Conceptually Grounded Quantification*

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April 30, 2025

Abstract
This paper shows how to lift an extensional logic for contextually relativized quantification to an intensional format. The result is shown to provide for a method of conceptually grounded quantification, or quantification through conceptual windows, as it is also called here. The method is shown to formally and methodologically improve upon currently available approaches to cross-modal quantification.

1 Contextually Relativized Quantification

Contextual relativization plays an important part in the, context-sensitive, interpretation of natural language quantifiers, including definite noun phrases and also indefinite ones. In (Dekker 2024b) I have presented an extensional logic for contextually relativized quantification. In the system presented there quantifiers are dressed with indices, pre-superscripted, that serve to identify and distinguish the various, possibly distinct, domains that the quantifiers are taken to range over. A zero index is assumed (stipulated) to indicate the most general domain, one that all other domains are considered subsets of. The proof rules are given in the format of a Fitch-style natural deduction system. The deduction rules are all classical, except, of course, those for the quantifiers.

In the proposed system the rules for the context-sensitive quantifiers differ only minimally from the classical ones. The proof rules for the *introduction* and *elimination* of a relativized existential quantifier are displayed in figure 1. The existential introduction rule is relativized to the indices (contexts) in which a free variable is declared to have a value; the existential elimination rule in converse initiates a sub-derivation in which such a declaration is issued. That is to say, if something, z, has been established to be ϕ, and if it has also been established that this z lives in context i—"counts as in i and declared"—, then we can conclude that something in i is ϕ. Conversely, if it has been established that something in context i is ϕ, and if we are able to conclude that ψ holds whenever

* This note is one in a series of notes aiming to contribute to a formal logical framework which allows us to present and study natural language meanings. The direct predecessor of this note is one on the logic, model- and proof-theory, of contextually relativized quantification in an, extensional, first order framework.

Figure 1: Deduction Rules for Relativized Quantifiers

$\exists$-Introduction (I$\exists$)

$\vdots$

m. $[z/x]\phi$

$\vdots$

n. ${}^{i}\exists x\phi$ [I$\exists$, m]

Variable z must count as in i and declared at line m.

(I use $[z/x]\phi$ to indicate the formula obtained from ϕ by replacing all free occurrences of x in ϕ by z. Variable z must be free for x in ϕ.)

$\exists$-Elimination (E$\exists$)

$\vdots$

l. ${}^{i}\exists x\phi$

$\vdots$

m. $[z/x]\phi$ [ass.]

$\vdots$

n-1. ψ

n. ψ [E$\exists$, l]

Variable z may not occur free in any assumptions, ${}^{i}\exists x\phi$ or ψ. It counts as in i and declared from line m to n.

anything in i is ϕ, then ψ, of course, must hold. These rules are actually the familiar ones, except for the condition of being declared, in a context i.

Various authors have argued that quantification in natural language is not just restricted by extensional constraints on a given domain of quantification, but that it is governed by intensional constraints, also characterizing the properties of and relations between the individuals in there. (Reimer 1998, for instance.) Kuroda, Recanati and Bonomi have, accordingly, proposed to not employ restricted domains of individuals, but whole '*mini-worlds*' as contexts. (Kuroda 1982; Recanati 1987; Bonomi 1998) According to Recanati a context can also be a '*situation*', or a way in which it is represented. (Recanati 1996) None of the authors mentioned have, however, engaged in incorporating any such notion of context in a fully fledged formal framework.

This paper therefore sets out to upgrade the extensional system of contextually relativized quantification to an intensional one. Quantifiers are taken to range over the individuals that can be seen through conceptual windows, as the context are conceived of here. I will show how this approach may help us to adequately render the relations between, on the one hand, various modal realms and intentional states, and, on the other, the individuals and situations that these possibilities and states can be taken to relate to.

I will proceed as follows. In the following section I present the proof rules for a basic modal logic. These are supposed to capture, all and only, intuitively motivated modal principles. Section §3 defines some constraints on our contexts, so that they can be properly conceived of as conceptual windows. Section §4 I presents a general format for the regimentation of conceptual windows in practice, so that they sustain the cross-modal inferences that we encounter in natural discourse and reasoning. In section §5 I present the model-theory underlying our deduction system, backing it up somewhat further with some reflections on the (non-)logical status of certain modal-metaphysical principles. The final section (§6) finally shows how the approach developed here provably improves upon three others types of approach that are currently available.

I have to issue a subtle warning before we proceed. In the extensional system it is safe, and classical, to assume that all free variables, and also individual constants, just count as declared (existing) in the default, unrestricted, context. This is an assumption that we have give up when we turn to a modal system. There is no intuitive reason to assume, as a logical principle, that every thing necessarily exists.

2 Basic Modal Logic

In order to account for any intensional features of contextually relativized quantification, the initially extensional first order logic has to be turned into an intensional one, and the language of the above mentioned first order predicate logic is therefore extended with, indexed, modal operators '$\Diamond^k$' and '$\Box^k$'. The modal operators are decorated with indices to identify, and distinguish, the various modalities that we may find in natural reasoning, such as the various modal bases and mental states that we may want to characterize in intentional discourse. It is assumed that the zero index serves to indicate a universal modality that subsumes all the others.

The following definition presents the required supplement to the definition of the first order language provided in (Dekker 2024b).

Definition 1 (Language of $\mathcal{MPL}^{\mathcal{CGQ}}$) *for (...) and* $k \in \mathbb{N}$

$$\phi ::= (\ldots) \mid \Diamond^k\phi \mid \Box^k\phi$$

Reflecting on what can be considered possible and necessary at all, we can subtract some general, familiar, modest and intuitive, modal logical principles. In the first place it is assumed that possibility and necessity are each other's dual, in whatever type of modality ^{k}.

$$\vdash \neg\Box^k\phi \leftrightarrow \Diamond^k\neg\phi \qquad (DUAL)$$

Something that is not possible is necessarily not, so, likewise, something that is not necessary is possibly not. Secondly, as indicated, something that is possible according to any specific modality (k) is a possibility sui generis.

$$\vdash \Diamond^k\phi \rightarrow \Diamond^0\phi \qquad (U)$$

Thirdly, it is fairly standard to assume that what is actual is possible, and that what is possibly possible is also just possible.

$$\vdash \phi \rightarrow \Diamond^0\phi \qquad (T)$$

$$\vdash \Diamond^0\Diamond^0\phi \rightarrow \Diamond^0\phi \qquad (4)$$

Given the duality of $\Diamond$ and $\Box$, (T) and (4) imply that whatever is necessary is actual, and that every necessity is necessarily a necessity. Fourthly, I assume that a possibility is necessarily a possibility.

$$\vdash \Diamond^0\phi \rightarrow \Box^0\Diamond^0\phi \qquad (5)$$

As may be known, the three principles (T), (4) and (5), conveniently simplify the logical space of possibilities so that every true modal proposition is itself both possibly and necessarily true.

Finally, any modal space is required to obey logical truths and principles.

$$\textit{if } \phi_1, \ldots, \phi_n \vdash \psi, \ \textit{then } \Box^k\phi_1, \ldots, \Box^k\phi_n \vdash \Box^k\psi \qquad (\textit{DN})$$

This schematic rule of *distributed necessitation* (*DN*) combines the familiar principle of necessitation with the familiar distribution law. Principles that are stipulated to be generally valid can be called upon any time in our inference system, one taken to properly extend that of (Dekker 2024b).

It should be pointed out that there is a subtle, but significant, constraint on how to properly use the (*DN*) rule. If, relative to any premise $\Box^k\phi_j$, a variable counts as in any context ^{i} and declared there, it does not necessarily counts as declared when we evaluate ϕ_j in an inference that is supposed to support the necessitation rule. (Although it still counts as being in context ^{i}.) In order to count as being in i and declared it may be in need of support from an explicit existence assumption. This assumption may show up as a restriction on the intended conclusion.

As has already been noted, I do not think logic should force us to assume that everything necessarily exists, so I do not want to exclude the possibility that terms, variables as well as individual constants, fail a referent. For this reason it is convenient to have a notation for when they do have one, and use this for instance to restrict the scope of modal operators.

Notation Convention 1 (Existence and Restriction)

$$^i\mathsf{E}t \ := \ ^i\exists z\, t{=}z$$
$$\Box_x\, \phi \ := \ \Box(^0\mathsf{E}x \rightarrow \phi)$$

To express that a term refers, or actually has a referent in some context i, I use the existence predicate 'E' already employed in the extensional framework of (Dekker 2024b). Using this predicate we can employ '$\Box_x\, \phi$' to express a necessary property of x, saying that x necessarily has ϕ if it exists at all.

It is quite generally assumed that for an atomic formula to be true, its constituent terms must have a referent. If such a formula is derived, or assumed, it therefore allows existential generalization over its constituent terms.

E-Introduction (IE)

$$\begin{array}{ll} & \vdots \\ \text{m.} & AT(t) \\ & \vdots \\ \text{n.} & {}^{i}\mathsf{E}t \quad [\mathrm{I}\mathsf{E}, \mathrm{m}] \end{array}$$

Term t must count as being in i at line m. A term always counts as being in 0.

The, schematic, formula '$AT(t)$' here is short for any atomic one, possibly an identity statement, that hosts the term 't'. Typically, the negation of any atomic formula does not support such an existential inference. which means that I hereby adopt what is called a *negative free logic.* (Nolt 2020)

The above basically settles the logical skeleton of our system, except for the logic of identities and their cross-modal validity. The deduction rules for the identity sign are those taken from (Dekker 2024b).

Self-Identity (=)

$$\begin{array}{ll} & \vdots \\ \text{n.} & x = x \quad [=] \end{array}$$

The variable x must count as declared at line n.

(Here I use $[t/z]\phi$ to indicate the formula obtained from ϕ by replacing all free occurrences of z in ϕ not in the scope of a modal operator by t.)

Leibniz (L)

$$\begin{array}{ll} & \vdots \\ \text{l.} & t_1 = t_2 \\ & \vdots \\ \text{m.} & [t_1/z]\phi \\ & \vdots \\ \text{n.} & [t_2/z]\phi \quad [\mathrm{L, l, m}] \end{array}$$

The terms t must be free for the variable z in ϕ

We can establish the self-identity of an object if it counts as existing, i.e., if it is declared. As is probably desired, to be self-identical, to be something that exists, and to exist, all three turn out to be equivalent in the current system. The latter two, '$\exists z\, t{=}z$' and '$\mathsf{E}t$', are equivalent by definition. Both can directly be derived from the first, '$t{=}t$', by the E introduction rule (IE). And by the $\exists$ elimination rule (E$\exists$) we can derive identity from existence, as follows.

$$\begin{array}{lll} 1. & \exists z\, t{=}z & [\text{ass.}] \\ 2. & \quad t{=}z & [\text{ass.}] \\ 3. & \quad t{=}t & [\text{L, 2, 2}] \\ \hline 4. & \quad t{=}t & [\mathrm{E}_{\exists}, 1] \end{array}$$

Analogous observations may serve to show that the notion of *being declared in context* i and *existence in* i also amount to one and the same thing.

The Leibniz rule can be taken to state, quite indubitably, that if something x has a certain property, then something y has that property if it is the same thing. One may think that this should hold for modal properties, too, but there is a little snag. The substitution of terms that we are allowed to make according to the Leibniz rule has to be restricted to terms not in the scope of a modal operator. In the statement of this rule, $[t/x]\Diamond\phi =_{df} \Diamond\phi$, and for $\Box\phi$ likewise. The reason is that we cannot assume any direct *logical* relation between (what we know about) x or t in some conceived possibility, and (what we know about) x or t in what we assume to be actual reality.

When we, for instance, say that it is possible, or necessary, that x is F, the term 'x' figures under a modal operator, and there is no logical guarantee that it is used, there, to just talk of who actually is x.[1] In order to know anything about who or what x possibly is, for any x figuring under any modal operator, we must know something about who or what x necessarily is, and this can only be given to us by means of other, modal, assumptions. The actual identity of some x and y may for this reason not be sufficient for concluding that it is possible that y is F if it is possible that x is. For the desired conclusion to follow we need to know that x and y are necessarily identical, because then the intended conclusion can be derived by classical modal reasoning.

In this section I have presented the basics of a modal logic which I tend to qualify as, fairly, uncontroversial, and I have preserved the, I believe also unobjectionable, rules for contextually relativized quantification from (Dekker 2024b). The proposed modal characterization, or presentation, of individuals is kept minimal. This is a deliberate choice. I think one should generally strive to keep a logic free from any metaphysical dogma. I also think that, of course, a logic should be able to acknowledge such assumptions, and allow one to state them explicitly as assumptions that one can choose to live and reason with, or not live and reason with. The next two sections shows how we can go about that.

3 Defining Conceptual Windows

I have had to restrict the application of the Leibniz rule above because of our, assumed, lack of direct knowledge of individuals in non-actual possibilities. However, we all do, naturally, engage in talk and thought about objects in possibilities. How is this possible? This type of talk is actually facilitated by the ways in which individuals are presented in some context ^{i}, or, as I will also phrase it, by the ways in which they are seen through that window. While contexts, in

1. No matter how different their philosophical inclinations, Gottlob Frege, Willard Van Orman Quine and Jaakko Hintikka have all denied that the actual referent ("gewöhnliche Bedeutung") of a term is at stake in such a modal ("ungerade", "opaque") context. (Frege 1892, p. 28; Quine 1980, §1; Hintikka 1969b, §VI) Please mind that we are not talking about somebody who is actually x in some different, *real and actual*, *circumstances*, but about somebody, who is actually x, in some, *non-actual*, *possibility*. Saul Kripke has, famously, argued that there is sameness of reference across possibilities, but this is claimed to be the result of a, model-theoretic, stipulation. It is not a fact that can be established by empirical means, neither by the principles of a deduction system. See (Dekker 2024c) for discussion.

the extensional setting of (Dekker 2024b), only serve to constrain the domains of individuals that quantifiers are taken to range over, in the current intensional setting these contexts enable us to present and regiment the ways in which such individuals are conceived of.

Let us consider an example from the literature. Kripke insists that we can stipulate that we talk about Nixon in alternative, non-actual, possibilities.[2] Our contexts can be used to make such a stipulation explicit. We can specify a way in which Nixon is seen, as he is seen through some particular window i. Assume, for the sake of simplicity, that *being Nixon* can be rendered as *being* n, being the individual Nixon. (Note that being Nixon should not be equated with being named "Nixon".) We may then formulate Kripke's stipulation as follows.

$$^{i}\exists x(n{=}x \wedge \Box_x\, n{=}x) \qquad (N)$$

With such a stipulation context i presents something (x) as necessarily being Nixon. (So long as he exists, that is.) Looking through this window ^{i} one may now speculate about what would have happened to Nixon, thus conceived, if, for instance, the presidential debate in 1960 had been broadcasted on the radio. If we consider this a genuine, even if counterfactual, possibility, one may be inclined to conclude that, in that, counterfactual, case he would have won the elections. Given the way one has then conceived of *him*, it will have to be *Nixon* that is the one that would have won in such a counterfactual possibility.

Another example, a window provided by Quine, presents a view on some individual Ortcutt, as a man seen on the beach, say, even if not as a man seen to be Ortcutt. (Quine 1956) Some agent Ralph can be reported to have a belief about Ortcutt that he is supposed to have seen that way. Ralph need not realize it is Ortcutt he is having this belief about. Perhaps he even believes it is not Ortcutt. Even so he knows it is the man seen on the beach which he is having beliefs about, and actually, Quine tells us, this is Ortcutt. Such a window j on Ortcutt can be formally regimented as follows.

$$^{j}\exists x(o{=}x \wedge \Box_x\, {}^{j}\imath z\,(Bz)(x{=}z)) \qquad (Q)$$

This formula has it that context j supplies a view on the real individual Ortcutt as someone with the said property B, that of being the man seen on the beach. Intuitively, in any possibility where x exists it has this property. If someone next says of Ralph that he believes of Ortcutt, seen this way, that he is a spy, this entails that he believes the man seen on the beach to be a spy.

Notice that this window ^{j} on Ortcutt implies nothing whatsoever about who or what Ortcutt necessarily is or does. He just happens to actually be the

2. "There is no reason why we cannot *stipulate* that, in talking about what would have happened to Nixon in a certain counterfactual situation, we are talking about what would have happened to *him*." (Kripke 1981, p. 44) "[I]t is *because* we can refer (rigidly) to Nixon, and stipulate that we are speaking of what might have happened to *him* (under certain circumstances), that 'transworld identifications' are unproblematic (. . .)." (Kripke 1981, p. 49)

individual seen on the beach that Ralph may happen to have mistaken beliefs about, possibly even the belief that he is not Ortcutt. And even if the window does not allow us, or Ralph, to conceive of x, i.e., Ortcutt, as not being the man on the beach, it easily allows us to conceive of x as being a man that, seen from another window, could just as easily not have been on the beach.

These two examples serve to give an indication of how our modal reasoning, and the intentional states of individuals that we describe, can be understood to be related to individuals and situations in the real world. Our contexts can be said—assumed, or stipulated—to provide an objective view, or conception, of individuals in the actual world. When we do so, we normally assume that these individuals are objectively conceived of as individuals that are *clearly* and *distinctly* given.[3] That is to say that our contexts, or conceptual windows, satisfy certain integrity constraints. I propose three of them here.

It is first assumed that every window i is *clear*, or *transparent* if one wants, meaning that everything one sees through it is real.

$$\vdash {}^{i}\forall x\, {}^{0}\mathsf{E}x \tag{E}$$

This principle says that if something exists in context i, it actually exists. Second, saying that things are seen *distinctly* through a window means that we are not confused about the (non-)identity of the individuals thus seen.

$$^{i}\forall x^{i}\forall y(x{\neq}y \rightarrow \Box\, x{\neq}y) \tag{D}$$

There is no possible confusion of the identity of two actually distinct individuals, as seen through this window. This is not to say that one can have different windows on the same two individuals, the identity of which, thus conceived, can elude us. It is just to say that the various individuals are conceived of as distinct, for as far as they are seen through window ^{i}. I will assume that (D) and EX are necessarily true for most windows ^{i}, even if not as principles of logic.

Just assuming (E) and (D) does not suffice to exclude the possibility that a context ^{i} supplies a view on individuals that do not exist, but that would be, or become, visible if conceived from other, non-actual, possibilities. The following extensionality principle serves to exclude that an actual window host such *phantom* objects.

$$^{i}\forall x\Box(^{i}\exists x\top \rightarrow \mathsf{E}x) \tag{EX}$$

This principle has it that if a window supplies a view on any individual in any accessible possibility at all, then we can see there everything that we can see here already. Since our generic modality $\Box$ is S5, (D) and (EX) bring about that a context ^{i}, and all conceptions in it, count as declared for the same possibilities, those where one can see anything through ^{i}.[4]

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3. These phrases are actually borrowed from Descartes. I give a formal characterization of these notions below, without any pretense that it captures what Descartes had in mind.
4. Since our default modality $\Box$ is S5, (D) and (EX) jointly imply that

$$^{i}\forall x^{i}\forall y(x{=}y \rightarrow \Box_{x}\, x{=}y) \tag{NI}$$

4 Regimenting Conceptual Windows

We often think and talk of mental or intentional states that an agent entertains regarding particular objects and individuals in the real world, and this can be done accurately only if we thereby relate of the ways in which these objects and individuals are present to her. An object that an agent is familiar with one way may be known to have certain properties, when conceived of that way, and not when the object is presented to her in some other way. In the literature such cases are typically described by the characterization of an objective situation, some actual state of affairs that certain objects are participating in, and such that these objects are objectively known, or *seen*, to have certain properties and stand in certain relations. Assuming that we can see the individuals that way, which is like we assume the characterized agent sees them, it makes sense to attribute to these agents certain mental states regarding the individuals *seen that way*. Conceptual windows can be taken to provide for precisely these views on such situations, and our modal language is particularly suited to regiment the ways in which individuals are seen through them.

We can in general regiment —or stipulate if one wishes— what can be seen through a conceptual window using the following, schematic, format.

$$^{i}\exists\vec{x}(\phi(\vec{x}) \wedge \Box_{\vec{x}}\psi(\vec{x}))\ ^{v} \qquad (EW)$$

v. Here, $\vec{x}$ must be read as a non-empty sequence of variables $x_1 \dots x_j$, $^{i}\exists\vec{x}$ must be read as a series of corresponding existential quantifiers $^{i}\exists x_1 \dots {}^{i}\exists x_j$, $\phi(\vec{x})$ as expressing a way in which the $\vec{x}$s are related, $\Box_{\vec{x}}$ as restricting the set of accessible possibilities to those where the $\vec{x}$s exist, and $\psi(\vec{x})$ as expressing the ways in which the $\vec{x}$s are seen, through ^{i}, to be related.

In this format the factual condition $\phi(\vec{x})$ may typically involve the claim that the $\vec{x}$ are all distinct, that no other individuals are seen through ^{i} (an exhaustiveness condition), and further actually identifying properties like, perhaps, equations of the various $\vec{x}$ with actually existing individuals. The condition $\psi(\vec{x})$ may specify how these individuals are objectively seen through ^{i}, with certain properties that perhaps enable one to identify each one of them, as seen through ^{i}, as standing in a row, from left to right, or as listed by a definition or enumeration.

Relative to a situation that may have been regimented in the way indicated, we can characterize modalities *de re*, and the intentional states of our world mates regarding the individuals that we can see there. It is common practice to render this in the format of a *de re* attitudinal or mental state description, one that we can now relativize to the window through which the individuals are seen. Such a characterization consists in a description of the contents of a state of an agent, or other body carrying modal content, in relation to the individuals as they are seen there, in the following schematic way.

The principles thus express the quite generic modal assumption that Saul Kripke makes, but for the fact that it is relativized, here, to the individuals seen through one particular window, and only in so far as they are seen that way. Kripke's assumption is not in any like way restricted, however, except, arguably, for the existence condition.

$$ {}^{i}\exists\vec{z}(\chi(\vec{z}) \wedge \Box^{a}\, \xi(\vec{z}))\ ^{w} \qquad (DR) $$

w. Here, ${}^{i}\exists\vec{z}$ is a non-empty sequence of existential quantifications binding the variables $z_1 \ldots z_l$, $\chi(\vec{z})$ serves to identify or characterize the actual $\vec{z}$ as being χ, and $\Box^{a}\, \xi(\vec{z})$ serve to render agent a's belief or desire that they are or be ξ.

In the scenario discussed by Quine, Ralph is attributed the belief, of Ortcutt, that he is a spy. This can be specified according to the suggested format.

$$ {}^{j}\exists x(o{=}x \wedge \Box^{r}\, Sx) \qquad (R) $$

As mentioned above, without any knowledge of any context, a belief attribution like this does not tell us anything about how Ralph thinks of Ortcutt. It only says that Ralph believes that someone, who is actually Ortcutt, is a spy. But, like we said, we can characterize, independently, relative to which envisaged real situation it is that we take Ralph to believe this. The situation has been presented to us by Quine himself, and we have rendered what Quine told us in (Q) above. Assuming context ^{j} to be a genuine window satisfying (E), (D) and (EX), the attribution (R) and the characterization of the situation (Q) that Ralph's belief must be understood to be relative to, jointly entail that Ralph believes that the man seen on the beach is a spy, where this man is actually Ortcutt.[5]

It may have to be emphasized that Ralph's beliefs are rendered, here, in, I believe, an intuitive and appealing way. A context is used here not merely as an aid in the contextual interpretation of certain sentences, but, intuitively, it is thought of as a real, objective, component in our description and understanding of other agents' intentional activities. Key to our understanding of, e.g., Ralphs belief about Ortcutt, is that it puts him en rapport with a certain real man, in a way that we can also so to speak conceive of him. We assume, or pretend we assume, that the situation that Quine has presented to us is, or was, an actual one, and that we can all relate to it. It, thus, also helps us explain why and how we can use the intentional state attribution as a contribution to an explanation of what Ralph can be expected to say and do. Perhaps he is inclined to alert a local police officer on the very same beach. The described intentional state is assumed to be intrinsically related to, and tied to, situations that are thought of as real, and which are not '*in the mind*', so to speak.

This way of understanding the belief attribution neatly aligns with actual ordinary practice. In Quine's explanation of the situation there is no mention whatsoever of what exactly is the form and structure of Ralph's cognitive state,

5. The reader may enjoy performing the, easy, but laborious, derivation from the premises ${}^{j}\exists x(o{=}x \wedge \Box_{x}\, {}^{j}\imath z\,(Bz)(x{=}z))$ (Q) and ${}^{j}\exists x(o{=}x \wedge \Box^{r}\, Sx)$ (R) to the conclusion ${}^{j}\exists x(o{=}x \wedge \Box^{r}\, {}^{j}\imath z\,(Bz)(x{=}z \wedge Sz))$, thereby also employing the stated assumptions about $\Box$ and ^{j}. Under these assumptions it can also be shown, more generally, that a regimentation (EW) and attribution (DR) entail that ${}^{i}\exists\vec{x}\ {}^{i}\exists\vec{z}(\phi(\vec{x}) \wedge \chi(\vec{z}) \wedge \Box^{a}(\vec{x}{\supseteq}\vec{z} \wedge \psi(\vec{x}) \wedge \xi(\vec{z})))$, for some condition $\vec{x}{\supseteq}\vec{z}$ in which every one of the $\vec{z}$s is equated with one of the $\vec{x}$s, provided that $\phi(\vec{x})$ includes the above-mentioned exhaustiveness condition.

or which mental sentence it really is that figures in his head, or how it could be possible at all that such a sentence would come to be about Ortcutt. We deem these issues, often raised in the philosophical literature, practically unanswerable, and essentially irrelevant. Notwithstanding the, by itself intriguing discussions in (Kaplan 1968), it seems no solutions to these issues are needed, at all, in our actual understanding of these kind of attribution ascriptions in natural language. It has, for instance, sufficed for Quine to just sketch a supposed real and objective situation, and characterize Ralph's beliefs as, so to speak, his attitude in response to it. After Quine has presented this situation, we can all four of us, Quine, Ralph, you and me, so to speak peek through this window, and understand that, according to Quine, Ralph believes that the man seen through it is a spy.

Methods of individuation have been shown to be crucial, too, for our understanding the *knowing who* and *knowing which* constructions that we find in natural language. These constructions have seemed to be rather perplexing for many formal and philosophical approaches to knowledge and belief, but Maria Aloni has shown that they can be understood best in relation to the various way in which individuals are or can be conceived of. I want to outline, here, how the conceived solution can be formulated in our framework, without the aberrations that Aloni's approach invites. (In the final section we will discuss Aloni's approach more in particular.)

According to a basic, and fairly classical, insight in the meaning of *Wh*-constructions one can be said to know *who danced* only if one knows of everyone who danced that she did.[6] However, as Aloni, among others, has pointed out, to know of someone that she danced can only appropriately be attested if it is relativized to the way in which the dancer is conceived of, it is relative to a conceptualization of the domain. Thus, if one is just looking at the dance floor, one can just see who dances, so know who dances, while one can at the same time be said to not know who dances, if one does not know the identity, from some other relevant perspective, of those who dance.

Phrasing things in our terminology, a person, Rebecca, can have a look at the dance floor through some window ^{i}, in which everyone is just seen to dance or not dance: ${}^i\forall x(\Box_x Dx \vee \Box_x \neg Dx)$. Rebecca may also know that the dancing room is occupied with just the people from her class, everyone of which she knows through some window ^{c}, and so that ${}^i\forall x\, {}^c\exists x$. Even so, while Rebecca can be said to know ^{i}who dances, just by having seen the dancers through window ^{i}, and also know of all these dancers that they are classmates of hers, she may fail to know ^{c}who dances, in so far as she is unable to figure out who, from her classmates, is dancing. The following, thus, is entirely consistent.

$${}^i\forall x\,(Dx \to \Box^r Dx) \qquad {}^i\forall x\,(Dx \to \Box^{r\,c}\exists x) \qquad {}^c\forall x\,(Dx \to \neg\Box^r Dx)$$

6. And also only if one knows of everyone who did not dance, that she didn't. We pass over this part of the meaning, here, though.

In this situation Rebecca can point out the dancers on the dance floor, demonstrating that she knows ^{i}who dances, while she may not be able to name anyone of those pointed out, even if she does know ^{c}everyone of her classmates by name.

This type of analysis is not just an ad hoc fix, but it neatly generalizes. To know *who* dances with *whom* is to know of every pair of dancers that they dance with each other: $^i\forall x^j\forall z(Dxz \rightarrow \Box^k Dxz)$, again, relative to some window through which the x's and z are identified. Note that these windows need not be the same. This is particularly relevant when it comes to *Who_is_who*-constructions. It is already been established above that the identity of individuals within the scope of one window is assumed to be settled, and therefore everybody who peeks through the window knows ^{i}who is ^{i}who. (Because everybody knows $x{=}x$ if x exists.) However, there are always legitimate questions about the identity of individuals seen from one window, with those seen from another. Therefore, even if windows i and j have to supply a view on the same sets of individuals, there still can be complete ignorance of the question ^{i}who is ^{j}who. The following is also perfectly consistent.

$$\Box^0\forall x(^i\mathsf{E}x \leftrightarrow {}^j\mathsf{E}x) \quad {}^i\forall x\, {}^j\forall z(\Diamond x{=}z \wedge \Diamond x{\neq}z)$$

As for a final example, there may be a window ^{i} on, say, the players of the team of FC Barcelona, as we see them on a group photo, a window ^{j} on the same team that consists in the list of names with their squad numbers, and a window ^{k}, when we see the players playing on the field. When we witness, looking through ^{k}, one of the players scoring a goal, we normally know ^{k}who scored. However, perhaps we fail to identify this person with a name from the list, so that we can be said to not know ^{j}who scored, and also not ^{i}who scored. It can also be known, to a real fan, ^{i}who is ^{j}who. He then knows the squad numbers and names, by means of which the players are seen through ^{j}, of the persons pictured on the photo, i.e., as they are presented through ^{i}. Even so the fan may fail to know ^{j}who is ^{k}who, because, for instance, it is foggy on the field. The sketched state of the fan (f) can be characterized as follows.

$$^i\forall x\, {}^j\forall y\, (x{=}y \rightarrow \Box^f x{=}y) \wedge {}^j\forall y\, {}^k\forall z\, (\Diamond^f x{=}y \wedge \Diamond^f x{\neq}y)$$

5 Conceptually Grounded Quantification

The reader may perhaps feel able to appreciate our proof theory better if it is accompanied by a suitable interpretation, so that's why we talk semantics in this section. To this purpose, and as is relatively common, we start with formally postulating a set of possibilities $\mathcal{W}$, aka '*possible worlds*', and a set of possible instantiations $\mathcal{U}$, aka '*possible objects*'. (The familiar denominators need not be taken literally.) It is also common practice to adopt a family of accessibility relations $\mathcal{R}^k$ over $\mathcal{W}$, rendering *what* is possible in *which* possibility according to modality k. The default, assumed most general, relation $\mathcal{R}^0$ can be adequately thought of as the universal relation, even if we only need to require it to be an equivalence relation defining equivalence classes of possibilities, classes that one, so to speak, cannot escape from. For this reason it is also stipulated that $\mathcal{R}^k \subseteq \mathcal{R}^0$, for any k.

What are suitable contexts in the intensional setting? In the extensional setting contexts are conceived of as sets of individuals so as a first shot one might now think of contexts as functions from possibilities to sets of individuals. It is, however, more appropriate to think of contexts instead as sets of functions from possibilities to individuals, sets of '*individual concepts*', so called. (Again, the familiar denominator should not be interpreted literally.) Such sets convey more information, as they have *individuating functions* as their elements, and these sets of functions can be conceived of as *methods of individuation*, in the sense of Hintikka.[7] Note one major, and obvious, difference, between our contexts and Hintikka's methods of individuation. Our contexts are typically partial, they present possibly only a part of the assumed domain of individuals, and they involve concepts of individuals not necessarily existing. So the concepts themselves may be partial functions.

Formally a context $\mathcal{C}^i$ is just any set of, possibly partial, functions from $\mathcal{W}$ to $\mathcal{U}$. In any world v its projection gives the individuals '*seen*' in it in v.

$$D_v^i := \{c_v \mid c \in \mathcal{C}^i\} \qquad (P)$$

(I systematically follow the practice of writing f_v for the realization, or value, $f(v)$ of any function f in a possibility v.) Since window ^{0} must be understood to provide the default, unrestricted, window on the world, it is taken to define the real domain of the world, and all windows are assumed to provide a view on individuals existing there only.

$$D_v^i \subseteq D_v^0 = D_v \qquad (O)$$

In the interpretation of our language, everything is made intensional, i.e., everything extensional is made functionally dependent on possibilities in $\mathcal{W}$. Thus, in a model of our language, we employ an interpretation function $\mathcal{I}$ that assigns an extension to the non-logical constants of our language in each possibility, and so that $\mathcal{I}_v$ constitutes an extensional model for any $v \in \mathcal{W}$. So $\mathcal{I}_v(R) \subseteq D_v^j$ is a set of j-tuples of individuals in D_v, for any relational constant R of arity j, and any possibility v. For any individual constant a, $\mathcal{I}_v(a) \in D_v$, if it has any value at all. We define a model for our language as follows.

7. "Since variables bound to quantifiers range over individuals, a method of individuation is an indispensable prerequisite of all quantification into modal contexts. A quantifier that binds (from the outside) a variable occurring in a modal context does not make any sense without such a method of individuation, and its meaning is relative to this method." (Hintikka 1969a, p. 168–9) "As I have put it elsewhere, members of **F** [a method of individuation, a set of individuating functions, PD] do not only involve a 'way of being given' as Frege's senses do, but also a *way of being individuated*." (Hintikka 1969b, p. 40).

Definition 2 (Models of $\mathcal{MPL}^{\mathcal{CGQ}}$) *A model $\mathcal{M}$ of $\mathcal{MPL}^{\mathcal{CGQ}}$ is a quintuple*

$$\langle \mathcal{W}, \{\mathcal{R}^k\}, \mathcal{U}, \{\mathcal{C}^i\}, \mathcal{I} \rangle$$

the five components of which are as detailed above.

The formulas of our language are evaluated in such models and relative to a possibility and relative to a variable assignment function. We write $\mathcal{M}, v, g \models \phi$ to indicate that ϕ is satisfied in model $\mathcal{M}$, relative to possibility v and variable assignment g. An assignment function g interprets any variable x as an individual conception $g(x)$, the value $g(x)_v$ of which is an object in a point of evaluation v, if it has any value at all. The interpretation of our terms, relative to these three parameters, is defined as follows.

$$\begin{aligned} [t]_{M,v,g} &= g(t)_v \text{ if } t \in V \\ &= \mathcal{I}_v(t) \text{ if } t \in N \end{aligned}$$

With the parameters set, we say that an atomic formula is satisfied relative to any possibility v if its constituent terms denote objects that have the properties ascribed, like being one and the same object, or like standing in a certain relation. If any one of the terms does not have a value in v, these formulas are assumed to be not satisfied there. (As said, I adopt a "negative free logic", so-called. Nolt 2020)

Definition 3 (Interpretation of $\mathcal{MPL}^{\mathcal{CGQ}}$)

$$\begin{array}{rll} \mathcal{M}, v, g \models t_1 = t_2 & \textit{iff} & [t_1]_{M,v,g} = [t_2]_{M,v,g} \\ \mathcal{M}, v, g \models Rt_1 \ldots t_j & \textit{iff} & \langle [t_1]_{M,v,g}, \ldots, [t_j]_{M,v,g} \rangle \in \mathcal{I}_v(R) \\ & \vdots & \\ \mathcal{M}, v, g \models {}^i\exists x\phi & \textit{iff} & \textit{there is } c \in \mathcal{C}^i\textit{: } c_v \in D_v \textit{ and } \mathcal{M}, v, g[x/c] \models \phi \\ \mathcal{M}, v, g \models {}^i\forall x\phi & \textit{iff} & \textit{for all } c \in \mathcal{C}^i\textit{: if } c_v \in D_v \textit{ then } \mathcal{M}, v, g[x/c] \models \phi \\ \mathcal{M}, v, g \models \Diamond^k\phi & \textit{iff} & \textit{there is } w\textit{: } \mathcal{R}^k vw \textit{ and } \mathcal{M}, w, g \models \phi \\ \mathcal{M}, v, g \models \Box^k\phi & \textit{iff} & \textit{for all } w\textit{: if } \mathcal{R}^k vw \textit{ then } \mathcal{M}, w, g \models \phi \end{array}$$

Quantifiers are interpreted in a classical way, but relative to the context they are indexed for. Modal operators are also interpreted in the usual fashion, in the modal domain that they are indexed for. I have left out the clauses for the propositional connectives because they are entirely classical.

Let us briefly inspect how and to what extent, the semantics from section 5 reflects the proof theoretic principles from the earlier sections. First, it must be obvious from their semantic definition that the modal operators $\Diamond^k$ and $\Box^k$, are each others duals, as are the existential and universal quantifiers ${}^i\exists x$ and ${}^i\forall x$.

Second, as is well-known in the modal field, the principles (T), (4) and (5) require the accessibility relation $\mathcal{R}^0$ to be an equivalence relation, so that it is a universal relation over the possibilities in the equivalence class of possibilities that the actual world resides in. Since the other modalities are restricted to this default modality, the language does not afford one to escape from the class of possibilities that actuality is supposed to be in. That the various accessibility relations $\mathcal{R}^k$ are included in the default one $\mathcal{R}^0$ has been recorded already by the earlier principle (U) $(\Diamond^k\phi \rightarrow \Diamond^0\phi)$.

Third, the transparency principle (E) is guaranteed to be valid by the model theoretic assumptions (O). If an individual conception in any window has a value in a possibility, so if it exists, this implies that it exists in the domain of that world, i.e., it is the value of a concept in the window $\mathcal{C}^0$. In short, ${}^i\forall x\, {}^0\exists z\, t{=}z$ (E) is valid.

Finally, observe that the principles of distinctness (D) and extensionality (EX) have not been built as such into the semantics, because we have chosen to employ them as additional constraints on conceptual windows. It can be observed, however, that the two principles jointly require a context $\mathcal{C}^i$ to consist of concepts that are all defined, i.e., exist, in exactly the same set of possibilities, and in each of these possibilities, distinct concepts always have distinct values. Typically, there is a bijection between $\mathcal{C}^i$, D^i_u and D^i_v, for any u and v for which $\mathcal{C}^i$ is defined.

I generally assume that the windows that we actually employ satisfy distinctness and extensionality, because this seems to be the way we think we see things: *clearly* and *distinctly*. The default window $\mathcal{C}_0$, however, provides a view on the things that there are, and only the things that there are, because what one sees through it constitutes the definition of what exists. This, so to speak *ontological*, window on things is therefore, by definition, clear, or transparent. Should we assume it to satisfy distinctness, and extensionality too?

It is not obvious, at least not to me, that such an ontological view should satisfy these two principles, too, i.e., that they should be taken to be a property of what things essentially are, or, in somewhat more charged words, of the way God sees things. I gladly leave it to qualified philosophers to try and make sense of, and take a stance on, such questions. For our mainly logical purposes, however, I believe one should prefer to not let our logic arbitrate these questions and not exclude any logical possibilities. This reserved attitude is also inspired by a somewhat Kantian warning by Jaakko Hintikka, which I cannot but agree with in spirit, even if perhaps not in tone:

> *The idea that the identification of individuals between different worlds is somehow given to us by the grace of logic or God or some other authority is an example of the worst kind of arbitrary metaphysics.*
>
> (Hintikka 1996, p. 126)

6 Covers, Generators and Counterparts

In the preceding sections I have sketched how we can reason about modal identities, and intentional relations with objects in reality, through the use of our contexts, or conceptual windows, and also how these contexts can be moderated so that the right contextual conclusions can be drawn, proof- but also model-theoretically. Three types of proposals have relatively recently become fashion-able in the philosophical and linguistic literature and that can be considered rival methods of cross-modality quantification. There are proposals to conduct modal quantification through *conceptual covers*, to employ *generated concepts*, and to think of all such quantification as being mediated by Lewisian *counterpart relations*. In the next three subsections I compare our conceptual windows approach with the three mentioned ones, in rather general terms, arguing that ours is empirically at least as good as the others, and generally preferable on foundational grounds. The considered rival alternatives charge us with, unwarranted, metaphysical and/or cognitive representational commitments, while the approach presented here is metaphysically uncharged, and comes without any cognitive-psychological commitments.

6.1 Conceptual Covers

In a series of papers Maria Aloni and others have promoted a method of cross-modal quantification mediated by conceptual covers.[8] This approach, like ours, employs individual concepts, functions from worlds to individuals, to identify individuals in possibilities. Quantifiers are analyzed using *conceptual covers*, sets of such concepts, that are said to provide a perspective on the domain of individuals. Two features of the conceptual covers approach are crucial. First it is assumed that there is a fixed domain of individuals, that is the same across all possibilities, and second, for any individual in any possibility, a cover has to host a unique concept that identifies the individual in there. It is fairly easily established, formally, that all the results of a conceptual cover approach can be accomplished with ours and it can furthermore be argued that it better not be done using conceptual covers, but, rather, with conceptual windows.

My first point is that Aloni's conceptual covers are, effectively, a very special type of conceptual windows. Covers can be conceived of as conceptual windows satisfying certain special constraints. First, distinctness (D) and extensionality EX) must be required to hold for the ontological window ^{0} too, and it is also assumed that necessarily something exists. By these assumptions the domains of all possibilities are thereby rendered substitution variants of one another, i.e., they are virtually (logically) the same. Second, for a window ^{c} to figure as a conceptual cover it has to host a concept of every individual in every possibility, so that $^0\forall x\, ^c\exists x$, the converse of our transparency principle E, is universally valid. If we impose these constraints on our conceptual windows, they effectively figure as conceptual covers.[9] All this serves to say, in a nutshell, that conceptual

8. Aloni 1997; Aloni 2001; Aloni 2005; Schwager 2007; Aloni & Roelofsen 2011; Aloni & Port 2015; Aloni 2018; Kalpak 2020. Linguistic applications other than those discussed here include a treatment of functional nouns (Schwager), epistemic indefinites (Port), and concealed questions (Kalpak, Roelofsen).

9. I have said 'effectively', twice, because the proof-theory cannot guarantee satisfaction of the model-theoretic constraint that the accessibility relation is a total one, in stead of just an equivalence relation, and that the various possibilities have one and the same domain of individuals in common, in stead of just bi-similar domains. Established modal logical wisdom

windows enable one to do whatever conceptual covers enable one to do, and that we can prove this.

The second point I wanted to make directly connects up with the first. In order to make conceptual windows behave like conceptual covers we have to make the mentioned assumptions. But the assumptions are, by themselves, questionable and actually quite dubious. In Aloni's models it is for instance required that literally everything exists in literally every possibility. Not only does this make every existence a necessary existence, but because this is turned into a logical 'fact' it is also something that cannot be doubted by any agent in the model. This, actually surprising, consequence is not in any way given any independent motivation for.[10] Actually none of the above mentioned assumptions are required for any of the mentioned applications to work. We can actually employ conceptual windows to do the very same things that conceptual covers have been used for, without turning into them into conceptual covers, so without making the questionable assumptions. (See (Dekker 2024a) for further details and elaborate illustrations.) The *only* reason for making these assumptions, then, is that they constitute a sine qua non for the very definition of a conceptual cover. Since, as said, we actually do not need any such conceptual covers, there is therefore no need to make these assumptions.

There are also more tangible reasons to avoid the use of conceptual covers. In all practical applications of the conceptual covers framework the intended models are always very strictly constrained and this has certain quite unwelcome consequences. As said, by definition, every single concept in a conceptual cover must identify an object in all possibilities in a model. Not only does this make every so determined individual a necessary existent individual, as we have already observed, but its identifying properties must also be assumed to be instantiated in every possibility. Thus, when Aloni for instance calls on concepts like, quite typically, *the ace of spades*, *the winning card*, *the card on the left*, then throughout the model, in every possibility, there must be some such unique ace of spades, a unique winning card, a unique card on the left, etc. As a consequence, the models cannot allow for there being any agents who do not know that there is this ace of spades, winning card, etc. Thus if some agent is said to know, or even to not know, which card is the winning card, then it follows that everybody knows there is a winning card.

In order to prevent these disturbing consequences it has been suggested, e.g., by Aloni (p.c.), that the method of quantification through conceptual covers should be dressed up with a method of quantifier domain restriction. However, some serious reflection on how such domain restriction should be get to work,

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has it that these 'deviations' are spurious, logically speaking, and also linguistically.

10. Timothy Williamson has provided an argument yielding the conclusion that existence is necessary. (Williamson 2002) His reasoning is sound, of course, but it relies on certain assumptions about propositions, truth, and existence that are surely not uncontestable. I therefore see no reason to render these, arguably metaphysical, assumptions into principles of logic, that by definition no one can disagree with.

formally, in Aloni's system, reveals that this is not without serious pitfalls.[11] Moreover, and actually quite ironically, the previous parts of this note may have served to show that a suitable intensional logic of domain restriction, independently motivated itself, already gives us the conceptual windows that render Aloni's conceptual covers superfluous. Concisely put, conceptual covers do not give us just what we need, they force assumptions on us that we do not want, while domain restriction, i.e., quantification though conceptual windows, gives us everything we need.

Like I said, more on the positive side, Aloni has been the first to provide for a promising take on *knowing who*-constructions. If practiced, however, her use of conceptual covers must suffer from the above-mentioned problems also in this domain. We have seen above that conceptual windows allows us to suitably model distinct views one may have on, say, the team of FC Barcelona. It is, however, hard to see how this should be done if we employ conceptual covers. If we were to restrict the domain of our intended models to just that team this would imply that the team necessarily exists, that there is nothing else, and that everybody knows that. These are of course no tolerable consequences of just the analysis of a certain linguistic construction.

Instead, one might think of somehow focussing on the concepts in a conceptual cover that have the players of FC Barcelona as their actual values. If we would do so, however, there is no guarantee that these concepts denote players of FC Barcelona in all possibilities. This appears to be wrong, too. One knows who of the players of Barcelona, seen one way, is who of the players seen another way, only if one knows that they are the players of Barcelona, seen one way or the other. To remedy this, one may, instead, require that the relevant conceptual covers have concepts necessarily depicting the players of FC Barcelona, but this implies, again, within the conceptual covers framework, that the whole team necessarily exists, and that this is known to everybody. This is actually the same problem that we wanted to solve to begin with.

All this is not to say, of course, that no cure of these problems is possible in a conceptual cover framework, but it is quite likely that it is going to require additional assumptions and stipulations which are ad hoc, if not entirely dubious. Moreover, and again, there is also no need to try and do so. Instead of adding constraints to the conceptual covers framework, and simply by withdrawing its foundational assumptions, we can present appropriate analyses of all the cases that the framework has been used for.

11. I don't have the space here to elaborate on the details, but such an exercise will require us to rethink the notion of a conceptual cover itself, and actually come up, instead, with something much more like the kind of conceptual windows argued for in this paper. See (Dekker 2012, p. 63*ff*) for a first attempt.

6.2 Concept Generators

Since early this century modal identity issues have also been approached with a method using *concept generators*, often so called.[12] [13] In this type of approach proper names, and other terms, are assumed to have an object as their semantic value, but in modal contexts the syntax may trigger a *concept generator* CG that turns the denoted object into a concept of that object. Such a concept, in stead of the object, then plays a further semantic role in the evaluation of the clause in the scope of the modal. Since different concept generators can be assumed to be at stake, one and the same referent may be turned into different conceptions of it, and a seemingly necessary statement of the identity of an object (o) with itself (o) can be turned into a contingent statement of the identity of the value of one conceptions $CG_i(o)$ with that of another $CG_j(o)$. It is often claimed an advantage that this approach allows for a, said, *in-situ* analysis of names or other terms in the modal constructions in which they occur: they deliver their value within the scope of the modal operators under which they occur, but for the fact that it is not their usual value, an object, but the concept that it helps to generate.

The use of individual concepts is an unmistakable feature that the concept generator approach shares with our method of quantification through conceptual windows. It seems to be also obvious that whatever one is able to get by a semantics with concept generators, can be gotten by employing quantification through conceptual windows. One, basically, only needs to assume that any generated concept is available in the window that the associated existential quantifier quantifies through. In terms of expressiveness and empirical coverage a conceptual windows approach can therefore achieve whatever has been accomplished in a concept generator approach.[14] The main difference consists in some metaphysical assumptions that the generator approach is built upon, and in the architecture through which the required readings of natural language expressions are derived

First, the concept generator approach has been built on the assumption that names (and possibly other terms) are rigidly denoting.[15] Only because these

12. Percus & Sauerland 2003; Anand 2006; Charlow & Sharvit 2014; Pearson 2015; Deal 2018, among various others; c.f., also, Lederman 2014; Holliday & Perry 2014.

13. The authors of the latter paper are apparently unaware of the concept generator approach, but the philosophical and formal intentions of both approaches are surprisingly similar. First, both essentially build on Kripke's, model-theoretic, stipulation that names are rigid designators. Second, propositional attitude ascriptions are supposed to come with unarticulated constituents, those denoting "generated concepts" in the one type of approach, and "cognitive fixes", "roles" in the other. Third, in both approaches, these constituents are existentially quantified, so unavailable for further specification.

14. There are some subtle extensions and modifications of the concept generator approach, e.g., the use of properly indexical concepts, and the treatment of *double bind readings*, so-called. See (Dekker 2024a) for how to accommodate these modifications in our system.

15. Actually, it is developed with the aim, among others, of undoing the quite unwelcome effects of assuming rigid designation in particular epistemic contexts.

terms are assumed to, rigidly, denote one and the same object in all possibilities, can we speak of something like *the generated concept*. Without this assumption, the generators might apply to different objects in different possibilities, thereby yielding possibly different conceptions of it in different possibilities, and the analysis would face the treat of being entirely opaque. Holliday and Perry are apparently aware of this. While the last mentioned authors "do not mean to suggest that there is a consensus on the proper semantics for alethic modal predicate logic." (Holliday & Perry 2014, p. 592), they do motivate their approach claiming that to "violate the [alethic] rigidity of names" is something "no one wants." (Holliday & Perry 2014, p. 613). Since the latter claim is quite obviously false, see footnote 18, this mere fact actually challenges the ground on which their paper is built, and the same goes for the concept generator approach.

Like I said, a claimed benefit of the concept generator approach is that it allows for this *in situ* analysis of the relevant terms, whereas our's, in the current set up, requires the use of existential quantifiers whose scope needs to be established, something which one may deem costly and unnecessary. Here I am tempted to say, in reply, first, that the said *in situ* interpretation of terms does not entirely come for free in the concept generator approach, and that it is somewhat euphemistic to consider it an *in situ* analysis of *de re* attitude ascriptions. The method of concept generation has to allow for the possible introduction of an arbitrary number of concept generator variables in the logical syntax, operators abstracting over them, and existential closure over the abstractions, at the main sentence level, and a multiply ambiguous entry for all relevant modal and propositional attitude verbs.[16] Moreover, the *only* way that the generated concepts can be said to relate to the *res* that they are 'generated from', consists in the fact that they are existentially quantified at the main sentence level—so not in situ—, and that there, relative to the 'actual' world of evaluation—so not in situ— the subject of the ascribed attitude is supposed to stand in some relation of acquaintance with the *res* in question.[17]

I would like to add that the formulation of a compositional, *in situ*, semantics of natural language is premature as long as we are still in the process of deciding on the logical architecture of the system in which the semantics should be formulated. After all, compositionality is most often considered a methodological constraint. Probably any preferred assignment of semantic values can be given a compositional formulation, and the only real question is how much the compositional reformulation of such an assignment costs, in the sense of, e.g., how (in-)appropriate the assumptions are that it requires one to make on, say,

16. The latter feature is one that the concept generator approach shares with the approach advocated in (Quine 1956). For Quine, an ascribed attitudinal relation can be any n+1-ary relation between individuals, where n is the number of terms that are read *de re* in the attribution. In the concept generator approach it is, essentially, thought of as a relation between an object and n concepts.

17. This is the basic idea mostly employed. (Percus & Sauerland 2003, p. 236–8), see also (Charlow & Sharvit 2014, pp. 322–4), and (Pearson 2015; Deal 2018).

the syntactic level. (Janssen 1997, p. 419, 457, 461) It may furthermore stem hopeful that our, non-local, quantificational interpretation of terms can also be obtained, in situ, in a multi-dimensional interpretation architecture, as I have demonstrated in (Dekker 2008).

There is, finally, a substantial difference between the approach that we advocate and the concept generator approach. The latter advocates the ad hoc generation of concepts, but eventually the generated concepts are existentially quantified. (Percus & Sauerland 2003, p. 230, 36, 37) This implies that one actually cannot really speak of *the concept* invoked in a specific attitude description, because there can be various of them, and it is therefore also hard to see how a multitude of generated concepts could be specified further, and systematically related to each other, in any non-ad hoc manner. As we have seen, our windows, actually allow for precisely this. Windows moderate the presentation the individuals seen through them in tandem, and, as we have seen, they allow us to give further, independent, specifications of the ways in which the individuals are seen there.

6.3 Counterpart Theory

Various authors have proposed, in contradiction with Kripke's proposals, that objects in one possibility, say actual reality, cannot exist in another possibility, e.g., any counterfactual possibility, and that cross-world reference and quantification should be seen to be mediated by (sets of) *counterpart relations*.[18] Counterparts play a role similar to the one concepts from our windows play: both regiment the possible values of variables in possibilities other than those in which the variables are declared, or quantified. The formal methods employed to achieve this are structurally very closely related, even if not so closely related conceptually. Let me expand on the first point first.

We can correlate counterpart relations and conceptual windows, roughly, as follows. We can say that an individual has a certain counterpart in another possibility (under a certain counterpart relation) if, and only if, they are both the value of one and the same conception (seen through a certain conceptual window). This equation may hold modulo two reservations that we have to make. The reservations relate to two subtle differences between the two sorts of constructions. The first is that a conceptual window may afford a view on only a part of the domain of a possibility, while a counterpart relation always connects all individual objects, at least with themselves. This difference does not, however, affect any cross-possibility identities. The second is that a counterpart relation is possibly asymmetric, while the relation of being two possible values of one and the same conception is, of course, symmetric. While our conceptual windows are, for this reason, more restricted, model-theoretically speaking,

18. Lewis 1968, in the first place, of course, and, e.g., Kocurek 2018; Ninan 2018, fairly recently. Significantly, Lewis' *second postulate* has it that "Nothing is in two worlds," p. 114, thereby actually refuting the generalization from (Holliday & Perry 2014), mentioned above in footnote 6.2, motivating crucial ideas from the concept generator approach.

The difference is subtle, and arguably irrelevant for us. Lewis' reason for allowing asymmetric counterpart relations is that he thinks of a counterpart as the object in a possibility most resembling the object it is a counterpart of, and this condition is intuitively not necessarily symmetric. (Lewis 1968, p. 114, 116) However, this way of thinking of counterparts builds on the, I believe rather charged, metaphysical, assumption that it makes any sense to speak of the *factual* resemblance of two entirely possible, *non-actual objects*. If we choose to not make such, metaphysical, claims, there seems to be no ground for allowing counterparthood to be possibly not symmetric.

Besides the mentioned, minor, differences, counterpart relations and conceptual windows can be suitably translated one into the other. Since we have not assumed that the domains of our possibilities are actually disjoint, while Lewis does, we first have to make sure that our domains are. We can do so modeling our domain of 'possible' objects as pairs consisting of a possibility and an object that is considered an object in that possibility, as in (Ninan 2018). The space of possible objects that are taken to stand in the counterpart relation thus is equated with the set $\mathcal{V} := \{\langle v, d\rangle \mid d \in D_v\}$. Given a counterpart relation L over $\mathcal{V}$ we can then define a corresponding conceptual window ${}^{L}C$, and given a window $C \subseteq (\mathcal{W} \hookrightarrow \mathcal{U})$ we can define a corresponding counterpart relation ${}^{C}L$, as follows.

$$
{}^{L}C = \{c \mid \forall v, w \textit{ if } c_v, c_w \textit{ are both defined then } \langle\langle v, c_v\rangle, \langle w, c_w\rangle\rangle \in L\}
$$
$$
{}^{C}L = \{\langle\langle v, c_v\rangle, \langle w, c_w\rangle\rangle \mid c \in C \textit{ and } c_v, c_w \textit{ are both defined }\}
$$

It can be shown that, if we ignore the intra-world counterpart relation, ${}^{({}^{L}C)}L$ equals L, provided that L is symmetric.[19] This may serve to show that, modulo the mentioned reservations, a rendering of counterparts through windows preserves all the information relevant in the counterpart framework.

It may be observed that ${}^{L}C$, as defined, does not generally constitute the proper type of conceptual window that we normally use, and that satisfy our constraints on windows. There are entirely spurious redundancies which we can remove by selecting the *largest concepts* in that set by an operator ‡, so that $\ddagger C = \{c \in C \mid \textit{ there is no } c' \in C : c \subset c'\}$. It is easily seen that also then, and under the mentioned conditions, ${}^{(\ddagger{}^{L}C)}L$ equals L. But we can now also establish that, if a window C satisfies distinctness (D) and extensionality (EX), then also $\ddagger^{({}^{C}L)}C$ equals C.[20] All this may serve to show that a choice between counterpart relations and conceptual windows is largely immaterial, and may have to be arbitrated on other, methodological, grounds.

19. If $\langle v, d\rangle$ and $\langle w, e\rangle$ stand in a counterpart relation L, and vice versa, then there is a concept c in ${}^{L}C$ that is fully defined by $c_v{=}d$ and $c_w{=}e$. By definition, then, $\langle\langle v, d\rangle, \langle w, e\rangle\rangle \in {}^{({}^{L}C)}L$. In converse, if $\langle v, d\rangle$ and $\langle w, e\rangle$ stand in the counterpart relation ${}^{({}^{L}C)}L$, then there must be $c \in {}^{L}C$ such that $c_v{=}d$ and $c_w{=}e$. By definition, then, $\langle\langle v, d\rangle, \langle w, e\rangle\rangle \in L$.

20. If C satisfies (D) and (EX), then ${}^{C}L$ in an equivalence relation on $\{\langle v, c_v\rangle \mid c \in C\}$. The equivalence classes induced by that equivalence relation are the (set-theoretic renderings of) the concepts in both $\ddagger^{({}^{C}L)}C$ and C.

For the above, formal, considerations we can also counter a recent argument from Dilip Ninan, against the conceptual covers framework, that it does not allow one to successfully handle a specific case that he claims a framework with counterpart relations can. (See Ninan 2018, §4.2.) I have already shown elsewhere, elaborately, that the specific case that Ninan discusses can be handled, entirely satisfactorily, in our framework using conceptual windows. (Dekker 2024a, §4.6) It appears, however, that Ninan is actually and more particularly worried that, on a conceptual covers approach like that of Aloni, we must, in actual practice, assume there to be a contextually given set of concepts, individuating all relevant entities in a given situation, each one "mentally singling out" a given object. Ninan argues that in the cases he has sketched some such assumption seems to be totally out of place. I agree that this may indeed be an unwelcome consequence of how the system of conceptual covers is often conceived of and presented, but Ninan's argument is entirely mis-directed if one discards with Aloni's metaphysical and representational commitments, which we wanted to dispense with anyway. Like I said, the cases made up by Ninan can be handled totally satisfactorily in a conceptual windows framework, and it can be done, and actually is done, entirely proof-theoretically. This means that, for the purpose of establishing the right inferences, no conception of any contextually given object is needed. There are no such 'objects' that the proof-theory is concerned with. Surely, then, Ninan has established no point whatsoever against a framework that employs conceptual windows.

Let me now turn to my second point, that the methods of using of counterpart relations and that of using conceptual windows are conceptually distinct. There surely is an issue with how Lewis' counterpart relations are conceived. Lewis, and his disciples, endorse the idea that there are possible worlds, thought of as complete and total universes. They properly include possible individuals, which are considered to exist, but not actually, and so that they also allow one to speak of objects in one world "closely resembling" objects in other worlds "in important respects", and also to say things like "the color of o in v is identical to the color of o' in v'." (Ninan 2018, p. 454) This kind of modal realism conflicts with Kripke's insight that possible worlds are not like distant countries, that we can investigate using, say, a philosopher's telescope. Kripke warns against the idea that we can judge that Nixon might have won the 1964 elections only after observing someone, in some possible world, to be most closely resembling Nixon and winning those elections there. (Kripke 1981, p. 44/49)

While Lewis' views are, for these and similar reasons, often and systematically met by "incredulous stares", as Lewis says himself, he has extensively argued that he has not encountered conclusive arguments against it. Regarding his type of modal realism, he also judges that "the price is right, high as it is", but "[m]odal realism ought to be accepted as true. The theoretical benefits are worth it." Cautiously he adds "Provided, of course, that they cannot be had for less." (Lewis 1986, §2.8) Since modal realism is clearly a matter of philosophical dispute, and since we can, arguably, do whatever Lewis does, without committing to any such form of modal realism, we can have everything for less, so that any such price is too high.

7 Conclusion

Let me summarize what has been established in this paper. I have turned a quite simple logical method of *contextually relativized quantification* into a method of *conceptually grounded quantification* by framing the first within an intensional framework and making only some most minimally required logical assumptions. It has been shown that this method of contextually grounded quantification allows us to do whatever rival methods enable one to do by means of *conceptual covers*, *concept generators*, or *counterpart relations.*

These rival frameworks are all tied to their own specific presuppositions or dogmata. Conceptual covers are of no use without the assumption of a fixed *universal domain* of necessary existents, because without that assumption a conceptual cover is impossible by definition. The use of concept generators only make sense if one commits oneself to the use of *rigid designators*, a model-theoretic property of terms that cannot be secured by any proof-theoretic means. The use of counterparts finally commits one to the there being *objective modal facts* that relate spatially, temporally, and causally unconnected entities. Such 'facts' can not be empirically grounded, and they are no logical truths.

It is surely true that to get our own logical machinery to work in actual practice, we have to make, every now and then, various assumptions. However, the required assumptions can all be stated, explicitly, in our framework, and arguably, these are the kinds of assumptions we all make when we inquire about the state of the world, what we know about it, and how deal with it.

When we stipulate that London is experienced a certain way by some agent Pierre, we give an objective rendering of how London then is conceived of, by Pierre, according to Kripke. The various situations relating Pierre to London in order to understand his, we judge conflicting, beliefs about the city, are presented to us by Kripke himself in an objective manner. We use our windows only to present what Kripke has presented to us, the charitable readers of his paper. And due the distinct windows through which London is seen, we can all understand what famous Pierre's beliefs are about it. To this end we do not have to inspect his mind, which is difficult with a made up person, nor do we have to investigate any non-actual possibilities, and see what is going on there.

Summing up, all we need is a decent proof-theory, and a formally proper *re*-presentation of what people think and say. The framework presented in this paper, and not one of the alternatives, gives us that, and nothing more.

References

Aloni, Maria 1997. Quantification in Dynamic Semantics. In: Paul Dekker, Martin Stokhof & Yde Venema (eds.) *Proceedings of the Eleventh Amsterdam Colloquium*. Amsterdam: ILLC, University of Amsterdam, pp. 73–78.

Aloni, Maria 2001. *Quantification under Conceptual Covers*. Ph.D. thesis, ILLC/Department of Philosophy, University of Amsterdam, Amsterdam.

Aloni, Maria 2005. Individual Concepts in Modal Predicate Logic. *Journal of Philosophical Logic* 34, pp. 1–64.

Aloni, Maria 2018. Knowing-who in quantified epistemic logic. In: Hans van Ditmarsch & Gabriel Sandu (eds.) *Jaakko Hintikka on knowledge and game theoretical semantics*, Dordrecht: Springer. 109–129.

Aloni, Maria & Angelika Port 2015. Epistemic Indefinites and Methods of Identification. In: Luis Alonso-Ovalle & Paula Menendez Benito (eds.) *Epistemic Indefinites: Exploring Modality Beyond the Verbal Domain*, Oxford: Oxford University Press. 117–140.

Aloni, Maria & Floris Roelofsen 2011. Interpreting Concealed Questions. *Linguistics and Philosophy* 34(5), 443–478.

Anand, Pranav 2006. *De De Se*. Ph.D. thesis, MIT, Massachusetts.

Bonomi, Andrea 1998. Indices and Contexts of Discourse. *Lingua e Stile* XXXIII, pp. 471–485.

Charlow, Simon & Yael Sharvit 2014. Bound 'de re' pronouns and the LFs of attitude reports. *Semantics and Pragmatics* 7(3), pp. 1–43.

Deal, Amy Rose 2018. Compositional paths to *de re*. In: Katherine Blake Sireemas Maspong, Brynhildur Stefánsdóttir & Forrest Davis (eds.) *Proceedings of SALT 28*. DMLL Publications, Cornell University, pp. 622–48.

Dekker, Paul 2008. A Multi-dimensional Treatment of Quantification in Extraordinary English. *Linguistics and Philosophy* 31(1), pp. 101–127.

Dekker, Paul 2012. *Dynamic Semantics*. Berlin: Springer.

Dekker, Paul 2024a. *Conceptual Windows Afford Views on Satisfying Individuals*. Technical Note Series X-2024-02, ILLC/Department of Philosophy, Amsterdam.

Dekker, Paul 2024b. *A Logic for Contextually Restricted Quantification*. Technical Note Series X-2024-05, ILLC/Department of Philosophy, Amsterdam.

Dekker, Paul 2024c. *On Presenting Objects in Propositions*. Technical Note Series X-2024-04, ILLC/Department of Philosophy, Amsterdam.

Frege, Gottlob 1892. Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik* NF 100, pp. 25–50.

Hintikka, Jaakko 1969a. On the logic of perception. In: Norman S. Care & Robert M. Grimm (eds.) *Perception and Personal Identity*. Cleveland: The Press of Case Western Reserve University, pp. 140–175. Reprinted in: J. Hintikka, 1969, *Models for Modalities*, pp. 151–83, Reidel, Dordrecht.

Hintikka, Jaakko 1969b. Semantics for Propositional Attitudes. In: John Whitney Davis, Donald J. Hockney & W.K. Wilson (eds.) *Philosophical Logic*,

Dordrecht: Reidel. pp. 21–45.
Hintikka, Jaakko 1996. World Lines and their Role in Epistemic Logic. In: Peter I. Bystrow & Vadim N. Sadovsky (eds.) *Philosophical Logic and Logical Philosophy*, Dordrecht: Kluwer AP. pp. 121–137.
Holliday, Wesley & John Perry 2014. Rules, Rigidity, and Quantification in Epistemic Logic. In: Alexandru Baltag & Sonja Smets (eds.) *Johan van Benthem on Logic and Information Dynamics*, Heidelberg: Springer. pp. 591–629.
Janssen, Theo M. V. 1997. Compositionality. In: Johan van Benthem & Alice ter Meulen (eds.) *Handbook of Logic and Language*, Amsterdam: Elsevier. pp. 417–473.
Kalpak, Hana 2020. The semantics and pragmatics of nouns in concealed questions. *Semantics and Pragmatics* 13(7), pp. 1–24.
Kaplan, David 1968. Quantifying In. *Synthese* 19(1/2), pp. 178–214.
Kocurek, Alexander W. 2018. Counteridenticals. *The Philosophical Review* 127(3), 323–369.
Kripke, Saul 1981. *Naming and Necessity*. Oxford: Blackwell. Paperback edition of Kripke, 1980, the revised and enlarged edition of Kripke, 1972, in: Donald Davidson and Gilbert Harman (eds.), *Semantics of Natural Languages*, Reidel, Dordrecht.
Kuroda, Sige-Yuki 1982. Indexed Predicate Calculus. *Journal of Semantics* 1, pp. 43–59.
Lederman, Harvey 2014. Fine-grained semantics for attitude reports. *Semantics and Pragmatics* 14(1), pp. 1–33.
Lewis, David 1968. Counterpart Theory and Quantified Modal Logic. *Journal of Philosophy* 65, pp. 113–126.
Lewis, David 1986. *On the Plurality of Worlds*. New York: Basil Blackwell.
Ninan, Dilip 2018. Quantification and Epistemic Modality. *Philosophical Review* 127(4), pp. 433–85.
Nolt, John 2020. Free Logic. In: Edward N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*, Stanford: Stanford University. FALL 2021 edition.
Pearson, Hazel 2015. The interpretation of the logophoric pronoun in Ewe. *Natural Language Semantics* 23(2), pp. 77–118.
Percus, Orin & Uli Sauerland 2003. On the LFs of Attitude Reports. In: Matthias Weisgerber (ed.) *Proceedings of Sinn und Bedeutung VII*. Stuttgart: FB Sprachwissenschaft, pp. 228–42.
Quine, Willard Van Orman 1956. Quantifiers and Propositional Attitudes. *Journal of Philosophy* 53, pp. 177–187. Reprinted in: W.V. Quine, 1966, *The Ways of Paradox and Other Essays*, Random House, New York.
Quine, Willard Van Orman 1980. Reference and Modality. In: W.V.O. Quine (ed.) *From a Logical Point of View*, Originally from 1953.: Harper and Row, chapter IX. pp. 139–159.
Recanati, François 1987. Contextual Dependence and Definite Descriptions. *Proceedings of the Aristotelian Society* 87, pp. 57–73.

Recanati, François 1996. Domains of Discourse. *Linguistics and Philosophy* 19, pp. 445–475.

Reimer, Marga 1998. Quantification and Context. *Linguistics and Philosophy* 21, pp. 95–115.

Schwager, Magdalena 2007. Bodyguards under Cover: the Status of Individual Concepts. In: Tova Friedman & Masayuki Gibson (eds.) *Proceedings of SALT XVII*. CLC Publications, Cornell, 246–263.

Williamson, Timothy 2002. Necessary Existents. In: Anthony O'Hear (ed.) *Logic, Thought and Language*, Cambridge: Cambridge UP. 233–51.