

Why logical pluralists should be anti-exceptionalists

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Abstract

Beall and Restall's (2006) proposal sparked renewed interest in logical pluralism, but it hasn't gone uncriticized. The most widely-discussed objection to this view is the so-called collapse problem, which highlights some unpalatable consequences arising from the combination of an intra-theoretic pluralist framework with a traditional take on the properties that are essential for the notion of logicity. In this thesis I argue that, in order to avoid the collapse problem, pluralists of the same mold as Beall and Restall should depart from an orthodox perspective on logic. Instead, I propose that they should embrace some form of logical anti-exceptionalism, a position about logic whose core claim is that at least one of the properties that made up the 'exceptional' status of this discipline through its history should be rejected. In particular, I contend that their choice is between dropping either normativity or generality, and I do not fall short of assessing the implications of such metaphysical alterations.

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Chapter 1

Introduction

Logical pluralism is the metaphysical¹ stance according to which there are *at least two* correct logics.² What this claim precisely amounts to depends on a specification both of what a *logic* is and of what it means for a logic to be *correct*: the second section of this Introduction has the purpose of clarifying the precise terminological choices I commit to within this work.

Advanced for the first time by Carnap (1937), logical pluralism received a revival of interest after the joint work of Beall and Restall (2000, 2001, 2006), in which the two philosophers elucidate a version of logical pluralism that diverges significantly from that of its Carnapian origins. Namely, while Carnap thinks that different logics are required to be grounded on different languages, Beall and Restall argue that a plurality of logics can stem from one and the same language – that is, they present a form of *intra-theoretic* logical pluralism.³

¹I use the attribute ‘metaphysical’ because the thesis of logical pluralism concerns the essence of logic itself: according to pluralists, logic is not a unitary, monolithic tool; on the contrary, the fact that more than one logical system is taken to be correct implies that the fundamental *nature* of logic is that of a multi-faceted phenomenon. See Rush et al. (2014) for a further elucidation of the role that logical pluralism has for logic’s metaphysical status.

²Some would say *true*, *admissible*, *adequate* or *legitimate* logics. I chose the term ‘correct’ because, to my knowledge, it is the most used in recent literature. It is also less ambiguous when compared, for instance, with the attribute ‘true’.

³Actually, some (see, for instance, Hjortland, 2013) have argued that Beall and Restall’s logical pluralism is not really intra-theoretical, because the meaning of the logical symbols actually varies across the different logics. This is the so-called ‘meaning variance objection’ to logical pluralism, and while I think it is important to acknowledge its existence in the literature, in this thesis I will grant that it is indeed possible for Beall and Restall’s logics

The antagonist position *par excellence* of logical pluralism is logical monism: the idea that there is a unique correct logic. Undoubtedly, logical monism is the ‘default’ outlook on logic’s metaphysics, against which pluralism reacts (Clarke-Doane, forthcoming). For the sake of completeness, it is worth mentioning that there is a third way other than logical monism and pluralism: logical nihilism, according to which there is no correct logic *at all* (Russell, 2018).

As hinted by the previously mentioned divergence between Carnap’s and Beall and Restall’s frameworks, the ways in which logical pluralism got implemented in the literature are varied (Caret, 2021). While in the course of this work I touch upon several accounts of logical pluralism (Shapiro, 2014*b*; Caret, 2017; Blake-Turner and Russell, 2018), my starting point and the main target of my argumentation will be Beall and Restall’s so-called case-based logical pluralism. I made this choice because this version of logical pluralism has played a fundamental role in the philosophical debate of the past 20 years: due to its compelling innovativeness, it generated a substantial literature and it re-vitalized the interest in the fundamental question “how many logics are there?” (Russell, 2021). Long story short, it is currently the most discussed pluralist account.

Nonetheless, case-based logical pluralism is not without its flaws. In this thesis I will argue that pluralists of the same kind of Beall and Restall should revise part of their theory in order to overcome part of the criticisms they received. Namely, I claim that it would be congenial for them to commit to logical anti-exceptionalism, the view that at least some of the traditional properties of logic (e.g. apriority, analyticity, necessity, normativity, generality, etc.) ought to be abandoned (Martin and Hjortland, 2022).

In the remaining part of this introduction I will explain in detail what the aim of this thesis is. Then, I will clarify my terminological commitments by giving a definition of logic and by specifying what is the notion of logical correctness at stake in the proposal of Beall and Restall, which is also the one I refer to in this work. After that, I will elucidate what the claims of logical pluralism and logical anti-exceptionalism precisely amount to. Finally, I will delineate the structure of the thesis.

to stem from a unique language. This choice is justified by works like that of Ferrari and Orlandelli (2019), in which it is shown (from a proof-theoretic perspective) that developing a meaning-invariant logical pluralism of the same kind of that of Beall and Restall is a feasible enterprise.

1.1 Aim of the thesis

The aim of this thesis is to establish a connection between logical pluralism and logical anti-exceptionalism. In particular, I contend that logical pluralists of the same mold as Beall and Restall should give up on an exceptional view of logic, that is, a view in which logic possesses a peculiar status in virtue of a cluster of properties that are not part of the metaphysics of any other discipline. I argue for this claim by showing how dropping one of the traditional properties of logic provides a viable solution to the ‘collapse problem’ (also known as the ‘normativity objection’), a frequently raised challenge to the consistency of Beall and Restall’s view. The reason why case-based logical pluralism is susceptible to this criticism is that Beall and Restall combine the unorthodox thesis of logical pluralism with aspects about logic’s metaphysics that are deeply rooted in the traditional picture of this discipline. Namely, they argue that for a given system to be worthy of the label ‘logic’, it needs to be *necessary*, *formal*, and *normative*. Arguing via the collapse problem, I show how this attempt at departing from logical monism while still relying on this set of properties ultimately results in inconsistency. I then claim that in order for this inconsistency to be resolved, at least one of *normativity* and *generality* should be dropped; in fact, I show that rejecting either provides a good solution to the collapse problem. In advocating this argument, I also provide grounds for establishing a link between Beall and Restall’s pluralism and the anti-exceptionalism of logic (AEL) program. Namely, I claim that pluralists in the sense of Beall and Restall should commit to logical anti-exceptionalism, especially in its most recent formulation according to which AEL is not primarily about showing the continuity of logic with the sciences (Hjortland, 2017), but about questioning at least some of logic’s long-lastingly attributed properties (Martin and Hjortland, 2022). In fact, if my argument is convincing, it would become evident that logical pluralists have compelling reasons for embracing a picture of logic that deviates from its tradition.

This thesis has two main upshots. The first one consists in providing new inputs for the debate about the solutions that logical pluralists have available to overcome the collapse problem. The second and most relevant upshot is the contribution to the investigation concerning the relationship between logical pluralism and logical anti-exceptionalism. The thesis concerning the anti-exceptional status of logic is by itself independent from considerations about the number of correct logics out there. In fact, logical pluralism and

logical anti-exceptionalism have been formulated as two independent theses, and up to now there is not much research about their relation. Hjortland (2017) has argued that it could be convenient for the anti-exceptionalist to embrace logical pluralism (either in an extra- or intra-theoretic form).⁴ The question at stake in this thesis goes in the opposite direction: should logical pluralists be anti-exceptionalists? Beall and Restall certainly tried to support arguments that point towards a negative answer. Contrarily, the aim of this thesis is to show that logical pluralists should indeed be anti-exceptionalists.

1.2 What is a logic and when is a logic correct?

In order to make the theoretical perspective I adopt explicit, I now elucidate which interpretation of the two concepts that play a key role in any logical pluralism – *logic* and logical *correctness* – is at stake in this work.

With respect to the notion of *logic*, I follow Cook’s (2010a, p. 493) uncontroversial definition according to which a *logic* is any pair $\langle L, \Rightarrow \rangle$ where L is a formal language⁵ and \Rightarrow is a consequence relation holding between a set of statements from L (the premises) and another statement from L (the conclusion). Given that this definition of logic is highly formal (the members of L are strings of symbols) and natural languages are essentially informal,⁶ a question that spontaneously arises is: how are these two realms connected? Together with the tradition that legitimizes the use of logic as a tool for the analysis of subject matters that are not strictly formal, we assume that statements of natural languages can be adequately ‘regimented’ in virtue of possessing an underlying logical form⁷ that, once identified, can be part of

⁴However, not that of Beall and Restall, given that it is wedded to the insurgence of the collapse problem. It is not unlikely that Hjortland would have more enthusiasm for a version of case-based pluralism that is immune from this objection.

⁵That is, a nonempty set of primitive symbols coupled with a set of (usually recursive) formation rules that establish what are the requirements for a certain combination of primitive symbols to be ‘well-formed’.

⁶Just in the sense of non-rigorous: they contain ambiguities, figures of speech, vague expressions, and therefore it is not straightforward that they can be analyzed *logically*.

⁷The idea that there is a unique logical form bridging formal logic and natural language is controversial (Stich, 1975; Iacona, 2018). However, here I simply want to state that the discussion about logic at stake in this thesis is relevant for arguments in the form of natural language in virtue of them being suitable for formalization.

the formal statements contained in L (Tarski, 1956). This account of logic and of its relationship with natural language is aligned with the assumptions of Beall and Restall (2006).

The exact specification of what it takes for a logic to be *correct* is much more controversial, and it has been argued that different accounts of correctness lead to different versions of logical pluralism (Cook, 2010a; Passmann, 2021). I now provide three conceptions of logical correctness that are quite established in the literature, but it is important to acknowledge that there is not overall consensus about this notion. At the end of this section, I will locate the position of Beall and Restall within the range of theoretical possibilities that are available to logicians.

As a first instance of what logical correctness can amount to, let's examine Cook's (2010a) definition:

A logic (plus an identification and interpretation of logical vocabulary) is correct if and only if, for any way of interpreting the nonlogical vocabulary, the logic validates a particular argument if and only if the natural language statement corresponding to the conclusion of that argument is a logical consequence of the natural language statements corresponding to the premises of that argument. (Cook, 2010a, p. 495-496)

For a natural language statement – the conclusion – to be a logical consequence of a set of natural language statements – the premises – it is required that the simultaneous truth of all the premises *guarantees* the truth of the conclusion (*necessity*) and that such guarantee follows solely from the *logical form* of both the premises and the conclusion (*formality*). An alternative definition of logical correctness is provided by Haack (1978), who holds that “a logic is correct if the formal arguments that are valid in the formal system correspond to informal arguments that are valid in an extra-systematic sense” (Haack, 1978, p. 222), where ‘in an extra-systematic sense’ means ‘within natural language’. A third option is offered by Shapiro (2014b), who contends that all the logics which give rise to “legitimate structures, worth of mathematical study” (Shapiro, 2014b, p. 82) ought to be considered correct. It seems reasonable to argue that Cook and Haack are fundamentally aligned in identifying the correctness of a logic with the requirement for the valid formal arguments it comprises to be adequate codifications of the valid arguments that constitute their informal counterparts in natural language.

Differently, Shapiro holds that for a logic to be correct it needs to have a technical interest or fruitful applications in mathematics.

Sereni and Fogliani (2020) point out that the concept of correctness can diverge depending on what the ‘informal arguments’ that a logic is supposed to adequately codify are taken to be. Such ‘informal arguments’ could be identified with (i) the reasoning patterns that are judged to be valid according to agents’ intuitive ideas,⁸ (ii) the inferences that are valid according to some particularly relevant practice (e.g. mathematics) or (iii) the arguments that comply with some crucial philosophical requirements (e.g. necessity, formality, normativity). If we refer to this taxonomy, Cook’s definition turns out to be a combination of (i) and (iii), Haack’s concise definition of correctness seems to only commit to (i) and the focus of Shapiro’s conception appears to be on (ii).

From a more comprehensive perspective, following Passmann (2021, p. 12666), it can be exhaustively said that the various notions of correctness that have been presented in the literature satisfy different (sets of) desiderata, among which there could be axiomatisability, decidability, the existence of a good semantics, truth-preservation, necessity, formality, normativity, strength, the best suitability for mathematical reasoning, the best ability to model natural language phenomena, etc. As a result, the number of logics that fall under the label ‘correct’ is inversely proportional to the number of desiderata that are taken into account. In the case of logical pluralisms, the logics that satisfy the desiderata turn out to be more than one.

Since there is no universally agreed-upon notion of ‘logical correctness’, it is important to identify what is the particular conception of correctness at stake when it comes to case-based logical pluralism. With respect to Sereni and Fogliani’s (2020) taxonomy, Beall and Restall (2006) think that, in order to be correct, a formal logic ought to adequately codify informal arguments that can come *both* from agents’ intuitive ideas and from more technical practices (like mathematics); moreover, they think that in order to count as correct, a formal system needs to fulfill certain philosophical requirements: necessity, formality and normativity.⁹ That is, Beall and Restall’s notion of correctness is broad enough to comprise all the available hallmarks mentioned by Sereni and Fogliani. With respect to the possible desiderata for

⁸Where agents can be either laymen or trained logicians, and natural language is taken to be a medium to give manifestation to agents’ intuitive ideas.

⁹I will provide a more detailed explanation of what these requirements amount to in chapter 2.

correctness listed by Passmann, it seems to me that Beall and Restall hold that for a logic to be correct it needs to: (a) adequately model some kind of more or less technical practice (e.g. natural language conversations, mathematical reasoning, fictional discourse, etc.) and (b) satisfy the requirements of necessity, formality and normativity. The notion of correctness at stake in this work has now been specified.

1.3 Logical pluralism

This section is devoted to better elucidating the central ideas of logical pluralism. I offer a precise articulation of its main thesis in order to provide a solid groundwork for the subsequent discussion.

Logical pluralism, the position I focus on in this thesis, is not the only kind of pluralism that recently gathered attention and consensus: the increasingly widespread belief of there being different but equally legitimate approaches towards the same subject matter is leading scholars to adopt a more and more positive disposition towards pluralism in general, both as a methodology and as a substantial philosophical stance (Mandt, 1986). In addition to that concerning *logic*, the debates about *metaphysical*, *ontological* and *alethic* pluralisms have been significantly lively (Price, 1992; Turner, 2010; Pedersen and Wright, 2018; Russell, 2021), accompanied by a number of attempts to draw connections among them (Yu, 2018; Pedersen, 2008, 2014, 2020). While all pluralisms are compelled to face similar challenges (for instance that of secluding themselves from forms of uncontrolled relativism), each of them has its own specificities.

Let's now examine in further details what logical pluralism is. As previously noted, there is a plenitude of variations of this metaphysical position in the literature. Generally speaking, logical pluralism is the view that there is more than one correct logic. First of all, following Cook (2010*a*), we can single this position out from *logical relativism*: the idea of logic being a plural phenomenon amounts to saying that there is more than one correct account of logical consequence, while the thesis of logical relativism consists in holding that a certain correct account of logical consequence *is relative to* some variable, be it a language, a domain, or something else. There are logical pluralists who are also logical relativists, but in principle the two claims are independent. Additionally, logical pluralism should be distinguished from other trivial claims, like 'there is more than one logic that can be fruitfully

applied (in the general sense of ‘applied mathematics’)’ (mathematical application pluralism) and ‘there is more than one logic that can be fruitfully applied to philosophically interesting phenomena’ (philosophical logic pluralism).¹⁰

Following Cook (2010*a*) and Russell (2021), we ought to recognize an important difference between the first formulation of logical pluralism made by Carnap (1937) and that of Beall and Restall. Carnap’s pluralism is grounded on his tolerance for a (potentially unlimited) plenitude of linguistic frameworks. According to Carnap, a logician is entitled to choose the language which is ‘the best fit’ with the pragmatic concerns at stake in her practices (the kind of task she is dealing with, the goals that she wants to accomplish, etc.). Building on that language, she can design a formal system. Given that there is a wide range of languages that are a good fit for as many practices, there are also as many logics that can be build on those languages. This idea is concisely expressed in the following well-known passage from *The Logical Syntax of Language* (1937):

In logic there are no morals. Everyone is at liberty to build his own logic, i.e. his own language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (Carnap, 1937, Chapter 17)

This passage shows how close linguistic and logical tolerance are in Carnap’s view: perhaps he even thought that they amount to the same thesis (Russell, 2021). The underlying assumption here is that logics do not capture some fundamental fact about the logical consequence relation. For Carnap, they are just a matter of convention, of the free choice of the logician (Leitgeb and Carus, 2023). Carnap outlined a version of *extra-theoretic* logical pluralism, one which is also a form of logical relativism, because logics are depicted as relative to languages.

As already pointed out, Beall and Restall depart from the position of Carnap. They think that the notion of logical consequence carries an inherent ‘unsettledness’, which makes it a fragmented phenomenon: there are multiple senses in which ‘follows from’ can be used, each systematically captured by

¹⁰As Cook (2010*a*) himself specifies, the truth of these theses is not trivial *per se*, but it is trivial given the current status of logical research, in which different logics have been successfully employed for a multiplicity of purposes, both within and outside the field of mathematics.

a particular logic. However, crucially, all the admissible logics are built on one and the same language. This is why the pluralism of Beall and Restall is a form of *intra-theoretic* pluralism.

According to Cook (2010*a*), what makes this version of logical pluralism a substantial and controversial thesis is the idea of the multifaceted nature of logical consequence. This leads to the most interesting form of logical pluralism, namely, that there is more than one logic such that its corresponding logical consequence relation exactly preserves the truth from the premises to the conclusion of (formalized) natural language arguments.¹¹

For the sake of completeness, it is important to mention Shapiro’s (2014*a*) position within this debate. He points out that:

[I]n the philosophical literature, terms like “relativism” and “pluralism” are used in a variety of ways, and at least some of the discussion and debate on the issues appears to be bogged down because the participants do not use the terms the same way. (Shapiro, 2014*a*, p. 49)

While there is substantial agreement in the literature about case-based logical pluralism not falling under the spectrum of relativism, Shapiro argues that there is a sense in the logics endorsed by Beall and Restall bear a relative status. I will expand on this claim in chapter 4, section 2.

A more detailed picture of Beall and Restall’s logical pluralism will be given in chapter 2. In particular, it will become clear what “case-based” stands for. Before concluding this paragraph, it’s important to point out that there are way more versions of logical pluralism besides those of Carnap and Beall and Restall (Varzi, 2002; Russell, 2008; Field, 2009; Cook, 2010*a*; Shapiro, 2014*b*). Nonetheless, I think that the accounts of Carnap (1937) and Beall and Restall (2006) can be regarded as the two fundamental pillars within the literature, in virtue of their influence. A great number of positions (Blake-Turner and Russell, 2018; Caret, 2017; Pedersen, 2014) are largely

¹¹The adverb ‘exactly’ stands for the fact that, under this perspective of what it means for a pluralism to be interesting, there is a commitment about the inferences that are valid in the admissible logics being ‘really’ valid, and about the inferences that are invalid according to the admissible logics being ‘really’ invalid. If one were to require that, in order for a pluralism to be interesting, the admissible logics ought to ‘adequately’ preserve truth, this would entail a commitment only towards the idea that valid inferences are ‘really’ valid, and no strong commitment about invalid inferences being ‘really’ invalid, resulting in a pluralism that might not be actually interesting.

linked to them, both ideologically and genealogically; some are ‘updated versions’ of the frameworks of Carnap or Beall and Restall, designed to take into account and overcome particular criticisms.

1.4 Logical anti-exceptionalism

I now move to a deeper explanation of the second thesis about logic’s metaphysics that plays a key role in this work: logical anti-exceptionalism.

Logic has been traditionally conceived as an ‘exceptional’ discipline in virtue of special properties claimed to differentiate it from the other sciences in a substantial way. Necessity, aprioricity, analyticity, generality, normativity, and formality are only some of the characteristics that are thought to make up for the peculiarity of logic (Martin and Hjortland, 2022). However, such presumed exceptional status has been challenged. While the forerunner of an ‘anti-exceptional’ view of logic is Quine (1951), this position has found many more recent supporters, such as Maddy (2002), Priest (2005, 2016), Russell (2014, 2015) and Williamson (2014, 2013, 2017), in addition to many others (Hjortland, 2017; Russell, 2020; Wyatt and Payette, 2019; Arenhart, 2021; Martin and Hjortland, 2022).

The most organic and extensive critique to the traditional metaphysical picture of logic up-to-date has been put forward by Martin and Hjortland, who are gathering more and more compelling arguments against the idea that logic is a discipline carrying a special status. Their ‘Anti-exceptionalism of logic (AEL) program’ had as a manifesto the thesis that logical theories are continuous with sciences like mathematics and physics, and the methods employed in logic are continuous with the scientific method (Hjortland, 2017).

After five years, Martin and Hjortland (2022) radically reformulated the proposal of logical anti-exceptionalism. They argued that a great amount of clarificatory work was needed after the proliferation of widespread misunderstanding about AEL’s core, resulting in multiple distortions of its theses within current debates. According to them, rather than being about the idea that logic is continuous with the sciences, “AEL is better understood as the rejection of certain traditional properties of logic” (Martin and Hjortland, 2022, p. 1). The resulting outlook of AEL after this reformulation is that of a cluster of positions that, even if supported by different motivations, agree with the claim that *one or more* of the properties that are traditionally

attributed to logic do not actually hold.¹²

Moreover, Martin and Hjortland give special attention to the distinction between *metaphysical* and *epistemological* AEL. Metaphysical anti-exceptionalism is the denial that logic is ‘special’ because of the exceptional nature of logical facts, and that it is in virtue of this special status that logical laws are essentially different from those of other fields. This view about the nature of logic has traditionally set the basis for claims about logic’s formality, generality, normativity, necessity, analyticity and so on. Beall and Restall endorse both logical normativism and logical generalism, that is, the claims that logical laws are normative and general, respectively. These two theses are examples of how logic can be considered to be metaphysically exceptional. Since my aim is to challenge Beall and Restall’s commitment to normativity and generality, it follows that my primary aim is to draw a connection between case-based pluralism and (at least) the *metaphysical* level of AEL.

Epistemological AEL is instead the denial that logic is epistemologically privileged in virtue of the exceptional kind of *justifications* that set logical laws apart from others. Logical laws are in fact commonly considered foundational, self-evident or valid a priori. Epistemological AEL challenges the attribution of such properties to logical principles.

Even if in the recent history of logic the two views have been considered as a package, given that metaphysical anti-exceptionalists have been traditionally inclined to reject epistemological exceptionalism and vice versa, they are in principle distinct (Martin and Hjortland, 2022). To be very clear, I reiterate that in this thesis I argue for the fact that logical pluralists in the sense of Beall and Restall ought to be metaphysical anti-exceptionalists. Whether this implies something at the epistemological level is not an issue that will be addressed in this work.

Before ending this section, it is worth addressing a legitimate worry: isn’t it the case that logical pluralism fits within logical anti-exceptionalism by definition, given that it denies the standard view that there is only one correct logic? This question expresses a valid concern, because ‘being one and only’ is indeed a traditional property of logic. If logical anti-exceptionalism is the

¹²One might wonder whether giving up on only *one* of the traditional properties of logic is enough for a position on logic’s metaphysics to be called ‘anti-exceptional’. After all, the properties that are claimed to set logic apart from other disciplines are quite a few, and even if one of them is relinquished, logic preserves a great amount of its ‘exceptionality’. I think that this consideration puts pressure on the most recent definition of AEL, but in this thesis I will just accept it as it is.

rejection of one or more of the traditional properties of logic, it seems coherent to argue that logical pluralists are straightforwardly anti-exceptionalists. Nonetheless, I argue that the rejection of the uniqueness of the relation of logical consequence does not automatically make every instance of logical pluralism fall under anti-exceptionalism. The reason is that logic has not been considered special, compared to other disciplines, because of its ‘uniform’ nature. Traditional views about mathematics, physics and so on all hold that these disciplines tell a unique story about their laws and concepts. Monism didn’t play a role in distinguishing logical laws from those of the other sciences, and it did not contribute to the exceptional status of logic. Therefore, whether pluralism falls within anti-exceptionalism is not an idle question: the claim at stake in this thesis is not trivial.

1.5 Structure

In chapter 2 I outline Beall and Restall’s case-based logical pluralism and the challenge posed by the collapse problem. Then, I present one of the strategies that can be pursued in order to solve it: avoiding rivalry among logics. I conclude that this strategy seems to affect the other metaphysical properties of logic, and therefore dropping rivalry alone is not a viable solution. In chapter 3 I explore a second possible solution to the collapse problem: giving up on normativity. After outlining how the notion of logical normativity can be understood, I give some motivations for abandoning this property and I provide the example of a non-normativist version of Beall and Restall’s logical pluralism. Upon assessing this proposal and suggesting a way to improve it, I conclude that it successfully solves the collapse problem. In chapter 4 I analyze a third solution to the collapse problem: dropping generality. First of all, I explain in which sense logics can be said to be general and I put forward a challenge to the idea that Beall and Restall’s case-based pluralism actually preserves this property. Secondly, I present a non-generalist version of Beall and Restall’s framework and I evaluate it. As for the strategy of dropping normativity, I ultimately conclude that giving up on logical generality provides a good solution to the collapse problem. In the conclusion I sum up the results of the thesis and I take into account two possible objections to its outcome.

Chapter 2

Beall and Restall's Logical Pluralism

In this chapter I will spell out the characteristics of the version of logical pluralism that constitutes my focus and address one of the possible strategies to avoid the most discussed challenge to it, namely, the collapse problem. In the first section I will delineate the fundamental features of Beall and Restall's case-based logical pluralism. In the second section I will outline the collapse problem and identify the hallmarks of Beall and Restall's proposal that give rise to it. In the third section I will examine a proposed solution to the collapse problem: contriving a mechanism to avoid logical *rivalry*. In the final section I will contend that, in the ways in which it has been formulated in the debate, this strategy falls short of being the satisfactory resolution that its proponents claim it to be. The reason for this is that it is difficult (if not impossible) to drop rivalry without affecting some of the other metaphysical properties of logic. In short, giving up rivalry alone is not a viable strategy for the pluralist: more is required.

2.1 Case-based logical pluralism

Beall and Restall's (2006) proposal can be summarized as the conjunction of two theses (Russell, 2021). *First*, Beall and Restall embrace a generalized version of Tarski's Thesis about an argument's validity. In its original formulation, Tarski's Thesis is the claim that "[t]he sentence X follows logically from the sentences of the class K if and only if every model of the class K

is also a model of the sentence X” (Tarski, 1956, p.417). The generalized version that Beall and Restall endorse is the following:

Generalised Tarski Thesis (GTT): An argument is valid_x if and only if, in every case_x in which the premises are true, so is the conclusion. (Beall and Restall, 2006, p.29)

According to GTT, the concept of validity and the concept of case are constrained by a parameter, which expresses the belief in the existence of a multitude of *kinds* of cases and, a fortiori, of *senses* of validity.

Second, Beall and Restall hold that there are *at least two* admissible precisifications of the expression “ case_x ”, which lead to *at least two* acceptable senses for the validity predicate. Henceforth, validity is understood as truth preservation over “ cases_x ”, where different correct logics correspond to different precisifications of the range of ‘cases’.

Building on the Tarskian tradition, Beall and Restall think that the formal concept of logical consequence ought to represent its common usage in the language of everyday life. However, the way in which we use natural language can not be captured in a sharp way due to the wide range of our linguistic practices, and therefore logical consequence can’t be a one-size-fits-all phenomenon. On the contrary, it inevitably carries some degree of *unsettledness*. As it is the case for the Church-Turing Thesis, which aims at capturing the fuzzy notion of computability, this version of logical pluralism has among its aims that of making precise a concept that is inherently informal. Ferrari and Moruzzi (2017) efficaciously interpret Beall and Restall’s pluralism as a thesis of semantic indeterminacy of our concept of logical consequence – i.e. as a form of indeterminacy logical pluralism.

Given that the pre-theoretic notion of logical consequence doesn’t have sharp borders, its formalization ought to account for some *degree of freedom*. At the same time, however, such liberty should be regimented, and in Beall and Restall’s proposal the number of admissible types of logical consequence is narrowed down by the requirement of it being a *necessary, formal* and *normative* relation on sentences. The fulfilment of these properties is of great value for Beall and Restall, because it guarantees the continuation of their proposal with the core tradition of logic, that is, with the picture of this discipline that the imprint of history made into a time-honored standard, which they strongly want to adhere to.¹³

¹³Beall and Restall’s endorsement of these and only these as the properties which con-

For logical consequence to be necessary means that “the truth of the premises of a valid argument *necessitates* the truth of the conclusion of that argument” (Beall and Restall, 2006, p. 14). Necessity is therefore a property that concerns the transition from the premises to the conclusion of a valid argument. Specifically, it requires that in all the possible worlds in which the premises are true, the conclusion is also true.¹⁴

With respect to formality, Beall and Restall don’t settle on a unique definition, motivating this choice with the absence of a unified notion of formality within the logical tradition itself. Following MacFarlane (2000), they subscribe to three distinct characterisations of this property: (1) logic provides constitutive norms for thought as such, independently from considerations about the kind of propositional content at stake; (2) logic is indifferent to the identities of objects – that is, an argument will be valid independently from the particular reference of the denoting terms that appear in it; (3) logic is about the *form* of sentences, as opposed to their *content*; it is therefore not about the elements of sentences which pick out a referent in the world, but about those items that constitute the form of such sentences, like logical connectives, whose function is not that of referring to something but showing how two distinct claims are related to each other.

Moving to normativity, Beall and Restall define it as the property for which, given a valid argument, “you somehow go *wrong* if you accept the

stitute the settled core of logical consequence has been challenged. Paseau (2007) claims that the settled core of logical consequence has more substance. When reviewing Beall and Restall’s proposal, he argues that ‘they appear to have cut the settled core of logical consequence to the bone and stripped it of its many crucial constituents’ (Paseau, 2007, p. 393). More recently, Griffiths (2013) also put pressure on Beall and Restall’s selection of the settled features of consequence. He notices that ‘the test for whether or not a certain feature of consequence is part of the ‘settled core’ is empirical’ (Griffiths, 2013, p. 172). That is, according to Griffiths, necessity, formality and normativity are endorsed by Beall and Restall because they contend that, if we look at the state of the debate about them, a consensus about them characterizing logical consequence has been reached. Differently, properties like ‘axiomatisability’ are taken to be unsettled because there is still debate about them. However, Griffiths identifies a certain degree of arbitrariness in Beall and Restall’s endorsement of these exact properties. He argues that any of necessity, formality and normativity are any more settled than axiomatisability (or other supposedly unsettled properties), and presents many historical examples as evidence for this claim.

¹⁴An important clarification put forward by Beall and Restall concerns the distinction between *necessity* and *epistemic certainty*: the first is a claim like “if A is true, then B is true too”, but from A it need not to be *certain* that B, as epistemic certainty would require.

premises but reject the conclusion” (Beall and Restall, 2006, p. 16). Moreover, they claim that valid arguments ought to be used for the “rational assessment” of beliefs and theories, and they therefore constitute the (normative) benchmark with respect to which agents’ inferences ought to be judged.

In addition to specifying what they take to be the requirements for a meaningful logical pluralist view, Beall and Restall make explicit what are the precisifications of “case_x” that they consider legitimate, that is, they identify the logics that they believe to be admissible: they embrace *classical*, *relevant* and *constructive* logic.¹⁵ Classical logic corresponds to truth-preservation over *complete* and *consistent* models, intuitionistic logic corresponds to truth-preservation over *possibly incomplete* constructions and relevant logic corresponds to truth-preservation over *possibly incomplete and inconsistent* situations. None of these three qualifies as the *correct* or *right* account of logical consequence: each of them captures one of the *façades* of validity, accounting for the way in which we use “follows from” in a certain subset of our linguistic practices.

Ultimately, Beall and Restall claim that “the degree of success of any account of logical consequence will be the degree to which it *works*” (Beall and Restall, 2006, p. 28-29). This shows that the approach of the two philosophers is guided by pragmatic, instrumental considerations: the logics that they deem admissible are those that are useful for capturing a particular sense in which logical consequence is employed in our linguistic practices, and not by idealistic *a priori* concerns.

This account of logical pluralism is considered to be very attractive. After all, it is a reality-matching view that comes at little or no cost and offers a charitable interpretation of many debates within the field of philosophical logic (Russell, 2021). However, a number of arguments have been moved against it. One of them is the collapse problem.

¹⁵Notice that they are not very assertive about the endorsement of this precise cluster of logics - they say that their pluralist proposal is mainly aimed at designing a certain pluralist ‘framework’, whose content can then be filled in a variety of ways, depending on how many kinds of cases one judges to be admissible.

2.2 The collapse problem

The collapse problem has been formulated in many different ways within the literature. Its first outline was given by Williamson (1988), who argued for there being a substantial dispute between the classical and the intuitionist logician, one that could not be solved within a pluralist framework without incurring in normative problems.

As a matter of fact, both classical and intuitionist logicians treat $X \vdash A$ as meaning that you are committed to A in making the set of assumptions X . It would otherwise be unclear that they could recognize each other as engaged in *reasoning* at all; to speak of classical and intuitionistic *logic* would be to equivocate on the word ‘logic’. Suppose that there were distinct but equally legitimate ‘deducibility’ relations, one classical and one intuitionist, and that you discovered your beliefs to have a certain consequence in the sense of one but not in the sense of the other; should you accept that consequence or not? (Williamson, 1988, p. 112)

After that, the most well-known formulations of the normativity objection are those made by Priest (2005):

Let s be some situation about which we are reasoning; suppose that s is in different classes of situations, say, K_1 and K_2 . Should one use the notion of validity appropriate for K_1 or for K_2 ? We cannot give the answer ‘both’ here. Take some inference that is valid in K_1 but not K_2 , $\alpha \vdash \beta$, and suppose that we know (or assume) α holds in s ; are we, or are we not entitled to accept that β does? Either we are or we are not: there can be no pluralism about this. In fact, the answer is that we are. (Priest, 2005, p. 203)

And Read (2006):

[S]uppose there really are two equally good accounts of deductive validity, K_1 and K_2 , that β follows from α according to K_1 but not K_2 , and we know that α is true... It follows K_1 -ly that β is true, but not K_2 -ly. Should we, or should we not conclude that β is true? The answer seems clear: K_1 trumps K_2 . After all, K_2

does not tell us that β is false; it simply fails to tell us whether it is true... K_1 and K_2 are not equally good. K_1 answers a crucial question which K_2 does not. [This] question is the central question of logic. (Read, 2006, p. 194-195)

The main point of the argument is that the normative force of one or more logics is neutralized by a stronger or a weaker logic when they disagree on the validity of a certain argument.¹⁶ This is undesirable, because the attractiveness of pluralism is to have more than one logic to provide exemplary guidance to rational agents. However, the collapse problem shows that in the situations in which the plurality of logic is called into action due to the disagreement of two admissible consequences relations, one of the two turns out to actually contribute nothing. Even if the normative effects of all the logics that one deems admissible are taken into account, what is the logic which ought to be followed by a certain agent in her reasoning tasks? It seems that logical normativity is hardly compatible with a pluralist framework.

Like many others, I regard the collapse problem as a serious challenge for the logical pluralist. What strategies can be employed to tackle this issue? In order to even start thinking about a possible solution, it might be useful to individuate those features within Beall and Restall's view that, combined, give rise to this objection. According to Stei (2020c), the collapse problem in its most general version applies to a whole family of pluralist positions. Namely, those that present the conjunction of the following hallmarks, which Stei collectively calls *Global Consequence Pluralism* (GCP):¹⁷

¹⁶Notice that the normativity objection can lead to a collapse to either the *strongest* or the *weakest* logic, depending on the particular conception of normativity at stake: Beall and Restall claim that normativity is the property in virtue of which “you somehow go *wrong* if you accept the premises but reject the conclusion” of a valid argument, and this leads to a collapse to the strongest logic (in their case, that corresponds to a collapse of intuitionistic and relevant logics into classical logic). If one takes normativity to be regulated by a principle in which the logical facts that provide doxastic guidance are constituted by invalidities, e.g. “if an argument is not valid, then you should not use it as a model for reasoning”, then the collapse is towards the weakest logic.

¹⁷The collapse problem has been sometimes interpreted as an objection that puts the logical pluralist in front of an *aut-aut*: in light of the inconsistency it outlines, pluralists must choose between pluralism and normativity (Evershed, 2021). According to such views, the two necessary and sufficient hallmarks of the collapse problem are (1) a plurality of logical consequence relations within one language and (2) normativity. Stei's proposal, to which I commit for this work, diverges from such accounts by identifying a broader set of conditions as those that are necessary and sufficient for the rise of the collapse

- a. There is more than one correct logical consequence relation within one language.
- b. Logical consequence is general.¹⁸
- c. There is rivalry between different logical consequence relations.
- d. Logical consequence is normative.

Many of the versions of logical pluralism in the literature (Carnap, 1937; Varzi, 2002; Cook, 2010*a*; Shapiro, 2014*b*) do not fall within GCP; however, Beall and Restall’s (2006) logical pluralism surely does.

To sum up, the collapse problem for Beall and Restall’s account consists in the fact that (i) logics are normative and general (hallmarks (b) and (d)), and (ii) there are arguments that turn out valid according to admissible-logic₁ but not according to admissible-logic₂ (hallmarks (a) and (c)), from which it follows that (iii) agents are sometimes motivated to reason in conflicting ways given the inconsistent directives prescribed by the various logics that they consider legitimate (and this outcome is unacceptable).

In Russell’s (2018) concise words:

[...] if pluralism is true, then sometimes the answer to the question: ‘is this argument valid?’ is ‘Yes. And no.’ Assuming that this means that the answer to normative questions like ‘ought I to believe the conclusion of the argument?’ and ‘ought I to infer the conclusion from the premises?’ can’t ever be ‘Yes. And no.’, logical pluralism must have made a mistake. (Russell, 2018, p. 373)

In order to avoid the undesirable consequences that the collapse problem brings about, at least one among (a)-(d) ought to be discarded.

problem. I think that a more in-depth analysis should be carried out to verify whether the following four hallmarks are indeed all *necessary* and *sufficient* for this objection, and whether and to which extent they are interrelated. In this thesis I touch upon this issue by answering questions like “Can there be generality without normativity?” (section 3.5) and “Can there be normativity without generality?” (section 4.6), but I do not provide an exhaustive and definitive take on the topic.

¹⁸In his paper, Stei uses the attribute ‘global’. However, he then interchanges it with ‘general’, which explains my substitution of ‘global’ with ‘general’ for this list. I made this choice to have uniform terminology throughout the thesis.

Since the rejection of (a) would bring us back to logical *monism* and I want to find a solution that refrains from the relinquishment of a pluralist perspective on logic *tout court*, giving up on (a) is not an option.

The only hallmark whose rejection does not seem to commit Beall and Restall’s pluralism to anti-exceptionalism is (c). In fact, logical rivalry is not one of the traditional properties of logic that determines the exceptional status of this discipline. Quite the opposite: the concept of logical rivalry enters into play only in pluralist frameworks,¹⁹ and the traditional view of logic is not that of a pluralist discipline. Therefore, in principle, dropping rivalry does not affect the alleged exceptional status of logic in any way. However, I will argue that the solutions falling within the family of GCP *minus* (c) that have been offered in the literature have significant issues. In particular, some of their shortcomings seem to suggest that it is not possible to discard logical rivalry without affecting any of the other hallmarks. In the next section I examine the outcomes of this strategy in detail, pointing out why I think that they are not convincing.

2.3 Dropping rivalry

Rejecting logical rivalry while maintaining all the other hallmarks of GCP might *prima facie* seem a very appealing solution to the collapse problem. In particular, it seems to be a strategy that allows to solve the problem without any consequence on the traditional metaphysics of logic. In this section, I challenge this alleged strategy. After explaining the two main senses in which logics can be said to be rivals, I examine two proposals which have been presented as instances of case-based pluralism that successfully avoid the normativity objection by dropping logical rivalry while leaving the other hallmarks of Beall and Restall unchanged.

2.3.1 In what sense are logics rivals?

Determining what exactly logical rivalry (or disagreement) amounts to – and therefore when two logics are authentic rivals – is an arduous enterprise (Stei, 2020*a*; Barrio et al., 2021). For the sake of this discussion, I follow the concept of logical rivalry employed by Stei (2020*c*) for his third hallmark. According to this interpretation, the idea of logical rivalry expresses the fact

¹⁹In monism there is no rivalry because the one true logic has no competitors.

that the pluralist positions which are members of the GCP family entail the emergence of a ‘competition’ among the logical consequences relations that are deemed admissible.

Rivalry can be said to be located either at the *semantic* or at the *applicational* level of the logics involved. Rivalry located at the semantic level corresponds to semantic disagreement. Semantic disagreement obtains in case two logics L_1 and L_2 share the logical vocabulary and its meaning, but then there are arguments which are licensed by L_1 but not by L_2 (for instance, if we take L_1 to be classical logic and L_2 to be relevant logic we have that L_1 licenses ‘ $\phi \wedge \neg\phi \models \psi$ ’ but that is not the case for L_2). In this case, there is a straightforward sense in which L_1 and L_2 are rivals: they sometimes disagree on the validity of the same inferences.

A historically established example of a pluralist framework that avoids semantic disagreement is Carnap’s account. In fact, the core idea of Carnap’s logical pluralism is that distinct logics are different because their *languages* are different, and the meanings of the connectives are determined by the logic they are part of. This implies that when a classical and an intuitionistic logician disagree about the validity of the *tertium non datur* principle ($\phi \vee \neg\phi$), their divergence ultimately rests on a variation in the meaning of negation and disjunction (i.e. not on some fact about validity), and therefore there is no *genuine* dispute between them (Quine, 1970). According to this picture, the classical and the relevant logicians simply talk past each other in virtue of them employing different languages. This is the *meaning-variance thesis* about logical disputes, and although it has been deemed attractive by philosophers like Haack (1978) and Carnap (1937), pluralists like Beall and Restall (2006) try to avoid it at all costs. The reason for striving towards a view that is immune from *meaning-variance* is that *meaning-variance pluralism* is not considered to be such an interesting position: since either the meaning of ‘valid’ or that of the logical connectives varies across logical theories, there is no real conflict between validity-attributions. As Hjortland (2013) puts it, “the overarching idea is to think of logical pluralism [...] as a plurality of consequence relations (and derivability relations) within one and the same logical theory” (Hjortland, 2013, p.2) – that is, what many logicians find interesting lays within the boundaries of *intra-theoretic pluralism*.

If we refer to the definition of semantic rivalry given above, case-based logical pluralism turns out to present such feature. However, Beall and Restall contend that when two logics give different verdicts about some argument form like ‘ $\phi \wedge \neg\phi \models \psi$ ’ considered ‘in the abstract’, that is, independently

from an application, they are not *really* rivals.²⁰ They simply specify a different *façade* of the concept of logical consequence, without truly competing. According to this view, the relationship between classical and relevant logic, which disagree about ‘ $\phi \wedge \neg\phi \models \psi$ ’, should be described in the following terms:

These two accounts of consequence are *different* but, with respect to the chief question of Logic (what arguments are valid?), they are not rivals. There is no sense in calling the two accounts *rivals* with respect to whether such and so argument is valid. Qua answers to Logic’s chief question, the two accounts do not compete. (Beall and Restall, 2006, p. 44)

What Beall and Restall consider to be the only meaningful sense of logical rivalry enters into play when logics are *applied*.

Indeed, rivalry among logics can also be claimed to be located at the *applicational* level. Stei explains that, given two logics L_1 and L_2 that can be applied to a domain D , applicational rivalry arises when L_1 and L_2 do not give the same verdicts for all arguments pertaining D . More precisely:

[T]here is [applicational] rivalry among L_1 and L_2 iff there is at least one argument A_n in domain D such that A_n is valid-in- L_1 but not valid-in- L_2 . In these cases, L_1 sanctions the move from the premises of A_n to its conclusion, while L_2 does not. (Stei, 2020b, p. 416)

This is the type of logical antagonism Beall and Restall commit to. They claim that “it is only at [...] the level of application, that *rivalry* between logics arises” (Beall and Restall, 2006, p. 36). According to their view, relevant and classical consequence compete at the level of the application because, for instance, “relevant consequence seems better for modelling inconsistent or incomplete theories; [while] classical consequence seems to perform downright badly in that role” (Beall and Restall, 2006, p. 59).

Within the literature there are at least two compelling versions of case-based pluralism that avoid the collapse problem in virtue of rejecting logical

²⁰It is useful, here, to think about Priest’s (2005) distinction between *pure* and *applied* logics.

rivalry (and, supposedly, nothing else). I am referring to Caret’s (2017) logical contextualism²¹ and Tajer’s (2022b) idea of a plurality of bridge principles characterizing the normativity force of the logics that one deems admissible. In the following part of this chapter I will analyze these two options in detail and provide an explanation for why this strategy doesn’t seem to work *in general*, and not only in the two instances that were just mentioned.

2.3.2 Caret’s logical contextualism

A compelling attempt to preserve the consistency of Beall and Restall’s pluralism without giving up on generality and normativity comes from the work of Caret (2017), which at the same time initiates the debate about a *contextualisation* of logic(s) that is an alternative to that theorized by Shapiro (2014b). Inspired by the idea – now widespread within the field of epistemology²² – that the operator ‘knows’ includes a parameter for an *epistemic standard*, Caret (2017) claims that ‘is valid’ carries a parameter for a *deductive standard* which, in turn, gives the appropriate extension to the validity attributions made in a certain context.²³ When sufficiently overt,²⁴ the deductive standard is supposed to *distinctly* select the logic that is the most suitable for a particular *conversational context*, that is, the context originated (at least) by “the structural assumptions of determinacy or consistency, our epistemic interests and our attention” (Caret, 2017, p. 756). This mechanism enables the pluralist to avoid the threat of collapse because of there being no more applicational rivalry between distinct logics, given that each context is mapped to one and only one logic.

My criticism towards this account stems from an asymmetry between epistemic and logical contextualism so construed. According to epistemic contex-

²¹Caret’s account may be interpreted as entailing a rejection of (b), given that Beall and Restall consider ‘contexts’ as something that already involves a downsizing of the notion of generality they commit to. This is, for instance, Goßens’s (2022) take on the issue. However, I *primarily* consider Caret’s contextualism as a rejection of (c), and it is not immediately evident that this view also entails the negation of (b). Nevertheless, I will argue that ultimately it does.

²²See Cohen (1988), DeRose (1992), Lewis (1996).

²³An in-depth analysis and critique of Caret’s logical contextualism was developed in collaboration with Sabina Domínguez Parrado in a joint paper that was supervised by Robert Passmann as part of a Master of Logic individual research project.

²⁴I think that the issue of whether in our actual logical or linguistic practices it happens regularly that the deductive standard at stake is easily intelligible is somewhat problematic.

tualism, each conversational context selects an epistemic standard, altering the extension of ‘knows’ accordingly; analogously, in logical contextualism each conversational context is said to select a deductive standard, altering the extension of ‘is valid’ accordingly (Caret, 2017, p. 752). An important aspect of every contextualist account is the specification of the features of the context that select and shape the relevant standard that, subsequently, determines the content of the expression that is being analyzed (indexical contextualism) or its extension (non-indexical contextualism). Caret claims that “the structural assumptions of determinacy or consistency, our epistemic interests, and our attention are some of the factors that plausibly help determine a deductive standard” (Caret, 2017, p. 756). I argue that the set of features which are claimed to select a deductive standard is vague and under specified: a more explicit delineation of the mentioned items is absent in Caret (2017), yet required for the sake of clarity. It might be, however, that the way in which a certain deductive standard gets selected is not as ‘transparent’ as the way in which a particular standard is selected in other kinds of contextualism.

Take the example of epistemic contextualism. What determines the epistemic standard that specifies the (more or less strict) sense in which the operator ‘know’ is used is determined by the accuracy standard required by the situation of utterance: at a scientific conference, ‘I know’ will be employed according to very demanding requirements, while in convivial situations it is reasonably allowed to say ‘I know’ without the need to meet overly strict requirements. Going back to contextualism about validity, what could we say for the kind of contextual elements that might play a role in determining the content of the predicate ‘valid’? Caret claims that his contextualism is concerned with the *conversational context*, hence the items that determine the context ought to be selected among some pragmatic conversational features.²⁵ However, defining a conceptual apparatus with a clear descriptions of the properties of the various conversational features and of the dimensions across which they can vary seems a very difficult enterprise. Moreover, grasping which of such properties and dimensions are actually instantiated in a certain situation appears epistemically demanding: certainly less straightforward than understanding what is the epistemic standard of a certain sit-

²⁵Notice that identifying the conversational context as the framework which determines the deductive standard is not the only option: in Shapiro’s (2014*b*) contextualism, that role is played by the *mathematical* context: the context determining the meaning of “valid” is the mathematical structure. I examine Shapiro’s contextualism more in depth later on.

uation. A conversational context determining a clear deductive standard could be, for instance, a talk at a conference with the title “Topics in intuitionistic mathematics”, or a workshop with the title “Semantics for relevant logics”. However, contexts that are this explicit about a certain deductive standard are extremely rare: in the majority of conversational contexts the features that might be relevant for determining the deductive standard of a certain conversational context remain underspecified. According to Stenning and van Lambalgen (2008), the difficulty in determining what is the relevant deductive standard at stake in a certain situation seems to account for the underwhelming results of some of the psychological tasks commonly employed to assess agents’ rationality. For instance, consider the Wason’s selection task, in which participants are given four double-sided cards (they all present a letter on one side and a number on the other) of which they can only see the sides showing “A”, “K”, “4”, “7”, and they have to decide which cards to turn in order to verify “If there is a vowel on one side, then there is an even number on the other side”. Stenning and van Lambalgen claim that an often overlooked possible explanation of why agents perform very poorly in the Wason’s selection task is the unawareness of the deductive standard involved in the experiment. In fact, multiple logical forms can be assigned to the conversational context in which the natural language conditional occurs. For instance, the conditional can be interpreted either descriptively or deontically. More broadly, the conditional involved in the experiment is taken to have the semantics of the classical logic material conditional, while it is not universally agreed that, in daily conversations, the classical semantic interpretation of the conditional matches its intuitive meaning. In conclusion, it appears much more difficult to gain an awareness of the conversational features that determine the deductive standard which, in turn, determines the content of the predicate “valid” than to determine the elements that play a role in determining other types of context (for instance, epistemic contexts). It remains to be explored whether this issue can be solved or it has to be accepted as an inevitable feature of a logical contextualism that is largely grounded on the pragmatic level of arguments.

The difference between logical and epistemic contextualism has also been pointed out by Steinberger, who claims that “epistemic contextualism enjoys at least a *prima facie* intuitive pull. The same cannot be said for Caret’s proposal. [...] We simply do not ordinarily recognize contexts that select for different stricter or laxer logical standards and so for weaker and stronger logics” (Steinberger, 2019a, p. 13).

There is another aspect of Caret’s (2017) proposal that I want to challenge: one of the alleged upshots of conversational logical contextualism so formulated is that it preserves the picture of logic as a *topic neutral* discipline, in virtue of conversational contexts being independent from considerations about *domains*. In fact, Caret puts great effort in highlighting that his contextualist proposal doesn’t undermine topic neutrality, which has traditionally been regarded as one of the main characteristics of logic; he claims that “for the contextualist about validity, [...] context has nothing to do with subject matter, it is simply conversational context along broadly Kaplanian lines” (Caret, 2017, p. 756). Therefore, his view apparently doesn’t regress into a form of topic-relativism about logic: it is not the subject matter that determines the deductive standard, but a certain set of purely pragmatic conversational items. I claim that it is far-fetched to hold that this contextualist proposal is independent of the notion of domain. Consider the following conversation between two agents, Alvin and Boram, that Caret himself uses to better explain his proposal:²⁶

Alvin: I think the Liar sentence is true.

Boram: How do you arrive at that conclusion?

Alvin: Well, the alternative is absurd, which entails that the Liar is true.

Boram: Couldn’t the truth-value of the Liar be indeterminate?

Alvin: That’s a fair point, I guess nothing entails that the Liar is true or false.

I contest that in this linguistic exchange the ‘attention’ and the ‘epistemic interests’ of the speakers are independent from the topic of the conversation – namely, the Liar paradox. I argue, instead, that it is quite the opposite: it appears plausible to claim that Alvin and Boram’s attention and epistemic interest are determined by the fact that they are having a discussion precisely about the Liar paradox, and their knowledge of the Liar sentence being a debated and difficult-to-handle issue in logic has a consistent influence on how they navigate their conversation and switch the deductive standards at stake in it.

I also want to go back to the examples I made earlier about contexts in which the deductive standard at stake is clear: a conference with the

²⁶This conversation is employed in Caret (2017) as an example in which a *switch* in the deductive standard occurs. In particular, the deductive standard changes from classical to intuitionistic logic.

title ‘Topics in intuitionistic mathematics’ and a workshop with the title ‘Semantics for relevant logics’. I argue that ‘intuitionistic mathematics’ and ‘relevant logics’ are, after all, ‘subject matters’ that influence the deductive standard. The topic of the conversation plays a role in these situations too.

In light of these considerations, I claim that rejecting applicational rivalry through this form of logical contextualism affects logics’ generality.

2.3.3 Tajer’s plurality of bridge principles

Another proposal that is presented as an updated version of cased-based pluralism with the upshot of solving the collapse problem by discarding rivalry is Tajer’s (2022*b*) account. Tajer develops a pluralist framework in which the normativity of different logics is regulated by distinct *bridge principles*, i.e. principles that connect logical entailment and epistemic norms.

The notion of bridge principle was originally introduced by MacFarlane (2004), who defined it as a principle that articulates a relation between facts about logical consequence and norms governing the doxastic attitude and behavior of agents. Usually, a bridge principle has the following form: “if ϕ implies ψ then [specific epistemic demand]”. Bridge principles can vary along some dimensions: the scope of the deontic operator that is commonly used to express a certain epistemic demand, its modality (ought, having a reason, etc.), its polarity (whether the epistemic demand is a ‘positive’ directive, like ‘you have a reason to believe’, or a negative one, like ‘you ought not to believe’), and whether the epistemic constraint requires the agent to be aware of the logical implication that constitutes the antecedent of the principle. Admitting the existence of these principles might seem undesirable at first, due to concerns about parsimony. However, this ontological enlargement has the very valuable upshot of clarifying the relationship between logical laws and the normative influence they exert on rational agents: logical facts are not normative *per se*, but in virtue of these principles, which connect them to norms of belief. While the tool of bridge principles is now quite well-recognized within the literature (Steinberger, 2019*b*), the question as for which bridge principles best characterize the normative constraints held by a certain logic is still open.

Tajer (2022*b*) designs the following bridge principle for classical logic (CL):

If $A_1, \dots, A_n \models_{CL} B$, then you have reasons to (either not believe A_1, \dots , or

not believe A_n , or believe B).

Then, he suggests the following principle for FDE (a particular kind of relevant logic):

If $A_1, \dots, A_n \models_{FDE} B$, then you ought to (either not believe A_1, \dots , or not believe A_n , or believe B).

The difference between these two bridge principles is the modality of the epistemic constraint. The principle for classical logic features ‘have reasons to’, while that for relevant logic features ‘ought’.

Tajer’s proposal consists in minimizing the overarching normative effects of classical logic by means of a weak bridge principle (one which makes use of the ‘*having reasons to*’ operator), while the normative effects of relevant logic are enhanced through a strong bridge principle (one that involves the ‘*ought*’ operator, which is stronger than the ‘*having reasons to*’ operator). Consequently, when an inference is valid according to both relevant and classical logic, the normative force of relevant logic prevails, while when an inference is valid only in classical logic it is only the normative force of classical logic that is triggered. This solution bypasses the collapse problem because it prevents the directives of the stronger logic from always trumping those of the weaker ones: all the admissible logics have the potential to exert their normative strength, in a way that avoids conflict between them. No logic is condemned to be always silent.

This proposal can be located in the broader debate about what it means for a logic to do genuine normative work. For instance, Blake-Turner (2021) claims that one of the mistakes of both the proponents and the opponents of the collapse problem consists in assuming that the normative work of a logic must be cashed out in terms of all-or-nothing notions like entitlement or obligation. Blake-Turner suggests that the normativity objection can “be resisted by allowing a logic to do genuine normative work that is nondecisive, for instance by generating *pro tanto* reasons” (Blake-Turner, 2021, p. 4100). Similarly, Ferrari and Moruzzi (2017) assert that an adequate account of the guidance that logic is expected to provide agents with in ordinary contexts of reasoning could be expressed in terms of a criterial, non-guiding normative force.²⁷ If one rejects the idea that the bridge principles connecting logical principles with doxastic attitudes employ a modality that can only carry

²⁷They do not explain in depth what they mean with this expression (“criterial, non-

non-negotiable epistemic obligations, the collapse problem may be avoided. In particular, Blake-Turner suggests that the notion of normativity at play in bridge principles should be that according to which the validity of a certain argument *provides reasons to base* the belief of a certain conclusion on some already accepted premises. He considers this to be a viable alternative to taking the normative force of bridge principles to express all-or-nothing obligations. While Blake-Turner seems to favour a pluralist framework in which the bridge principles of all the admissible logics share the same modality, Tajer adopts different principles for different logics. By doing so, Tajer avoids a possible collapse into the weaker logic, which might instead affect a proposal in which all bridge principles are weakened down to the same degree.

The first criticism I raise against Tajer's solution is that it is too *ad hoc*: associating weaker logics with stronger bridge principles and *vice versa* certainly prevents the collapse problem, but it seems that the bridge principles governing the normative force of the various logics are designed with the precise aim of avoiding this particular objection. In order for this view to be appealing there should be some *independent* reasons for coupling the various logics with those specific bridge principles. Tajer claims that we can take the logical weakness of a certain logic as a motivation for a stronger normativity; however, the justification for us to accept such motivation makes sense only in the perspective of providing a solution for the collapse problem. Actually, one might concede that there are some intuitive reasons for endorsing a framework in which, the stronger a logic is, the less normative force it should exert. After all, the fact that a strong logic allows a comparatively larger number of inferences may be considered a good reason to adopt a cautious attitude when following the directives it provides. Vice versa, the guidance provided by a weak logic might raise less concerns: the few arguments it predicts to be valid are probably supported by well-grounded reasons.²⁸ While I maintain that the dimensions (scope, modality, polarity etc.) of the bridge principles capturing the normative force of the admissible logics should be shaped according to factors that go beyond the intuition just presented, and

guiding normative force'), but they likely mean something along the same lines of Blake-Turner. That is, they think that the normative force of logic may be taken to simply provide a criterion to reason in a certain way, as opposed to providing unquestionable guidance.

²⁸I thank Thomas Schindler for suggesting this perspective about the justification of Tajer's couplings between logics and bridge principles expressing their normative force.

among which ‘providing a solution to the collapse problem’ should not be a priority, one might grant that these sort of justifications constitute convincing support for Tajer’s proposal.

Even so, this strategy has another shortcoming. The point I want to put forward is that this solution doesn’t solve the collapse problem independently from the other three hallmarks identified by Stei. In particular, I argue that this solution significantly affects the notion of normativity at stake in hallmark (d). As I will show in more details in the next chapter, Beall and Restall commit to the notion of normativity that is rooted in the logical tradition. While there are certainly some divergences among how the various logicians conceived the normative force of logic,²⁹ they all thought of normativity as providing all-or-nothing obligations: if an agent reasons in alignment to valid logical principles, she reasons correctly. Vice versa, if in her deliberations she violates some logical principles, she reasons incorrectly. This is to say, the idea of normativity at stake in the strategy presented by Tajer (and also that endorsed by Blake-Turner) does not conform to that of Beall and Restall and to that identified by Stei as one of the GCP’s hallmarks that give rise to the collapse problem. This means that the notion of normativity employed by Tajer does not conform to the conception of normativity rooted in the logical tradition that conceives logic as exceptional. After all, the type of normative force that some of the admissible logical laws turn out to exert in this proposal does not differ much from the normative force of the laws of disciplines like physics. I conclude that Tajer’s proposal fits within logical anti-exceptionalism, and therefore it is not actually an instance of GCP minus (c) as it seems *prima facie*.³⁰

2.4 Take-aways

In the previous section I have been challenging Caret’s contextualism and Tajer’s plurality of bridge principles on two fronts. First of all, I pointed out some of the internal weaknesses the two proposals. Namely, I criticized the

²⁹An example of such differences can be found in section 4.6, where I present the difference between the Fregean and the Carnapian notions of logical normativity.

³⁰Notice that the criticism I pointed out affects Tajer’s proposal only when conceived as an instance of Beall and Restall’s case-based logical pluralism that gives up *only* on rivalry. It is not to be excluded that Tajer would disagree with such interpretation of his account, or that he would not consider the claims I presented as a drawback.

lack of intuitiveness of Caret’s contextualism and the ad hocness of Tajer’s account. Secondly, I claimed that they do not solve the collapse problem without affecting the other hallmarks. In particular, I have argued that Caret’s contextualism bears on the notion of domain, and therefore it is not completely topic-neutral (i.e. it does not preserve logical generality), and that Tajer’s account involves a downsizing of the traditional notion of normativity that makes this notion fall out of the scope of the sense in which it is used by Beall and Restall and by Stei (2020c) (i.e. it does not match the canonical, full-fledged normativity at stake in hallmark (d)). This means that committing to these proposals entails committing to a form of anti-exceptionalism. Thus, if regarded as appealing strategies, these proposals actually turn out to provide reasons for the pluralist to endorse logical anti-exceptionalism.

This being said, the fact that two of the proposals presented as instances of “GCP minus (c)” have the shortcomings that were just mentioned does not rule out that a new proposal with the same argumentative structure may provide a compelling solution to the collapse problem without affecting the traditional metaphysics of logic. However, it appears very demanding to provide a solid theoretical foundation for a pluralism in which logics are not applicational rivals without affecting the generality or the normative force of those very logics. In a framework predicting that all admissible logics get to play a role without conflict, it seems extremely hard to account for the mechanism pairing logics with the appropriate situations of use without relying on considerations about domains or without weakening the normative bond between logic and belief. Moreover, an additional struggle for such a framework would consist in doing so in a way that is intuitive and faithful to actual linguistic and logical practices – in other words, without any ad hoc elements. How would such a proposal look like? The only idea that occurs to me is that of an intra-theoretic pluralism in which logics are general and normative, that is, they all apply to every possible domain and exert an all-or-nothing normative force, but they give the exact same verdicts about the validity of all possible arguments when applied to any possible domain. This framework does not give rise to the collapse problem because the directives provided by the admissible logics never clash, but it also arguably falls short of philosophical interest: for all intents and purposes, the logics coincide.

If these considerations are taken to provide enough reasons to contend that the general strategy of giving up on rivalry without affecting generality or normativity is not viable, then the main claim of this thesis could be

reinforced. The aim of this work is showing that dropping either generality or normativity (i.e. committing to an anti-exceptional view of logic) is *sufficient* to solve the collapse problem. However, if rejecting logical rivalry alone is ruled out *tout court* as a possible solution to this objection, it turns out that giving up on either generality or normativity is also a *necessary* move for those who want to solve the collapse problem and, at the same time, preserve an interesting intra-theoretic pluralist framework. Since I do not yet have a strong position about this matter, I limit myself to outlining this perspective without explicitly endorsing it.

Let's take stock. In this chapter I have shown that rejecting (c) is not sufficient to restore the consistency of Beall and Restall's pluralism in a satisfactory way: the theoretical instability that the collapse problem exposes within case-based logical pluralism has to be resolved by affecting the two remaining hallmarks, (b) and (d). The next two chapters are devoted to examining the implications of their rejection.

Chapter 3

Dropping normativity

In the previous chapter I explained how the collapse problem impacts case-based logical pluralism. Then, I presented Stei's list of the hallmarks that, combined, give rise to the collapse problem within Beall and Restall's view. I then embarked on a pursuit to find a strategy that enables the pluralist to overcome the collapse problem. I decided to proceed by analyzing the consequences of rejecting each of Stei's hallmarks. The rejection of hallmark (a), 'there is more than one correct logical consequence relation within one language', leads to the abandonment of pluralism *tout court*. Since I want to find a solution to the collapse problem within the framework of (intra-theoretic) pluralism, dropping (a) is not a viable option. Moreover, I have shown that the rejection of hallmark (c), 'there is rivalry between different logical consequence relations', does not resolve the inconsistency brought about by the collapse problem without affecting the remaining hallmarks, (b) and (d). At this point, the two remaining options correspond to dropping either *generality* or *normativity*.

This chapter is focused on the consequences of abandoning normativity. In the first section I provide some evidence for the historical relevance that normativity played for the traditional picture of logic, and I also present Steinberger's (2019*b*) recent survey of what it means for logic to be normative. In the second section I report some compelling arguments supporting the thesis that logic is not normative, and I delineate the account of logical pluralism designed by Blake-Turner and Russell (2018), which constitutes an instance of Beall and Restall's pluralism *without* normativity. In the third section I present the objection of Stei (2020*b*) to Blake-Turner and Russell's proposal, whose main point is the idea that *non-normativist* logical

pluralism is not actually immune to the collapse problem. I then challenge Stei's criticism and provide alternative grounds for a pluralist framework without normativity. In the fourth section I conclude that giving up on normativity is a good solution to the collapse problem. In the fifth and final section I contend that dropping normativity does not affect logical generality.

3.1 In what sense is logic normative?

Holding that logic is *normative* is one of the forms that metaphysical exceptionalism about logic can take. Logical normativists typically claim that logic is normative for thought, in the sense that “claims about validity or consistency become understood as claims about how an agent *ought* to infer or believe” (Martin and Hjortland, 2022, p. 148).

The idea that logic is normative is firmly rooted in its tradition. For Kant, logic instructs us on “how [understanding and thinking] ought to proceed” (Kant, 1800/1992, p. 16). Frege also regards logic as a “normative science” (Frege, 1897/1979, p. 228), one whose laws “prescribe universally how one should think if one is to think at all” (Frege, 1893/1903/2009, p. xv). According to the detailed analysis of Steinberger (2017), for Frege logic is normative in a three-folded sense: its laws *describe* the laws of thought, *prescribe* how one ought to think and reason and *constitute* thought as such (if judgement, inferences and beliefs are not aligned with the laws of logic they do not count as such).

To mention a more recent account of logical normativism, Leech (2015) claims that the facts underpinning logical theories are fundamentally different from those that provide the foundation for the claims of other theories: they are *normative facts*. Leech further claims that logic is both *constitutive* and *normative* for thought: she claims that logical laws are “normative laws, evaluability in light of which is constitutive of thought” (Leech, 2015, p. 17), and therefore “any account of the nature of logical laws needs to take note of the inescapability or rational indubitability of logical principles” (ibid, p. 26).

As it was repeatedly stressed earlier, Beall and Restall conform to the normativist tradition: they hold that necessity, formality and normativity are the three core features of logical consequence, and that therefore “any account of logic must take account of them” (Beall and Restall, 2006, p. 14).

In his wide-ranging overview of the concept of logical normativity, Stein-

berger (2019*b*) claims that the so-called ‘Normativity Thesis’ can be spelled out as the idea that “logic [...] [plays] a normative role [...] in reasoning – that is, in the ways we go about forming and revising our doxastic attitudes – and in our evaluations of the reasoning of others” (Steinberger, 2019*b*, p. 5). He further argues that this thesis is actually three-folded, and he identifies three ways in which logic can be said to be normative:

1. logical norms might provide first-personal *directives* that guide the reasoner;
2. logical norms might serve to make third-personal *evaluations*, setting standards or ideals by which to assess an agent’s doxastic state for its logical cogency; or, finally,
3. logical norms might play the role of third-personal *appraisals* by which we criticize, blame, or otherwise hold accountable an agent for her doxastic conduct.³¹

(Steinberger, 2019*b*, p. 7)

This distinction serves this work to both clarify what is the broad range of meaning covered by the concept of ‘logical normativity’ and also to help me pinpoint what is the precise area of this spectrum on which this thesis is focused. With respect to the latter purpose, recall that Beall and Restall claim that:

We use arguments we take to be valid to judge inferences. [...] [This] undergirds the use of deductive inference in the rational assessment of beliefs and theories, arguments and hypotheses. (Beall and Restall, 2006, p. 16)

While the normativist tradition of logic comprises accounts for which logic is normative also in the sense of giving agents first-personal guidance for how they ought to think or in the sense of laying the basis to attribute praise or blame to rational agents in light of their doxastic conduct, the most straightforward way to think about logical normativity is that to which Beall and Restall (but also Leech and Steinberger themselves) adhere to: providing a way to *evaluate* reasoning. Beall and Restall focus on logic being

³¹The difference between evaluations and appraisals is subtle. While the former are merely judgments about agents’ logical skills, the latter involve an assessment of agents’ *value* based on reasoning abilities.

normative in (at least) the sense of setting a golden standard against which agents can assess their own doxastic states and those of other agents from a third-personal perspective.

3.2 Against logical normativity

The idea that logic is a normative discipline has not gone uncriticized. Among the first who challenged this claim there is Russell (2020), who argues that “we can add ‘normativity’ to the list of distinguishing properties logic was once thought to have, but which have turned out to be illusory” (Russell, 2020, p. 372), in explicit alignment with the project of logical anti-exceptionalism.

First of all, Russell distinguishes three ways in which a theory can be entangled with the normative. The strongest one is when a theory counts as a theory of a certain kind (e.g. logical) precisely because it is normative. The second degree of normative entanglement is that according to which a certain theory entails normative conclusions; for instance, a logical theory can be taken to be descriptive *per se*, and yet bear a certain amount of normative force for the way in which agents ought to reason. Thirdly, a theory can be entangled with the normative in the ‘weakest’ degree, that is, the theory has normative consequences only alongside other normative assumptions. For instance, the arithmetical fact that $2+3=5$ doesn’t by itself entail that when you have 2 euros in one pocket and 3 euros in the other you should not believe that you have 7 euros in your pockets; such conclusion is reached in conjunction with the *non-arithmetical* normative fact that “one ought not to believe false things”. In this sense, the theory of arithmetic doesn’t carry normative consequences on its own, but only in combination with other widely accepted normative assumptions. While the way in which Beall and Restall think about logic’s normativity fits within the first and the second level of entanglement with the normative,³² according to Russell logic is entangled with the normative merely to the extent of

³²The agreement with the first degree of entanglement with the normative comes from the fact that, for Beall and Restall, it is *necessary* for a logic to be normative in order to be admissible. They therefore agree with the idea that “a logic won’t *count* as a logic unless it is normative” (Russell, 2020, p. 379). Moreover, from their definition of normativity as the property according to which “you somehow go *wrong* if you accept the premises but reject the conclusion” (Beall and Restall, 2006, p. 16), they also seem to agree with the fact that logic has normative consequences for the way in which agents ought to reason.

this third, very weak degree of normative entanglement: it is only with the support of common normative commitments concerning truth and falsity that logical facts have normative consequences. More precisely, the particular and common-sensical normative assumption that Russell identifies as enabling logic to have normative effects is “you ought not to accept things as true if they are not true” (Russell, 2020, p. 380)³³.

After putting forward this thesis, Russell further points out that for what concerns its degree of entanglement with the normative, logic shares such status with other paradigmatically descriptive scientific theories, including those of physics and mathematics. However, contrary to those of these other disciplines, logical theories have long been misconceived as normative. The reason is that, while it is fairly easy to see that physics is descriptive because the realm of the things it describes – the physical world – is easy to identify and delimit given its immediate availability to the senses, it is not clear what logic is about, that is, it is not straightforward to pinpoint the kind of phenomena logic is supposed to describe. In order to fill this theoretical gap, Russell puts forward the thesis that “logic is the study of patterns of truth-preservation on truth-bearers” (Russell, 2020, p. 382): the ‘object’ that logic describes is how truth is preserved over arguments.

Russell’s conclusion have been shared by other philosophers (for instance, by Labukt, 2019), but the idea that logic is normative is still very rooted in the logical canon.

Building on Russell’s analysis, Blake-Turner and Russell (2018) reiterate the statement that, considered in isolation, logical theories are descriptive: they describe patterns of truth-preservation over sentences by providing sets of claims about which argument forms are valid or invalid. They call such claims ‘E-sentences’, where ‘E’ stands for ‘entailment’. On top of that, Blake-Turner and Russell hold that logical theories possess a ‘superstructure’: deductive theories often assume the existence of additional objects and properties and make use of auxiliary assumptions, whose status (of these objects, properties and assumptions) is determined by specific philosophical views.

³³Notice that this tripartition of logic’s entanglement with the normative has little to do with the three-folded account of logical normativity given by Steinberger (2019*b*). While Steinberger focuses on the question “if logic is normative, what does it norm?”, Russell is concerned about the ‘source’ of the normativity force carried by a certain theory. In this sense, Russell is answering the question “where does the normative status of a logical theory come from?”.

Then, Blake-Turner and Russell argue that what determines the accuracy³⁴ of an E-sentence (e.g. ‘ $\phi \wedge \neg \phi \models \psi$ ’) generated by a certain superstructure are descriptive facts: the objects and the properties posited by the superstructure ought to exist and their behavior has to be successfully captured, and the auxiliary assumptions in the superstructure ought to be proper formalizations of natural language sentences. Given that the accuracy of E-sentences is a descriptive matter, logical theories turn out to be descriptive as well. By means of this account, Blake-Turner and Russell provide a more comprehensive elaboration of Russell’s original thesis according to which logical theories are not entangled with the normative in any ‘strong’ sense, that is, neither by definition nor by virtue of having some normative consequences. Instead, they are normative for how rational agents ought to reason only *when combined with* widely accepted background norms.³⁵

3.2.1 Non-normativist logical pluralism

In light of these considerations, Blake-Turner and Russell (2018) developed three non-normativist versions of Beall and Restall’s logical pluralism. The first one is *flat-footed logical pluralism*, in which the constraints for admissible logics are limited to necessity and formality. However, this view has the downside of allowing trivial logics³⁶ into the set of admissible logics, and it is therefore not an ideal solution. A better option for the pluralist consists in replacing the normativity requirement with either a *pragmatic* or an *epistemic* constraint. In the former case, dubbed *pragmatic logical pluralism*, unpalatable logics would be excluded from the set of admissible logics, but tying the metaphysics of logic to what is useful for humans could turn out to be too constraining for a logician. The accounts just discussed show that the pluralist has a number of options for designing a logical framework without normativity. However, the preferred solution of Blake-Turner and Russell, and the account they actually commit to, consists in replacing normativity

³⁴Accuracy stands for a combination of “being true” and “not missing out any important aspect of the world that would go into determining logical consequence” (Blake-Turner and Russell, 2018, p. S4863).

³⁵Similarly to the principle advocated by Russell (2020), Blake-Turner and Russell mention the very basic normative principle according to which “one ought to believe only true things” (Blake-Turner and Russell, 2018, p. S4868).

³⁶For instance, a logic allowing models in which \perp (falsum) is true, where \perp entails every sentence buildable in that logic. For example, a classical contradiction. In such a logic any sentence is a consequence of any other set of sentences whatsoever.

with the *epistemic constraint* according to which admissible consequence relations ought to be suited to meeting some set of epistemic goals. This *telic logical pluralism* has the upshot of not admitting trivial logics, because they do not concur to meeting any epistemic goal at all, and *usefulness* turns out to be only one among a broader set of goals that a logic can be aimed at fulfilling.³⁷ Actually, the notion of ‘being the logic that is best suited to meeting the epistemic goal X’ may be said to be normative, hence it may seem that this view fails in excising normativity once and for all. However, while according to logical normativism the truths of logic have normative power *eo ipso*, telic pluralism shifts the normative burden from logic to *epistemology*, providing an account in which logic *per se* is normatively inert. Moreover, since for Blake-Turner and Russell the concept of ‘epistemic goal’ is a purely descriptive feature, their take on the notion of ‘being the logic that is best suited to meeting the epistemic goal X’ is also that of a non-normative requirement.

Blake-Turner and Russell are fundamentally in agreement with Beall and Restall in opposing the idea of validity as a one-size-fits-all notion. Combining their descriptive account of logic with their pluralism, it follows that there is no such thing as *tout-court* validity: a certain principle is valid *with respect* to its superstructure,³⁸ and a certain superstructure is accurate if it says true things about natural languages *with respect to* some epistemic goals. Moreover, their framework permits the selection of the same logics that Beall and Restall deem admissible without resorting to normativism. In fact, by taking the set of epistemic goals to include (i) truth-preservation, (ii) relevant truth-preservation and (iii) demonstrable truth-preservation, one ends up endorsing classical, relevant and intuitionistic logic. Basically, Blake-Turner and Russell claim to have designed an account “that captures the attractive features of Beall and Restall’s pluralism, without requiring a commitment to logic’s normative status” (Blake-Turner and Russell, 2018, p. S4875). Most importantly for the purposes of this work, they also argue that the move of removing normativity from the metaphysical composition of logic provides a solution to the collapse problem: if the pluralist rejects the normative status

³⁷In order to preserve the intuition that the primary purpose of logic is related to reasoning or argumentation (Priest, 2005), one could restrict the epistemic goals logic can aim at to the ones that pertain to such practices; this allows to preserve a strong link between logic and reasoning without the obligation to endorse normativity.

³⁸That is, with respect to the assumptions concerning the objects and the properties involved in the logical theory.

of logic, she is not bound to “explain how all of her endorsed logics can successfully impose normative constraints on deliberators” (Blake-Turner and Russell, 2018, p. S4871).

To my knowledge, the telic version of logical pluralism put forward by Blake-Turner and Russell is the only systematic and detailed instance of Beall and Restall’s pluralism *minus* hallmark (d) (i.e. case-based pluralism without normativity) in the literature. Nonetheless, there are other pluralist accounts that are an instance of GCP without (d): Haack’s (1978) global logical pluralism, Hjortland’s (2013) intra-theoretical pluralism and Restall’s (2014) proof-theoretic pluralism. In the upcoming section I will present several objections to the proposal put forth by Blake-Turner and Russell, in order to assess the extent to which it does indeed constitute a viable option for the pluralist. Part of them might reasonably apply to some of the instances of GCP without (d) listed above, but I will not enter in any detailed discussion about this matter.

3.3 Assessment

In this section I evaluate this solution to the collapse problem by pointing out its possible flaws. First of all, I present the work of Stei (2020*b*) and explain his claim that the collapse problem is not once and for all defeated by excising normativity from pluralism. After that, I challenge his arguments. In conclusion, I present additional reasons for questioning Blake-Turner and Russell stake on logic and put forward alternative grounds to argue for a descriptive conception of logic.

3.3.1 The reemergence of the collapse problem

The major upshot of non-normativist logical pluralism that Blake-Turner and Russell (2018) explicitly mention is that logical pluralists (including Beall and Restall) do not need to worry about objections from the normativity of logic (e.g. the collapse problem), because, after all, logic is not normative.

Challenging the alleged upshot of Russell’s solution, Stei (2020*b*) argues that the non-normativist version of Beall and Restall’s pluralism outlined in Blake-Turner and Russell faces the re-emergence of the collapse problem: the weakest degree of entailment with the normative possessed by logic is enough

for the objection to go through.³⁹

In particular, Stei argues that even the logician who takes logic to be entirely descriptive acknowledges that valid arguments may have normative consequences; what is different from the two stronger degrees of entailment with the normative is that such consequences arise when logic is combined with general doxastic norms whose location is *external* to logic. Stei stresses how this shift of the normative burden from logic to epistemology doesn't provide any compelling solution for how agents ought to reason when distinct logics disagree about the validity of an argument and therefore provide different directives. He presents his criticism through two main objections.

First and foremost, Stei claims that, given that universal epistemic norms like “you ought not to accept things as true if they are not true” (Russell, 2020, p. 380) or “one ought to believe only true things” (Blake-Turner and Russell, 2018, p. S4868) are so general and foundational that it can be reasonably claimed that they are ubiquitous,⁴⁰ they are certainly in place in those situations in which two logics disagree. Let's consider an agent endorsing a logical pluralism in which both classical and intuitionistic logic are admissible, and who is deliberating as to whether to infer P from $\neg\neg P$: in Blake-Turner and Russell's framework, if the epistemic goal at stake is truth-preservation simpliciter, P has to be believed, whereas if the epistemic goal in place is demonstrable truth-preservation, there is no obligation for the agent to believe P . Stei points out how this leads to a paradoxical utterance *à la* Moore like ‘ P but I don't believe P ’: after all, the agent knows that P is true by his expertise of classical logic, but then he might not be inclined to believe it if the epistemic goal at stake is that of intuitionistic logic. Aside

³⁹Stei also notices that the framework of telic pluralism, in which different logics aim at different epistemic goals, is not a prerogative of the pluralist. In fact, some monists hold that while classical logic is the one and only correct account of logical consequence in virtue of it being precisely aimed at *truth-preservation* (and truth-preservation is the one and only thing that logical consequence should be about, in their view), they also acknowledge that there are other logics with other epistemic purposes, which however do not classify as good candidates for capturing the concept of logical consequence. These remarks highlight that if one considers appealing a framework in which epistemic goals play such a fundamental role, there is no need to buy the whole package of logical pluralism, because monism is compatible with a telic view too.

⁴⁰Blake-Turner and Russell seem aligned with this claim. They call such principles ‘common normative commitment[s] concerning truth and falsity’ (Russell, 2020, p. 10), and ‘basic tenets of epistemic normativity’ (Blake-Turner, 2021, p. 17), and therefore it is really implausible that they think about them as restricted in scope.

from the fact that this outcome is undesirable because of its paradoxical nature, Stei also claims that it is not straightforward at all that epistemic goals can ‘trump’ general epistemic norms: there seems to be something weird in an epistemic goal that blocks general epistemic norms and leads a subject to believe, for instance, false propositions.

The second criticism that Stei puts forward is that “it is not clear that what a person ought to believe always depends on her epistemic goals” (Stei, 2020*b*, p. 174), because in real-life situations we do not really take them into account when trying to achieve some argumentative goal. In order to exemplify this point, Stei uses the following citation from Kelly (2003):

[I]n arguing for my conclusions in this paper, I think of myself as attempting to provide strong reasons for believing my conclusions, and *not* as attempting to provide strong reasons for believing my conclusions for those who happen to possess goals of the right sort. (Kelly, 2003, p. 621)

This is to say, in our real-life argumentative practices we are only concerned with putting forward truth simpliciter, and we do not really consider more fine-grained goals like demonstrable truth-preservation or relevant truth-preservation. Going back to the previous example, it is implausible that when having to decide whether P follows from $\neg\neg P$, the goal of demonstrable truth-preservation can completely trump the general epistemic norm of ‘one ought to believe only true things’, and therefore license the agent to be in a state in which she actually does not believe P .

To conclude, here is a general formulation of the collapse problem for non-normativist logical pluralists:

Suppose that, except for the rejection of the normativity constraint, L_1 and L_2 are both admissible instances of GTT. Suppose, further, that $\Gamma \models_1 \phi$ but that $\Gamma \not\models_2 \phi$, and that we know all sentences in Γ . Suppose there is a general epistemic norm N_1 that, in combination with L_1 yields normative pressure to accept ϕ . Finally, let us grant that L_2 is the best logic when it comes to following some different epistemic norm N_2 . On the current analysis, even if we do not achieve the epistemic goal or value associated with N_2 , we also don’t seem to violate any constraints if we accept Γ and do not reject ϕ . (Stei, 2020*b*, p. 175)

If one were to accept Stei’s criticism according to which the inconsistency of Beall and Restall’s original view re-emerges even when logic is not taken to be inherently normative, it is hard to see how giving up on normativity could help the pluralist solve the collapse problem. Before committing to such pessimistic perspective, I try to argue against Stei’s objections.

3.3.2 Possible solutions

The first criticism that Stei brings up against telic pluralism is that it licences paradoxical utterances like ‘ P and I don’t believe P ’ in face of inferences like ‘ $\neg\neg P \models P$ ’. To begin with, such objection relies on the idea that a pluralist would be forced, for some compelling motives, to *assert* the conjunction of P and ‘I don’t believe P ’, and it is not straightforward at all why she should be so obliged.⁴¹ In fact, while the pluralist (who accepts both classical and intuitionistic logic) can reasonably be said to be compelled to accept P given that the E-sentence ‘ $\neg\neg\phi \models \phi$ ’ belongs to his comprehensive set of beliefs in virtue of his knowledge of classical logic, the epistemic status of ‘I don’t believe P ’ is context-sensitive, and the instantiation of a certain epistemic goal is part of such context. This is to say, the truth of the first conjunct is evaluated in a context-independent way, while the second one makes sense only in case the goal of demonstrable truth-preservation is in place. If we consider the conjunction on context-insensitive grounds, the second conjunct turns out to be not assertable. Alternatively, if we evaluate the conjunction taking into account the situatedness of the logician in a context in which the epistemic goal of demonstrable truth-preservation is in place (for instance, that of practicing constructive mathematics), then the first conjunct is not assertable. This doesn’t make the logician irrational, because, after all, ‘ $\neg\neg\phi \not\models \phi$ ’ is also one of the E-sentences that are part of her overall set of credencies. To better illustrate my point, I designed the following example. Consider a pluralist who endorses both classical and relevant logic, and therefore holds both ‘ $\models (\phi \wedge \neg\phi) \rightarrow \psi$ ’ (from classical logic) and ‘ $\not\models (\phi \wedge \neg\phi) \rightarrow \psi$ ’ (from relevant logic) among her comprehensive set of beliefs. If we evaluate it according to a context-independent standard, the conjunct ‘It rains and it doesn’t rain, so Juliet survives’ ought to be accepted because of the endorsement of classical logic, and ‘I don’t believe

⁴¹I thank Colin Caret for putting forward this point, and Thomas Schindler for helping me formulate this criticism in an effective way.

that if it rains and it doesn't rain, Juliet survives' is not assertable. By contrast, in the context of reading the tragedy of *Romeo and Juliet*, given that the epistemic goal at stake is that of relevant truth-preservation,⁴² 'It rains and it doesn't rain, so Juliet survives' won't be accepted, and 'I don't believe that if it rains and it doesn't rain, Juliet survives' will be, instead, assertable. In conclusion, the conjunction 'It rains and it doesn't rain, so Juliet survives and I don't believe that if it rains and it doesn't rain, Juliet survives' is not assertable, because the two conjuncts are grounded on two different epistemic standards.⁴³ Finally, notice that this solution can be used by other kinds of pluralists too, to which analogous Moore-style utterances could be objected. This strategy clears out this kind of paradoxes by showing that the two clashing conjuncts are legitimated by different grounds, and they are therefore not assertable in combination. In fact, this strategy is in alignment with the very intents of all pluralist frameworks:⁴⁴ allowing the endorsement of a multitude of conceptions of a certain notion while granting the possibility, for the pluralist, to employ them only in the appropriate circumstances.

As for Stei's second objection, I argue that it relies on the assumption that the only meaningful role (or at least the most prominent one) undertaken by logical consequence is that of truth-preservation simpliciter; the citation taken from Kelly (2003) is used to stress that point. This entails that Stei embraces some kind of hierarchy among different senses of validity and the corresponding epistemic goals: some of them – namely, classical logical consequence and its associated goal of truth-preservation simpliciter – are more important than others, due to their prominent instantiation rate in real-life situations. This is a fair view of things,⁴⁵ but it contradicts the very foundational assumptions of Beall and Restall's pluralism, for which the different accounts of the relation of logical consequence are equals: considered by themselves (independently from specific areas of applications), they are *on*

⁴²Here I follow Beall and Restall idea that relevant logic is the most appropriate for fictional discourse.

⁴³Resorting to contextual-epistemology for solving paradoxes is a rather popular strategy. For a survey of some attempts to handle the preface paradox in which epistemological context-sensitivity comes into play, see Kauss (2021).

⁴⁴At least those bearing on metaphysical considerations, that is, those stemming from the idea that a certain property can't be characterized univocally.

⁴⁵In fact, the alleged intuitiveness of the idea that logical consequence captures truth-preservation simpliciter grounds a great part of the monist accounts of logic in which classical logic is taken to be the one true logic.

a par. Analogously, Blake-Turner and Russell conceive goals like demonstrable truth-preservation and relevant truth-preservation as full-fledged goals for logic (and not as subordinates of truth-preservation simpliciter), because they commit to the same assumptions of Beall and Restall. Moreover, if we admit that the epistemic goal of demonstrable truth preservation can trump that of truth preservation simpliciter, it could be reasonably said to at least affect (if not suspend) the general epistemic norm that ‘one only ought to believe true things’ to the extent that the sense of ‘true’ at stake in it becomes that of demonstrable truth preservation. To better illustrate this point, I consider again the example of a pluralist that endorses both classical and relevant logic. While she engages in the activity of reading *Romeo and Juliet*, the sense of ‘true’ at stake in the general norm of belief according to which “one ought to believe only true things” is not that of truth-preservation simpliciter. In fact, it is not clear that it makes sense for her to believe the sentence ‘It rains and it doesn’t rain, so Juliet survives’: it seems more reasonable to say that the epistemic goal of relevant truth preservation affects the alleged general norm of belief, turning it into “one ought to believe only *relevantly* true things”.

Finally, for what concerns the re-emergence of the collapse problem in a version that applies to non-normativist pluralism, it could be avoided if one were to accept my counterarguments to Stei’s objections. In fact, if the epistemic goal at stake in a given circumstance is taken to affect the general epistemic norm that an agent has to conform to, then the normative pressure to accept or not the debated verdict about ϕ ’s validity can be either in place or not, depending on which epistemic goal is actually instantiated. Even if this argument were to be considered not enough to block the re-emergence of the collapse problem, it seems that this new formulation of the objection is not as threatening as the original collapse problem. In Stei’s version of the objection, the source of conflict is not internal to the framework of case-based logical pluralism without normativity, but it comes from an epistemological source. Therefore, the non-normativist pluralist is not forced to accept that the diagnosis of the collapse problem comes from her incorrect views about logic. The inconsistency arises from a combination of logic *plus* general norms of belief, and those are not located on the logical level. Of course, this version of the collapse problem still points to some undesirable consequences of Beall and Restall’s pluralist framework without normativity (if one were not to accept the arguments I put forward earlier in this section), but the challenge of this objection is not directed to the very idea of logical consequence of

their proposal.⁴⁶

3.3.3 Alternative grounds for a non-normativist pluralism

After providing some counterarguments to Stei’s criticism of Blake-Turner and Russell’s telic pluralism, I want to address what I consider to be a pressing issue in this account: the reasons given in favour of logic’s descriptiveness coming from the work of Russell (2020). To mention one more time Russell’s theoretical move, she argued for a shift of the normative burden of logic from logic *itself* to epistemology, claiming that it is from universal epistemic norms like “you ought not to accept things as true if they are not true” (Russell, 2020, p. 380) or “one ought to believe only true things” (Blake-Turner and Russell, 2018, p. S4868) that the normative strength of logic comes from. However, I think that it is not optimal to argue for the descriptiveness of logic in this way. In fact, one could use the same sort of argument to remove the normative force from all the subject areas that are commonly thought to be normative. For instance, it could be said that ethical theories are not normative *per se*, but in virtue of a ‘commonsense idea’ external to ethics⁴⁷ according to which “doing the good thing is always desirable”. Analogously, the rules of chess can be said not to be normative *by themselves*, but in virtue of an intersubjective, societal norm according to which “one ought to play games obeying to their rules”.

This unpalatable outcome is the reason why I suggest that it is better to support the thesis of logic’s descriptiveness in an alternative way. As a premise to my argument, I want to stress that it should be acknowledged that all theories have both a normative and a descriptive dimension. The physical theory of relativity has certainly the purpose of providing an accurate representation of time and space, and in this sense it has an eminent descriptive purpose.⁴⁸ Nonetheless, there is a sense in which physical theories are normative as well: they set the standard for which kind of statements about time and space ought to be regarded as true. In the same way, logic always carried

⁴⁶One of the options is to accept the conflict of epistemic norms causing this new collapse problem. Analogously, moral dilemmas are bad, but they might be sometimes unavoidable. I thank Colin Caret for suggesting me this parallelism.

⁴⁷That is, located at a more intuitive, foundational level than that of ethics.

⁴⁸Physical theories, like theories pertaining to other sciences, are in fact commonly taken to be descriptive.

both a normative and a descriptive dimension. However, for the largest part of its history, the normative aims were prominent: logic was studied with the main purpose of finding out what the fundamental laws of reasoning are, in accordance to which all human theorizing should have been conducted.⁴⁹ But if we adopt a *practice-based* perspective in the spirit of Martin (2022) and look at how logic is practiced nowadays, it is quite plausible to contend that the most prolific and successful area of logical studies is that in which formal systems are used as a *modeling* tool, similarly to how models are used in science. According to Glanzberg (2021), “logics function surprisingly like animal models” (Glanzberg, 2021, p. 225): they provide a controlled environment to study a certain phenomenon with the purpose of comprehending it better. In this sense, contemporary logical practice is very much in alignment with a descriptive conception of logic, in which inference patterns are not studied with the aim of providing *norms*, but controlled frameworks to enhance the comprehension of some targeted (metaphysical, linguistic, cognitive...) occurrences.⁵⁰ This is to say, in many contemporary applications of logic, the descriptive intentions trump the normative ones. As an example, let’s consider the work conducted in the field of formal semantics at the Institute of Logic, Language and Computation (ILLC) of Amsterdam in recent years. Various logical systems have been developed in order to account for specific phenomena of natural language, like anaphoric pronouns, questions or free-choice phenomena (just to mention a few), which have been modelled with a version of predicate logic with anaphora (Dekker, 2000), inquisitive logic (Ciardelli and Roelofsen, 2011) and bilateral state-based modal logic (Aloni, 2023), respectively. Within all these areas of research, the prominent role of logic is that of providing a formal model that accurately captures (i.e. describes) some linguistic occurrences. Again, it is not the case that the normative dimension of logic is completely eliminated. First of all, the process of ‘regimentation’ of natural language into formal languages is a normative enterprise; also, once a logic gets established as the best way to account for a particular natural language phenomenon, it does dictate the norm for how to reason about it. But again, this is a very minimal sense in which a theory can

⁴⁹See, for instance, Steinberger (2017)’s exegesis of Frege’s thought.

⁵⁰This idea of logics as models is already present in Cook (2010*a*). He claims that different models may emphasize one aspect or another: “Different logics, viewed as models of various linguistic [phenomena], are correct relative to different theoretical goals, or relative to different ways of simplifying, idealizing, or precisifying the phenomena in question.” (Cook, 2010*a*, p. 500-501)

be normative. Moreover, it does not set logic apart from the other sciences.

If logics are taken to have an eminently descriptive role in virtue of the arguments that I just presented, the collapse problem is still avoided. The reason is that, under this perspective, logical validities do not force a rational agent to believe the consequences of a valid argument, even if its premises are part of her beliefs. The normative link between logic and belief does not exert the same force it has in Beall and Restall's proposal. The pluralist who conceives logics as descriptive models is free to ponder them and adopt the one she prefers, in virtue of an attitude of epistemic humility that allows her to recognize that there can be different acceptable reasons for employing this or that logical system for a certain task.

3.4 Take-aways

In this chapter I presented an instance of GCP without normativity, I counterargued against its alleged criticisms and suggested a way to improve its grounds. Most importantly, I showed why attributing a primarily descriptive function to logic is a good solution to the collapse problem.

Notice that, in a non-normativist view, logics can still be rivals. In fact, there can be situations in which two logics compete for the role of formal system that is the best suited to meeting a certain epistemic goal. Even within the logics-as-models approach, there can be multiple logics concurring to be the best way of modelling a given phenomenon.

A less straightforward issue is whether dropping normativity has some consequences for logics' generality. I will use the remaining of this chapter to answer this query, which sheds further clarity on whether there are some theoretical obligations (e.g. giving up on generality) for the pluralist that rejects normativity.

3.5 Can there be generality without normativity?

Once we abandon the full-fledged idea of logical normativity that makes logic *exceptional*, are there any consequences for the generality of logic? Let's tackle this question starting from a worry raised by Russell (2020): the normativity of logic provides an all-encompassing background against which to

evaluate the ‘rightness’ of formal systems: assuming that there is a fact of the matter about how we ought to reason and that logic is normative for reasoning, the ‘best’ logic is that which better captures the laws of correct thinking. When we shift to a descriptive conception of logic, *in virtue of what* can we say that a logic is right? If we admit that an answer to this question necessarily involves matters of domain, then it is hard to see how logic’s generality can be preserved. In Russell’s words, advocates of logic’s normativity say that this property “is the best hope for interpreting a non-theory-relative concept of logical consequence” (Russell, 2020, p. 379). However, I argue that rejecting normativity doesn’t force pluralists to abandon generality too. In fact, we already dived into an account that gives support to such claim: Blake-Turner and Russell’s telic pluralism. While endorsing a descriptive view of logic, this proposal doesn’t undermine the topic-neutrality of logic. Its criterion for logics’ *rightness* is that of ‘being the best suited to meeting a certain epistemic goal’, and epistemic goals do not inherently carry any domain-relativity.

However, there is a potential danger to this claim that is worth spelling out. If we admit that Blake-Turner and Russell’s framework involves a conception according to which the suitability of epistemic goals is tied to particular contexts and those contexts bear on some notion of domain, then it does not seem far-fetched to argue that, after all, telic pluralism does not preserve generality. For instance, it can be said that since the epistemic goal of relevant truth preservation is the best suited for the practice of engaging with fictional discourse, and since fictional discourse can be identified as a particular domain, this pluralist account is indeed domain-specific. In response to that, I claim that the notion of *suitability* at stake here is a fairly loose one: it does not bear on any strong claim of strict correspondence between epistemic goals and domains. While it might be true that the epistemic goal that suits fictional discourse the most is that of relevant truth preservation, such relation is non-binding. Nothing prohibits the telic pluralist to instantiate the goal of truth-preservation simpliciter when reading a novel. In the spirit of Tajer (2022a)’s methodological pluralism for logic, the telic pluralist can commit to the following view:

[V]arious philosophical problems and approaches can involve completely different kinds of evidence and epistemic values. There is no “methodology of logic” that applies to every debate. Logical research involves a set of different methodologies, and it is im-

portant to understand how those methodologies work. [...] The adoption of a specific methodology depends mostly on the philosophical problem that logicians want to solve [...]. Moreover, in some cases the same problem can give rise to different methodologies. (Tajer, 2022a, p. 14-15)

From this perspective, the telic pluralist is not forced to drop logics' generality. However, one could wonder if this is the most natural or 'convenient' position for the non-normative pluralist, but since the question under investigation is whether dropping normativity *implies* dropping generality, I leave this question for further work.

To conclude, identifying an instance of non-normativist pluralism that is not necessarily committed to dropping logics' generality is enough to support a positive answer to the question: "Can a non-normativist account of logic preserve the generality of logical laws?". Therefore, I can now move to the next chapter and investigate the consequences of dropping generality.

Chapter 4

Dropping generality

In Chapter 3 I analyzed the implications of dropping normativity for Beall and Restall's framework, drawing the conclusion that such strategy provides a good solution to the collapse problem.

In this chapter I will examine the last option available to the case-based pluralist to block the normativity objection (if one commits to Stei's (2020*c*) diagnosis of its sources): rejecting logics' generality, hallmark (b) from section 2.2.

The first section is aimed at spelling out in more details in what sense logic can be said to be general. In the second section I will defend the claim that Beall and Restall's pluralism does indeed preserve generality. In the third, I will consider Pedersen's (2014) interpretation of Beall and Restall's proposal, which provides a version of Beall and Restall's pluralism without generality. I will then present the main upshots of a non-generalist account of logic, some of the objections it has to deal with and their respective possible resolutions. After that, in the conclusive section I will take stock of the achieved results and argue that logical normativity can be preserved even if generality is abandoned.

4.1 In what sense is logic general?

First of all, it is necessary to shed some clarity on the terminological choices at stake. Following MacFarlane (2000), I take 'generality' to be kindred in meaning to the notion of 'formality':

[G]enerality and topic-neutrality turn out to be unclear in pre-

cisely the same ways as formality. Thinking through the senses in which logic might be said to attend only to “form,” abstracting from subject matter or content, and thinking through the senses in which logic might be said to be “general” or “topic-neutral” are really the same enterprise. All of these terms have the same range of meanings. (MacFarlane, 2000, p. 20)

I will argue that these notions are trifurcated in precisely the same way as formality. By the “generality” (or “topic-neutrality”) of logic, I will show, one might mean either 1-formality, 2-formality, or 3-formality. (MacFarlane, 2000, p. 69)

Beall and Restall themselves endorse MacFarlane’s three-folded account of formality (that also applies to the concept of ‘generality’), according to which:

1. To say that logic is 1-formal is to say that its norms are constitutive of concept use as such (as opposed to a particular kind of concept use). 1-formal laws are the norms to which any conceptual activity – asserting, inferring, supposing, judging, and so on – must be held responsible.
2. To say that logic is 2-formal is to say that its characteristic notions and laws are indifferent to the particular identities of different objects. 2-formal notions and laws treat each object the same (whether it is a cow, a peach, a shadow, or a number). Mathematically, 2-formality can be spelled out as invariance under all permutations of the domain of objects.
3. To say that logic is 3-formal is to say that it abstracts entirely from the semantic content or “matter” of concepts – that it considers thought in abstraction from its relation to the world and is therefore entirely free of substantive pre-suppositions.

(MacFarlane, 2000, p. 51)

Nonetheless, both the claim that ‘formality’ and ‘generality’ (but also ‘topic-neutrality’) share the same extension and the thesis that their meaning can be accounted for via the previous tripartition are not universally agreed-upon ideas. While MacFarlane’s analysis received great appreciation in the

literature, setting a terminological and conceptual common ground on this matter seems an unachievable goal.

For instance, in their list of the traditional properties of logic, Martin and Hjortland (2022) give generality and formality different meanings. Following the Kantian and the Fregean tradition, they interpret ‘generality’ as 1-formality, and ‘formality’ as 2-formality.

For the purposes of this thesis, I fix the interpretation of generality as 1-formality. The first reason is that this interpretation is aligned with the way in which leading anti-exceptionalists use the term, as pointed out earlier. Secondly, this is the notion of generality at stake in Stei’s (2020*c*) explanation of hallmark (b): he highlights that generality is the distinguishing criterion between pluralism and relativism, and reminds the reader that Beall and Restall are committed to the ‘universal applicability of logical consequence’, understood as the idea that ‘logical consequence applies under any condition whatsoever’ (Beall and Restall, 2006, p. 16). Thirdly, Beall and Restall themselves claim that their pluralism only admits ‘all-purpose logics’, and that this is what makes their framework distinct from relativism. To further stress this point, Beall and Restall (2006) specify the following:

We do not take logical consequence to be relative to languages, communities of inquiry, contexts, or anything else. We do not take logic to be relative in this way. We are pluralists about logical consequence because we take there to be a number of different consequence relations, each reflecting different precisifications of the pre-theoretic notion of deductive logical consequence. This is a pluralism, not a relativism. (Beall and Restall, 2006, p.88)

According to MacFarlane, 2-formality is concerned with showing the independence of logical laws from the identity of particular objects, and 3-formality is about the independence of logic from arguments’ semantic content. However, Martin and Hjortland’s account, Stei’s analysis and Beall and Restall’s quote are focused on the independence of logic from particular domains of application, be they characterized as languages, contexts or analogous notions, and this is exactly the kind of independence involved in 1-formality.

4.2 Are Beall and Restall’s logics really general?

The claim that the logics deemed admissible in Beall and Restall’s framework can uphold a meaningful sense of generality has been challenged by some commentators. For instance, Keefe (2014) claims that Beall and Restall’s proposal faces a dilemma. On the one hand, Beall and Restall embrace a full-fledged conception of the *generality* of logical consequence. On the other hand, they claim that different logics give the right answers about consequences in different types of applications, and this appears to be a down-sizing of the generality of logic. According to Keefe, the idea underlying the second part of Beall and Restall’s (2006) book, titled “Logics”, where logics and their applications are discussed individually, is that “the right logical consequence relation varies with (and is thus relative to) different contexts” (Keefe, 2014, p. 1384). More specifically, Beall and Restall claim that *classical* logic is appropriate for (i) the analysis of (classical) mathematics and (ii) semantics; *relevant* logic is suitable for (i) inconsistent, non-trivial theories, (ii) fictional discourse, (iii) truthmaking and (iv) situation semantics; *constructive* logic is suitable for (i) mathematical reasoning and (ii) constructive warrant. In Keefe’s perspective, this really seems to undermine the unconstrained all-purpose scope of logic they advocate.

Moreover, Shapiro (2014a) is right in claiming that Beall and Restall’s ‘Generalised Tarski Thesis’ (GTT) is an instance of what Wright (2008) calls *folk-relativism*, whose slogan is “there is no such thing as simply being ϕ ”. In Shapiro’s words, “[i]f ϕ is relative, in this sense, then in order to get a truth-value for a statement of the form “ a is ϕ ”, one must implicitly or explicitly indicate something else”. According to this criterion, simultaneity and length are relative, but also predicates of personal taste like “tasty” and “fun”. The truth-value of statements involving simultaneity and length is relative to the reference frame of a certain observer, and the truth-value of assertions like “this pizza is delicious!” or “this videogame is amusing!” is relative to the preferences of a particular individual. The notion of folk-relativism can be vividly captured by the “general relativistic schema” put forward by Swoyer in an old edition of the entry ‘Relativism’ in the *Stanford Encyclopedia of Philosophy*:

(GRS) Y is relative to X .

In Shapiro’s interpretation, in the case of Beall and Restall’s GTT the de-

pendent variable Y is logical consequence, and the independent variable X is a certain kind of case; therefore, logical consequence turns out to be *relative to* a kind of case. Burgess also raises analogous concerns:

I'm not entirely sure how the authors can avoid the charge that their view is relativist. For pluralism to be true, one logic must be determinately preferable to another for one clear purpose while determinately inferior to it for another. If so, why then isn't the notion of consequence simply purpose relative? The authors officially deny this claiming that their pluralism of co-existence applies only to logics that meet the spare notion of admissibility defined above; competition between admissible logics emerges as soon as we start discussing applications. I don't think that this can possibly be right. (Burgess, 2010, p. 521)

In order to defend Beall and Restall's proposal from Burgess's objection, I am now going to point out an aspect of case-based logical pluralism that contravenes the idea of it being application-relative. What I consider to be the most charitable interpretation of case-based logical pluralism is that the 'talk' about applications is *not* directed at binding the correctness of a certain logic to a particular application.⁵¹ Rather, Beall and Restall mention several applications for each of the logics they deem admissible in order to help the reader intuitively understand how a certain 'deductive standard' plays a role in our linguistic practices.⁵² Basically, they give reasons for considering the various logic admissible by elucidating how they work 'in practice', but the logic-purpose relationship is not binding: even if a certain sense of 'follows from' is commonly employed for a particular application, that does not mean that such correspondence is *necessary*. It is useful to think about this aspect of Beall and Restall's proposal from a (late) Wittgensteinian perspective: the source of the well-established albeit non-necessary links between logics and applications is located in our shared practices. When we engage with fictional discourse it is common to expect that the premises of an argument are relevant to its conclusion, and such expectation is rooted in our socio-cultural *milieu*. However, the 'rules' governing our shared practices

⁵¹Even if there is undoubtedly some ambiguity about this, which led other logicians to read Beall and Restall's proposal differently. Later on I will examine the interpretation Pedersen (2014), who understood case-based pluralism as a version of domain-specific pluralism.

⁵²I have to thank Colin Caret for making me see this.

can be challenged: someone could try to write a novel where the relevance of the premises for the conclusion of an argument is not always safeguarded. Still following Wittgenstein (1953), that’s because rules and practices have a fundamentally *open* nature, which allows individuals to diverge from them without being ‘wrong’.

To conclude this discussion, I argue that in case-based pluralism the admissible logics are probably as general as they can be. In Beall and Restall’s framework, logics are indeed *dependent* on the particular type of semantic structure at stake (as Shapiro points out), but once that the *kind* of cases is fixed (i.e. be it classical models, constructions or situations), the validity of an argument is independent from domains, contexts, applications and so on. After putting forward the thesis that Beall and Restall’s pluralism is actually an instance of relativism, Burgess himself claims that “perhaps pluralism is relativism but relativism of such a harmless kind that to use that word to promote it would dramatise the position too much” (Burgess, 2010, p. 521). After all, maybe it is constitutively impossible to endorse a pluralist framework and, at the same time, refrain from such minimal degree of relativism.

4.3 Against logical generalism

The idea that logical laws ought to be universally applicable has been challenged. Within the literature directed at addressing the issues brought about by the collapse problem, one of the strategies consists in rejecting logics’ generality by adopting a *local* conception of the relation of logical consequence: under this perspective, the validity of a certain argument form becomes tied to particular domains.⁵³ According to local views of logical consequence, in order for a certain logical law to be valid it does not have to be applicable across the board (where the ‘board’ is unrestricted). Validity is evaluated with respect to a particular domain of application, or ‘subject matter’. Local views of the logical consequence relation have been defended by authors like DeVidi (2011), Kissel (2018), Pedersen (2014), and Shapiro (2014b). According to these so-called *domain-specific* logical pluralists, the correctness of a certain logic depends on the domain of discourse; while classical logic

⁵³Domains are sometimes called ‘contexts’, but for matters of simplicity I adopt uniform terminology here. Additionally, I think that there are many instances in which ‘domains’ and ‘contexts’ actually denote the same thing.

might be right for a certain domain, the correct logic for another domain could be intuitionistic or relevant logic. While the way of identifying the different domains varies, all the localist accounts mentioned above have in common the rejection of the idea that logical consequence is global in scope; they argue for the relativity of different logical systems to different purposes, different linguistic frameworks, different metaphysical domains, or different mathematical structures, respectively. It is important to notice that these localist proposals are all combined with a pluralist view about logic. It is in fact very hard to imagine why a monist should restrict the validity of logical laws to one or more particular domains. To my knowledge, no one defends such metaphysical combination (i.e. localist monism).⁵⁴

4.3.1 Non-generalist logical pluralism

A non-generalist version of Beall and Restall’s pluralism has been endorsed by Pedersen (2014), who argues in favor of a three-layered pluralist ‘package’. According to his proposal, at the most foundational level there is metaphysical pluralism, the idea that reality is divided into domains that have fundamentally different properties; for instance, he contends that the fact that there are both mind-dependent and mind-independent objects is rooted in the structure of reality. Thus, some propositions pertain to do-

⁵⁴However, there is a version of non-generalist view about logic that is compatible with monism: Wyatt and Payette’s (2019) logical particularism. After citing substantial literature in which counterexamples have been advanced even towards some among the most well-established argument schemes, Wyatt and Payette argue that the solutions which try to preserve the generality of logic at all costs fall short. Their central thesis is that the nihilist challenge according to which “there is no correct logic” has to be neutralized not by arguing that there are, after all, laws that are indiscriminately applicable, but by revising the conception of logic according to which it is essential to the nature of this discipline to provide universal laws. Instead, logical particularism commits to a fundamentally different perspective over the nature of logical laws: when in force, even in a restricted domain of application, they are not required to be completely exceptionless. The existence of argument instances that constitute counterexamples to logical laws is not a threat to their very status of laws. In this sense, this strategy is distinct from the strategy underlying most forms of *local* logical pluralism, but compatible with them. In fact, the main idea of logical particularism is that exceptions to logical laws are admissible, whereas that is not necessarily the case for local pluralism. Hofweber (2007) put forward a similar view, according to which the best way to solve semantic paradoxes is to consider laws of logic as something similar to generics, that is, as statements that express generalizations but also have exceptions (e.g. “Tigers are striped”).

mains whose objects carry the former metaphysical status (e.g. those involving talk of mathematical entities as conceived by the intuitionists) and other propositions belong to domains whose objects have the latter kind of existence (e.g. those reporting brute physical facts). In turn, the subject matter of a given proposition (i.e. the part of reality that the proposition is concerned with) determines the features of the truth property relevant to it, yielding a pluralism on the alethic level. For instance, Pedersen argues that the truth property which is the most suited for propositions concerned with intuitionistic mathematics is ‘superwarrant’, while the appropriate one for propositions about the physical world is correspondence. In fact, since correspondence truth is roughly the idea that for a proposition to be true it needs to be isomorphic (or congruent) to reality, such requirement might be too demanding for intuitionistic mathematics, according to which mathematical entities are mind-dependent constructions (i.e. they are not real in the sense of mathematical realism), but not for propositions concerning brute facts about the physical world. On the contrary, ‘superwarrant’ is an antirealist truth property according to which a proposition is true just when believing it is warranted at some stage of inquiry and would remain warranted without defeat at every successive stage of inquiry. Since this truth property bears on the doxastic commitments of the agent(s) dealing with the proposition under examination, it is compatible with an antirealist view of mathematical entities, and therefore with the sphere of intuitionistic mathematics. Moving to Pedersen’s perspective on the relation between alethic and logical pluralism, he holds that the specific features of a certain truth property play a crucial role in determining the logic that is appropriate for a particular metaphysical domain. With respect to Beall and Restall’s framework, this means that the truth property pertaining to a given domain determines how cases – in the sense relevant to the Generalized Tarski Thesis – are selected and construed for arguments with premises and conclusion belonging to that domain. In particular, Pedersen claims that:

The truth properties of some domains have the feature of being epistemically constrained and go hand in hand with cases that deliver intuitionistic logic, while the truth properties of other domains have the feature of being epistemically unconstrained and go hand in hand with cases that deliver classical logic. In short, alethic pluralism yields logical pluralism. (Pedersen, 2014, p. 262)

Basically, the fact that reality is split into a taxonomy of different domains (metaphysical pluralism), provides grounds for a plurality of ways to assess the truth of propositions involving objects pertaining to those domains (alethic pluralism), and this plurality of truth properties plays a crucial role in the selection of a particular logic (logical pluralism). While Pedersen explicitly states that “[m]etaphysical pluralism connects with alethic pluralism, and alethic pluralism supports logical pluralism” (Pedersen, 2014, p. 275), he does not explain in detail the precise relationship between the three pluralisms that constitute his ‘package’; he remains particularly vague with respect to the connection between alethic and logical pluralism. Nonetheless, I take his view to be sufficiently spelled out to constitute a good example of Beall and Restall’s pluralism without generality.

As I argued for earlier, Beall and Restall’s proposal does not set up any substantial correspondence between applications and logics, and this has triggered the reactions of some commentators: Burgess and Keefe wonder whether case-based logical pluralism should be considered as a relativistic view. In Pedersen’s domain-specific version of Beall and Restall’s framework, the reason why a particular precisification of “case_{*x*}” is more appropriate for a certain purpose has a very well-grounded answer: a certain logic is suited for a certain application in virtue of the metaphysical features of the domain involved in that field of application. Contrary to how I described it with respect to Beall and Restall’s original proposal (see Section 4.2), in Pedersen re-interpretation of case-based logical pluralism the logic-application relationship is *necessarily* rooted in the metaphysical structure of reality, and therefore strictly binding.

Rejecting generality in this way allows the pluralist to avoid the collapse problem because each domain corresponds to a certain logic, and this one-to-one correspondence between areas of discourse and formal systems entails that there is no conflict among different logics when it comes to provide agents with directives pertaining to how to reason about a particular argument.

4.4 Assessment

As attractive as it might seem, local logical pluralism has been the target of various criticisms. I dedicate this section to the objections towards the notion of *domain* brought forward by Keefe (2018) and to some of the possible answers that the domain-specific pluralist can advance to them. I also present

Shapiro’s (2014*b*) version of the notion of domain, which provides a possible answer to Keefe’s arguments in virtue of its independence from the alleged metaphysical divides of reality involved in Pedersen’s proposal.

4.4.1 Issues of employing domains

The very idea of employing domains in a pluralist account of logic raises a number of problems. Keefe (2018) raises two objections against domain-specific logical pluralism that have been previously raised against domain-specific truth pluralism. First of all, “[i]f different logics govern different domains, what logic governs arguments that involve statements from more than one different domain?” (Keefe, 2018, p. 432). This is the ‘mixed inference challenge’ for domain-specific pluralisms, and it had originally been formulated by Tappolet (1997) and Williamson (1994).⁵⁵ For instance, consider the following disjunction: “the Mona Lisa is the most beautiful painting in the world or water is CO₂”: which logic should govern the mixed inference “the Mona Lisa is the most beautiful painting in the world or water is CO₂, *but* water is not CO₂, and *therefore* the Mona Lisa is the most beautiful painting in the world”? Domain-specific logical pluralism does not provide a straightforward answer to this question.

The source of the second objection towards domain-specific pluralism – the ‘demarcation problem’ – is the lack of a well spelled-out procedure of domain delimitation. Since this kind of pluralist framework heavily relies on the notion of domain, its proponents ought to state a clear definition of what a domain is, together with straightforward instructions about how to demarcate each domain and determine when a proposition belongs to a particular domain.

To my knowledge, domain-specific pluralists have not yet provided a clear definition of ‘domain’, nor have they established how one ought to trace the boundaries of domains. Pedersen (2014) roughly states that domains are ‘what reality is divided into’. He further specifies the nature of domains by mentioning ‘the empirical world’, ‘the law’ and ‘mathematics’ as salient examples. He also claims that these domains are not merely verbal labels, but they track genuine differences in the metaphysical nature of reality. According to Lynch (2009), atomic propositions (those who capture basic facts, that is, obtaining state of affairs) necessarily belong to one and only do-

⁵⁵It also seems reminiscent of the Frege-Geach paradox.

main. In his words, “belonging to a particular domain is a feature an atomic proposition at least, has in virtue of being the sort of proposition it is. [...] [T]herefore, belonging to a particular domain is an essential fact about an atomic proposition” (Lynch, 2009, p. 80). Lynch also explains what determines the ‘membership’ of a proposition to a particular domain: it’s “the kind of concepts (moral, legal, mathematical) that compose the proposition in question” (Lynch, 2009, p. 80). In short, it seems that for domain-specific pluralists like Lynch and Pedersen reality is divided into domains that possess peculiar metaphysical properties. Every atomic proposition captures a fact pertaining to one and only one of these domains, and the proposition-domain correspondence is determined by the subject matter of the concepts that are part of the proposition. Let’s now consider an instance of the demarcation problem that poses a challenge to this account. If we consider the proposition ‘7309 is a prime number’, it is unproblematic to place it in the mathematical domain. However, the proposition ‘7309 is a large number’ is troublesome: in spite of being supposedly atomic, it contains concepts of two different kinds: ‘7309’ and ‘number’, mathematical concepts, and ‘large’, a vague concept. The proposition ‘7309 is a large number’ is therefore in between the domain of mathematics and that of vagueness.⁵⁶ Keefe (2018) vividly states this issue:

Whereas domains such as mathematics, morality and perhaps humor might be thought to be individuated by their subject-matter, logically challenging phenomena such as vagueness, future contingents and empty names surely aren’t unified by their subject-matter. Vague language can be about anything, for example, so the phenomenon and its logical repercussions cuts across several domains. (Keefe, 2018, p. 440)

In the next section I will examine a number of strategies that the domain-specific pluralists can adopt to overcome these objections.

4.4.2 Possible solutions

First of all, I will analyze solutions that remain aligned to the idea of domain shared by Pedersen and Lynch. After that, I will consider more radical solutions, like Keefe’s suggestion to give up the notion of domain completely

⁵⁶The fact that a ‘domain of vagueness’ actually exists is debated.

and that of Shapiro consisting in a profound re-thinking of how domains ought to be conceived.

With respect to the ‘mixed inference problem’, one viable strategy consists in embracing a principle of *logical modesty*, as suggested by Lynch (2008) in defense of his domain-specific alethic pluralism. The idea is that, when more than one logic is available for the assessment of an argument given the involvement of multiple domains, the intersection of the logics governing each of the domains that are relevant for the argument should be endorsed. To illustrate this solution, let’s consider again the previous example, and suppose that the domain of aesthetics is governed by intuitionistic logic, and that of chemistry by classical logic. Then, in virtue of logical modesty, the disjunction “the Mona Lisa is the most beautiful painting in the world or water is CO₂” should be evaluated with intuitionistic logic. However, Steinberger (2019*b*) points out how this solution threatens the domain-specific logical pluralist with another collapse into the weakest logic: if arguments involving multiple domains are always evaluated with respect to the intersection of the logics governing each of the involved domains, why should one refrain from adopting the intersection of all the logics available as the One True Logic and applying it to all domains? Moreover, what if the intersection of the logics involved in a certain argument is empty?⁵⁷ An alternative solution could be that of admitting a ‘domain of mixed inference’ for those arguments that range over two or more domains. However, since for the domain-specific logical pluralist a particular domain is individuated by the subject matter of the propositions that compose it, how should such ‘mixed domain’ be canvassed? This solution seems to undermine the mechanism of ‘domain delimitation’ that is at the very core of the domain-specific proposal.

For what concerns the ‘demarcation problem’, the domain-specific pluralist could answer to the criticism by claiming that ‘7309 is a large number’ and analogous propositions are not atomic, or by ruling out problematic domains such as those of ‘vague expressions’, ‘contingent futures’ and so on. After all, they do not have a perspicuous subject matter. These kind of disputes will benefit from further clarity once the domain-specific pluralists will dissipate the previously diagnosed fuzziness about the notion of domain they employ.

Overall, the combination of the ‘mixed inference problem’ with the ‘demarcation problem’ sheds a quite negative light on a logical pluralism grounded

⁵⁷This would not be a problem for Beall and Restall, since the intersection of the logics they endorse cannot be empty.

on the notion of domain. In light of these difficulties, Keefe suggested that a *contextual* account of logical pluralism could be preferable. While for domain-specific logical pluralists the intension of the validity predicate depends on the *content* of a certain inference (the *topic* at stake), for context-specific logical pluralists it is the *context* in which an argument is situated that determines the meaning of the relation of logical consequence.⁵⁸ I already pointed out that I find it hard to provide a satisfactory notion of context when it comes to logical contextualism – that is, a notion of context that gives an intuitive account of how deductive standards get selected in our practices. In fact, when I examined Caret’s (2017) contextualist proposal in subsection 2.3.2, I argued that this framework actually involves a topic-dependent conception of contexts, contrary to Caret’s claims.

Another proposal that is sometimes described as an instance of logical contextualism is Shapiro’s eclectic logical pluralism, according to which different mathematical contexts require different logics. More specifically, each *consistent* mathematical structure is coupled with an appropriate logic. Notice that this position is considered a form of *domain*-specific pluralism by Shapiro himself (see Shapiro and Kissel, 2020). However, differently from Lynch and Pedersen, Shapiro employs mathematical domains, whose boundaries are usually much easier to draw, because they are well-defined and consistent mathematical structures. For the same reason, they cannot be mixed to give rise to inferences that range across different structures. In conclusion, this notion of domain may be very appealing in virtue of its upshot of avoiding Keefe’s objections, but also in virtue of its intuitiveness and precision.

4.5 Take-aways

In this chapter I scrutinized Beall and Restall’s claim that the logics they admit in their framework are ‘all-purpose logics’, concluding that their logics are as general as they can be for a pluralist proposal. After that, I presented an instance of case-based pluralism without logics’ generality, I spelled out some of its criticisms and pointed out Shapiro’s notion of domains as a way

⁵⁸It is worth mentioning that in the discussion about local logical pluralism context and domains are not always sharply distinguished. For instance, in the work of Arenhart (2021) the expressions ‘domain’ and ‘context’ are used interchangeably, testifying that he does not acknowledge a substantial divide between the two.

to overcome those criticisms. Most importantly, I showed why adopting a localist view over logics' 'jurisdiction' is a good solution to the collapse problem.

Notice that, in a non-generalist view, logics can still be rivals. As Steinberger (2019a) points out, it can be that two logics (or more), even if non-general, are suitable for a single domain, and therefore give rise to an instance of intra-domain applicational rivalry. In order to avoid this issue, a localist account needs to require that there is at most one logic suitable for a particular domain. Otherwise, more than one logic might have to compete against each other within one domain, and a domain-relative version of the collapse problem could apply. For instance, consider the following formulation of a *local* version of the collapse problem:⁵⁹

Let D be some domain; suppose that L_1 and L_2 are two admissible logics for that domain. Should one use the notion of validity of L_1 or of L_2 ? We cannot give the answer 'both' here. Take some inference, $\alpha \vdash \beta$, that is valid in L_1 but not L_2 and suppose that we know (or assume) α holds in D ; are we, or are we not entitled to accept that β does?

Lynch and Pedersen both claim that there is a one-to-one correspondence between logics and domains, but mixed inferences could still constitute a threat to their frameworks: in the case of an argument that ranges across domains, it is possible that the logics pertaining the various involved domain provide conflicting directives to the reasoner, triggering a new collapse problem. Shapiro's conception of domains as mathematical structures does not allow mixed arguments, because each structure is, by definition, self-contained. However, even his view is subject to a potential objection. Consider the mathematical field of arithmetic: the logic associated to this mathematical domain can be either classical (giving rise to Peano arithmetic) or intuitionistic (giving rise to Heyting arithmetic), and the logician embracing both classical and intuitionistic arithmetic may have to face an intra-domain version of the normativity objection.

Long story short, here are the conditions for the examined local pluralisms to overcome the collapse problem once and for all. If local pluralists along

⁵⁹This instance of the collapse problem was originally formulated by Sabina Domínguez Parrado for the joint paper we wrote at the end of a Master of Logic individual research project supervised by Robert Passmann. Here, I slightly modified it to fit in the conceptual and terminological context of this thesis.

the lines of Lynch and Pedersen guarantee the one-to-one correspondence between logic and domains and find a way to handle mixed inferences, they successfully solve the collapse problem in all its formulations. With respect to Shapiro's account, it can be said to efficaciously solve all kinds of collapse problem if we endorse his idea of mathematical structures (domains) coming hand in hand with a certain logic. Shapiro claims that mathematical structures are *characterised by* a logic, that is, mathematical theories come already with a logic. Under this perspective, one cannot really think of 'arithmetic' independently from either Peano or Heyting axiomatization, that is, independently from either classical or intuitionistic logic. To strengthen this point even more, Arenhart (2021) suggests that pluralists *a lá* Shapiro should think of logics as *a priori* ingredients of mathematical structures.

In the final section of this chapter I will examine whether local pluralists are compelled to giving up on normativity.

4.6 Can there be normativity without generality?

Whether there is any interesting sense in which logic is normative once that the generality of logic is rejected is the question at the center of the paper 'Logical pluralism and normativity' by Shapiro and Kissel (2020), who both endorse domain-specific logical pluralism, albeit in two distinct formulations. Starting from Steinberger's (2017) exegesis of the thought of Frege and Carnap on the normative status of logical laws, Shapiro and Kissel draw the conclusion that domain-specific logical pluralism is not compatible with Frege's view on logical normativity. Frege takes logic to be normative in three ways: its laws are *descriptive* of the laws that govern thought, *prescriptive* for thinking and reasoning and *constitutive* for thought as such. The only metaphysical stance about logic that can meet the requirements of this kind of normativity is a strong monism, for which the one and only admissible set of laws that our thinking ought to conform to is independent from the practice we are pursuing or the theory we are developing. Logical pluralism is intrinsically unsuited to describe, prescribe and constitute the laws of the *single* realm of thoughts, because it is by definition the rejection of the idea that there is only one logic governing all deductive reasoning.

After ruling out the thesis that non-generalist logical pluralism can be

normative in a Fregean sense, Shapiro and Kissel try to evaluate whether their pluralist accounts are compatible with Carnap’s conception of normativity. Since Carnap advocates a logical pluralism in which the goodness or badness of a logic is relative to the logician’s aim, that is, a framework allowing to freely choose the logical system which better serves certain theoretical ends on pragmatic grounds, he needs to tell a story about normativity different from that of Frege. One may argue that Carnap thought that logic is normative in a *volutarist* sense: we are required to stick only to the logical norms we have selected to use. This way of conceiving normativity still means that the laws of certain logics are constitutive and descriptive for thought, albeit only within certain practices and not across the board as Frege thought. “In a sense, Frege has it that the laws of logic are categorical imperatives. Carnap has them as hypothetical imperatives tied to this or that theoretical purpose” (Shapiro and Kissel, 2020, p. 398). As mentioned in the previous section, Shapiro and Kissel hold that there is a legitimate logic for each coherent mathematical system, which is also the *appropriate* logic for reasoning within that particular mathematical system (or ‘context’). The Carnapian conception of normativity (as Steinberger portrays it) is compatible with Shapiro and Kissel’s non-generalist versions of logical pluralism. At the end of their paper, Shapiro and Kissel make their proximity to Carnap’s ideas even more explicit:

[T]he norms implicit in these practices [e.g. classical analysis, intuitionistic analysis, inconsistent mathematics...] are different from each other. Each of the logics is truth-preserving in its domain, in the sense that if the premises of [a] valid argument (in the indicated logic) are true in the domain, then so is its conclusion. None of the logics, or perhaps only the weakest of them, is truth preserving across the board. (Shapiro and Kissel, 2020, p. 408)

In his paper ‘Logical Pluralism and Logical Normativity’, Steinberger (2019a) also acknowledges that in domain-specific logical pluralism the normative authority of the endorsed logics is confined to their proper jurisdiction. Therefore, Steinberger also agrees that normativity can be preserved on non-generalist accounts, but it is necessarily relativized to domains.

In this section we showed that it is possible to preserve a genuine idea of normativity even when logical generality is dropped. This means that the rejection of generality does not, per se, entail the rejection of normativity. In

the subsection 3.4.1 of the previous chapter, I argued for the contrapositive of this claim: once that normativity is dropped, a general conception of logic can be maintained. Nonetheless, given that there are good reasons to abandon *both* generality and normativity, it is of course a viable option, for the pluralist, to reject these two properties at the same time.

Chapter 5

Conclusion

Logical pluralism is an attractive position, and the renewed popularity it gained after the advent of its case-based version is still far from fading away. In this thesis I tried to highlight that, in spite of its attractiveness, there are theoretical costs associated with Beall and Restall’s proposal. In particular, the collapse problem is a thorny issue. I argued that, in order to avoid the collapse problem, a compromise with the logical tradition should be made. More specifically, pluralists ought to drop at least one of the traditional properties of logic: the choice ranges between generality and normativity.⁶⁰ The fact that rejecting either generality or normativity provides the pluralist with a solution to the collapse problem is the main reason why logical pluralists should be anti-exceptionalists. Highlighting these ‘theoretical costs’ may help the logician in her choice about whether embracing logical pluralism is a good decision or not.

While in this thesis I have been focusing only on case-based logical pluralism, I think that the conclusion I arrived at can be generalized to all the pluralist positions that fall within GCP: after all, they are all wedded to the inconsistency highlighted by the collapse problem, and for the same reason (the combination of the four hallmarks identified by Stei, 2020c). Arguably, the arguments against the possibility of only rejecting rivalry and about the advantageous outcomes of dropping either normativity or generality apply beyond the boundaries of Beall and Restall’s account. However, a detailed explanation about how to carry out this generalization is not something I

⁶⁰At this point it is worth noting that, for reasons that are independent from the collapse problem, the fact that Beall and Restall include *necessity* among the constraints of logics’ correctness has also been challenged (see Bueno and Shalkowski, 2009).

dive into in this thesis; it is matter for further work.

In this final chapter I examine a final, possible concern: are we still talking about logic when we remove either normativity or generality from the metaphysics of this discipline? After answering this question, I make some concluding remarks.

5.1 Is non-generalist or non-normative logic still logic?

At the end of this investigation, a reasonable worry is whether a conception of logic that excludes either normativity or generality (or both) from the properties that make up for the very *essence* of logicality is acceptable.⁶¹ The operation of metaphysical re-definition that seemed necessary for logical pluralism to successfully withstand the collapse problem might involve a metaphysical cost that is too high to pay. As MacFarlane (2000, 2004) reports, the number of philosophers who identified the property that sets the boundary of logic with either generality or normativity is large. In particular, for what concerns normativity, he says:

Logic is often said to provide norms for thought or reasoning. Indeed, this idea is central to the way in which logic has been demarcated as a discipline, and without it, it is hard to see how we would distinguish logic from the disciplines that crowd it on all sides: psychology, metaphysics, mathematics and semantics. (MacFarlane, 2004, p. 1)

For what concerns the generality of logic, both Frege and Kant regard it as the defining feature of logic, so that if a certain law turns out to fail in some

⁶¹Notice that this concern arises only for those who use *principled* demarcations to set logic out from other disciplines, and thus claim that a combination of properties is a necessary and sufficient condition for logicality. The scenario is less troublesome for those who identify logicality by means of some *pragmatic* demarcations: what makes logic *logic* within these perspectives is then the ability to do a certain job, which can be, for instance, that of providing a “framework for the deductive systematization of scientific theories” (Warmbröd, 1999, p. 516). Since according to these positions there are no necessary nor sufficient properties that characterize logic, giving up on some of the attributes that have distinguished logic from other disciplines in its historical tradition does not undermine the fact that one is still talking about the same discipline.

domain, then it is not, by definition, a logical law. Beall (2017) also claims that if a certain law only holds for some theories, then it is by definition extra-logical. In MacFarlane's words:

The other non-technical property of logic that is often invoked in demarcation projects is its distinctive generality or topic-neutrality. Logic is not supposed to be about anything in particular; it is distinguished by its lack of any special subject matter. (MacFarlane, 2004, p. 20)

Given that the concept of logic is different from that of *gold* – a natural kind whose essence can be studied through things that can be observed in the empirical world, like the number of protons in the nucleus of a gold atom – it is not so straightforward to come up with a largely revised picture of logic and claim that such a thing is indeed still worthy of the label 'logic'. As MacFarlane puts it:

Logic is not a natural kind concept. It does not play a role in laws of nature, and so the natural order of the world cannot take up the slack between our ways of grabbing onto the concept and the concept itself. Thus we must attend more closely to the ways in which our predecessors marked out the subject if we want to ensure that we are talking about the same thing. This is not to say that there is no room to criticize our predecessors' conceptions. We want to be able to say that our predecessors were wrong (at least in part) about both the scope and the essential characteristics of logic. But our model for such criticism cannot be our criticism of our predecessors' conceptions of gold. The concept of logic, I suggest, is more usefully compared with legal concepts such as negligence, property, or contract. The correct application of these concepts requires much more sensitivity to past usage and past theory than does the correct application of gold. That is why judges must attend studiously to precedent. And that is why an investigation of the nature and bounds of logic must attend to the tradition of demarcating logic. (MacFarlane, 2000, p. 23)

This concern can be summed up through the following question: to which extent do we want to be essentialists about logic? I think that there is a very

good answer to this issue. Following Martin (2022), one could claim that it would be better to adopt a more *practice-based* approach for the way in which we shape our idea of logic: giving more importance to certain time-honored metaphysical assumptions about the nature of logic than to the ways in which logic is actually practised by experts is likely to be detrimental to our philosophical investigation. An account of logic that preserves normativity and generality at all costs may turn out to be more based on what we want logic to look like given our preconceptions rather than on a reflection of the reality of the research in the field. In this sense, a picture of logic that is committed to the tradition of this discipline up to this degree may be excessively idealised. According to Martin, we should embrace a “practice-based turn” in logic, which I already presented in chapter 3 as an alternative foundation to that of Blake-Turner and Russell for the descriptive role of logic.⁶² In virtue of this approach, which gives adequate importance to the way in which logic is actually practiced, one could observe that logicians are not really worried about designing and employing logics that are not ‘all-purpose’ (DeVidi, 2011; Shapiro, 2014b; Pedersen, 2014; Kissel, 2018). In a similar fashion, the ‘logics-as-models’ approach that emphasises the descriptive role of logic has an important number of supporters (Cook, 2010b; Glanzberg, 2021; Arenhart, 2021). Therefore, one should not worry too much about abandoning a picture of logic according to which this discipline is normative and/or general, a picture that was probably adequate for the way in which logic has been practiced for many years. However, things have changed. This “practice-based” perspective helps us being more favorable towards abandoning an exceptional view of logic in favour of an anti-exceptional one. Of course, this should not mean that ‘everything goes’, and that one ought to adopt a tolerant standpoint towards whichever bizarre view of logic. Before including a certain account of logic among the admissible ones there should be inter-subjective agreement about it in (at least) a consistent part of the community of logicians. In a nutshell, when we assess our notion of logicality we need to have “a more detailed consideration of the activities of the practitioners” (Martin, 2022, p. 270). Moreover:

⁶²This kind of ‘turn’ has already been made in other sciences, i.e. mathematics. (see, for instance, Mancosu, 2008). In Martin’s words, “[p]rominent uses of the approach can be found in contemporary philosophy of science and philosophy of mathematics, with each containing research programmes built around the approach, in the form of the philosophy of scientific practice (PSP) and the philosophy of mathematical practice (PMP), respectively” (Martin, 2022, p. 269).

[I]n virtue of the propensity within contemporary philosophy of logic to build idealised accounts of logic based upon certain pre-suppositions about [...] logic's properties, what tends to result is an essentialist account of logic's aims and methods, which gives the impression that logic as a field is diachronically homogeneous. Just like the empirical sciences and mathematics, however, logic is a social enterprise with continually changing priorities and techniques. (Martin, 2022, p. 272)

In conclusion, pluralists should not worry too much about giving up either normativity or generality: there are compelling arguments supporting that choice, even from a meta-metaphysical perspective.

5.2 The actual aims of Beall and Restall's proposal

At this point, it might be charitable towards Beall and Restall to acknowledge that the main point of their proposal is not that of giving a precise account of what counts as a correct logic. That is, their interest does not consist in convincingly identifying the precise properties that set out the admissible logics from the non-admissible ones. This is perhaps why the notions of necessity, formality and normativity they provide are not explained in depth, but merely 'sketched'. They probably did not spend much time figuring out what kind of objections may have arisen from their formulation of those properties. In fact, the main thesis that they wanted to put forward is arguably that 'logical consequence' is vague, unsettled, and it can therefore be *precisified* in a variety of ways, in virtue of the multitude of approaches towards validity that we can adopt in all the distinct practices we engage with. In light of these considerations, the obstinacy of the critics in attacking the view because of the inconsistency brought about by the collapse problem may be criticized, in turn, for missing the major point of Beall and Restall's proposal.

This being said, I personally think that it is valuable to insist on the inconsistency highlighted by the normativity objection and attempt to provide a solution to it. After all, even if the collapse problem may be regarded as a criticism that misses the main thesis of Beall and Restall's framework, it nonetheless generated a heated debate that goes beyond case-based logical

pluralism.

5.3 Concluding remarks

Whether the argument presented in this thesis is convincing will depend on the meta-metaphysical stance one wishes to adopt about logic’s metaphysics. On the one hand there is the ‘liberal’ perspective of anti-exceptionalists, whose purpose is providing arguments for freeing logic from the burdensome influence of its tradition. On the other hand there is the ‘conservative’ standpoint of philosophers like Beall, whose outlook on logic was concisely summarized through MacFarlane’s quotes about the reasons why the metaphysical foundations of logic that were built by our predecessors cannot be modified at will.⁶³ Depending on what one considers to be the best middle ground between these two perspectives, different solutions can be embraced. Some among the possible strategies are: (1) dropping generality or normativity (or both) and defeating the collapse problem *tout court*, accepting a picture of logic that is very different from that of its tradition; (2) making an argument to show that the collapse problem is not such a serious problem for the pluralist; (3) resorting to logical monism, avoiding at the same time the collapse problem and a possible affront to logic’s orthodoxy. This is to say, there is a number of ways one might try to resist the argument that I have presented in this thesis. Nonetheless, I think (and I hope the reader agrees) it has been worthwhile to investigate the compatibility of Beall and Restall’s proposal and logical anti-exceptionalism, and to assess the positive outcomes of dropping either generality or normativity for a pluralist framework belonging to the GCP family. After all, I think it is valuable to aim at an account that preserves the theoretical richness that a plurality of logics brings about. The recent revival of logical pluralism’s popularity is tied to “the recognition that logical systems can be applied [...] with myriad purposes” (Martin, 2022, p. 279). Beall and Restall’s framework carries the spirit of this acknowledgement, without embracing it across the board: even

⁶³The point brought about by MacFarlane is also relevant for the discussion concerning logical anti-exceptionalism alone: within the cluster of positions that fall under this label there are some which reject properties of logic that have traditionally been deemed *essential* for logic, like generality or normativity, and properties that do not carry the same historical importance. Depending on how much one agrees with MacFarlane, there might be subsets of anti-exceptionalist positions that could be considered inadmissible in virtue of undermining the very essence of logic, and others that are less controversial.

if Beall and Restall do not deny that there can be more ‘cases’ suitable for further admissible instances of GTT, they limit themselves to Kripke models, constructions and situations. I hold that it is this ‘navigating in between’ a liberal and a conservative view of logic that leads to the most controversial aspects of their account, including the collapse problem. Beall and Restall recognize that there can be several applications of logic, but they remain close to an epicenter of more “canonical applications”, such as those pertaining to mathematics and some kinds of semantics. Their commitment to the historical tradition of logic also underlies their choice of necessity, formality and normativity as essential features of logicity. However, these metaphysical properties of logic are coming more and more at odds with the practices of logicians. As I pointed out throughout this thesis, new formal systems are developed and studied for modelling an increasingly diversified range of phenomena, and some of these areas of study have very short historical roots. A great group of experts within the logicians’ community emphasizes the *descriptive* role of logic and points towards a *domain-relative* understanding of the discipline.

Instead of relying so much on the traditional picture according to which logic carries an exceptional status, I claim that pluralists cast in the same mold as Beall and Restall should embrace a more radical and practice-based form of pluralism, by renouncing to either the normativity or the generality of logic (or both). In this work I have tried to argue for this claim by showing that rejecting at least one of these properties also constitutes the best, and perhaps only, solution to the collapse problem.

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