Reconciling positionalism and antipositionalism

Joop Leo

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Abstract

Positionalism and antipositionalism, two apparently opposing views on relations, give different answers to the question how things can be arranged one way rather than another. In positionalism, relations come with positions to which objects may be assigned; in antipositionalism relations have no positions, but relations consist of a network of complexes interrelated by substitutions. In this paper, a new version of positionalism is proposed, and it is shown that—contrary to what the names suggest—positionalism and antipositionalism are essentially two sides of the same coin.

1 Introduction

Abelard's loving Eloise is obviously not the same as Eloise's loving Abelard. A distinguishing feature of non-symmetric relations, like the love relation, is that they admit of *differential application*, i.e., they may apply to the same things in multiple ways. A crucial question is, what makes differential application possible? How can things be arranged one way rather than another?

The answers given depend on the view on relations one adheres to. There are three basic accounts of relations: the *standard view*, the *positionalist view*, and the *antipositionalist view*.

In brief, the standard view says that the arguments of a relation come in a linear order, e.g., Abelard comes first and Eloise comes second in Abelard's loving Eloise. The positionalist view says that a relation comes with positions to which arguments may be assigned, e.g., for the love relation we have the positions *Lover* and *Beloved*. The antipositionalist view says that a relation is a network of complexes interrelated by substitutions, e.g., substituting Anthony for Abelard and Cleopatra for Eloise in Abelard's loving Eloise gives the complex of Anthony's loving Cleopatra.

In his seminal paper 'Neutral relations', Kit Fine made clear that the standard view and the positionalist view give rise to problems (Fine 2000). His answer was a new view on relations, the antipositionalist view. However, the antipositionalist view has also been heavily criticized (Donnelly 2016; Gaskin & Hill 2012; MacBride 2007; 2014; Orilia 2011). In my opinion, however, the criticisms arise from a fundamental misunderstanding of the position. In this paper

I want to clarify some of the misconceptions. In particular I will show that positionalism and antipositionalism are not really opposite views.

For simplicity I will assume throughout the paper that all relations are of finite degree.

2 Views on relations

The views presented here contain some aspects that have not been described before. For the positionalist view we make a distinction between thick and thin positionalism, where only in thick positionalism objects may occupy positions.

A note in advance: in Leo (2013), I made a sharp distinction between relational states and relational complexes, and conceived of relational complexes as a structured perspective on relational states. I argued that a state may have more than one corresponding complex. For example, the state of Abelard's loving Eloise corresponds not only with a complex from the binary love relation with two relata, but (among others) also with a complex from the unary relation of loving Eloise with one relatum. For the argumentation in this paper relational states do not play an essential role. However, occasionally I will not only talk about relational complexes but about relational states as well.

2.1 Standard view

The standard view assumes that the arguments of a relation always come in a given linear order. For example, in each instance of the love relation one of the arguments comes first and the other comes second. One might also say that relations have a direction. In the instance aRb of a relation R the relation runs from a to b, and in bRa the relation runs in the opposite direction. Different directions make differential application possible.

A nice feature of the standard view is that it corresponds straightforwardly with natural and most formal languages. For example, for the relation *loves*, we have a direct match with linguistic expressions of the form '__ loves __'.

Unfortunately, there are also problems with the standard view. In the states 'out there' there is no linear order or direction between the arguments. The linear order is just a representational artifact. Already in 1913 Russell rejected the idea that all relations have a 'natural' direction. For example, this is not the case for right and left, up and down, and greater and less (Russell 1984, p. 87).

This problem may also be formulated in different terms. The standard view makes it plausible that for each binary relation R there is a *converse relation* R', where aRb holds iff $bR'a$ holds. For example, for the relation *on top of*, we have the converse relation *beneath*, where the state of a's being on top of b is the same as the state of b's being beneath of a. We would like to regard this state as a relational complex consisting of a *single* relation in combination with the two relata. However, this relation can neither be *on top of* nor *beneath*, because there is no good reason to choose one over the other (Fine 2000, pp. 3–4).

2.2 Positionalism

According to positionalism, each relation comes with a collection of positions to which objects may be assigned and with no intrinsic order between the positions. Such an assignment results in a relational complex. We distinguish two forms of positionalism: *thick positionalism*, which is the 'normal' positionalist view, and *thin positionalism*, a new variant introduced in this paper.

Thick positionalism

In thick positionalism, a relation comes with positions to which objects may be assigned. Such an assignment may result in a relational complex with objects *occupying* positions.

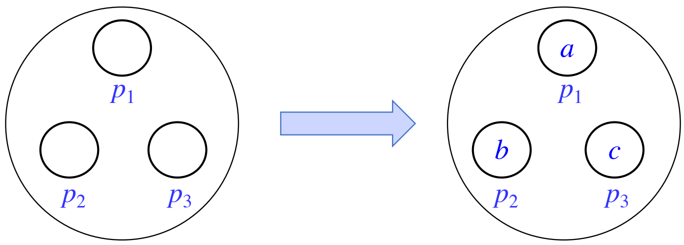

Figure 1: Thick positionalism.

As pointed out by Fine, a problem with this view is that symmetric relations like the adjacency relation have distinct complexes that intuitively should be the same (Fine 2000, p. 17). We would, for example, like to regard a's being next to b as the same complex as b's being next to a. But suppose that the adjacency relation has two positions *Next* and *Nixt*. Then assigning a to *Next* and b to *Nixt* gives a complex which is distinct from the complex obtained by assigning b to *Next* and a to *Nixt* if in the complexes objects occupy positions. In one complex, a occupies *Next* and b occupies *Nixt*, and in the other complex it is the other way around.[1]

Thin positionalism

In thin positionalism, a relation comes with positions for which objects may be *substituted*. Such a substitution may result in a relational complex with *occurrences* of the objects involved.

[1]A proposed way out is to allow objects of a symmetric relation to occupy the same position. This is already done in (Russell 1984, p. 146), and later in Orilia (2011) and in Dixon (2018). Such an approach works for the adjacency relation and many other symmetric relations, but it fails for relations where the objects are arranged clockwise in a circle (Fine 2000, p. 17, note 10). Another nice example of a relation for which it fails is playing tug-of-war (MacBride 2007, pp. 42–43).

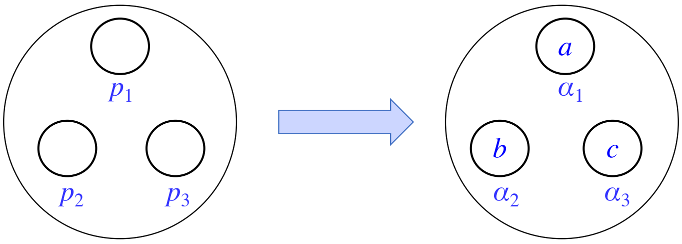

Figure 2: Thin positionalism.

Positions are not boxes in which you can put an object; rather they are substitutable places in a structure or form. The relevance of this distinction can be illustrated with an example.

For the adjacency relation with positions *Next* and *Nixt* substituting a for *Next* and b for *Nixt* results in a complex with an occurrence of a and an occurrence of b. The complex is the same as the one that we get when we substitute b for *Next* and a for *Nixt*. It is as if the positions disappear once we assign objects to them.[2] So we don't get too many complexes as in thick positionalism. This makes thin positionalism preferable over thick positionalism.

A relation itself is viewed as an entity and its positions as occurrences of some kind of entity. Though it is not essential, positions might perhaps best be seen as occurrences of arbitrary objects. What is essential is that we may substitute objects for positions. The result of a substitution (if any) is a complex with occurrences of the objects substituted for positions.

The notions of substitution and occurrence are taken as primitive.

In Appendix A a general composition principle for substitutions is given. In the principle substitution is conceived of as an operation on occurrences of entities within an entity.

We will assume that thin positionalism endorses the Composition Principle in Appendix A..

The Composition Principle does not speak about complexes and positions for which objects may be substituted, but about entities and occurrences of entities for which entities may be substituted. However, because positions are conceived of as occurrences of some kind of entity, and because objects can be substituted for positions, the principle applies in a straightforward way to thin positionalism.

Composition principle of thin positionalism.
Let s be a substitution of objects for the positions of a relation R resulting in a complex ξ. Then there is a surjective map μ from the positions of R to the occurrences of objects in ξ such that

1. μ maps every position p to an occurrence of the object substituted by s for p,

[2] A comparison could be made with assigning values to variables. Take the formula $x+y=5$. Then assigning 2 to x and 3 to y results in $2+3=5$, where in the result the variables are no longer present.

2. for every substitution s' in ξ, s' results in a complex ξ' iff $\mu \cdot s'$ is a substitution for the positions resulting in ξ',

where $\mu \cdot s'$ denotes the substitution that maps each position p to the object substituted by s' for $\mu(p)$.

If s is taken as a substitution in a complex, then a similar statement holds.

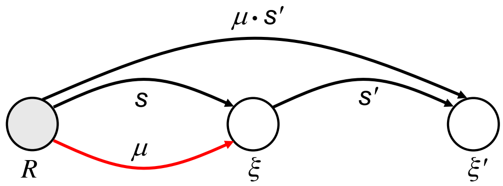

Figure 3: Composition Principle of Thin Positionalism.

We call μ a *co-map* of substitution s.

The Composition Principle of Thin Positionalism has the interesting consequence that substitutions in complexes can be derived from the substitutions for the positions and their co-maps.

A single substitution of objects for positions may have more than one co-map. For example, if for a symmetric relation like the resemblance relation substituting an object a for both positions p, p' results in a complex with two occurrences of a, then this substitution has two co-maps; one that maps p to an occurrence α and p' to an occurrence α', and another that maps p to α' and p' to α.

One could in principle allow that a co-map μ is not injective. For example, one could argue that for the love relation with positions *Lover* and *Beloved*, substituting Narcissus for both positions results in a complex with just one occurrence of Narcissus.

If for a given substitution s of objects for positions a co-map μ is not injective, then we say that the substitution results in a *coalescence* of occurrences.

We call a relation *coalescence-free* if it has no coalescence of occurrences. So each complex of an n-ary coalescence-free relation will have n occurrences of objects. If the love relation is coalescence-free, then the complex of Narcissus' loving Narcissus would have one occurrence of Narcissus in the role of lover and another one in the role of beloved.

As we have seen, the adjacency relation is symmetric in a strict sense. Switching the arguments does not change the complex. More generally, we say that R is *strictly symmetric* if there is a non-identity permutation π of its positions such that for every substitution s for the positions resulting in a complex ξ, substitution $\pi \cdot s$ results in ξ as well.[3]

[3]This definition of strict symmetry is not completely satisfactory in combination with an ontology that is only committed to complexes that actually obtain. In that case, the love relation would according to this definition be strictly symmetric if people would only love themselves. However, by assuming that every substitution resulting in a complex comes with a specific set of one or more co-maps, a more robust definition of strict symmetry can be given by adding the condition that s comes with a co-map μ and $\pi \cdot s$ with a co-map μ' that

Thin positionalism may appear to be more complicated than thick positionalism. Nevertheless, I think it is a much more natural view than thick positionalism. Having relational complexes in the world as a result of substituting objects for positions seems to make more sense than having complexes 'out there' containing objects in a kind of boxes, called positions.

2.3 Antipositionalism

Relational complexes have constituents. But this does not necessarily mean that we can directly speak about *how* these constituents occur in a given complex. According to antipositionalism, the structure of a relation can be fully expressed in terms of structure preserving connections between its complexes. There is no need to say anything about the internal structure of the complexes. This may sound a bit vague, so let us look at an example.

For the love relation, one of the complexes could be Paris' loving Helen. In this complex we have one occurrence of Paris and one of Helen. By *substituting* Venus for the occurrence of Paris and Adonis for the occurrence of Helen we get the complex of Venus' loving Adonis. With this substitution corresponds a structure preserving map between the occurrences of Paris and Helen in Paris' loving Helen and the occurrences of Venus and Adonis in Venus' loving Adonis. By taking all possible substitutions into account, we get a network of interrelated complexes.[4]

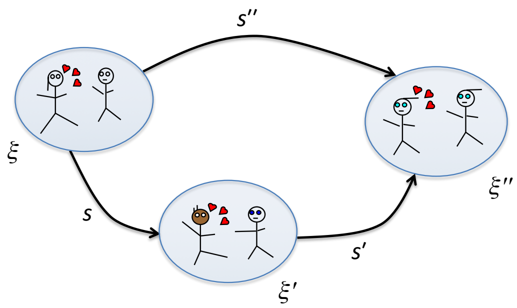

Figure 4: Antipositionalism.

Networks like this are conceived of as relations. Isomorphic relations are not necessarily identical, as the monadic relations of having a heart and having a kidney make clear.

As in thin positionalism, the notions of substitution and occurrence are taken as primitive. Likewise, we assume that antipositionalism endorses the Composition Principle in Appendix A.

To make the Composition Principle appropriate for antipositionalism, we only have to make a slight change in terminology. Instead of using a phrase like 'a

is distinct from μ. With this addition, the love relation will in no case be labeled as strictly symmetric if every substitution resulting in a complex comes with only one co-map.

[4]In Fine's paper 'Neutral relations', objects are substituted directly for objects in a complex, and not for occurrences of objects. However, Fine said (private communication, 2005) that in 'Neutral Relations' he was, for simplicity, ignoring the fact that substitution is properly done on occurrences, as is made clear in Fine (1989).

substitution of entities for the occurrences of entities in an entity ξ' we say 'a substitution of objects for the occurrences of objects in a complex ξ'.[5]

Composition principle of antipositionalism.
Let s be a substitution of objects for the occurrences of objects in a complex ξ resulting in a complex ξ'. Then there is a surjective map μ from the occurrences of objects in ξ to the occurrences of objects in ξ' such that

1. μ maps every occurrence α in ξ to an occurrence of the object substituted by s for α,

2. for every substitution s' in ξ', s' results in a complex ξ'' iff $\mu \cdot s'$ is a substitution in ξ resulting in ξ'',

where $\mu \cdot s'$ denotes the substitution that maps each occurrence α in ξ to the object substituted by s' for $\mu(\alpha)$.

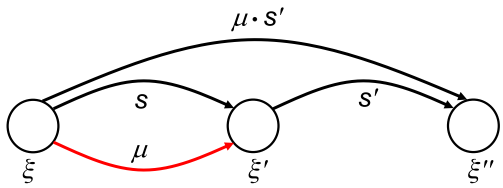

Figure 5: Composition Principle of Antipositionalism.

We call a map μ with this property a *co-map* of substitution s.

We call a complex an *initial complex* if any complex of the relation can be obtained from it by a substitution. If a relation has an initial complex, then it follows from the Composition Principle of Antipositionalism that for any complex ξ of the relation the substitution in ξ that maps each occurrence α to the object of α results in ξ itself.[6]

More principles could be given. An interesting, but controversial one says that all complexes of a relation are connected via a substitution. This may not hold for certain relations of variable degree, like the relation of *forming a circle*. It is not obvious how to characterize for such relations the unity of its complexes.

Like thin positionalism, antipositionalism does in principle not exclude a *coalescence* of occurrences, i.e., two or more occurrences of objects in a complex may be mapped to the same occurrence of an object in another complex. For example, substituting Narcissus for the occurrence of Paris as well as for the occurrence of Helen in the complex of Paris' loving Helen could result in a complex with one occurrence of Narcissus.

A coalescence of occurrences is very natural for set-like relations. For the relation of *forming a group* we may want the complex for the group consisting of Athos,

[5] I do not presuppose that there is a distinction between entities and objects, but it is common to say that a relational complex has (occurrences of) objects as relata.

[6] To prove this, let ξ_0 be an initial complex and s_0 a substitution in ξ_0 resulting in ξ. If μ_0 is a co-map of s_0, and s a substitution in ξ that maps each occurrence α to the object of α, then $\mu_0 \cdot s$ is the same substitution as s_0. So, by condition 2 of the Composition Principle of Antipositionalism, s results in ξ itself.

Porthos, and Aramis to have three occurrences and the group of Batman and Robin to have two occurrences. If this is the case, then the second complex may be obtained from the first by a substitution, but there is no substitution the other way around.

Also for the ternary relation R where $Rabc$ is the complex of *a's loving b and b's loving c* it may seem natural to assume that a coalescence of occurrences can take place. For substituting in $Rabc$ the object a for c gives the complex $Raba$, and substituting in $Rabc$ the objects b, a, b, for the occurrences of a, b, c gives the complex $Rbab$. These complexes are obviously empirically indistinguishable, but if a coalescence of occurrences is allowed they can be identical (cf. Leo (2010), pp. 147–148).

It should be noted that not always all complexes in a relation are empirically distinguishable. This is obvious for mathematical relations, but it is also the case for some other relations, like the conjunction of the binary love relation with the unary relation of loving d, where d is a fixed object.[7] For this relation, the conjunction of a's loving d with d substitutable and b's loving d with d fixed is a complex that is distinct from the conjunction of b's loving d with d substitutable and a's loving d with d fixed, but the two complexes are empirically indistinguishable (cf. Leo (2013), p. 364).

Under antipositionalism, different substitutions in a complex may result in the same complex, which is a defining characteristic of *strictly symmetric* relations. For the adjacency relation, for example, we have the complex of a's being adjacent to b. Substituting in this complex b for (the occurrence of) a and a for (the occurrence of) b gives the same complex. This means that in the network of the relation we have a map from each complex to itself that switches the two objects involved.

One may worry that antipositionalism is less able to identify complexes than positionalism because in antipositionalism we don't have positions with meaningful names like *lover* and *beloved*. However, in antipositionalism we could give occurrences equally meaningful names like *lover in complex* ξ and *beloved in complex* ξ. Besides, names can be freely chosen; in both views on relations the meaning of names do not play a constitutive role.

There are alternative antipositionalist accounts possible. One could, for example, assume that any complex has for each object at most one occurrence. Then there is not really a need to talk about occurrences and one can simply substitute objects for objects in complexes.

3 Intertranslating the views

In this section the translatability from positionalism to antipositionalism and vice versa will be examined. Particular attention will be given to the question whether the translations respect the Composition Principle in Appendix A. By examining the translations back and forth, we get a clear picture of the relative expressive power of positionalism and antipositionalism.

[7]The conjunction of two relations is a relation whose complexes are conjunctions of the complexes of the original two relations. See Leo (2013) for a detailed definition.

3.1 From positionalism to antipositionalism

Can a positionalist express himself in antipositional terms? We will describe what kind of networks of interrelated complexes a thick and a thin positionalist can construct, and discuss whether these networks are all acceptable for an antipositionalist as networks of relations.

From thick positionalism to antipositionalism

Let us first assume you are a thick positionalist. Let R be a relation with positions $p_1, \ldots, p_n$. Then you can simply create a network of interrelated complexes as follows. Let ξ be the complex obtained by assigning $a_1, \ldots, a_n$ to $p_1, \ldots, p_n$. Identify the pairs $\alpha_i = \langle \xi, p_i \rangle$ with occurrences of objects in ξ. If ξ' is the complex obtained by assigning $b_1, \ldots, b_n$ to $p_1, \ldots, p_n$, then define the assignment of $b_1, \ldots, b_n$ to $\alpha_1, \ldots, \alpha_n$ as a substitution in ξ resulting in ξ'.

By repeating the construction for every assignment of objects to the positions of R, you get a network of complexes interrelated by substitutions.

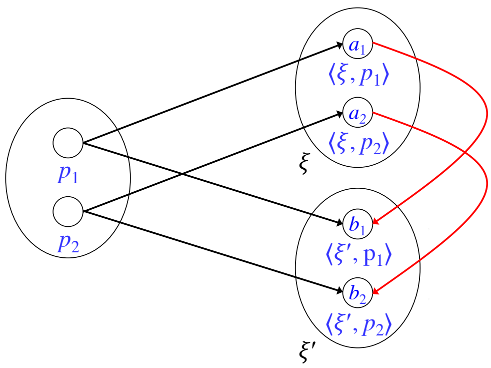

Figure 6: Translating thick positionalism to antipositionalism.

It is easy to verify that the resulting network of complexes satisfies the Composition Principle of Antipositionalism.

The construction is adequate for non-symmetric relations, but not for symmetric relations since in thick positionalism different assignments of objects to positions always result in different complexes.

A way out could be the use of equivalence classes of complexes to express strict symmetry of relations. The equivalence classes could be identified with what the antipositionalist regards as complexes. There is, however, a complication; not for every relation, occurrences of objects can be defined *non-arbitrarily* in set theory in terms of positions, complexes, and objects. This will be discussed in the last part of this section.

From thin positionalism to antipositionalism

Now assume you are a thin positionalist. Let again R be a relation that comes with a set of positions. Without any adjustment, the complexes of the relation already form a network of complexes interrelated by substitutions—at least, if there are complexes. So, for the translation, we just retain the network of complexes.

The network of complexes satisfies the Composition Principle of Antipositionalism. But is it always acceptable as a relation for the antipositionalist?

If the relation R is not coalescence-free, then it might happen that not all the complexes are interrelated by substitutions. For example, let R be a ternary relation with only two assignments to its positions p_1, p_2, p_3 resulting in a complex, namely a, a, b and a, b, b, respectively. If the resulting complexes both have only two occurrences, then the complexes cannot be connected via a substitution.

It may be questionable whether an antipositionalist would regard such a network of complexes with unconnected parts as a relation. If not, then a thin positionalist who allows coalescence of occurrences could have relations for which an antipositionalist has no counterpart.

It is also possible that the thin positional relation has no complexes. So also in this case a thin positionalist has relations for which there is no antipositional counterpart.

In all other cases, the relations of the thin positionalist do have an antipositional counterpart.

Identifying occurrences

As I said in Section 2.2, a thick positional relation may have distinct complexes that intuitively should be the same. In translating such a relation to thin positionalism or antipositionalism, we may want to translate such similar complexes to the same complex. If so, then the question is how to define the occurrences of objects for the reconstructed complexes. In particular, we may ask whether the occurrences can be defined in a *non-arbitrary way* in terms of the positions, complexes, and objects of the original or the reconstructed relation.

If in the reconstructed complexes each object occurs at most once, then occurrences may simply be defined as ordered pairs $\langle \xi, a \rangle$, with ξ a reconstructed complex and a an object. But if we want the reconstructed relation to be coalescence-free, we have to distinguish different cases.

For coalescence-free relations without strict symmetry, we can define occurrences in a complex ξ as ordered pairs $\langle \xi, p_1 \rangle, \ldots, \langle \xi, p_n \rangle$, with $p_1, \ldots, p_n$ the positions of the relation. This is the translation depicted above in Figure 6.

For coalescence-free relations with complete strict symmetry, we can define the occurrences of an object a in a complex ξ as triples $\langle \xi, a, 1 \rangle, \ldots, \langle \xi, a, k \rangle$, where k is the number of positions to which a is assigned to obtain ξ.

However, for some other strictly symmetric coalescence-free relations, we cannot define occurrences for certain complexes in a non-arbitrary way in terms of positions, complexes and objects within the context of set theory. This is, for example, the case for a quaternary cyclic relation for which the complexes may be depicted as four objects equally spaced on a circle and such that rotating them over 90° gives the same complex.

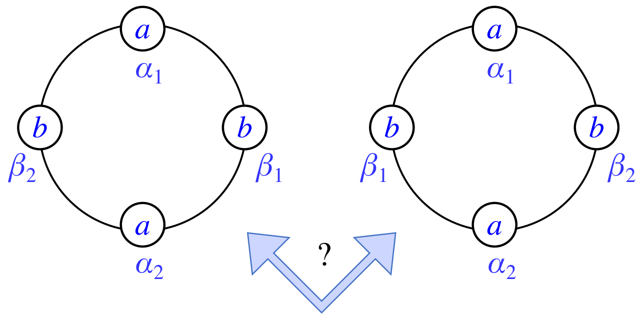

Figure 7: Occurrences cannot be reconstructed in a non-arbitrary way.

The proof is given in Appendix B.2.

3.2 From antipositionalism to positionalism

The name 'antipositionalism' suggests that the view is against positions, but it is certainly not against a reconstruction of this notion within the confines of its theory.

Reconstructing positions

According to (Fine 2000, p. 29) the antipositionalist can reconstruct positions as abstracts with respect to the equivalence relation *co-positionality*, where object a in state s is *co-positional* to object b in state t if s results from t by a substitution in which b goes into a (and vice versa). But this reconstruction is not satisfactory for cyclic relations, where the objects are arranged clockwise in a circle, because for such relations all objects in a state are co-positional with each other, and therefore we would get just one position (Leo 2008a, p. 357).

Here we will follow a different approach. Let R be an antipositional relation with an initial complex ξ_0 (i.e., a complex from which any complex of the relation can be obtained by a substitution). Then we could treat the occurrences of objects in ξ_0 as positions, but there are more elegant approaches; one makes use of abstraction and the other of subtraction.

Suppose that we may *abstract* from the nature of the objects of the occurrences. Then, by simultaneously abstracting in ξ_0 from the nature of the objects of all occurrences, we get a kind of skeleton complex.[8] What remains of the occurrences an antipositionalist may call the positions of the relation.

[8]Abstracting from the nature of the objects may be understood as a Cantorian abstraction (cf. Fine (1998)).

Instead of abstracting from the nature of the objects of the occurrences, we may perhaps also simultaneously *subtract* the objects from the occurrences. If so, then the result is again a skeleton complex with 'empty' occurrences that can be taken as positions.

In my view the operation of abstraction and the operation of subtraction are both quite natural. It's hard to say what is the best choice. An advantage of abstraction is that it does not necessarily commit you to the existence of additional entities. It may be seen as just a way of speaking about a class of complexes (cf. Russell (2009), pp. 33–34).[9] In favor of subtraction it may be argued that substitution is in fact a two-step operation, where in step one objects are subtracted and in step two objects are added. If so, then subtraction is an operation we implicitly already had.

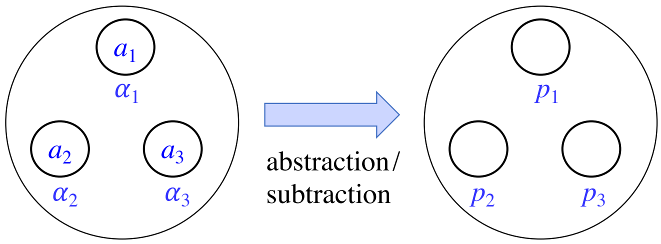

Figure 8: Translating antipositionalism to positionalism.

From antipositionalism to thick positionalism

We start with an antipositional relation R with an initial complex ξ_0, and assume that the operation of abstraction or subtraction yields a skeleton complex ζ with reconstructed positions, each corresponding with exactly one occurrence of an object in ξ_0. Then there is a bijection π from the occurrences in ξ_0 to the positions in ζ.

For an assignment f of objects to the positions, we define as resulting complex (if it exists) the complex obtained by the substitution $\pi \cdot f$ in ξ_0 together with the positions being occupied by the assigned objects.

The translation may give more complexes than in the original relation. For example, if R is the adjacency relation, then the corresponding positional relation has two positions p_1, p_2, and for a's being adjacent to b it has two complexes, one with p_1, p_2 being occupied by a, b, and another with p_1, p_2 being occupied by b, a.

[9]The occurrences of objects in an initial complex can collectively be used as a representation of the positions, and all such representations together form a non-arbitrary representation of the collection of positions. But it should be noted that, as a consequence of what is proved in Appendix B.3, it is not always possible for an antipositionalist to identify the positions individually in a non-arbitrary way with an equivalence class.

From antipositionalism to thin positionalism

For translating antipositionalism to thin positionalism, we follow the same route, except that we simply use the original complexes as the complexes for the positional relation. So we start again with an initial complex ξ_0, and we assume that by abstraction or subtraction we obtain reconstructed positions and a corresponding bijection π from the occurrences in ξ_0 to the positions. Then, for any assignment f of objects to the positions, define as resulting complex (if it exists) the complex obtained by substitution $\pi \cdot f$ in ξ_0.[10]

This completes the translation. To be acceptable for a thin positionalist, the reconstructed relation must satisfy the Composition Principle of Thin Positionalism.

This can be proved as follows. Let ξ_0, π be as in the translation, and let f be a substitution of objects for the reconstructed positions resulting in a complex ξ. Then substitution $\pi \cdot f$ in ξ_0 results in ξ as well. Let μ be a co-map of $\pi \cdot f$. Then, by the Composition Principle of Antipositionalism, for every substitution s' in ξ,

> s' results in an entity ξ' iff $\mu \cdot s'$ is a substitution in ξ_0 resulting in ξ'.

By the reconstruction of the positional relation, $\mu \cdot s'$ is a substitution in ξ_0 resulting in ξ' iff $\pi^{-1} \cdot (\mu \cdot s')$ is a substitution for the positions resulting in ξ'. So, because $\pi^{-1} \cdot (\mu \cdot s') = (\pi^{-1} \cdot \mu) \cdot s'$,

> s' results in ξ' iff $(\pi^{-1} \cdot \mu) \cdot s'$ is a substitution for the positions resulting in ξ'.

From this fact and the observation that $\pi^{-1} \cdot \mu$ is a surjective map from the positions to the occurrences of objects in ξ mapping each position p to an occurrence of the object substituted by f for p, it follows that $\pi^{-1} \cdot \mu$ is a co-map of f. This completes the proof.

If a relation has more complexes from which all of its complexes can be obtained by substitution, then any of them could be chosen for abstracting from the nature of the objects of the occurrences. As you might expect, the reconstruction of a positional relation is essentially independent of the choice of ξ_0. More specifically, the reconstructed sets of positions may perhaps be different for different choices of ξ_0, but it is not difficult to show that the reconstructed relations are all the same up to isomorphism.

Nevertheless, there is a subtle complication; in set theory the positions cannot always be reconstructed 'neutrally', i.e., without an arbitrary choice in terms of the basic ingredients of antipositionalism. This will be shortly discussed at the end of this section.

A serious restriction of the given reconstructions is that it only works for relations with an initial complex. But there might be more sophisticated reconstructions that also work for certain relations without initial complexes. However, for relations with a variable number of objects in different instantiations, like

[10] Although $\pi \cdot f$ is just a map from the occurrences in ξ_0 to objects, I identify it here with a substitution in ξ_0.

the relation of *forming a circle*, there may not be equivalent positional relations. This might mean that antipositionalism is a richer theory that offers more possibilities than positionalism.

Identifying positions

An interesting question is whether for any antipositional relation with an initial complex a reconstruction of positions can be made with no arbitrary choices.

For relations without strict symmetry a non-arbitrary reconstruction of positions is possible. We can, for example, identify a position for such a relation with the equivalence class of occurrences of objects in initial complexes that can be mapped to each other by co-maps.

For strictly symmetric relations this reconstruction does not work. For some strictly symmetric relations there is simply no reconstruction of positions possible in set theory without an arbitrary choice. This is, for example, the case for a quaternary cyclic relation for which the complexes may be depicted as four objects equally spaced on a circle and such that rotating them over 180° gives the same complex, but rotating them over 90° gives a different complex when the objects are not all the same.

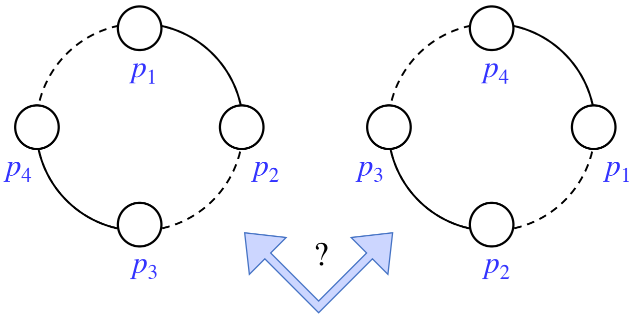

Figure 9: Positions cannot be reconstructed in a non-arbitrary way.

The proof that for this relation no non-arbitrary reconstruction of positions is possible is given in Appendix B.3.

3.3 Translations back and forth

That positionalism and antipositionalism are translatable into each other is nice, but it doesn't say that much. With translations relevant information can in principle get lost. Therefore it is very interesting to investigate if translations back and forth yield a structure that is isomorphic to the original relation. If this is the case, then the translation is really good.

First we translate back and forth starting from a positional relation, and then we translate back and forth starting from an antipositional relation.

From positionalism to antipositionalism and back again

We have the following results:

Claim 1. For a thick positional relation, the translation to antipositionalism and back gives a reconstructed relation that is the same as the original relation, up to isomorphism.

This is easy to see. The translation to antipositionalism gives a coalescence-free network of reconstructed complexes without any strict symmetry, where the reconstructed complexes correspond one-to-one with the original complexes. By translating it back to thick positionalism we get a structure of reconstructed complexes and positions that matches the original relation, up to isomorphism.

Claim 2. For a thin positional relation with at least one coalescence-free substitution for the positions, the translation to antipositionalism and back gives a reconstructed relation that is the same as the original relation, up to isomorphism.

We may prove this claim as follows. The translation to antipositionalism retains all complexes and the substitutions between them. Because the original relation has at least one coalescence-free substitution for the positions, it has an initial complex ξ_0. We use ξ_0 for the reconstruction of the positions. Then, for some bijection τ from the reconstructed positions to the original positions, any s with co-map μ is a substitution for the reconstructed positions iff $\tau \cdot s$ with co-map $\tau \cdot \mu$ is a substitution for the original positions. This proves the claim.

From antipositionalism to positionalism and back again

A minimal requirement for a translation of an antipositional relation R to positionalism and back again to result in essentially the same relation as the original one is that R has an *initial complex*, i.e., a complex from which any complex of the relation can be obtained by a substitution.

Claim 3. For an antipositional relation with at least one initial complex, the translation to thick positionalism and back gives a reconstructed relation that is the same as the original relation, up to isomorphism if and only if the original relation is coalescence-free and without any strict symmetry.

We prove this as follows. Assume that R is an antipositional relation with an initial complex ξ_0. Furthermore assume that R is coalescence-free and without any strict symmetry. Translating R to thick positionalism gives complexes being a combination of the original complexes and positions being occupied by the assigned objects. Because R is without any strict symmetry, these reconstructed complexes correspond one-to-one with the original complexes. Because R is coalescence-free, translating back to antipositionalism gives a network of complexes in which the complexes have occurrences that correspond one-to-one to the occurrences in the complexes of R. From this it follows that the reconstructed relation is the same as the original relation, up to isomorphism.

The "only if" part of the claim follows because the translation of a thick positional relation to antipositionalism always gives a coalescence-free relation without any strict symmetry. This completes the proof.

Claim 4. For an antipositional relation with at least one initial complex, the translation to thin positionalism and back gives a reconstructed relation that is the same as the original relation.

The proof is straightforward. By translating from antipositionalism to thin positionalism, the original complexes and the substitutions between them are fully retained. Translating back to antipositionalism gives as a result again the original relation.

4 Conclusion

In this paper we compared positionalism and antipositionalism. The main conclusion is that, contrary to what the names suggest, the views are not really opposites of each other. In fact, a specific form of positionalism, which I called thin positionalism, is very similar to antipositionalism.

In thin positionalism as well as in antipositionalism substitution is taken as a primitive operation. In thin positionalism we have substitution of objects for positions of a relation, and in antipositionalism we have substitution of objects for occurrences of objects in relational complexes.[11] Substitution is in both cases used to characterize the structure of a relation.

As we have seen, the translations back and forth show that there is a very close relationship between thin positionalism and antipositionalism. The class of thin positional relations with at least one coalescence-free assignment of objects to its positions matches perfectly with the class of antipositional relations with at least one initial complex; they are translatable into each other without any loss of information.

In summary, the relationship between thin positionalism and antipositionalism may be expressed as follows:

1. both views rely upon the notion of substitution, which I regard as a fundamental operation for expressing relatedness between complexes;

2. the main difference between the views is that in positionalism the relatedness between complexes is expressed via positions and in antipositionalism it is expressed directly between complexes;

3. the views are for a significant range of relations translatable into each other in a natural way with complete preservation of structure.

What about the standard view? Relations of the same significant range could also be translated from the standard view back and forth to positionalism and

[11] In thin positionalism we also have substitutions between complexes, but, as we saw, a thin positional relation is completely determined by the substitutions for the positions.

antipositionalism. However, in this case the end result is not necessarily isomorphic with the original relation. The reason is that in translating from the standard view to positionalism or antipositionalism some constitutive information—namely the order of the arguments—is lost. This puts the standard view apart from positionalism and antipositionalism

It may go too far to say that thin positionalism and antipositionalism are essentially the same. In thin positionalism, a relation is seen as a universal and positions belong to the fundamental furniture of the world, whereas in antipositionalism no ontological commitment to relations as universals and to positions is needed.[12]

Because antipositionalism is apparently less demanding with respect to ontological commitments, I am inclined to regard it as the preferable view. Furthermore, a strong feature of antipositionalism is that it may accept relations with a variable number of objects involved in the complexes, as in the relation of *forming a group* and *forming a circle*, for which the positionalist may have no equivalent counterpart.

But there may perhaps be reason for not jumping to the conclusion that antipositionalism is in every way superior, because a positionalist may accept relations with no complexes and relations for which the translation to antipositionalism yields an unconnected network of complexes. Such relations may be unacceptable for an antipositionalist.

Despite the differences, I consider the agreement between positionalism and antipositionalism as fundamental. The analysis given in this paper shows that the views are essentially two sides of the same coin. Therefore I regard the name 'antipositionalism' as misleading. A better name might be 'apositionalism'.

Acknowledgments
Many thanks to Kit Fine, Fraser MacBride, and Jan Plate for their valuable comments.

A Substitution principles

In (Fine 1989, pp. 235–238), Fine made a start for developing a *general theory of constituent structure*. The key notion of the theory is the operation of *substitution*. A substitution takes an entity ξ and a map from the *occurrences* of entities in ξ to entities as input, and gives an entity as a result (if any).

Fine gave the following example of a basic principle for the theory:

> If F' is the result of substituting E' for the occurrence e of E within F, then there is an occurrence e' of E' within F' such that the result of substituting any expression E'' for e' within F' is identical to the result of substituting E'' directly for e in F.

[12] As Kit Fine pointed out to me, whether this means that the two views are genuinely distinct depends upon one's willingness to draw a distinction between a kind of entity being basic or derivative within one's ontology.

The notions of substitution and occurrence are taken as primitive.

Because we may simultaneously substitute entities for occurrences, I propose the following more general principle.

Composition principle.
Let s be a substitution in an entity ξ resulting in an entity ξ'. Then there is a surjective map μ from the occurrences of entities in ξ to the occurrences of entities in ξ' such that

1. μ maps every occurrence α in ξ to an occurrence of the entity substituted by s for α,

2. for every substitution s' in ξ', s' results in an entity ξ'' iff $\mu \cdot s'$ is a substitution in ξ resulting in ξ'',

where $\mu \cdot s'$ denotes the substitution that maps each occurrence α in ξ to the entity substituted by s' for $\mu(\alpha)$.

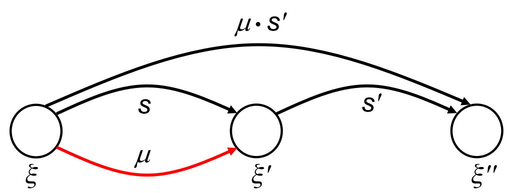

Figure 10: Composition Principle for Substitutions.

We call a map μ with this property a *co-map* of substitution s.

B Neutral reconstructions

In this appendix we show two things: (1) for some relations, occurrences of objects cannot be reconstructed set theoretically in a non-arbitrary way in terms of basic notions of thick positionalism, and (2) for some relations, positions cannot be reconstructed set theoretically in a non-arbitrary way in terms of basic notions of antipositionalism.

We will not give a precise definition of non-arbitrariness, but we will give formal definition of *neutrality* that obviously any non-arbitrary construction in set theory should fulfill. This notion of neutrality, which was introduced Leo (2008b), is interesting in its own right since it may be more generally applicable for showing that certain things cannot be modeled in set theory in a non-arbitrary way.

All reconstructions in this appendix are understood to be within the context of standard set theory with urelements. Other modeling media may provide more possibilities.

B.1 The notion of neutrality

I will define neutrality in the context of set theory with urelements A. The idea is as follows. Let X and Y be sets. Suppose that Y is constructed in a non-arbitrary way on the basis of X. Let π be a permutation of the urelements for which replacing in X each occurrence of each urelement a by $\pi(a)$ doesn't change the set. Then—since all urelements are set-theoretically indiscernible—replacing in Y each occurrence of each urelement a by $\pi(a)$ doesn't change this set either.

If Y has the property that each permutation of the urelements that keeps X unchanged also keeps Y unchanged, then we say that Y is *neutral* with respect to X.

The notion of neutrality may in principle be used to show that certain things cannot be constructed in a neutral way with respect to other things, and we will do that in the next sections, but first we give a formal definition of neutrality.

Let $\mathsf{V}[A]$ be the cumulative hierarchy with urelements A. Any function $u : A \to A$ can be lifted to a function $\widetilde{u} : \mathsf{V}[A] \to \mathsf{V}[A]$ in an obvious way:

$\widetilde{u}(a) = u(a)$ for any $a \in A$,

$\widetilde{u}(X) = \{\widetilde{u}(x) \mid x \in X\}$.

We may regard $\widetilde{u}(X)$ as the result of replacing in X each occurrence of each urelement a by $u(a)$.

Definition B.1. For $X, Y \in \mathsf{V}[A]$ we say that Y is *neutral with respect to* X if for any bijection $u : A \to A$,

$$\widetilde{u}(X) = X \quad \Rightarrow \quad \widetilde{u}(Y) = Y.$$

So if $A = \{a, b\}$, then any set in $\mathsf{V}[A]$ is neutral with respect to $\{a\}$, but $\{a\}$ is not neutral with respect to $\{a, b\}$.

I do not claim that the definition of neutrality completely characterises non-arbitrariness of a set-theoretic construction, but it should be clear that any non-arbitrary construction of Y on the basis of X will be neutral with respect to X.

B.2 Reconstructing occurrences

We will show that not for every positional relation the occurrences of objects in the complexes can be neutrally reconstructed in terms of positions, complexes, states, and objects.

Let R be a positional relation for which the states may be depicted as four not necessarily distinct objects equally spaced on a circle and such that rotating them over 90° gives the same state.

A set-theoretical positional model for R is a tuple $\mathcal{M} = \langle C, S, O, P, \Gamma, \Omega \rangle$, with complexes C, states S, objects O, positions P, a map Γ from O^P to C, and a map Ω from C to S, where Γ maps assignments of objects to positions to complexes, and Ω maps complexes to their corresponding states.[13]

We assume that C, S, O, and P are mutually disjoint sets of urelements, and that O has at least four objects.

The symmetry of R can be expressed in terms of the model $\mathcal{M}$ as follows.

The set P can be written as $\{p_1, p_2, p_3, p_4\}$ such that for the permutation group G generated by the map taking p_1, p_2, p_3, p_4 to p_4, p_1, p_2, p_3, we have for every $f, g \in O^P$, $\Omega(\Gamma(f)) = \Omega(\Gamma(g))$ iff $g = f \circ \pi$ for some $\pi \in G$.

Let us now try to reconstruct a coalescence-free thin positional or antipositional model for R with the same states as in R and for each state just *one* corresponding reconstructed complex.

For every reconstructed complex ξ with four distinct objects a, b, c, d we may define its occurrences non-arbitrarily as pairs $\langle \xi, a\rangle, \langle \xi, b\rangle, \langle \xi, c\rangle, \langle \xi, d\rangle$, but, if each complex has four occurrences, then no neutral reconstruction of all occurrences is possible with respect to $\mathcal{M}$. This can be shown as follows.

Select two objects a and b. Let $\delta : O \to O$ switch the objects a and b and leave all other objects unchanged. Define $u : C \cup S \cup O \cup P \to C \cup S \cup O \cup P$ by:

$$u(x) = \begin{cases} \delta(x) & \text{if } x \in O, \\ \Gamma(\delta \circ f) & \text{if } x = \Gamma(f) \text{ for some } f \in O^P, \\ \Omega(\Gamma(\delta \circ f)) & \text{if } x = \Omega(\Gamma(f)) \text{ for some } f \in O^P, \\ x & \text{otherwise.} \end{cases}$$

It is not difficult to see that u is a bijection, $u \circ u = \mathsf{id}_{C \cup S \cup O \cup P}$, and $\widetilde{u}(\mathcal{M}) = \mathcal{M}$.

Let *Rabab* be the state with objects a, b, a, b arranged in a circle (in that very order) and let $\alpha_1, \beta_1, \alpha_2, \beta_2$ be the occurrences of a, b, a, b in the corresponding reconstructed complex ξ_{abab}.

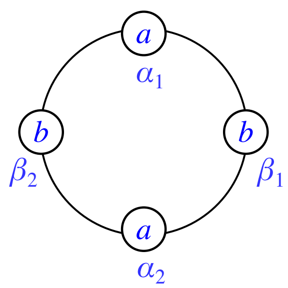

Figure 11: The occurrences cannot be reconstructed in a neutral way.

Now suppose that the occurrences are neutrally reconstructed with respect to $\mathcal{M}$. More specifically, suppose R has a coalescence-free thin positional or antipositional reconstruction $\mathcal{N}$ in $V[C \cup S \cup O \cup P]$ such that $\widetilde{u}(\mathcal{N}) = \mathcal{N}$.

[13] O^P denotes the set of functions from P to O.

Then, because $u(Rabab) = Rabab$ and u switches a and b, $\widetilde{u}(\alpha_1)$ must be an occurrence of b in the reconstructed complex ξ_{abab}. So, either $\widetilde{u}(\alpha_1) = \beta_1$ or $\widetilde{u}(\alpha_1) = \beta_2$.

Let c, d, e, f be distinct objects in O and let ξ_{cdef} be the complex obtained by substituting c, d, e, f for $\alpha_1, \beta_1, \alpha_2, \beta_2$ in ξ_{abab}. From $u(Rabab) = Rabab$ and $\widetilde{u}(\mathcal{N}) = \mathcal{N}$ it follows that substituting c, d, e, f for $\widetilde{u}(\alpha_1), \widetilde{u}(\beta_1), \widetilde{u}(\alpha_2), \widetilde{u}(\beta_2)$ in ξ_{abab} results in ξ_{cdef} as well.

From this it follows that $\widetilde{u}$ must preserve the relative order of the occurrences in ξ_{abab}. This means that either

$$\widetilde{u} \text{ maps } \alpha_1, \beta_1, \alpha_2, \beta_2 \text{ to } \beta_1, \alpha_2, \beta_2, \alpha_1$$

or

$$\widetilde{u} \text{ maps } \alpha_1, \beta_1, \alpha_2, \beta_2 \text{ to } \beta_2, \alpha_1, \beta_1, \alpha_2.$$

In both cases $\widetilde{u}(\widetilde{u}(\alpha_1)) \neq \alpha_1$. But, since $\widetilde{u} \circ \widetilde{u} = \widetilde{u \circ u}$,[14] this contradicts that $u \circ u = \mathsf{id}_{C \cup S \cup O \cup P}$.

So we conclude that if each state has just one reconstructed complex and each complex has four occurrences, then the occurrences cannot be neutrally reconstructed with respect to $\mathcal{M}$.

B.3 Reconstructing positions

In a similar way as we did for occurrences, we can prove that not for every relation positions can be neutrally reconstructed in terms of the notions of antipositionalism.

I will show this again for a cyclic relation, but not for the same one. For an antipositional relation that holds of objects a, b, c, d when a, b, c, d are arranged in a circle (in that very order) it is possible to reconstruct the positions in a non-arbitrary way. I leave this as an exercise.[15]

Let R be an antipositional relation for which the states may be depicted as four distinct objects equally spaced on a circle and such that rotating them over 180° gives the same state, but rotating them over 90° does not give the same state.

We assume that each state of R has just one corresponding complex.

The symmetry of R can be expressed as follows.

For every complex ξ the occurrences of objects can be written as $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ such that for the permutation group G generated by the map taking $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ to $\alpha_3, \alpha_4, \alpha_1, \alpha_2$, we have for every substitution s, t in ξ, if s results in a complex, then $\xi \cdot s = \xi \cdot t$ iff $s = \pi \cdot t$ for some $\pi \in G$.

In Corollary 7.8 of Leo (2008b) it is shown that for this relation positions cannot be neutrally reconstructed in terms of the notions of antipositionalism if

[14] More generally, for functions $u, v : A \to A$, with A a set of urelements, $\widetilde{u} \circ \widetilde{v} = \widetilde{u \circ v}$. We prove this by $\in$-induction: (i) If $x \in A$, then $\widetilde{u \circ v}(x) = u \circ v(x) = u \circ \widetilde{v}(x) = \widetilde{u} \circ \widetilde{v}(x)$. (ii) Let $x \in \mathsf{V}[A]$ and assume $\widetilde{u \circ v}(z) = \widetilde{u} \circ \widetilde{v}(z)$ for every $z \in x$. Then $\widetilde{u \circ v}(x) = \{\widetilde{u \circ v}(z) \mid z \in x\} = \{\widetilde{u} \circ \widetilde{v}(z) \mid z \in x\} = \widetilde{u}(\{\widetilde{v}(z) \mid z \in x\}) = \widetilde{u} \circ \widetilde{v}(x)$. So, by $\in$-induction, $\widetilde{u \circ v} = \widetilde{u} \circ \widetilde{v}$.

[15] A clue to the solution can be found in Example 6.5 of Leo (2008b).

substitution is directly done on objects. Here we show that it is also impossible when substitution is done on occurrences.

A set-theoretical antipositional model for R is a tuple $\mathcal{M} = \langle C, S, O, \mathsf{Oc}, \Pi, \Theta, \Omega \rangle$, with complexes C, states S, objects O, occurrences Oc, a map Π from Oc to O, a partial map Θ from $C \times O^{\mathsf{Oc}}$ to C, and a map Ω from C to S, where Π maps occurrences to their objects, Θ represents the substitutions in complexes of objects for occurrences, and Ω maps complexes to their corresponding states.

We assume that C, S, O, and Oc are mutually disjoint sets of urelements, and that each occurrence occurs in only one complex. Furthermore, we assume that O has at least four objects, and that R holds for any selection of four distinct objects in O in any order, but not for any other selection.

We call two states *siblings* if each can be obtained from the other by rotating the objects over 90°. Furthermore, we call two complexes *siblings* if their corresponding states are siblings, and we call two occurrences *siblings* if they are occurrences of the same object in complexes that are siblings. Note that by our assumptions each state, each complex, and each occurrence has exactly one sibling.

Define $u : C \cup S \cup O \cup \mathsf{Oc} \to C \cup S \cup O \cup \mathsf{Oc}$ by:

$$u(x) = \begin{cases} \text{sibling of } x & \text{if } x \in S, \\ \text{sibling of } x & \text{if } x \in C, \\ \text{sibling of } x & \text{if } x \in \mathsf{Oc}, \\ x & \text{otherwise.} \end{cases}$$

It is not difficult to see that u is a bijection, $u \circ u = \mathsf{id}_{C \cup S \cup O \cup P}$, and $\widetilde{u}(\mathcal{M}) = \mathcal{M}$.

Let $P = \{p_1, p_2, p_3, p_4\}$ be reconstructed positions for R and let Ψ be a partial map from O^P to S that maps assignments of objects to positions to corresponding states. We may assume that the assignment of distinct objects a, b, c, d to p_1, p_2, p_3, p_4 results in the same state as the assignment of c, d, a, b to p_1, p_2, p_3, p_4. We denote this state as $Rabcd$.

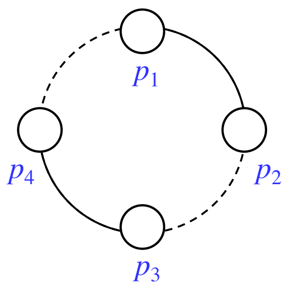

Figure 12: The positions cannot be reconstructed in a neutral way.

Now suppose that P and Ψ are neutral with respect to $\mathcal{M}$.

Let f be an assignment of a, b, c, d to p_1, p_2, p_3, p_4. Then $\widetilde{u}(f)$ is the assignment of a, b, c, d to $\widetilde{u}(p_1), \widetilde{u}(p_2), \widetilde{u}(p_3), \widetilde{u}(p_4)$, and

$$\begin{aligned}
\Psi(\widetilde{u}(f)) &= (\widetilde{u}(\Psi))(\widetilde{u}(f)) && \text{because } \widetilde{u}(\Psi) = \Psi \\
&= \widetilde{u}(\Psi(f)) && \text{because if } g\colon x \mapsto y, \text{ then } \widetilde{u}(g)\colon \widetilde{u}(x) \mapsto \widetilde{u}(y) \\
&= \text{sibling of } \Psi(f) && \text{by the definition of } u \\
&= Rdabc.
\end{aligned}$$

From this it follows that $\widetilde{u}$ preserves the relative order of the positions. This means that either

$$\widetilde{u} \text{ maps } p_1, p_2, p_3, p_4 \text{ to } p_2, p_3, p_4, p_1$$

or

$$\widetilde{u} \text{ maps } p_1, p_2, p_3, p_4 \text{ to } p_4, p_1, p_2, p_3.$$

In both cases $\widetilde{u}(\widetilde{u}(p_1)) \neq p_1$. But, since $\widetilde{u} \circ \widetilde{u} = \widetilde{u \circ u}$, this contradicts that $u \circ u = \mathsf{id}_{C \cup S \cup O \cup \mathsf{Oc}}$.

So we conclude that positions for R cannot be neutrally reconstructed with respect to $\mathcal{M}$.

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