Conceptual Windows Afford Views on Satisfying Individuals

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Abstract

This paper presents a Modal Predicate Logic with conceptually restricted quantification. It provides a framework for the modeling of a variety of *de re* modalities and *knowing who* reports in a realist and literally transparent way. The framework also allows us to make proper sense of talk about things non-existent.

Keywords: Modal predicate logic, contextually restricted quantification, conceptual windows, definite descriptions, *de re* belief, knowing *who*, *Londres*, *Ortcutt*, *Vulcan*.

1   Introduction

Philosophers, linguists and logicians have a shared interest in the interplay between reference and modality. One object of common attention is the knowledge that we have of real individuals, including ones that have previously existed. We all assume we have loads of such knowledge, but how does that come about and what does it consist in? Individuals are out there in the real world, one might think, and knowledge, or belief, is something about our states of mind. And if some real individual were known with our minds, how is it at all possible that we can confuse her with some other individual, given that that other individual is obviously distinct from the first one? Nevertheless, the possibility of such a confusion is all over the place.

This paper builds on the familiar assumption, a firm conviction as a matter of fact, that we are jointly knowledgeable of a common world that is assumed to exist independently of ourselves. It comes to us in bits and pieces, though. We will use the metaphor of *conceptual windows* for what figure, formally, as the kinds of things that we take to enable and provide a view on such an external world, a view that is almost all of the time a limited one. Our knowledge of the world not only comes from what we, thus, see, but also from the connections that we make between the things we can see.

We are all familiar with the planet earth, in a wide variety of ways. We identify the earth that we walk on and travel around, with the planet which we learn about in the textbooks that present our solar system in our primary schools, and also with a major subject in the discussions about climate change.

These are surely not supposed to be distinct objects! We know these are various
ways of seeing one and the same thing, through various conceptual windows,
that is. We also happen to know that the planet Venus that we see in the
textbook pictures of our solar system, is identical to the heavenly body that we
can detect in the sky in the evening, the one we have learned is called "Hesperus",
and identical to the subject of certain disputes in the philosophy of language
and mind. Everybody must also be familiar with failures to, on occasion, make
such identifications. Most dramatic are the well-known cases in which one may
mistake a close relative, say one's own father, with a complete stranger that one
slaughters, or in which one does not see, before she fits her glass slipper, that
one is currently facing one's own dream princess of the ball.

This paper aims to present a formal framework to adequately describe
and model such knowledge, and the lack thereof, a framework that licenses us
to make the proper kinds of inferences that we want to make, and that does so
without inducing unwarranted philosophical assumptions. The paper presents a
version of Modal Predicate Logic that accommodates a method of conceptually
restricted quantification through conceptual windows. The first section presents
the syntax and semantics of our modal predicate logical framework. In the sec-
ond section we show how the use of *conceptual windows* serves to to properly
extend and significantly improve upon Maria Aloni's fairly successful method of
perspective relative quantification through *conceptual covers*. (Aloni 2005b) We
show how it can be put to use to model *de re* modalities, so-called, and *knowing
who*--reports. In the third section the system is further contrasted with that of
Aloni and with some recent alternative approaches to the phenomena. We finally
sketch a straightforward and promising extension that allows us to make sense
also of talk about things non-existent. The appendix gives the characteristic
definitions of the underlying proof-theory.

2   Definitions

The method of conceptually restricted quantification, quantification through
conceptual windows, is cast in a language $\mathcal{L}$ of first order modal predicate logic
that is built up, in the usual way, from a set of variables $x, y, \ldots \in \mathcal{V}$, sets
of relational constants $R, S, \ldots \in \mathcal{R}^j$ of any arity j, and the logical constants
$=, \neg, \wedge, \rightarrow, \exists, \forall, \Diamond, \Box$. (Hughes & Cresswell 1996)

Our language includes a quantificational, or declarative, treatment of
names $\mathsf{N} \in \mathcal{N}$, so that $\mathsf{N}x\,\phi$ is a formula declaring N to be an individual x,
so-named, such that ϕ.[1] A further distinctive feature is that our quantifiers
and modal operators are superscripted with indices $0, 1, \ldots$. For both types

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1. For our purposes there is no need to deal with individual constants that more or less
logically speaking figure as proper names of individuals. Mirroring the use of proper names
in natural language, we want to allow for the possibility of names being the name of more
than one real individual, or of no real object at all. This is, of course, not standard, but it
is not unorthodox either, and it is less problematic than the proof-theoretically unwarranted

of operators the index 0 plays the role of the default index and this index is therefore usually omitted. The indices on quantifiers other than 0 play the same role as the indices on quantifiers in Maria Aloni's language of quantification under conceptual covers. (Aloni 2005b) The indices serve to distinguish various conceptual windows one can have on the domain. Indices on modal operators serve to distinguish among the variety of modalities that there are. (Kratzer 1991; Blackburn, de Rijke & Venema 2001) The default modalities $\Diamond^0$ and $\Box^0$ are taken to have the absolute or universal interpretation, in a sense to be clarified below. Otherwise the modal indices serve to indicate the possibilities and necessities according to various other more particular types of modal bases, or according to specific intentional states of individual agents or collections of them (doxastic, bouletic, . . .). Summing up:

Definition 1 (Language of $\mathcal{MPL^{CRQ}}$)
Formulas ϕ are defined in Backus-Naur style, where $x \in \mathcal{V}$, $R \in \mathcal{R}^j$, and $\mathsf{N} \in \mathcal{N}$:
$$\phi ::= x_1{=}x_2 \mid Rx_1 \ldots x_j \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \rightarrow \phi) \mid \mathsf{N}x\phi \mid {}^i\exists x\phi \mid {}^i\forall x\phi \mid \Diamond^k\phi \mid \Box^k\phi$$

The models for our language host a set $\mathcal{W}$ of possibilities, and a set $\mathcal{U}$ of possible individual realizations. These possibilities and possible realizations are not to be thought of as real entities, but, rather, as theoretical posits, points of coordination, that serve to structure our own thought and talk about possibilities and things.[2]

Talk about individuals and their properties is facilitated by a family of sets of individual conceptions. Individual conceptions are taken to be, possibly partial, functions from $\mathcal{W}$ to $\mathcal{U}$, and any such set $\mathcal{C}_i$ of such functions provides for what we call a *Conceptual Window* on the world. Formally these sets figure as *methods of individuation* in the sense of (Hintikka 1969), but for the fact that they are perspectival and restricted here, in a way to be clarified further in the next section. The primary window $\mathcal{C}_0$, God's window if you want, is taken as the default and figures as a *domain projection*. It is taken to define the domain D_v of any possibility $v \in \mathcal{W}$, since the projected domain D_v of possibility v is defined to be the range of possible objects 'seen' through $\mathcal{C}_0$ in v.

$$D_v := \{c_v \mid c \in \mathcal{C}_0\} \tag{D_v}$$

(We systematically follow the practice of writing c_v for the realization, or value, $c(v)$ of an individual concept c in a possibility v.) It is assumed that any other

window can only see individuals actually existing.[3]

$$\text{for any } i, \text{ and for all } c \in \mathcal{C}_i : c_v \in D_v, \text{ if defined} \qquad (T)$$

Models also accommodate an indexed family $\{\mathcal{R}_k\}$ of accessibility relations. The primary one $\mathcal{R}_0$ is supposed to be universal, or equally likely an equivalence relation which then is also assumed to subsume all the other accessibility relations. (The required assumptions are made explicit in the appendix.) These other accessibility relations are, as is usual, taken to give us the sets of possibilities in which particular modal bases or states are satisfied.

Models finally host an interpretation function $\mathcal{I}$ over $\mathcal{W}$ associating with each possibility $v \in \mathcal{W}$ an interpretation $\mathcal{I}_v$ such that $\langle D_v, \mathcal{I}_v \rangle$ is an extensional model. Any such $\mathcal{I}_v$ is standard but for the fact that names N are interpreted, in any possibility, as a set $\mathcal{I}_v(\mathsf{N}) \subseteq D_v$, possibly empty, of individuals understood to be named N, or if you want, *be* N.[4] Putting things together:

Definition 2 (Models of $\mathcal{MPL}^{\mathcal{CRQ}}$) *A model $\mathcal{M}$ of $\mathcal{MPL}^{\mathcal{CRQ}}$ is a quintuple*
$$\langle \mathcal{W}, \mathcal{U}, \{\mathcal{C}_i\}, \{\mathcal{R}_k\}, \mathcal{I} \rangle$$
the five components of which are as described above.

The formulas of our language are evaluated in a model and relative to a possibility and relative to a variable assignment function following a Tarski-style satisfaction definition. (Tarski 1956) We here define more in particular whether $\mathcal{M}, v, g \models \phi$, i.e., whether ϕ is satisfied in model $\mathcal{M}$, relative to possibility v and variable assignment g. An assignment function g here interprets any variable x as an individual conception $g(x)$, the value $g(x)_v$ of which is an object in a world of evaluation v, if it has any value at all. Atomic formulas are satisfied if its constituent terms denote objects that have the properties ascribed, like being one and the same object, or like standing in a certain relation. If any one of the terms does not have a value in v, these formulas are assumed to be not satisfied

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3. Actually this is not much of a constraint, because, as we will see, the semantics stated below would simply ignore any such $c_v \notin D_v$. Having such $c_v \notin D_v$, that is things that do not exist, would not yield any philosophical or technical problems either, but it would distract our focus by raising questions that don't make actual sense. (Quine 1948) Note, however, that there may always be individual conceptions not defined for a world v, even in God's window $\mathcal{C}_0$. They will eventually play a role when we come to think and talk of things non-existent, but for all our real world modal reasoning they are arguably irrelevant.
4. Such a more predicativist conception of names is at least as old as (Russell 1919, p. 174-5; Kneale 1962; Burge 1973), and it has re-gained currency fairly recently. (Bach 1981; Geurts 1997; Matushansky 2008; Fara 2015). Note that it also motivates the standard *DRT*-treatment of names, in which names show up as predicative terms in the logical language, even though the benefits of such a view have, to our opinion, never been cashed out. (Kamp 1981; Kamp, Van Genabith & Reyle 2011). We have chosen, here, to equivocate the property of being *Dick Nixon* with that of being named "*Dick Nixon*", even if we should, of course, choose to distinguish between the two, when the need arises.

in v.[5] The other clauses defining the satisfaction relation may, we trust, speak for themselves.

Definition 3 (Interpretation of $\mathcal{MPL^{CRQ}}$)

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\begin{aligned}
\mathcal{M}, v, g &\models x_1 = x_2 &\quad\textit{iff}\quad& g(x_1)_v = g(x_2)_v \\
\mathcal{M}, v, g &\models Rx_1\ldots x_j &\quad\textit{iff}\quad& \langle g(x_1)_v, \ldots, \ldots g(x_j)_v \rangle \in \mathcal{I}_v(R) \\
\mathcal{M}, v, g &\models \neg\phi &\quad\textit{iff}\quad& \mathcal{M}, v, g \not\models \phi \\
\mathcal{M}, v, g &\models (\phi \wedge \psi) &\quad\textit{iff}\quad& \mathcal{M}, v, g \models \phi \textit{ and } \mathcal{M}, v, g \models \psi \\
\mathcal{M}, v, g &\models (\phi \rightarrow \psi) &\quad\textit{iff}\quad& \textit{if } \mathcal{M}, v, g \models \phi \textit{ then } \mathcal{M}, v, g \models \psi \\
\mathcal{M}, v, g &\models \mathsf{N}x\phi &\quad\textit{iff}\quad& \textit{there is } c \in \mathcal{C}_0\colon c_v \in \mathcal{I}_v(\mathsf{N}) \textit{ and } \mathcal{M}, v, g[x/c] \models \phi \\
\mathcal{M}, v, g &\models {}^i\exists x\phi &\quad\textit{iff}\quad& \textit{there is } c \in \mathcal{C}_i\colon c_v \in D_v \textit{ and } \mathcal{M}, v, g[x/c] \models \phi \\
\mathcal{M}, v, g &\models {}^i\forall x\phi &\quad\textit{iff}\quad& \textit{for all } c \in \mathcal{C}_i\colon \textit{if } c_v \in D_v \textit{ then } \mathcal{M}, v, g[x/c] \models \phi \\
\mathcal{M}, v, g &\models \Diamond^k\phi &\quad\textit{iff}\quad& \textit{there is } w\colon \mathcal{R}_k vw \textit{ and } \mathcal{M}, w, g \models \phi \\
\mathcal{M}, v, g &\models \Box^k\phi &\quad\textit{iff}\quad& \textit{for all } w\colon \textit{if } \mathcal{R}_k vw \textit{ then } \mathcal{M}, w, g \models \phi
\end{aligned}
$$

This almost concludes the presentation of the system of $\mathcal{MPL^{CRQ}}$. We finally need to agree on appropriate notions of validity and entailment. These are defined in the usual fashion, for sentences, formulas without free variables.[6]

Definition 4 (Validity in $\mathcal{MPL^{CRQ}}$) *A sequence of premises $\phi_1, \ldots, \phi_n$ entail a formula ψ, $\phi_1, \ldots, \phi_n \models \psi$ iff for all $\mathcal{M}, v$ if $\mathcal{M}, v \models \phi_1 \ldots \mathcal{M}, v \models \phi_n$ then $\mathcal{M}, v \models \psi$. We say that ψ is valid iff $\models \psi$.*

3   Applications

Before we turn to the proper and substantial use of conceptual windows in the description of our daily practices, it is expedient to first clarify our view on the use of proper names, and our method of contextually, extensionally, restricted quantification.

3.1   Proper Names

Our treatment of names is somewhat non-standard, so it may be worthwhile to spend a few words on it, and explain that it is less committing than other more familiar views, in the sense that these other views can be accommodated by explicit assumptions, if one finds the need to do so. The adoption of our minimal and modest stance is not a prerequisite for the more substantial applications that we will discuss in subsequent sections, but it does help an easier grasp of them, and avoid misunderstandings, which otherwise might be likely to arise.

As indicated above, our framework employs names in quantified, or declarative, constructions of the form $\mathsf{N}x\,\phi$, saying that some x, so-named, is ϕ. Such constructions can be understood by, or translated back into, a more 'standard'

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5. We, thus, adopt a "negative free logic", so-called, as in e.g., Burge 1974; Sainsbury 2005.
6. The appendix provides details about the proof theory.

predicate logical formula $\exists x(x = n \land \phi)$, or just $[n/x]\phi$, if n could serve as the logically proper name of the one and only individual satisfying $\mathsf{N}x\,\phi$. Our rendering is preferred because it does not commit to the existence of some unique n, when this has not been explicitly declared. Our system could of course be rendered more classical in this respect, if, for any name N, we do make the following stipulation:

$$\Box\exists x\forall y(\mathsf{N}z\,y{=}z \leftrightarrow x{=}y) \tag{1N}$$

This would amount to saying that necessarily N exists, and necessarily there is only one N. Obviously this would render our use of names more classical, but equally obviously (1N), while perhaps convenient for certain technical purposes, does not appear to be very reasonable upon reflection.

Some readers may be worried that uses of a sentence like *Ann sees Ben.* are normally not assumed to be about some or other Ann and some further unspecified Ben, but can be presupposed to be about a certain Ann and Ben that the interlocutors are familiar with. It may be interesting to note, in reply, that to voice such a worry one must already acknowledge the possible existence of various Anns and Bens in the first place, like the predicativists and we ourselves do. More interestingly, the worry can then also be suitably met by allowing such uses of names to be moderated by conceptual windows, that the interlocutors jointly assume to be looking through, and which may provide a view on only one Ann and only one Ben. Even so, notice that such an assumption need not be necessarily satisfied, of course. Many actual situations may serve to show that also such assumptions may misfire. Also in such cases, however, the language proposed here appears to be very well equipped to describe what is going then.

Further ideas about the use and understanding of names, extensively discussed in the literature, can be specified explicitly as well, more particularly those about how named individuals show up in modal contexts. One may be tempted to think of, e.g., Nixon in a non-actual possibility as being the real Nixon, with some properties stripped off. But which properties are stripped off, and which properties remain? Perhaps a property that remains is the property of being Nixon, but what property is that? It is certainly not the property of being named Nixon. Actually these questions are a matter of a lot of debate, but these constitute a metaphysical debate, rather than a logical one.

We prefer to approach this issue from an oppositie direction. Suppose we consider a non-actual possibility, in which there is an individual x who is actually Nixon. What do we, logically speaking, know about that individual x in that possibility? Surely we know that he has all the properties that someone who is actually Nixon necessarily has. But what are these properties? There are of course the trivial properties that our logic ascribes to any object, like being self-identical: $x{=}x$, and to exist: $\exists z\,x{=}z$. Our logic does not, however, by itself, provide any further, and substantial, answers to this question.

Our logic does allow us to state further *assumptions* that we may make about the properties that individuals necessarily have, so also someone x who

is actually Nixon. Perhaps we assume that he is necessarily blessed with a soul or animus, and that he is necessarily gifted with the capacity of reason. If we furthermore assume that these two predicaments constitute sufficient and necessary reasons for any object to be human, we may also conclude our x in the considered possibility to be human, by classical modal reasoning. In general, however, no further conclusions can be reached, if not ensuing from explicitly stated assumptions, and classical logical reasoning.

One might also wish to maintain the Kripkean assumption that, e.g., Nixon is necessarily Nixon. This can be made even more Kripkean requiring Nixon to necessarily be Nixon *if he exists*. (Kripke 1981, p. 48) To do so it proves convenient to first introduce two notation conventions, one abbreviating the restriction of the evaluation of modalities to possibilities in which a certain individual exists, and one abbreviating the attribution of existence itself.

Utility 1 (Existence Restrictions)

$$^{c}\mathsf{E}x := {}^{c}\exists y\, x{=}y \qquad\qquad (\mathsf{E}x)$$
$$\Box_x \phi := \Box(\mathsf{E}x \to \phi) \qquad\qquad (\Box_x)$$

A formula $\Box_x F x$, thus defined, says that in all possibilities in which it exists x is F. Thus, F can be said to be a necessary property of x, without this entailing that x necessarily exists. Using this convention, the following formula now says, in a more suitable fashion, that being Nixon is a necessary property of the man.

$$\mathsf{N}x\, \Box_x \mathsf{N}y\, x = y \qquad\qquad (\mathsf{N}_x\Box_x)$$

It is indeed convenient that we can make such an assumption explicit. Some people, not just a few, endorse it and the belief in such assumption has been so firm that it has tempted them to present it as a principle of logic. We believe it is convenient that we can make it explicit as an assumption, because, in the first place, this allows us to also not make the assumption, and because we are convinced it makes sense to indeed also not do so. After all, it is an assumption, or dogma, undeniable only for those who believe in it.

Our treatment of names is not only free from unwarranted existence assumptions, it also, obviously, handles them as scope bearing devices, which comes with specific advantages. To see this, first notice that a sentence like "*Ann sees Ben.*" will be rendered here as $\mathsf{A}x\, \mathsf{B}y\, Sxy$, saying that there is *Ann* and there is *Ben*, and that the first sees the second. Since names have scope now, a negated sentence like "*Ann does not see Pegasus.*" can now be rendered as $\mathsf{A}x\, \mathsf{P}y\, \neg Sxy$ but also, for instance, as $\mathsf{A}x\, \neg\mathsf{P}y\, Sxy$. The latter representation says that there is *Ann* and no *Pegasus* that she sees, one of the kinds of things we do want to say. We can now also assert that *Pegasus* does not exist, or that no *Pegasus* exists, simply by negating its declaration: $\neg\mathsf{P}x\, x{=}x$, or just $\neg\mathsf{P}$, for short.[7]

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7. Note that it may make kind of sense to wonder *which Pegasus* is said to not exist. There

3.2 Contextually Restricted Quantification

The current system extends more standard modal predicate logics in two significant ways. It allows us to regiment *what* we see through a particular conceptual window, and also *how* we see it. The first feature allows us to impose extensional constraints on conceptual windows so as to naturally incorporate the kind of contextually restricted quantification relatively familiar from the literature. (Westerståhl 1984; Stanley & Szabó 2000) To achieve some such, we can, for instance, stipulate that $\forall x({}^c\exists y\, x{=}y \leftrightarrow Kx)$ thereby determining all other quantifications made using window c to be restricted to the K's, the Kardashians, say. Employing the above notation convention we could simply have stated $\forall x({}^c\mathrel{\rlap{\vDash}}x \leftrightarrow Kx)$. This say more transparently that being a Kardashian is extensionally equivalent to being existent in c, or to being seen through window c. When we subsequently speak of "^{c}the blonde", suggestively rendered as "${}^c\imath x Bx$", this locution apparently denotes Khloé, the unique blond in c, which is the unique blond among the Kardashians.

To be somewhat more specific, an expression of the form "the A" or $\imath x\phi x$ is understood, upon its Russellian analysis, to denote the unique individual that is A, or that satisfies ϕ, respectively.[8] Its meaning is contextually explained so that a sentence of the form "The A B" says that there is a unique A and that it is B. Formally:

Utility 2 (Russellian Description)

$$ {}^c\imath x\phi\,\psi := {}^c\exists x({}^c\forall y([y/x]\phi \leftrightarrow x{=}y) \wedge \psi) \qquad (R) $$

For most uses of definite descriptions in natural language, such a Russellian analysis has often been observed to only make sense if they are evaluated relative to contextually supplied domains of quantification, or so that in particular the universal quantifier in (R) is suitably restricted. (Neale 1990) Our windows are able to model precisely that. Thus, if it is said that ^{c}*the blonde smiles*, this can be rendered as ${}^c\imath x Bx\, Sx$ which upon its Russellian expansion comes out as ${}^c\exists x({}^c\forall y(By \leftrightarrow x{=}y) \wedge Sx)$. If, as is required by this formula, there is indeed such a unique blond Kardashian in c, Khloé, the description can be understood as a term that denotes that individual. More generally any form of quantification can thus be rendered extensionally contextual, by (i) relating the quantifier to some conceptual window c and (iii) before or after that defining to_be_seen_in_c to be equivalent with satisfying whatever contextual requirement one may want to impose on it.

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does exist an air company so-named, a gay-magazine, and there is also this "mythological creature" so-named, a horse with wings. (Wikipedia Contributors 2022) We may want to say only of the latter that it does not exist. In the last section we indicate how we may make sense of also this type of talk of things non-existent, without any commitment to their existence.

8. Existence and uniqueness of such an individual are thereby asserted, according to Russell, or presupposed, according to Strawson, both among many others. (Russell 1905; Strawson 1950) There is no need to enter the enduring and rather convoluted debate about this here.

As long as there are no constraints on the windows that we have, the fact that some window r provides a view on an individual with property F, does not imply that that individual is seen through another window s as well. Typically, $^r\exists x\phi \not\models {}^s\exists x\phi$. But equally typically, within one and the same window all the usual laws of logic hold, of course. For instance: $^r\exists xFx, {}^r\forall y(Fy \to Gy) \models {}^r\exists zGz$. If there is a cat in a window r, and if all cats in that window r meow, then that cat in the window meows. In the appendix we provide the natural deduction principles that govern our logic of contextually restricted quantification.

Any facts about individuals seen through a window are of course plain facts about these individuals, because all individuals seen are individuals that exist. The following axiom holds by the way our semantics is defined:

$$\Box^c\forall x \Eltx x \tag{E}$$

This implies that our windows are *transparent*. If a window c provides a view on a cat ($^c\exists xCx$), then there just is a cat ($\exists yCy$) and if it also so happens that all cats meow ($\forall z(Cz \to Mz)$), then also this cat, in c, meows ($^c\exists x(Cx \wedge Mx)$). So, everything that is said to be seen through a window is guaranteed to exist. We can of course also stipulate, for some particular window c, that everything that there is is seen through it.

$$\Box\forall x\, {}^c\Eltx x \tag{U}$$

Doing so would render the window extensionally vacuous, but not intensionally. The window would be rendered vacuous extensionally speaking because for all non-intensional formulas ϕ we would find $^c\exists x\phi$ to be equivalent with $\exists x\phi$. It would not be intensionally vacuous, however, because for modal formulas ϕ, $^c\exists x\phi$ may still depend on *the way in which* the individuals are seen through c, which can be relevantly different from the way in which they are actually given, through the default window 0. Like we said, the use of windows not only allows us to regiment *what* we see through a particular conceptual window, but also *how* we see it, and it is this application that we turn to in the next paragraphs.

3.3 Regimenting Conceptual Covers

Maria Aloni has convincingly argued that it is the ways in which individuals are conceived of that proves relevant in the evaluation of modal and other intensional discourse. (Aloni 2005b) We may have a perspective under which we see an individual, and believe of her, the real individual, that she is a full professor, and another perspective under which we believe, without any inconsistency, the very same individual to be an unemployed musician. Aloni's conceptual covers provide ways of conceiving of individuals that may serve to characterize these beliefs, and our conceptual windows do likewise. The latter, however, also allow us to explicitly regiment *how* we actually see things, and they to do so in a much more flexible way; a way that allows us to properly generalize, and improve upon,

the way in which this is done through conceptual covers. To understand why, and how, it is instructive to first see how we can generically remodel, in our system, these uses of conceptual covers—and actually all of them.

Conceptual covers are actually a very special type of conceptual windows, and we can also specify explicitly when a conceptual window c may figure as one. A set of individual conceptions C is a conceptual cover according to (Aloni 2005b) if, and only if, every single thing in every possibility is seen by a single concept in C. We can make a conceptual window c behave this way by means of two postulates. The first is the postulate (U) above that requires the window to be universal, so that it provides a view on everything. The second postulate stipulates that the window allows us to see the individuals distinctly.[9]

$$\Box^c \forall x^c \forall y(x{=}y \to \Box x{=}y) \ \ and \ \ \Box^c \forall x^c \forall y(x{\neq}y \to \Box x{\neq}y) \qquad (D)$$

This second postulate should be taken to say that there are no two conceptions in c of one and the same object, and that any two distinct objects seen through c are necessarily distinct, as long as they are viewed through c. These two constraints serve to ensure that the window c hosts exactly one conception of any individual in the domain. If we adopt the postulates (U) and (D) for all conceptual windows, then they all effectively behave like conceptual covers.[10]

This paper could end here, if we were happy with assuming (U) and (D), because then we could stipulate these principles and directly copy all of Aloni's results into the current framework. However, we claim there are good reasons not to be happy with assuming (U) and (D), without any qualification. We will show, first, that they are not needed to achieve the same results any way, and, second, that we can achieve the same results in a more accurate fashion.

To begin with, we do not need to, and we do not want to, impose the requirement (U) that windows allow us to see all individuals. Conceptually, philosophically as well as empirically, the requirement appears to be quite unwarranted. Aloni's conceptual covers are thought of as perspectives on a given and established domain of all possible individuals in a model. If they serve to model the attribution of propositional attitudes, like beliefs and desires, something which they apparently are used for, then these domains must comprise everything that anyone

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9. I use $\neq$ here to indicate the non-identity of the realization of two conceptions. Recall that $x{=}y$ is false also when at least one of x and y does not exist. With $\neq$ we want to exclude that possibility, so that a formula $x{\neq}y$ is, thus, actually short for $\mathsf{E}x \wedge \mathsf{E}y \wedge \neg x{=}y$.

10. Some calculation reveals that if a window c satisfies (U) and (D) then all (accessible) possibilities have the same number of individuals, and $\mathcal{C}_c$ moreover provides unique individuations of all these individuals in every (accessible) possibility. The postulates ensure that the conceptual windows in any satisfying model are conceptual covers, according to the definition of Aloni, if we can assume the accessibility relation R is indeed universal. If the relation is not universal, but still an equivalence relation, as required, our windows figure as Aloni's covers relative to each one of the blocks in the induced partitions of $\mathcal{W}$. Established modal logical wisdom has it that this only apparent restriction does not in any way constrain the results obtained, because one cannot ever look into portions of a modal universe beyond what the equivalence relation relates us to.

can believe or desire to be there, and every cover must come with a conception of any such thing, before the interpretation of any quantified or existential statement can be given. We don't think there is any ground for such an assumption. On the contrary, we think we can make sense of the quantifiers without any such determinate assumptions about the domain of quantification. As we will see in due course, if it is only in order to remodel any of Aloni's applications, there is certainly no need to do so.

Notice, furthermore, that (D) indirectly implies that every actually existing thing, as seen through a window, *necessarily* exists. We, however, don't think that necessary existence belongs to the ordinary conception of things, neither does it belong to the conception of seeing things through a window. We do take it to be a welcome property of windows that the things we see through it actually exist and that they are relatively distinct, that is distinct as conceived through that window. A, suitable and harmless, relativization of (D) therefore renders the identity of things conditional upon their existence.

$$\Box^c \forall x^c \forall y(x{=}y \rightarrow \Box_x\, x{=}y) \ \ \text{and} \ \ \Box^c \forall x^c \forall y(x{\neq}y \rightarrow \Box_x x{\neq}y) \qquad (D_x)$$

A window c satisfying (D_x) allows us to see all things seen through it distinctly, without rendering them necessarily existent. Assuming (C) as well as (D_x), then, makes that a window c allows us to see certain things, "clearly and distinctly" — to borrow a phrase from Descartes —, but not necessarily all things, and in all circumstances.

As the reader may be pleased to verify, assuming (D_x) for window c furthermore implies that in any satisfying model and possibility v, the individual conceptions in window $\mathcal{C}_c$ that are realized in v all have one and the same subset S of accessible possibilities in which they are realized.[11] These are the possibilities that the individuals, thus conceived, so to speak 'live in'. They all do so. Notice that this does not prohibit the identification of these individuals with real individuals conceived from a different window, and living in yet different possibilities. We can for instance conceive of one actual individual, Ortcutt, as a man seen on the beach, and conceived this way, through this beach window, say, he is necessarily a man on the beach and necessarily distinct from all other man seen on the beach through that window. But Ortcutt, and all the others, can also be conceived from other windows, and under those other conceptions they may have none of the attributes necessary for seeing them through the beach window. We see Ortcutt, the man on the beach, or the man in various other guises, but for this we do not need to see a *Ding an Sich*.

Actually this brings us to showing how we can actually remodel Aloni's results without assuming the troublesome universal assumption (U), and with our restricted version (D_x) of relativized distinctness.

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11. The realization of one individual conception x in c in an accessible possibility w drags along with it the realization of other conceptions in c; and this dragging along is mutual.

3.4   The Ortcutts of Quine's Ralph

The first example that we want to discuss consists in the famous case of a character Ralph, who has been reported to hold conflicting beliefs about some individual Ortcutt. (Quine 1956) First it is observed that Ralph, relative to a certain situation, believes of Ortcutt that he is a spy. He has observed Ortcutt under certain suspicious circumstances, with a brown hat on his head, and such that he was led to conclude that the person observed had to be a spy. Independently he has been acquainted with the same man, as a man he met on the beach yesterday, and so that he came to believe that that man is surely not a spy. Since it concerns one and the same person, Ortcutt, both beliefs cannot be correct, of course, but we cannot blame Ralph for not knowing this impossibility. He simply fails to know that the person witnessed on the one occasion is the same person as the person seen on the other.

Let us inspect and analyze one way in which Ralph's state of information may be described. First it can be said that Ralph believes of Ortcutt that he is a spy, fairly transparently rendered through the following formula.[12]

$$^{s}\bigcirc x \, \Box^{r}(\mathbb{E}x \land Sx) \qquad\qquad (^{s}\Box^{r})$$

This formula says that Ortcutt is seen through some window s and Ralph believes that Ortcutt, thus conceived, is a spy.[13] This does not, by itself, tell us very much yet.[14] It says that window s provides a view on Ortcutt so that we can portray Ralph as seeing the world as one in which an individual seen that way is a spy. Notice that we are not, here, referring to some individual conception that Ralph has of the man, some private mental representation, and we think quite rightly so. How would we have come to know anything about that? The most appealing fact about the individual conceptions in our windows is that they are thought of as public modes of presentation of individuals, that we assume we can share. It is this, assumed, shared and public character that underscores our ability to embark on the way Ortcutt is seen. We want to emphasize the significance of this way of looking at things, because this is exactly how Ralph's case has been put forward in the logico-linguistic literature. When Quine tells us about how Ralph conceives Ortcutt, Quine presents Ortcutt in a way in which we imagine we can in principle also conceive him.[15]

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12. In this formula a convenient contextually restricted naming device has been employed, $^{s}\bigcirc x \, \phi$, which is short for $^{s}\exists x(\bigcirc x \land \phi)$, where $\bigcirc x$ in turn abbreviates $\bigcirc z \, x{=}z$, that x is Ortcutt.
13. Upon a more adequate understanding intentional attitudes like beliefs are essentially indexical in the sense of (Perry 1979; Lewis 1979). Conceptual windows should actually be understood to be essentially indexical, too. These windows are not, however, essentially *de se*—or *de nobis*—, like our intentional attitudes are, but they are, essentially, *de te*. Their contents do not emanate from an indubitable Cartesian or Fregean thinking of theirs, but necessarily rely on a Peircean viewing of them. The contents of the window are centered not on the very window, but on its viewer—so on you, or on me, so long as we actually look through them. We will return to this issue in section 4.3.
14. Recall the points made about Nixon above.
15. It may be interesting to note that Ortcutt, as well as Ralph, have most probably sprung

As indicated, Quine has, indeed, subsequently explained the situation somewhat further, explaining that Ortcutt is here conceived of as he was on the first occasion reported. It is assumed we have Ortcutt, seen through a window as the individual that is a man with a brown hat, observed under suspicious circumstances, etc. If we abbreviate these properties by means of the predicate BH, then all this can be made explicit as follows.

$$^s\bigcirc x \,\square_x\, {}^s\imath y\, BHy\, x{=}y \qquad\qquad ({}^s\square_x)$$

Informally, this says that we see Ortcutt ($\bigcirc$) through s with the identifying property that, in s, he is the man with the brown hat, etc. The individual must have the property of being the man with the brown hat, because that is how he is conceived of here. This does not, of course, say that Ortcutt himself necessarily has this property. Neither is the man with the brown hat in s necessarily Ortcutt. Being the man with the brown hat is just the way in a certain individual is identified in s. In the actual world, this man so identified happens to be Ortcutt, but this is not part of the definition of the man seen.

Window s is supposed to present an actual situation to us, as well as to Ralph, a situation as it has been reported to us by Quine. Interestingly, Quine's $({}^s\square^r)$ and $({}^s\square_x)$ independently but jointly entail that Ralph believes that the man with the brown hat is spy. To see this, we must first make explicit the silent assumption that Quine's reports relate of a unique Ortcutt in s. Formally:

$$^s\imath y(\bigcirc x\, y{=}x)(\mathsf{E}y) \qquad\qquad ({}^s1\bigcirc)$$

In s, there is a unique individual named Ortcutt. We can write this more conveniently as $^s\imath y \bigcirc y\, \mathsf{E}y$. Then we can establish that:

$$({}^s\square_x), ({}^s1\bigcirc), ({}^s\square^r) \quad \models \quad \square^r\, {}^s\imath x\, BHx\, Sx$$

The conclusion from the three premises is that Ralph believes that the man so identified through s is a spy.[16] As has already been indicated, we do not assume that we can see in s that it is Ortcutt we see there. That is something that we typically can fail to see, even though it is something that we —you, me, and Quine— are all assumed to know. Significantly, however, Ralph does not know this. For of course anyone can be able to see some individual Ortcutt, while not seeing that it is Ortcutt.

The remainder of Quine's story can be re-presented analogously. Ralph is also reported to believe of Ortcutt that he is not a spy $({}^t\square^r)$, where Ortcutt is seen from a different window t. This view is explained further by means of $({}^t\square_x)$, which provides information on how Ortcutt is conceived of there.

$$^t\bigcirc x\, \square^r(\mathsf{E}x \wedge \neg Sx) \qquad\qquad ({}^t\square^r)$$

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from Quine's mind. Yet we all participate in the pretense of conceiving of them as real. We will not dwell upon this point here, but it is relevant for the last paragraph of the final section.
16. This inference can be seen to be valid, perhaps more easily, using the proof-theoretic principles from the appendix. As is usual, the proof is tedious, but by no means difficult.

$$^t\!\bigcirc x \,\square_x\, {}^t\!\imath y\, MBy\, x{=}y \qquad\qquad (^t\!\square_x)$$

The predicate MB here is assumed to abbreviate the relevant descriptive information by means of which Ortcutt, according to Quine, is identified as the man seen on the beach. This may include the fact it is Ortcutt, but this does not need to be so. We assume, in addition, that the two situations feature one and the same Ortcutt:

$$^s\!\imath y \,(\bigcirc x\, x{=}y)\, (^t\!\imath z\, (\bigcirc x\, x{=}z)\, (y{=}z)) \qquad\qquad (^{st}1\bigcirc)$$

Informally, $(^{st}1\bigcirc)$ says that the y which is the one and only Ortcutt seen in s is identical to the z which is the one and only Ortcutt seen in t.[17] The three assumptions jointly, but independently, entail that Ralph believes that the man on the beach is not a spy.

$$(^t\!\square_x), (^{st}1\bigcirc), (^t\!\square^r) \quad\models\quad \square^r\, {}^t\!\imath x\, MBx\, \neg Sx$$

Needless to say, we hope, that the witnessed two entailments do by no means imply any inconsistency on our part, or on that of Ralph. Neither does it require us to make any assumptions about how things look like in Ralph's head. For while we ourselves do employ the concept of individual conceptions, these have here been presented as public ways in which, we assume, individuals are presented to us as real. Formally this is as a matter of fact also the way in which individual conceptions have been employed in the work of Aloni, and, arguably Hintikka.

3.5   The Londons of Kripke's Pierre

The present framework can likewise deal with the case of Pierre, who, in a somewhat analogous fashion, is said to hold conflicting beliefs about London. The first half of the story goes like this. Pierre's behavior, verbal and otherwise, indicates that the city advertised in certain magazines and travel brochures that he read in Paris, has a great attraction on him. The city is London, and it is reported therefore that he believes that London is pretty. We here assume that we —Kripke, his readers, and Pierre— agree that we can look at the world in a certain way in which we see London the way in which it is presented through the magazines and brochures. Such a conceptual window (r) supplies a view on London as a city which is attractive to visit, and also so that we can truly report that:

$$^r\!\llcorner x \,\square^p\, \llcorner u(x{=}u \wedge Pu) \qquad\qquad (^r\!\square_p)$$

There is a London in r, and Pierre believes it is a London that is pretty. We indeed even assume that Pierre can be said to know the city is named $\llcorner$ because *our* Pierre, unlike Kripke's, has been told that "*London*" is the English version of "*Londres*". Looking at London this way people normally, but not inflexibly, accept something like $(^r\!\square_p)$ as giving a true report of Pierre's state, and this

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17. We can more transparently render $(^{st}1\bigcirc)$ by means of $^s\!\imath y\, \bigcirc y = {}^t\!\imath z\, \bigcirc z$.

judgement can be supported further by learning more about the, public, way in which London is presented here.

$$^{r}\llcorner x \,\Box_x(^{r}\imath u\, \phi\, x{=}u \wedge {}^{r}\imath u \llcorner u\, x{=}u) \tag{$^{r}\Box_x$}$$

Assuming distinctness (D_x), this assumption says that it is, from the perspective of window r, a defining characteristic of London to have the property of being ϕ—where ϕ may summarizes the traits defining London, possibly in relation to Pierre, and according to the brochures.[18] The first report and this additional assumption entail, in the current framework, that Pierre believes, is assumed to, e.g., assent to, some London with these characteristics to be pretty.[19]

The second half of Pierre's story involves a window on London the way in which he experienced it when he lived there under miserable circumstances, a view from a window s in which London appears as a city not really nice to live in. Pointing out of his window, there, in London, Pierre might for instance sincerely assent to the claim "*That city, London, is not pretty.*" Again we assume that we —all of us again– agree that there is this public way of identifying a city named London, and that that city is believed not to be pretty.

$$^{s}\llcorner x\, \Box^{p}\llcorner v(x{=}v \wedge \neg Pv) \tag{$^{s}\Box_p$}$$

As above, we can further expand upon the way London is conceived here. From this window s the city that we know to be London is identified by certain characteristics assumed to be abbreviated by the formula ψ.

$$^{s}\llcorner x\, \Box_x(^{s}\imath v\, \psi\, x{=}v \wedge {}^{s}\imath v \llcorner v\, x{=}v) \tag{$^{s}\Box_x$}$$

It follows that Pierre believes the city with these characteristics, the London in s, to be not pretty. Of course we assume, or know, that both views of London through the windows r and s involve conceptions of one and the same real city.[20] However, there is no inconsistency in this. Pierre might perhaps face an inconsistency, if he were to realize that the two windows provide a view on one and the same London. For all he knows, however, he ought to conclude that the two Londons are distinct.[21]

$$\Box^{p}\, {}^{r}\imath x \llcorner x \neq {}^{s}\imath y \llcorner y \tag{$\Box^{p}2\llcorner$}$$

Kripke has, somewhat obsessively perhaps, asked his readers: "Does Pierre, or does he not, believe that London is pretty? I know of no answer to *this* question

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18. This is by no means to say that London itself necessarily has these characteristics, nor that there necessarily should be some such thing. This is different in Aloni's system, a point that we return to below.
19. As $\Box\phi$ generally implies $\Box^k\phi$, $(^{r}\Box_x)$ and $(^{r}\Box_p)$ jointly imply that $^{r}\llcorner x\Box^{p}\llcorner y(x{=}y \wedge Py \wedge \phi)$.
20. We must therefore also live with the fact that the property of being a city attractive to visit (ϕ) must be consistent with the property of being a city not nice to live in (ψ)—-or we should have to give up one of these characteristics as identifying the real London.
21. I here employ again the transparent notation from footnote 17.

that seems satisfactory. (...)" (Kripke 1979, p. 259) There is, however, a simple and proper response to Kripke's question, totally along the lines of (Aloni 2005b, p. 35). It reads: "That depends!" One cannot unconditionally answer the question, since the question is underspecified when it has not been determined which view on London we assume we share with Pierre. If we consider London as it is seen through window r, the answer would have to be "*Yes, he believes London is pretty.*" If we consider London through s it would have to be "*No, he believes London is not pretty.*" On our own more generic view on London, the question most probably has to be that he fails to have any pertinent beliefs about it. The answer must vary with the conception that we prefer to adopt.

Kripke has expanded his question above further with the following somewhat puzzling remark.

> *(...) It is no answer to protest that, in some* other *terminology, one can state 'all the relevant facts.'* (Kripke 1979, p. 259)

The remark is puzzling, because what else than yet any other description could one long for? As a logician Kripke must agree with us that if there is any ambiguity or indeterminacy in his own question, then such an indeterminacy should have to be resolved, before any unambiguous answer can be supplied. When all the relevant facts are specified, they must include the perspective that we adopt, and an answer can be given. And, actually, this is something that we have done in this paper, in the way Aloni has done it before, providing a formal language in which the question, under whatever specification, does get a proper reply, and which can be motivated both model-theoretically, as well as proof-theoretically.

Maria Aloni aptly observed that Kripke envisaged so-called *de dicto* readings of the relevant attributions of a belief to Pierre. (Aloni 2005b, fn. 27) Such readings would imply an inconsistency on behalf of Pierre, it has been said, but only if it is assumed that one and the same name can only figure as a name of one and the same thing. This assumption, standard in classical first order logics, is however one we have chosen not to make. The case of Pierre in fact shows that it is actually advantageous to give up this assumption, from both a proof- as well as a model-theoretical perspective. We can, thus, make good sense, too, of the idea that *the two Londons of Pierre are in reality indeed one and the same city,* without thereby having to postulate any realms of non-actual objects. For it can throughout be assumed to be a plain fact that there is only one London in the whole wide world. Allowing ourselves the transparent notation from footnote 17, we could simply have it that:

$$^r\imath u \lfloor u \;=\; ^s\imath v \lfloor v \;=\; \imath x \lfloor x \tag{1$\lfloor$}$$

The Londons in r and s actually are identical to the one and only real London.

3.6 *Knowing Which* Constructions

The next application will, for certain reasons that will become clear below, be slightly more detailed. On various occasions Maria Aloni has discussed the

following scenario.

> *In front of you lie two face-down cards. One is the Ace of Hearts, the other is the Ace of Spades, but you don't know which is which. You are playing the following game. You have to choose one card: if you choose the Ace of Spades, you win (...), if you choose the Ace of Hearts, you lose (...).* (Aloni 2008, p. 191)

Aloni observes that the player in the scenario (i.e., *you*) can, on the one hand, be said to know which card is the winning card. You are assumed to know it is the ace of spades because you have learned from the above description that that is the winning card. On the other hand, there is also good reason to say you do not know which card is the winning card. For, obviously, you do not know whether it is the card on the left, say, or the card on the right. If you would know, the game would not be very 'exciting'. Is this a puzzle about belief? (Kripke 1979) Aloni analyses the issue and concludes it is not. We can only establish whether or not you know which card is the winning card once we agree what is the relevant perspective to look at things. If the question serves to check whether you know the setting of the game, then it is important that you know that you have to pick the right one among hearts and spades. But if the question is what you have to do, whether you know which card to turn, you indeed don't know the answer to the relevant question, which is whether the winning card is the one on the left or on the right.

Aloni has moreover provided some necessary ingredients to formally characterize the cards case, which we will now rephrase in the terms of our own system. First it is established that we focus on two cards.[22]

$$\exists!2x(Cx \wedge {}^c\!\models\!x)$$

It is precisely two cards that we are concerned with in c, i.e., that we 'see' through c. Next, three pairs of conceptions of the two cards are made available, through windows r, s and t, respectively. We do so by stipulating, for any window w that is either r, s, or t:

$$\forall x(({}^c\!\models\!x \wedge Cx) \leftrightarrow {}^w\!\models\!x)$$

So for each of these three windows we find that to be seen through it is equivalent to being one of these two cards in c. As above, we assume distinctness (D_x) of these windows, so this implies that all three windows host exactly two conceptions of the two cards in c. We furthermore make the following assumptions about the three windows. Window r shows the basic situation in which one of these two cards is put forward as winning and the other as losing. Window s shows one of the cards to be the ace of spades, while the other appears to be the ace of hearts. Window t presents the two cards on the table, one on the left, and one on the right. We can formally represent this information as follows.

$${}^r\exists x \Box_x W^i x \wedge {}^r\exists x \Box_x L^o x \quad \text{and} \quad {}^s\exists y \Box_y Sy \wedge {}^s\exists y \Box_y Hy \quad \text{and} \quad {}^t\exists z \Box_z L^e z \wedge {}^t\exists z \Box_z R^i z$$

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22. Pretty signficantly, unlike Aloni, we have no urge to assume that the whole universe consists of only these two cards. As usual, $\exists!nx\phi$ says that there are precisely n x so that ϕ.

Each pair of predicates is assumed to be a pair of contraries, so the unary predicates in the formulas above can as well be assumed to be uniquely denoting, so that one could read, e.g., $W^i x$ as saying that x is the winning one. The following, perhaps slightly misleading, picture may be helpful.[23]

$$\{☺, ☹\}_r \quad \{♥, ♠\}_s \quad \{⬅, ➡\}_t$$

We finally have a player whose cognitive situation is characterized using $\square^a$, as follows. This agent a, let us call her *Anne*, is assumed to be familiar with the three windows. This is secured by stipulating for any window w that is either r, s or t that:

$$^w\forall x \square^{a\,w} \mathsf{E} x$$

Anne is more in particular also supposed to know that the three windows provide a view on one the same set of cards. That is to say, for any two windows v and w which each can be either r, s, or t:

$$\square^{a\,v}\forall x^w \mathsf{E} x$$

Our agent Anne was also assumed to know the basic situation, that the winning card is the ace of spades. (I here employ the picture of a concept as an abbreviation of the definite description expressing it.)

$$\square^a(☺ = ♠)$$

However she does not know whether the winning card is the card on the left, or on the right. According to her, both possibilities are not ruled out.

$$\Diamond^a(☺ = ⬅) \quad and \quad \Diamond^a(☺ = ➡)$$

Let us return to the original issue. Consider the following sentence, and the question whether it is true or not.

<div align="center"><i>Anne knows which is the winning card.</i>   (A)</div>

Aloni here adopts and elaborates Groenendijk and Stokhof's sophisticated treatment of interrogative complement constructions. (Groenendijk & Stokhof 1984, we will return to their analysis in the next paragraph.) According to their analysis a statement like (A) can be rendered formally as follows.

$$^w\forall x(☺ = x \to \square^a ☺ = x) \wedge {}^w\forall x(☺ \neq x \to \square^a ☺ \neq x) \qquad (\square^a ☺)$$

This says that if something is the winning card, Anne knows it is, and if it is not, Anne knows it is not. But this of course depends on the way in which the cards

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23. The picture may be slightly misleading, because, formally, there can be other conceptions in a window that, however, are no conceptions of things actually existing. Since we here actually only deal with things that do actually exist, this simplification is harmless.

are conceived of. In our case, as well as in that of Aloni, a suitable choice of the cover or window w settles the question whether $(\square^a ☺)$ is true or not. Since, obviously and necessarily $☺ = ☺$ and $☺ \neq ☹$, it is easily established that $(\square^a ☺)$ is true if we look at the issue through window $w = r$. This is trivial, of course, because what Anne would then be said to know would be that the winning card is the winning card, and everybody who knows there is a winning card of course knows that the winning card is the winning card. Formula $(\square^a ☺)$ is, however, not redundant if we look at the question through the other windows.

Consider window s. Since we have that $(☺ = ♠)$ and $\square^a(☺ = ♠)$, and also $(☺ \neq ♥)$, and since $\square^a(☺ = ♠)$ implies $\square^a(☺ \neq ♥)$, it must be obvious that the sentence is true when the window w is s. This is correct, since, as we have seen, the fact that if one knows that the ace of spades is winning, one can be said to know ^{s}which is the winning card. When we look at $(\square^a ☺)$ through window t, however, the issue is not whether or not spades or hearts is winning, but it is which card to choose, the one on the left or the one on the right. Now we find that while, actually $(☺ = ⬅)$, as it so happens, it is not the case that $\square^a(☺ = ⬅)$, and we likewise find that $(☺ \neq ➡)$ while not $\square(☺ \neq ➡)$. It must be obvious, therefore, that $(\square^a ☺)$ is false, so conceived. Again, this is as required. Our player, Anne, is supposed not to know whether the left or the right card is winning, so she does not know ^{t}which is the winning card.

We have engaged in this detailed exposition in order to show two things in particular. The first is that each and every aspect of Aloni's analysis can, like we said, be reproduced in our own terminology, using conceptual windows, rather than conceptual covers. For the interested reader it also serves to point out that we have been able to formulate all the required details of the analysis through assumptions formulated in our formal language itself, and not in the meta-language that Aloni needs to resort to so as to characterize and define her covers. The second thing that the exposition must be meant to show is, more importantly, this. All the things that we said and stipulated and that were required to hold about what is true about things in our windows, do not entail anything about the things seen beyond the scope of these windows. Significantly, the assumptions that we necessarily made do not need to hold throughout the whole space of possibilities. Designated subsections of our models do need to supply us with sets of possibilities that satisfy the relevant agent's beliefs, and that also satisfy the individual conceptions that we need to employ. It need, however, never be assumed that there are no possibilities that do not comply with these assumptions of ours. Such freedom, which is most desirable, is excluded from the conceptually covered models of Aloni. In her models there can be no worlds in which there is no winning card, because otherwise a cover containing the conception of a winning card would simply be unavailable. As a consequence, every agent in an Aloni model knows that there is winning card, and a card of spades, etc. Such aberrations are obviously absent from our treatment using conceptual windows.

3.7 Knowing *Who is Who*

In the seminal work of Jeroen Groenendijk and Martin Stokhof, to be said to know *who*, *what*, or *whether* ϕ, is to be said to know the full and true true propositional answer to the embedded question. (Groenendijk & Stokhof 1984) One can be said to know whether ϕ if the value of ϕ in one's doxastic alternatives equals that of ϕ in reality. One can also be said to knows *who F*, if ones beliefs settle the question who actually are F, and one likewise can be said to know *who Rs who* if ones beliefs correctly settle the extension of the relation R. Some such has been taken to require that the extension of F, or R, in one's doxastic alternatives corresponds to, or even equals, the extension of F in reality.[24]

Groenendijk and Stokhof's treatment has been challenged by some philosophical and linguistic puzzles, that relate to the issue of how individuals actually existing in reality can be equated with, and dissociated from, things in possibilities that are merely conceived. It has appeared that such problems cannot properly be solved by merely *stipulating* cross-possibility identities, not even though this, in the footsteps of (Kripke 1981), seems to have become relatively standard practice in much philosophical-logical work. Actually, such Kripkean stipulations generate puzzles of their own, for how can one not know that a equals b, if a actually equals b, and therefore necessarily so? In more linguistically oriented work such stipulations have therefore been given up.

Inspired by logico-linguistic observations from, among many others, (Sosa 1970; Fraassen 1979; Bonomi 1995), and building on the foundational work in quantified modal logic of (Hintikka 1969) and (Lewis 1968), Maria Aloni has rather solidly established that such cross-possibility identities should be evaluated relative to particular methods of individuation, which she takes to be her conceptual covers. (Aloni 2005a) According to this approach, to know who F is understood to so to speak be able to establish the extension of F relative to a particular conceptualization of the domain. If we translate this into our own terminology: to establish the extension of F through a particular conceptual window. An agent can be said to know who are F if she is principle able to provide a correct and complete reply to our question which individuals are F, when conceived through some window, which is commonly accessible to us.

Questions of the form *Who is Who?* are particularly interesting, because if it queries the identity of objects twice seen through one and the same conceptual window, the question must obviously be trivial. In that case a true and complete answer to the question would be *Everything is identical to itself, and to nothing else*, and that is trivial, because it actually restates the distinctness assumption (D_x) above. Once we have different windows on the same domain, however, the question is by no means trivial. Let us digress and discuss one example, also handled in (Dekker 2012).

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24. In a two-dimensional modal framework, in which $\ddagger\phi$, under whatever modal operator, denotes what ϕ denotes in the actual world, this analysis of these constructions is perspicuously characterized by means of $\Box\,?\phi := \Box\,(\ddagger\phi = \phi)$.

Suppose we focus on the 11 players in the team of FC Barcelona in 2011. Employing the predicate B for the property of being one of them, our extensional focus is given by some window c.

$$\forall x(Bx \leftrightarrow {}^c\mathsf{E}x)$$

The individual members of the team can be identified in various ways. We may have a copy of a team shot taken just before the match. This constitutes one window, r, through which we can see all of the eleven players. There is also a list of names of the players, which may figure as window s. And then there are the squad numbers by means of which all the players can be identified uniquely, too, and this provides window t on the team. We may safely assume that we know that these are three windows on the 11 players, so that, for w equal to r or s or t, we have:

$$\forall x({}^c\mathsf{E}x \leftrightarrow {}^w\mathsf{E}x)$$

In a gloss, *if something is one of the 11 players player of Barcelona, we see it through w, and if we see something through w, it is one of the 11 players of Barcelona.* Since we have our distinctness postulate (D_x), and since any particular kind of possibility is a possibility, identity questions about individuals seen through one and the same window turn out trivial. For if an agent a has any access to the windows at all, i.e., if ${}^w\forall x \Box^a {}^w\mathsf{E}x$, for any w that is r or s or t, then the following formula turns out trivially true, for any such w as well.

$${}^w\forall x {}^w\forall y(x{=}y \to \Box^a x{=}y) \quad and \quad {}^w\forall x {}^w\forall y(x{\neq}y \to \Box^a x{\neq}y)$$

This implies that by seeing through, e.g., window t, agent a knows that the person with squad number n is the person with squad number n and that persons with squad numbers n and m, with n and m distinct, are different persons.

Besides this, there are more substantial things that one may be said to know, or not to know. When visiting the match, agent a may have been supplied with a leaflet informing her which squad numbers the individual players have. With, not without, this information agent a can be said to know ^{s}who is ^{t}who.

$${}^s\forall x {}^t\forall y(x{=}y \to \Box^a x{=}y) \quad and \quad {}^s\forall x {}^t\forall y(x{\neq}y \to \Box^a x{\neq}y)$$

Another leaflet may provide the names with the faces. This should serve to inform our agent ^{r}who is ^{s}who.

$${}^r\forall x {}^s\forall y(x{=}y \to \Box^a x{=}y) \quad and \quad {}^r\forall x {}^s\forall y(x{\neq}y \to \Box^a x{\neq}y)$$

If one fails this type of information, however, then it is possible that one can know that Messi scored, without knowing that it was Messi who scored. One may have witnessed that the player with squad number 10 made a goal, and fail to know that that is Messi. So it can be the case that:

$${}^t\mathsf{M}x \, \Box^a Sx \quad and \quad {}^s\mathsf{M}x \, \neg\Box^a Sx \quad and \ even \quad \neg{}^s\exists x \Box^a Sx$$

However, once one knows the numbers with the names, this possibility is ruled out. If one knows that Messi is the player with squad number 10, then one knows that #10 scored iff one knows that Messi scored.

These examples may serve to show, once more, how the knowledge reported in such *who-is-who*-constructions can be adequately rendered in our system, using the techniques employed by Maria Aloni, but without the untenable demand that there is nothing in the world besides these eleven players, or the further assumption that we know how to identify each one of the individual players among all individuals in all conceivable possibilities. The latter demand would, in turn, imply that the players necessarily exist, and the properties by means of which they are identified are necessarily instantiated, and everybody is deemed to know that. The use of conceptual windows, in stead of conceptual covers, has no such implications. These same kind of properties, which are left implicit in the last example, are necessary to identify the individual players in the windows, and beyond the scope of the relevant windows those properties may be left uninstantiated. They are properties that you, agent a, and we ourselves, take the objects to have, but nothing about this is assumed to be necessary and universally known.

4   Discussion

4.1   Conceptual Windows and Covers

It may seem that the present approach to *de re* attitude ascriptions, like that of Aloni, belongs to what may be conceived of as a family of conceptualist, or representationalist approaches. (Edelberg 1992; Kaplan 1968; Kamp 2015; Maier 2009; and, arguably the Concept Generator approach discussed in section 4.2.) While this might be justified to a small extent in so far as Aloni sometimes calls concepts "representations", nothing would be less appropriate than to so characterize any one of the two. Without any intention to deny their diversity, conceptualist or representationalist approaches have in common that they crucially relate to concepts and representations of the particular agents whose attitudes are described. Most explicitly this is done in the seminal (Kaplan 1968). In Kaplan's analysis the truth of *de re* attitude reports is rendered dependent on properties of the concepts or representations that an agent has. They must, e.g., be *vivid* representations and stand in a certain kind of causal relation to the object represented. (Kaplan 1968, p. 201–3)

However, as Aloni has extensively argued, the question whether or not a specific conception of an individual can be deemed appropriate to license a *de re* attitude ascription, depends on features of the discourse context in which the attitude ascription is made, on the purposes of the discourse, say, and the interests and perspectives of the interlocutors. (Aloni 2005b) It is not dictated by specific properties of some concept of the agent that is being described. Such a particular concept that the agent may be assumed to entertain does, or does

22

not, have these properties, so there seems to be no way in which the same belief of one agent can be described as being *de re*, and also not *de re*. The major conclusion that emerges from the examples that Aloni draws from the literature is that this, however, easily happens. Depending on the conception of things shared by *the interlocutors* a certain attitude can be appropriately characterized as being *de re*, while by adopting another perspective this might render such an attribution entirely mistaken. Kripke's "puzzle about belief" is a classical and prime example, of course, and so is Aloni's own example of the cards situation, discussed in section 3.5 above.

If one looks at the examples discussed in the literature, the conceptions involved in attitude reports are assumed to be public to the interlocutors, and not a private characteristic of an agent. Otherwise it would have to remain a mystery how one could ever establish that the representation that an agent entertains is, say, *sufficiently vivid*. Such representations are theoretical entities, that can be postulated to be there, but they are not observed, discovered and described. We can of course *assume* or *stipulate* that they are there, and we can *assume* that they are, say, *vivid*, but this cannot constitute anything like an empirical condition on the correctness of our ordinary, colloquial, belief ascriptions.

All this is not of course to deny that our analyses, the one presented here and the one presented by Aloni, do employ conceptions of objects. However, these are assumed, here, to *present* objects to us, not *re*-present them. This, subtle, distinction is, we believe, significant. The way we conceive of it is that representations need additional machinery to come to represent what they represent. They need an organism or practice or community that assigns the representations an interpretation. Individual conceptions are understood to present objects and situations. They can do so successfully, and then the presented objects exist; or they can be unsuccessful, and then no such objects exist. In either case, its reference, if any, is determined. This view can be appropriately called a Husserlian or a Fregean one, except for the fact that we do not posit and commit to a realm of meanings, or *Sinne*, so-called. (Frege 1892)

So conceived, our take on *de re* attitude reports can perhaps be considered more radically realist, and non-representational, than that of Aloni. Aloni focuses explicitly on perspectives which are described as conceptualizations of a domain, close to a representational outlook on things. We have here deliberately invoked the term "windows", and emphasized the solid assumption that what we see through a window actually exists. Such a take on things has indeed informed our understanding of *de re* belief reports. What, one might have wondered, could motivate us to, so to speak, relate someone's inner life, her private thoughts, to something external and real? We think there is no viable approach towards answering this question by studying the particular cognitive state of an individual, and search for proper representations of particular things in there. Instead, or so we believe, we find ourselves engaged with the real world around

us, and assume that we, like our world mates, are aware and knowledgeable of the things in there. We *assume* that other agents are consciously aware of things that we are aware of, and judging from the way they act and talk, we make assumptions about how these things appear to them. We essentially fail any direct evidence for how they do in fact see and conceive of these things, but we do build on the, supposed shared, assumption that we, all of us, can in principle phrase, in our own language, the way things appear to our mates. We will briefly come back again to this in section 4.4

The views exposed here align well with the ways in which Quine and Kripke have actually presented the beliefs of Ralph and Pierre to the academic audience. They did not give any description of the cognitive-psychological constitution of the agents, but they started out setting the stage, where the two agents were presented as being engaged with certain situations, which were supposed to be real life ones. In the presentation of these real life situations there was supposed to be some actually existing Ortcutt, and the real city of London, that subsequently figured as the objects of the reported beliefs. There was never any reference to a concept or representation, which was subsequently found out to actually represent these objects.

It thus appears that, in the case of a *de re* belief attribution, we understand we face a reality that is assumed to be shared, and that we assume we experience snapshots of. Crucial is not the idea or conception that we have of it, but whatever it is that we have a conception of.

We believe that our conception of conceptual windows improves, philosophically, on that of a conceptual cover, and, as we have already argued above, the empirical results obtained by means of conceptual covers can yet be maintained and sharpened. Conceptual covers have been employed in the literature rather successfully for a variety of purposes. Further linguistic applications involve the treatment of functional nouns (Schwager 2007), epistemic indefinites (Aloni & Port 2015), and concealed questions (Aloni & Roelofsen 2011; Kalpak 2020). The results so obtained can, as we have seen, be copied directly in the present framework, and the present approach will prove again to be superior whenever restrictions on the domain of quantification are required. The latter is expected to be almost always the case.

4.2   Windows and Concept Generators

Fairly recently an approach to the phenomena discussed here has been developed, for good linguistic reasons, one that is also stated in terms of individual concepts or conceptions. In this subsection we want to briefly contrast our approach with this seminal approach of (Percus & Sauerland 2003), and profile the relative philosophical-logical merits of both proposals, without in any way attempting to seriously meet any linguistic merits of theirs.

(Percus & Sauerland 2003, P&S, henceforth) successfully meets a major, classical, and well-known, challenge in the compositional analysis of attitude

reports. The challenge, and the way in which they meet it, can be most straight-forwardly illustrated by means of the following example, which is entirely fictive except for the assumption that Phosphorus and Hesperus are actually one and the same planet Venus.

> *Plato believes that Hesperus orbits around Phosphorus.* (P)

It is a common and natural assumption that the string *Hesperus orbits around Phosphorus* makes up a subsentence and a constituent of (P), which lies in the scope of, or is dominated by, the verb *believe* in (P). It is also quite common and natural to assume that the terms *Hesperus* and *Phosphorus* have their regular meaning in (P), which is intuitively seen to consist in establishing a reference to, in this case, the object Venus. Under these assumptions it seems we are invited to interpret (P) as saying that Plato believes that Venus orbits around itself, which most people think it doesn't say. Venus doesn't orbit around itself, things cannot orbit around themselves, and Plato surely didn't think so. Even so, people think (P) can nevertheless be true.

P&S respond to the challenge by reserving room, in the logical forms that (P) admits, for what they call *concept generators*. Most generally, concept generators are functions that apply to an object and that deliver ('generate') a concept of it. Passing over some, pertinent, details for the time being, (P) is interpreted as roughly saying:

> *Plato believes that CG_i(Hesperus) orbits around CG_j(Phosphorus).* (P-CG)

where CG_i yields a conception that Plato has of Hesperus, and where CG_j yields a conception that Plato has of Phosphorus. Now even though Hesperus and Phosphorus are one and the same planet, Venus, the two, different, generators may yield different concepts. In this way both our initial assumptions can be maintained: the subsentence can be interpreted *in situ*, as it is called, and both terms in it have their ordinary and intuitive reference. Nevertheless, the sentence does not thus ascribe an awkward belief to Plato, but one that he may very well have. For instance, it may be a belief that the heavenly body that Plato witnesses in the evening orbits around the heavenly body that he witnesses in the morning. This is not at all an awkward or non-sensical thing to believe, it is fairly sophisticated.

There are quite a few things about P&S's analysis worth expanding upon, and about some of them we will below, but for our more philosophical-logical concerns one point needs to be emphasized first. It should be observed, and emphasized, that the referent(s) of the terms *Phosphorus* and *Hesperus* play no role whatsoever in the belief that (P-CG) is meant to attribute to Plato. Surely the interpreter of the sentence must so to speak employ the referent so that it can be taken to deliver him the concepts that the concept generators yield for that particular referent, i.e., Venus, but it is the delivered concepts that figure

in the attributed belief, not the referent that the concepts are generated from.[25] It is therefore completely accurate to reformulate the intended interpretation of (P-CG) by saying that there is Venus, two conceptions $y = \mathrm{CG}_i(\mathrm{Venus})$ and $z = \mathrm{CG}_j(\mathrm{Venus})$ of it, and that Plato believes that y orbits around z. This is no more to say than that P&S' (P-CG) by and large corresponds to our own:

$$^e\vdash y\, {}^m\mathsf{P}z \,\square^p\, OAyz \qquad\qquad \text{(P-CW)}$$

whereby it is assumed—and actually can be stipulated—that the Hesperus seen in e (*the evening*) and the Phosphorus seen in m (*the morning*) actually are one and the same object, viz., Venus.[26]

We believe it is not unreasonable to suppose that the contents of the LFs that sentences according to P&S analysis admit can all be modeled—modulo some qualifications which we indulge upon below—within the framework that we have stated above. What are these qualifications? There are essentially two, one about the *in situ* interpretation of the theoretically problematic terms in (P), and one about the indexical nature of the generated concepts. We will discuss the latter qualification in the next subsection.

As said, one of the main results of P&S's approach is that it provides for an *in situ* analysis of the terms that establish the *res* in *de re* attitude reports. For the purpose of giving a compositional analysis of natural language this is without any doubt an advantage over the representations that we present, in which the relevant terms appear to be interpreted as having scope over the attitudinal operators. Even so, upon both accounts, the terms involved are interpreted relative to *the actual world*, so-called, while the given concepts are evaluated in whatever possibility induced by the attitudinal operators. So upon both accounts it is an issue how, and to what extent, syntactic structure does drive compositional interpretation.

We do not want to speculate too much about how our own analysis can be rendered "more compositional", but just want to mention that conceiving of compositionality as a methodological principle, the question is not whether a compositional analysis can be given, but only how, and at what cost, and this is a challenge we gladly leave for another occasion. Here we only want to suggest that it may be possible that some kind of two- or more-dimensional account of interpretation might come to the rescue. Notice that the P&S's own uses of "*world variables*" actually are one way of given a two- or more-dimensional approach an explicit formulation. In previous work it has moreover been shown how certain terms can be interpreted as escaping from scope islands, without thereby

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25. This observation should comply with the intentions of the authors, because otherwise the belief attributed to Plato might boil down to something like "*The so_and_so by means of which I conceive of Hesperus orbits around the such_and_such by means of which I conceive of Phosphorus*", and we would be facing the original problem again.
26. In our *re*-presentation (P-CW) it is moreover assumed that whatever concept that CG_i generates is available in window m and the one generated by CG_j likewise in window e.

violating scope island constraints, *in situ*, that is, also in a multi-dimensional setting. (Dekker 2008). Sure enough, a certain amount of work would remain to be done to see what is a most viable approach.

It may be observed as well that the P&S' *in situ* analysis also does not come for free. Analyzing (P), roughly, as (P-CG) requires quite some additional work at the syntax-semantics interface. We witness the intrusion of any arbitrary number of CGs in logical form, abstractions over them, a multiply typed interpretation of attitude verbs, a type that varies with the numbers of CGs abstracted over, and the postulation of a silent existential closure operator in the scope of the attitude verbs. All such is not, strictly speaking, uncontroversial.

Let us also mention one additional reason why our approach might possibly be preferred. Once we have characterized someone's beliefs regarding an individual relative to some conceptual window, we are able to independently state further assumptions about what that individual in that window looks like and how it is conceived of. It is hard to see how some such can be done on the generated concept approach. There are two problems, then. Technically, we have no independent access to the generated concepts, because the concept generators eventually are existentially quantified. It therefore requires a non-trivial adjustment of the system of interpretation to make the actual concepts formally available for further specification.[27] The second is of a more philosophical nature. As indicated above, our conceptual windows are assumed to be common good. P&S' concepts and generators are, however, pictured as individual mental entities. It could be hard, though, to see how we could make proper, non-trivial, sense of talk about properties of individual mental representations. We will reflect somewhat more on this issue below, but only after we have discussed another, perhaps more important, contribution that P&S' proposal makes: the use of indexical concepts.

4.3   Indexical Windows

Quite an attractive feature of P&S' account, and also one that is very adequately spelled out, is that they employ what they call, *acquaintance based concepts* and, likewise, *acquaintance based concept generators*. Acquaintance based concepts serve to identify individuals relative to others, in particular relative to the concept-holder, so to speak.[28] An essential characteristic of such concepts,

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27. An independent motivation for assuming existential closure comes from the authors treatment of example (Z_r) below, but, as we will see, we can handle that without assuming any internal existential closure operation.

28. A more generic motivation for these indexical concepts may be that it is intuitively hard, if not impossible, to come up with pure concepts (representations, descriptions, . . .) that all by themselves serve to identify particular individuals. One might think that we are all able to define *the King of England* by that very description of him, but that would assume we would be able to define England in the first place. But then, which, world independent, description is going to give us England? Of course there is a huge amount of literature that one can bring to bear on this issue, but the question just raised may be considered to be sufficiently alerting and thus help to also sufficiently appreciate the indexical, and Cartesian, reply that we will

and of the correlated beliefs, is that they accommodate a center, or *self*. To believe that one is smart is to believe to be in world in which *self* is smart; and a concept of one's husband is a concept of a person who is the husband of *self*.

These sketchy intuitions can be adequately modeled, after (Lewis 1979), by parametrizing the intensional objects that we have at our disposal. The belief to be smart is modeled by employing the property of being smart, which is, formally, a relation that holds between pairs of individuals a and possibilities v such that a is smart in v.[29] To stand in the belief relation with that property must be understood to consist in self-ascribing that property. One thinks of one-self as being such an individual in such a world v. An indexical individual concept is, likewise, modeled as a function, not from mere possibilities to objects, but from centered possibilities to objects, where centered possibilities again are pairs $\langle a, v \rangle$, conceived of as a possible subject (self, or center) a in possibility v. An acquaintance-based concept like "one's husband" is, thus, modeled by a function that to each centered possibility $\langle a, v \rangle$ assigns the object which is the husband of self (a) in v, if any.

I will not here give an exposition of how such Lewisian indexicality is actually implemented in P&S's system, but instead sketch how it could be incorporated in that of our own. As said, these indexical concepts can be thought of as functions to individuals, not from plain possibilities v, but from centered possibilities $\langle a, v \rangle$, conceived of as possibilities in which a is being the center in v.[30] If this is a concept in a window r, then one might quite aptly understand this as the concept of an individual that the viewer sees through r.[31] For the time being, in the remainder of this subsection, we will assume concepts, and, hence, conceptual windows, to be indexical in this sense.[32]

If we make our windows indexical they as a matter of fact are even more *window*-like then they were before. If you see a picture of a rose, this now can be taken to by given by a window w (not: a picture) quasi-formally characterized by ${}^{w}\exists x \square_x$ (*x is the rose* you *see*). In every possibility $\langle a, v \rangle$ where it is defined, it is the unique rose that a sees in v.[33] If we also add to our language an indexical variable i, defined by $g(i)_{\langle a, v \rangle} = a$, for any indexical possibility i, then we can express this, formally, as ${}^{w}\exists x \square_x \imath z (Rz \wedge Siz)(x{=}z)$.

Indexical windows are a nice tool to play around with. We could, e.g., dictate our windows to figure like P&S's *acquaintance-based* concept generators,

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be dealing with here. *England* is the country that *my oldest daughter* lives in.

29. Equivalent, formally, to a function that associates individuals a with the sets of worlds v in which that individual is smart.

30. It is surely a prerequisite that, for any such indexical possibility $\langle a, v \rangle$, $a \in D_v$.

31. Recall the observation from footnote 13 above.

32. Formally we would not even need to make any essential changes in the semantics presented in section 1, except for the fact that every time we employ a possibility v there, it will be understood to be replaced now by a centered possibility $\langle a, v \rangle$. Since we didn't do anything with the *self*s in that section, the addition of the a's is entirely innocent.

33. It would be even more helpful if we provided more parameters, like temporal ones. For the time being we however prefer to not complicate things with temporal dependencies.

by making the following, model-theoretic stipulation. For any such window ab and for any indexical concept $c \in \mathcal{C}_{ab}$, it can be required that $\forall \langle a, v \rangle$ in the domain of c, there is some acquaintance relation AR such that x is the unique individual that a stands in the AR relation with in v. Such would ensure that an agent that is ascribed some belief *de re* will also think of the *res* as something she is directly acquainted with.[34] We think such would indeed correspond to a kind of an assumption that we often make. It is questionable, however, whether we should introduce such a stipulation. Not only is it dubious whether we always do, or should, make such an assumption, but also because the stipulation may be questionable itself. In order to make the stipulation explicit one is required to quantify over relations, so it would directly condemn us to the realm of higher order logics. Moreover, the whole notion of an *acquaintance relation* is not very well defined. When Lewis discusses acquaintance, he presents an inventory of some suitable acquaintance relations, but decided to abstain from giving a definition.[35] Now, of course, any attempts of defining what direct acquaintance really is, is a respectable philosophical enterprise. However, it seems that giving such a definition should not belong to the task of a linguist attempting to model the natural language practice of ascribing *de re* reports. To account for these practices, it seems that it just may suffice if we are in principle able to give, for every specific *de re* report that we may want to account for, an explanation of the specific kind of acquaintance relation that can be assumed to support it. Actually this is also what P&S do. They indeed suffice with mentioning and listing the kinds of relations playing up in particular examples.[36]

It may be observed, moreover, that our system with conceptual windows does not at all force one to assume that there is only one, specific, acquaintance relation active every time, or that all agents are committed to see the same things in one and the same window. It is actually entirely easy to construct one window on London through which Pierre sees a pretty London, while the same window present a dirty London to Ralph. This possibility makes it actually easy for us to make proper sense of an example attributed to Ede Zimmermann, one of a type that constituted a main motivation for P&S to resort to existential quantification over acquaintance relations. (Percus & Sauerland 2003, fn. 14, and p. 234*ff*) The example relates to a scenario in which every contestant is having a *de re* presentation of himself, without realizing it is himself he is having a belief about. The following sentence arguably can be true in that scenario, provided that every contestant believes that the person that is presented to him wins, even if none of the contestants has any idea of whether she herself wins.

Every contestant thinks he wins.$^{\ulcorner}\forall x(Cx \to \Box^x Wx)$ (Z$_r$)

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34. We would have to add that the agent is also actually acquainted with the *res* that way.
35. Lewis (Lewis 1979, p. 541-2) gives a collection of samples, observes that it involves "an extensive causal dependence (. . .) of a sort apt for the reliable transmission of information," and observes that this is not actually a necessary neither a sufficient condition.
36. Such as, for instance, *one's husband, the reviewer of one's article, the first (second, third, . . .) candidate one hears*, etc. (Percus & Sauerland 2003, p. 231/5)

We can in principle even, easily, give a further specification of the way in which every contestant thinks of himself in this attribution, which is just the way in which *we* present the contestants through window r. We can also refrain from doing so. There are no mysteries or challenges here.

Interestingly, there is also the quite sophisticated possibility of defining a *de se* window, one which provides those who look through it with knowledge of themselves. Suc a window s can be most succinctly defined as follows.

$$^{s}\forall x \Box^{x} \, x{=}i$$

This stipulates that every person in s knows who she herself is. What does this mean? It means that if we say, e.g., that every contestant in s believes she is the winner, then she has the *de se* belief that she is the winner. For, obviously:

$$^{s}\forall x \Box^{x} \, x{=}i \quad ^{s}\forall x(Cx \to \Box^{x} \, Wx) \quad \models \quad ^{s}\forall x(Cx \to \Box^{x} \, Wi)$$

Interestingly, merely by our choice of window, and by the stipulations that we can explicitly make, we can interpret a formula such as $\forall x \Box^{x} \, Wx$ as either *de re*, or *de se.* This choice is, apparently, available in English and also in our system. And it is independent of any explicit *de se* device, which is apparently available in Italian, arguably not in English, but available again also in the indexical version of our system.

4.4   Representations Revisited

As they say themselves, Percus and Sauerland present a "way of adapting Lewis' analysis of 'de re' belief", where the latter's "account of belief *de re* is broadly similar to Kaplan's" as Lewis says himself. (Percus & Sauerland 2003, p. 230; Lewis 1979, p. 539) All appear to commit to the idea that there is a fact about the (internal) mental state of the believer that renders a belief *de re* or not. As indicated above, Aloni has argued fairy extensively, and convincingly, we think, against such an approach. We do not want to repeat these arguments here, but we do like to emphasize one issue once more. Eventual facts about (other) people's internal mental states are notably obscure and inscrutable. All evidence we have comes from their external behavior, verbal as well as non-verbal, and the assumptions we make to make sense of it. This practice involves *our* explanations of *public* facts, not facts about what goes on privately in people's minds, unobservable to us.[37] The question whether beliefs and other mental states are appropriately construed *de re* does arguably not depend on facts about the (internal) states of the agents *per se*, but upon the perspective we, the ascribers, and the consumers of the ascriptions, adopt to understand the state and behavior of the agents so described.

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37. Brains scans might eventually prove the presence or existence of certain concepts and representations in the mind in the lab, but arguably such data have so far never been employed in the attitude ascriptions that we actually make in real life.

The view adhered to here is a realist, not representationalist one. As already has been mentioned above, the underlying idea is that when one ascribes a belief to someone acting in an actual and shared reality, whether the belief be *de re* or not, one provides a characterization of that person's behavior, non-verbal as well as verbal, as one that would make sense if that reality and the things in it were in a certain condition, as presented by the belief reported. Such involves our presentation of that virtual state of reality, not, or not directly, that of the holder of the ascribed belief. It is one that we think we would provide as a description of the world if we saw the world we think she sees it. Such a conception of belief reports represents that of Quine, summarized thus:

> When we ascribe a belief in the idiom 'x believes that p' (...) [w]e reflect on the believer's behavior, verbal and otherwise, and what we know of his past, and conjecture that we in his place would feel prepared to assent, overtly or covertly, to the content clause.

> The language is that of the ascriber of the attitude, though he projects it empathetically to the creature in the attitude. (...) The cat is purportedly in a state of mind in which the ascriber would say 'A mouse is in there'. (Quine 1992, p. §27)

Also Lewis apparently held that the *de re* nature of a *de re* belief does not reside purely in the mental state of the believer. "[O]ther-ascriptions of properties are not further beliefs alongside the self-ascriptions, but rather are states of affairs that obtain partly in virtue of the subject's self-ascriptions and *partly in virtue of facts not about his attitudes*." (Lewis 1979, p. 543, emphasis mine.) We take these observations to imply that what, on the occasion of a *de re* report, the factual acquaintance relations are, may be none of our logical or linguistic business. As Maria Aloni has argued, it is a matter of pragmatics. (Aloni 2005a)

4.5 Double Bind Readings

Simon Charlow and Yael Sharvit have shown how the Concept Generators from Percus and Sauerland predict there to be readings of sentences that one might not have expected in the first place, and that can, apparently, be argued to be real. (Charlow & Sharvit 2014) Consider the following sentence.

$$\text{John believes that every student likes her mother.} \tag{CS}$$

The relevant, theoretically implied, reading we can most adequately present by rendering it, in our own terminology, as follows:

$$^r\forall x(Sx \rightarrow {}^s\exists z(x{=}z \land \ \Box^j\imath y\, Mzy\, Lxy)) \tag{CS-CG}$$

Motivation for this "interpretation" can be found in a scenario in which all the students are explicitly presented twice, and in which this *John* has come to believe of each student, presented the one way, likes the mother of the same student, presented the other way. As Charlow and Sharvit observe, the Concept

Generators approach predicts such a reading of (CS) because in the logical form of the sentences there is a trace of the quantifier every student, as well as a coindexed pronoun, both in the scope of John's belief, and because both elements, according to the generator approach, may independently trigger a generated concept. In our rendering this has been mimicked by duplicating the variable for the students, so as to allow each of the students to be seen through two windows, r and s. The situation that we have characterized by means of (CS-CG) actually matches the situation that Charlow and Sharvit describe quite closely, even though it does not equally directly match the surface form of (CS). However, as argued, it does come out to be described by one of the Logical Forms that Percus and Sauerland advocate.

One may notice that it does appear to require some brute, but explainable, force, to come to render (CS) the way indicated by (CS-CG), but, as we argued above, so do the readings that Percus and Sauerland allow for. Their logical form for (CS) involves the operation of two concept generators within the logical form of this sentence, plus an abstraction over them, and their subsequent existential closure, all of this guided by a, polymorphic, syntactic and semantic analysis of the verb *believe*. One may notice, as well that the very same type of analysis, even more easily, predicts a true reading of the following sentence:

Ralph believes that Ortcutt is not identical to himself (NSI)

Using the mechanisms staged in (Charlow & Sharvit 2014), this sentence can be interpreted in a way that we would render as follows.[38]

$$^q\bigcirc x\ ^s\exists z(x{=}z \wedge \square^r x{\neq}z) \qquad\text{(NSI-CG)}$$

We hope the reader is familiar enough with our framework by now to see that (NSI-CG) not only can be possibly true, but that it even must be judged to be true in the situation presented by Quine. There is Ortcutt, seen one way, and there is the same individual, presented another way, and Ralphs beliefs imply that the individual presented the first way is not the man presented the other way. However, we suspect that many people will not that easily be inclined to say that such constitutes a genuine reading of (NSI), even if one can manage to interpret it as *somehow true* in that situation. Likewise, while we might come to accept (CS) as true in a certain situation, no matter how ingeniously construed, we believe that one should admit that interpreting it that way does at least require a *tour de force*, something that the somewhat deviant logical representation (CS-CG) might be indicative of. One may finally observe that one might even more easily consider the very same sentence to be plainly *false* in the very same motivating situation, just as much as one may be most readily inclined to judge (NSI) to be false in the situation presented by Quine, too.

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38. Charlow and Sharvit's example (5b) actually has the form of (NSI), and the sentence is indeed claimed to have the corresponding "bound de re" reading.

What can we conclude from this? If one wants to insist that something like (CS-CG) and (NSI-CG) present a good way of interpreting sentences like (CS) and (NSI), then we have an argument for adopting the involved type flexible logical forms that Percus and Sauerland and Charlow and Sharvit argue for, but, if so, also one that would motivate construing the sentence the way we did. If, on the other hand, one were to judge these readings as degenerate, they can only count against the concept generator theory, or challenge it, so as to find plausible independent constraints to prohibit such readings.

4.6   Windows and Counterparts

Recently, Dilip Ninan has, with good reason we think, criticized Aloni for presenting her conceptual covers as providing ways of thinking of an independently given domain, thereby rendering it truly conceptual, or even representational. (Ninan 2018, p. 458) As indicated above, one may doubt whether the latter applies, appropriately, to Aloni's system, but it is surely not true for the system with conceptual windows that we have presented here. Our windows serve to present, or define, a domain, rather then conceptualize a given one. This is also why we have talked about a *domain projection* in section 1. Perhaps it is this point that makes our approach immune to a problem that Ninan has raised for a conceptual cover analysis.

The problem that Ninan observes starts from the following observation: "[W]e often possess a way of thinking of a group without that way of thinking decomposing into a set of ways of thinking of the objects in the group." (op. cit., p. 464) Note that the problem that follows does not, or not obviously, have to do with knowledge of groups per se, or or collections, or sums, of individuals, because these might be incorporated in the system by extending the language, and the models, so as to allow for talk of plural individuals or some such, and this does not, or not obviously, relate to the problem that Ninan addresses. The problem is that Aloni, according to Ninan, needs to assume that for an agent to have any knowledge of such a group of individuals, the agent must possess the means to individuate each one of them individually. The latter requirement, then, is something that we by assumption or stipulation may consider not to be the case, while this does not prohibit true ascriptions of knowledge about the domain. If this is correct, it would imply that Aloni's account of knowledge ascriptions cannot account for such cases.

While we are, again, not entirely sure that this way of looking at *conceptual covers* is entirely correct, we are quite confident that such criticism does not apply to the method of conceptually restricted quantification that we have advocated in this paper. Our argument will be of a constructive nature. We will present, in the terminology employing *conceptual windows*, the case that Ninan has come up with, and show that it yields the results we want, and not those that we do not want. In presenting this case, we however have to slightly remodel it, because, like most most of Ninan's paper, it is concerned with *epistemic modalities*, a particular type of modality that we have not discussed here.

We can however state Ninan's case in our terms, by casting it in terms of our doxastic modalities. We will henceforth employ our $\Box^n$ as standing, not so much for Ninan's epistemic *Must*, but as short for "*Ninan believes*", or "*Ninan knows*". We will most of time assume that we ourselves share, with Ninan, this kind of knowledge or belief, a kind of shared knowledge or belief that seems to be characteristic of the intuitive epistemic modality that Ninan himself has dealt with.

The basic situation is that there are a hundred tickets, and we propose to focus on them by seeing them through a window indexed t. There are 50 red ones, and there are 50 blue ones, and this piece of knowledge can be characterized by saying we have a window on the 100 cards, through we see them as 50 red and 50 blue ones. This windows is labeled ^{c}, for 'color'. Note that we so far assume we know nothing about any of the cards seen through c, besides the fact that they belong to the set of hundred tickets seen through t, that they are all distinct, and, that we know of each cards, which color it has, and nothing more. Al the tickets are also viewed through another window, where we find 50 cards with a circle on them, and 50 with a square. The window is labelled ^{f}, for 'form'. Looking through window f we know we see a hundred tickets, we know of each ticket seen that it is one of the hundred seen through t, and through c, but we don't know which one. We know each ticket that we see through c what form it has, and that it is distinct from all the others. (The mentioned pairs of properties are obviously supposed to be exclusive.)

Formally, in our terminology, we have three windows, t, c and f, and for any w equal to c or f, we have:

$$\exists!100x\,{}^t{\in}x \qquad \forall x({}^t{\in}x \leftrightarrow {}^w{\in}x)$$

This is to say that we, with Ninan, see a hundred items through t, and that c and f provide a view on the same set of things. It is also stipulated that we all know they are tickets, and that we see all these tickets through w, where w is equal to c or f.

$$^t\forall x\,\Box^n Tx \qquad \Box^n\,{}^t\forall x\,{}^w{\in}x$$

It is stipulated moreover that there not only is this distribution of colors and forms over the tickets, but that we actually see 50 of them being red, and fifty being blue, when we look through c. Likewise we also actually see 50 of them having a circle, and 50 a square, when we look through f.

$$^c\exists!50x\,\Box^n Rx \text{ and } {}^c\exists!50x\,\Box^n Bx$$

$$^f\exists!50x\,\Box^n Cx \text{ and } {}^f\exists!50x\,\Box^n Sx$$

These are to say that if one would pick out any arbitrary ticket from what we see through c, we would know it is red or blue. Likewise we would know the form on the ticket if it were arbitrarily chosen from what we see through f. From the assumptions made it already follows that we also know that there are fifty tickets

with a circle on them when we look through the color window c: $\Box^n{}^c\exists!50xCx$. But given only these assumptions, we don't know which ones. Each red or blue ticket seen this way might have a circle and might have a square. Surely, once we know, or guess, about fifty of them that they indeed have a square, we know the others have a circle, but, as said, we need to make additional assumptions (further knowledge, or guesses) to establish this.[39]

The punch-line in the situation is that we all know, with Ninan, that there is a unique winning ticket and that it is blue.

$$\Box^n \imath x\, Wx\, Bx$$

Now Ninan presents the following line of reasoning, where we read "*might*" as "Ninan conceives / We conceive it possible that."

(13) Any circular ticket might be the winning ticket. $(\forall x(Cx \to \Diamond^n Wx))$
(14) Any square ticket might be the winning ticket. $(\forall x(Sx \to \Diamond^n Wx))$
(15) Every ticket is circular or square. $(\forall x(Tx \to (Cx \vee Sx)))$
(4) Any ticket might be the winning ticket. $(\forall x(Tx \to \Diamond^n Wx))$
(16) Any red ticket might be the winning ticket. $(\forall x(Rx \to \Diamond^n Wx))$

The problem, Ninan says, is that, first, (13)–(15) each seem to be true; second, that (4) appears to be a logical consequence of the three; and, third, that (16) again appears to be a logical consequence of (4). However, Ninan says, the conclusion (16) seems to be at odds with the assumption that, for as far we we and Ninan know, a red ticket cannot be the winning ticket, because the winning ticket, we know, is blue.

There is not, however, any problem, we think. If we judge all five sentences above through the general window t the conclusion ${}^t\forall x(Rx \to \Diamond^n Wx)$ follows from the premises, simply because neither we, nor Ninan needs to know of ^{t}any ticket that it is red, and also not of any red ticket that it is red, so that so far as we know, any red ticket can be just any ticket, and possibly the winning ticket. Formally, if we make no further assumptions than those stipulated, ${}^t\forall x\,(\Diamond^n Rx \wedge$

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39. These results are most easily established formally following the model-theoretic method. First recall that we are assuming distinctness (D_x). From ${}^t\forall x\,\Box^n Tx$, and the last four assumptions, it directly follows that all hundred individual conceptions in t, c and f are defined in the possibilities not excluded by Ninan. By distinctness, it follows that these moreover provide the whole ranges of things seen through t, c and f, also in Ninan's possibilities. By $\Box^n\,{}^t\forall x^w\Sqsubset x$ it, then, also holds that in each of Ninan's possibilities the things seen through t, c and f are the same set of items. Since in each possibility they constitute a set of 100 tickets, neatly divided into 50 red and 50 blue ones, and 50 have a circle and 50 have a square, this establishes the indicated kind of knowledge. For in each possibility there are 50 red and 50 blue tickets seen through any of the windows.

However, it is only when seen through the color window that, in all conceived possibilities, it is the same 50 individual conceptions that yield the red ones, and the other individual conceptions that yield the blue ones. Looking through the other windows any individual conception may yield a red ticket in one possibility, and a blue ticket in another. All that matters is that the whole set of conceptions yields a fifty-fifty distribution in each possibility. QED.

Notice that we can of course establish the same results proof-theoretically, but also that this would be rather laborious. One would need a whole lot of variables!

$\diamond^n Bx$) so if we say that any ticket r seen through t, even if r is actually a red one, we don't need to know it is red, and we can consider the possibility that it is blue, i.e. possibly a winning ticket.

So why would Ninan object to this conclusion? Why do we feel an inclination that this seems wrong? Well, if he were to take part in our conversation, and if we indeed talk about *the red tickets*, we would most probably have adopted a view through *the color window*, and then indeed we would know of any red ticket that it cannot possibly a winner. And indeed $^c\forall x(Rx \to \diamond^n Wx)$ is inconsistent with the situation sketched. For it follows from our assumptions that, looking through this window, we, and Ninan, see all red tickets as red tickets, $^c\forall x(Rx \to \Box^n Rx)$. Since we know that being red is inconsistent with being blue, this implies $^c\forall x(Rx \to \neg\diamond^n Wx)$, in apparent contrast with (16), but not in real contrast with it.

That (16) is false when it is evaluated using the colour window c, is not in conflict with the premises, which were considered true, but true evaluated using window t. But we can back-track the falsity of conclusion (16) relative to c, and establish that (4) must be false, too, when evaluated using c. And this is correct. When one sees 50 red and 50 blue tickets, and one knows the wining ticket is a blue one, then of course one cannot maintain that any ticket might be the winning one. So, backtracking again, one of the premises (13)–(15) must be false relative to c. This is not premise (15), because it is true, and known to be true, no matter seen through which window. The problem originates from premises (13) and (14), if they are construed relative to the color window. They are indeed both false then, pending no further assumptions.

Imagine, for instance, we know what we are said to know, and we are looking at a space with 50 red and 50 blue tickets. And someone comes by and says *Any circular ticket might be the winning ticket.* Our intuition is that you would hedge. For you have no idea which are the circular tickets among the red and blue ones that you see. And if an arbitrary circular ticket might be one of the red tickets that you see, you would for sure know that it is not the winner. Again, when you see the red and blue tickets, half of the candidates are already known to be excluded as a winner, the red ones, if there is any circular one among them —and there might be a circular one among them— could surely not possibly be a winning ticket.[40]

Summing up, Ninan's case does not seem to raise any problem, if we follow the presentation sketched here. We can naturally account for the proper logical consequences, and also for the, only apparent, contradiction, and the appearance of it, if we carefully mind the ways we look at things. Ninan may still be right that upon Aloni's account there might be a complication, viz., that an explanation like the one given here would only be possible in terms of her

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40. My intuition about this is strong, but I know that thinking and working on many of these and related issues for years, and also gradually having developed a "*theory*", is something that pretty much messes one's proper and original intuitions. I leave it therefore to the readers to establish their own judgement, which hopefully is not so much a theoretically infected one!

framework of conceptual covers if we had "*demonstrative* modes of presentation" of each one of the individual tickets, which we can be said to fail, because, for instance, "we might not be in a position to mentally single out any of the tickets" (p. 467). We think we agree, but it is unclear to us whether Aloni's approach would indeed require some such, intuitively. It surely does not constitute an issue for us, however, because the worry last mentioned does not arise in our framework. The observations that we have made about the statements in (13)—(16) were indeed semantically, model-theoretically, motivated, but they are also valid following our proof theory, a sketch of which is given in the appendix. The fact that we in principle can give the whole account above by proof-theoretic means, so without any mention of satisfying models, must suffice to show that nowhere any individual ticket, real or model-theoretic, has to be actually individuable.

Somewhat independent from the above discussion, it may be conjectured that Ninan's own counterpart alternative will eventually be the same in expressive potential as our own approach, a point that perhaps better be substantiated at another occasion. Pending such a comparison, what we would deem less attractive about Ninan's own alternative is that it commits itself to a Lewisian metaphysics which postulates a whole variety of *cross-possibility counterpart relations* "having the same number", or having the same color," etc. This Lewisian theory only works if we assume there to be *modal facts* such as the following:

> [*T*]*he color of o in v is identical to the color of o′ in v′.* (Ninan 2018, p. 470)

What kinds of facts do we draw from here? The requirement here reads is as if it is taken from a travel guide, describing and comparing various holiday resorts. But we just don't think of (non-actual) possibilities as hosting facts that a travel agent in principle is able to compare and find out. In this respect we do agree with Kripke, who observed:

> *A possible world isn't a distant country that we are coming across, or viewing through a telescope. (. . .) 'Possible worlds' are stipulated, not discovered by powerful telescopes. (. . .) I repeat: Generally, things aren't 'found out' about a counterfactual situation, they are stipulated.* (Kripke 1981, p. 44/49)

We can totally agree with *thinking of possible objects* as having the same color, or with *stipulating them* to be that way. Ninan's Lewisian approach is, however, committed to having possible objects that as a matter of fact be that way. We find it difficult to commit to that.

4.7 Windows on Non-Existent Objects

It has been emphasized in the previous exposition that the windows in our models are transparently extensional, but there is good reason to not deny any non-extensional use of them. When we characterize, e.g., the doxastic states of our world-mates, we do so employing a natural, and a formal, language that we can understand as describing things un-real, viz, the situations we think they

think to be in, perhaps mistakenly. Our characterizations, and our windows, allow us to 'see things', in a pregnant sense. We may say that while there are actually no things unreal, we however do assume we have *a conception of* things unreal. And when we say that someone believes S, then S presumable presents some possibility, which we conceive or understand, and which very well may be not actual. Such an attribution provides a window on something not real, that is, say, something like an intentional window. This may sound perplexing, philosophically speaking, and the phenomenon may not at all be understood, naturalistically speaking, but we think it is constitutive part of a very common and shared practical assumption. It may be taken to simply boil down to the, fundamental, but colloquial, assumption that we understand what a sentence S says, even if we, upon reflection, have no idea of what the locution "*understanding S*" actually means.

These abstract 'philosophical' reflections can be rendered somewhat more concrete as follows. The reader may be familiar with the fact that some one and half century ago it has been *hypothesized* that there would be a planet, called "Vulcan", the orbit of which would have been contained in that of Mercury. We conceive of this as a possibility, even if it is one that is now known to be not actual. It is easy, though by itself not very informative, to characterize the possibility using a modality $\Box^u$, reading, roughly, *according to the hypothesis of Urbain Le Verrier* as follows:

$$\Box^{u\ w}\bigvee v\ \Box_v\ \chi$$

This line reads that according to the hypothesis of Urbain —here u—, there exists a v named *Vulcan* and so that it (v) has χ as essential characteristics, when seen through window w. Here, χ is taken to abbreviate the conditions that according to Le Verrier obtain, including those that support the supposedly defining characteristics of Vulcan.

In a sense, then, there is talk, here, of a possible object. It is seen, with χ as its essential characteristics, through a window w, which here figures as an intentional window on it. If we were in a world conceived possible by Urbain Le Verrier, we would be able to see, through this very window w, a Vulcan with these characteristic properties; for $^w\bigvee v\ \Box_v\ \chi$ would have to hold there. Somehow, however, we also cannot see it, because, as we assume, our world is not actually like Le Verier thinks it is, and, also by assumption, there is no such Vulcan to be seen through w. This must be all reasonable, but it is somewhat worrying, that it seems we are currently unable to actually express, in our formal language, that this Vulcan does not exist, neither that it would have the characteristic properties ascribed to it. Something must be done if we want to approximate or render the apparent practices of the users of our natural language, that obviously do engage in thought and talk about possible objects, even if this type of *thought* and *talk about* is construed intentionally.

Instead of wanting to say, literally, of Vulcan, that it does not exist, we might think we could suffice, following (Quine 1948), with simply asserting

that there is, in reality, no such thing, with these defining characteristics. But even if this were appropriate, it may be difficult, or even impossible, to do so, because Vulcan may have been defined just as the thing seen through w, if w provided a view on only one object. We would then ourselves fail any such defining characteristics.[41] But we can do better than that. Two adjustments may serve to improve the situation. The first adjustment consists in a more suitable characterization of Verrier's hypothesis, one that is already possible within the system we have defined here in this paper; the second adjustment consists in a proper, but we think independently motivated, extension of it.

In the first place we can improve our sketch of Verrier's conception of Vulcan by so to speak in dragging him out of the solipsist slumber that we up until now have pictured him to be possibly in. We can after all situate his hypothesis relative to an accurate window that we maintain on our actual solar system. Let us define window s to figure as some such window:

$$^s\exists\vec{x}\,\Box_{x_0}\,\phi$$

Here we have used $^s\exists\vec{x}$ as short for $^s\exists x_0 \ldots {}^s\exists x_8$, and we let ϕ abbreviate $\mathsf{SUN}x_0 \wedge \mathsf{MERCURY}x_1 \wedge \mathsf{VENUS}x_2 \wedge \mathsf{EARTH}x_3 \wedge \ldots$. The idea is that, through s, we see the sun and the planets in our solar system and the formula ϕ is moreover assumed to picture them as distinct objects, which have their generally acknowledged orbits, etc. It is these assumed real objects that we can construe Le Verriers hypothesis relative to, in the following way.

$$^s\exists\vec{x}(\Box_{x_0}\phi \wedge \Box^{u\,w}\exists\vec{z}\,{}^w\!\vee v\,\Box_v(\vec{x}{=}\vec{z} \wedge \psi))$$

We have now used $^w\exists\vec{z}$ as short for $^s\exists z_0 \ldots {}^s\exists z_8$, $\vec{x}{=}\vec{z}$ abbreviates the conjunction of equations $x_i{=}z_i$ for all nine indices i, and ψ summarizes the essential characteristics of Vulcan, this time relative to the $\vec{z}$, which are, by the preceding equations, the $\vec{x}$, which are seen as the objects in our actual solar system. Window w, thus defined, provides the view on the solar system as *Le Verrier* hypothesized it. It can be conceived of as an intentional window, because it presents a possibly non-existent object, but this hypothetical object is presented among a system of objects that are indeed rendered here as being real. From something like the above we may be able to conclude, e.g., that:

$$^s\mathsf{SUN}x_0\ {}^w\mathsf{MERCURY}x_1\Box^{u\,w}\!\vee v\,OBx_0vx_1)$$

(Here $OBuvw$ is taken to abbreviate that v orbits between u and w.) Verriers hypothesis, then, actually locates the orbit of his hypothetical object between the sun and that of Mercury, that is, the real sun and the real planet Mercury, that we see through s.

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41. Note that window w may be a private window, in the sense that what is seen through it in the world according to Verrier may be entirely distinct and independent from anything that we in the actual world see through it.

So much for the first adjustment; yet more remains to be done. We want to be able to actually express something that has remained implicit in the above presentation. This is not just the fact that some such Vulcan does not exist, but more particularly the fact that, as we normally assume, *this Vulcan* does not exist, for there are various things called "Vulcan" that do and do not exist. (Wikipedia Contributors 2018) This point intrinsically relates to the intricate topic of thought and talk about the non-existent. Let us briefly characterize our take on this issue, which is not an original one, and the way we can make it formal, which is something that is claimed to be original.

      Many people agree that we cannot refer to Vulcan, because we cannot refer to something that does not exist, but, upon a little reflection, this explanation is just as paradoxical as the intuitive claim, about Vulcan, that it does not exist. Many philosophers, and linguists for that matter, have therefore proposed to, after all, render Vulcan some *kind of* existence.[42] We intuitively want to resist such a temptation, and insist on the non-existence of Vulcan. Closer to our intuitions then come certain views formulated by Cartwright, Sainsbury, Crane and Moltmann, who do not aim to credit intentional objects with any actual being, but conceive of them as having only intentional being, as things (merely) thought and talked about. (Cartwright 1960; Sainsbury 2005; Crane 2012; Moltmann 2015)[43] These 'objects' can be *grasped*, conceptually, and *referred to*, semantically, without some such requiring the existence of an actual referent. In our communicative exchanges they can be accorded the status of a discourse referent. Discourse referents are mere means that help us coordinate all kinds of discourse, including intentional (speculative, fictional, . . .) discourse, without ontological commitments to the existence of possible objects, or fictional characters, or what have you.[44]The authors mentioned here all sketch, informally,

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42. Whether it be, for instance, full fledged Meinongian being, like for instance Parsons and Berto advocate, or plain existence in other possible worlds (as portrayed by Lewis), or existence as an abstract fictional character as Van Inwagen and even Kripke apparently propose. (Parsons 1980; Berto 2012; Lewis 1986; Kripke 2013; van Inwagen 2001)

43. *"Objects of thought are not, as such, entities. An object of thought is just anything which is thought about (. . .). Some objects of thought exist, and some do not. (. . .) When an object of thought exists — for example, when I think about the planet Neptune — then the object of thought simply is the thing itself (Neptune itself). When the object of thought does not exist, it is nothing at all [Husserl 1900-01, 99].*" (Crane 2012, 57–8)

"Positing intentional objects (. . .) does not mean taking unsuccessful acts of reference to in fact be successful, referring to intentional objects. Rather intentional objects are 'pseudo-objects' entirely constituted by unsuccessful or pretend acts of reference itself (and acts they are coordinated with). (. . .) Intentional objects are not part of the ontology; they are mere projections of intentional acts, which is why they have the status of nonexistents. Intentional objects thus are not peculiar types of objects that are by nature nonexistent." *"[I]ntentional objects (. . .) can be made available only by the presence (. . .) of intentional acts on which the intentional objects depend.*" (Moltmann 2015, p. 145/166)

44. *"[D]oes not what I have said (. . .) require that dragons be even though they don't* exist*? Must not dragons have some* MODE OF BEING, *exist in some* UNIVERSE OF DISCOURSE*? To these rhetorical questions it is sufficient to reply with another: What,* beyond the fact that it can be referred to, *is said of something when it is said to have some mode of being or to exist*

an ontologically non-committal approach to analyzing and understanding talk about possible and fictional entities, and we believe that we can also actually formally account for this, basically on the basis of the system presented in this paper.

In a dynamic semantics, discourse reference is accounted for by keeping track of the, Tarskian, extensional, 'witnesses' of terms satisfying the formulas of a language. In (Dekker 2012) such a method has been generalized to modal and intentional discourse, licensing intentional discourse reference.[45] The kinds of phenomena under discussion are familiar from the literature, and are typically of the following kind.. Even if a linguistic context is not extensional, and does not allow reference back to items which are expected to be introduced in that non-extensional context, merely because there isn't anything that is asserted to be there in the first place, the tentatively introduced items do remain available at some intentional level. Since they have been introduced at least intentionally (possibly, hypothetically, speculatively, . . .), they therefore remain intentionally available through subsequent intentional efforts. Motivating examples include the following.

(i) *John hopes to catch* [a fish]^x. *Mary plans to prepare it$_x$ for dinner.*

(ii) [A wolf]^y *might have come in. It$_y$ would have eaten you first.*

(iii) *Hob thinks* [a witch]^v *has blighted Bob's mare, and Nob wonders whether she$_v$ (the same witch) killed Cob's sow.*

(iv) *The army were planning to build* [a bridge]^w, *and the rebels already prepared for blowing it$_w$ up.*

While there is quite some discussion in the linguistic literature about the precise analysis, and, consequently, the truth-conditions, of these sequences of sentences, we believe that what should minimally be accounted for is that, in each example, the two coordinated modalities should receive coordinated satisfaction conditions. That is to say that if, regarding, e.g., example (i), for John's reported hope and Mary's reported plan to be jointly realized, there must be a fish that John caught and Mary prepared. Of course, this does not mean that there is some actual fish, that John hopes to catch, and that Mary plans to prepare. Likewise, in example (ii), for the reported belief of Hob to be true, and Nob's wonder to be simultaneously resolved, there must be a witch that blighted Bob's mare, and such that Nob knows whether or not she killed Cob's sow. This kind

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in a universe of discourse?" (Cartwright 1960, p. 639)

"Discourse referents are what make possible the use of definite expressions in speculations which leave open whether they have referents." "[O]nce we appreciate the ubiquity in our thought and talk of reference [intentional, PD, possibly 'empty'] and related notions, as opposed to reference [to an actual referent, PD], the arguments that seemed to favor realism about fictional entities lose all persuasive value. We can happily combine commonsensical realism about fictions (novels, plays), which of course really exist, with irrealism about the fictional characters, people and places they portray, which typically do not [exist, PD].* (Sainsbury 2021, p. 46/58)

45. A proof theory for such an extensional system is provided in (**?**); (**?**) presents the proof theory for this intentional offspring.

of coordination of the reported attitudes can be mediated by means of intentional discourse referents, conceptions of individuals that can be supposed to be realized only in the circumstances said to be satisfying these attitudes.

The only tools required for these kinds of pieces of discourse consist, on top of the system presented in this paper, in the evaluation of the relevant sentences relative to witnesses, individual concepts, which provide actual values in actual reality in the case of ordinary, extensional discourse, but also, possibly, values in non-actual possibilities jointly considered by various agents, or by agents in possibilities considered by agents etc. Thus, the first sentence of example (i) can be considered true relative to a concept c from sone window w, if c_w is a fish that John catches in any possibility w that satisfies John's hopes. For the subsequent sentence to be true this concept c must also yield something that Mary prepared in any possibility that realizes Mary's plans. As said, these are kind of the minimal truth conditions that any analysis of example (i) should put forward. They are minimal, but we believe structurally sufficient. For, in the first place, it seems to be particularly hard to argue for structurally stronger truth conditions, for all types of sequences like (i); in the second place, it is easy to, on occasion, strengthen such readings by additionally imposing further conditions on the window through which the fish is 'seen'. As we have seen, our language is well equipped to express such additional assumptions.

Once such a mechanism of discourse reference is in place, we can return to the example involving Vulcan, and pick up on the intentional conception of Vulcan that has been introduced in there and assert that it does not exist, i.e., that it has no value in the actual world. The original example $\square^{u\ w}\vee v\,\square_v\,\chi$ will turn out true relative to some conception c, from window w, if c delivers a planet named Vulcan, with all its essential characteristics, in any possibility matching the hypothesis of Urbain le Verrier. This concept can be picked up by a pronoun v in subsequent discourse, and we may observe or claim that it actually has no value, by saying it doesn't exist: $\neg\mathsf{E}v$. Actually, and perhaps surprisingly, this is all there is to it. If asked, but who or what is this v that you deny existence of? One may reply by saying, again, that Urbain le Verrier hypothesized a certain planet with particular characteristics, e.g., by means of $\square^{u\ w}\vee z\,\square_z\,\chi$, and then answer the question that v is that hypothesized planet: $v{=}z$. Surely no ordinary denoting description (or $\imath$-term) will do as an answer, because it would require the actual existence of the thing. A locution like "^{w}the planet that Le Verrier hypothesized" can, we think, only be taken to mean, literally, that there is an actual planet that Le Verrier hypothesized, and this is not what he did. What such a phrase, as an answer to the above question, must be taken to say is, rather, that Le Verrier hypothesized some planet, and that we are 'referring' to *that* hypothetical planet. We can so to speak 'point' at it in a formula, or in a picture, but not in reality.[46]

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46. And if we, for such a purpose, want to allow for reference and quantification over possible objects, that is literally all possible objects, then we face the danger that we increase the

5   Conclusion

Maria Aloni has presented a treatment of, among others, *de re* belief ascriptions and *knowing who* constructions using a system of conceptual covers that successfully applies also to various other logic-philosophical phenomena. In this paper we have presented a more refined system of *Modal Predicate Logic with Conceptual Restricted Quantification* that preserves all the successes of Aloni's applications, that substantially increases its expressive power, and that does without some rather troublesome assumptions that Aloni makes. Troublesome are the assumptions that there not just is, but actually has to be, one rigidly fixed domain of individuals, that everybody is supposed to know this, and that for any of the model's applications to be successful, substantial assumptions from the interlocutors must be taken to hold of necessity; none of the agents discussed in the model is able to even have any doubt about their truth. Considering the examples that are dealt with, these are unwarranted assumptions, with damaging implications.

The gain in expressive power consists in the fact that we can use the formal language to characterize and regiment our conceptual windows. The relevant properties of conceptual windows thus do need not to be specified in the meta-semantics, but all results can be argued for and obtained by classical proof-theoretical means. This is not to say that Aloni's system fails a proof theory; it does have one. It, however, fails the possibility to express the kinds of contingent assumptions one needs to make to successfully apply the model to specific cases.

The system moreover has the potential, a possibility that we wanted to mention in particular, of explicitly incorporating the use of intentional windows, a use that the system already implicitly provides. Our discourses not only provide transparent windows on an external reality, but also provide an intentional view on domains of things non-existent. It is thus able, in principle, to make sense of talk about things non-existent, without attributing them any sort of existence, besides a purely ideal one.

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expressive strength of our language to that of a second order language, with the result that no complete proof theory is possible any longer. See (Hughes & Cresswell 1996, pp. 335–42, 348) for relevant discussion.

Contents

Appendix

Contextually Restricted Quantification The proof theory for extensional, contextually restricted, quantification, can be obtained by, essentially, relativizing the rules of quantification for an inclusive logic, a logic that allows empty domains, and one that does not have to deal with individual constants. (As said, we have adopted a quantificational treatment of names.) Since our restricted quantifiers are superscripted with indices indicative of their domain of quantification, we keep track of these domains by means of superscripts on the variables that are bound this way. For this purpose we assume that, for any domain of quantification c, we have a sufficient (infinite) supply of individual variables $^{c}x_1, \ldots, {}^{c}x_i, \ldots$.[47] Since we have an inclusive logic, existential generalization and universal instantiation are restricted to what are called *declared* variables here. Variables are declared in subderivations serving the use of existentially quantified formulas and the introduction of universally quantified ones. Free variables are not otherwise allowed in any assumptions.

The rules, with the required restrictions, read as follows.[48]

∃-Introduction (I∃)

$$\vdots$$

m. $[^{c}z/x]\phi$

$$\vdots$$

n. $^{c}\exists x\phi$ $\qquad$ [I∃, m]

The variable ^{c}z must count as declared at line m.

∃-Elimination (E∃)

$$\vdots$$

l. $\quad ^{c}\exists x\phi$

$$\vdots$$

m. $[^{c}z/x]\phi$ $\quad$ [ass.]

$$\vdots$$

n-1. ψ

n. $\quad \psi$ $\qquad$ [E∃, l]

The variable ^{c}z may not occur free in assumptions, ϕ or ψ. The variable counts as declared from line m till n.

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47. We also have to keep track of the names under which individuals are introduced, so for any such name N we also assume a sufficient (infinite) supply of individual variables $^{n}z_1, \ldots, {}^{n}z_j, \ldots$. The rules dealing with names will be a variant of the existential rules and we will not actually specify them here.

48. These natural deduction rules here closely correspond to those presented in (Francez 2014, §2). Our system differs in essentially three ways. First, our logic does not make any existential assumptions, so, e.g., it does not follow from $\forall x\phi$ that $\exists x\phi$, as it does in Francez' system. Second, his contexts essentially restrict the domain of quantifiers by explicitly constraining the values of the variables bound. Hence, the system does not allow any substitution of bound variables. Most importantly, third, Francez' contexts always consist of all and only those individuals satisfying a certain, monadic, description. Our contexts can be defined much more specifically. We can stipulate, for instance, that a context c consists of precisely two individuals that love each other, so that we can appropriately speak of the lover in c and the beloved one in c. No such specification of contexts is possible in Francez' system. A more detailed comparison will be left for another occasion.

Defining $\forall x\phi$ as $\neg\exists x\neg\phi$, we can generate, and validate, the rules for $\forall$.

$\forall$-Introduction (I$\forall$)
$\vdots$
m. $^{c}z = {}^{c}z$ [ass.]
$\vdots$
n-1. $[^{c}z/x]\phi$
n. $^{c}\forall x\phi$ [I$\forall$]
The variable ^{c}z may not occur free in assumptions, or in ϕ. The variable counts as declared from line m till n.

$\forall$-Elimination (E$\forall$)
$\vdots$
l. $^{c}\forall x\phi$
$\vdots$
n. $[^{c}z/x]\phi$ [E$\forall$, l]
The variable ^{c}z must count as declared at line n.

A complete system of natural deduction system hosts (i) the usual rules for the usual propositional operators; (ii) rules that allow one to infer the existence of the arguments of atomic predications, including identity statements, but not of their negations; (iii) rules that infer the self-identity of declared variables; (iv) a Fregean version of the Leibniz rule, rendering the extensional indiscernibility of identicals; and finally (v) a set of modal principles, cf., below.

Besides the usual extensional logical principles, the following axioms govern the *modal inferences* licensed in this paper.

$$\vdash \neg\Box^{k}\phi \leftrightarrow \Diamond^{k}\neg\phi \tag{DUAL}$$

(Something not necessary is possibly not.)

$$if \vdash \phi, \; then \vdash \Box^{k}\phi \tag{N}$$

(Possibilities obey logical truths.)

$$\vdash \Box^{k}(\phi \to \psi) \to (\Box^{k}\phi \to \Box^{k}\psi) \tag{D}$$

(Possibilities obey logical principles.)

$$\vdash \Box\phi \to \Box^{k}\phi \tag{C}$$

(If something is any specific kind (k) of possibility, it is a possibility.)

$$\vdash \Box\phi \to \phi \tag{T}$$

$$\vdash \Box\phi \to \Box\Box\phi \tag{4}$$

$$\vdash \Diamond\phi \to \Box\Diamond\phi \tag{5}$$

(As is well-known, the latter three axioms make the accessibility relation $\mathcal{R}^{0}$ an equivalence relation, so that it is a universal relation over the possibilities in the equivalence class of possibilities that the actual world resides in.)

Soundness and completeness of the system of rules can be shown along the lines of (Aloni 2005b, appendix), drawing from the fact that the system here mostly consists in the dropping of assumptions made there.

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