Formalizing the FLINT Ontology

Building an action-oriented formal language
for the interpretation of normative texts

MSc Thesis *(Afstudeerscriptie)*

written by

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Abstract

This thesis provides a full formalization of the formal language for the interpretation of normative texts of the TNO Norm Engineering Project. The FLINT language is built to give a standardized representation of the interpretations of normative texts (laws, contracts, guidelines etc.).

The approach that FLINT takes towards modeling norms is an action-oriented approach. The idea is that norms tell us when certain normative actions can be performed, and what the consequences of those actions are. When a normative action is carried out, this results in a transition between normative states. These normative states describe facts that are of normative importance, such as whether an agent has a duty to perform an action.

With the use of the FLINT formalization one should be able to answer the following key questions in a normative state: what can/should I do to others, what can/should others do to me, under what circumstances can/should we do that, and, most importantly, what happens when we do that? The FLINT formalization provides a computational theory of norms, upon which a computationally implementable language of norms can be based.

At the start of this project, FLINT was still a semi-formal language, consisting only of an ontology. This thesis project built a full formalization of the FLINT ontology by describing the language in a syntax, and giving this language meaning with a formal semantics.

This thesis therefore contributes towards the literature on the formalization of norms. The formalization, taking the FLINT ontology as a starting point, ensures that a systematic comparison with other formalizations of norms can be made.

"La liberte est le droit de faire ce que les lois permettent."

— MONTESQUIEU
DE L'ESPRIT DES LOIS (1748)

Contents

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Chapter 1

Introduction

The *Rule of Law* is a core legal concept that requires the law to be (1) certain, so that anyone should be able to understand the law (2) predictable, so that it is clear which actions lead to what outcomes under the law and (3) reasonable, government officials should apply the law in a reasonable manner (Waldron, 2008, 2010).

In one of the largest scandals in the Dutch political history, the so called 'childcare benefits affair', these principles were grossly offended.[1] A large group of parents was unfairly accused of committing fraud with their applications for childcare benefits. The Dutch childcare benefits law was interpreted by government officials in such a way that small formal mistakes were enough to demand a total refund of the received benefits. The obligation to refund a large sum of money at once, resulted in many of the unfortunate parents going into debt, destabilizing their lives.

In 2021 the analysis of this scandal was published in the report 'Unprecedented injustice' (Kinderopvangtoeslag, 2021). This report called for a variety of measures to ensure this won't ever happen again.

1.1 (Semi)-automated decision-making tools

One of the areas where principles of the *Rule of Law* currently are in jeopardy, is the clarity of the reasoning process of (semi)-automated decision-making tools used by governmental institutions. These tools are necessary as large amounts of decisions have to be made every day.[2] The decision-making process should be understandable for all parties involved, especially citizens. To make the reasoning process of (semi)-automated decision-making tools more clear, the norms upon which the decisions are grounded should be encoded into the IT-systems in a systematic and correct manner. This is actually a difficult task, because normative texts are *prima facie* not easy to understand. Several different interpretations of normative texts can exist. This can be problematic if there is no documentation available about which interpretation is eventually encoded into a decision-making tool (van Doesburg et al., 2016; van Engers & van Does-

[1] "A black page in Dutch governmental history" (Kabinet, 2021)

[2] This is not only an issue for governmental institutions but for any large organization with its business processes encoded into IT-systems (Van Doesburg & Engers, 2016).

burg, 2016). When there is no link between (1) the sources of normative texts, (2) the interpretations of normative texts, and (3) the codifications of these interpretations into IT-systems, three issues arise:

1. No transparency of the reasoning in the decision-making process:
   If there is no systematic way to get from the source of a normative text to the codification into an IT-system, the decision-making process becomes opaque. When explaining the decision-making process of an IT-system, the system should be able to show which interpretations of normative texts are ground for this decision. The interpretations of normative texts provide reasons for why certain decisions are taken. When we can only show how an IT-system reaches a certain output but cannot provide reasons based on interpretations of norms, the decision making process misses normative substantiation.

2. Possible inconsistencies in the interpretations of the law:
   Due to the fact that the interpretation process is a layered procedure, mistakes might occur with the codification of not fully formalized interpretations of normative texts. It should be prevented that IT-experts are unknowingly taking extra interpretation steps, because there is no intermediate step linking the normative source to a fully formalized interpretation such that this interpretation can be encoded.

3. Possible outdated interpretations:
   When legal changes are made by law or because of new interpretations of old normative texts, the interpretations encoded into IT systems should also be updated. However, when it is unclear which lines of code depend on the interpretation of which normative texts, these updates can easily be missed. There is no link that tells us which lines of code to update when a particular normative text is replaced or interpreted differently.

One solution to overcome these issues is a standardized representation of the interpretation of normative texts that can be used to encode norms into the IT-system. A standardized interpretation of normative texts enhances the necessary normative coordination among stakeholders (citizens, businesses, government).

1.2 The Norm Engineering Project

The TNO Norm Engineering Project is tasked with creating such a standardized model. This standardized model has to be both theoretically sound and practically useful. To ensure theoretical soundness TNO is building onto research of van Doesburg and van Engers (van Doesburg & van Engers, 2019a; van Doesburg et al., 2016; van Engers & van Doesburg, 2016), who together developed a semi-formal language for modeling interpretations of normative texts called FLINT (Formal Language for the Interpretation of Normative Texts). The FLINT language is used within the action-oriented approach of the Calculemus protocol (van Doesburg et al., 2016), see figure 1.1.

The idea of the Calculemus protocol is that regulated tasks that are suitable for automated decision making, such as the grant of social benefit, can be divided into several actions. These actions are governed by norms that describe

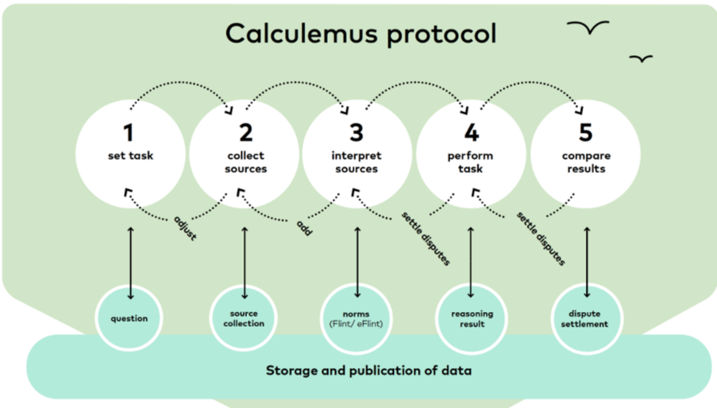

Figure 1.1: 5 steps of the Calculemus protool ("Inwilligen aanvraag verblijfsvergunning voor bepaalde tijd", 2024)

under what circumstances an action can be carried out and what the consequences of that action are. Legal norms are the result of the interpretation of one or more normative texts (Governatori et al., 2021). We call this an action-oriented approach because, rather than explaining an agent's normative status in *deontic* terms of obligations, permissions, and prohibitions, the Norm Engineering Project is focused on determining someone's normative status in terms of possible courses of action.

According to figure 1.1 the FLINT language is part of the third step of the Calculemus protoocol, the part in which normative sources are interpreted. In figure 1.2, one can see how the FLINT language represents an interpretation of the normative texts. FLINT is the representation of the norms that regulate the actions that are part of the task procedure. The yellow marked areas in figure 1.2 show how the different concepts relate to each other. The interpretation of the normative text is included in the norm governing the action:

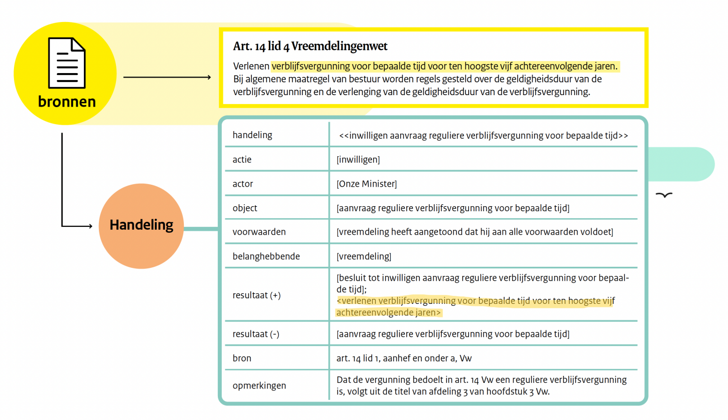

Figure 1.2: normative source, action and interpretation ("Inwilligen aanvraag verblijfsvergunning voor bepaalde tijd", 2024)

The fact that the explicit focus of the Calculemus protocol is on modeling procedural tasks, asks for specific questions that the interpretations of the normative texts should answer. The formalization of the norms should not look at formalizing normative texts in isolation, but in the context of the actions that are necessary to carry out said task. A model of the FLINT language should therefore be able to answer the following key questions after each action in the task procedure: what can/should I do to others, what can/should others do to me, under what circumstances can/should we do that, and, most importantly, what happens when we do that?

The current version of the FLINT language was built as a first step towards a formalization of the representation of norms in terms of answers to these questions (Breteler et al., 2023). It consists of an ontology in which the core concepts of the interpretation of a normative source are distinguished. The construction of the ontology will be discussed in chapter 2.

As a semi-formal model of interpretations, this current version of the FLINT language still lacks a few desirable properties:

1. Give an accurate description of the expressive power of the language (thereby also showing its limitations)

2. The ability to reason about norms

3. The ability to reason about scenarios

4. Verification of consistency of the norms

Logic, as the formalized description of reasoning, can be used for all of these desiderata (Di Bello, 2007; Markovich, 2020).

1.3 Thesis structure

To attain the aforementioned desired properties, this Master of Logic thesis sets out to build a logic that is a full formalization of the current semi-formal FLINT language, called L_{FLINT}. It builds onto the work of van Gessel, 2024. The thesis takes a constructive approach to building the logic, retaining the current FLINT concepts within the logic. The explicit goal for the logic is therefore:

To provide complete formalizations of norms with the use of the current FLINT concepts, such that the key questions of the action-oriented approach of the Calculemus protocol can be answered in a given scenario. In this way, this logic should provide a computational theory of norms as a first step towards computational norm implementation.[3]

To reach this goals the structure of this thesis will be as follows: in chapter 2 we'll provide the theoretical background for the other chapters of this thesis. Chapter 3 contains an overview of relevant developments in the field of logic and law and various summaries of related work. In chapter 4 we'll start with the

[3]Different uses for FLINT interpretations are mentioned in earlier works (van Doesburg & van Engers, 2019a): 1. Explainability to users 2. Validaton by legal experts 3. Basis for computational implementation in (semi)-automated decision making tools. We explicitly focus on this third goal, which will have consequences for our modeling choices.

construction of the logic. First a propositional logic is constructed, L^{prop}_{FLINT}, as a variant of Propositional Dynamic Logic (PDL) without the concept of duties. Soundness and completeness proofs for the logic are presented. An extension of the syntax and semantics is then given so that duties are incorporated in the logic. This chapter can be viewed as providing a logically robust foundation upon which a first-order version that is able to represent all the FLINT concepts can be built. Chapter 5 will extend L^{prop}_{FLINT} toward this first-order version, L^{fo}_{FLINT}, and run through a scenario of playing the game of tic-tac-toe. In chapter 6 limitations of the modeling approach and concepts not included in the logic are discussed. The conclusion is brought forward in chapter 7 and will reflect upon the main takeaways of the construction of the logic, the logic's place within the field of norm modeling and the next steps towards improvement of the formalization.

By building a logic that contains the FLINT concepts and can answer the key questions of the action-oriented approach, this thesis should not only contribute to the Norm Engineering Project, but also towards the literature on the formalization of norms. The full formalization ensures that a systematic comparison between L_{FLINT} and other logics of norms can be made. In the broader picture, this thesis will contribute towards the efforts to reconcile (semi)-automated governmental decision-making processes with the *Rule of Law* principles.

Chapter 2

Theoretical background

In this chapter we'll first provide the reader with a couple of explanations for some of the most important technical terms used throughout this thesis. We'll then put forward the theoretical background for the FLINT language. In the third section we'll describe the FLINT language as it is currently constituted and give an example of FLINT in action. As this thesis builds on work done by the researchers of TNO, this chapter will largely be a synthesis of Breteler et al. (2023), van Doesburg and van Engers (2019a, 2019b), van Doesburg et al. (2016), and van Engers and van Doesburg (2016).

2.1 Definitions

The focus of this thesis is on formalizing norms. As will become clear in chapter 3, the literature on the formalization of norms is vast. Throughout the literature some important normative concepts repeatedly show up. This section is therefore designed to clearly state which normative concepts and what interpretations of these normative concepts will be used in this thesis. The interpretations that we use, taken together, clearly picture our action-oriented perspective for modeling norms.

2.1.1 Norms

In the introduction we already stated that "legal norms are the result of the interpretation of one or more normative texts." This definition is derived from the definition of Peczenik (1989), in which he states that "a legal norm n is the result of the interpretive process of one or more legal provisions p1, . . . , pn." The difference between our definition and Peczenik's is that we want to make explicit that we don't only look at provisions of regular legal sources such as the law or jurisprudence, but at any normative text, for example, company guidelines.

In the definition of norms, an important distinction is made between norms and legal provision. In Governatori et al. (2021) a norm is contrasted with a legal provision as being "equivalent to one or more provisions plus the activity of their interpretation." This contrast should be kept in mind throughout this thesis. A provision refers to one specific article, e.g. article 6:162 of the Dutch Civil

Code. A norm can be constructed out of the interpretation of multiple provisions. Our formalizations of norms are therefore most of the time not one-to-one formalizations of provisions, but rather combinations of the interpretations of different provisions. This puts this thesis at some distance of work trying to formalize ‘the law (Di Bello, 2007; M. J. Sergot et al., 1986), in which the goal is to stay as close as possible to the text of the law. In earlier work van Doesburg et al. (2016) expressed the goal of reaching an isomorphism between normative texts and the FLINT formalization. This is not something we strive for in this thesis. A norm in FLINT is represented “in terms of normative acts and the pre- and postconditions of these acts” (Breteler et al., 2023). The conditions which should hold before an action can be performed and the conditions that hold after an action is performed. The presented interpretations of normative texts will be linked to the normative texts they are derived from, but this does not have to be an isomorphic relation. The FLINT formalizations of norms are not isomorphic with those normative texts because they allow for interpretative additions of legal experts. One could argue that reaching an isomorphism does not do justice to the open nature of the law and the necessary interpretative procedure that has to be carried out to determine the content of the law. The fact that L_{FLINT} accounts for this open nature and interpretative procedure could be one of the main advantages of this approach over more classical approaches to norm modeling.

2.1.2 Normative judgments

The judgements made in the FLINT language can be considered as normative judgments or legal facts (Governatori et al., 2021). Various types of legal facts are distinguished by Governatori et al. (2021). The types of legal facts which we will deal with for the FLINT language are either *qualificatory* statements, ascribing normatively relevant qualities to a person or an object, or *deontic* duty statements, stating that actions are obligatory. Deontic concepts are permission, obligation (duty), and prohibition. Of the deontic concepts only the duty concept is directly modeled in FLINT. In the literature also the class of *Potestative* judgements is distinguished. These judgements are relevant for FLINT, but are modeled indirectly. Potestative judgements attribute powers, meaning that you can change your own or other people’s normative relations.

2.1.3 Normative systems

As a result of the focus on modeling procedural tasks„ the perspective we take on norms in this thesis is that norms don’t exist in isolation. Norms exist as action steps of the procedure of a regulated task. We view the norms governing these action steps as forming a *normative system* (Breteler et al., 2023). The idea of a normative system, first proposed by Alchourrón and Bulygin (1971), is that facts, situations, are related to deontic consequences. In this thesis we slightly modify this idea by having actions relate situations towards new situations and specifically changes of duties. A FLINT normative system can therefore be understood as a, “a set of norms and mechanisms that systematically interplay for deriving the qualificatory judgements and deontic duty relations in force in a given situation” (modified towards FLINT from (Governatori et al., 2021)).

2.1.4 Regulative and constitutive norms

An important taxonomic distinction in the literature is that of regulative versus constitutive norms (Governatori et al., 2021). There has been ample debate on how this distinction should be explained (Grossi & Jones, 2013). The interpretation that we'll adhere to in this thesis is that it depends on the rules' capacity for guidance (Placani, 2017). Regulative norms require people to behave according to the norm. If the maximum speed limit is 90 kilometers per hour, this rule demands that you don't drive faster than 90. Constitutive norms don't provide agents with a reason to behave according to the norm, but rather show how something is brought into 'legal existence'. The canonical form of constitutive norms is 'X counts as Y in context C'. For instance, performing the wedding ritual counts as getting married. Being married has further legal consequences, and that is why we say that something new has been brought into existence.

This distinction is important to us because it will become clear later in this thesis that because of FLINT's action-oriented approach difficulties arise when trying to model constitutive norms that don't make an explicit reference to actions. We'll therefore readdress this issue in chapter 6, the limitations of L_{FLINT}.

2.2 Searle's layers of normative system

FLINT is used to make the interpretation of sources of norms explicit. More specifically, it should be possible to use FLINT to extract the normative position at a normative state of an individual within the normative system, when a scenario is given. A scenario is a series of actions, events, and observations that may or may not be compliant with the norms in the normative system. When we look at our definition of a normative system, it becomes clear that a normative position in FLINT can be viewed as those legal facts and duty relations that hold with regard to an agent, and the preconditions of actions that they satisfy.

To determine this normative position within a normative system, an interconnected process through different layers of a normative system has to be carried out. This separation of different layers of a normative system within our theory is based on the work of John Searle Searle, 1969. The FLINT language operates within what John Searle calls the 'institutional reality' layer of a normative system. To understand what this means for the function of the FLINT language in a normative system, we'll take a closer look at the different layers that are distinguished in van Doesburg and van Engers (2019a):

1. Sources of norms: The sources of norms layer describes the normative texts in natural language. We want to 'translate' these normative texts into formal interpretations so that we can then encode these translated norms into computational models. This layer is used to highlight the components, structure, and referential mechanisms of normative sources that should be included in a formal interpretation.

2. 'Social Reality':
The 'Social Reality' layer describes the constituency of society and the behavior of people in it. It can contain any empirical, 'brute', fact. Facts from social reality can be qualified as institutional facts. These facts then become part of a legal scenario.

3. ‘Institutional Reality’:
   This layer can be divided into two parts: it describes the formal interpretation of the sources of norms in the sources of norms layer. Furthermore, it formalizes which qualified facts from the social reality layer hold as part of a normative scenario. Institutional facts and acts are those facts and acts contained in a normative source or that are part of a qualified normative scenario. The requirement to have an official border-crossing document to enter the Netherlands is an institutional fact, as it is stated in the law. Similarly, the granting of a temporary residence permit is an institutional act (van Doesburg & van Engers, 2019a). The ‘brute’ facts of a court case within that context count as institutional facts.

   It is said that constitutive norms relate social reality, the physical layer, to institutional reality, the institutional layer (Grossi & Jones, 2013). In this thesis we'll often need our formalizations to make use of this relation. To ensure that we can encode our formalizations in IT-systems we'll need to formalize some of our institutional facts in terms of 'brute facts'. This is only done in case this relation is governed by constitutive norms. So we can still assume that FLINT only operates in the “institutional reality” layer.

The three layers are connected by the processes that are carried out within a normative system: interpretation, qualification, and assessment.

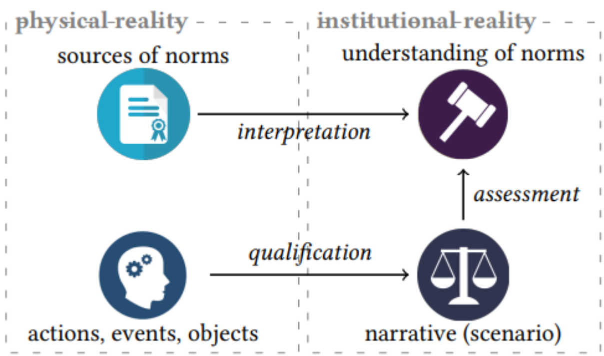

Figure 2.1: Searle's layers of reality (van Engers & van Doesburg, 2016)

In this diagram you can recognize the source, institutional and social reality layers. The physical reality is made up out of the source and social reality layers. The institutional reality contains the qualified social reality, the scenario, and the understanding of the norms, the interpretation. The processes that connect the different layers can be used to carry out an audit of the workings of a normative system. It can be tested how the normative system actually operates.

The assessment of a scenario against the understanding of norms is where normative decision making occurs. With the Norm Engineering Project we want to make clear how this decision comes about. As can be seen in the diagram, this can only be done when the institutional abstractions made over the physical reality are made explicit. Two abstractions from the physical reality towards the institutional reality are made. First, the interpretation of the sources of norms. This shows that a standardized interpretation model is necessary to create the desired link between the normative source and the encoded rules in a (semi)-automated decision-making tool. Second, the understanding of the case itself as a scenario, the qualification (van Doesburg et al., 2016). With FLINT we aim to make the institutional abstraction of the sources of norms towards an interpretation explicit, such that we can check a given formalized scenario against this interpretation.

To achieve this goal, it is necessary to determine which concepts should be contained in an interpretation of a normative text.

2.3 The Hohfeldian legal relations

The theoretical foundation of the conceptualization of norms in the FLINT language is Wesley Newcomb Hohfeld's theory of legal relations (Hohfeld 1919). In his theory Hohfeld observes that the 'legal position' of an individual is always a legal relation with another individual (van Binsbergen et al., 2020). Hohfeld distinguishes four different types of legal relations:

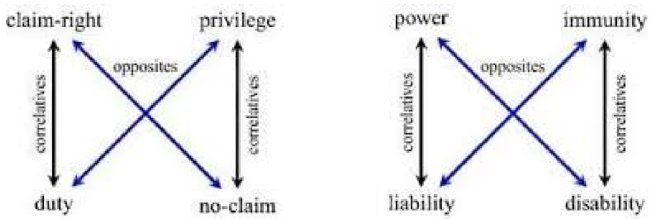

Figure 2.2: Hohfeld's legal relations (Markovich, 2020)

We'll illustrate the meaning of each of these legal relations by a mother-child analogy:

1. Duty - Claim-right:
   A duty is an obligation of an agent towards another agent. An agent a has a claim when another agent has an obligation toward agent a.

   You can have the duty to do the dishes towards your mother. Your mother then has the claim towards you that you do the dishes.

2. Privilege - No-claim:
   Liberty is the concept that you don't have a duty towards someone else. No-claim means that you can't claim that another agent carries out a duty towards you.

   A privilege is that you can do whatever you want on Friday night and your mother won't be able to say you have a duty to do your homework, she has no claim towards you.

3. Power - Liability:
   Power for Hohfeld is the ability of an agent to change/create a normative relation. Liability means that your normative relation can be changed.

   Your mother might have the power at any moment to make you clean your room. You are then liable for this change of duty, even when you are gaming.

4. Immunity - Disability:
   Immunity means that someone else does not have the power to change your normative relations. Disability means you can't change the normative relations of someone else.

   When you leave the house you're now immune to the creations of duties by your mother. She is no longer able to place a duty on you.

According to Hohfeld all legal relations can be reduced to one of these four types. These relations can only exist in pairs. They describe relations between two persons, with each person holding one of the respective rights of the pair (van Doesburg et al., 2016). This is an important aspect of Hohfeld's theory, his theory is explicitly relational (Markovich, 2020). Rights and duties, but also powers and liabilities, never exist on their own. There is always a relation between two agents. Any theory correctly representing Hohfeld's theory should explicitly include the relational aspect of all types of Hohfeldian rights.

The first two relations, the Power-Liability, Immunity-Disability pairs, are 'generative' in nature. They can create new legal relations. The latter two relations, Duty-Claim-right, Liberty-No-claim pairs, are 'situational' in nature. They can only be created or terminated by the generative pairs. The claim-right group are deontic modalities, whereas the power group are 'potestative' rather than 'deontic' modalities. The implication of this distinction is that when we do something without permission we can expect a penalty, whereas if we do something without power, we regard the act as having never been constituted.

Part of the relation aspect of Hohfeld's theory is that when an actor exercises a 'power' (performs a certain action), this impacts the actor with the correlative 'liability' position. The 'liable' actor is bound to the effects of the exercised power (performed action) (van Binsbergen et al., 2020; van Doesburg et al., 2016). We need to distinguish here between Searlean power and Hohfeldian power (Markovich, 2020). Searlean power refers to institutional power as power to perform a 'normal' action, which we all recognize counting as an institutional action. Hohfeld's power focuses on the effects of performing institutional actions, the changes in normative relations. The Hohfeldian perspective is in the law. Taking Hohfeld's perspective, one does not have to focus on the change of brute facts (social reality) to institutional reality, but only on the change of the normative relations. In this thesis we'll adhere to the interpretation of Hohfeldian power that, for a power to exist, a normative relation needs to be changed (created or terminated) in comparison to the previous normative state. In FLINT the Hohfeldian legal relations are perceived as normative relations. Again, focusing on the idea that normative relations can be found in any normative source (e.g. company guidelines).

Furthermore, for FLINT the four Hohfeldian relations are reduced to two, the Duty-Claim-right and Power-Liability relations. van Doesburg and van Engers (2019a) claims this can be done because, in line with Kocourek (1930), the authors view the Liberty-No-claim and Immunity-Disability relations as absent Duty-Claim-right and Power-Liability relations. This perspective is not completely in line with other formalizations of Hohfeld's framework that we'll discuss in the related work chapter. Since the FLINT ontology is modeled for positive actions, i.e. actions that can be performed, it is unclear how a duty to refrain from performing an action fits within the action-oriented approach. To model the Privilege-No-claim relation it is necessary that not only positive duties are absent, but also negative duties. The deontic notion of permission is defined as the absence of a duty to refrain. Without the inclusion of the notion of refrainment, we are not able to model the Privilege-No-claim relation in L_{FLINT}.

In the next section it will be shown how the theories of Hohfeld and Searle are represented in the current FLINT ontology.

2.4 Description of FLINT

Inspired by Searle, the FLINT language is a representation of the interpretation part of the institutional reality layer. Its goal is to provide a standardized form of interpretation of normative sources that relies upon the Hohfeldian conceptualization of legal relations. These theories that are foundational to the FLINT language are represented within the FLINT ontology.

2.4.1 FLINT ontology

Ontologies are used to describe the core concepts of a specific subject or domain and shows how these concepts are related to each other. An ontology therefore provides a key first step towards a full standardized formalization of the interpretation process of normative sources. Together Breteler et al. created the FLINT ontology, figure 2.3.

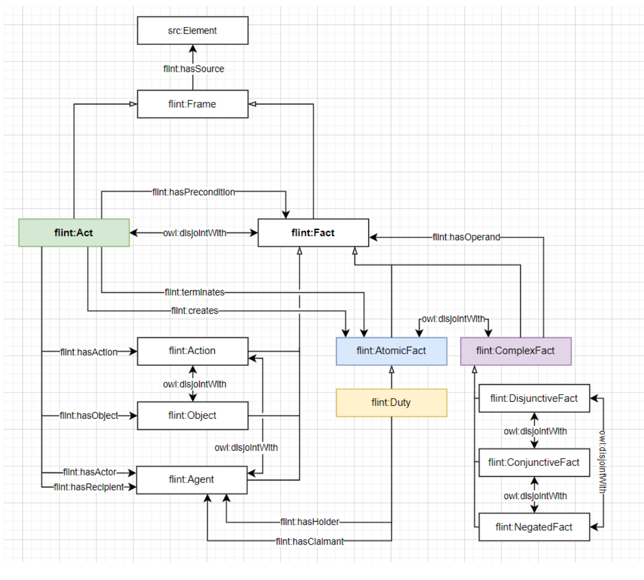

Figure 2.3: FLINT ontology (Breteler et al., 2023)

The FLINT ontology consists out of 13 classes, related to each other by subclass relations and properties. Arrows with white tips represent subclass relations. The act frame and fact frame are subclasses of the frame class. Labeled arrows with black tips connect the domain and range of the properties indicated by the label.

An interpretation of a normative source in FLINT is built up out of instances of the 13 classes of the FLINT ontology. The idea of the FLINT ontology is that we have *frames*: containers with bundled information (Breteler et al., 2023). Except for the source class, all other classes represent such frames, which is because they are in a subclass relation towards the frame class.

We see that the fact frame contains several subclasses. Fact frames describe the state of the normative system. This includes several concepts: first, propositions which are true or false relative to a state. Propositions can be complex or atomic. Complex propositions are negated, conjunctive, or disjunctive facts. Second, agents and objects that play a role in the normative system. Third, actions, things an agent can do (Breteler et al., 2023). Another subclass of fact frames is the duty class. Duties are a special kind of fact. The duty fact captures that a certain action is expected from an actor in that normative state. Since a duty is part of a Hohfeldian Duty-Claim-right relation it must always have a holder and a claimant. That's why the holder and claimant properties have the duty as the domain and agent(s) as their range.

Act frames describe actions that agents might take, which result in a transition between states of the normative system. The act frame is connected to the fact frame via the properties of the act frame. We see that actions can have facts as preconditions through the hasprecondition property, and that actions can also create or terminate facts. The creation or termination of facts we call the postconditions of facts. The properties hasActor, hasRecipient, hasObject, hasAction have to be viewed as specifications describing the action. Who performed the action, who received it, which object underwent a transformation, and how was this done.

We see that of the Hohfeldian concepts only duties are contained explicitly in the ontology. Other concepts, such as claims, the Power- Liability relation and Immunity- Disability relation must be inferred. Claims should be viewed as the counterparts of duties. If a duty exists for a towards b, then a claim exists for b from a. Hohfeld's Power-Liability concept is incorporated in FLINT's representation of the institutional reality by the idea that institutional facts contained in the institutional reality function as preconditions, such that they can give rise to the power of an actor to perform an institutional action that changes a duty fact. Immunity and Disability relations can be inferred from the absence of the preconditons of power-inferring actions or the fact that a duty relation was already created/terminated, so the exercise of power would have no effect. As stated before, it is unclear how the Privilege-No-claim relation fits into the FLINT ontology.

We say that an agent has the 'ability' to perform an action in the FLINT ontology when the preconditions of an action are satisfied. This means that ability in the FLINT ontology concerns both facts and duties. Since we find ourselves in the institutional reality layer of Searle, this ability has to be viewed as 'legal' ability (Herzig et al., 2011). The ability to change legal facts. Also, because actions change duties, our version of ability partly overlaps with Hohfeldian power.

The FLINT ontology has to be viewed separately from the work in the Norm Engineering Project on EFLINT (van Binsbergen et al., 2020). EFLINT is a computer-implementable language version of the action-oriented approach of the calculemus protocol. The FLINT ontology is a first step towards a computational theory of norms. The FLINT ontology could therefore provide a theoretical basis for EFLINT, but over the years these two tracks of the Norm Engineering Project of TNO have diverged. As this thesis focuses on formalizing the FLINT ontology, the relation to van Binsbergen et al. (2020) is limited. One benefit of fully formalizing the FLINT ontology is that it could function as a starting point towards converging the two tracks again.

2.4.2 FLINT interpretation of library regulations

An interpretation of a normative source, a norm, is built up out of a collection of act and fact frames. A legal expert needs to make the link with the source of the norm in natural language and FLINT explicit, by representing the frames with specific instances of the frame types included in the ontology. A relatively simple example of a representation of norms with the use of the FLINT ontology is the library example in Breteler et al. (2023):

1. Books can be loaned to library members if they have no outstanding fines.
2. Library members who borrow books are obligated to return them.

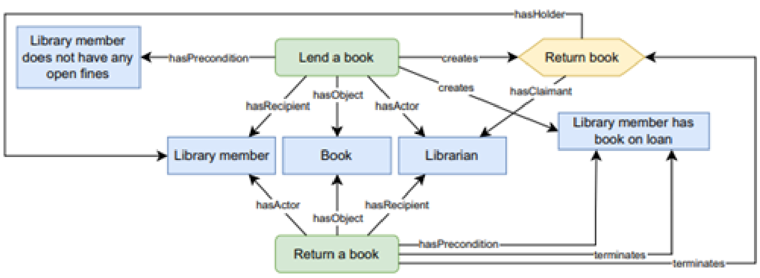

Figure 2. Representation in FLINT of the regulations for lending and returning books. Blue rectangles are instances of *AtomicFact*, green rounded rectangles are instances of *Act*, yellow hexagons are instances of *Duty*. Actions are omitted for readability.

Figure 2.4: FLINT-frame library example (Breteler et al., 2023)

We see in figure 2.4 that the act and fact frames together represent the rules of the library. The preconditions for lending a book are distilled from the first rule. The actor should be a librarian and the recipient a library member. If you perform the action of lending the book, a few postconditions are created, resulting in a transition to a new normative state in accordance with rule number two. In this normative state the library member has the book on loan, but also has the duty to return the book. When returning the book, the library member terminates this fact and duty. In this way, the member actually returns to the original normative state in which it is possible to lend a book, and there is no duty to return that book.

The frame instances of the FLINT ontology provide information on the level of 'types'. A library member is a type of which any actual member can be an 'instance'. Such instances are not contained in the FLINT ontology. One of the current objectives of the Norm Engineering project is therefore to also build a scenario ontology. With the current semi-formal model we can formally express the general conditions of norms, but we can only informally infer what this means for a specific agent in a concrete situation. Because the concepts included in the ontology are not given a formal semantics, we cannot formally reason with this semi-formal model. We can only describe the information it contains and informally infer what this would mean in a scenario. We cannot apply norms to normative scenarios. For computational implementation an exact semantics should be specified. This semantics is then able to show exactly what it means to be a duty in L_{FLINT} and how such a duty can be created or terminated by the performance of an action.

2.5 From a semi-formal model towards L_{FLINT}

The goal of L_{FLINT} is to be able to represent full formalizations of interpretations such as the library example. By constructing a modal logic representing these interpretations, we can build graphs of possible normative states. In the actual normative states, certain institutional facts and normative positions hold, and from this world we can reach several other possible worlds by performing institutional acts when the required preconditions of that act are met. A performance of an action is then a normative transition between normative states. As a result, we can express the normative action space available to a certain agent within the actual world. This will be all those institutional acts for which the preconditions are met. In this way we can give a formalized representation of the institutional reality. As such, L_{FLINT} will allow us to answer the key questions of the action-oriented approach.

This chapter has provided the necessary background knowledge and definitions of important concepts to understand this thesis, but before we move on to constructing L_{FLINT}, we first place the goals for our logic within the long tradition of norm modeling in the related work chapter.

Chapter 3

Related work

This chapter begins with a short overview of relevant developments in the field of logic and law, before we address the formalizations of legal concepts most relevant to the FLINT approach. The choice for the logics presented is based upon their relation to the Hohfeldian theory of normative positions and their ability to answer the key questions defining the FLINT approach: what can/should I do to others, what can/should others do to me, under what circumstances can/should we do that, and what happens when we do that?

To answer these questions we cannot make use of mere formalizations describing the legal code, but a useful formalization should also help to understand which rights and duties are entailed by the application of norms. This thesis therefore has other goals than work focused on representing the written law as closely as possible, such as the famous attempt of M. J. Sergot et al. (1986) to formalize the British Nationality Act and an earlier Master's of Logic thesis by Di Bello (2007), formalizing the Italian Civil Code with event calculus.

Another line of work that will not be discussed in this chapter is the work on modeling specific kinds of norms. Examples of the purpose of these specific models are for instance what we call 'access and usage control models' (Sileno & Pascucci, 2020; van Binsbergen et al., 2020) In line with the Calculemus protocol, we are interested in modeling all sorts of norms as long as they are part of a task procedure. This means that our language is not limited to certain forms of normative provisions or legal fields.

Where EFLINT can be compared to languages based on event calculus van Binsbergen et al., 2020, L_{FLINT} will focus on a modal analysis of norms. To answer the key questions, we need to be able to determine the normative action space at a certain normative state (Breteler et al., 2023). Event calculus can model events that have taken place before (Di Bello, 2007; van Binsbergen et al., 2020). The occurrence of an event can then initiate and terminate what are called 'fluents', which we can view as facts. Although such a model of norms would be able to model the postconditions of actions, it does not explicitly capture the preconditions of actions. There is no modal operator in event calculus expressing that an action can happen at a certain state. No special language and semantics for handling actions (Shanahan, 1999). This is exactly what we need for L_{FLINT} to be able to express the available normative action space of an agent.

In summary, we want a model of interpretations of normative texts that:

- is based upon Hohfeldian theory of normative positions
- focuses on norms rather than representations of normative texts
- treats a variety of normative domains and sources
- can answer the calculemus protocol key questions through a modal analysis

The most relevant literature on formalizations of norms will therefore be found in work springing from the development of Standard Deontic Logic (SDL) (von Wright, 1951) formalizing the Hohfeldian normative positions, and logics that have incorporated a dynamic operator to model the action perspective of the Calculemus protocol.

We split up this part of the related work section into two parts. In the first part we summarily touch on the development of SDL and discuss work that formalizes Hohfeldian relations but doesn't treat actions as a concrete building block of the logic. We do this to put L_{FLINT} in the tradition of SDL, but at the same time show why it is necessary to deviate from this tradition to be able to answer the key questions of the action-oriented approach. In the second part we will discuss logics that not only model the Hohfeldian legal relations, but also include actions as first-class citizens within the logic. Therefore, these logics form the most relevant comparisons for L_{FLINT}.

3.1 Logic and law

The connection between logic and law provides an interesting dichotomy between logicians. For some, its relation is blatantly obvious, for others some extra explanation is needed to see the uses of logic in such a practical field as the legal one (Governatori et al., 2021; Grossi, 2011).[1] Logic and law enthusiasts can refer their more skeptical colleagues to the long-standing history of the relation between law and logic, famously dating back to works of Leibniz and Bentham.[2] These early works already handled questions related to modeling normative concepts.

Originally the main purpose for logic in law has been the "representation of law in a clear and unambiguous manner" (Markovich, 2018). The symbolic representation of laws can help to:

- Reveal syntactic ambiguities (Allen, 1957)
- Define the meaning of legal terms (e.g. 'duty')
- Analyze normative positions and relations created by norms

[1]Of note is my personal experience, when, in the process of applying for the Master of Logic, I asked MoL students whether it would be possible to combine my interests in both law and logic, and them being rather suspicious about the possibilities.

[2]See Hilpinen and McNamara (2013) and Governatori et al. (2021) for extensive contributions on this topic.

Di Bello (2007) points at another important use of a formal system that represents the law:

- A formal system can be used for reasoning tasks, "such as consistency checking, computation of implicit inferences, and instance enumeration" (Di Bello, 2007)

Here again we see that logic is the right tool for the desiderata for L_{FLINT} of the Norm Engineering Project: reasoning with norms, consistency checking and evaluating expressive power.

The emergence of artificial intelligence in the legal field has led to an increased interest in applications of logic to the law. Bench-Capon and Prakken (2008) provide an overview of the early developments of the connection of logic, AI and law. For a more recent overview, one can look at Rotolo and Sartor (2023a) and Rotolo and Sartor, 2023b. The current developments of the field have to be viewed as products of the counter-movement towards the critique that the connection of logic and law faced early on. Two forms of criticism can be distinguished, a more radical line of criticism and a moderate version (Governatori et al., 2021).

The moderate critique version argues that logic can only partly capture the concept of legal reasoning. This critique long relied on the limitations of logics that could only describe legal reasoning in terms of judicial syllogisms. But there is of course a wide variety of logics available, each with its own machinery, that can describe reasoning that is significantly more complex than simple consequence relations. Several logics have been developed that are able to deal with the more complex aspects of legal reasoning (Governatori et al., 2021).

The radical criticism of the connection between logic and the law fundamentally says that logic cannot be applied to normative reasoning. A fundamental problem for a logic of norms is that norms by themselves do not have a truth value. The repercussions this has for normative reasoning are exemplified by the Jörgensen dilemma (Hilpinen & McNamara, 2013; Markovich, 2018). For his dilemma Jörgensen first presents the following seemingly valid inference pattern:

1. If x causes damage wrongfully to y, then x has to pay for it (to y)
2. a caused damage wrongfully to b

$\therefore$ a has to pay for it (to b)

Although at first hand this seems like a regular valid inference pattern, the problem according to Jörgensen is, that premise 1 and the conclusion are not sentences that can hold a truth value in classical logic. As a result, the conclusion can also not be a valid inference since valid inferences are about true and false sentences. So in the Jörgensen dilemma there is an invalid inference pattern, where we expect it to be valid. There are three ways of solving the Jörgensen dilemma (Markovich, 2018):

1. rules of inferences should be extended to be applicable to norms
2. norms should be translated or reduced to propositions, descriptive sentences

3. we should accept that there is no notion of a valid inference that could be applied to norms.

Georg Von Wright proposed a solution along the lines of option 2 that led to a renewed development of a branch of logic that has been fundamental to logic and law developments. Von Wright founded *Standard Deontic Logic* (von Wright, 1951). SDL is a "logic of norm-satisfaction", such that sentences like premise 1 and the conclusion can hold truth values. The importance of this work cannot be understated. According to Hilpinen and McNamara (2013, p. 39) "It is the most widely known, well-studied system, and central in the accelerated historical development of the subject over the last 50 or so years. As such, it serves as a historical comparator, where various important developments in the subject were explicit reactions to its perceived shortcomings, and even when not, sometimes can be fruitfully framed as such."

It is this logic and its offspring related to the Hohfeldian theory that we'll discuss in the next few sections.

3.2 The SDL tradition

3.2.1 Standard Deontic Logic

Deontic logic is probably the first logic that comes to the mind of a logician for the formalization of rights and duties. The word deontic comes from the Greek word déon, which means 'what is binding' or 'proper' (Hilpinen & McNamara, 2013). Leibniz referred to the deontic notions of obligation, permission, and prohibition, as the "modalities of the law" and found that basic principles of alethic modal logic could be applied to these modalities (Hilpinen & McNamara, 2013). Obligation could be viewed as a normative necessity, and permissibility could be viewed as a normative possibility. In the 1950's Georg Von Wright breathed new life into this idea with his classic paper 'deontic logic' (von Wright, 1951). Von Wright defined modal operators for the normative concepts of obligations (O), permissions (P) and prohibitions (F). In this way it can be said of a norm whether it is true or false. A norm can be said to be true if it is actually a norm of our normative system, and false when it is a non-existent rule (von Wright, 1968). With a few alterations, the logic developed in von Wright (1951) has become known as Standard Deontic Logic (Hilpinen & McNamara, 2013).

Von Wright acknowledged that his earlier work had limitations and further developed SDL by incorporating a complex logic of action (von Wright, 1963, 1968). His system, however, was still relatively unrefined and notably lacks the expressiveness to properly model Hohfeldian theory. Addressing this gap, several logics were developed that tried to formalize not only rights and duties but all of Hohfeld's 'fundamental legal conceptions' and how they are interconnected by relations of pairs of agents and actions by these agents. These Hohfeldian logics built onto SDL and fall under the theory of normative positions (M. Sergot, 2013).

3.2.2 Formalizations of normative positions - Hohfeldian logics

The first attempt to use a modal logic to formalize Hohfeld's theory using SDL as its base was made by Stig Kanger (Kanger, 1971, 1972). His work was then further developed by Lars Lindahl (Lindahl, 1977, 1994). For an in-depth discussion of the logics of Hohfeld's normative positions, see M. Sergot (2013). The Kanger-Lindahl theory, as their combined efforts are now named, notably lacked the relational perspective and a 'legal' power, both cornerstones of the Hohfeldian framework. The most recent formalization of the Hohfeldian normative positions addresses this issue, it is the work of Reka Markovich (Markovich, 2018, 2020). In her work Markovich focuses specifically on formalizing the Hohfeldian theory of rights, working as closely as possible to Hohfeld's intentions. This work can therefore be used to see how L_{FLINT} incorporates Hohfeldian theory and where it differs.

Standard Deontic Logic forms the base of Markovich's logic. It is extended with a seeing-to-it-that operator ($\exists$) to model that obligations are about actions. Some extra operators are introduced to properly capture the meaning of the Hohfeldian relations:

- F - a state of affairs
- C - compensation for a state of affairs
- $P_x \rightarrow_y$ - x has power over y
- $\Box$ and $\Diamond$ - the it's necessary and it's possible modal operators.

Markovich mentions that previous formalizations of Hohfelds relation have to deal with a 'loss of direction' problem, where these logics fail to specify the counterpart in the Hohfeldian relation Markovich, 2020. As we'll see, in her formalization all Hohfeldian relations will include an explicit one-to-one directed relation by virtue of a directed arrow, $_x \rightarrow_y$.

Markovich models each of the Hohfeldian legal relations in the following way:

- Duty - Claim-Right:

$$O_x \rightarrow_y \exists xF \leftrightarrow \Box\,(\neg\exists xF \rightarrow CR_y\exists j\exists xCF)$$

  An agent x has an obligation towards agent y that y sees to it that the state of affairs F is reached, if and only if, it is necessary that, if x does not see to it that the state of affairs F is reached, then y has a claim towards the judiciary that x compensates the state of affairs F.

- Privilege - No-claim

$$\neg O_x \rightarrow_y \exists xF \leftrightarrow \Diamond\,(\neg\exists xF \wedge \neg CR_y\exists j\exists xCF)$$

  An agent x does not have obligation towards agent y that x sees to it that the state of affairs F is reached, if and only if, it is possible that x does not see to it that the state of affairs F is reached, and y does not have a claim towards the judiciary that x compensates the state of affairs F.

- Power - liability

  $\mathrm{P}_x \rightarrow_y E_x F_c \leftrightarrow (E_x F_c \rightarrow (O_y \rightarrow_v E_y F \vee \neg O_y \rightarrow_v E_y F \vee P_y \rightarrow_v E_y F \vee \neg P_y \rightarrow_v E_y F))$

  Where F_c tells us that the action is an action created by a constitutive norm. And F, F', F", F"' are independent states of affairs.

  This equation then says that x has the power over y that x can perform an institutional action, if and only if, if x performs the institutional action, a normative relation is changed.

- Immunity- Disability

  $\neg P_x \rightarrow_y E_x F_c \leftrightarrow \Diamond (E_x F_c \wedge \neg (O_y \rightarrow_v E_y F \vee \neg O_y \rightarrow_v E_y F \vee P_y \rightarrow_v E_y F \vee \neg P_y \rightarrow_v E_y F))$

  This equation tells us that x does not have the power over y that x can perform an institutional action, if and only if, it is possible that x performs the institutional action and no change of a normative relation occurs.

Although Markovich's formalization of norms is built to capture Hofheld's theory of normative positions, it notably does not treat actions as first-class citizens within the logic. In action logic theory, STIT-operator based action logics are result-based action logics (Herzig et al., 2018). This means that in STIT-logics we only care about the fact that some action has resulted in the desired state. We see that this state is represented by the F operator in the formalizations of the Hohfeldian relations. However, in L_{FLINT} we also care about the actions themselves. We don't want to focus only on the result, but also the means by which the result has been reached, the type of action. We consider the performance of specific actions (institutional actions) to bring about a transition from one normative state to the other. These specific actions have specific pre- and postconditions. Something that can't be modeled in logics not giving space to actions as means to a result. In the next section we'll therefore look at logics that have incorporated a dynamic action operator that shows which actions result in what consequences.

3.3 Action-oriented logics

This section looks at those logics that have not only modeled the Hohfeldian legal relations, but also regard actions as first-class citizens of the logic. We'll take a closer look at the construction of each of these logics. We describe their formalization of each of the Hohfeldian relations. Although the FLINT ontology does not explicitly model the Privilege-No-claim, we'll also describe the formalizations of these relations as a point of reference for possible extensions of L_{FLINT}. Since we are interested in answering the key questions of the calculemus protocol, we also specifically look at how these logics can represent pre- and postconditions of actions in normative states.

3.3.1 PDL formalizations of norms

Meyer, 1988 was the first to my knowledge to apply the tools of PDL to SDL. His dynamic variant of deontic logic, PDeL, had the objective of providing a solution towards the appearance of paradoxes in many variants of SDL. By strictly separating actions from assertion, PDeL by design prevents many of these paradoxes from occurring.[3]

While PDeL stays close to the original ideas of SDL, in more recent applications of PDL in modeling norms, the 'classic' deontic operators (obligation, permission, forbidden) are omitted. This work is from the authors van Eijck, Xu and Ju, and will be introduced in the next subsections. Their approach more closely resembles the approach we'll take with L_{FLINT}.

van Eijck et al. (2023)

van Eijck et al. (2023) created a Logic for Dynamics of Legal Relations (DyLeR) à la Hohfeld using dynamic propositional logic. In their work the world is viewed as a collection of atomic facts. Only the concepts of claims, duties, and powers are modeled directly. They motivate this choice with three different reasons:

- They consider claims, duties and powers as the most central to the dynamics of legal relations.
- The other Hohfeldian conceptions are definable in terms of claims, duties, and powers. Similar to the claim made in van Doesburg and van Engers, 2019a.
- Hohfeld's conception of these legal relations is not accurate for all legal contexts. For instance in tort law, liability means a responsibility for payment rather than being a correlative for power.

These motivations can also be used as arguments for the construction of the FLINT ontology, which, in fact, only models duties explicitly.

The claim, duty, and power concepts are functions within the DyLeR model. The claim function makes use of another function that expresses an agent's ability to change the the truth value of atomic propositions, an ability function. Abilities in this model are related to the ability to change facts of the 'physical reality' layer of Searle. The following functions construct DyLeR models:

- **Ability:** Abilities let agents perform actions that change the truth value of atomic propositions (facts). Change from one possible world to another is modeled by flipping the truth values of atomic propositions from true to false or vice versa. A function with as input one world and one agent gives the ability to flip the truth value of one specific atomic proposition to that specific agent in that specific world.
- **Claim:** Claims and duties describe the static rights of agents in a world. In DyLeR claims and duties are modeled as a claim from one agent to another (the duty holder) that the truth value of an atomic fact should be flipped. In the model there is a function from worlds and agent pairs to atomic facts, saying in which worlds which agents are in a Duty-Claim relation with respect to the truth value of certain facts. Claim-changing actions themselves do not change facts. The duty holder should therefore in principle have the ability to perform the fact-flipping action.

[3] Since L_{FLINT} also makes a distinction between actions and assertion, the language should also provide solutions towards paradoxes in SDL. This investigation is outside the scope of this thesis.

- **Power:** A power is a function from worlds and agent pairs to atomic facts. The function assigns in which worlds which agents have certain powers. In DyLeR powers are only related to claims, not truth values. Powers can change claims but not facts. This means that the exercise of a power does not change the truth value of a fact, but rather the claim of an agent towards (possibly) another agent to change that fact. Powers hold in specific worlds. Changes of legal facts can therefore change which powers are available because this changes what world we are in. There are two power functions in DyLeR, because van Eijck, Xu, and Ju make a distinction between outward and inward powers. This has to do with the way a claim is defined by Xu, Ju, and van Eijck as the duty to change a truth value. You can create/withdraw a claim against someone to let that person change the truth value of an atomic proposition (outward power), or you create/withdraw a duty for yourself to change the truth value of an atomic proposition (inward power).

The idea of modeling legal relations with PDL is a promising approach. The idea is captured that changes through actions of the facts of the world will also affect the different Hohfeldian rights. However, some of the chosen simplifications for the model do not match the concepts of the FLINT ontology.

The preconditions are not explicitly modeled in the logic. They can also not be inferred. The functions constructing the DyLeR model directly impose in which worlds which abilities, claims, and powers will hold. These actions are not dependent on specific facts being true, but rather on which world we are in. This means we are only able to represent legal scenario's in DyLeR of which we know exactly how they should play out, assigning abilities, claims, and powers to the worlds in which they should hold. In L_{FLINT} we not only want to describe scenarios, but also represent norms themselves.

Postconditions are separated into the postconditions of fact-changing actions and claim-changing actions. These two types of postconditions are taken together in the FLINT ontology. There is no symbol in the logic to say that something is a pre- or postcondition of an action.

Also, the idea of performing duties by flipping facts seems to be a too simplified notion. It might be that a duty can be terminated by different actions rather than swapping one specific truth value. Since claim-changing actions do not change facts in DyLeR, a claim-changing action does not result in a change of worlds but rather in a change of model. In the FLINT ontology duties are viewed as special facts. In contrast to DyLeR, for our formalization it would be more in line with the FLINT ontology to let claim-changing actions change the world we are in.

The assumption is made in the article that claim-changing actions do not change powers. Meaning that if you had the power to make/cancel a claim and you exercised that power, you can always revert back. E.g. if you invite someone to your wedding you can uninvite this person and reinvite this person endlessly. Although this seems plausible for an invitation, in general this idea does not work in a legal system. One important principal in legal systems is that one should be able to know his or her legal position. So it should not be possible for someone else to continuously change this position. E.g. if you were in a car accident, and you are liable for damages to a car of person X, this person should not be able to claim these damages, revoke the claim and then claim the damages again. Our logic should model claims as terminable.

Ju and Xu (2023)

Xu & Ju refine their earlier work in Ju and Xu (2023). The explicit goal of this paper is again to formalize duties and powers by using propositional dynamic logic. They call their language the Multi-agent Logic for Duties and Powers in Private Law (MLDP). In this work duties are perceived to be part of the state of the world, similar to the FLINT ontology. They are atomic duty propositions. The meaning of duties in the language is that one agent has a duty towards another agent to make a fact true. Together with the atomic fact propositions they make up the 'static' view of the world. This static view can change in two ways: (1) natural events change the facts, e.g. cargo loss of a shipping container due to a storm at sea, or (2) the performance of actions. Changing the static view also leads to a change of duties and powers. Powers are dependent on the static view and as such can change by any event occurrence or action performance. Powers are modeled as the ability to make duties true. Duties are themselves part of the static view, but can also change by the occurrence of an event or performance of an action. Xu & Ju categorize four different types of action performance that lead to a change of duties:

- An action exercises a power. This means that a certain fact is made true that makes a duty proposition true.
- The action breaches an existing duty. This automatically generates a new duty to restore this breach. Possibly after involvement of the judiciary that establishes a duty has been breached (Markovich, 2020). In law this is called a delict; e.g., breaking someone's porcelain vase leads to a duty to compensate the damage.
- The action fulfilling an existing duty. By delivering a good, the duty to deliver the good is extinguished.
- Ontic actions creating institutional facts such that duties arise. Actions within the 'physical reality' layer of Searle. Writing a song creates intellectual property and the duty for other people not to copy it.

Xu and Ju model these normative changes by transitions between what they call 'moments'. In these moments certain facts are true, certain duties hold. Furthermore these moments contain four functions that model the ability to change facts of agents and whether they can exercise power:

- The ability function refers to the ability of agents to make facts true with ontic actions according to which facts are true in a moment.
- The power function relates the state of the world at that moment, defined by the facts and duties, to powers. That is the power to make a duty true. A duty is defined as a directed obligation to make a proposition true. This function can be viewed as merely listing all available powers at a certain moment. That is, we have a list of all duties that can be changed.

- The exercise function, as the name suggests, means that a power can be exercised by making a proposition true. So for the list of powers defined by the power function, the exercise function lists the way to exercise these powers. The function shows for all the duties that can be changed according to the power function how to change them.

- The consequence function is there to show that besides the execution of a power, duties can also change in other ways. By performing a certain action making a proposition true, which can be in the exercise function but not necessarily, a duty is made true or false. The consequence function therefore also models duty changes because of a breach, fulfillment, or other actions that change duties.

Notably these four functions are the same for any moment in the model of MDLP. They function globally over all moments, expressing that abilities and powers always exist relative to the same conditions in a normative state. The 'moments' can be concatenated in a serial tree of moments in time, connected by transitions through actions of making propositions true or false.

Preconditions of actions in this model are the input facts of the ability function. Postconditions are the output of the ability function, plus those duties that are made true because of a fact change according to the consequence function. There is no explicit symbol within the logic to denote that facts are preconditions, or that facts and duties are postconditions.

The setup of MLDP aligns very well with the setup of the FLINT approach. The key questions are questions that need to be answered from the perspective of the normative state we are in at a certain time. The 'moment' model of Xu & Ju captures this idea. The FLINT approach would also like to be able to guide citizens or other stakeholders through a legal administrative process. The model of concatenated moments serves this purpose.

Conceptually, we'll however make some different choices. The FLINT ontology models ability as the satisfaction of the preconditions. These preconditions can be any kind of fact, so also duties. There is therefore no need in L_{FLINT} to model ability as pertaining only to the change of 'regular' facts. This also means that we rather model actions in L_{FLINT} as having both facts and duties as postconditions. We don't want to include duties as postconditions as a derivative of making facts true, but model them directly. The consequence function is therefore also ubiquitous, we need one function that can express all postconditions.

Furthermore, the idea that powers need to be expressed explicitly by a power function, and also a separate power exercise function, is not in line with the fact that powers need to be inferred within the FLINT ontology. Such a function will therefore be left out of L_{FLINT}

The final limitation of MLDP is that its propositional nature limits its capability to model norms. The propositions that are pre- or postconditons cannot contain variables over which we can quantify. Without quantification there are two ways of modeling pre- and postconditions of actions. An action can have general propositions as pre- and postconditions. Such as the postcondition of loaning a book that the library member has a duty towards the librarian to return the book. Such a formalization of a norm cannot be applied to concrete scenarios. Another option is that the propositions of pre- and postconditions express a concrete case, but then we lose the general normative nature of pre- and postconditions of actions. In L_{FLINT} we want to be able to operate at both levels at the same time.

3.3.2 A semi-first-order formalization of norms

The logic Sileno and Pascucci (2020) develops, differs from the PDL logics in that it is a semi-first-order version of the Hohfeldian legal relations. It includes agents and objects in the logic but no form of quantification. The motivation for Sileno & Pascucci to develop this logic is similar to the motivations for L_{FLINT}: "any decision-making of autonomous components/agents requires relatively reliable expectations of the behavior of the other components/agents, as well as of the measures holding to maintain these expectations, e.g. penalties in case of violations. Making these expectations clear can be seen as the main function of normative artefacts." In their paper Sileno & Pascucci address the issues there are with modeling norms with deontic logic. They say that deontic logics primarily focus on describing wrong or correct 'situations'. Deontic logics therefore primarily focus on outcomes, where (semi)-automated task procedures the focus should also be on which behavior is allowed for agents and what actions they are capable of performing. Sileno & Pascucci call these the internal and external perspectives of normative agents. Agents are acting (internal) and observing consequences of actions (external). Sileno & Pascucci's modal analysis of the deontic and potestative relations of Hohfeld's framework is an attempt to unite these perspectives.

The logic is built with actions as primary building blocks. An action performance is modeled by the predicate *performs*, with an agent and action as input. Sileno & Pascucci make use of what they call a 'refined action-type, meaning that the object that is involved in the instantiation of the action is included in the description of the action. "E.g., the objects 'taxes' and 'bank transfer' in the notion of 'paying taxes with a bank transfer"' (Sileno & Pascucci, 2020). The performances of actions can result in configurations denoted by S. These configurations are possible partial descriptions of objects at a certain point in time. They perceive the world in which the agents operate as evolving over time. The structure of the model is therefore a tree ordered by a precedence relation, from left (past) to right (future). Nodes that are vertically aligned are simultaneous nodes. The effects of the performance of an action then are expressed by formulas with modal operators operating on conditionals with the act performance as antecedent and a configuration as consequent. Two modal operators are introduced: $\Box$ and $\boxplus$: $\Box$ is the necessity operator for all alternative simultaneous nodes. $\boxplus$ is the necessity operator for all alternative successor nodes. The hasability predicate is a good example of the interplay of the vocabulary of the language:

$$\text{hasability}(\text{x},\, \alpha, S_\varphi) \stackrel{\text{def}}{=} \Diamond\text{performs}(x,\alpha) \wedge \neg\text{is}(*, S_\varphi) \wedge \Box\,[(\text{performs}(x,\alpha) \wedge \neg\text{is}(*, S_\varphi)) \rightarrow \boxplus : \text{is}(*, S_\varphi)]$$

This formula informally reads that an agent has the ability to produce a configuration S_φ that holds, if x can perform α and it is not currently the case that S_φ, and in all such cases in the following normative state configuration S_φ will be the case. The asterisk for the is predicate means that all objects in the state are in that configuration. E.g. when it's raining, it's raining with respect to all of the objects in the state.

The deontic notions are all modeled in terms of obligation: $xO_y\varphi$ means that φ is obligatory for x with respect to y. Prohibition is a duty to refrain $xF_y\varphi \overset{\text{def}}{=} xO_y\neg\varphi$ and permission is not having a duty to refrain $xP_y\varphi \overset{\text{def}}{=} \neg xO_y\neg\varphi$. The Hohfeldian legal relations can then be modeled as follows (Sileno & Pascucci, 2020):

Deontic Terms:

- **Duty:**
$$xDTy(\alpha) \overset{\text{def}}{=} xO_y\text{performs}(x,\alpha)$$
- **Claim-right:**
$$yCRx(\alpha) \overset{\text{def}}{=} xDTy(\alpha)$$
- **Privilege (Liberty):**
$$xPRy(\alpha) \overset{\text{def}}{=} xP_y\text{performs}(x,\alpha) \wedge xP_y\neg\text{performs}(x,\alpha)$$
- **No-Claim:**
$$yNCx(\alpha) \overset{\text{def}}{=} xPRy(\alpha)$$

Potestative Terms:

- **Power:**
$$xPOWy(\alpha,\varphi) \overset{\text{def}}{=} \text{has ability}(x,\alpha,S_yO_x\varphi)$$
- **Liability:**
$$yLBLx(\alpha,\varphi) \overset{\text{def}}{=} xPOWy(\alpha,\varphi)$$
- **Disability:**
$$xDISy(\alpha,\varphi) \overset{\text{def}}{=} \text{has disability}(x,\alpha,S_yO_x\varphi)$$
- **Immunity:**
$$yIMMx(\alpha,\varphi) \overset{\text{def}}{=} xDISy(\alpha,\varphi)$$

The deontic terms are self-explanatory. We see that for Sileno & Pascucci power is concerned with a change of duty of (possibly) another agent towards the acting agent. The predicate has disability is a conjunction of two forms of not having an ability. For Sileno & Pascucci a disability not only concerns not having the ability, but it also means there is no controllability. Meaning that if the action is performed the change will never occur.

The use of predicates and the inclusion of agents as well as objects make the logic quite expressive. The formalizations of the Hohfeldian legal relations are clear and Sileno & Pascucci claim that difficult legal constructions such as delegation can be modeled in the logic as well (Sileno & Pascucci, 2020). The formalizations provide the higher-granularity Hohfeld's theory of normative positions has to offer. Comparing it to the FLINT ontology, the logic shares the idea that actions concern both agents and object.

The use of refined action types by Sileno & Pascucci points at an important modeling choice for L_{FLINT}. The actions in normative texts are often refined types, but where in the formalization do you express this refinement. Do you consider completely refined types such that an action is paying taxes with a bank transfer, or do you just model paying taxes, allowing for the possibility to perform this action with different objects.

Sileno & Pascucci don't mention pre- and postconditons explicitly, but one can infer from the structure of the has ability predicate that pre- and postconditions should be modeled as conditionals within a normative state. Preconditions would take the form:

$$(\text{is}(y, S\varphi) \rightarrow \Diamond\text{Performs}(\text{x},\ \alpha)) \wedge (\text{is}(y, S\varphi) \rightarrow \Box\Diamond\text{Performs}(\text{x},\ \alpha))$$

.
Postconditions would be formalized as:

$$(\text{Performs}(\text{x},\ \alpha) \rightarrow \boxplus\text{is}(y, S\varphi)) \wedge (\Box(\text{Performs}(\text{x},\ \alpha) \rightarrow \boxplus\text{is}(y, S\varphi))$$

. However, there is no way to express in the logic that these formulas are the specific pre- or postcondition for the refined actions in any normative state. It can only express these preconditions and postconditions locally.

Furthermore, because the configurations are partial descriptions of specific objects, the postconditions can only be modeled with regard to a specific object configuration. So the postconditions only model that a specific object is in the configuration. So a postcondition could be that you, as a specific agent in the domain, are in the configuration that you paid taxes. But to express that this is a postcondition for anyone who has paid taxes, we would have to state the implication for each agent in the domain. Again, this means that this logic is not able to operate at both a normative and scenario level at the same time.

3.3.3 A dynamic epistemic formalization of norms

Dong and Roy (2021) call the Hohfeldian distinction between deontic and potestative rights, static and dynamic rights. The dynamic side is about what they call the 'legal competences', power and immunity, and the corresponding liability and no-power rights. The article mainly focuses on how to model these legal competences. For the modeling of static rights Dong & Roy make use of Markovich's work (Markovich, 2020). The logic Dong and Roy have built is therefore a generalization and extension of this work.

The logic Dong and Roy built, is based upon work in the field of dynamic epistemic logic (van Ditmarsch et al., 2007). There are models of static situations and models of actions. Static models can be 'updated' when combining them with the action models.

The static models of Dong and Roy are models built up out of sets of possible worlds related to each other by a preference relation. This preference relation is there to show the comparative 'legal ideality' of the different worlds with respect to the legal relation of one agent towards another agent. Preference relations are reflexive and transitive. Duties are defined in the logic as directed conditional obligations to see to it that a formula φ becomes true, and are understood as being true in the most ideal worlds. So an agent i has an obligation towards an agent j to see to it that φ conditional on some formula ψ: $O_j \rightarrow_i DO_j(\varphi/\psi)$. The Privilege - No-claim relation is modeled as $\neg O_j \rightarrow_i DO_j(\neg\varphi/\psi)$

An action model in the logic of Dong and Roy exists out of a set of actions, a preference relation between these actions and two functions expressing what the pre- and postconditions of actions are. The precondition function takes as input an action from the action set and returns a formula in the language.

An update of a static model with an action model is made possible by making use of functions in the action model expressing the pre- and postconditions of actions. The precondition function maps actions to formulas of the language and the postcondition maps actions and atomic formulas to truth values. The result of a combination of a static model and an action model is a new static model which can differ from the original model with respect to the set of worlds, valuations and preference relations. The new set of worlds, valuations, and preference relations are determined by the pre- and postconditions functions of the action model. If the preconditions are met in the static model, these worlds are retained in the updated model. The postcondition function determines the valuation of the propositions in the new static model. The preference relation between worlds are there when either there is a preference relation between actions of the action model, or the actions in the action model are equivalent and there already was a preference relation in the static model. Two worlds are equivalently preferential if this was the case in the original static model.

A dynamic operator is introduced into the language to show the effects of an action. With this dynamic operator the legal competences of power and immunity can be given a semantics. They end up choosing a local definition of power and immunity, exercising a power therefore means actually changing a normative position in a certain world w, and immunity means that no action can change the normative position in that world:

Let M be a preference-action model and w a state in it such that $M, w \models T(j, k, \psi/\varphi)$ where $T(j, k, \psi/\varphi)$ is an arbitrary normative position between j and k.

- Agent i has a power against agents j, k regarding $T(j, k, \psi/\varphi)$ at M, w, if and only if, agent a can perform one of the actions in the action set in the current normative state and the normative position changes:

$$M, w \models \bigvee_{a \in A} \langle A_i, a \rangle \neg T(j, k, \psi/\varphi)$$

- Agents j, k have an immunity against agent i regarding $T(j, k, \psi/\varphi)$ at M, w, if and only if, after a performance of any action in action set by agent a the normative position of j and k will stays same:

$$M, w \models \bigwedge_{a \in A} [A_i, a])T(j, k, \psi/\varphi)$$

The logic of Dong & Roy might be the most different of all presented logics compared to the FLINT ontology in that duties and the other Hohfeldian relations are modeled through conditionals and preference relations and that we have different models for actions and static situations. However, the logic most closely resembles the ontology in how it treats pre- and postconditions. We see that to decide which formulas function as pre- and postconditions for an action, we construct functions taking actions as input and providing formulas as output. Dong & Roy are also very clear in that they consider actions as changing both legal facts and normative relations at the same time. So while the conceptualization of the normative positions is of little importance to our construction of L_{FLINT}, we should incorporate the idea that pre- and postconditions are determined apart from normative states.

3.4 On our way to L_{FLINT}

All of the presented logics contain aspects that reflect the FLINT ontology. For Sileno and Pascucci (2020) it is the inclusion of agents and objects variables within the logic and relate actions to objects in the language. In Ju and Xu (2023) the authors take duties as primitive facts and also incorporate the idea that pre- and postconditions of actions don't change according to which normative state you are in, the globality of pre- and postconditions in a normative system. This last idea is modeled in a way that most closely resembles the FLINT ontology by Dong and Roy (2021). The pre- and postcondition functions are determined unrelated to normative states, and the postcondition function changes both facts and duties at the same time. These construction ideas will provide inspiration for the start of our work on L_{FLINT} in the next chapter.

Because the presented articles focused on modeling Hohfeldian legal relations, they all provided direct formalizations of these concepts. In the FLINT ontology only duties are a concrete concept within the ontology. A representation of the FLINT ontology therefore does not have to model these concepts explicitly. This makes the construction of L_{FLINT} relatively less complex in this regard.

However, a most complicating factor is that the FLINT ontology is designed to be able to model norms. Norms are always general in the sense that they apply to all citizens and objects of a domain. This means that we should build a logic that can quantify over all agents and objects in the domain, so that we capture the general function of norms. The final version of L_{FLINT} should therefore operate at the level of first-order logic.

Before we'll go first-order, we start by building a propositional version of L_{FLINT} in chapter 4, L^{prop}_{FLINT}. This propositional variant of L_{FLINT} is built to highlight what we believe is the most important aspect of a logic of a normative systems, the fact that pre- and postconditions of actions always remain the same in any normative state.

Chapter 4

A normative logic of pre- and postconditions

In this chapter we'll start with the propositional construction of L_{FLINT}, L^{prop}_{FLINT}, building onto the work of van Gessel (2024). In the first section we'll provide a conceptualization of the logic to motivate our construction choices. We'll then create a first PDL version of L^{prop}_{FLINT} that does not contain duties. We provide soundness and completeness proofs for this logic. L^{prop}_{FLINT} can be viewed as a logic of pre- and postconditions of actions that is different from the classical way of modeling pre- and postconditions in PDL. This is done to fit the normative context of L_{FLINT}, and its results could have broader applications than only in the legal field. It can be used in any normative context in which you want to keep the pre- and postconditions of operations constant. Since the goals of L_{FLINT} remain in the legal realm, we add duties to our logic. We conclude by showing why L^{prop}_{FLINT} with duties is not yet sufficient.

4.1 Conceptualization

L_{FLINT} is a language designed to formalize the FLINT approach towards the modeling of norms. The core concepts of the FLINT approach are included in the FLINT ontology. L_{FLINT} by design will contain more features of legal reasoning than the ontology, such as the ability to reason about norms due to the addition of inference rules to the formal system. At the same time L_{FLINT} will explicitly not include certain aspects of the legal realm that are not part of the FLINT ontology. Important legal features like time or the notion of refrainment, having a duty to not do something, are left out. L_{FLINT} is not supposed to be an all encompassing formalization of legal systems with all their peculiarities, other than is intended for instance by Governatori et al. (2021) with defeasible logic. The reason for this is that the initial goal of this thesis is to use the formalization of the FLINT ontology to be precise about its expressive power, to point out what can and what cannot be expressed by the language. Concepts not included in L_{FLINT} can therefore, as for now, also not be reasoned about. A list of important concepts from the literature that are not included in L_{FLINT} will be presented in Chapter 6. This list serves as an overview of issues that may be relevant for the further development of the FLINT approach.

The logic chosen as the basis for formalizing the FLINT approach is propositional dynamic logic (Fischer & Ladner, 1977). This choice was made after considering the relevant comparisons of Hohfeldian legal relations in the related work chapter and because its machinery most closely resembles the FLINT approach. In comparison to other action logics, such as STIT logics, PDL is not only focused on a resulting state of affairs, but also on the means, in other words 'the actions' by which this state of affairs is reached (Herzig et al., 2018). PDL at its core is a modal logic for the execution of programs, historically stemming from the work of Hoare (Hoare, 1969). Hoare triples are connections of preconditions, programs, and postconditions (Troquard & Balbiani, 2023). Normally PDL contains 'complex' actions. Complex actions are, for instance, actions that express two actions at the same time or multiple actions in a sequence. We simplify PDL by leaving such actions out of our syntax. We step-by-step build a variant of PDL that will result in L^{prop}_{FLINT}. We extend the simplified version of PDL in two steps; first, we'll differentiate between the way PDL reformulates the pre- and postcondition connections and the FLINT approach towards pre- and postconditions, resulting in a logic of pre- and postconditions; in the second step we'll introduce the duty concept into the logic. By separating these two steps, we (1) create a logic that could have broader applications and (2) are able to clearly explain our conceptualization of the duty concept and what changes its addition brings to our system.

4.2 L^{prop}_{FLINT}

In line with the FLINT ontology, the core idea of L_{FLINT} is that a norm contains (complex) institutional facts as preconditions for an institutional act and (complex) institutional facts as postconditions of an institutional act. Hoare triples, as the foundation of PDL, treat these pre- and postconditions together (Hoare, 1969). A 'Hoare triple', $\{A\}\alpha\{B\}$, says that if the precondition A is in place, then when action α is performed, B will always be the postcondition. This is translated into PDL by the conditional A $\rightarrow [\alpha]$B (Troquard & Balbiani, 2023). Here $[\alpha]$ functions as the dynamic operator, stating that it is necessary that after α is performed, B holds.

In L_{FlINT} we'll deviate from these conceptions of pre- and postconditions by taking pre- and postconditons apart. This is done because we are in the context of the modeling of norms and there are several features of norms that are not properly expressible by PDL. Instead of wanting to lay out when and how a program will run, we want to describe what a norm says. The PDL conditional is not able to do this because the PDL conditional doesn't adequately describe the way pre- and postconditions function within normative contexts:

- Necessary and sufficient preconditions:
  PDL cannot express the distinction there is in the law between necessary and sufficient preconditions (Bench-Capon & Prakken, 2008). In the law, necessary preconditions are those conditions that have be true for the overall preconditions of a norm to hold. Without these conditions being true, the overall preconditions of a norm are not fulfilled. Sufficient preconditions are those conditions that, if fulfilled, immediately make sure that the overall preconditions of a norm are fulfilled. L_{FLINT} will be able to express both notions within the language.

- Exact postconditions:
Postconditions of the PDL conditional are not yet exact. They don't have to express all of the postconditions an action. In standard PDL, if for any model and for any state A $\rightarrow$ [α]B, and also for any model and for any state $B \rightarrow C$, then we also have A $\rightarrow$ [α]C. This means that both B and C are postconditions of A, but we have no way of determining that these are all of the postconditions of A, or that there are possibly more. Also B and C could represent disjunctive formulas, meaning that a postcondition might hold in different ways. Modeling norms in such a way conflicts with the principle of the rule of law that the law has to be predictable. We therefore constrain postcondition formulas in L_{FLINT} to non-disjunctive formulas. By having exact postconditions, the L_{FLINT} description of norms adheres to the determinate character of norms that you exactly all of the consequences of an action and make sure that these consequences cannot be indeterminate.

- Globality:
The truth value of the Hoare conditional is local, its truth value in a model depends on the values of A and $[\alpha]B$ at a certain state in a model. What pre- and postconditions for actions are in one state could change for a different state. The important idea that is changed for the L_{FLINT} logic of pre- and postconditions is that, not only do we take pre- and postcondition apart, they are also global notions. This means that in any state within our formal system the formulas that express the pre- and postcondition of an action will be the same. This is in line with the idea that norms determine fixed outcomes. Only those institutional facts change that are under the scope of the postconditions of the norm.

We'll describe norms in the FLINT approach as the conjunction of preconditions for an action and the postconditions of that action, hence whether a norm holds is dependent on both the preconditions and postconditions being stated correctly. This conjunction will entail the conditional A $\rightarrow$ [α]B, but will also express features of norms such as their globality, exact postconditions, and sufficient and necessary preconditions.

By changing those features of PDL that don't fit within the normative context, L_{FLINT} makes sure pre- and postconditions function in the way that would be expected of norms. Consequently, we provide a logic for pre- and postconditions that could have broader applications in contexts like the normative context in which one would like to make sure that the pre- and postconditions of operations always remain the same. In the next few sections we'll lay out the syntax and semantics of L^{prop}_{FLINT}.

4.2.1 Syntax

Language

We formulate a propositional variant of Flint without duties. Let Φ be a countable set of atomic formulas and Π be a countable set of actions.

Vocabulary

1. Logical constants: $\bot$, $\neg$, $\vee$, $\wedge$, $\rightleftharpoons$, $\rightharpoonup$, $\leftharpoondown$, $\blacktriangleright$

2. Propositional letters: p, q, r $\in \Phi$
3. Actions in uppercase bold italic letters: $\boldsymbol{ACTION} \in \Pi$
   For action variables we make use of $\alpha, \beta,$ etc.
4. Auxiliary symbols: (,), [,]

Grammar

We define the syntax of Flint in four steps. The reason for this procedure is that when talking in the L_{FLINT} language , we should be careful about which aspect of the normative system we are describing: postconditions, states, normative features, or the whole system. Postconditions, states and normative features don't allow for all tools of the vocabulary being used. This is because each of these formulas describe different parts of a normative system. The postconditions need to be determinate, so can't allow disjunctive formulas. The state-formulas can be disjunctive, but should describe stable facts, so that we can't allow an action operator in these formulas. Both postconditions and state-formulas are used as input for normative formulas. For this reason, we restrict the formulas that can talk about these aspects of the normative system.

We start by defining the most restricted set of formulas in the FLINT-language and work our way down to general formulas that describe the FLINT-language. The most restricted set of formulas in the FLINT-language is the set of postcondition-formulas. These are formulas that can be used as exact postconditions of actions in FLINT. We then define the set of state-formulas that can function as preconditions of formulas. These formulas describe the institutional facts of the institutional reality.

For these first two sets of formulas we see that we use facts to describe pre- and postconditions. This is done in this manner because we don't want formulas about pre- or postconditions to be pre- or postconditions themselves. E.g. we don't want to be able to express that a formula ψ says that φ is a precondition of action α, whilst ψ is a precondition of β. Which would mean that the precondition of β is that φ is a precondition of α.

Because we limit the complexity of state- and postcondition-formulas, we can define the set of normative-formulas to give us representations of norms in terms of pre- and postconditons. These formulas express which state-formulas are preconditions of an action and which postconditions an actions has.

Finally, we define the overall set of formulas of the FLINT language, which can be used to describe scenarios in which institutional actions are performed.

Postcondition-formulas

Postconditions of actions are those formulas that changed truth-values by transitioning from one state to another because of the action. These formulas need to be determinate to know exactly which changes occurred, therefore disjunctions are not allowed (Di Bello, 2007; Dong & Roy, 2021). By specifying the postconditions, all the information is expressed one needs to know to construct the transition from the old state to the new state. The set of postcondition-formulas is defined as follows:

- Every atomic formula is a postcondition-formula

- Every negation of an atomic formula is a postcondition-formula
- If φ and ψ are postcondition-formulas then so is $(\varphi \wedge \psi)$,
- Nothing else is a postcondition-formula

For example, $(p \wedge r)$ is a postcondition-formula, but $(p \vee r)$ is not. A further constraint on the set of postconditon formulas is that for any arbitrary atomic formula p, no postcondition-formula can contain both p and $\neg p$. $p \wedge \neg p$ is not a postcondition formula.[1] In short, a postcondition-formula is therefore a literal or a conjunction of consistent literals. We define the following relation between postcondition-formulas:

$$\varphi \approx \psi \iff \varphi \text{ and } \psi \text{ contain the same literals, e.g. } (p \wedge \neg q) \wedge r \approx (p \wedge r) \wedge \neg q$$

State-formulas

State-formulas describe the facts of a normative state. We use the formulas of classical propositional logic to represent these facts. The set of state-formulas is defined as follows:

- Every atomic formula is a state-formula
- $\perp$ is a state-formula
- If φ is a state-formula then so is $\neg\varphi$
- If φ and ψ are state-formulas then so is $(\varphi \wedge \psi)$
- If φ and ψ are state-formula formulas then so is $(\varphi \vee \psi)$
- Nothing else is a precondition-formula

Both $(p \wedge r)$ and $(p \vee r)$ are state-formulas, but $[\alpha]\varphi$ isn't.

Normative-formulas

The normative formulas describe the normative features of a normative system. These features describe when actions are allowed to be performed, the preconditions of actions, and what postconditions are the result of a performance of an action. The normative formulas therefore describe connectives between actions and facts. Three different types of preconditions are distinguished (Bench-Capon & Prakken, 2008), plus the exact postcondition:

- $\rightharpoonup$ - sufficient precondition:
  The $\rightharpoonup$ symbol is the sufficient precondition connective. When certain facts are sufficient preconditions for performing an action, the action can be performed in a normative state when those preconditions are true, but it is not necessarily the case that if these facts are not true in the same normative state that the agent can't perform that action. E.g. if φ and ψ are both sufficient preconditions for α, then if φ is not true, but ψ is, α can still be performed.

[1] This is done to avoid contradictions as postconditions, which would lead to impossible actions. In principal we we want to be able to perform all actions.

- $\leftharpoondown$ - necessary precondition:
  The $\leftharpoondown$ symbol represents the necessary precondition connective. When certain facts are necessary preconditions for performing an action, an action can only be performed in a normative state when those preconditions are true, but it is not necessarily the case that if these facts are true in the normative state that the agent can perform that action. E.g. if φ and ψ are both necessary preconditions for α, then if either φ or ψ is false, α cannot be performed. φ and ψ must always be true in order to be able to perform α. But it could the case that there are more necessary preconditions for performing α then φ and ψ. This would mean that the fact that φ and ψ are true is not sufficient for performing α.

- $\rightleftharpoons$ - necessary and sufficient precondition:
  We define $\varphi \rightleftharpoons \alpha$ as a shorthand for $\varphi \leftharpoondown \alpha \wedge \varphi \rightharpoonup \alpha$. The $\rightleftharpoons$ symbol is the necessary and sufficient precondition connective. When we state that a combination of formulas are necessary and sufficient preconditions for performing an action, this means that all of these formulas are necessary preconditions for an action, and that taken together they are sufficient for performing that action. This also means that if φ and ψ are both necessary and sufficient preconditions, they must be logically equivalent.

- $\blacktriangleright$ - exact postcondition:
  The $\blacktriangleright$ symbol is the exact postcondition connective. This connective is used to describe the act-fact-connection that after an act is performed, the atomic facts linked to the action by the exact postcondition connective are always true in the next state. More importantly, all other atomic facts don't change their truth value. E.g. if p and q are false in some normative state, and $\alpha \blacktriangleright p$ is true, then after performing α, p will be true in the next normative state, but q will remain false.

The set of normative-formulas is then defined as follows:

- If φ is a state-formula and α an action, then the following are normative-formulas:
  - $\varphi \leftharpoondown \alpha$
  - $\varphi \rightharpoonup \alpha$
  - $\varphi \rightleftharpoons \alpha$
- If α is an action and φ a postcondition-formula then $\alpha \blacktriangleright \varphi$ is a normative-formula.
- Nothing else is a normative-formula

For example, $(p \vee r) \leftharpoondown \alpha$ is a normative-formula, while $\neg(\alpha \blacktriangleright \neg p)$, $(p \leftharpoondown \alpha) \vee (\alpha \blacktriangleright q)$ and $\alpha \blacktriangleright (\beta \blacktriangleright p)$ are not.

Definition norm of L_{FLINT}:
A norm of L_{FLINT} is a collection of normative-formulas expressing the sufficient and necessary preconditions for a certain action, and the exact postconditions for that same action.

FLINT-formulas

We now define the syntax of FLINT-formulas:

- Every atomic formula is a formula
- Every normative-formula is a formula
- $\bot$ is a formula
- If φ is a formula then so is $\neg\varphi$
- If φ and ψ are formulas then so is $(\varphi \wedge \psi)$
- If φ and ψ are formulas then so is $(\varphi \vee \psi)$
- If φ is a formula and α an atomic action then $[\alpha]\varphi$ is a formula
- Nothing else is a formula

Compared to standard PDL we do not define any operations on actions, so only atomic actions are allowed. The material implication is defined as usual.

4.2.2 Semantics

Before we define our model of a normative system, we define two functions, *Pre* and *Post*:

- *Pre* a function from the set of actions into state-formulas
- *Post* a function from the set of actions into postcondition-formulas

These functions will determine the pre- and postconditions for all actions in our language. In this way we replicate how a legal system functions. Once all the rules are in place, we can reason about the way they interact.

Our L_{FLINT} model is the model we use to evaluate these interactions. Within this model $Pre(\alpha)$ and $Post(\alpha)$ will be metalinguistic abbreviations of the output sentences of the *Pre* and *Post* functions. E.g. $Pre(\alpha)$ could stand for $(p \vee q) \wedge \neg r$ and $Post(\alpha)$ for $(p \wedge q) \wedge \neg r$. We'll now present our L_{FLINT} model.

Model

We evaluate formulas relative to a model $M = \langle W, R_\alpha, V \rangle$ with:

- W a nonempty set of states
- R_α a mapping from the set Π into binary relations on W
- V a mapping of Φ into subsets of W

We impose the following constraints on R and V. For all actions α:

- If $Pre(\alpha)$ is false in w according to classical truth-functional propositional logic, then it is not the case that $wR_\alpha v$ for any $v \in W$

- If $Pre(\alpha)$ is true in w according to classical truth-functional propositional logic, then there is at least one $v \in W$ such that the following holds for all atomic formulas p:
  - If p is a conjunct of $Post(\alpha)$, then $v \in V(p)$
  - If $\neg p$ is a conjunct of $Post(\alpha)$, then $v \notin V(p)$
  - If neither p nor $\neg p$ occurs in $Post(\alpha)$, then $v \in V(p)$ just in case $w \in V(p)$

  For those $v \in W$ such that the above holds wR

We will view the states in W as states in a normative system, and we consider a relation R_α as indicating what can happen when an action is performed in a given state: namely, reaching one of the states connected by R_α to the current state. We view these results as deterministic. We know exactly what will happen after an action is performed. This is in line with the predictability of the law principle and this principle has been incorporated by other works on action logics and the law as well (Di Bello, 2007; Dong & Roy, 2021). This is possible due to the fact that L_{FLINT} is only concerned with institutional reality. Actions have effects in physical reality that are not predetermined by norms, but we are not concerned with these changes of facts.

Truth conditions

The truth value of a formula relative to a state in a model is determined as follows:

$$
\begin{aligned}
M, w \models p &\iff w \in V(p) \\
M, w \not\models \bot & \\
M, w \models \neg\varphi &\iff M, w \not\models \varphi \\
M, w \models \varphi \vee \psi &\iff M, w \models \varphi \text{ or } M, w \models \psi \\
M, w \models \varphi \leftharpoondown \alpha &\iff \text{for all } v \in W : M, v \models Pre(\alpha) \to \varphi \\
M, w \models \varphi \rightharpoonup \alpha &\iff \text{for all } v \in W : M, v \models \varphi \to Pre(\alpha) \\
M, w \models [\alpha]\varphi &\iff \text{for all } v \in W \text{ such that } wR_\alpha v : M, v \models \varphi \\
M, w \models \alpha \blacktriangleright \varphi &\iff Post(\alpha) \approx \varphi
\end{aligned}
$$

The important clauses are discussed in more detail below.

Actions

A formula of the form $[\alpha]\varphi$ is true in a state just in case φ is true in all states connected to the current state by the relation R_α. This intuitively means that 'φ will be true after α. Note that this is not the same as a postcondition because it applies locally to a state.

Preconditions

A formula of the form $\varphi \leftharpoondown \alpha$ is true just in case if the preconditions of α are true, then φ must be true. Intuitively, a formula of this form expresses 'φ is a necessary precondition of α'. The direction of the harpoon arrow is used to indicate that if α can be performed, φ is true.

We also define an operator for sufficient preconditions: a formula of the form $\varphi \rightharpoonup \alpha$ is true just in case if φ is true, then the preconditions of α must be true. The direction of the harpoon arrow now shows that if φ is true, α can be performed.

The truth conditions for these formulas are global because we want to express that no matter what normative state we are in, the same norms and therefore the same preconditions hold.

Postconditions

A formula of the form $\alpha \blacktriangleright \varphi$ can be read as 'φ is the postcondition of α, all else remains the same'. This formula is not truth-functional because it is not possible to give truth conditions for $\alpha \blacktriangleright \varphi$ based on the truth of φ.[2] The truth value of the formula depends on the constituency of φ and whether that matches the output of $Post(\alpha)$. We use $Post(\alpha)$ as a metalinguistic abbreviation for some formula φ that is the output of the $Post$ function with α as input. Therefore if we have $Post(\alpha) = (p \wedge q) \wedge r$, all different configurations of the literals p, q and r are postcondition-formulas such that $\alpha \blacktriangleright (p \wedge q) \wedge r$. For instance also $\alpha \blacktriangleright (q \wedge r) \wedge p$.

4.2.3 Proof system

We obtain a simple proof system by realizing that $\leftharpoondown$ and $\rightharpoonup$ become expressible as abbreviations if we introduce a global modality G with the following semantics:

$$M, w \models G\varphi \iff \text{for all } v \in W : M, v \models \varphi$$

Because we only defined atomic actions, the logic of $[\alpha]$ is a normal modal logic which relates to G in a well understood way.

[2] The semantics of $\blacktriangleright$ is formulated in such a way that a formula of the form $\alpha \blacktriangleright \varphi$ never follows from any set of $\blacktriangleright$-free formulas. In a sound proof system this formula can therefore also never be derived. We see that this is a desirable property because norms do not depend on which actions, in fact, can be performed. A similar approach could have been adopted for $\rightleftharpoons$, but we chose not too to because of its relation to $\leftharpoondown$ and $\rightharpoonup$ as subformulas.

Axioms and rules

- All propositional tautologies
- **Global operator axioms:**
  1. $G\varphi \to \varphi$
  2. $G\varphi \to GG\varphi$
  3. $\varphi \to G\neg G\neg\varphi$
- **Action relation axioms:**
  1. $G\varphi \to [\alpha]\varphi$
  2. $((\varphi \rightharpoonup \alpha) \wedge \varphi) \to \neg[\alpha]\bot$
  3. $((\varphi \leftharpoondown \alpha) \wedge \neg\varphi) \to [\alpha]\bot$
  4. $(\alpha \blacktriangleright \varphi) \to [\alpha]\varphi$
  5. $(\alpha \blacktriangleright \varphi) \to G((p \to [\alpha]p) \wedge (\neg p \to [\alpha]\neg p))$,
     for p not occuring in φ
- **Precondition axioms**
  1. $(\varphi \leftharpoondown \alpha) \leftrightarrow G(Pre(\alpha) \to \varphi)$
  2. $(\varphi \rightharpoonup \alpha) \leftrightarrow G(\varphi \to Pre(\alpha))$
- **Postcondition axioms:**
  1. $(\alpha \blacktriangleright \varphi) \to (\alpha \blacktriangleright \psi)$,
     for $\varphi \approx \psi$
  2. $(\alpha \blacktriangleright \varphi) \to \neg(\alpha \blacktriangleright \psi)$,
     for $\varphi \not\approx \psi$
  3. $(\alpha \blacktriangleright Post(\alpha))$

Rules (for $\boxtimes \in \{G, [\alpha]\}$):

1. $\varphi, \varphi \to \psi \ / \ \psi$
2. $\varphi \ / \ \boxtimes\varphi$
3. $\boxtimes(\varphi \to \psi) \ / \ \boxtimes\varphi \to \boxtimes\psi$

4.3 Soundness and completeness proofs

4.3.1 Soundness

The soundness of the proof system can be checked by checking the soundness of the individual axioms and rules. $[\alpha]$ is the smallest normal modality, while G is $S5$. Axiom $G\varphi \to [\alpha]\varphi$ encodes their relation: R_α is a subset of the global accessibility relation. That the axioms regarding $\leftharpoondown, \rightharpoonup, \blacktriangleright$ are sound follows from the constraints on the model:

- Global operator axioms:
  - $G\varphi \to \varphi$:
    Assume, $M, w \models G\varphi$, by definition of truth in an interpretation for any $v \in W, M, v \models \varphi$, therefore also $M, w \models \varphi$

- $\varphi \to G\neg G\neg\varphi$:
Assume $M, w \models \varphi$. Now suppose for contradiction that $M, w \models \neg G\neg G\neg\varphi$, by definition of truth in an interpretation there is a v in w s.t. $v \not\models \neg G\neg\varphi$, by definition of truth in an interpretation that $v \models G\neg\varphi$, by definition of truth in an interpretation for any $u \in W$, $M, u \models \neg\varphi$. But we assumed $M, w \models \varphi$ and $w \in W$. Contradiction. Therefore $M, w \models G\neg G\neg\varphi$

- $G\varphi \to GG\varphi$
Assume $M, w \models G\varphi$, by definition of truth in an interpretation for any $v \in W, M, v \models \varphi$. We need to show that $M, w \models GG\varphi$, which by definition of truth in an interpretation means that for any $v \in W, M, v \models G\varphi$, meaning for any $u \in W, M, u \models \varphi$.

  Since we assumed $M, w \models G\varphi$ we have that for any $u \in W, M, u \models \varphi$. Since we have for any $u \in W, M, u \models \varphi$, we have that for any $v \in W, M, v \models G\varphi$. Since for any $v \in W, M, v \models G\varphi$, $M, w \models GG\varphi$

• Action relation axioms

  - $G\varphi \to [\alpha]\varphi$:
Assume $M, w \models G\varphi$, by definition of truth in an interpretation for any $v \in W, M, v \models \varphi$. Now assume for any arbitrary $v \in W$ such that $wR_\alpha v$ and show that $M, v \models \varphi$. We assume for some arbitrary v that $wR_\alpha v$. We had that for any $v \in W, M, v \models \varphi$, so also for this $v \in W$ such that $wR_\alpha v$ in particular. Since v was arbitrary, $M, w \models [\alpha]\varphi$

  - $((\varphi \rightharpoonup \alpha) \land \varphi) \to \neg[\alpha]\bot$:
Assume $M, w \models (\varphi \rightharpoonup \alpha) \land \varphi$. Because of $\varphi \rightharpoonup \alpha$ by definition of truth in an interpretation we have that for any $v \in W, M, v \models (\varphi \to Pre(\alpha))$, so $M, w \models (\varphi \to Pre(\alpha))$. Since $M, w \models \varphi$ by assumption and $M, w \models (\varphi \to Pre(\alpha))$, we have by the truth definitions of $\to$ that $M, w \models Pre(\alpha)$ is true. By constraints on the model if $Pre(\alpha)$ is true there is at least one v s.t. $wR_\alpha v$, therefore $M, w \models \neg[\alpha]\bot$

  - $((\varphi \leftharpoondown \alpha) \land \neg\varphi) \to [\alpha]\bot$
Assume $M, w \models (\varphi \leftharpoondown \alpha) \land \neg\varphi$. Since $(\varphi \leftharpoondown \alpha)$ is true, by definition of truth in an interpretation we have that for any $v \in W, M, v \models (Pre(\alpha) \to \varphi)$. Therefore $M, w \models (Pre(\alpha) \to \varphi)$. Since we assumed $M, w \models \neg\varphi$, $Pre(\alpha)$ must be false. Then, by constraints on the model, there is no v s.t. $wR_\alpha v$, therefore $M, w \models [\alpha]\bot$

  - $(\alpha \blacktriangleright \varphi) \to [\alpha]\varphi$:
Assume $M, w \models (\alpha \blacktriangleright \varphi)$, by definition of truth in an interpretation $\varphi \approx Post(\alpha)$. This means that if p is a conjunct of $Post(\alpha)$ it is a conjunct of φ and if $\neg p$ is a conjunct of $Post(\alpha)$ it is a conjunct of φ. Therefore, both formulas are true in the exact same worlds. In other words for any v if $M, v \models Post(\alpha)$, $M, v \models \varphi$. Therefore if $Post(\alpha)$ is true in all of the R_α accessible worlds, φ is true in all of the R_α accessible worlds.

    We need to show that for any arbitrary $v \in W$ such that $wR_\alpha v$, $M, v \models \varphi$. We assume some arbitrary v s.t. $wR_\alpha v$, by the constraints on the model for v, if p is a conjunct of $Post(\alpha)$, $p \in V(p)$ and if $\neg p$

is a conjunct of $Post(\alpha)$, $p \notin V(p)$. Therefore $M, v \models Post(\alpha)$, and since v was arbitrary this is the case for any v s.t. $wR_\alpha v$.

Since $\varphi \approx Post(\alpha)$, the same goes for φ.

So we have that $M, w \models [\alpha]\varphi$

- $(\alpha \blacktriangleright \varphi) \rightarrow G((p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p))$ for p not occurring in φ :
Assume $M, w \models (\alpha \blacktriangleright \varphi)$, by definition of truth in an interpretation $\varphi \approx Post(\alpha)$, such that if p is a conjunct of $Post(\alpha)$ it is a conjunct of φ and if $\neg p$ is a conjunct of $Post(\alpha)$ it is a conjunct of φ.

  Now assume p does not occur in φ therefore not in $Post(\alpha)$.

  First assume p is true in w and show that for any arbitrary v s.t. $wR_\alpha v$ that p is true in v. By constraints on the model for any v s.t $wR_\alpha v$ if p is not a conjunct of Post(α) and p is true in w, then p is true in v. So also for $wR_\alpha v$. Therefore $G(p \rightarrow [\alpha]p)$

  Now assume $\neg p$ is true in w and show that for any arbitrary 'v s.t. $wR_\alpha v$ that $\neg$ is true in v. By constraints on the model for any v s.t $wR_\alpha v$ if p is not a conjunct of Post(α) and $\neg$ p is true in w, then $\neg p$ is true in v. So also for $wR_\alpha v$. Therefore $G(p \rightarrow [\alpha]\neg p)$

  Therefore $M, w \models G((p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p))$

- Precondition axioms

  - $(\varphi \leftharpoondown \alpha) \leftrightarrow G(Pre(\alpha) \rightarrow \varphi)$
  Assume $M, w \models (\varphi \leftharpoondown \alpha)$, by definition of truth in an interpretation if and only if, $M, w \models G(Pre(\alpha) \rightarrow \varphi)$. Therefore $M, w \models (\varphi \leftharpoondown \alpha) \leftrightarrow G(Pre(\alpha) \rightarrow \varphi)$
  - $(\varphi \rightharpoondown \alpha) \leftrightarrow G(\varphi \rightarrow Pre(\alpha))$
  Assume $M, w \models (\varphi \rightharpoondown \alpha)$, by definition of truth in an interpretation if and only if $M, w \models G(\varphi \rightarrow Pre(\alpha)$. Therefore $M, w \models (\varphi \rightharpoondown \alpha) \leftrightarrow G(\varphi \rightarrow Pre(\alpha))$

- Postcondition axioms

  - $(\alpha \blacktriangleright \varphi) \rightarrow (\alpha \blacktriangleright \psi)$,
  for $\varphi \approx \psi$:

    Assume $M, w \models (\alpha \blacktriangleright \varphi)$, by definition of truth in an interpretation $\varphi \approx Post(\alpha)$. Assume $\varphi \approx \psi$, then also $\psi \approx Post(\alpha)$, by definition of truth in an interpretation $M, w \models (\alpha \blacktriangleright \psi)$
  - $(\alpha \blacktriangleright \varphi) \rightarrow \neg(\alpha \blacktriangleright \psi)$.
  for $\varphi \not\approx \psi$: Assume $M, w \models (\alpha \blacktriangleright \varphi)$, by definition of truth in an interpretation $\varphi \approx Post(\alpha)$. Assume $\varphi \not\approx \psi$ then $\psi \not\approx Post(\alpha)$. By contraposition of the definition of truth in an interpretation, we have that $M, w \models \neg(\alpha \blacktriangleright \varphi)$
  - $(\alpha \blacktriangleright Post(\alpha))$:
  Assume $M, w \models Post(\alpha)$, $Post(\alpha) \approx Post(\alpha)$, therefore by definition of truth in an interpretation $M, w \models (\alpha \blacktriangleright Post(\alpha))$

4.3.2 Completeness

We claim that the proof system is also complete. We'll first sketch the proof to give the intuitive idea and then prove it formally:

1. Well start by constructing a canonical non-standard model. This is a structure $\langle W^c, R^c_\alpha, R^c_G, V^c\rangle$ which is different from a regular model in that it includes an explicit accessibility relation R_G for the modality G instead of it being global.

2. We prove the truth lemma: for all maximally consistent sets of Γ : $\varphi \in \Gamma \iff M^c, \Gamma \models \varphi$

3. We use the truth lemma to show that if $\Gamma \nvdash \psi$, then there exists a maximally consistent set $\Gamma' \supseteq \Gamma \cup \{\neg\psi\}$ such that $M^c, \Gamma' \models \varphi$ for all $\varphi \in \Gamma$ but $M^c, \Gamma' \not\models \psi$.

4. We then show how to construct a regular model out of the canonical non-standard model. This amounts to taking the R^c_G-equivalence class of Γ' as the domain of states.

   - We show that the model satisfies the constraints on R and V regarding the preconditions in Pre and the postconditions in $Post$.
   - We show that this model makes the exact same set of formulas true.

5. We have thereby shown that if $\Gamma \nvdash \psi$, there exists a model that satisfies Γ and not ψ, so that $\Gamma \not\models \psi$.

Step 1: construction of the canonical model

We construct the following canonical model:

- We define R^c_G as follows: $\Gamma R^c_G \Delta \iff \{\varphi \mid G\varphi \ \in \Gamma\} \subseteq \Delta$
- We define R^c_α as follows: $\Gamma R^c_\alpha \Delta \iff \{\varphi \mid [\alpha]\varphi \in \Gamma\} \subseteq \Delta$
- We define V^c as follows: $\Gamma \in V(p) \iff p \in \Gamma$
- The truth conditions for the canonical model for the non-standard symbols of our language are as follows:

$$
\begin{aligned}
M^c, w \models \varphi \leftharpoondown \alpha &\iff \text{for all } v \in W \text{ s.t. } wR_Gv : M^c, v \models Pre(\alpha) \to \varphi \\
M^c, w \models \varphi \rightharpoondown \alpha &\iff \text{for all } v \in W \text{ s.t. } wR_Gv : M^c, v \models \varphi \to Pre(\alpha) \\
M^c, w \models G\varphi &\iff \text{for all } v \in W \text{ such that } wR_Gv : M^c, v \models \varphi \\
M^c, w \models \alpha \blacktriangleright \varphi &\iff Post(\alpha) \approx \varphi
\end{aligned}
$$

Step 2: Truth Lemma

We cannot perform a straightforward induction on the complexity of formulas and prove the truth lemma for our formal system of L_{FLINT}. This is the case because the standard inductive step that we use for the complexity of formulas by number of connectives/operators is not sufficient when it comes to defining the complexity of formulas with the necessary and sufficient precondition symbols as dominant connective. The truth conditions of necessary and sufficient preconditions are expressed by formulas that contain both a global operator and an implication. Under the normal complexity measure of the number of connectives/operators the complexity of $G(Pre(\alpha) \to \varphi)$ is $2 + (c(\varphi) + c(Pre(\alpha))$, which is greater than that of $\varphi \leftharpoondown \alpha$, which is $1 + c(\varphi)$. Using the normal complexity measure we can therefore not perform the standard inductive step in our truth lemma proof.

To perform the necessary induction for the truth lemma proof, we need to make use of a different complexity measure.

Complexity measure:
We need to make use of a complexity measure c that has the following properties:

1. If ψ is a proper subformula of φ, then $c(\varphi) > c(\psi)$
2. $c(\varphi \leftharpoondown \alpha) > c(G(Pre(\alpha) \to \varphi))$
3. $c(\varphi \rightharpoonup \alpha) > c(G(\varphi \to Pre(\alpha))$

The following non-standard complexity measure has these properties:

$$\begin{aligned}
&c(p) = c(\bot) = 1 \\
&c(\neg\varphi) = 1 + c(\varphi) \\
&c(\varphi \circ \psi) = 1 + \max(c(\varphi), c(\psi)) \quad \text{for } \circ \in \{\wedge, \vee, \to\} \\
&c(\Box\varphi) = 1 + c(\varphi) \quad \text{for } \Box \in \{G, [\alpha]\} \\
&c(\varphi \circ \alpha) = 3 + \max(c(\varphi), c(Pre(\alpha))) \quad \text{for } \circ \in \{\leftharpoondown, \rightharpoonup\} \\
&c(\varphi \blacktriangleright \alpha) = 1 + c(\varphi)
\end{aligned}$$

By inspection of the definition, it is immediately visible that c has both of the properties. Consequently, we get the result that we needed, complexity $G(Pre(\alpha) \to \varphi) < \varphi \leftharpoondown \alpha$ and $G(\varphi \to Pre(\alpha)) < \varphi \leftharpoondown \alpha$.

Since because of syntactic constraints normative formulas cannot contain normative formulas, the complexity measure is not circular and so is well defined.[3]

Truth lemma:
Let φ be an arbitrary modal formula. Then $M^c, w \models \varphi$ if and only if $\varphi \in w$ for all $w \in W^c$.

Base case
By definition of V^c we have that $\Gamma \in V^c(p) \iff M^c, \Gamma \models p \iff p \in \Gamma$

Inductive step
Inductive hypothesis: for all sentences ψ less complex than, φ $M^c, \Gamma \models \psi \iff \psi \in \Gamma$. For the proofs we'll make use of w and v to represent the maximally consistent sets.

[3]E.g. it is not possible that $(\varphi \rightharpoonup \alpha) \rightharpoonup \beta$ is a formula such that our complexity measure becomes circular.

Proof. φ : for $\boxtimes \in \{G, [\alpha]\}$

$M^c, w \models \boxtimes\varphi$ if and only if $\boxtimes\varphi \in w$:

- $(\rightarrow)$ Suppose $M^c, w \models \boxtimes\varphi$
  - By definition of truth in an interpretation for all $v \in W$ such that $wR_{\boxtimes}v$: $M^c, v \models \varphi$
  - Using the existence lemma in Blackburn et al. (2002) we can say that for any world $w \in W^c$, if $\neg \boxtimes \neg\varphi \in w$ then there is a state $v \in W^c$ such that $wR^c_\alpha v$ and $M^c, v \models \varphi$
  - By contraposition of the existence lemma, we get that if there is no state $v \in W^c$ such that $wR_{\boxtimes}v$ and $M^c, v \models \varphi$, then $\boxtimes\neg\varphi \in w$
  - Now since we had that for all $v \in W$ such that $wR_{\boxtimes}v$: $M^c, v \models \varphi$, we have that there is no world v s.t. $wR_{\boxtimes}v$ and s.t. $M^c, v \models \neg\varphi$, we get that $\boxtimes\varphi \in w$
- $(\leftarrow)$ Now, suppose $\boxtimes\varphi \in w$
  - By definition of $R^c_{\boxtimes}$ $\varphi \in v$ for any v s.t. $wR_{\boxtimes}v$
  - By inductive hypothesis for all $v \in W$ such that $wR_{\boxtimes}v$: $M^c, v \models \varphi$
  - By definition of truth in an interpretation $M^c, w \models \boxtimes\varphi$

□

Proof. $\varphi : \varphi \leftharpoondown \alpha$,
$M^c, w \models \varphi \leftharpoondown \alpha$ if and only if $\varphi \leftharpoondown \alpha \in w$

- $(\rightarrow)$ Suppose $M^c, w \models \varphi \leftharpoondown \alpha$
  - By definition of truth in an interpretation for all $v \in W$ s.t. $wR_G v, M^c, v \models \text{Pre}(\alpha) \rightarrow \varphi$
  - By definition of truth in an interpretation $M^c, w \models G(\text{Pre}(\alpha) \rightarrow \varphi)$
  - By the induction hypothesis, we have that $G(\text{Pre}(\alpha) \rightarrow \varphi) \in w$. We can use the induction hypothesis here because we have defined our complexity measure such that $c(\varphi \leftharpoondown \alpha) > c(G(Pre(\alpha) \rightarrow \varphi))$.
  - By axiom $\varphi \leftharpoondown \alpha \leftrightarrow G(Pre(\alpha \rightarrow \varphi)$, and the modus ponens inference rule we have that $\varphi \leftharpoondown \alpha \in w$
- $(\leftarrow)$ Now, suppose $\varphi \leftharpoondown \alpha \in w$
  - By axiom $(\varphi \leftharpoondown \alpha) \rightarrow G(Pre(\alpha) \rightarrow \varphi)$ and modus ponens, we have that $G(\text{Pre}(\alpha) \rightarrow \varphi) \in w$.
  - By the induction hypothesis, we have that $M^c, w \models G(\text{Pre}(\alpha) \rightarrow \varphi)$.
  - By definition of truth in an interpretation, we have that $M, w \models (\varphi \leftharpoondown \alpha)$.

□

Proof. $\varphi : \varphi \rightharpoonup \alpha$,
$M^c, w \models \varphi \rightharpoonup \alpha$ if and only if $\varphi \rightharpoonup \alpha \in w$

- $(\rightarrow)$ Suppose $M^c, w \models \varphi \rightharpoonup \alpha$
  - By definition of truth in an interpretation for all $v \in W$ s.t. $wR_Gv, M, v \models \varphi \rightarrow Pre(\alpha)$
  - By definition of truth in an interpretation $M, w \models G(\varphi \rightarrow Pre(\alpha))$
  - By the induction hypothesis, we have that $G(\varphi \rightarrow Pre(\alpha)) \in w$. We can use the induction hypothesis here because we have defined our complexity measure such that $c(\varphi \rightharpoonup \alpha) > c(G(\varphi \rightarrow Pre(\alpha)))$.
  - By axiom $\varphi \rightharpoonup \alpha \leftrightarrow G(\varphi \rightarrow Pre(\alpha))$, and modus ponens we have that $\varphi \rightharpoonup \alpha \in w$
- $(\leftarrow)$ Now, suppose $\varphi \rightharpoonup \alpha \in w$
  - By axiom $(\varphi \rightharpoonup \alpha) \rightarrow G(\varphi \rightarrow Pre(alpha))$ and modus ponens, we have that $G(\varphi \rightarrow Pre(\alpha)) \in w$.
  - By the induction hypothesis, we have that $M^c, w \models G(\varphi \rightarrow Pre(\alpha))$.
  - By the definition of truth in an interpretation, we have that $M^c, w \models (\varphi \rightharpoonup \alpha)$.

□

Proof. $\varphi : \alpha \blacktriangleright \varphi$
$M^c, w \models \alpha \blacktriangleright \varphi$ if and only if $\alpha \blacktriangleright \varphi \in w$:

- $(\rightarrow)$ Suppose $M, w \models \alpha \blacktriangleright \varphi$
  - By definition of truth in an interpretation $\varphi \approx Post(\alpha)$
    By axioms $\alpha \blacktriangleright Post(\alpha)$ and $(\alpha \blacktriangleright \varphi) \rightarrow (\alpha \blacktriangleright \psi)$ for $\varphi \approx \psi$, we have that $\alpha \blacktriangleright \varphi \in w$
- $(\leftarrow)$ Now, suppose $\alpha \blacktriangleright \varphi \in w$
  - By axiom $(\alpha \blacktriangleright Post(\alpha))$ we have that $\alpha \blacktriangleright Post(\alpha) \in w$
  - Suppose $\varphi \not\approx Post(\alpha)$, then by our assumption $\alpha \blacktriangleright \varphi$ and axiom $(\alpha \blacktriangleright \varphi) \rightarrow \neg(\alpha \blacktriangleright \psi)$ for $\varphi \not\approx \psi$, we have that $\neg(\alpha \blacktriangleright Post(\alpha))$, but $(\alpha \blacktriangleright Post(\alpha))$ is an axiom, so we have a contradiction.
  - Therefore if $\alpha \blacktriangleright \varphi \in w$, then $\varphi \approx Post(\alpha)$ and by definition of truth in an interpretation $M, w \models \alpha \blacktriangleright \varphi$

□

Step 3: Completeness of the canonical model

Completeness: If $M^c, \Gamma \models \psi$ then $\Gamma \vdash \psi$

By contraposition of the completeness theorem we get that if $\Gamma \nvdash \psi$ then $M^c, \Gamma \not\models \psi$

Proof. If $\Gamma \nvdash \psi$ then $M^c, \Gamma \not\models \psi$

Suppose we take a set Γ s.t. $\Gamma \nvdash \psi$. We can extend this set to a maximally consistent set Γ' by the Lindenbaum lemma (which can be found in Blackburn et al., 2002, chapter 4). Since Γ' is maximally consistent and $\Gamma \subseteq \Gamma'$, $\Gamma' \nvdash \psi$. By properties of maximally consistent sets $\psi \notin \Gamma'$. By the truth lemma $M^c, \Gamma' \not\models \psi$. Now since $\Gamma \subseteq \Gamma'$, $\Gamma \not\models \psi$

□

Step 4: construct FLINT model out of Canonical model

We now construct a standard FLINT model out of the non-standard canocial model. This amounts to taking the R_G-equivalence class of Γ' as the domain of states. We denote this equivalence class by $W^{\Gamma'}$. First we show that R^c_G is an equivalence relation on the set of worlds W^c:

Proof. G is an equivalence relation on the set of worlds:

- Reflexivity: for any $w \in W^c$, $wR^c_G w$
  Suppose for an arbitrary $w \in W^c$ that $G\varphi \in W$, by our axiom $G\varphi \to \varphi$ and the modus ponens inference rule we have $\varphi \in w$. By definition of R^c_G, we have that $wR^c_G w$.

  Since w was arbitrary we have that for any $w \in W^c$, $wR_G w$.

- Symmetry: if $wR^c_G v$, then $vR^c_G w$ for any arbitrary $v, w \in W^c$
  Suppose $wR^c_G v$, therefore $G\varphi \in w \to \varphi \in v$ for an arbitrary formula φ. Now suppose that $G\varphi \in v$. We want to show that $\varphi \in w$.

  Assume $\varphi \notin w$, by axiom $\varphi \to G\neg G\neg\varphi$ and modus ponens we get that $G\neg G\neg\neg\varphi \in w$ meaning $G\neg G\varphi \in w$. By our hypothesis $G\varphi \in w$ implies $\varphi \in v$. Therefore since $G\neg G\varphi \in w$, $\neg G\varphi \in v$. But we had assumed that $G\varphi \in v$. Contradiction. Therefore $\varphi \in w$. Since $\varphi \in w$, we have that if $G\varphi \in v$ then $\varphi \in w$. By definition of R^c_G, $vR^c_G w$.

  Since v, wwere arbitrary we have that if $wR^c_G v$, then $vR^c_G w$ for any arbitrary $v, w \in W^c$.

- Transitivity: if $wR^c_G v$ and $vR^c_G u$, then $vR^c_G u$ for any arbitrary v, w, u $\in W^c$
  Suppose $wR^c_G v$ and $vR^c_G u$, therefore $G\varphi \in w$ implies $\varphi \in v$, and $G\varphi \in v$ implies $\varphi \in u$, Now suppose that $G\varphi \in w$ and show that $\varphi \in u$.

  Since $G\varphi \in w$, the axiom $G\varphi \to GG\varphi$ and modus ponens, $GG\varphi \in w$. Since $G\varphi \in w$ implies $\varphi \in v$, $G\varphi \in v$. Since $G\varphi \in v$ implies $\varphi \in u$, $\varphi \in u$.

  Therefore if $G\varphi \in w$ then $\varphi \in u$. By definition of R^c_G, $wR^c_G u$.

  Since v, w and u were arbitrary we have that if $wR^c_G v$ and $vR^c_G u$, then $vR^c_G u$ for any arbitrary v, w, $u \in W^c$

□

In our standard FLINT model we do not have an explicit accessibility relation for the modality G between worlds. G is global. This model then is a structure: $\langle W^{\Gamma'}, R_\alpha^{\Gamma'}, V^{\Gamma'}\rangle$:

- $W^{\Gamma'}$ is the R_G^c-equivalence class of Γ' on W^c
- We define $R_\alpha^{\Gamma'}$ as follows: $\Gamma R_\alpha^{\Gamma'} \Delta \iff \{\varphi \mid [\alpha]\varphi \in \Gamma\} \subseteq \Delta$
- We define $V^{\Gamma'}$ as follows: $\Gamma \in V(p) \iff p \in \Gamma$

We prove that this is a structure that represents a L_{FLINT}^{prop} model by showing that the model satisfies the constraints on R and V regarding the preconditions in Pre and the postconditions in $Post$. We then show that this model makes the exact same set of formulas true as M^c.

Constraints on the model:

For this part of the proof we'll need to make use of two theorems:

Theorem 1. $Pre(\alpha) \leftharpoondown \alpha$

Proof. $Pre(\alpha) \to Pre(\alpha)$ is a tautology and therefore a theorem. By inference rule $\varphi \to \Box\varphi$ if φ is a theorem. We therefore have $G(Pre(\alpha) \to Pre(\alpha))$, and by definition of truth in an interpretation we have that $Pre(\alpha) \rightharpoondown \alpha$ □

Theorem 2. $Pre(\alpha) \rightharpoondown \alpha$

Proof. $Pre(\alpha) \to Pre(\alpha)$ is a tautology and therefore a theorem. By inference rule $\varphi \to \Box\varphi$ if φ is a theorem, we have $G(Pre(\alpha) \to Pre(\alpha))$, by definition of truth in an interpretation we have that $Pre(\alpha) \rightharpoondown \alpha$ □

1. **No relation constraint:**
   If $Pre(\alpha)$ is false in w according to classical truth-functional propositional logic, then it is not the case that $wR_\alpha^{\Gamma'}v$ for any $v \in W$

   Proof. Suppose $M^{\Gamma'}, w \not\models Pre(\alpha)$

   - $M^{\Gamma'}, w \not\models Pre(\alpha)$, therefore by definition of truth in an interpretation $M^{\Gamma'}, w \models \neg Pre(\alpha)$
   - By axioms $((\varphi \leftharpoondown \alpha) \wedge \neg\varphi) \to [\alpha]\bot$, and theorem 1, $Pre(\alpha) \leftharpoondown \alpha)$, and $M^{\Gamma'}, w \models \neg Pre(\alpha)$, we have that $M^{\Gamma'}, w \models [\alpha]\bot$
   - Because of the truth lemma we have that $[\alpha]\bot \in w$
   - Now assume $wR_\alpha^{\Gamma'}v$ for an arbitrary v
   - By definition of $R_\alpha^{\Gamma'}$, since $wR_\alpha^{\Gamma'}v$ and $[\alpha]\bot \in w$, $\bot \in v$, contradiction
   - Since v was arbitrary there is no v s.t. $wR_\alpha^{\Gamma'}v$

   □

2. **Constraints on existing relations**
   If Pre(α) is true in w according to classical truth-functional propositional logic, then there is at least one $v \in W$ such that the following holds for all atomic formulas p:

(a) If p is a conjunct of $Post(\alpha)$, then $v \in V(p)$

(b) If $\neg p$ is a conjunct of $Post(\alpha)$, then $v \notin V(p)$

(c) If neither p nor $\neg p$ occurs in $Post(\alpha)$, then $v \in V(p)$ just in case $w \in V(p)$

For those $v \in W$ such that the above holds $wR_\alpha^{\Gamma'}v$

Proof. Assume $M^{\Gamma'}, w \models Pre(\alpha)$. Therefore $Pre(\alpha) \in w$

(a) Show that there exists at least one v s.t. $wR_\alpha^{\Gamma'}v$
   - By axioms $((\varphi \rightharpoonup \alpha) \wedge \varphi) \rightarrow \neg[\alpha]\bot$ and theorem 2 $Pre(\alpha) \rightharpoonup \alpha$, modus ponens, and our assumption $M^{\Gamma'}, w \models Pre(\alpha)$, we have that $M^{\Gamma'}, w \models \neg[\alpha]\bot$.
   - Therefore there is some $v \in W$ s.t. $wR_\alpha^{\Gamma'}v$

(b) Show that for any arbitrary v for which $wR_\alpha^{\Gamma'}v$ the constraints on R and V hold
   - Constraint (a) and (b)
     * By axioms $((\varphi \rightharpoonup \alpha) \wedge \varphi) \rightarrow \neg[\alpha]\bot$ and $(Pre(\alpha) \rightharpoonup \alpha)$ and our assumption $Pre(\alpha)$, we have that $\neg[\alpha]\bot \in w$
     * By axioms $(\alpha \blacktriangleright \varphi) \rightarrow [\alpha]\varphi$ and $(\alpha \blacktriangleright Post(\alpha))$ we have that $[\alpha]Post(\alpha) \in w$
     * By the definition of the canonical relation $R_\alpha^{\Gamma'}$ and since $[\alpha]Post(\alpha) \in w$, we have that $Post(\alpha) \in v$ for any v s.t. $wR_\alpha^{\Gamma'}V$
     * Now since $Post(\alpha) \in v$, by the canonical definition of V, if p is a conjunct of $Post(\alpha)$, then $v \in V(p)$ and if $\neg p$ is a conjunct of $Post(\alpha)$, then $v \notin V(p)$
   - Constraint (c)
     * Assume p and $\neg p$ do not occur in $Post(\alpha)$
     * By axiom $(\alpha \blacktriangleright \varphi) \rightarrow G((p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p))$ for p not occurring in φ and axiom $(\alpha \blacktriangleright Post(\alpha))$, we have that $G((p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p)) \in w$
     * By axiom $G\varphi \rightarrow \varphi$ we have that $(p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p) \in w$
     * Suppose that $w \in V(p)$, then $p \in w$
     * Then since $(p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p) \in w$, by modus ponens $[\alpha]p \in w$. Then by the definition of $R_\alpha^{\Gamma'}$, for any arbitrary v for which $wR_\alpha^{\Gamma'}$, $p \in v$. Therefore by the canonical definition of V, $v \in V(p)$
     * Now suppose $w \notin V(p)$, then $p \notin w$, therefore by properties of maximally consistent sets $\neg p \in w$ and since $(p \rightarrow [\alpha]p) \wedge (\neg p \rightarrow [\alpha]\neg p) \in w$, by MP $[\alpha]\neg p \in w$. Then by the canonical R_α definition, for any arbitrary v for which $wR_\alpha^{\Gamma'}$, $\neg p \in v$. Therefore by the canonical definition of V, $v \notin V(p)$
     * Therefore if p and $\neg p$ do not occur in $Post(\alpha)$, $v \in V(p)$ just in case $w \in V(p)$

$\square$

Model makes exact same set of formulas true

To show $M^{\Gamma'}, w \models \varphi \iff M^c, w \models \varphi$.
We assume w to be an arbitrary world in $W^{\Gamma'}$.

Base case:
φ is p: V is the same as V^c for both models so automatically if $M^{\Gamma'}, w \models \varphi$, $w \in v(p)$ so also $M^c, w \models \varphi$.

Inductive hypothesis: If ψ is of lesser complexity than φ, we have that $M^{\Gamma'}, w \models \varphi \iff M^c, w \models \varphi$

Inductive step:
The proofs for the propositional cases are straightforward and will be left out. We show here the proofs for the non-standard elements of our language:

- $[\alpha]\varphi$
  - $\rightarrow$
    - Assume $M^{\Gamma'}, w \models [\alpha]\varphi$
    - Iff $M^{\Gamma'}, v \models \varphi$ for any v $\in W^{\Gamma'}$ s.t. $wR^{\Gamma'}_{\alpha}v$
    - For any arbitrary $v \in W^c$, s.t. $wR^c_{\alpha}v$, $wR^c_G v$, since $wR^c_a v \subseteq wR^c_G v$ by axiom $G\varphi \rightarrow [\alpha]\varphi$.
    - So for any arbitrary v in W^c s.t. $wR^c_{\alpha}v$, $v \in W^{\Gamma'}$
    - So $v \in W^{\Gamma'}$ for any v s.t. $wR^c_{\alpha}v$, and therefore for any v such that $wR^{\Gamma'}_{\alpha}v$, $wR^c_{\alpha}v$.
    - Therefore it must be the case that $M^{\Gamma'}, v \models \varphi$ for any v s.t. $wR^c_{\alpha}v$
    - By the induction hypothesis if $M^{\Gamma'}, v \models \varphi$ for any v s.t. $wR^c_a v$ then $M^c, v \models \varphi$ for any v s.t. $wR^c_a v$
    - By definition of truth in an interpretation $M^c, w \models [\alpha]\varphi$
  - $\leftarrow$
    - Assume $M^c, w \models [\alpha]\varphi$
    - Iff $M^c, v \models \varphi$ for any $v \in W^c$ s.t. $wR^c_{\alpha}v$
    - Since $W^{\Gamma'} \subseteq W^c$, for any arbitrary $v \in W^{\Gamma'}$ s.t. $wR^c_{\alpha}v$, $M^c, v \models \varphi$
    - by definition of $wR^{\Gamma'}_{\alpha}v$ (which is the same as $wR^c_a v$ but restricted to $W^{\Gamma'}$), for any arbitrary $v \in W^{\Gamma'}$ s.t. $wR^{\Gamma'}_{\alpha}v$, $M^c, v \models \varphi$
    - By the induction hypothesis if $M^c, v \models \varphi$ for any v s.t. $wR^{\Gamma'}_{\alpha}v$ then $M^{\Gamma'}, v \models \varphi$ for any v s.t. $wR^{\Gamma'}_{\alpha}v$
    - By definition of truth in an interpretation $M^{\Gamma'}, w \models [\alpha]\varphi$
- $G\varphi$
  - $M^{\Gamma'}, w \models G\varphi$ iff $M^{\Gamma'}, v \models \varphi$ for any v in $W^{\Gamma'}$
  - Iff $M^c, v \models \varphi$ for any v in $W^{\Gamma'}$ by induction hypothesis

– Since $W^{\Gamma'}$ is an equivalence class of R^c_G, for all v $\in W^{\Gamma'}$ we have that wR^c_Gv. And for any $v \notin W^{\Gamma'}$, it is not the case that wR^c_Gv.
– Therefore for all $v \in W$ s.t. wR^c_Gv, $M^c, v \models \varphi$
– Iff by definition of truth in an interpretation $M^c, w \models G\varphi$

- $\varphi \leftharpoondown \alpha$
  - $M^{\Gamma'}, w \models \varphi \leftharpoondown \alpha$ iff $M^{\Gamma'}, v \models (Pre(\alpha) \rightarrow \varphi)$ for any v in $W^{\Gamma'}$
  - Iff $M^c, v \models (Pre(\alpha) \rightarrow \varphi)$ for any v in $W^{\Gamma'}$ by induction hypothesis
  - Since for all $v \in W^{\Gamma'}$ we have that wR^c_Gv, $M^c, v \models (Pre(\alpha) \rightarrow \varphi)$ for any v in $W^{\Gamma'}$ iff $M^c, v \models (Pre(\alpha) \rightarrow \varphi)$ for any v in wR^c_Gv
  - Iff $M^c, w \models \varphi \leftharpoondown \alpha$

- $\varphi \rightharpoonup \alpha$
  - $M^{\Gamma'}, w \models \varphi \rightharpoonup \alpha$ iff $M^{\Gamma'}, v \models \varphi \rightarrow Pre(\alpha)$ for any v in $W^{\Gamma'}$
  - Iff $M^c, v \models \varphi \rightarrow Pre(\alpha)$ for any v in $W^{\Gamma'}$ by induction hypothesis
  - Since for all v $\in W^{\Gamma'}$ we have that wR^c_Gv, $M^c, v \models \varphi \rightarrow Pre(\alpha)$ for any v in $W^{\Gamma'}$ iff $M^c, v \models \varphi \rightarrow Pre(\alpha)$ for any v in wR_Gv
  - Iff $M^c, w \models \varphi \rightharpoonup \alpha$

- $\alpha \blacktriangleright \varphi$
  - $M^{\Gamma'}, w \models \alpha \blacktriangleright \varphi$ Iff $\varphi \approx Post(\alpha)$
  - Iff $M^c, w \models \alpha \blacktriangleright \varphi$

Step 5: completeness of L^{prop}_{FLINT}

Completeness: If $M^{\Gamma'}, \Gamma \models \psi$ then $\Gamma \vdash \psi$

We again make use of the contraposition of the completeness theorem, $\Gamma \nvdash \psi$ then $M^{\Gamma'}, \Gamma \not\models \psi$, to prove completeness for L^{prop}_{FLINT}.

Proof. Suppose $\Gamma \nvdash \psi$ and that we extend Γ to a maximally consistent set Γ'. We know that because of completeness of the canonical model in step three we have that $M^c, \Gamma' \models \varphi$ for all formulas $\varphi \in \Gamma'$. Now since $\Gamma \subseteq \Gamma'$ and $\Gamma \nvdash \psi$, $\Gamma' \nvdash \psi$ and by properties of maximally consistent sets, $\psi \notin \Gamma$. By the truth lemma for the canonical model $M^c, \Gamma' \not\models \psi$. Now since $M^{\Gamma'}$ makes the exact same set of formulas true as M^c, $M^{\Gamma'}, \Gamma' \not\models \psi$. Because $\Gamma \subseteq \Gamma'$, $M^{\Gamma'}, \Gamma \not\models \psi$. Therefore if $\Gamma \nvdash \psi$ then $M^{\Gamma'}, \Gamma \not\models \psi$. □

4.4 Duties

Before we lay out first-order FLINT in the next chapter, we'll first take a small intermediary step and introduce duties into our formal system. We take this step because we want to emphasize duties as a central concept within the FLINT approach. In FLINT frames duties are presented as special facts that are part of normative states. A duty expresses that a duty holder has an obligation to a claimant to perform an act. This duty stays in place until:

- the act itself is performed
- some other act is performed of which the postconditions state that this duty is terminated

Notice that this is a limited view of the concept of duties itself and resolving duties. Duties can be more than the obligation to perform an act, 'ought-to-do' duties. There are also 'ought-to-be' duties, enforcing an obligation upon the duty holder to bring the world into a certain state. Parties agree on a certain result that needs to be achieved. In the legal literature this distinction is called an obligation of means versus an obligation of result.

Furthermore, a resolution of a duty can happen because of the actions of other people and even because of natural events. For instance, a real-world event like a natural disaster can be cause for a force majeure that rules out existing duties. We'll put more attention on L_{FLINT}'s limited perspective in this regard in chapter 8.

We'll represent this conceptualization of duties in our syntax and semantics in the next few subsections. We'll keep the formalization of duties simple for now, leaving out the explicit relational perspective of Hohfeld on duties. The relational nature of duties will be represented within the formal framework of first-order FLINT. This is done because it will allow us to introduce agents alongside objects in the logic. For duty-flint this means that a scenario can only be described in which it is clear from context for whom the duty to perform an action holds.

4.4.1 Syntax

We extend the syntax with the duty concept as follows:

- If α is an action, then $\Box\alpha$ is a formula that expresses that there is a duty to perform α.

We treat formulas of the form $\Box\alpha$ as atomic formulas. This means that $\Box\alpha$ and $\neg\Box\alpha$ may also appear in Flint-formulas, even as conjuncts in postcondition-formulas. We do this because duties in the FLINT ontology are modeled as facts as well. To give an accurate formalization of the FLINT ontology, duties should therefore be treated in the same way.

4.4.2 Semantics

On a semantic level $\Box$-formulas are treated as atoms of L_{FLINT} as well. This means that the valuation function V determines directly whether a $\Box$-formula is true or false in a state: $M, w \models \Box\alpha \iff w \in V(\Box\alpha)$ The constraints on R_α and V also apply to $\Box$-formulas.

The semantics can therefore be understood as follows: duties are either true or false at a state, like light switches on a board that are either on or off. You can think of it as if each state has its own to-do list, encoding which actions should be performed according to which duties are active. The to-do list keeps track of which actions to perform, but does not determine which action is next. Depending on the postconditions of an action, after an action is performed, the to-do list will be the same or different in the next state. If an action on the to-do list is performed, we assume that this duty is resolved, and in the next state this duty will be off the to-do list.

To model this correctly we do need to put an extra direct constraint on the construction of the *Post* function. For each action input, the output sentence of *Post* contains the negation of the duty to perform that action. E.g. $\neg\Box\alpha$ should be a conjunct of $Post(\alpha)$. We add $[\alpha]\neg\Box\alpha$ as an axiom to be able to prove in our system that this constraint holds.

4.4.3 Proof system

To represent the semantics of duties we extend the proof system with the following axioms:

- $(\alpha \blacktriangleright \varphi) \rightarrow G((\Box\beta \rightarrow [\alpha]\Box\beta) \wedge (\neg\Box\beta \rightarrow [\alpha]\neg\Box\beta))$ for $\Box\beta$ not occurring in φ. This axiom reflects that $\Box$-formulas behave just like atoms semantically.

- $[\alpha]\neg\Box\alpha$. This axiom reflects that duties are always resolved by the performance of the duty itself.

4.4.4 Towards first-order FLINT

We now have a language that is expressive enough to formalize norms such that they resemble FLINT frames. We have action symbols, duties, and propositions to represent pre- and postconditions. The actor, recipient and object frames of the FLINT-frame can be inferred from the description of the propositions and actions. However, the propositional nature of our current language still severely limits us in describing the actual meaning of FLINT frames. With the machinery of propositional duty FLINT we are not able to reason about norms and scenarios at the same time and capture the 'general' nature of the law.

To show this we can look at a self-made simplified example of the Dutch traffic rule for speeding, "if you drive faster than the speed limit, a police offer can write out a fine".[4] Out of this rule we can construct a FLINT-frame: first we look at the action of the rule, which is writing out a fine. The object of the action is the fine itself. We then look at the preconditions of writing out a fine. We can see that someone must have driven faster than the speed limit, and you have to be a police officer to write out the fine. One postcondition of writing out a fine is, of course, that a fine is given. Within the rule there seems to be an important missing postcondition, the duty for the speedster to pay the fine. However, the concept of a fine is part of another rule that obligates a person that receives a fine to pay the fine. We can therefore add this duty to the rule with reference in FLINT to the other source of law.

- Action: ***FINING***

- Object: fine

[4]Constructed from the following sources: Reglement verkeersregels en verkeerstekens 1990 (RVV 1990) en Wet administratiefrechtelijke handhaving verkeersvoorschriften

- Precondition:
  - a maximum speed violation has been registered- s (speeding)
  - police officer administrating the violation - p
- Postcondition:
  - a fine for speeding is given - f
  - there is a duty to pay the fine - $\Box\boldsymbol{PAYTHEFINE}$

We can then represent this FLINT-frame in propositional duty FLINT by the following pre- and postcondition-formulas:

- $(s \wedge p) \rightleftharpoons \boldsymbol{FINING}$
- $\boldsymbol{FINING} \blacktriangleright (f \wedge \Box\boldsymbol{PAYTHEFINE})$

For this FLINT interpretation to hold any value we must assume that the fine for speeding is given by the police officer to the person who drove too fast. However, this interpretation does not follow formally. Formally, setting the truth conditions of the precondition propositions to true makes these propositions true for the entire normative state. But a norm is supposed to apply only to those people and those circumstances that meet its preconditions. In propositional duty-FLINT we can't specify who the actor is that writes out a fine and who the person is that is fined. Another consequence of the propositional nature of our current formal system is that we cannot describe two different cases of someone speeding. We also can't express that any police officer that determines a violation of the speeding limits, at any time in the future, will be able to write out a fine to the person who drove too fast.

With propositional duty FLINT we can therefore only represent toy examples in which we assume to know who and what the pre- and postconditions apply to. But with FLINT we want to do more than that. We want to present an interpretation of norms, such that, if we are given a scenario, we can reason with the general norms how this scenario would play out. In the next chapter, we therefore make use of the expressiveness of first-order logic to link pre- and postconditions to specific agents and circumstances for scenarios and capture the generality of the norms in their descriptions.

Chapter 5

First-order FLINT and examples

We have now arrived at the final version of L_{FLINT}, first-order FLINT (L^{fo}_{FLINT}). The expressiveness of first-order logic is necessary to capture the generality, or what we can call the foreseeing nature, of the law. By this we mean to say that norms talk about what would happen if the institutional preconditions are fulfilled and the corresponding institutional actions performed. We cannot predict exactly which norms will apply to us in the future; however, we should be able to predict what will happen to us if they do in fact apply. The first-order expressiveness furthermore allows us to specify that preconditions hold with respect to specific agents and objects. The postconditions of an institutional action will then also refer to the legal situation of those specific agents and objects. This means we can present scenario's and norms together.

We'll first present our first-order variant of FLINT and how we envision we could reason with the formal system. Soundness and completeness proofs are left for future work. Using L^{fo}_{FLINT} we'll then make a representation of FLINT frames and show how these representations can be used to guide us through an actual scenario with the assumption that our propositional proofsystem applies to L^{fo}_{FLINT}.

5.1 L^{fo}_{FLINT}

5.1.1 Syntax

Vocabulary

We'll now make use of the first-order vocabulary of individual agents, individual variables, individual symbols and predicates. We assume that the domain of L^{fo}_{FLINT} is finite. This assumption simplifies the transition from propositional FLINT to L^{fo}_{FLINT} and can be justified by the fact that we can assume that the number of people and resources in our world are finite:

1. Individual variables (infinitely many): x, y, z, etc. When needed suitable variable letters for object variables can be used, so f for field.

2. Non-logical symbols:
   - Individual symbols (finitely many): a finite amount of letters appropriate to denote specific objects and agents, d for agent Dean and t for agent Thom, t_1 for their first game of tic-tac-toe.
   - Predicates in uppercase letters, regular font (infinitely many): PREDICATE
3. Logical constants:
   - Logical truths: $\bot$
   - Connectives: $\neg$, $\vee$, $\wedge$, $\rightleftharpoons, \rightharpoonup, \leftharpoondown, \blacktriangleright, =$
   - Quantifiers: $\exists, \forall$
4. Actions in bold uppercase italic letters with variable or individual subscripts for the relevant agents and objects (finitely many): $\boldsymbol{ACTION}_{xyz}$ Every action will contain *at least* three variables that denote two agents (actor and recipient) and one object. The first subscript will always have to be interpreted as the actor of an action, the second subscript as the recipient and the third subscript as the object. For action variables we use α_{xyz} and β_{xyz}
5. Auxiliary symbols: (,), [,]

5.1.2 Grammar

Definition Term of L^{fo}_{FLINT}:
The terms of L^{fo}_{FLINT} are the individual agents, the individual variables, and individual constants.

Atoms of L^{fo}_{FLINT}:

- A formula in L_{FLINT} is an atomic formula if it consists of a predicate of degree n followed by n terms (e.g. Pxy) or if it is a formula of two terms connected by the identity sign, so $t = t'$
- Duty: $\Box_x \alpha_y$ is an atomic formula. Meaning that there is a duty of x towards y to perform α

Bound and free occurrences of variables in $\mathbf{L}_{FLINT}$:
An occurrence of a variable x in a formula φ of L^{fo}_{FLINT} is bound if and only if it is within an occurrence in φ and φ is of the form $\forall\alpha\varphi$ or $\exists\alpha\varphi$. Otherwise, it is free.

Definition sentence of L^{fo}_{FLINT}:
A sentence of L^{fo}_{FLINT} is a formula of L^{fo}_{FLINT} in which no free variables occur, other than within the scope of *normative formulas*. For example $Pa \wedge ((Px \wedge Sy \wedge Tz) \rightleftharpoons \alpha_{xyz})$ is a sentence of flint that could describe that an agent named Adrian is a police officer and for any x, y, z, x being a police officer together with the fact that y was speeding and that z is a speeding ticket are the necessary and sufficient preconditions for the police officer x to write out ticket z to the person

who went over the speed limit on the motorway, y. $Px \wedge ((Px \wedge Sy \wedge Tz) \rightleftharpoons \alpha_{xyz})$ is not a sentence of FLINT because x is a free variable outside the scope of a normative formula. We allow variables to be free in normative formulas because normative formulas are assumed to apply equally to all individuals, and therefore they will be implicitly bound in the truth conditions.

Formulas

The formulas of L^{fo}_{FLINT} syntactically will be constructed similarly to L^{prop}_{FLINT}. There are a few important variations on the construction of formulas:

- Precondition formulas can contain quantifiers. So $\forall xyz(Pxyz) \leftharpoondown \alpha_{xyz}$ is a formula of L^{fo}_{FLINT}. Also $\exists xyz(Pxyz) \leftharpoondown \alpha_{xyz}$ is a formula of L^{fo}_{FLINT}. This is done because we need to be able to leave open precondition formulas for different configurations of agents and objects.
- The action operator does not contain individual variables. So $[\alpha_{xyz}]\varphi$ is not a formula of L^{fo}_{FLINT}. This is done because the action operator will always refer to an action carried out by a specific configuration of agents and objects. So an action operator will always refer to a specific 'instance' of a 'type' of action. $[\alpha_{qrs}]\varphi$ then says that the action type α is performed by actor q, recipient r, with object s.

5.1.3 Semantics

The entire construction of L^{fo}_{FLINT} is based on the idea that preconditions and postconditions determine the norms of a normative system. For L^{prop}_{FLINT} it was clear that these pre- and postconditions are always defined before the construction of our model. For L^{fo}_{FLINT} we need to be very explicit about the way our *Pre* and *Post* functions work, so that we make sure all outputs are defined. We know that we want to have determinate postconditions and that only under the circumstances relative to the concrete agents, object, and context of the governing norms the preconditions can hold. The modeling difficulty for our *Pre* function that arises is that we need to be specific about which agents and which objects satisfy the preconditions of an action. For our *Post* function the difficulty is that we need to specify that the only sentences that change truth value in the next normative state are sentences referring to the agents and objects involved in the action. The output of the *Pre* and *Post* functions should be in accordance with the input. In other words, you should only be allowed to perform an action if the preconditions hold with respect to you, and the postconditions of that action that refer to the actor of the action should change your position in the new world. This can only be done if the variables connected to actions in the *Pre* and *Post* functions represent concrete agents and objects. We achieve this by using substitution in the truth conditions of the preconditions and in the constraints on R_α.

We constrain the *Pre* and *Post* function such that the input actions can only contain variable subscripts, no agents or constants; The only free variables in the output formula can be those variable subscripts of the input action. The action inputs for the *Pre* and *Post* functions are 'action types'

So α_{dtz} cannot be an input, since d is the individual Dean and t the individual Thom. α_{xyz} can be an input to an action. The output of α_{xyz} can be any formula φ such that φ does not contain any free variables outside x, y and z. This means that the Pre and $Post$ function are implicitly quantified over all possible configurations of agents and objects. This will be reflected in the truth conditions of the necessary and sufficient preconditions and the quantifiers. For exact postconditions the truth conditions allow for formulas that contain free variables because we look at the construction of $Post(\alpha)$ to determine the truth of an exact postcondition formula.

With this being said we can now define our Pre and $Post$ function just like in L^{prop}_{FLINT}:

- Pre a function from the set of actions into state-formulas with x, y, z as free variables (possibly more, dependent on the amount of action subscripts)
- $Post$ a function from the set of actions into postcondition-formulas with x, y, z as free variables (possibly more, dependent on the amount of action subscripts)

Model

Since we keep the domains of our agents and objects finite, we are still able to evaluate formulas relative to a model with a valuation function. We get $M = \langle W, R\alpha_{qrs}, V\rangle$:

- W a non-empty set of worlds
- $R\alpha_{qrs}$ is a function from the action symbols and sequences of individuals and objects into binary relations on W
- V is a function mapping the atomic sentences in L^{fo}_{FLINT} to a subset of W, such that a sentence φ is true in a world w, when:[1] $w \in V(\varphi)$

  Since we have a finite domain we can say that $\forall x\, \varphi$ is true just in case $\varphi[x/a]$ is true for every a in L^{fo}_{FLINT}. $\exists x\, \varphi$ is true just in case $\varphi[x/a]$ is true for at least one a in L^{fo}_{FLINT} .

We impose the following constraints on R and V. For all actions α_{qrs}:

- If $Pre(\alpha_{qrs})$ is false in w according to classical truth-functional predicate logic, then it is not the case that $wR\alpha_{qrs}v$ for any $v \in W$.
- If $Pre(\alpha_{qrs})$ by agents and objects is true in w according to classical truth-functional predicate logic, then there is at least one $v \in W$ such that the following holds for all atomic sentences φ:
  - If all free variables in $Post(\alpha)$ are substituted by q, r and s, and φ then is a conjunct of $Post(\alpha)$, then $v \in V(\varphi)$
  - If all free variables in $Post(\alpha)$ are substituted by q, r and s, and $\neg\varphi$ then is a conjunct of $Post(\alpha)$, then $v \notin V(\varphi)$
  - If all free variables in $Post(\alpha)$ are substituted by q, r and s, and neither φ nor $\neg\varphi$ occurs in $Post(\alpha)$, then $v \in V(\varphi)$ just in case $w \in V(\varphi)$

[1] We use a valuation function V modified from Gamut (1991)

For those $v \in W$ such that the above holds $wR\alpha_{qrs}v$

The construction of the constraints reflects that for each configuration of agents and objects, there is a specific action instantiation that has its own R_α relation. $Pre(\alpha)$ and $Post(\alpha)$ reflect general pre- and postconditions. After substitution these sentences represent that pre- or postcondtions hold with respect to specific agents and individuals. Consequently, these sentences determine whether an R_α relation between worlds exists.

Truth conditions

M, w $\models \varphi \leftharpoondown \alpha_{xyz} \iff$ for all $v \in W : M, v \models \forall xyzG(Pre(\alpha_{xyz}) \rightarrow \varphi)$

M, w $\models \varphi \rightharpoondown \alpha_{xyz} \iff$ for all $v \in W : M, v \models \forall xyzG(\varphi \rightarrow Pre(\alpha_{xyz}))$

M, w $\models \forall x\varphi \iff M, w \models \varphi[x/a]$ for all agents and objects $a \in L^{fo}_{FLINT}$

M, w $\models \exists x\varphi \iff M, w \models \varphi[x/a]$ for some agent or object $a \in L^{fo}_{FLINT}$

Because the domain is finite $\forall$ formulas can be replaced by large conjunctive formulas. $\exists$ formulas can be replaced by large disjunctions. Instead of having pre- and postconditions for action types, we could also have individual action 'instances' and let the *Pre* and *Post* functions assign pre- and posconditions to each of these instances. This would take away from the idea of having general norms, but this idea can be used to compare L^{fo}_{FLINT} to its propositional counterparts.

5.1.4 The expressive power of L^{fo}_{FLINT}

The construction of ${}^{fo}_{FLINT}$ provides a full formalization of the concepts in the FLINT ontology. We have pre- and postcondition formulas including duty atoms that, together with the agents and object individual symbols in the language, can represent the fact frame. The act frame is represented by the actions themselves. Actions have subscripts for the actor, recipient, and object, so that the *hasactor*, *hasrecipient* and *hasobject* property relations in the ontology are modeled. We formalize the *hasaction* property also by the action itself. The *hasprecondition* and *haspostcondition* properties are represented by the normative formulas. The properties *hasholder* and *hasclaimant* are formalized by having duties contain subscripts for agents in a holder-claimant relationship.

L^{fo}_{FLINT} also adds some properties and concepts to L_{FLINT} that were not yet included in the ontology. As we have given a formal semantics of the concepts of L_{FLINT} we can now reason about norms and scenarios. We can now say which norms and facts are true. Because of the addition of a dynamic action operator, we are able to reason about whether an action can be performed, and what happens if we perform an action to which a norm applies. Other concepts that are added are the notions of necessary and sufficient preconditions.

Since the ontology does not explicitly model the Hohfeldian legal relations this is also not the case for L^{fo}_{FLINT}. Only the duty - claim relation is defined by $\Box_x \alpha_y$. We can only describe the Power - Liability and Immunity - Disability relation by describing how this relation would manifest itself in L^{fo}_{FLINT}. This is the case because action operators do not take variables as subscripts and so we can't generally define these potestative relations. We can only express what happens if an action instance is performed. This is modeled in this way because the action operator tells us what is true in the next normative state after performing an action. Only sentences can hold truth values. An action operator with free variables would not be able to express a relation between worlds, because it would not know which worlds are related. The Privilege - No-Claim relation cannot be modeled at all because we don't consider actions of refrainment.

To see how L^{fo}_{FLINT} can now formalize interpretations of normative texts represented with the ontology concepts, we make use of a toy-example of the classic tic-tac-toe game.

5.2 Toy-example: tic-tac-toe

The machinery of L^{fo}_{FLINT} allows us to represent interpretations modeled with the ontology concepts. In this thesis we'll make use of one of the first toy-examples of a set of rules represented with the FLINT ontology, an old-fashioned game of tic-tac-toe. This toy-example will now be used to show L^{fo}_{FLINT} in action.

The set of rules of tic-tac-toe, taken from the website annex.exploratorium.edu ("Rules for tic-tac-toe", 2024) as source, are as follows:

1. The game is played on a grid that's 3 squares by 3 squares.

2. You are X, your friend (or the computer in this case) is O. Players take turns putting their marks in empty squares.

3. The first player to get 3 of her marks in a row (up, down, across, or diagonally) is the winner.

4. When all 9 squares are full, the game is over. If no player has 3 marks in a row, the game ends in a tie.

Figure 5.1 is a visualization of how the rules of tic-tac-toe are interpreted in the FLINT ontology. Remember that each norm is a collection of act and fact frames, so since there are four actions, we count four norms within this figure. We'll represent these FLINT frames one-by-one in our L^{fo}_{FLINT} formalization. Since norms are defined as a collection of the normative-formulas expressing the sufficient and necessary preconditions and exact postconditions of an action in L^{fo}_{FLINT}, each formalization of the tic-tac-toe rules will take this form.

L^{fo}_{FLINT} is designed so that we have a logical language that is able to reason about norms and scenarios at the same time. As a scenario progresses, different rules will become applicable at different times. However, we want to make sure that as the scenario progresses we are still talking about the same game of tic-tac-toe, with the same players and 3x3 grid. We are always operating within

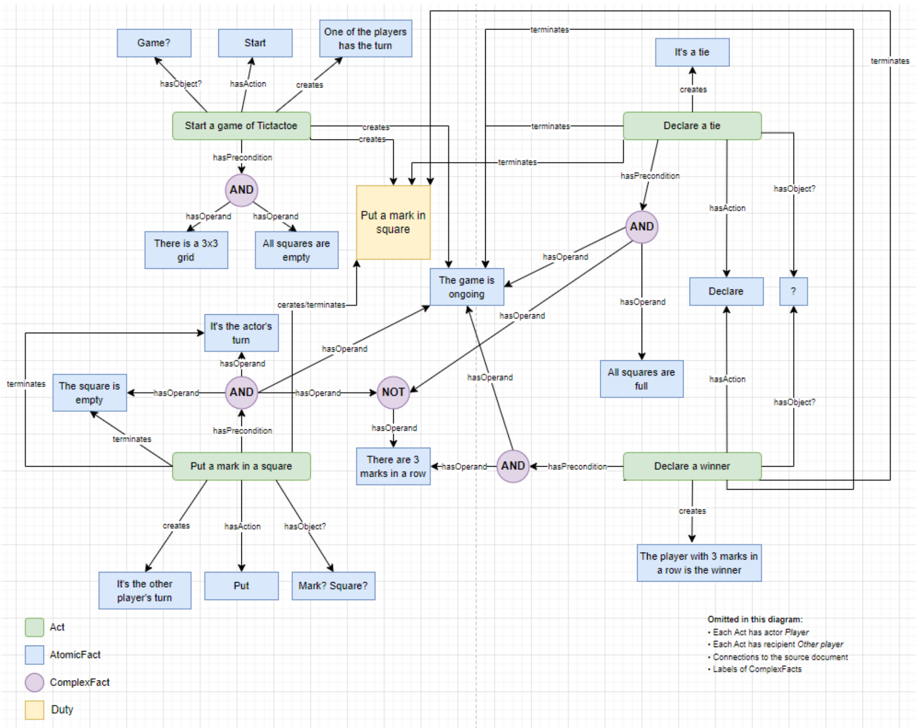

Figure 5.1: tic-tac-toe in FLINT-frames

the context of the tic-tac-toe rules. In other words, we don't want to have that the preconditions of a norm accidentally hold for agents that were not involved in this specific game of tic-tac-toe. One can imagine if two different games of tic-tac-toe are started simultaneously, but with different players, that the same norm will apply at the same time to both scenarios. We need to be able to differentiate between these scenarios.

To ensure we prevent any such coincidence we make an important modeling choice, we assign a few predicates a 'global function'. Global in the sense that they apply to all rules of tic-tac-toe as preconditions. We use these global predicates to denote all the objects in the game that we need to be present for any rule of tic-tac-toe to make sense. It is of course implicitly assumed for all the rules of tic-tac-toe that we are in the same context. The global predicates describe this context.

For tic-tac-toe the global predicates are as follows:

- It's a game of tic-tac-toe, variable t, with a 3x3 grid, variable g, that has 9 squares, variables f_1 to f_2- $TIC-TAC-TOEtgf_1f_2f_3f_4f_5f_6f_7f_8f_9ar$
- The game of tictactoe has 2 players - $HASPLAYERtx \wedge HASPLAYERty \wedge x \neq y$
- There are nine squares - $f_1 \neq f_2 \neq f_3 \neq f_4 \neq f_5 \neq f_6 \neq f_7 \neq f_8 \neq f_9$

Together the global predicates form the following sentence:
$\exists tgf_1f_2f_3f_4f_5f_6f_7f_8f_9xy((TIC{-}TAC{-}TOEtgf_1f_2f_3f_4f_5f_6f_7f_8f_9 \wedge HASPLAYERtx \wedge HASPLAYERty \wedge (x \neq y) \wedge (f_1 \neq f_2 \neq f_3 \neq f_4 \neq f_5 \neq f_6 \neq f_7 \neq f_8 \neq f_9))$

This formula sets the systematic conditions that always need to be in order to play the game of tic-tac-toe. It reads that it's a game of tic-tac-toe with a 3x3 grid and 9 squares, the game has 2 players who are not the same agents, and the nine squares are nine different objects. So that we don't have to repeat this very large formula every time, we use the abbreviation $GLOBAL$ for this formalization in the preconditions of the rules. The scope of the $\exists$ symbol is adjusted to cover all formulas before the precondition symbol. The variables x, y and t are left out of the $GLOBAL$ for the precondition formulas, because these free variables are also variables of the different actions, which means that in the precondition formulas these variables will be implicitly universally quantified. When there is a deviation from the original figure, an explanation is provided.

Di Bello (2007) states three requirements for good modeling:

- (R1) An adequate modeling sanctions only the right or intended inferences.
- (R2) An adequate modeling is textually precise
- (R3) A good formalization should be maintainable: as the texts of the law

Both R1 and R2 are reasonable requirements for our model as well. However, since Di Bello (2007) is focused on modeling legal provisions and not norms, we should slightly rephrase R2: (R2') an adequate modeling is textually precise as far as the interpretations of the text allow this. This is a necessary addition because, as mentioned before, interpretations of legal provisions can add legal facts that are not directly referenced in the text.

Also, since we want L^{fo}_{FLINT} to be a computational theory of norms, we need to add a fourth rule: (R4) formalizations should be disambiguated as much as possible. By this we mean that formalizations should only contain facts that can be verified or instantiated in a computational implementation. A consequence is that some institutional facts should be represented in terms of 'brute' facts, so that the normative system functions coherently. An example is the constitutive norm for winning the game when you have three marks in a row. To be able to relate the actions of putting marks in a square to eventually winning the game of tic-tac-toe, the postconditions of putting a mark in a square should function as preconditions for winning the game. This can only be done if 'three in a row' is formalized by three separate individual claims to squares that are in a row. Because in L_{FLINT} we consider only qualified normative scenarios, these seemingly 'brute' facts are still considered to be part of the institutional reality layer of Searle.

The next section will show exactly how we pay attention to these requirements.

5.2.1 Rule 1

Rule 1: the game is played on a grid that's 3 by 3 squares:

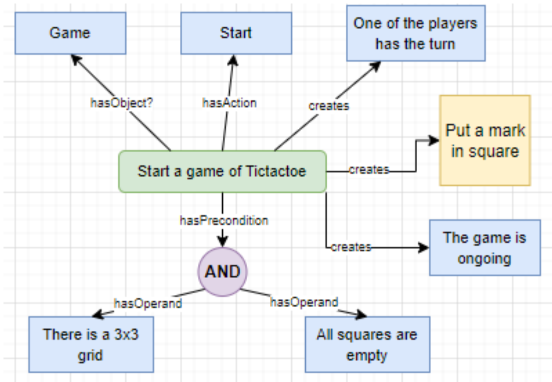

Figure 5.2: Rule 1: starting a game of tic-tac-toe

Action: Starting the game - $\boldsymbol{STARTING}_{xyt}$

Preconditions:

- $GLOBAL$
- All squares are empty - $EMPTY_{f_1} \wedge EMPTY_{f_2} \wedge ... \wedge EMPTY_{f_9}$

Postconditions:

- One of the players has the turn - $TURNy \wedge \neg TURNx$

  One of the players has the turn is an indeterminate postcondition. This should not be possible in FLINT. It is therefore now specified to be the recipient of the action of starting the game that will get the first turn of the game. Because there can't be two turns at the same time, it is also specified that the actor now does not have the turn.

- The game is ongoing - $ONGOINGt$
- The recipient has the duty of putting a mark in a square - $\Box_y \boldsymbol{PUTTING}_x$

Resulting in the following L^{fo}_{FLINT} representation of "the game is played on a grid that's 3 by 3 squares":

- $((GLOBAL \wedge (EMPTYf_1 \wedge EMPTYf_2 \wedge ... \wedge EMPTYf_9) \rightleftharpoons \boldsymbol{STARTING}_{xyt})$
- $(\boldsymbol{STARTING}_{xyt} \blacktriangleright (ONGOINGt \wedge TURNy \wedge \neg TURNx \wedge \Box_y \boldsymbol{PUTTING}_x))$

5.2.2 Rule 2

Rule 2: You are X, your friend (or the computer in this case) is O. Players take turns putting their marks in empty squares:

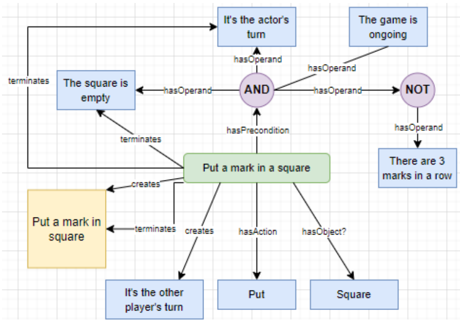

Figure 5.3: Rule 2: putting a mark in a square

Action: Putting a mark in a square - $\boldsymbol{PUTTING}_{xytf}$
For this action we use an extra subscript to denote an extra free variable. Because you can put a mark in any one of the nine squares, the action instance has to denote exactly in which square the mark is put.

Preconditions:

- $GLOBAL$
- There is an empty square - $(f = f_1 \vee f = f_2 ... \vee f = f_9) \wedge EMPTYf$
- It's the actor's turn - $TURNx$
- There are not 3 marks in a row - $\neg((CLAIMxf_1 \wedge CLAIMxf_2 \wedge CLAIMxf_3) \vee (CLAIMxf_4 \wedge CLAIMxf_5 \wedge CLAIMxf_6) ...... \vee (CLAIMyf_7 \wedge CLAIMyf_8 \wedge CLAIMyf_9))$
  By the three dots (...) we mean that all the possible configurations in which three in a row can be achieved are contained in the formula. We'll make use of $INAROW$ to represent $((CLAIMxzf_1 \wedge CLAIMxf_2 \wedge CLAIMxf_3) \vee ...(CLAIMyf_7 \wedge CLAIMyf_8 \wedge CLAIMyf_9))$. Therefore, there are not 3 marks in a row can be formalized as ¬ INAROW

  The precondition "not in a row" is added as a 'defeasible condition'. When there are three marks in a row the players should not continue putting marks in squares. A winner should be declared. If the preconditions of the 'winning' rule are satisfied, the preconditions of putting a mark in square should not be satisfied. We also see that "in a row" is explained in terms of claims to squares that are positioned in a row. We see here how constitutive norms are included in the formalizations.
- The game is ongoing - $ONGOINGt$

Postconditions:

- The other player has the turn - $TURNy \wedge \neg TURNx$

- The claimed square is not empty $\neg EMPTYf$
- The actor has claimed a square - $CLAIMxf$

  We see that representing norms in L^{fo}_{FLINT} can help to specify which pre- and postconditions are necessary. In the figure it is not mentioned that the square that is claimed is not empty anymore, but this is an important condition if we want to be able to determine when somebody has won the game or when the game is tied. For these actions we need to have the precondition fulfilled that either three squares in a row are claimed, or that all squares are claimed without a player having three in a row. We see here that the norms of tic-tactoe are not taken in isolation, but as part of the normative system of the task procedure.

- The recipient has the duty towards the actor to put a mark in a square - $\Box_y \boldsymbol{PUTTING}_x$
- The duty of the actor towards the recipient to put a mark in a square is terminated $\neg\Box_x \boldsymbol{PUTTING}_y$

Resulting in the following L^{fo}_{FLINT} representation of "you are X, your friend (or the computer in this case) is O. Players take turns putting their marks in empty squares":

- Precondition:
  $((GLOBAL \wedge (f = f_1 \vee f = f_2 ... \vee f = f_9) \wedge EMPTYf \wedge TURNx \wedge \neg INAROW) \rightleftharpoons \boldsymbol{PUTTING}_{xytf})$
- Postconditon:
  $(\boldsymbol{PUTTING}_{xytf} \blacktriangleright (TURNy \wedge \neg TURNx \wedge CLAIMxf \wedge \neg EMPTYf \wedge \neg\Box_x \mathbf{PUTTING}_y \wedge \Box_y \boldsymbol{PUTTING}_x))$

5.2.3 Rule 3

Rule 3: the first player to get 3 marks in a row is the winner:

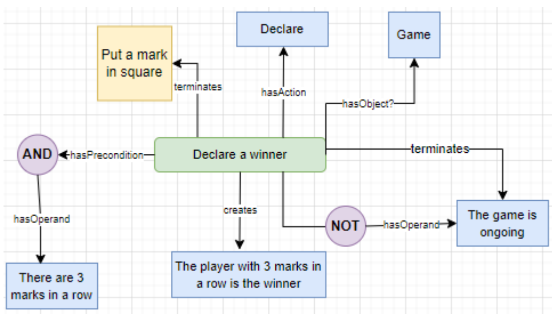

Figure 5.4: Rule 3: declare a winner

Action: Declare a winner - $\boldsymbol{WINNING}_{xyt}$

Preconditions:

- $GLOBAL$
- There are 3 marks in a row - $((CLAIMxf_1 \wedge CLAIMxf_2 \wedge CLAIMxf_3) \vee (CLAIMxf_4 \wedge CLAIMxf_5 \wedge CLAIMxf_6)......\vee (CLAIMyf_7 \wedge CLAIMyf_8 \wedge CLAIMyf_9))$
- The game is ongoing - $ONGOINGt$

Postconditions:

- There is a winner - WINNERr
- The game is not ongoing - $\neg ONGOINGt$
- Both players do not have a duty to put a mark in a square anymore - $\neg\Box_x \boldsymbol{PUTTING}_y \wedge \neg\Box_y \boldsymbol{PUTTING}_x$

Resulting in the following L^{fo}_{FLINT} representation of "the first player to get 3 marks in a row is the winner":

- $(GLOBAL \wedge INAROW \wedge ONGOINGt) \rightleftharpoons \boldsymbol{WINNING}_{xyt}))$
- $(\boldsymbol{WINNING}_{xyt} \blacktriangleright (\neg ONGOINGt \wedge WINNERr \wedge \neg\Box_x \boldsymbol{PUTTING}_y \wedge \neg\Box_y \boldsymbol{PUTTING}_x))$

5.2.4 Rule 4

Rule 4: when all 9 squares are full the game is over. If no player has 3 marks in a row the game ends in a tie:

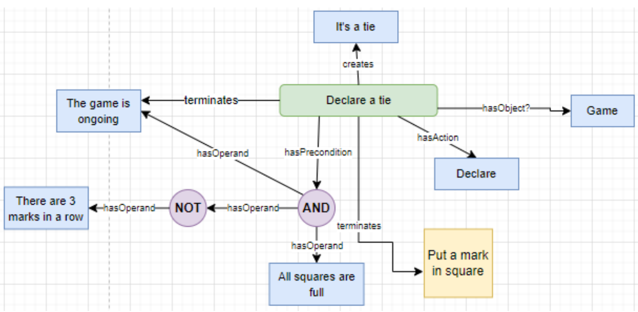

Figure 5.5: Rule 4: Declare a tie

Action: Declare a tie - $\boldsymbol{TYING}_{xyt}$

Preconditions:

- $GLOBAL$

- All squares are full- $(\neg EMPTY_{f1} \wedge \neg EMPTY_{f2} ... \wedge \neg EMPTY_{f9})$
- There are not three marks in a row - see rule 2
- The game is ongoing - $ONGOINGt$

Postconditions:

- It's a tie - $TIExyt$
- The game is not ongoing - $\neg ONGOINGt$
- Both players do not have a duty to put a mark in a square anymore - $\neg\Box_x \boldsymbol{PUTTING_y} \wedge \neg\Box_y \boldsymbol{PUTTING_x}$

Resulting in the following L^{fo}_{FLINT} representation of "when all 9 squares are full the game is over. If no player has 3 marks in a row the game ends in a tie":

- $(GLOBAL \wedge (\neg EMPTY_{f1} \wedge \neg EMPTY_{f2} ... \wedge \neg (EMPTY_{f9})) \wedge \neg INAROW) \rightleftharpoons \boldsymbol{TYING_{xyt}})$
- $(\boldsymbol{TYING_{xyt}} \blacktriangleright \neg ONGOINGt \wedge TIExyt \wedge \neg\Box_x \boldsymbol{PUTTING_y} \wedge \neg\Box_y \boldsymbol{PUTTING_x}))$

5.2.5 The normative system of tic-tac-toe

The collection of the four tic-tac-toe norms together forms its normative system, a complete formalization of 5.1. Now within that normative system scenarios can be played, i.e. games of tic-tac-toe can be played. We can view playing tic-tac-toe as a task procedure, actions are taken step-by-step. Due to the first-order nature of L^{fo}_{FLINT}, when presented with qualified facts of a game of tic-tac-toe, the formalizations of the norms of tic-tac-toe can tell us which actions are available to specific agents in a normative state and what would happen if those actions were actually performed. In the next section we'll see how we can use these reasoning steps to model a scenario of a game of tic-tac-toe.

5.3 Scenario of tic-tac-toe

In this section we'll parse through a possible scenario of a game of tic-tac-toe and see how we can apply the norms of tic-tac-toe to the scenario. The idea behind FLINT is that we transition from one normative state to another in which the truth conditions of formulas are influenced by the normative actions carried out in the previous normative state. We'll therefore first describe which conditions must hold in our initial normative state and then show what effect the transitions from one normative state to the other have on which formulas are true or false in each respective normative state. The semantics of actions are such that the performance of an action makes true all of the literals in the postcondition of that action.

5.3.1 Scenario

We'll have the following informal interpretations of the individuals and object symbols in our language:

- a_1 and a_2 = player 1 and player 2
- g_1 = a 3x3 grid
- f_1, f_2, f_3, ... , f_9 = all the fields of the 3x3 grid square. Numbered from left to right.
- t_1 = a game of tic-tac-toe

Normative state 1:

To start a game of tic-tac-toe the following preconditions of this action must hold:

- $GLOBAL$
- $EMPTY f_1 \wedge EMPTY f_2 \wedge .... \wedge EMPTY f_9$

Both players of this game of tic-tac-toe can now start this game, since for both the preconditions of starting a game hold.

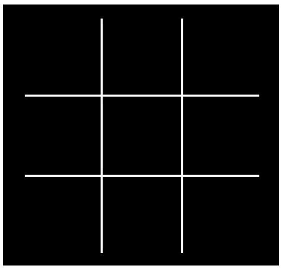

Figure 5.6: Initial state tic-tac-toe

Normative state 2:

Player 2 starts the game by performing $\boldsymbol{STARTING}_{a_2 a_1 t_1}$. Performing this action results in a transition of normative state 1 to normative state 2. The norm for starting a game is:

- $((GLOBAL \wedge (EMPTY f_1 \wedge EMPTY f_2 \wedge ... \wedge EMPTY f_9) \rightleftharpoons \boldsymbol{STARTING}_{xyt})$
- $(\boldsymbol{STARTING}_{xyt} \blacktriangleright (ONGOINGt \wedge TURNy \wedge \neg TURNx \wedge \Box_y \boldsymbol{PUTTING}_x))$

Following the semantics of actions this means the following formulas become true in state 2 compared to state 1:

- $TURNa_1$
- $ONGOINGt_1$
- $\Box_{a_1} \boldsymbol{PUTTING}_{a_2}$

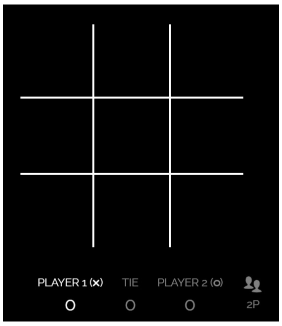

Figure 5.7: Player 1 has the first turn

Normative state 3:

Now player 1 puts his mark in square f_5, performing $\boldsymbol{PUTTING}_{a_1 a_2 f_5 f_5}$ resulting in a transition of normative state 2 to normative state 3. The norm for putting a mark in square is:

- $((GLOBAL \wedge (f = f_1 \vee f = f_2 ... \vee f = f_9) \wedge EMPTYf \wedge TURNx \wedge \neg INAROW) \rightleftharpoons \boldsymbol{PUTTING}_{xytf})$
- $(\boldsymbol{PUTTING}_{xytf} \blacktriangleright (TURNy \wedge \neg TURNx \wedge CLAIMxf \wedge \neg EMPTYf \wedge \neg \Box_x \mathbf{PUTTING}_y \wedge \Box_y \boldsymbol{PUTTING}_x))$

Following the semantics of actions this means the following formulas become true in state 3 compared to state 2:

- $\neg EMPTY_{f_5}$
- $TURNa_2$
- $\neg TURNa_1$
- $CLAIMa_1 f_5$
- $\neg \Box_{a_1} \mathbf{PUTTING}_{a_2}$
- $\Box_{a_2} \boldsymbol{PUTTING}_{a_1}$

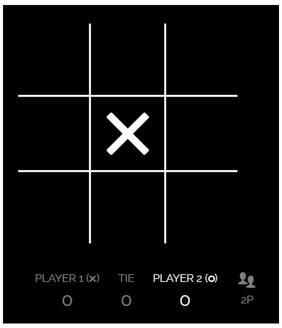

Figure 5.8: Player 1 put a mark in square f_5, player 2 has the turn

Normative state 8:

We fast forward to a situation in which player 1 has three squares in a row:

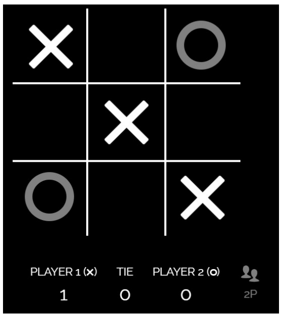

Figure 5.9: Player 1 is the winner

Meaning the following preconditions of declaring a winner are now fulfilled:

- $INAROW$
- $ONGOINGt_1$

Player 2 therefore declares player 1 to be the winner by performing $\boldsymbol{WINNING}_{a_2 a_1 t_1}$. This results in a transition of normative state 7 to normative state 8. The norm for declaring a winner is:

- $(GLOBAL \wedge INAROW \wedge ONGOINGt) \rightleftharpoons \boldsymbol{WINNING}_{xyt}))$
- $(\boldsymbol{WINNING}_{xyt} \blacktriangleright (\neg ONGOINGt \wedge WINNERr \wedge \neg \Box_x \boldsymbol{PUTTING}_y \wedge \neg \Box_y \boldsymbol{PUTTING}_x))$

Following the semantics of actions this means the following formulas become true in state 8 compared to state 7:

- $\neg ONGOINGt_1$
- $WINNERa_1$
- $\neg \Box_{a_2} \boldsymbol{PUTTING}_{a_1}$
- $\neg \Box_{a_1} \boldsymbol{PUTTING}_{a_2}$

We can see that there are no duties on the to-do list anymore as all duties to put a mark in a square are terminated. This shows that by declaring the winner of the game, the game ends and the task of playing a game of tic-tact-toe is completed.

Final normative state tie:

Now let's imagine a scenario in which the players kept playing until there were no more squares empty but also there were no 3 squares in a row claimed by the same player. Then these conditions would hold:

- $TURNa_2$
- $\neg EMPTY_{f_1} \wedge \neg EMPTY_{f_2} ... \wedge \neg EMPTY_{f_9}$
- $\neg INAROW$

Which of course means that the precondition for declaring a tie is fulfilled. So after performing declaring a tie, $\boldsymbol{TYING}_{a_2 a_1 t_1}$, with the norm for declaring a tie being:

- $(GLOBAL \wedge (\neg EMPTY_{f1} \wedge \neg EMPTY_{f2} ... \wedge \neg (EMPTY_{f9})) \wedge \neg INAROW) \rightleftharpoons_{\boldsymbol{TYING}} {}_{xyt})$
- $(\boldsymbol{TYING}_{xyt} \blacktriangleright (\neg ONGOINGt \wedge TIExyt \wedge \neg \Box_x \boldsymbol{PUTTING}_y \wedge \neg \Box_y \boldsymbol{PUTTING}_x))$

Following the semantics of actions the following formulas become true:

- $\neg ONGOINGt_1$
- $TIEa_2 a_1 t_1$
- $\neg \Box_{a_2} \boldsymbol{PUTTING}_{a_1}$
- $\neg \Box_{a_1} \boldsymbol{PUTTING}_{a_2}$

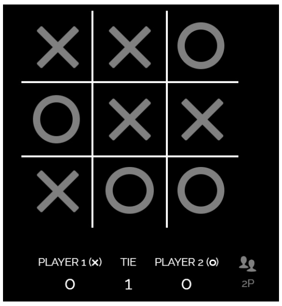

Figure 5.10: A tie

5.3.2 Reflections on FLINT scenario

The examples in the last section show that we are able to describe a scenario of tic-tac-toe being played out with FLINT. We'll now reflect on how the modeling choices affect the way a scenarios can be played out in L^{fo}_{FLINT}.

Duties are modeled as obligations from one agent towards another agent for the performance of an act without further specification of what this act entails. So no specific object is denoted which the duty relates to. So the duty of putting a mark in a square is ${}_x\boldsymbol{PUTTING}_y$, not ${}_{xf}\boldsymbol{PUTTING}_y$. This is done to keep the postconditions determinate. In tic-tac-toe, after the action of starting the game, you don't receive a duty to put a mark in a specific square but in one of the nine squares. It would therefore not be appropriate to add a free variable subscript to the action of starting the game that can be instantiated for a specific square. The duty has to stay generic at this point. This example applies to all actions that have duties as pre- or postconditions of which the object of the duty is not determined by the action.

The nature of the game of tic-tac-toe changes slightly because of the modeling choices made in the representation of the rules of tic-tac-toe. Normally winning or tying a game is not an action, but a qualification of the state of the world that occurs as soon as one of the players puts a third mark in a row.

Furthermore, there are also important normative questions in relation to real-world scenarios where players don't play by the rules, such as when is a duty violated? This is a question that can't be modeled in $L_{foFLINT}$. Even more so, we can't model any 'illegal' actions, because we only allow actions of which the preconditions are fulfilled to change normative states.

Why these issues arise in our formalization and what this means for the expressive power of our language, we'll discuss in the next chapter of this thesis, the limitations of L_{FLINT}.

Chapter 6

Limitations

The main benefit of the FLINT approach towards modeling interpretations of norms is that it provides us with a clear procedure of which actions can be carried out in compliance with the law at a given time, and which actions should be carried out (duties). This is helpful because rather than explaining the norm in an abstract manner, FLINT shows concretely what we can do and what we should be doing in a normative state. However, there are certain limitations to the expressivity of L_{FLINT}. There are limitations that are inherent to the FLINT approach, and limitations of the current construction of the FLINT ontology, the list of concepts that are currently included in FLINT. In this chapter we aim to highlight a few of the most pressing limitations apparent from the examples in the previous section and as seen in the literature.[1] For each of these limitations we'll provide examples of modeling issues. Where possible we'll try to suggest a solution towards these modeling issues, other times it will be made clear that a limitation is outside the scope of the goals of FLINT.

6.1 Approach limitations

6.1.1 Action perspective

FLINT's action perspective limits or complicates the modeling of norms as interpretations of legal provisions that don't exactly fit this perspective. As represented in L^{fo}_{FLINT} a norm is formalized as the collection of preconditions and exact postconditions of an institutional action. The assumption is therefore made in FLINT that norms always refer to some kind of act. However, often the legal provisions of which we derive the norms don't contain a direct reference to a performance of a certain act. How should we construct FLINT-frames when the legal provisions we are representing don't directly mention actions?

For regulative norms it is simple to reconstruct what type of behavior a rule should apply to. If a legal provision says that the maximum speed limit on the highway is 90 kilometers per hour, then the regulative norm tells us that we should not perform the action of driving faster than 90 kilometers per hour.

[1]Section 2 of Governatori et al., 2021 and chapter 3 of Di Bello, 2007 provide useful overviews of recurrent modeling issues

The action of speeding can then be linked to the source through the words " The maximum speed is 90 kilometers per hour".

For constitutive norms, the most straightforward solution is to formalize the norm as part of the precondition for an action. For instance, the legal rule that "18 years of age counts as age of majority" can be modeled directly as a precondition for the institutional action of voting. If we take $ADULTx$ to mean that you are over 18 years old, then we have $ADULTx$ as a necessary precondition for voting: $ADULTx \leftharpoondown VOTING_x$.

For other constitutive norms this approach doesn't always work. A prime example is rule 3 of our game of tic-tac-toe, "the first player to get 3 marks in a row is the winner." There is no specific act that brings about the fact that you are the winner of a game of tic-tac-toe.[2] In our FLINT-frame and L^{fo}_{FLINT} representation, we have a workaround for this issue by creating the additional action of declaring a winner. This has the advantage that we make very explicit that someone has become the winner of the game of tic-tac-toe and that the game has now ended (which is a postcondition of this act). With this action we complete the task of playing the game of tic-tac-toe. The disadvantage of modeling the 'winning rule' in this manner is that we stray away from the way the rule is written and its natural interpretation. It's simply a fact of the game that someone has become the winner of tic-tac-toe after getting 3 marks in a row. In real life we don't need to perform any action to create this fact. If we would actually require this action in real life, we could imagine a scenario in which the losing player never declares the winner.

One solution to this problem is that we model the ability of enforcing institutions, such as the judiciary, to punish agents when they don't abide by the rules. Not following the rules then serves as a precondition for the performance of punitive actions by such institutions. For this solution it is important that it is clear from context when a violation of the rules occurs, such that the preconditions of punitive actions are satisfied. This is not always possible in FLINT. We'll further address this issue in Section 6.1.3.

Another option is to hand over the authority to determine victory to a referee. A third party, which we can reasonably expect to always carry out its tasks, verifies that there are three marks in a row. This option is actually in line with how an online game of tic-tac-toe would be played. We need to keep in mind that the goal of this thesis is to provide a computational theory that can be used as a basis for computer implementation of norms. In a computational process, a computer will first verify if there are three marks in a row and then declare a winner of the game. To formalize this correctly, the formalization of the rules of tic-tac-toe will have to allow a third party (the computer) to perform the action of declaring a winner. It is possible to model the involvement of third parties in L_{FLINT} by adding extra variable subscripts to actions.

Every time we model a norm as an interpretation of sources of legal texts in FLINT, we have to fit the interpretation into the action perspective. The considerations above show how this can be done for different types of norms. The consequence of modeling norms that don't directly fit the action perspective is, that their representations don't reflect the original text. However, this is not within the goals of FLINT. For FLINT the representations should provide the

[2] Unless of course you consider it the act of putting a mark in square such that you get 3 marks in a row as a specific sort of act. But this would result in hopeless overspecification of actions.

right inferences, be maintainable and disambiguate the source text so that it can be computationally implemented.

6.1.2 Layered nature of the law

The law is by nature a layered system. The difficulty with modeling the interpretation norms of such a system is that the conditions of certain legal provisions within the system also apply to other legal provisions in the system. Some of these conditions are implicitly assumed to hold for all provisions within the system. We call these conditions here ‘systematic conditions’. Other conditions are directly enforced by cross-referencing another legal provision. Finally, some legal provisions are in a hierarchical order, meaning some norms only apply when others don’t. Since our action-oriented approach represents all preconditions to perform a certain action, all these conditions should be contained in a proper formalization.

System conditions

Almost all legal provisions must be interpreted within the context of the legal act or rule book that they are in. This context is often set by a few preliminary legal provisions in such an act, creating ’system conditions’. What we mean by system conditions is reflected by the global predicate we use in the tic-tac-toe example. For all the other rules it is assumed that we are still in the context of a game of tic-tac-toe. The tic-tac-toe game is played on a 3x3 grid with 2 players, as the ‘starting a game’ rule states. The squares of the game are squares on this 3x3 grid. So when we interpret rule 2, ”putting a mark in a square”, the square must be a square on this 3x3 grid.

In our formalization we repeat the global predicate for each norm of tic-tac-toe to ensure that the norms operate within the same context. This means that the first step towards a proper formalization of norms with L_{FLINT} is to determine which systematic conditions must hold for all rules, so that the global predicate can be constructed. Again, the result is a formalization that doesn’t adhere to the source text but does provide us with a correct interpretation.

Cross-referencing

Normative texts often refer to other normative texts. Our tic-tac-toe example does not make use of cross-referencing but Di Bello, 2007 gives the following examples of how cross-referencing might appear in normative texts ”‘refn shall apply’, ‘according to the definition of . . . contained in refn’, ‘provided the provisions given by refn have been satisfied’, etc.’” The result of cross-referencing is that the conditions in a different legal provision also apply to the legal provision that makes use of cross-referencing. For our approach of modeling norms this means that we should include all these conditions in one frame.

To illustrate this we’ll use the following adapted example from Di Bello (2007):

- (A) Those who were born in the Netherlands, shall acquire Dutch citizenship.
- (B) Article A applies also to those who were adopted by parents of which one parent was born in the Netherlands.

How should we model the action of acquiring Dutch citizenship? If we regard acquiring citizenship as one and the same action, it's clear that we get a disjunctive precondition. One can acquire Dutch citizenship in multiple ways: either you were born in the Netherlands or you were adopted by parents of which one parent was born in the Netherlands.

Modeling norms in this way could mean that we get very large disjunctive preconditions. The more ways there are to satisfy the preconditions of an action, the larger the disjunctive precondition gets. A solution could be that we separate these legal provisions into different norms, applying to different actions (frames). So for our example we would get acquiring Dutch citizenship (A) and acquiring Dutch citizenship (B). The postconditions are the same, but the formalization is now more succinct and closer to the original text.

This can only be done for provisions that provide alternative preconditions. If extra (necessary) preconditions are contained for an action in a different legal provision, these preconditions should always be included in the norm for that action.

Defeasibility

Sometimes, when the preconditions of one norm are fulfilled, this overrides the applicability of another norm. We can see this in our tic-tac-toe example by the addition of the precondition that 'there are not 3 marks in a row' to rule number 2, 'putting a mark in a square'. This precondition is added to ensure that if there is already a winner of the game of tic-tac-toe, the players cannot meaningfully keep putting marks in squares. The rule of winning the game is hierarchically superior to the rule of putting a mark in a square.

In the case of tic-tac-toe the hierarchical structure of its normative system amounts to the addition of one defeasiblity condition. But one could imagine that in larger normative systems this hierarchical structure could get increasingly complex. Formalizations would have large preconditions with many defeasibility conditions. Without an hierarchical order of norms incorporated into the language this is unavoidable. Within L_{FLINT} we do have a stylistic solution to make the formalizations more succinct. We could use a negation of our metalinguistic abbreviation $Pre(\alpha)$ to express that the norm of action α is a defeasible condition for another norm.

For a logic that explicitly models defeasibility see Governatori et al., 2021.

6.1.3 Duty concept

In the section on duties in Chapter 4 we already stated that the conceptualization of duties within FLINT is limited. In FLINT we treat duties as a primitive fact stating that one agent has an obligation towards another agent to perform an action. Positive duties tell us that an act should be performed. However, what we mean by performing an action can vary according to the types of duty we consider. It also goes without saying that there are duties to refrain from performing an action. Furthermore, because of the action-oriented approach, it is unclear how we model not performing a duty as constituting a violation of this duty.

Duty types

In L_{FLINT} we consider active duties to be a sort of to-do list for the duty holders. The action needs to be performed. The question of how this action should be performed cannot be modeled. This is problematic when it comes to duties that explicitly mention that a duty needs to be performed in a certain way. The GDPR for instance says that data processors have a duty to process the data *carefully.* A possible solution is that we think of such actions as special actions. Adding the description of the performance conditions to the action itself. We would talk about carefully processing data as one action and not carefully processing data as another action.

Another difficulty is modeling duties that don't mention the performance of an action but a result. In principle these duties can be rephrased as actions (Governatori et al., 2021), however, resolving result duties is different from performance-based duties. As soon as the result is reached the duty terminates, no matter how the result came about. This is something that we can't model in L_{FLINT} because this would lead to indeterminate postconditions. When a specific configuration of facts is the intended result and this result is reached by accident by an action, the result duty terminates. But since we have determinate postconditions for each action, this would not be the case. Currently such duties can therefore not be modeled in FLINT.

Refrainment

Another issue with regard to the action-oriented approach of FLINT is that this approach makes it difficult to treat the idea of refraining from action as a duty. We currently don't provide semantics saying that an agent has the duty to not perform an action. This is a large limitation for L_{FLINT} as refrainment is necessary to model the Hohfeldian Privilege - No-claim relation.

To formalize the notion of refrainment one could think of adding the formula $\Box\neg\alpha$ to the language, which we would treat similarly to $\Box\alpha$ as an atomic proposition. The issue here is that it is unclear how to treat the consequences of performing α when one had a duty not to. Other than for $\Box\alpha$, an axiom stating that the performance of α takes away the duty wouldn't work for $\Box\neg\alpha$. Rather than removing the duty, this action would result in a violation of an obligation. A duty not to perform an action can only be taken away by a change of the circumstances by which it would now be allowed to perform that action again. Be it permission by the counterparty, passing of time, or some other event. Since we only model normative state transitions by the performance of actions in L^{fo}_{FLINT}, a duty to not perform an action can only be taken away by some other action. The passing of time and the occurrence of another event cannot be modeled. We can therefore not generically express in our semantics when a duty of refrainment ceases to hold.

It is furthermore unclear how we would perceive the meaning of $\Box\neg\alpha$ in a vacuum. Do we regard a duty to refrain as an addition to a 'to-not-do-list'? What would be the benefit of such a list? There are many things we shouldn't do according to the law, so a to-not-do list can be very large. An argument could be made that refrainment is not such an important notion within the task procedure framework of the Calculemus protocol. The key questions also ask us what we can/should do, not what we shouldn't do. To model a task procedure we should focus on which task should in fact be performed.

However, violation of refrainment could have serious consequences, also within a task procedure. For instance, if you are trying to publish a paper in a specific scientific journal, it can be a requirement that you refrain from handing in this paper for publication to other journals at the same time. For normative coordination among stakeholders in a task procedure, it might be quite helpful to know what they shouldn't do. To have a task-specific to-not-do list. The fact that we can't generically specify when a duty of refrainment ends might also not be such a problem. Since we have knowledge of the postconditions of the actions in the normative system of our task procedure, we would know which specific actions will end refrainment duties. We would just include $\Box\neg\alpha$ into our system without the addition of an axiom expressing when a duty of refrainment ends.

Violation

The to-do-list metaphor for duties in L_{FLINT} creates the idea that a duty stays active until the action is performed. In the semantics of duties there is no notion of when a duty is violated. In reality the different types of duties mentioned in this section do always determine violation conditions. A duty cannot even exist if it can't be violated.

In general, positive duties to perform an action are often constrained by time limits. A duty needs to be performed within a certain time frame. In our tic-tac-toe example we expect the player who has the turn to perform the duty of putting a mark in a square in a reasonable amount of time. For refrainment, a violation is performing the forbidden action.

The current inclusion of duties in L_{FLINT} as primitive facts means that the semantics of duties do not contain its violation conditions. However, we could model violations in a different way. Not directly detecting a violation of a duty when it happens, but using the violation conditions as preconditions for a punitive action. For these violation conditions it must always be a precondition that a duty still exists, otherwise there can be no violation of the duty. So the duty must always be on the to-do list for preconditions of the punitive action to hold.

As the exact preconditions for punitive actions often contain time, duty violations remain an issue in the current L_{FLINT} version. In Section 6.2.1 we'll address this further.

6.2 Structural limitations

6.2.1 Time

Time is of the essence for many norms. Norms often refer to the notion of time (Governatori et al., 2021). Libraries have terms such that you have to return a book on loan before the end of that term. Just as contracts often have clauses in which a duty is supposed to be carried out before a certain end date. In the Netherlands there was a law that would prevent construction workers from starting their work earlier than 07:00 in the morning.

Besides temporal parameters determining the content of rules, there is also the meta-function of time in determining the validity, applicability and period of effectiveness of norms (Di Bello, 2007):

- Validity: the validity of norms refers to the period in which a norm is part of a normative system.
- Applicability: specifies a certain period as a criterion for which the norm can have an effect.
- Effectiveness: the period in which the norms have legal effects.

We take a slightly modified example from Di Bello (2007) to illustrate these notions. We have a norm about having rights to social benefits that was enacted in 2012, was applicable to people born between 1975-1980, produced effects only after 2020, and was repealed in 2024. This law was then valid for 12 years from 2012-2024, with an applicability period from 1975-1980 and a period of effectiveness of 4 years from 2020-2024. Di Bello (2007) mentions that these concepts don't always come apart as in this example. More often than not, the period of validity coincides with the applicability and effectiveness.

Both time as a content parameter and meta-parameter are not contained in the FLINT ontology by itself and, therefore, also not in L^{fo}_{FLINT}. This means that we are limited in our representation of time in our formalization.

If we take the example of time as a content parameter of having to bring back a book on loan within 30 days, we see that we have a duty with a deadline as a postcondition of loaning a book. However, with L^{fo}_{FLINT} we have no way of imposing a time constraint directly on the duty and no way of initiating a timer of 30 days. We could impose the 30-day limit as a precondition for the punitive act of giving a fine for not returning the book $30t$ or $t = 30$, but without a timer being initiated by the act of loaning a book, we'll have no way of knowing whether this precondition is fulfilled in a scenario. As such, we cannot provide IT services with an implementable interpretation if time is a content parameter.

With regard to the meta-time parameters, we are not so much interested in modeling validity, as either a norm is valid and part of the normative system or it is not part of the normative system and we don't have to model it. We should be concerned with modeling applicability and effectiveness. Both the applicability period and the effectiveness period should be included in the preconditions of norms as they decide whether the performance of the action will result in a change of the normative state at all. Looking at the example, if you are not born between 1975-1980, no benefit can't be granted. And if the year was not yet 2020, the granting of the benefit would not yet have effect.

Some notion of time should therefore be included in an updated formalization of the FLINT ontology and L_{FLINT}.

6.2.2 Calculation

The fact that we need to incorporate time into the construction of L_{FLINT} automatically means that a calculation must also be added to the machinery. Calculation is necessary to express periods of time or differences between different time stamps. When the loan period is 30 days, we need to be able to calculate when exactly the loan period ends. That is, of course, the date of loan + 30 days.

Calculation can also be used in legal provisions to express the value of certain institutional facts. We can think of fiscal rules or other laws in which income plays a role. The TNO Norm Engineering Project has worked on a use case of the 'Dutch participation act' in which the height of income was a precondition for the grant of a benefit. Income in the participation act was a sum of different factors (income, taxes, etc.). Therefore, if we want to be able to model computationally implementable norms of legal terms that are defined to be the sum of different values, calculation is a necessary addition to L_{FLINT}.

Currently the TNO Norm Engineering project is already working on incorporating calculation.

6.2.3 Events

Actions are not the only cause of changes of facts in our world. Sometimes events change what's true and what's not. This can be natural events, like a snowstorm that takes away a kid's duty to go to school that day. Or events not governed by norms, such as moving out of your house.

The fact that the configuration of the world changes obviously has an effect on our normative state. When different facts are true, different preconditions are satisfied and, as such, the normative action space of an agent changes.

We could say that we don't model events and just require to look at action. But this would mean that we are not able to adequately describe scenarios anymore.

6.2.4 Complex actions

Currently, we can only consider scenarios in L_{FLINT} in which one action within the task procedure is always performed after the other, never at the same time. Hence, modeling a game of rock-paper-scissors in L_{FLINT} is quite complicated. A solution would be to say that the different configurations of rock-paper-scissors are all different types of actions. But this is of course not an ideal solution.

It remains an open question whether L_{FLINT}'s inability to model simultaneous actions explicitly will be a big issue. We are looking at task procedures, and it is often the case that such procedures are, in fact, step-by-step procedures. Only the postconditions of one action make sure the preconditions of the following action are satisfied. Also, task procedures are often limited in scope. This means that there might only be a few actions that can be taken simultaneously. The solution of creating special composite actions for such scenarios might not be such a problem then.

6.3 Developing FLINT

The different types of limitations of L_{FLINT} teach us different things. The limitations due to concepts currently not included in L_{FLINT}, point at clear ways of improving its expressive power by incorporating these concepts into the language. The inherent limitations should be viewed in a different light. These inherent limitations show that the action-oriented approach might not be suitable for the modeling of all kinds of norms, and, as such, for answering all types of normative questions. This should be taken into account when developing the language further. It would therefore also be interesting to try to model norms that intuitively don't suit the action-oriented approach to see how far the action-oriented approach can be stretched. In the discussion, reflections on the uses of the current construction of L_{FLINT} will be addressed.

Chapter 7

Conclusion

In the final chapter of this thesis we'll summarize its most important ideas. It is divided into three sections: the first section summarizes the goals of the thesis and main takeaways. We'll then discuss for which normative questions L_{FLINT} can now be used. In conclusion, directions for future research are discussed.

7.1 Summary

The goal of this thesis was to provide a full formalization of FLINT, the formal language for the interpretation of normative texts, based on the action-oriented approach of the Calculemus protocol. A full formalization was necessary to be able to specify the semantics of the concepts included in the FLINT ontology, reason with these concepts about norms and scenarios, and make verification of consistency of norms possible. These properties of the formal system allow us to formalize the interpretations of normative texts such that we can answer the key questions of the action-oriented approach of the Calculemus protocol in a given scenario: what can/should I do to others, what can/should others do to me, under what circumstances can/should we do that, and most importantly what happens when we do that? As such, L_{FLINT} provides a computational theory of norms that can be compared to other formalizations.

The construction of L_{FLINT} is based upon the theories of Hohfeld and Searle, and the FLINT ontology. Its place within the literature is therefore alongside logics of norms that include actions as first-class citizens and model the Hohfeldian legal relations. The presented logics notably lacked the expressiveness to model the general applicability of norms. The purpose of L_{FLINT} therefore has been to build a logic that is able to formalize both concrete scenarios and general norms. Therefore, this thesis is also largely an exploration of the difficulties that arise when formalizations of general norms are applied to concrete scenarios.

To properly lay out what a logic of norms entails, first a proposition version of L_{FLINT} has been built, which we call L^{prop}_{FLINT}. This version could be viewed as a logic of global pre- and postconditions. We believe that this logic may have broader applications in contexts similar to the legal field, in which pre- and postconditions of actions always remain the same. Subsequently, the L^{prop}_{FLINT} version of L_{FLINT} was extended to a version with duties, where the limitations of the propositional nature of L^{prop}_{FLINT} in applying norms to concrete scenarios were emphasized.

Finally, to express the general applicability of norms, a first-order logic was constructed L^{fo}_{FLINT}. By applying L^{fo}_{FLINT} to interpretations of rules of tic-tac-toe represented with the concepts of the FLINT ontology, it was exemplified how one can go from a semi-formal model of a norm towards a full formalization. These full formalizations of the tic-tac-toe rules were then used to run through a scenario of the game actually being played. This showed that L^{fo}_{FLINT} can answer the key questions of the Calculemus protocol and provide a computational theory of norms.

The limitations of L_{FLINT} described in Chapter 6 provide insights into which important normative concepts cannot be expressed in L_{FLINT}. In the next subsection we discuss what this means for the usefulness of L_{FLINT} for answering normative questions.

7.2 Discussion

L_{FLINT} is a logic of norms, but at the same time the limitations make clear that L_{FLINT} is not perfectly suitable to model all kinds of norms and answer all kinds of legal questions. How should we use L_{FLINT}?

Because L_{FLINT} can operate both at a normative and a scenario level, L_{FLINT} can provide a direct link between normative sources and the codification of these norms into IT systems. In IT systems, the code should be applicable to concrete scenarios. At the same time, the code should not lose its link to the general normative source. L_{FLINT} provides a standardized interpretation to ensure this link.

The key questions indicate that L_{FLINT} is focused on what is relevant to an agent at a particular normative state. The modal analysis of norms in L_{FLINT} allows this agent to reason about hypothetical scenarios. This qualifies L_{FLINT} for modeling legal administrative procedures, which are mostly step-by-step, such as the procedures for social benefits or residence permit grants. L_{FLINT} can also help normative coordination among stakeholders because it models the available normative action space. What becomes clear is that L_{FLINT} is suitable for answering questions about future normative behavior.

For other normative questions L_{FLINT} is not directly suitable. The current version of L_{FLINT} is not able to look back and tell you how a normative state has been reached. This can be relevant for post hoc analyses of procedures, such as determining which actions in a (semi-)automated decision making tool brought the agents into the normative state.

Furthermore, L_{FLINT} only allows actions to be performed of which the preconditions are fulfilled, this means we can't model the performance of norm-breaking actions. For the purpose of (semi-)automated decision making tools this is probably desirable, because you don't want to allow illegal actions.

Detection of the violation of norms is currently not possible in L_{FLINT}. Violation conditions can be modeled as preconditions of punitive actions, but it would be preferable that the language can express exactly which violation occurred.

With respect to the determination of its normative position, an agent can only refer to its duty to-do list in L_{FLINT}. The other Hohfeldian legal relations are not explicitly modeled. As these relations are the foundational theory for L_{FLINT}, this can be seen as an important shortcoming. Especially compared to the Hohfeldian logics presented in the related work chapter, which all define these relations within the logic.

Whether this really matters is a philosophical question that is also asked by Ju and Xu, 2023: "Exercise of powers causes changes of legal facts. What if the law just specifies the consequences of behaviors? Why is the notion of powers needed?" Is power a mere normative artifact used to describe a difference between actions that change only facts and actions that also change duties? If this is the case, it seems good enough that L_{FLINT} can tell you the consequences of actions without telling you whether you are in a Power - Liability relation or not. But then the question arises: how Hohfeldian is L_{FLINT} in the end?

The discussion above makes clear that whether the current construction of L_{FLINT} can provide normative answers depends on the normative questions you have. Clearly determining which questions L_{FLINT} should answer is, therefore, the first task for the TNO Norm Engineering Project to further develop L_{FLINT} as a computational theory for norms. While doing this, the inherent limitations of the action-oriented approach highlighted in Chapter 6 should be kept in mind.

7.3 Future work

Several different lines of future work can be explored. First of all, it is desirable to provide a complete and sound proof system for L_{FLINT}^{fo}. A more extensive systematic comparison of L_{FLINT} and related work could then provide further insights into the advantages and disadvantages of the system. Such a comparison could look not only at related work on formalizations of Hohfeldian theory but also at formalizations of norms that are focused more on computational implementation. This could help to review the suitability of L_{FLINT} as providing a computational theory for the computational implementation in EFLINT.

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