A Logic for Contextually Restricted Quantification

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Abstract
This note presents, motivates and details, a logic —model and proof theory— for a first order language with contextually restricted quantification.

1 Introduction

It can hardly be denied that the interpretation of natural language expressions is context dependent, and that the interpretation of various kinds of noun phrases —quantifying noun phrases, definite descriptions, as well as indefinite descriptions— can be contextually determined in substantial ways.[1] The philosophical and linguistic literature consequently features intriguing discussions about the nature and location of such contextual restrictions.[2] This present note is not, however, concerned with the philosophical reflection on this contextual influence, nor with the peculiarities of contextual restriction in the syntax and semantics of natural languages. The note is concerned with providing a formal language that allows us to formulate the dependencies and study their logical impact. There are, to my knowledge, only a few previous proposals for such a formalization, and these are, arguably, not entirely conclusive.[3]

In order to obtain a logic of context dependent quantification this note first presents a language of first order predicate logic with contextually restricted quantification, and next supplies it with both a model- and a proof theory. The system will be illustrated by means of some linguistic applications, and it is also shown that the logical system properly extends and properly improves upon,

1. Westerståhl 1985; von Fintel 1994; Roberts 1995; Neale 1990; Stanley & Szabó 2000; Schwarzschild 2002; Martí 2003; Glanzberg 2006; Etxeberria & Giannakidou 2010
2. There is the question whether contextual dependence should be marked in the syntax, explicitly, or implicitly, and where, and also whether such data are cross-linguistic valid. (Neale 1990; Reimer 1998; von Fintel 1994; Stanley & Szabó 2000; Etxeberria & Giannakidou 2010). There are also foundational questions such as whether contexts are sets, properties or situations perhaps(Westerståhl 1985; Reimer 1998; Recanati 1996; Stanley & Szabó 2000; Schwarz 2012), and whether the determination of the context is objective or intentional. (Gauker 1997; Bianchi 2006)
3. Kuroda 1982; Bonomi 1998; Francez 2014

in particular, Nissim Francez' previous proposal for a proof system that is very much alike in spirit.

2 Contextual Restriction in Predicate Logic

Contextually restricted quantification will be cast in a language PL of first order predicate logic that is built up, in the usual way, from a set of variables $x, y, \ldots \in V$, individual constants $n, m \in N$, sets of relational constants $R, S, \ldots \in R^j$ of arbitrary arity j, and the logical constants $\neg, \wedge, \rightarrow, =, \exists, \forall$.[4] A distinctive feature is that our quantifiers are (pre-)superscripted with indices $0, 1, \ldots$. The indices on the quantifiers serve to distinguish various contexts, or contextual restrictions, on the domain of quantification.[5]

The predicate logic language for extensionally restricted quantification can be defined, as follows, in Backus-Naur style.

Definition 1 (Language of PL^{ERQ}) *for $x \in V$, $n \in N$, $R \in R^j$ and $i \in \mathbb{N}$*

$$t ::= x, n$$
$$\phi ::= Rt_1 \ldots t_j \mid t_1{=}t_2 \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \rightarrow \phi) \mid {}^{i}\exists x\phi \mid {}^{i}\forall x\phi$$

One may read ${}^{i}\exists x\phi$ as saying that in context i there is an x such that ϕ, and ${}^{i}\forall x\phi$ as saying that for all x in context i ϕ holds.

In the extensional system presented in this note, contexts are just sets of individuals from a domain D, which is equated with a default context C_0. All contexts consist of individuals actually existing.

$$\textit{for any } i\textit{: } C_i \subseteq C_0 = D$$

Models for our language consist of such an indexed collection $\{C_i\}$ of such contexts, and an interpretation function I giving an extensional interpretation of the individual and relational constants in our language. So if $n \in N$, $I(n) \in D$, and if $R \in R^j$, then $I(R) \subseteq D^j$ is a set of j-tuples of individuals.

Definition 2 (Models of PL^{ERQ}) *A model M for PL^{ERQ} is a pair $\langle\{C_i\}, I\rangle$, with $\{C_i\}$ and I as detailed above.*

The formulas of our language are evaluated in a model and relative to a variable assignment function. A Tarski-style satisfaction relation $M, g \models \phi$ is defined, which reads that ϕ is satisfied in model M and relative to variable assignment

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4. I would rather favor a declarative, quantificational, treatment of names $\mathsf{N} \in \mathbf{N}$, so that $\mathsf{N}x\,\phi$ is a formula declaring N to be an individual x, so-named, such that ϕ. I will not, however, detail or motivate such a treatment any further here.

5. This method of indexing, as originally proposed in (Westerståhl 1985), aligns with many of the sources mentioned in the previous footnotes, but for the fact that not all authors agree that the index is associated with the quantifier. It appears to me that there are no convincing reasons for putting them elsewhere, logically speaking.

g. Atomic formulas are satisfied if its constituent terms denote objects that have the properties ascribed, like standing in a certain relation, or like being one and the same object. The clauses dealing with the propositional connectives $\neg$, $\wedge$ and $\rightarrow$ are standard. We will comment on the clauses for the quantifiers below.

Definition 3 (Interpretation of PL^{ERQ})

$$
\begin{array}{rl}
M, g \models Rt_1 \ldots t_j & \textit{iff } \langle g(x_1), \ldots, g(x_j)\rangle \in I(R) \\
M, g \models t_1 = t_2 & \textit{iff } g(x_1) = g(x_2) \\
M, g \models \neg\phi & \textit{iff } M, g \not\models \phi \\
M, g \models (\phi \wedge \psi) & \textit{iff } M, g \models \phi \textit{ and } M, g \models \psi \\
M, g \models (\phi \rightarrow \psi) & \textit{iff if } M, g \models \phi \textit{ then } M, g \models \psi \\
M, g \models {}^{i}\exists x\phi & \textit{iff there is } d \in C_i\textit{: } M, g[x/d] \models \phi \\
M, g \models {}^{i}\forall x\phi & \textit{iff for all } d \in C_i\textit{: } M, g[x/d] \models \phi
\end{array}
\qquad
\begin{array}{rl}
[t]_{M,g} & = g(t) \textit{ if } t \in V \\
 & = I(t) \textit{ if } t \in N
\end{array}
$$

Suitable notions of validity and entailment are defined in the usual fashion.

Definition 4 (Validity in PL^{ERQ}) *A sequence of premises $\phi_1, \ldots, \phi_n$ entail a formula ψ, $\phi_1, \ldots, \phi_n \models \psi$, iff for all M and g, if $M, g \models \phi_1 \ \ldots \ M, g \models \phi_n$, then $M, g \models \psi$. We say that ψ is valid iff $\models \psi$.*

It is useful to be able to state of the value of a variable, declared in one context, that it also figures in another. The following notation convention helps to express precisely this:

Utility 1 (Existence Restrictions) ${}^{i}\!E x := {}^{i}\exists y\, x{=}y$

Like we said, all individuals, in any context C_i, are stipulated to exist in the default context. So here is our first, schematic, theorem:

Theorem 1 (Transparency of Contexts, Model-Theoretic)

$$\models {}^{i}\forall x\, {}^{0}\!E x$$

3 Contextual Restriction at Work

The current system extends standard predicate logic in that it allows us to model the kind of contextually restricted quantification familiar from the literature and also to express constraints on, or properties of, those contexts. For our more logical purposes, it may be worthwhile to observe, that, relative to a fixed domain, whether it is restricted or not, all familiar inferences are valid. I hope this is intuitively clear, and, if not, it can be proved formally relatively easily. However, if multiple quantifiers relate to possibly different domains, then, without any further constraints, hardly any conclusions follow, besides, of course, the universally valid ones.

Consider the following line of reasoning. If ${}^{i}\forall x(Fx \rightarrow Gx)$ and ${}^{j}\exists yFy$, then ${}^{k}\exists zGz$. If there are no restrictions on the quantifiers, so if $i{=}j{=}k{=}0$, then this pattern of reasoning is valid, of course. Also if the three quantifiers are restricted, but restricted to the same context, so if $i{=}j{=}k$, the very same pattern of reasoning is valid, too. But if the first premise quantifies over a contextual domain C_i that is independent from the contextual domain C_j of the second premise, then—without any further constraints—, nothing follows. If, in Italy, say, every scholar knows Dante, and if, in France, there is a scholar, this does not logically imply that, in Germany, someone knows Dante. Some such could only be inferred with additional premises, including those that tells us more about the three contexts involved. This brings us to the second benefit of our system.

The present system enables us to delineate the contexts quantification can be relative too. We can, for instance, stipulate that ${}^{0}\forall x(Cx \leftrightarrow {}^{i}\mathsf{E}x)$ thereby determining all quantified expressions that depend on context i to be restricted to the C's, the Canadians, say, or to whatever condition one might want it to be related too.[6] With such stipulations, restricted quantifier readings familiar from the literature can be made fully sense of. Every time a domain restricting context is specified or referred to in the literature, we can formulate that constraint as an assumption in our language.

It is important to observe, however, that we can also do without such stipulations. If someone says "Everybody danced, and nobody complained." this is most naturally construed as quantifying twice about a contextually restricted domain of individuals C_i which we may know nothing about, but for the fact that it is said to be one such that none of them did not dance, and everyone of them did not complain. The two formulas ${}^{i}\forall xDx$ and $\neg{}^{i}\exists xCx$ express precisely this. The two statements can, if need be, be supplemented further by additional assumptions or claims. For instance, it is not at all irrelevant to assume that ${}^{i}\exists x\, x{=}x$, because otherwise the two previous claims would be vacuously true. One could also possibly add, more substantially, for instance, that, in i, there are the people who visited the party last night. But note that while such supplements are, of course, indispensable, pragmatically speaking, they, equally obviously, need not be supplied by the formulas or linguistic expressions themselves.

Restricted domains also allow us to make proper sense of definite, and also indefite, noun phrases. An expression of the form "the A" or "$\imath x\phi x$" can be understood, upon its Russellian analysis, to denote the unique individual that is A, or that satisfies ϕ.[7] Its meaning can be explained in full sentential constructions so that sentences of the form "The A B" say that there is a unique A and that it is B. Formally:

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6. Note that such a stipulation would then also allow us to eliminate the context dependence of any quantified expression ${}^{i}\mathrm{Q}x\phi$ by replacing it by an unrestricted quantifier ${}^{0}\mathrm{Q}x\phi'$, where ϕ' is obtained from ϕ by adding the explicit restriction that Cx.

7. Existence and uniqueness of such an individual are thereby asserted, according to Russell, or presupposed, according to Strawson. (Russell 1905; Strawson 1950, among many others.) There is no need to enter the enduring and rather convoluted debate about this here.

Utility 2 (Russellian Description)

$$^{i}\imath x\phi\,\psi := {}^{i}\exists x({}^{i}\forall y([y/x]\phi \leftrightarrow x{=}y) \wedge \psi)$$

For most uses of definite descriptions in natural language, such a Russellian analysis has often been observed to only make sense if they are evaluated relative to contextually supplied domains of quantification, or so that in particular the universal quantifier is suitably restricted. (Neale 1990) Our contexts model precisely that. Thus, if it is said that ^{i}*the blonde smiles*, this can be rendered as $^{i}\imath xBx\,Sx$ which upon its Russellian expansion comes out as $^{i}\exists x({}^{i}\forall y(By \leftrightarrow x{=}y) \wedge Sx)$. If, as is required by this formula, there is indeed such a unique blond in i the description can be understood as a term that denotes that individual.

More generally any form of quantification can thus be rendered contextual, by (i) relating the quantifier to some contextual domain C_i and (ii) possibly define to be in C_i to be equivalent with satisfying whatever contextual requirement one may want to impose on it. Roger Schwarzschild has argued that it is linguistically expedient to have such a restricted interpretation of indefinite descriptions, too. (Schwarzschild 2002)

4 Proof Theory for Contextually Restricted Quantification

The proof theory for our language can be presented in the form of a natural deduction system along the lines of (Fitch 1952). Such a system defines the *derivability of a conclusion* ψ from a, possibly empty, series of premises, formally: $\phi_1, \ldots, \phi_n \vdash \psi$.

A *derivation* here is most generally an *enumerated* and *labeled list* of *formulas*, in which each formula is either an explicit assumption, or a formula derived from what has been established earlier in the list according to the natural deduction rules which will be specified in due course. The labels always indicate by what rule a formula is inserted on a line, and, if relevant, with unambiguous reference to the preceding lines that provide the input required for applying the rule. The rules ensure that what is obtained on a line is, if not an assumption, something that logically follows from what has been established before.

As we will see, certain rules may, as it is called, discharge specific assumptions. What is withdrawn then, on some line n, is the last pending assumption, on some previous line m, and it means that the formula on that line m, and those on the lines below it—until, but not including, the current line n—, are no longer available for use below line n. In a visual display they are then *bracketed out*. All assumptions in a derivation that are not discharged this way are pending, and better be explicitly included among the initial list of premises.

More formally, but still somewhat incompletely, we can now define:

Definition 5 (Derivability in PL^{ERQ}) *A conclusion ψ is derivable from a series of premises $\phi_1, \ldots, \phi_n$, $\phi_1, \ldots, \phi_n \vdash \psi$, iff there is a valid derivation:*

$$
\begin{array}{lll}
1. & \phi_1 & [\text{ass.}] \\
 & \vdots & \\
n. & \phi_n & [\text{ass.}] \\
 & \vdots & \\
 & \psi &
\end{array}
$$

without any pending assumptions after line i.

The definition is said to be "incomplete" because the natural deduction rules have not been specified yet. The required specification of the classical rules is given in appendix A; the rules dealing with contextually restricted quantification are presented and discussed in the remainder of this section.

The proof rules for contextually restricted quantifiers are obtained by relativizing the familiar rules of quantification to the contexts they are meant to be sensitive to. The rules of *existential generalization* and *universal instantiation* have to be related to variables that are secured to inhabit the contexts that quantification is relative to. Such a guarantee comes from the rules for *universal generalization* and *existential instantiation*, which induce sub-derivations in which variables are declared to inhabit the contexts quantification is relative to. Guiding notion is that of a variable x that, in certain parts of the proof, counts as being declared in context i.

The rules, with the necessary restrictions and licenses, read as follows.

∃-Introduction (I∃)

$$
\begin{array}{lll}
 & \vdots & \\
m. & [z/x]\phi & \\
 & \vdots & \\
n. & {}^{i}\exists x\phi & [\text{I}\exists,\ m]
\end{array}
$$

Variable z must count as declared in i at line m.

∃-Elimination (E∃)

$$
\begin{array}{lll}
 & \vdots & \\
l. & {}^{i}\exists x\phi & \\
 & \vdots & \\
m. & [z/x]\phi & [\text{ass.}] \\
 & \vdots & \\
n\text{-}1. & \psi & \\
n. & \psi & [\text{E}\exists,\ l]
\end{array}
$$

Variable z may not occur free in other assumptions, ϕ or ψ. It counts as declared in i from line m to n.

If something, z, has been established to be ϕ, and if it has also been established that this z lives in context i, then we can conclude that something in i is ϕ. Conversely, if it has been established that something in context i is ϕ, and if we are able to conclude that ψ holds whenever anything in i is ϕ, then ψ, of course, must hold. These rules are actually the familiar ones, save for the restriction to being in i. Observe that if we, as usual, consider the quantifiers interdefinable, so that ${}^{i}\forall x\phi$ equals $\neg{}^{i}\exists x\neg\phi$, we can validate, the following rules for ∀.[8]

8. Appendix C shows how this is accomplished.

∀-Introduction (I∀)

$$\begin{array}{lll} & \vdots & \\ \text{m.} & z = z & \text{[ass.]} \\ & \vdots & \\ \text{n-1.} & [z/x]\phi & \\ \hline \text{n.} & {}^{i}\forall x\phi & [\text{I}\forall] \end{array}$$

Variable z may not occur free in other assumptions, or in ϕ. It counts as declared in i from line m till n.

∀-Elimination (E∀)

$$\begin{array}{lll} & \vdots & \\ \text{m.} & {}^{i}\forall x\phi & \\ & \vdots & \\ \text{n.} & [z/x]\phi & [\text{E}\forall,\ \text{m}] \end{array}$$

Variable z must count as declared in i at line n.

If we can conclude that z is ϕ on the mere assumption that it is there in context i, then we can conclude that every z in i is ϕ. This is, of course, the standard logical inference rule, but now relativized to contexts. Conversely, if everything in i is ϕ, then so is any z, provided that z has been established to be in i.

In our extensional framework it is assumed that all variables count as declared in the default context, and the same goes for individual constants. This assumption grounds our second theorem.

Theorem 2 (Transparency of Contexts, Proof-Theoretic)

$$\vdash {}^{i}\forall x\, {}^{0}Ex$$

The very same assumption also entails that, if there is no dependence on contexts other than the default one, our system of rules is classical. Soundness and completeness of our calculus is thereby easily established. (See appendix B for an outline of the proof.)

The alert reader may note that the notion of a variable or name being declared in a context, and that of having a value, or existing, in a context, do not entirely coincide. This, however, is a spurious distinction in the proof theory. If a variable x or name n is not declared in a context C_i, and if it is equated with a variable y that is declared in that context, then we can effectively count the variable or name as being declared in that context, too. The Leibniz inference rule guarantees that whatever holds of y, holds of x, or of n, and vice versa. So if we have established that y exists in i, then so does x, or n, for that matter. As a consequence, if a context C_i is known to be subsumed by another context C_j, so that, semantically, $C_i \subseteq C_j$,[9] this does not entail that a variable x that counts as declared in i also counts as declared in j, but it does entail that it can be equated with some variable y that is declared in j, and so that if we can deduce anything about y, because it belongs to context j, it will also ipso facto hold about x, and vice versa.

Note, as well, that when x counts as declared in i, we have $x{=}x$ from which we can deduce ${}^{i}\exists y\, x{=}y$ which equals ${}^{i}\mathsf{E}x$; conversely, when ${}^{i}\mathsf{E}x$, then,

9. The relevant situation is rendered by the assumption that ${}^{i}\forall x\, {}^{j}\mathsf{E}x$.

effectively, x may count as being declared in i in a subsequent sub-derivation. The same goes for individual constants. When ${}^{i}\mathsf{E}n$, then, effectively, n counts as declared in i. The rules for ${}^{i}\exists$-introduction and ${}^{i}\forall$-elimination may, thus, indirectly, apply to individual constants, or *names*, too, that is, as long as they are stipulated to have a value in i. Note, finally, that if $\mathsf{E}n$ (i.e., $\exists y\, n{=}y$), then by $\exists$-elimination and the identity rules we have that $n{=}n$.

$$\begin{array}{lll}
1. & \exists y\, n = y & [\text{ass.}] \\
\ulcorner 2. & \quad n = y & [\text{ass.}] \\
\mid 3. & \quad n = n & [\text{L, 2, 2}] \\
\hline
4. & \quad n = n & [\text{E}_{\exists}\text{, 1}]
\end{array}$$

This is why it suffices to have the Identity Rule license self-identity of variables only: indirectly it applies to individual constants, too.[10]

5 Francez' Calculus of Restricted Quantification

In Francez 2014, §2, a proof theory is proposed for contextually restricted quantification in a first order language that is similar to the one proposed here, but regimented differently. Nissim Francez defines a calculus issuing whether $\Gamma \vdash_c \phi$, i.e., when ϕ is derivable from Γ in context c, where this context c constrains the possible values that quantified variables in Γ and ϕ may have. A context c is defined as a collection of formulas, each one of which has one free variable only, and the formulas are thereby, collectively, and proof-theoretically, taken to restrict the possible ranges of the variables quantified over in Γ and ϕ.

Without going into details, it may, I hope, be obvious from this circumscription that whatever results from Francez' system, can be captured in ours in a rather transparent way. For any quantified expression $\mathsf{Q}x\phi$ in Francez' system, if it is employed in a deduction involving any context c there, it can be equated with one in ours as ${}^{i}\mathsf{Q}x\phi$, if we add the explicit assumption $\forall x(Cx \leftrightarrow {}^{i}\mathsf{E}x)$, where Cx equals whatever Francez' context c brings to bear on the possible values of x. If things work in Francez' system the ways things are supposed to work classically, then a translation along these lines should capture all of it.

Francez presents his system to argue for, and favor, a proof-theoretic approach to natural language semantics, as against a model-theoretic one. Even though I am quite sympathetic to this enterprise, I fear the contextually restricted interpretation of quantifiers does not constitute a convincing argument. The model-theory we have defined in section 2 is sound and complete relative to

10. The Leibniz Rule could also be stated for identified variables only, if we had adopted the quantificational, declarative, treatment of names mentioned above.

the natural deduction system that we presented in section 4, so it should be immaterial logically speaking which method to adopt.[11]

Francez also claims that it is a benefit of his proof system that it obeys what he calls the *CIP*, the *context incorporation principle*:

> *[F]or every quantified sentence S depending on a context c, there exists a sentence S', the meaning of which is independent of c, s.t. the contextually restricted meaning of S is equal to the meaning of S'. Thus, the effect of a context can always be internalized. The current model-theoretic accounts of contextual domain restriction do not satisfy CIP, in that they imply intersection of some extension with an arbitrary subset of the domain that need not be the denotation of any NL-expression.* (Francez 2014, p. 249)

I find it hard to make proper sense of this. Surely it is convenient if we are able to express everything we want to express in a context independent way. One might, with good reason, call this *Frege's ideal*. It is, however, at best, a convenient result, if a result at all. In Francez' system, however, it is a built-in assumption, one that arguably limits the scope of its work. Contexts are, in Francez' system, *defined* by his open formulas, so, in Francez' approach, unspecifiable contexts do not exist *by definition*. This is a point that is surely not independently argued for in the paper, and actually it might be a severe limitation, as we will argue below.

As against model theoretic semantics (*MTS*) approaches to context dependence, Francez also levels the following objection.

> *(...) [T]he* consequences *that can be drawn from the contextually varying meaning of an (affirmative) sentence, namely (affirmative) sentences* entailed *by a sentence with contextually varying meaning, which themselves have meanings varying with context, are hardly ever considered in MTS-based discussions.* [p. 256]

I believe the rather straightforward semantics spelled out in the second section of the current note proves this objection to be mistaken.

In the system presented in section 2 of this note, it is possible to give partial specifications of a context, for instance that a context C_i contains two individuals one of which loves the other.[12] Such enables us to legitimately speak of *the lover* in C_i and *the beloved one* there. Such are typically the kind of things we need to know about contexts, and most of the times perhaps all we need to know. Such a partially specified context, is, however, not available in Francez' system. Likewise, it can be useful to know that one context C_i is included in an other one C_j, as expressed by ${}^i\forall x {}^j\exists x$, even if we don't know how the two context are actually specified. Again, it is impossible to model some such in Francez' system.

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11. Normally the notions of soundness and completeness are defined the other way around, but in this case it seems to be more appropriate to establish this converse relation.

12. We can, e.g., stipulate, or assume, that ${}^i\exists x {}^i\exists y(Lxy \wedge \neg Lyx \wedge {}^i\forall z(z{=}x \vee z{=}y))$.

There are, moreover, some logical problems with the proposal by Francez. Francez' contexts essentially restrict the domain of a quantifier by explicitly constraining the values of the variables bound and for this reason the system does not allow any substitution of bound variables. The fact that we cannot always allow substitution of bound variables is, logically speaking, problematic, to put it relatively mildly. Relatedly, there is the problem that the specifications of the contexts must be interpreted in a context independent way themselves, otherwise there is no obvious way to save and rescue the cherished CIP. It is not clear, however, how we must regiment this use of assumed context independent assumptions, in an inference format that is meant to interpret them context dependently.

There are, to my knowledge, only two other proposals for a logic dealing with restricted quantification, Kuroda 1982 and Bonomi 1998. Kuroda's project is, nominally, similar to mine, in that he advocates an indexed predicate calculus, the indices in which serve to be indicative of varying contexts of interpretation. However, his aims pertain to non-extensional types of discourse, that involve other worlds, or '*mini-worlds*', and he does not actually present any formal proposal.[13]

Bonomi's project is also close to mine, when he observes, for instance, that "(...) different quantifiers in the same sentence may refer to different domains. For this purpose, a set **I** of indices is added to the usual vocabulary of the first order language." (Bonomi 1998, p. 471–2). The indices are supposed to relate to contexts which are full-blown, but partial, submodels of an extensional model. Unfortunately, Bonomi also offers only a "sketch of a formalism". (Bonomi 1998, § 6) It is not obvious how it should be spelled out.[14]

6 Conclusion

In this note I have presented a calculus for contextually restricted quantification, and sketched some applications. I have detailed a model-theoretic interpretation, as as well as a proof theory, the suitability of which is outlined in the appendix. The proof system was also compared with a proof calculus proposed by Nissim Francez, which is similar in outline and spirit, but which is heavily restricted by the assumption that its contexts are syntactically defined; Francez' system was also suggested to be superior to any model-theoretic approach, a suggestion I think failed solid motivation.

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13. "[T]he (...) paper is intended only to set out a programme, not as a summary of an accomplished work. (...) No semantic rule in the strict sense of formal logic is formulated. (Kuroda 1982, p. 45-6)

14. The semantics suggested in the paper involves a specification of some partial *extensions* and *counterextensions* of the relation expressions and, apparently, an evaluation of the formulas that is partial, with a positive and negative dimension. Footnote 2 does refer to some earlier work, "formerly discussed from a different standpoint," but these references are by no means conclusive, either.

My main, principal and principled, objection to Francez' proposal is that it is not just built on a convenient and simplifying assumption, but it is essentially relying on it. Without a syntactic specification of contexts, there simply are no contexts in Francez' system—just none, by definition—, while in natural and logical practice we may still be able to reason with contexts that are only partially specified, or not even specified at all.

Two pressing issues have not been addressed in this note. First, as Craige Roberts, among many others, has argued, natural language discourse can be used to change contexts, also those that affect domains of quantification. (Roberts 1995) Moreover, various theories of dynamic semantics have emphasized obviously anaphoric uses of definite descriptions, so uses which also ask for a contextual determination. It must for now remain an open question how this type of discourse dynamics has to be incorporated in our system.

Second, Sige-Yuki Kuroda, Kai von Fintel and Jason Stanley and Zoltán Szabó have, among various others, argued that contexts can be quantified, perhaps indirectly. (Kuroda 1982; von Fintel 1994; Stanley & Szabó 2000) Von Fintel discusses the following example, attributed to Irene Heim.

(24) Only one class was so bad that no student passed the exam.

Apparently this must be read as saying that in one class all students, in that class, failed, while in any other class at least one student, in that class, passed. The remedy, consists in allowing for a kind of functional dependencies, in the proposals by these authors indicated by compound indices. Thus, they propose to (partially) formalize (24) as (25).

(25) Only one class x was so bad that $\text{no}_{f(x)}$ student passed the exam.

The idea here is that the function f assigns a domain of quantification to any x that is a class, and more in particular the set of individuals belonging to that class. It may go without saying that the very same method can also be incorporated in our system.

In my first footnote I mentioned several other pressing issues relating to context dependence and this note is obviously incapable of fully answering them. I nevertheless hope that a decent and well-defined classical formal framework as presented in this note will help in guiding subsequent discussions along tractable and verifiable directions.

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Appendix A. Proof Rules for the Propositional Connectives

The derivation rules for propositional logic are displayed in a schematic form and their use is briefly explained.

Assumption (Ass)	
$\vdots$	[Any assumption can be made any time.]
n. ϕ [ass.]	

The propositional connectives are governed by rules of elimination (aka 'use') and introduction. If a conjunction has been established, say at some previous line m, then either conjunct ('left' or 'right') can be inferred from it.

$\wedge$-Elimination (E$\wedge_l$)

	$\vdots$	
m.	$(\phi \wedge \psi)$	
	$\vdots$	
n.	ϕ	[$E_{\wedge_l}$, m]

$\wedge$-Elimination (E$\wedge_r$)

	$\vdots$	
m.	$(\phi \wedge \psi)$	
	$\vdots$	
n.	ψ	[$E_{\wedge_r}$, m]

Of course, the previously established conjunction may not be an assumption that is withdrawn, or depend on assumptions that have been withdrawn in the meantime. One may derive the conjunction $(\phi \wedge \psi)$ of any two previously established propositions ϕ and ψ.

$\wedge$-Introduction (I$\wedge$)

	$\vdots$	
l.	ϕ	
	$\vdots$	
m.	ψ	
	$\vdots$	
n.	$(\phi \wedge \psi)$	[$I_\wedge$, l, m]

Again, the previously established propositions may not depend on assumptions that are withdrawn. (I will henceforth refrain from making this proviso explicit.)

$\rightarrow$-Elimination (E$\rightarrow$)

	$\vdots$	
l.	$(\phi \rightarrow \psi)$	
	$\vdots$	
m.	ϕ	
	$\vdots$	
n.	ψ	[$E_\rightarrow$, l, m]

$\rightarrow$-Introduction (I$\rightarrow$)

	$\vdots$	
m.	ϕ	[ass.]
	$\vdots$	
n-1.	ψ	
n.	$(\phi \rightarrow \psi)$	[$I_\rightarrow$]

there are no pending assumptions between lines m and n.

An implication $(\phi \to \psi)$ established at some line l can be used if its antecedent ϕ has been established as well, say, at line m, and it licenses one to conclude to its consequent ψ, at line n. We conclude to such an implication by hypothetically assuming its antecedent ϕ on some line m, and then, in the context of that assumption (and in the context of the lines above m) validly derive some conclusion ψ. If this has been established then—upon withdrawing that assumption, and everything that is based on it—one may conclude that $(\phi \to \psi)$, in that very same context again.

¬-Elimination (E¬)

$\vdots$

l. $\neg\phi$

$\vdots$

m. ϕ

$\vdots$

n. $\bot$ [E¬, l, m]

¬-Introduction (I¬)

$\vdots$

m. ϕ [ass.]

$\vdots$

n-1. $\bot$

n. $\neg\phi$ [I¬]

there are no pending assumptions between lines m and n.

A negation $\neg\phi$ excludes that ϕ, so if we establish $\neg\phi$, on some line l and ϕ on some line m, then we reach a dead end, marked by the falsum ('$\bot$') in the negation elimination rule. If such a falsum ('$\bot$') marks the dead end of a line of hypothetical reasoning, we may conclude that the assumption that it is based upon is excluded, so that we can conclude to the negation of that assumption.

¬¬-Elimination (E¬¬)

$\vdots$

m. $\neg\neg\phi$

$\vdots$

n. ϕ [E¬¬, m]

If it is excluded that ϕ is excluded, as $\neg\neg\phi$ says, we cannot but agree to accept ϕ, even if we fail a direct proof of it. (For the latter reason the rule is not universally agreed upon.) For the first order system, we also need two, classical, rules dealing with identity.

Leibniz (L)

$\vdots$

l. $t_1 = t_2$

$\vdots$

m. $[t_1/z]\phi$

$\vdots$

n. $[t_2/z]\phi$ [L, l, m]

[The terms t_1 and t_2 must be free for the variable z in ϕ.]

Self-Identity (=)

$\vdots$

n. $x = x$ [=]

The variable x must count as declared at line n.

Appendix B. Normalization in the Extensional Calculus

We define a *normalization*, or *decontextualization*, of our contextually restricted quantifiers by making their context-dependence explicit in the language. For this purpose we avail ourselves of a set of one place predicates $\{P_i\}$, not used otherwise, that we can assume to be uniquely related to the contexts $\{C_i\}$ that we employ. We define the normalization ϕ' of any formula ϕ, by a simple recursive definition, which hosts only two characteristic clauses. Assuming that P_i is the one place predicate uniquely associated with index i, these are the following.

$$^i\exists x\phi' = \exists x(P_i x \wedge \phi') \qquad ^i\forall x\phi' = \forall x(P_i x \to \phi')$$

It may be obvious that, for any formula ϕ, the normalization ϕ' contains no contextual restrictions other than the default ones, so that the formula is classical, and has a classical model- and proof theory.

We can use the normalization procedure to show soundness and completeness of our system, in outline, through the following series of equations.

Theorem 3 (Soundness and Completeness of PL^{ERQ})

$$\phi_1, \ldots, \phi_n \models_{rqpl} \psi \text{ iff}_a\ \phi'_1, \ldots, \phi'_n \models_{rqpl} \psi' \text{ iff}_b\ \phi'_1, \ldots, \phi'_n \models_{cpl} \psi' \text{ iff}_c$$
$$\phi'_1, \ldots, \phi'_n \vdash_{cpl} \psi' \text{ iff}_d\ \phi'_1, \ldots, \phi'_n \vdash_{rqpl} \psi' \text{ iff}_e\ \phi_1, \ldots, \phi_n \vdash_{rqpl} \psi$$

The first equivalence (iff_a) holds that the normalization procedure is effectively meaning preserving in the sense that if, and only if, any model $M = \langle\{C_i\}, I\rangle$ satisfies any formula ϕ it can be transformed into a model M' that models ϕ' by adding, to M, the interpretation $I(P_i) = C_i$, for any context $C_i \in \{C_i\}$.

The second equivalence (iff_b) follows from the fact that the semantics of our language with no other contextual restriction than the default one is classical. The third equivalence (iff_c) follows from the soundness and completeness of classical logic. The fourth equivalence (iff_d) follows from the fact that our proof theory for the context free fragment of the language is classical.

The last equivalence (iff_e) holds because any application of a proof rule for a context-dependent quantifier can be replaced by an application of the corresponding proof rule for the normalized quantifier, with the proviso that any time a variable z was taken to count as declared in a context i, it is now associated with the condition $P_i z$. Thus:

Normal ∃-Introduction (I∃)

	$\vdots$	
m.	$[z/x]\phi'$	
	$\vdots$	
n.	$P_i z$	[R, decl. z]
n′.	$[z/x](P_i x \wedge \phi')$	[I∧, n, m]
n″.	$\exists x(P_i x \wedge \phi')$	[I∃, n′]

Normal ∃-Elimination (E∃)

	$\vdots$	
l.	$\exists x(P_i x \wedge \phi'$	
	$\vdots$	
m.	$[z/x](P_i x \wedge \phi')$	[ass.]
m′.	$P_i z$	[E∧, m]
m″.	$[z/x]\phi'$	[E∧, m]
	$\vdots$	
n′.	ψ'	
n″.	ψ'	[E∃, l]

Appendix C. Derivability of the Rules for ∀ from those for ∃

∀-Introduction (via E∃)

	$\vdots$	
m.	${}^i\exists x \neg\phi$	[ass.]
m′.	$[z/x]\neg\phi$	[ass.]
m″.	$z = z$	[=]
	$\vdots$	
n″-1.	$[z/x]\phi$	
n″.	$\bot$	[E¬, m′, n″-1]
n‴.	$\bot$	[E∃, m]
n⁗.	$\neg{}^i\exists x \neg\phi$	[I¬]

Variable z may not occur free in other assumptions, or in ϕ. It counts as declared in i from line m′ till n‴.

∀-Elimination (via I∃)

	$\vdots$	
m.	$\neg{}^i\exists x \neg\phi$	
	$\vdots$	
n.	$[z/x]\neg\phi$	[ass.]
n′.	${}^i\exists x \neg\phi$	[I∃, n]
n″.	$\bot$	[E¬, m, n′]
n‴.	$[z/x]\neg\neg\phi$	[I¬]
n⁗.	$[z/x]\phi$	[E¬¬, n‴]

Variable z is assumed to be declared in i at line n.