Bernard Bolzano, *Wissenschaftslehre*, Seidelsche Buchhandlung, Sulzbach, 1837

Draft of a Topoi 'untimely review'

We live in an academic world where mass-producing books has become the norm, and even young authors feel that their unripe thoughts, jotted down below the age of 50, deserve a volume, an artistic cover, and a professional publicity machine supplied by a mega-factory like Elsevier or Springer. But every now and then, a real book appears that stands out, and *Wissenschaftslehre* (or in the English translation that will appear close on its heels, *Theory of Science*) is definitely one of these. The author Mr. Bernard Bolzano is an unusual character. He is a philosopher of religion, but endowed with an original mathematical mind whose acute observations have already attracted the attention of major scientists: it is rumored that his name may soon be attached to a basic theorem capturing the essential continuity of the real number line. But this book is surely his masterpiece, collecting the thoughts of a lifetime on logic and methodology of science. And just as the best beer is produced today, not in mega-factories, but in micro-breweries, this book is published by a local bookseller in a Bavarian country town, showing the continuing sparkle and tenacity of intellectual life outside of The Matrix of modern universities and their academic-industrial complex.

Simply put, the book is a breath of fresh air in an overheated stuffy room. In this review, I will focus on Mr. Bolzano's thoughts about logic, even though he offers much more than that to readers interested in theory of science and general philosophy. Modern logic has become more and more technical, cutting itself loose from its broader origins as the study of reasoning, and philosophers of logic slavishly play up to this trend by devising ever more arcane criteria of 'logicality' that apply only to a small elite of 'logical constants', making it harder and harder for new themes to enter the field. Refreshingly, Mr. Bolzano does none of this. He resolutely ignores received wisdom in logic textbooks, and deftly avoids entanglement in the scholasticism of our modern age. Instead, he just goes back to what logic is about, and rethinks it afresh.

Let me start with the idea of logical consequence, and logical form. Mr. Bolzano starts with a simple but highly original observation, namely, that one can only judge validity of a given inference if we first *decide on its form*, stating which parts of its linguistic structure we want to consider as *variable*, subject to replacements, and which parts as

constant, having a fixed meaning. The criterion of validity is then that an inference from *P(X)* to *C(X)*, where the sequence of variables *X* denotes all the relevant variable parts in premises and conclusion is valid if *every substitution of suitable terms for the X that makes P(X) true also makes C(X) true*. There is a small intriguing proviso on this definition that I will discuss later. On this view, many traditional problems dissolve in a new freedom. We can follow standard textbooks, where the only constant parts are the usual 'logical constants', but we can equally well fix the meaning of other *functional words* [a much broader category] in natural language. Take the case of comparatives like "taller". If we fix its meaning, then, John is taller than Paul, and Paul is taller than Mary will imply that John is taller than Mary, even when we treat 'John', 'Paul' and 'Mary' as variable place-holders for objects. But we can also extract more generality than this, fixing only the meaning of the comparative construction, and noting that the following inference is still valid: "*x A-er than y, y A-er than z*, and therefore *x A-er than z*". Of course, we can also do what modern logic textbooks do, and gloat over the invalidity of the still more general form "*x R y, y R z*, and therefore *x R z*" – but this ignores the essential structure of the inference. In other words, logical-inferential potential lives all across natural language, once we look in Mr. Bolzano's style at the rich repertoire of function words in daily use.

One might feel that this startling innovation plays down the role of the traditional logical constants like Boolean connectives and quantifiers a bit too much. Still I myself mainly felt a sense of liberation and exhilaration in looking at what is key to inference in new ways, outside of the closed world of permutation invariance that makes logic the most rarified content-free discipline of all. But maybe Mr. Bolzano will one day offer us a view on what makes our old heroes special. On the other hand, I must also remind the reader that benign neglect of received topics can be one of the greatest forces for progress in our field. We all still remember the immense intellectual benefits to our community of the 1990 twenty-five-year moratorium on discussing the principle of *Compositionality*, whose renewal is coming up in 2 years' time at the next World Congress of Logic, Methodology and Philosophy of Science. As Wittgenstein said so well, in the end, ladders are for throwing away.

But let us now focus on the notion of valid consequence itself. The logically educated reader will immediately see analogies with Alfred Tarski's traditional notion of valid consequence, and wonder: did Mr. Bolzano perhaps borrow from this earlier European

tradition without explicit acknowledgment? Indeed, I think that there may be some sleuthing to do here, but there is also a crucial difference. Unlike modern logic with its mathematical universe of totally abstract models, Mr. Bolzano chooses to stay close to interpreted natural language as we normally use it, and still finds the generality required for validity in replacing expressions with fixed meanings by other expressions of the same kind. In this way, validity becomes relative to definable predicates in a given concrete situation or model – a style of definition that cuts across current boundaries of truth versus validity. I am sure contemporary logicians will find this new level of defining notions interesting, once they get over the shock of heterodoxy.

It is truly amazing that today in 2013, more than a century after the birth of modern logic, highly original things can still be said about the core notions of logic, and that by a relative amateur trained in theology. And there is more to all this. Mr. Bolzano does not quite agree with modern notions of consequence, and requires something extra. The premises of a valid inference should be *consistent*, being true under at least one substitution: that is he uses an Aristotelean universal quantifier over substitutions. Of course, he is well aware that this introduces special features. For instance, though he does not discuss the often-touted fact that this notion of consequence, unlike that of Tarski, makes the valid laws of predicate logic non-axiomatizable, it is easy to predict the answer of a theologian like Mr. Bolzano. What does it matter if the road ahead is narrow and difficult, if it is the *right* road?

What Mr. Bolzano does discuss is another type of deviation, such as the fact that not every strengthening of premises still supports the original conclusion, the way that new likes may invalidate old ones. Though he does not mention the currently fashionable non-monotonic logics that drop standard 'structural rules' of consequence, at least, not in so many words, Mr. Bolzano is clearly sympathetic towards them. Indeed, his book states a large number of interesting observations on formal properties of consequence relations, and what is more, these come in a highly original style, since on his view, we must use a richer format for assertions, indicating their variable and constant parts. In this way, inferences can involve premises with explicitly marked different variable/constant divisions, supporting sophisticated conclusions that go beyond consequence in fixed formalisms as normally studied by contemporary logicians. For instance, we can now ask questions like what is 'the most general conclusion' from given premises in terms of minimal constant parts. Clearly, there is a

lot of work to be done in developing a theory along these trails blazed by Mr. Bolzano, since language choice and inference now go together.

While the above is nothing short of amazing, some modern readers may still want to chant their standard mantra: 'Where are the completeness theorems'? It is true that Mr. Bolzano's book does not develop meta-theory in the contemporary logician's sense – but let us not forget that this particular obsession was not around in acknowledged classics of logic such as Frege's *Begriffsschrift*. One can be a pioneer without engaging in court rituals. Perhaps Mr. Bolzano also feels justified in leaving out such results because he senses that, for the new systems he proposes, one can presumably always hire some eager young Dutch logician to do that work at a modest academic fee.

But maybe I have catered too much to the tastes of logicians already. For, unlike them, Mr. Bolzano does not believe that there is a 'one size fits all' notion of consequence. Instead, he believes that there is a multitude of natural styles of reasoning: ranging from general-purpose to specialized expert mathematical or philosophical argument. Indeed, he sees the demands on philosophical reasoning as the highest, since they are not just about truth preservation, but also about transmitting *relevance*, that elusive but crucial quality of not just 'correct' but *good* reasoning that is 'to the point'. This quality fits in a current trend toward making questions and issues crucial components of logical theory today as devices for maintaining relevance in discourse, and Mr. Bolzano's perspective will no doubt provide welcome additional motivations.

One might say that in this emphasis on different legitimate reasoning styles. Mr. Bolzano has jumped onto a modern bandwagon, that of 'logical pluralism'. Maybe so – but his book is liberating, once more, in being much less beholden to intuitionism or relevant logic as the main paradigms for pluralism – and of course, in all this, making the role of the language so crucial is a stroke of genius all of Mr. Bolzano's own.

I have said at the start that I cannot do justice to everything in Mr. Bolzano's magnum opus. One particular striking thing is that he treats logic together with the *methodology of science*, topics that he sees as belonging together: scientific reasoning is just ordinary reasoning continued by other means. To show that this makes sense, his catalog of basic reasoning styles also includes notions of *probabilistic* consequence, so crucial to the empirical sciences: showing how logic and probability are a natural match instead of fated competitors – something that should be clear to anyone interested in scientific

reasoning. In doing so, once again, Mr. Bolzano disregards old disciplinary boundaries, and sides resolutely with minority views like Rudolf Carnap's that logic and philosophy of science form a natural unity. This is a courageous stand. Only a few isolated followers of Carnap are left today, clinging precariously to the foothills of the Northern Alps. When the last Humboldt grant runs out, what will be left?

But there is no cause for despondency in the work reviewed here. While some people feel that modern logic and methodology have run into a maze of epicycles with ever-diminishing returns, Mr. Bolzano's book puts the lie to this pessimism. A subject that can still produce original work of this caliber, has a great future ahead.

I need to wind up. *Wissenschaftslehre* is a Big Book in every sense, running to some 1700 pages. This far exceeds the attention span of a modern referee, who can perhaps handle 10 pages in Latex unisex style. But it can be excused easily by anyone who knows the publication pressures of the German-style Habilitation system, that has not changed much since the time around 1950 when Brand Blanshard explained how German professors prefer not to argue, but to stifle opposition "by the massive yardage of their collected 'Werke'". And there is also a saving grace. Somewhere among his myriads of methodological rules for scientific thinking, Mr. Bolzano mentions the following brief and attractive maxim: "Avoid Prolixity". Surely, this light humoristic touch of opposition to his academic environment shows us where his heart truly lies.

I end on a personal-political note that may not really fit with a pure review like this. Mr. Bolzano's father is Italian, his mother is Czech, though he writes his scientific work in German, like all of us today, now that Germany has become the economic and political mainstay of the European Union. While this much is obvious, however, he also displays a feature that is decidedly less so. While we all think in terms of a Northern Europe and a Southern Europe today, separated by that cold dividing line called the Alps, Mr. Bolzano thrives in Prague, the capital of recently liberated Czechia, while his intellectual horizon seems solidly Austrian, rather than German. What this suggests to me is that there is room for more than two poles in a United Europe, with a special place for its *central regions*. We may not see it happen in our lifetime, but I, for one, would not be surprised if, with multi-cultural talents like this storming onto the scene, Central Europe will one day become a hotbed of its own in philosophy, science – and why not, maybe even the arts.

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