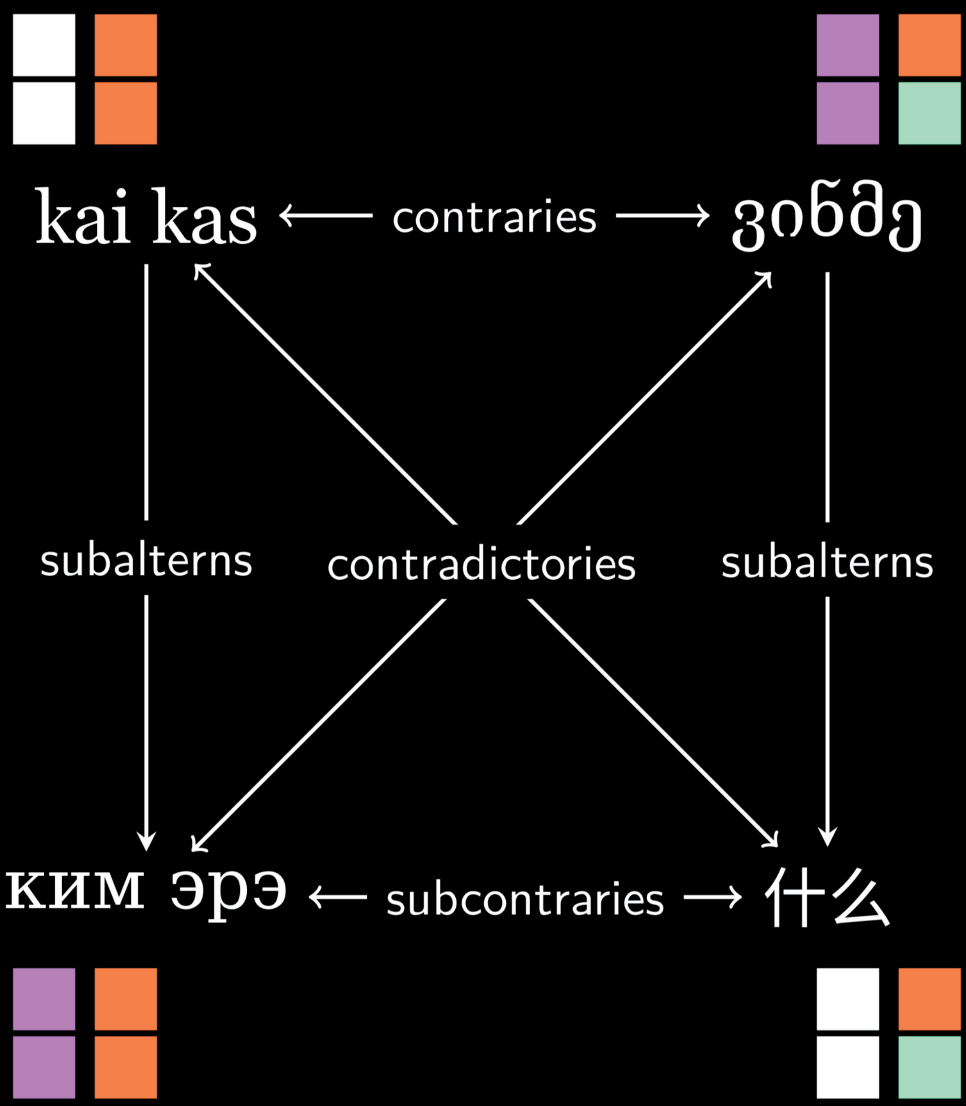
INDEFINITES AND THEIR VALUES



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Indefinites and their values

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Marco Degano
Amsterdam, August 2024

Chapter 1

Prologue

Bob:	*Alice,* ***someone*** *is outside and rang the bell. Do you know him?*
Alice:	*Yes, he's* ***someone*** *I met yesterday at the book club. His name is Josh.*
Bob:	*Oh, every time you go to the book club, you meet* ***someone*** *new. What's his story?*
Alice:	*He's* ***someone*** *from Wisconsin who just moved to town. I have never met* ***someone*** *from Wisconsin before.*
Bob:	*Well,* ***someone*** *from the book club and from Wisconsin is* ***someone*** *we should definitely get to know.*

The dialogue above between Bob and Alice showcases different uses of the English indefinite pronoun *someone*. How indefinite the indefinite *someone* is varies significantly in each case. In Bob's initial statement, *someone* refers to a specific individual unknown to Bob. Alice's response clarifies that she knows this person. In the former case, we say that the indefinite has a 'specific unknown' use, while in the latter it has a 'specific known' use. Bob's remark on the third line does not seem to refer to a specific individual, but it rather establishes a relationship between the event of going to the book club, and meeting a person there, where this person varies each time. We say that in this case the indefinite has a 'non-specific' use. Alice's second use of *someone* hints at partial knowledge about the person, suggesting different degrees of acquaintance. When *someone* interacts with negation, it gives rise to a meaning closer to English *anyone*. Finally, Bob's last statement features *someone* twice: first to indicate an arbitrary person from the book club and Wisconsin, and second to refer to this person.

This dialogue demonstrates the diversity in the uses of the English indefinite *someone*. Not surprisingly, while English can express all these differences using a single form, different languages employ various forms: indefinites associated with the speaker's knowledge like Lithuanian *kai*, indefinites associated with the speaker's lack of knowledge like German *irgend-*, indefinites that can only convey

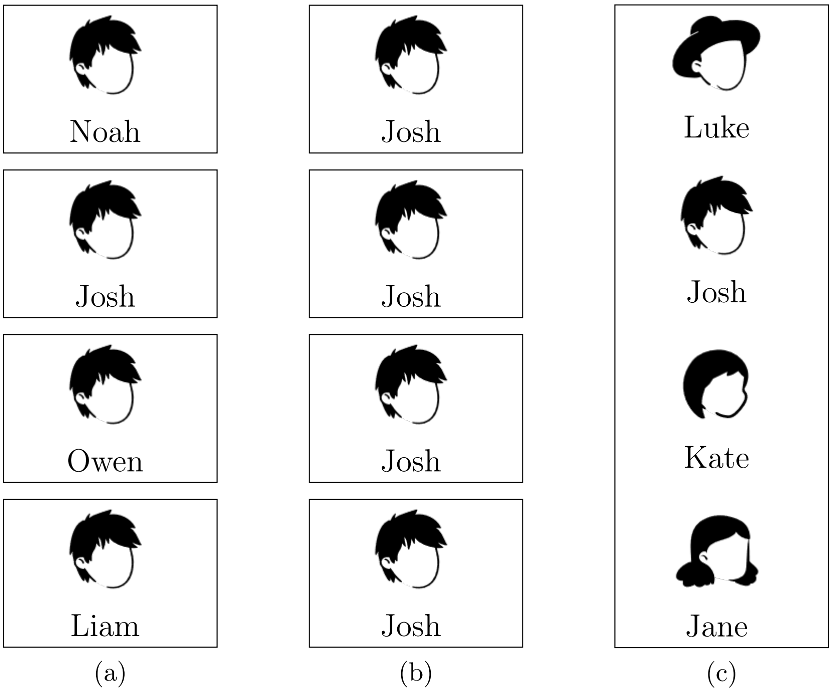


Figure 1.1: Indefinites and their values.

non-specificity like Georgian *me*, and indefinites that can only occur with negation like Italian *alcuno*, among many others.

The main idea behind this work combines two fundamental insights: first, indefinites are associated with a range of values; second, speakers may entertain different possibilities for the state of the affairs in the actual world, reflecting what they know and what they do not know. To illustrate this, Bob's first use of *someone* would be compatible with the picture in Figure 1.1a, where each box corresponds to a possible value for the person who is outside, limited to four for illustrative purposes. In this sense, Bob does not know who that person is, as it could be Noah, Josh, Owen, or Liam. By contrast, Alice is aware of who is outside, and the value of *someone* is constant across all her epistemic possibilities, as in Figure 1.1b. The use of *someone* by Bob in the third line aligns with the picture in Figure 1.1c, where the indefinite is associated with all the values corresponding to the people Alice met at the book club. Note that, in this case, these are not 'possible' values but rather all the actual values that the indefinite receives for the different times Alice went to the book club.

To formalize this characterization, we rely on what we call two-sorted team semantics (2TS). Team semantics has found various applications in linguistics and beyond. The underlying idea is that indefinites are associated with a variable, and a team is composed of a set of assignment functions that assign a value to this variable. The 'two-sorted' part in 2TS means that, in addition to domain-variables

x ranging over individuals, we include world-variables v ranging over possible worlds (ways the world might be). This allows us to represent the information state or epistemic state of the speaker as a collection of possible worlds.

To illustrate this, consider the schematic representations in Table 1.1, which depicts three possible teams T_1, T_2 and T_3. The first column lists the assignments within each team. The variables are shown in the first row, with subsequent rows displaying the values assigned by each assignment. Here, v is a world-variable encoding the information state of the speaker, and x is a domain-variable encoding the values of the indefinite. For instance, the team T_1 contains four assignments i_1, i_2, i_3 and i_4. The assignment i_1 assigns the possible world v_1 to the world-variable v and 'Noah' to the domain-variable x for the indefinite.

T_1	v	x
i_1	v_1	*Noah*
i_2	v_2	*Josh*
i_3	v_3	*Owen*
i_4	v_4	*Liam*

(a)

T_2	v	x
j_1	v_1	*Josh*
j_2	v_2	*Josh*
j_3	v_3	*Josh*
j_4	v_4	*Josh*

(b)

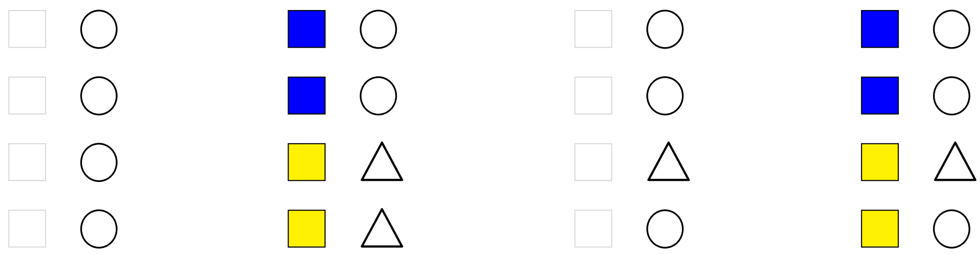
T_3	v	x	z
k_1	v_2	*Luke*	*May*
k_2	v_2	*Josh*	*June*
k_3	v_2	*Kate*	*April*
k_4	v_2	*Jane*	*March*

(c)

Table 1.1: Teams and Variables. Specific Unknown, Specific Known and Non-specific.

Table 1.1a corresponds to the scenario in Figure 1.1a. Each box in Figure 1.1a represents a different possible value for the indefinite, represented in the team by means of the value of x given a value for v. Similarly, Table 1.1b and Figure 1.1b, depict a situation where Alice knows the value of x, which remains constant across all assignments in the team. Table 1.1c stands for Figure 1.1c, where in this case the value of x is associated with more values in a single epistemic possibility, which we can take to be the world representing the actual state of affairs. The value of the indefinite thus varies based on the occasion z on which Alice met x at the book club.

One of the goals of this work is to provide a formal rendering of the distinctions illustrated in Table 1.1. Consider the four pictures in Figure 1.2, which display different combinations of colours and shapes. In picture (a), all objects are circles, regardless of their colour. In picture (b), the shape is fixed relative to the colour: a circle for blue and a triangle for yellow. In the latter case, we can say that the shape depends on the colour or that the shape is a function of the colour. Pictures (c) and (d) encode in a sense the opposite conditions of the first two. To make the shape not fixed, the minimal case in (c) is sufficient, where the shapes vary and the colour is irrelevant. In (d), one colour which is associated with more than one shape, as it is the case for the yellow colour, is sufficient to break the dependence of the shapes on the colours seen in (b).



(a) (b) (c) (d)

Figure 1.2: Constancy and Variation Conditions.

It is not difficult to translate these correspondences between colours and shapes into correspondences between variable values, thereby capturing the contrasts displayed in Table 1.1. For instance, picture (a) in Table 1.2 corresponds to a constant value for the indefinite as in the team in Table 1.1b, which represents specific known cases. A combination of (b) and (c) allows us to encode the team for specific unknown in Table 1.1a, with (b) requiring the value of x to depend on the value of v; and with (c) requiring that there are different values of x. The condition in (d) where the shape changes for at least one colour ('yellow') captures non-specific cases, like in Table 1.1c, as the value of x changes for at least one value of v (v_2, the only value in that case).

These conditions, informally presented using colours and shapes, have been formally studied in dependence logic, which extends first-order logic by incorporating various notions of dependence between variables. Not surprisingly, dependence logic has found applications in areas where reasoning about dependencies is crucial, such as database theory (e.g., query optimization), computer science (e.g., program verification), game theory (e.g., games of imperfect information), causal inference, and linguistics.

This thesis aims to explore further applications of teams semantics and dependence logic to formal semantics, focusing particularly on indefinites. The advantage of a rigorously defined formal system is that it makes clear predictions and lays the groundwork for further extensions of the framework.

We will demonstrate how this approach addresses classical puzzles involving indefinites, such as anaphora, exceptional scope, ignorance and free choice inferences.

We will explore how the basic conditions in Figure 1.2 can be used to capture different kinds of indefinites. The core idea, again, is that different indefinites impose different conditions on the variables they are associated with. We will see how 2TS allows making some language universal claims on the distribution of indefinites cross-linguistically.

Moreover, indefinites often exhibit a high rate of semantic change, and we will investigate how different developmental paths of indefinites can be explained and predicted by studying how the conditions on their values change over time.

Indefinites, thus, have significant *value*, and this does not merely express the fact that they are a legitimate area of study.

Organization of the thesis

This dissertation is designed as a monograph, and ideally, it should be read from beginning to end. However, alternative reading paths are possible. Chapter 2 serves as a concise introduction to indefinites and formal semantics, particularly for phenomena of (non-)specificity. Chapters 3 and 4 constitute the core of the thesis, with Chapter 3 focusing on the formal foundations of two-sorted team semantics (2TS) and Chapter 4 exploring its applications. Both chapters can also be read independently, as Chapter 4 provides informal explanations of some key components of 2TS. Chapter 5, 6, 7, and 8 each address a particular type of indefinite, and readers may choose to focus on a single chapter, consulting Chapter 3 and Chapter 4 when needed. Chapter 9 examines indefinites in sign languages and is best read together with the initial sections of Chapter 4.

Chapter 2: Background and Core Puzzles. In this chapter, we lay down the core terminological distinctions, overview previous approaches in the literature on indefinites, and set up the main puzzles and empirical phenomena we investigate. We focus in particular on indefinites and scope, epistemic specificity, anaphora and cross-linguistic distinctions in marked indefinites.

Chapter 3: Two-sorted Team Semantics. In this chapter, we establish the foundations of 2TS. We discuss the role of teams and sorts in more detail, and define the basic components of 2TS, including different formal conditions on the values of the variable for the indefinite. Having a system which is rigorously defined allows to make clear predictions and ease possible extensions of the framework. This chapter is in part an elaboration of the framework presented in Aloni and Degano (2022), which finds here a more definitive form.

Chapter 4: Indefinites Across Languages. In this chapter, we discuss how 2TS accounts for the typology of (non-)specific indefinites and explain why certain types of indefinites are not attested in terms of complexity and failure of convexity. We also discuss how 2TS deals with modality and negation. Finally, we dedicate one section to the diachronic development of indefinites, highlighting attested diachronic changes and possible predictions. The first part of this chapter takes as a starting point the work done in Aloni and Degano (2022).

Chapter 5: Epistemic Indefinites. This chapter focuses on epistemic indefinites, which are indefinites which signal the speaker's lack of knowledge with respect to the value of the indefinite (so-called ignorance inferences). We discuss the predictions of 2TS, also in relation to previous accounts in the literature. We dedicate one section to the interaction between plurality and ignorance inferences.

Chapter 6: Non-specific Indefinites. This chapter focuses on specific indefinites, which are indefinites which only receive non-specific uses. We relate one interesting relationship between the property of Locality in dependence logic to the distribution of non-specific indefinites. We compare this class of indefinites with so-call dependent indefinites and indefinites which display a negative polarity behaviour. We present a dynamic system of 2TS which can account, among various things, for anaphora. Part of this chapter will be presented at SuB 29 (*Sinn und Bedeutung 29*, Noto, 2024).

Chapter 7: Specific Indefinites. This chapter focuses on specific indefinites, which are indefinites which only receive specific uses. This chapter is relevant as it allows us to make a connection with some of the previous approaches to indefinites and scope and in particular to choice-functional ones. We present a novel perspective on specificity which revisits the proposal made in Chapter 3 and offer both a pragmatic and a semantic explanation to specificity. We also include an overview of various classes of specific indefinites cross-linguistically.

Chapter 8: Free Choice Indefinites. This chapter is dedicated to free choice indefinites. We account for their distribution, and we dedicate a consistent section to the diachronic development of this class of indefinites. We also comment on the insights that 2TS offers with respect to the relationship between universal quantifiers and free choice indefinites. Part of this chapter relates to the work of Degano and Aloni (2021) and includes material presented at FoDS 7 (*Formal Diachronic Semantic 7*, Budapest, 2022) and TbiLLC 2023 (*Tbilisi Symposium on Language, Logic and Computation*, Telavi, 2023).

Chapter 9: Indefinites and Sign Languages. This chapter investigates the realization of indefinites in sign languages and attempts a connection with the way indefinites are accounted in 2TS.

As outlined above, part of this dissertation is also based on Maria Aloni and Marco Degano (2022). "(Non-)specificity across languages: constancy, variation, v-variation". In: *Semantics and Linguistic Theory*. Vol. 32, pp. 185–205. The study in Aloni and Degano (2022) was conceptualized through joint discussions between Maria Aloni and Marco Degano. The writing of the paper was carried out by Marco Degano.

Chapter 2

Background and Core Puzzles

The study of indefinites has played a pivotal role in philosophy, logic, and linguistics, often leading to new theoretical insights or the development of novel formal tools.[1]

Indefinites are associated with many linguistic and philosophical puzzles, and different formal accounts have emerged seeking to capture their properties. Notwithstanding the vast empirical and theoretical landscape, there appears to be no overall agreement on what constitutes an indefinite. The characterization of an indefinite often depends on the specific empirical puzzles being examined and the theoretical framework in which indefinites are situated.

In this chapter, we will clarify the notion of an indefinite from an empirical and formal perspective in Section 2.1 and Section 2.2, respectively. We will then outline some of the core linguistic puzzles pertinent to the aims of the present work in Section 2.3.

2.1 The Empirical Status of Indefinites

There appears to be no working definition or classificatory distinction that universally captures what an indefinite is. A circular definition might describe indefinites as nominal expressions that express indefinite reference, but what counts as indefinite reference is of course theory-dependent. In this section, we offer

[1]Without the claim of being comprehensive, here are some key contributions: Bertrand Russell's paper *On Denoting* in 1905 and his theory of definite versus indefinite descriptions; the referential versus quantificational debate of indefinites (Donnellan 1978; Wilson 1978; Fodor and Sag 1982); the role of indefinites in categorical grammars (Montague 1973) and Generalized Quantifier Theory (Barwise and Cooper 1981); type-shifting (Partee 1986); the dynamic turn and the anaphoric potential of indefinites (Heim 1982; Kamp 1984; Groenendijk and Stokhof 1991), the view of indefinites as choice functions (Reinhart 1997; Kratzer 1998; Winter 1997); the study of marked indefinites across languages (Farkas 2002b; Kratzer and Shimoyama 2002; Chierchia 2013).

some examples of indefinites based on previous literature and typological studies, setting up the main terminological distinctions of the present work.

We follow the conventional distinction between indefinite pronouns, distinguished among different semantic categories, like English *somebody* for the semantic category 'person', and indefinite determiners, like English *some*, combining with nouns. Unless explicitly stated, we will refer to this class of items as indefinites in general.

Across languages, there are two main types of indefinites: indefinites which exhibit morphological similarity to interrogatives (e.g., Georgian *raghats* 'something' formed by the interrogative *ra* 'what' together with the suffix *ghats*) and those related to generic nouns like *thing*/*body* or the numeral *one* together with an indefinite marker (e.g., English *somebody* or *someone*). According to the typological work of indefinites by Haspelmath (1997, 2013), these two classes comprise 85% of the languages considered in the aforementioned studies.[2]

Indefinite pronouns typically occur in a series formed by an indefinite marker, which can be interrogative or generic-noun based. Different series in a language are associated with different distributions and uses. Their morphological makeup (e.g., affixes, particles, reduplication) contributes to their enriched meaning. We will refer to such indefinites as *marked indefinites*, where markedness refers to an underlying distinction in distribution and uses. For instance, Table 2.1 displays three indefinite series in Polish: the general *-ś*-series, comparable to English *some*, the free choice *-kolwiek*-series, comparable to English *any*, and the negative *ni*-series, comparable to English *no*, where *żaden*, an expression from a different root, is used for the negative determiner. In this case, we would say that *-kolwiek* is marked with respect to its free choice uses, since it can only have such usages compared to the general *-ś*. We will subsequently revisit the notion of 'marked indefinite', particularly with regard to the contrasts pertinent to the present work.

A related class of items concerns so-called indefinite articles, such *a book* in English. In many languages, the numeral 'one' is the source of the indefinite article (e.g., Italian *un(o)* 'one') (Dryer 2013; Givón 1981), while in other languages, indefinite articles are not related to the numeral 'one', which may still admit generic-like readings. In what follows, we will use the label 'plain indefinite' to refer to such indefinites. The relationship between indefiniteness and the

[2]In particular, 60% of the languages have interrogative-based indefinites, while 25% have generic-noun-based indefinites. Other languages employ dedicated expressions unrelated to interrogative or generic nouns, and other languages make use of a mix of interrogative-based and generic-noun-based indefinites. Importantly, the notion of an indefinite marker for generic-noun-based indefinites should deserve better scrutiny. For instance, the Italian *qualcuno* 'someone' is classified as a generic-noun-based indefinite due to the presence of *uno* 'one'. However, the indefinite marker *qualc* derives from *qual* + *che*. The former, *qual*, comes from Latin *qualis* (an interrogative with the meaning 'of what kind'); the latter, *che*, comes from Latin *quis* (an interrogative with the meaning 'who'/'what'). This implies that the Italian *qualcuno* shows a strong affinity with interrogative words, even though it is classified as generic-noun-based indefinite in Haspelmath (2013)'s typological work.

Semantic category	Interrogative	-*ś*-series	-*kolwiek*-series	*ni*-series
Person	*kto*	*kto-ś*	*kto-kolwiek*	*ni-kto*
Thing	*co*	*co-ś*	*co-kolwiek*	*ni-co*
Quality	*jaki*	*jaki-ś*	*jaki-kolwiek*	*ni-jaki*
Place	*gdzie*	*gdzie-ś*	*gdzie-kolwiek*	*ni-gdzie*
Time	*kiedy*	*kiedy-ś*	*kiedy-kolwiek*	*ni-kiedy*
Manner	*jak*	*jak-ś*	*jak-kolwiek*	*ni-jak*
Determiner	*który*	*który-ś*	*który-kolwiek*	*żaden*

Table 2.1: Polish Indefinite Series (Haspelmath 1997, p. 271).

numeral 'one' will be revisited in Chapter 7, when discussing so-called 'specific indefinites'. It should also be noted that some languages do not have a definite or an indefinite article, relying on the context to disambiguate the (in)definiteness of bare nouns.

Lastly, there are indefinite constructions which do not fit in any particular category, but they have been studied for their particular relevance in linguistics and formal semantics. For example, the English construction *a certain book* has been examined in the context of indefinites and scope, while *a different book* has been studied alongside its counterpart expression *the same book* (Barker 2007). We call these indefinites *special constructions*.[3]

These terminological distinctions are summarized in Table 2.2, together with relevant examples.

	Unmarked Indefinite	Marked Indefinite	Plain Indefinite	Special Construction
English	*someone*	*anyone*	*a book*	*a certain book*
Italian	*qualcuno*	*qualunque*	*un libro*	*un certo libro*
Dutch	*iemand*	*wie dan ook*	*een boek*	*een bepaald boek*

Table 2.2: Main Terminological Distinctions.

2.2 The Formal Status of Indefinites

As highlighted at the beginning, the study of indefinites has often led to the emergence of novel theoretical insights, resulting in various perspectives on their

[3]We do not classify these indefinites as 'marked indefinite', since we consider a marked indefinite to have specific morphological makeups.

formal representation. Not surprisingly, in their introductory handbook chapter on indefinites, Brasoveanu and Farkas (2016) observe that the most appropriate way to present indefinites is to provide a broad summary of solutions to the various problems they raised in the literature. We will take a similar approach for the contrasts examined in this work in Section 2.3. Before that, we will attempt some broad remarks on the formal status of indefinites. In Chapter 3 and Chapter 4, we will illustrate how all these components are integrated in 2TS.

2.2.1 Indefinites as Existential Quantifiers

One of the earliest significant discussions involving indefinites is Russell (1905)'s theory of definite and indefinite descriptions. According to Russell, indefinite descriptions like (1-a) involve existential quantification, as in (1-b). By contrast, definite descriptions like (2-a) contain an additional uniqueness requirement.[4]

(1) a. A desk is black.
 b. $\exists x(D(x) \wedge B(x))$

(2) a. The desk is black.
 b. $\exists x(D(x) \wedge \forall y(D(y) \rightarrow x = y) \wedge B(x))$

This conceptualization paved the way to classical generalized quantifier theory starting from the work of Montague (1973) and further developed by Barwise and Cooper (1981) and Keenan and Stavi (1986), which exerted a significant influence in the formal semantics tradition. Under this account, indefinites and definites are both subtypes of generalized quantifiers with the latter having a uniqueness requirement as opposed to the former.

To make this more explicit, in generalized quantifier theory (see e.g. Westerståhl 2019 for an overview), a basic determiner can be viewed as a relation between two sets of entities. For instance, (1) corresponds to a relation between the set of desks and the set of black things. More formally, we can view such quantifiers as binary relations over subsets of a universe of individuals which we call M, given a model $\mathcal{M}$.

In particular, an indefinite like *a desk* or *some desk* in example (1-a) can be viewed as requiring that the set of desks intersected with the set of black things is non-empty (i.e., that there is something which is both a desk and black). Thus, given two sets $A \subseteq M$ and $B \subseteq M$, indefinites like *a* or *some* amount to (3-a).

Definites, on the other hand, presuppose that there is a unique object satisfying a certain property. For instance, in (2), that there is a unique desk and this

[4]Frege's early perspective on this matter is that expressions with the definite article point to an *object*, while expressions with an indefinite article indicate a *concept*. This view is expressed in Frege (1950, $51) or in Frege, Geach, and Black (1951). It is not immediate how this characterization could extend to uses of an indefinite like in 'A man is walking.'.

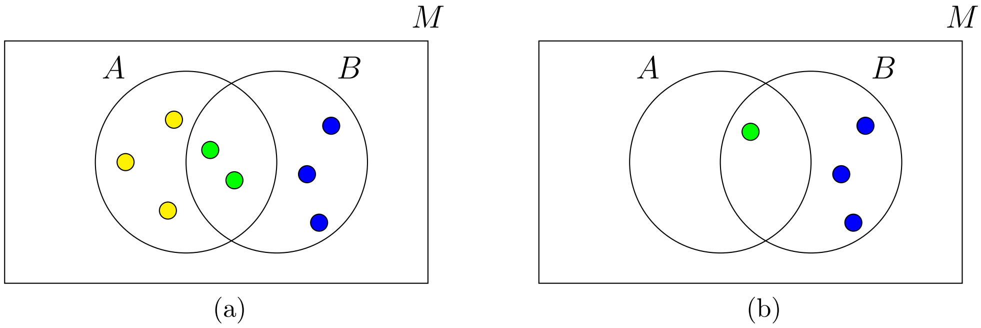


Figure 2.1: Indefinites versus Definites in Generalized Quantifier Theory.

is black. Formally, this can be captured as in (3-b), adapted from Barwise and Cooper (1981), where the failure of uniqueness leads to undefinedness.

(3) a. $a_{\mathcal{M}}[A, B]$ iff $A^{\mathcal{M}} \cap B^{\mathcal{M}} \neq \varnothing$

b. $the_{\mathcal{M}}[A, B]$ iff $\begin{cases} A^{\mathcal{M}} \subseteq B^{\mathcal{M}} & \text{if } |A^{\mathcal{M}}| = 1 \\ \text{undefined} & \text{otherwise} \end{cases}$

Generalized quantifiers are useful for analysing more complex and nuanced expressions about quantities (e.g., *most*, *many*, *exactly n*, *infinitely many*, ...) and have led to an influential research agenda (van Benthem 1984; van Benthem and Meulen 1985; Peters and Westerståhl 2006; Keenan and Westerståhl 2011; Szymanik 2016). What is relevant for our purposes is that under this view, indefinites are treated as existential quantifiers, and they differ from definites in being non-unique.

2.2.2 Indefinites as Choice Functions

While the previous approach takes indefinites to be existentials, another perspective is to view indefinites as arbitrary witnesses of a formula. This view has been proposed by David Hilbert and his school in the early half of the 20th century within the context of the so-called *Epsilon Calculus* (Hilbert and Bernays 1939). This approach has then found several applications in the treatment of indefinites in linguistics in Egli and Heusinger (1995), Viol (1999), and von Heusinger (2000).

In Hilbert's epsilon calculus, a term-forming operator ϵ is used to construct a term like $\epsilon_x A$, which corresponds to some x which satisfies A, if there is one and an arbitrary object otherwise. As a result, the case in (4), which received the logical rendering in (4-a) in the previous approach, is now analysed as (4-b) using epsilon terms.

(4) A desk is black.

a. $\exists x(D(x) \wedge B(x))$

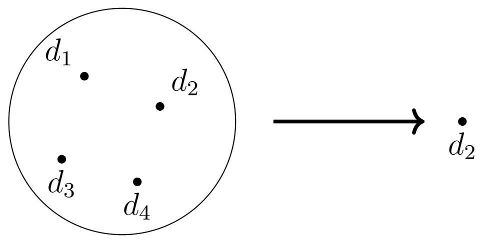


Figure 2.2: Indefinites as Choice Functions.

b. $B(\epsilon_x D)$
c. $\exists f\ B(f(D))$

There is of a natural correspondence between the ϵ operator and choice functions analyses of indefinites (Reinhart 1997; Winter 1997; Kratzer 1998), as pointed out, for instance, by Gratzl and Schiemer (2017). Given a set in the powerset of M, a choice function assigns an element from that set or an arbitrary element in M if that set is empty. We can schematically represent this as follows:

$$f(A) = \begin{cases} d \in A^{\mathcal{M}}, & \text{if } A^{\mathcal{M}} \neq \varnothing \\ d \in M, & \text{otherwise} \end{cases}$$

We will return to choice functions in Chapter 7, but here we note that indeed we can equivalently view Hilbert's terms $\epsilon_x A$ as $f(d \in M : d \in A^{\mathcal{M}})$ for some choice function f. Most importantly, indefinites are not taken to be existential quantifiers in the sense delineated in the previous section, rather they seem to be devices to refer to an (arbitrary) element.

2.2.3 Indefinites and Dynamics

While the classical treatment of indefinites as existential quantifiers focused on the definite versus indefinite distinction, another perspective is offered by dynamic semantic approaches (Karttunen 1977; Kamp 1984; Heim 1982; Groenendijk and Stokhof 1991; Dekker 1993; Groenendijk, Stokhof, and Veltman 1996). In this view, definites signal familiarity in the discourse, while indefinites introduce novel information. An indefinite in most of these approaches introduces a variable that can be bound outside the syntactic scope of the indefinite, allowing for proper anaphoric relationships with pronouns beyond the scope of the indefinite. The minimal contrasts in (5) suffice to illustrate this point.

(5) A book$_x$ is on the desk.

a. It$_x$ is heavy.
b. The book$_{x/?y}$ is heavy.
c. A book$_{\#x/z}$ is heavy.

Given the sentence in (5), some possible continuations are displayed in (5-a–c), assuming that the indefinite introduces the variable x. An explicit pronominal element like *it* clearly refers to the book previously introduced. The use of the definite is also associated with the book introduced, which is now 'familiar' in the discourse. A less immediate reading would also associate *the book* with another book salient in the discourse or context. Finally, another instance of the indefinite does not allow referring to the previous instance of *a book*, but rather it introduces a novel variable in the discourse.

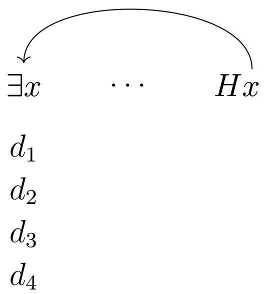


Figure 2.3: Indefinites and Dynamics.

Different treatments and extensions of this basic idea and the basic frameworks cited above have emerged over the years (van den Berg 1996; Aloni 2001; Dekker 2004; Nouwen 2003; Brasoveanu 2007; Roelofsen and Dotlačil 2023).[5] The key takeaway is that, under this view, indefinites contribute novel information to the discourse.

2.3 The Core Puzzles

In this section, we consider some core puzzles and distinctions that will be relevant in subsequent chapters. Specifically, we will focus on the scope of indefinites in Section 2.3.1, the interaction between indefinites and epistemic inferences in Section 2.3.2, and marked indefinites in Section 2.3.3. While each of these topics merits extensive discussion, we will concentrate on the core observations here and address additional empirical and theoretical points as they become relevant throughout the dissertation.

2.3.1 Indefinites and Scope

A seminal puzzle concerning indefinites is their ability to take scope freely with respect to other operators. Example (6) is a canonical case illustrating the scope

[5] Note that in most of these views, indefinites still range over ordinary individuals, but formulations integrating a dynamic notion of meaning and choice functional analyses of indefinites are possible (von Heusinger 2000).

flexibility of indefinites with respect to other operators (Fodor and Sag 1982; Farkas 1981; Reinhart 1997).

(6) a. If someone reads this dissertation, Marco will be happy.
✓(if > ∃) ✓(∃ > if)
b. If everyone reads this dissertation, Marco will be happy.
✓(if > ∀) ✗(∀ > if)

Example (6-a) is ambiguous between two readings. In the first, *someone* takes scope over the *if*-clause. In the second, *someone* is interpreted within the *if*-clause. For a universal quantifier like in (6-b), only the latter reading is available. In other words, (6-b) cannot be interpreted as conveying that for every x, if x reads this dissertation, Marco will be happy.

The availability of such readings is particularly remarkable, as the indefinite occurs in the antecedent of a conditional, which is commonly assumed to be a syntactic island. This implies that the reading in which the indefinite scopes over the *if*-clause cannot be immediately captured by syntactic movement. Fodor and Sag (1982) proposed that an indefinite like *someone* is ambiguous between a quantificational and a referential reading. In the reading where the indefinite appears to receive scope outside its syntactic environment, the indefinite must be interpreted referentially, thus receiving the widest scope possible.

While Fodor and Sag (1982)'s proposal accounts for the contrasts in (6), Farkas (1981) noted that such account cannot deal with intermediate readings of indefinites when multiple operators are present in the sentence. For instance, in example (7), the indefinite occurs in a syntactic island formed by the relative clause. The sentence admits a reading where the indefinite *a student* is interpreted at an intermediate position between the two universal quantifiers, which is not compatible with Fodor and Sag (1982)'s widest scope proposal.

(7) Every teacher had to read every essay that was written by a student.
a. Narrow Scope: every teacher > every essay > a student
b. Intermediate Scope: every teacher > a student > every essay
c. Wide Scope: a student > every teacher > every essay

Importantly, we underline that the structures in (6) and (7) make it difficult to argue that syntactic movement is a viable explanation. This implies that any potential analysis should maintain the indefinite in situ.

These observations led to an influential research agenda examining the scope of indefinites and solutions to these empirical observations. Some influential proposals include choice functional analyses, which we will explore in Chapter 7, and analyses resulting from insights from independence-friendly logic (Brasoveanu and Farkas 2011), which we review in Chapter 4 and which have been instrumental in the development of `2TS`.

In this work, we will adopt the term (scopal) specificity to refer to wide scope readings and (scopal) non-specificity to refer to non-wide scope readings. This clarificatory remark is important in light of the distinctions between scopal specificity, partitive specificity, and epistemic specificity discussed in Farkas (1994), which we outline here before moving to the next section.

Scopal specificity relates to the distinction we just outlined. Partitive specificity pertains to the possibility of indefinites having as possible values a subset of the possible values of a previously introduced referent or a salient restricted domain in the discourse. For instance, in (8), the domain over which the indefinite *someone* ranges is clearly restricted to the people in the lift.

(8) There were many people in the lift and someone fainted.

We will not address partitive specificity in these introductory remarks, even though the framework we will develop can potentially account for such data. In the next section, we will focus on epistemic specificity. However, as mentioned, we will reserve the term specificity for scopal specificity. We will thus use the label ‘known’ for epistemic specificity and ‘unknown’ for epistemic non-specificity.

2.3.2 Indefinites and Knowledge

In examples like (9), the indefinite *a book* can be interpreted in two salient ways: (i) the speaker knows which book; (ii) the speaker does not know which book. We call the former the ‘known’ reading of the indefinite and the latter the ‘unknown’ reading of the indefinite.

(9) A book received good reviews.

Most importantly, the known vs unknown contrast can combine with scopal specificity. To see this, consider the example in (10), where the indefinite *a book* is occurring within the scope of the universal quantifier *every student.* There are three salient readings: two in which the indefinite has a wide scope reading with respect to the universal quantifier, and this specific book can be known or not known to the speaker. And there is also a third reading where the indefinite possibly co-varies with each student.[6]

(10) Every student gave good reviews to a book.
  a. Specific known: There is a book such that every student gave to this book good reviews. The speaker knowns which book.
  b. Specific unknown: There is a book such that every student gave to this book good reviews. The speaker does not know which book.

[6] Under this reading, the value of the indefinite is not known in the sense that it is not fixed. But it is easy to think of more complex forms of knowledge, like knowing the mapping between each student and the book. These readings can be properly modelled, but the core distinctions are the ones displayed in (10).

c. Non-specific: Every student gave good reviews to a book, possibly a different one for each student.

This implies that a satisfactory analysis should be able to account for the integration of scope and epistemic distinctions in a comprehensive fashion.

2.3.3 Marked Indefinites

We have observed that indefinites are associated with different scope and epistemic readings. All the examples we considered involved simple indefinites like *a book* or *some book*, which, in principle, allowed all possible readings.

As discussed in Section 2.1, indefinites vary significantly in form and meaning across languages (Haspelmath 1997; Kratzer and Shimoyama 2002; Farkas 1997, 2002b,a; Jayez and Tovena 2002; Partee 2005; Yanovich 2005; Ebert and Hinterwimmer 2012; Chierchia 2013; Alonso-Ovalle and Menéndez-Benito 2015). We will return to this point in more detail in Chapter 4. Here, we illustrate two relevant examples: the German *irgend-* and the Russian *-nibud'*.

German *irgend-* cannot receive 'known' readings, and it is thus incompatible with the 'guess who?' continuation in (11):

(11) Irgendein Student hat angerufen. #Rat mal wer?
some student has called. guess who?
'Some (unknown) student called. #Guess who?'

Russian *-nibud'* is infelicitous in episodic contexts and can only be interpreted non-specifically in interaction with other operators:

(12) a. #Ivan včera kupil kakuju-nibud' knigu.
Ivan yesterday bought which-INDEF. book.
b. Kazhdyj student včera kupil kakuju-nibud' knigu.
every student yesterday bought which-INDEF. book.
'Every student bought some (non-specific) book yesterday.'

Typological research has shown that indefinites exhibit diverse distributions across languages. One significant study is Haspelmath (1997)'s typological analysis of indefinite pronouns. Haspelmath (1997) developed a semantic map where nodes represent different functions of indefinite pronouns. We will return to the notion of 'function' in Chapter 3.

Figure 2.4 provides an example for Russian. This language is illustrative as it features various indefinites with some core distinctions relevant to our study. The pertinent functions central to this work are specific known, specific unknown, and non-specific. As shown in the map in Figure 2.4, *koe-* can only be used for specific unknown. The indefinite *-to* can be used for both specific unknown and non-specific, while the indefinite *-nibud'* is restricted to non-specific uses.

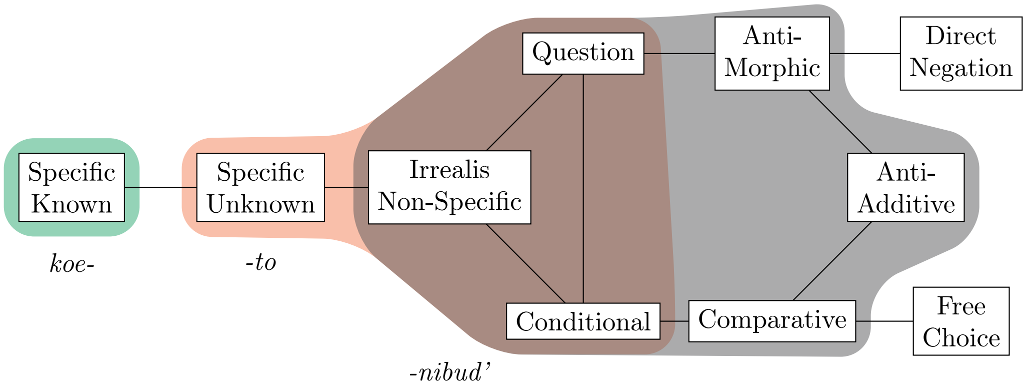


Figure 2.4: Haspelmath map of Russian *koe-*, *-to* and *-nibud'*.

An important question is how these forms relate to each other and what semantic tools are needed to capture them comprehensively. Kratzer and Shimoyama (2002) and Kratzer (2005) provide an account within the theory of Alternative Semantics, originally developed for questions (Hamblin 1973). In this framework, expressions denote sets of alternatives, and indefinites denote sets of individual alternatives. These alternatives combine with other elements in the clause until an operator selects them, determining their distribution. For instance, Menéndez-Benito (2005) and Aloni (2007) show that free choice indefinites are associated with a universal [∀] operator over propositional alternatives. Alternative Semantics offers several advantages, including a direct parallelism between questions and indefinites and an account of negative concord, polarity, and free choice indefinites.

Another approach is proposed by Alonso-Ovalle and Menéndez-Benito (2010) and Chierchia (2013), who derive the enriched meaning of marked indefinites via pragmatic inferences resulting from operations of exhaustification. For instance, Chierchia (2013) suggests that indefinites are associated with different types of alternatives (scalar, domain, degree, . . .) and generate relevant readings through various exhaustification operations on these alternatives. This approach establishes interesting connections between polarity phenomena and indefinites and clarifies the relationship between indefinites and disjunction.

Throughout this work, we will revisit some of these approaches and assess their relationship with our analysis. Here, we point out that these approaches cannot fully address the interaction between scopal and epistemic specificity contrasts, as outlined in previous sections, and they do not provide a systematic account of functions on the left part of the Haspelmath map with respect to their cross-linguistic variation.

A relevant insight comes from Farkas and Brasoveanu (2020), who distinguish between anti-variation determiners, which impose stability on the values of the indefinite, and pro-variation determiners, which impose variability. This observation will be of crucial importance for the development of `2TS` in the next chapter. The task ahead is to formalize stability/variability in terms of scope versus stability/variability in terms of knowledge.

Several interesting questions arise: What is the most common distribution of marked indefinites, and why are some not realized? How do indefinites and marked indefinites interact with negation? How do (non-)specificity distinctions interact with multiple operators? What is the diachronic relationship between these types of indefinites? How do we account for other types of indefinites, such as free choice indefinites? Most of these questions will be addressed in the following chapters.

2.4 Conclusion

This chapter set the stage for the next chapters in this work. We began by empirically exploring the landscape of indefinite forms and establishing some key terminological distinctions. We then highlighted a number of perspectives on indefinites: (i) indefinites as existential quantifiers; (ii) indefinites as choice functions; (iii) the dynamicity of indefinites. All these elements will play a role in `2TS`. In particular, (i) we will treat indefinites as existential quantifiers; (ii) the incorporation of dependence atoms will allow us to associate indefinites with functions; (iii) the introduction of new variables will be modelled upon similar notions that have been discussed in dynamic semantics, and a dynamic version of `2TS`, as we will see, can be easily given.

Chapter 3

Two-sorted Team Semantics (2TS)

In this chapter, we present the core components of two-sorted team semantics (2TS), the foundational framework of this dissertation. 2TS is a team semantics, where formulas are evaluated with respect to sets of assignments. It is termed 'two-sorted' because it includes both a sort for individuals and a sort for worlds, and it captures the relationships between different variables in the team using dependency conditions from the tradition of dependence logic.

In Section 3.1, we will explore the team-based nature of 2TS, the role of world variables, and explain how teams represent the information states of speakers. Section 3.2 will discuss how teams can be extended with new discourse information and introduce the dependence and the variation conditions, also known as atoms. In Section 3.3, we will present the semantic clauses of 2TS. Finally, in Section 3.4, we will introduce two additional atoms - inclusion and independence atoms - which will be crucial for further applications of 2TS. Additionally, we will discuss alternative notions of existential quantifiers from the literature, specifically the inquisitive and independence existentials.[1]

3.1 Teams and Second Sort

Traditional logical systems, such as classical propositional logic, modal logic, or first-order logic, interpret formulas with respect to *single evaluation points*. In contrast, team semantics interprets formulas with respect to sets of points rather than individual ones. These evaluation points can be valuations (as in propositional team logic Yang and Väänänen 2017), assignments (as in first-order

[1]Part of this chapter is based on Maria Aloni and Marco Degano (2022). "(Non-)specificity across languages: constancy, variation, v-variation". In: *Semantics and Linguistic Theory*. Vol. 32, pp. 185–205. In particular, Section 3.2 and Section 3.3 (adapted and expanded). The study was conceptualized through joint discussions between Maria Aloni and Marco Degano. The writing of the paper was carried out by Marco Degano.

team semantics Galliani 2021a; Väänänen 2007a), or possible worlds (as in team-based modal logic Aloni 2022; Lück 2020). This *set* of evaluations is typically referred to as a *team*. As we will see, extending the interpretation procedure to sets of assignments is useful in cases where the relationships between these assignments are of interest.

The first formulation of a team semantics is attributed to Wilfrid Hodges (Hodges 1997), who provided a team semantics for the independence-friendly logic of Hintikka and Sandu (1989). It should be noted, however, that early intuitions about the role of sets of evaluations points and the avenues they open were present in different forms in previous work. To a certain extent, the transition from propositional logic and single valuations into modal logic and Kripke models can be subsumed under the same tendency (Galliani and Väänänen 2014). For approaches in the field of formal semantics, some forms of dynamic semantics (Groenendijk and Stokhof 1991; van den Berg 1996; Groenendijk, Stokhof, and Veltman 1996; Veltman 1996), based on early systems which shared a similar team-like approach (Heim 1982; Kamp 1984), modelled formulas as relations between assignments or sets of assignments, or as functions from an information state to another.[2]

In this section, we will present how the team layer is encoded in 2TS and the role of worlds as second sort in Section 3.1.1. We will then discuss how teams represent information states of speakers (or relevant agents) in Section 3.1.2.

3.1.1 Team Layer and Worlds

2TS is a first-order team semantics where teams are sets of assignment functions. For a simple example, consider the team T depicted in Table 3.1. T consists of four variable assignment functions: i_1, i_2, i_3, and i_4. Table 3.1 shows the values these assignments assign to the variables x and y. In this simplified team, it holds that $y = x^2$, but not that $x = y$. This example illustrates how teams can encode relationships among variable assignments, as the fact that the value of y is the square of the value of x.

We will work with a *two-sorted* first-order framework, with two sorts of entities, individuals in D and possible worlds in W, with variables ranging over each set. For the sake of example, $P(x, w)$ is a formula where x is an individual variable, while w is a world variable.

Integrating modal or 'intensional' information in a first-order system involves numerous design choices motivated by logical, philosophical, and linguistic considerations (Gamut 1991a,b; Cresswell 1990). One standard approach is to define truth at a world relative to an assignment function, typically formalized using

[2] The dissertation of Martin van den Berg (van den Berg 1996) deserves an important mention. In van den Berg (1996, ch. 5), the concept of a state as sets of assignments is introduced and a particular notion of dependence among variables in a state is discussed. We will return to this in Section 3.4 and in Chapter 6.

T	x	y
i_1	1	1
i_2	2	4
i_3	3	9
i_4	4	16

Table 3.1: Team $T = \{i_1, i_2, i_3, i_4\}$. In this and subsequent tables, assignments are indicated in the leftmost column (sometimes omitted), and the variables in the team are shown in a grey row.

world-assignment pairs. The approach we take here is more liberal in a sense, as we assume that all reference to worlds is made through variables.

In particular, as said, this approach is inspired by the Two Sorted Type Theory (Ty2) formulation of Montague's intensional logic discussed in Gallin (1975).[3] This leads us to adopt the view that formulas will be evaluated with respect to a world variable.[4]

A two-sorted language is quite expressive, allowing us to formulate a variety of statements that might not have a natural language counterpart but are of philosophical significance. For instance, consider the distinction between $P(x, w)$, which roughly requires x to satisfy P in w, and $P(x)$, which is more akin to across-world predication.

We define the language of our logical system as follows. In the rest of this section, we will clarify the underlying idea behind a two-sorted team semantics and the language defined below.[5]

3.1.1. Definition (Language). Given a first-order signature σ (composed of individual constants $c \in \mathcal{C}$, and predicates $P^n \in \mathcal{P}^n$ with $n \in \mathbb{N}$), and individual variables $z_d \in \mathcal{Z}_d$ and world variables $z_w \in \mathcal{Z}_w$, the terms and formulas of our language are defined as follows:[6]

$$t ::= c|z_d|z_w$$

$$\phi ::= P(\vec{t})|\neg P(\vec{t})|t = t'|\neg t = t'|dep(\vec{z}, \vec{z})|var(\vec{z}, \vec{z})|\phi \vee \psi|\phi \wedge \psi|\exists_{strict} z\phi|\exists_{lax} z\phi|\forall z\phi$$

[3]Effectively, this reduces an intensional theory to an extensional one by having two sorts of individuals.

[4]Sometimes, we adopt the notational convention, in our examples but in the definition of our language, to add the world variable as the last 'argument' separated by a semicolon (e.g., $P(x; w)$). This is particularly relevant for cases involving more than one individual variable.

[5]The attentive reader might have already noticed that negation is only defined for first-order literals and identity. We are assuming that all formulas are in negation normal form (i.e., all negations occur in front of atomic formulas). Negation is a complex issue in dependence logic for reasons of expressive power and definability (Väänänen 2007a; Burgess 2003). We will return to it in Section 4.8 of Chapter 4.

[6]$\vec{t}$ stands for an arbitrary sequence $t_1, \dots, t_n$.

A variable x in a formula is *bound* if it occurs in the scope of a quantifier, and otherwise it is *free*. We indicate the set of all free variables of a formula ϕ with $Free(\phi)$. A closed formula or a *sentence* is a formula without free variables.

3.1.2. Definition (Two-sorted model). A two-sorted model is a triple $M = \langle D, W, I\rangle$ composed of a domain of individuals $Dom_d(M) = D$, a domain of worlds $Dom_w(M) = W$, and an interpretation function I assigning an element of D to every individual constant symbol and sets of n-tuples constructed from W and D to every n-ary predicate symbol.

A two-sorted first-order team is a set of assignments mapping world variables to elements of W and individual variables to elements of D. We first define a variable assignment and then a team.[7]

3.1.3. Definition (Variable Assignments). Given a two-sorted first-order model $M = \langle D, W, I\rangle$ and a finite set of variables $Z = Z_d \cup Z_w$, an assignment i is a function with domain Z s.t. $i = i_d \cup i_w$ for some $i_d \in D^{Z_d}$ and $i_w \in W^{Z_w}$. For any variable z_* and any element e_* with $* \in \{d, w\}$, we write $i[e_*/z_*]$ for the assignment function with domain $Z \cup \{z_*\}$ s.t. for all variable symbols $l \in Z \cup \{z_*\}$:

$$i[e_*/z_*](l) = \begin{cases} e_* & \text{if } l = z_* \\ i(l) & \text{otherwise} \end{cases}$$

For every assignment i, every sequence $\vec{e} = e_1, \ldots, e_n$ and $\vec{z} = z_1, \ldots, z_n$, we write $i[\vec{e}/\vec{z}]$ as an abbreviation for $i[e_1/z_1] \ldots [e_n/z_n]$.

A team in our framework is, as said, a set of variable assignments. We define the notion of a team in Definition 3.1.4 and give an illustration in Table 3.2.

3.1.4. Definition (Team). Given a two-sorted first-order model $M = \langle D, W, I\rangle$ and a set of variables $Z = Z_d \cup Z_w$, a team T over M with domain $Dom(T) = Z$ is a set of assignments i with domain Z.

We also introduce the following notions and operations on teams that will prove to be useful in the rest of this work.[8]

3.1.5. Definition (Projection). Given a team T and a sequence of variables $\vec{z}$ constructed from $Dom(T)$, the projection of T with respect to $\vec{z}$, $T(\vec{z})$, is defined as:

$$T(\vec{z}) = \{i(\vec{z}) : i \in T\}$$

[7] To keep the definitions general, we indicate the sort in the subscript. z_d and z_w will be individual and world variables respectively. Similarly, e_d will be an element of D and e_w an element of W. When the type of variable is clear from the context, we often omit the subscript.

[8] Here and in the following, we write $i(\vec{z})$ as an abbreviation for $i(z_1)i(z_2)\ldots i(z_n)$ for a relevant sequence of the relevant length.

T	v	x
i_1	v_1	d_1
i_2	v_2	d_2

Table 3.2: Example of a two-sorted first order team $T = \{i_1, i_2\}$ with domain $dom(T) = Z = \{v, x\}$ over a model M with $D = \{d_1, d_2, \ldots, d_n\}$ and $W = \{v_1, v_2, \ldots, v_n\}$.

3.1.6. Definition (Subteam). Given a team T, a model M and of sequence of variables $\vec{z}$ constructed from $Dom(T)$, a sequence of entities constructed from $Dom_d(T)$ and $Dom_w(T)$, the subteam of T where $\vec{z} = \vec{e}$, $T_{\vec{z}=\vec{e}}$, is defined as:

$$T_{\vec{z}=\vec{e}} = \{i \in T : i(\vec{z}) = \vec{e}\}$$

3.1.7. Definition (Restriction (Galliani 2012a)). Given a team T and a set of variables $V \subseteq Dom(T)$, the restriction T with respect to V, $T_{\upharpoonright V}$ is defined as

$$T_{\upharpoonright V} = \{i_{\upharpoonright V} : i \in T\}$$

where $i_{\upharpoonright V}$ is the assignment i' with domain V s.t. $i(z) = i'(z)$ for all $z \in V$.

T	v	x	y
i_1	v_1	d_1	d_1
i_2	v_1	d_2	d_1
i_3	v_2	d_3	d_2
i_4	v_2	d_4	d_2

(a)

$T_{v=v_1}$	v	x	y
i_1	v_1	d_1	d_1
i_2	v_1	d_2	d_1

(b)

$T_{\upharpoonright\{v,y\}}$	v	y
i_1	v_1	d_1
i_3	v_2	d_2

(c)

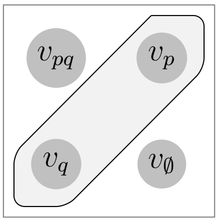
Table 3.3: Illustration of Subteam $T_{v=v_1}$ and Restriction $T_{\upharpoonright\{v,y\}}$ for team T. For projection, $T(v) = \{v_1, v_2\}$.

3.1.2 Teams as Information States

We view teams as representing the information states of speakers. This characterization is typical of dynamic semantics (Groenendijk, Stokhof, and Veltman 1996; Veltman 1996). It is also a feature of recent team-based frameworks like inquisitive semantics, developed by (Ciardelli, Groenendijk, and Roelofsen 2018); and Bilateral State-based Modal Logic (BSML), developed by (Aloni 2022).[9] For

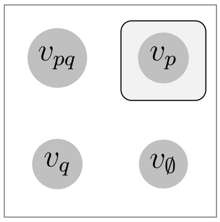
[9] Representing the information of an agent by means of a set of relevant possibilities stems from the early work of Hintikka (1962) and it is a typical modelling assumption in logics which deal with knowledge and belief, where such notions are captured by modal operators over a Kripke structure (van Benthem 2003; Baltag, Ditmarsch, and Moss 2008).

T	p	q
v_p	1	0
v_q	0	1



(a)

T	p	q
v_p	1	0



(b)

Figure 3.1: Illustration of teams as information state for a propositional case.

illustrative purposes, let's consider the basic case of an information state in a propositional setting. Here, teams are sets of possible worlds, which are valuations for propositional letters. Table 3.1 provides some illustrations.

Teams represent the information states of speakers. For the team in (a), the speaker excludes the possibility that the actual world is v_{pq} or $v_\emptyset$ but considers both v_p and v_q possible. For the team in (b), the speaker considers only v_p possible. Consequently, in the team in (a), the speaker does not know whether p holds, whereas in (b), the speaker knows p.

In the case of 2TS, we work with sets of variable assignments. We define the notion of an initial team as the team where only factual information (information about the actual world of the speaker) is represented. We use $v \in Z_w$ as a special variable encoding information about the actual world.

3.1.8. Definition (Initial Team). A team T is *initial* iff $Dom(T) = \{v\}$.

The possible values that v receives in different assignments across the team represent different ways the actual world might be (epistemic possibilities). Intuitively, a team where v receives only one value has maximal information, as the speaker is certain about the state of affairs in the actual world.

3.1.9. Definition (Team of Maximal Information). Given a team T such that $v \in Dom(T)$, T has *maximal information* iff $i(v) = j(v)$ for all $i, j \in T$.

Table 3.4a is an example of an initial team. The team in Table 3.4a conveys that the epistemic possibilities the speaker entertains are v_1, v_2, up to v_n. As stated, only factual information is represented, since the domain of the team consists solely of the variable for the actual world v. Operations of assignment extensions introduced by quantifiers add variables encoding discourse or modal information to the team.

As said, teams encode the information state of the speaker. For instance, in Table 3.4b the speaker is certain about - or *knows* - the value of x, since x is constant across all their epistemic possibilities. However, the speaker does not know the value of y. World variables, like w, will be used to model modals or attitudes verbs, as we will see in Chapter 4.

T	v
i_1	v_1
i_2	v_2
...	...
i_n	v_n

(a)

T	v	x	w	y	...
i_1	v_1	a	w_1	b_1	...
i_1	v_2	a	w_2	b_2	...
...	...	a	...	...	...
i_n	v_n	a	w_n	b_n	...

(b)

Table 3.4: Initial team in (a). Team as Information State for the first-order case in (b).

3.2 Information Growth and Variable Dependencies

We have discussed the notion of a team as a set of variable assignments, specifically how teams encode the speaker's information state. As we will further explore in Chapter 4, new discourse information can be added to the team when evaluating a sentence starting from the initial team. This is achieved by extending the team with variables, which also allows characterizing different dependency relationships between the values of the variables across different assignments. In Section 3.2.1, we will define relevant notions of extensions of a team. In Section 3.2.2, we will introduce two important conditions that establish how variables' values are related to one another: the dependence atom and the variation atom.

3.2.1 Assignment Extensions

Our assignment extensions are based on similar operations discussed in dynamic and team semantics (Groenendijk and Stokhof 1991; Dekker 1993; Aloni 2001; Väänänen 2007b; Galliani 2012b). We present here the relevant definitions for `2TS` and later consider alternative options that have been discussed in the literature.

3.2.1. Definition (Universal Extension). Given a model $M = \langle D, W, I\rangle$, a team T and a variable z_* with $* \in \{d, w\}$, the universal extension of T with z_*, $T[z_*]$ is defined as follows:

$$T[z_*] = \{i[e_*/z_*] : i \in T \text{ and } e_* \in Dom_*(M)\}$$

Universal extensions consider all assignments that differ from the ones in T only with respect to the value of z_*. Table 3.5b is an example, assuming the initial team in Table 3.5a and a domain D of two individuals. Note that universal extensions are unique.

3.2.2. Definition (Strict Functional Extension). Given a model $M = \langle D, W, I\rangle$, a team T and a variable z_* with $* \in \{d, w\}$, the strict functional extension of T

with z_*, $T[f_s/z_*]$ is defined as follows:

$$T[f_s/z_*] = \{i[f_s(i)/z_*] : i \in T\}, \text{ for some strict function } f_s : T \to Dom_*(M)$$

Strict functional extensions assign only one value to the variable for each assignment in the original team T. Table 3.5c shows one of the four possible examples for the initial team in Table 3.5a and a domain D of two individuals.

3.2.3. Definition (Lax Functional Extension). Given a model $M = \langle D, W, I \rangle$, a team T and a variable z_* with $* \in \{d, w\}$, the lax functional extension of T with z_*, $T[f_l/z_*]$ is defined as follows:

$$T[f_l/z_*] = \{i[e_*/z_*] : i \in T \text{ and } e_* \in f_l(i)\}, \text{ for some lax function}$$
$$f_l : T \to \wp(Dom_*(M)) \backslash \{\varnothing\}$$

Lax functional extensions amount to assign one or more values to the variable for each original assignment in T. Table 3.5d shows one of the nine possible examples, assuming again the initial team in Table 3.5a and a domain D of two individuals.

v	T
v_1	i_1
v_2	i_2

(a)

v	y	$T[y]$
v_1	$\to d_1$	i_{11}
	$\to d_2$	i_{12}
v_2	$\to d_1$	i_{21}
	$\to d_2$	i_{22}

(b)

v	y	$T[f_s/y]$
v_1	$\to d_1$	i_{11}
v_2	$\to d_2$	i_{22}

(c)

v	y	$T[f_l/y]$
v_1	$\to d_2$	i_{12}
v_2	$\to d_1$	i_{21}
	$\to d_2$	i_{22}

(d)

Table 3.5: Illustration of Initial Team and Extensions. Initial Team in (a), universal y-extension (b), strict functional y-extension in (c) with f_s s.t. $f_s(i_1) = d_1$ and $f_s(i_2) = d_2$, and lax functional y-extension (d) with f_l s.t. $f_l(i_1) = \{d_2\}$ and $f_l(i_2) = \{d_1, d_2\}$.

3.2.2 Dependence and Variation Atoms

Team semantics frameworks are often equipped with dependence atoms - expressions which impose conditions of dependence on the variables' values given by the different assignments (Väänänen 2007a; Galliani 2021a).[10] The core of 2TS makes use of the *Dependence Atom* in Definition 3.2.4 and the *Variation Atom* in Definition 3.2.5. In Section 3.4.1 we will consider other atoms which play a role in further applications of 2TS.

3.2.4. Definition (Dependence Atom).
$M, T \models dep(\vec{z}, \vec{u}) \Leftrightarrow$ for all $i, j \in T : i(\vec{z}) = j(\vec{z}) \Rightarrow i(\vec{u}) = j(\vec{u})$

3.2.5. Definition (Variation Atom).
$M, T \models var(\vec{z}, \vec{u}) \Leftrightarrow$ there is $i, j \in T : i(\vec{z}) = j(\vec{z})\ \&\ i(\vec{u}) \neq j(\vec{u})$

The first atom in Definition 3.2.4 says that if any two assignments of the team agree on the value of $\vec{z}$, they also agree on the value of $\vec{u}$ (i.e. the value of $\vec{u}$ is *dependent* on the value of $\vec{z}$ in T). The variation atom in Definition 3.2.5 corresponds to the Boolean negation of the definition of Dependence Atom above, and as such it encodes the failure of functional dependence.[11] It is valid when there is at least a pair of assignments in T for which the value of $\vec{u}$ varies and $\vec{z}$ is the same. Table 3.6 displays a team of three assignments together with some illustrations.

T	x	y	z	l
i	a_1	b_1	c_1	d_1
j	a_1	b_1	c_2	d_1
k	a_3	b_2	c_3	d_1

$dep(x, y)$ ✓	$var(x, z)$ ✓
$dep(\varnothing, l)$ ✓	$var(\varnothing, x)$ ✓
$dep(xy, z)$ ✗	$var(x, y)$ ✗

Table 3.6: Dependence and Variation atoms - Illustrations.

In Table 3.6, we have that $dep(x, y)$, since for any assignment i, j and k, the value of x determines the value of y. But we do not have $dep(xy, z)$ (consider for example i and j: $i(xy) = j(xy)$, but $i(z) \neq j(z)$). It also holds that $var(x, z)$

[10]Note that dependence relations can also be modelled without resorting to teams (Baltag and van Benthem 2021). The team like nature of dependence logic and the related assignment extensions, however, will allow modelling more easily the addition/update of discourse information in the team.

[11]The variation atom was mentioned in Galliani (2012b) as a possible way to model failure of dependence. In dependence logics, a stronger version of the variation atom is typically considered:

3.2.6. Definition (Variation Atom (Stronger Version)).
$M, T \models VAR(\vec{z}, \vec{u}) \Leftrightarrow$ for all $i \in T$ there is $j \in T : i(\vec{z}) = j(\vec{z})\ \&\ i(\vec{u}) \neq j(\vec{u})$

Note in fact that $VAR(\vec{z}, \vec{u})$, unlike $var(\vec{z}, \vec{u})$, is downwards closed like $dep(\vec{z}, u)$, which typically simplifies the study of the underlying logic. Recently, Väänänen (2022) employed the stronger variation atom, called *anonymity atom* in his work, to model the notion of anonymity in database theory. See also Yang (2022) for some metatheoretical results on the propositional fragment of these logics.

since $i(x) = j(x)$ but $i(z) \neq j(z)$. A case which will be of importance later are atoms where the first argument is the empty sequence $\varnothing$: constancy atoms of the form $dep(\varnothing, l)$ which is valid when l receives the same value across all assignments; and variation atoms of the form $var(\varnothing, y)$, which is valid when y receives different values across at least a pair of assignments.

Dependence atoms have been studied have been studied in the context of database theory and a set of (complete) axioms, Armstrong's Axioms (Armstrong 1974), characterize the basic properties of dependence atoms (since the variation atom is the Boolean negation of the dependence atom, one might also determine some properties for the variation atom). We will not be concerned with such characterization here, but we highlight the importance of one (derived) property, namely that if $dep(x, y)$ holds then $dep(xz, y)$ holds, meaning that if y depends on x then y depends on xz. This will become relevant in the applications of the framework that we will consider in the next chapters.

3.3 Semantic Clauses of 2TS

We now give precise rules for semantic clauses of the formulas of our language (Hodges 1997; Väänänen 2007a; Galliani 2012b).

3.3.1. DEFINITION (Semantic Clauses). Given a model M and a team T over M, a formula ϕ over the signature of M (i.e., M is a suitable model for ϕ) and $Free(\phi) \subseteq Dom(T)$ (i.e., T is a suitable team for ϕ), we define the satisfaction relation of ϕ in T, denoted by $M, T \models \phi$, inductively on ϕ as follows:

$$
\begin{array}{lcl}
M, T \models P(t_1, \ldots, t_n) & \Leftrightarrow & \forall j \in T : \langle j(t_1), \ldots, j(t_n)\rangle \in I(P^n) \\
M, T \models \neg P(t_1, \ldots, t_n) & \Leftrightarrow & \forall j \in T : \langle j(t_1), \ldots, j(t_n)\rangle \notin I(P^n) \\
M, T \models t_1 = t_2 & \Leftrightarrow & \forall j \in T : j(t_1) = j(t_2) \\
M, T \models \neg t_1 = t_2 & \Leftrightarrow & \forall j \in T : j(t_1) \neq j(t_2) \\
M, T \models dep(\vec{z}, \vec{u}) & \Leftrightarrow & \text{for all } i, j \in T : i(\vec{z}) = j(\vec{z}) \Rightarrow i(\vec{u}) = j(\vec{u}) \\
M, T \models var(\vec{z}, \vec{u}) & \Leftrightarrow & \text{there is } i, j \in T : i(\vec{z}) = j(\vec{z})\ \&\ i(\vec{u}) \neq j(\vec{u}) \\
M, T \models \phi \wedge \psi & \Leftrightarrow & M, T \models \phi \text{ and } M, T \models \psi \\
M, T \models \phi \vee \psi & \Leftrightarrow & T = T_1 \cup T_2 \text{ for teams } T_1 \text{ and } T_2 \text{ s.t. } M, T_1 \models \phi \\
 & & \text{and } M, T_2 \models \psi \\
M, T \models \forall z\, \phi & \Leftrightarrow & M, T[z] \models \phi \\
M, T \models \exists_{\text{strict}} z\, \phi & \Leftrightarrow & \text{there is a strict function } f_s \text{ s.t. } M, T[f_s/z] \models \phi \\
M, T \models \exists_{\text{lax}} z\, \phi & \Leftrightarrow & \text{there is a lax function } f_l \text{ s.t. } M, T[f_l/z] \models \phi
\end{array}
$$

A first order literal is satisfied in a team T iff it is satisfied in all assignments in T. We allow negation only on first-order atoms and we assume that formulas are always in negation normal formal. We will return to negation in Section 4.8 of Chapter 4. A team T satisfies a conjunction $\phi \wedge \psi$ iff T satisfies ϕ and satisfies ψ. A team T satisfies a disjunction $\phi \vee \psi$ iff T is the union of two subteams, each satisfying one of the disjuncts.[12] We use the universal extension for the universal quantifier, and the strict and lax functional extensions for the strict and lax existentials.

If ϕ is satisfied in all suitable models M and all suitable teams over T, we say that ϕ is *valid*. We then define the notion of entailment as follows:

3.3.2. Definition (Entailment). A formula ϕ entails a formula ψ, in symbols $\phi \models \psi$, iff for all suitable models M and all suitable teams T such that $M, T \models \phi$, we have $M, T \models \psi$.

We say that ϕ and ψ are *equivalent* and we write $\phi \equiv \psi$ when $\phi \models \psi$ and $\psi \models \phi$.

As we discussed, there are teams of special importance in 2TS, initial teams, whose domains contain only v, and likewise, as we will further see in Chapter 4, there are formulas of special importance, namely those that have only v as free variable. We then say that ϕ is a 2TS-v formula when $Free(\phi) = \{v\}$ and we label such formulas ϕ_v. We then define a restricted notion of entailment (and equivalence) over initial teams for such formulas.

3.3.3. Definition (Entailment (restricted)). A formula ϕ_v entails a formula ψ_v, in symbols $\phi_v \models_v \psi_v$, iff for all suitable models M and all suitable initial teams T such that $M, T \models \phi_v$, we have $M, T \models \psi_v$.

It is interesting to observe that, except for the variation atom, all formulas in 2TS are downwards closed ($T \models \phi$ and $T' \subseteq T$ imply $T' \models \phi$). The variation atom, instead, is upwards closed ($T \models \phi$ and $T \subseteq T'$ imply $T \models \phi$), and therefore also union-closed ($T \models \phi$ and $T' \models \phi$ imply $T \cup T' \models \phi$). We note that for downwards closed formulas, the strict and lax existentials are equivalent. The latter statement follows from the fact that it is easy to construct a strict function from a lax one; and from a lax one, by downwards closure, a strict one. In the next sections, we will see that the variation atom and its interaction with the two existentials will make a distinction and play an important role.[13]

[12] We are employing the so-called split or tensor disjunction (Väänänen 2007b), which over the *dep* and *var* free fragment gives classical logic. We will return to disjunction in Chapter 10.

[13] Dependence logic, which does not include the variation atom, is equivalent, over sentences, to existential second-order logic Σ^1_1. The addition of the variation atom is safe in the sense of Galliani (2021b), as it does increase the expressive power over sentences when added to the logic.

3.4 Additional Atoms and Existentials

We conclude by mentioning some other notions and variants of existential quantifier that are relevant to 2TS: inclusion and independence atoms in Section 3.4.1, and inquisitive and independence existentials in Section 3.4.2.

3.4.1 Inclusion and Independence Atoms

We define here two other atoms which will become relevant in the rest of this work. The first is the *Inclusion Atom* introduced by Galliani (2012b) and also studied in Yang (2014).

3.4.1. DEFINITION (Inclusion Atom).
$M, T \models \subseteq (\vec{z}, \vec{u}) \Leftrightarrow$ for all $i \in T$, there is a $j \in T : i(\vec{z}) = j(\vec{u})$

Definition 3.4.1 says that the values of $\vec{z}$ are also values of $\vec{u}$. In fact, we can simply represent the condition in Definition 3.4.1 by requiring that $T(\vec{z}) \subseteq T(\vec{u})$. We give some illustrations in Table 3.7. In Table 3.7, $\subseteq (x, y)$ holds since any value for x (namely, d_1 and d_2) is also a value of y. Similarly, $\subseteq (xz, xy)$ holds, since any value for xz (namely, $d_1 d_2$ and $d_2 d_4$) is also a value for xy. But it does hold that $\subseteq (y, x)$, since for instance d_3 is not a value for x.

x	y	z
d_1	d_1	d_2
d_1	d_2	d_2
d_2	d_3	d_4
d_2	d_4	d_4

$\subseteq (x, y)$ ✓
$\subseteq (xz, xy)$ ✓
$\subseteq (y, x)$ ✗

Table 3.7: Illustration of Inclusion Atom.

The second atom is the *Independence Atom* introduced in Grädel and Väänänen (2013) in the context of independence logic.

3.4.2. DEFINITION (Independence Atom).
$M, T \models ind(\vec{z}, \vec{u}) \Leftrightarrow$ for all $i, i' \in T$, there is $i'' \in T : i''(\vec{z}) = i(\vec{z})$ and $i''(\vec{u}) = i'(\vec{u})$

Definition 3.4.2 models complete independence between $\vec{z}$ and $\vec{u}$, meaning that knowing the value of $\vec{z}$ in an assignment of the team does not convey any information with respect to the possible values of $\vec{u}$. In fact, a more intuitive way to represent the condition expressed by $ind(\vec{u}, \vec{z})$ in T is that it must hold that $T(\vec{u}) \times T(\vec{z})$ is equal to $T(\vec{u}\vec{z})$, assuming a correspondence between ordered pairs and sequences.

A more general form of the independence atom is $ind_{\vec{z}}(\vec{u}, \vec{l})$, whose definition is given in Definition 3.4.3. It says that for any possible value of $\vec{z}$, $\vec{u}$ and $\vec{l}$ are independent.

3.4.3. DEFINITION (Independence Atom (general form)).
$M, T \models ind_{\vec{z}}(\vec{u}, \vec{l}) \Leftrightarrow$ for all $i, i' \in T$ such that $i(\vec{z}) = i'(\vec{z})$ there exists $i'' \in T$: $i''(\vec{z}) = i(\vec{z})$, $i''(\vec{u}) = i(\vec{u})$, and $i''(\vec{l}) = i'(\vec{l})$

In this case, it similarly holds that for all $\vec{e} \in T(\vec{z})$, we have that $T_{\vec{z}=\vec{e}}(\vec{u}\vec{l}) = T_{\vec{z}=\vec{e}}(\vec{u}) \times T_{\vec{z}=\vec{e}}(\vec{l})$.

x	y	z	l
a_1	b_1	c_1	d_1
a_1	b_2	c_1	d_2
a_2	b_1	c_1	d_2
a_2	b_2	c_2	d_2

$ind(x, y)$ ✓ $ind(z, l)$ ✗
$ind(x, z)$ ✗ $ind_x(z, l)$ ✓
$ind_z(x, y)$ ✗

Table 3.8: Illustration of Independence Atom.

3.4.2 Inquisitive and Independence Existentials

In Section 3.2.1, we have discussed two notions of existentials: strict and lax existential. In this section, we consider two other notions that have been discussed in the literature and can be defined in a team-based system. In what follows, we will define these quantifiers as ranging only over D. The first is the *Inquisitive Existential* and the second is the *Independence Existential*. One could define separate notions of team extensions and define the existentials based on those. In what follows, we will define the extension in the definition of these quantifiers themselves.

The *Inquisitive Existential* has been considered in the context of inquisitive semantics (Ciardelli, Groenendijk, and Roelofsen 2018; Ciardelli 2022):

3.4.4. DEFINITION (Inquisitive Existential). $M, T \models \exists^1 x\phi \Leftrightarrow M, T[d/x] \models \phi$, where $T[d/x] = \{i[d/x] : i \in T\}$ for some $d \in D$

Such existential is definable in 2TS, as one can show that for any formula ϕ, the following holds:

3.4.5. FACT. $\exists^1 x\phi \equiv \exists_l x(\phi \wedge dep(\varnothing, x)) \equiv \exists_s x(\phi \wedge dep(\varnothing, x))$

The *Independence Existential* has been first considered in van den Berg (1996) and has received quite a lot of attention in the linguistic literature, in particular in dynamic semantic for plurals (Nouwen 2003), where new variables are added to the team when an existential is evaluated, similarly to the current assignment extensions, and one option is to make sure that the new variable is independent of the previous ones.

3.4.6. Definition (Independence Existential). $M, T \models \exists^{ind} x\phi \Leftrightarrow M, T[E/x] \models \phi$, where $T[E/x] = \{i[e/x] : i \in T \text{ and } e \in E\}$ for some $E \in \wp(D)\backslash\{\varnothing\}$

As discussed, the independence existential was introduced in the context of dynamic semantics, where existential quantifiers introduce new variables that are required to be independent of the previous ones. In fact, assuming that such variables consist of $Dom(T)/\{x\}$ and $\vec{y}$ is a sequence constructed from all the variables in such set, one can express $\exists^{ind} x\phi$ by alternatively requiring that $\exists_l x(\phi \wedge ind(x, \vec{y}))$.

We provide some illustrations in Table 3.9.

T	v
i_1	v_1
i_2	v_2

(a)

T	v	x
i'_1	v_1	a
i'_2	v_2	a

(b)

T	v	x
i'_1	v_1	a
i''_1	v_1	b
i'_2	v_2	a
i''_2	v_2	b

(c)

Table 3.9: Illustration of Inquisitive and Independence Existential. We give an illustration of the possible extensions given the team in (a). (b) shows a possible extension for the inquisitive existential with $d = a$, and for the independence existential with $E = \{a\}$. (c) shows a possible extension for the independence existential with $E = \{a, b\}$.

3.5 Conclusion

In this chapter, we have laid the groundwork for this dissertation by introducing the two-sorted team semantics (2TS) framework. We have shown that 2TS departs from traditional single-point evaluation systems by interpreting formulas with respect to sets of assignments, called teams. We have then discussed the notion of a team as an information state of the speaker. We have shown how new discourse information can be added to the team using different types of assignment extensions. We have then introduced two crucial atoms, dependence and variation, which express the relationships between the values of variables across different assignments in the team. The stage is now set to apply 2TS to the analysis of natural language phenomena.

Chapter 4

Indefinites Across Languages

This chapter will focus on the variety of readings associated with indefinites across languages. Our primary focus will be on the distinctions between scopal specific/non-specific uses and known/unknown uses of indefinites. We will investigate how various languages have developed lexicalized forms with restricted distributions pertaining to these uses, and how 2TS can offer a comprehensive characterization of this diversity.

In Section 4.1, we will introduce the notion of specific known, specific unknown and non-specific uses of indefinites and outline how different languages lexicalize these distinctions. In Section 4.2, we will review the basic idea of treating teams as information states and apply this to the modelling of indefinites in Section 4.3, where we will propose that indefinites are strict existential, using the terminology introduced in Chapter 3, and they are evaluated in situ. Section 4.4 discusses how the three uses mentioned above are encoded in 2TS and how this captures the typological variety of indefinites. Section 4.5 discusses the phenomenon of partitive specificity. Section 4.6 overviews different notions of implication and sets some empirical desiderata. Section 4.7 examines how modality can be analysed in 2TS, distinguishing between epistemic and deontic modality. Section 4.8 discusses negation. Finally, Section 4.9 considers how 2TS can shed some light on diachronic changes observed in the domain of indefinites.[1]

[1]Part of this chapter is based on Maria Aloni and Marco Degano (2022). "(Non-)specificity across languages: constancy, variation, v-variation". In: *Semantics and Linguistic Theory*. Vol. 32, pp. 185–205. In particular, Section 4.1, Section 4.2, Section 4.3, Section 4.4.1, Section 4.4.2, Section 4.7 (adapted and expanded), and Section 4.8.4 (adapted and expanded). The study was conceptualized through joint discussions between Maria Aloni and Marco Degano. The writing of the paper was carried out by Marco Degano.

4.1 Typology of (Non-)specific Indefinites

As outlined in Chapter 2, indefinites are associated with a variety of readings and forms (Haspelmath 1997; Farkas 2002b; Partee 2005; Jayez and Tovena 2002; Ebert and Hinterwimmer 2012; Chierchia 2013; Alonso-Ovalle and Menéndez-Benito 2015). This chapter will focus on two core distinctions: scopal specific/non-specific uses and known/unknown uses of indefinites. Specifically, we will use the term *specific* to refer to wide scope uses of an indefinite, and the term *non-specific* to refer to non-wide-scope uses, which can include narrow scope and intermediate scope interpretations in the presence of multiple operators. In Section 4.1.1, we will introduce these distinctions in detail and provide relevant examples. In Section 4.1.2, we will classify different types of indefinites based on the distributional restrictions with respect to these uses.

4.1.1 Specific Known, Specific Unknown, Non-specific

We will distinguish between specific known (SK), specific unknown (SU), and non-specific (NS) uses of indefinites. These distinctions are based on previous typological work by Haspelmath (1997), who adopts this terminology. Example (1) illustrates these contrasts for the English indefinite *someone*:

(1) a. Specific known (SK): Someone called. I know who.
b. Specific unknown (SU): Someone called. I do not know who.
c. Non-specific (NS): John needs to find someone for the job.

Cross-linguistically, languages have developed lexicalized forms with restricted distributions with respect to the uses in (1). For instance, the German *irgend-* is incompatible with SK, as the infelicitous continuation in (2) shows:

(2) Irgendein Student hat angerufen. #Rat mal wer?
some student has called. guess who?
'Some (unknown) student called. #Guess who?'

Another relevant example is the Russian *-nibud'*, which is not allowed in episodic contexts and can only be interpreted non-specifically:

(3) *Ivan včera kupil kakuju-nibud' knigu.
Ivan yesterday bought which-INDEF. book.
'Ivan bought some [non-specific] book yesterday.'

As discussed in Chapter 2, Haspelmath (1997) examined the distribution of indefinites in 40 languages and developed a functional map of indefinites with nine main uses/functions. Importantly, the functions are organized in an implicational way: a certain item always expresses functions which are contiguous (i.e., connected by a line) on the map.[2] Figure 4.1 presents a semantic map for the German indefinite *irgend-*, with the grey area indicating the possible functions available for *irgend-*.

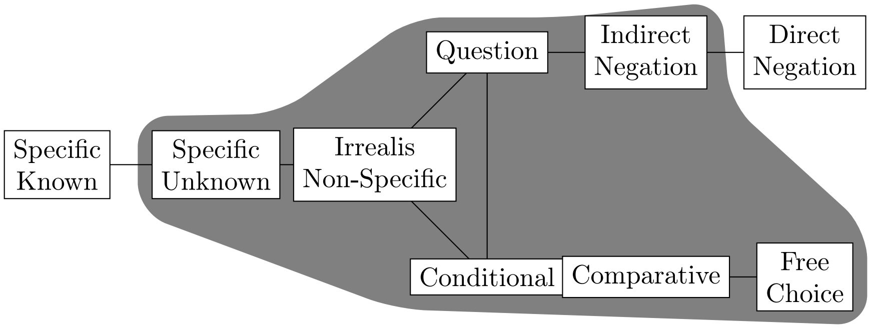


Figure 4.1: Haspelmath's map for German *irgend-*.

It is worth noting that Haspelmath (1997) employs the term *function* in a manner that some might interpret more generally classify as *use*. Specifically, according to Haspelmath (1997), a function can be characterized by both syntactic and semantic elements. Syntactically, this includes the infelicity of NS indefinites like *-nibud'* in episodic contexts, as illustrated in (3). Semantically, the obligatory ignorance component of *irgend-*, as shown in (2). The term 'function' in linguistics is associated with various interpretations, and we want to emphasize to the reader that, in this work, it corresponds to the 'uses' of indefinites, as exemplified in (1). Specifically, it refers to the SK, SU, and NS uses that we have discussed here.

In this regard, we emphasize that our notion of specificity is essentially 'syntactic' as it pertains to scopal specificity. Indefinites with exclusively specific uses presuppose the existence of their referent, meaning they can be paraphrased using a *there*-insertion construction. These indefinites can also introduce discourse referents, allowing for continuations with appropriate pronominal expressions. Conversely, indefinites that only permit non-specific uses are ungrammatical in episodic contexts and require a licensing operator, such as a modal or a universal quantifier.

4.1.2 Indefinite Types

As the examples in (2) and (3) illustrate, indefinites can display different functional restrictions. Combinations of SK, SU, and NS lead to seven possible types of indefinites, summarized in Table 4.1 along with relevant examples. In Table 4.1,

[2]Haspelmath (1997) restricted his analysis to indefinite pronouns and determiners formed with indefinite markers (e.g., the English *some-* or *any-*) that occur in a series (e.g., *some-thing*, *some-where*, ...). This excludes expressions such as the English *a certain*, which, however, have a specific-like flavour.

TYPE	FUNCTIONS SK	SU	NS	EXAMPLE	#/40
(i) unmarked	✓	✓	✓	Italian *qualcuno*	27
(ii) specific	✓	✓	✗	Georgian *-ghats*	7
(iii) non-specific	✗	✗	✓	Russian *-nibud'*	11
(iv) epistemic	✗	✓	✓	German *irgend-*	8
(v) specific known	✓	✗	✗	Russian *koe-*	5
(vi) SK + NS	✓	✗	✓	unattested	0
(vii) specific unknown	✗	✓	✗	Kannada *-oo*	1

Table 4.1: Possible Types of Indefinites.

we introduce some naming conventions for the types of indefinites we will examine in this work. Importantly, the distinctions presented here consider only the three uses discussed at the beginning of this section. At the end of this section, we will explore whether other possible types of indefinites within the domain of (non-)specificity can be conceived and how they should be analysed. In subsequent chapters, we will also explore other types of indefinites beyond the SK, SU, and NS uses.

Unmarked indefinites do not have any restrictions; specific indefinites admit only specific uses (SK and SU); non-specific indefinites admit only NS uses; and epistemic indefinites allow for both SU and NS uses. The last two types warrant further remarks. Type (vi), encoding SK and NS but not SU, is unattested in the data collected by Haspelmath (1997). Type (vii), which admits only SU uses, is very infrequent: out of the 40 languages examined by Haspelmath (1997), only one, Kannada, exhibits such an indefinite.[3]

Table 4.2 displays some within-language distinctions. An important question is whether a trade-off between the number of marked indefinites and the functions they cover can be established. Generalizations are challenging due to the limited amount of data.[4] In the case of Russian, we observe that there are two marked indefinites[5] to express NS: the epistemic *-to*, which also admits SU uses, and the non-specific *-nibud'*, which only admits non-specific uses. However, Russian speakers tend to select *-nibud'* for NS and *-to* for SU. Why then has *-to* maintained

[3]Kannada is a Dravidian language spoken mainly in Karnataka in southwestern India. Kannada is a determinerless language, and as such, bare nouns are ambiguous between definite and indefinite uses. It is possible that this ambiguity facilitated the development of a specific form with unknown uses since the definite form already encodes familiarity with the referent. However, this does not seem to be the case for other determinerless languages. For more on the uses of Kannada bare nouns, see Srinivas and Rawlins (2021).

[4]Moreover, there are equivalent expressions (e.g., *a specific*) that, although not being indefinites, have meanings similar to some of the marked indefinites considered here.

[5]Russian also has other indefinites with non-specific uses, which are not included here as they are commonly considered to be tied to different registers.

LANGUAGE	INDEFINITE	FUNCTIONS SK	SU	NS	TYPE
Italian	un qualche	✗	✓	✓	epistemic
	qualcuno	✓	✓	✓	unmarked
Russian	koe-	✓	✗	✗	specific known
	-to	✗	✓	✓	epistemic
	-nibud'	✗	✗	✓	non-specific
Japanese	-ka	✓	✓	✓	unmarked
Turkish	bir	✓	✓	✓	unmarked
	herhangi	✗	✓	✓	epistemic
German	etwas	✓	✓	✓	unmarked
	irgend	✗	✓	✓	epistemic
Georgian	-ghats	✓	✓	✗	specific
	-me	✗	✗	✓	non-specific
Ossetic	-dær	✓	✓	✗	specific
	is-	✗	✗	✓	non-specific
Kazakh	bir	✓	✓	✓	unmarked
	älde	✓	✓	✗	specific
Kannada	-oo	✗	✓	✗	specific unknown
	-aadaruu	✗	✗	✓	non-specific

Table 4.2: Marked Indefinites Across Languages.

its NS uses and not become a specific unknown indefinite? As we will see, 2TS will provide an interesting answer to this question.

4.2 Teams and Knowledge

In Chapter 3, we introduced 2TS. We review here the basic components as it concerns the way in which the known/unknown contrast described in the previous section receives a formal treatment in 2TS.

As said, formulas in 2TS are evaluated upon teams, which are sets of assignment functions. Recall also that 2TS is a two-sorted system in which formulas are evaluated with respect to a world variable, where $v \in Z_w$ is a special variable encoding information about the actual world.

In Section 3.1.2 of Chapter 3, we defined the notion of initial team, as one whose domain contains only the variable for the actual world v. This means that initial teams contain only factual information (information about the actual world of the speaker). We discussed how teams represent information states of speakers, since the possible values that v receives in different assignments across the team represent different ways the actual world might be (epistemic possibilities). Recall also that teams where v receives only one value are of maximal information, since the speaker is sure about the state of affairs in the actual world.

We illustrate this again in Table 4.3, where we consider a domain $D = \{a, b\}$ of two individuals, and we assume that worlds in v differ with respect to the property 'read the dissertation'. As a reminder, the first row indicates the variables present in the domain of the team, and the rows below show the values assigned by the assignments in the team.

Table 4.3a is a team of 'minimal' information, in the sense that the speaker is completely unaware of who read the dissertation: it could be both a and b; it could be a; it could be b or it could be that no one read the dissertation. Table 4.3b is a team of partial information: the speaker is unsure whether a or b read the dissertation, but they exclude some of the cases in Table 4.3a. In Table 4.3c the speaker is maximally informed, and they are certain that only b read the dissertation.

T	v
i_1	v_{ab}
i_2	v_a
i_3	v_b
i_4	$v_\varnothing$

(a)

T	v
i_1	v_a
i_2	v_b

(b)

T	v
i_1	v_b

(c)

Table 4.3: Initial Teams. Values for v are constructed over the property 'read the dissertation' with a domain $D = \{a, b\}$ of two individuals. (a) represents a team of minimal information (or no information). (b) represents a team of partial information. (c) represents a team of maximal information, meaning that the speaker knowns that only b read the dissertation.

Only factual information (i.e., information about the actual world) is represented in initial teams, as the domain of the team consists only of the variable for the actual world v. Operations of assignment extensions introduced by quantifiers add variables, encoding discourse or modal information, to the team.

This leads to the idea that a given sentence in natural language is felicitous when the corresponding rendering in 2TS, which could lead to the abovementioned assignments extensions, is supported by an initial team, as defined in Definition 4.2.1. If there is no initial team, then the sentence is predicted to be infelicitous. This will play a role, for instance, in explaining the infelicity of non-specific indefinites in episodic contexts.

4.2.1. Definition (Felicitous sentence). Given a 2TS formula ϕ we say that ϕ is felicitous iff there is an initial team T over a model M such that $M, T \models \phi$.

4.3 Indefinites as Strict Existentials

As we have seen in Chapter 2, semantic theories differ in their formal treatment of indefinites. In the present account, we treat indefinites as existential quantifiers. Indefinites introduce a new variable in the team, and such a variable may receive different values across assignments.

In particular, we will model them as strict existentials, as introduced in Chapter 3. As a reminder, this means that only one new variable value for assignment is introduced.[6] We give a simple example in (4), where we C stands for 'went to the cinema'.

(4) a. Someone went to the cinema.
b. $\exists_s x(C(x, v))$

Assuming an initial team where the speaker is unsure whether both a and b went to the cinema or only b went to the cinema, this leads to two possible strict extensions of T with x, as depicted in Table 4.4. By contrast, if the existential in (4-b) had been analysed by means of the lax quantifier, a lax extension would have allowed for selecting more than one new variable value per assignment and thus the additional extension depicted in Table 4.4.

T	v
i	v_{ab}
j	v_b

(a) Initial Team

$T[f_s/x]$	v	x
i'_1	v_{ab}	a
j'	v_b	b

(b) Strict Extension I

$T[f_s/x]$	v	x
i'_2	v_{ab}	b
j'	v_b	b

(c) Strict Extension II

$T[f_l/x]$	v	x
i'_1	v_{ab}	a
i'_2	v_{ab}	b
j'	v_b	b

(d) Lax Extension

Table 4.4: Illustrations. Initial team T in (a). (b) and (c) are strict extension of T with x. (d), and also (b) and (c), are lax extensions of T with x.

As noted in Chapter 3, the two notions of existential - strict and lax - are equivalent for downwards closed formulas, and the example in (4) is a clear instance of this (see Semantic Clauses of 2TS in Definition 3.3 of Chapter 3). However, remember that the variation atom in 2TS is not downwards closed. Our choice is to treat indefinites as strict existentials, meaning that only the configurations in (b) and (c) in Table 4.4 are allowed. As we will see, this design choice will play a key role in explaining the distribution of indefinites.

[6] Note that indefinites receive one value given a value for v in an initial team where only v is present in the team. If there are other variables in the team with different assignments assigning the same value to v, but different values to other variables (i.e., they are different assignments), then the variable introduced by the indefinite can be associated with more than one value given a fixed value for v.

4.3.1 Indefinites and Scope

As we discussed in Section 2.3, the problem of indefinites and exceptional scope has been central to the formal treatment of indefinites. We mentioned that Brasoveanu and Farkas (2011) exploit ideas from independence friendly-logic (Hintikka 1986, 1996) to give an account of the exceptional scope of indefinites. Given the conceptual similarity between the approaches, it is not surprising that the present approach can also offer an account of indefinites and scope.

In 2TS, dependence atoms allow us to easily capture the different scope readings by specifying how the indefinite's variable might co-vary with other operators. For instance, a sentence like (5) is ambiguous between three different readings, depending on the scope of *a doctor* with respect to the universal quantifiers.[7]

As a base case, we assume a team of maximal information (i.e., the value of v is fixed). As shown in Table 4.5, $dep(v, x)$ yields a wide scope interpretation where the value of x is constant (since v is constant); $dep(vy, x)$ yields the intermediate reading where the value of x depends only on the first universal quantifier; and $dep(vzy, x)$ yields narrow scope where the value of x depends on both universal quantifiers. Note that, for instance, as it is the case in first-order logic rendering in square brackets below, the corresponding atom for wide scope, $dep(v, x)$ entails the corresponding atom for the weaker readings $dep(vy, x)$ and $dep(vzy, x)$.

(5) Every kid$_z$ ate every food$_y$ that a doctor$_x$ recommended.
  a. Wide scope [$\exists x/\forall z/\forall y$]: $\forall z \forall y \exists_s x(\phi \wedge dep(v, x))$
  b. Intermediate scope [$\forall y/\exists x/\forall z$]: $\forall z \forall y \exists_s x(\phi \wedge dep(vy, x))$
  c. Narrow scope [$\forall z/\forall y/\exists x$]: $\forall z \forall y \exists_s x(\phi \wedge dep(vzy, x))$

As said, this approach is conceptually similar to Brasoveanu and Farkas (2011) and leads to the generalization in (6).[8] In Brasoveanu and Farkas (2011), dependence relations are encoded in the meaning of the existential. For instance,

[7]We give some concrete instantiation of the three readings. In the wide scope reading, there is a particular doctor (say Dr. Malcom), such that every kid ate every food that Dr. Malcom recommended. In the intermediate scope reading, for every kid, there is a doctor, say the pediatrician of each kid, such that all kids ate every food that their doctor recommended. In the narrow scope reading, the sentence is true also in cases of total co-variation between the doctors and the foods, meaning that one kid might have eaten different foods recommended by different doctors.

[8]The generalization in (6) overgenerates. Unavailable readings can be ruled following strategies similar to Brasoveanu and Farkas (2011). For instance, it is clear that we cannot have a dependence on the first quantifier, without also being dependent on the intermediate one. One might introduce an ordering of the possible values of $\vec{y}$ based on the surface order of the quantifiers. The system can be amended further to explain the intricacies of natural language with respect to scope, and in Chapter 7 we will return to some of these issues. We believe that Brasoveanu and Farkas (2011) showed how this can be done within a dependence-like framework, and we do not pursue this any further, as we hope that we showed that this is achievable and, as our main concerns here are the typological variety of indefinites and the integration of epistemic readings.

ignoring the role of v, in the case of (5), the narrow scope interpretation would be captured by $\exists^{z,y}x$, which in their account means that the value of x is fixed relative to no variable.

(6) Indefinites & Scope
An unmarked/plain indefinite $\exists_s x$ in syntactic scope of $O_{\vec{z}}$ allows all $dep(\vec{y}, x)$, with $\vec{y}$ included in $v\vec{z}$:

$$O_{z_1} \ldots O_{z_n} \exists_s x(\phi \wedge dep(\vec{y}, x))$$

In 2TS, dependence relations are not part of the meaning of the existential, but they are evaluated as separate clauses by means of dependence atoms. This allows us to work with a uniform entry for existentials and with a better-behaved logical system. For instance, Brasoveanu and Farkas (2011) need to define a non-standard clause for universal quantification, which is not needed here. Moreover, the current formalization, as we will see in Chapter 7, will allow us to better appreciate the relationship with choice-functional approaches to indefinites and scope.

v	z	y	x
v_1	...	...	b_1
v_1	...	...	b_1
v_1	...	...	b_1
v_1	...	...	b_1

WS: $dep(v, y)$

v	z	y	x
v_1	a_1	...	b_1
v_1	a_1	...	b_1
v_1	a_2	...	b_2
v_1	a_2	...	b_2

IS: $dep(vx, y)$

v	z	y	x
v_1	a_1	c_1	b_1
v_1	a_1	c_2	b_2
v_1	a_2	c_3	b_3
v_1	a_2	c_4	b_4

NS: $dep(vxz, y)$

Table 4.5: Indefinites and Scope.

4.3.2 Indefinites and Ignorance

In the example considered in the previous section in Table 4.5, we worked with a team of maximal information (i.e., the value of v was fixed). In the context of Table 4.5, this means that, for a given choice of z and y, the value of x is known. This does not have to be the case.

For instance, in the context of (5), there could be a wide scope reading where it is uncertain which specific doctor is being referred to. The speaker may not know if this doctor is b_1 or b_2. This would correspond to a team similar to the one for wide scope in Table 4.5, but with additional assignments, where v is assigned to v_2 and x to b_2.

To model epistemic distinctions, we need to distinguish between full specificity (specific known) and what we called specific unknown: a specific individual, but epistemically not determined. We can capture the difference using possible worlds representing epistemic possibilities. In the former case, the specific individual will be constant across all epistemically possible worlds, while in the latter it will vary. We will make this more formal and give some illustrations in the next section.

4.4 Variety of Indefinites

In this section, we discuss how 2TS can encode the variety of indefinites considered in Section 4.1. Towards this end, we will first explain how the SK, SU, and NS uses are captured in 2TS in Section 4.4.1, and then discuss how to model the restricted distribution of these uses in indefinites in Section 4.4.2. We will explore how the rendering of the different indefinites mirrors the convexity of the functions in Haspelmath's map in Section 4.4.3. Section 4.4.4 introduces the Dependence Square of Opposition, which will be useful to understand the various relationships between indefinite types, and Section 4.4.5 explores other possible indefinite types that can be defined in the system with respect to (non-)specific usages.

4.4.1 SK, SU and NS in 2TS

We will now proceed to distinguish between the SK, SU, and NS uses discussed at the beginning of this chapter. To do so, we will introduce the following conditions to make the logical rendering of these uses easier.

		v	x			v	x
constancy	$dep(\varnothing, x)$	$\dots$	d_1	v-constancy	$dep(v, x)$	v_1	d_1
		$\dots$	d_1			v_2	d_2
		v	x			v	x
variation	$var(\varnothing, x)$	$\dots$	d_1	v-variation	$var(v, x)$	v_1	d_1
		$\dots$	d_2			v_1	d_2

Table 4.6: Constancy and Variation Conditions.

Constancy means that the variable x is mapped to the same individual in every assignment, while *variation* ensures that there is at least one pair of assignments in which x receives different values. Their v-counterparts relativize these notions to the variable for the actual world v: *v-constancy* means that the value of x is constant within an epistemic possibility, whereas *v-variation* guarantees that there is at least one epistemic possibility in which x receives different values. With these conditions, we can logically characterize the specific known, specific unknown, and non-specific uses.

The logical rendering of the various uses is shown in (7) together with some illustrations in Table 4.7, where x is the variable associated with the indefinite. SK is captured by constancy, ensuring speaker knowledge; SU is captured by

v-constancy, ensuring specificity, and variation, ensuring unknownness; NS is captured by v-variation, which, as we will see will, ensures scopal non-specificity.

(7) a. SK: $dep(\varnothing, x))$ [constancy]
 b. SU: $dep(v, x) \wedge var(\varnothing, x))$ [v-constancy + variation]
 c. NS: $var(v, x))$ [v-variation]

		v	...	x
Specific Known	constancy $dep(\varnothing, x)$	v_1	...	d_1
		v_2	...	d_1
		v	...	x
Specific Unknown	v-constancy $dep(v, x)$ + variation $var(\varnothing, x)$	v_1	...	d_1
		v_2	...	d_2
		v	...	x
Non-specific	v-variation $var(v, x)$	v_1	...	d_1
		v_1	...	d_2

Table 4.7: Logical Rendering of SK, SU and NS.

4.4.2 Variety of Indefinites

We now have all the components necessary to capture the variety of marked indefinites discussed in Section 4.1. As anticipated, we claim that marked indefinites come with particular restrictions regarding the constancy and variation conditions examined in the previous section. We summarize our proposal in Table 4.8.[9]

TYPE	FUNCTIONS			REQUIREMENT	EXAMPLE
	SK	SU	NS		
(i) unmarked	✓	✓	✓	none	Italian *qualcuno*
(ii) specific	✓	✓	✗	$dep(v, x)$	Georgian *-ghats*
(iii) non-specific	✗	✗	✓	$var(v, x)$	Russian *-nibud'*
(iv) epistemic	✗	✓	✓	$var(\varnothing, x)$	German *irgend-*
(v) specific known	✓	✗	✗	$dep(\varnothing, x)$	Russian *koe-*
(vi) SK + NS	✓	✗	✓	$dep(\varnothing, x) \vee\!\!\!/\, var(v, x)$	unattested
(vii) specific unknown	✗	✓	✗	$dep(v, x) \wedge var(\varnothing, x)$	Kannada *-oo*

Table 4.8: Marked Indefinites.

Unmarked indefinites, like English *someone*, do not have specific requirements and can, in principle, express all the functions we considered. Specific indefinites

[9] We would like to mention Champollion, Bledin, and Li (2017), a recent relevant work which integrates dependence logics and dynamic plural logic. Champollion, Bledin, and Li (2017) adopts a rigidity requirement comparable to our $dep(\varnothing, x)$ to distinguish between rigid and lax quantification in Champollion, Bledin, and Li (2017)'s terminology.

are associated with 'v-constancy': the referent of the indefinite is the same in a given world but can vary between worlds. The opposite condition, 'v-variation', characterizes non-specific indefinites. Epistemic indefinites require 'variation': the referent of the indefinite must vary, possibly within the same world. 'Constancy' leads to specific known: a unique individual across all worlds.

Now, let us turn to the last two types in Table 4.8, which require a more detailed explanation. The type 'specific known + non-specific' cannot be subsumed under a single atom. It requires that the referent satisfies either 'constancy' or 'v-variation', which are incompatible with each other.[10] Therefore, this type can only be captured by a (Boolean) disjunction of atoms, explaining the difficulty of finding a lexicalized indefinite encoding opposite meanings.[11] To our knowledge, no language encodes this meaning in a specific form. Moreover, type (vi) constitutes a clear violation of convexity, a constraint typically assumed in lexicalizations (Gärdenfors 2014; Steinert-Threlkeld and Szymanik 2020; Steinert-Threlkeld, Imel, and Guo 2023; Enguehard and Chemla 2021). The next section will address how convexity is encoded in the system.

The last type, specific unknown, requires two atoms: 'v-constancy' for specificity and 'variation' for unknown. Crucially, only one language among those examined by Haspelmath (1997) possesses such an indefinite. We claim that complexity is the reason. Specific unknown requires two atoms, making its lexicalization less likely to occur.

This analysis also allows us to address the question at the end of Section 4.1.2. Russian has a dedicated indefinite for NS uses (*-nibud'*) and an epistemic indefinite (*-to*) that expresses both NS and SU. In practice, speakers almost always select *-to* for SU and *-nibud'* for NS. The preferential use of SU for *-to* arguably has a pragmatic root: speakers are aware that there is an alternative form with only NS uses. Nonetheless, Russian maintains *-to* as an epistemic indefinite, as turning *-to* into a specific unknown would increase its complexity in the sense delineated here. This represents an interesting balance between the language user and the language system.

4.4.3 Semantic Convexity

The notion of convexity plays a key role in several lexicalization patterns across different domains, such as colour terms (Gärdenfors 2000, 2014; Jäger 2007,

[10] Note that $dep(\varnothing, x)$ implies $dep(v, x)$, which contradicts $var(v, x)$.

[11] To express such a combination of functions, we would need a Boolean/global/inquisitive notion of disjunction: $M, T \models \phi \mathbin{\vee\!\!\!/} \psi \Leftrightarrow M, T \models \phi$ or $M, T \models \psi$. Note that $\mathbin{\vee\!\!\!/}$ is definable in 2TS by means of dependence atoms (Fan Yang, pc.):

$$\phi \mathbin{\vee\!\!\!/} \psi \equiv \exists x \exists y (dep(\varnothing, x) \wedge dep(\varnothing, y) \wedge (x = y \wedge \phi) \vee (x \neq y \wedge \psi))$$

2010), generalized quantifiers (van Benthem 1984; Steinert-Threlkeld and Szymanik 2020), and modals (Steinert-Threlkeld, Imel, and Guo 2023).[12]

The underlying idea is that the meaning of expressions should denote a convex 'region' provided a suitable notion of meaning space. Convexity would be violated when gaps are present in the underlying 'region' that expressions denote.[13] To illustrate this concretely, consider the case of modified numerals or generalized quantifiers. In this domain, there are no expressions that lexicalize meanings like 'more than five or less than two,' which are intuitively, and formally, non-convex determiners. In the domain of generalized quantifiers, where a determiner is represented by a relation between two sets, a constraint on convexity can be represented as in Definition 4.4.2, adapted from van Benthem (1984). For the determiner 'more than five or less than two,' it is easy to construct a counterexample by taking $A = \{d_1, \ldots, d_{10}\}$, $B_1 = \{d_1\}$, $B = \{d_1, d_2\}$, and $B_2 = \{d_1, \ldots, d_6\}$.[14]

4.4.1. Definition (Convexity in Generalized Quantifiers (van Benthem 1984)). A determiner Q is convex iff for all M with $A, B, B_1, B_2 \subseteq M$ such that $B_1 \subseteq B \subseteq B_2$, $Q_M(A, B_1)$ and $Q_M(A, B_2)$ imply $Q_M(A, B)$.

The question to be addressed is what would constitute a suitable convex meaning space for indefinites, particularly for the functions considered in the present work. Figure 4.2 orders our atoms according to the degree of variation (from constancy to v-variation) in a way that is compatible with Haspelmath's original ordering in his semantic map.[15]

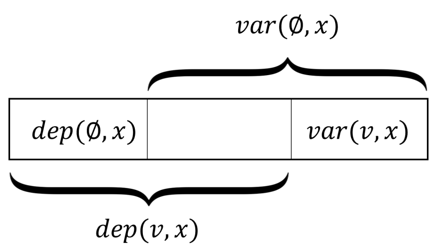


Figure 4.2: Meaning Space of Marked Indefinites.

[12]See also Enguehard and Chemla (2021) for an interesting proposal connecting convexity and exhaustification.

[13]In geometry, a convex region (or convex set) is a subset of a Euclidean space that has the property that, for any two points within the region, the line segment connecting them lies entirely within the region. For example, a triangle forms a convex region, while a star shape does not.

[14]van Benthem (1984) uses the term *continuity* instead of *convexity*, following earlier literature. We prefer the term convexity in alignment with convex functions, as opposed to continuous functions.

[15]The conditions in Table 4.6 can be considered the most basic representation of constancy and variation requirements in the variables' assignment values, and in this sense, they constitute minimal meaning elements of the meaning space of indefinites.

In 2TS, we can define a suitable notion of convexity on a set of teams as in Definition 4.4.2:

4.4.2. DEFINITION (Convexity over Teams). A set of teams P is convex iff for all T, T', T'' such that $T \subseteq T' \subseteq T''$, if $T \in P$ and $T'' \in P$, then $T' \in P$.

It is easy to show that the Boolean union of the formulas associated with the SK and NS cells in our map, as in (8), defines a property that does not satisfy convexity. A counterexample is given in Figure 4.3.

v	x
v_1	d_1

(a) T

v	x
v_1	d_1
v_2	d_2

(b) T'

v	x
v_1	d_1
v_1	d_2
v_2	d_2

(c) T''

Figure 4.3: Failure of Convexity for SK + NS. In the teams above, it holds that $T \subseteq T' \subseteq T''$. Moreover, $T \models \mathit{dep}(\varnothing, x) \veebar \mathit{var}(v, x)$, since $T'' \models \mathit{dep}(\varnothing, x)$. $T'' \models \mathit{dep}(\varnothing, x) \veebar \mathit{var}(v, x)$, since $T \models \mathit{var}(v, x)$. But $T' \not\models \mathit{dep}(\varnothing, x)) \veebar \mathit{var}(v, x)$.

(8) SK + NS: $\mathit{dep}(\varnothing, x) \veebar \mathit{var}(v, x)$

However, this is not the case for the other two possible combinations, which define convex sets of teams. The former does so because $\mathit{dep}(v, x)$ is downwards closed, and the latter because $\mathit{var}(\varnothing, x)$ is upwards closed:

(9) SK + SU: $\mathit{dep}(\varnothing, x) \veebar (\mathit{var}(\varnothing, x) \wedge \mathit{dep}(v, x)) \equiv \mathit{dep}(v, x)$

(10) SU + NS: $(\mathit{var}(\varnothing, x) \wedge \mathit{dep}(v, x)) \veebar \mathit{var}(v, x) \equiv \mathit{var}(\varnothing, x)$

This gives us a principled explanation for the specific ordering among functions assumed in Haspelmath's original map, namely SK-SU-NS. A natural constraint on implicational maps is that properties expressed by contiguous cells must satisfy convexity. If we had ordered the functions differently, such as SK-NS-SU or SU-SK-NS, this constraint would not have been satisfied. We can then provide a more grounded explanation for the absence of indefinites that lexicalize only the SK and NS functions as a violation of a convexity constraint.

4.4.4 Dependence's Square of Opposition

We dedicate this section to an interesting parallelism between our dependence and variation conditions and Aristotle's Square of Opposition. Figure 4.4 displays the traditional Aristotle's Square of Opposition, which is a collection of logical relations between four main categorical propositions.[16] The corners are traditionally considered to be propositions, but Figure 4.4 displays the corresponding determiners (e.g., *Every A is B* for *Every*). Typically, only three corners of the square correspond to simple lexical items across languages. For instance, English lexicalizes *every*, *some*, and *no*, but not *not every* as a simple determiner.[17]

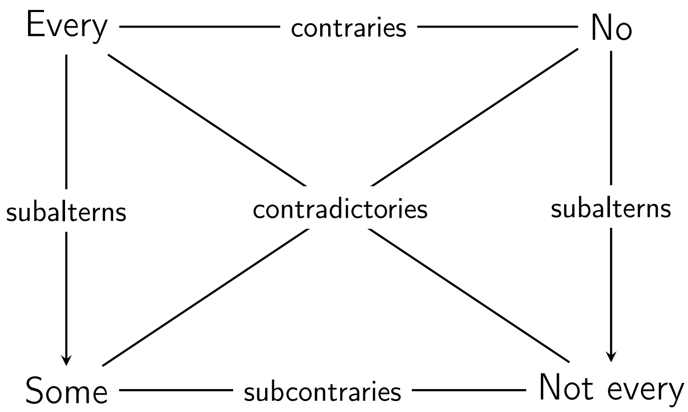


Figure 4.4: Aristotle's Square of Opposition.

Interestingly, our dependence conditions along the dimensions of (v-)constancy and (v-)variation give rise to the same logical relationships observable in the standard Aristotelian square. Figure 4.5 displays our 'Dependence Square of Opposition'. This is expected, as the dependence and variation atoms are the Boolean negation of each other. Crucially, each corner corresponds to one of the lexicalized marked indefinites discussed in the previous section.

In the traditional Aristotelian Square, each corner corresponds to the four basic ways categorical propositions can be formed. Similarly, the Dependence Square of Opposition corresponds to the four basic ways marked indefinites can be formed. Moreover, we note the absence of the indefinite 'SK + NS' and 'specific

[16]We remind the reader of the classical terminology:

- Contraries: Two propositions are *contraries* iff they can be both false, but not both true.
- Contradictories: Two propositions are *contradictories* they cannot be both true and they cannot be both false.
- Subcontraries: Two propositions are *subcontraries* iff they cannot both be false but can both be true.
- Subalternation: A proposition p subalternates a proposition q iff p implies q.

Note that the relationships in Figure 4.4 hold assuming that *Every* and *No* have existential import, while *Some* and *Not Every* do not.

[17]A similar pattern can be observed in the domain of temporal adverbs. English lexicalizes *always*, *never*, and *sometimes*, but has no corresponding adverb in the lower right corner of the square.

unknown', reinforcing the idea that indefinites present in the Square are simpler and more frequent, while the others are unattested or rare.[18]

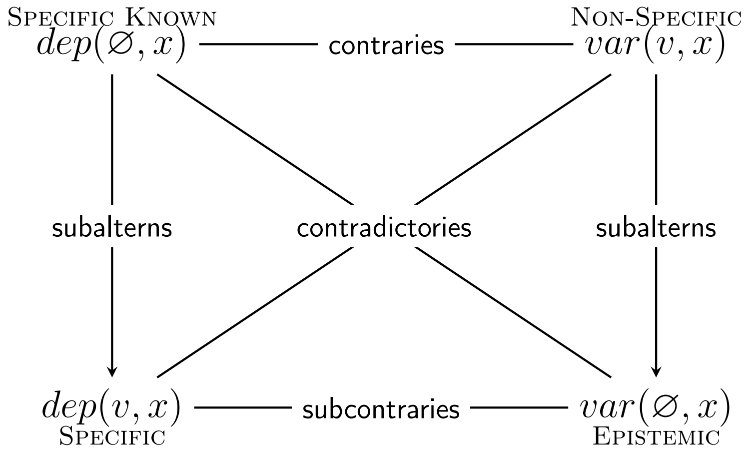


Figure 4.5: Dependence Square of Opposition.

4.4.5 Additional Types of Indefinites

We have already seen that non-specific indefinites cannot receive the widest scope possible with respect to other operators. This means that they are compatible with narrow scope and possibly intermediate scopes as well. Are there indefinites that can only receive the narrowest scope possible? From an empirical viewpoint, there appears to be no such indefinite. From a formal viewpoint, such indefinites can be modelled by a requirement of the form $dep(v\vec{z}, x)$, where $\vec{z}$ is a sequence of all the variables whose syntactic scope contains the indefinite.[19]

Similarly, there might be indefinites that can co-vary only with respect to a certain variable sort. For instance, an indefinite which can co-vary and receive narrow scope with a bona fide quantifier like *every*, but not with a modal like *must*.[20] In such a case, the variables in $\vec{z}$ can only belong to the sort for individuals. Again, there appears to be no such indefinite. In Chapter 6 we will consider the case of dependent indefinites, which are indefinites that cannot be licensed by modals but are licensed by bona fide quantifiers. However, such indefinites are infelicitous in modal contexts, where the requirement $dep(v\vec{z}, x)$ restricted to

[18] An open question is why the lower right corner of the 'Dependence Square of Opposition' is lexicalized, unlike the cases we mentioned before.

[19] A variation of this is an indefinite which only receives the narrowest scope and is incompatible with the stronger wide and intermediate scope readings. Such an indefinite would be captured by the additional contribution of variation atoms. Again, there appear to be no indefinites with such a distribution.

[20] As we will see in Section 4.7, universal modals can be analysed as universal quantifiers over world variables.

individual variables would predict that such indefinites receive wide scope with respect to the modal. It thus seems that, at least regarding dependence and variation atoms, the kind of dependence allowed in marked indefinites can only contain the empty variable or the variable for the actual world, besides, of course, the variable of the indefinite.

We end this section with a note on compositionality in 2TS. Compositionality is a fundamental topic to logic and natural language (Janssen and Partee 1997; Szabó 2022; Pagin and Westerståhl 2010a,b; Hodges 2001; Westerståhl, Baltag, and Benthem 2021). We can think of compositionality as requiring that the meaning of a complex expression is determined by the meanings of its parts and the way they are combined.[21]

The issue of compositionality is relevant to us, as we assumed that indefinites are associated with atoms, which might include variables from other operators in the sentence. We would like to draw a distinction between atoms associated with lexical marked indefinites and atoms encoding the scope of the indefinites. As concerns the former, we observe that the only reference to other variables is v, the variable for the actual world. This is unproblematic, as such variable simply encodes the epistemic state of the speaker. Regarding atoms for scope, we argued that an indefinite is associated with $dep(v\vec{y}, x)$, where $\vec{y}$ is a sequence of variable of constructed from operators having syntactic scope above the indefinite, and possibility other constraints on the order of such operators. However, if we maintain a bottom-up compositional procedure $\vec{y}$ cannot be already part of the indefinite. This might suggest that scope readings are not determined compositionally, but they are the result of a processing operation which takes place at a later stage once the relevant operators have been introduced. In Chapter 7, we will argue for an approach along these lines, where the default reading of indefinites will be the one without any dependence atoms encoding scope, yielding the narrowest scope possible, and the addition of dependence atoms for other scope readings incurs in a pragmatic cost.

4.5 Partitive Specificity and Inclusion Atoms

So far, our discussion concerned scopal specificity (specific vs. non-specific) and epistemic specificity (known vs. unknown). Another relevant notion of specificity is partitive specificity. An indefinite is partitive if the range of its value is a subset of a previously introduced referent. Partitivity can be overt, as in example (11-a), with languages employing various partitive constructions. However, partitivity can also be covert, as in (11-b), where the indefinite *a student* refers to the

[21] From a formal perspective, team semantics was developed by Hodges (1997) as a reaction to Hintikka's claims that independence-friendly logic is non-compositional Hintikka (1973, 1996, 2006). Hodges (1997)'s team semantics respects some core principles of compositionality in certain respects (see Westerståhl, Baltag, and Benthem 2021 for a discussion).

previously mentioned group of two women who worked on the paper.

(11) a. John is reading one of Henry James' novels.
b. Two women worked on the paper. A student was the leading author.

It is clear that this notion of partitive specificity finds a natural treatment in a team-based framework. The idea, for the cases in (11) is that the values of the variable associated with the indefinite should be a subset of the values of the variables of the previously introduced referent. The inclusion atom $\subseteq (\vec{x}, \vec{y})$ discussed in Section 3.4 of Chapter 3 readily accounts for this idea.

As concerns the examples in (11), a schematic analysis would require that if the values of 'Henry James' novels' are encoded by the variable x and the indefinite 'one' is associated with the variable y, then $\subseteq (y, x)$ must hold. Clearly, a proper analysis should be dynamic. In fact, the system of van den Berg (1996) contains an operation of variable introduction that mimics the contribution of the inclusion atom with respect to a previous variable in the discourse.

In Chapter 6 we will present a dynamic version of 2TS, but we do not pursue this any further as we believe that the insights that van den Berg (1996) discussed in his work can be directly imported in 2TS. In Section 4.7, we will examine an application of inclusion atoms more pertinent to this work - modality.

4.6 Implication

Team semantics frameworks allow for expressing a variety of connectives. Different notions of implication have been studied. Below, we offer some possible notions proposed in the literature (Yang 2014; Abramsky and Väänänen 2009; Kontinen and Nurmi 2011):

4.6.1. Definition (Classical/Material Implication).
$M, T \models \phi \rightarrow_C \psi \Leftrightarrow$ if $M, T \models \phi$, then $M, T \models \psi$

4.6.2. Definition (Intuitionistic Implication).
$M, T \models \phi \rightarrow_I \psi \Leftrightarrow$ for all $T' \subseteq T : M, T' \models \phi$, we have $M, T' \models \psi$

4.6.3. Definition (Singleton Implication).
$M, T \models \phi \rightarrow_S \psi \Leftrightarrow$ for all $i \in T : M, \{i\} \models \phi$, we have $M, \{i\} \models \psi$

4.6.4. Definition (Maximal Implication).
$M, T \models \phi \rightarrow_\forall \psi \Leftrightarrow$ for all $T' \subseteq T$ such that $M, T' \models \phi$ and T' is maximal, we have $M, T' \models \psi$

Being *maximal* amounts to the following:

4.6.5. DEFINITION (Maximal Team). Given a model M, a team T, and a formula ϕ, $T' \subseteq T$ maximally satisfies ϕ in M iff $M, T' \models \phi$ and there is no T'' such that $T' \subsetneq T'' \subseteq T$ and $T'' \models \phi$

Before examining indefinites, we consider one simple case where it is apparent that material implication $\rightarrow_C$ does not work. Consider a team T such that $T(v) = \{v_1, v_2\}$, where v_1 is a world where Sue is happy, but Mary is not in Amsterdam, and v_2 is a world where Sue is not happy. Provided a suitable model, the material implication would make (12-b) vacuously supported in such a team, as $\phi(v)$ does not hold. All the other clauses for implication do not encounter this problem.

(12) a. If Sue is happy, then Mary is in Amsterdam.
b. $\phi(v) \rightarrow \psi(v)$

When considering the role of implication in (indicative) conditionals, there are many empirical observations that might help adjudicate between different notions of implications. The focus of the present discussion will be on indefinites, particularly on occurrences of marked indefinites in conditional antecedents and in the main clause of the conditional. Specifically, we will consider epistemic indefinites, modelled by $var(\varnothing, x)$, and specific known indefinites, modelled by $dep(\varnothing, x)$.

As the above conditions already suggest, finding a suitable notion of implication is not immediate, as one condition is upwards closed and the other is downwards closed. Let us consider the simple case in which indefinites occur in the main clause of a conditional, particularly an epistemic indefinite. (13-a) says that if we are in a situation where John is happy, then someone passed the course and the speaker does not know who. Consider the team where v_a is a world where a passed the course and John is happy, and likewise for v_b. Ideally, such a team should support (13-b). Clearly, the intuitionistic $\rightarrow_I$ and the singleton $\rightarrow_S$ do not work, as variation cannot be satisfied in singleton or empty subteams.

(13) a. If John is happy, then some student (epistemic) passed the course.
b. $\psi(v) \rightarrow \exists_s x(\phi(x, v) \land var(\varnothing, x))$

The maximal implication $\rightarrow_\forall$ does not suffer this problem, as we can evaluate the consequent with respect to the whole (maximal) team.

Next, we will examine another example, leading us to adopt a different clause for implication based on the maximal one. We now consider the behaviour of indefinites in conditional antecedents, specifically the case of specific known indefinites. (14-a) should be judged as true in cases where there is a specific student, and not just anyone, who passed the course and John is happy.

(14) a. If a certain student (specific-known) passed then course, then John is happy.
b. $\exists_s x(\phi(x, v) \land dep(\varnothing, x)) \rightarrow \psi(v)$

However, consider a team T such that $T(v) = \{\overline{v_a}, v_b\}$, where $\overline{v_a}$ is a world where student a passed the course, but John is not happy, whereas v_b is a world where student b passed the course and John is happy. Clearly, there are two maximal teams supporting the antecedent of (14-b), one for the portion of the team that agrees on a and one for the portion of the team that agrees on b. However, only the latter supports the consequent, and thus (14-b) is not supported.

This leads to adopting a weaker notion of maximal implication, where it suffices that *some* maximal teams supporting the antecedent support the consequent, rather than all:[22]

4.6.6. Definition (Weak Maximal Implication).
$M, T \models \phi \rightarrow_{\exists} \psi \Leftrightarrow$ for **some** $T' \subseteq T$ s.t. $M, T' \models \phi$ and T' is maximal, we have $M, T' \models \psi$

The semantic clause for weak maximal implication $\rightarrow_{\exists}$ states that a formula $\phi \rightarrow_{\exists} \psi$ holds when there is a maximal team that supports both the antecedent and the consequent.

Note that if a formula ϕ is closed under unions, then there is at most one maximal team $T' \subseteq T$ satisfying ϕ. In particular, all *dep*-free formulas are closed under unions. For formulas that are also *var*-free, downwards closure guarantees the existence and uniqueness of the maximal team, being trivially supported in the empty team. However, this is not the case for $var(\vec{z}, \vec{u})$ in general, where such a team may not exist. Importantly, when the maximal team is unique, the notions of maximal implication $\rightarrow_{\forall}$ and weak maximal implication $\rightarrow_{\exists}$ are equivalent.

4.7 Modality

2TS is a two-sorted predicate logic, with also variables for worlds. We can therefore analyse modals as (lax) quantifiers over worlds ($\Diamond_w \sim \exists_{l(ax)} w; \Box_w \sim \forall w$). Necessity modals will be analysed as universal quantifiers over worlds, and existential/possibility modals as lax existential quantifiers over worlds.[23]

In the context of 2TS, the accessibility relation R is a binary relation whose denotation is given by the assignment function I in the model. This means that we might take a universal modal like $\Box_w \phi$ as $\forall w(R(v, w) \rightarrow \phi[v/w])$ and an existential modal like $\Diamond_w \phi$ as $\exists_l w(R(v, w) \wedge \phi[v/w])$. For handling multiple modal operators, one might index the modal with a variable in the domain of the team for the first argument of R, which we took to be v for simplicity. For the

[22] The relevance of the maximal implication has also been investigated in the philosophical literature on indicative conditionals (Kolodny and MacFarlane 2010).

[23] As will see in Chapter 6, since lax quantification allows for branching extensions, we will be able to capture the availability of non-specific indefinites under possibility modals.

universal case, note that the antecedent of the implication is *dep* and *var* free, so both $\rightarrow_\forall$ and $\rightarrow_\exists$ will yield the same results.[24]

We have seen how this framework captures universal and existential modality. Kratzer (1986) and many others distinguish between two broad classes of modality: epistemic modals, compatible with what the speaker knows, and root/deontic modals, compatible with a set of circumstances or normative rules. For instance, the necessity modal *must* can be used epistemically, as in 'Sue must be home' or deontically, as in 'Sue must pay a fine.'.

One important feature of epistemic modals are so-called epistemic contradictions, which arise in formulas of the form $\neg\phi \wedge \Diamond\phi$:

(15) a. #It is not snowing, and it might be snowing.
b. $\neg S(v) \wedge \exists_l w\ S(w)$

As said, epistemic modality is related to the epistemic state of the speaker. And crucially, in this system, we already have a way to characterize the epistemic state of the speaker: the variable for the actual world v. As a result, we would like epistemic modals to be restricted to worlds over which v ranges. Deontic modality, on the other hand, is related to particular normative rules or desires which do not necessarily coincide with the state of affairs in the actual world. As a result, we would like deontic modality to range over worlds compatible with such norms, but not necessarily worlds over which v ranges.

Recall that the underlying idea of the framework is that the dependencies in the values of the variable introduced by an indefinite across different assignments help us model scopal and epistemic effects in indefinites. Similarly, the relationship between *world* variables can be used to model the difference between epistemic and deontic modality.

Since epistemic modals range only over worlds compatible with the speaker's epistemic state (the values of v), we propose that an epistemic modal introducing a variable w also triggers the restriction $\subseteq (w, v)$. By contrast, deontic modals are relational, meaning that for each world, different normative rules are possible.

[24]It is interesting to compare these notions of modality with those proposed in dependence logic for the propositional setting. We adapt here the definitions from Hella et al. (2014). A Kripke team semantics for modal logic can be given by $M = \langle W, R, V\rangle$, a normal Kripke model, and $T \subseteq W$. Regarding $\Box$:

$M, T \models \Box\phi$ iff $M, T' \models \phi$ for $T' = \{w \in W : \exists v \in T : R(v, w)\}$

Extending this to the first-order case could be done by a team extension with w such that for each assignment $i \in T$ is extended with w for each value s.t. $R(v, w)$. This is the same as having a universal extension and considering the maximal subteam where $R(v, w)$ holds.

$\Diamond$ is defined as follows:

$M, T \models \Diamond\phi$ iff $M, T' \models \phi$ for some T' s.t. $\forall v \in T \exists w \in T' : R(v, w)$ and $\forall w \in T' \exists v \in T : R(v, w)$.

The first clause ensures that each world in T sees some other world, and the second clause ensures that no world in T' is unseen. This is what a lax functional extension together with the requirement that $R(v, w)$ gives us.

To illustrate this, consider the basic cases in (16-b) and (17-b):[25]

(16) Epistemic Existential Modal
  a. John might be in Paris.
  b. $\exists_l w\ (\subseteq (w, v) \wedge \phi(w))$

(17) Deontic Existential Modal
  a. John is allowed to be in Paris.
  b. $\exists_l w\ (R(v, w) \wedge \phi(w))$

The table below displays some possible lax extensions for (16-b) and (17-b). For epistemic modality, the condition $\subseteq (w, v)$ guarantees that the worlds introduced by the functional extension will always be a subset of the values for v. For deontic modals, as illustrated in the examples in Table 4.9, it might not be the case that every world has access to the same set of 'normative-valid' worlds, and thus a world-dependent accessibility relation is needed. In other words, we are here proposing that epistemic modals are global, since they globally look at the epistemic state encoded by v, while deontic modals are relational, in line with several accounts of epistemic and deontic modality.

v
v_1
v_2
v_3

(a)

v	w
v_1	v_1
v_1	v_2
v_2	v_2
v_3	v_1

(b)

v	w
v_1	w_1
v_2	w_1
v_3	w_3
v_3	w_4

(c)

Table 4.9: Epistemic and Deontic Modals.

This treatment of epistemic modals readily captures epistemic contradictions like (15-b). Clearly, if a statement does not hold in the epistemic possibilities in v, then it will also not hold in the worlds introduced by an epistemic modal, since they are always a subset of the values of v.

Importantly, since we introduced modals as quantifiers, we can capture non-specific and specific readings of indefinites in modal environments by requiring the variable of the indefinite to possibly depend on the variable of the modal:

[25] One may of course add the pertinent constraints on the accessibility relation R for epistemic modals and maintain a uniform analysis.

(18) John wants to read a book.

a. Non-specific: John wants to read a book
$\forall w(R(v,w) \to \exists_s x(\phi(x,w) \wedge dep(vw,x)))$

b. Specific: There is a book x such that John wants to read x
$\forall w(R(v,w) \to \exists_s x(\phi(x,w) \wedge dep(v,x)))$

4.8 Negation

The 2TS semantic clauses presented in Section 3.3 have negation for literals and identity, and we thus assumed that all sentences are in negation normal form. In this section, we will discuss a general clause for negation. Different notions of negation have been investigated in dependence logic. A motivating factor behind this research agenda is that adding the so-called classical or Boolean negation in the language greatly increases the expressive power of the logic, leading to full second-order logic which is not completely axiomatizable by effective means (Väänänen 2007a). The aim of this section is to find a suitable notion of negation for 2TS which is also compatible with the empirical picture of (marked) indefinites and negation.

4.8.1 Negation and Scope

An important question we need to address is the syntactic configuration we should allow when an indefinite is negated. This issue is relevant for cases like the one shown in (19). The most salient reading for (19) is a narrow scope reading, where John read no book. However, a wide scope reading, where there is a specific book that John didn't buy, is also available. The latter reading can be made more salient by a continuation like 'John didn't buy a book because it was too expensive' where the pronominal element *it* forces the wide scope reading.[26]

(19) John didn't buy a book.

a. $[\neg > \exists]$ $\neg\exists x(\texttt{book}(x) \wedge \texttt{buy}(j,x))$

b. $[\exists > \neg]$ $\exists x(\texttt{book}(x) \wedge \neg\texttt{buy}(j,x))$

While the configuration in (19-b) is not incompatible with any island effects that negation is known to trigger, and in general, we have observed that indefinites can receive exceptional scope, we assumed that indefinites should always be evaluated in situ, in relation to the other operators in the sentence. Scope readings were explained not through movement, but rather through dependence atoms. We want to maintain this assumption for negation as well, so the only

[26]English *some book*, in contrast to the plain indefinite *a book*, is typically considered a positive polarity item, meaning that it can only receive the reading in (19-a). We will return to the possible difference between *some* and *a(n)* in Chapter 7.

logical form that we will admit is (19-a). In what follows, we will not only discuss how 2TS predicts the different readings of (19), but also consider the behaviour of marked indefinites under negation, keeping again in mind that the only logical form we admit is the following:

(20) a. John didn't buy INDEF book.
b. $\neg\exists_s x(\phi(x, v) \wedge \text{ATOM})$

A non-specific indefinite, like Turkish *herhangi*, gives rise to a narrow scope reading (i.e., the intended (19-a) reading), while a specific known indefinite, like Russian *koe*, gives rise to a specific reading (i.e., the intended (19-b) reading). While there is no available data for specific indefinites (indefinites with specific known and specific unknown uses), we assume here that they behave like specific-known ones in terms of scope, but they also licence a wide scope unknown reading.

(21) Herhangi bir şey gör-me-di-m
HERHANGI thing see NEG-PST-1SG
'I didn't see anything' (Haspelmath 1997, p. 286)

(22) Ivan ne chital koe-kakuyu knigu.
Ivan not read.PST KOE-KAKUYU.ACC book.ACC
'Ivan did not read a specific book.'

As concerns epistemic indefinites, it appears that this class is associated with both readings, a 'wide scope' (specific unknown) reading, and a narrow scope (non-specific) reading. We will describe the distribution of epistemic indefinites and negation in more detail in Chapter 5.[27]

We summarize the expected predictions in Table 4.10. For ease of illustration, we will consider the teams depicted in Table 4.11 in the last row of Table 4.10.

Type	Atom	WS-known	WS-unknown	NS
specific-known	$dep(\varnothing, x)$	✓	✗	✗
specific	$dep(v, x)$	✓	✓	✗
epistemic	$var(\varnothing, x)$	✗	✓	✓
non-specific	$var(v, x)$	✗	✗	✓
		T_3, T_4	T_2, T_4	T_4

Table 4.10: Marked Indefinites and Negation. The last row in the table indicates the teams in Table 4.11 compatible with such readings.

[27]The empirical distribution of epistemic indefinites is quite complex, and some indefinites in this class admit only one of these readings, especially when interacting with clausemate sentential negation. Some languages resort to specific constructions to distinguish between the two readings. In Spanish, the epistemic indefinite *algún* under a higher negation typically takes narrow scope when it is postnominal, and it typically takes wide scope when it is prenominal.

v
v_{ab}
v_b
$v_\varnothing$

(a) T_1

v
v_a
v_b

(b) T_2

v
v_b

(c) T_3

v
$v_\varnothing$

(d) T_4

Table 4.11: Basic Cases for Negation. v_a is the world in which John bought book a, $v_\varnothing$ is the world in which John bought no books, and so on.

As pointed out, in the following discussion, we will assume that negation always takes higher scope, as in (20). The desired behaviour of marked indefinites under negation should emerge from the interaction of a suitable notion of negation and the atom associated with each indefinite form. In the following, we will explore three different notions of negation: Boolean Negation, Dual Negation, and Intensional Negation.

4.8.2 Boolean Negation

We start by looking at the case of Boolean negation, defined in Definition 4.8.1. We refer to it as Boolean negation, following Väänänen (2007a) and Hintikka (1996), as it states that *it is not the case that* ϕ holds in T for a model M:

4.8.1. DEFINITION (Boolean Negation).

$$M, T \models \neg_B \phi \Leftrightarrow M, T \not\models \phi$$

This notion of negation has been first discussed in Hintikka (1996) within the context of independence-friendly logic. It roughly corresponds to a weak form of rejection or, to a certain extent, to a metalinguistic form of negation. For instance, while $P(x)$ states that x has the property P in all assignments of the team, $\neg_B P(x)$ states that x does not have the property P in some (not necessarily *all*) assignments of the team.[28]

The Boolean negation in Definition 4.8.1 appears to be ill-suited to model the negation of our dependence and variation atoms in interaction with the existential. The problem has to do with the 'weak' nature of the Boolean negation. For instance, a specific known indefinite with the $dep(\varnothing, x)$ atom is supported in all cases in which it is not the case that John does have a book, and we know which

[28] Hintikka (1996, 2002) claims that this notion of negation is compatible with sentence initial 'not' in English and cannot be embedded. This also relates to the way negation is treated in the game-theoretic semantics framework employed by Hintikka - independence friendly-logic. (Dual) negation, as we will see, standardly serves as a way to switch the verifier and falsifier roles. Instead, the Boolean negation $\neg_B \phi$ can only be used globally to express the fact that the verifier does not have a winning strategy.

one. So an initial team in which the value of x is not known, as in T_2, would be supporting. Moreover, we observe that non-specific indefinites are supported in all the cases we are considering here. The reason is that negation is not introducing any operator in the team, and it is thus effectively treating configurations like (20-b) alike episodic contexts, where the v-variation condition $var(v, x)$ is never supported. We summarize the predictions for the relevant cases in Table 4.12.

Type	Atom	Expected Predictions	Predictions
specific-known	$dep(\varnothing, x)$	T_3, T_4	T_1, T_2, T_4
specific	$dep(v, x)$	T_2, T_3, T_4	T_1, T_4
epistemic-i	$var(\varnothing, x)$	T_4	T_1, T_3, T_4
epistemic-ii	$var(\varnothing, x) \wedge dep(v, x)$	T_2, T_4	T_1, T_3, T_4
non-specific	$var(v, x)$	T_4	T_1, T_2, T_3, T_4

Table 4.12: $\neg_B \exists_s x(\phi(x, v) \wedge \text{ATOM})$

4.8.3 Dual Negation

An alternative notion of negation is presented in Definition 4.8.2, which we refer to as Dual Negation. The Dual Negation has again its origin in the game-theoretic semantics of Hintikka (1996), where, as said before, it serves a mechanism to switch the verifier and falsifier role. The Dual Negation is the typical negation assumed in Dependence Logic, where it is defined by requiring that for a first-order literal π, $M, T \models \neg\pi$ if and only if for all $i \in T$, $M, \{i\} \not\models \pi$ and in terms of de Morgan's laws and double negation elimination. Here we give the semantic clauses in a dual form, with support and an anti-support clauses (Hintikka 1996; Väänänen 2007a).[29]

4.8.2. Definition (Dual Negation). Given a suitable model M and a suitable team T over M, a formula ϕ, we define the support relation of ϕ in T, denoted by $M, T \models \phi$, and the anti-support relation of ϕ in T, denoted by $M, T \dashv \phi$, inductively on ϕ as follows:

[29] Since we have two notions of existential, which are not equivalent for upwards closed formulas, we need to carefully understand the relationship between support and anti-support clauses of existentials and universals. We might assume that both anti-support clauses of the existentials lead to the anti-support of the universal, keeping the duality between existential and universal quantifiers. Another option is to have the strict existential anti-supported when there are no strict functions which support it (even though there could be lax ones). Here, we simply consider the case of the strict existential, as this notion of negation will not be the one we adopt.

$M,T \models P(t_1,\ldots,t_n)$	$\Leftrightarrow$	$\forall j \in T:\ \langle j(t_1),\ldots,j(t_n)\rangle \in I(P^n)$
$M,T \dashv P(t_1,\ldots,t_n)$	$\Leftrightarrow$	$\forall j \in T:\ \langle j(t_1),\ldots,j(t_n)\rangle \notin I(P^n)$
$M,T \models t_1 = t_2$	$\Leftrightarrow$	$\forall j \in T:\ j(t_1) = j(t_2)$
$M,T \dashv t_1 = t_2$	$\Leftrightarrow$	$\forall j \in T:\ j(t_1) \neq j(t_2)$
$M,T \models \phi \wedge \psi$	$\Leftrightarrow$	$M,T \models \phi$ and $M,T \models \psi$
$M,T \dashv \phi \wedge \psi$	$\Leftrightarrow$	$T = T_1 \cup T_2$ for teams T_1 and T_2 s.t. $M,T_1 \dashv \phi$ and $M,T_2 \dashv \psi$
$M,T \models \phi \vee \psi$	$\Leftrightarrow$	$T = T_1 \cup T_2$ for teams T_1 and T_2 s.t. $M,T_1 \models \phi$ and $M,T_2 \models \psi$
$M,T \dashv \phi \vee \psi$	$\Leftrightarrow$	$M,T \dashv \phi$ and $M,T \dashv \psi$
$M,T \models \exists_s z\, \phi$	$\Leftrightarrow$	there is a strict function f_s s.t. $M,T[f_s/z] \models \phi$
$M,T \dashv \exists_s z\, \phi$	$\Leftrightarrow$	$M,T[z] \dashv \phi$
$M,T \models \forall z\, \phi$	$\Leftrightarrow$	$M,T[z] \models \phi$
$M,T \dashv \forall z\, \phi$	$\Leftrightarrow$	there is a strict function f_s s.t. $M,T[f_s/z] \dashv \phi$
$M,T \models \neg\phi$	$\Leftrightarrow$	$M,T \dashv \phi$
$M,T \dashv \neg\phi$	$\Leftrightarrow$	$M,T \models \phi$

More relevant to the current discussion are the clauses for the dependence and variation atoms. We will consider two versions. The former originates from the work of Väänänen (2007a) and is defined as below.[30]

$M,T \models dep(\vec{z},\vec{u})$	$\Leftrightarrow$	for all $i,j \in T: i(\vec{z}) = j(\vec{z}) \Rightarrow i(\vec{u}) = j(\vec{u})$
$M,T \dashv dep(\vec{z},\vec{u})$	$\Leftrightarrow$	$M,T \models \bot$
$M,T \models var(\vec{z},\vec{u})$	$\Leftrightarrow$	there is $i,j \in T: i(\vec{z}) = j(\vec{z})\ \&\ i(\vec{u}) \neq j(\vec{u})$
$M,T \dashv var(\vec{z},\vec{u})$	$\Leftrightarrow$	$M,T \models \top$

Table 4.13: Dual Negation for Dependence and Variation Atom - Version 1.

In this formulation, a dependence atom is anti-supported only in the empty team, while the variation atom is anti-supported in all teams.[31] These anti-support clauses are incompatible with the intended predictions, as summarized in Table 4.14. For a first order formula α, we have that for all non-empty teams T,

[30] Note that due to the variation atom, which requires the existence of assignments in the team, strong and weak notions of tautologies and contradictions may arise, similarly to what occurs in logics which employ an atom requiring the team to be non-empty, as in BSML (Aloni 2022). In what follows, we will assume that $M,T \models \bot$ iff $T = \varnothing$, where $\varnothing$ is the team which contains no assignment and $M,T \models \top$ for any team T.

[31] The anti-support clause for the dependence atom in the dependence logic tradition (Väänänen 2007a) is meant to preserve downwards closure. Moreover, if we apply the notion of negation used for first-order literals, we obtain that $M,T \models \neg dep(\vec{z},\vec{u})$ iff $\forall i \in T : M,T \not\models dep(\vec{z},\vec{u})$ iff $T = \varnothing$. Applying this to the variation atoms leads to $M,T \models \neg var(\vec{z},\vec{u})$ iff $\forall i \in T : M,\{i\} \not\models var(\vec{z},\vec{u})$ iff $T = \top$, since variation requires the existence of two distinct assignments.

$M, T \models \neg\exists_s(\alpha(x, v) \wedge dep(\vec{z}, \vec{u}))$ iff $M, T \models \forall x(\neg\alpha(x, v) \wedge \neg dep(\vec{z}, \vec{u}))$ iff $M, T \models \forall x(\neg\alpha(x, v) \vee \bot)$ iff $M, T \models \forall x(\neg\alpha(x, v))$. Similarly, $M, T \models \neg\exists_s x(\alpha(x, v) \wedge var(\varnothing, x))$ iff $M, T \models \top$.

Type	Atom	Expected Predictions	Predictions
specific-known	$dep(\varnothing, x)$	T_3, T_4	T_4
specific	$dep(v, x)$	T_2, T_3, T_4	T_4
epistemic-i	$var(\varnothing, x)$	T_4	T_1, T_2, T_3, T_4
epistemic-ii	$var(\varnothing, x) \wedge dep(v, x)$	T_2, T_4	T_1, T_2, T_3, T_4
non-specific	$var(v, x)$	T_4	T_1, T_2, T_3, T_4

Table 4.14: $\neg_{D_1}\exists_s x(\phi(x, v) \wedge \text{ATOM})$

Alternatively, we have already discussed that the variation atom was defined as the Boolean negation of the dependence atom. As a result, we might take anti-support clauses of the dependence and variation atoms to the corresponding Boolean negation.

$$M, T \models dep(\vec{z}, \vec{u}) \quad \Leftrightarrow \quad \text{for all } i, j \in T : i(\vec{z}) = j(\vec{z}) \Rightarrow i(\vec{u}) = j(\vec{u})$$
$$M, T \dashv dep(\vec{z}, \vec{u}) \quad \Leftrightarrow \quad \text{there is } i, j \in T : i(\vec{z}) = j(\vec{z}) \;\&\; i(\vec{u}) \neq j(\vec{u})$$

$$M, T \models var(\vec{z}, \vec{u}) \quad \Leftrightarrow \quad \text{there is } i, j \in T : i(\vec{z}) = j(\vec{z}) \;\&\; i(\vec{u}) \neq j(\vec{u})$$
$$M, T \dashv var(\vec{z}, \vec{u}) \quad \Leftrightarrow \quad \text{for all } i, j \in T : i(\vec{z}) = j(\vec{z}) \Rightarrow i(\vec{u}) = j(\vec{u})$$

Table 4.15: Dual Negation for Dependence and Variation Atom - Version 2.

While this alternative formulation might seem more intuitive and could be independently motivated, it does not significantly help with the interaction of indefinites and atoms we are considering here. For dependence atoms, a formula of the form $\forall x(\neg\alpha(x, v) \vee var(v, x))$ is always supported as long as there are more than two individuals in the domain. For variation atoms, a formula $\forall x(\neg\alpha(x, v) \vee dep(v, x))$ is supported only in team T_4, since one subteam needs to be constant with respect to T and the other one needs to make $\alpha(x, v)$ false.[32] The latter is a good prediction, as it correctly captures the narrow scope readings of indefinites under negation, but it cannot deal with wide scope readings.

[32] A further undesired prediction that we are not considering here is epistemic-i would also support cases in which $T(v) = \{v_a, v_\varnothing\}$ for instance.

Type	Atom	Expected Predictions	Predictions
specific-known	$dep(\varnothing, x)$	T_3, T_4	T_1, T_2, T_3, T_4
specific	$dep(v, x)$	T_2, T_3, T_4	T_1, T_2, T_3, T_4
epistemic-i	$var(\varnothing, x)$	T_4	T_1, T_2, T_3, T_4
epistemic-ii	$var(\varnothing, x) \wedge dep(v, x)$	T_2, T_4	T_4
non-specific	$var(v, x)$	T_4	T_4

Table 4.16: $\neg_{D2}\exists_s x(\phi(x, v) \wedge \textsc{atom})$

4.8.4 Intensional Negation

We now turn to a different notion of negation, which appears to be well-behaved in interaction with marked indefinites. Recall that our framework is two-sorted with a special variable for the actual world, v. It is thus possible to view negation as a particular kind of quantification over worlds. We thus, we adopt an intensional notion of negation (Brasoveanu and Farkas 2011; Berto 2015), which we define in Definition 4.8.3:

4.8.3. Definition (Intensional Negation).

$$\neg_I \phi(v) \Leftrightarrow \forall w(\phi[v/w] \rightarrow v \neq w)$$

Definition 4.8.3 says that when ϕ does not hold in the actual world, it must be the case that for all worlds w in which ϕ holds, w must be different from the actual world.[33]

Note that intensional negation in Definition 4.8.3 contains an implication. In light of our discussion in Section 4.6, we adopt the weak maximal implication $\rightarrow_\exists$. As we will see, this will give us the corrects predictions in a way which is parallel to the case of indicative conditionals discussed in Section 4.6, highlighting the parallelism between negation and implication.

We observe that for classical formulas (formulas without the dependence or the variation atom), the intensional notion of negation in Definition 4.8.3 and the dual negation in Definition 4.8.2 are equivalent over initial teams. Note that classical formulas are downwards closed and closed under union. So there is always a maximal team satisfying $\phi[v/w]$ above, and it is unique.

4.8.4. Fact. Let ϕ be any *dep*-free and *var*-free 2TS formula. Then $\neg_D \phi \equiv_v \neg_I \phi$

[33] Non-identity is defined as in the semantic clauses of 2TS. Here we use $v \neq w$ in place of $\neg v = w$.

$$M, T \models x \neq y \Leftrightarrow \forall i \in T \text{ s.t. } i(x) \neq i(y)$$

Proof:
The proof is by structural induction on ϕ, and follows from the semantic clauses of $\neg_I$ and $\neg_D$. We focus on the basic case. For the case of negation, it is important to note that $\neg_D\neg_D\phi \equiv \phi$. Let M be arbitrary and T an arbitrary suitable initial team over M.

(i) Let ϕ be a first-order literal. ($\Rightarrow$) Assume $M, T \models \neg_D\phi$. Then $M, \{i\} \not\models \phi$ for all $i \in T$. Let $X = T[w]$ be the universal w-extension of T, and $T' \subseteq X$ the maximal team satisfying $\phi[v/w]$. If $T' = \varnothing$, then $v \neq w$. Otherwise, by maximality there is no $j \in T'$ s.t. $j(v) = j(w)$ and thus $v \neq w$.
($\Leftarrow$) Conversely, assume $M, T \models \neg_I\phi$. Let $i \in T$ be arbitrary and suppose $M, \{i\} \models \phi$. Then there exists $j \in T'$ with T' the maximal team of $T[w]$ satisfying $\phi[v/w]$ s.t. $j(v) = j(w)$. But then it is not the case that $v \neq w$, which is impossible. Hence $M, \{i\} \not\models \phi$.

□

As concerns the interaction of negation and marked indefinites, some remarks on the 'for some' versus 'for all' distinction in the definition of maximal implication we discussed in Section 4.6 are in order. For non-specific indefinites, the formula in the antecedent of the conditional will be union-closed, and thus this difference is trivialized, as long as there are at least two individuals in the domain D. However, the *for some* clause will play a role when an atom like $dep(\varnothing, x)$ leads to more than one maximal supporting team (i.e., compatibly with different possible constants values for x.)

Let's then consider cases like (23), again under the assumption that *a certain* stands for a specific known indefinite, triggering $dep(\varnothing, x)$:

(23) a. John does not have a certain book.
b. $\forall w(\exists_s x(\phi(x, w) \wedge dep(\varnothing, x)) \rightarrow v \neq w)$

The formula in (23-b) should be supported when the initial team is $\{w_\varnothing\}$[34], corresponding to a world where John read no book), or $\{w_a\}$ (John read book a and not b) or $\{w_b\}$ (John read book b and not a). But not by $\{w_{ab}\}$, corresponding to a world where John read both book a and book b. This is precisely what we predict. When the initial team is $\{w_\varnothing\}$, both maximal teams satisfying the antecedent (i.e., $\exists_s x(\phi(x, w) \wedge dep(\varnothing, x))$) support the consequent (i.e., $v \neq w$). For $\{w_a\}$, we have two maximal teams satisfying the antecedent, but only the one which maps x to b also supports the consequent. For $\{w_{ab}\}$, none of the maximal teams satisfying the antecedent supports the consequent. We illustrate this in Table 4.17.

Let's now examine the interaction between non-specific indefinites and negation. To facilitate the analysis, we consider the example in (24), with 'some-*nibud*'' as a placeholder for a non-specific indefinite.

[34] We use $\{w_\varnothing\}$ to indicate the initial team where the value of v is $w_\varnothing$. Given that there are no other variables in an initial team, we can also represent it by means of the projection $T(v) = \{w_\varnothing\}$.

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	a
$w_\varnothing$	w_{ab}	a

(a) Supporting

v	w	x
w_a	$w_\varnothing$	a
$\mathbf{w_a}$	$\mathbf{w_a}$	a
w_a	w_b	a
w_a	w_{ab}	a

(b) Non-Supporting

v	w	x
w_{ab}	$w_\varnothing$	a
w_{ab}	w_a	a
w_{ab}	w_b	a
$\mathbf{w_{ab}}$	$\mathbf{w_{ab}}$	$\mathbf{a}$

(c) Non-Supporting

v	w	x
$w_\varnothing$	$w_\varnothing$	b
$w_\varnothing$	w_a	b
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

(d) Supporting

v	w	x
w_a	$w_\varnothing$	b
w_a	w_a	b
w_a	w_b	b
w_a	w_{ab}	b

(e) Supporting

v	w	x
w_{ab}	$w_\varnothing$	b
w_{ab}	w_a	b
w_{ab}	w_b	b
$\mathbf{w_{ab}}$	$\mathbf{w_{ab}}$	$\mathbf{b}$

(f) Non-Supporting

Table 4.17: Illustrations for (23). Worlds differ with respect to which books John has. In $w_\varnothing$ John has no book, in w_a John has only book a, and so on. The maximal teams satisfying the antecedent in (23-b) are depicted in grey.

(24) a. John does not have some-*nibud'* book.
  b. $\forall w(\exists_s x(\phi(x,w) \wedge var(v,x)) \rightarrow v \neq w)$

Crucially, in this case, there is only one maximal team satisfying the antecedent. The variation atom $var(v,x)$ is trivialized, and the resulting reading is simply a negated existential, which is supported only for the initial team $\{w_\varnothing\}$.

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

(a) Supporting

v	w	x
w_a	$w_\varnothing$	a
$\mathbf{w_a}$	$\mathbf{w_a}$	$\mathbf{a}$
w_a	w_b	b
w_a	w_{ab}	b

(b) Non-supporting

Table 4.18: Supporting and Non-supporting teams for (24-b).

4.9 Marked Indefinites and Diachrony

We have seen how 2TS can account for the variety of indefinites. Indefinites (Haspelmath 1997; Gianollo 2019) tend to constitute a highly dynamic environment from a diachronic perspective. The functional distribution of indefinites in a given language changes over time more frequently compared to other parts of the grammar.

Type	Atom	Expected Predictions	Predictions
specific-known	$dep(\varnothing, x)$	T_3, T_4	T_3, T_4
specific	$dep(v, x)$	T_2, T_3, T_4	T_2, T_3, T_4
epistemic-i	$var(\varnothing, x)$	T_4	T_4
epistemic-ii	$var(\varnothing, x) \wedge dep(v, x)$	T_2, T_4	T_2, T_4
non-specific	$var(v, x)$	T_4	T_4

Table 4.19: $\neg_I \exists_s x(\phi(x, v) \wedge \text{ATOM})$

An influential tradition called formal diachronic semantics (Deo 2015b) combines principles from formal semantics and diachronic linguistics to study how meaning changes over time. The underlying idea is that the historical development of linguistic constructions can inform formal models, helping to determine which models can better handle diachronic developments. Conversely, formal systems can provide insights into predicted and expected diachronic developments. In recent years, a growing number of works have emerged in this tradition (Eckardt 2006; Gianollo 2019; Deo 2015a; Beck and Gergel 2015; Beck 2020).

Regarding indefinites, several questions arise: examining the grammaticalization patterns of various indefinite forms, developing models that account for the distribution of indefinites within and across languages, studying the interaction of indefinites with related constructions conveying similar meanings, and investigating how marked indefinites turn into unmarked ones, and vice versa. In this section, we focus on outlining the main predictions and observations that stem from the characterization of marked indefinites in 2TS.[35]

In this regard, it is insightful to revisit the Dependence Square of Opposition presented in Section 4.4.4, and reproposed here. Several remarks are worth noting. First, we do not expect an indefinite associated with a certain atom to turn into its contradictory, as this would imply that the new form conveys all and only the functions that the previous form did not. This excludes changes from specific known $dep(\varnothing, x)$ to epistemic $var(\varnothing, x)$ and vice versa, as well as from non-specific $var(v, x)$ to specific $dep(v, x)$ and vice versa.

Second, a change among contraries is also problematic for two reasons. On the one hand, the functional gain resulting from such a change does not align with the underlying convexity of the meaning space of such functions, as discussed in Section 4.4.3. For instance, developing from specific known into non-specific

[35] This implies that we are not considering any systematic contrast with respect to the whole array of indefinite forms in a given language. For instance, if a language displays both a non-specific and an epistemic indefinite, then it would be unlikely for the non-specific indefinite to turn into an epistemic one since a dedicated form already covers the latter. Eventually, it would be beneficial to integrate the insights that 2TS offers with respect to the formal characterization of (non-)specific indefinites, and models that investigate indefinite pronouns from a more systematic perspective like Denić, Steinert-Threlkeld, and Szymanik (2022).

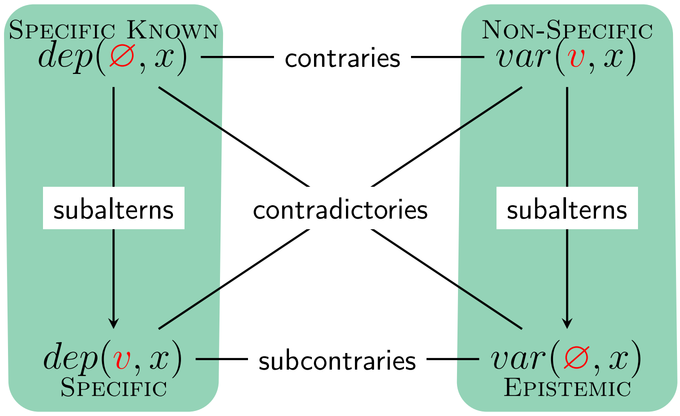


Figure 4.6: Dependence Square of Opposition. The green shades corresponding to subalternation indicate semantic weakening. The red colour highlights the concreteness ($\varnothing$) vs. abstractness (v) factor.

ignores the intermediate specific unknown function. On the other hand, if a specific known $dep(\varnothing, x)$ turns into a non-specific $var(v, x)$, the latter entails $var(\varnothing, x)$, which contradicts the original form. Thus, changes among contraries are not expected to occur.

Third, we consider the relation of subalternation, which corresponds to entailment and thus to weakening among atoms. Semantic weakening has been central in diachronic changes (Heine 1997; Traugott and Dasher 2002; Hopper and Traugott 2003; Heine 2017; Lehmann 2015; Bybee 2017) and is often referred to as semantic bleaching in the context of content words, where an expression acquires a more general meaning (e.g., the change from Latin *verum* 'true' to English *very*). We thus expect this to be a strong factor favoring changes from non-specific $var(v, x)$ to epistemic $var(\varnothing, x)$ and from specific known $dep(\varnothing, x)$ to specific $dep(v, x)$. Although the other direction, corresponding to strengthening, is less common, it is not excluded (Heine 2017).

Fourth, regarding subcontraries, the convexity of functional gains is not violated. However, for such changes to occur, an indefinite must both gain and lose a function.

Fifth, we focus on the role of the variable for the actual world v. Worlds are more abstract entities than individuals, and languages often change from more concrete to more abstract concepts (Heine 1997; Port and Aloni 2021). In 2TS, the distinction between known and unknown is captured by a variable v that ranges over worlds. Without this variable, or with a fixed value of v, $dep(\varnothing, x)$ and $var(\varnothing, x)$ would express the specific vs. non-specific contrast. The addition of world variables allows for the distinction between known and unknown, leading to the addition of $dep(v, x)$ and $var(v, x)$. Thus, we expect changes from $dep(v, x)$ to $dep(\varnothing, x)$ and from $var(v, x)$ to $var(\varnothing, x)$. We call this factor abstractness/concreteness.

Old Indefinite	New Indefinite	⇒/⇐	Functional Gain	Concreteness	Example
specific known $dep(\varnothing, x)$	specific $dep(v, x)$	⇒	+1	$\varnothing \to v$	unattested
specific known $dep(\varnothing, x)$	unmarked	⇒	+2	-	unattested
specific $dep(v, x)$	specific known $dep(\varnothing, x)$	⇐	-1	$v \to \varnothing$	unattested
specific $dep(v, x)$	epistemic $var(\varnothing, x)$	neither	-1;+1	$v \to \varnothing$	unattested
specific $dep(v, x)$	unmarked	⇒	+1	-	*English *one*
epistemic $var(\varnothing, x)$	specific $dep(v, x)$	neither	-1;+1	$\varnothing \to v$	unattested
epistemic $var(\varnothing, x)$	non-specific $var(v, x)$	⇐	-1	$\varnothing \to v$	*Italian *alcun(o)*
epistemic $var(\varnothing, x)$	unmarked	⇒	+1	-	*Icelandic *nokkur*
non-specific $var(v, x)$	epistemic $var(\varnothing, x)$	⇒	+1	$v \to \varnothing$	German *irgend-*
non-specific $var(v, x)$	unmarked	⇒	+2	-	unattested

Table 4.20: Factors driving diachronic change from Old Indefinite to New Indefinite. The ⇒/⇐ stands for semantic weakening/semantic strengthening. Weakening is the expected directionality. *Functional Gain* indicates how many functions were gained or lost. Concreteness represents the factor discussed in the main text. In the Example column, a * indicates that the indefinite is in line with respect to that change, but further remarks are needed.

Lastly, we consider how a marked indefinite might turn into an unmarked one concerning (non-)specificity. The issue is general, but we focus on the four types of (non-)specific indefinites central to 2TS. Changing from specific known $dep(\varnothing, x)$ to unmarked requires gaining two additional functions/uses. The same applies to changing from non-specific $var(v, x)$ to unmarked. Changes from specific $dep(v, x)$ to unmarked and from epistemic $var(\varnothing, x)$ to unmarked require only one functional gain. In the former case, this development also neutralizes the abstractness factor, as there is no atom tied to quantification over worlds.

Concerning concrete examples, we summarize the discussion of this section in Table 4.20, together with an 'Example' column. For instance, a change from specific to unmarked could be represented by English *one*, though it cannot be classified as an indefinite pronoun in the sense of Haspelmath (1997). We will return to this point in Chapter 7. The change from non-specific to epistemic satisfies all constraints and is quite common (e.g., Port and Aloni 2021 for German *irgend-* or Foulet 1919 for French *quelque*). We will return to this point in Chapter 5. Epistemic indefinites might also turn into unmarked. This could be exemplified by Icelandic *nokkur*, as we will further discuss in Chapter 5, but this is by no means a frequent development. The change from epistemic to non-specific is exemplified by Italian *alcun(o)*, which used to have specific unknown uses that are now lost (Gianollo 2019). A similar development occurred for Dutch *enig* (Hoeksema 2010). However, the non-specificity of both these indefinites is tied to negation. We will return to this point in Chapter 6. Notably, all these changes are in line with the constraints and hypotheses stemming from the `2TS` formalization, as evidenced by Table 4.20. One case which is not attested is from specific to specific known. This development is in line with some of the constraints, but it is a case of strengthening and not of weakening: we might conjecture that weakening is a strong requirement, as the other case in which strengthening possibly occurred concerned negation, as we will further discuss in Chapter 5.

4.10 Conclusion

In this chapter, we have examined the variety of readings associated with indefinites across languages. We have focused on the distinctions between specific/non-specific and known/unknown uses of indefinites. We have investigated how various languages have developed lexicalized forms with restricted distributions pertaining to these uses. We have proposed that indefinites are strict existentials, evaluated in situ, and that their variety can be captured by dependency and variation atoms. In the next chapters, we will delve into different indefinites types and discuss their distribution and the predictions of `2TS` in more detail.

Chapter 5

Epistemic Indefinites

> Ogni individuo, pure il meno intelligente e l'infimo dei paria, fino da bambino si dà **una qualche** spiegazione del mondo. E in quella si adatta a vivere. E senza di quella, cadrebbe nella pazzia.
>
> 'Every individual, even the least intelligent and the lowest of the pariahs, from childhood gives **some** explanation **or other** of the world. And in that explanation, they adapt to live. Without it, they would fall into madness.'
>
> Elsa Morante, *La Storia*

Indefinite expressions in everyday language often signal the speaker's lack of knowledge about the referent. Across many languages, we observe lexicalized indefinite forms that specifically convey a lack of knowledge. These are known as epistemic indefinites and include examples like German *irgend-*, Italian *un qualche*, Spanish *algún*, Russian *-to*, Finnish *-kin*, and many more.[1]

In this chapter, we will revisit our treatment of epistemic indefinites proposed in Chapter 4 and explain their distribution in Section 5.1. We will then focus in Section 5.2 on a comparison with the so-called implicature account, where the ignorance component of epistemic indefinites is derived as an implicature. Section 5.3 is dedicated to the role of negation and its interaction with epistemic indefinites, while Section 5.4 covers the free choice uses of epistemic indefinites. In Section 5.5, we show how the present account can be integrated with conceptual covers, which have been shown to be important in the analysis of epistemic indefinites. Section 5.6 deals with the puzzling behaviour of plural forms of epistemic

[1] The term 'epistemic indefinites' originates from the specificity distinctions (scopal, epistemic, partitive) introduced by Farkas (1994). This class of indefinites has received significant attention in the literature under various labels, such as ignoratives Karceoski (1941), modal indefinites with partial variation Alonso-Ovalle and Menéndez-Benito (2010), and (subtype of) existential free choice indefinites Chierchia (2013), among others.

indefinites, while Section 5.7 covers indifference readings. We conclude in Section 5.8 with some diachronic remarks on this class of indefinites.[2]

5.1 Ignorance and Variation

In Chapter 4, we proposed that epistemic indefinites should be characterized by the atom $var(\varnothing, x)$, which we refer to as the Variation Condition in (1).

(1) The Variation Condition
Epistemic indefinites are associated with the variation condition $var(\varnothing, x)$.

There are two properties shared by all epistemic indefinites that any theory should account for: (i) they are associated with an indefeasible ignorance inference in episodic contexts, and (ii) they can co-vary with another operator. The Variation Condition in (1) accounts for both (i) and (ii). We discuss the first point in Section 5.1.1 and the second point in Section 5.1.2.

5.1.1 Ignorance Inferences

The hallmark of epistemic indefinites is their indefeasible ignorance inference in episodic contexts.[3] For instance, the 'namely' continuation in (2) combined with the Italian *un qualche* results in oddity. A similar behaviour is observed for the German *irgendein* in (3) when asking the hearer to guess which student called.

(2) Maria ha sposato un qualche dottore (#cioè Ugo).
Maria has married un qualche doctor (#namely Ugo)
Maria married some doctor, namely Ugo.'

(3) Irgendein Student hat angerufen. #Rat mal wer?
some student has called. #guess who?
'Some (unknown) student called. #Guess who?

The variation condition $var(\varnothing, x)$ accounts for this ignorance effect. For episodic sentences like (4), $var(\varnothing, x)$ gives rise to the ignorance component of epistemic indefinites. In particular, the strict existential will ensure that there is only one value for x for each world for v, given that only v is in the domain of the

[2]Part of this chapter is based on Maria Aloni and Marco Degano (2022). "(Non-)specificity across languages: constancy, variation, v-variation". In: *Semantics and Linguistic Theory*. Vol. 32, pp. 185–205. In particular, one paragraph (adapted and expanded) included in Section 5.1.2. The study was conceptualized through joint discussions between Maria Aloni and Marco Degano. The writing of the paper was carried out by Marco Degano.

[3]In such contexts, epistemic indefinites can also give rise to 'indifference' readings, where speakers use an epistemic indefinite to signal that the identity of the referent is not relevant or important, even if they know the referent's identity. We will return to indifference readings in Section 5.7.

initial team, and the variation component $var(\varnothing, x)$ will ensure that the value of x is not constant across all epistemic possibilities of the speaker (i.e., the speaker does not know the value of x).

(4) a. Maria ha sposato un qualche dottore.
Maria has married UN QUALCHE doctor.
Maria married some doctor.'
b. $\exists_s x(\phi(x,v) \wedge var(\varnothing, x))$

The inference we are interested in is of the following form: $\exists x\phi(x) \rightsquigarrow \exists y(\Diamond\phi(y) \wedge \exists z(\Diamond\phi(z) \wedge y \neq z))$ with $\Diamond$ being an epistemic modal. We can then prove something like the former for any classical formula ϕ. Here we give a proof for the basic case where ϕ is a literal. Note that we are assuming that both domains in the model contain at least two entities.[4]

5.1.1. FACT (Ignorance Inference). $\exists_s x(\phi(x,v) \wedge var(\varnothing, x)) \models_v \exists_s y(\exists_w w(\phi(y,w)) \wedge \exists_s z(\exists_{w'} w'(\phi(z,w')) \wedge y \neq z))$

Proof:
The proof is by structural induction on ϕ. We focus on the basic case. Let M be arbitrary and T an arbitrary suitable initial team over M.

(i) Let ϕ be a first-order literal. Suppose that $M, T \models \exists_s x(\phi(x,v) \wedge var(\varnothing, x))$. Then for some strict function $f_s : T \to D$, there exist $i_1, i_2 \in T[f_s/x]$ s.t. $M, \{i_1\} \models \phi(x,v)$ and $M, \{i_2\} \models \phi(x,v)$ with $d_1 = i_1(v) \neq i_2(v) = d_2$ for some $d_1, d_2 \in D$ and $v_1 = i_1(v) \neq i_2(v) = v_2$ for some $v_1, v_2 \in W$. Let $T' = T[f_s/y]$ be the strict extension of T with y and $T'' = T'[f_l/w]$ be the lax extension of T' with w s.t. for all $j \in T''$ $j(x) = d_1$ and $j(w) = v_1$. Then $M, T'' \models \phi(x,w) \wedge \subseteq (w,v)$. A similar constructive procedure for d_2 gives $M, T'' \models \exists_s z \exists_l w'(\subseteq (w,v) \wedge \phi(z,w'))$. Since $d_1 \neq d_2$, it follows that $x \neq y$.

□

Interestingly, we can model degrees of ignorance by generalizing the variation atom from Chapter 3.[5] Instead of requiring the value to change across at least two assignments, as in the case of $var(\varnothing, x)$, we can generalize variation to level k and retrieve $var(\varnothing, x)$ when k is 2:

5.1.2. DEFINITION (Generalized Variation).
$var_k(\vec{u}, \vec{z}) \Leftrightarrow$ there is $i \in T : |\{j(\vec{z}) : j \in T \text{ and } i(\vec{u}) = j(\vec{u})\}| \geq k$

An equivalent formulation would be to require that for some $\vec{e} \in T(\vec{u})$, we must have $|T_{\vec{u}=\vec{e}}(\vec{z})| \geq n$. In the case of $var_k(\varnothing, x)$, such atom amounts to requiring

[4]We are using the notion $\models_v$ of restricted entailment over initial teams as defined in Definition 3.3.3 in Chapter 4.

[5]See Väänänen (2022) for a similar operation in the context of database theory for the generalized variation atom (to level k) of the stronger variation atom discussed in Section 3.2.2 of Chapter 3.

that in the supporting team $|T(x)| \geq k$. As said, the previous variation atom corresponds to $k = 2$. A different k could indicate a different degree of ignorance regarding the value of x. The extreme case in which k is equal to the cardinality of the domain D would correspond to a context in which the speaker is completely ignorant with respect to the value of x.

5.1.2 Co-variation Uses

Epistemic indefinites also exhibit what is known as a co-variation reading when embedded under universal quantifiers or other quantificational operators (Alonso-Ovalle and Menéndez-Benito 2010). The sentence in (5) illustrates this for the Spanish *algún*. Example (5) is compatible with a situation in which each professor dances with a different student.

(5) Todos los profesores están bailando con algún estudiante.
all the professors are dancing with algún student.
'Every professor is dancing with some student.'

Note that the 'ignorance reading', where every professor is dancing with a specific unknown student, is still available in cases like (5), although this reading is less salient. A similar contrast is observed in German with *irgendein*, as shown in (6):

(6) Jeder$_y$ Student hat irgendein$_x$ Buch gelesen.
every student has irgendein book read

a. Ignorance: There is a particular book x which every student y read. The speaker does not know which one.
b. Co-variation: For every student x, there is a book y s.t. x read y.

We claim that the availability of both readings in (6) can be readily captured by the variation condition $var(\varnothing, x)$. The difference in (6) is due to the different scope configurations of the indefinite with respect to the universal quantifier.

More concretely, the crucial fact is that the two readings reflect the different scope of the indefinite, which is handled by dependence atoms. Consider the example in (7) and the teams in Table 5.1. The indefinite can receive both wide scope, modelled by $dep(v, x)$, and narrow scope, modelled by $dep(vy, x)$. When the indefinite receives wide scope, $var(\varnothing, x)$ ensures that the value of x changes across different epistemic possibilities. When the indefinite receives narrow scope, the value of x can vary with respect to y and thus does not need to vary with respect to v. This explains the disappearance of the ignorance effect in co-variation readings.[6]

[6]Note that $dep(vy, x)$ in (7-b) is modelling narrow scope, but it is not forcing a co-variation readings. Similarly, wide scope configurations entail narrow scope ones. This is why in (7), we prefer to use the label non-specific rather than co-variation.

(7) Jeder$_y$ Student hat irgendein$_x$ Buch gelesen.
every student has irgendein book read

a. SPECIFIC UNKNOWN: $\forall y \exists_s x(\phi(x, v) \wedge dep(v, x) \wedge var(\varnothing, x))$
b. NON-SPECIFIC: $\forall y \exists_s x(\phi(x, v) \wedge dep(vy, x) \wedge var(\varnothing, x))$

v
v_1
v_2

(a)

v	y	x
v_1	b_1	a_1
v_1	b_2	a_1
v_2	b_1	a_2
v_2	b_2	a_2

(b)

v	y	x
v_1	b_1	a_1
v_1	b_2	a_2
v_2	b_1	a_1
v_2	b_2	a_2

(c)

Table 5.1: (a) Initial team; (b) Ignorance; (c) Co-variation.

Note that similar contrasts extend to the case of modals, as illustrated in (8), and similarly for necessity modals. When the form *qualche* 'some' without *un* 'a/one' is used, which behaves similarly to English plural 'some' but the noun does not exhibit plural morphology, non-specific readings appear to be the only ones available. We will return to this data point in Section 5.6.

(8) Giovanni può giocare con un qualche compagno.
Giovanni can play with UN QUALCHE schoolmate.

a. SPECIFIC UNKNOWN: There is a particular schoolmate x such that Giovanni can play with x. The speaker does not know which one.
b. NON-SPECIFIC: Giovanni can play with a schoolmate, anyone would do.

5.2 Interlude: the Implicature-based Approach

As discussed at the beginning of this chapter, epistemic indefinites are quite widespread across languages and have received significant attention in the formal semantics literature. A prominent approach treats the ignorance or epistemic component as a form of quantity implicature (Alonso-Ovalle and Menéndez-Benito 2010; Chierchia 2013). Implicature-based approaches rely on two basic components: (i) a mechanism to generate alternatives, and (ii) a form of pragmatic reasoning or implicature computation over these alternatives. In episodic contexts, an additional component is needed: a covert doxastic operator, which we represent as $\Box_s$, corresponding to 'the speaker believes that'. We illustrate this with the example in (9).

(9) John danced with ALGÙN student.

a. $\Box_s(\exists x(x \in f(\texttt{student}) \land \texttt{danced}(j, x)))$
b. Anti-singleton constraint: $\mid f(\texttt{student}) \mid > 1$

In (9), we outline the account proposed by Alonso-Ovalle and Menéndez-Benito (2010) and Alonso-Ovalle and Menéndez-Benito (2017), which relies on a function f to generate alternatives. Specifically, f takes a set as an argument and returns a subset of that set as an output. Alonso-Ovalle and Menéndez-Benito (2010) and Alonso-Ovalle and Menéndez-Benito (2017) claim that the Spanish epistemic indefinite *algún* is associated with the anti-singleton constraint in (9-b), which blocks the possibility of having a singleton set as an output of f. For instance, the function f could return the set $\{d_1, d_2, d_3\}$ from the set of students. This corresponds to (10), with alternatives given in (10-a), (10-b), and (10-c).

(10) $\Box_s(\exists x(x \in \{d_1, d_2, d_3\} \land \texttt{danced}(j, x)))$
a. $\Box_s(\exists x(x \in \{d_1\} \land \texttt{danced}(j, x)))$
b. $\Box_s(\exists x(x \in \{d_2\} \land \texttt{danced}(j, x)))$
c. $\Box_s(\exists x(x \in \{d_3\} \land \texttt{danced}(j, x)))$

The ignorance effect is then derived as a standard quantity implicature: the speaker chose to utter the weaker (10) because the stronger alternatives in (10-a), (10-b), and (10-c) are false. This results in the desired ignorance reading: it is not the case that the speaker believes that 'John danced with d_1', it is not the case that the speaker believes that 'John danced with d_2', and it is not the case that the speaker believes that 'John danced with d_3'.

The resulting inferences are compatible with a situation of partial ignorance, in line with the behaviour of epistemic indefinites. For instance, the inferences in (10-a)-(10-c) are compatible with the speaker believing that John did not dance with d_1 (i.e., $\Box_s \neg \texttt{danced}(j, d_1)$). Importantly, the covert doxastic operator is needed for the implicature approach to work; without it, a plain contradiction would arise with the asserted sentence if the implicature derivation is carried out.

The implicature approach offers a fairly intuitive way to model epistemic indefinites and reasoning about alternatives. Additionally, it potentially explains nuanced distinctions between epistemic indefinites and indefinites in general by using different types of alternatives (Alonso-Ovalle and Menéndez-Benito 2010; Chierchia 2013; Alonso-Ovalle and Menéndez-Benito 2015; Alonso-Ovalle and Menéndez-Benito 2017). As we will see in Section 5.3, ignorance readings disappear in downward-entailing environments, such as negation. The implicature approach readily explains such disappearance, as ignorance effects result from quantity reasoning.

Interestingly, our variation condition $var(\varnothing, x)$ has a similar effect to the anti-singleton constraint in (9-b): the value of the indefinite must differ in at least a pair of assignments. However, unlike Alonso-Ovalle and Menéndez-Benito (2017), we do not derive the ignorance effect as an implicature but as part of the meaning of the indefinite, given the strict notion of the existential and our intensional framework. This also accounts for its indefeasibility and the oddness of 'guess who' continuations, a data point that implicature approaches cannot immediately account for unless they assume that implicatures are obligatory in this case. Moreover, our framework integrates the non-specific or co-variation uses of epistemic indefinites in a more general theory of indefinites and scope.

5.3 Negation

We have examined how our account predicts both ignorance and co-variation uses. We now proceed to consider the role of negation and downward-entailing environments in general. As noted by Alonso-Ovalle and Menéndez-Benito (2015), epistemic indefinites differ cross-linguistically with respect to their behaviour in negative or downward-entailing environments.

The general trend is that the ignorance component of epistemic indefinites disappears in downward-entailing environments. For instance, the Italian epistemic indefinite *un qualche*, under indirect negation in examples like (11-a) and (11-b), is naturally interpreted as a negative polarity item. While interpretations with *un qualche* outside negation are possible in certain contexts, they are much less prominent.

(11) a. Dubito che Mario abbia letto un qualche libro.
I-doubt that Maria has read UN QUALCHE book.
I doubt that Mario has read any books.
b. Nessuno ha un qualche strumento per aprire la scatola.
nobody has UN QUALCHE device how open the box.
'Nobody has any tool to open the box.'

Under sentential negation, however, the empirical picture is somewhat different: some epistemic indefinites do indeed display a negated existential behaviour like Danish *nogen* in (12):[7]

[7]The Danish indefinite pronoun *nogen* comes from an expression whose meaning is roughly 'I don't know who', similarly to Old English *nathw-* (Slade 2015a). Note that it is not a strict NPI like Italian *alcuno*, as it can also be used in positive episodic contexts with an ignorance inference:

(i) Nogen ringede. ?Gæt hvem?
NOGEN called. guess who

Importantly, when used in episodic contexts like (i), the degree of obligatory ignorance associated with *nogen* appears to be weaker than that of the German *-irgend*. Moreover, *nogen* is preferentially used in questions with a non-specific interpretation (e.g., 'Is there *anyone* here?') and under negation with a negated existential reading. This suggests that *nogen* is transitioning towards a non-specific indefinite that is restricted to downward-entailing environments and questions. We will return to this point in Chapter 6.

(12) Lars talte ikke med nogen.
Lars talked not with NOGEN
'Lars didn't talk to anyone.'

Some epistemic indefinites are infelicitous under sentential negation. An example is German *irgend-* in (13):[8]

(13) #Ich hab' nicht irgendwas gelesen.
I have not IRGENDWAS gelesen.

Other epistemic indefinites receive a positive-polarity interpretation (e.g., Spanish *algún* or Italian *un qualche*), where the indefinite must be interpreted outside negation. For instance, while the usage of the Italian *un qualche* is typically odd in contexts like (14-a) and (14-b), as claimed by Aloni and Port (2015), the continuation within brackets makes the overall sentence felicitous under a reading where *un qualche* is interpreted over negation.

(14) a. Nessuno ha risposto a una qualche domanda (e
Nobody has answered to UNA QUALCHE question (and
l'insegnate ha redarguito tutta la classe).
the-teacher has scolded whole class)
'No one answered some question (and the teacher scolded the whole class).'

b. Mario non ha comprato un qualche libro (perchè era troppo costoso).
'Mario didn't buy some book (because it was too expensive).'

In Section 4.8 of Chapter 4, we discussed the role of negation in our system and argued for an intensional notion of negation. We also discussed how this notion accounts for the behaviour of epistemic indefinites under negation. Here, we review the basic facts of our notion of intensional negation in interaction with epistemic indefinites. Note that in the following, we work with schematic examples.

We first consider the negated existential behaviour of epistemic indefinites. A schematic example is illustrated in (15) together with a supporting and a non-supporting team in Table 5.2. (15-b) is supported only if the initial team has $w_\varnothing$ as value for v (i.e., John did not read any book). In the other cases, the maximality of the antecedent makes $v \neq w$ false. Note that the maximal team supporting the antecedent of (15-b) is unique. To further elaborate on this, in plain episodic contexts, epistemic indefinites were felicitous only in initial teams compatible with the speaker's partial ignorance. By contrast, (15-b) is only supported by initial teams where the speaker knows that John has no book.

I am grateful to Søren Knudstorp for confirming the Danish judgments.

[8]Prosodic stress can redeem that sentence with a 'just not any' reading.

(15) a. John does not have *irgend*-book.
b. $\forall w(\exists_s x(\phi(x,w) \wedge dep(vw,x) \wedge var(\varnothing,x)) \rightarrow v \neq w)$

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

(a)

v	w	x
w_a	$w_\varnothing$	b
$\mathbf{w_a}$	$\mathbf{w_a}$	$\mathbf{a}$
w_a	w_b	b
w_a	w_{ab}	a

(b)

Table 5.2: Illustration for (15). Worlds differ with respect to which books John read. In $w_\varnothing$ John read no book, in w_a John read only book a, and so on. The maximal team of the antecedent of (15) is depicted in grey. The initial team T such that $T(v) = \{w_\varnothing\}$ (i.e., John read no book) supports (15-a), as $v \neq w$ holds in the extended team in (a). The initial team T such that $T(v) = \{w_a\}$ (i.e., John read only book a) does not support (15-a), as $v \neq w$ does not hold in the extended team in (b).

Note that for the negated existential/narrow scope reading, the indefinite in (15) is interpreted non-specifically with respect to the variable introduced by the intensional negation. This is captured in (15-b) by the dependence atom $dep(vw,x)$.

As discussed, epistemic indefinites can also receive a specific-unknown reading, where negation is interpreted 'outside the scope' of negation. One of the basic principles and advantages of 2TS is that everything is interpreted in-situ. The negated specific-unknown reading is captured by assuming that the variable x of the indefinite is interpreted specifically with respect to the variable of the negation w by means of $dep(v,x)$.

(16) a. John does not have *irgend*-book.
b. $\forall w(\exists_s x(\phi(x,w) \wedge dep(v,x) \wedge var(\varnothing,x)) \rightarrow v \neq w)$

5.4 Free Choice

So far, our focus has been on the canonical behaviour of epistemic indefinites concerning their ignorance readings, co-variation uses, and behaviour under negation. However, the German *irgend*- also displays free choice behavior when stressed and under a modal.[9] The inclusion of free choice uses for *irgend*- will require us

[9]The availability of free choice is not attested for other well-known epistemic indefinites, although recent work by Cao (2023) within the team semantics formal framework developed here has shown that the Mandarin *wh*-indefinite *shenme* patterns with *irgend*- concerning the

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	a
$w_\varnothing$	w_{ab}	a

v	w	x
w_a	$w_\varnothing$	b
w_a	w_a	b
w_a	w_b	b
w_a	w_{ab}	b

Table 5.3: Illustration for (16). A maximal team of the antecedent of (16) is depicted in grey. Note that there are two possible maximal teams given that $dep(v, x)$ must hold: one where x is mapped to a and one where x is mapped to b. The initial team T such that $T(v) = \{w_\varnothing\}$ (i.e., John read no book) supports (16-a), as $v \neq w$ holds in the extended team in (a), as well as for the other maximal team. The initial team T such that $T(v) = \{w_a\}$ (i.e., John read only book a) supports (16-a), as $v \neq w$ holds in the extended team in (b), but not for the maximal team where x is mapped to a. Note, however, that given the definition of weak maximal implication, one maximal team is sufficient.

to reconsider our assumption that epistemic indefinites simply associate with $var(\varnothing, x)$. However, this revision aligns with the diachronic development of *irgend-*, as we will see in Section 5.8.

(17) Mary muss IRGENDEINEN Arzt heiraten.
Mary must irgend-one doctor marry.
'Mary must marry a doctor, any doctor is a permissible option'.

To properly characterize a free choice reading, we use the strong version of the generalized variation atom introduced earlier to model the degree of variation:[10]

5.4.1. DEFINITION (Generalized Variation).
$VAR_n(\vec{x}, y) \Leftrightarrow$ for all $i \in T : |\{j(y) : j' \in T$ and $i(\vec{x}) = j(\vec{x})\}| \geq n$

We propose that the meaning associated with free choice is $VAR_{|D|}(v, x)$: in all epistemic possibilities of the speaker, every value for x is a possible option. Importantly, it seems that free choice readings arise when *irgendein* is stressed (Haspelmath 1997; Aloni and Port 2015), and we claim that the role of stress is precisely to strengthen the variation to level $|D|$:

availability of free choice uses.

[10]The generalized variation atom in Definition 5.4.1 is equivalent to:

$$\forall i \in T \exists j_1 \ldots \exists j_n \in T : \bigwedge_{1 \leq m \leq l \leq n} (i(\vec{x}) = j_m(\vec{x}) \text{ and } j_m(y) \neq j_l(y))$$

This generalized atom is based on the stronger version of variation mentioned before. (Väänänen 2022). By requiring that $\exists i \in T$ instead of $\forall i \in T$, we can obtain the generalized version of the weaker variation atom we originally considered.

(18) Mary musste$_w$ irgendeinen$_x$ Mann heiraten.
Mary had-to irgend-one man marry.

a. SPECIFIC UNKNOWN:
$\forall w \exists_s x\, (\phi \wedge dep(v,x) \wedge var_2(\varnothing, x))$

b. NON-SPECIFIC:
$\forall w \exists_s x\, (\phi \wedge dep(vw,x) \wedge var_2(\varnothing, x))$

c. FREE CHOICE:
$\forall w \exists_s x\, (\phi \wedge dep(vw,x) \wedge VAR_{|D|}(v, x))$

Note that the variation condition needed to obtain free choice is v-variation $var_{|D|}(v,x)$, and not $var_{|D|}(\varnothing,x)$, which would be satisfied simply if we can find $|D|$-many-values of x across all epistemic possibilities of the speaker. This departs from our original minimal assumption that epistemic indefinites associate with $var(\varnothing,x)$. However, we note that among epistemic indefinites, the *irgend* type, which also has displays free choice readings, is quite rare.[11]

Free choice inferences are of the form $\Box\exists_s x\phi \rightsquigarrow \forall x \Diamond \phi$. We can prove a statement along these lines for any classical formula ϕ. In the proof, we focus on the basic case. Note, however, that the condition $VAR_{|D|}(v,x)$ does not make sense for complex sentences, as the variation requirement cannot hold for the entire domain of the model but only with respect to a restriction of the domain relevant for the noun phrase. This is an implicit assumption we are making, but it would require proper treatment from a compositional point of view.

5.4.2. FACT.

$$\forall w(R(v,w) \to \exists_s x(\phi(x,w) \wedge VAR_{|D|}(v,x))) \models_v \forall x \exists_l w(R(v,w) \wedge \phi(x,w))$$

Proof:
The proof is by structural induction on ϕ. We focus on the basic case. Let M be arbitrary and T an arbitrary suitable initial team over M.

(i) Let ϕ be a first-order literal. Suppose that $M, T \models \forall w(R(v,w) \to \exists_s x(\phi(x,w) \wedge VAR_{|D|}(v,x)))$. This implies that for some $f_s : T[w] \to D$ for all $d \in D$, we can find an $i \in T' = T[w][f_s/x]$ such that $i(x) = d$ and $M, \{i\} \models \phi(x,w)$. Let $T[x]$ be the universal extension of T with x. Then we can construct a lax functional extension of $T[x]$ with w such that an assignment $j \in T[x]$ is mapped to $\{i(w)\}$ based on f_s above when $j(v) = i(v)$ and $j(x) = i(x)$. Hence, by construction, $M, T \models \forall x \exists_l w(R(v,w) \wedge \phi(x,w))$.

□

Finally, a relevant factor to note is that the free choice readings of *irgend*- are generally licensed by deontic modals, rather than epistemic ones. The different logical forms for epistemic vs. deontic modals are sketched in (19) and discussed more in detail in Section 4.7 of Chapter 4.

[11]The Mandarin *shenme*, as said, also displays free choice uses (see among others, Li 1992; Cao 2023).

(19) a. Epistemic: $\forall w(\subseteq (w,v) \to \exists_s x(\phi(x,w) \wedge var(\varnothing, x)))$
b. Deontic: $\forall w(R(v,w) \to \exists_s x(\phi(x,w) \wedge var(\varnothing, x)))$

Importantly, while deontic modals are relational, epistemic modals are subject to a restriction in the form of an inclusion atom $\subseteq (w,v)$, which ensures that epistemic modals range over possible values for the actual world/epistemic state of the speaker, encoded by v.

In this regard, we note that *irgend-*, when used under its SU reading, is typically associated with partial variation (i.e., $k < |D|$), as the speaker is generally not completely ignorant about all possible values for the referent of the indefinite. However, a free choice reading under an epistemic modal would require the speaker to be in a state of total epistemic variation/ignorance, as epistemic modal range over the epistemic state of the speaker given $\subseteq (w,v)$. The latter is not the typical context for the use of *irgend-*. This suggests why deontic modals are the typical licensors of free choice readings for *irgend-*.

5.5 Interlude: Conceptual Cover

In Section 5.2, we explored the implicature approach as a prominent alternative to account for epistemic indefinites. In this section, we review another approach, the conceptual cover account proposed by Aloni and Port (2015), which offers the advantage of capturing more nuanced distinctions regarding the ignorance component of epistemic indefinites. We will first illustrate this proposal and then describe how it can be integrated into the current framework.

The fundamental component of Aloni and Port (2015) is the notion of a conceptual cover, introduced by Aloni (2001), and the corresponding method of identification of the indefinite. We illustrate this with the example in (20). Suppose you are looking at two cats. The cat on the left is grey and is the mother of the cat on the right, who is black and is the kitten. Their owner told you they are called Coco and Pepper, but you do not know which is which.

(20) You know which cat is the mother.

Intuitively, the truth of (20) depends on the method of identification: you know that the cat on the left is the mother, and you know that the grey cat is the mother, but you do not know whether Coco or Pepper is the mother. This has been formalized by Aloni (2001) with the notion of a conceptual cover.[12]

5.5.1. DEFINITION. Conceptual Cover Given a set of possible worlds W and a domain of individuals D, a conceptual cover CC based on (W, D) is a set of functions $W \to D$ such that:

[12] Note that there might be good reasons to not require that concepts form a cover, as done by Dekker (2023). For the purposes of this chapter, we will adhere to Aloni (2001)'s original treatment.

$$\forall w \in W \forall d \in D : \exists! c \in CC : c(w) = d$$

For instance, we can represent the conceptual covers of the cat scenario described earlier as shown in Figure 5.1.

$$\begin{array}{cccc} & & o & \\ & & o_1 & o_2 \\ v_1 & \rightarrow & a & b \\ v_2 & \rightarrow & a & b \end{array} \qquad \begin{array}{cccc} & & n & \\ & & n_1 & n_2 \\ v_1 & \rightarrow & a & b \\ v_2 & \rightarrow & b & a \end{array} \qquad \begin{array}{cccc} & & d & \\ & & d_1 & d_2 \\ v_1 & \rightarrow & a & b \\ v_2 & \rightarrow & a & b \end{array}$$

(a) (b) (c)

Figure 5.1: Illustration of three conceptual covers based on $D = \{d_1, d_2\}$ and $W = \{v_1, v_2\}$. (a) is a conceptual cover composed of two concepts o_1 and o_2. It is a rigid cover, since each concept maps every world to the same individual. The o indicates that these concepts correspond to 'ostension'. So o_1 will be the function $left$ and o_2 the function $right$. Similarly, for n (naming), n_1 is *Coco* and n_2 is *Pepper*; and for d (description) d_1 is *mother* and d_2 is *kitten*.

We now proceed to integrate conceptual covers in 2TS. The first element to consider is that in Aloni (2001)'s original treatment, a set of worlds upon which the concepts of a cover are defined, as in Definition 5.5.1, is given. Due to the two-sorted nature of 2TS, we will define conceptual covers relative to a world variable, typically the variable for the actual world v. Given a team, we then construct our cover based on $(T(w), D)$ with $T(w) \subseteq W$ for some $w \in Dom(T)$.

The integration of conceptual covers into 2TS is achieved by assuming a different notion of strict functional extension, which incorporates a world variable to determine the cover and an index n to determine the relevant cover (e.g., ostension, naming, description, etc.). As in Aloni (2001), we assume that concepts are given by a function $\mathcal{C}$, which takes an index and returns a concept.

5.5.2. Definition (Extension under cover). Given a model $M = \langle D, W, I \rangle$, a team T, a variable $w \in Dom(T)$, and a variable x, the extension under cover in w of T with x, $T^w_n[c/x]$ is defined as follows:

$$T^w_n[c/x] = \{i[c(i(w))/x] : i \in T\}, \text{ for some } c \in \mathcal{C}(n) : T(w) \rightarrow D$$

Using this notion of extension, we redefine the semantic clause for the (strict) existential:

5.5.3. Definition (Semantic Clause for Existential Under Cover).

$$M, T \models \exists x_n^w z\, \phi \quad \Leftrightarrow \quad M, T_n^w[c/z] \models \phi \text{ for some } c \in \mathcal{C}(n)$$

This means that we are not changing our dependence and variation atoms. With this in mind, let us consider the sentence in (21) with the Italian epistemic indefinite UN QUALCHE.

(21) Un qualche gatto sta facendo le fusa.
UN QUALCHE cat is doing the purrs
'Some cat is purring.'

In the context outlined earlier, (21) is felicitous if the cat is identified by naming, which is the only non-rigid cover.

Given a suitable model, it holds that for the non-rigid cover in (22-a), variation holds, but it does not for the rigid ones, as variation cannot be satisfied.

(22) a. $M, T \models \exists x_n^v z(\phi(x, v) \wedge var(\varnothing, x))$
b. $M, T \not\models \exists x_o^v z(\phi(x, v) \wedge var(\varnothing, x))$
c. $M, T \not\models \exists x_d^v z(\phi(x, v) \wedge var(\varnothing, x))$

Moreover, the specular phenomenon also holds. Specific-known indefinites, represented by *a certain* in (23) for the sake of example, will be supported only by rigid covers.

(23) A certain cat is purring.
a. $M, T \not\models \exists x_n^v z(\phi(x, v) \wedge dep(\varnothing, x))$
b. $M, T \models \exists x_o^v z(\phi(x, v) \wedge dep(\varnothing, x))$
c. $M, T \models \exists x_d^v z(\phi(x, v) \wedge dep(\varnothing, x))$

The attentive reader might have noticed that this approach faces a problem, which also constitutes an issue for Aloni and Port (2015)'s account, namely the availability of non-specific readings. Consider the example in (24):

(24) Every student likes UN QUALCHE cat.
$\forall y \exists x_n^v z(\phi(x, v) \wedge var(\varnothing, x))$

We have observed that a non-specific reading where the indefinite co-varies with the students is possible for epistemic indefinites. However, the logical rendering in (24) does not allow for this: the value of x is fixed by the extension under cover in Definition 5.5.2, which assigns values to x based on v. This implies that when v is fixed, x is fixed as well. This would predict that (24) has only specific unknown readings.

We propose the following solution. Concepts will not be functions from worlds to individuals, but from sequences of values to individuals. While this might initially seem philosophically puzzling, it is a way to model the fact that discourse information can lead to more fine-grained distinctions regarding the set of worlds and concepts we are considering. In fact, one might construct from a sequence of values a corresponding set of worlds which acts as an input for the concept function:

5.5.4. Definition (Extension under cover (general)). Given a model $M = \langle D, W, I \rangle$, a team T, a sequence of variables $\vec{z}$ constructed from $Dom(T)$, and a variable x, the extension under cover in $\vec{z}$ of T with x, $T^{\vec{z}}_n[c/x]$ is defined as follows:

$$T^{\vec{z}}_n[c/x] = \{i[c(i(\vec{z}))/x] : i \in T\}, \text{ for some } c \in \mathcal{C}(n) : T(\vec{z}) \to D$$

This modification preserves the facts we discussed before for the basic case and allows us to capture non-specific readings when epistemic indefinites interact with other operators, as illustrated below:

(25) Every student likes un qualche cat.
$\forall y \exists x^{vy}_n z(\phi(x, v) \wedge var(\varnothing, x))$

Lastly, some languages impose restrictions on the usage of indefinites based on specific methods of identification. For instance, Slade (2015b) observes that Sinhala, an Indo-Aryan language primarily spoken in Sri Lanka, contains two indefinites formed by a *wh*-element combined with the particles *hari* and *də*.

The former, *wh*+*hari*, can be used only when the referent cannot be visually identified (ostension), but it might be identified in other ways. This implies that when interpreted under the ostension cover o, $var(\varnothing, x)$ must hold. The latter, *wh*+*də*, cannot be used when the referent is not visually identifiable, but it may remain unidentified by non-visual means (e.g., naming). This means that when interpreted under the ostension cover o, $dep(\varnothing, x)$ must hold, and also $var(\varnothing, x)$ when interpreted under some non-ostension relevant cover k.

These restrictions in Sinhala underscore the importance of integrating conceptual covers into our formal framework. By doing so, we can more accurately capture the nuances and constraints that different languages impose on the use of indefinites based on the method of identification.

Another approach to integrate conceptual covers in 2TS, and perhaps closer to Aloni (2001)'s original account, is to recognize that constancy atoms in the system are meant to capture knowledge (i.e., the value of x is that same across all worlds). As such, we define a notion of knowledge under cover by means of a separate dependence atom $dep_k(\varnothing, x)$ which checks that the values assigned to x in the team correspond to one concept in a cover k:

5.5.5. Definition (Knowledge under cover). $M, T \models dep_k(\varnothing, x) \Leftrightarrow \forall i \in T \exists c \in \mathcal{C}(k) : c(i(v)) = i(x)$

Likewise, ignorance will be captured by

5.5.6. Definition (Ignorance under cover). $M, T \models var_k(\varnothing, x) \Leftrightarrow \forall i \in T \neg \exists c \in \mathcal{C}(k) : c(i(v)) = i(x)$

This implies that an indefinite like *wh+də* (which requires visual identifiability and it remains unidentified by non-visual means) can be captured by (26), where $dep_o(\varnothing, x)$ guarantees that there is one concept in the ostension cover which matches the value of x and $var_k(\varnothing, x)$ that for some other cover k there is no concept which matches the value of x.

(26) $\exists_s x(\phi(x, v) \wedge dep_o(\varnothing, x) \wedge var_k(\varnothing, x))$

5.6 Ignorance and Plurality

Our discussion so far has focused on epistemic indefinites with singular morphology. However, the interaction of epistemic indefinites with plurals and plural morphology is quite complex. There appears to be a contrast between two types of epistemic indefinites: (i) epistemic indefinites that lose their ignorance inferences in their plural form; (ii) epistemic indefinites that maintain an ignorance inference, but with respect to a plural sum of individuals.

In Section 5.6.1, we focus on type (i). In Section 5.6.2, we address type (ii). Finally, in Section 5.6.3, we discuss how plurality should be encoded in 2TS.

5.6.1 The Disappearance of Ignorance

As concerns the first class, consider the case for Spanish *algunos* or Italian *alcuni*.[13] For instance, the example in (27) does not convey any ignorance with respect to the identity of the boys who are absent.[14]

[13] These are the plural forms of Spanish *algún* and Italian *alcuno*. The latter, in its singular morphology, can only be used under negation. However, singular uses with ignorance inferences were possible in Old Italian, as well as in the Latin form *aliquis* from which it originated (Gianollo 2019).

[14] Martí (2008) observes that Spanish *algunos* lacks indeed the ignorance component of its singular counterpart, but it is associated with a previously established discourse entity, as opposed to the Spanish plural indefinite *unos*, which does not come with such requirement. While the Italian *alcuno* could give rise to such effect, the referential link is by no means required. For instance, (i) can be read as a simple existential statement, with no reference to previously introduced boys or to boys who are not in the room.

(i) Nella stanza ci sono alcuni ragazzi.
In-the room there are ALCUNI boys.
'There are some boys in the room.'

(27) Alcuni ragazzi sono assenti. Nella fattispecie, Ludovico e
alcuni boys are absent. concretely, Ludovico and
Camillo sono assenti.
Camillo are absent.
'Some boys are absent. Concretely, Ludovico and Camillo are absent.'

Importantly, Italian also has a 'plural' form of the epistemic indefinite we previously considered in this chapter: *un qualche*. Consider the contrasts in (28). A singular plain indefinite like *un libro* 'some book' in (28-a) can be used in contexts where the speaker knows exactly which book fell from the bookcase. By contrast, the epistemic indefinite *un qualche libro* 'UN QUALCHE book' in (28-b) cannot be used in such contexts and implies that the speaker does not know which book fell. The form *qualche libro* 'QUALCHE book' in (28-c) implies that a small but indeterminate number of books fell from the bookcase, similar to English *a few* or plural *some*. The intriguing aspect of (28) is that (28-c) is semantically plural, conveying a multiplicity of books, but syntactically singular, as it exhibits singular morphology. This contrasts with Italian *alcuni* in (28-d), which is morphologically plural.

(28) a. Un libro è caduto dall'armadio.
a book is fallen from-the closet.
'A book fell from the closet.'
b. Un qualche libro è caduto dall'armadio.
UN QUALCHE book is fallen from-the closet.
'A book fell from the closet.'
c. Qualche libro è caduto dall'armadio.
QUALCHE book is fallen from-the closet.
d. Alcuni libri sono caduti dall'armadio.
ALCUNI books is fallen from-the closet.

The difference between (28-c) and (28-d) is subtle. The intuition is that (28-d) tends to be used when the speaker has a specific set of books in mind, while *qualche* might be used when the speaker is less focused on the exact books. This intuition, however, is weak and may be influenced by the fact that the Italian singular form *alcuno*, unlike in Spanish, can only be used under negation as an NPI. In contexts of maximal information like (29), both forms are acceptable with no difference in meaning.

(29) Context: Giovanni has 10 friends. Bob, Sue and Mary will come to the party, but the others will not.
a. Alcuni amici di Giovanni verranno alla festa.
ALCUNI friends of Giovanni will-come to-the party
b. Qualche amico di Giovanni verrà alla festa.
QUALCHE friend of Giovanni will-come to-the party

'Some friends of Giovanni will come to the party.'

The example in (29) also shows that, similarly to Spanish *algunos*, plural forms do not carry any obligatory ignorance inference. Moreover, multiplicity inferences associated with *qualche* disappear in the antecedents of conditionals, in questions, and similar environments, as noted by Zamparelli (2007), and illustrated in the examples in (30). In (30-a), finding even one restaurant open is sufficient to consider oneself lucky. Similarly, (30-b) allows for a positive 'yes' answer even if the hearer read just one book during the holidays.[15]

(30) a. Se trovi qualche ristorante aperto, puoi ritenerti fortunato.
if find QUALCHE restaurant open, can consider lucky.
'If you find any restaurant open, you can consider yourself lucky.'
b. Hai letto qualche libro durante le vacanze?
have read QUALCHE book during the holidays?
'Did you read any book during the holidays?'

Regarding the Italian *alcuni* and Spanish *algunos*, the status of multiplicity inferences is more contentious. This difference appears to be more significant compared to the nuanced contrast between (28-c) and (28-d). Specifically, the multiplicity inferences associated with the plural form *alcuni* in (30-a) and (30-b) seem less cancellable.[16]

It is also interesting to examine the behaviour of these items in combination with collective predicates or collective readings. Even though *qualche* is semantically plural, it is incompatible with group-like readings as evidenced by (31-a) and

[15] As concerns the behaviour in downward entailing environments, the judgements are less clear as singular *alcuno* is used with NPI readings.

In indirect negation, they both admit NPI readings:

(i) a. Dubito che Giovanni abbia letto qualche libro.
doubt that Giovanni has read QUALCHE book.
b. Dubito che Giovanni abbia letto alcuni libri.
doubt that Giovanni has read ALCUNI books.
'I doubt that Giovanni has read any books.'

Under sentential negation, only specific readings, most salient for the variant with *alcuni* appear to be admitted:

(ii) a. Giovanni non ha letto qualche libro.
Giovanni not has read QUALCHE book.
b. Giovanni non ha letto alcuni libri.
Giovanni not has read ALCUNI book.
'Giovanni didn't read some books.'

[16] For bare plurals, these environments are typical cases to test the cancellability of multiplicity inferences. However, the behaviour of complex NPs and particularly morphologically plural epistemic indefinites appears different.

(32-a), adapted from (Zamparelli 2007), suggesting that *qualche* is incompatible with plural sums. In contrast, parallel cases with *alcuni* in (31-b) and (32-b) are acceptable:

(31) a. ??Qualche studente è un gruppo
QUALCHE student is a group.
b. Alcuni studenti sono un gruppo.
ALCUNI students is a group.
'Some students are a group.'

(32) a. ??Qualche studente ha formato un gruppo.
QUALCHE student has formed a group.
b. Alcuni studenti hanno formato un gruppo.
ALCUNI students have formed a group.
'Some students formed a group.'

Similarly, collective predicates appear incompatible with *qualche* in (33-a), but possible with *alcuni* in (33-b):

(33) a. ?Qualche studente si è riunito di fronte all'aula.
QUALCHE student REFL is gathered in front of-the.classroom
b. Alcuni studenti si sono riuniti di fronte all'aula.
ALCUNI students REFL are gathered in front of-the.classroom
'Some students gathered in front of the classroom.'

A similar contrast is observed in cases like (34-a) and (34-b). (34-a) can only be interpreted as stating that a small number of students individually wrote a paper (rather than, for instance, delivering a presentation). The addition of 'together' results in oddness. In contrast, collective readings are available for *alcuni* in (34-b):

(34) a. Qualche studente ha scritto un articolo (?assieme).
QUALCHE student has written a paper (together)
b. Alcuni studenti hanno scritto un articolo (assieme).
ALCUNI students have written a paper (together)
'Some students wrote a paper (together).'

5.6.2 The Plural Appearance of Ignorance

By contrast, the German *irgend*-series induces an obligatory ignorance inference even when inflected in its plural form (Alonso-Ovalle and Menéndez-Benito 2010; Alonso-Ovalle and Menéndez-Benito 2017). This is illustrated in examples (35) with the plural indefinite *irgendwelchen* and in (36) with *irgendzwei*, where *irgend* combines with the numeral *two*. Note that in example (36), the speaker conveys ignorance about which two students passed the course. In example (35), the speaker might also be ignorant about the exact number of students. It is worth mentioning that *irgendzwei* is not a common German expression and might sound odd, but it is a permissible form.

(35) Juan wohnt mit irgendwelchen Studenten aus dem Institut
Juan lives with IRGENDWELCHEN students in the department
zusammen #und zwar mit Peter und Sally.
together, #namely with Peter and Sally
'Juan lives with some students in the department, namely Peter and Sally.' (from Alonso-Ovalle and Menéndez-Benito 2010)

(36) Irgendzwei Studenten haben (die Prüfung) bestanden.
irgend-two student have the exam passed
'Irgend-two students passed (the exam).'

Regarding multiplicity inferences, they are clearly obligatory for *irgendzwei Studenten*. For *irgendwelchen Studenten*, in constructions like (37-a), the use of *irgendwelche* has a non-specific reading similar to the English *any open restaurants*, where no multiplicity seems to surface, despite the plural morphology. (37-b) requires some emphasis, as the use of *ein* or bare plurals are more natural alternatives to convey the intended meaning.

(37) a. Wenn du irgendwelche offenen Restaurants findest, kannst
if you IRGENDWELCHE open restaurants find, can
du dich glücklich schätzen.
you yourself lucky consider
'If you find any open restaurants, you can consider yourself lucky.'

b. Hast du irgendwelche Bücher während der Ferien gelesen?
have you IRGENDWELCHE books during the vacation read
'Have you read any books during the vacation?'

Finally, similar to the Italian *alcuni*, both *irgendzwei* and *irgendwelche* permit collective and group-like readings. However, they also allow distributive readings whenever the context and the predicate allow for it.

(38) a. Irgendwelche Studenten haben eine Gruppe gebildet.
IRGENDWELCHE students have a group formed
'Some students have formed a group.'

b. Irgendzwei Studenten haben eine Gruppe gebildet.
IRGENDZWEI students have a group formed
'Two students have formed a group.'

The discussion in this and the preceding sections can be summarized in Table

5.4. We will return to multiplicity inferences at the end of the next section.

	Plural morphology	Obligatory Ignorance Inference	Collective Readings Allowed
Italian *qualche*	✗	✗	✗
Italian *alcuni*	✓	✗	✓
German *irgendwelche*	✓	✓	✓

Table 5.4: Illustration of various epistemic indefinites with plural meaning. Note that Spanish *algunos* exhibits the same behaviour as Italian *alcuni* with respect to the distinctions in the table.

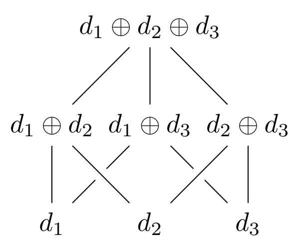
5.6.3 Plurality in 2TS

In this section, we examine the behaviour of epistemic indefinites and plurality, focusing on how plurality should be represented in 2TS. We will explore three possible approaches to modelling plurals in 2TS, presenting each informally before discussing their implications for epistemic indefinites in plural form.

First, while singular indefinites are *strict* existentials, we may consider plural indefinites as *lax* existentials. Second, we could treat indefinites as strict existentials but over a domain of plural individuals.[17] Third, we may adopt a hybrid approach, where indefinites are lax existentials over a pluralized domain.

Before discussing which notion best captures the behaviour of epistemic indefinites and plurality, we offer some general remarks. A team semantics framework already encodes plurality in the evaluation procedure, and early approaches to plurals have leveraged this aspect to model plurality (van den Berg 1996). We can distinguish between encoding plurality at a structural level (interpreting formulas as sets of assignments) and at a domain level (allowing plural individuals in the domain). These distinctions reflect what Brasoveanu (2011) called evaluation

[17] This domain is constructed, as is standard, from a basic domain of entities E such that $D = \wp(E) \setminus \{\varnothing\}$ (Landman 1991; Link 1983; Scha 1984). Structures of the kind $\langle D, \subseteq, \cup \rangle$ form a join-free semilattice, as shown below, where we refer to $\{d_1, d_2\}$ with Link's notation $d_1 \oplus d_2$.



v	x
v_1	d_1
v_1	d_2
v_1	d_3

(a)

v	x
v_1	$d_1 \oplus d_2 \oplus d_3$

(b)

v	x
v_1	d_1
v_1	$d_1 \oplus d_3$
v_1	$d_2 \oplus d_3$

(c)

Table 5.5: (a) Plurals as lax existentials; (b) Plurals as strict plural existentials; (c) Plurals as lax plural existentials.

plurality and domain plurality, highlighting their linguistic applications in related frameworks.

In 2TS, having a variable for the actual world allows teams to represent information states, rather than merely singular or multiple values for variables. Moreover, as previously mentioned, lax and strict semantics for the existential are equivalent for downwards closed formulas. However, these differ with the addition of variation atoms, such as those associated with epistemic indefinites.

Note also that in all these approaches, all options in Table 5.5 are compatible with 'singular' values.[18]

We now move on to the main topic of this section: epistemic indefinites. Our proposal is that the Italian *qualche* should be modelled as a lax existential (Table 5.5a), *irgendwelche* as a strict plural existential (Table 5.5b), and Italian *alcuni* / Spanish *algunos* as a lax plural existential (Table 5.5c).

The first option, modelling indefinites as lax existentials, creates an interesting parallelism between plurality and the variation $var(\varnothing, x)$ condition. While the strict and lax existentials are equivalent for downwards closed formulas, in episodic contexts, $var(\varnothing, x)$ is compatible with a plural interpretation. More concretely, consider the configuration in (39). While an epistemic indefinite in singular form is not licensed by a team of maximal information where v is constant, such a team could potentially license (39). However, the $var(\varnothing, x)$ condition requires that more than one element in the domain satisfy the formula. This explains why the ignorance effect is not an obligatory component of such indefinites: the role of $var(\varnothing, x)$ is not to signal ignorance but to signal plurality in such cases. This also explains the 'small-quantity' plural inferences associated with the cases in (28-c), as the only requirement in teams of maximal information is to have a minimum of two individuals acting as witnesses for the formula, as illustrated in the extended team in Table 5.6.

[18]Plural nouns are typically associated with a so-called 'multiplicity inference', as they are generally used in situations involving more than one entity. One perspective is that this inference is of a pragmatic nature (among others, Sauerland 2003; Sauerland, Anderssen, and Yatsushiro 2005; Spector 2007). The fact that a speaker uses a plural form, instead of the more informative singular one, leads to the inference that they are referring to more than one entity.

(39) a. Qualche studente ha passato l'esame
QUALCHE student has passed the-exam
'Some student passed the exam'.
b. $\exists_l x(\phi(x, v) \wedge var(\varnothing, x))$

v	x
v_1	d_1
v_1	d_2

Table 5.6: Illustration of extended team for (39). Lax existentials allow for branching extensions.

In contexts relevant for testing multiplicity inferences, the indefinite is interpreted non-specifically, and the variation condition becomes vacuous. In this sense, multiplicity inferences either disappear or become vacuous. This phenomenon, previously illustrated in the case of negation, also extends to a lax treatment.

For Italian *qualche*, this explains the apparent discrepancy between its syntactic singularity and its semantic plurality. Although the domain over which *qualche* ranges consists of singular individuals, the variation condition introduces a semantic plural interpretation. The fact that *qualche* does not access plural individuals is further evidenced by its incompatibility with group-like predicates and the unavailability of collective readings, as discussed in the previous section.

The second option, modelling indefinites as strict existentials over a pluralized domain, readily captures the behaviour of *-irgend*. Here, the variation condition $var(\varnothing, x)$ requires the value of x to be associated with at least two different plural individuals.

Consider the case of *irgendzwei* in the example below, where $2(x)$ indicates that the cardinality of x must be 2. The example is supported when the resulting team contains at least two 'plural individuals' that are 'different' as shown in Table 5.7.[19]

(40) a. Irgendzwei Schüler haben (die Prüfung) bestanden.
irgend-two student have the exam passed
'Two students passed (the exam).'
b. $\exists_s x(2(x) \wedge \phi(x, v) \wedge var(\varnothing, x))$

This approach ensures that ignorance inferences are obligatory in plain episodic contexts, as desired. It also allows for collective readings since the domain in-

[19]Note that under a regular notion of inequality, $d_1 \oplus d_2$ and $d_2 \oplus d_3$ would count as different since they are two different sets. Preliminary empirical observations suggest that *irgendzwei* would also be acceptable in such cases. If not, each atomic element would need to be distinct.

v	x
v_1	$d_1 \oplus d_2$
v_2	$d_3 \oplus d_4$

Table 5.7: Illustration of extended team for (40). Strict plural existentials allow only non-branching over a pluralized domain, predicting ignorance together with variation.

cludes plural individuals.[20]

Finally, let's consider the last option, treating indefinites as lax plural existentials. This approach captures the behaviour of Italian *alcuni.* The difference with *irgendwelche* lies in the fact that a lax existential explains the disappearance of ignorance inferences, similar to the first approach. The difference with *qualche* is that having an existential ranging over plural individuals explains the availability of collective readings.[21]

	Plural domain	Obligatory Ignorance Inference	Collective Readings Allowed
Lax Existential	✗	✗	✗
Strict Plural Existential	✓	✗	✓
Lax Plural Existential	✓	✓	✓

Table 5.8: Extensions with plural meaning and predictions.

Finally, we offer some remarks regarding multiplicity inferences. As discussed in the previous sections, from an empirical standpoint, these inferences are easily cancelled with *qualche* across different environments. They can also be cancelled with *irgendwelche*, though their status with *alcuni* is unclear. From a formal perspective, all three approaches summarized in Table 5.8 predict that multiplicity inferences are not obligatory. We have already addressed why these inferences are present in plain episodic contexts and the role of variation. However, it is noteworthy to consider the implications of explicitly incorporating them into the semantics, given that we have different treatments of plurality.

[20]Note that under this treatment, distributive readings need to be obtained by a dedicated distributive operator.

[21]Depending on the analysis of collective predicates, variation cannot be satisfied in teams of maximal information and thus gives rise to an ignorance inference for collective readings. This is incorrect for Italian *alcuni.* A solution is to use a strict existential quantifier over a plural domain and redefine variation to require that the variable's value is plural. This approach maintains the current predictions for *alcuni*, but it implies that multiplicity inferences cannot be cancelled, and the judgments here are unclear.

If plurals are treated as lax existentials, this would imply that the lax extension maps only to non-singleton sets (i.e., they always generate branching configurations). In contrast, if we allow for a plural domain, we can impose constraints on the cardinality of the plural individual. This approach seems appropriate if one considers these requirements as part of the semantics of plurals. This might also explain the contrasts observed in Table 5.8, at least for the Italian case, where multiplicity inferences easily disappear with *qualche* (morphologically singular) but not with *alcuni* (morphologically plural).[22]

5.7 Indifference Readings

A puzzling phenomenon involving epistemic indefinites concerns so-called indifference readings, as opposed to the canonical ignorance readings. For instance, a speaker might utter the sentence in (41-a) as a report of the news of the day, even in contexts in which they are fully aware of which politician was shot. The intended inference is that it does not matter which politician was shot, but rather that the shooting happened.

These kinds of inferences have often been overshadowed by the focus on 'knowledge' in philosophy and linguistics, and the role of teams as information states in 2TS follows this tradition.

(41) a. Hanno sparato ad un qualche politico.
they-have shot to UN QUALCHE politician.
'Some politician was shot.'
b. Hanno sparato ad un politico.
they-have shot to a politician.
'Some politician was shot.'

We propose two possible ways to account for indifference inferences.

The first explanation relies on a reconceptualization of the 2TS framework. Instead of having v as the variable for the actual world, we may require assignments in a team to denote the possible alternatives for the values of the indefinite. This means that, independently of which alternative is the actual one, each of them is considered relevant by the speaker. The speaker is indifferent as there is no unique alternative acting as the relevant value for the indefinite, which is what $var(\varnothing, x)$ guarantees.

The second explanation links this inference to the other agents in the conversation, rather than the speaker itself. In particular, we propose that a speaker

[22]As mentioned in the previous footnote, another possibility worth exploring is that multiplicity inferences arise because the variation component of the indefinite ensures that the value of x is not singular in a given assignment of the team, and this would explain the contrast between *irgendwelche* and *alcuni*. This, however, would depart from the original formulation of the variation atom.

utters (41-a) in contexts where informing other agents in the conversation that a politician was shot does not lead them to identify which politician was shot. In this regard, the variation component $var(\varnothing, x)$ is not related to the speaker's information state, but rather to the other agents in the conversation. The indifference results from a pragmatic inference from the listener's perspective. Since the speaker uttered a sentence which will never lead the listener to identify who was shot, it must mean that this information was not relevant for the present context, and in this sense the speaker is indifferent with respect to communicating who was shot.

This calls for a dynamic update system, which we will discuss in Chapter 6, which can account for the role that new information brings into the context. For now, let's assume that teams encode the information present in the context. For an initial context where it has not been settled if a, b, or none was shot, the variation component will ensure that the context will not settle who was shot, but it will exclude the possibility that no one was shot. This implies that, even though the speaker might be aware of who was shot, the speaker is leading the conversational context where the listener cannot determine who was shot. By contrast, a plain indefinite would be compatible with an update leading to a context that settles who was shot.

(a)

C	v
i_1	v_a
i_2	v_b
i_3	$v_\varnothing$

$\rightarrow \quad \exists_s x(\phi(x, v) \wedge var(\varnothing, x)) \quad \rightarrow$

C'	v	x
j_1	v_a	a
j_2	v_b	b

(b)

C	v
i_1	v_a
i_2	$v_\varnothing$

$\rightarrow \quad \exists_s x(\phi(x, v)) \quad \rightarrow$

C'	v	x
j_1	v_a	a

Figure 5.2: Context Updates.

This predicts that using an epistemic indefinite in a context where the listener knowns that either a was shot or no one was shot (i.e., $C(v) = \{v_a, v_\varnothing\}$) is not possible, even with an indifference flavour. This prediction appears to be borne out.

5.8 Diachrony: from Non-specific to Epistemic

We conclude this section by considering the diachronic development of this class of indefinites. Recall from Section 4.9 of Chapter 4 that we predict a diachronic development characterized by weakening from non-specific to epistemic. In particular, we will examine the development of the German *irgend-* series and the French *quelque*, highlighting the parallelism with Italian *(un) qualche*.

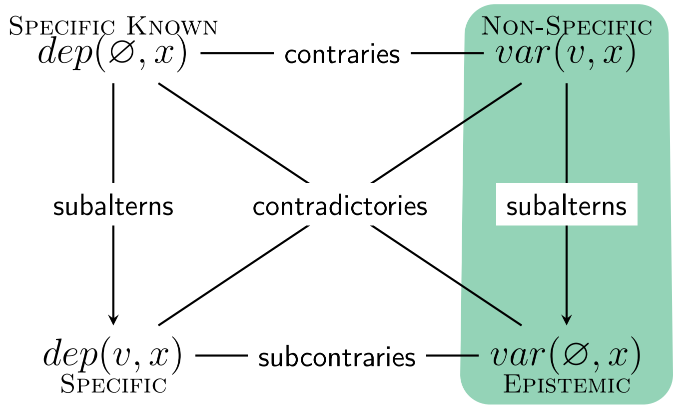


Figure 5.3: Dependence Square of Opposition. From Non-specific to Epistemic.

The development of the German *irgend-* series has been empirically examined by Port and Aloni (2021) and Fobbe (2004). *Irgend-* originated from a particle with locative meaning and, once it acquired its indefinite status, was initially used non-specifically and only later acquired its specific unknown usages.

Regarding the French epistemic indefinite *quelque*, it is generally agreed (Foulet 1919; Jayez and Tovena 2011) that *quelque* originated from correlative concessive constructions formed by *quel* 'which' in combination with *que* 'that'. This is similar to the initial usages of the Italian free choice indefinite *qualsiasi* ('anything') from *qual (si) sia* 'which it is', which we will explore in Chapter 8.[23] This original universal and non-specific meaning associated with such concessive constructions subsequently weakened to the specific unknown usages of epistemic indefinites.

The Italian *(un) qualche* is morphologically similar to *quelque*, as it is also formed by a combination of the *wh*-element *which* together with the relative *that*. We can thus conjecture that this item shared a similar developmental trajectory. Crucially, while this item displayed specific uses in early Italian, free choice/non-specific uses were also attested in early Italian.[24]

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[23]Such usages are still possible in current French, as in expressions like (i):

(i) Sous quelque forme que ce soit
under QUELQUE form that this is
In any form whatsoever

[24]The following examples illustrate the non-specific behaviour of *qualche* in Old Italian. In contemporary Italian, a free choice indefinite is preferred for such constructions.

(i) . . . che e' chiamavano propriamente aiuti, di qualche sangue o paese e' si fossero, . . .
. . . that they properly called helpers, of QUALCHE blood or country they were, . . .

We aim to account for this trend from non-specific to epistemic uses in a unified manner. As stated, the explanation we offer here is that this phenomenon can be understood in terms of semantic weakening. As we have seen, non-specificity is captured in the framework by the variation condition $var(v, x)$, while epistemic indefinites are associated with $var(\varnothing, x)$. Given this treatment of non-specificity in terms of variation, $var(v, x)$ indeed entails $var(\varnothing, x)$. Semantic weakening thus finds a natural explanation in terms of logical entailment within the formal framework.

We note that the precise development of each indefinite could vary significantly across languages and may involve various syntactic reconfigurations, ultimately leading to the current distribution of the indefinite.[25] Here, we offer some general observations based on our formal system.

We have observed that the German *irgend-* transitioned from a non-specific indefinite $var(v, x)$ to an epistemic one $var(\varnothing, x)$ by also acquiring specific unknown usages. However, the case of *irgend-* is more complex. Port and Aloni (2021) note that the free choice uses of *irgend-* emerged only later.[26] If this is the case, it would mean that $var(v, x)$ strengthened into $VAR_{|D|}(v, x)$, which is how we analyzed free choice in Section 5.4. This would imply that while generally the non-specific form weakened, in certain environments *irgend-* strengthened into $VAR_{|D|}(v, x)$ due to its original non-specific meaning. As discussed in Section 4.9 of Chapter 4, the expected directionality is of weakening, but strengthening phenomena are also attested in semantic change.

We conclude this section by considering two questions. First, can epistemic indefinites turn into unmarked ones? Second, can epistemic indefinites turn into non-specific ones, reversing the weakening directionality just described? We will answer the first question and postpone the second one to Chapter 6, dedicated to non-specific indefinites.

Recall that epistemic indefinites are associated with $var(\varnothing, x)$. To turn into an unmarked one, they would undergo weakening, but not directly among atoms. This means that $var(\varnothing, x)$ would become $var(\varnothing, x) \mathbin{\vee\!\!\!/} dep(\varnothing, x)$. The latter would then be lost, as it trivially covers the entire meaning space of marked indefinites. Such an operation is not impossible, as discussed in Section 4.9 of Chapter 4, but it is certain more complex than weakening among simple atomic conditions.

> ...that were properly called helpers, of whatever blood or country they were, ...
> (Vincenzo Borghini, *Discorsi*, ca. 1550)

[25] For instance, in the case of French *quelque*, Foulet (1919) notes that the development of this item as a determiner likely originated from the idiom *à quelque paine* ('whatever pain it might cause').

[26] However, it should be noted that the original locative particle from which *irgend-* originates is *io-wergin*, where *io* was an adverb meaning 'always', from which *je* in contemporary German derives. As such, universal/free choice uses might have already been present at the locative stage.

In this regard, it may be instructive to consider two families of descendants of an early indefinite form. First, the Romance descendants of Latin *aliquis*, which was an epistemic indefinite (Gianollo 2019). These include Italian *alcuno*, Spanish *alguno*, Portuguese *algum*, Catalan *algun*, and French *aucun*. As discussed in Gianollo (2019, 2020), the Spanish, Portuguese, and Catalan forms remained epistemic indefinites. The Italian *alcuno* can only be used in negative contexts, with the same distribution as a strict NPI, and the French *aucun* is a negative concord item used in combination with negation. None of these turned into an unmarked indefinite.

The second family we consider are the North Germanic descendants of Old Norse *nǫkkurr*. This term derives from an original expression meaning 'I don't know who' and had the distribution of an epistemic indefinite.[27] The North Germanic descendants are Swedish *någon*, Danish *nogen*, Faroese *nakar*, Icelandic *nokkur*, and Norwegian *noen*. In this case, these indefinites are mostly used under negation or in questions, similar to English *anyone*, but they also admit specific unknown usages. One exception is Icelandic *nokkur*, which is typically used in reports with a meaning similar to English *a certain* or English *one*.[28] If this is the case, it would suggest that Icelandic *nokkur* is on its way to becoming an unmarked indefinite, even though the other descendants did not, highlighting the potential low frequency of such diachronic change.

5.9 Conclusion

In this chapter, we considered the class of epistemic indefinites, focusing on their inherent ignorance inferences and co-variation uses. We compared the 2TS approach to the implicature view and examined the effects of negation and polarity. Additionally, we provided an account for the free choice uses of *irgend-*.

We incorporated conceptual covers to model nuanced distinctions in the ignorance component of epistemic indefinites. Furthermore, we analysed the complex interaction of epistemic indefinites with plural forms and plural morphology. We proposed different ways to model plurals in 2TS and discussed how each approach

[27] https://onp.ku.dk/onp/onp.php?o57831

[28] Consider, for instance, the example below. The usage of the epistemic indefinite *un qualche* in Italian would be quite unnatural. The most natural translation is with *un certo* 'a certain'.

(i) **Maður nokkur**, sem tók þátt í að steypa leiðtoga Afríkuríkis af stóli, sagði í viðtali við bandaríska tímaritið *Time* um nýju stjórnina: "Þetta var útópía sem endaði strax í algerri ringulreið."
One man, who participated in overthrowing the leader of an African country, said in an interview with the American magazine *Time* about the new government: "This was a utopia that immediately ended in total chaos."

https://www.jw.org/is/bÃşkasafn/tÃŋmarit/g201309/bera-mÃştmÃęli-Ãąrangur/

accounts for the behaviour of epistemic indefinites in plural forms. Clearly, the latter treatment of plurality should be embedded in a more general theory of plurality.

Regarding the diachronic development of indefinites and the conceptual cover analysis, it would be interesting to investigate whether it is possible to identify a developmental trajectory among epistemic indefinites tied to specific covers. Additionally, it would be relevant to link epistemic indefinites, which signal the speaker's ignorance, to evidentials, which indicate the source and reliability of the information being conveyed. Notably, we have seen that it is possible to have epistemic indefinites tied to the ostension cover. However, in the case of evidentials, it has been noted (see e.g. Saratsli and Papafragou 2023 for an overview) that evidentiality is more often associated with indirect evidence (e.g., reported speech) rather than direct evidence (e.g., visual identifiability).

Chapter 6

Non-specific Indefinites

vimedovnebi, rom **odesme vinme** am q'velapers mikhvdeba.
'I hope that **someone**, **someday**, will make sense of all this.'

UNKNOWN

Non-specific indefinites are indefinites that only allow for scopally non-specific uses, and never take the widest possible scope. Paradigmatic examples include Russian *-nibud'*, Georgian *-me*, Greek *típota.* A similar class of indefinites, known as dependent indefinites, has also been discussed in the literature. This class encompasses reduplicated indefinites like Hungarian *egy-egy* and distributive particles used with indefinite determiner phrases, such as Romanian *câte* and Russian *po.*

In Section 6.1, we will outline our core approach to non-specific indefinites and explain how we account for their distribution. This discussion will lead us to explore the relationship between the property of *Locality* in 2TS and our licensing predictions. Section 6.2 will delve into the relationship between non-specific and dependent indefinites, outlining the main empirical landscape and existing accounts. We will pay particular attention to Russian, which displays both a non-specific indefinite, *-nibud'*, and a dependent indefinite, *po.* In Section 6.2.3, we will introduce the 'informational dependence' atom which will be used to model dependent indefinites. We will examine its relationship with the variation atom. Next, in Section 6.3, we will propose a dynamic account of 2TS that will help capture some differences between non-specific indefinites and other classes of indefinites. Finally, in Section 6.4, we will conclude with some remarks concerning the diachronic development of non-specific indefinites, continuing our analysis from where we left off in Section 5.8 of Chapter 5.[1]

[1]Part of this chapter is based on Maria Aloni and Marco Degano (2022). "(Non-)specificity across languages: constancy, variation, v-variation". In: *Semantics and Linguistic Theory.* Vol. 32, pp. 185–205. In particular, some paragraphs (adapted and expanded) included in

6.1 Meaning and Licensing

As discussed in Chapter 4, non-specific indefinites are associated with the non-specific function. We captured this by assuming that they trigger the v-variation condition $var(v, x)$, which requires that the value of x is non-constant in at least one epistemic possibility of the speaker.

(1) THE v-VARIATION CONDITION
Non-specific indefinites obligatory trigger the v-variation condition $var(v, x)$

6.1.1 Non-specific Indefinites: Licensing & Variation

Non-specific indefinites cannot occur freely in episodic sentences, but they need to be licensed by an operator. This operator can be a (distributive) quantifier, a modal, an attitude verb, ... Examples (2) and (3) illustrate the case of Russian *-nibud'*. In (2), *-nibud'* leads to infelicity in an episodic context, while the universal quantifier *kazhdyj* 'every' in (3) redeems *-nibud'* from infelicity.

(2) #Ivan včera kupil kakuju-nibud' knigu.
Ivan yesterday bought which-INDEF. book.
Intended: 'Ivan bought some [non-specific] book yesterday.'
$\exists_s x(\phi(x, v) \wedge var(v, x))$

(3) Kazhdyj student včera kupil kakuju-nibud' knigu.
every student yesterday bought which-INDEF. book.
'Every student bought some book yesterday.'
$\forall y \exists_s x(\phi(x, v) \wedge var(v, x))$

To explain the infelicity of non-specific indefinites in episodic contexts, two main components of 2TS discussed in Chapter 3 are important. First, the notion of an initial team, a team whose domain contains only the variable for the actual world v. Second, the conditions under which a sentence is felicitous: when there is an initial team that supports it. These two components, together with the $var(v, x)$ requirement for non-specific indefinites, are enough to explain cases like (2) and (3).

To see this, we will work with the initial team in Table 6.1a, where, for the sake of illustration, we assume that v is non-constant. Consider the logical rendering of an episodic sentence like (2). The strict existential may lead to an extension like the one in Table 6.1b. However, this is not enough to satisfy $var(v, x)$, since there must be a pair of assignments in which x differs and v is fixed. As we discussed, the definition of the strict existential rules out 'branching' extensions. This implies that we cannot have any initial team which supports (2). By defining a sentence

Section 6.1.1. The study was conceptualized through joint discussions between Maria Aloni and Marco Degano. The writing of the paper was carried out by Marco Degano.

as felicitous if it can be supported by an initial team, our analysis predicts the infelicity of (2).

Let us examine what happens when an operator (e.g., a universal quantifier) intervenes and licenses the non-specific indefinite, as in (3). The universal quantifier leads to a universal y-extension of the initial team as in Table 6.1c. We can then have a strict individual x-extension as in Table 6.1d, which satisfies $var(v, x)$ in (3), provided a suitable model where $\phi(x, v)$ holds.

v
v_1
v_2

(a)

v	x
v_1	d_1
v_2	d_2

(b)

v	y
v_1	a_1
v_1	a_2
v_2	a_1
v_2	a_2

(c)

v	y	x
v_1	a_1	d_1
v_1	a_2	d_2
v_2	a_1	d_2
v_2	a_2	d_2

(d)

Table 6.1: Licensing of non-specific indefinites. (a) Initial Team. (b) Strict functional x-extension. (c) Universal y-extension. (d) Universal y-extension + strict functional x-extension.

Observe that the v-variation atom $var(v, x)$ is satisfied in teams like the one shown in Table 6.1d, even though the value of x does not vary in v_2. In fact, $var(v, x)$ holds as long as there is at least one epistemic possibility where the value of the variable for the indefinite is not determined. We believe that this partial variation corresponds to the received empirical distribution of this class of indefinites. Using the stronger variation atom, mentioned in footnote 11 of Chapter 3 and repeated below, would account for variation across all epistemic possibilities.

6.1.1. Definition (Variation Atom (Stronger Version)).
$M, T \models VAR(\vec{z}, \vec{u}) \Leftrightarrow$ for all $i \in T$ there is $j \in T : i(\vec{z}) = j(\vec{z})$ & $i(\vec{u}) \neq j(\vec{u})$

Importantly, other indefinites cannot license non-specific ones. Consider, for instance, the case of epistemic indefinites, which are associated with the variation atom $var(\varnothing, x)$. Given the initial team in Table 6.1a, an epistemic indefinite could lead to an extension like the one in Table 6.1b, where the value of x differs in v_1 and v_2. However, in such a team, $var(v, x)$ would not be satisfied for the reasons discussed earlier. In general, indefinites of any type are strict existentials and do not allow for branching extensions that are necessary to license the $var(v, x)$ condition of non-specific indefinites.

Before moving on to the next section, we observe an interesting parallelism between non-specificity and the lax treatment of plurality we discussed in Section 5.6 of Chapter 5. In both cases, the indefinite is allowed to range over multiple values. In the former case, it is due to the v-variation condition, which also leads to some distributional restrictions, whereas in the latter case the multiplicity is encoded at the level of the assignment extensions and in this sense it is not restricted.[2]

6.1.2 Infelicity and Locality

In this section, we explore the relationship between the notion of *Locality* in a team based system and the infelicity of non-specific indefinites in episodic contexts.

The property of *Locality* in Definition 6.1.2 has received some attention in the Dependence Logic tradition (Väänänen 2007a; Galliani 2012a). Definition 6.1.2 says that the satisfaction of a formula ϕ is only determined by the variables occurring free in the formula ϕ, $Free(\phi)$.

6.1.2. Definition (Locality). Given a suitable model M, a suitable team T, and a formula ϕ, let V be a set of variables such that $Free(\phi) \subseteq V \subseteq dom(T)$ and $T' = T \upharpoonright V$ be the restriction of T to the variables in V. Then:

$$M, T \models \phi \Leftrightarrow M, T' \models \phi$$

Dependence Logic, which only contains dependence atoms, satisfies *Locality* (Väänänen 2007a). However, the additional upwards closed variation atom in 2TS, in combination with the strict existential, leads to a failure of Locality.[3]

To illustrate this, consider the formula $\psi := \exists_s x\ var(v, x)$, which only contains v as free variable (i.e., $Free(\psi) = \{v\}$). The team T in Table 6.2a supports ψ, as we can extend T with x in such a way that $var(v, x)$ holds. However, its restriction T' to the free variables in ψ in Table 6.2b does not, since the strict existential will not allow for branching extensions.

Adopting only a lax semantics for the existential would preserve *Locality*, but it would not yield the correct licensing of (non-specific) indefinites. In fact, the failure of *Locality*, as Table 6.2b suggests, is particularly revealing for our previous claims. Specifically, when there are no other free variables except v, the restriction

[2]Quite interestingly, the lax plural epistemic indefinite *qualche* can be used under attitudes as in (i) to convey non-specificity similarly to *-nibud'* in (ii), while in the other cases the competition with the singular form *un qualche* is strong and plural readings are preferred.

(i) Giovanni vuole cantare qualche canzone.
Giovanni wants sing QUALCHE song
'Giovanni wants to sing some (non-specific) song.'

(ii) Ivan khochet spet' kakoj-nibud' romans.
Ivan wants sing KAKOJ-NIBUD' romance
'Ivan wants to sing some (non-specific) romance.'
(from Padučeva 1985)

[3]See e.g., Galliani (2012b) for a similar remark concerning inclusion atoms, which are also upwards closed.

T	v	y
i_1	v_1	a_1
i_2	v_1	a_2

(a) T

T'	v
i_1	v_1

(b) $T' = T \upharpoonright \{v\}$

Table 6.2: Locality - Illustration.

of the team to its free variables will effectively lead to an initial team, which is precisely when non-specific indefinites are not licensed.

6.1.3 Negation and Modality

Non-specific indefinites are licensed not only by universal quantifiers but also by other operators such as negation and modals. In Chapter 4, we introduced the notion of intensional negation in 2TS and discussed our treatment of modality. Here, we show that our approach aligns with the predictions for non-specific indefinites. In the following, we take *-nibud'* as a placeholder for a non-specific indefinite.

First, non-specific indefinites receive a narrow scope/negated existential reading under negation (and in downward-entailing environments in general) without additional enriched meaning. Second, modals, including those with existential flavor, can license non-specific indefinites.

For negation, we consider the basic case of clausemate negation. However, it is important to note that cross-linguistically, not all non-specific indefinites are licensed by clausemate sentential negation. This includes *-nibud'*, which, while not acceptable under clausemate sentential negation, is acceptable in other NPI-licensing contexts, such as the antecedents of conditionals. This issue is known as the Bagel problem (Pereltsvaig 2004). Pereltsvaig (2004) observed that in the case of Russian *-nibud'*, the context of clausemate sentential negation creates 'a bagel hole' with respect to the downward-entailing environments where *-nibud'* is licensed. We follow Pereltsvaig (2004) in proposing that this occurs because of polarity-sensitive lexical items that only appear in clausemate sentential negation (e.g., dedicated *n*-words like the Russian *ni*-pronouns), which are preferred in such contexts due to lexical competition.

We will illustrate the licensing of non-specific indefinites under sentential negation using the simplified example in (4). Similarly to the case of epistemic indefinites, the v-variation atom is upwards closed, ensuring that the only case in which (4-c) is supported by an initial team is when $T(v) = \{v_\varnothing\}$, with $v_\varnothing$ being the world in which John did not read any book.

(4) a. John did not read book-nibud'.
b. $\neg\exists_s x(\mathtt{book}(x, v) \wedge var(v, x) \wedge \mathtt{read}(j, x, v))$
c. $\forall w(\exists_s x(\mathtt{book}(x, w) \wedge var(v, x) \wedge \mathtt{read}(j, x, w)) \rightarrow v \neq w)$

As concerns modal licensing, universal modals clearly license non-specific indefinites in the same way that universal quantifiers do. Therefore, it is more relevant to consider existential modals. In (5), we schematically represent an opaque reading of the indefinite under an existential modal. The crucial assumption is that we model existential modals using lax quantification. This approach allows for branching extensions, allowing $var(v, x)$ to be satisfied. Again, note that lax existential quantification would be equivalent to strict quantification for downwards closed formulas. However, the variation atom's contribution makes a significant difference in predictions. If existential modals were treated as strict quantifiers over worlds, they would fail to license non-specific indefinites for the same reason that other indefinites, which are strict existentials over individuals, cannot license non-specific indefinites.

(5) a. John might read book-nibud'.
b. $\exists_l w \exists x(\texttt{book}(x, w) \wedge var(v, x) \wedge \texttt{read}(j, x, w))$

6.2 Dependent indefinites

The main examples discussed in Haspelmath (1997)'s typological work, which formed the basis for the generalizations in Chapter 4, concerned non-specific marked indefinites, typically formed from *wh*-elements. A related class of indefinites, which has received discrete attention in the literature concerns so-called dependent indefinites (Farkas 1997; Brasoveanu and Farkas 2011; Henderson 2014; Balusu and Jayaseelan 2013; Kuhn 2019; Farkas 2021).

6.2.1 Distribution

In the following, we will distinguish between non-specific indefinites and dependent ones. The latter can be further subdivided into two main classes: reduplicated numerals/articles and distributive particles modifying indefinite determiner phrases. We provide examples in (6): (6-a) illustrates the non-specific indefinite *-nibud'* in Russian; (6-b) the reduplicated article *egy-egy* in Hungarian; and (6-c) the distributive particle *câte* in Romanian. In all these cases, the sentence is considered false on its wide scope reading, where every boy read the same book. This is the core semantic feature common to these types of indefinites. However, as we will see, non-specific and dependent indefinites exhibit different distributions.

(6) a. Kazhdyy mal'chik prines kakuyu-**nibud'** knigu.
every boy brought which-nibud book.SING.ACC
b. Minden fiú hozott **egy-egy** könyvet.
every boy brought a-a book
c. Fiecare băiat a adus **câte** un carte.
every boy has brought câte a book

	Episodic	Distributive DP	Modal	Plural DP
NON-SPECIFIC	✗	✓	✓	(✓)
DEPENDENT	(✓)	✓	✗	✓

Table 6.3: Simplified distribution of non-specific and dependent indefinites.

'Every boy brought a book' [false if every boy read the same book]

Reduplicated indefinites and distributive-like particles have sometimes been treated as a general phenomenon under the name of dependent indefinites (Farkas 1997; Henderson 2014). While we distinguish between reduplication and particle modification, it appears that the two constructions share similar distributions and uses. Both, in fact, encode some form of distributivity. Distributive particles are often related to distributive quantifiers like the English *each*. For instance, the Romanian *câte* derives from the Latin preposition *cata*, which was used in a distributive sense similar to the English *by*. In other languages, *cata* combined with the numeral *unum* (lit., 'one'), resulting in forms like *cadauno* in Italian and *chacun* in French, which function as distributive quantifiers and can also appear postnominally, similar to English *each*. Likewise, reduplication is a common strategy to indicate distributivity (Gil 2013).

Farkas (2021) provides a comprehensive overview of several dependent indefinites in different languages. We summarize the general distributional patterns of non-specific and dependent indefinites in Table 6.3.[4] As discussed, non-specific indefinites are not allowed in episodic contexts. While dependent indefinites are typically not licensed in episodic contexts, some exhibit auto-licensing. We illustrate this with the Hungarian *egy-egy* from Farkas (1997). In (7), the sentence implies that the event of a student failing occurs more than once, with the students possibly co-varying with the event of failing.[5]

(7) Egy-egy diák megbukik de ez ritkán fordul elo.
a-a student fails but this seldom comes up
'Occasionally, a student fails but this happens rarely.'

Distributive quantified determiner phrases and adverbs of quantification typically licence both non-specific and dependent indefinites. Modals licence non-specific indefinites but not dependent ones. Plurals, when interpreted distributively, licence both types of indefinites. One salient difference is that dependent

[4]Farkas (2021) distinguishes between simple indefinites morphologically marked as dependent and dependent indefinites where the determiner is a numeral. We are treating these two classes as the same. As for non-specific indefinites in Table 6.3, the predictions concern Russian *-nibud'*. It remains to be seen whether the class of non-specific indefinites is stable with respect to these contrasts.

[5]Note that in Hungarian, the reduplicated indefinite article gives rise to auto-licensing, but reduplicated numerals do not.

indefinites seem to force a distributive reading of the plural, while non-specific indefinites do not; if licensed, they typically occur with a dedicated distributive marker or a dependent indefinite itself, as we illustrate in the next section for the case of Russian.

6.2.2 Russian *-nibud'* and *po*

Russian presents an interesting case study as it features both a *wh*-based non-specific indefinite *-nibud'* and a dependent indefinite *po*.[6] The two constructions can co-occur in the same sentence, providing valuable empirical data for studying their interaction. In previous work by Pereltsvaig (2008), *-nibud'* was classified as a dependent indefinite. For the reasons we discussed earlier, we are treating it as a non-specific indefinite.

The non-specific *-nibud'* is a suffix that combines with *wh*-interrogatives of different semantic categories to form an indefinite pronoun (e.g., *kto-nibud'* for 'someone'). It can also combine with the interrogative determiner *kakój* (e.g., *kakój-nibud' knígu* for 'some book'). *Po* is a preposition with various uses in Russian.[7] In the relevant context, *po* can appear with bare nouns, numerals, and *wh*-indefinites. When used with numerals, the dative case is applied to the numeral 'one', while the accusative case is used for other numerals.

Both *po* and *-nibud'* are infelicitous in episodic contexts, as illustrated in (8-a) and (8-b).

(8) a. #Ivan vzyal kakuyu-nibud' knigu.
Ivan took which-nibud book.SING.ACC
b. #Ivan vzyal po knige.
Ivan took po book.SING.DAT

Both are licensed by distributive quantifiers like *kazhdyy* ('every'), as well as by adverbs of quantification such as *vsegda* ('always'). The intended reading of the examples in (9) is that Ivan reads a different book every day.

(9) a. Ivan vsegda chitayet kakuyu-nibud' knigu v den'.
Ivan always reads which-nibud book.SING.ACC in day
'Ivan always reads some book in a day.'
b. Ivan vsegda chitayet po knige v den'.
Ivan always reads po book.SING.ACC in day
'Ivan always reads some book in a day.'

[6] I thank Jenia Khristoforova and Miriam Rey for their help and insights with the Russian data.

[7] An interesting related use is the combination of *po* with days of the week or times of day to indicate the regular occurrence of an event (e.g., *po četvergám*, lit. po Thursday, for 'every Thursday').

Adverbs of quantification licence *po* also when it combines with numeral expressions. This contrasts with the Hungarian type, where indefinites with a numeral as determiner and marked as dependent are not licensed by adverbs of quantification (Farkas 2021).

(10) Ivan chasto poseshchal po dva seminara v semestr.
Ivan frequently attended po two seminar.PLR.GEN in semester
'Ivan frequently attended two seminars in a semester' [false under the reading there are two specific seminars which Ivan frequently attended in a semester.]

Interestingly, modals license *-nibud'*, but not *po*. We illustrate this for both existential and universal modals in (11) and (12), respectively. This suggests that *po* aligns with the dependent indefinite type outlined earlier.

(11) a. Mozhet Ivan chitat' kakuyu-nibud' knigu.
maybe Ivan read which-nibud book.SING.ACC
'Ivan might read some book.'
b. #Mozhet Ivan chitat' po knige.
maybe Ivan read po book.SING.DAT

(12) a. Ivan dolzhen chitat' kakuyu-nibud' knigu.
Ivan must read which-nibud book.SING.ACC
'Ivan must read some book.'
b. #Ivan dolzhen chitat' po knige.
Ivan must read po book.SING.DAT

Next, we consider the universal quantifier *vse* ('all'), which typically admits collective interpretations, with distributive readings arising only when *vse* is heavily stressed (Pereltsvaig 2008). Under *vse*, *-nibud'* in (13-a) is not felicitous, while *po* in (13-b) is licensed under the typical reading of such indefinites. A similar pattern is observed for bare plural nouns under a definite interpretation and numerals.

(13) a. #Vse mal'chiki chitali kakuyu-nibud' knigu.
all boy.PLR.NOM read kakuyu-nibud' book.SING.ACC
b. Vse mal'chiki chitali po knige.
all boy.PLR.NOM read po book.SING.DAT
'All boys read some book.' [false if they read the same book.]

(14) a. #Mal'chiki chitali kakuyu-nibud' knigu.
boy.PLR.NOM read which-nibud book.SING.ACC
b. Mal'chiki chitali po knige.SING.DAT
boy.PLR.NOM read po book.
'The boys read a book.' [false if they read the same book.]

(15) a. #Dva mal'chika chitali kakuyu-nibud' knigu.
two boy.PLR.GEN read which-nibud book.SING.ACC
b. Dva mal'chika chitali po knige.
two boy.PLR.GEN read po book.SING.DAT
Two boys read some book. [false if they read the same book.]

Importantly, when *po* co-occurs with *-nibud'* in the environments just discussed in (13), (14) and (15), *-nibud'* is no longer infelicitous. We illustrate this in (16) for the case of the combination with numerals in (16).

(16) Vse mal'chiki chitali po kakoi-nibud' knige.
all boy.PLR.NOM read po which-nibud book.SING.ACC.
'All boys read some book.' [false if they read the same book.]

The data outlined present clear challenges. We need a theory that predicts (i) the infelicity of both *po* and *-nibud'* in episodic contexts, (ii) the unavailability of modal licensing for *po*, (iii) the unavailability of (definite) plural licensing for *-nibud'*, and (iv) the interaction between *-nibud'* and *po*.

6.2.3 Informational Dependence

As discussed in Farkas (2021), previous analyses of dependent indefinites can be subsumed under two main accounts: the *Dependent Variable* account (Farkas 1997; Brasoveanu and Farkas 2011; Farkas 2021) and the *Evaluation Plurality* account (Henderson 2014). In the former account, dependent indefinites need to co-vary with respect to the values of another variable. In the latter, they are associated with a set of assignments across which their value must vary. Note that the latter approach aligns with our v-variation atom for non-specific indefinites.

In this section, we will introduce a condition on the indefinite's variable values, which we call *informational dependence*. We will show that such condition is not generally equivalent to the v-variation condition for non-specific indefinites. Specifically, these two conditions are equivalent in the environments where both types of indefinites are licensed. However, they diverge when the licensing conditions differ, as outlined in Table 6.3. This approach allows us to account for both the core 'co-variation' meaning of dependent and non-specific indefinites, as well as their distinct distributions.

Informational Dependence

We introduce a new condition, which we call informational dependence, defined in (17).

(17) Informational Dependence
$M, T \models info\text{-}dep_v(y, x)$ iff $\exists v_1 \in T(v) : \exists d_1, d_2 \in T(y) : T_{vy=v_1d_1}(x) \neq T_{vy=v_1d_2}(x)$

This notion is closely connected with the independence atom $ind_{\vec{z}}(\vec{x}, \vec{y})$ we discussed in Section 3.4.2 of Chapter 3. In fact, $info\text{-}dep_v(y, x)$ is equivalent to $\neg_B ind_v(y, x)$, the Boolean negation of the corresponding independence atom, and we could have defined more generally informational dependence atoms in such a way.[8] To give some illustrations, $info\text{-}dep_v(y, x)$ does hold for the team depicted in Table 6.4a, but it does not for the teams in Table 6.4b and 6.4c.

v	y	x
v_1	a_1	d_1
v_1	a_1	d_2
v_1	b_1	d_1

(a)

v	y	x
v_1	a_1	d_1
v_1	a_1	d_2
v_1	b_1	d_1
v_1	b_1	d_2

(b)

v	y	x
v_1	a_1	d_1
v_1	b_1	d_1

(c)

Table 6.4: Illustrations. $info\text{-}dep_v(y, x)$ is satisfied in (a), but not in (b) and not in (c).

It is informative to explain why we labelled (17) as informational dependence, as opposed to the dependence atoms we originally considered. The condition $info\text{-}dep_v(y, x)$ states that for some value of v, knowing the value x conveys some information about the value of y (i.e., x 'depends' on y in v). For instance, for the team in Table 6.4a, knowing that the value of x is d_2 already informs about the value y, namely a_1. Not surprisingly, independence atoms have been used to model many key concepts central to information theory Grädel and Väänänen (2013).

Note that informational dependence differs from the notion of functional dependence conveyed by dependence atoms. Specifically, the corresponding dependence atom neither entails nor is entailed by the informational one. However, this notion of informational dependence has been extensively discussed in the linguistic tradition, particularly in dynamic semantics approaches (van den Berg 1996; Nouwen 2003), as we have briefly discussed in Section 3.4.2 of Chapter 3.

[8] The general form is given in Definition 6.2.1. This atom has also been studied by Galliani (2015), who showed that it is first-order definable.

6.2.1. Definition (Informational Dependence).
$M, T \models \text{info-dep}_{\vec{z}}(\vec{y}, \vec{x})$ iff there exist $i, i' \in T$ such that $i(\vec{z}) = i'(\vec{z})$, it holds that for all $i'' \in T$, $i''(\vec{x}\vec{z}) \neq i(\vec{x}\vec{z})$ or $i''(\vec{y}\vec{z}) \neq i(\vec{y}\vec{z})$

Informational Dependence and Licensing

We now proceed to outline how the informational dependence condition accounts for the core distribution of dependent indefinites. We will discuss an interesting parallelism between the informational dependence condition for dependent indefinites and the variation condition for non-specific indefinites.

In line with the Dependent Variable account, we propose that dependent indefinites are associated with the informational dependence atom with respect to the operator upon which they depend.

(18) DEPENDENT INDEFINITES
Dependent indefinites obligatory trigger the informational dependence atom $info\text{-}dep_v(y, x)$ with y being the operator upon which they depend.

Dependent indefinites cannot be licensed in episodic contexts. This is evident from the fact that there is no operator y which could saturate the first argument of the atom. Moreover, assuming that in episodic contexts the informational dependence condition degenerates to $info\text{-}dep(\varnothing, x)$ as in (19), we predict that there is no initial team supporting it. Trivially, we cannot find any $d_1, d_2 \in T(\varnothing)$ for any team T.

(19) $\exists_s x(\phi(x, v) \wedge info\text{-}dep(\varnothing, x))$

In passing, we observe that the $info\text{-}dep_v(y, x)$ requirement could be linked to the auto-licensing behaviour of some dependent indefinites, as a way to saturate their y argument with a covert variable in the context (e.g., quantification over events).

Let us consider when an intervening operator licenses a dependent indefinite. We predict that the value of the indefinite cannot be constant with respect to the licensing operator.

(20) $\forall y \exists_s x(\phi(x, v) \wedge info\text{-}dep_v(y, x))$

Considering (20), we observe that the only case which would make x independent of y is when x is mapped to the same value in all assignments, given that indefinites can only give rise to strict extensions. By requiring x to be non-independent (i.e., informationally dependent) on y, we predict the desired non-constant behaviour.

In fact, this point highlights an interesting parallelism between the informational dependence condition and the v-variation condition for non-specific indefinites. While $info\text{-}dep_v(y, x)$ and $var(v, x)$ are clearly not equivalent and encode different conditions in general, the interaction between a licensing operator and the strict existential makes the formulas in (21-b) equivalent for initial teams.

(21) a. $info\text{-}dep_v(y, x) \not\equiv var(v, x)$

b. $\forall y \exists_s x(\phi(x, v) \wedge \mathit{info}\text{-}\mathit{dep}_v(y, x)) \equiv_v \forall y \exists_s x(\phi(x, v) \wedge \mathit{var}(v, x))$

In Section 6.2, we observed one important difference between non-specific and dependent indefinites: the former can be licensed by modals, while dependent indefinites cannot. For the unavailability of modal licensing, we follow the *extensional dependency condition* proposed by Farkas (1997), which states that y in $\mathit{info}\text{-}\mathit{dep}_v(y, x)$ cannot be a world variable. Representing dependent indefinites by means of $\mathit{info}\text{-}\mathit{dep}_v(y, x)$ makes it more natural to impose conditions on the value of y, as opposed to the variation requirement $\mathit{var}(v, x)$ for non-specific indefinites.

Lastly, we comment on the interaction between multiple operators and dependent indefinites. As an illustration, we consider the case in (22), where a dependent indefinite occurs under the scope of a universally quantified determiner phrase, its licensing operator, with a universal modal intervening in an intermediate position. The most salient reading is the one represented in Table 6.5b, where we have co-variation with respect to the first quantifier, which is in line with (22-b). The prediction of (22-b) is that a dependent indefinite should not be licensed under a wide scope known reading of the indefinite, as represented in Table 6.5b. However, we also predict that dependent indefinites cannot be used for scopally non-specific readings of the kind in Table 6.5c, where the *set* of possible books does not co-vary with respect to each individual. In this reading, the set of possible books which each student must read is fixed for each individual and in this sense informationally independent of each value of y. By contrast, such reading would in principle be possible with a non-specific indefinite, as compatible with the v-variation condition.[9]

(22) a. Everyone must read a-a book.
b. $\forall y \forall w \exists_s x(\phi(x, v) \wedge \mathit{info}\text{-}\mathit{dep}(y, x))$

Interaction with Plurality and Additional Operators

The remaining data that needs to be explained concerns the interaction with plurality. In Section 5.6 of Chapter 5, we discussed different options for modelling plurality in the system, such as modelling plurals as lax existentials over a singular domain or as lax existentials over a plural domain. In what follows, we will assume that plurals are strict existentials over a pluralized domain. We will explore how

[9] The judgments are not clear on whether the reading in Table 6.5c is possible, perhaps due to the complexity of the example and the salience of the reading in Table 6.5a. If such reading is indeed possible, we would need to assume that the informational dependence condition for cases like (21-a) is $\mathit{info}\text{-}\mathit{dep}(yw, x)$, which would allow teams like in Table 6.5c, but still disallow the one in Table 6.5b. This would mean that while word variables cannot be the licensor of dependent indefinites, they still play a role in determining how they co-vary with respect to the licensor operator.

v	y	w	x
v_1	d_1	w_1	b_1
v_1	d_1	w_2	b_1
v_1	d_2	w_1	b_2
v_1	d_2	w_2	b_2

(a)

v	y	w	x
v_1	d_1	w_1	b_1
v_1	d_1	w_2	b_1
v_1	d_2	w_1	b_1
v_1	d_2	w_2	b_1

(b)

v	y	w	x
v_1	d_1	w_1	b_1
v_1	d_1	w_2	b_2
v_1	d_2	w_1	b_1
v_1	d_2	w_2	b_2

(c)

Table 6.5: Intermediate scope reading in (a), wide scope reading in (b); narrow scope reading in (c). Due to the presence of a world variable, dependent indefinites are only compatible with (a), while non-specific indefinite are compatible with both (a) and (c).

much the current system can explain under this assumption.[10]

One of the motivating factors behind this account of plurality is that it allows us to explain the infelicity of non-specific indefinites under plurals. For instance, in (23), *two students* will still be associated with a strict extension mapping each assignment to a plural individual (e.g., $\{d_1, d_2\} = d_1 \oplus d_2$). This implies that no branching extension is generated and $var(v, x)$ is not satisfied. By contrast, treating plurality via lax extension would have predicted the felicity of (23-a).

(23) a. #Two students read book-*nibud'*
b. $\exists_s y \exists_s x(\texttt{student}(y, v) \wedge |y| = 2 \wedge \texttt{book}(x, v) \wedge \texttt{read}(x, y, v) \wedge var(v, x))$

As concerns the non-distributive quantifier *vse-all* or distributively interpreted definite plurals, we follow the standard approach of analysing such expressions with a maximality operator defined as follows:[11]

(24) Maximality Operator
$M, T \models M_x^{\vec{z}}(\phi) \Leftrightarrow M, T[f_s/x] \models \phi$ for some strict function $f_s : T \to \wp(D)\backslash\{\varnothing\}$ and there is no strict function $f'_s : T \to \wp(D)\backslash\{\varnothing\}$ s.t. $(M, T[f'_s/x] \models \phi$ and there is $i \in T$ s.t. $f_s(i) \subset f'_s(i))$

The operator in (24) says that given a value for $\vec{z}$, the value associated with x must be the maximal set satisfying ϕ.

(25) #All boys read book-nibud'.
$M_y^v(\texttt{boy}(y, v) \wedge \exists_s x(\texttt{book}(x, v) \wedge \texttt{read}(yx, v) \wedge var(v, x)))$

We then predict that (25) is indeed infelicitous for the same reasons of (23).

[10] In particular, given a pluralized domain $\wp(D)\backslash\{\varnothing\}$ constructed from D we will assume that singular noun phrases must be singletons, whereas plurals can denote any subset of the pluralized domain.

[11] We are effectively analysing *all* x as a strict existential over a pluralized domain together with a maximality requirement on the value of x.

Most importantly, the default reading of (25) will be collective. In fact, *vse-all* and numerals are typically non-distributive in Russian, explaining the infelicity of *-nibud'* for cases like (13), where *-nibud'* occurs under *vse*. We obtain distributive readings by means of a dedicated distributive operator.

(26) Distributive Operator
$M, T \models \delta_z(\phi)$ iff $M, T[z/\delta_z] \models \phi$ with $T[z/\delta_z] = \{i' : \exists i \in T$ and $i' = i[\{a\}/z]$ with $a \in i(z)\}$.

To illustrate this, consider the example in (27). The default reading would be the collective one in (27-a), and the distributive is generated by the contribution of the $\delta_y(\cdot)$ operator. A relevant question is what is triggering this operator. In English, *all* appears to give rise to both collective and distributive readings, while Russian *vse* is strongly non-distributive (Pereltsvaig 2008), and prosodic stress might be needed to allow for distributive readings.

(27) All boys read a book.
a. $M^v_y(\texttt{boy}(y, v) \wedge \exists x(\texttt{book}(x, v) \wedge \texttt{read}(yx, v)))$
b. $M^v_y(\texttt{boy}(y, v) \wedge \delta_y(\exists x(\texttt{book}(x, v) \wedge \texttt{read}(yx, v))))$

We have thus shown how this approach accounts for the infelicity of *-nibud'* also in plural construction. However, in Section 6.2.2, we observed that *po* is licensed in such contexts. As it is, the default collective behaviour of such constructions would predict infelicity also for *po*.

This leads to the hypothesis that *po* does not only contribute the informational dependence atom $info\text{-}dep_v(y, x)$, but also a dedicated distributive operator, as in (28). This aligns with the suggestion made in Kuhn (2017), where dependent indefinites also introduce a distributive operator on the same variable they depend on.[12]

(28) All boys read *po* book.
$M^v_y(\texttt{boy}(y, v) \wedge \delta_y(\exists x(\texttt{book}(x, v) \wedge \texttt{read}(yx, v) \wedge info\text{-}dep_v(y, x))))$

Importantly, the addition of *po* redeems *-nibud'* from ungrammaticality, without change in meaning: the variation condition from *-nibud'* is trivial given the dependence requirement from *po*.

(29) All boys read *po* book.
$M^v_y(\texttt{boy}(y, v) \wedge \delta_y(\exists x(\texttt{book}(x, v) \wedge \texttt{read}(yx, v) \wedge info\text{-}dep_v(y, x))))$

We have explored minimal contrasts between *po* and *-nibud'*. To advance our understanding of the relationship between non-specific and dependent indefinites,

[12] A relevant factor is that while *po* in isolation can occur and give rise to the indented reading, the presence of postnominal *kazhdyj* 'each' in combination with *po* appears more natural. In this sense, *po* can be though as exhibiting distributive concord with *kazhdyj* (Kuhn 2019).

we believe that further investigations into the interaction between non-specificity and distributivity across languages should encompass not only indefinites but also other structures that give rise to similar meanings. We will revisit this point in the conclusions.

6.2.4 Indefinites and Negative Polarity

In Chapter 5 we observed that Italian *alcun(o)*, an epistemic indefinite in Old Italian, functions like a strict negative-polarity item in current Italian:

(30) Mario non ha letto alcun libro.
Mario not has read ALCUN book.
'Mario has not read any book.'

In Section 6.2 we introduced dependent indefinites and argued in which sense the informational dependence condition they introduce makes them dependent on *another variable*. We would like to suggest that indefinites acting as negative polarity items can be an expression of the same phenomena, where now the dependence is on the variable introduced by negation.

Recall in fact, as discussed in Section 4.8 of Chapter 4 that negation introduces an unrestricted universal quantification over the set of worlds W in the model. In particular, consider the following schematic representation, where we are representing this dependence requirement by means of $info\text{-}dep_v(w, x)$.

(31) a. John did not read $\text{INDEF}_{\text{NPI}}$ book.
b. $\forall w(\exists_s x(\phi(x, w) \wedge info\text{-}dep_v(w, x)) \rightarrow v \neq w)$

We point out three important remarks. First, the world variable introduced by negation is unrestricted, as it ranges over the whole domain W of the model, unlike world variables for modals or attitudes which come with an accessibility relation or similar requirements like inclusion atoms. This makes more plausible to assume a form of dependence on such a variable. This ties the presence of the negative polarity indefinite to the variable for negation, and predicts that in other environments such indefinite cannot occur. Second, the $info\text{-}dep_v(w, x)$ is effectively excluding wide scope readings outside negation, as there must be covariation between w and x. Third, given the latter and the maximality requirement from the maximal implication, discussed in Section 4.6 of Chapter 4, we obtain the correct narrow scope, negated existential, reading of the indefinite, similarly to non-specific indefinites under negation.

Another approach that 2TS offers is to view negative polarity indefinites as a restricted form of non-specificity. In particular, while non-specific indefinites are captured by $var(v, x)$, this class of non-specific indefinites would be captured by $var(v, wx)$, where w is again the unrestricted variable introduced by negation, and we may assume that $var(v, wx)$ is undefined in the absence of negation which is introducing w. Note, in fact, that $var(v, x)$ entails $var(v, wx)$.

6.3 Interlude: The Dynamicity of Indefinites

In this section, we outline how the 2TS framework adopted so far can be made dynamic. In particular, we will first consider some elements that motivate a dynamic treatment, including the status of dependencies atoms in Section 6.3.1, the role of anaphora for marked indefinites in Section 6.3.2, and appositive constructions in Section 6.3.3. We will then present our dynamic framework in Section 6.3.4 and explain how this accounts for the three points mentioned above in Section 6.3.5.

6.3.1 The Status of Dependencies Atoms

In this section, we comment on the status of the enriched meaning of marked indefinites. The issue might be tangential to a dynamic account, but ultimately, we will see how a dynamic account could be relevant in this regard.

First, let us consider the behaviour of indefinites which are marked for their epistemic component: epistemic indefinites with $var(\varnothing, x)$ and specific known indefinites with $dep(\varnothing, x)$. In (32-a) and (32-b), we give some examples for the Italian epistemic indefinite *un qualche*, and for the Russian specific known indefinite *koe-*.

If no one is knocking at the door, both (32-a) and (32-b) are considered false. By contrast, if someone is knocking at the door, and the speaker does not know whom, (32-a) is good, while (32-b) appears to be odd rather than false. A specular contrast occurs in a context where the speaker is aware of the identity of who is knocking at the door, where now (32-a) is odd, but not false, and (32-b) good.

(32) a. Una qualche persona sta bussando alla porta.
UN QUALCHE person stay knocking at-the door
'Someone is knocking at the door'
b. Koe-kto stuchitsya v dver'.
KOE-who knocking in door.
'Someone is knocking at the door'.

It thus appears that the enriched meaning of marked indefinites does not affect the truth conditions of a sentence, in so far as the latter are defined as being true and false, and when the additional component of marked indefinites is not satisfied the sentence appears to be odd, rather than false.

Let us now consider the behaviour of non-specific indefinites. We have already pointed out a crucial feature of non-specific indefinites: they are infelicitous in episodic contexts. Moreover, if we look at their behaviour under a universal quantifier as in (33), there appears to be a contrast with the case of epistemic and specific known indefinite discussed above. In particular, (33), uttered in a context where every boy read the same book, is again not false, but rather odd. As we will, a dynamic system will help us address these contrasts.

(33) Kazhdyy mal'chik chital kakuyu-nibud' knigu.
every boy read which-nibud book.SING.ACC
'Every boy read some book or other.'

6.3.2 Anaphora and Discourse Referents

As mentioned in Chapter 2, coreferential discourse anaphora has been the primary motivation for a dynamic treatment of indefinite noun phrases:

(34) A man_x is in the park. He_x is whistling.

One crucial aspect that is relevant to us is the interplay between anaphora and marked indefinites. The system should predict that the enriched meanings and restricted distributions associated with marked indefinites will also apply to their anaphoric uses. For instance, if we consider a non-specific indefinite like *-nibud'*, which is not allowed in episodic contexts, the use of a pronoun referring to the indefinite in subsequent discourse should not be allowed in episodic contexts as well. We thus expect the use of *it* as in examples like in (35) to be infelicitous:

(35) a. Mary wants to sing some-*nibud'* song_x at her birthday.
b. *John listened to it_x yesterday.

Interestingly, parallel phenomena seem to occur for indefinites and scope readings. For instance, a plain indefinite under a universal quantifier as in (36-a) is compatible with both a narrow scope/specific and wide scope/non-specific reading, and this is reflected in the availability of a singular and plural pronoun, respectively. A non-specific indefinite in (36-b) appears to be compatible only with plural pronouns, while a specific known indefinite in (36-c) is compatible only with a singular one.

(36) a. Every student read some book_x. It_x/They_x was/were recommended by the teacher.
b. Every student read some-*nibud* book. They_x were recommended by the teacher.
c. Every student read some-*koe* book. It_x was recommended by the teacher.

6.3.3 Two-dimensional Meaning and Appositives

A well-known distinction in linguistic theory is the one between AT-ISSUE content and NON-AT-ISSUE content. The former roughly corresponds to the primary meaning, the content that is central to a conversation. The latter conveys secondary, non-at-issue information. A canonical example illustrating this distinction are appositive clauses. A well-known case is given in (37), where the main clause represents the AT-ISSUE content, while the appositive phrase 'a postman' contributes to the NON-AT-ISSUE meaning.

(37) John, a postman, is married to Sue.

We will develop a system that can account for the distinction between these kinds of meaning contents and their integration into the final overall meaning. Minimally, our account will handle appositive cases like (37). However, much of the literature has focused on cases of plain indefinites. We will see to what extent our analysis can generalize to account for marked indefinites. We now give an overview of some novel puzzles we will account for.

In what follows, we will focus on Italian to show the key contrast with epistemic indefinites. Consider the examples in (38), which involve the usage of two indefinites, one in the main clause and one in the appositive clause.

(38) a. Una persona, un ingegnere, sta correndo.
a person, an engineer, stay running
'A person, an engineer, is running.'
b. #Un ingegnere, una persona, sta correndo.
an engineer, a person, stay running
'An engineer, a person, is running.'

While (38-a) is felicitous and conveys the fact that an engineer is running, its counterpart in (38-b), where the indefinite in the main clause contains more information than the indefinite in the appositive clause, is odd. The order in which discourse information is presented seems to play a key role, and this possibly calls for a dynamic treatment. Note that the usage of the epistemic indefinite *un qualche* in the appositive clause makes the sentence in more natural:

(39) Un ingegnere, una qualche persona, sta correndo.
an engineer, UNA QUALCHE person, stay running
'A engineer, some person or other, is running.'

It is interesting to observe that similar contrasts can also be observed in the case of proper names, as it can be seen in (40). While (40-a) is felicitous, the usage of a proper name in an appositive, whose anchor is an epistemic indefinite as in (40), seems odd.[13]

[13] Note that the second example in (i) can be redeemed if the appositive makes it clear that the proper name 'Giovanni' is just an attribute of the possible students who called rather than a way to single out a specific student:

(40) a. Giovanni, un qualche studente, ha telefonato.
Giovanni, UN QUALCHE student, has called.
'Giovanni, some student or other, called.'
b. #Un qualche studente, Giovanni, ha telefonato.
UN QUALCHE student, Giovanni, has called.
'Some student or other, Giovanni, called.'

The task that looks ahead is therefore the following: we need to develop an account which (i) models appositive constructions; (ii) deals with the combinations of different types of indefinites in the main clause and in the appositive clause. The focus, in particular, will be on the Italian epistemic indefinite *un qualche* and on proper names.

6.3.4 A Dynamic Team Semantics

There can be different implementations of dynamic systems in semantic frameworks. The two dominant views are relational and update systems: in relational systems formulas denotes relations over assignment functions, while in update systems they denote functions over sets of assignments. A fully relational system of 2TS can be easily given, since the notion of assignment extensions used to capture quantifiers naturally extends to a relation between sets of assignments. As we have seen, 2TS is a team-based system. The way we capture this team layer is by taking formulas to denote relations over sets of assignment functions, while in standard dynamic predicate logic (Groenendijk and Stokhof 1991) formulas denote relations over assignment functions. This is, in fact, the same strategy proposed by van den Berg (1996).

The language is the same as 2TS. The semantic clauses for the fully relational version can be given as in Definition 6.3.1, where the strict and lax existential and universal quantifiers are defined upon the notion of strict and lax functional extensions introduced in Chapter 3.

An important remark is that we are making universal quantification dynamic and binding relations with the variable introduced by the universal extensions are possible, unlike in standard dynamic predicate logic where universal quantification is static. The rationale behind this choice is that universal quantifiers can in principle introduce plural discourse referents and instead of banning binding relations at all, an alternative approach would be to add constraints on the kind of discourse referents such extension introduces.[14]

(i) Un qualche studente, di nome Giovanni, ha telefonato.
UN QUALCHE student, of name Giovanni, has called
'Some student, named Giovanni, called.'

[14]It is of course possible to render such clause static by requiring that $T = T'$ and $\exists X : \langle T[z], X\rangle \models \phi$

6.3.1. Definition (Semantic Clauses for Dynamic 2TS). Given a suitable model M and a formula ϕ, we define the satisfaction relation of ϕ in an input team T and an output team T', denoted by $M, \langle T, T' \rangle \models \phi$, inductively on ϕ as follows:

$M, \langle T, T' \rangle \models P(t_1 \dots t_n)$	iff	$T = T'$ and $\forall i \in T : \langle i(t_1), \dots, i(t_n) \rangle \in I(P)$
$M, \langle T, T' \rangle \models \neg P(t_1 \dots t_n)$	iff	$T = T'$ and $\forall i \in T : \langle i(t_1), \dots, i(t_n) \rangle \notin I(P)$
$M, \langle T, T' \rangle \models z = u$	iff	$T = T'$ and $\forall i \in T : i(z) = i(u)$
$M, \langle T, T' \rangle \models \neg z = u$	iff	$T = T'$ and $\forall i \in T : i(z) \neq i(u)$
$M, \langle T, T' \rangle \models \phi \wedge \psi$	iff	$\exists X : M, \langle T, X \rangle \models \phi$ and $M, \langle X, T' \rangle \models \psi$
$M, \langle T, T' \rangle \models \phi \vee \psi$	iff	$T = T'$ and $\exists X, Y : M, \langle T_1, X \rangle \models \phi$ and $M, \langle T_2, Y \rangle \models \psi$ for some T_1, T_2 s.t. $T = T_1 \cup T_2$
$M, \langle T, T' \rangle \models \exists_s z\ \phi$	iff	$\exists X : X = T[f_s/z]$ and $M, \langle X, T' \rangle \models \phi$ for some strict function f_s
$M, \langle T, T' \rangle \models \exists_l z\ \phi$	iff	$\exists X : X = T[f_l/z]$ and $M, \langle X, T' \rangle \models \phi$ for some lax function f_l
$M, \langle T, T' \rangle \models \forall z\ \phi$	iff	$M, \langle T[z], T' \rangle \models \phi$

We can define a standard notion of support and entailment for a formula ϕ in a team:

6.3.2. Definition (Support). Given a formula ϕ, a suitable model M and a suitable team T over M, T supports ϕ, in symbols $M, T \models \phi$, iff there exists T' s.t. $M, \langle T, T' \rangle \models \phi$

6.3.3. Definition (Entailment). A formula ϕ entails a formula ψ, in symbols $\phi \models \psi$, iff for all suitable models M and all suitable teams T, T' such that $M, \langle T, T' \rangle \models \phi$, we have $M, T' \models \psi$.

As it was the case for 2TS, we initially assume that all formulas are in negation normal form, and we later discuss the role of negation. In particular, we will return to negation in the next section when discussing anaphora.

One advantage of this fully relational system is that it is easy to see the correspondence between this system and the static version. In fact, similar remarks to the correspondence between dynamic predicate logic and predicate logic in terms of satisfaction conditions, studied in Groenendijk and Stokhof (1991), carry over to this system.

Before moving on to the next section, we would like to propose an alternative dynamic system which allows for the update of world information. Such a system would be, only in this respect, eliminative, as some assignments might not be preserved in the output team T'. For instance, given a model $M = \langle D, W, I \rangle$, a first-order literal like $F(a, v)$ will denote all team pairs $\langle T, T' \rangle$ such that in the output state T' all assignments make $F(a, v)$ true.

$$M, \langle T, T' \rangle \models P(t_1 \dots t_n) \text{ iff } T' = \{i \in T : \langle i(t_1), \dots, i(t_n) \rangle \in I(P)\}$$

T	v
i_1	v_{ab}
i_2	v_a
i_3	v_b
i_4	$v_\varnothing$

$\rightarrow \quad F(a, v) \quad \rightarrow$

T'	v
i_1	v_{ab}
i_2	v_a

Table 6.6: Illustration for $F(a, v)$.

The advantage of such a system is that it can better capture the dynamics of a conversation between speakers and hearers, as discussed at the end of Chapter 5, and notions like coherence and consistency can be formulated as in Groenendijk, Stokhof, and Veltman (1996).

6.3.4. Definition (Semantic Clauses for (Update) Dynamic 2TS). Given a suitable model M and a formula ϕ, we define the satisfaction relation of ϕ of an input team T and an output team T', denoted by $M, \langle T, T' \rangle \models \phi$, inductively on ϕ as follows:

$M, \langle T, T' \rangle \models P(t_1 \dots t_n)$	iff	$T' = \{i \in T : \langle i(t_1), \dots, i(t_n) \rangle \in I(P)\}$
$M, \langle T, T' \rangle \models \neg P(t_1 \dots t_n)$	iff	$T' = \{i \in T : \langle i(t_1), \dots, i(t_n) \rangle \notin I(P)\}$
$M, \langle T, T' \rangle \models t_1 = t_2$	iff	$T' = \{i \in T : i(t_1) = i(t_2)\}$
$M, \langle T, T' \rangle \models \neg t_1 = t_2$	iff	$T' = \{i \in T : i(t_1) \neq i(t_2)\}$
$M, \langle T, T' \rangle \models \phi \wedge \psi$	iff	$\exists X : M, \langle T, X \rangle \models \phi$ and $M, \langle X, T' \rangle \models \psi$
$M, \langle T, T' \rangle \models \exists_s z\ \phi$	iff	$\exists X : X = T[f_s/z]$ and $M, \langle X, T' \rangle \models \phi$
$M, \langle T, T' \rangle \models \exists_l z\ \phi$	iff	$\exists X : X = T[f_l/z]$ and $M, \langle X, T' \rangle \models \phi$

To model the notion of support, we define the notion of survival of an assignment in a team based on Dekker (1993) and Groenendijk, Stokhof, and Veltman (1996).

6.3.5. Definition (Survival, based on Dekker 1993). Given a team T, an assignment i, we say that i survives in T, defined as follows, and we write $i \prec T$.

Similarly, given a team T and team T', we say that T survives in T', defined as follows, and we write $T \prec T'$.

$$\begin{array}{lll} i \prec T & \text{iff} & \exists j \in T : i \subseteq j \\ T \prec T' & \text{iff} & \forall i \in T : i \prec T' \end{array}$$

6.3.6. Definition (Support). Given a formula ϕ, a suitable model M and a team T over M, T supports ϕ, $M, T \models \phi$, if and only if $\exists T' : M, \langle T, T' \rangle \models \phi$ and $T' \prec T$.

If we want to define universal quantification similarly to the previous dynamic framework, we need some more elaborate operations.

Given a team T and an assignment i, we define $X_{i \sim T}$ as the portion of the team X to the variables in the domain of T with respect to the values assigned by i: $\{j \in X : j(z) = i(z) \text{ for all } z \in Dom(T)\}$. For instance, if T is the initial team where only v is in the domain of T, $X_{i \sim T}$ reduces to $X_{v=i(v)} = \{j \in X : j(v) = i(v)\}$ for a given i. The general version allows for occurrences of embedded universal quantifiers. This allows us to require that for a formula like $\forall z \phi$ only the original assignments which preserve the universal extension after the evaluation ϕ are kept. The semantic clause for the universal quantifier can then be given as below, and we give an illustration in Table 6.7:

$$M, \langle T, T' \rangle \models \forall z\ \phi \quad \text{iff} \quad T' = \{i \in X : X_{i \sim T}(z) = T[z]_{i \sim T}(z)\} \text{ with } X \text{ s.t. } M, \langle T[z], X \rangle \models \phi$$

T	v
i_1	v_{ab}
i_2	v_a
i_3	v_b
i_4	$v_\varnothing$

$\rightarrow \forall z \rightarrow$

$T[z]$	v	z
i_1	v_{ab}	a
i'_1	v_{ab}	b
i_2	v_a	a
i'_2	v_a	b
i_3	v_b	a
i'_3	v_b	b
i_4	$v_\varnothing$	a
i'_4	$v_\varnothing$	b

$\rightarrow F(x, v) \rightarrow$

X	v	z
i_1	v_{ab}	a
i'_1	v_{ab}	b
i_2	v_a	a
i'_3	v_b	b

$\rightarrow$

T'	v	z
i_1	v_{ab}	a
i'_1	v_{ab}	b

Table 6.7: Illustration for $\forall z F z$.

While this version of a dynamic 2TS could be used for certain applications, in what follows we will mainly consider the fully relational system we considered at the beginning of this section, which appears to be more well-behaved or easily definable.

6.3.5 Applications

We now proceed to discuss some applications concerning the cases discussed in the previous sections.

The Status of Dependencies Atoms

In Chapter 3, we discussed that 2TS employs dependence and variation atoms to account for the different functional uses of indefinites. One possibility is to add such clauses as tests similarly to the first-order atoms in the semantic clauses we discussed before:

$$M, \langle T, T' \rangle \models dep(\vec{z}, \vec{u}) \text{ iff } T = T' \text{ and } \forall i, j \in T : i(\vec{z}) = j(\vec{z}) \Rightarrow i(\vec{u}) = j(\vec{u})$$

$$M, \langle T, T' \rangle \models var(\vec{z}, \vec{u}) \text{ iff } T = T' \text{ and } \exists i, j \in T : i(\vec{z}) = j(\vec{z}) \ \&\ i(\vec{u}) \neq j(\vec{u})$$

This approach would be entirely parallel to the static framework for a case like (41-b). As such, then, it would not thus capture the oddness of uttering (41-a) in contexts where there is some man who is walking, but the speaker does not know who. In such a case, no output team which satisfies the variation atom $var(\varnothing, x)$, as (41-b) would merely be not supported.

(41) a. Some (epistemic) man is walking.
  b. $\exists_s x(\phi(x, v) \wedge var(\varnothing, x))$

We then look a different way of capturing dependencies atom. Another approach is to treat them as post-suppositions. We will outline what a post-supposition in a dynamic system is, implement it in 2TS, and motivate such analysis.

Post-supposition impose constraints that need to be satisfied on the output team. If such a constraint is met, then the output state is maintained. By contrast, pre-suppositions impose constraints that need to be satisfied on the input team.

The rationale behind this choice is that dependence and variation atoms impose conditions on the value of the variable of the indefinite x, variable which is introduced by the existential. We propose that post-suppositions can be used to capture the oddness of marked indefinites when their enriched meaning is not satisfied. A similar approach has been defended by Aloni (2023), who uses the notion of post-suppositions within a propositional update semantics to model so-called neglect-zero inferences. Henderson (2014) and Brasoveanu (2013) also discuss the role of post-suppositions and its linguistic applications in a related dynamic framework.

As discussed, the dependencies conditions for marked indefinites can be encoded as post-suppositions.[15]

[15] To deal with undefinedness, we would need to partialize the semantics. For instance, one

$$M, \langle T, T' \rangle \models [\phi]^{\psi} \text{ iff } \begin{cases} M, \langle T, T' \rangle \models \phi, & \text{if } M, T' \models \psi \\ \text{undefined} & \text{otherwise} \end{cases}$$

The way post-suppositions interacts with marked indefinites is as follows. First, for the case of marked indefinites ψ will be $dep(\varnothing, x)$ for cases in which the value of x is known to the speaker and $var(\varnothing, x)$ for cases in which the value of x is not known to the speaker, as defined above. Second, we will assume that post-supposition apply globally at the sentence level. Later, we will revisit this when considering negation.

This amendment would account for the data discussed in Section 6.3.1, and in particular it offers a way to model oddness, as it occurs in epistemic indefinites uttered in a context where the speaker knows the identity of the referent.

(42) a. Someone (epistemic) is walking.
b. $[\exists_s x\ W(x, v)]^{var(\varnothing, x)}$

As concern (42-a), there are cases where the sentence is judged as false, namely when no one is walking. But there are also cases when the sentence is judged odd/undefined. This would be compatible with an initial team where the value of x is constant. In such team, under a pertinent model, $W(x, v)$ would hold, but $var(\varnothing, x)$ would not, leading to undefinedness.

Let us consider the case of a non-specific indefinite interacting with another operator. Non-specific indefinites are only admitted in readings where the indefinite does not receive the narrowest scope possible.

(43) a. Everyone read some (non-specific) book.
b. $[\forall y \exists_s x(B(x, v) \wedge R(, v) \wedge dep(v, x))]^{var(v,x)}$
c. $\forall y \exists_s x(B(x, v) \wedge R(y, x, v) \wedge var(v, x))$

If we were to treat the variation atom $var(v, x)$ for non-specific indefinites as a post-supposition, we would predict that uttering (43-a) in a context in which everyone read the same book is odd rather than false. This is captured in (43-b), where we are requiring the indefinite to receive wide scope encoded by means of the dependence atom $dep(v, x)$. Clearly, $dep(v, x)$ is incompatible with $var(v, x)$, as the latter corresponds to the Boolean negation of the former. Note, however, that if we are in a context where the indefinite receives narrow scope, captured by $dep(vy, x)$, it would be possible to find output teams T' where $var(v, x)$ holds and x is defined.

However, we have observed that, in the case of non-specific indefinites, it appears that most speakers judge the sentence in (43-a) false, rather than unde-

might add pertinent acceptance, rejection and *definiteness* clauses to the semantics, and redefine support accordingly. Clearly, given the role that undefinedness plays, the system should be weak Kleene.

fined. This leads to the logical rendering in (43-c), where the variation is treated as a test under the first approach we outlined. This leads to the conclusion that different marked indefinites are encoded in different way that are reflected in this dynamic framework. While atoms encoding epistemic distinctions, like $dep(\varnothing, x)$ or $var(\varnothing, x)$, are better treated as post-suppositions, atoms encoding scopal specificity, like the non-specific $var(v, x)$ just outlined and $dep(v, x)$, are better analysed as tests.

This difference is important, as it might reflect the different status of the enriched meaning of marked indefinites. While indefinites encoding epistemic distinctions are associated with inferences resulting from a post-supposition, indefinites marked for scopal (non)-specific appear to be integrated in the semantics and evaluated as conjuncts like in (43-c). This difference will play a role in the way indefinites develop with respect to these functions, as we will further see in Section 6.4, since $dep(\varnothing, x)$ and $var(\varnothing, x)$ are more close to a form of pragmatic inference, whereas $dep(v, x)$ and $var(v, x)$ are integrated into the semantics of the indefinite under this view.

Anaphora and Discourse Referents

Cases of anaphora with plain indefinites are accounted as in standard dynamic systems. (44) shows an example:

(44) a. Some man is in the park. He is whistling.
b. $\exists_s x(M(x, v) \wedge P(x, v)) \wedge W(x, v)$

Let us consider the behaviour of marked indefinites. Recall that for non-specific indefinites, discourse referents like in (45) are not allowed. Note, however, that singular pronominal uses are allowed if used non-specifically, as in (45-c), where the modal is allowing the non-specific reading to be licensed.

(45) a. Mary wants to sing some-*nibud* song$_x$ at her birthday.
b. #John listened to it$_x$ yesterday.
c. It$_x$ must be a rock song.

We propose that the use of *it* comes with a singularity condition. We define singularity and plurality conditions of a variable x with respect to $\vec{z}$ in Definition 6.3.7 and Definition 6.3.8. The former says that, given a value for $\vec{z}$, x receives only one value. The latter says that, given a value for $\vec{z}$, x receives must receive more than one possible value.

6.3.7. Definition (Singularity). $M, \langle T, T' \rangle \models singular_{\vec{z}}(x)$ iff $T = T'$ and $\forall i \in T$, $|\{j(x) : j \in T \text{ and } i(\vec{z}) = j(\vec{z}\}| = 1$

6.3.8. Definition (Plurality). $M, \langle T, T' \rangle \models plural_{\vec{z}}(x)$ iff $T = T'$ and $\forall i \in T$, $|\{j(x) : j \in T \text{ and } i(\vec{z}) = j(\vec{z}\}| > 1$

In particular, the relevant singularity condition for cases like (45-b) is $singular_v(x)$, as the pronominal element 'it' occurs in an episodic context, as represented in (46).

(46) $\forall w \exists_s x(\phi(x, w) \wedge var(v, x)) \wedge \psi(x, v) \wedge singular_v(x)$

When the pronominal element *it* occurs within a modal and is interpreted non-specifically, like in (45-c), we assume that the relevant singularity condition which needs to be checked is $singular_{vw}(x)$. This means that the pronominal element is interpreted within the modal, as in (47).

(47) $\forall w \exists_s x(\phi(x, w) \wedge var(v, x)) \wedge \psi(x, v) \wedge singular_{vw}(x)$

X	v	w	x
i_1	v_1	w_a	a
i_2	v_1	w_b	b
i_3	v_1	w_c	c
i_4	v_1	w_d	d

X	v	w	x
i_1	v_1	w_a	a
i_2	v_1	w_b	b
i_3	v_1	w_c	c
i_4	v_1	w_d	d

Table 6.8: The portion of the team where the singularity condition is being checked is surrounded by a box. On the left, we give an illustration for $singular_v(x)$ which is not satisfied, on the right for $singular_{vw}(x)$ which is satisfied.

The system also extends to cases like (36) before and (48), when an indefinite occurs under a nominal quantifier, and both singular and plural forms are allowed. A wide scope reading is compatible with 'it' and thus with $singular_v(x)$. Similarly, a plural 'they' is associated with $plural_v(x)$. Clearly, in such cases, it cannot be that all the students read the same book.

(48) Every student read some book$_x$. It$_x$/They$_x$ was/were recommended by the teacher.

We conclude with a case which can be handled, but it showcases the limitations of the present system. Consider the example in (49).

(49) Every student$_y$ read some book$_x$. It$_x$ was their$_y$ favourite.

The usage of 'their' is not difficult to explain, as $plural_v(y)$ holds as long as there is more than one student. The usage of 'it' is more difficult to account for. Clearly, $singular_v(x)$ does not necessarily hold, as the favourite book of one student could be different from the favourite book of another student. What appears to be happening is that 'it' is interpreted distributively regarding the students. As such, the relevant condition which needs to be checked is $singular_{vy}(x)$.

Examples like (49) show that while the framework with the *singular* and *plural* conditions is able to explain the presence of singular or plural pronominal forms, the usages of the latter need to be evaluated case by case, rather than within a general systematic theory.

We end this section with some remarks on negation. Adapting the standard notion of dynamic negation (Groenendijk and Stokhof 1991) to a team-based system, is immediate:

$$M, \langle T, T' \rangle \models \neg\phi \text{ iff } T = T' \text{ and } \neg\exists K : M, \langle T, K \rangle \models \phi$$

The above semantic clause takes negation as a test and checks there is no pair $\langle T, K \rangle$ supporting the formula. There are two issues with this notion of negation for the applications we have in mind. First, it is in many respects similar to the Boolean negation ($M, T \not\models \phi$), which, as we have seen in Section 4.8 of Chapter 3, is not suitable to model the interaction between indefinites and negation.

Second, such a notion is externally static, and it thus does not correctly capture the anaphoric potential of indefinites under negation. To illustrate this, consider the sentence in (50), which can be associated with a narrow scope/negated existential reading ('John didn't buy any book') or with a wide scope reading ('there is a book which John didn't buy'). It appears to be that for each case, a discourse referent linked to *a book* is possible: a plural for the former reading, and a singular one for the latter.

(50) John didn't buy a book$_x$.
   a. They$_x$ were (all) too expensive.
   b. It$_x$ was too expensive.

The challenge is to explain the data in (50) while maintaining the indefinite in situ. In Section 4.8 of Chapter 3, we proposed a notion of negation which can readily account for such narrow scope/wide scope readings: intensional negation $\neg_I$. The latter was based on a particular notion of implication called weak maximal implication and universal quantification over worlds, where the indefinite is introduced in the antecedent of the implication. It is of course possible to render such notion of implication dynamic.

$$M, \langle T, T' \rangle \models \phi \rightarrow_{\exists} \psi \quad \text{iff} \quad \text{there exist } X, Y : X \subseteq T \text{ and } M, \langle X, Y \rangle \models \phi \text{ and } X \text{ is maximal and } M, \langle Y, T' \rangle \models \psi$$

Moreover, to capture the anaphoric behaviour of indefinites under negation, the variables introduced in the antecedent must be available in subsequent discourse. Given the above clause, this is indeed the case, both for a singular discourse referent, which matches with wide scope reading of the indefinite, and for a plural discourse referents, which matches with the multiplicity of values introduced by the narrow scope reading. We refer the reader to Section 4.8 of Chapter 3 for some illustrations.

While this works in principle for negation, assuming such a clause of implication for conditionals in general overgenerates drastically. While there are cases of anaphora when an indefinite occurs in the consequent of the conditional (cases that would be accounted by the clause above), arguing for anaphora in conditional antecedents is simply empirically wrong. We thus acknowledge that either we assume a special clause of implication for negation like the previous one and another one for conditionals, which need to be static. Alternatively, this analysis of negation in terms of intensional negation and in situ requirement must be reconsidered.

Before moving on to the next section, we observe that specific known indefinites give rise to wide scope readings under negation. We have observed that these indefinites, associated with $dep(\varnothing, x)$, are accounted in terms of post-suppositions. If so, post-suppositions cannot be maintained global as in (51-a), since the correct behaviour is obtained by the interaction of the maximality requirement with $dep(\varnothing, x)$ in the antecedent of the implication, as in the configuration in (51-b).

(51) a. $[\neg\exists_s x(\phi(x, v))]^{dep(\varnothing,x)}$
b. $\neg[\exists_s x(\phi(x, v))]^{dep(\varnothing,x)}$

Two-dimensional Meaning and Appositives

We now analyse the problem considered in Section 6.3.3: the interplay between indefinites and appositive constructions. As a base case, consider the example in (52).

(52) a. John, a postman, runs.
b. $\langle R(j, v), P(j, v)\rangle$

As discussed in Section 6.3.3, we assume that appositives contribute to two components, represented here by $\langle \phi_{\text{AT-ISSUE}}, \phi_{\text{NON-AT-ISSUE}}\rangle$. In the case of (52-a) the AT-ISSUE component is $R(j, v)$, while $P(j, v)$ is the NON-AT-ISSUE component. The rendering in (52-b) is the canonical way in which appositive constructions are represented. We might have also chosen to represent the NON-AT-ISSUE component as $P(x, v)$ and later require, as we shall see, that the integration of the two dimensions requires identification between the term in the appositive and the term in the main clause by means of $x = j$. Since this will have no impact on our formalization, we will assume that the term specified in the NON-AT-ISSUE component refers to a term in the main clause, which acts as its so-called anchor.

In what follows, we settle two questions underlying the representations in (52-b): (i) how proper names should be treated in a team-based system; (ii) how the AT-ISSUE component should be integrated with the NON-AT-ISSUE one.

As concerns the first question, recall that the present system is intensional with a variable for the actual world v representing the speaker's epistemic possibilities. We propose that a notion of rigidity of proper names compatible with the present

v	w	j
v_1	w_1	d_1
v_1	w_2	d_1
v_2	w_3	d_2
v_2	w_4	d_2

Table 6.9: Illustration for (53).

framework is assuming that $dep(v, j)$ holds for any name j, meaning that proper names refer to the same individual in a particular epistemic possibility of the speaker. Formally, this implies that we cannot treat j as a constant, as this would require the value of j to be fixed by the interpretation function of the model, but as a free variable with the additional requirement $dep(v, j)$. This view is compatible with the value of proper names to differ across the epistemic possibilities of the speaker. We illustrate this with the example in (53) and the illustration in Table 6.9, which conveys that the speaker is ignorant about who John is (it could be d_1 or d_2), but the value of John does not change across the epistemic possibilities of the speaker.

(53) a. John can pass the exam.
b. $\exists_l w P(j, w)$

As concerns the point (ii), different account of appositive constructions have been proposed within a dynamic system (among others, Nouwen 2007; AnderBois, Brasoveanu, and Henderson 2015). Here we assume that appositives should be interpreted as a dynamic conjunction, which is in line with unidimensional accounts of appositives and NON-AT-ISSUE content as given in Schlenker (2010).

(54) MERGING
$\mu(\langle \phi_{\text{AT-ISSUE}}, \phi_{\text{NON-AT-ISSUE}} \rangle) \Leftrightarrow \phi_{\text{AT-ISSUE}} \wedge \phi_{\text{NON-AT-ISSUE}}$

In general, we take the contribution of the NON-AT-ISSUE to be non-trivial (i.e., $\phi_{\text{AT-ISSUE}} \not\models \phi_{\text{AT-ISSUE}} \wedge \phi_{\text{NON-AT-ISSUE}}$). If such a requirement is not met, then some redundancy effect leads to considering the use of the appositive unexpected.

Consequently, the case above is simplified to (55-c):

(55) a. John, a postman, is running.
b. $\langle R(j, v), P(j, v) \rangle$
c. $R(j, v) \wedge P(j, v)$

Let us consider how the analysis extends to cases involving epistemic indefinites as in (56).

(56) a. John, some (epistemic) postman, is running.
b. $\langle R(j, v), [P(j, v)]^{var(\varnothing, j)} \rangle$

c. $R(j,v) \wedge [P(j,v)]^{var(\varnothing,j)}$

The rendering in (56-c) predicts the desired reading, namely that the speaker knows that John is a postman, but they cannot identify which postman exactly.

We conclude, similarly to the previous section, with one example that can be handled in the present framework, but it requires some additional explanation. In particular, we observe that while (57-a) appears to be fine, (58-a) appears to be rather odd. As it stands, both representations in (57-c) and (58-c) are not a problem. The former specifies that John called and that he is a student, but the value of j must not be constant across all epistemic possibilities of the speaker, signalling that the speaker does not know exactly who John is. The latter is effectively equivalent, assuming that the contribution of the appositive is $x = j$.

(57) a. John, some (epistemic) student, called.
b. $\langle C(j,v), [S(j,v)]^{var(\varnothing,j)} \rangle$
c. $C(j,v) \wedge [S(j,v)]^{var(\varnothing,j)}$

(58) a. #Some (epistemic) student, John, called.
b. $\langle [\exists_s x(S(x,v) \wedge C(x,v))]^{var(\varnothing,x)}, x = j \rangle$
c. $[\exists_s x(S(x,v) \wedge C(x,v))]^{var(\varnothing,x)} \wedge x = j$

To account for the contrast between (58-a) and (57-a), we propose that the contribution of the appositive in the latter case is to *identify* the student with John. In such a case, the value of the variable j does not merely satisfy $dep(v,j)$, but rather $dep(\varnothing,j)$, signalling that the speaker refers to a particular individual. Consequently, this would be incompatible with the variation condition $var(\varnothing,x)$ and $x = j$.

Note also that given the non-triviality requirement we mentioned above, this would account for the contrasts in (38), where the unavailability of (38-b) can be explained by the fact that being an engineer entails being a person, but not in the case of (39), where the contribution of variation from the epistemic indefinite blocks the entailment.[16]

6.4 Diachrony: Shades of Non-specificity

In each chapter, we have dedicated the final section to discussing the diachronic development of each indefinite form. Some aspects of the diachronic development concerning non-specificity have already been addressed in previous chapters dedicated to other indefinites or to 2TS in general. Here, we offer two additional

[16]Note that we have assumed that the contribution of the epistemic indefinites is a post-supposition in the logic renderings above. Alternatively, we can treat them as conjunctions, disregarding for the moment the discussion in Section 6.3.5. Some further elements which should be considered is whether a post-supposition treatment makes different predictions when the appositive is embedded and if merging operations can occur locally.

remarks.

First, in Section 5.8 of Chapter 5, we examined the relationship between non-specific indefinites and epistemic indefinites, noting how the v-variation condition $var(v,x)$ transitions to simple variation $var(\varnothing,x)$ through weakening. This weakening trend is further illustrated by the case of the non-specific Russian indefinite *-nibud'*. The latter originated from a construction with a free choice meaning (Penkova 2021) similar in distribution to English *whatever*. In Chapter 5.8, we discussed how free choice is captured by the total variation $VAR_{|D|}(v,x)$, where every value is considered possible. Importantly, assuming that the team is non-empty, $VAR_{|D|}(v,x)$ entails $var(v,x)$. We conjecture that the development of non-specific indefinites from an original free choice meaning represents a gradual weakening process, starting from full variation and reducing to the minimal requirement of making the indefinite vary within a value for v, which might subsequently reduce to $var(\varnothing,x)$, as pointed out in Chapter 5 for the case of epistemic indefinites.

$VAR_n(v,x)$

v	x
v_1	d_1
v_1	d_2
v_1	$\ldots$
v_1	d_n

$\Rightarrow$

$var(v,x)$

v	x
v_1	d_1
v_1	d_2
v_1	d_2
v_1	d_2

$\Rightarrow$

$var(\varnothing,x)$

v	x
$\ldots$	d_1
$\ldots$	d_2
$\ldots$	d_2
$\ldots$	d_2

Table 6.10: Weakening path from total v-variation to v-variation to variation.

The second remark concerns the question raised at the end of Chapter 5: can epistemic indefinites turn into non-specific ones, reversing the weakening directionality? Generally, this does not occur based on the available data, but there is an environment where such a change appears to be attested: negation.

We have already observed that Latin *aliquis*, an epistemic indefinite, retained this status in Old Italian but evolved into an indefinite with the same distribution as a negative polarity item, similar to English *anyone*, under negation. A similar development occurred for Dutch *enig* (Hoeksema 2010). Initially, Dutch *enig* appeared in positive contexts in both modal environments and episodic sentences as an epistemic indefinite. Today, Dutch *enig* is restricted to downward-entailing environments. We aim to account for these developmental paths in a unified manner.

In Section 6.2.4, we observed that NPI indefinites can be viewed as a specific kind of non-specific indefinites, licensed only by the variable for negation. There are two ways to conceptualize this phenomenon: one in which non-specificity is encoded by $var(v,wx)$, requiring the unrestricted variable w introduced by negation to be present in the variation atom, and another in which the indefinite is a particular kind of dependent indefinite of the form $info\text{-}dep_v(w,x)$, dependent on the variable for negation. We have also pointed out the close relationship between the two requirements.

At first glance, one might hypothesize that the direction of entailment is reversed in negative environments. However, our treatment of negation does not immediately lead us to this conclusion. Consider an epistemic indefinite under negation as in (59-a) and a non-specific indefinite under negation as in (59-b).

(59) a. $\neg_I \exists_s x(\phi(x,v) \land var(\varnothing, x))$
b. $\neg_I \exists_s x(\phi(x,v) \land var(v, x))$

As discussed in Section 4.8 of Chapter 4, in the absence of a dependence atom, (59-a) is equivalent to (59-b) because the variation atom is trivial under negation. However, epistemic indefinites also allow for a specific reading when combined with the dependence atom $dep(v, x)$ in (59-a), a reading not available for non-specific indefinites due to the clash with $var(v, x)$ in (59-b). We might thus conjecture that the potential ambiguity of an epistemic indefinite under negation paved the way for a particular kind of non-specific indefinite, restricted to downward-entailing environments.[17]

6.5 Conclusion

In this chapter, we have discussed how 2TS accounts for non-specific indefinites by examining their meaning and licensing conditions, explaining how the variation atom is crucial in explaining their infelicity in episodic contexts. Furthermore, we discussed how non-specific indefinite are related to dependent indefinites, examining the case of Russian which exhibits both a non-specific indefinite *-nibud'* and a dependent indefinite *po*. We also provided a dynamic version of 2TS, discussing different possibilities for integrating dependence atoms in such a system and examining some applications, like anaphora and appositives.

Regarding the distinction between non-specific and dependent indefinites, our emphasis was on the contrast observed in Russian between *-nibud'* and *po*. It is evident that this discussion needs to be situated within a broader cross-linguistic perspective. For instance, no other language has been identified that features both types of indefinites. Future studies should not be confined to mere indefinite 'forms', as languages may employ diverse constructions to signal dependence or non-specificity, such as explicit distributive quantifiers or certain plural constructions.

[17] A relevant point is that we would expect such specialization in use to also occur for cases of epistemic indefinites embedded under universal quantifiers. The ambiguity raised by an epistemic indefinite under negation might be more salient, but more importantly, the variable w for negation is unrestricted, making it more plausible to hypothesize such strengthening with respect to such environments.

For example, we noted how Romanian *câte* shares a common root with the Italian distributive quantifier *ciascuno*, underscoring the importance of integrating cross-linguistic generalizations concerning how languages signal both covert and overt distributivity (see e.g., Chapter 9 in Champollion 2017 and Zimmermann 2002). Similarly, in Italian, non-specificity in embedded contexts can be conveyed by the 'plural' form of the epistemic indefinite *(un) qualche*, as discussed in Chapter 5. Furthermore, it would be useful to differentiate between determiner and determinerless languages, as languages may then employ different structures to denote specificity and non-specificity.

In summary, achieving cross-linguistic generalizations concerning the distinction between non-specific and dependent indefinites should be anchored in a comprehensive theory of non-specificity and distributivity.

Regarding the dynamic system we considered, an open question remains concerning the correct projection behaviour of marked indefinites in various environments (and not just the epistemic indefinites type we considered when dealing with appositives), particularly in light of the post-supposition analysis, which would need to be revised in light of the empirical data.

Chapter 7

Specific Indefinites

> There is, nevertheless, **a certain** respect and a general duty of humanity that ties us, not only to beasts that have life and sense, but even to trees and plants.
>
> Michel de Montaigne, *Of Cruelty* [English translation]

In Chapter 2, we introduced the problem of indefinites and exceptional scope. Chapter 4 explored how a dependence logic-based account can address this issue. In this chapter, we will start in Section 7.1 by revisiting a seminal approach to the problem: choice functional analyses of indefinites. The comparison with our current account will shed some light on their similarities and differences. In Section 7.2, we will then examine indefinite expressions with a specific meaning like English *a certain*, which are known to give rise to so-called functional readings. We will consider the relationship between these functional uses and the specific(-known) marked indefinites discussed in Chapter 4. In light of this discussion, in Section 7.3 we will revisit our initial characterization of specific indefinites and link the dependence atoms to a salient function in the discourse either via a pragmatic principle or by introducing explicit dependence atoms over functions. In Section 7.4, we will provide an outline of different classes of indefinites encoding specific meanings. To allow for cross-linguistic generalization, we will maintain a broad perspective in this section, but we will also show how the account can be adapted to capture fine-grained distinctions within a given language. We will conclude in Section 7.5 with some diachronic remarks on specific indefinites, as well as the interaction with the other types of indefinites.

7.1 Simple and Generalized Choice Functions

Recall our discussion in Chapter 4 where we observed that indefinites can escape islands and take scope freely. This surprising behaviour has led several authors to propose that indefinites should involve quantification over choice functions rather than standard first-order existential quantification (Reinhart 1997; Kratzer 1998; Winter 1997; Matthewson 1998; Chierchia 2001; Schlenker 2006). Specific implementations of this approach vary: we begin with a working definition of a choice function.

We can read Definition 7.1.1 as describing a choice function over the (characteristic sets of the) predicates of our language. For any given non-empty predicate P, a choice function c selects an element from P.

7.1.1. Definition (Choice Function).
Given a set of non-empty sets $\mathcal{P}$, a choice function is a mapping $c : \mathcal{P} \to \cup\mathcal{P}$ such that $\forall P \in \mathcal{P} : c(P) \in P$

For instance, (1-a) corresponds to the standard first-order treatment of the sentence in (1), while (1-b) makes use of choice functions. In (1-b), the function f selects an element from the set of zebras which is then applied to the predicate R. (1-a) and (1-b) have almost the same truth conditions. However, the role of the empty set is important. In fact, according to Definition 7.1.1, the empty-set is not a possible argument for the choice-function, since we assumed that all sets are non-empty. Other definitions would simply map the empty-set to an arbitrary individual. We will set aside this issue for the present discussion, as the implementation we will in the end adopt does not run into this problem.[1]

(1) A zebra ran.
   a. $\exists x(R(x) \wedge Z(x))$
   b. $\exists c(R(c(Z))$

Reinhart (1997) proposed that indefinites are ambiguous between a standard quantificational first-order reading and a choice functional reading, the latter being associated with exceptional scope uses. Other authors (Winter 1997) assumed that indefinites are always choice functional, which, for ease of presentation, is the view we outline in what follows. The recurrent example in (2) showcasing wide, intermediate and narrow scope reading can thus be obtained by existentially closing the choice function at the relevant level of the syntactic derivation.

(2) Every kid$_x$ ate every food$_z$ that a doctor$_y$ recommended.
   a. Wide scope
      ($\exists c$) every kid ate every food that c(*doctor*) recommended.
   b. Intermediate scope
      every kid ($\exists c$) ate every food that c(*doctor*) recommended.

[1]The underlying issue is that a non-empty set for cases like (1) should lead to falsity rather than presupposition failure (namely, the presupposition that the denotation of the noun-phrase is non-empty). See e.g. Winter (1997) for a possible solution.

c. Narrow scope
every kid ate every food that $(\exists c)$ $c(doctor)$ recommended.

While some authors have argued that choice functions overgenerate unattested readings (see e.g., Schwarz 2011), it has been claimed that simple choice functions alone cannot account for all cases (Schlenker 2006; Chierchia 2001). This has led to the adoption of the more powerful formalism of Generalized Skolem functions, where the function also contains a sequence of variables as argument.[2]

7.1.2. Definition (Generalized Skolem Function).
Given a set of non-empty sets $\mathcal{P}$ and $\vec{Z}$ be the set of all sequences formed from a set of variables Var, a Generalized Skolem function a mapping $F : \vec{Z} \times \mathcal{P} \to \cup\mathcal{P}$ such that $\forall P \in \mathcal{P}\ \forall \vec{z} \in \vec{Z} : F(\vec{z}, P) \in P$

Note that when $\vec{z}$ is the empty sequence, we obtain again choice functions along the lines of what we previously discussed. It is interesting to note that, for the cases that we discussed, the use of Generalized Skolem functions makes it possible to maintain a 'global' use of existentials over functions. For instance, the intermediate reading of (2) can now be represented globally without existentially closing the function at an intermediate level, but by adding the relevant variable to the function:

(3) Intermediate scope
$(\exists F)$ every kid$_x$ ate every food$_y$ that $F(x, doctor)$ recommended.

7.1.1 Choice Functions and Dependence Atoms

The use of Generalized Skolem Functions bears a strong resemblance to the treatment of scope and dependence atoms proposed in Chapter 4.

To see this, we will first make a connection of our dependence atoms with second-order quantification over functions. As observed in Chapter 3, dependence logic is equivalent to existential second-order logic. This correspondence can be seen more concretely in the examples below. Consider the case in (4) and (5). Example (4-a) corresponds to a 'narrow scope' reading captured by $dep(y, x)$. Recall that $dep(y, x)$ encodes a functional dependence of x on y (i.e., $x = f(y)$ for some function f). Example (4-b) is an equivalent representation in second-order logic, where f is a unary function from the domain of the model to itself. Here, we explicitly show the functional dependence among the variables. It is important to note that in this context, f is a second-order function from individuals to individuals, not a choice function as defined in Definition 7.1.1.

[2]The reason behind the term 'Skolem' is that the approach is close to the process of Skolemization in logic: every first-order formula can be converted in Skolem normal form (i.e., a formula in prenex normal form with only universal quantifiers). For instance, $\forall y \exists x P(x, y)$ is equivalent to second-order formula $\exists f \forall y P(f(y), y)$, where f takes the name of Skolem function, which is in turn equisatisfiable to $\forall y P(f(y), y)$.

(4) a. $\forall y \exists x(P(x,y) \wedge dep(y,x))$
b. $\exists f \forall y(P(f(y),y))$

A similar reasoning applies to the 'wide scope' representations in (5), where $f()$ denotes a 0-ary function.

(5) a. $\forall y \exists x(P(x,y) \wedge dep(\varnothing,x))$
b. $\exists f \forall y(P(f(),y))$

Consider now the case of Generalized Skolem functions. The representations in (6) illustrate this. The Generalized Skolem function in (6-c) extends the second-order functional approach seen in (5). Here, the function F is more complex, taking both a set and an individual variable as arguments. One might then wonder if the need of such complex object is really required, as we discuss below.

(6) a. $\forall x(P(x) \rightarrow \exists z(Q(z) \wedge R(x,z) \wedge dep(x,z)))$
b. $\exists f \forall x(P(x) \rightarrow (Q(f(x)) \wedge R(x,f(x))))$
c. $\exists F \forall x(P(x) \rightarrow (R(x,F(x,Q))))$

These representations lead to several theoretical considerations. The functional approaches in (6-b-c) are global, treating the indefinite itself as an object corresponding to a function. In contrast, the use of dependence atoms in (6-a) separates the existential component of the indefinite, which remains in situ, from the functional relationship with other variables, encoded by the dependence atoms.

Before discussing functional uses of indefinites and their relationship with specific indefinites, let's consider some predictions and an issue that arises with the global approach of (Generalized Skolem) functions.

First, we look at an example from Schlenker (2006) that motivated Generalized Skolem functions. We will show how our account captures such cases using second-order functions. The example, originally about a 'syntax class,' has been adapted to a 'cooking class' in (7).

(7) Context: *In our cooking class, every student has one particular dish they struggle with—Alice can't get soufflés right, Bob always burns the risotto, etc. Before the final cooking assessment, I say:*
a. If each student improves in some dish, nobody will fail the assessment. Intended Reading: There is a certain distribution of dishes per student such that if each student improves in the dish assigned to him/her, nobody will fail the assessment.
b. $\exists F \forall y(\phi(y,F(D,y)) \rightarrow \psi)$
c. $\forall y \exists_s x((\phi'(y,x) \wedge dep(y,x)) \rightarrow \psi)$
d. $\exists f \forall y(\phi'(y,f(y)) \rightarrow \psi)$

(adapted from Schlenker 2006)

Schlenker (2006) uses this example to advocate for Generalized Skolem functions, as simple choice functions cannot capture the reading in (7-a). (7-b) requires that the choice of each dish relates to each student. In 2TS, ignoring world variables, this is captured as in (7-c), whose second-order equivalent is (7-d). This shows that these readings can be captured by second-order quantification, without needing complex objects like Generalized Skolem functions.

Global accounts for Generalized Skolem functions have been criticized for lacking arbitrary existential closure, necessitating a more flexible approach which does not maintain the function to be always global (Chierchia 2001; Schlenker 2006). Consider the examples in (8-a) and (8-b). Example (8-a) has an intermediate reading, and (8-b) can be used to deny such a reading. To obtain the intermediate reading in (8-b), negation must take scope over the Generalized Skolem function, meaning it cannot remain global.

(8) a. Every linguist studied every solution that some problem has.
b. Not every linguist studied every solution that some problem has.

The question for us is how cases like (8) are handled in 2TS. For (8-a), the sentence has three usual readings: wide scope in (9-a), intermediate scope in (9-b), and narrow scope in (9-c). The intermediate scope is the most salient, specifying a function mapping every linguist z to some problem x such that z studied all solutions y that x has.

(9) a. $\forall z \forall y \exists_s x(\phi(z, y, x; v) \wedge dep(\varnothing, x))$
b. $\forall z \forall y \exists_s x(\phi(z, y, x; v) \wedge dep(z, x))$
c. $\forall z \forall y \exists_s x(\phi(z, y, x; v) \wedge dep(zy, x))$

As concerns negation, Chierchia (2001) observes that (8-b) would be true in a case in which there is a linguist such there is no problem such that this linguist studied all its solutions. This reading is not immediately accounted for in 2TS, where (8-b) would be analysed using intensional negation as described in Section 4.8 of Chapter 4:

(10) a. $\forall w(\forall z \forall y \exists_s x(\phi(z, y, x; v) \wedge dep(\varnothing, x)) \rightarrow v \neq w)$
b. $\forall w(\forall z \forall y \exists_s x(\phi(z, y, x; v) \wedge dep(z, x)) \rightarrow v \neq w)$
c. $\forall w(\forall z \forall y \exists_s x(\phi(z, y, x; v) \wedge dep(zy, x)) \rightarrow v \neq w)$

(10-a) states that there is a problem x such that every linguist *did not* study all its solutions. (10-b) states that for every linguist z, there is a problem x such that z did not study all its solutions. (10-c) is a narrow scope reading, stating that there is no problem for which every linguist studied all its solutions.

These results align with our treatment of specificity but do not capture the reading Chierchia (2001) is after for (8-b). This is why Chierchia (2001) suggests allowing negation to take scope over the Generalized Skolem function. However, we can capture this behaviour in 2TS without revising our analysis of scope. Instead of intensional negation, (8-b) would be compatible with the Boolean negation $\neg_B$ of the intermediate reading:

(11) a. $\neg_B \forall z \forall y \exists_s x(\phi(z,y,x;v) \wedge dep(z,x))$
b. there is no function f s.t. $\forall z \forall y(\phi(z,y,f(z);v)$

(11-a) effectively corresponds to (11-b), requiring that there is no function f mapping every linguist to a problem for which every solution was studied, the reading we are after. This is significant, as Hintikka (1996, 2002) claimed that the role of initial *not*, as in (8-b), is precisely to express Boolean negation.

7.2 Functional Uses and Specific Indefinites

In Chapter 4, we broadly described two classes of specificity: specific known and specific indefinites. Our discussion was limited to so-called marked indefinites, implying that expressions like the English *a certain* were not included. The use of *a certain* in (12-a) gives rise to a specific-like meaning, whereas its counterpart with a plain indefinite in (12-b) is most likely associated with a non-specific reading.

(12) a. John wants to buy a certain house in Manhattan.
b. John wants to buy a house in Manhattan.

In this section, we consider English *a certain* and compare it with indefinites marked for specificity, suggesting that there is common ground that calls for a unified analysis.

7.2.1 Functional Uses

The question to be addressed is to what extent such specific indefinites behave similarly to cases with *a certain* as in (12). In Chapter 4, we claimed that specific known and specific indefinites were associated with the $dep(\varnothing, x)$ and $dep(v, x)$ conditions, respectively. The example in (12) could suggest that these two classes of indefinites behave similarly, as *a certain* in (12-b) induces a specific interpretation compatible with the scope behaviour of indefinites marked for specificity. Moreover, *a certain* appears to be compatible with specific-unknown uses, as illustrated in (13), suggesting that $dep(v, x)$ should be the correct condition for *a certain*.

(13) A certain student gave me a good teaching score. I have no idea who she is.

Given our treatment of scope, we have a clear prediction: both $dep(\varnothing, x)$ and $dep(v, x)$ are only compatible with wide scope readings. However, English *a certain* often gives rise to so-called functional readings, as in (14), where the continuation *his*$_i$ *mother* with a pronoun referring to *every man*$_i$ makes salient a function associating each man with his mother.

Assuming that such readings are treated as narrow scope readings, this would clash with the two dependence conditions, which predict only wide scope readings.

(14) Every man$_i$ likes a certain woman - namely, his$_i$ mother.

An important point is to determine if (14) genuinely represents narrow scope cases and if specific known and specific indefinites can license such readings. The next section addresses this issue for the Russian specific known marked indefinite *koe-*.

7.2.2 Specific Indefinites and Scope

Martí and Ionin (2019) studied the scopal behaviour of Russian indefinites in a series of experiments. In particular, Martí and Ionin (2019) considered the specific known *koe-*, the epistemic *-to*, and the non-specific *-nibud'*. Here, we focus on their results pertaining to the specific known *koe-*. Martí and Ionin (2019) examined which readings are available by paraphrasing a sentence containing an indefinite with an intended interpretation and asking if such a reading would be possible. Example (15) illustrates this, where a sentence with the indefinite *koe-* under a universal quantifier is paraphrased in three possible ways: wide scope in (15-a); functional narrow scope in (15-b); and narrow scope in (15-c). While Martí and Ionin (2019) do not treat the context in (15-a) as functional, it can also be viewed as representing the constant function mapping each doctor to the same patient.

In the experimental results of Martí and Ionin (2019), wide scope readings and functional narrow scope readings were accepted by most participants, while narrow scope readings with no function triggered by the context had low acceptance rates.

(15) Každyj doktor osmotrel **koe-kakogo** pacienta.
every doctor examined koe-wh patient
'Every doctor examined some patient.'

a. Točnee, vse doktora osmotreli pacienta, kotoryj privlek
more precisely all doctors examined patient which attracted
vseobščee vnimanie svoimi neobyčnymi simptomami.
everyone's attention self's unusual symptoms
<u>WS context:</u>
'That is, all the doctors examined the patient who attracted everyone's attention with his unusual symptoms.'

b. Točnee, každyj doktor osmotrel samogo bol'nogo pacienta
more precisely every doctor examined most sick patient
v ego otdelenii.
in his unit.

functional NS context:
'That is, every doctor examined the sickest patient in his unit.'

c. Točnee, vse doktora osmotreli raznyh pacientov.
more precisely all doctors examined different patients
NS context (not functional):
'That is, all the doctors examined different patients.'

These results support the view that functional uses are possible for specific known indefinites like *koe-*. Martí and Ionin (2019) also discuss similar cases for intermediate scope configurations, showing that *koe-* indeed licenses such readings, which are typically functional. Thus, *koe-* indefinites seem to pattern with English *a certain* in this regard.

As previously mentioned and illustrated in (13), English *a certain* allows for specific unknown uses in plain episodic contexts, while Russian *koe-* is associated with speaker identifiability. Is this difference reflected in functional readings? Let us consider again the example in (14).

On the one hand, there could be readings where the underlying function is known (e.g., *his mother*), but the speaker does not know who the mothers are (i.e., the set of mothers changes from world to world).[3] On the other hand, there could be readings where the underlying function linking the men to the women is not known (e.g., *his mother*, *his daughter*, etc.). It appears that while English *a certain* licenses the latter reading in certain contexts, Russian *koe-* does not.

In general, the results of Martí and Ionin (2019) are puzzling for the treatment of scope outlined in Chapter 4. Both (15-b) and (15-c) are narrow scope configurations and would be captured by the same dependence atoms, leaving unexplained why (15-b) is deemed acceptable while (15-c) is not.

This comparison between English *a certain* and marked indefinites like Russian *koe-* shows that their distribution is similar, despite some differences. This calls for a unified analysis of specific indefinites that can account for these similarities and differences in scope and functional readings.

7.3 Dependence Atoms and Specificity

In this section, we present two possible accounts for the findings outlined in the previous section. The first account, discussed in Section 7.3.1, revisits our original approach to scope, suggesting that dependence atoms are not mandatory for indefinites but are regulated by pragmatic factors. The second account, in Section 7.3.2, explores the distinction between existential quantification over individuals and existential quantification over functions. Section 7.3.3 compares these two

[3] A potential issue is that once a function is specified or salient in the context, the relevant question for knowledge is whether someone is a mother or not, rather than more fine-grained levels of identification.

perspectives.

7.3.1 Dependence Atoms and Pragmatics

In Chapter 4, we proposed a treatment of scope wherein each indefinite has both an existential component and a 'dependence' component, encoding the relationship of dependence with other variables in the domain. Specifically, we assumed that each indefinite is associated with $dep(v\vec{y}, x)$, where $\vec{y}$ represents a sequence of variables formed by operators within the syntactic scope of the indefinite, and v represents the variable for the actual world.

Consider the simplified example in (16) where we have an existential quantifier, representing an indefinite, under a universal quantifier. We assume a team of maximal information where v plays no role and can thus be omitted. (16-a) does not contain any dependence atom, (16-b) with $dep(y, x)$ corresponds to a narrow scope configuration, and (16-c) with $dep(\varnothing, x)$ represents a wide scope (inverse-scope) configuration. Our original account assumed that (16-b) and (16-c) were the only possible simplified logical forms.

(16) a. $\forall y \exists x(P(x, y))$
b. $\forall y \exists x(P(x, y) \wedge dep(y, x))$
c. $\forall y \exists x(P(x, y) \wedge dep(\varnothing, x))$

Now, we observe that (16-a) is equivalent over initial teams to (16-b), without the presence of the dependence atom as the syntax (i.e., the order of the quantifiers) makes $dep(y, x)$ trivial. The equivalence between (16-a) and (16-b) sets the path for the pragmatic view we will endorse here. The configuration in (16-b) contains the additional component of the dependence atom. We have already seen in the previous section that dependence atoms can be equivalently seen as second-order functions.

We propose thus that dependence atoms are triggered only when there is a salient function present in the context, whereas the default reading of a plain indefinite would be the one determined by its syntactic position.

For example, (16-a) corresponds to a plain narrow scope reading, while (16-b) represents a functional narrow scope reading. Similarly, the inverse scope reading could be associated with a 0-ary function as a device to signal that the value x is fixed.

We propose the generalization in (17), associating dependence atoms with a salient function in the context.

(17) Dependence Atoms and Salience
A dependence atom $dep(\vec{z}, x)$ is associated with salient function with input $\vec{z}$ and output x.

(i) $dep(\varnothing, x)$: constant (0-ary function);

(ii) $dep(v, x)$: individual concept (function from worlds to individuals);

(iii) $dep(\vec{y}, x)$: function from individuals to individuals;

(iv) $dep(\vec{z}, x)$ [with $\vec{z} \subseteq v\vec{y}$]: complex function mapping to individuals.

For (i), when $\vec{z}$ is empty, $dep(\varnothing, x)$ means that x is equal to a 0-ary function, which can be seen as a constant. It is triggered in contexts where it is salient that the value of x is fixed and does not depend on any other operator.[4]

For (ii), the function is an individual concept - a function from worlds to individuals - associated with contexts where the speaker possibly does not know the value of x. (iii) corresponds to the standard functional reading, a function from (a sequence of) individuals to individuals. (iv) generalizes (iii), allowing multiple mappings from $\vec{y}$ to x depending on the value of v.

We claim that cases (i) to (iv) are associated with an additional pragmatic cost. For cases in (i) and (ii), when the scope is not immediately determined by the context but by subsequent discourse, speakers will default to the simple narrow scope configuration in (16-a), and the addition of dependence atoms for functional readings incurs a cost. This aligns with experimental literature showing that inverse scope readings are typically associated with increased processing cost (Tunstall 1998; Anderson 2004; Brasoveanu and Dotlacil 2015).[5]

We propose that unmarked indefinites are by default not associated with any dependence atom, yielding the ordinary quantificational (narrow scope) interpretation. They are compatible with all types of dependence atoms as in type (iv), based on a salient function in the context. Marked indefinites, by contrast, are obligatorily associated with one of the dependence atoms in (i)-(iv). For instance, we propose that Russian *koe-* is associated with $dep(\vec{y}, x)$. In episodic contexts, the default reading will be $dep(\varnothing, x)$, signaling that the value of x is constant. Under other quantifiers over individuals, functional readings might arise, but ordinary narrow scope readings would not be possible.

We will return to other types of indefinites and the consequences of this pragmatic view in Section 7.4. Before that, the next section outlines an alternative account.

[4]Alternatively, we may propose that in this case, the function is linked to the speaker s, who has a specific individual in mind. If s is a constant (variable), then $dep(s, x)$ would guarantee that x is constant in the team.

[5]Brasoveanu and Dotlacil (2015) argue that the increased processing cost is due to 'model structure reanalysis', compatible with our perspective, as the order of quantifiers follows the surface structure. The expected reading follows the surface structure, while inverse scope requires reanalysis, here captured by our dependence atom $dep(\varnothing, x)$. Similar considerations may apply to functional readings.

7.3.2 Dependence Atoms over Functions

We have discussed the relationship between specificity markers like *a certain* and functions. Such usages of *a certain* signal that the speaker has a specific function in mind, as opposed to a specific referent. For instance, in (14), the function under consideration maps a man x to his mother $f(x)$. We propose that (i) existential quantification is over function variables, not individual variables; (ii) dependence atoms are also defined for function variables, with relevant conditions $dep(v, f)$ or $dep(\varnothing, f)$, signalling that the function, not the value of the individual variable, is specific/fixed.[6]

Consider the simplified example discussed in the previous section, where again we omit the v variable and assume a team of maximal information. One possible approach is to have existentials and dependence atoms over individuals in (18-a-b) to account for the regular scope of the indefinite, while (18-c), equivalent to (18-a), represents the functional reading.

(18) a. $\forall y \exists x (P(x, y) \wedge dep(y, x))$
b. $\forall y \exists x (P(x, y) \wedge dep(\varnothing, x))$
c. $\forall y \exists f (P(f(y), y) \wedge dep(\varnothing, f))$

The advantage of this approach over the pragmatic view is that we do not alter the account of scope presented in Chapter 4, and functional readings are handled by a separate mechanism. Moreover, we can maintain that specific indefinites are still associated with a constancy atom, which can be over individuals or over functions. An alternative view is to maintain that specific indefinites are always choice functional, and for the case in (18), they can give rise to wide scope readings when the function is 0-ary or to functional narrow scope readings when the function is 1-ary.[7] The difficulty in accepting such proposal is that it would imply that the wide scope reading of specific indefinites is functional, while the wide scope reading of unmarked indefinites is a consequence of atoms encoding scope.

To be precise, we need to add second-order functions to our language, modify our interpretation and assignment functions accordingly, and add relevant semantic clauses to quantifiers over functions.

We illustrate the latter point below for the case of the strict extension, which is the one we adopted in the previous examples for the existential quantifier $\exists f$. Note that we are only adding functions of arbitrary arity over the first sort, individuals.

[6]To interpret dependence atoms over functions, two functions f and f' are equivalent when they have the same domain, codomain, and for every d in the domain, $f(d) = f'(d)$.

[7]In fact, $\exists f(P(f()) \wedge dep(\varnothing, f))$ is equivalent to $\exists x(P(x) \wedge dep(\varnothing, x))$ since 0-ary function variables can equivalently be seen as individual variables in second-order logic.

7.3.1. Definition (Strict Functional Extension (for functions)). Given a model $M = \langle D, W, I \rangle$, a team T and a functional variable f of arity n, the strict functional extension of T with f, $T[f_s/f]$ is defined as follows, where $Dom_f(M)$ contains all the functions $D^n \to D$.

$$T[f_s/f] = \{i[f_s(i)/f] : i \in T\}, \text{ for some strict function } f_s : T \to Dom_f(M)$$

While not directly relevant for our current discussion, which is on the applications of such a system, we offer a few remarks about extending a team-based system with functions in the language. First, since we have not introduced second-order quantification over predicates, we should maintain identity between terms in the language. Second, we noted that the addition of dependence atoms over individual variables leads to a fragment of second-order logic. Similarly, allowing dependence atoms over both individual variables and functional variables leads to a fragment of a higher-order logic (upper bounded by existential third-order logic), as we can now express functional dependencies between functions and variables as well as between functions and other functions.

We end this section with two remarks concerning the two views, pragmatic and functional, outlined in these sections. First, we can consider the case in which specificity is encoded by the dependence atom $dep(v, x)$. The corresponding functional atom would be $dep(v, f)$. Interestingly, we can express a higher-order ignorance reading which the pragmatic view would not immediately account for.

(19) Every man likes a certain woman.
$\forall y(\texttt{man}(y; v) \to \exists_s f(\texttt{woman}(f(y); v) \wedge \texttt{like}(y, f(y); v)) \wedge dep(v, f))$

(19) would allow the function to change from world to world. This could encode that the speaker is not aware of the functional relationship between the set of women and the men (e.g., *his* mother, *his* sister, ...), even though the set of women could possibly be the same across all worlds. This approach shows the flexibility and expressive power of the functional view over the pragmatic one, particularly in handling higher-order dependencies and expressing nuanced interpretations of specificity and ignorance. At the same time, it is worth reflecting if increasing the expressive power of our language is worth doing if the relevant examples are relatively marginal.[8]

[8]Under the pragmatic view, $dep(vy, x)$ can formally capture such higher-order contexts, but it requires maintaining a unique function, encoded by $dep(vy, x)$, as opposed to allowing the function to change. In fact, $dep(vy, x)$ allows for the same set of women and the same man to have different values depending on the value of v. We prefer the representation in (19), which allows the function to differ across assignments, as it more closely represents the underlying idea behind the higher-order reading we are considering.

7.3.3 Pragmatic vs Functional View

In this section, we compare the pragmatic and functional views presented in the previous two sections. The pragmatic view has a clear advantage over the functional view in terms of parsimony, as it avoids introducing higher-order quantification over functions and relates dependence atoms to functional readings. In the functional view, standard dependence atoms are responsible for the different scope readings of an indefinite with respect to other operators, while in the pragmatic view, the scope is determined by the absence of such dependence atoms or by assuming the presence of a relevant function in the context. In what follows, we will consider how the pragmatic view can handle the interaction between indefinites and modality, as well as negation, effectively a type of modality in `2TS`.

In the examples above, we always considered functions determined by individual variables. The question is whether examples involving modal quantification can give rise to similar functional readings. Let us consider the example in (20), where for simplicity we assume maximal information, and we ignore the role of v. Under the pragmatic view, the non-specific reading of sentences like (20) would be represented by (20-a), whereas the functional view would capture it by (20-b). Considering the pragmatic view, is (20-b) a plausible functional parsing (i.e., a function from John's obligations to the set of books)? It does not appear to be so. A ban on world variables would not work, as individual concept functions (i.e., $dep(v, x)$) are in principle possible. In the functional view, this functional reading cannot be generated, as quantification over functions is possible only for functions that have individual variables as arguments.

(20) John must read some book.
  a. $\forall w \exists_s x(\phi(x, v))$
  b. $\forall w \exists_s x(\phi(x, v) \wedge dep(w, x))$

The pragmatic view may indeed argue that functional readings, captured in this view by dependence atoms, are only possible for individual variables, and $dep(\varnothing, x)$ or $dep(v, x)$ are special cases. This would also explain why type (iv) in (17) is difficult to parse. The problem with this argument comes from examples like (21).

(21) John must$_w$ read every$_y$ book that some$_x$ professor recommended.
  a. $\forall w \exists_s x(\phi(x, v) \wedge dep(\varnothing, x))$
  'there is a professor x s.t. for every world w compatible with John's obligations, John read every book y that x recommended.'
  b. $\forall w \exists_s x(\phi(x, v) \wedge dep(w, x))$
  'for every world w compatible with John's obligations, there is a professor x s.t John read every book y that x recommended.'
  c. $\forall w \exists_s x(\phi(x, v) \wedge dep(wy, x))$ 'for every world w compatible with John's obligations, John read every book y s.t. there is a professor x that

recommended y.

The sentence in (21) is associated with the three possible readings in (21-a-b-c). In the functional view, these three readings result from scope and are not associated with any underlying function. The pragmatic view can handle (21-a) and (21-c) by assuming the absence of a dependence atom. However, (21-b), which appears to be a salient reading of (21), remains unaccounted for.[9]

In Section 4.8 of Chapter 4, we discussed how the notion of intensional negation can account for the behaviour of specific indefinites under negation. The treatment presented in that section also extends to the behaviour of specificity markers like English *a certain*, assuming a parallel behaviour of dependence atoms. Moreover, it can account for the interaction of functional readings and negation, as in (22), as we discussed in Section 7.1. Both the pragmatic view in (22-a) and the functional view in (22-b) predict that the sentence in (22) holds for initial teams where every boy does not like his dentist.

(22) Every boy doesn't like a certain doctor. Namely, his dentist.
a. $\forall y \forall w(\exists_x x(\phi(x, v) \wedge dep(y, x)) \rightarrow v \neq w)$
b. $\forall y \forall w(\exists_s f(\phi(f(y), v) \wedge dep(\varnothing, f) \rightarrow v \neq w)$

Finally, we also consider the difference between a plain indefinite like *an exercise* and a 'some' indefinite like *some exercise* in (23). The former allows for a negated existential reading under negation, whereas *some book* is typically taken to be a positive polarity item and only compatible with specific readings under negation.

(23) a. John didn't complete an exercise.
b. John didn't complete some exercise.

Taking negation as a modal allows us to capture the difference, again ignoring for simplicity the role of v, between a specific-like reading as in (24-a) and a negated existential reading as in (24-b). Under this view, the polarity of *some* can be expressed by requiring that it cannot co-vary with respect to the variable introduced by negation, ruling out the parsing in (24-b). The problem with the pragmatic view is that it does not generate dependence atoms for narrow scope configurations like (24-b), and thus it cannot explicitly disallow such configurations. Under this view, one would need to argue that *some* is always interpreted (functionally) as in (24-a), but this would be incompatible with, for instance, narrow scope readings of *some* under universal quantifiers over individuals.

(24) a. $\forall w(\exists_x x(\phi(x, v) \wedge dep(\varnothing, x)) \rightarrow v \neq w)$

[9] Again, the pragmatic view may resort to the argument that such a reading is attested in contexts when there is a salient function from John's obligations to the set of books. It is, however, not immediately clear how this 'intensional' salience can be triggered.

b. $\forall w(\exists_x x(\phi(x, v) \wedge dep(w, x)) \rightarrow v \neq w)$

In conclusion, the pragmatic view is admittedly a parsimonious theory and offers an elegant explanation to functional readings, but it does not fully generalize to the interaction between indefinites and modal operators.

7.4 Classes of Specific Indefinites

In the previous sections, we considered the behaviour of English *a certain* and Russian *koe-* and argued that, at least for cases of maximal information, they displayed a parallel behaviour.

However, the ways in which languages encode specificity can vary. The Table 7.1 below outlines a possible classification, drawing distinctions between the pragmatic view (Section 7.3.1), where dependence atoms signal salient functions in the context, and the functional view (Section 7.3.2), where dependence atoms can range over functions themselves.

Type	Pragmatic View	Functional View	Example
Specific Known	$dep(\varnothing, x)$	$dep(\varnothing, x)$	German *gewiss*
Functional Specific Known	$dep(\vec{y}, x)$	$dep(\varnothing, f)$	Russian *koe-*
Specific	$dep(v, x)$	$dep(v, x)$	Polish *pewien*
Functional Specific	$dep(v\vec{y}, x)$	$dep(v, f)$	English *a certain*

Table 7.1: Flavours of Specificity.

For the Specific Known type, both the pragmatic view and the functional view associate such indefinites with $dep(\varnothing, x)$. This would mean that the indefinite always receives the widest scope and the referent is known. The pragmatic view would need to argue in this case that there is a trivial 0-ary function, or, as we discussed, link the function to the speaker. This type of indefinite can be exemplified by German *gewiss*, which always seems to receive the widest possible scope and the referent is known to the speaker, according to (Ebert, Ebert, and Hinterwimmer 2013).

The Functional Specific Known is a type of indefinite which, besides allowing for wide scope known readings, also allows for functional narrow scope uses, as discussed in examples like (15). In the pragmatic view, this would mean that in the absence of operators or when no (0-ary) function is available in the context, $dep(\vec{y}, x)$ reduces to $dep(\varnothing, x)$. By contrast, in contexts with a salient function, $dep(\vec{y}, x)$ could possibly represent such a function. In the functional view, existentially quantifying over functions would take care of such functional interpretation. When the function is absent in the context, the function would have arity 0, and thus the function variable would be then equivalent to an individual variable.[10]

The Specific type is conceptually similar to the Specific Known type, with the addition that wide scope unknown readings are also allowed. Polish *pewien* appears to display such behaviour in allowing uses where the referent is known by the speaker[11], as well as uses where the referent is not known by the speaker.[12]

Lastly, the Functional Specific also allows for functions that possibly change from world to world, indicating that the speaker is not aware of which functional relationship is relating the indefinite to other variables in the discourse. One possible example was discussed in Section 7.3.2 with English *a certain*, where the underlying function associated with the indefinite is unknown to the speaker. Under the pragmatic view, this could be captured by assuming that the world variable is an argument of the function (e.g., $f(v_1, d_1) \neq f(v_2, d_1)$), where the functional view allows the function to change from assignment to assignment (e.g., $f_1(d_1) \neq f_2(d_1)$), which, even though equivalent to the former, comes closer to the higher-order ignorance reading we are considering.

In the final paragraphs of this section, we consider the case of Finnish, which can signal specificity in various ways.[13] Finnish conveys specificity through three prominent means: (i) the indefinite *eräs*, which functions as both a pronoun and a determiner; (ii) the adjective *tietty* (derived from *tietää* 'to know'); and (iii) the numeral *yksi* 'one', which can also be used non-specifically like the English *one*.

In episodic contexts, *eräs* and *tietty* are typically associated with specific known reading. *Yksi* can be interpreted as both specific known and unknown. Notably, the epistemic indefinite *joku* cannot be used when the referent is known.

(25) a. #Eräs / #tietty / yksi mies / joku soitti sinulle. Mutta en
ERÄS / TIETTY / YKSI man / JOKU called you but not
tiedä kuka se oli.
know who it was
'Someone called you. But I don't know who it was.'

---

[10] Alternatively, one may posit an ambiguity between functional uses where the indefinite introduces a function, and ordinary uses where the indefinite is interpreted as a standard existential quantifier over individuals.

[11] One further salient difference between German *gewiss* and Polish *pewien* is that the former requires identifiability by the speaker, while *pewien* also allows identifiability by other relevant agents. I thank Tomasz Klochowicz for the comments on the Polish data.

[12] For instance, Polish *pewien* is commonly used to translate the common English expression *once upon a time* to indicate a specific, but likely unknown, time point in the past.

[13] I am grateful to Aleksi Antilla for judgments and comments on the Finnish data.

b. Eräs / tietty / yksi mies / #joku soitti sinulle. Arvaa
ERÄS / TIETTY / YKSI / JOKU man called you. guess
kuka (se oli).
who (it was)
'Someone called you. Guess who (it was).'

Importantly, only *yksi* allows for functional specific readings. In cases like (26), *eräs* is excluded, while *tietty* is less preferred or degraded.

(26) Jokainen mies rakastaa #erästa / ?tiettyä / yhtä naista, nimittäin
every man loves ERÄSTA / TIETTYÄ / YHTÄ woman, namely
äitiään.
his.mother
'Every man loves a certain woman, namely his mother.'

This suggests that *eräs* should be classified as a specific known type based on the classifications in Table 7.1, allowing for specific known, but being incompatible with functional readings, unlike English *a certain*.

7.5 Diachrony: Specific Indefinites and Specific Meanings

Markers of specificity, such as the English *certain*, are common across languages. However, *wh*-based marked specific indefinites are relatively rare. A notable observation is that specific known indefinites, like Russian *-koe* or Lithuanian *kai*, exhibit simple morphology. In contrast, other marked indefinites often incorporate specialized particles with more complex meanings.[14] This suggests that plain indefinites might have historically become associated with specific functional contexts, integrating the dependence condition into the indefinite's form. This process would explain the morphological simplicity of specific indefinites, arising from the contextual specialization of unmarked indefinites. Additionally, in the pragmatic framework outlined earlier, dependence atoms incur a pragmatic cost, potentially leading to the fossilization of these atoms within a dedicated form.

In Section 4.9 of Chapter 4 we noted that some diachronic patterns can be explained by semantic weakening in terms of logical entailment. Specifically, the $dep(\varnothing, x)$ atom for specific known indefinites uses entails the $dep(v, x)$ atom for specific indefinites. For marked indefinites, no trace of this change was found. It is likely that some weakening occurred for expressions like *a certain*, whose original meaning was arguably associated with full knowledge, but no available

[14] For example, Haspelmath (1997) notes that the Russian *-koe* is the neuter form of the interrogative *koj* ('which'). Similarly, the Lithuanian *kai* translates to 'when'.

data seems to support this claim.[15]

As discussed in Section 4.9 of Chapter 4, another perspective on possible constraints in language change is that the representation of *known* versus *unknown* requires variables ranging over a domain of *abstract* entities. The emergence of abstract concepts typically occurs later in grammaticalization processes compared to concrete ones (Traugott and Dasher 2002; Heine 1997). While in a language without world variables, the contrast between $dep(\varnothing, x)$ and $var(\varnothing, x)$ accounts for the difference between specific and non-specific, the use of world variables is necessary to express also the known vs unknown distinction (with now $var(\varnothing, x)$ vs $dep(\varnothing, x)$ standing for unknown vs known and $var(v, x)$ and $dep(v, x)$ standing for specific and non-specific). Assuming thus that individual quantification precedes world quantification leads to the following two predictions:

1. Non-specific $var(v, x) >$ Epistemic $var(\varnothing, x)$;
2. Specific $dep(v, x) >$ Specific known $dep(\varnothing, x)$.

As mentioned in in Section 4.9 of Chapter 4, the transition from non-specific to epistemic can be explained by both weakening and concreteness factors, which support the commonality of this diachronic path. Furthermore, the absence of a change from specific known to specific as a form of weakening can be explained by its conflict with the second constraint on concreteness.

Cross-linguistically, specific known indefinites are attested and examined in the literature, but specific indefinites are rare or at least it is more difficult to categorize an indefinite as truly specific as opposed to specific known.[16] We offer some remarks on the possible reasons behind this infrequent occurrence. First, epistemic indefinites, which allow for specific unknown uses in episodic contexts, are very common cross-linguistically, even in languages with dedicated non-specific indefinites. This implies there is no need to develop another form covering the specific-unknown function if another indefinite already covers it.

Second, an indefinite with only known uses can only be used in contexts where the speaker is maximally informed, which do not license specific unknown uses. One possibility is that some specific known indefinites allowed for a shift in the relevant agent responsible for knowledge, leading to the emergence of 'specific' uses, even though this means that those indefinites are still 'known', albeit by a different agent.[17]

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[15]Even though Latin lacked articles, Latin *certus* was already associated with specific unknown meanings.

[16]While Haspelmath (1997) claims that some of them are attested, it was not possible to confirm his data. For instance, Haspelmath (1997) observes that Georgian *-ghats* is a specific indefinite. This could have been an interesting case study, as *-ghats* derives from the particles *-gha* ('only') and *-c(a)* ('also'). However, Georgian *-ghats* does not allow for specific known readings and can be used in non-specific contexts. Thus, it must be classified as an epistemic indefinite. I am grateful to Nino Amiridze, Zurab Baratashvili and Keti Chilaia for these observations.

[17]This could be related to the distinction between German *gewiss* tied to the speaker and

Lastly, we consider a class of indefinites connected with the specific indefinites discussed here: those derived from the numeral *one*. Givón (1981) observed that many languages use the numeral *one* as an indefinite and proposed a three-stage diachronic path: (i) quantification (numeral one) > (ii) referentiality/denotation > (iii) genericity/connotation.

Stage (ii) roughly corresponds to scopal specificity, while (iii) corresponds to scopal non-specificity, with further subdivisions for genericity not considered here. Heine (1997) further divided stage (ii) into sub-stages, where the case in which the identity of the referent is known to the speaker precedes the stage where the identity is not known. This aligns with the atomic weakening proposed here. However, specific unknown readings could be licensed even by the stage where *one* expresses only a quantitative claim.

A key question is how the development from (ii) to (iii) occurs, where a specific indefinite also acquires non-specific uses. One possible trajectory concerns the functional uses discussed in this chapter. We could conjecture that one-related indefinites began to be used in environments that license functional readings. Over time, this functionality might have been neutralized, allowing non-specific uses in contexts like attitudes.

7.6 Conclusion

In this chapter, we began by revisiting choice functional analyses of indefinites, highlighting their similarities and differences with the dependence logic-based account. We then examined the functional readings of specific indefinites. In light of this discussion, we revised our original characterization of specific indefinites. We presented two possible accounts: the pragmatic view and the functional view. The pragmatic view proposes that dependence atoms are not mandatory but are regulated by pragmatic factors, while the functional view distinguishes between existential quantification over individuals and functions. We compared these views, emphasizing their strengths and weaknesses in handling various linguistic phenomena. Additionally, we proposed different classes of indefinites that encode specific meanings, maintaining a broad perspective for cross-linguistic generalization. We concluded with diachronic remarks on specific indefinites and their interaction with other types of indefinites.

The generalization of different types of specific indefinites mentioned in Section 7.4 deserves further scrutiny. The empirical picture should be more thoroughly substantiated, and the experimental research mentioned in this chapter should be extended to different types of specific indefinites.

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bestimmt whose knowledge can also be tied to other agents, as discussed in Ebert, Ebert, and Hinterwimmer (2013).

Chapter 8

Free Choice Indefinites

> ...perché **qualunque volta** sia presente l'effetto, necessariamente vi è anco quella causa.
>
> '...because **any time** the effect is present, that cause is necessarily also there.'
>
> GALILEO GALILEI, *Il Saggiatore*

The topic of free choice (FC) has been a pivotal theme in formal semantics, starting from the work of Kamp (1973) and von Wright (1968) on free choice disjunction, which gave rise to an influential research agenda (among others, Zimmerman 2000; Fox 2007; Goldstein 2019; Bar-Lev and Fox 2020; Aloni 2022).

In (1-a) a disjunction embedded under a deontic modal gives rise to a so-called free choice permission, where the speaker is giving freedom of choice to the hearer's choosing between an apple and a pear. A similar effect occurs for indefinites: (1-b) with the plain indefinite 'a fruit' can be taken as an invitation to take any fruit.

(1) a. You can eat an apple or a pear.
   $\rightsquigarrow$ You can eat an apple and you can eat a pear.
  b. You can eat a fruit.
   $\rightsquigarrow$ You can eat an apple, and you can eat a pear, and ....

Importantly, FC inferences of plain indefinites are cancellable, as shown by the bracketed continuation in (2-a).

(2) a. You can take a book from the bookcase. (Do not take *Wuthering Heights*.)
  b. FC pragmatic inference: *Every book from the bookcase is a possible option.*

Languages have developed lexicalized indefinite forms, which go by the name of FC indefinites, to explicitly encode such FC enriched meanings (e.g., Italian *qualunque*, Dutch *wie dan ook*, Japanese *daredemo*, Hebrew *kol*, ...). A canonical example is the English determiner *any* in (3). Unlike the plain indefinite in (2), the infelicity of the continuation in (3-a) suggests the FC inference is part of the conventional meaning of *any*.

(3) a. You can take any book from the bookcase. (#Do not take *Wuthering Heights*.)
b. Conventional meaning: You can take a book from the bookcase and every option is a permitted one.

In Section 8.1 we will outline the core distribution of FC indefinites and introduce the total variation atom which accounts for the meaning and distribution of FC indefinites. In Section 8.2, we summarize the grammaticalization path of three types of FC indefinites and in Section 8.3, we illustrate how our formal account can shed some light on some aspect of grammaticalization processes. In Section 8.4, we examine the relationship between universal quantifiers and FC indefinites.

8.1 Meaning and Licensing

We outline the core distribution of FC indefinites in Section 8.1.1, and in Section 8.1.2 we propose that FC indefinites associate with a total variation condition, accounting for their distribution. In Section 8.1.3, we offer some remarks on the distinction between existential and universal quantifiers.

A methodological remark is in order. FC indefinites can occur in many environments (e.g., comparatives, conditionals, imperatives, questions, ...) which would require an extensive analysis on their own. The guiding questions of this section follow the dominating theme of this dissertation: which conditions on variable assignments do FC indefinites impose, and to what extent does this account for their meaning and distribution?

8.1.1 Distribution

The distribution of free choice indefinites has been extensively discussed in the literature (Chierchia 2013; Dayal 1998, 2004; Menéndez-Benito 2010). A common observation is their unavailability in episodic environments. This occurs in plain episodic statements like (4-a), as well as under an operator like a universal quantifier in (4-b).[1]

[1] Importantly, FC indefinites are allowed in episodic contexts in so-called subtrigging configurations (Dayal 1998; Aloni 2007). For instance, in (i) the relative clause ‘that was recommended to him’ is redeeming the FC indefinite ‘any book’ from infelicity. The resulting reading is a universal one, where John bought all the books that were recommended to him.

(4) a. #John bought any book yesterday.
b. #Everyone bought any book yesterday.

FC indefinites show a distinctive behaviour under modals. They are licensed under possibility modals, like (5-a), which gives John freedom of choice with respect to which book he can buy. The behaviour of FC indefinites and universal modals like in (5-b) is more complex. While some analyses predict FC indefinites to be ungrammatical in such environments (Chierchia 2013), other accounts highlight how the empirical picture is after all not so clear (Menéndez-Benito 2010; Giannakidou 2001).[2]

(5) a. John is allowed to solve any exercise.
b. ?John must solve any exercise.

The sentence in (5-b) admits only a strong reading according to which John must solve all the exercises, and not the weak one typically associated with FC. We are thus distinguishing between the contrasts in (6-a) and (6-b). In the weak reading, we are giving freedom of choice with respect to which exercise John must solve (to pass the assignment). In the weak reading, we are requiring that John must solve all the exercises (to obtain full marks). Under the assumption that (5-b) is judged as felicitous, the only available reading seems to be strong one in (6-a).[3]

(6) John must solve any exercise.
a. Strong reading: John must solve all the exercises.
b. Weak reading: John must solve an exercise and every option is a permitted one.

Importantly, under imperatives, the difference between strong and weak reading appears to be more prominent (Giannakidou 2001). (7-a), displayed on a screen monitor, is not an invitation to push all keys while (7-b) appears, by contrast, to be an order to confiscate all the guns.

(7) a. Push any key.
b. Confiscate any gun.

One interesting construction which is worth considering is so-called supplemental

(i) John bought any book that was recommended to him.

[2]Note also that in generic readings, FC indefinites are fully compatible with universal modals:

(i) Any student must be diligent.

[3]The reason for the availability of the strong reading could be attributed to covert subtrigging (i.e., 'any exercise there is'). We do not necessarily commit to such analysis, and we simply highlight that the strong reading is available.

any (Dayal 2004; Giannakidou 2001; Jennings 1994), where *any* occurs as a supplement (or appositive), of a plain indefinite in the main clause. Notably, *any* is licensed in the appositive, even if the modal in the main clause is universal.

(8) You must solve an exercise, any exercise.

One further case in which the behaviour of FC indefinites under universal modals is different concerns the distinction between universal FC indefinites like English *any* or Italian *qualunque* and existential FC indefinites like German *irgendein* or Italian *uno qualunque* (Chierchia 2013). As discussed above, universal modals do not license universal FC indefinites, but they are compatible with existential FC indefinites. We have already discussed several examples for *irgendein* in Chapter 5. Example (9) illustrates this point for Italian *uno qualunque* which is formed by the universal FC counterpart *qualunque* and the article/numeral *uno* 'a/one'.

(9) Giovanni deve risolvere un qualunque esercizio.
Giovanni must solve UN QUALUNQUE exercise.
'Giovanni must solve an exercise. Any exercise would do.'

Finally, we comment on the relationship between FC indefinites and negative polarity items. It is well-known that English *any* displays both a FC and a NPI behaviour, as in (10).

(10) John didn't eat any biscuit.

However, cross-linguistically, this is not necessary the case. In contrast, based on the survey conducted by Haspelmath (1997), it appears that in general indefinites with dedicated FC uses are not allowed under sentential negation. (11) illustrates this for Italian, where in place of FC indefinite a neg-word like *nessuno* 'no' or a strict NPI like *alcuno* is preferred.[4]

(11) #Giovanni non ha mangiato qualsiasi biscotto.
Giovanni not has eaten QUALSIASI biscotto.
Intended: 'Giovanni didn't eat any biscuit.'

[4] However, in indirect negation NPI-like uses appear to be possible in Italian:

(i) Dubito che Luca abbia qualsiasi cosa a che fare con la vicenda.
I-doubt that Luca has QUALSIASI thing to that do with the matter
'I doubt Luca has anything to do with the matter.'

Moreover, prosodic prominence on the indefinite can redeem Italian FC indefinites under negation, giving rise to a negated existential reading.

(ii) Luca non ha QUALSIASI COSA a che fare con la vicenda.
Luca not has QUALSIASI thing to that do with the matter.
'Luca has nothing to do with the matter.'

At the end of the next section, we will return to this issue and discuss our predictions with respect to negation.

8.1.2 Total Variation

In Chapter 5, we introduced the notion of generalized variation atom $VAR_n(\vec{z}, \vec{u})$, which we report again below in Definition 8.1.1. Roughly, Definition 8.1.1 says that fixing a value for $\vec{z}$, we can find at least n values for $\vec{u}$.[5]

8.1.1. Definition. Generalized Variation

$$M, T \models VAR_n(\vec{z}, \vec{u}) \Leftrightarrow \textit{ for all } i \in T : |\{j(\vec{u}) : j \in T \textit{ and } i(\vec{z}) = j(\vec{z})\}| \geq n$$

In particular, we argued that the meaning associated with FC corresponds to $VAR_{|D|}(v, x)$ in (12). The latter says that for any given value v_i of v, x must receive all possible values of D in the restriction of team with respect to v_i. An equivalent formulation is that for all $u \in T(v)$, we must have that $T_{v=u}(x) = D$.[6]

(12) Total Variation Condition

$$M, T \models VAR_{|D|}(v, x) \quad \Leftrightarrow \quad \text{for all } i \in T : |\{j(x) : j \in T \text{ and } i(v) = j(v)\}| = |D|$$

To illustrate this, consider the case in (13), where the modal introduces a lax functional extension and the FC indefinite a strict functional extension together with the $VAR_{|D|}(v, x)$ requirement, as illustrated in Table 8.1.[7] The logical rendering in (13) requires that in each epistemic possibility of the speaker, the variable x introduced by the indefinite ranges over all possible values, in line with the intended meaning of FC indefinites.

(13) a. John can take anything.
b. $\exists_l w \exists_s x(R(v, w) \wedge \phi(x, w) \wedge VAR_{|D|}(v, x))$

---

[5] We note that the generalized variation atom in Theorem 8.1.1 is downward closed, unlike the variation atom. Moreover, it can be proved that the logic resulting from adding the generalized variation atom is equivalent to inclusion logic (Galliani 2012b).

[6] Note that since we are taking n to be to $|D|$, the requirement $\geq |D|$ really corresponds to $= |D|$. Relatedly, there is a connection between this the generalized variation atom in Definition 8.1.2 and the Totality defined below, which says that any combination of values for $u_1, \ldots, u_n$ is possible.

8.1.2. Definition. Totality Atom

$$M, T \models All(u_1, \ldots, u_n) \Leftrightarrow T(\vec{u}) = D^n$$

Clearly, if a team satisfies $VAR_n(\vec{z}, u)$, it also satisfies $All(u)$, but not vice versa.

[7] To illustrate the behaviour of $VAR_{|D|}(v, x)$ we are considering an initial team with two possible values for v in Table 8.1. However, sentences like (13) are typically uttered in a context of maximal information, where v only receives one value.

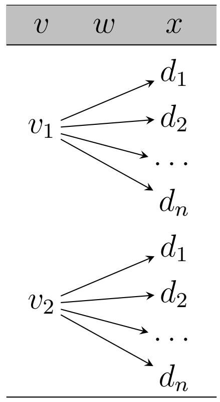


Table 8.1: Illustration of $VAR_{|D|}(v, x)$. Fixing a value for v, x must receive all possible values in the domain D.

The FC condition readily accounts for the distribution of FC indefinites. We start by considering their infelicity in episodic context in (14):

(14) a. #John took any book.
b. $\exists_s x(\phi(x, w) \wedge VAR_{|D|}(v, x))$

The sentence in (14-b) is predicted infelicitous, as there is no initial team which can satisfy (14), since indefinites are strict existential and do not allow for the kind of branching extension needed to satisfy $VAR_{|D|}(v, x)$. The prediction is parallel to non-specific indefinites, which indeed share a similar variation condition. By contrast, an intervening modal can generate the relevant branching extension which makes possible to satisfy $VAR_{|D|}(v, x)$, as discussed above.

In Section 8.1.1, we have observed that FC indefinites are not licensed by nominal universal quantifiers. We give a schematic representation in (15-a). As it stands, the logical rendering in (15-b) predicts that (15-a) is felicitous with the reading that each student took a book each and each book was taken by a student.

To account for the apparent ungrammaticality of (15-a), we propose that the variation condition also comes with a requirement of the form $VAR_{|D|}(v\vec{y}, x)$, where $\vec{y}$ is a possible empty sequence of all the non-world variables introduced in the discourse. This allows us to explain the ungrammaticality of (15-a), as the additional requirement of $VAR_{|D|}(vy, x)$ makes (15-c) satisfiable only when the branching occurs within a single value for y.

(15) a. Every student took any book.
b. $\forall y \exists_s x(\phi(x, v) \wedge VAR_{|D|}(v, x))$
c. $\forall y \exists_s x(\phi(x, v) \wedge VAR_{|D|}(vy, x))$

We have already seen that other types of indefinite are sensitive to the kind of operator which licenses them. In particular, we discussed in Chapter 6 how dependent indefinites cannot be licensed by modals, and we have implemented this in our informational dependence atom. Moreover, this restriction correctly predicts the behaviour of free choice in other environments. Let us consider a case where a modal interacts with a universal quantifier. The intended reading for (16-a) is that for each student x, x can take any book. This is indeed the prediction that (16-b) gives us.[8]

(16) a. Every student can take any book.
b. $\forall y \exists_l w \exists_s x(R(v,w) \wedge \phi(x,w) \wedge VAR_{|D|}(vy,x))$

Finally, in Section 8.1.1, we observed that English *any* is not only a FC indefinite, but it also functions as a NPI under negation. In other languages, there is a clear demarcation between FC indefinites and NPIs. The prediction of the current system is that, given our notion of intensional negation, $VAR_{|D|}(v,x)$ for a case like (17) will result in a NPI reading. In particular, assuming that $W = \{v_{ab}, v_a, v_b, v_\varnothing\}$ is constructed over the property P with respect to two individuals in the domain, we have that $\neg\exists_s x(P(x,v) \wedge VAR_{|D|}(v,x))$ will be supported by a relevant model M only for initial teams T s.t. $T(v) = \{v_\varnothing\}$.

Importantly, given our notion of maximal implication in the definition of intensional negation, the strength of the variation atom in obtaining NPI readings does not matter:

8.1.3. FACT (Free Choice and Negation). For any positive dependence-free formula ϕ.

$$\neg\exists_s x(\phi \wedge VAR_{|D|}(v,x)) \equiv_v \neg\exists_s x(\phi \wedge VAR_2(v,x))$$

These observations point to the following remarks. First, we can explain the common denominator between FC and negative polarity in the case of *any*. Second, given Fact 8.1.3, we conjecture that languages might prefer atoms of simpler complexity as NPI, since the additional meaning of total variation is effectively vacuous under negation. This would account for languages which distinguish between FC indefinites and indefinites with a negative polarity use which do necessarily carry total variation.

8.1.3 Universal and Existential Free Choice

An undesirable prediction of the current system is that it does not distinguish between universal FC indefinites, like Italian *qualunque*, and existential FC indefinites, like German *irgendein* or Italian *uno qualunque*, which we discussed in Chapter 5 and in Section 8.1.1. Both of them are associated with the variation condition $VAR_{|D|}(v,x)$).

[8] Arguably, here $VAR_{|D|}(vy,x)$ does not operate at the level of the domain of the model, but rather on the set of books, assuming the denotation of the latter is constant across worlds.

Yet we saw that their distribution is different under universal modals. While existential FC indefinites give rise to a free choice reading under universal modals, the behaviour of universal FC indefinites is different: if licensed, they can only receive a strong reading.

(17) a. John must take QUALUNQUE book
$\rightsquigarrow$ John must take all the books.
b. John must take UNO QUALUNQUE book
$\rightsquigarrow$ John must take a book. Any book would do.

In light of this difference, we propose an amendment to our original account: universal FC indefinites are lax existential quantifiers, whereas existential FC indefinites are strict existential quantifiers. This predicts that universal FC indefinites are compatible with both readings, while existential FC only with weak readings. We give some illustration for (18) in Table 8.2.

(18) a. John must take QUALUNQUE book.
$\forall w(R(w,v) \rightarrow \exists_l x(B(x;v) \wedge T(j,x;v) \wedge VAR_{|D|}(v,x)))$
b. John must take UNO QUALUNQUE book.
$\forall w(R(w,v) \rightarrow \exists_s x(B(x;v) \wedge T(j,x;v) \wedge VAR_{|D|}(v,x)))$

v	w	x
	w_1	d_1
	w_2	d_2
v_1	$\dots$	$\dots$
	w_n	d_n

(a)

v	w	x
		d_1
		d_2
v_1	w_1	$\dots$
		d_n
	w_2	$\dots$

(b)

Table 8.2: Illustration of Weak and Strong Readings for (18). The team in (a) represents a weak (free choice) reading while (b) a strong (universal) reading.

Allowing universal FC indefinites to be lax existentials, rather than strict ones, has however some consequences. First, as we said, universal FC indefinites now are not only compatible with strong (universal) readings, but also with weak (free choice) ones. Moreover, in episodic contexts, we predict that universal FC indefinites are no longer infelicitous, but they generate strong universal readings, which is what we observe, as discussed, when subtrigging rescues the indefinite in episodic contexts.

The distinction between lax and strict also allows capturing cases of supplemental *any*, as in (19). Following the analysis of appositives discussed in Section 6.3 of Chapter 6, we propose that the contribution of the appositive is the total variation condition. Since the existential in the main clause is a strict one, this allows for weak free choice readings.

(19) a. You must solve an exercise, any exercise.
b. $\langle \forall w \exists_s x \phi(x,w), VAR_{|D|}(v,x) \rangle$

Finally, we discuss the behaviour of existential FC indefinites in combination with numerals, like in (20).

(20) a. John can take any two books.
b. $\rightsquigarrow$ John can take a set of two books, and every combination is a permitted one.

While universal FC indefinites were analysed as lax quantifiers, plural existential FC indefinites still involve strict quantification, but with reference to a plural domain. The striking contrast also appears in the fact that universal FC indefinites do not exhibit plural morphology (e.g., the Italian *qualsiasi libro* 'any book'), while existential FC indefinites do (e.g., the Italian *due qualsiasi libri*, lit. 'two any books'). This in line with the treatment of plurality and epistemic indefinites discussed in Chapter 5. We thus analyse cases as in (21), where total variation condition has been generalized to deal with a plural domain in (22).

(21) John can take any two books.
$\exists w \exists_s x (R(v,w) \wedge B(x;v) \wedge |x| = 2 \wedge T(j,x;v) \wedge VAR_{|D_2|}(v,x))$

(22) TOTAL VARIATION CONDITION OVER PLURALS
$M,T \models VAR_{|D_n|}(v,x) \Leftrightarrow$ *for all* $i \in T : |\{j(x) : j \in T$ *and* $i(v) = j(v)\}| = |D_n|$ *with* $D_n = \{d \in D : |d| = n\}$

8.2 Grammaticalization of Free Choice Indefinites

While several works have been dedicated to formal analyses of FC indefinites and others to diachronic analyses of such indefinites, little attention has been paid to the relationship between formal analyses and diachronic findings. The issue is particularly relevant for FC indefinites, as grammaticalization processes of novel FC indefinites tend to happen rather frequently, making it possible to study their developmental path. In Section 8.2.1 we present three main types of FC indefinites based on the classificatory remarks in Haspelmath (1997) and each of the following sections deals with one of these. At the end, we will conclude with some remarks concerning these diachronic findings, setting the stage for the formal diachronic semantics analysis in the section that follows.

8.2.1 Types of Free Choice Indefinites

The grammaticalization process of FC indefinites has received a significant attention in the linguistic literature (Haspelmath 1997; de Vos 2010; Pescarini 2010; Company Company and Loyo 2006; Degano and Aloni 2021; Halm 2021). In an influential work on indefinites, Haspelmath (1997) proposed different trajectory pathways for the functional development of indefinites. With regard to FC indefinites, his generalizations lead to a distinction between three main types of *wh*-based FC indefinites, summarized in Table 8.3 together with their hypothesized grammaticalization.[9]

Type	Construction	Origin	Examples
Type I: The 'it may be' type	wh + (subjunctive mood of) to be	Parametric concessive conditional clause	French *qui que ce soit* (who that may be), Croatian *ko bilo* (who it be), Dutch *wie dan ook* (who even also), Italian *qualsiasi* (who it may be)
Type II: The 'want/pleases' type	wh + (subjunctive mood of) + to want / it pleases or similar construction	Non-specific free relative clause	Latin *quivis* (who you want), Spanish *cualquier(a)* (who it may want), Italian *qualsivoglia* (who it may want)
Type III: The 'no matter' type	wh + expression with no matter' meaning	Weak grammaticalization from original expression	French *n'importe qui* (it does not matter who), Dutch *onverschilling wie* (indifferent who)

Table 8.3: Main FC indefinites based on Haspelmath (1997).

The generalizations established in Table 8.3 call for further scrutiny for two reasons. First, Haspelmath (1997) was able to reach the subdivisions in Table 8.3 due to the relatively syntactic-morphological transparency of the FC indefinites he considered. The latter means that their origin can be easily inferred from the form of the indefinite (see 'Example' column in the table). Empirical studies reconstructing the development of such items are therefore in order.

Second, we might be interested in understanding if the different types and trajectory patterns outlined in Table 8.3 are related to different distributions and uses of such indefinites. Considering the stable distribution of FC indefinites cross-linguistically, the answer appears to be negative. However, different types of FC indefinites might end up having the same distribution for independent reasons or their diachronic development could simply not be related to their distribution.

[9]The generalizations of Table 8.3 do not concern the case of non-*wh*-based FC indefinites, like *any*.

In the forthcoming subsections, we will delve into the grammaticalization process of one item from each type listed in Table 8.3, and discuss related diachronic studies.

8.2.2 Type I

We expect that this kind of FC indefinites originates from concessive or unconditional constructions headed by the corresponding *wh*-phrase. Example (23) is an illustration. We will refer to 'what(ever) the reason is' as the adjunct of the unconditional, while to 'I will be happy' as the main clause of the unconditional.

(23) *What(ever) the reason is*, I will be happy.

Degano and Aloni (2021) reconstructed the diachronic path of Italian *qualsiasi*, showing that the early uses involved a concessive construction. In particular, the diachronic development of *qualsiasi* aligned in many respects with one of the Dutch FC indefinite *wie dan ook*, also a Type II indefinite, studied by de Vos (2010) and Aguilar-Guevara et al. (2011). We therefore give some simplified examples for the case of Dutch.

The first stage corresponds to an unconditional construction like (24). The unconditional is headed by a *wh*-element, typically in combination with other elements (e.g., the particles *dan* 'then' *ook* 'also' in the case of *wie dan ook*), which will then be part of the grammaticalized indefinite. As outlined in Table 8.3, this kind of indefinites typically lexicalizes also the verb *to be* in the main clause of the unconditional. However, this did not occur for Dutch *wie dan ook*.

(24) *Concessive Conditional Clause*
Wie dan ook the reader of this paper is, the author will be happy.
Whoever reads this paper, the author will be happy.

In the second stage, the *wh*-element and related material appear as an appositive, often marked by two commas. In particular, in the diachronic data concerning the development of such items, there appears to be two salient appositive constructions. The first, which we call 'referential' appositive, corresponds to a structure like (25) where the anchor of the appositive is a 'referential expression' (e.g., a proper name, a definite, . . .). The resulting interpretation corresponds to what we call 'ignorance' reading.

(25) John, *wie dan ook*, passed the exam.
<u>Ignorance:</u> John passed the exam and the speaker does not know who John is.

The second, which we call 'non-referential' appositive, corresponds to a structure like (26), where the anchor is a non-referential expression (e.g., a plain indefinite, a plural, a generic-like expression, . . .). A modal is most often present in the main clause. The resulting interpretation corresponds to what we call 'free choice' reading. Ignorance readings should also be available if the indefinite is interpreted specifically.

(26) A student, *wie dan ook*, can pass the exam.
Free Choice: Any student can pass the exam.

It is important to note that while it appears that these appositives were differentiated with respect to ignorance and free choice readings, the distinction in terms of anchors between (25) and (26) is not easily discernible in the data. Such division was mainly chosen because of the parallelism which we can observe with similar constructions with English *whoever* in (27) and (28).

When the main clause is episodic, as in (27), the only available reading is an 'ignorance' one for both referential and non-referential expressions. Clearly, the indefinite *a student* in (27-b) is interpreted specifically in episodic contexts.

(27) a. John, whoever he is, passed the exam.
b. A student, whoever he is, passed the exam.

By contrast, when the main clause contains a modal, a referential appositive as in (28-a) is always interpreted with an ignorance reading, while a non-referential one as in (28-b) is ambiguous between an ignorance reading, when *a student* is interpreted specifically, and a free choice reading, the latter being the most salient reading.

(28) a. John, whoever he is, can pass the exam.
b. A student, whoever he is, can pass the exam.

Importantly, English *whoever* is already a grammaticalized FC indefinite, and it cannot occur by itself in the appositive. As said, the examples in (25) and (26) are schematic. In these intermediate stages, the appositive often undergoes some form of syntactic simplification. For instance, the change from a full unconditional structure to a reduced one [*whoever he is* > *whoever*] with only the material which will be part of the final form of the indefinite.

Finally, the item is integrated into the main clause as a full-fledged determiner or pronoun:

(29) *Wie dan ook* can pass the exam.
Free Choice: Anyone can pass the exam.

8.2.3 Type II

In this section, we will focus on the 'want/pleases' type II of FC indefinites. The original hypothesis of Haspelmath (1997) is that indefinites of this type originate from so-called non-specific free relative clauses, like the one in (30):

(30) You may take *what(ever) you want to take.*

A sentence like (30) has a clear universal reading which resembles very closely the meaning of FC indefinites (e.g. 'You may take anything'). It is not so surprising, as Haspelmath (1997, ch. 3.3.3) notes, that languages which lack marked FC indefinites usually resort to constructions similar to (30) to express free choice.

We will focus on two particular indefinites of the 'want/pleases' type: the Spanish *cualquier(a)* in Section (30) and the Italian *qualsivoglia* in Section (31).

The Spanish *cualqueir(a)*

The Spanish *cualquier(a)* is composed by the *wh*-item *cual* 'which/who' together with *quier(a)*, the 3rd singular person of the verb 'to want'. According to Company Company (2016), the grammaticalization path of the indefinite *cualquier(a)* starts from a non-specific free relative as in (31-a), followed by an intermediate phrasal compound constructions in (31-b), leading to the formation of a FC indefinite as in (31-c).

(31) a. Haga en 'el cual castigo quiera.
Do on him which punishment want-3.PRES.SUBJ
b. Haga en 'el cual quiera castigo.
Do on him which want-3.PRES.SUBJ punishment.
c. Haga en 'el cualquier(a) castigo.
Do on him any punishment.

The data related to Spanish *cualquier(a)* in Company Company (2009) and Company Company (2016) appears to be based on sparse examples. In this regard, the diachronic corpus study by Aguilar-Guevara et al. (2011) tried to study the development of *cualquier(a)*, but the data in the corpora they considered displayed the final form in (31-c) since the first available examples.

As it stands, the pattern in (31) aligns with the hypothesis proposed by Haspelmath (1997). In the next section, we will now consider another indefinite of the 'want/pleases' type: the Italian *qualsivoglia*.[10]

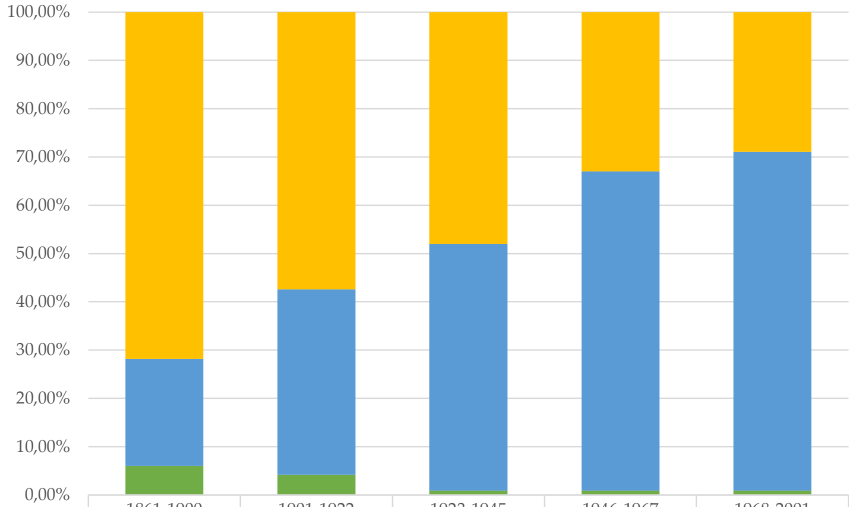
The Italian *qualsivoglia*

Qualsivoglia is formed by the *wh*-interrogative pronoun *qual(e)* ('who' / 'which'), together with the expression *si voglia*, the impersonal form of the 3rd singular person of the verb *to want* in subjunctive mood.

It should be noted that in current Italian, *qualsivoglia* is rarely used, and such indefinite has been very peripheral in the past two centuries. The chart in Figure 8.1 shows that, at least a century ago, *qualsivoglia* was more common than today,

[10] Data related to the following section can be found at the following repository: `https://osf.io/8kjna/`.

even though still considerably less frequent than *qualunque* and *qualsiasi*, which later became the most widespread Italian indefinite.



	1861-1900	1901-1922	1923-1945	1946-1967	1968-2001
Qualunque	71,83%	57,39%	48,01%	32,97%	28,94%
Qualsiasi	22,16%	38,45%	51,07%	66,11%	70,16%
Qualsivoglia	6,02%	4,15%	0,93%	0,92%	0,90%

Figure 8.1: Diachronic relative frequency of *qualunque, qualsiasi, qualsivoglia* based on DiaCORIS.

To study the diachronic development of Italian *qualsivoglia*, we consulted the Italian historical dictionary by Battaglia and Barberi Squarotti (2002) the text corpus OVI, which contains Italian texts written before the 1400 (a total of 3000 texts for 30 million occurrences). We did not only search for the occurrences of *qualsivoglia*, but also the free form *qual si voglia* and the second singular person *qual ti vogli(a)*, which would be more in line with a grammaticalization emerging from a non-specific free relative.

The OVI corpus displays some occurrences of the form *qualsivoglia* since the first early data. In particular, in (32) *qualsivoglia* occurs in a comparative construction in combination with *altra* 'other', which clearly shows the fully grammaticalized indefinite status of such item.

(32) E certo è viepeggiore gloria gloriarsi, e reputarsi, e credere di essere spirituale, che **qualsivoglia** altra vanagloria corporale.
'And certainly it is a worse arrogance to boast, and to consider oneself, and to believe that one is spiritual, than **any** other physical vainglory.'
(Domenico Cavalca, *Disciplina degli spirituali*, 1341)

The early presence of these examples does not align with the observations made in the historical dictionary by Battaglia and Barberi Squarotti (2002), which posits that *qualsivoglia* originated in the 15th century. One issue regarding the data we considered is that examples with the whole form *qualsivoglia* are not from the original source, but texts from the 18th century which report the original source, which may lead to an incorrect transcription. Furthermore, we observe that later forms that were found in the original texts of the 15th century still displayed plural inflection (i.e., *qualsivogliano*), suggesting that the indefinite was not fully grammaticalized at that stage. If that is the case, looking at early data might still be instructive.

A common construction is the combination of *qual* with the verb *to want* and the subjective form of the verb *to be* in concessive clauses:

(33) e per questo si comprende, **quale voglia essere** il nostro operare, se piacere vogliamo a Dio
and for this CL understand, which want be the our doing, if like want to God
'and for this we understand, whatever our behaviour is, if we want to be liked by God.
(Agnolo Torini, *Brieve collezzione della miseria della umana*, 1374)

In some cases, there were ambiguous usages between non-specific free relative and concessive structures:

(34) ella passa di bellezza t[utte] l'altre dame; e **sia qual si voglia**.
and she surpass of beauty every the-other ladies; and be who CL wants.
(*La Tavola Ritonda o l'Istoria di Tristano*), <1400)

a. and she surpasses the beauty of all the other ladies, and let be the others who(ever) you want.
b. and she surpasses the beauty of all the other ladies, whoever they are.

The example in (34) shows a context where *qual si voglia* can be interpreted as belonging to a non-specific free relative clause (34-a) or as the subject of a concessive conditional sentence (34-b), where *qual si voglia* behaves more closely to other grammaticalized FC indefinites. It might be thus possible that this kind of constructions facilitated the grammaticalization of *qualsivoglia* as an indefinite.

Based on our discussion, we hypothesize that the pattern emerging from the alleged free relative construction is the following:

(35)

Chiedi	qual	dono	si	voglia.
Ask	which	gift	CLITIC	wants-3.PRES.SUBJ

(36)

Chiedi	un	dono,	sia	qual	si	voglia.
Ask	a	gift,	it-be-3.PRES.SUBJ	which	CLITIC	wants-3.PRES.SUBJ

(37) Chiedi un dono, qual si voglia.
Ask a gift, which CLITIC wants-3.PRES.SUBJ

(38) Chiedi qual si voglia dono.
Ask any gift.

(39) Chiedi qualsivoglia dono.
Ask any gift.

We start with a non-specific free relative clause in (35). The latter is then embedded in a concessive construction in (36), which acts ambiguously as described in (34). The construction *sia qual si voglia* and variants thereof become quite standard and *sia* is eventually dropped. We conjecture an appositive phase in line with the development of Type I indefinites.[11] The resulting apposition is then reinterpreted at a nominal level. Finally, the indefinite *qual si voglia* undergoes a morphological process of compounding, typical of grammaticalization phenomena.

In sum, we cannot exclude that Type II FC indefinites emerged from non-specific free relatives, as Haspelmath (1997) conjectures, but concessive constructions appear to have played an important role. Further evidence from this comes from the fact that in Italian *qualsivoglia* contains the third-person form and not the second-person form, which we would expect if such an indefinite emerged from free relatives.[12]

8.2.4 Type III

As concerns Type III, we offer some remarks on the French *n'importe qu-* series, based on the work by Pescarini (2010). This indefinite is formed by the negative marker *ne* with elision of the vowel, together with the 3rd singular person of the verb *importer* ('to matter') and an indefinite determiner (like *quel*) or a pronoun (like *quoi*). In the former case, it behaves as a FC indefinite determiner; in the latter as a FC indefinite pronoun. In what follows we focus on the first

[11]The evidence of this apposition phase is very scarce, since we found only one clear example of such case, displayed below. It might be therefore possible that the relative/concessive construction was directly reinterpreted as an indefinite.

(i) et questo meo potter mai no me manca, / et pur, qual voglia, di cotesto parla
and this my power never not to-me miss, / and yet, QUAL VOGLIA, of this speaks
'And this power of mine never fails me, / and yet, whatever it is, speaks of this.'
(Jacopo Gradenigo, *Gli Quatro Evangelii concordati in uno*, 1399)

[12]Relatedly, Halm (2021) shows that Hungarian *akárki*, formed from the Old Hungarian imperative form of the verb *akar* 'to want' together with the particle *ki* 'even' shows that unconditional constructions played a role in Old Hungarian. See also Szabolcsi (2019).

construction, with an example in (40).

(40) N'importe quel chat est un chasseur.
not-matter what cat is a hunter
'Any cat is a hunter.'

The origin of its FC status is weak grammaticalization from the original expression 'it does not matter which'.[13]

Based on the study carried out by Pescarini (2010), we propose the following grammaticalization path. We start with a no matter concessive expression in (41). Such construction can then appear in an appositive after its anchor in (42). Importantly, this stage facilities a form of syntactic change which is relevant to the grammaticalization of the indefinite. In fact, in (43), the preposition *de* 'of' displays higher attachment, indicating that *n'importe* is integrated at a determiner level and lost its original verbal status. Finally, (44) shows the final FC indefinite form.

(41) N'importe de quel grade, ils s'agenouillent tous devant Napoléon
not-matter of which rank, they kneels all in-front-of Napoleon.
'No matter what rank, everyone kneels in front of Napoleon.

(42) Au soldat, n'importe de quel grade, doit partir.
a soldier, not-matter of which grade, needs leave.
'The soldier, no matter what rank, must leave.'

(43) Au soldat, de n'importe quel grade, doit partir.
a soldier, of not-matter which grade, needs leave.
'The soldier, no matter what rank, must leave.'

(44) N'importe quel soldat doit être en bonne santé.
Not-matter which soldier must be in good health
'Any soldier must be in good health'.

The remarks that we offered in these sections highlight that two constructions, among others, played an important role in the development of indefinites: unconditional-like and appositives structures.[14] Clearly, language change is a complex phenomenon which involves many factors. The precise developmental

[13] It is weakly grammaticalized because the insertion of some prepositions between the frozen verbal phrase and the true determiner phrase is still possible, as well as some rare cases where the verb *importer* occurs with the imperfective form *importait* (Pescarini 2010).

[14] We have already noted that for Type II indefinites, non-specific free relative uses might also constitute a source for the meaning of free choice. At the same time, we have also noted how concessive constructions appeared to have had a role in the grammaticalization process of such type of FC indefinite. One of the limits of the present work is that we are not providing an explicit analysis of non-specific free relative uses.

trajectory of an indefinite in a given language can be subject to constraint which are language-specific and studies looking at the development of a particular indefinite form are welcome, as they are to contribute to the general debate. In what follows, we will offer some insights that our current formalism gives us, with particular regard to unconditionals and appositives.

8.3 Formal Diachronic Analysis

In the previous sections, we have examined how FC indefinites can be analysed in 2TS and determined the kind of relationship that the variable they introduce brings about. In this section, we describe how the diachronic development of FC indefinites outlined in Section 8.2 can be adequately understood given our formal treatment of FC indefinites.

8.3.1 Meaning Interfaces in Language Change

The general plan of our proposal in sketched in Table 8.4. In particular, we argue that total variation $VAR_{|D|}(\varnothing, x)$ is originally a pragmatic inference stemming from the unconditional. The appositive phase as a *conventionalization* bridge where $VAR_{|D|}(\varnothing, x)$ gets strengthened into $VAR_{|D|}(v, x)$, which is then *integrated* into the semantic content of the indefinite.

PHASES	TOTAL VARIATION
1. Unconditional	Pragmatic inference $VAR_{|D|}(\varnothing, x)$
	↓ **conventionalization**
2. Appositive	Conventional NON-AT-ISSUE $VAR_{|D|}(\varnothing, x)$
	↓ **strengthening**
	Conventional NON-AT-ISSUE $VAR_{|D|}(v, x)$
	↓ **integration**
3. Indefinite	Conventional AT-ISSUE $VAR_{|D|}(v, x)$

Table 8.4: Main Components of Formal Diachronic Semantics Analysis.

8.3.2 Unconditional

Recall that the first phase of the diachronic development we are concerned involved unconditional constructions. The schematic representation of this phase is illustrated below:

(45) a. *Wie dan ook* reads this paper, the author will be happy.

b. $\Phi \Rightarrow \psi$

We will assume that the antecedent of unconditionals is associated with the alternatives of an interrogative clause, in line with several previous approaches to unconditionals (Rawlins 2008a, 2013; Ciardelli 2016). We will thus work with the representation in (45-b), where Φ stands an interrogative clause. We use $Alt(\Phi)$ for the alternatives of Φ.

An important question is how to define the set of alternatives $Alt(\Phi)$. Different theoretical options and theories of questions have been explored in the literature, and our proposal is not meant to adjudicate between them. A question like (46-a) is associated with two salient readings, a mention-some reading which is resolved if an individual is walking and a mention-all reading which asks to completely specify who is walking. These two possible representations are given in Figure 8.2.

(46) a. Who is walking?
b. $?xW(x, v)$

One way to generate the corresponding structure is to assume that the denotation of a question like (46-a) contains all the maximal initial teams T' over a model M satisfying (46-b). These are the alternatives of $?xW(x, v)$, $Alt(?xW(x, v)$. In 2TS, it is possible to express such inquisitive meanings. We have already observed in Chapter 3 that the inquisitive existential can be defined. For instance, assuming that $?xW(x, v)$ is $\exists_s x(W(x) \wedge dep(\varnothing, x))$ leads to the mention-some case in (a) in Figure 8.2.[15]

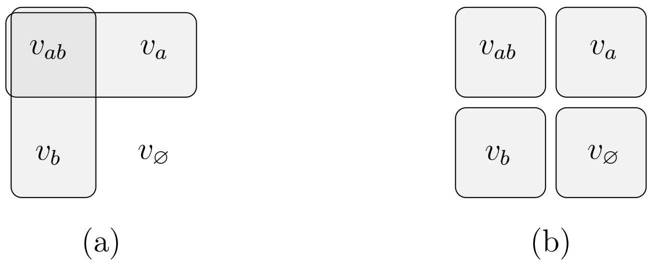


Figure 8.2: Alternatives of (46-a). Mention-some reading in (a); mention-all reading in (b). In the figures, we only consider the values of $T'(v)$ for each alternative T'.

Our proposal for unconditionals follows closely the lifting approach of conditionals to unconditionals put forward by Ciardelli (2016).[16] The additional

[15]The mention-all case in (b) can be generated by taking $?xW(x, v)$ to be $\forall x(W(x, v) \mathbin{\rotatebox[origin=c]{90}{$\geqslant$}} \neg W(x, v)) \mathbin{\rotatebox[origin=c]{90}{$\geqslant$}} \exists_s x(W(x, v) \wedge dep(\varnothing, x) \wedge \forall y(W(y, v) \leftrightarrow x = y))$. Note that inquisitive disjunction is definable in 2TS, as remarked in Chapter 4.

[16]We will not be concerned with cases in which questions appear in the consequent of the unconditional, even though the analysis can be generalized also to such cases.

element in our analysis is the integration of the epistemic state of the speaker, encoded by the initial team T. Definition (47) states that an unconditional is supported when all alternatives of the unconditional adjunct intersected with the initial team support the consequent.[17]

(47) UNCONDITIONAL (preliminary)

$$M, T \models \Phi \Rightarrow \psi \Leftrightarrow \text{ for all } T' \in \mathit{Alt}(\Phi),\ M, T \cap T' \models \psi$$

Consider for instance the case in (48) in

(48) a. Whoever comes to the party, I will be happy.
b. $?xC(x, v) \Rightarrow H(v)$

The intuition is that when uttering (48-a), the speaker does not know who comes to the party. However, this is not captured by the treatment of unconditionals in (47). For instance, consider an initial team where the speaker knows that only a comes to the party (i.e., $T(v) = \{v_a\}$). Then, under a suitable model, this would result in supporting the unconditional, since the empty set resulting from the intersection with $\{v_a\}$ would vacuously support the consequent.[18]

To fix this, we assume that $T \cap T'$ must be non-empty, preventing vacuous satisfaction:

(49) UNCONDITIONAL

$$M, T \models \Phi \Rightarrow \psi \Leftrightarrow \text{ for all } T' \in \mathit{Alt}(\Phi),\ M, T \cap T' \models \psi \text{ and } T \cap T' \neq \varnothing$$

We consider two additional points. First, unconditional typically presuppose that there is one individual satisfying the antecedent (Rawlins 2008b). The addition of this existence presupposition implies that under a treatment like (a) in Figure 8.2, the alternative corresponding to $T(v) = \{v_\varnothing\}$ is excluded. Second, unconditionals are typically associated with an exclusivity requirement which would exclude alternatives which include cases like $\{v_{ab}\}$ (Rawlins 2008b). This means that the possible alternatives for an unconditionals are as in (b) in Figure 8.2 without the teams related to $\{v_{ab}\}$ and $\{v_\varnothing\}$.[19]

Given these requirements, it follows that an initial team T which supports an unconditional of the form in (48) supports also (50). In other words, an initial

[17] There might be cases in which the antecedent of the unconditional does not range over the epistemic state of the speaker, but over a different set of possibilities. To account for this, one might generalize (47) by intersecting the alternatives T' with the team extended with the relevant world variable ranging over a different set of possibilities.

[18] We are again using the notation $\{v_a\}$ to indicate the initial teams which assigns v_a to v.

[19] Such alternatives can be constructed by taking the unconditional antecedent to express the interrogative clause corresponding to $\exists_s x(W(x, v) \land \mathit{dep}(\varnothing, x) \land \forall y(W(y, v) \leftrightarrow x = y))$ (which unique individual is walking?). Arguably, however, both the existence presupposition and the exclusivity requirement should be modelled independently.

team supports an unconditional when we are in a situation where any individual might satisfy the antecedent.

(50) $M, T \models \exists_s x(R(x, v) \wedge VAR_{|D|}(\varnothing, x))$

We classify the $VAR_{|D|}(\varnothing, x)$ condition as a form of 'pragmatic' inference, as the inference in (50) follows from the non-empty requirement operative in the unconditional, and it is parallel with the pragmatic inferences of indicative conditionals being compatible with the epistemic state of the speaker (Stalnaker 1975).

8.3.3 Appositive

We now move on to analyse the second phase of diachronic development outlined in Section 8.2. Recall that our discussion led to the generalization of two types of appositives, depending on their anchor, with the following structure.

(51) Referential Appositive
John, *wie dan ook*, passed the exam.
Ignorance: John passed the exam and the speaker does not know who John is.

(52) Non-Referential Appositive
A student, *wie dan ook*, can pass the exam.
Free Choice: Any student can pass the exam.

In Section 6.3 of Chapter 6 we discussed how appositive constructions can be analysed by assuming that they contribute to a separate non-at-issue component, and account for constructions like (51) and (52) as $\langle \phi_{\text{at-issue}}, \phi_{\text{non-at-issue}} \rangle$. We also discussed that proper names are not treated as constants but as variables with the requirement $dep(v, j)$ for any name j, meaning that proper names refer to the same individual in a particular epistemic possibility of the speaker. We also discussed that the at-issue and non-at-issue components are integrated by means of dynamic conjunction $\phi_{\text{AT-ISSUE}} \wedge \phi_{\text{NON-AT-ISSUE}}$.

Appositives and Proper Names

We now move on to analyzing the contribution of what we called 'referential appositives', as in (53). In such constructions, the resulting meaning is of 'total' ignorance: (53) conveys that John passed the exam and that the speaker is completely unaware of who John is. Recall that in the previous stage, the unconditional was associated with a pragmatic inference of the form $VAR_D(\varnothing, x)$. We claim that at this stage the total variation condition is now the contribution of the appositive at a non-at-issue level, as illustrated in (53-a) and (53-b).

(53) John, *wie dan ook*, passed the exam.

a. At issue: $P(j, v)$
b. Non at-issue: $VAR_{|D|}(\varnothing, j)$

The resulting integration between the at-issue and non-at-issue component is equivalent to $P(j, v) \wedge VAR_{|D|}(\varnothing, j)$, corresponding to the desired total ignorance reading, since $VAR_{|D|}(\varnothing, j)$ requires an initial team which comprises all possible values for j, as illustrated in Table 8.5.

v	j
v_1	d_1
v_2	d_2
...	...
v_n	d_n

Table 8.5: Illustration for (53).

An important question is how the original unconditional construction turned into an appositive one. Most likely this involved unconditional constructions of the form 'John$_1$, *wie dan ook* he$_1$ is, passed the exam.' If these constructions are interpreted as full unconditionals, we again observe that the requirement they impose on the epistemic state of the speaker is $VAR_{|D|}(\varnothing, j)$.

Appositives and Non-Referential Expressions

The account presented in the preceding section also extends to appositives with contain non-referential expressions, like plain indefinites. This is illustrated in (54).

(54) A student, *wie dan ook*, can pass the exam.
a. At issue: $\exists_l w \exists_s x\ \phi(x, w)$
b. Non at-issue: $VAR_{|D|}(\varnothing, x)$

As said in the previous section, these configurations typically involved a modal in the main clause, here captured by the lax functional extension in (54-a). The presence of the modal is particular important, as it allows the $VAR_{|D|}(\varnothing, x)$ condition to be satisfied in different 'types' of teams corresponding to different readings. To see this, consider the teams represented in Table 8.6, which all satisfy the $VAR_{|D|}(\varnothing, x)$ condition. The first team in (a) represents a case of total ignorance, and it parallels the case discussed in the previous stage. The last team (c) represents the free choice reading, where every element in the domain is possible in a given epistemic possibility (v_1 in this case). The intermediate team in (b) is a case of non-specific ignorance, where the speaker is uncertain whether d_1 up to d_m are possible options or d_{m+1} up to d_n are possible options. We observe that this reading is admittedly difficult to parse and would require a quite contrived context.

v	w	x
v_1	w_1	d_1
v_2	w_2	d_2
...	...	...
v_n	w_n	d_n

(a)

v	w	x
v_1	w_1	d_1
v_1	...	...
v_2	...	...
v_2	w_n	d_n

(b)

v	w	x
v_1	w_1	d_1
v_1	w_2	d_2
v_1	...	...
v_1	w_n	d_n

(c)

Table 8.6: Illustration for (54).

Recall that in our account, FC indefinites are associated with $VAR_{|D|}(v, x)$ and not with $VAR_{|D|}(\varnothing, x)$. The condition $VAR_{|D|}(v, x)$ says that the variation must occur after having fixed a value for v. In particular, only (c) also satisfies the stronger $VAR_{|D|}(v, x)$. We might then conjecture that a strengthening of $VAR_{|D|}(\varnothing, x)$ to $VAR_{|D|}(v, x)$ occurred, making configurations like (c) more prominent for different reasons.

First, $VAR_{|D|}(v, x)$ could play a disambiguating role as regards the scope of the indefinite with respect to the modal. Note, in fact, that $VAR_{|D|}(v, x)$ is incompatible with wide scope readings of the indefinite.[20] Second, we conjecture that the strongest possible meaning gets lexicalized and $VAR_{|D|}(v, x)$ is stronger than $VAR_{|D|}(\varnothing, x)$.

8.3.4 Free Choice

In the last phase, our item behaves as a full-fledged determiner, and the variation condition is integrated in the at-issue meaning of the indefinite.

(55) a. *Wie dan ook* can pass the exam.
b. $\exists_l w \exists_s x(\phi(x, w) \wedge VAR_{|D|}(v, x))$

Importantly, the presence of a modal as a distinctive feature of FC indefinites could be the result of the previous appositive phase. In fact, non-specific uses where $VAR_{|D|}(\varnothing, x)$ can be strengthened to $VAR_{|D|}(v, x)$ are only possible in (modal) embedded contexts.

To recap, we have outlined the grammaticalization path of FC indefinites, showing how the semantic account is related to the diachronic development of such class of indefinites:

[20] In 2TS, the scope of an indefinite is handled by dependence atoms. For the case in (54) and the scope of the indefinite with respect to the modal, $dep(v, x)$ would correspond to wide scope, while $dep(vw, x)$ would correspond to narrow scope. Importantly, $dep(v, x)$ is contradictory with $VAR_{|D|}(v, x)$.

1. 'Pragmatic' inference $VAR_{|D|}(\varnothing, x)$
2. NON-AT-ISSUE meaning $VAR_{|D|}(\varnothing, x)$
3. Strengthening of NON-AT-ISSUE meaning to $VAR_{|D|}(v, x)$
4. AT-ISSUE meaning $VAR_{|D|}(v, x)$

In particular, we have argued that NON-AT-ISSUE content in (2) and (3) acts as a conventionalization bridge for the integration of an originally pragmatic inference into at-issue semantic content.

8.4 Free Choice and Universal Quantifiers

Haspelmath (1997, 1995) observes that several languages have universal quantifiers which are morphologically similar to FC indefinites in that they are formed by a *wh*-element together with other indefinite markers. For instance, Romanian *fiecare* 'everyone' is formed by the *wh*-element *care* 'who/which' and *fie* the 3rd subjunctive form of *to be*. This suggests that such quantifier might be derived from FC indefinites.

Given our discussion of universal FC indefinites, it is not difficult to see that for initial teams, Fact 8.4.1 holds: lax quantifiers with total variation (i.e., universal FC indefinites) and universal quantifiers are equivalent.

8.4.1. FACT. $\exists_l x(\phi(x, v) \wedge VAR_{|D|}(v, x)) \equiv_v \forall x\ \phi(x, v)$

We have already observed that in episodic contexts, universal FC indefinites are typically infelicitous, but they can give rise to a universal-like meaning. To account for this reading, we proposed to treat universal FC indefinites as lax existential quantifiers. It might be possible that the equivalence in Fact 8.4.1 facilitated the reanalysis of the variation component into the universal extension required by the universal quantifier. Such reanalysis could be driven by the factor that when embedded, lax quantification generates both strong and weak readings, as discussed in this chapter, while universal quantification would only be compatible with the strong reading. Given the derivational trajectory from free choice to universal, this would imply that only the strongest meaning is preserved, which is a common derivational line when an ambiguity is present.

Strict	Lax	Universal
$\exists_s x(\phi(x, v) \wedge VAR_{|D|}(v, x))$	$\exists_l x(\phi(x, v) \wedge VAR_{|D|}(v, x))$	$\forall x\ \phi(x, v)$

Table 8.7: From FC Indefinite to Universal Quantifier.

Our observations explain the shift from universal FC indefinites to universal quantifiers. This shift, as said, likely occurred with items like *fiecare* in Romanian, which show elements derived from the constructions from which FC indefinites originated (e.g., unconditionals), as discussed in Section 8.2.

Haspelmath (1997) points out that in general the change from universal to FC indefinite occurs more rarely, even though Haspelmath (1997, p. 156) mentions some cases, like Hebrew *kol*, which derived from the Proto-Seminitic **kull* expressing totality or Turkish *herhangi* which was formed from the early *her* 'every'.

While the shift from a strict to a lax semantics was motivated by avoiding infelicity and generating possible parses, there seems to be no motivation to reinterpret a universal quantifier, which is unambiguous in all contexts, into a lax existential quantifier that would introduce ambiguity in embedded contexts. This could suggest why the derivational line 'universal quantifier > free choice' is less common.

In this regard, it is worth intersecting our discussion with the work of Beck (2017, 2020), which proposes a universal semantic cycle based on data from Old English. Beck (2017, 2020)'s analysis relies on an Alternative Semantics for indefinites (Hamblin 1973; Kratzer and Shimoyama 2002). The logical rendering in (56) is not supposed to be interpretable, but rather a sketch on the underlying analysis.

(56) BECK'S UNIVERSAL CYCLE

1.Covert universal quantification over alternative propositions

Whoever Ellen supervises, she needs a bigger office.

$$\forall\phi \in \Phi(\phi \rightarrow \psi)$$

2.Universal quantification over individual alternatives

Ellen will supervise whoever.

$$\forall e(e \in D \rightarrow \phi(e))$$

3.Lexical universal quantification over individuals

Ellen will supervise everyone.

$$\forall x\ \phi(x)$$

4.Group denoting DP with possible universal distributive readings

Everyone gathered at the town square.

[adapted from Beck 2020]

In our discussion, stage (1) is associated with universal quantification over teams, rather than over alternative propositions. Stage (2) cannot be directly captured in team semantics, as Alternative Semantics also allows quantification over individual alternatives. However, we might argue that the total variation atom of FC indefinites effectively requires the variable to be associated with all elements in a given domain, without explicit universal quantification. Stage (3) corresponds to the universal quantifier as captured by the universal extension of team. As a result, the shift from (2) to (3) aligns with our description of universal quantifiers derived from FC indefinites.

Beck (2020) also considers the stage in (4), where the item is now interpreted collectively, and a distributive operator is needed to generate distributive readings. In 2TS, as discussed in Section 6.2.3 of Chapter 6, such case would be captured by strict quantifiers over a pluralized domain with a maximality and distributive operator, reflecting the loss of 'genuine' universal quantification. The data we discussed in the present chapter, however, is not informative with respect to this latter change.

8.5 Conclusion

In this chapter, we examined FC indefinites, focusing on their licensing conditions and grammaticalization paths. We proposed that FC indefinites are associated with a total variation condition, which accounts for their free choice reading. To distinguish between universal and existential FC indefinites, we exploited the distinction between lax and strict existential quantification in 2TS. We discussed diachronic data concerning FC indefinites and provided a formal diachronic analysis of their grammaticalization process. Our analysis suggests that the total variation condition initially arises as a pragmatic inference from unconditional constructions and is gradually strengthened and integrated into the semantic content of FC indefinites. Finally, we examined the formal and diachronic relationship between FC indefinites and universal quantifiers in 2TS.

In future work, it would be relevant to further expand on the points discussed in Section 8.4 regarding the relationship between universal quantification, free choice, and distributivity. For instance, Latin displayed both a distributive quantifier like *omnis* 'every' and a collective one like *totus* 'all/the whole'. Almost all Romance languages maintained the form *totus* (e.g., Italian *tutti*, French *tous*, Spanish *todos*, Romanian *toți*), but only Italian retained the form *omnis* with Italian *ogni*. In the other cases, distributive quantification is constructed with the forms derived from *cata* 'by' together with *unum* 'one', which we have already discussed when considering dependent indefinites and Romanian *câte* (e.g., Italian *ciascuno*, French *chaque*, Spanish *cada*), while Romanian uses the form *fiecare*, which as discussed above possibly derives from an early FC indefinite. While the data is arguably not fully available, it would be instructive to reconstruct how the form *omnis* was lost and study the different paradigms across Italian, French-Spanish, and Romanian.

We have modelled free choice indefinites as indefinites associated with the total variation condition. In recent work, Bledin (2024) proposes an account of FC indefinites in terms of arbitrary objects (Fine and Tennant 1983; Horsten 2019). For instance, an arbitrary person is an abstract entity associated with a range of concrete persons, which are the values of the arbitrary person. There is a close relationship with our total variation requirement, where the indefinite can have as value all the possible individuals in the domain. It would be valuable to understand how arbitrary objects can be added to a team-based system or compared with the premises of such a system.

Chapter 9

Indefinites and Sign Languages

SOMEONE, *Italian Sign Language*

The empirical focus of this work, as well as much of formal semantics, has predominantly been on spoken languages. However, in recent years there has been a growing interest in sign languages (SLs) and the intersection of formal semantics with sign language studies (among others, Barberà 2015; Kuhn 2015; Davidson 2022; Schlenker 2018; Schlenker, Lamberton, and Kuhn 2023). In this chapter, we will explore how indefinites are encoded in SLs and how the topics discussed in this work relate to SLs.

In Section 9.1 we introduce some basic terminology and basic concepts of sign language linguistics. In Section 9.2, we will provide an overview of how indefinites are expressed in SLs, focusing in particular on indefinite pronouns. In Section 9.3, we will examine how the semantic diversity of indefinites in spoken languages, as discussed in previous chapters, can be represented in SLs. Finally, in Section 9.4, we will discuss the appropriateness of a team semantics approach for modelling SLs, and how the `2TS` framework developed here can be instrumental in the proper analysis of indefinites in SLs. We conclude in Section 9.5 with some points for future study.[1]

[1] I am really grateful to Raquel Veiga Busto for comments and discussion on an early version of this chapter.

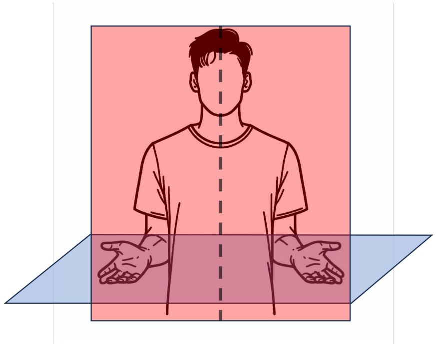


Figure 9.1: Signing Space (two-dimensional rendering). The blue shade corresponds to the horizontal plane, and the red shade to the frontal plane. A dotted line divides the ipsilateral side from the contralateral side.

9.1 The Signing Space

In this section, we provide some general terminological remarks and basic concepts of sign language linguistics that will be relevant in the subsequent sections.

The signing space refers to the three-dimensional area in which signers produce their signs. It consists of three main components: (i) the horizontal plane, which is perpendicular to the body and where most of the signs are produced, (ii) the frontal plane, which runs from just above the head to about the waist level, and (iii) the midsagittal plane, which extends forwards and backwards.

We represent signs with small capitals to convey their meanings (e.g., HOUSE for the corresponding sign). For compound signs, we use a circumflex accent to separate morphemes (e.g., HOME^WORK). Signs produced higher in the plane are denoted as $[\text{SIGN}]_{\text{up}}$.[2] Signs can be produced on the ipsilateral side of the signing space (the area closer to the dominant hand), or on the contralateral side (the area opposite to the dominant hand). Figure 9.1 illustrates the horizontal and frontal planes.

The horizontal and frontal planes are crucial for the distinctions we will examine. However, the midsagittal plane is also important as it allows for the production of signs that require movement away from or towards the body.

SLs utilize the signing space for grammatical purposes. For example, in American Sign Language (ASL), so-called directional verbs, also called agreement verbs, incorporate the signing space to indicate the subject and object: a verb like 'to give' can show who is giving to whom based on the direction of the movement (Valli et al. 2011; Padden and Humphries 1990). Signers use different areas of the signing space to represent different roles or entities. By shifting their body position slightly to one side or the other, signers can indicate a change in speaker or character, which is vital for narrative structure.

[2] We assume that $_{\text{up}}$ in $[\text{SIGN}]_{\text{up}}$ means above shoulder level. It is also possible to introduce different categorical distinctions [low, middle, high].

Signers can establish referential points, called referential *loci* or *r-loci* (Lillo-Martin and Klima 1990), within the signing space primarily through pointing signs, which are indicated by IX in the glosses.[3] *Loci* are used to refer back to specific people, objects, or places, helping to maintain clarity and coherence in discourse.

Finally, while not pertaining to the signing space, we note that signers can also include non-manual features (facial expressions, head movements, etc.), which are essential for conveying both grammatical information and emotional nuance. As we will see, these features will play an important role in our discussion. When relevant, we indicate non-manuals using overlines.

9.2 Indefinites in Sign Languages

In this section, we present an overview of how indefinite forms are encoded in various SLs. The aim is to offer a general perspective on SLs and indefinites.

Regarding what are traditionally called indefinite and definite articles, SLs typically lack overt determiners that distinguish between definite and indefinite usages (Neidle and Nash 2012). Noun phrases are often realized as bare or in combination with the use of pointing signs. The use of the prenominal or postnominal position of the sign frequently plays a role in distinguishing between definite and indefinite interpretations. For instance, in American Sign Language (ASL), the position of the pointing sign contributes to the distinction between indefinite and definite interpretations, as illustrated in (1) from MacLaughlin (1997, p. 117) mentioned in Neidle and Nash (2012). A pointing sign used before the noun is compatible only with a definite interpretation, while a pointing sign used after the noun is compatible with both definite and indefinite interpretations.

(1) a. IX$_3$ MAN ARRIVE
‘The/that man is arriving.’
*‘A man is arriving.’
b. MAN IX ARRIVE
‘A/the man there is arriving.’

As concerns indefinite pronouns, to address this issue from a cross-linguistic viewpoint, we gathered some preliminary evidence from the website *SpreadTheSign*, a multilingual sign language platform, which allows looking up signs for individual words and sentences in different SLs.[4] We searched, based on the English pronouns, for the forms *someone*, *somebody* (‘person’ semantic category) and *something* (‘thing’/‘object’ semantic category) in isolation and in relevant exam-

[3] Person values are indicated with numbers, applicable to pointing signs and predicates.

[4] *SpreadTheSign* can be accessed at `https://www.spreadthesign.com/`. Data and images were collected with permission.

ples. A spreadsheet containing the relevant data collected can be found in the following repository: `https://osf.io/w48eh/`.[5]

In what follows, we report our findings for the 'person' pronominal form. Unless otherwise stated, the examples are pictures from *SpreadTheSign*. Given the limited extent and quality of the data available on *SpreadTheSign*, these findings should be supported by in-depth language-specific studies. Whenever possible, we tried to substantiate our claims by considering sign language corpora, dictionaries, or data available in previous work. The latter are mentioned in the text when considered. In this regard, at least from a lexical point of view, the data collected from *SpreadTheSign* proved to be reliable.

It appears that SLs resort to four basic strategies, which we summarize as follows together with some illustrations in Table 9.1. Importantly, the subdivisions below are meant to capture general strategies of indefinite pronoun formation. This does not imply that SLs resort to only one of them. Similar to what happens in spoken languages, different forms are admitted within a specific language.

(i) Sign SOMEONE. Some SLs have a dedicated sign SOMEONE used as an indefinite pronoun, lexically distinct from the sign for the interrogatives WHO or WHICH. In Sign Language of the Netherlands (NGT), the sign SOMEONE can be realized with the little finger on the ipsilateral side. In Croatian Sign Language (HZJ), the sign SOMEONE appears as if two pronominal forms are realized on both sides with two hands, with up and down movements, probably a form of reduplication.

(ii) Sign PERSON. The sign PERSON is very similar across sign languages: it is realized by forming a small space between the thumb and the index fingers on the ipsilateral side and moving down, presumably referring to the body of a person. It is typically combined with other signs like the determiner determiner SOME or the sign ONE. In Greek Sign Language (ENG), a moving PALM-UP sign is concatenated with the sign PERSON.[9] This appears to be a quite common strategy across SLs.

[5] It was unclear how the data was collected on *SpreadTheSign*. It could be that signers were asked to sign a word or a sentence given the spoken language counterpart. Or, signed forms could have been translated or matched to the corresponding spoken language. Due to the frequent mouthing, and given the objective of the platform of keeping the same or similar sentence in different languages, the first option seems more plausible.

[6] Example from *NGT SignBank* `https://signbank.cls.ru.nl/` (Crasborn et al. 2020).

[7] Example from Veiga Busto, Degano, and Roelofsen (2024).

[8] Example from Branchini and Mantovan (2020).

[9] The PALM-UP sign displays a wide array of uses in SLs, ranging from particle-like usages, discourse marker, interrogative and indefinite usages, and as a consequence several works have been dedicated to describe its uses in many SLs (see Cooperrider, Abner, and Goldin-Meadow (2018) for an overview). It also occurs as a common gesture in many spoken languages. It is interesting to note that this sign can be used both as a marker of absence of knowledge and of interrogative usages.

LANGUAGE	EXAMPLE	GLOSSES	LINK
Sign Language of the Netherlands (NGT)[6]		SOMEONE	https://edu.nl/rm7bh
Croatian Sign Language (HZJ)		SOMEONE	https://edu.nl/dxawh
Greek Sign Language (ENG)		PALM-UP^PERSON	https://edu.nl/ku9bx
Catalan Sign Language (LSC)[7]		WHO^IX$_{3pl[up]}$	https://edu.nl/733xw
Italian Sign Language (LIS)[8]		SOMEONE	https://edu.nl/e3wh4

Table 9.1: Realization of the indefinite pronoun 'someone' across SLs. Following the distinctions outlined above NGT, and HZJ display a type (i) indefinite; ENG a type (ii); LSC a type (iii); and LIS a type (iv).

(iii) Sign WHO/WHICH. Several SLs use the sign WHO/WHICH with indefinite as well as interrogative usages. This occurs in Catalan Sign Language (LSC), where the sign WHO can often concatenate with the 3rd person plural pronominal form (Barberà and Quer 2013; Barberà 2015). The sign WHO can also occur in isolation (Veiga Busto, Degano, and Roelofsen 2024), aligning with what has been referred to as quexistential in spoken languages (Hengeveld, Iatridou, and Roelofsen 2023). Russian Sign Language (RSL) (Kimmelman 2018) also exhibit such affinity. In German Sign Language (DGS), the sign SOMEONE [https://edu.nl/xrgbj] displays a close affinity to the sign for WHICH [https://edu.nl/e8wq8], but with different reduplication patterns and orientation parameters. A possible way to disambiguate between the interrogative and existential meaning appears to be reduplication of the sign or mouthing.[10]

(iv) Sign ONE. The sign ONE can be used to express indefinites, similarly to what we observe in spoken languages, as discussed in Chapter 7. This sign can occur in isolation or combined with other signs as a determiner. In Table 9.1, we provide an illustration for Italian Sign Language (LIS). We return to this category below.

The last category (iv) warrants additional remarks. Notably, Mantovan and Geraci (2018) seem to differentiate between the sign ONE for the numeral 'one' and the sign SOMEONE for the indefinite 'one'/'someone'. While ONE is generally produced with the index finger in the upper position of the ipsilateral side without any movement, as depicted in the static figure above, the indefinite form is often accompanied by tremoring movement, as the video in the EXAMPLE column shows. A similar distinction is observed in Turkish Sign Language (TİD). Spoken Turkish uses the indefinite *biri* 'someone', derived from *bir* 'one'. These distinctions appear to be realized also in TİD. According to the *Contemporary Turkish Sign Language Dictionary* (Makaroğlu and Dikyuva 2017), the numeral form is produced without any movement [`https://edu.nl/d4cyb`], whereas the indefinite form involves a tremoring motion akin to the LIS case [`https://edu.nl/fa3ga`].[11] Moreover, some signs which show an affinity with interrogatives and would belong to type (iii) are also related to the numeral one. For instance, this is the case for British Sign Language (BSL). According to the *BSL SignBank* (Fenlon et al. 2014), both SOMEONE [`https://edu.nl/7jpvf`] and WHO [`https://edu.nl/7fckg`] are signed with the dominant '1' hand with circular motion, even though the numeral ONE is typically, but not necessarily (e.g., the LIS example discussed before), signed at a different position.

We also emphasize that the above list does not exhaust all the ways indefinite reference is expressed in SLs but offers some relevant subdivisions. For instance, indefinites can also be constructed by determiners like English *some*. The subdivisions above, however, mostly concern pronominal forms. Furthermore, as mentioned, particular uses of pointing signs can themselves give rise to indefinite readings. Furthermore, there might be signs for specific indefinite-like meanings. For instance, some SLs employ the sign OTHER to convey the English equivalent of 'someone else' or the non-specific use of 'someone/anyone'.[12]

[10] See Kimmelman (2018) for some preliminary remarks in Russian Sign Language (RIS) and Veiga Busto, Degano, and Roelofsen (2024) for LSC. Reduplication or mouthing are arguably not obligatory to induce an existential interpretation. Determining in which contexts they occur is an open question.

[11] The dictionary can be accessed at `https://tidsozluk.aile.gov.tr/en/` with a Turkish IP address.

[12] For instance in Danish Sign Language (DTS), which uses the sign ANDRE 'others' as in (i),

Finally, it is interesting to note that in certain cases, indefinites are typically related to a region or a portion of the signing space, while definite expressions are often associated with a specific point in the signing space, as is the case for ASL (MacLaughlin 1997; Neidle and Nash 2012). In this sense, indefinites show a close affinity to plural marking. Plurality is often encoded in SLs through circular or arc movements, reduplication, or multiple realizations of the same sign in different loci (Pfau and Steinbach 2021). These strategies seem to be visible, though not always, in the domain of indefinite pronouns, as discussed above for the case of reduplication and circular/tremoring movements. A relevant question would be to determine for which usages this affinity is particularly strong and how it is realized.[13]

9.3 Variety of Indefinites

In the previous section, we broadly overviewed how indefinite pronouns can be realized in a variety of SLs. The guiding motivation behind the development of 2TS was the observation that languages cross-linguistically exhibit a variety of indefinites with different distributions. In particular, our focus was on scopally specific (specific vs. non-specific) and epistemically specific (known vs. unknown) usages. A natural question is whether and how this variety applies to the domain of SLs.

We remind the reader of the distinction between manual and non-manual features. Manual features include handshapes, movements, and locations of hands and arms. Non-manual features or markers (NMMs) involve everything else signers use with their upper body, such as facial expressions, head tilts, shrugs, and mouthing words.

We conjecture that there could be three possible ways in which manuals interact with NMMs concerning indefinite forms. First, SLs may display phonologically realized signs or specific combinations of manual signs that can only be associated with certain functional readings, corresponding to the marked indefinites discussed in previous chapters.

Second, NMMs could play a key role in determining the correct functional interpretation of an indefinite. While the manual form might remain constant,

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taken from the official dictionary of DTS (Kristoffersen and Troelsgård 2010).

(i) IX_1 FORGET DEODORANT OTHERS THERE-IS [https://edu.nl/amjgm]
'I forgot deodorant. Does **anyone** have one?'

The full dictionary entry can be found by searching ANDRE at https://tegnsprog.dk/.

[13]One interesting case is the HZJ example mentioned in Table 9.1. As we will discuss, we believe that such realizations are possible for non-specific usages. For instance, based on the data from *SpreadTheSign*, which again can offer only a surface-level perspective, such cases occur in non-specific usages of indefinites, like in polar questions of the form 'Is there *someone/anyone* here who speaks German?'. We will return to this point in Section 9.4.3.

dedicated NMMs could give rise to specific known, specific unknown, and non-specific uses as discussed in this work. It is thus possible that certain NMMs might become conventionalized into the meaning of the indefinite, paving the way for marked forms due to the presence of NMMs.

Third, we may have a combination of both strategies: it may be possible that only certain combinations of manuals with NMMs are felicitous, giving rise to restricted forms.

To address this issue from a cross-linguistic viewpoint, we again gathered some preliminary evidence from the website *SpreadTheSign* and relied on previous literature.[14]

The ultimate goal, rather than describing in full detail cross-linguistic distinctions - which, though crucial, would be very laborious - is to offer some general remarks on the status of marked indefinites in SLs. We will focus on the aspects most relevant to the theoretical framework we developed, `2TS`. We will begin in the next section by delineating why a team semantics framework is suitable for modelling SL phenomena.

9.4 Team Semantics and Sign Languages

In this section, we discuss why a team semantics framework, and in particular `2TS`, is a suitable system to model formal semantics phenomena with respect to SLs. We will start by considering the role of variables and the relevance of a team assigning values to indefinites across a set of assignments. We will then focus on the class of epistemic, non-specific and specific (known) indefinites.

9.4.1 Variables and Partivity

`2TS` is a team semantics framework where formulas are interpreted with respect to a set of variable assignments. Initial teams only contain the variable for the actual world v and the addition of new variables models the growth of discourse information. One of the central aspects of `2TS` is the ability of variables to receive different values across assignments, naturally encoding relationships of dependence among variables. This offers two significant advantages.

First, as we have already observed, the use of *(referential) loci* in the signing space is prominent in SLs and has been argued to be a faithful representation of variables in a logical system (Schlenker 2018), particularly with respect to

[14]Regarding *SpreadTheSign*, it also allows looking up sentences in different SLs. This makes it possible to compare different constructions across various SLs. Importantly, the target words/sentences were also given in the original language, making it possible to detect if a marked form was used in the target word/sentence, assuming that this effectively how the data was collected (see footnote 5). However, since signers can resort to different structures not involving indefinites, in our considerations we took the restrictive choice to include only examples that contained overt signed forms of indefinites.

the phenomena of anaphora and binding, which we discussed in Chapter 6. The phenomenon is arguably more complex and loci have also been analysed as clitics, as features and as 'featural variables' (Schlenker 2018). Here, we simply outline this parallelism to underscore its relevance to the contrasts we are interested in.

Second, the notion of inclusion atom $\subseteq (\vec{x}, \vec{y})$ that we defined in Section 3.4.1 of Chapter 3 can be used to capture cases where the speaker refers to a referent previously introduced in the discourse or some phenomena of partitive specificity. This appears to be overt and visible in SLs. For instance, consider the following example in ASL from Schlenker (2018). In (2), a large locus corresponding to the set of all students is indicated by IX$_{\text{arc-ab}}$ in (2). The signing space is used to indicate relevant subsets of the students: IX$_{\text{arc-a}}$ for the students who came to class (2) and asked good questions (2-b) and IX$_{\text{arc-b}}$ for the students who stayed at home (2-a).

(2) POSS-1 STUDENT IX$_{\text{arc-ab}}$ MOST IX$_{\text{arc-a}}$ $_{\text{a-}}$CAME CLASS
'Most of my students came to class.'
$\exists_l x(\phi(x, v) \wedge \subseteq (x, y))$

a. IX$_{\text{arc-b}}$ $_{\text{b-}}$STAY HOME
'They stayed home'.
$\psi_1(z, v) \subseteq (z, x)$

b. IX$_{\text{arc-a}}$ $_{\text{a-}}$ASK$_1$ GOOD QUESTION
'They asked me good questions'
$\psi_2(x, v)$

c. IX$_{\text{arc-ab}}$ SERIOUS CLASS
'They are a serious class'
$\psi_3(y, v)$
(example from Schlenker 2018, p. 170)

Importantly, assuming a lax treatment of plurality, where plurals are captured by allowing the values of the variable to vary within a given value for v, our notion of inclusion atoms of the form $\subseteq (\vec{x}, \vec{y})$ can be used to model such subset relations, in a way similar to what Schlenker (2018) proposes.

Moreover, we have a formal definition of the inclusion atom $\subseteq$ and the constraints that follow appear to be valid in the signing space as well. These constraints are the same as those for the subset order $\subseteq$ relation over the subsets of a set. For instance, every locus is a sublocus of itself (i.e., $\subseteq (\vec{x}, \vec{x})$ for any $\vec{x}$); transitivity of loci (i.e., $\subseteq (\vec{x}, \vec{y})$ and $\subseteq (\vec{y}, \vec{z})$ imply $\subseteq (\vec{x}, \vec{z})$); antisymmetry of loci (i.e., $\subseteq (\vec{x}, \vec{y})$ and $\subseteq (\vec{y}, \vec{x})$ imply $x = y$).

Not surprisingly, Schlenker (2018) coined the term 'visible meaning' to indicate that certain formal features are visible or overt in SLs. Given the role that variables and relationships among variables play in SLs, a natural question is whether SLs can be informative with respect to the dependency conditions that constitute the enriched meaning of marked indefinites.

Recall, in fact, that in 2TS, specific known indefinites are captured by constancy - $dep(\varnothing, x)$ - requiring the value of the indefinite to be the same across all epistemic possibilities of the speaker. This is compatible only with the team (a) in Table 9.2. Epistemic indefinites are captured by variation - $var(\varnothing, x)$ - requiring the value of the indefinite to vary across all assignments. This condition is compatible with the team in (b) for specific unknown uses and with the team in (c) for non-specific uses. Non-specific indefinites are captured by v-variation - $var(v, x)$ - requiring the value of v to differ within a given epistemic possibility of the speaker. This condition is only compatible with the team in (c). Note also that for such a condition to be licensed, at least another variable (i.e., an operator in the sentence) needs to be introduced, indicated by dots in Table 9.2.

T_1	v	x
i_1	v_1	a
i_2	v_2	a
i_3	v_3	a
i_4	v_4	a

(a)

T_2	v	x
i_1	v_1	a
i_2	v_2	b
i_3	v_3	c
i_4	v_4	d

(b)

T_3	v	...	x
i_1	v_1	...	a
i_2	v_1	...	b
i_3	v_1	...	c
i_4	v_1	...	d

(c)

Table 9.2: Three teams T_1, T_2, T_3 with different conditions on the variable x.

The question that we will address in the sections that follow is to what extent such relationships among variable assignments are 'visible', to borrow Schlenker (2018)'s terminology, in the domain of marked indefinites in SLs.

9.4.2 Epistemic Indefinites

One of the types of indefinites which has received considerable attention in sign language linguistics are indefinites that convey uncertainty or lack of knowledge on the part of the speaker. Several studies (Barberà and Quer 2013; Barbera and Cabredo Hofherr 2019; Mantovan and Geraci 2018; Kimmelman 2018) focused on so-called 'impersonal' constructions, a broad category that includes all cases where the indefinite is, in their terminology, not referential. For the indefinites' distinctions considered in the present work, 'impersonal' constructions encompass both specific unknown and non-specific usages of indefinites.

In episodic contexts involving specific unknown indefinites, non-manual markers play a prominent role in indicating the speaker's lack of knowledge. These NMMs include furrowed eyebrows, lowered mouth corners, sideward or upward gaze, slightly raised chin, and raised shoulders. An example from Veiga Busto, Degano, and Roelofsen (2024) in LSC illustrates this point. In (3), the form WHO^ONE with lowered mouth corners and furrowed eyebrows, as depicted in Figure 9.2, signals a specific unknown use of the indefinite.

Figure 9.2: WHO with non-manual features of mouth corners downs and furrowed eyebrows. Example from Veiga Busto, Degano, and Roelofsen (2024).

cd, bf

(3) IX$_1$ SEEM WHO.prs OUTSIDE IX. [https://edu.nl/d4mn6]
‘There seems to be someone outside.’

These NMMs appear to be quite stable cross-linguistically for the limited number of languages we considered. For example, in RSL (Kimmelman 2018) and LIS (Mantovan and Geraci 2018) furrowed eyebrows and lowered mouth corners are strongly associated with such readings. However, the presence of these NMMs does not seem to be obligatory in the studies mentioned above.

Regarding the presence of dedicated manual forms or combinations of manuals and non-manuals, Mantovan and Geraci (2018) shows that in LIS, the combination of raised eyebrows and mouth corners down with the sign SOMEONE (i.e., ONE with tremoring) and the sign PERSON induces unknown readings. While these NMMs are optional for the sign SOMEONE, they are more strongly associated with the sign PERSON.

One possible explanation for this pattern is that indefinite forms used in unknown contexts typically include some form of tremoring or in-situ reduplication of the base form, especially for indefinites based on the numeral ‘one’. For instance, as said, the sign SOMEONE in LIS resembles ONE with the addition of a tremoring movement. This modulation of the sign might signal lack of knowledge. However, such tremoring would be difficult to apply to the sign PERSON from an articulatory point of view or in any case it would then be difficult to interpret the sign, which is why signers more often resort to NMMs for this form.[15]

[15] Note however that Mantovan and Geraci (2018) mention that SOMEONE can be used also when the referent is referential, presumably referring to a specific known reading. This would be indeed compatible with the distribution of Italian *qualcuno* ‘someone’, assuming that there is a parallelism between LIS and spoken Italian in this regard - which does not have to be the case. But it would be in contrast with the proposal that tremoring on the sign always signals lack of knowledge. However, Mantovan and Geraci (2018) do not explicitly mention if in such cases, the sign is accompanied by tremoring movement. In general, it would also be valuable to determine if both SOMEONE and PERSON display the same distribution in non-specific contexts (e.g., attitude verbs, conditional antecedents, . . .).

Recall that in our framework, an epistemic indefinite is represented by the variation condition $var(\varnothing, x)$. This condition is compatible with both specific unknown usages and non-specific ones. In the former, the value of the indefinite varies across the value of v, while in the latter the values vary within a given v. Given the significance of variable values in SLs, as discussed in Section 9.4.1, is this the variation condition also reflected in SLs?

We offer two remarks. First, this condition appears to be encoded by the NMMs mentioned above. One difficulty in assessing this claim is that 'impersonal' constructions typically encompass both specific unknown and non-specific uses. It is essential to distinguish between contexts in which the indefinite is used in specific unknown cases and non-specific cases. If a non-manual is used in both cases, it would cover the space of epistemic indefinites. We have reasons to believe that not all NMMs appear in all contexts. For instance, while lowered mouth corners are predominant, the role of the eyebrows might differ: furrowed in specific unknown usages but raised or neutral in non-specific cases like conditional antecedents (Dachkovsky and Sandler 2009; Pfau and Quer 2010). This claim requires further empirical research.

Second, we conjecture that signs can also be 'modulated' to give rise to enriched meanings. One such case was that the additional tremoring, which could signal that the referent's value is not fixed, similar to what our variation condition achieves.[16]

In conclusion, there are two components central to the realization and interpretation of epistemic indefinites in SLs: NMMs like lowered mouth corners and sign modulations like tremoring movement. An open question is to determine the whole array of NMMs and sign modulations associated with a lack of knowledge. For instance, are lowered mouth corners always accompanied by furrowed eyebrows? Does reduplication also play a role for some signs? Such questions are, of course, language-dependent, but some patterns might also be cross-linguistically stable.

We could not detect dedicated signs, besides complex constructions, that can only be used as epistemic indefinites.[17] However, it would be valuable to study if and to what extent NMMs can be conventionalized in certain signs, leading to

[16]Note also that epistemic indefinites in SL can take the form of complex signs. For example, in LSC, the sign for WHO can be concatenated with $\text{IX}_{\text{-3pl}}$ or the sign SOME, typically produced by extending the fingers of the dominant hand rapidly from the index to the pinky (Veiga Busto, Degano, and Roelofsen 2024). These signs could be intuitively associated with the variation condition.

[17]For instance, Danish has the complex determiner *en eller anden*, equivalent to English *some or other*, which functions as an epistemic indefinite. *SpreadTheSign* contained a dedicated sign realized with both thumb and index finger [`https://edu.nl/evfuk`], which is also indicative of the variation condition. However, the official dictionary for Danish Sign Language (Kristoffersen and Troelsgård 2010) lists this sign as meaning 'yes and no' or correlative constructions like 'both ... and', 'either ... or'. The entry can be found at `https://tegnsprog.dk/` by searching for *enten-eller*.

marked forms of signs in combination with NMMs.[18]

9.4.3 Non-specific Indefinites

Non-specific indefinites are indefinites which are ungrammatical in episodic contexts and do not admit wide scope readings.

Importantly, we observe that for spoken languages which exhibit a non-specific indefinite, their sign language 'counterpart' does not typically maintain such form in a parallel way. This is expected, since they are two different languages. However, elements of the spoken language used in the same community or geographic area might exert some influence on the sign languages which are used there. For instance, Russian forms indefinites for the semantic category of person by the *wh*-item *kto* 'who', together with the prefix *koe-* for specific known, the affix *-to* for epistemic, and the affix *-nibud'* for non-specific. It appears that in RSL (Kimmelman 2018), the *wh*-item *kto* is maintained as a quexistential with possible mouthings being *kto* or the epistemic *kto-to*. The forms *kto-nibud'* (non-specific) or *koe-kto* (specific known) do not appear to be overtly realized or mouthed. How is then non-specificity encoded in SLs?

There appear to be two possible ways to encode non-specificity. First, the use of height in the frontal plane of the signing space. Barberà (2015) showed that for signs not bound to a dedicated location in the signing space, a specific interpretation arises in the neutral/lower region, while a non-specific interpretation is associated with a higher (above shoulders) region.

The role of height could lead to the hypothesis that signs for indefinites that are always signed higher are non-specific indefinites. However, there are two concerns. First, Barbera and Cabredo Hofherr (2019) observed that the sign SOME, as well as ONE, signed high in the frontal plane, is compatible with specific unknown readings.

Second, one needs to determine if this specificity is scopal (it arises in interaction with other operators) or partitive (it signals a restriction to the domain of interpretation of the indefinite). In this regard, Davidson and Gagne (2022) observe that for ASL, the sign SOMEONE, signed as the numeral 'one', gives rise to different interpretations depending on the height at which it is signed. If signed neutrally or low, it is understood to encompass a restricted domain. However, when signed higher, it encompasses a larger domain. This indicates that, at least in ASL, the height of the signing space is related to partitive specificity rather than scopal specificity.[19]

[18] A remark is in order. There are manual forms signalling speaker uncertainty, like the PALM-UP sign we discussed before. But these apply generally and not to indefinites specifically. Moreover, signers might resort to more complex constructions to indicate their lack of knowledge, like spoken languages do with dedicated parentheticals: 'Someone - I don't know who - called.'.

[19] Importantly, as also noted by Davidson and Gagne (2022), there are cases in which the two

Another way to encode non-specificity involves the horizontal plane of the signing space, bearing some affinity with plurality. For instance, in the data elicited by Veiga Busto, Degano, and Roelofsen (2024), under a possibility modal, the sign WHO, which acts as a quexistential, can be concatenated with IX-b_{straight} to give rise to a non-specific interpretation of the indefinite.[20]

(4) IX-2 INVITE CAN WHO^IX-b_{straight}. [`https://edu.nl/bfrmb`]
'You can invite anyone.'

This could resemble our v-variation condition, which ensures the indefinite is associated with more than one possible value in the actual world.[21]

Further evidence for this comes from the use of indefinites in polar questions, where indefinites are arguably interpreted non-specifically. For instance, in Chilean Sign Language, the sentence 'Is there someone here who speaks German?' is realized by multiple realizations of ONE with each hand followed by reduplication with movement.[22]

We underline again that in our framework, non-specific indefinites are captured by $var(v, x)$, ensuring that the value of the indefinite varies within a given v. In this sense, non-specific indefinites are closely tied to certain plurals, encoded similarly by means of lax extensions.

For non-specificity, the usage of the signing space appears more relevant than a dedicated set of non-manuals associated with such usages. The latter might still be present, and given the remarks of the previous section, some might be shared with specific unknown usages since epistemic indefinites cover both specific unknown and non-specific uses, though there appears to be no prior work focusing particularly on non-specific contexts and NMMs.

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notions of specificity, partitive and scopal, coincide. For instance, under a treatment of scopally specific readings as singleton indefinites (Schwarzschild 2002). This is why testing cases with more than one operator with the availability of intermediate readings could be relevant.

[20] In example (4), 'b' refers to the configuration of the hand (B-handshape). One important point is if examples like (4) should be parsed with single usages of an indefinite, given the presence of WHO or as a more complex construction like 'You can invite someone, anyone'.

[21] We have acknowledged that RSL does not have a dedicate non-specific indefinite, like Russian does with *kto-nibud'*. Still, *SpreadTheSign* lists one realization by one Belarusian signer (Belarusian has a non-specific indefinite similar to the Russian type) which realize the non-specific indefinite with the *wh*-item *kto* 'who' followed by two pointing signs on both sides of the space. [`https://edu.nl/36a7j`] In this regard, while in RSL or Bulgarian Sign Language the form *kto-nibud'* is arguably not realized as in *SpreadTheSign*, it is still relevant that the signer resorted to that particular construction we discussed, or that this form was mapped to *kto-nibud'*, depending on how the data was collected.

[22] `https://edu.nl/9muyp`
The use of multiple realizations of ONE for such cases is also possible in Catalan Sign Language (Raquel Veiga Busto, pc). A relevant point is the connection between these signed forms of the indefinite and plural indefinites, as the latter are known to exhibit multiplicity inferences that disappear in questions or under negation. Note that in spoken Spanish and Chilean Spanish, the singular form *alguien* 'someone' is used in such contexts.

9.4.4 Specific Known

Specific indefinites are indefinites that admit both specific and specific unknown readings, but not non-specific usages. Specific known indefinites are those that only admit specific known uses. Both types of indefinites should thus admit only wide scope readings, although, as observed in Chapter 7, they can also license functional narrow scope readings. Moreover, we noted that there are few cases of specific indefinites cross-linguistically for reasons we discussed in Chapter 7. As a result, here we focus on specific known in particular, but we acknowledge that specific (wide scope) readings of indefinites, both known and unknown, are typically realized in the lower region of the frontal plane. This might be due to partitive specificity rather than scopal specificity, but it has been noted that restrictions to singleton domains can be a way to model wide scope readings.

There are essentially no studies dedicated to indefinites with specific known uses. One reason might be that when the referent is known to the speaker, signers might resort to demonstratives, more definite-like constructions, or the definite use of pointing signs.

Kimmelman (2014, p. 56) notes that in NGT and RSL a combination of lowered eyebrows and a wrinkled nose, typically referred to as the 'you know' expression, can be used together with nouns in cases where the referent is known and familiar to both the speaker and the addressee. Relatedly, Barberà (2015) observes that squinted eyes as a marked of definiteness are associated with familiarity in the discourse.[23] On the assumption that at least some of the features for familiarity are shared with specific known usages, it appears that what is relevant for what counts as known are NMMs or combinations of manuals and NMMs.

9.4.5 Free Choice

Several SLs appear to have a dedicated sign for indefinites conveying FC. Recall that we have analysed FC indefinites using the total variation condition $VAR_{|D|}(v, x)$, which ensures that the variable for the indefinite ranges over all possible values in a given domain for a fixed value of v, representing a complete form of non-specificity.

Our characterization of FC aligns well with the overt realization of FC indefinites in many SLs. For instance, in Chilean Sign Language (LSCh), FC indefinites are realized by moving both hands in the upper position of the signing space and opening them towards the signer. In Finnish Sign Language (FinSL), the *wh*-item for *who* is followed by a sign spanning the entire horizontal axis with an open hand. In Italian Sign Language (LIS), the sign is produced by moving both hands

[23] However, specific known indefinites are used when the speaker knows the identity of the referent, and the listener is aware of this, but they use an indefinite expression as the referent is somehow not relevant to the discourse. Therefore, there appears to be no need for familiarity to both the speaker and the listener.

towards each other on the horizontal plane.

Generally, signs for 'anyone' are produced with both hands moving across the horizontal plane and occasionally include multiple reduplications with movement.

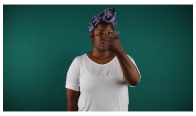
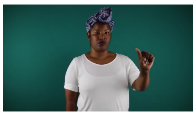
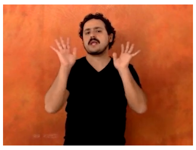
LANGUAGE	EXAMPLE	GLOSSES	LINK
Italian Sign Language (LIS)	 	QUALSIASI	`https://edu.nl/ku9bx`
American Sign Language (ASL)[24]	 	ANY	`https://edu.nl/69jhk`
Finnish Sign Language (FinSL)[25]	 	KUKAAN	`https://edu.nl/e33v3`
British Sign Language (BSL)		ANYBODY	`https://edu.nl/w7m36`
Chilean Sign Language (LSCh)		CUALQUIERA	`https://edu.nl/pe4yt`

Table 9.3: Realization of FC indefinites across SLs. The Finnish example is formed by WHO concatenated with a straight pointing sign formed with both hands.

Recall that the condition $VAR_{|D|}(v, x)$ discussed in Chapter 8 on free choice indefinites, conveys that x ranges over all possible values in a given domain. As shown, this appears to be reflected in the way FC indefinites are realized in the signing space. This highlights again the strong connection between the variable's values and the signing space.

FC indefinites also exhibit a similarity to universal quantifiers. In Chapter 8, we discussed the formal relationship between FC indefinites and universal quantifiers, noting how the latter are sometimes diachronically derived from FC indefinites. In SLs, distributive quantifiers like the English *every* are typically realized through reduplication, while quantifiers like *all* involve the realization of large circular loci encompassing all individuals in the relevant domain.[26] An interesting

[25] Example taken from ASL SignBank (Hochgesang, Crasborn, and Lillo-Martin 2017–2024).

[26] This contrast between these two types of universal quantification is visible in LIS as well as in Russian Sign Language (RSL) (Kimmelman 2017).

research question would be to determine which type of universal quantification FC indefinites most closely resemble, and if there are cases where signs for FC indefinites have developed into universal quantifiers, or vice versa.

9.4.6 The Status of Marked Indefinites

We have observed that for epistemic indefinites, and presumably for specific known ones, the role of non-manual markers or sign modulations (e.g., tremoring affecting the sign ONE) seem to be more prominent. In contrast, markers of non-specificity appear to be more visible in the signing space, utilizing height or the horizontal plane.

This observation aligns with the fact that specific known and epistemic indefinites were captured by constancy $dep(\varnothing, x)$ and variation $var(\varnothing, x)$, respectively. By contrast, non-specific indefinites tied the variation to the variable for the actual world using $var(v, x)$. Given the role that variables play in the signing space, as discussed in Section 9.4.1, we might conjecture that this is reflected by the more prominent use of the signing space to mark non-specificity.

An additional point concerns the status of the enriched meaning of marked indefinites. In Section 6.3 of Chapter 6, we observed that while specific known and epistemic indefinites are considered odd when their additional meanings are not satisfied in the context, non-specific indefinites tend to be judged as false rather than odd. We repeat below some relevant patterns: while an epistemic indefinite, as in (5), is odd in a context where the speaker knows the identity of who is knocking at the door, (6) is false, rather than odd, in a context where every boy read the same book. A similar remark applies to FC indefinites, which clearly lead to falsity when free choice is not granted.

(5) Una qualche persona sta bussando alla porta.
UN QUALCHE person stay knocking at-the door
'Someone is knocking at the door'
$[\exists_s x\ \phi(x; v)]^{var(\varnothing,x)}$

(6) Kazhdyy mal'chik chital kakuyu-nibud' knigu.
every boy read which-nibud book.SING.ACC
'Every boy read some book.'
$\forall y \exists_s x(\phi(x, y; v) \wedge var(v, x))$

In Chapter 6, we explained these differences by assuming that the atoms $dep(\varnothing, x)$ and $var(\varnothing, x)$ are better analysed as post-suppositions in the dynamic version of 2TS, leading to undefinedness when not satisfied. By contrast, the v-variation atom $var(v, x)$ must be evaluated as a test. This implies that while the former were more pragmatic-like inferences, scope readings are better accounted as a 'semantic' phenomenon. Under this treatment, $dep(\varnothing, x)$ and $var(\varnothing, x)$ were global operators, while $var(v, x)$ was evaluated within the scope of the indefinite, as in (5) and (6) respectively.

This distinction appears to be reflected in the domain of SLs: NMMs associated with speaker ignorance tend to spread over the entire sentence, as in example (4), and not just the indefinite (Barberà 2015; Veiga Busto, Degano, and Roelofsen 2024), whereas features associated with non-specificity like height or productions of signs in wider areas of the horizontal plane must be, of course, localized within the scope of the indefinite.

Importantly, in Section 6.3 of Chapter 6, we observed that in certain environments, like negation or conditional antecedents, post-suppositions should be embedded rather than evaluated globally. It would be valuable to determine if such distinctions are also visible in the scope of the NMMs in these relevant environments. Moreover, it could be interesting to further investigate these differences in terms of NMMs features for epistemic distinctions (known vs unknown) vs features for scope distinction (specific vs non-specific) in acceptability judgements to determine if the contrast false/odd carries over to SLs as well.

9.5 Conclusion

In this chapter, we have explored how indefinites are realized in SLs, illustrating various realizations across several SLs. Our examination aimed to determine the extent to which SLs exhibit the diversity of indefinites observed in spoken languages, guided by the idea that some typological distinctions in spoken languages may also be present in SLs. This approach was inspired by Haspelmath (1997)'s typological work on indefinites, which proposed that indefinites occupy regions on a functional map, which we reproduce here in Figure 9.3.

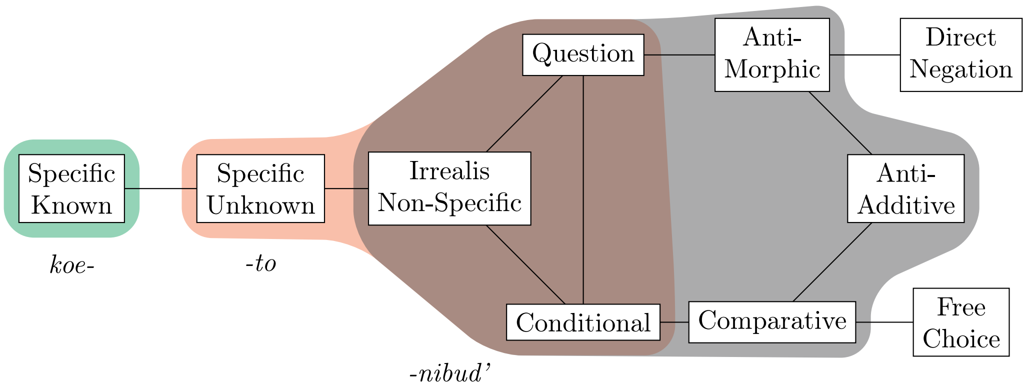


Figure 9.3: Haspelmath map of Russian *koe-*, *-to* and *-nibud'*.

We sought to understand whether a map as the one in Figure 9.3 could be applied to SLs. The answer was not immediate, since we found that the meaning of indefinites in SLs results from various lexical forms, non-manual markers

(NMMs), sign modulations, and interactions with the signing space (e.g., high vs. low realization of signs in the frontal plane).[27]

In general, we have observed that for epistemic indefinites (which can convey both specific unknown and non-specific uses), both sign modulations like tremoring movements on the sign and NMMs seem to play a key role, potentially conveying the variability or lack of knowledge associated with these readings. Specific known indefinites are less studied, but some evidence suggests that NMMs associated with familiarity or givenness may be relevant. For non-specific indefinites, the use of the signing space, such as height or production of signs in wider areas of the horizontal plane, seems more relevant, potentially reflecting the v-variation condition with respect to the variable for the actual world proposed in the theoretical framework. Free choice indefinites in SLs are often realized through signs that span a large portion of the signing space, potentially aligning with the total variation condition proposed for this semantic class. This suggests a division of labour between epistemic distinctions and scopal distinctions, a division also reflected in the implementation of these underlying conditions modelling these indefinites, as discussed in the previous section.

It is thus crucial to empirically investigate the distribution of indefinites in specific SLs. This would allow for a comprehensive inventory of indefinite forms in a particular language and facilitate the study of the effects of different NMMs on each form. In the long term, careful examination across languages might lead to a better understanding of how non-manuals, manuals, and the signing space interact with each other. This suggests that a characterization like the one in Figure 9.3 for spoken languages will not be as simple as mapping lexical items to uses. Rather, it should involve multi-level components like manual signs, non-manual markers, movement, height, and so on, all interacting with each other.

We would like to point out two concrete future directions of study. First, while the present work has focused on specific, epistemic, non-specific, and free choice indefinites, languages exhibit a much richer variety. For instance, in Chapter 6, we considered the class of dependent indefinites. These indefinites share some distributional patterns with non-specific indefinites, but they also exhibit notable differences. Kuhn (2017) discusses the close connection between dependent indefinites and distributivity. Studying how non-specificity, as delineated in this chapter, relates to distributivity would thus be valuable. Second, regarding free choice indefinites, we discussed in Chapter 8 the distinction between existential free choice indefinites and universal free choice indefinites. Investigating

[27]Importantly, this could simply indicate that we have not yet fully determined the correct subdivision to accurately model the underlying structure of SLs. For instance, the Haspelmath map represented in the figure is unlikely to reflect the full picture of spoken languages when factors like prosody are considered, or when the inventory of expressions encoding indefinite reference, as opposed to just indefinite pronouns, is taken into account. The challenge, therefore, is to find the right representations for SLs that would provide a pristine picture of indefinites in SLs. I thank Raquel Veiga Busto for pointing this out to me.

how this distinction is encoded in SLs and studying all the environments in which free choice indefinites appear is an interesting empirical question and potentially informative for semantic analyses as well.

Chapter 10

Conclusion

In this dissertation, we have explored the topic of indefinites and (non-)specificity. We began by observing the cross-linguistic variety of indefinites, focusing on the distinctions between specific and non-specific readings, as well as between known and unknown readings. We then introduced the framework of two-sorted team semantics (2TS) and demonstrated how it can account for these distinctions.

We examined how different types of indefinites can be characterized in terms of these conditions and how this characterization can explain their distribution. Our analysis showed that the framework predicts universal patterns of lexicalization and provides insights into why certain indefinite types are not realized. Additionally, we discussed the relationship between 2TS and the diachronic development of (non-)specific indefinites, leading to possible predictions. We also explored how the framework could be extended to account for other types of indefinites, such as dependent and free choice indefinites. Furthermore, we investigated the realization of indefinites in sign languages and argued that a team semantics approach is suitable for modelling the relevant phenomena.

In this chapter, we first briefly look at the interplay between indefinites and questions in Section 10.1. We then explore the relationship between indefinites and disjunction in Section 10.2. These sections serve as a reference point for a more comprehensive treatment of indefinites in natural language within the context of 2TS.

10.1 Indefinites and Questions

The present work has primarily focused on simple examples to demonstrate how a formal system can model (non-)specificity and indefinites. However, indefinites can occur in more complex environments, and a comprehensive theory must account for these occurrences in a unified manner. For instance, we must consider the interplay between indefinites and (counterfactual) conditionals, the role of (non-specific) indefinites in comparatives, the relationship between weak indefinites and plurality, and more. Although these topics have been extensively investigated in formal semantics, the focus has often been on plain or simple indefinites. One of the goals of this dissertation was to show that considering the variety of indefinite forms and their enriched meanings could provide valuable insights into the semantic analyses of these topics. This calls for a research agenda that can account for the variety of indefinites across different environments within a unified framework.

One particular area that warrants further investigation is the relationship between indefinites and questions. As discussed in Chapter 9, several languages employ the same word for both interrogative and existential/indefinite uses. These elements are known as *quexistentials* (Hengeveld, Iatridou, and Roelofsen 2023). The following examples from Hengeveld, Iatridou, and Roelofsen (2023) illustrate this point for the Dutch *wat* ('what/something'), which can be used as a question word, as in (1), or as an indefinite, as in (2). Note that in the latter case, the indefinite *iets* ('something') is also possible.

(1) Wat heeft Miranda gegeten?
WAT has Miranda eaten
'What has Miranda eaten?'

(2) Miranda heeft wat gegeten.
Miranda has WAT eaten
'Miranda has eaten something.'

We conjecture that when this happens, the indefinite is of the epistemic kind, signaling that the speaker does not know the identity of the referent.[1] This suggestion, however, seems to conflict with the logical representation of questions in a team-based setting. In inquisitive semantics (Ciardelli, Groenendijk, and Roelofsen 2018; Ciardelli 2022), a question like (1) would be represented by means of the inquisitive existential, which we know is equivalent to our notion of existential combined with constancy $dep(\varnothing, x)$, as in (3-a). Within the context of 2TS, we can consider a question like (1) to raise an issue corresponding to the set of maximal (initial) teams supporting (3-a) (i.e., Miranda ate d_1, Miranda ate d_2, and so on). This, however, seems to contrast with the logical rendering of an epistemic indefinite, which requires variation $var(\varnothing, x)$.

(3) a. $\exists_s x(\phi \wedge dep(\varnothing, x))$
b. $\exists_s x(\phi \wedge var(\varnothing, x))$

[1] This is particularly evident for Mandarin *shenme*, which is also a quexistential, and as an existential, it carries an obligatory ignorance inference (Li 1992; Chen 2018; Cao 2023). Cao (2023) proposes a theory within the context of 2TS developed in this work, which can account for the distribution of *shenme*, also in questions. This treatment captures the empirical behaviour of *shenme* in Mandarin, but it rests on several assumptions about the underlying semantics of *shenme*.

We propose that while the issue raised by a question should be tied to (3-a), the variation component ensures that in the initial team, it is not settled what Miranda has eaten, meaning that the initial team must support (3-b). In this way, we distinguish between the role of a question, which asks for a specific value, and the variation component, which ensures that the initial team upon which the question is asked does not already determine precisely what Miranda ate. These two elements align with how questions are used in ordinary conversation and should play a role in any formalization.[2]

Moreover, it would be valuable to investigate the use of marked indefinites in (polar) questions properly. For instance, to account for contrasts like (4-a) and (4-b).

(4) a. Are you looking for a specific book?
 b. Did you read any book?

One puzzling observation concerns the behavior of non-specific indefinites in polar questions. Specifically, we have noted that non-specific indefinites are infelicitous in episodic contexts like (5) due to the strict notion of existential and the variation condition $var(v, x)$. However, they are licensed in the corresponding polar question in (6).

(5) #Kto-nibud' zvonil mne
 KTO-NIBUD' called me
 Intended: 'Someone called me.'

(6) Zvonil li mne kto-nibud'?
 called Q me KTO-NIBUD'
 Did anyone call me? (from Haspelmath 1997)

Under a simple treatment of polar questions, the issue raised by (6) would correspond to the set of maximal (initial) teams satisfying (5) and the one satisfying its negation. However, there is no initial team supporting (5) given our treatment of non-specific indefinites. To account for this contrast, we would like to maintain the same logical rendering for non-specific indefinites by means of $var(v, x)$, which cannot be supported for cases like (5), but still explain why this does not affect their presence in polar questions.

[2]This relates to the treatment provided in the dynamic inquisitive semantics framework proposed by Roelofsen and Dotlačil (2023). In this framework, both an interrogative *wh*-phrase and an indefinite introduce a discourse referent. Additionally, the interrogative raises an issue that requires identifying one entity that possesses the properties associated with the introduced discourse referent.

10.2 Indefinites and Disjunction

The focus of this work was on indefinites, and did not cover in detail the topic of disjunction, which can be considered the propositional counterpart of indefinites treated as existential quantifiers. In this section, we outline some remarks concerning both the theoretical relationship between existential quantifiers and disjunction as an operator and the empirical aspect between indefinites and disjunction in natural language.[3]

First, we have observed that two different notions of existential can be given for the existential quantifier: strict and lax. The same holds for the split or tensor disjunction in the semantic clauses of `2TS`, where strict disjunction contains the additional requirement that the intersection of the two subteams must be non-empty:

10.2.1. Definition (Strict Disjunction).
$M, T \models \phi \vee_s \psi$ iff there exist $T_1, T_2 \subseteq T$ with $T_1 \cup T_2 = T$ and $T_1 \cap T_2 = \varnothing$ s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

10.2.2. Definition (Lax Disjunction).
$M, T \models \phi \vee_l \psi$ iff there exist $T_1, T_2 \subseteq T$ with $T_1 \cup T_2 = T$ s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

Clearly, over a finite domain, it holds that the strict and lax existentials are equivalent to a series of disjunctions over the whole domain. Here and in what follows we will assume that ϕ and ψ are classical formulas.

10.2.3. Fact (Strict and Lax Disjunction and Existential).

1. $\exists_s x\ \phi(x) \equiv \phi(d_1) \vee_s \phi(d_2) \vee_s \cdots \vee_s \phi(d_n)$
2. $\exists_l x\ \phi(x) \equiv \phi(d_1) \vee_l \phi(d_2) \vee_l \cdots \vee_l \phi(d_n)$

We have observed that across languages, indefinites exhibit a lot of variety, and we have argued that this variety can be captured with the help of the dependence and variation conditions. Two questions arise: (i) do languages also exhibit a variety of disjunctions?; (ii) if so, how can such diversity be modeled?

First, we will explore how the conditions imposed on indefinites can be translated to the domain of disjunction. We have already considered in Chapter 3 the inquisitive existential $\exists^1 x\phi$, which corresponds to the combination of the strict or lax existential together with constancy. Similarly, the global or inquisitive disjunction is treated likewise as in Definition 10.2.4, leading to Fact 10.2.5. Note that in this case, the distinction between lax and strict is not relevant, as $dep(\varnothing, x)$ is downwards closed.

[3] Disjunction has been extensively studied in team semantics and dependence logic. Some of the definitions included in this section come from Fan Yang's 2023 ESSLLI course *Logics of Dependence and Independence.*

10.2.4. Definition (Global Disjunction).
$M, T \models \phi \mathbin{\vee\!\!/} \psi$ iff $M, T \models \phi$ or $M, T \models \psi$

10.2.5. Fact (Global Disjunction and Existential).
$\exists^1 x\phi(x) \equiv \exists_l x(\phi(x) \wedge dep(\varnothing, x)) \equiv \exists_s x(\phi(x) \wedge dep(\varnothing, x)) \equiv$
$\equiv \phi(d_1) \mathbin{\vee\!\!/} \phi(d_2) \mathbin{\vee\!\!/} \ldots \mathbin{\vee\!\!/} \phi(d_n)$

Similarly, we can define a global notion of disjunction relativized to v for $dep(v, x)$. In what follows, we always assume that $v \in Dom(T)$.

10.2.6. Definition (v-Global Disjunction).
$M, T \models \phi \mathbin{\vee\!\!/}_v \psi$ iff for all $w \in T(v)$, $M, T_{v=w} \models \phi$ or $M, T_{v=w} \models \psi$

10.2.7. Fact (v-Global Disjunction and Existential).
$\exists_l x(\phi(x) \wedge dep(v, x)) \equiv \exists_s x(\phi(x) \wedge dep(v, x)) \equiv \phi(d_1) \mathbin{\vee\!\!/}_v \phi(d_2) \mathbin{\vee\!\!/}_v \ldots \mathbin{\vee\!\!/}_v \phi(d_n)$

We now consider the variation conditions. A notion of so-called *Relevant Disjunction* has been proposed in the literature. Here we call it *Variation Disjunction*. It can be defined upon the strict and lax disjunction, which in this case lead to distinct semantic clauses. It comes with the additional requirement that both subteams must be non-empty. In Definition 10.2.8, we use the strict notion of disjunction.

10.2.8. Definition (Variation Disjunction).
$M, T \models \phi \vee_s^{\text{var}} \psi$ iff there exist $T_1, T_2 \subseteq T$ with $T_1 \cup T_2 = T$ and $T_1 \cap T_2 = \varnothing$ and $T_1 \neq \varnothing$ and $T_2 \neq \varnothing$ s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

There is a parallelism between this notion of disjunction and our variation atom. In particular, the following holds:

10.2.9. Fact (Variation Disjunction and Existential).

1. $\exists_s x(\phi(x) \wedge var_n(\varnothing, x)) \equiv \phi(d_1) \vee_s^{\text{var}} \phi(d_2) \vee_s^{\text{var}} \cdots \vee_s^{\text{var}} \phi(d_n)$
2. $\exists_s x(\phi(x) \wedge var_2(\varnothing, x)) \equiv \phi(d_1) \vee_s^{\text{var}} \phi(d_2)$ for some $d_1, d_2 \in D$

Similarly, we can relativize this notion to the variable v. Given the way $var(v, x)$ is defined, we are requiring the second clause to hold for some $w \in T(v)$. To find the corresponding notion of disjunction for $VAR(v, x)$ we discussed in Chapter 3, the second clause over all $w \in T(v)$ would have sufficed.

10.2.10. Definition (v-Variation Disjunction).

$$M, T \models \phi \vee_s^{v\text{-var}} \psi \text{ iff}$$

1. for all $w \in T(v)$, there exist $T_1, T_2 \subseteq T_{v=w}$ with $T_1 \cup T_2 = T_{v=w}$ and $T_1 \cap T_2 = \varnothing$ s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$
2. for some $w \in T(v)$, there exist $T_1, T_2 \subseteq T_{v=w}$ with $T_1 \cup T_2 = T_{v=w}$ and $T_1 \cap T_2 = \varnothing$ and $T_1 \neq \varnothing$ and $T_2 \neq \varnothing$ s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$

We summarize the different notions of disjunctions we considered in Table 10.1. Note that, similarly to the relationship between dependence and variation atoms, it holds that $\phi \mathbin{\vee\!\!/} \psi \models \phi \mathbin{\vee\!\!/}_v \psi$ and $\phi \vee_s^{v\text{-var}} \psi \models \phi \vee_s^{\text{var}} \psi$.

T_1	v	$\dots$
i_1	v_1	$\dots$
i_2	v_1	$\dots$
i_3	v_2	$\dots$
i_4	v_2	$\dots$

(a) $\mathbin{\vee\!\!/}$

T_2	v	$\dots$
i_1	v_1	$\dots$
i_2	v_1	$\dots$
i_3	v_2	$\dots$
i_4	v_2	$\dots$

(b) $\mathbin{\vee\!\!/}_v$

T_2	v	$\dots$
i_1	v_1	$\dots$
i_2	v_1	$\dots$
i_3	v_2	$\dots$
i_4	v_2	$\dots$

(c) $\vee_s^{\text{var}}$

T_4	v	$\dots$
i_1	v_1	$\dots$
i_2	v_1	$\dots$
i_3	v_2	$\dots$
i_4	v_2	$\dots$

(d) $\vee_s^{v\text{-var}}$

Table 10.1: Illustration of Disjunctions. The case in (a) is also compatible with $\mathbin{\vee\!\!/}_v$. The case in (d) is also compatible with $\vee_s^{\text{var}}$.

We now consider how these alternative notions of disjunction might be relevant to modelling the behaviour of disjunction in natural language. First, as concerns the variation disjunction $\vee_s^{\text{var}}$, it has been observed that some (complex) disjunctions are associated with obligatory ignorance inferences. This is the case for the Russian *to li* ... *to li* construction (Ivlieva 2016). Similarly, languages without a dedicated disjunction tend to express disjunction through an uncertainty marker (e.g., the equivalent of English *perhaps*), as seen in *Wari'*, a language spoken by the Wari' people in Brazil (Mauri 2008). To maintain the parallelism with indefinites, we refer to this class of disjunctions as 'epistemic'. (7) illustrates a schematic representation, where these disjunctions semantically encode ignorance inferences. Notably, given our notion of epistemic modality discussed in Chapter 4, the statement in (7-b) holds over initial teams.

(7) a. Mary is in Paris or$_{\text{epi}}$ in London.
$\rightsquigarrow$ Mary might be in Paris and Mary might be in London.
b. $\phi \vee_s^{\text{var}} \psi \models_v \Diamond\phi \wedge \Diamond\psi$

Disjunctions can also differ in terms of scope. Consider the contrasts in (8), which can be associated with a narrow scope reading of disjunction in (8-a) and a wide scope reading in (8-b). Following the terminology used for indefinites, we use the term 'specific' for wide scope and 'non-specific' for narrow scope.

(8) Everyone read the Castle or the Trial.
  a. [$\forall > \vee$] for each x, x read the Castle or the Trial.
  b. [$\vee > \forall$] for each x, x read the Castle or for each x, x read the Trial.

Dawson (2020) observes that Tiwa, a Tibeto-Burman language, has two disjunctive markers: *ba* for non-specific uses of disjunction and *khí* for specific uses.[4] This suggests that *ba* might correspond to the v-variation disjunction $\vee_s^{v\text{-var}}$ and *khí* to the v-global disjunction $⩔_v$, which requires wide scope. Importantly, Dawson (2020) discusses some examples where *ba* also occurs in episodic contexts without any operator. However, $\vee_s^{v\text{-var}}$, like non-specific indefinites, is predicted to be infelicitous in episodic contexts. Therefore, the empirical picture does not fully align with our predictions, but here we aim to highlight the correspondence between non-specificity in indefinites and disjunctions.

Finally, concerning the global disjunction $⩔$, we propose that such disjunction is realized in so-called interrogative disjunction. It is well-known that languages distinguish between standard disjunction and interrogative disjunction, which can only occur in questions (Haspelmath 2004). For instance, Finnish differentiates between the standard disjunction *tai* and the interrogative one *vai*. The latter gives rise to an alternative question (a question that presents a number of options employing disjunctions, from which the hearer is expected to choose). Mandarin also displays the interrogative disjunction *háishi* (Erlewine 2024). Example (9) displays the Finnish interrogative disjunction *vai* in an alternative question, where the possible answers are 'coffee' or 'tea' (and not 'both' or 'neither').

(9) Haluatko kahvia vai teetä?
  want coffee or tea
  'Do you want coffee or tea?' [alternative question]

Importantly, in inquisitive semantics (Ciardelli, Groenendijk, and Roelofsen 2018), alternative questions are precisely captured by $⩔$. In 2TS, we can potentially model the issue raised by (9) as containing all the maximal (initial) teams supporting $\phi_{\text{coffee}} ⩔ \phi_{\text{tea}}$. Seemingly, there seems to be a contrast with the corresponding $dep(\varnothing, x)$ associated with specific known indefinites, which require the speaker to know the value of the referent. However, the correspondence is indeed insightful, as it reflects the different roles constancy plays for declaratives and questions: for declarative uses of indefinites, $dep(\varnothing, x)$ signals a specific value satisfying the indefinite; for questions, interrogative disjunction $⩔$ asks for a specific choice among the alternatives. These observations call for a unified theory that can account for the behaviour of marked indefinites and marked disjunctions in both declaratives and questions.

[4] Dawson (2020) does not consider intermediate readings. Thus, it remains an open question whether *ba* receives the narrowest scope possible or if intermediate readings are allowed. Here, we assume the latter.

Table 10.2 summarizes the different classes of disjunctions discussed in this section and their relationship with the classes of indefinites investigated in the previous chapters.

ATOM	DISJ.	CLASS	INDEFINITE	DISJUNCTION
$dep(\varnothing, x)$	$\mathbin{\vee\!\!\!/}$	specific known	Lithuanian *kai*	Finnish *vai*
$dep(v, x)$	$\mathbin{\vee\!\!\!/}_v$	specific	Ossetic *-dær*	Diwa *khí*
$var(\varnothing, x)$	$\vee_s^{\text{var}}$	epistemic	German *irgend-*	Russian *to li* ... *to li*
$var(v, x)$	$\vee_s^{v\text{-var}}$	non-specific	Russian *-nibud'*	Diwa *ba*

Table 10.2: Variety of Indefinites and Disjunction Across Languages.

We have reached the end of this dissertation. We hope the reader found this work sufficiently *specific* to be considered a valuable contribution, yet *non-specific* enough to foster new questions and further research.

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Samenvatting

Indefinieten en hun waarden

Stel je voor dat je de verdediging van dit proefschrift bijwoont en de zin uitspreekt: 'Iemand leest dit proefschrift.'. Je kunt 'iemand' gebruiken om te verwijzen naar een vriend die voor je zit, die je kent. Maar je kunt 'iemand' ook gebruiken om te verwijzen naar een persoon in het publiek, die je niet kent. Formele semantiek bestudeert de *betekenis* van natuurlijke taal, en doet dit door middel van formele logische benaderingen. Dit proefschrift richt zich op een ogenschijnlijk klein maar belangrijk aspect van natuurlijke taal: indefinieten, zoals het Nederlands 'iemand'.

In het besproken voorbeeld is de *waarde* [Engels: value] van 'iemand' in het eerste geval vast, terwijl in het tweede geval de *waarde* varieert tussen alle mogelijke opties waarvan je denkt dat deze persoon bedoeld zou kunnen zijn. Belangrijk is dat, terwijl Nederlands 'iemand' in beide gevallen toestaat, verschillende talen specifieke vormen van indefinieten (gemarkeerde indefinieten) gebruiken die alleen kunnen worden gebruikt als je de referent kent, en vormen die alleen kunnen worden gebruikt als je de referent niet kent.

Dit proefschrift richt zich op deze en soortgelijke contrasten, met name op de zogenaamde scopale en epistemische specificiteit. In dit domein vertonen gemarkeerde indefinieten een verscheidenheid aan vormen en betekenissen in verschillende talen, wat verschillende vragen en onderzoeksdoelen oproept.

Hoe kunnen we formeel rekening houden met deze onderscheidingen tussen indefinieten? We ontwikkelen een formeel systeem, twee-gesorteerde teamsemantiek [Engels: two-sorted team semantics] (`2TS`), dat uit verschillende tradities put: teamsemantiek, afhankelijkheidslogica [Engels: dependence logic] en twee-gesorteerde logica [Engels: two-sorted logic]. Een indefiniet wordt geassocieerd met een variabele over een set van variabele toewijzingen, een team, dat zijn mogelijke waarden encodeert. Deze waarden kunnen verschillende afhankelijkheidsrelaties hebben met andere operatoren in de zin, wat de verschillende betekenissen en distributies van verschillende indefinieten weerspiegelt.

Wat zijn de waargenomen types van (niet-)specifieke indefinieten in verschillende talen? We laten zien hoe 2TS kan uitleggen waarom bepaalde types indefinieten worden waargenomen terwijl andere zeldzaam of niet-waargenomen zijn in termen van complexiteit en hoe gemarkeerde indefinieten een convex betekenisspectrum vormen. We bespreken hoe de waargenomen gemarkeerde indefinieten duidelijk kunnen worden weergegeven door een oppositievierkant, het Afhankelijkheidsvierkant van Oppositie genoemd. Bovendien demonstreren we hoe 2TS adequaat de verschillende klassen van gemarkeerde onbepaalde voornaamwoorden kan uitleggen.

Welke diachrone veranderingen zijn mogelijk onder gemarkeerde indefinieten? We demonstreren hoe 2TS adequaat enkele waargenomen diachrone paden kan uitleggen en andere kan uitsluiten. De 2TS formalisering geeft helder verschijnselen van semantische verzwakking weer middels implicatie. We analyseren ook verschijnselen van grammaticalisatie die vrije keus indefinieten betreffen, en verklaren ook hun distributie.

Hoe worden indefinieten gerealiseerd buiten gesproken taal? We onderzoeken de realisatie van indefinieten in gebarentalen en pleiten voor de geschiktheid van 2TS bij het modelleren van de relevante verschijnselen.

Tot slot hebben we vastgesteld hoe de formalisering van indefinieten en (niet-)specificiteit, geleverd door 2TS, waardevolle inzichten biedt in dit taalkundige domein. We hopen dat dit proefschrift heeft aangetoond dat de studie van indefinieten allesbehalve indefiniet is.

Abstract

Indefinites and their values

Imagine attending the PhD defence of the author of this dissertation and uttering the sentence 'Someone is reading this dissertation.'. You can use 'someone' to refer to a friend sitting in front of you, whom you know. But you can also use 'someone' to refer to a person in the audience, whom you do not know. Formal semantics studies the *meaning* of natural language, and it does so by means of formal logical accounts. This dissertation focuses on a seemingly tiny yet significant aspect of natural language: indefinites, such as the English 'someone'.

In the example discussed, the *value* of 'someone' is fixed in the former case, while in the latter case its *value* varies according to all possible options you consider this person might be. Importantly, while English allows 'someone' in both cases, different languages employ dedicated forms of indefinites (marked indefinites) that can only be used if you know the referent and forms that can only be used if you do not know the referent.

This dissertation focuses on these and similar contrasts, particularly on so-called scopal and epistemic specificity. In this domain, marked indefinites exhibit a variety of forms and meanings across languages, raising several questions and research goals.

How can we formally account for these distinctions between indefinites? We develop a formal system, two-sorted team semantics (`2TS`), that draws from different traditions: team semantics, dependence logic, and two-sorted logic. An indefinite is associated with a variable over a set of variable assignments, a team, encoding its possible values. These values can have different dependency relationships with other operators in the sentence, reflecting the meaning and distribution of different indefinites.

What are the attested types of (non-)specific indefinites cross-linguistically? We show how `2TS` can explain why certain types of indefinites are attested while others are rare or unattested in terms of complexity and how marked indefinites form a convex meaning space. We discuss how the attested marked indefinites can be perspicuously represented by a Square of Opposition, called the Dependence Square of Opposition. Furthermore, we demonstrate how 2TS can adequately account for different classes of marked indefinites.

What diachronic changes are possible among marked indefinites? We demonstrate how 2TS can adequately explain some attested diachronic paths and rule out others. The 2TS formalization transparently represents phenomena of semantic weakening in terms of entailment. We also analyse phenomena of grammaticalization involving free choice indefinites, accounting also for their distribution.

How are indefinites realized beyond spoken language? We investigate the realization of indefinites in sign languages and argue for the suitability of 2TS in modelling the relevant phenomena.

In conclusion, we have established how the formalization of indefinites and (non-)specificity provided by 2TS offers valuable insights into this linguistic domain. We hope that this dissertation has shown that the study of indefinites is anything but indefinite.

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