

# Some Extensional Term Models For Combinatory Logics and $\lambda$ - Calculus -

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## 1 Errata (*Corrections are in Red*)

- p.4, 1.1.4.I :

$$\begin{array}{ll} 1. xM = y[x/y]M & \text{If } y \notin FV(M) \cup BV(M) \\ 2. (xM)N = [x/N]M & \text{If } BV(M) \cap FV(N) = \phi \end{array}$$

- p.17, line 5,  
not have *the original theorem as a corollary.*
- p.27,  
*line 3:*  
to concepts ~~like~~ *such as* equality, quantification, etcetera.  
*line 21:*  
terms of the  $\lambda$ -calculus are *then called*  $\lambda$  terms.
- p.29, line 13:  
In order to obtain a reverse interpretation, we *first need* a
- p.55, line 21:  
of the  $M_i$  and then (\*\*) would *also* hold.
- p.62, line 8 of subcase 1.2 :  
Hence since  $CL_{\omega'} \vdash M \approx_{\alpha} N$ ,  $\alpha \neq 0$  and  $M_2$  is closed, it follows from 2.2.8.2) and 2.2.6 that
- p.64, line 19:  
We *give here* a modification of our original construction, due to Scott.
- p.68, line 5:  
we again make use of the underlining *technique.*

- p.71, 2.5.10, Lemma:

1.  $\lambda \vdash L \simeq M \Leftrightarrow [L \equiv M \text{ or } L \equiv \overline{M}] L \simeq M$
2.  $\lambda \vdash L \simeq \lambda x M \Leftrightarrow [\exists M' L \equiv \lambda x M' \text{ and } \lambda \vdash M \simeq M'] \text{ or } L \equiv \lambda x M$
3.  $\lambda \vdash L \simeq MN \equiv [\exists M' N' L \equiv M' N' \text{ and } \lambda \vdash M \simeq M', \lambda \vdash N \simeq N'] \text{ or } L \equiv \overline{MN}$
4.  $\lambda \vdash M \simeq M' \text{ and } \lambda \vdash N \simeq N' \Rightarrow \lambda \vdash [x \setminus N] M \simeq [x \setminus N'] M'$

Use  $\lambda \vdash M \simeq M' \equiv |M| \equiv |M'|$  like in  $A_{11}$

- p.74, line 11:  
If  $\lambda + \text{ext} \vdash M \geq N$  and if  $x$  is a variable not **occurring** in
- p.79, 2.5.26, Lemma, 3) :

If  $Z$  is a subterm occurrence of  $L$  such that ~~there is no~~ **the** corresponding subterm occurrence  $Z'$  of  $L'$  is **not** simple then  $Z$  has some line in  $L'$ .

- p.129, Definition 2, I, 1. :

$$1. \frac{M \geq_1 M'}{(\lambda x M) \geq_1 \lambda y [x \setminus y] M'} \quad \text{If } y \notin FV(M') \cup BV(M')$$

- p.130 Lemma 5: If  $\lambda' \vdash M \geq_1 M'$  and  $\lambda' \vdash N \geq_1 N'$ , then  $\lambda' \vdash [x/N] M \geq_1 [x/N'] M'$ . **Given that**  $BV(N') \cap FV(N') = \phi$ ,  $BV(MM') \cap FV(N') = \phi$  and  $x \notin BV(M')$

Proof:

Induction on the length of proof of  $M \geq_1 M'$  using the sublemma:

If  $x \neq y$ ,  $y \notin FV(N_1)$ ,  $x \notin BV(M)$ , then  $[x/N_1]([y/N_2]M) \equiv [y/[x/N_1]N_2]([x/N_1]M)$ .

The proof of the sublemma proceeds by induction on the structure of  $M$ .

- p.131 case 2. line 4

$$M_3 \equiv \lambda y' [x/y'] M'' \text{ where } \lambda \vdash M \geq_1 M'' \text{ and } y' \notin FV(M'') \cup BV(M'')$$