixed patterns were not very far from the way human beings across cultural boundaries would tend in fact to argue or rationally derive conclusions from the available data or evidence or premises.

On the level of categories, we have seen that the Nyāya used many similar basic categories as the Mohists. We have seen that they were based on perceptual similarities, though, like the Mohists, they regarded such similarities as potentially misleading since there is no complete identity between things. We have seen that the Nyāya considered categories (genera) to be established through the knowledge of comparison from perceived to perceived. Moreover, they considered perception as the primary means of knowledge.

On the level of language, we have seen the central role that Sanskrit grammar played in theorizing and logical thinking in the Indian tradition. Instead of constructing a formal language from scratch, a technical language derived from natural language enabled them to inherit the natural structure and context sensitivity of Sanskrit and avoid many paradoxes inherent in purely formal approaches. We have seen that the Nyāya considered the meaning of words to be conventional, based on normative, interpersonal invariances, and that false knowledge arises from applying the wrong category

Top	
Top-down, deductive	Inference, based on vyāpti
Conceptual analogies	Example
Hypothetical	Verbal testimony: unseen
Bottom	
Bottom-up, inductive	Establishing vyāpti
Perceptual analogies	Comparison
Actual	Verbal testimony: seen
Interaction	
• Inference relying on comparison • "unconscious mixture of [induction and deduction]" (Matilal, 1998)	

Figure 4.1: Correspondences to the adaptive process.

label to a perceived thing. Furthermore, we have seen verbal testimony as a means of knowledge based on the reliability of the speaker.

On the level of argumentation, we have seen that the Indian tradition had a rich debate culture with strict rules providing invariant conditions for fair disputation, gradually giving rise to a more invariant technical language and more sound forms of reasoning. We have also seen the difference between descriptive and argumentative language use and the Nyāya's standard five-part argument form, appealing to a concrete instance for support.

On the level of logic, we have seen inference as a means of knowledge from perceived to unperceived, relying on a notion of pervasion—a complete invariance relation of co-occurring properties typical of logical thinking. We have also seen the paradox that arose, following the later Buddhist philosopher Dignāga's deductivist approach, in trying to establish such a relation.

As for the adaptive process, I identified inference, going from perceived to unperceived and potentially leading to errors, as corresponding to the predictive, top-down part: applying a fallible model to predict a yet unseen thing. The comparison of the example to the subject of the inference, based on the relation stated in the vyāpti, corresponds to conceptual analogies. While comparison as a means of knowledge, from perceived to perceived, based on a high degree of similarity between well-known things (e.g., a cow and a bos gavaeus), and forming the basis for general notions, corresponds to the inductive, bottom-up part and perceptual analogies. We have also seen an interaction between them, the reliance of inference on comparison, and more generally, a mixture of inductive and deductive approaches.

Chapter 5

Plato and Aristotle in Ancient Greece

From the earliest surviving texts, the use of analogies and metaphors was widespread in ancient Greece. Lloyd (1966) points out that analogies were used for many purposes. Such as in moral discussions to determine personal actions and influence others' based on consequences of similar actions in the past. They were also used to explain the unfamiliar, such as internal psychological states or specific natural phenomena, and to anthropomorphize inanimate objects, animals, or gods. Similarities between things were often taken as signs of supernatural connections or omens to predict and influence the future. The early, pre-Socratic philosophers started using analogies more systematically (Lloyd, 1966). Rejecting anthropomorphizing and analogies as explanations for individual phenomena, they instead abstracted from particular instances to more general accounts. Analogies served as the basis for many theories and were combined into rough predictive models. Yet they were still often considered sufficient as explanations for the phenomena being studied. After the early philosophers, more critical views of analogies appeared. In this chapter, I want to focus on a small selection of the works of Plato and Aristotle that are relevant in this regard. Plato (427–347 BCE) was critical of the reliability of analogies, pointing out the potential deceptiveness of resemblances between things. However, he continued using them throughout his dialogues for persuasive, didactic, and explanatory purposes. Aristotle (384–322 BCE) was even more critical of using analogies in reasoning and as explanations, often instead trying to find underlying general laws to explain the similarities between things (Lloyd, 1966). Nevertheless, he gave the first formalization of analogical arguments. As with the previous two chapters, I will approach the subjects discussed from the perspective of the model of Chapter 2. I will start with the argumentative context, specifically Plato's dialectic, followed by a discussion of the use of analogy, and the two analogical argument forms studied by Aristotle.

5.1 Dialectics

Since I do not discuss just one comprehensive text in this chapter, its structure will be somewhat different. The topics discussed by Plato and Aristotle are spread out over many different works. I want to consider a small selection from the work of both because each provides a different relevant perspective in relation to the texts discussed in the previous two chapters and with regard to the model of Chapter 2. For Plato, I want to focus on his dialectics, starting in this section and continuing into the next with a discussion of models. For Aristotle, I want to focus on two analogical argument forms he identified and his remarks on similarity more generally; both in section 5.3. The comprehensive documentation of the use of analogy in ancient Greece by Lloyd (1966) has helped to select some of the relevant works for consideration. First, however, I want to briefly consider Plato's discussion of language and knowledge in connection with the topics discussed in the previous two chapters and as leading to the dialectical method.

In Plato's *Cratylus*, Socrates discusses the 'correctness of names,' when a thing is considered correctly named, with Hermogenes and Cratylus. Hermogenes is convinced that correctness is determined by convention, the correct name for a given thing being whatever is agreed upon within a community. Cratylus, on the other hand, argues that things get their name 'by nature,' the names being in some way like the things they name, which can be discovered through the study of etymology, and the likenesses of the letters making up words to the things they name. We can see here a correspondence to the normative, top-down and the descriptive, bottom-up tendencies of the adaptive process at the level of language. Socrates calls both views into question, finding that Hermogenes' view is too relative and arbitrary (in other words, too variable) while Cratylus' view is too absolute and rigid (or invariable). Eventually, he concludes that it has to be both: "I myself prefer the view that names should be as much like things as possible, but I fear that defending this view is like hauling a ship up a sticky ramp, as Hermogenes suggested, and that we have to make use of this worthless thing, convention, in the correctness of names" (*Cratylus*, 435c).[18] There are parallels here to the Mohists, who were likewise occupied with the correct naming of things, particularly with kind names, and the Nyāya, who considered the meaning of words to be determined by convention. Socrates then considers the use of names: "a name is a tool for giving instruction, that is to say, for dividing being" (*Cratylus*, 388b–c). Contrary to the Mohists, he argues that knowledge of names is of no real importance. Since it only reveals the views of the first 'rule-setters' (the people who first introduced our words)[19] on the nature of reality, seemingly pointing to the belief that everything is constantly changing. Whereas, Socrates argues, we can only have knowledge of anything that stays the same: "Indeed, it isn't even reasonable to say that there is such a thing as knowledge, Cratylus, if all things

are passing on and none remain" (*Cratylus*, 440a). Hence, we see here a conception of knowledge based on invariance. What matters, according to Socrates, is the unchanging (invariant) nature of things behind the words, which we can only examine through abstract thought. This view leads to Plato's theory of Forms, "eternal, nonphysical, quintessentially unitary entities [...] standing immutably in the nature of things as standards on which the physical world and the world of moral relationships among human beings are themselves grounded" (Cooper and Hutchinson, 1997, p. xiii), as developed in dialogues such as *Symposium*, *Phaedo*, and *Republic*.

As Cooper and Hutchinson (1997, p. xvii) note, this theory of Forms is further developed in later dialogues: from Forms being simple abstract unities, from which things in the physical world "acquired their name by having a share in them" (*Phaedo*, 102b), to a conception of Forms as divided wholes in *Phaedrus*, which can be known about through the method of collection and division, as used in *Sophist*, *Statesman*, and *Philebus*. Collection involves taking various kinds together under a single name. Division involves making successive conceptual cuts into more and more specific kinds. Both are used in combination until the kind that is sought after is found. In *Sophist*, this "dividing things by kind" and "adequately discriminating a single form spread out all through a lot of other things" is described as having expertise in dialectic (*Sophist*, 253d–e), which is said to be the primary concern of the philosopher. This collecting and dividing of things by kind is clearly a form of categorization: "each thing is to be understood through a full, lively awareness of its similarities and differences in relation to other things" (Cooper and Hutchinson, 1997, p. 236). Like natural categories, the way things are divided in dialectics is not arbitrary but restricted to some extent by the nature of reality. As Socrates makes clear in *Phaedrus*, one has to "cut up each kind according to its species along its natural joints, and to try not to splinter any part, as a bad butcher might do" (*Phaedrus*, 265e). Furthermore, collection can be seen as a bottom-up procedure, collecting kinds into more general kinds, while division is a top-down procedure, going from more general kinds to more specific ones, corresponding to both sides of the adaptive process. There is also an obvious parallel here to the Mohist conception of disputation or distinction-drawing, which is, however, mainly focused on the bottom-up extension of kinds. In *Sophist* and *Statesman*, this method of collection and division is further developed, and the importance of models to determine the proper kind is discussed, which I consider next.

5.2 Analogy

In *Sophist* and *Statesman*, an unnamed visitor from Elea discusses the method of division, with Theaetetus and a young man named Socrates, respectively (not Socrates from the Socratic dialogues), to define what a

sophist is and what a statesman is. The Eleatic visitor points out the importance of models (*parádeigma* παράδειγμα) for finding certain kinds through the method of division: "It's a hard thing, my fine friend, to demonstrate any of the more important subjects without using models" (*Statesman*, 277d). There is a clear parallel here to the use of models to determine kinds through distinction-drawing in the Mohist Canons, but there is a difference in how they are used. For example, in *Sophist*, the Eleatic visitor demonstrates the method of division by first trying to find a simpler kind: the angler—a type of fisherman (218e). But when they find this kind through successive divisions into more specific kinds, the Eleatic visitor points out that the angler is similar to the sophist. In fact, they belong to the same kind, in that they are both hunters (221d). Except where an angler hunts water creatures, a sophist hunts terrestrial creatures (trying to sell the knowledge of virtue to wealthy youths). So here we see not the use of models to make analogies to perceptually similar things, but rather to make conceptual analogies. Other uses of models to determine kinds also show this. In *Statesman*, the art of statesmanship is explored by analogy to the art of weaving (305e–311c). The Eleatic visitor argues that there are people of two different natures with opposing tendencies. On the one hand, there are people who embody the virtue of moderation, the "quiet and moderate" (307b), who are gentle and orderly, live a quiet life, and try to find peace wherever they can, but who are at the mercy of those who attack them (307e). On the other hand, there are people who embody the virtue of courage, the "excessive and manic" (307c), who are brave and vigorous, but who make many enemies, draw others into war, and destroy their own lands (308a). Both tend to seek out those who are like them and to not like those who are unlike them. The art of the statesman, the Eleatic visitor argues, is in weaving together these two dispositions—the brave and the moderate—pairing people of opposing tendencies into a "smooth and 'fine-woven' fabric" (311a) to establish order and balance in their cities.

Lloyd (1966) has argued that models seem to have three functions, as used in *Sophist* and *Statesman*: (i) *practice*, for practicing the method of division specifically, (ii) *didactic*, teaching by leading someone from something familiar to something unfamiliar but similar, and (iii) *discovery*, providing a procedure for extending knowledge from simple to complex subjects. In *Statesman*, the Eleatic visitor proposes to illustrate the use of models through the use of a model (277d–278d). He gives the example of children learning to read and write. They might be able to distinguish individual letters in the shortest syllables but not yet recognize them and make mistakes when trying to identify the same letters in other syllables. The Eleatic visitor argues that the easiest and best way to make them recognize the letters in the other syllables is to "take them first back to those cases in which they were getting these same things right, and having done that, to put these beside what they're not yet recognizing. By comparing them, we demonstrate

that there is the same kind of thing with similar features in both combinations" (278b). He then generalizes this example to the "individual 'letters' of everything" and the "long 'syllables' of things" to point out the universal nature of this technique, to grasp the unfamiliar by the familiar and the complex by the simple.

While Plato uses analogies throughout the dialogues, he also makes numerous cautionary remarks about their reliability in arguments. In various places, he points out that similarities can be deceptive. In *Sophist*, when Theaetetus sees a similarity between the thing they are discussing and a sophist, the Eleatic visitor replies: "And between a wolf and a dog, the wildest thing there is and the gentlest. If you're going to be safe, you have to be especially careful about similarities, since the type we're talking about is very slippery" (231a). In *Phaedrus*, Socrates argues that those who have an accurate knowledge of how things are similar and different can "shift from one thing to its opposite" (262a) through small steps, and so deceive others, making them falsely believe they are the same. Plato also, at various points, argues that similarities need to be verified, such as in *Republic*, where justice in the ideal city, a more familiar subject, is used as a model for understanding justice in the individual (434e–435a):

> So, let's apply what has come to light in the city to an individual, and if it is accepted there, all will be well. But if something different is found in the individual, then we must go back and test that on the city. And if we do this, and compare them side by side, we might well make justice light up as if we were rubbing fire-sticks together. And, when it has come to light, we can get a secure grip on it for ourselves.

Clearly, Plato was aware of the potential for analogies to be unreliable and misleading. Still, as Lloyd (1966, p. 400) has pointed out, he kept using them throughout the dialogues and often did not follow his own advice to verify the similarities involved.

Aristotle was even more critical of analogies and metaphors in reasoning. In *Posterior Analytics*, he rejects the use of metaphors in arguments, as well as their use in definitions: "And if one should not argue in metaphors, it is clear too that one should not define either by metaphors or what is said in metaphors; for then one will necessarily argue in metaphors" (II.13 97b 35). In *Metaphysics*, he criticizes Plato's theory of Forms: "to say that they [the Forms] are patterns [models] and the other things share them is to use empty words and poetical metaphors" (I.9 991a 20). As Lloyd (1966, p. 412) has pointed out, despite being critical of analogies, Aristotle, like Plato, continued using them in many places to make arguments and to establish theories and explanations. Though he did do so more cautiously than earlier philosophers, pointing out differences between the things being

compared and often trying to find a common cause or general law underlying them. However, like Plato, he did not always follow his own standards.

5.3 Argument Forms

We have seen Plato's dialectics and method of collection and division and the use of models to determine kinds as a form of argumentation. More generally, in Plato's Socratic dialogues, Socrates often uses analogies to other specific, familiar cases to make arguments and draw out the implications of the positions of others. In many cases, he scrutinizes both sides of a debate equally through such analogical arguments, such as between the conventional and the natural view of language in *Cratylus*, as we saw in section 5.1. Although implicit in the case of Socrates, this reminds of the standards for fair analogical argumentation as set out in the Mohist Canons and the rules for debate in the Indian tradition. For the remainder of the chapter, I want to look at the use of analogies and similarity in arguments as discussed by Aristotle. Bartha (2010) has noted that Aristotle identified two forms of argument that can be considered to be analogical: "argument from example" (παράδειγμα, *parádeigma*, paradigm) and "argument from likeness" (ὁμοιότης, *homoiótēs*, similarity). I will consider these in turn.

Argument from Example (*parádeigma* παράδειγμα). This argument form is discussed in both *Rhetoric* and *Prior Analytics*. In *Rhetoric*, Aristotle mentions it as a type of rhetorical proof and that "it has the nature of induction, which is the foundation of reasoning" (II.20 1393a 25–26). It is divided into two varieties: those based on "actual past facts," and those based on the "invention of facts." The former are *historical parallels*, and the latter are further divided into two subtypes: *illustrative parallels* and *fables* (II.20 1393a 22–30). Aristotle notes that illustrative parallel was the type of argument used by Socrates,[20] and that fables are "suitable for addresses to popular assemblies" (II.20 1394a 2). He argues that these two types of invented comparisons are easier to make than finding parallels between actual past events: "all you require is the power of thinking out your analogy, a power developed by intellectual training" (II.20 1394a 5–6). However, historical parallels are more valuable because "in most respects the future will be like what the past has been" (II.20 1394a 8). Here we can see a rough correspondence to more conceptual analogies with illustrative parallels and fables based on invented facts, and more perceptual analogies with historical parallels based on actual past facts, i.e., actual things in past experience. But it is a rough correspondence since historical parallels are often higher-level conceptual comparisons, and historical facts are not generally based on personal experience.

After introducing the different types, Aristotle goes on to describe the use of argument by example. He considers its use in combination with an

enthymeme, a type of syllogism used in rhetoric that does not have to be formally complete. If there is no enthymeme available to prove a certain point, we can use an argument from example instead. However, if we can make the point through an enthymeme, an example can still serve as supporting evidence. Aristotle points out that in the latter case, the example should follow, not precede, the enthymeme. Otherwise, it will make the argument seem inductive, and many examples will be needed: "if you put your examples first you must give a large number of them; if you put them last, a single one is sufficient; even a single witness will serve if he is a good one" (II.20 1394a 14–16). There is a parallel here to the use of examples in the Nyāya Sūtras and the Indian tradition of logic more generally: the practice of giving a concrete instance of a general rule to show its applicability. Lloyd (1966) points out that the enthymeme is called a 'rhetorical syllogism,' while the example is equivalent to a rhetorical induction (p. 407), hence corresponding to the top-down and bottom-up tendencies of the adaptive process.

According to Bartha (2010), in his treatment of the argument from example, Aristotle gives the first formal analysis of an analogical argument. He characterizes the argument form as going from one or more similar cases to a general proposition, which is then deductively applied to a new case (II.25 1402b 15–18), i.e., it is effectively an argument based on an inductive step followed by a deductive step (Bartha, 2010). In *Prior Analytics*, the following example is given of a historical parallel (II.24 68b 39–69a 8):

> [L]et A be evil, B making war against neighbours, C Athenians against Thebans, D Thebans against Phocians. If then we wish to prove that to fight with the Thebans is an evil, we must assume that to fight against neighbours is an evil. Conviction of this is obtained from similar cases, e.g. that the war against the Phocians was an evil to the Thebans. Since then to fight against neighbours is an evil, and to fight against the Thebans is to fight against neighbours, it is clear that to fight against the Thebans is an evil.

Lloyd (1966) notes that in *Rhetoric*, Aristotle discussed argument from example from the perspective of rhetorical use, as a persuasive rather than demonstrative argument form, whereas in *Prior Analytics* it is analyzed instead in comparison to the deductive syllogism that he is establishing (p. 407). In *Prior Analytics*, Aristotle identifies argument from example as going not from particular to general (inductive) or from general to particular (deductive), but from particular to particular, with one of the particulars being familiar (II.24 69a 14–16). According to Aristotle, argument from example differs from a 'complete' induction—the enumeration of all particular cases to establish a general proposition—because it does not consider all particulars and is followed by a deductive step (II.24 69a 16–19). Since it

does not consider all particular cases, Aristotle considers it a weaker argument form than a 'complete' induction or syllogism.

In terms of the adaptive process, argument from example shows the interaction of induction and deduction in a single argument, which amounts to an analogical argument in Aristotle's analysis. It would seem that this is at odds with my aim to show that analogies can be seen as more fundamental, underlying both induction and deduction. In fact, as Lloyd (1966, p. 410) notes, Aristotle was "trying to reduce all other modes of argument to the syllogism in order to show the more fundamental nature of that form of reasoning." One difference to notice is that perceptual and conceptual analogies, as introduced in Chapter 2, are considered only comparisons, not argument forms on their own (although they can be used in arguments as we have seen in all three traditions). More importantly, the reconstruction of this argument form as an isolated sequence of induction and deduction, although it neatly lines up the steps from 'particular to general' and 'general to particular' to result in 'particular to particular,' is somewhat artificial. It would be more natural, taking the Mohist perspective of applying models, even for inferences, to think of 'making war against Phocians is evil' as a prototype for an already established general notion that 'making war against neighbors is evil.' There would then simply be a comparison with this prototype to determine whether a new instance matches the pattern, hence a form of categorization. We might be reminded of a more general notion by remembering a specific instance, but this does not itself constitute an induction. The general notion must have formed originally through an inductive process of generalizing from particulars, as Aristotle notes himself, "[c]onviction of this is obtained from similar cases," but this does not have to be part of the argument.

Argument from Likeness (*homoiótēs* ὁμοιότης). In *Topics*, Aristotle recommends the study of similarities as it is useful for inductive arguments, hypothetical deductions—what Lloyd (1966, p. 409) calls 'syllogisms based on a hypothesis'—and the rendering of definitions (I.18 108b 7–27). It is useful for inductive arguments because it is not easy to "induce the universal" without knowing the similarities between cases. It is useful for hypothetical deductions because it is generally admitted that "among similars what is true of one is true also of the rest." We first form the hypothesis that 'if something is true in similar cases, it is also true in the case under consideration.' Then, when we prove it for the similar cases, we will also, as far as the strength of the hypothesis goes, have demonstrated it for the new case. In these two uses of the study of similarities, we can see perhaps most clearly a correspondence to the use of analogical comparison as the basis for both the bottom-up inductive part and the top-down, deductive part of the adaptive process. Finally, studying similarities is useful for rendering definitions because citing what is common to all cases of a genus constitutes a definition for that genus. Lloyd (1966) has noted that the use

of similarities to determine a genus in this way is similar to Plato's method of collection (p. 410).

Aristotle notes that argument from likeness is similar to induction but differs from it because no universal is established (VIII.1 156b 9–16), making its conclusion only plausible, hence again weaker than syllogism or 'complete' induction. Similarly, argument from likeness is also distinguished from argument from example, in that the former does not establish a universal as an intermediate step and so does not require acquaintance with a universal that covers both cases. As Lloyd (1966) has pointed out, though, many of Aristotle's inductions in practice are not supported by an explicit enumeration of all particulars, but only by a few similar cases (p. 411).

Finally, and most importantly, Aristotle identified two types of similarity when comparing different things (I.17 108a 7–17):

> Likeness should be studied, first, in the case of things belonging to different genera, the formula being: as one is to one thing, so is another to another (e.g. as knowledge stands to the object of knowledge, so is perception related to the object of perception), or: as one is in one thing, so is another in another (e.g. as sight is in the eye, so is intellect in the soul, and as is a calm in the sea, so is windlessness in the air). [...] We should also look at things which belong to the same genus, to see if any identical attribute belongs to them all, e.g. to a man and a horse and a dog; for in so far as they have any identical attribute, in so far they are alike.

According to Hesse (1965), these two types of similarity lead to two types of analogical comparison: in terms of a *similarity in the relation* between things of different genera, and in terms of *properties in common* between things of the same genus (p. 330). She has called these "formal analogy" and "substantive analogy," respectively, and she notes that many of Aristotle's analogies rely on a combination of both. The former is what we more commonly think of as an analogy and is closely related to the meaning of the original Greek *analogia*, a proportional relation of some kind, based on a conceptual invariance. The latter is more like a regular comparison, based on a perceptual invariance. These correspond directly to the two types of analogies identified in section 2.3.3, and thus the two sides of the adaptive process. Comparison between things of the same genus corresponds to perceptual analogy and is classificatory. Comparison between things of different genera, on the other hand, corresponds to conceptual analogy: making new connections between things and explaining something in terms of something else. This correspondence will come back in the next chapter.

5.4 Conclusion

The use of analogies was widespread in the ancient Greek tradition as the basis for both explanation and prediction. They were used as supernatural explanations and omens by early writers, and as models for moral action. Early philosophers relied on them for rough explanatory accounts and predictive models for natural phenomena. Plato used analogies in the form of comparisons to models for determining kinds. And for Aristotle, they served as a first step to more rigorous accounts, based on general laws, to explain and predict phenomena.

On the level of categories, we have seen Plato's method of collection and division to determine kinds, like natural categories to some extent limited by the nature of reality, needing to cut species along their "natural joints." We have seen the use of models as prototypes to compare against the kind that is sought after. And we have seen the examining of similarities for defining and determining a genus. On the other hand, we have also seen that both Plato and Aristotle warned about the deceptiveness of perceptual similarities when comparing things.

On the level of language, we have seen the opposing views of things getting their name by convention or by nature, and Socrates' conclusion in Plato's *Cratylus* that it has to be a combination of both. We have also seen Socrates describe names as a tool for dividing being, leading to Plato's method of collection and division, and his assertion that knowledge relies on things staying the same, i.e., invariance.

On the level of argumentation, we have seen Plato's dialectic and Socrates equally questioning opposing views through analogical arguments. We have seen Aristotle's two analogical argument forms, one based on a combination of induction and deduction, the other on similarities of either one of two kinds. Both argument forms were considered by Aristotle from a rhetorical perspective and judged as weaker than 'complete' induction and syllogism. We have also seen that Aristotle considered the study of similarities to be useful for both inductive as well as hypothetical deductive arguments.

In this chapter, we have not specifically looked at the level of logic. Given the limited scope of the thesis, much had to be left out in favor of a focus on analogies, such as discussions of knowledge like in the other traditions, the logical notions used throughout Plato's dialogues, and Aristotle's discussions on categories, language, rhetoric, and logic.

As for the adaptive process, I identified Plato's use of models for determining kinds in abstract thought as corresponding to conceptual analogies. We have seen that Aristotle recommended the study of similarities for both inductive arguments and hypothetical deductions, corresponding to analogies underlying the bottom-up and top-down tendencies of the adaptive process. Furthermore, I identified the two types of analogical comparison based on Aristotle's two types of similarities as corresponding to the two

Top	
Conceptual analogies	Models (method of collection and division) Arg. from likeness: similarity in relation
Hypothetical	Arg. from example: invented facts
Bottom	
Perceptual analogies	Arg. from likeness: properties in common
Actual	Arg. from example: actual past facts
Interaction	
• Argument from example based on induction and deduction • Likeness underlying both induction and hypothetical deduction	

Figure 5.1: Correspondences to the adaptive process.

types of analogies of the model: comparisons between things of the same genus, based on the properties they have in common, corresponds to perceptual analogies, while comparisons between things of different genera, based on similarity in relation, corresponds to conceptual analogies.

Chapter 6

Further Discussion of the Model

In the previous three chapters, we have seen how many of the concepts of the model introduced in Chapter 2 were present and played a role in early stages of the development of explicit logical reasoning across three different intellectual traditions.

In this chapter, I want to return to a discussion of the model, reflecting on what we have learned from the ancient traditions in relation to it, and also consider classical logic in light of it. Modern logic has advanced far beyond the confines of classical logical notions. However, the classical logical system of first-order logic still forms a paradigm for logical systems (Ferreirós, 2001), providing a solid foundation for many endeavors in science, mathematics, and intuitively in our everyday thinking. Therefore, I want to briefly consider classical logic from the perspective of the model.

For most of the thesis, I have considered the model for adaptive processes from the perspective of cognition, as the prototypical example for them. I will argue that as a simple model for cognition, it has broad applicability and can help to keep a few essential aspects of our thinking at the forefront of our mind to avoid common confusions, cognitive biases, and a false sense of certainty and rigidity in our thinking.

However, I also want to point out that the main components of the model—finiteness, invariance, and a continuous process of constructing and applying invariances based on an analogical kind of mechanism—are more widely applicable and go beyond cognition and the study of logic alone. At the end of the chapter, I will briefly argue that there are other instances of adaptive processes, such as legal systems and the scientific method.

6.1 Finiteness: Allowing Novelty and Uncertainty

Although we have not seen finiteness explicitly mentioned in the texts of the ancient traditions discussed in the previous three chapters, I consider it a foundational component of the model. I hope to make this clearer in the present chapter. We have seen a notion of the infinite reflected in, for example, the Mohist doctrine of all-inclusive care (or universal love), something approaching the notion of a universal with vyāpti in the Indian tradition, and the absolute in Plato's ideal Forms. As proponents of early rationalistic, logical schools in their respective traditions, this is no surprise since logic deals in absolutes and universal principles. We did furthermore see finiteness implicitly in the early logical traditions, in their consideration of various kinds of errors in judgment. Such as in the unreliability of perception, the misapplication of labels to things at the level of language, the incorrect extending of categories, and the appealing to the wrong explanation for an inference. In all these expressions of fallibility, we can see an admission of our limited capacities as finite beings. Keeping our finiteness in mind provides a natural counterweight to the reflexive assumption that we can know some things with absolute certainty because we cannot imagine them to be otherwise.

In terms of the proposed model, our finiteness has a simple consequence: *all invariances that we are familiar with are based on finite experience.*[21] The explanatory and predictive model we have of the world necessarily does not fully capture reality, yet we invariably mistake our model of reality for reality itself. In the words of Korzybski (1994): "the map is *not* the territory."

6.2 Invariance: A Tendency towards Generality

In this section, I want to further discuss invariance. First, I will reflect on what we saw in the early logical traditions in the previous three chapters. Then I want to briefly consider some classical logical notions in terms of the model. At the end of the section, I will put the concept of invariance in a broader context by briefly considering some manifestations of this concept that go far beyond what has been considered in this thesis.

The early logical traditions. Categorization, as we have seen, played an important role in the early logical traditions, and the use of prototypes to determine or extend categories can be seen in each tradition. Categorization was central to all argumentation for the Mohists; their reasoning relied on comparisons to models to draw distinctions and extend kinds. The Nyāya similarly were concerned with forming categories through the use of comparison between perceptually similar things. For Plato, the use of models to determine kinds played a similar role. However, his method of collection and division was to be engaged in through abstract thought, and

we can see a corresponding use of conceptual analogies to determine kinds. Language likewise played an important role. We have seen no discussion of the graded structure of categories. This makes sense if we consider that the concern in each of the traditions was with the correct naming of things, i.e., the application of discrete labels in natural language. Language essentially flattens the graded structure of natural categories by using discrete labels for them (section 2.2.2). We have also seen that both the Mohists and Nyāya identified misapplying a label to a thing as a source of the error in judgment, arising from the conventional nature of language. When it comes to argumentation, in all three traditions, logical thinking and theorizing developed in the context of debate, in the making of consistent arguments for their views towards others, and explicitly defining specific terms they used. In each tradition, we have seen argument forms that relied on making analogical comparisons. With regard to logic, we have seen the universal notions of the Mohists' all-inclusive care, the Nyāya's notion of vyāpti, and Aristotle's 'complete' induction and syllogism. All these are aimed at achieving invariance to some absolute degree.

Classical logic. As we saw in section 2.2.4, Tarski defined the logical constants as those "invariant under permutations," as staying the same through uniform transformations of the universe of discourse. In other words, logic is concerned with an ideal of absolute invariance in abstraction, making it universally applicable and context-independent. I want to briefly consider classical logic from the point of view of what has been discussed, from the perspective of natural categories, category structures, and our finiteness. As we consider logic to be among the most invariant of things, and as more invariance translates to deeper entrenchment in our category structures, a natural possibility suggests itself: what if our logical laws and notions are simply those that are either among the most deeply entrenched in our category structures, or inherent characteristics of the process of categorization itself? In either case forming a foundation for our conceptual thinking. I want to consider this possibility for a moment from the perspective of the model for adaptive processes. Ferreirós (2001) has argued that the classical logical system of first-order logic forms the paradigm example for modern logic systems, and that our current notion of classical logic is sometimes assumed to be ahistorical, as if predetermined. However, he argued that what we now consider to be part of first-order logic was not inevitable as a logical unity. It has developed into this form due to certain historical contingencies. Ferreirós considered the logical connectives to be natural components of any logical system, but the choice of including universal and existential quantification, specifically, as not inevitable. In fact, he argued, it can be traced back to a paradigm set by Aristotle. Had it been the rival Stoic logic in ancient Greece that was taken as a paradigm, which did not have a similar notion of quantification, then what we would consider the paradigmatic system for logic today might have been quite different and

perhaps mostly sentential in nature.

Indeed, the logical connectives do seem to have an undeniable self-evident status. They could, perhaps, be seen as naturally arising from the process of categorization itself. Since categories are based on drawing boundaries for membership (whether definite or gradational), negation could be seen as arising from the notion of 'what is not considered a member' for a certain category, disjunction as arising from the dividing of a category into subcategories (comparable to Plato's method of division), and conjunction as arising from the taking together of different categories (comparable to Plato's method of collection). As abstract notions made explicit under these names, as 'logical connectives' and in everyday language as the words 'and,' 'or,' and 'not,' they could be seen as part of the fabric of categorization itself, if we take mental categorizing as primary. Considering the connectives as directly arising from how we categorize is much like a set-theoretic definition of the connectives (or perhaps Peirce's alpha graphs, Shin and Hammer, 2013). However, unlike natural categories with a graded structure, sets are clearly defined and delineated entities, like Aristotelian categories. Thus the logical connectives as binary notions defined in terms of sets are abstractions and simplifications of corresponding notions for natural categories.

The logical connective of implication can logically be defined in terms of negation and disjunction (or conjunction). However, it might perhaps more intuitively have arisen as an abstraction from instances where one kind of thing is followed by another (much like Hume's notion of 'constant conjunction'). To use an example from the Nyāya, we might conjecture that there is a connection when we repeatedly see ants carrying off their eggs in a certain way followed by rain. As we make this connection between ants carrying off their eggs and rain more often, it becomes more established, forming a category encoding the general rule that if ants carry off their eggs in this way, it will be followed by rain. When we see a new instance of ants carrying off their eggs, it will be categorized as an instance of this category (by comparison to its prototype), and we start expecting rain as a consequence, as it was for other members of this category. Perhaps, abstracting from all such cases (noticing an analogy between them, like Whitehead's example of the discovery of number from different groups of things, see section 2.3), we could naturally get an abstract logical notion of implication.

Now, as Ferreirós (2001) has noted, there is a great semantic difference between the logical connectives connecting individual sentences and quantifiers which introduce a notion of a universe of discourse: "from an intuitive semantic standpoint, it is quite dubious that the different logical particles form a 'natural unity'. This parallels what happens at the metalogical level, with the sentential calculus being decidable while the predicate calculus is not" (p. 451). We could perhaps consider quantification as an abstraction from making statements about categories. The notion of a universe of discourse would then be an abstraction from specific categories. Existen-

tial quantification would simply be an abstraction of cases where we talk about a particular instance of a category. Universal quantification would be an abstraction from expressions about all members of a category. However, given the binary nature of classical logic, such operations flatten natural categories with a graded structure based on an (intransitive) relation of similarity, to Aristotelian categories with all-or-nothing membership based on a (transitive) relation of equality. An instance for an existential quantifier will be considered a complete member of a particular set, whereas in a natural category there is a degree of membership depending on how much an instance deviates from the prototype of that category. All instances falling under a universal quantifier, likewise, are considered exactly the same and interchangeable for its purpose. Whereas members of a natural category are similar to each other but not exactly the same. Hence the simplifying nature of such a logical operation in comparison to how we naturally conceive of groups of things, both cognitively and in everyday language use. Even if in most cases we will consider something to be a complete member of a certain category (something which our discrete natural language words already reinforce), there are many cases where things are not so clear-cut. The difference is most apparent with boundary cases of members of a natural category, where membership of a category is uncertain, or when something is considered a member of a category by some people but not by others, leading to philosophical paradoxes when put into binary logical form. Still, these quantifiers could be seen as a rather straightforward generalization from how we cognitively and linguistically categorize.

What about logical laws, such as the law of non-contradiction or the law of excluded middle? Perhaps we could think of the law of non-contradiction as arising from a need for consistency in our communication and coordination with others, especially in argumentative language use. As we all seem to be familiar with contradicting ourselves. As we saw, language and argumentation require a degree of invariance in the form of consistency to be effective and to have a shared meaning. It requires that we keep certain things the same and not vary in the communication of our views and the justification of our actions. Hence there is pressure not to say or do something what contradicts what we have said or done earlier, so as to be intelligible and reliable to others.

Other logical laws, such as the law of excluded middle, might be thought of as simply among the most entrenched in our category structures, based on the most evidence in our experience. It seems natural for things either to be the case or not to be the case. Since this seems to pervade all of our experience, and so many other things rely on it, we cannot imagine it to be otherwise; it seems the most certain and is the most resistant to change. However, by analogy to physical laws, we might have an intuitive grasp of how things are at the familiar human-scale level, like Newtonian physics was, whereas there might be a more general, more fundamental model that can

account for the same things and more, like Einstein's relativity theory. Perhaps we can also think of notions such as indeterminacy and superposition at the quantum-theoretical level as challenging the belief that things either are or are not, thus challenging the law of excluded middle.[22] That we might have fallible beliefs about logical truths, like we have fallible beliefs about anything else, has been argued before by Quine, Haack (1978, p. 232), and others. That does not, of course, invalidate the usefulness of a model that seems to accord with our everyday experience, just as Newtonian physics is still useful and more intuitive as a model for many purposes. Nevertheless, even what we consider to be the most certain and fundamental, such as certain logical truths, may turn out not to be the most fundamental.

It is not clear then, that what we commonly consider to fall under classical logic is necessarily an inevitable unity and completely immutable. Nor that logical truths derived from it are infallibly, necessarily true in some absolute sense. Instead, we might think of logical notions and laws as the Mohists did, as models to compare against new instances. And given our finiteness, as necessarily simplifying and fallible models.

Invariance as ubiquitous concept. Throughout this thesis, I have considered many kinds of invariance, such as Gibson's invariants in the stimulus flux, Tarski's invariance under permutation of logical constants, the Nyāya notion of vyāpti or invariable concomitance, the notion of pervasion in both the Mohist and Nyāya traditions, and Hume's notion of constant conjunction. However, invariance, things staying the same and keeping things the same, as a fundamental concept goes far beyond what has been discussed in this thesis.[23] We can think of many cognitive biases arising from invariances. And can we not think of trust as being based on keeping things the same, thus being reliable and predictable to other people? Can we not think of boredom as too many things staying the same, jolting us back into action to keep exploring and seeking out new stimuli, to keep our model of reality evolving? Can we not think of some forms of depression as a more pathological, entrenched expression of boredom that does not trigger a response to seek out novelty, keeping too many things too rigidly the same, resulting in a static worldview that seems unchangeable? Can we not think of the control over a narrative in public or groups of people being held the same as becoming more and more entrenched, as there is no counter-evidence for which the model needs to be revised. As it becomes more entrenched, it becomes more difficult to be challenged or imagine an alternative to it. If we take the discussion on logic above, such a narrative literally becomes relatively closer to laws of logic in terms of entrenchment and certainty. It becomes more logical, more unquestionable. The fewer deviating experiences we have, the more certain we become of its correctness. As it starts to inform our decisions and the way we live, the more other things become reliant on it, making it more of a foundation for our general sense-making so that we cannot consider a revision to it without a serious crisis of meaning.

Clearly, then, we can go too far in keeping things the same, and be misled by things staying the same too rigidly. Korzybski (1994) identified equating different things as the chief source of our misunderstanding the world as it is. He effectively argued that in order to avoid doing so, we should consider each thing uniquely on its own. He argued against the use of the word "is" in the sense of an identity relation. However, truly considering each thing uniquely on its own is unrealistically demanding given our finite capacities, and there would be no way to distinguish between individual things without considering certain things as the same, as the Nyāya argued. We cannot function without invariances. We need to find a balance between generality and specificity, which is what the final section of this chapter is about.

6.3 The Adaptive Process: Finding a Balance

In this section, I first briefly reflect on what we saw in the early logical traditions in the previous three chapters with regard to the adaptive process, and identify two sources of novelty for adaptive processes. Second, I will consider an argument made by Wisdom that both induction and deduction are reducible to comparisons between cases, i.e., analogies, as further support for the argument that analogical comparison provides a basic mechanism for both sides of the adaptive process (section 2.3). Third, I will further consider induction and deduction as isolated logical notions as well as understood in terms of each other as part of a continuous process. Finally, I will argue for the need to find a balance between these two opposing tendencies, and consider other instances of adaptive processes beyond cognition that illustrate this balance.

The early logical traditions. In all three of the early traditions, we have seen both aspects of the adaptive process. With the Mohists, we have seen on the one hand the incremental extending of kinds, primarily through perceptual analogies based on perceptual invariances, and on the other hand the use of more conceptual analogies in explanations, for the giving of reasons; an analogue, as Fraser (2020b) has pointed out, to making arguments. With the Nyāya, we have seen on the one hand a similar use of comparison based on a high degree of similarity as a basis for genera, again perceptual analogies, and on the other hand, more conceptual analogies in the use of examples as part of inferences, to illustrate the invariance relation involved. Finally, Aristotle noted the use of similarities in both induction and hypothetical deduction, and distinguished two types of similarity that directly correspond to both types of analogy. On the one hand, he identified comparisons between things of the same genus based on common properties, which corresponds to perceptual analogies based on common categorical connections, to inductively extend established categories. On the other hand, he identified comparisons between things of different genera based on a

similarity in relation, which corresponds to the use of conceptual analogies to make new connections between different categories, suggesting different hypotheses to explain past experience and predict future experience.

Corresponding to both sides of the adaptive process, then, we can identify two sources of novelty. On the one hand, there is the genuine novelty we encounter in experience, which we incrementally make sense of through perceptual analogies to things we already know. On the other hand, novelty arises from recombining known things in new and creative ways through conceptual analogies. It makes sense that our approach to genuinely new experience is more incremental and cautious, trying to explain the unknown in terms of what is closest to it in what we already know, while our recombination of known things, to explore alternative accounts of our past experience and suggest different predictive models for future experience, can be more unrestrained and radical in imagination. However, perhaps we could consider the encountering of something new in our experience that poses a serious challenge to our current model, that we cannot adequately categorize using our existing category structures, as what triggers the making of new conceptual connections. That it is then that we try to recombine known things in different ways to suggest a new category to accommodate the new thing, by focusing on different similarities between things and thus categorizing them differently.

Induction and deduction. As a complement to the arguments made in Chapter 2 for analogy as an elementary mechanism underlying the adaptive process, I want to briefly consider an argument made by Wisdom (1991). He distinguished between deductive argument, inductive or analogical argument, and case-by-case procedure. The difference between the latter two, he argued, is in the fact that the case-by-case procedure, essentially an argument by parallels, can consider actual as well as imaginary cases for comparison, whereas inductive or analogical argument is only concerned with comparison between actual cases. In terms of the adaptive process, Wisdom's analogical or inductive argument corresponds to the inductive, bottom-up part of the process, concerned with actual experience, whereas the case-by-case procedure corresponds to perceptual and conceptual analogies, concerned with either concrete, actual cases or abstract, imaginary cases. Wisdom argued that both induction and deduction are ultimately reducible to case-by-case procedure, i.e., comparison between instances. To illustrate this, he gave the example of a child trying to solve an arithmetical problem involving the squaring of numbers (p. 47). The child's parents use different strategies to make the child see the solution, which Wisdom called the 'mother procedure' and the 'father procedure.' The mother starts with a very simple example involving the squaring of 2 and then step-by-step works up to higher numbers until approaching the numbers of the problem. This procedure corresponds to the bottom-up, inductive part of the adaptive process. The father takes the opposite approach. He cites a general arith-

metical principle and points out that it applies to the problem. As Wisdom noted, short and conclusive, more like what we would recognize as a proof. This procedure corresponds to the top-down, deductive part of the adaptive process. But when the child asks for the grounds of the principle, the father cites a more general principle, of which the child again asks the same question, and the father cites a still more general principle. This goes on in the same way until eventually, the father says that it is self-evident and proceeds to give example instances of the principle in the same way the mother did. This, Wisdom argued, shows that "the father's procedure does as much as the mother's procedure, *but no more*." Both, in the end, come to the case-by-case procedure. He recognized the differences in characteristics of both procedures, that they both are necessary and have different uses, but that they both ultimately rest on the same, more fundamental process of comparing instances. He also recognized the opposite but interconnected tendencies of both: "proof is the process of learning what we mean, carried out in reverse" (p. 50). As a result, he argued that it is not only deductive arguments that can be good arguments, nor that arguments based on comparing instances are always bad, "whether a piece of reasoning is good or bad is not determined by whether it comes in the end to this case-by-case procedure, since all reasoning, good or bad, does so" (p. 56). As Yalden-Thomson notes in the introduction, Wisdom shows that "[d]eductive argument, because of its brevity, fails to draw explicitly to our attention specific cases of the sort upon which the force of the proof ultimately rests, whereas the case-by-case procedure can do this most vividly" (p. xvii).[24]

If induction and deduction are studied in isolation and from the level of abstraction of logic, as they often are, they lead to all kinds of paradoxes. A purely logical account of induction is notoriously elusive, while deduction on its own can also lead to paradoxical claims. We have seen the problem of establishing general principles or universals in the Indian tradition. Dignāga's deductivist approach to knowledge led to the paradox that all instances have to be perceived for the general rule used in inference to be established, including the instance that is the subject of the inference, making inference redundant.[25] A similar example is given by Wisdom (1991) when he discusses the example with the child: the child asks the father whether the instance under consideration is included in the general principle or not. If it is, then the argument is circular; if it is not, then the argument is invalid (p. 49).[26]

If, however, induction and deduction are taken in combination, in more general terms as opposing or complementary tendencies, and as two aspects of a single, continuous process, as I take them in this thesis, they start to make sense. The deductive, top-down aspect by applying our mental model of the world provides direction for what to focus on in inductive generalizations, giving direction to new inquiries, decisions, and actions. The inductive, bottom-up aspect, in turn, provides the grounds, the evidence, for

the construction, testing, and elaboration of the model. Such a continuous process involving induction and deduction is, as we have seen, well known and, for example, comparable to the interaction of inductive experiment and deductive theory in the scientific method. What, however, if this also applies to logic itself and logic can be seen as arising out of such a process? Like the logical notions considered in the previous section, we can perhaps see the abstract logical notions of induction and deduction as arising from these two tendencies of the adaptive process, abstracting them from processes we engage in on an unconscious level, but like the other logical notions making them overly simplifying models of how we actually reason. We could then clearly see classical logic as focused only on the 'deductive' aspect in isolation. In the early logical traditions considered in this thesis, both aspects of the process we have seen to be present. If we take logic in this broader context, as perhaps the most established foundation in a constantly evolving process, and like the Mohists treat all principles, including deductive logical principles, as models, and ultimately fallible models, we can avoid many problems that we run into if instead we consider them to be absolute because we cannot imagine them to be otherwise.

Finding a balance. As we consider the constant interaction of these opposing tendencies, from the specific to the general and the general to the specific, it is important to find a balance between them. Classical logic, with its binary notions of true or false, all or nothing, existential or universal quantification, is concerned with absolutes and focuses on the most general. The other extreme, the completely undifferentiated 'stimulus flux' (Gibson, 1967) or 'much-at-onceness' of experience (James, 1916), the most specific, is too much for our finite capacities to absorb in its entirety and make sense of as a whole. What we need is a middle position, which we find in our natural tendency to categorize, our grouping things together. We have seen this throughout the thesis. Descartes (1641/2008) understood humans as finite beings, with incrementally growing understanding, to be somewhere between nothing and the infinity of an all-encompassing God.[27] The Mohists focused on kind names as the basis for their reasoning instead of on personal ('one') or all-reaching ('all') names. The Nyāya argued for the necessity of finite genera, the grouping of individuals, for words to have meaning, regarding individuals on their own without them to be indistinguishable and thus part of the whole and infinite. And we saw the incremental growing of category structures through the adaptive process. In all of these, analogical comparison plays a mediating role, allowing us to group things together based on similarities despite their inevitable differences.

For most of the thesis, I have considered adaptive processes from the perspective of cognition. I want to argue, however, that the general structure of the proposed model is more widely applicable. The main components—finiteness, invariance, and a continuous process of constructing and applying invariances based on an analogical kind of mechanism—provide a minimal

model for adaptive processes in general. If we take these general characteristics as a model for adaptive processes, we can ask what else can be considered an adaptive process by comparing it to this model. There are some that we might consider human constructs and in some sense extensions of cognition as an adaptive process; I will consider legal systems, the scientific method and philosophy below. The levels of invariance of language, argumentation and logic were introduced for cognition as an adaptive process, but for other adaptive processes we might consider invariance only at the primary level of categorization of some kind.

6.3.1 Law

There is an extensive literature on analogical reasoning in legal settings, especially in case law. In case law, the precedents set by earlier cases play a central role in deciding new cases. This is enshrined in the legal doctrine of *stare decisis*, Latin for 'staying by things decided,' meaning that similar cases should be decided in the same way, with the decisions made in earlier cases informing decisions on new cases. This doctrine thus expresses reliance on a type of invariance, of keeping things the same. Precedents set by earlier cases can be advanced by prosecutors or defendants to argue that the case under consideration is relevantly similar, and precedents advanced by the opposing side can be critiqued as critically different from the current case. If the judge considers a case relevantly similar to a precedent, it receives a similar verdict; if it is considered to be critically different from precedents, the case will be distinguished from earlier ones, setting a new precedent for future cases that are similar to it.

In this way, the legal system of case law can be seen as an adaptive process. The process of comparing new cases to precedents is essentially a form of categorization. Precedents, as typical cases, act as prototypes for their respective categories of similar cases. The invariance underlying such categories could be seen as the commonalities between the similar cases and the type of verdict they have received. This extending of 'categories' of similar cases by precedents corresponds to the bottom-up, inductive aspect of adaptive processes, and the comparisons to precedents can be seen as classificatory analogies. The case law records thus established as a 'model,' in turn, are applied to new cases and direct attention to which 'categories' are considered relevant for comparison. This corresponds to the top-down aspect of adaptive processes of applying established invariances (or generalizations) to new input. Unlike the role of imagination and creative analogies in the case of cognition, the 'category structures' of case law are relatively more static, as no new connections are made between existing categories beyond this incrementally growing categorization. New 'categories' of cases are established only when a case is found to be critically different from precedents and is distinguished as a new kind of case. Furthermore, in case

law, as well as legal systems more generally, the legal process relies on a finite set of laws and case law records to coherently explain past cases and which are applied by extrapolation to adjudicate over a potentially infinite number of possible future cases.

An important point here is that this mechanism of *stare decisis*, of analogical reasoning in case law, has been argued as a mediating process to strike a balance between conservative and progressive social values (Bartha, 2010, p. 246). It makes judicial decision-making predictable, preserving consistency and fairness by deciding similar cases in the same way, while still allowing the law to gradually evolve, incrementally adapting to changing norms in society through the ability to distinguish cases when critically different from earlier ones.

A further analogy can be made between legal laws and logical laws. Legal laws inform how judicial decisions are being made more generally, as normative rules that are more deeply entrenched in the 'legal category structures' than the categories formed by the case records. A nation's constitution, in this analogy, is then the most entrenched, the most invariant ('kept the same'), and the most resistant to challenges and revision as it underlies all else as the foundation.[28]

6.3.2 Science

Analogies also obviously play a vital role in science. The use of models to explain one thing by analogy to another are ubiquitous in science, and analogy has often been noted as an important source for hypotheses. Some even argue that all scientific discoveries have been made either by accident or through analogy; Bartha (2010) notes that chemist Joseph Priestley held such a view. Some of the most well-known scientific discoveries were made by analogies to familiar, everyday things. Such as Archimedes' "Eureka" moment in the bathtub,[29] Newton's falling apple, and Einstein imagining riding on a beam of light.

In section 2.3, we saw that the adaptive process directly corresponds to a familiar representation of the scientific method, as a constant interaction between inductive experimentation, to test and support theories by generalizing from experience and categorizing observed phenomena, and deductive theorizing, seeking a consistent explanation for the observed phenomena and suggesting hypotheses, which in turn give direction to new experiments. The scientific method can in a sense be seen as a more rigorous and formalized, collaborative extension of our natural explorative behavior as embodied in our cognition. In this way, we can think of the scientific method as another instance of an adaptive process, one that is again based on a finite set of theories or laws (e.g., physical laws) to explain past phenomena and which are applied by extrapolation to predict a potentially infinite number of possible future phenomena.

As we saw in the previous section, analogical reasoning in case law has been argued to strike a balance between conservative and progressive social values. Bartha (2010, p. 252) has made an elaborate analogy between case law and science to show that analogical reasoning in science similarly leads to a balance between conservative and progressive epistemic values, between a stable, coherent, and simple foundation on the one hand, and explorative, creative innovation on the other, allowing our scientific understanding to gradually evolve over time. We can also think of Leonardo da Vinci, who famously tried to find a balance between inductive and deductive methods in scientific inquiries (Ackerman, 1978).

Finally, on a larger scale of science as a whole, there is also a connection between analogical reasoning and Kuhn (1970)'s notions of 'normal science' and 'scientific revolution,' as Bartha (2010, p. 306) has suggested. These also roughly correspond to the two sides of adaptive processes. Normal science corresponds to the bottom part of an adaptive process, incrementally making predictive and classificatory analogies based on existing scientific theories and paradigms. Scientific revolution corresponds to the top part of an adaptive process, making more radical creative explanatory analogies by making new connections between existing categories, challenging the established 'scientific category structures' more generally, thereby reconceptualizing established theories and paradigms.

6.3.3 Philosophy

As James (1916) observed: "Philosophy in the full sense is only *man thinking*, thinking about generalities rather than about particulars" (p. 15). As such, as an extension of our cognition, or perhaps a specific application of it, philosophy could also trivially be considered as an adaptive process, in its development and application of philosophical theories. However, philosophy, as ancestor of natural philosophy (i.e., science), is unlike cognition on its own and like science in that it is for the most part collaborative in nature, with philosophers arguing over particular theories. Indeed, Haack (1978) has argued, like James, that philosophy is continuous with science: "instead of insisting on a sharp demarcation, I hold that philosophy differs from the natural sciences rather in degree of abstraction and generality" (p. 116). Because of its generality, corresponding to a focus on the top part of the adaptive process, Wisdom (1991) has argued that "philosophy stands in need of *documentation*: we should try to produce evidence that the confusions which concern us are active in nonphilosophical discourse" (p. 4). In other words, we need to take into account the bottom part of the adaptive process, trying to find concrete examples in experience for the philosophical problems we grapple with in abstraction. We saw in section 2.2.3, that formally defined terms, which are prevalent in philosophy, often have a more specific and rigid meaning than their counterparts in everyday language use. And

while they have the potential to crystallize ideas, they can also lead to paradoxes that arise only in abstraction.

The opposing tendencies of the two aspects of adaptive processes can be seen more generally in philosophical theorizing. I discussed in section 2.3 how both Russell (1912/2001) and James (1916) noted that there are two general tendencies in philosophy; some people are more inclined towards one side, while others are more inclined towards the other. On the one hand, there is the empiricist perspective, which focuses more on generalizing from specific instances, which corresponds to the inductive aspect. On the other hand, there is the rationalist perspective, which focuses more on applying general or universal principles to specific instances, which corresponds to the deductive aspect. Although they leaned in different directions, both James and Russell acknowledged that both tendencies are needed for a balanced understanding of cognition.

In epistemology, there has traditionally been a dispute between the opposing theories of foundationalism and coherentism. Foundationalism, with its focus on hierarchy grounded in experience and causal relations, can be seen as more inductive. While coherentism, with its focus on consistency in abstraction and logical relations, can be seen as more deductive. Haack (1993) has proposed a balance between these with her theory of foundherentism, incorporating both what she calls 'experiential anchoring' and 'explanatory integration,' and arguing that a satisfactory theory for the justification of beliefs has to be partly causal, partly 'quasi-logical.' Through integrating these opposing views she resolved many paradoxes traditionally plaguing both one-sided views.

Beyond these there are many examples in philosophy that manifest either one or both of the opposing tendencies of adaptive processes.

Chapter 7

Conclusion

In this thesis, I have considered a minimal model for adaptive processes consisting of three parts: finiteness, invariance, and a process of a continuous interaction between an inductive-like aspect of forming invariances and a deductive-like aspect of applying invariances. I have tried to show that analogies, making one-to-one comparisons, form a fundamental mechanism underlying this process through our ability to recognize and construct invariances. I have attempted to show signs of the presence of both aspects of this process in the early development of logical thinking in three different intellectual traditions. And, I have considered classical logic in terms of this process. What have we learned from these considerations?

First, keeping in mind our finiteness forces us to realize that we have to continually mediate between extremes. On the one hand, we cannot consider each thing uniquely; it would be too much to process and there would be no way of distinguishing between things. As the Nyāya argued, particulars would be indistinguishable from one another without finite general notions. On the other hand, we cannot grasp the whole of reality either, since we are only a small part of it. Hence we have to mediate between the general and the specific. It also reminds us that our understanding of anything is necessarily based on finite experience, incomplete and fallible. Despite the obviousness of the aforementioned, we often live our lives convinced that we have a full grasp of certain things or know them with absolute certainty. If instead we take our finiteness, our necessarily fallible understanding of the world and other people, as a foundation for our thinking, bearing in mind our finiteness continually—as a first axiom or filter as it were for all of our reasoning—we will become better adapted and gain a better understanding of the world and ourselves. We will be more open to new experiences and views and value our own views and beliefs at a particular point in our lives less. We will become more humble and less likely to judge others for their views and beliefs at a particular point in their lives. We might allow more uncertainty and consciousness of our not knowing into our lives.

Second, we have seen the pervasiveness of the fundamental notion of invariance throughout our sense-making of any kind. We passively notice things staying the same in our experience—forming the basis for natural categories. We actively keep things the same on top of this—in the form of signs and symbols in language, for communication and argumentation, and ultimately in the abstract form of logic. From a finite perspective, finding things staying the same and keeping things the same is a way to construct explanatory and predictive models out of the complexity of our experience and to communicate efficiently. However, we sometimes tend to take such invariances as too rigid, perhaps even as absolute, independent of experience as in the case of classical logic. The Mohists remind us that we can see all reasoning as making comparisons to different types of models. The Nyāya show the importance of concrete examples to demonstrate the applicability of abstract rules to specific situations. Plato and Aristotle point out the different functions of analogies and the merits of studying similarities.

Third, we have seen that analogies provide the basis for noticing and constructing invariances, for perceiving things staying the same and keeping things the same, by treating different things as the same in specific respects. Incremental perceptual analogies provide the basis for inductive generalization from specific instances to general notions, i.e., categories. These general notions provide the basis for deductive application of them to new instances in experience. Conceptual analogies allow us to recombine things we already know in new and creative ways, to consider alternative explanations for past experience and come up with new hypotheses to test for predicting future experience. We have seen both tendencies in the early logical traditions of ancient China, ancient India, and ancient Greece. Moreover, we have seen that classical logic focuses on the deductive aspect in isolation, ignoring its broader roots. The continuous nature of the adaptive process reminds us to find a balance between opposites grounded in too much invariance or too much variance, between generality and specificity. Acknowledging our finite and incrementally growing understanding, we can recognize that our views at any one point in time are probably subject to change. Seeking out novelty and uncertainty can help us to maintain a balance and evolve.

Finally, I considered other instances of adaptive processes, beyond the prototypical example of cognition. James (1916) argued that philosophy is nothing other than a particular expression of our thinking. Haack (1978) argued that philosophy is continuous with science. Bartha (2010) argued that science involves a similar mediating process between opposing tendencies as in case law. All this points to a similar process involved throughout. Equipped with this model, can we not imagine there to be still more instances of adaptive processes, as extensions of our cognition or otherwise, perhaps even as other naturally arising processes? What would it say about sense-making beyond human cognition? This, then, is an attempt to make a new connection, to establish a new category, in search of new cases.

Notes

Chapter 2: Proposal of the Model

1. Levels of invariance should not be confused with levels of abstraction. While higher levels of abstraction tend to stay more constant in their meaning, there is not necessarily a complete correspondence, as things at low levels of abstraction can still be highly invariant.

2. Turner (1988) makes a distinction between our broader 'conceptual systems' and the more specific 'category structures' embedded within it, seeming to suggest that we make analogies in our broader conceptual systems, and arguing that new connections suggested by analogies first "become shallowly entrenched in our conceptual systems, relative to our more deeply entrenched category structures" (p. 5), before they potentially become entrenched in and alter the category structures. For the purposes of this thesis and the simplicity of the model, I simply use 'category structures' to refer to the whole, with degrees of entrenchment ranging from novel analogical connections to deeply entrenched categorical connections.

3. This does not mean that all natural categories are named. We can unconsciously categorize certain experiences and recognizing instances of it without having a name for it, without being able to express it in language because there might not be a word for it yet that has a shared meaning with other people, or we might not realize that there exists a word for it based on other people's similar experiences.

4. We need only consider Hume's concluding paragraph (1748/2007, p. 120, *E* 12.34):

   > When we run over libraries, persuaded of these principles, what havoc must we make? If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, *Does it contain any abstract reasoning concerning quantity or number?* No. *Does it contain any experimental reasoning concerning matter of fact and existence?* No. Commit it then to the flames: For it can contain nothing but sophistry and illusion.

5. The respective titles of their books describing their views on this, *Some Problems of Philosophy* by James, *The Problems of Philosophy* by Russell, is telling in this regard.

6. There is another correspondence here to Descartes, who distinguished between a "passive faculty of perception, that is, of receiving and taking knowledge of the ideas of sensible things," and an "active faculty capable of forming and producing those ideas," arguing that the former would be useless without the latter (1641/2008, p. 270). This corresponds to the interaction between passively perceiving things staying the same through perceptual analogies based on perceptual invariances on the one hand, and actively keeping things the same through conceptual analogies based on conceptual invariances on the other.

Chapter 3: Mohist Canons in Ancient China

7. Fraser (2020a), supplement *Textual History and Philological Issues*:

   > Unhappily, given their importance to our understanding of early Chinese thought, the later Mohist writings are among the most obscure and unreliable in all the ancient literature. Their complex content, technical terminol-

> ogy, extreme terseness, and difficult grammar render many passages vague, ambiguous, even impenetrable. This obscurity is due partly to the nature of the texts, which were probably a set of notes for school members familiar with their content, not a treatise for wide distribution. The interpretive difficulties are compounded by damage and corruption.

8. All translated passages of the Mohist Canons are from Fraser (2020b).

9. We could see in canon A5 and its explanation ("Knowing is connecting. […] Like seeing."), in making an analogy between knowing and seeing as both being a form of connecting of some kind, an interesting connection to both sides of the adaptive process: 'knowing is connecting' corresponds to connecting different things in thought through conceptual analogies, while 'like seeing' hints at an analogous making of connections between perceptually similar things in experience through perceptual analogies.

10. This suggests that the Mohists had a sense of the logical structure of language, of similarly structured sentences having a similar logical content. In this sense a connection could be made to the focus on natural language grammar as a basis for logical argumentation and theorizing in the Indian tradition, as we will see in Chapter 4. It also lends support to the idea of levels of invariance building on each other, with purely syntactic logical theorizing arising out of argumentative language use, as in the model proposed in Chapter 2. It gives pause for thought that the Mohists were most cautious regarding this particular type of argument form because the invariances are purely syntactic, detached from the semantic content of the compared expressions.

Chapter 4: Nyāya Sūtras in Ancient India

11. An interesting parallel here is that, like thinkers from the School of Names in the Chinese tradition (Fraser, 2020d), the Nyāya seem to have originally been criticized by other schools and considered sophists, for focusing on superficial logical form without proper regard for the authority of the Vedic scriptures. As translator Vidyābhuṣana points out in the introduction to the Nyāya Sūtras, the orthodox community of Brâhmanas, wanting to establish an organized society, were focused on rituals as guide to action, much like Confucius in ancient China. While the Nyāya can in fact be seen as seeking to establish standards for debate and inference, like the Mohists. There is another parallel in the subsequent dominance of Buddhism in ancient India and of Confucianism in ancient China, overshadowing the Nyāya and Mohist schools, respectively.

12. All translated passages of the Nyāya Sūtras are from Vidyābhuṣana (1913).

13. The development of a systematic interpretation and use of Sanskrit can be traced back all the way to *Aṣṭādhyāyī*, the tremendously influential grammatical treatise of Pāṇini, and the oldest fully extant linguistics and grammar text of any language. It is highly technical and complex, using a metalanguage and meta-rules.

    As Bajaj (2011) notes, much like Euclid's *Elements* provided a paradigm example for axiomatized formal theories in the Western tradition, Pāṇini's *Aṣṭādhyāyī* analogously provided a paradigm example for the Indian method of theory construction.

    Matilal (1998, p. 14) similarly notes:

> [H]istorically, from the time of the Greeks, the mathematical model played an important part in the development of logic in the West. In India, it was grammar, rather than mathematics, that was dominant, and logical theories were influenced by the study of grammar.

Aṣṭādhyāyī was a monumental achievement (Bajaj, 2011, p. 19):

> It provides a complete characterisation of valid Sanskrit utterances, a characterisation more thorough than what has been possible for any other language so far, by devising a system of description which enables one to generate and analyse all possible meaningful utterances. […] it is this systematic analysis of the Sanskrit language, which enabled Indians to develop a precise technical language of logical discourse.

14. The means of knowledge identified by the Sânkhyas—perception, inference, and verbal testimony—roughly correspond to the Mohists' personal knowledge, knowledge by explanation, and knowledge by hearsay. The Nyāya considered comparison as a fourth distinct means of knowledge; for the Mohists, all reasoning amounted to comparisons to models, according to Fraser (2020c).

15. Matilal (1998, pp. 29–30):

> The most common form of the substantive suffix in Sanskrit is *-tva* or *-tā* (comparable to English '-ness' or '-hood'). This mechanism of substantivization turns both adjectivals and nominals into words expressing the so-called abstract locatables. And a locatee-word can easily be turned into an adjectival by the use of possessive suffixes, *-vat*, *-mat* and *-in*. Sanskrit logicians use this double mechanism of substantivizing and possessive suffixes to assimilate the usual subject-predicate sentences into their locus-locatee model. […] Sanskrit logicians argue that the two operations—use of possessive suffix and substantivization—are reciprocal to each other. Hence, $x+vat+tva = x$ […] This means that as locatees or *dharmas*, it does not make a difference whether we say 'fire-possessing-ness' or 'fire.'

16. Rosch (1976) also investigated the role of expertise in natural categorization, how conceptual boundaries are drawn differently for categories in a certain domain that one has expertise in. When trying to make a convincing argument, it makes sense to rely on (exemplars of) categories that are well-established for most people, regardless of expertise.

17. An additional advantage could be that an example instance demonstrates that the reason and the to-be-inferred properties do, and thus *can*, actually occur together somewhere.

Chapter 5: Plato and Aristotle in Ancient Greece

18. All translated passages of Plato and Aristotle are from Cooper and Hutchinson (1997) and Barnes (1995), respectively.

19. "Rule-setters" is a fitting name for 'the people who introduced words,' from the perspective of words as referring to categories that encode general notions or rules, and the conventional nature of language.

20. Aristotle also credits Socrates to be "the first to raise the problem of universal definitions […] it was natural that Socrates should seek the essence. For he was seeking to deduce, and the essence is the starting-point of deductions. […] two things may be fairly ascribed by Socrates—inductive arguments and universal definition, both of which are concerned with the starting-point of science" (*Metaphysics*, XIII.4 1078b 17–28).

Chapter 6: Further Discussion of the Model

21. The idea of finite experience obviously extends beyond our personal experience to our collective experience as a species and even to all life on Earth, which on a cosmic timescale (an idea extrapolated from finite experience) has only existed and experienced reality for an exceedingly brief period of time.

22. That is, unless we take a deterministic interpretation of quantum-theoretical phenomena such as the pilot wave theory, which elegantly reconciles the wave–particle duality, and thereby indeterminacy, by postulating the simultaneous presence of both particles *and* waves, with so-called 'pilot waves' guiding particles.

23. The interaction of the two types of invariance extends into interpersonal contexts. If we decide to *keep something the same*, in our speech or our actions, then other people will experience this as *something staying the same*, just like any other thing staying the same in their environment; we add to the regularities in the environment for others and ourselves.

24. Wisdom also argued that there are only three 'operators' in philosophical disputation: "You might as well say…," "But this is different," and "Exactly so" (1991, pp. 19, 160). This corresponds to (analogical) arguments based on similarities, differences, and equality.

25. Here we can see perhaps most clearly the real, predictive value of inference, and logic more generally, by going beyond what is personally known, actual, and perceived, into the unknown, hypothetical, and abstract. The point is not that inference is redundant because it adds nothing new, but that in truth it serves a predictive function, making certain simplifying assumptions about things staying the same, allowing us to draw conclusions (perhaps more accurately described as 'predictions') beyond our direct experience.

26. We saw another instance of considering induction and deduction in isolation in Chapter 5 (p. 55), in Aristotle's analysis of analogical arguments as consisting of a single inductive step followed by a deductive step, as if separable from previous inductive experience.

27. Descartes (1641/2008, pp. 250–251):

> I observe that there is not only present to my consciousness a real and positive idea of God, or of a being supremely perfect, but also, so to speak, a certain negative idea of nothing, in other words, of that which is at an infinite distance from every sort of perfection, and that I am, as it were, a mean between God and nothing, or placed in such a way between absolute existence and non-existence, that there is in truth nothing in me to lead me into error, in so far as an absolute being is my creator; but that, on the other hand, as I thus likewise participate in some degree of nothing or of non-being, in other words, as I am not myself the supreme Being, and as I am wanting in many perfections, it is not surprising I should fall into error.

28. There is an interesting concern here with how constitutional legal rules can be revised in terms of themselves, known as the paradox of self-amendment (Suber, 1990). This brings up issues reminiscent of logical paradoxes such as the Liar paradox, though Suber notes that logical validity and legal validity are not the same, as "legal validity is a matter of power and social practice, not abstract correctness." Still, it is interesting to contemplate taking the analogy back to logic from the perspective of adaptive processes, how logic can revise itself, if everything being argued for relies on it.

29. From the Greek *heureka* meaning "I have found (it)," coming from the same root as heuristic (*heuriskein*), which, of course, is one of the functions of analogy.

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