WHAT VAGUE OBJECTS ARE LIKE*

We cannot trace with complete precision the outline of a cat, the limits of a city, or the edges of a cloud. Might cats, cities, clouds and other ordinary things really be vague, in the sense that they lack sharp boundaries? It is widely thought that they just cannot be. Vague things, it is thought, would be very queer: they would have a shady presence, being somehow neither quite there nor not there, and they would have vague identities, being somehow neither quite identical to nor different from other things. The idea that anything might really be vague is thought to be more or less unintelligible.

In fact, I shall argue, the hypothesis that ordinary things are vague can be taken quite seriously. It is intelligible, since vague things need not have any sort of shady presence or dodgy identity after all. It is plausible, since there is sometimes no fact of the matter whether one thing is a part of another. And it is readily acceptable, since indefinite parthood is substantially compatible with received ideas about parts and wholes, as set out in classical mereology. Arguably, as we shall see, this hypothesis offers us our best shot at preserving sensible everyday ideas about what there is in the world.

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I shall proceed as follows. Section 1 contrasts the hypothesis that ordinary things are vague with the orthodox view that vagueness is all a matter of thought and language. Section 2 explains why it has been thought that there must be something mysterious about the presence and identities of vague things, and why it is wrong to think so. Section 3 shows how it can happen that there is no matter of fact whether one ordinary thing is a part of another. Section 4 categorizes vague objects and motivates the introduction, in section 5, of a class of abstractions, fuzzy boundaries. The final section 6 uses these boundaries to construct a world of vague abstract objects whose presence and identities are quite definite. The logic of this world is not classical but, as we shall see, its part-whole theory very nearly is.

1. Vague objects and representations

Take Tibbles, the cat. The one loose whisker used to be firmly attached and soon it will drop off for good. Meanwhile it is a _questionable_ part, neither definitely in nor out. Or take the city of Toronto. The tree in the outskirts is a questionable part and so the city, like the cat, has no sharp boundary. Or take just about any ordinary material object. Look closely and there is always something tearing away or coming loose. Look even more closely and there are always microscopic particles wearing off at the edges or evaporating away. The world seems to be full of objects that lack sharp boundaries and in this sense really are vague.

Even so there might be no vague objects. Instead of a vague cat there might just be many precise quantities of animal tissue that almost completely overlap, differing around the edges by the odd whisker or hair. In this case each of these precise quantities has a claim to be called ‘Tibbles’ but none has a special claim, and this name fails to pick out any one candidate in particular. Questionable parts of Tibbles are then just things that are parts of some candidates but not others. Similarly there might just be many precisely delimited localities, equally deserving to be called ‘Toronto’, some including the questionable tree, some not. In this case the names ‘Toronto’ and ‘Tibbles’ are vague. They fail to pick out any one particular thing. But there is no vague city and there is no vague cat.

Orthodoxy has it that there are indeed no vague objects. Words, thoughts, pictures and other representations are vague, but apart from them the world is precise. Bertrand Russell was the first to say that it is so. He argued that to think the world is vague is to mistake properties of thoughts for properties of things, and blamed Idealism for cultivating confused habits of thinking that can lead to this sort of mix-up. One might as well think, he scoffed, that the world is also muddleheaded.[1] There is no mistaking the appeal of this idea. Certainly the relation

[1] Bertrand Russell, “Vagueness,” Australian Journal of Philosophy and Psychology, 1 (1923): 84-92.

between representations and what they represent is somewhat indefinite, and if this fact by itself can explain the phenomena of vagueness then it might seem idle to imagine any other vagueness in the world. It might seem simpler and better to suppose that, apart from representations in thought and language, all things are precise.

But this orthodoxy is hard put to explain some of the phenomena. Take an ordinary material object, say Tibbles.[2] If vagueness is all a matter of representation then there is no vague cat. Instead there are many precise cat candidates that only differ around the edges by the odd whisker or hair. Since there is a cat, Tibbles, and since this orthodoxy leaves nothing else for her to be, one of these cat candidates must then be a cat. But if any is a cat then also the next one must be a cat, so small are the differences between them. So all of the cat candidates must be cats. The sensible idea that vagueness is all a matter of representation seems to entail that wherever there is a cat there are a thousand and one of them, all prowling about in lockstep or curled up together on the mat. But that is absurd. Cats and other ordinary things can come and go one at a time.[3]

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[2] I shall use 'object' broadly to cover organisms, cities and abstract entities, as well as ordinary material objects.

[3] Versions of this puzzle were suggested by Peter Unger in "The Problem of the Many," Midwest Studies in Philosophy, 5 (1980): 411-468, and by Peter Geach in Reference and Generality, third edition (Ithaca NY: Cornell University Press, 1980), on p. 215.

Someone might try to contain the outbreak of cats by explaining that none of the candidates is a cat, but that is no levelheaded thing to do. It follows that there is no cat, not one, and by implication that there are no ordinary things at all in the world: no babies, no bath water, nothing. Commonsense balks.[4] One might instead explain that although there are many different candidates they are all one and the same cat.[5] Or that just one of them is a cat but there is no saying which, or that although all the candidates are different cats this is no matter since, so very similar, they are _almost_ one and the same cat.[6] These explanations do not fly in the face of commonsense; but there are many of them and none is obviously right. One starts to wonder, with all of this explaining away that orthodoxy has to do, whether the hypothesis that ordinary objects are vague might square up better with sensible ideas about what there is in the world.[7]

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[4] But Peter Unger does not balk: “There are No Ordinary Things,” _Synthese_, 41 (1979): 117-154.

[5] Peter Geach illustrates the notion of relative identity with his version of the problem of the many cats (_op. cit_.).

[6] As David Lewis explains in “Many, but Almost One,” in K. Campbell, J. Bacon and L. Reinhardt (_eds_.) _Ontology, Causality, and Mind: Essays on the Philosophy of D.M. Armstrong_ (Cambridge: Cambridge University Press, 1993), pp. 23-42.

[7] Others who think that various sorts of things might really be vague include Arthur Burks, “Empiricism and Vagueness,” _The_

2. Vague existence and identity

One obstacle to accepting the hypothesis that ordinary objects are vague has been the notion that vague objects must be very queer. In particular, it has been thought that they must have some sort of shady presence, and indefinite identities. In this section, we shall see that there is no good reason to think so.

The notion that vague objects must have a shady presence has an origin in the notion that if there are to be questionable parts then composition must be vague, in the sense that there is sometimes no matter of fact whether it takes place. A part of a

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Journal of Philosophy, 43 (1945): 477-486; Kit Fine, “Vagueness, Truth and Logic,” Synthese, 30 (1975): 265-300, f. 10; Mark Sainsbury, "Concepts without Boundaries," in R. Keefe and P. Smith (eds.) Vagueness: A Reader (Cambridge Mass.: MIT Press, 1987), pp. 251-264; Michael Tye, “Vague Objects,” Mind, 99 (1990): 535-557; John A. Burgess, “Vague Objects and Indefinite Identity,” Philosophical Studies, 59 (1990): 263-287; Peter van Inwagen, Material Beings (Ithaca: Cornell University Press, 1990); Eddy Zemach, “Vague Objects,” Nous, 25 (1991): 323-340; Richard Heck, “That There Might be Vague Objects (So Far As Concerns Logic),” The Monist, 81, 2 (1998): 274-296; Peter Simons, "Does the Sun Exist? The Problem of Vague Objects," in T. Rockmore (ed.) Proceedings of the XX World Congress of Philosophy, Vol. 2: Metaphysics (Bowling Green: Philosophy Documentation Center, 1999), pp. 89-79; Michael Dummett, "Is Time a Continuum of Instants?" Philosophy, 75 (2000): 497-515.

whole is something whose fusion with some other things makes up this whole. So, one might think, when there is no matter of fact whether one thing is a part of another, that must be because there is no matter of fact whether the questionable part and some other things compose to make it up.[8] But then, one might think, there can be no matter of fact whether something, a composition of the questionable part and these other things, exists.[9] Now <u>this</u> idea is genuinely mysterious. How can something neither quite be nor not be there? Must we imagine that the presence of vague objects is somehow a matter of degree, like the intensity of a beam of light?

There is no need, for such thinking is mistaken. In fact there can be vague objects though composition is precise, and indeed completely unrestricted. To see how, suppose that largely overlapping with Tibbles there is a cat candidate - say Tibbles

[8] Vann McGee claims that an argument for unrestricted composition "presupposes" that there is no vagueness in the world. See his “Kilimanjaro,” <u>The Canadian Journal of Philosophy</u>, 23 (1998): 141-163, p. 163. Peter Van Inwagen argues that questionable parthood and vague composition go together in his <u>op. cit</u>., <u>e.g</u>. p. 228. David Lewis seems to assume that unrestricted composition is at odds with worldly vagueness in <u>On the Plurality of Worlds</u> (Oxford: Blackwell, 1986). There, having rejected the idea of vague composition, he claims that the only intelligible account locates vagueness in thought and language. See p. 212.

[9] Compare van Inwagen, <u>op. cit</u>., pp. 272-273

minor - that is just like Tibbles except that the one whisker is a definite non-part of her. Since the whisker is a questionable part of Tibbles, we might suppose that there is no matter of fact whether the whisker manages to compose with Tibbles minor to make up Tibbles. Let it be so. The crucial point is that composition can take place even so. The whisker can still compose with Tibbles minor to make up <u>something</u>, it is just that the something cannot be Tibbles but must be something else - say Tibbles <u>major</u> - of which this whisker is a definite part, not a questionable part. The suggestion is that questionable parthood need not restrict composition at all, and the abstract vague world of section 6 will bear it out. Although the things within this world have fuzzy boundaries, there is always a matter of fact whether any given ones compose because they always do. Composition is completely unrestricted there, and nothing has any sort of shady presence.

A second mystery surrounding vague objects has been the idea that there can be no saying which are which, or how many there are. The very <u>identities</u> of vague objects must be vague. This idea led Russell to brush off the hypothesis that there are vague objects with the remark that “things are what they are, and there is an end of it,”[10] and it led Gareth Evans to argue more fully that since the identities of things are always definite, the hypothesis that there are “objects about which it is a <u>fact</u> that

[10] Russell, <u>op. cit</u>.

they have fuzzy boundaries” is incoherent.[11] The objects of the vague world of section 6 will refute this argument since not only their presence but also their identities are quite definite. But this matter ought not to hang over the discussion until the end. In the rest of this section, I shall show that the argument from definite identities is inconclusive.

I shall not call into question what I take to be the insight at the center of both Russell's remark and Evans' argument, that there is always a fact of the matter whether any given things are one and the same. That there might sometimes be no fact of this matter really is hard to imagine. Things are identical to themselves but different from other things, and it is hard to see how, somehow between itself and everything else, there can be anything left for a vague object to be indefinitely identical to. Talk about vague objects with indefinite identities really does not appear to be talk about anything at all.

Antirealists, thinking truth is bound up with methods for finding out, might disagree. For instance, a mathematical intuitionist might think that there is sometimes no matter of fact about the identities of choice sequences, conceived as entities that are not fully determinate in advance.[12] Perhaps,

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[11] Gareth Evans, “Can There be Vague Objects?” Analysis, 38 (1978): 208. See also Nathan Salmon, Reference and Essence (Oxford: Basil Blackwell, 1982), pp. 243-246.

[12] L.E.J. Brouwer described the construction by means of choice sequences of what one might take to be indefinitely equal real

despite appearances and arguments, the idea of indefinite identities is at least coherent.[13] Be this as it may, we shall have no use for it here. For a long time the suspicion has lingered that the idea that the world is vague has something to do with Idealism. A later section argues that ordinary material objects can be vague and, although the discussion there does not require it, to ward off suspicion that some sort of antirealism is involved we shall do well to rule out indefinite identities from the start. This is of course not to say that identity statements are always either true or false; they will still have truth-value gaps, as they must, when singular terms do not determinately denote.

The main problem with the argument from definite identities is just that there is no reason to think that things with fuzzy boundaries must have indefinite identities. Evans did not even try to establish this crucial matter; perhaps it did not occur to him that having a fuzzy boundary and having an indefinite identity might be different things.[14] However this may be, the

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numbers, in "Über Definitionsbereiche von Functionen," Mathematische Annalen, 97 (1927): 60-75.

[13] As Terence Parsons and Peter Woodruff argue in "Worldly Indeterminacy of Identity," Proceedings of the Aristotelian Society, 95 (1995): 171-191

[14] Mark Sainsbury was the first to distinguish them in print, in "What Is a Vague Object?" Analysis, 49, 2 (1989): 99-112.

omission hides a crucial difficulty with his argument that comes to light as soon as we try to make good.

To establish that fuzzy boundaries make for indefinite identities someone might reason as follows. Comparing a vague Tibbles to a precise Tibbles-candidate we see that they exactly agree as far as the definite parts and non-parts of Tibbles are concerned. So any difference between them must be among the questionable whiskers and hairs. But then, since there is no saying whether these are parts of Tibbles, there is no saying whether there is any difference at all between Tibbles and the candidate. In other words, there is no matter of fact whether Tibbles and the candidate are one and the same.

This reasoning overlooks something. If vagueness is a real characteristic of things, in particular of cats, then the possible differences between Tibbles and the Tibbles-candidate are not confined to the questionable whiskers and hairs, and in fact there is an obvious difference between them after all: Tibbles is vague but the candidate is not vague. There can be no question whether they are identical, for they definitely are two different things, not one, and nothing in between.[15] Evidently we

[15] Leibniz' law says that if _o_=_p_, then whatever is true of _o_ is also true of _p_. Here I use the contrapositive, which might seem unjustified. Truth must pass from the antecedent of an instance of Leibniz' law to the consequent, but where there can be truth-value gaps perhaps falsity need not pass the other way: arguably, the consequent might be false while the antecedent has a truth-

have completed the argument from definite identities only by setting aside the possibility that the vagueness of things is real. Since its reality is just what is at issue, this argument begs the question.

The argument from definite identities might be construed as an attempt to show that nothing is vague in the special sense of being indefinitely identical to something. Some have asked whether objects can be vague in this special sense,[16] and although Evans speaks of objects with “fuzzy boundaries” perhaps he too had this in mind. Be this as it may the argument, so construed, remains inconclusive. It does not even touch the question whether things can be vague in the core sense that they lack sharp boundaries.

In summary, we can agree with Russell that things are what they are. They are identical to themselves, different from other things, and indefinitely identical to nothing. Still the world might be vague since, for all anyone has said, also things without sharp boundaries can have definite identities. Also they can be "what they are." And there is an end of the argument that, since identities are definite, the world must be precise.

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value gap. Crucially, no one can say so in defense of Evans' argument. It uses the contrapositive of Leibniz' Law to establish that identities are always definite (op. cit.).

[16] E.g. Francis J. Pelletier, in “Another Argument Against Vague Objects,” The Journal of Philosophy, 86 (1989): 481-492, p. 492

3. Questionable parts

In this section, I shall argue that ordinary material objects can have questionable parts. I shall begin by contrasting them with certain special material objects that apparently cannot have any. I have in mind scatterable objects picked out by expressions like ‘some Styrofoam' and ‘all the soap in China', which we shall call quantities of matter.[17]

We can grasp the notion of questionable parthood by thinking of parts that have come loose and will be lost. But this will not help us to grasp the notion of questionable inclusion among quantities of matter because these cannot gain or lose parts. Take say a quantity of soap, and separate some from the rest. Afterwards the soap that you began with will still include the separate part; it will still include it even if you send the separate part somewhere else altogether, in which case some of the soap will end up there and some will remain here. You can scatter a quantity of something but, short of destroying it, you cannot keep some of it from being included in all of it. Quantities of anything – of matter, space, time or what have you - have their parts essentially.

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[17] Helen Morris Cartwright discusses these objects at length in “Quantities,” Philosophical Review, 79 (1970): 25-42. See also Tyler Burge, "Truth and Mass Terms," The Journal of Philosophy, 69 (1972): 263-282, and Dean Zimmerman, "Theories of Masses and Problems of Constitution," Philosophical Review, 104, 1 (1995): 53-110.

This is one reason to think there must be more to part-whole relations among material objects than just inclusion among quantities of matter. Unless ordinary thinking is very mistaken a cat can lose a whisker, and when it does the cat continues to exist although the whisker is no longer a part of it. Part-whole relations among organisms and other material objects, unlike the relation of inclusion among quantities of matter, are temporally variable. A further reason to think there must be more to part-whole relations than material inclusion is that the matter of one object can be included within that of another without their being related as part to whole. Among the parts of a cat we count a head and a tail, but we do not count the material content of an arbitrarily marked out region within the cat. We do not count say an arbitrary fusion of cat parts, a head-tail, if there is such a thing.

It seems that, besides the eternal relation of inclusion among quantities of matter, there is a fuller and temporally variable part-whole relation among ordinary material objects. English provides a handy distinction: an arbitrarily marked off portion of cat is "some" of the cat and "part" of it, its matter included in that of the cat, but it is not fully "a part" of the cat.[18] Plausibly, if something is to be a part of the cat then, as well as being some of the cat, it must play a suitable role in the

[18] This observation is Richard Sharvy's, in "Mixtures," Philosophy and Phenomenological Research, 44 (1983): 607-624.

life of the cat - as the head and the tail taken separately both do, but the head-tail does not.

Someone might say that there is no such temporally variable part-whole relation at all. Causal relationships among things are apt to change from time to time so, one might think, we honor an eternal part of the cat as “a part” of the cat if and when it plays a suitable role in the life of the cat, but that is it. The alleged temporary parts are just eternal parts temporarily playing suitable roles. If this is right, though, then cats are no ordinary things. If a cat does not lose the whisker that drops off or the carbon-dioxide molecule in respiration then it is a highly scattered being. The cat is partly on the mat, partly floating around freely, partly built into the shrubbery, and by maturity it will have taken on the roughly the same size and shape as the entire biosphere. But this is absurd. Organisms can gain and lose parts.

This is why it is plausible that organisms are vague. Organisms gain and lose their parts <u>gradually</u>. Since there is no precise moment at which the whisker quits its role in the life of the cat and drops off, it is plausible that it does not instantly change from a part to a non-part but is, for a time, a questionable part of the cat. Indeed it is plausible that organisms, maintaining themselves in continuous metabolic flux, are much more vague than meets the eye. A carbon-dioxide molecule in respiration also takes a while to quit the small part that it plays in the life of the cat. Plausibly it too is, for a time, a questionable part of the cat.

How can material objects be vague if, as appears to be the case, quantities of matter are precise? Not, we have assumed, by being indefinitely identical to quantities of matter. But there is another way. Suppose there is a special relation of constitution, other than identity, in which say a statue stands to a quantity of bronze, or a cat to a quantity of animal tissue.[19] Then material objects can be vague if they are indefinitely constituted by quantities of matter without being indefinitely identical to them. In this case there is a formal parallel between the vagueness of representations, and on the other hand the vagueness of indefinitely constituted objects. The name 'Tibbles' is vague if there is no matter of fact which of several candidates it picks out; the cat Tibbles is vague if, having questionable parts, there is no matter of fact which of

[19] Those who separate constitution from identity include Tyler Burge, in op. cit., p. 278 and in "Mass Terms, Count Nouns and Change," Synthese, 31 (1975): 459-478; David Wiggins in Sameness and Substance (Oxford; Blackwell, 1980), pp. 30-35; Jonathan Lowe, in "The Problem of the Many and the Vagueness of Constitution," Analysis, 48 (1982): 27-30; Mark Johnston in "Constitution is not Identity," Mind, 101 (1992): 89-105; and Dean Zimmerman in op. cit.

several candidates constitutes her.[20] If neither quantities of matter nor identities are vague, the idea that ordinary material objects are vague is about as good as the idea that they are constituted by quantities of matter but not identical to them.

Just how good this is depends on another question. The distinction between constitution and identity goes with the doctrine that ordinary material things are continuants, persisting through time and meanwhile undergoing changes. It is with this distinction that one can explain how it is that a continuant cat is made up of different quantities of matter at different times. One who accepts the doctrine that ordinary material objects are continuants ought to have no special difficulty with the idea that they can be vague. The alternative is that material objects persist through time only in the improper sense that, like processes and events, they have earlier and later temporal parts.[21] If material objects have temporal parts there is no need to distinguish constitution from identity. We can explain that an object has a variable makeup insofar as it is identical-for-a-while to each of several different quantities

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[20] Vague constitution has been proposed before as a solution to the problem of the many. See Jonathan Lowe, op. cit., and Mark Johnston, op. cit., p. 101.

[21] This is the view of Goodman and Quine, “Steps Toward a Constructive Nominalism,” Journal of Symbolic Logic, 12 (1947): 105-122. See also Quine, Word and Object (Cambridge MA: MIT Press, 1960), p. 171.

of matter, having a different temporal part in common with each one. Then it is hard to see how material objects can be vague. They must be as precise as the quantities of matter to which they are, for a while, identical.

In conclusion, the hypothesis that ordinary material objects are vague fits best within the doctrine that they are three-dimensional continuants. The doctrine of temporal parts might perhaps be made to admit vague objets anyway, by adding on a distinction between constitution and identity, but such a construction is liable to lack integrity. It might also be made to admit them by allowing that there are vague quantities of matter, or that identity is indefinite; then objects might be identical-for-a-while to vague quantities, or vaguely-identical-for-a-while to precise ones. Also these ideas are unattractive.

Let us now put organisms to one side, turning instead to social entities such as cities, nations, clubs and families. I shall argue that they can be vague because of indeterminacy in the relations of <u>belonging</u> in which a city stands to its environs and a nation to its territories, and the relations of <u>membership</u> in which a club or family stands to the people that make it up. Interestingly, we can speak of belonging, membership and parthood in much the same terms. The Cook Islands “belong to” New Zealand or else they are “a part of” that nation; someone “belongs to” a family or club or else he is “a member of” or “a part of” it. Without going into how these matters are related outside of idiom, I shall also speak of them in the same terms.

We shall concentrate on possession. Possession is sometimes questionable because it is established on the balance of _prima facie_ claims. If you happen to find some trivial item then the fact that you did makes it yours – unless someone else finds it at the same moment, in which case the two of you will have competing claims and it will be questionable whether the item is yours. Similarly, what made the Cook Islands a part of New Zealand was a legal act of annexation, undertaken in 1901 after petitioning by the chiefs of the larger islands. As it happened parthood was duly established on the balance of _prima facie_ legal reasons, but questionable parthood could have resulted had there been a conflict of reasons. We can imagine a case. Suppose that, having quarreled, half of the chiefs had petitioned New Zealand for annexation but the other half Australia, and suppose that each country had proceeded in a way that, had it proceeded alone, would have resulted in its having sovereignty over the Cook Islands. Then there would have been conflicting claims to sovereignty and the Cook Islands would have been a questionable part of New Zealand and also of Australia.

This explains how various materially extended things can have fuzzy boundaries. In the following section, I shall distinguish among varieties of vague things according to how deeply their vagueness is compounded, and how deeply it is embedded within their part-whole structure. This will motivate the notion of a fuzzy boundary in section 5, and the construction of an abstract vague world in section 6.

4. Varieties of vague objects

Natural change is continuous. Just as there is no precise moment at which say a molecule becomes a non-part of the cat, there is none at which it becomes a questionable part, either. As well as a first-order penumbra of questionable parts, of things so to speak in limbo between parthood and non-parthood, a vague object can have second-order penumbrae of objects that questionably are questionable parts, in limbo either between parthood and questionable parthood, or between questionable parthood and non-parthood. There can be third-order penumbrae of objects in limbo between penumbrae of lower orders, and so on. Ranking vague objects according to the maximal orders of their penumbrae, we can distinguish between objects of different orders of vagueness.

The continuous nature of change is one source of higher-order vagueness. Where parthood is established on the balance of reasons, as it seems to be in the case of countries, and where questionable parthood is the result of conflicts between reasons, conflicts between higher-order reasons are a further source. Suppose there are secondary reasons that can resolve conflicts between primary reasons, and with them questions about parthood, by excluding primary reasons from the balance of reasons.[22] In this case, whether there is a conflict between primary reasons

[22] Joseph Raz introduced “exclusionary reasons” in Practical Reason and Norms (London: Hutchinson, 1975).

and whether one thing is a questionable part of another will turn on which primary reasons go on the balance. Where higher-order reasons conflict among themselves it can be questionable which ones go on the balance, and therefore questionable whether one thing is a questionable part of another.

This discussion of higher-order vagueness brings us now to the notion of a sharpening of a vague object, which is to play a central role in the coming sections. Penumbral objects have an unclear status, in limbo between penumbrae of lower orders (we count the categories of parts and non-parts as zeroth order penumbrae). Now take any vague object and imagine hypothetically clarifying the status of those objects within its highest-order penumbrae, by distributing them among suitable penumbrae of lower orders. For each way of doing this we can imagine a slight sharpening of the object in question, an object that is less vague by one order of vagueness. Do not imagine that a slight sharpening is right there, largely overlapping with the vague object whose sharpening it is. The vague object could have had clearer relations to the objects within its penumbrae, and had the world been slightly different – had the whisker been more firmly attached, had the chiefs been less quarrelsome – it would have had clearer relations to them. A sharpening of a vague object is a possible object that does have clearer relations to them.[23]

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[23] Slight sharpenings do not bring with them a new problem of the many. The cat is here with us in this possible world. Her many slight

We can say that the sharpenings of a thing are just it but in another possible worlds.[24] Or else we can say that they are its counterparts there.[25] The alternatives are familiar and so is the use to which we shall put sharpenings, in the interpretation of *de re* modal claims. I shall not say more about the nature of sharpenings here because it does not matter, in the end, whether there are any such things. In the following section, I shall introduce a class of abstract objects, the fuzzy boundaries of vague cats, countries and other things. Some of these boundaries are sharper than others. What really matters is that we can evaluate claims about how things would be, were they to have boundaries that are sharper than their actual boundaries. If the correct treatment of such counterfactual claims does not call for possible objects, then all is well and good. Then sharpenings will be suggestive fictions with a purely heuristic role.

Let us now turn to another matter. Both her head and a protein molecule are parts of Tibbles, but they are parts of different levels. A cat falls apart immediately into head, torso and tail, but only after much further dismemberment do we finally reach

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sharpenings are distributed among as many other possible worlds. Nowhere are there many overlapping cat sharpenings.

[24] Compare Saul Kripke, “Naming and Necessity,” in Donald Davidson and Gilbert Harman (*eds*.), *Semantics of Natural Languages* (Dordrecht: Reidel, 1972).

[25] Compare David Lewis, “Counterpart Theory and Quantified Modal Logic,” *The Journal of Philosophy*, 65 (1968): 113-126.

molecules, atoms and so on. Her head is an immediate part of a cat; molecules, atoms and so on are not. Of course, there is no uniquely correct way to render things into their immediate parts, and these in turn into theirs, and so on: there are more ways than one to carve up a cat. In what follows, I presuppose some or other hierarchy of parts.

Let us call a questionable immediate part any part of an object that, intuitively speaking, has what it takes to be an immediate part except that it is a mere questionable part. It is a questionable part that would be an immediate part if it were a part of the object in question; it is an immediate part of all sharpenings of which it is a part. Then we can distinguish as follows between two sorts of vague objects, according to the level at which vagueness appears within their part-whole hierarchies. An object is immediately vague if it has questionable immediate parts. The vagueness of such an object is manifest at the top level of its parts. Otherwise an object is immediately precise. Any vagueness is to be found only at lower levels of the part-whole hierarchy.

For instance Tibbles, like any normal cat, is immediately precise. Her head is firmly stuck on and so are her paws, tail and so on; she does have questionable parts, but they are just whiskers, hairs, molecules and other things further down the part-whole hierarchy. None of her questionable parts would be, were it a part of Tibbles, a top-level part. She has no questionable immediate parts. Typically a country is also immediately precise. At the borders there are questionable clods of earth and clumps of grass, but there is nothing that, were the borders sharpened up, could be counted among major geographical regions, infrastructure systems and so on as an immediate part of the country. There are no questionable immediate parts. New Zealand, in the imaginary case of the territorial dispute about the Cook Islands, is immediately vague. Among the questionable parts of this nation there is a major geographical region, the Cook Islands, which would be an immediate part, if it were a part.

5. Fuzzy boundaries

Since the whisker is coming loose it is incorrect to assert:

> The whisker is a part of Tibbles' head.

It is also incorrect to assert:

> The whisker is a part of Tibbles.

Suitable "many valued" truth tables can account for this, by allowing these sentences to go with truth-value gaps, neither true nor false.[26] But truth tables cannot account for the fact that it is correct to assert:

> If the whisker is a part of Tibbles' head, then it is a part of Tibbles.

[26] See Steven Blamey "Partial Logic," of D. Gabbay and F. Guenthner (eds.) Handbook of Philosophical Logic, Vol. 3 (Dordrecht: Reidel, 1986), pp. 1-70.

since there is no truth-functional difference between this sentence and another, gotten by substituting other constituents with truth-value gaps. Acceptable truth tables do not call it true because they do not call all of them true. So what makes this sentence assertible? If not truth, what does it have that its constituents and all of the unassertible substitution instances lack?

It has super-truth, truth no matter how completely sharp boundaries are drawn for everything, whisker, head and cat. In this section, I shall introduce a class of abstract objects: fuzzy boundaries, some sharper than others. At the end of this section, I shall use these boundaries and their sharpenings to illustrate the supervaluation of claims about vague objects.[27] In the final section, I shall use them to construct an abstract domain of vague objects and to answer questions about vague composition, existence and identity.

We can think of a sharp boundary as the vanishingly thin outline of a precise object, perhaps as a suitable set of

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[27] Henryk Mehlberg first made such a proposal in connection with vague singular terms in The Reach of Science (Toronto: University of Toronto Press, 1958). Bas van Fraassen coined the term 'supervaluation' in “Singular Terms, Truth-Value Gaps, and Free Logic,” The Journal of Philosophy, 63 (1966): 481-495. Kit Fine's op. cit. is the most sophisticated supervaluational treatment.

points.[28] And we can think of a <u>fuzzy</u> boundary as the more or less thick and fuzzy outline of a vague object, within which a range of vanishingly thin outlines can be drawn. That is, we can think of it as a <u>constraint</u> on the drawing of sharp boundaries. To begin with, we can make this idea precise by thinking of a fuzzy boundary as a set of sharp boundaries, the ones that it allows to be drawn. To sharpen a fuzzy boundary is then just to choose from among its elements.

This conception is familiar and good, but it is in two ways too simple. Firstly, fuzzy boundaries so conceived do not reflect the higher-order vagueness of objects. A suitable elaboration is well known, though. We can let fuzzy boundaries be <u>sets</u> of boundaries, sharp or fuzzy. Then the order of an object's vagueness will be reflected in the degree of nesting of its boundary.[29]

A second way in which this conception is too simple is that it does not require fuzzy boundaries to reflect the part-whole structure of objects. To see why they ought to, imagine sharpening up the boundaries of some vague objects by counting questionable parts either in or out. Clearly we must consider together any objects that are related as parts and wholes since,

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[28] Compare Richard Cartwright, "Scattered Objects," in his <u>Philosophical Essays</u> (Cambridge, MA: MIT Press, 1987), pp. 171-186.

[29] Compare Kit Fine's notion of an "n-th order boundary," <u>op. cit.</u> page 293.

for instance, to count a loose whisker a part of Tibbles' head just _is_ to count it a part of Tibbles. So far, though, there is nothing to ensure this. We seem to be quite free to choose from the boundary of the head an alternative that includes the whisker, but from the boundary of the cat an alternative that excludes it. To remove the illusion of such freedom we shall further elaborate boundaries by adding internal boundaries, the boundaries of parts. Later this will suitably constrain the sharpening of boundaries.

Let us begin with a stock of simple boundaries. These are to be the boundaries of mereological atoms and we shall assume that they are sharp. Now, taking simple boundaries as our basis, we shall inductively define successive levels of fuzzy boundaries. At each successor level there are two sorts of complex boundaries. There are non-empty sets of boundaries of lower levels, marked as _sharp collections_. These are to be the boundaries of immediately precise objects. And there are other non-empty sets of boundaries of lower levels, marked as _vague collections_. These are to be the boundaries of immediately vague objects.

What is it for an object to _have_ a boundary? Mereological atoms have basic boundaries, and only they have them. An object has a given sharp collection as its boundary if and only if this object is immediately precise, each of its immediate parts has a boundary within this collection, and each boundary within this collection is the boundary of one of its immediate parts. And an object has a given vague collection as its boundary if and only if this object is immediately vague, each of its slight sharpenings has a boundary within this collection, and each boundary within this collection is the boundary of one of its slight sharpenings.

Suppose Tibbles is immediately precise and, simplifying for the sake of the illustration, that her immediate parts are just head, torso and tail. Suppose also that each of these immediate parts is immediately vague, having whiskers or hairs as its questionable immediate parts. Then, picturing sharp collections as rectangles and vague collections as ellipses, Tibbles' boundary is, as far as the outer two levels of structure are concerned, as in figure 1. Continuing to oversimplify, suppose her head has just a single questionable immediate part, the whisker. Then it has just two slight sharpenings, with and without. Secondly, let us declare the other parts of Tibbles' head (two ears, another five whiskers and so on) to be honorary atoms. The boundaries of the slight sharpenings of the head are then as pictured in figures 2a and 2b. Finally, the boundary of Tibbles' head is the vague collection of figure 3, a detail from figure 1.

Now let us turn to the “drawing” of sharper boundaries. I shall describe incremental sharpening functions that, when applied repeatedly to fuzzy boundaries, yield completely sharp boundaries, within which there are no vague collections. Informally, the idea is to work from the outside of boundaries inwards, as we go replacing each vague collection with our choice from among its elements. Choices are to be consistent: we shall always choose the same element from any given vague collection, irrespective of the context in which we run up against it. When we run up against a sharp collection we pass straight to its elements, since to sharpen up an immediately precise object is just to sharpen up its immediate parts and then to put the results back together as they were. When we reach a basic boundary we stop, since these are already sharp.

Technically, a way of sharpening boundaries is a choice function $\underline{f}$ that maps vague collections onto their elements: for each vague collection $\underline{V}$, $\underline{f}(\underline{V})\in\underline{V}$. For example, the choice function $\underline{f}$ of figure 4 maps the boundary of Tibbles' head onto the boundary of the sharpening of which the loose whisker is a part. Another, say $\underline{g}$, maps this boundary onto the other alternative, without the whisker. To extend choice functions to arbitrary boundaries we put $\underline{f}(\underline{B}) = \underline{B}$ if $\underline{B}$ is a basic boundary, and $\underline{f}(\underline{S}) = [\underline{f}(\underline{s}):\underline{s}\in\underline{S}]$ if $\underline{S}$ is a sharp collection ('[' and ']', like rectangles, enclose sharp collections).

In figure 5, $\underline{f}$ maps Tibbles' boundary onto a precise boundary within which the boundary of the whisker is embedded. Here, oversimplifying, the result of a single application of $\underline{f}$ to the boundary of Tibbles' head is completely sharp. In general it takes longer to achieve complete sharpness but we always get there in the end, since each application of $\underline{f}$ reduces the degree of nesting of vague collections. Finite iteration maps any boundary _B_ onto a completely sharp boundary _f_*(_B_), the _complete sharpening_ of _B_ by _f_.[30]

Take some vague objects. We have seen how to sharpen up their boundaries slightly using choice functions, and we can suppose that there are, in other possible worlds, objects that have the slightly sharper boundaries. Call these possible objects _slight sharpenings_ by these functions of these objects. Finite iteration leads to completely sharp boundaries and _complete sharpenings_. Now we can allow that a claim is super-true of some objects if it is true no matter how they all get sharpened up. Letting super-truth of claims about completely sharp objects be truth according to some antecedently given evaluation scheme, super-truth passes from claims about completely sharp objects to claims about vague objects of ever increasing orders of vagueness, one order at a time:

Super-truth condition: A statement about some vague objects is _super-true_ (_super-false_) if and only if, for every choice function, it is super-true (super-false) of their slight sharpenings by that function.

For instance:

The whisker is a part of Tibbles' head.

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[30] For any sharp collections $\underline{S}_i$, let $\cup_i\underline{S}_i$ be the sharp collection [$\underline{s}$: for some $\underline{i}$, $\underline{s}\in\underline{S}_i$]. Now for any choice function $\underline{f}$, $\underline{f}(\cup_i\underline{S}_i) = \cup_i\underline{f}(\underline{S}_i)$. Provided $\cup_i\underline{S}_i$ is a boundary (the $\underline{S}_i$ must be bounded in their complexity, say finite in number) $\underline{f}^*(\cup_i\underline{S}_i) = \cup_i\underline{f}^*(\underline{S}_i)$. We shall need this fact in the next section.

is neither super-true nor –false. The complete sharpening of the whisker by the earlier choice function f is a part of the complete sharpening of Tibbles' head by f, but the same does not hold for the complete sharpenings by g. For the same reason:

> The whisker is a part of Tibbles.

has a super-truth gap.[31] But the "penumbral truth":

> If the whisker is a part of Tibbles' head, then it is a part of Tibbles.

is super-true. This is just because any sharpening that like f counts the whisker a part of the head must also count it a part of the cat, since the head is a part, indeed an immediate part, of the cat. Choice functions respect part-whole structure.

Fuzzy sets, having their elements to a greater or lesser degree, are, like my fuzzy boundaries, a kind of vague abstract object.[32] Some think they can contribute to an understanding of the composition and identity of vague objects.[33] But there is a

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[31] I assume that 'a part of' expresses a relation among completely precise objects that is at least as inclusive as the ancestral of the immediate part-whole relation. Any immediate part of … an immediate part of an object is to be a part of it.

[32] Lofti Zadeh, "Fuzzy Sets," Information and Control, 8 (1965): 338-353.

[33] Van Inwagen uses fuzzy sets to explain vague composition in his op. cit., pp. 221-224. Jack Copeland uses them to explain vague identity in "Fuzzy Logic and Vague Identity," The Journal of Philosophy, 94 (1997): 514-534.

familiar and crucial difficulty with the use of fuzzy sets to explain vague composition. Standard many-valued interpretations of fuzzy logic are truth functional.[34] Such a treatment runs afoul of penumbral truth.

What of the vagueness of singular terms? Supposing neither Tibbles nor her head is vague, but instead the terms 'Tibbles' and 'Tibbles' head' are, we can still say that the penumbral sentence is super-true. Now we shall mean by this true on all ways of resolving the reference of these terms. But we can say so only if we can somehow coordinate the resolution of singular terms which, like these two, pick out objects related as parts to wholes (or which come to do so, as a result of resolution). The idea is familiar in outline but the details have still to be filled in.[35] One cannot simply mimic this treatment since, unlike these two, singular terms do not in general reflect the part-whole structure of things.

Taken at face value, supervaluation does something amazing: it reconciles vagueness with classical logic, finding truth-value gaps where, in view of the super-truth of the law of the excluded middle, it might seem that there can be none. But maybe this is just a trick, worked by passing off as truth something that

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[34] Joseph Goguen, "The Logic of Inexact Concepts," Synthese, 19 (1969): 325-373.

[35] See Hartry Field, “Quine and the Correspondence Theory”, Philosophical Review, 83 (1974): 200-228; and see Kit Fine’s notion of “penumbral connection” in his op. cit.

really is no such thing. Arguably truth comes to a disjunction of two sentences only if one of them is true, but a disjunction can be super-true though neither is. Truth is disquotational, but super-truth is not.[36]

I suggest that super-truth is not really truth but a surrogate. The assertibility of a sentence turns its truth value if it has one, but otherwise on its super-truth value. Indeed, it appears that we cannot identify truth with super-truth if we insist that identities are definite. In any interesting vague world there will be some things that have some but not all complete sharpenings in common. If super-truth were truth, then the sentence expressing the identity of any two such objects would, incorrectly, have a truth-value gap.

This completes the discussion of fuzzy boundaries. In the following section, I shall use these boundaries to construct an abstract world of vague objects. We shall see that there need not be anything mysterious about vague objects. These ones do not have any sort of shady presence or vague identities. In fact, as we shall see, the idea that there are vague objects is compatible with received ideas about parts and wholes, as set out in classical mereology.

6. Vague Mereology

[36] Compare Timothy Williamson, Vagueness (London and New York; Routledge, 1994), p. 162

What is the part-whole theory of vague objects? We shall take the classical theory as our starting point. Letting 'part of' be primitive, one thing overlaps with another if they have some part in common. One thing is a fusion of some others if and only if any further thing overlaps with it, the fusion, if and only if this further thing overlaps with one or other of the things fused. A fusion is pieced together from the things fused, with no part coming out of the blue. The first three basic propositions of classical mereology are:

> Transitivity: The parts of the parts of anything are parts of it.
>
> Unrestricted composition: Any things have a fusion.
>
> Unique composition: Any things have a unique fusion.

A proper part of a thing is any part except the whole thing. Mereological atoms cannot be identified from their proper parts since they have none; but, according to the fourth basic proposition, everything else can be identified from its proper parts:

> Extensionality: If any given things have proper parts, and they have the same proper parts, then these things are identical.[37]

Classical mereology does not say much about things. It does not say whether they have temporal as well as spatial parts. Nor does it say whether they are made up of atoms or can be divided

[37] For a thorough survey of classical mereology see Peter Simons, Parts: a Study in Ontology (Oxford: Clarendon, 1987).

indefinitely. But it might be thought that classical mereology says that things are precise. Certainly it has been thought, as I noted in section 2, that if things have questionable parts then composition must be vague, and thus restricted. It might also seem that if things have questionable parts there ought to be two things with the same proper parts but different questionable parts, refuting extensionality. In this section, I shall show that the four basic propositions of classical mereology do <u>not</u> say that the world is precise.

I shall proceed by validating mereological propositions within an abstract domain of vague objects. Two ideas underlie the construction of this domain. The first is to take fuzzy boundaries themselves as objects - as vague objects that are their own boundaries. The second is to generalize the part-whole and fusion relations from precise to vague objects using a sort of ontological pendant of supervaluation. One thing - precise or vague - will be a (generalized) part of another if it will be a part no matter how both are made completely sharp. A (generalized) fusion of some things will be something that is pieced together from them no matter how everything in question is made completely sharp. That is, it will be a thing such that any complete sharpening of it is a fusion, in the classical sense, of the corresponding complete sharpenings of the things fused. Proper definitions will follow. Meanwhile, notice that I shall not spell out generalized fusion, in analogy to the classical notion, in terms of the generalized part-whole relation. Instead

I shall spell out both generalized notions in terms of sharpenings and the classical part-whole relation.

Accordingly we now need some precise objects with a classical fusion operation, some vague objects, and sharpening functions that map the vague onto the precise objects. Firstly the:

Precise objects: These are non-empty sharp collections of simple boundaries. The part-whole relation among them is set-theoretic inclusion ($\subseteq$) and fusion is union ($\cup$).

Clearly the part-whole theory of these objects is classical. In defining the vague objects we shall run together all fuzzy boundaries that, no matter how they are made completely sharp, turn out to be of the same simple stuff. To compare the stuff of *sharp* boundaries, (nested) sharp collections of simple boundaries, we shall use an homogenizer $\underline{H}$. This is a function that turns any sharp boundary into a precise object, in the above sense, by adding or removing structure as needed. For any simple boundary $\underline{B}$ we put $\underline{H}(\underline{B}) = [\underline{B}]$; and for any sharp boundary $\underline{S}$, we put $\underline{H}(\underline{S}) = \cup_{\underline{s}\in\underline{S}}\underline{H}(\underline{s})$. We say that boundaries $\underline{B}$ and $\underline{C}$ are *equivalent* if and only if for every choice function $\underline{f}$: $\underline{H}(\underline{f}^*(\underline{B})) = \underline{H}(\underline{f}^*(\underline{C}))$. $\underline{B}^{\approx}$ is the class of all boundaries equivalent to $\underline{B}$. Now we have the:

Vague objects: These are equivalence classes $\underline{S}^{\approx}$ of sharp collections, $\underline{S}$.

It remains first to define sharpening functions that map vague onto precise objects, and then to extend the part-whole and fusion relations from precise to vague objects. Vague objects will take their sharpenings from their representatives: for any choice function f we let $f^*(S^{\approx})$, the *complete sharpening* of $S^{\approx}$ by f, be $H(f^*(S))$. Notice that $f^*(S^{\approx})$ is always a precise object, in the sense of this construction. We generalize the part-whole relation by maximizing definite parthood and non-parthood. Thus $S^{\approx}$ is a (*definite*) *part* of $T^{\approx}$ if and only if for each f: $f^*(S^{\approx}) \subseteq f^*(T^{\approx})$, and $S^{\approx}$ is a (*definite*) *non-part* of $T^{\approx}$ if and only if for each f: $f^*(S^{\approx}) \not\subseteq f^*(T^{\approx})$. If neither a part nor a non-part, then $S^{\approx}$ is a *questionable part* of $T^{\approx}$. Clearly these objects really do have questionable parts. For example, letting A and B be basic boundaries, it can be seen that $[A]^{\approx}$ is a questionable part of $[<B,[A,B]>]^{\approx}$. (‘<’ and ‘>’, like the earlier ellipses, mark vague collections.) For, letting f and g be such that $f(<B,[A,B]>) = [A,B]$ but $g(<B,[A,B]>)= B$, we see that $f^*([A]^{\approx}) = [A] \subseteq [A,B] = f^*([<B,[A,B]>]^{\approx})$, but $g^*([A]^{\approx}) = [A] \not\subseteq [B] = g^*([<B,[A,B]>]^{\approx})$.

Although these objects are vague there is nothing shady about their presence. They are fully there. There is nothing vague about their identities, either: these objects are identical to themselves, different from other things and they are indefinitely identical to nothing. They refute Evans' argument from definite identities that there cannot be any vague objects.

Let us say that an object is *utterly* vague if there is no sharp line between *any* of its penumbral categories: not between its parts and its non-parts, not between its parts and its questionable parts, and so on. The vague objects of this construction have sharp sets of parts, questionable parts and non-parts, and are not utterly vague. The construction could be made to admit higher-order vagueness (most of the necessary technical infrastructure is already there) but the fundamental point would remain. The objects would still have sharp sets of absolutely uncontroversial parts and non-parts - of things that are in no sort of limbo at all.[38] Even so, the vague objects of this construction suit my purposes quite well. The idea that they exist is intelligible, since they can be modeled with familiar set-theoretic means. It remains to be seen whether the idea of utterly vague objects can be made as intelligible.

Let us now verify the basic propositions of mereology. We shall consider metalinguistic versions first. Formal versions follow as consequences in the Technical Annex. To start with there is:

Transitivity: If $\underline{S}^{\approx}$ is a part of $\underline{T}^{\approx}$ and $\underline{T}^{\approx}$ is a part of $\underline{U}^{\approx}$, then $\underline{S}^{\approx}$ is a part of $\underline{U}^{\approx}$.

I assume it is sufficient to show that any three objects that verify the antecedent must also verify the consequent. Suppose $\underline{S}^{\approx}$ is a part of $\underline{T}^{\approx}$, and $\underline{T}^{\approx}$ is a part of $\underline{U}^{\approx}$. Take any choice function $\underline{f}$. Then $\underline{f}^*(\underline{S}^{\approx}) \subseteq \underline{f}^*(\underline{T}^{\approx}) \subseteq \underline{f}^*(\underline{U}^{\approx})$ so, by transitivity of $\subseteq$ and arbitrary choice of $\underline{f}$, $\underline{S}^{\approx}$ is a part of $\underline{U}^{\approx}$.

Given that we must reckon with truth-value gaps, transitivity might be thought to require in addition that any three objects falsifying the consequent of an instantiation will also falsify its antecedent. But transitivity fails on such a double-barreled

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[38] For reasons that Mark Sainsbury sets out in his op. cit. p. 255

interpretation of 'if..., then...'. For instance, letting $\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$ be different atoms, choose $\underline{S}^{\approx} = [\langle\underline{B},[\underline{A},\underline{B}]\rangle]^{\approx}$, $\underline{T}^{\approx} = [\langle\underline{C},[\underline{B},\underline{C}]\rangle]^{\approx}$, and $\underline{U}^{\approx} = [\langle\underline{D},[\underline{C},\underline{D}]\rangle]^{\approx}$. Then, as can easily be seen, $\underline{S}^{\approx}$ is a non-part of $\underline{U}^{\approx}$. These three objects falsify the consequent of the relevant instantiation. But it is neither the case that $\underline{S}^{\approx}$ is a non-part of $\underline{T}^{\approx}$, nor the case that $\underline{T}^{\approx}$ is a non-part of $\underline{U}^{\approx}$. To falsify a conjunction is to falsify a conjunct, so these objects fail to falsify the antecedent of this instantiation.

No two objects are each other's parts. Clearly, if $\underline{S}^{\approx}$ is a part of $\underline{T}^{\approx}$ and $\underline{T}^{\approx}$ is a part of $\underline{S}^{\approx}$, then $\underline{S}^{\approx}=\underline{T}^{\approx}$. (Again, objects that falsify the consequent need not falsify the antecedent: $[\langle\underline{A},\underline{B}\rangle,\langle\underline{C},\underline{D}\rangle]^{\approx}$ and $[\langle\underline{A},\underline{D}\rangle,\langle\underline{C},\underline{B}\rangle]^{\approx}$ are different objects, but each is a questionable part of the other.)

Now let us generalize the fusion relation and verify the remaining basic propositions of mereology. We say that $\underline{F}$ is a (*definite*) (*generalized*) *fusion* of some objects $\underline{S}_1^{\approx}$, $\underline{S}_2^{\approx}$, $\underline{S}_3^{\approx}$, … if and only if for all $\underline{f}$: $\underline{f}^*(\underline{F})$ is a classical fusion of $\underline{f}^*(\underline{S}_1^{\approx})$, $\underline{f}^*(\underline{S}_2^{\approx})$, $\underline{f}^*(\underline{S}_3^{\approx})$,…. $\underline{F}$ is a (*definite*) (*generalized*) *non-fusion* of $\underline{S}_1^{\approx}$, $\underline{S}_2^{\approx}$, $\underline{S}_3^{\approx}$, … if and only if for all $\underline{f}$: $\underline{f}^*(\underline{F})$ is not a classical fusion of $\underline{f}^*(\underline{S}_1^{\approx})$, $\underline{f}^*(\underline{S}_2^{\approx})$, $\underline{f}^*(\underline{S}_3^{\approx})$,…. Otherwise, $\underline{F}$ is a *questionable* (*generalized*) *fusion* of $\underline{S}_1^{\approx}$, $\underline{S}_2^{\approx}$, $\underline{S}_3^{\approx}$, ….

Let us for the meantime restrict attention to vague objects with representatives whose complexity lies below some given finite upper bound. Then we have:

Unrestricted Composition: Any vague objects $\underline{S}_i^{\approx}$ have a fusion, namely $(\cup_i\underline{S}_i)^{\approx}$.

This follows directly from the fact that for any $\underline{f}$, $\underline{f}^*((\cup_i\underline{S}_i)^{\approx}) = \cup_i \underline{f}^*(\underline{S}_i^{\approx})$. This fact follows in turn from the earlier observation that for any $\underline{f}$ and sharp collections $\underline{S}_i$, $\underline{f}^*(\cup_i\underline{S}_i) = \cup_i \underline{f}^*(\underline{S}_i)$, together with the easily verified further observation that $\underline{H}(\cup_i\underline{S}_i) = \cup_i\underline{H}(\underline{S}_i)$. The restriction to $\underline{S}_i^{\approx}$ such that the $\underline{S}_i$ are of limited complexity ensures that $\cup_i\underline{S}_i$ is also a suitable boundary, and that $(\cup_i\underline{S}_i)^{\approx}$ is an object. Without this restriction we cannot always be sure that $\cup_i\underline{S}_i$ is even a boundary; it will be if the $\underline{S}_i$ are finite in number, though, so even without this restriction there are always finite fusions.

Here is a rigorous version of the informal example of Section 2, which suggests that questionable parthood need not restrict composition. Let $\underline{W}$ be the boundary of Tibbles' loose whisker, and let $\underline{T}^-$ be the boundary of the rest of Tibbles. Let 'Whisker' be $[\underline{W}]^{\approx}$, let 'Tibbles minor' be $[\underline{T}^-]^{\approx}$, let 'Tibbles' be $[<\underline{T}^-,[\underline{T},\underline{W}]>]^{\approx}$, and let 'Tibbles major' be $[\underline{T}^-,\underline{W}]^{\approx}$. Then Whisker is a questionable part of Tibbles, and Tibbles is a questionable fusion of Whisker and Tibbles minor. Even so Whisker and Tibbles minor fuse to make something: Tibbles major.

The third basic proposition of classical mereology is:

<u>Unique Composition</u>: If $\underline{F}^{\approx}$ and $\underline{G}^{\approx}$ are generalized fusions of the $\underline{S}_i^{\approx}$, then $\underline{F}^{\approx} = \underline{G}^{\approx}$.

This can be seen as follows. Letting $\underline{F}^{\approx}$ and $\underline{G}^{\approx}$ be generalized fusions of the $\underline{S}_i^{\approx}$, it follows from the properties of classical fusions that for any $\underline{f}$, $\underline{f}^*(\underline{F}^{\approx}) = \underline{f}^*(\underline{G}^{\approx})$, since whatever overlaps with either also overlaps with the other. So for every $\underline{f}$:

$\underline{H}(\underline{f}^*(\underline{F})) = \underline{f}^*(\underline{F}^{\approx}) = \underline{f}^*(\underline{G}^{\approx}) = \underline{H}(\underline{f}^*(\underline{G})$, and so $\underline{F}^{\approx} = \underline{G}^{\approx}$.

Existence, composition and identity are all definite, so there is a classical principle that cannot hold true. Classically, one object is a proper part of another if and only if there is a difference: an object that does not overlap with the proper part, whose fusion with it is equal to the whole. This principle asserts a link between parthood and, on the other hand, existence, composition and identity. There can be no such link here, where parthood is sometimes questionable but these other matters are always definite.

Finally, let us see that the vague domain is extensional. It is sufficient to show that everything confirms to a:

Principle of proper parts: If some vague object has proper parts, and each of its proper parts is a part of some second object, then the object in question is also a part of this second object.

Extensionality follows immediately since no two objects are each other's parts. We can demonstrate this principle within the full domain of vague objects, with representatives of arbitrary complexity.[39] Notice that, even so, extensionality remains substantially at odds with mereological vagueness, since it fails if we restrict attention to the sorts of basic parts that we ordinarily have in mind. Two vague objects might have the same

[39] For reasons that have already been explained, we can now only be sure to have finite fusions.

atoms or molecules as parts, say, but different atoms or molecules as questionable parts. The less familiar proper parts that make a difference between any two vague objects with proper parts are provided by the following lemma:

<u>Lemma</u>: If $\underline{S}^{\approx}$ is a proper part of $\underline{T}^{\approx}$, then $[\langle\underline{S},\underline{T}\rangle]^{\approx}$ is also proper part of $\underline{T}^{\approx}$.

There are demonstrations of this Lemma and the Principle of Proper Parts in the Technical Annex. The upshot of this section is that the mereology of vague objects, unlike their logic, can be very nearly classical.

Michael Morreau,
University of Maryland at College Park

Figures:

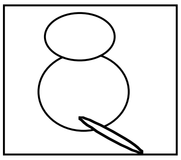

figure 1

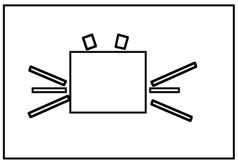

figure 2a

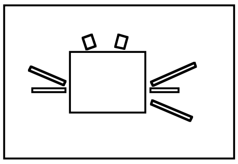

figure 2b

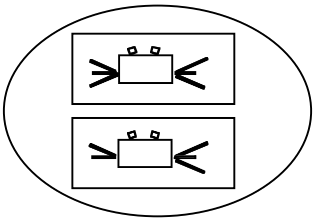

figure 3

f:

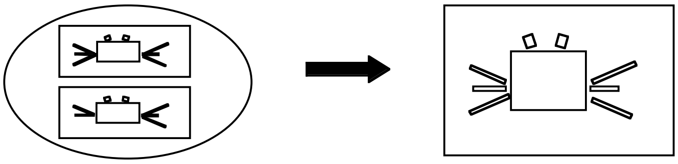

figure 4

f:

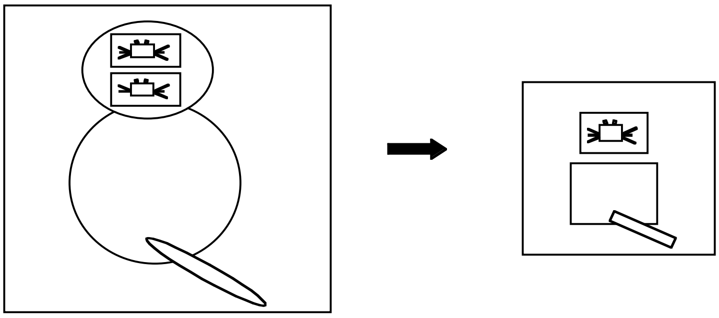

figure 5

Technical Annex to "What Vague Objects are Like"

Demonstration of the Lemma

Let V be the vague collection $<S,T>$. Then for any f, either $f^*(V) = f^*(S)$, or $f^*(V) = f^*(T)$. That $[V]^{\approx}$ is a part of $T^{\approx}$ is clear: for any f, $f^*([V]^{\approx}) = H(f^*([V])) = H([f^*(V)])$ = either $H(f^*(S))$ or $H(f^*(T))$ = either $f^*(S^{\approx})$ or $f^*(T^{\approx})$. Either way, since $S^{\approx}$ is a part of $T^{\approx}$, $f^*([V]^{\approx}) \subseteq f^*(T^{\approx})$. To see that $[V]^{\approx} \neq T^{\approx}$, since $S^{\approx} \neq T^{\approx}$ we can choose f such $H(f^*(S)) \neq H(f^*(T))$. Let $f_{[V/S]}$ be the choice function that differs from f, if at all, only insofar as $f_{[V/S]}(V) = S$. Since by the well-foundedness of boundaries V appears neither within S nor within T, we now have: $H(f_{[V/S]}^*([V])) = H([f_{[V/S]}^*(S)] = H[f^*(S)] = H(f^*(S)) \neq H(f^*(T)) = H(f_{[V/S]}^*(T))$. So $[V]^{\approx} \neq T^{\approx}$. This completes the demonstration of the lemma.

Demonstration of the Principle of Proper Parts

Supposing it is not the case that $T^{\approx}$ is a part of $U^{\approx}$, choose f such that $H(f^*(T)) \not\subseteq H(f^*(U))$. Supposing $T^{\approx}$ has a proper part, since U has a finite degree of nesting it is possible with repeated use of the lemma to find a proper part $[V]^{\approx}$ of $T^{\approx}$ such that T is an element of the vague collection V, but V does not occur within U (nor, by the well-foundedness of boundaries, does it occur within T). Now: $f_{[V/T]}^*([V]^{\approx}) = H(f_{[V/T]}^*[V]) = H[f_{[V/T]}^*(V)] = H(f_{[V/T]}^*(V)) = H(f_{[V/T]}^*(T)) = H(f^*(T)) \not\subseteq H(f^*(U)) = H(f_{[V/T]}^*(U)) = f_{[V/T]}^*(U^{\approx})$. So $[V]^{\approx}$, though a proper part of $T^{\approx}$, is not a proper part of $U^{\approx}$. This verifies the principle of proper parts, and with it extensionality.

Formal Mereology

Formal versions of the basic propositions of mereology follow from the informal versions. Take a first-order language with identity, a binary relation symbol $\leq$ (for "... is a part of...") and a ternary Σ ("... is a sum of ... and ..."). We also need a sentential operator Δ ("it is definitely the case that..."). Given a domain D of objects, an interpretation I maps each relation symbol R onto a pair $I^+(R)$ and $I^-(R)$ of suitable relations on D. For any I, $I^+(=)$ is identity and $I^-(=)$ is its complement, difference. One interpretation J extends another, I, if for each R, $I^+(R) \subseteq J^+(R)$ and $I^-(R) \subseteq J^-(R)$.

Given a set S of interpretations we introduce, for each $I \in S$, a partial evaluation function v_I that maps sentences onto determinate truth values, T and $\perp$. The clauses of the truth definition concerning atomic sentences, quantifiers and connectives other than $\rightarrow$ are standard in partial logic.[40] The remaining clauses are:

$v_I(\varphi \rightarrow \psi) = T$ iff for each J that extends I:
if $v_J(\varphi) = T$ then $v_J(\psi) = T$.

$v_I(\varphi \rightarrow \psi) = \perp$ iff $v_I(\varphi) = T$ and $v_I(\psi) = \perp$.[41]

[40] See Blamey op. cit., page 3.

[41] This two-sided intuitionistic interpretation of $\rightarrow$ was introduced by R.H. Thomason in "A semantical study of constructible falsity," Zeitschrift für mathematische Logik und Grundlagen der Mathematik 15 (1969): 247-257.

$v_I(\Delta\varphi) = T$ iff for each $J \in S$, $v_J(\varphi) = T$.

$v_I(\Delta\varphi) = \bot$ iff for some $J \in S$, $v_J(\varphi) = \bot$.

With this truth definition evaluations are monotonic: if J extends I and $v_I(\varphi)$ is a determinate truth value, either T or $\bot$, then $v_J(\varphi)$ is the same value. An interpretation $I \in S$ is the base of S if each $J \in S$ extends I; if I is the base of S then $v_I(\varphi)=T$ iff for some - indeed each - $J \in S$, $v_J(\Delta\varphi)=T$.

Let us now interpret the language within a domain comprising the earlier precise and vague objects. First, at the base we put an I such that $I^+(\leq) = \{(d_1,d_2)$: for all choice functions $\underline{f}$, $\underline{f}*d_1 \subseteq \underline{f}*d_2\}$, and $I^-(\leq) = \{(d_1,d_2)$: for all $\underline{f}$, $\underline{f}*d_1 \not\subseteq \underline{f}*d_2\}$. We put $I^+(\Sigma) = \{(d,d_1,d_2)$: for all $\underline{f}$, $\underline{f}*d = \underline{f}*d_1 \cup \underline{f}*d_2\}$, and $\underline{I}^-(\Sigma) = \{(d,d_1,d_2)$: for all $\underline{f}$, $\underline{f}*d \neq \underline{f}*d_1 \cup \underline{f}*d_2\}$. This interpretation I captures informal truths about the domain. In particular, assuming that for each d within this domain there is (or can be introduced) a suitably interpreted individual constant $\underline{d}$, we have: $v_I(\underline{d}_1 \leq \underline{d}_2)=T$ iff d_1 is a part of d_2, and: $v_I(\Sigma(\underline{d},\underline{d}_1,\underline{d}_2)=T$ iff d is a generalized fusion of d_1 and d_2. Now for each choice function $\underline{f}$ there is an interpretation, $I_{\underline{f}}$, that extends I: $I_{\underline{f}}^+(\leq) = \{(d_1,d_2)$: $\underline{f}*d_1 \subseteq \underline{f}*d_2\}$, $I_{\underline{f}}^-(\leq) = \{(d_1,d_2)$: $\underline{f}*d_1 \not\subseteq \underline{f}*d_2\}$, $I_{\underline{f}}^+(\Sigma) = \{(d,d_1,d_2)$: $\underline{f}*d = \underline{f}*d_1 \cup \underline{f}*d_2\}$, and $I_{\underline{f}}^-(\Sigma) = \{(d,d_1,d_2)$: $\underline{f}*d \neq \underline{f}*d_1 \cup \underline{f}*d_2\}$.

We can express the basic propositions of mereology as follows:

Transitivity: $\forall \underline{x},\underline{y},\underline{z}(\Delta \underline{x} \leq \underline{y} \wedge \Delta \underline{y} \leq \underline{z} \rightarrow \Delta \underline{x} \leq \underline{z})$

Unrestricted (binary) composition: $\forall \underline{x},\underline{y} \exists \underline{z} \Delta\Sigma(\underline{z},\underline{x},\underline{y})$

Unique (binary) composition:

$$\forall \underline{w}_1,\underline{w}_2,\underline{x},\underline{y}\ (\Delta\Sigma(\underline{w}_1,\underline{x},\underline{y}) \wedge \Delta\Sigma(\underline{w}_2,\underline{x},\underline{y}) \rightarrow \underline{w}_1=\underline{w}_2)$$

Finally, letting '$\underline{x}<\underline{y}$' abbreviate '$\underline{x}\leq\underline{y}\wedge{\sim}\underline{x}=\underline{y}$' there is:

<u>Extensionality</u>:

$$\forall\underline{x},\underline{y}((\exists\underline{w}\Delta\underline{w}<\underline{x} \wedge \forall\underline{w}(\Delta\underline{w}<\underline{x} \rightarrow \Delta\underline{w}<\underline{y}) \wedge \forall\underline{w}(\Delta\underline{w}<\underline{y} \rightarrow \Delta\underline{w}<\underline{x})) \rightarrow \underline{x}=\underline{y})$$

Now it follows from the informal versions of the propositions of mereology that these formal versions receive the value T at v_I (and at each other valuation of this space). In the case of extensionality, to take a representative example, it is sufficient to show that for any given $d_1,d_2\in D$ the following instantiation receives the value T:

$$(\exists\underline{w}\Delta\underline{w}<\underline{d}_1 \wedge \forall\underline{w}(\Delta\underline{w}<\underline{d}_1 \rightarrow \Delta\underline{w}<\underline{d}_2) \wedge \forall\underline{w}(\Delta\underline{w}<\underline{d}_2 \rightarrow \Delta\underline{w}<\underline{d}_1)) \rightarrow \underline{d}_1=\underline{d}_2.$$

To see that it does, consider any J that extends I such that: (a) $v_J(\exists\underline{w}\Delta\underline{w}<\underline{d}_1)=T$, (b) $v_J(\forall\underline{w}(\Delta\underline{w}<\underline{d}_1 \rightarrow \Delta\underline{w}<\underline{d}_2))=T$, and (c) $v_J(\forall\underline{w}(\Delta\underline{w}<\underline{d}_2 \rightarrow \Delta\underline{w}<\underline{d}_1))=T$. By (a) we can choose $d\in D$ such that $v_J(\Delta\underline{d}<\underline{d}_1)=T$. By the truth condition for Δ: $v_I(\underline{d}<\underline{d}_1)= T$, so d_1 has a proper part. Furthermore d_1 and d_2 have the same proper parts. If any given object d is a proper part of d_1, then $v_I(\underline{d}<\underline{d}_1)= T$, so $v_J(\Delta\underline{d}<\underline{d}_1)=T$, so by (b) $v_J(\Delta\underline{d}<\underline{d}_2)=T$, so $v_I(\underline{d}<\underline{d}_2)=T$, and so d is proper part of d_2. Likewise, by (c), any proper part of d_2 is also a proper part of d_1. By the informal version of extensionality therefore $d_1=d_2$, so $v_J(\underline{d}_1=\underline{d}_2)=T$. So v_I(<u>Extensionality</u>)=T, and similar demonstrations show that the other propositions of classical mereology also receive the value T.