

The Rules of Victorious Warriors

Logical and Game Theoretical aspects of Strategy Theory in ancient China
draft

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Abstract

Victorious warriors win first and then go to war, while defeated warriors go to war first and then seek to win. This phrase, attributed to Sun Tzu in his book “The Art of War”, is a strong recommendation for generals to analyse a combat scenario before actually engaging in combat. This analysis can be understood to require some form of game analysis.

In this paper we will discuss whether it is appropriate to ascribe a game theoretical perspective to the ancient Chinese strategical thinkers. In addition we search for indications in ancient Chinese history and literature or tradition whether Chinese warriors actually used game analytic principles in their strategic considerations. We also indicate aspects of these stories which illustrate interesting connections to game theoretical concepts.

This paper is a stand alone version of a draft chapter in the forthcoming handbook of logical thought in China¹.

1 Introduction

The purpose of this paper is to look into ancient Chinese works on strategy and warfare, with a focus on Sun Tzu, “The Art of War”, from a rather unorthodox perspective. Authors studying these topics in general are students in humanities (history, military science, philosophy, or linguistics). The

¹<http://www.springer.com/us/book/9783662438220>

background of the authors of this paper is quite different: mathematics, computer science, and logic.

We aim to illustrate to what extent the strategic theoretical works of the ancient Chinese, and the stories from (military) history and/or literature contain elements which are interesting from the perspective of logic and/or game theory.

The authors of this paper are not familiar with the Chinese language, therefore we are forced to use translated volumes, which, even when they agree on what the ancient authors want to convey, sometimes disagree on the phrases used, and occasionally, also on the precise location where some statements are to be found in the text. Help from our Chinese colleague, Yanjing Wang, (see the acknowledgement at the end of this paper) is very much appreciated.

1.1 Structure of this Paper

The first section presents a short inventory of the ancient Chinese strategy studies which we have investigated. Chinese military studies did not end after the third century AD, but since our prime interest is the ancient period preceding 300 AD, we have not included various works dating from the Tang Dynasty (618 - 907 AD) and later periods.

Next we present a concise description of Game Theory and why we consider it relevant for the purpose of this paper.

Section 4 gives an in-depth description of the most relevant of the old texts: Sun Tzu's *The Art of War*. We emphasize the hidden logical structure in this ancient text, using mind maps as an analysis tool.

In section 5 we look at a number of concrete stories from Chinese history and literature, indicating how these stories illustrate the recommendations of Sun Tzu being applied, and how information and other logical or game theoretical features surface in these stories.

Section 6 presents a survey on related work we have found in the literature; this deals with papers by other authors discussing game theoretical aspects of Sun Tzu and other stories from ancient Chinese military history.

Our conclusions are presented in section 7.

In the appendix we provide some general information about Chinese history for non-Chinese readers. Moreover we include a quantitative example of results of our analysis of the Sun Tzu text. Furthermore we discuss the changing nature of War which is relevant for placing this study in the right context.

2 Strategy Theory in Ancient China

During the Sung dynasty Chinese scholars identified a corpus of seven important classic texts on strategy theory, all translated and discussed in [37]. These texts are:

1. T'ai Kung's Six Secret Teachings (11 cent. BC), see also [43]
2. The Methods of the Ssu-ma (1000-700 BC)
3. **Sun Tzu the Art of War (515-512 BC)**
4. Wu-tzu (440-381 BC)
5. Wei Liao-tzu (350-240 BC)
6. Three Strategies of Huang Shih-kung (Han dynasty, around 0 AD)
7. Questions and Replies between T'ang T'ai-tsung and Li Wei Kung (Sung Dynasty, 1023-1063 AD)

Not included in this list of the seven classics are:

- Military sections of the Mozi (around 450 BC) [21]
- Sun Bin, the Art of War (around 360 BC) [34]

The best known of these ancient writings is the Art of War from Sun Tzu [1, 11, 16, 17, 27, 30, 35, 42, 48, 49]. This work has been written during the late Spring and Autumn period in China (770 - 476 BC). He is frequently confused with the later military strategist Sun Bin [34] who lived during the earlier part of the Warring States period (476 - 221 BC); in fact only the excavation in 1972 of a Han dynasty tomb at Yinqueshan provided conclusive evidence that there had been two authors living some 160 years apart and two different texts, both entitled *The Art of War*. The text of Sun Tzu originally was named "The Thirteen Chapters" and later "Sun Tzu's The Art of War".

Notwithstanding the fact that the oldest of these texts is attributed to a legendary author living 3100 years ago, the earliest written versions of the first four books date from the Warring States Period (476 - 221 BC), as is evident from the military equipment mentioned. The dates mentioned are approximate lifetimes of the legendary authors these works are being ascribed to. The chronological order in which the texts were written down consequently may be different from the order given above.

For example for the Ssu-ma text (2) dates in between 356-320 BC are given. For some of the texts it is even questionable whether they have a historical origin at all, or whether they are forgeries dating from later periods. The precise dating and the true identity of the authors of these works are subject of an intense discussion

among contemporary scholars; see the introductory chapter in [37]. We have no interest in contributing to this discussion.

Instead it is important to indicate in which way the text by Sun Tzu is outstanding among these texts.

Sun Tzu and the other Authors The T'ai Kung's Six Secret Teachings text (1) is structured as a dialogue between the king (King Wu) and his military advisor (T'ai Kung): the king poses a question describing some concrete situation and the advisor answers with an exposé how the king should solve this problem. Frequently the section is completed by an indication of approval from the king. The same pattern occurs in Wu-tzu (4) and in the T'ang T'ai-tsung (7), but also in the additional texts ascribed to Sun Tzu outside his famous thirteen chapters. In the other texts we primarily find the words of the advisor.

The emphasis of the topics under discussion is also different between these authors.

T'ai Kung (1) starts out with sections on good governance in the first two sections before discussing more concrete military case studies in the remaining four chapters.

The methods of the Ssu-ma (2) describe war as a gentlemen's game observing the warrior's code rules like not attacking before your opponent has completed his formation.

Wu-tzu (4) focusses on training and taking good care of the soldiers and their equipment.

Noticeable in the Wei Liao-tzu (5) is the focus on punishments in order to discipline the army.

Huang Shih-kung (6) again focusses on good governance leading to the awesomeness required for leading the army to victory.

Finally the T'ang T'ai-tsung text (7) extensively discusses the ideas of the earlier authors and investigates whether famous ancient generals and kings followed these recommendations.

As we will explain in more details below the text of Sun Tzu stands out by being structured in recurring patterns, and by discussing the topics at a far more abstract level, using categorizations and rules. Relevant features include the semi-structured format of the text, and the focus on being prepared before engaging in warfare: the ideal warrior may win a war without serious fighting by being well prepared, and when he knows both his opponent and himself.

Sun Tzu's text, though not involving either logic or game theory in a formal sense, exhibits features of a logical nature, and presents ideas and concepts which are, from contemporary perspective, related to game theory. These aspects are relevant since we want to investigate the logical aspects of the teachings of these ancient strategists.

In section 4 we will use mind maps as a presentation tool in order to visualise these features.

There are authors (in fields quite different from history, linguistics or logic) who characterize Sun Tzu to be an early founder of game theory, as can be observed from the hits produced on Google by a query on *Sun Tzu and Game Theory*. See for example [68, 69].

3 The Relevance of Game Theory

If game theory is defined as the *theory of agents in strategic interaction*, then strategy studies about warfare are game theory studies. In this way, many other studies, including the works of Teucidides [44], Machiavelli [26], and von Clausewitz [10] can be considered to be game theory, including the work of Sun Tzu.

From a scientific perspective, this is a rather inconclusive statement, because game theory is usually understood in a more narrow sense: the Mathematical theory developed by von Neumann & Morgenstern [45], Nash [31], and others, primarily aimed at economical sciences. A standard text which doesn't involve deep Mathematics is the textbook by Binmore [5].

Military applications of game theory are less frequent but seem to gain importance. We will look at the ancient Chinese texts primarily from this perspective. However, as we shall see, standard Game Theory is frequently insufficient for a correct representation of the strategic situations at hand, and modern extensions to game theory should be taken into consideration as well.

3.1 Logic and Game Theory

To modern logicians the disciplines of logic and game theory are very much intertwined. On one hand, logic is used to advance game theories and on the other hand, game theoretic principles are used as a tool-set in logic research. For more information on the long history logic and games have together see for example *Logic and Games* [20]. For a theoretical background of the connection between games and logic see *Logic in Games* [3] ².

3.1.1 A very short Introduction to Game Theory

Game theory can be defined as a mathematical theory aimed at describing the behaviour of agents in strategic interaction. Some basic notions in Contemporary Game Theory are:

- *Agents*: Players
- *Actions*: Moves
- *Strategies*: complete plans on how to move throughout the game
- *Outcomes*: positions where the game ends

²A concise review of this book recently appeared in the Bulletin of the ASL [39].

- *Preferences and Utilities*: express what players desire

The key idea is that players select actions according to some strategy which tells them which action they should perform given the state of the game. Starting from an initial situation, controlled by these actions, the game traverses a sequence of intermediate states, terminating in a final state called the outcome of the game. The preferences of the players indicate which outcomes are desired by the players. A standard way of expressing preferences is by associating with each outcome a numerical value called the *utility*; a higher utility value indicates that the outcome is preferred. It is generally assumed that players will aim for a more preferred outcome as far as it is in their power to do so: the players are *rational*.

More advanced notions in Game Theory include:

- *Extensive games versus Strategic games*: Extensive games traverse during play a sequence of states with in general only a single player having to move at this state. In a strategic game all players select a strategy which represents their action; the players all move simultaneously and the outcome state is reached in a single step.
- *Probabilistic aspects*: The result from an action (or a simultaneous set of actions) may be a stochastic event (E.G. when the action is throwing a dice or a coin flip). Alternatively the players may select their actions probabilistically from a given set of possible actions; this leads to the concept of *mixed strategies*
- *Incomplete information*: Players may be uninformed about the precise state of the game; this lack of information can be created at the start of the game (think of shuffling a deck of cards) or be the result of actions by players which are invisible to their opponents.
- *Solution concepts*: methods to analyse a given game leading to some prediction on how the players will behave while playing the game. Some of the better known solution concepts are:
 - *Nash Equilibrium*: a choice of strategies selected by the players with the property that no player can (unilaterally) obtain a better outcome by changing his strategy.
 - *Dominating strategies*: a strategy is dominating another strategy if it yields a better outcome regardless the actions chosen by the other players.

The Minimax and Maximin strategies are only mentioned in the context of the discussion of earlier work in [32] where only constant sum games are considered and where the two strategies will yield equal outcomes.

3.1.2 Implicit Assumptions for Game Theory

In game theory it is customary to assume several characteristics for the game and the players:

- *Common Knowledge of the Game and its rules:* Agents know the structure of the game and they know which moves are possible in which states, both for themselves and for their opponents. They know that the opponents have the same knowledge, and that they know that we know it as well (in fact common knowledge entails an infinite collection of nested statements of the form *I know that you know that I know that...*).
- *Common Knowledge of Preferences and Utilities:* the players not only know their own preferences but also those of their opponents, and are aware of the fact that the opponents know their preferences as well (again up to arbitrary depth).

One can doubt whether these assumptions are reasonable, particularly in the context of warfare. Common knowledge of the game would require that all parties are fully informed about all aspects of the terrain and have full knowledge about the composition and strengths of the opposing army. Secret weapons would not be allowed. Common knowledge of the utilities would require that parties have complete information on the policies and internal affairs of the enemy. Under these assumptions spying would be futile.

Instead, it can be investigated whether the results of game theory can be preserved while relaxing these unrealistic assumptions. One approach is to use those features in game theory which represent uncertainty to model the ignorance of the players. Incomplete information and probabilistic moves are the features involved. The problem is that one needs to know the ignorance of the players before one can model it in this manner.

3.1.3 Extended Game Theory

An alternative is to relax the assumptions in game theory themselves. Löwe et al. [25] show that game theoretical analysis is possible without assuming common knowledge of the utilities. Halpern and Rêgo [19] present a version of game theory where no common knowledge of the game structure is required. Another relevant approach is presented by Kaneko and Kline [23] who consider that a game is being learned while being played - an aspect certainly relevant for warfare.

As we will see in our discussion of the work of Niou and Ordeshook [32], many game theorists will not consider the writings of Sun Tzu as conforming to game theoretical concepts, because there is not a set of rules which are considered common knowledge for both parties. Actually, there are many games, recognized by everyone as a real game, that do not adhere to these rules. Even for a game like Chess, where it would theoretically be possible that both players know what the best next move would be, this is unrealistic in practice.

Both Game Theory and Sun Tzu's "Art of War" have received intense interest in business environments and many interpretations of Sun Tzu's text are applied to business practices or modern warfare, sometimes giving his text almost mythical powers, see for example the websites: [51] and [53].

If we constrain ourselves to a definition of Game Theory stating *analysis of strategies for dealing with competitive situations*, then, as cynical as it may sound, *war* is a game and Sun Tzu's analysis of war strategy is game theory. It is therefore understandable that many business researchers did not bother too much about being able to find the right game theoretical background. How far we can get while using one of these more extended and liberal interpretations of Game Theory is a subject for future research.

3.2 Theory of Mind.

Theory of Mind is a concept originating from the field of Psychology, defined in Wikipedia as: *Theory of Mind (often abbreviated as ToM) is the ability to attribute mental states — beliefs, intents, desires, pretending, knowledge, etc. — to oneself and others and to understand that others have beliefs, desires, and intentions that are different from one's own.*

In Game Theory, the use of *Theory of Mind* is more focussed toward the concept of reasoning from the standpoint of another person: "I think that you think that I think ...".

A quote from an ancient Chinese legend makes the concept more clear and it proves that this type of reasoning was indeed used in ancient China:

One day, Zhuangzi and Huizi are strolling on Bridge Hao.

Zhuangzi: "Look how happy the fish are just swimming around in the river."

Huizi: "How do you know they are happy? You are not a fish."

Zhuangzi: "And you are not me. How do you know I don't know the fish are happy?"

Huizi: "Of course I'm not you, and I don't know what you think; but I do know that you're not a fish, and so you couldn't possibly know the fish are happy".

This event is assumed to have happened during the Warring states period (403-211 AD), which means it is at least a century later than Sun Tzu who lived around 500 BC. This dating is relevant since, though Sun Tzu explicitly asks to reason about and prepare for your opponent, such an explicit reference to ToM reasoning can't be found in his text.

On the other hand, in the great Chinese classic *Three Kingdoms* [18] we do find explicit use of higher order *Theory of Mind reasoning*. This historical novel does describe a period of civil war occurring some seven centuries after Sun Tzu, and it was written down for the first time in the fourteenth century (a short synopsis of the events during the three kingdoms era is given in appendix A.1). So if we are

willing to accept that the events described in this novel are (in part) historical, it suggests that theory of mind reasoning was used in the third century AD.

A possible explanation for the lack of explicit ToM reasoning in Sun Tzu can be found in considering that the use of ToM reasoning may occur at different levels.

Van Benthem [2] distinguishes three levels of theory of mind reasoning:

1. *Using Theory of Mind*: Most adult persons are capable of using ToM, but often use it unconsciously.
2. *Understanding Theory of Mind*: At this level a person uses ToM consciously to determine his or her actions.
3. *Reasoning about Theory of Mind*: At this level it allows formulating a strategy in a game, and also to describe the nature of human empathy by philosophers or deficits in human behaviour by psychologists.

It is possible that the transition from the second to the third level of ToM reasoning in China did not occur before or during the lifetime of Sun Tzu.

4 The Logical Thinking of Sun Tzu in “The Art of War”

In this section we are looking more closely at Sun Tzu, “The Art of War”. As already remarked, Sun Tzu lived around 500 BC during the Chinese Warring States period. His very famous book is much more than a book about fighting in a war, it concerns itself primarily with winning without fighting and strategies for achieving that.

4.1 Translations of the Art of War into English

The history of the translations is interesting in itself, because most translators seem to be of the opinion that they improved significantly on what their predecessors did.

The first known translation into English was done by Captain E.F. Caltrop in 1905, with improvements in 1908 [7]. Lionel Giles must be the second translator, motivated to do this translation because he thought that the translation by Caltrop was “excessively bad” [16]. Giles has abundant notes and explanations in his text.

It would take about 50 years before another translation was made by Samuel B. Griffith and many would follow. Especially in business circles Sun Tzu became popular, which resulted in books with frivolous titles such as: “Sun Tzu was a Sissy” [4], “Sun Tzu- The Art of War for Managers” [28], and “Sun Tzu for Women” [8].

Other translations were done by: Cleary (1988), Ames (1993), Sawyer (1994), Wu Rusong (1999), Minford (2002), Lin Wusun (2003), and Ziegler (2010).

The translations are substantially different and it is difficult to decide whether the exact concepts from the old text are used. Moreover, the source text is of course written in ancient Chinese, yielding additional problems, also for modern native Chinese scholars.

We use mainly the translation into English by Lionel Giles, one of the oldest translations, made in 1910. This text is available as part of the Gutenberg project [15] and is therefore freely available without copyright restrictions. Although there are later translations based on more modern research and on Chinese texts that were found during later excavations, notably in 1972, we think that the Giles translation is most suitable for our purpose: the translation is highly structured and because of its unrestricted availability we can transform the text to highlight this structure and the rules behind it. We consulted newer translations to check that understanding of the text is not completely dependent on the Giles translation.

Finding the right meaning and translate it the right way remains tricky, as is illustrated by three different translations of a sentence in chapter 4:

- From the Computer Game “Civilization IV”, Military Tradition splash screen:
Victorious warriors win first and then go to war, while defeated warriors go to war first and then seek to win.
- From the translation of Andrew W. Zieger:

A winning army wins first and then seeks battle. A losing army first battles, and then seeks to win.

See also figure 1, to view the original Chinese text. Note that the Chinese text actually starts with *therefore*.



Figure 1: A winning army, excerpt of the translation by Andrew Zieger.

- From Lionel Giles, par. 15:

Thus it is that in war the victorious strategist only seeks battle after the victory has been won, whereas he who is destined to defeat first fights and afterwards looks for victory.

We can conclude from these examples that the word *bing* has been translated to single soldier, commander or the whole army. It can also mean weapons, or military. The use of the translation *warrior* in this context only appears in the Cleary edition [11] where it occurs in a commentary to the final sentence of chapter 1, ascribed to Zhang Yu, a commentator who lived during the Sung Dynasty (960-1279 AD). Therefore the quote in the Civilization IV game is actually a quote of an ancient commentator and not of Sun Tzu himself.

A comprehensive discussion of the diversity of the translations is given by Yang Ming [29].

4.2 The logical Structure of the thirteen Chapters of Sun Tzu

In this section we present a very short summary of the content of Sun Tzu’s “Art of War”. The purpose of the summary is to highlight the logical structure of the book and to make it easier for non-Chinese readers (who may not have read a complete version of the (translated) original text) to understand what is presented in this section.

The text of Sun Tzu’s *Art of War* is rather short and subdivided into 13 chapters. Already from the titles of the chapters it is clear that a strict procedural

sequence is followed for describing the military theories, from *planning* to *fighting* and all considerations to be taken into account.

Yang Ming [29] describes in her thesis, citing Li Ling, that ancient Chinese books were often divided in internal and external sections. The internal section is the most important part of a book. According to Li Ling, the first six chapters of *The Art of War* constitute the internal section and describe the fundamental military theories. The other seven chapters contain the application of these theories.

Similarly as for the content of each chapter, there is a great deal of discussion among translators what the exact wording of the chapter titles of “the Art of War” should be. We follow here the Giles translation.

4.2.1 The fundamental Part

In this part of the book, Sun Tzu explains military action at a strategical level.

1. Laying Plans

This chapter talks about the importance of making preparations, taking several factors into account, such as morale, circumstances and organization. He advises to assess which side has advantage in all of these factors. Sun Tzu states: Warfare is a way of deception, and subsequently he points out what can be done in various circumstances to deceive the opponent.

2. Waging War

This chapter is not about war itself, but about what is needed in preparation. Although there is an explicit list of how many chariots and troops are needed at the start of the chapter, it continues with a list of rules of when to attack and how. Emphasis is put on avoiding a long war, which according to Sun Tzu can never be beneficial. He stresses that material and food should be taken from the enemy, and not sent from home.

3. Attack by Stratagem

Stratagem means plan, deception, or ruse in the context of preparing for war. Therefore, again, this chapter is about planning. Indeed, in the Mair translation [27], the title for this chapter is “Planning for the Attack”, which has a less negative meaning than the word “stratagem”.

This chapter involves considerations that a peaceful solution is always superior to war, that an enemy should only be attacked when it is certain that you will win, never attack walled cities, and disobey the ruler when you must.

III-1. the best thing of all is to take the enemy’s country whole and intact;

III-2. Hence to fight and conquer in all your battles is not supreme excellence; supreme excellence consists in breaking the enemy’s resistance without fighting.

A famous quote of Sun Tzu can be found at the end of this chapter:

III-18. Hence the saying: If you know the enemy and know yourself, you need not fear the result of a hundred battles. If you know yourself but not the enemy, for every victory gained you will also suffer a defeat. If you know neither the enemy nor yourself, you will succumb in every battle.

4. **Tactical Dispositions**

We will look at this chapter in more detail later, when we use mind maps to analyse its text.

In the Mair translation [27], the title for this chapter is “Positioning”, which indicates more accurately what the content of this chapter is.

Sun Tzu explains in an abstract and poetic way what a great leader should do to maximize his chance for success in a war. The content can best be summarized with the following quote:

IV-17. In respect of military method, we have, firstly, Measurement; secondly, Estimation of quantity; thirdly, Calculation; fourthly, Balancing of chances; fifthly, Victory.

IV-18.
Measurement owes its existence to Earth;
Estimation of quantity to Measurement;
Calculation to Estimation of quantity;
Balancing of chances to Calculation;
and Victory to Balancing of chances.

Notice how in paragraph 18 each next item in the military method refers back to the previous one.

Two other interesting quotes from this chapter are:

IV-2. To secure ourselves against defeat lies in our own hands, but the opportunity of defeating the enemy is provided by the enemy himself.

and:

IV-15. Thus it is that in war the victorious strategist only seeks battle after the victory has been won, whereas he who is destined to defeat first fights and afterwards looks for victory.

5. **Energy**

Sun Tzu speaks here about combinations and variations.

V-1. The control of a large force is the same principle as the control of a few men: it is merely a question of dividing up their numbers.

V-10. In battle, there are not more than two methods of attack—the direct and the indirect; yet these two in combination give rise to an endless series of maneuvers.

6. Weak Points and Strong

In Mair's translation [27], the title of this chapter is *Emptiness and Solidity* and indeed this could be a literal translation, but as most of the chapter is about finding the weaknesses of the enemy and using ones own strengths, the title of the Giles translation seem more appropriate. An alternative translation (according to Yanjing Wang closer to the actual Chinese text) reads *The Fake and the Real*.

A quote:

VI-2. Therefore the clever combatant imposes his will on the enemy, but does not allow the enemy's will to be imposed on him.

A game theoretical important text fragment to consider is:

VI-28. Do not repeat the tactics which have gained you one victory, but let your methods be regulated by the infinite variety of circumstances.

VI-29. Military tactics are like unto water; for water in its natural course runs away from high places and hastens downwards.

4.2.2 The Practical Part

In this part, Sun Tzu describes more of applied tactics, such as various types of terrain that can be encountered, marching, how to attack by fire, and the final chapter which discusses the use of spies.

7. Maneuvering

After having assembled the army, nothing is more difficult than maneuvering. Maneuvering means moving the army or parts of it and Sun Tzu explains what can happen if you move a too large part of the army, or move too fast, and what happens if the terrain is not taken into account.

8. Variation in Tactics

This chapter talks about army positions and what to do in various circumstances. Sun Tzu states also that the commander has the highest authority on the battlefield and therefore:

VIII-3. There are ... commands of the sovereign which must not be obeyed.

Another illuminating quote from this chapter is:

VIII-11. The art of war teaches us to rely not on the likelihood of the enemy's not coming, but on our own readiness to receive him; not on the chance of his not attacking, but rather on the fact that we have made our position unassailable.

9. **The Army on the March**

This is a long chapter dealing with warfare in different conditions, places, what can happen while marching, and rules for commanding. The first line effectively sums it up:

IX-1. Sun Tzu said: We come now to the question of encamping the army, and observing signs of the enemy.

10. **Terrain**

As the title suggests, this chapter handles terrain:

X-1. Sun Tzu said: We may distinguish six kinds of terrain, to wit: (1) Accessible ground; (2) entangling ground; (3) temporizing ground; (4) narrow passes; (5) precipitous heights; (6) positions at a great distance from the enemy.

The next chapter will again talk about different types of terrain, somewhat repetitious. However, in this chapter the description is more situational. The chapter ends with another famous quote:

X-31. Hence the saying: If you know the enemy and know yourself, your victory will not stand in doubt; if you know Heaven and know Earth, you may make your victory complete.

11. **The Nine Situations**

This is the longest chapter in the book. It contains several miscellaneous subjects, starting with rules about how to fight in various types of terrain. The next subject is about how to care for and handle the soldiers and the army, followed again by a discussion about terrain.

12. **The Attack by Fire**

In this chapter Sun Tzu gives very specific advice how to attack with fire and he briefly also talks about water..

XII-1. There are five ways to attack by fire ...

XII-5. In attacking with fire, one should be prepared to meet five possible developments ...

The end of the chapter contains more general rules for the commander to follow.

13. The Use of Spies

This chapter presents some remarkable insight in the role of information and the way to obtain it. Sun Tzu describes five different types of spies whose roles vary between obtaining information from the enemy to planting false information with him.

XIII-5. Now this foreknowledge cannot be elicited from spirits; it cannot be obtained inductively from experience, nor by any deductive calculation.

XIII-6. Knowledge of the enemy's dispositions can only be obtained from other men.

In a later section the use of spies and how this relates to use of information will be discussed.

4.3 Mind Maps

After looking at the content of the “Art of War”, we introduce the use of *mind maps* to analyse and visualize the text and to expose the logic behind its meaning.

A *mind map* is a way to visualize information in an organizational and hierarchical structure. Visual methods to help brainstorming, problem solving, structuring of a subject and so on is quite old, but made popular again by Tony Buzan [6] in the 1970-ies.

Since then a multitude of computer tools to create mind maps were developed, such as *Freemind* [50]. Freemind allows to import semi-structured text and that is what we did with the “Art of War” in the Gutenberg edition of the translation by L. Giles.

4.3.1 Analyzing Text Using Mind Maps

A digital mind map is essentially text formatted in a tree structure. We tried to keep the formatting as simple as possible: although the tools allow pictures, colors, various shapes of nodes and more, we thought that this would not enhance the clarity of our analysis.

We did not add any text to the mind map, except for some categorizations, as explained below. Sometimes we removed text, when it seemed duplicate or not adding meaning after text restructuring.

The process of analyzing text using a mind map is roughly as follows:

- Import the original text, chapter by chapter, into a mindmap. We made a separate mind map for each chapter.
- The chapter title becomes the root node in the mindmap for that chapter.
- Each paragraph becomes a sub-node.
- Next, each sub-node is split into further sub-nodes. This is of course a subjective analytical process, but there were some rules to guide us.

- Guiding rules for finding sub-nodes are the occurrence of certain words such as *hence* or *therefore*.
- The occurrence of enumerations help in finding structure in the text.
- We made additional mind maps to show *enumerated concepts, rules* and *specific examples*.
- To provide better traceability of the regrouping of text and in the additional mind maps, we left indications of where the original text can be found. The Roman numerals indicate the chapter, and the numbers are the paragraphs as shown in the Giles translation.

The resulting mind maps are too large to be published in this paper, we therefore show an example of what a mind map looks like (do not expect it to be readable here), see figure 2.

We also show some partial maps with regroupings and several pieces of a mind map showing patterns that we found.

As a final example, we look at a mind map rendered as text in a way that this becomes readable on paper. See for this our rendering of chapter 4 in section 4.3.4.

We should stress here that our analysis is not finished, neither complete. It should be sufficient however, to prove that the Sun Tzu text contains a considerable amount of logical reasoning. The game theoretical aspects of “The Art of War” by Sun Tzu are discussed in later sections.

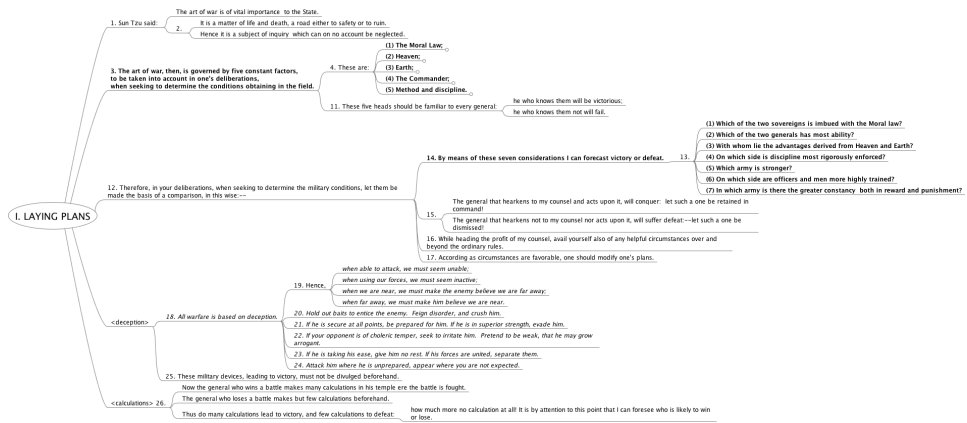


Figure 2: Mind map example showing chapter 1, Laying Plans

4.3.2 Categorizing Text

In our first iteration of structuring the mind maps, we did not change any text, we also did not reorder the text. In a next iteration we tried to add labelling, to

be able to categorize pieces of text, for example in the case that a definition or situation is described, such as this example in chapter 9 (The army on the march):

- IX - 7. In crossing salt-marshes, your sole concern should be to get over them quickly, without any delay.
8. If forced to fight in a salt-marsh, you should have water and grass near you, and get your back to a clump of trees. So much for operations in salt-marches.

is mapped as in figure 3.

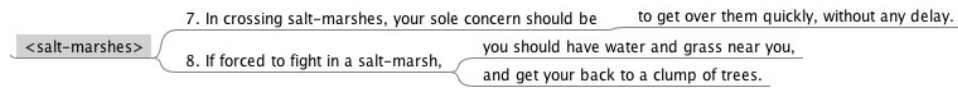


Figure 3: Chapter 9, Campaigning in salt-marshes

The figure shows that we added a label, <salt-marshes>, to the group of lines in paragraphs 7 and 8. We removed the text “So much for operations in salt-marches”. This text is useful in a narrative, but repetitive in a mind map.

If we continue our analysis for chapter 9, with the text in paragraphs 1-10, then we get a mind map as in figure 4.

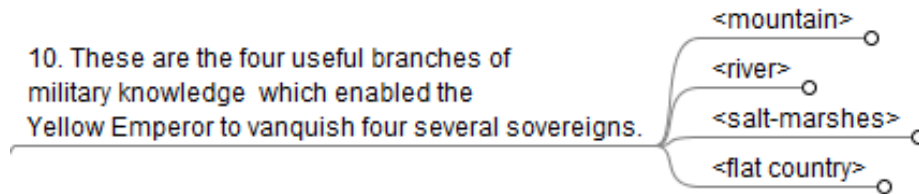


Figure 4: Chapter 9, Four rules for Campaigning

In figure 4, the lower nodes are collapsed, because we do not have space on the page show the complete text, but it makes clear that this text describes how to conduct warfare in four types of military terrain: *mountain*, *river*, *salt-marshes* and *flat country*.

From the paragraph number you can see that the definition starting with

10. These are ...

is placed in the original text **after** the descriptions in par. 1-9, while we put it as **root node** for the description of the various branches of military knowledge. This is done because Sun Tzu writes about “four useful branches”, which coincides with the four definitions just before paragraph 10, while in the text after this paragraph we could not find a clear enumeration of four things.

As a side note: We found that this text is one of the rare places where Sun Tzu explicitly mentions someone else in his advice (the Yellow Emperor is the legendary first emperor of China believed to have lived around 3000 BC). Terrain warfare is also covered in other chapters and it could be illuminating to cross-reference the advice given here with what is advised elsewhere. A next iteration of going through the mind maps could make this clear.

4.3.3 Patterns

From reading the abstract of the content of "The Art of War", or from reading a narrative translation, it is not easy to get a grip on the logical rules and game theoretical concepts that appear in the book. The mind maps helped us to find structure and we noticed several patterns that we analysed further searching through the text, using the specific words *Hence*, *Therefore* and *Thus*. Furthermore, we systematically looked for enumerations that are frequently found in the text.

From this analysis several patterns emerged, each will be described briefly here, with examples.

Rule

The occurrences of the words *Hence*, *Therefore* and *Thus*. Each of these words suggest that a rule could be found in the text.

A good example of a rule is in chapter 3, paragraph 8, shown here in figure 5.

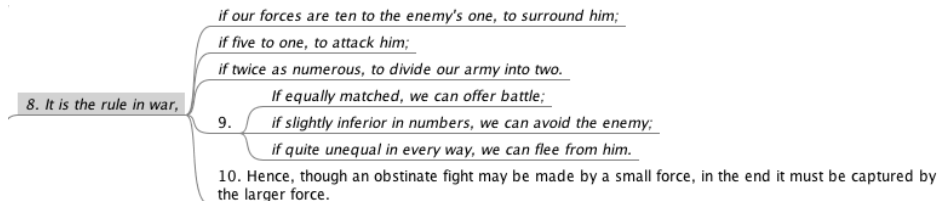


Figure 5: III-8. It is a rule in war...

In the translation of L. Giles there are 65 occurrences of *Hence*, *Therefore* and *Thus*. This is an approximation for the number of rules present, because on one hand, not all rules contain those words and on the other hand, some sentences, specifically those with *Thus*, cannot easily be interpreted as a rule.

Interesting is that the words have an even spread over the chapters, each containing 5, 6 or 7 instances, except for chapters 8 and 9, which only contain 1 and 2 respectively. The specific data can be found in appendix C.

Enumerated Concepts

We recognized the frequency of using enumerations by Sun Tzu ³. For example, the seven military considerations in chapter 1, the five essentials for

³Enumerations do occur in the other strategic classic texts also.

victory in chapter 3, the unlimited number of ways to combine the only two methods of attack, the nine varieties of ground, and so forth.

We collected these enumerations in a mind map. Figure 6 shows the highest level of this mind map.

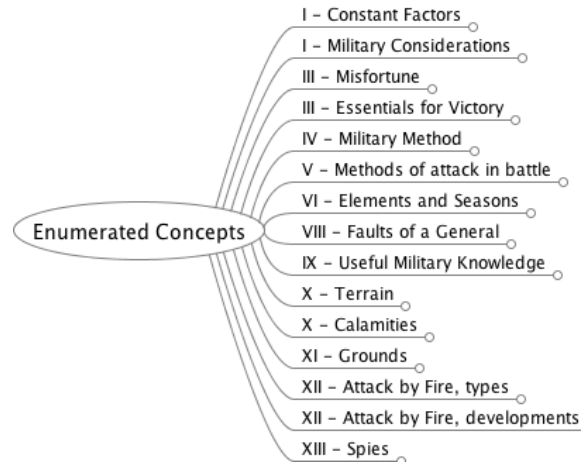


Figure 6: Overview Mind map of Enumerated Concepts

We cannot show an expanded view of all enumerated concepts in the mind map in figure 6, but an example is helpful how Sun Tzu uses these concepts. We show here an enumerated concept from chapter 1:

Right at the start of the book, there is a quite essential enumerated concept:

Sun Tzu says: I - 3. The art of war, then, is governed by five constant factors, to be taken into account in one's deliberations, when seeking to determine the conditions obtaining in the field. These are: ...

See figure 7 for the mind map showing the five constant factors.

Enumeration, Description, Rule

This is an extension of the enumeration concept, where the enumerations show a number of descriptions and then lead to a rule to follow.

We give a mind map example of this pattern from chapter 11, paragraph 1-14, see figure 8.

4.3.4 Reformatting Chapter 4, "Tactical Dispositions"

As a final example, we show chapter 4, in a reformatted and restructured way. This rendering of chapter 4 was arrived at by creating a mindmap and then restructure and categorize the text as described above.

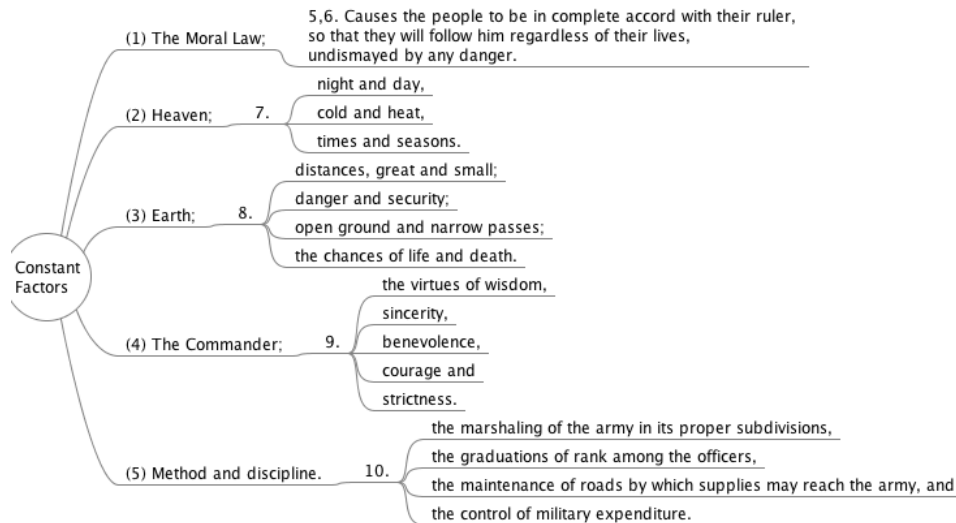


Figure 7: Mind map showing Constant Factors of chapter 1

We show this text because, as explained earlier, it is awkward to show larger mind maps. This formatting of the text tries to give a feeling for the structure of the mind map and what the results are of analyzing this chapter using a mind map.

The text is not altered, but we labelled groups of text. These labels are shown within the angle brackets <and >. By using nested lists, we added more structure.

As a result, we see the following subjects: *Good Fighter Qualities*, *Defensive and Offensive Tactics*, *Excellent Victory*, *Victorious Strategist*, *Military Method*, and *Victorious Army*.

The subject *Military Method*, is an example of an enumerated concept, referring back to chapter 1. To us, it seems out of place here. We have shown this quote before in 4.

The last subject in this list, talking about a *Victorious Army* is in fact a metaphorical description of the chapter.

By highlighting the sentences starting with **thus** and **hence** we see that these groups: *Good Fighter Qualities*, *Defensive and Offensive Tactics*, *Excellent Victory* and *Victorious Strategist*, have a reasoning with a conclusion. These are therefore instances of the **rule** pattern.

We find famous sayings in paragraphs 4 and 15.

4.3.5 The Final Reformatted Text

- 1. Sun Tzu said:

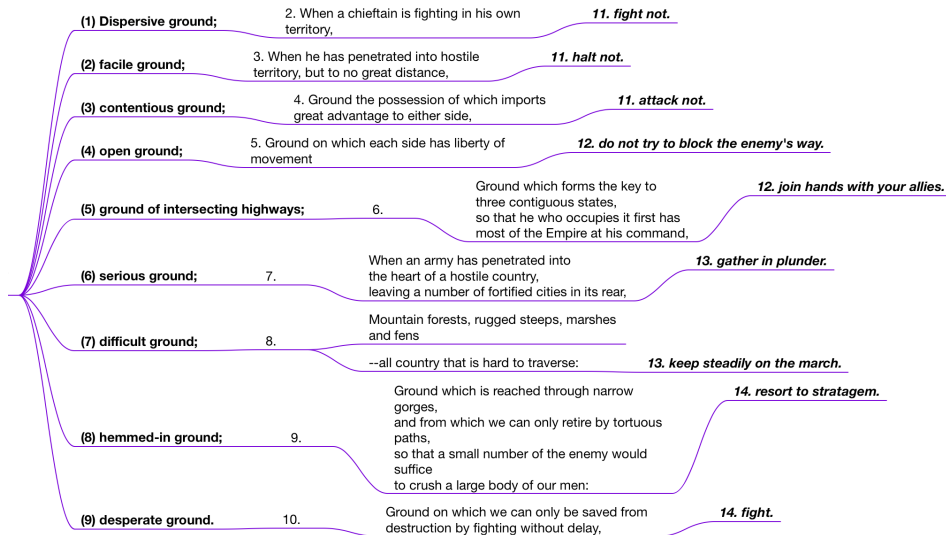


Figure 8: XI - 1. Sun Tzu said: The art of War recognizes nine varieties of ground.

- <Good Fighter Qualities>

1. The good fighters of old first put themselves beyond the possibility of defeat, and then waited for an opportunity of defeating the enemy.
2. – To secure ourselves against defeat lies in our own hands, – but the opportunity of defeating the enemy is provided by the enemy himself.
3. **Thus the good fighter is able to secure himself against defeat, but cannot make certain of defeating the enemy.**
4. **Hence the saying: One may know how to conquer without being able to do it.**

- <Defensive and Offensive Tactics>

5. – Security against defeat implies defensive tactics; – ability to defeat the enemy means taking the offensive.
6. – Standing on the defensive indicates insufficient strength; – attacking, a superabundance of strength.
7. – The general who is skilled in defence hides in the most secret recesses of the earth; – he who is skilled in attack flashes forth from the topmost heights of heaven.
7. **Thus on the one hand we have ability to protect ourselves; on the other, a victory that is complete.**

- <Excellent Victory>

8. To see victory only when it is within the ken of the common herd is not the acme of excellence.

9. Neither is it the acme of excellence if you fight and conquer and the whole Empire says, "Well done!"
 10. – To lift an autumn hair is no sign of great strength;
– to see the sun and moon is no sign of sharp sight;
– to hear the noise of thunder is no sign of a quick ear.
 11. What the ancients called a clever fighter is one who not only wins, but excels in winning with ease.
 12. **Hence his victories bring him neither reputation for wisdom nor credit for courage.**
- <Victorious Strategist>
 13. He wins his battles by making no mistakes.
 - Making no mistakes is what establishes the certainty of victory, for it means conquering an enemy that is already defeated.
 14. **Hence the skillful fighter puts himself into a position which makes defeat impossible, and does not miss the moment for defeating the enemy.**
 15. **Thus it is that in war the victorious strategist**
 - **only seeks battle after the victory has been won,**
 - **whereas he who is destined to defeat first fights and afterwards looks for victory.**
 16. The consummate leader cultivates the moral law, and strictly adheres to method and discipline;
 - **thus it is in his power to control success.**
 - <Military Method>
 17. In respect of military method, we have,
 - firstly, Measurement;** Measurement owes its existence to Earth;
 - secondly, Estimation of quantity;** Estimation of quantity to Measurement;
 - thirdly, Calculation;** Calculation to Estimation of quantity;
 - fourthly, Balancing of chances;** Balancing of chances to Calculation;
 - fifthly, Victory.** and Victory to Balancing of chances.
 - <Victorious Army>
 19. A victorious army opposed to a routed one, is as a pound's weight placed in the scale against a single grain.
 20. The onrush of a conquering force is like the bursting of pent-up waters into a chasm a thousand fathoms deep.

The resulting text may be less literary interesting than the original, but when we compared our text with a summary of the same chapter 4 in Wikipedia, we were surprised to find that this summary missed some of the points that our structure uncovered as important.

We quote here the summary of the chapter 4 in Wikipedia [52]:

Disposition of the Army explains the importance of defending existing positions until a commander is capable of advancing from those positions in safety. It teaches commanders the importance of recognizing strategic opportunities, and teaches not to create opportunities for the enemy.

From our analysis there are two points in this chapter that are missing from this summary:

- First, the conclusion that a strategist although making no mistakes himself, cannot win because the enemy makes no mistakes either.
- Second, excellent victories are not rewarded because they seem too easy.

There certainly is room for debate about the meaning of this chapter, that is made apparent through our mind mapping.

To complete our picture, we show a cut-out of the mindmap that was the basis of the reformatted text of chapter 4, shown above. See figure 9.

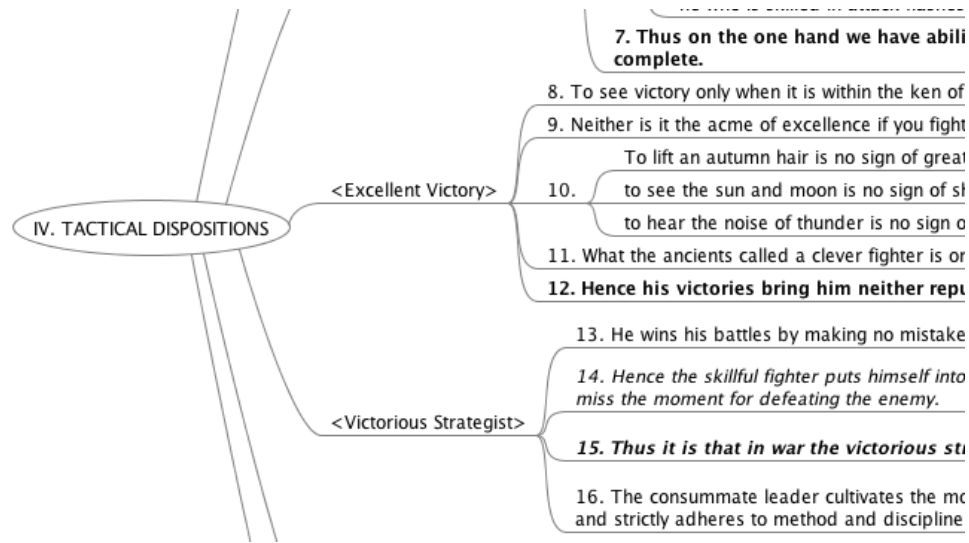


Figure 9: Part of the Mind Map of Chapter 4 after text analysis.

4.4 Discussion of Analysing the Text of the "Art of War"

In this part we tried to analyse "the Art of War" using mind maps with the purpose of illustrating the logical structure of Sun Tzu's book. In an ancient text that is not written explicitly as logic, this "proof" can only be an approximation.

It is an outstanding problem whether a similar transformation is possible when starting from the original text in Ancient Chinese. Our colleague Yanjing Wang considers this feasible, but we evidently need a Chinese collaborator versatile in Ancient Chinese literature and History in order to perform this task.

We did not do an exhaustive analysis, but our results so far show that more meaning and several patterns can be found more easily in this way, and that Sun Tzu's strategical reasoning can be shown with more clarity.

5 Information, its use, and non-violent Action: Examples from History and Literature

The ancient Chinese authors discuss information and its use at various places, providing a really modern insight in this matter. The very idea of warfare being based on deception requires to provide the enemy with false information and hiding the truth at the same time. A major difference with the concept of Theory-of-Mind reasoning which is also an essential ingredient of deception is that the Chinese authors are far more explicit when they discuss information and its use.

We indicate two specific fragments from the theoretical studies. Next we will consider a number of stories from history and literature which illustrate interesting game theoretical features.

5.1 T'ai Kung on Secret Letters

In section 25 of the third book (Dragon Secret Teaching) of the Six Secret Teachings (supposed to describe a dialogue which should have occurred more than 3000 years ago), we encounter an instance of the use of a cryptographic technique. The text also illustrates the question-reply-approval structure mentioned in section 1. Since, however, the first recorded written form of this text dates from the warring states era (after 476 BC), this yields no evidence that this form of cryptography actually has been used during the transition era between the Shang and the Chou dynasties, about 1500 years earlier. The same holds for the use of secret tallies as a type of code-book in section 24 of the same text.

Since it is a rather short section, I present the full text from the Sawyer translation in [37]

King **Wu** asked the T'ai Kung: "The army has been led deep into the territory of the feudal lords and the commanding general wants to bring the troops together, implement inexhaustible changes, and plan for unfathomable advantages. These matters are quite numerous; the simple tally is not adequate to clearly express them. As they are separated by some distance, verbal communications cannot get through. What should we do?"

The T'ai Kung said: "Whenever you have secret affairs and major considerations, letters should be employed rather than tallies. The ruler sends a letter to the general; the general uses a letter to query the ruler.

The letters are composed in one unit, then divided. They are sent out in three parts, with only one person knowing the contents. 'Divided' means it is separated into three parts. 'Sent out in three parts, with only one person knowing' means that there are three messengers, each carrying one part; and when the three are compared together, only then one knows the contents.

This is referred to as a 'secret letter'. Even if the enemy has the wisdom

of a Sage, they will not be able to recognize the contents”.
 “Excellent”, said King **Wu**.

Today we know that a two-part division of a message providing absolute secrecy can be implemented using the famous one-time pad scheme, where the public message is the exclusive-or sum of the plaintext message and a secret key message of the same length which has been transmitted by a safe channel previously (messages are assumed to be coded in binary).

Note that this is an instance of an early use of cryptographic techniques which is not mentioned in the historic survey part of the standard text on cryptography by Kahn [22].

5.2 Sun Tzu on Spies

The final chapter (chapter 13) of Sun Tzu’s The Art of War is dedicated to the use of spies. Given the huge expenses of maintaining an army during a campaign, Sun Tzu considers it immoral to abstain from gathering foreknowledge about the enemy for financial reasons:

2. Hostile armies may face each other for years, striving for the victory which is decided in a single day. This being so, to remain in ignorance of the enemy’s condition simply because one grudges the outlay of a hundred ounces of silver in honors and emoluments, is the height of inhumanity.

The only possible source for this knowledge is from other men:

- 5. Now this foreknowledge cannot be elicited from spirits; it cannot be obtained inductively from experience, nor by any deductive calculation.
- 6. Knowledge of the enemy’s dispositions can only be obtained from other men.

Sun Tzu distinguishes five types of spies, as described in paragraph 7 in chapter 13, see figure 10.

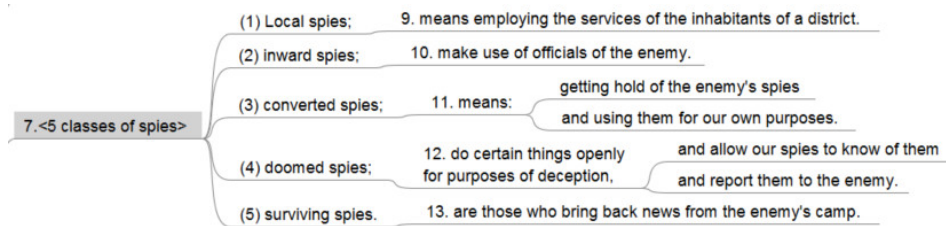


Figure 10: 5 Classes of Spies, Sun Tzu, Chapter 13, The Use of Spies

For Clarity, we show below a formatted transcript from the mindmap quote in fig. 10:

7.<5 classes of spies>

- (1) Local Spies; Means employing the services of the inhabitants of a district.
- (2) Inward Spies; Make use of officials of the enemy.
- (3) Converted Spies; Means: getting hold of the enemy's spies and using them for our own purposes.
- (4) Doomed Spies; Do certain things openly for purposes of deception, and allow our spies to know of them and report them to the enemy.
- (5) Surviving Spies; Are those who bring back news from the enemy's camp.

The remainder of chapter 13 discusses how to treat and pamper spies, and how the different types of spies should interact, see figure 11.

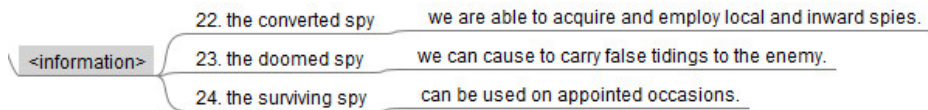


Figure 11: Information and Spies, Sun Tzu, Chapter 13, The Use of Spies

Particularly the way information is described as a transportable item in the case of the doomed spy represents a remarkable modern perspective on the concept of information in the book of Sun Tzu.

5.3 Information and Spies in the Romance of the Three Kingdoms

The Romance of the Three Kingdoms [18] is a historic novel describing events which occurred during a civil war in China at the end of the Han Dynasty during the years 168 – 280 AD. As a piece of literature it has had on Chinese culture an influence comparable to the works of Homer for Greek and Western culture. Part of the events described in the book are historic; others are inventions produced during the more than ten centuries separating the real events and the first written version of the book.

A short synopsis which may be useful for understanding this section is given in Appendix A.1.

5.3.1 Jiang Gan steals a Letter

A famous episode in chapter 45 is the story of a failed attempt by warlord Cao Cao to pacify his enemy Zhou Yu, general for the Southland, before the battle at Red Cliffs.

Cao Cao sends an old friend of Zhou Yu, Jiang Gan, as an envoy and spy to Zhou Yu in order to persuade Zhou Yu to surrender - Cao Cao controls at this time a stronger army.

Zhou Yu recognizes his old friend to be a spy, but hides this observation, treating Jiang Gan as a honoured guest instead. After a rich dinner party - no politics discussed - and ample amounts of wine, the two go together to sleep in the commanders tent.

During the night Jiang Gan discovers a letter written by two admirals of Cao Cao's fleet announcing that they will deliver Cao Cao's head to Zhou Yu. Jiang Gan steals this letter and hurries back to the other side of the river.

His mission to talk Zhou Yu into surrendering has failed but at least he has discovered a pending betrayal. Cao Cao doesn't hesitate and has his two admirals beheaded; however, as soon as the heads are delivered he realises that he has been tricked by Zhou Yu.

We see here an instance of an enemy spy being used unwittingly as a doomed spy against his employer. As is the case with many of the more interesting stories in the Romance of the Three Kingdoms, this story is labelled to be fictitious. Therefore it does not present a proof that spies were actually used in this way.

5.3.2 The battle at Tong pass

The next story (claimed to be historic [66]) shows how even non-information may be abused in order to set up your enemies against each other. It occurs in chapter 59 of the Romance of the Three Kingdoms.

After his defeat at Red Cliffs (208 AD), the attempts of Cao Cao to extend the territory he controls to the south come to halt. A few years later he attempts to get the Northwest regions under his control (211 AD).

Cao Cao's first opponent there is the local lord Han Sui, and his nephew Ma Chao who is commander of his army. Earlier Cao Cao has killed the father of Ma Chao and two of his brothers ⁴

Initially Cao Cao had tried to convince Han Sui to surrender and to deliver Ma Chao to him, but Han Sui refused. Han Sui goes to war against Cao Cao, but at Tong Pass, after some initial victories by Ma Chao, the two armies get stuck in a stalemate. Winter is arriving and the two parties start negotiating a truce.

Seeing no way to defeat Ma Chao, Cao Cao designs a plot to create animosity between Ma Chao and Han Sui.

⁴This element of the story is not historical; according to the official records Cao Cao had the father of Ma Chao executed only after Ma Chao had revolted against him.

During a negotiation in between battles, Cao Cao has a discussion with Han Sui; they discuss in public old events rather than the actual status. Ma Chao can hardly believe that no serious issues have been discussed.

Cao Cao then sends a letter to Han Sui filled with erasures. When asked for an explanation by Ma Chao, Han Sui can't explain why Cao Cao has produced such a sloppy document which seems to have no content at all.

Cao Cao and Han Sui have another discussion at the battle field, where Cao Cao expresses thanks for the earlier words of Han Sui.

These developments make Ma Chao more and more suspicious, and Han Sui feels it. Subsequently Han Sui figures out that defecting to Cao Cao is after all the wisest policy at this stage. However Ma Chao learns about Han Sui's intentions to defect to Cao Cao and to organize an murder attempt on Ma Chao.

Ma Chao attempts a pre-emptive strike on Han Sui, who is saved by Cao Cao, but Ma Chao escapes. So, having separated his enemies, Cao Cao becomes victorious at Tong Pass.

Ma Chao eventually will become one of the five tiger generals serving Liu Bei.

The curious fact that the ruse is based on a public exchange simulating the exchange of information where there is in fact nothing to be exchanged, has been observed by Chinese commentators as well, as is illustrated in the notes section in [18].

5.3.3 The Ambush at Huarong

A most peculiar event in the Romance of the Three Kingdoms describes the ambush staged at Huarong on the army of Cao Cao during his retreat after the lost battle at Red Cliffs. The ambush is staged by commander Guan Yu (Lord Guan) who implements a strategic plan designed by Zhuge Liang (Kongming). This story is remarkable since it explicitly involves third-order Theory-of-Mind reasoning, and it references Sun Tzu's teachings.

After his defeat at the battle of Red Cliffs, Cao Cao is retreating towards Huarong. Twice he passes a location which he indicates to be perfect for staging an ambush, had Zhuge Liang been clever enough to stage one. To his surprise, he is in fact ambushed at this very location.

Arriving at a junction between roads, Cao Cao, already severely damaged, has two options for continuing his retreat: he can follow a road through the valley, or take a longer more tortuous mountain trail. The scouts he has send out to recognize these options report that they observe smoke along the mountain trail, indicating the presence of the enemy.

Cao Cao argues that his opponent Zhuge Liang, a well known and intelligent strategist, has lit fire in the mountains in order to lure him (Cao Cao) into the valley, where he will be ambushed. Strictly in accordance with the teachings of Sun Tzu: *appear weak where you are strong and strong where you are weak*. However, Cao Cao overlooks the fact that Zhuge Liang has predicted that he will argue in this way (also pointing out that the reasoning by Cao Cao is in accordance with the teachings of Sun Tzu).

The result is that Cao Cao takes the mountain road and subsequently is ambushed by Guan Yu.

Cao Cao survives because Guan Yu has obligations towards him related to a previous encounter between him and Cao Cao.⁵

From a Game theoretical perspective the strategic situation is isomorphic to one of the simplest example games in Game Theory: Matching Pennies. Two players E and U both choose head or tails. E wins if the two choices are equal; otherwise U wins the game. If both players move simultaneously the game has no solution other than both players playing the mixed strategy of choosing by flipping a fair coin. In case the players move sequentially, it is evident that the second player wins because he can react on the move by his opponent.

By setting up a fire (having been instructed to do so by Zhuge Liang), Guan Yu actually grants the second move to Cao Cao, thus essentially giving away the game. However this analysis only works if we assume that the players are perfectly rational in a game theoretical sense. And in fact the players are not perfect; they are bounded reasoners.

The story evidently exhibits instances of Theory-of-Mind reasoning: Cao Cao, uses second order Theory-of-Mind reasoning (as he has done as well on several earlier occasions, for example when he indicates the precise locations for the earlier two ambushes), thus showing that his strategic reasoning goes beyond the basic rules of Sun Tzu. However, Zhuge Liang performs third order Theory-of-Mind reasoning, and thus outsmarts Cao Cao and wins the game. Also the references to the teachings of Sun Tzu are explicit. It is a historic fact that Cao Cao is the author of one of the most important commentaries on Sun Tzu.

The story also exhibits Zhuge Liang playing two strategic games at the same time: there is the short time game of damaging Cao Cao, and the long term game of strengthening the position of Liu Bei which requires Cao Cao to survive. Therefore his choice to have Guan Yu stage the third and final ambush is brilliant⁶.

⁵According to an ancient commentator this escape has also been foreseen by Zhuge Liang; even though Guan Yu has promised his head in case he would let Cao Cao escape, Guan Yu observes his obligations to the warrior code. The overall strategic situation makes it advantageous to not eliminate Cao Cao completely, since Liu Bei at this stage would be no match against Sun Quan and his general Zhou Yu. Therefore the plan also works perfectly with respect to this higher order issue.

⁶Combining two games in such a way that a single move is actually a move in two games at the same time is an operator on games which has not been investigated extensively in

5.3.4 Zhuge Liang borrows Arrows from Cao Cao

The events at Huarong are not a first example of Zhuge Liang playing two strategic games at the same time. A previous example is the famous ruse where Zhuge Liang collects arrows from the enemy.

Before the battle at Red Cliffs (208 AD) Zhou Yu assigns to Zhuge Liang the *mission impossible* task of procuring 100 000 arrows. Zhou Yu is almost certain that Zhuge Liang will fail this task, which will provide him with a sufficient excuse to get rid of this troublemaker. Surprisingly, Zhuge Liang doesn't only accept the task, but promises also to complete it within three rather than ten days.

In order to fulfill the task Zhuge Liang assembles a small fleet of 20 ships, covered by straw and feigns an attack at Cao Cao on the opposite side of the river at dawn in the fog. Cao Cao's troops are deceived and fire more than 150 000 arrows which are well preserved in the straw, and collected.

In this way Zhuge Liang both has saved his head against the evil plan of Zhou Yu, and also tricked Cao Cao.

The two stories about the ambush at Huarong and about producing the arrows are assumed to be fictitious. The battle at Red Cliffs and the defeat of Cao Cao are historic facts, but the events during his retreat are not. So the concepts mentioned in the story may have been conceived any time between the actual events in 208 AD and the recording of the Romance of the Three Kingdoms in the 14th century. This means that this story cannot be used as proof that theory-of-mind reasoning was consciously used during the warring state era or the Han Dynasty.

5.4 Diplomacy and covert Action

During the Warring States era, China was divided into seven major kingdoms. This means that whenever two kingdoms went to war, there were always several other kings available to form alliances. The actual course of history provides us with a chain of forged and broken alliances during this period, up to the time that China was unified when **Qin** had defeated the six other kingdoms in 221 BC.

One might expect therefore that building alliances and maintaining them would be an essential part of strategic theory. However, in Sun Tzu's "Art of War" the topic of building alliances is hardly mentioned.

We find the phrase alliance or related phrases only six times in the Giles translation. One occurrence is metaphorical, whereas a second one repeats one of the others. This leaves four primary occurrences, all in chapter 11 (the nine situations).

Game theory at present. The problematic aspect is not the structure of the state space, or the moves in the resulting game (which are easily obtained by a Cartesian product) but the structure of the new preference relation, which requires a solution of the difficult problem of preference aggregation.

12. On open ground, do not try to block the enemy's way.
On the ground of intersecting highways, join hands with your allies.
52. We cannot enter into alliance with neighbouring princes until we are acquainted with their designs. We are not fit to lead an army on the march unless we are familiar with the face of the country — its mountains and forests, its pitfalls and precipices, its marshes and swamps. We shall be unable to turn natural advantages to account unless we make use of local guides.
54. When a warlike prince attacks a powerful state, his generalship shows itself in preventing the concentration of the enemy's forces. He overawes his opponents, and their allies are prevented from joining against him.
55. Hence he does not strive to ally himself with all and sundry, nor does he foster the power of other states. He carries out his own secret designs, keeping his antagonists in awe. Thus he is able to capture their cities and overthrow their kingdoms.

For strategic recommendations on forming alliances and forms of non-violent subversive actions, we must look into the other six classics and other historical sources like the Records of the Grand Historian [41]. An excellent survey on this topic has been published by Sawyer [36]. This book illustrates the extent of spy-craft and diplomacy in Ancient China; the present paper is not the right place to investigate these issues more closely.

A direct and trivial form of subversive action is bribing the enemy, or its ministers and generals. Recommendations to do so already can be found in the Six Secret strategies, section 15 on Civil offence [37]:

Fourth, assist him in his licentiousness and indulgence in music in order to dissipate his will. Make him generous gifts of pearls and jade, and ply him with beautiful women

5.4.1 The Missions of Zi Gong and Zhuge Liang

A famous story involving diplomacy is the story of the mission of Zi Gong (described in [47] as an illustration of the third strategy: *Murder with a borrowed knife*), on behalf of Confucius during the late Spring and Autumn period.⁷ The story involves five different states (emphasized below) and many agents.

The story of the Mission of Zi Gong

The home state **Lu** of Confucius is on the verge of being attacked by **Qi** as part of a plan designed by Tian Chang, minister of **Qi**.

⁷Sawyer [36] also describes this story (naming the agent Tzu Kung) designating him to be the third spy in Chinese history; the first spy was involved in establishing the Shang dynasty over the Hsia in 1766 BC, whereas the second spy is the T'ai Kung who was instrumental in the establishing of the Chou dynasty over the Shang in 1045 BC.

As an envoy of **Lu** Zi Gong first convinces Tian Chang that it would be advantageous for him personally not to attack the weak state **Lu** but to make war against the mighty state **Wu** instead.

It is a win-win scenario: victory over **Lu** is easy but when it is done, the other ministers will reap the benefits. If the war against **Wu** succeeds Tian Chang's star will rise and if he loses the state of **Qi** will be sufficiently weakened so that Tian Chang may usurp its power.

Next he travels to **Wu** and convinces king Fu Chai of **Wu** to attack **Qi** thus providing Tian Chan with a good reason for his change of plans.

However, Fu Chai fears that he may be attacked, while at war with **Qi**, by his neighbours in the state **Yue**. Therefore Zi Gong travels to **Yue** and convinces the king of **Yue** to ally with **Wu** rather than to abuse this opportunity.

Returning to **Wu** he convinces Fu Chai to accept the support of **Yue** but not to allow the actual participation of the king of **Yue** himself.

As a final move Zi Gong travels to the neighboring state of **Jin** and informs its king about the impending war, and tells him to prepare his armies.

The result of these actions is that **Wu** attacks and conquers **Qi**; next Fu Chai makes peace with **Qi** and **Lu** and overconfidently attacks **Jin** where he is severely beaten by its well prepared army.

Lu is saved.

From a game theoretical perspective the win-win scenario presented to Tian Chang by Zi Gong is most interesting. Furthermore this story exhibits the fairy-tale like pattern where the hero of the story, in order to complete some task, needs the assistance of someone else who assigns yet another task to the hero, a pattern which is repeated several times.

The Mission of Zhuge Liang to Sun Quan Another famous instance of diplomacy is the legendary trip (described in the Romance of the three Kingdoms [18], ch. 43) of Zhuge Liang to Sun Quan, the king of Wu, in order to talk Sun Quan into an alliance with Liu Bei against Cao Cao.

Before being allowed to talk to the king Zhuge Liang first is subjected to a series of debates with Sun Quan's scholarly advisors; Zhuge Liang's logic and argumentation defeats all seven opponents in succession. The result is that Sun Quan no longer considers to subject to Cao Cao, but indeed starts to cooperate with Liu Bei.

From the perspective of logic it is interesting that Zhuge Liang has to win this debating game in order to succeed in his mission. ⁸

5.5 Sun Bin and the Horses of General Tian Ji

Sun Bin is the author of a second classic text entitled *The Art of War* [34]. For a long period he has been identified with Sun Tzu, but an excavation in a Han tomb in 1972 has confirmed that there have been two different authors, believed to have lived 140 years apart.

The text of Sun Bin exhibits a substantial overlap with the teachings of Sun Tzu. He uses different formats: dialogues with the king and general Tian Ji alternate with thematic surveys and enumerations. A pattern in Sun Bin which is not explicitly visible in Sun Tzu is that he occasionally compares the importance of various recommendations, as illustrated by the quote from [34] below:

King Wei asked Sun Bin, "Many scholars of Qi have advised me to strengthen the army, but they don't agree on how this can be done Some suggested that I practise a rule of benevolence ..., others recommended I distribute grain and provisions among the populace. Still others have suggested I let things run their course..."

To which Sun Bin replied, "None of these is the most crucial in strengthening the army."

King Wei asked, "What then is the most crucial?"

Sun Bin answered, "Make the country prosperous." When the king repeated these words, Sun Bin continued, "...it was by making the country prosperous that both King Wei and King Xuan defeated the neighbouring states..."

Other than his text Sun Bin is renowned for two famous stratagems in Classic Chinese history.

Strategy *Besiege Wei to Rescue Zhao* is listed as the second in the collection of 36 strategies [46, 47]. It is named after a campaign Sun Bin lead on behalf of Qi.

The state of Wei is attacking the state of Zhao and the latter asks for help from the state of Qi.

General Tian Ji, advised by Sun Bin, leads the army of Qi to support Zhao. Rather than supporting the army of Zhao, Sun Bin allows the Wei general Pang Juan to invade Zhao, but launches himself a campaign against the capital of Wei.

When Pang Juan learns about this attack he is forced to withdraw in order to defend his capital, and Sun Bin sets up an ambush on the retreating army and defeats it.

⁸In the recent film representing the history of the battle at Red Cliffs [65] it evidently was too hard to represent this debating game in the film; they turned it into a music contest instead.

Strategy 28 on the list of 36 strategies: *Climb up the roof and remove the ladder* is illustrated in [46] by a stratagem attributed to Sun Bin as well.

Ten years after the rescue of Zhao, the history is repeated when Wei invades the state of Hann, and Qi comes to the rescue of Hann by invading Wei. Rather than setting up an ambush straight away on the retreating Wei army, Sun Bin exploits the arrogance of the Wei commander Pang Juan (the same general he has defeated ten years before).

Pang Juan believes that the Qi soldiers are cowards, and Sun Bin confirms him in this belief by diminishing the number of camp fires in the Qi camp every day, thus giving the impression that the soldiers are defecting and fleeing.

Pang Juan, believing that he is chasing an army retreating in panic, pursues his enemy in double speed, and is subsequently ambushed.

Pang Juan is killed under a tree in which Sun Bin has carved a message stating *Pang Juan will die under this tree*. Passing at night Pang Juan observes the tree and sees that there is something written on it; he asks for a torch to read it, but this light springs the ambush. Seeing that he is defeated Pang Juan commits suicide under the tree.

A relevant fact is that Pang Juan is the man who originally, while Sun Bin was serving as an adviser for Wei betrayed him and had him jailed under false accusations. Sun Bin escaped by feigning madness. He eventually arrived in Qi where his wits were recognized by general Tian Ji.

From our perspective it is interesting to note that this story involves a prediction which is made true by being announced.

5.5.1 Tian Ji and the Horse Race

The advice which Sun Bin provided to Tian Ji is unrelated to warfare. Tian Ji was involved with regular horse race matches against the king, and he was losing substantial amounts of gold in the wagers involved in these races. The king simply had better horses. The horses were arranged in three groups: fast horses, ordinary horses and slow horses, and in each group the horses of the king were faster.

Sun Bin recommended to wager once again a substantial amount of gold, but then arrange the matches as follows: Tian Ji places his slow horse against the fast horse of the king, his fast horse against the ordinary one of the king, and his ordinary horse against the slow horse of the king. Thus he wins two out of the three bouts and therefore he wins the bet.

Sun Bin's achievement has been recognized as solving an instance of a combinatorial optimization problem some 24 centuries before this area in Operations Research became a branch of mathematics. Given the fact that Tian Ji could arrange his horses against a fixed ordering of horses from the King, it is a single-agent optimization problem rather than a multi-agent game solution.

It is remarkable that the general was enabled to determine the pairing of the horses; one would expect that this would be the prerogative of the king, giving the king a guaranteed win.

Another more plausible scenario is that both Tian Ji and the king select an order independently. In this case there are six possible matchings, only one of which will be winning for Tian Ji. This version of the game has been investigated by Leng and Parlar [24]. They show that under the standard interpretation of this game the solution of the game is that both Tian Ji and the King randomize amongst the six possible orderings. There are however alternative models of this game involving probabilistic outcomes of the races and more refined utilities where the game has more than a single equilibrium solution.

5.6 Conclusions about the use of information in these stories

This section contains a number of pieces of evidence on how the Chinese dealt with information and its use for strategic purposes in War and Diplomacy. Evidently our perspective is rather anachronistic, since we observe these ancient stories and texts from a contemporary perspective.

We have seen that the Chinese used a primitive type of code and of information hiding; this goes against the observation of Kahn [22] who claimed in his first edition that the Chinese didn't need cryptographic tools since the degree of literacy in the ancient time and the complexity of the Chinese alphabet by itself sufficed for hiding information in practice ⁹.

We have seen that Sun Tzu explicitly advocates the use of spies as a primary source for vital information which can't be obtained from oracles or similar sources. His concept of the doomed spy as a tool for transporting false information to the enemy presents an unusual modern perspective on the concept of information.

The various stories from the three Kingdoms era and history illustrate how some of these events relate to games, be it that they also indicate the shortcomings of standard game theory for modeling the actual strategic set-up in War or Diplomacy. The clearest shortcomings is the non-validity of common knowledge assumptions, but also the lack of an operator combining games in such a way that a single action can be considered to be a move in two games at the same time.

The story of the ambush at Huarong suggests that at some point in the history the Chinese did understand the importance of depth in Theory-of-Mind reasoning. It should be noted, however, that - even when the story could be labelled to be historic - it is not the first recorded instance of third order theory-of-mind reasoning in a military context. Cline [12] mentions an attack on the Canaanite city Megiddo by the Egyptian Pharaoh Tuthmose III in 1479 BC. Tuthmose attacks by a narrow and dangerous mountain road, rather than taking the easier accesses by the valley. His argument was that the Canaanites would have left this mountain access

⁹This suggests that the security of the system is based on its intrinsic complexity and ignorance of the system at the side of the enemy; however this goes against Kerckhoff's principle (see Kahn [22], ch. 8) that the security of a cryptographic system never should be based on its complexity or ignorance of the system.

undefended, since they would never believe that their opponent would be so stupid to choose a road which anybody would recognize to be vulnerable for an ambush. See [12], page 29.

In the next section on previous work we will encounter yet another example of game models in the analysis of the Open City strategy by Cotton and Liu [13].

The story of Sun Bin and his advice to Tian Ji for the Horse race strategy represents an instance of the solution to a combinatorial optimization problem pre-dating the birth of this area in Operations Research by almost 25 centuries.

6 Review of previous Work

For the purpose of this paper, previous work is researched that evaluates the work of Sun Tzu and other ancient Chinese writers from a perspective of *game theory*.

The first serious research of the Sun Tzu text related to game theory is done by Niou and Ordeshook [32], who claim that Sun Tzu came very close to understanding game theoretical concepts as we know them today. Other work we will describe in this section is the game-theoretical analysis provided by Cotton and Liu [13], about two famous military bluffs in Ancient China.

6.1 Sun Tzu and Games

The question arises how close the ancient Chinese came to inventing the fundamental concepts of today's Game Theory. Niou and Ordeshook [32] have identified several game theoretical concepts in Sun Tzu. We quote their conclusion:

The thesis of this essay is that he can be credited with having anticipated the concepts of dominant, minimax, and mixed strategies, but that he failed to intuit the full implications of the notion of equilibrium strategies. Thus, while he offers a partial resolution of the 'he-thinks-that-I-think' regresses, his advice remains vulnerable to a more complete strategic analysis.

6.1.1 Dominated Strategy

Niou and Ordeshook present several examples of recommendations which they consider to be instances of *Dominated Strategies*. These rules describe tactics one should always use in given circumstances or errors one should never make. For example in chapter 8 (Variation in Tactics):

2. When in difficult country, do not encamp.
In country where high roads intersect, join hands with your allies.
Do not linger in dangerously isolated positions.
In hemmed-in situations, you must resort to stratagem.
In desperate position, you must fight.

Another example is found in chapter 11, which provides rules for fighting on different types of ground. See our rendering as mind map in figure 8,

However, if we understand these rules as *default rules*, meaning that the rule should be followed if nothing unusual occurs, then exceptions will be allowed. Strictly speaking then, they are not instances of a dominated strategy, which should be followed without exceptions.

6.1.2 Minimax Strategy

Niou and Ordeshook present an analysis why it would be unreasonable to expect that Sun Tzu could have arrived at an equilibrium concept like the Nash equilib-

rium; he misses the required form of circular reasoning about the opponent reasoning about himself etc. This is directly linked to the absent explicit reference to Theory-of-mind reasoning we mentioned previously. However there exist relevant special cases where the reasoning of Sun Tzu will amount to recommending an equilibrium strategy, provided it exists.

These special cases originate from the fact that warfare in general is a zero-sum or constant-sum game: the gain of one party equals the loss of his opponent. If in such a game an equilibrium exists, it involves also an instance of a Minimax strategy.

Minimax and *Maximin* strategies are standard concepts in Game Theory. The Maximin strategy is obtained when the player considers for each of his possible actions the outcome resulting from the most unfavourable choice of actions by the opponents; next he selects an action maximizing the utility of the outcome obtained in this worst case scenario. The Minimax strategy is similar but one argues based on the possible actions of the opponents: for each action of the opponents the player determines what will his result when he chooses his best possible counteraction; next he assumes that the opponents will select strategies in such a manner that the resulting pay-off will be minimal.

For constant sum games minimizing the maximal damage you can suffer by an opponents move is equivalent to maximizing your minimal gain against possible opponents moves: Maximin = Minimax. However, for general games the concepts are not equivalent. See for example Binmore, chapter 6 [5].

Some rules of Sun Tzu can be interpreted as recommendation to minimizing your own loss assuming the worst case counter strategy of your opponent. See for example chapter 4 in the section on Good Fighter Qualities:

1. Sun Tzu said: The good fighters of old first put themselves beyond the possibility of defeat, and then waited for an opportunity of defeating the enemy.
2. To secure ourselves against defeat lies in our own hands, but the opportunity of defeating the enemy is provided by the enemy himself.
3. Thus the good fighter is able to secure himself against defeat,
4. Hence the saying: One may know how to conquer without being able to do it.

6.1.3 Mixed Strategy

As an example of a *Mixed Strategy* found by Niou and Ordeshook, consider this quote concerning *Tactical Dispositions* from Sun Tzu, in the translation of L. Giles, chapter 6 and rendered as mindmap in figure 12.

Sun Tzu states that you should never repeat yourself; Niou and Ordeshook recognize this as the concept of a *Mixed Strategy*. In a classical game theoretic sense, this should mean that Sun Tzu would use a random strategy, like throwing dice, to establish his next move. This seems rather unlikely, considering his contempt for oracles. On the other hand Sun Tzu explicitly recommends to be

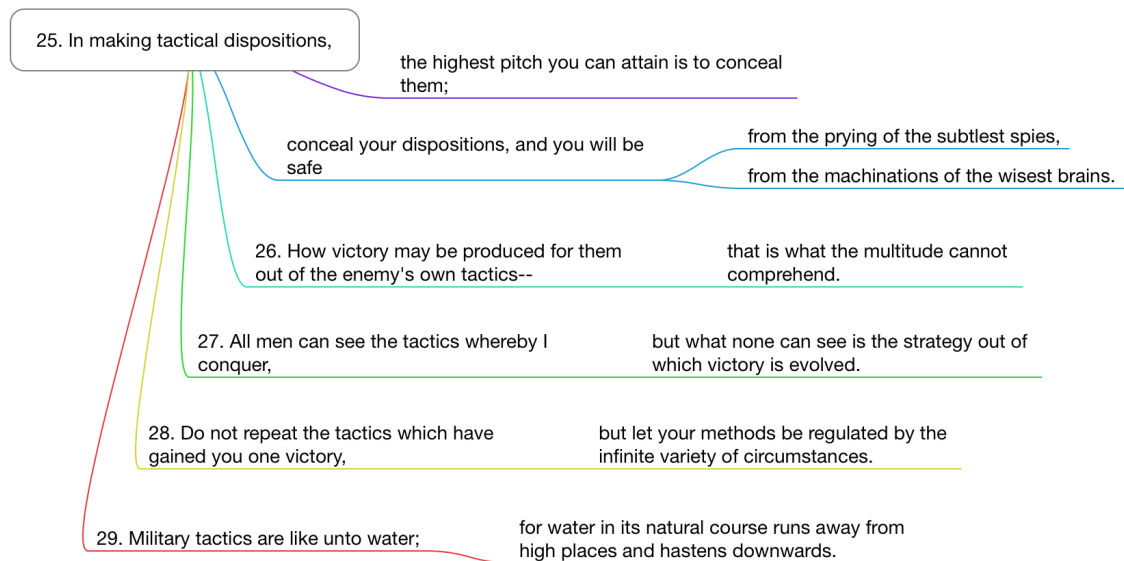


Figure 12: Tactical Dispositions, Sun Tzu: The Art of War, chapter 11, par. 25-29

variable in your choice of strategy. He connects this with the concepts *chen* (orthodox/expected/direct) and *ch'i* (unorthodox/unexpected/indirect). But also in this context he never explicitly refers to the strategic reasoning of the opponent.

Here is an example of the way Sun Tzu uses orthodox and unorthodox methods, from chapter 5:

5. In all fighting, the direct method may be used for joining battle, but indirect methods will be needed in order to secure victory.

10. In battle, there are not more than two methods of attack— the direct and the indirect; yet these two in combination give rise to an endless series of maneuvers.

11. The direct and the indirect lead on to each other in turn. It is like moving in a circle—you never come to an end. Who can exhaust the possibilities of their combination?

Sun Tzu evidently recognizes the importance of knowing your opponent, as illustrated by the famous quote:

So it is said that if you know others and know yourself, you will not be imperiled in a hundred battles; if you do not know others but know yourself, you win one and lose one; if you do not know others and do not know yourself, you will be imperiled in every single battle.

It is interesting to observe that Sun Tzu here omits the case specifically recommended by our Western antique knowledge: *Nosce Hostem; Know thy Enemy*; this phrase doesn't involve any knowledge about yourself.

6.2 The Horsemen and the Empty City

The analysis of Cotton and Liu [13] includes the legend of Li Guang and his 100 horsemen (144 BC), and the legend of Zhuge Liang and the Empty City (228 A.D.). The character of this paper differs from the Niou and Ordeshook paper since the authors are not looking for contemporary concepts in Sun Tzu; they use modern game theory and models to analyse the strategic situations in ancient history.

Both situations involve a defending party who is facing a superior force from the enemy, while the enemy is as surprised by the encounter as the defending party. In particular the enemy is uninformed of the actual strength of the defending party.

The Horsemen The story of 100 horsemen lead by Li Guang involves a group of horsemen which encounter a huge barbarian army in the plains. Rather than fleeing the general orders his troops to set up a camp without any defensive preparations, thus giving the impression that a larger army may be at a close distance for their protection. The barbarians leave the horsemen undisturbed.

Zhuge Liang and the Empty City Ruse The Empty City strategy (strategy 32 in the list of 36 proverbs [46, 47] is named after a famous episode from the three kingdoms era. In 228 Zhuge Liang finds himself with a small number of troops in the city Xicheng. Sima Yi approaches with a large army. Zhuge Liang orders the gates to be opened, all banners removed, and places himself on the city wall playing his zither. Sima Yi, fearing that there is an ambush inside the city, restrains from attacking and withdraws his troops ¹⁰.

Discussion The analysis of these strategic situations as a game involves both the features of probabilistic moves and incomplete information used in regular game theory. The game starts with a probabilistic move by nature yielding two normal start positions where the defending party is either weak or strong; this move represents the subjective uncertainty of the attacking party. Next the defending party decides to stay and prepare, or to withdraw. The third and last move is performed by the attacker when he decides to attack or withdraw. The attacker knows whether the defence is staying or withdrawing but remains uninformed whether the defence is weak or strong.

The utilities represent the facts that the attacker is defeated if and only if he attacks and the defence is strong. If the defending party withdraws he loses, but his loss is less severe than when he is defeated while staying and being weak.

¹⁰As is the case with other stories from the Romance, this event is historic, but the details are fictitious (E.G., Sima Yi can't have participated in the event).

The analysis shows that, depending on some condition on the precise utility values for the different outcomes, it indeed can be rational for both parties to behave as is reported in the stories: the defending party, while being weak, still stays and prepares while the attacker, fearing an ambush or a strong army nearby, withdraws.

For full details see the paper by Cotton and Liu.

The first recorded instance of a similar strategy occurred already in 666 BC during a war between Chu and Zheng ¹¹.

¹¹as explained on a web page by Siew Cheng Hoe which is no longer accessible; he mentions the Chinese page <http://baike.baidu.com/view/28157.htm> as another source

7 Conclusions

The goal of our project was to investigate the logical and game theoretical aspects of the strategy studies of Ancient China, and the events described in historical stories.

We have seen that the text of Sun Tzu, though not being a logic text from an orthodox contemporary perspective, is actually more structured than is usually stated. Our analysis using mind maps has made this structure clearly visible.

The other side of this question is more of a semantic nature. Sun Tzu has many rules, but we need our contemporary perspective in order to understand what these rules mean, and by what sort of logic they are to be read. For example, rules stating *you should always do this under these circumstances* only can be interpreted to recommend a dominating strategy only in case the rule is not to be understood as a default rule. In the study of Niou and Ordeshook the authors are rather generous in ascribing game theoretical concepts to Sun Tzu without investigating in depth how his rules should be read.

The question of whether the Ancient Chinese Strategy theoreticians recommend explicitly to reason about the opponent's reasoning has no definite answer. Theory-of-Mind reasoning was something the Chinese knew by the third century BC, but most likely they have applied it without a theoretical framework. Thus they have used it at some level in between level 2 and 3 in the classification of van Benthem [2].

The Ancient Chinese were more explicit about the role of information and the use of spies in order to obtain it. The stories presented in section 5 illustrate how information is used and misused in various events in battles and diplomacy. These stories illustrate moreover some interesting game patterns, like single actions having effects in two simultaneous games and games where you can't lose.

The game theoretical analysis by Cotton and Liu [13] of the empty city strategy represents a first example how game theory can shed a light on the interpretation of historic events. We can expect more studies of this nature in the future.

A Chinese History Timeline

For the purpose of our non-Chinese audience, here is a timeline of Chinese history, following [37]. We highlight the period in which Sun Tzu supposedly lived.

2205	-	1766 BC	Hsia Dynasty
1766	-	1045 BC	Shang Dynasty
1045	-	256 BC	Chou Dynasty
770	-	476 BC	Spring and Autumn Period
551	-	479 BC	Confucius
	Around	500 BC	Sun Tzu (or: Sun Zi, Sun Wu)
476	-	221 BC	Warring States Period
	Around	360 BC	Sun Bin
221	-	207 BC	Qin Dynasty –Unification of China
206 BC	-	220 AD	Han Dynasty
168	-	280 AD	Romance of the Three Kingdoms period
265	-	420 AD	Jin Dynasty
222	-	589 AD	Six Dynasties
589	-	618 AD	Sui Dynasty
618	-	907 AD	Tang Dynasty
907	-	957 AD	Five Dynasties
960	-	1279 AD	Sung Dynasty
1279	-	1368 AD	Yuan Dynasty
	Around	1370 AD	Presumed writing of the Romance of the Three Kingdoms
1368	-	1644 AD	Ming Dynasty
		1522 AD	First printed edition of the Romance of the Three Kingdoms
1644	-	1911 AD	Qing Dynasty
1911 AD	-	today	Modern era

A.1 Synopsis of the History of the Three Kingdoms Era

During the late Han Dynasty (206 BC – 220 AD) the Imperial court is weak and corrupted by the gang of ten eunuchs. Local Warlords have effective power; moreover in 184 AD a group of insurgents called the Yellow Scarfs are revolting.

The warlords repress the Yellow Scarfs revolt and eliminate the eunuchs. Subsequently they start competing for power. Warlord Dong Zhuo replaces the emperor by a weaker brother (189 AD).

Cao Cao becomes the dominant warlord in the state Wei; moreover he obtains full control over the last Han emperor (196 – 206 AD).

By 207 AD Cao Cao has eliminated most opponents; the main surviving competitors are his former ally Liu Bei and Sun Quan, the king of the Southlands (Wu).

Liu Bei, having suffered a sequence of defeats obtains the service of Zhuge Liang (Kongming), a very intelligent strategist.

Kongming suggests and forges an alliance between Liu Bei and Sun Quan; however both allies hardly thrust each other.

In 208 AD this alliance succeeds in defeating Cao Cao at the battle of Red Cliffs, terminating the southward expansion of Wei.

Liu Bei expands his influence into Riverlands (Shu), thus establishing a third Kingdom for himself (the other two kingdoms are Wei and Wu).

In 220 AD Cao Pi, the son of Cao Cao, disposes the last Han Emperor, which

brings the Han dynasty to an end.¹²

A sequence of battles follows without essential effects, but by 280 AD Wei has defeated both Wu and Shu, and China is reunified under the Jin Dynasty.

More details about this period can be found on the various Wikipedia pages about this era [56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67].

¹²Formally this is considered to be the actual start of the Three Kingdoms period, but the Romance describes the complete civil war.

B Miscellaneous Research data

In this section we have accumulated some data that was used for finding structure and rules in Sun Tzu, “The Art of War”.

B.1 Occurrences of Hence, Therefore and Thus

We show here a table of the occurrences of *Hence*, *Therefore* and *Thus* as found in the translation of L. Giles [15]. The roman numerals refer to the specific chapter, while the numbers refer to the paragraph within that chapter.

We used these findings to determine rules in the text of the book.

Chapter	Hence	Therefore	Thus
I	2,19	12	26
II	15	17	5,9,20
III	2,10,18	6	3,7,17
IV	4,12,14		3,7,15
V	21	14	19,23
VI	8,9,14	2,32	
VII	23,23	29	4,7,25
VIII	7		
IX		43	13
X	30,31		7
XI	23,31,55	11,46	20,25,34,35
XII	13,16,22		36,55,55
XIII	7,14,25,27		3,4,21

C Epilogue: On the Nature of War

Our study on the logical and game theoretical aspects of ancient Chinese Strategic thought covers ideas developed over a period of 25 centuries, starting from the transition era between the Shang and the Chou dynasties in the 11-th century BC to the Sung dynasty when the corpus of the seven classics was canonized around the 12-th century AD.

Strategic thought has not been confined to China, and neither has warfare. Moreover the nature of warfare has changed fundamentally during these periods. Some of the changes are even visible in the historical records and the texts of the authors themselves. An extensive discussion on the changing nature of war in the European tradition can be found in the introduction by A. Rapoport [33] to the English edition of the famous text by von Clausewitz *On War* [9, 10].

One major development is the change of military equipment during these centuries like the transition from Chariots to ridden Cavalry, and the introduction of the crossbow. We refer to the extensive studies by Sawyer [38] for more information on this topic.

A characteristic change which occurred during the Chou dynasty, Spring and Autumn period and Warring States era, is that war transformed from a gentleman's game into brutal massive battles.¹³

By way of illustration: inspecting the history of the Qin dynasty as described by the Grand Historian Sima Qian [40, 54], one can observe that the number of heads collected during battle (as far as specified in the text) crossed the 10 000 boundary after 500 BC. In the fourth century BC numbers like 80 000 are not uncommon. During the 56 years of the reign of king Zhaoxiang (306 - 251 BC) over one million heads were collected [40].

The gentleman's game nature of war during the early Chou dynasty is nicely illustrated in the first chapter of the Methods of the Ssu-Ma (the second text in the corpus of the seven classics). I cite from the translation by Ralph D. Sawyer [37].

In antiquity they did not pursue a fleeing enemy more than one hundred paces or follow a retreating enemy for more than three days, thereby making clear their observance of the forms of proper conduct [li]. They did not exhaust the incapable and had sympathy for the wounded and the sick, thereby making evident their benevolence. They awaited the completion of the enemy's formation and then drummed the attack, thereby making clear their good faith.

Compare this with what can be found in the Six Secret Strategies of T'ai Kung (section 52 in chapter VI):

The T'ai Kung said:"When the enemy has begun to assemble they can be attacked.

¹³similar transitions have been observed in the nature of war in Europe, as described in the introductions of [10, 26].

When the men and the horses have not yet been fed they can be attacked.

...

When they are fleeing they can be attacked.

When they are not vigilant they can be attacked.

When they are tired and exhausted they can be attacked.

....

Sun Tzu as well recommends to exploit weaknesses of the enemy whenever possible.

There are in the Ssu-Ma also instructions given to not abuse the enemy's civilians:

When you enter the offender's territory, do not violence to his gods; do not hunt his wild animals; do not destroy earthworks; do not set fire to buildings, do not cut down forests; do not take the six domesticated animals, grains, or implements. When you see their elderly or very young, return them without harming them. Even if you encounter adults, unless they engage you in combat, do not treat them as enemies. If an enemy has been wounded, provide medical attention and return him.

Sun Tzu is more pragmatic in this respect: it is better to conquer the territory of the enemy and his army intact than to destroy it, without an explicit prohibition against the latter.

This approach is quite distinct from what von Clausewitz recommends more than 2000 years later. For example, where Sun Tzu recommends to force the enemy into surrender rather than to destroy him: *Generally in war, the best policy is to take the enemy state whole and intact; to destroy it is not.*, chapter. 3, page 17 in [34], we find in von Clausewitz (Book I, chapter 1,3 page 102 in [10]):

Now, Philanthropists may easily imagine there is a skilful method of disarming and overcoming an enemy without causing great bloodshed, and that this is the proper tendency of the Art of War. However plausible this may appear, still it is an error which must be extirpated; for in such dangerous things as War, the errors which proceed from a spirit of benevolence are the worst.

The nature of war as described in the Romance of the three Kingdoms, is again more romantic. War is fought by armies which seem to consist of mercenaries bound by the warrior code of honour. Commanders may get involved in personal combat, but against the commanders of the enemy only, while they are not supposed to be disturbed in their fight by the ordinary troops. Aim is to subject the enemy, not to destroy him. If you can reach your goals without casualties so much the better, which doesn't exclude that battles may result in large damage and loss of human life. The defeated enemy troops may switch side after the battle and join the victorious general in the next fight.

Finally there is the issue whether the use of stratagems and tricks in warfare is morally acceptable or not. Sun Tzu definitely is in favour; if you can defeat the

enemy by deceit this is OK. You better win a war without serious fighting. In the first century AD Frontinus has collected a catalogue of various tricks known by the Romans of his age [14]. Machiavelli discusses stratagems in chapter 4 of his Art of War, basing himself mostly on events during the classical age, but also some more recent ones [26]. Some of his observations are close to the ones made by Sun Tzu, but it is extremely unlikely that Machiavelli knew about his Chinese colleague living some 2000 years before him.

However, for von Clausewitz stratagems and tricks are only to be used as a last resource solution if everything else fails (Book III, chapter 8, page 276 in [10]):

But the weaker the forces become which are under the command of Strategy, so much the more they become adapted for stratagem, so that the quite feeble and little, for whom no prudence, no sagacity is any longer sufficient at the point where all art seems to forsake him, stratagem offers itself as a last resource. The more helpless his situation, the more everything presses towards one single, desperate blow, the more readily stratagem comes to the aid of his boldness.

The Chinese had a more pragmatic solution for a last resource scenario; the final item on the classic list of thirty-six strategies of Ancient China [46] plainly reads *If all else fails, retreat*.

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References

- [1] Roger Ames(ed. and transl.), *Sun Tzu, The art of Warfare*, Ballantine Book, New York, NY, USA, 1993
- [2] Johan F.K.A.K. van Benthem, personal comm. at the 3rd Conference on the history of Logic in China, Tianjin, April 2014.
- [3] Johan van Benthem, *Logic in Games*, MIT Press, Cambridge, Mass, USA, 2014
- [4] Stanley Bing, *Sun Tzu was a Sissy*, Collins, New York, 2006
- [5] Ken Binmore, *Fun and Games; a Text on Game Theory*, Heath and Company, Lexington Mass. 1992
- [6] Tony Buzan, *How to Mind Map*, HarperCollins, 2002
- [7] Captain E.F. Caltrop, 1905 *The Book of War, by Sun Zu*, available on the Gutenberg project at <http://www.gutenberg.org/files/44024/44024-h/44024-h.htm>, last accessed 20160125
- [8] Chin-Ning Chu, *The Art if War for Women; Sun Tzu's Ancient Strategies and Wisdom for Winning at Work*, Doubleday, New York, 2007
- [9] Carl von Clausewitz, *Vom Kriege*, Reclam, Stuttgart, BRD, 1980
- [10] Carl von Clausewitz, *On War*, ed. Anatol Rapoport and transl. J.J. Graham, Penguin Classics, London, UK, 1968
- [11] Thomas Cleary(ed. and transl.), *Sun Tzu, The art of War*, , Shambala, Boston & London, UK, 1988
- [12] Eric H. Cline, *1177 B.C.; The Year Civilization Collapsed*, Princeton Press, New Jersey, USA, 2014
- [13] Christopher Cotton & Chang Liu, *100 Horsemen and the Empty City: A Game Theoretic Examination of Deception in Chinese Military Legend*, J. of Peace Research, 48 (2011) 217-233
- [14] Frontinus, *Stratagems*, transl. and ed. by Charles E. Bennet, Loeb Classical Library vol 174, Harvard Univ. press, Cmbridge, Mass. USA, 1925
- [15] Lionel Giles, 1910 *The Art of War, by Sun Zu*, available on the Gutenberg project at <http://www.gutenberg.org/files/17405/17405-h/17405-h.htm>, last accessed 20160125
- [16] Lionel Giles(trans.) & Dallas Galvin(ed.) ,*Sun Tzu, The art of War*, Barnes and Noble Classics, New York, NY, USA, 2003
- [17] Samuel B. Griffith(ed. and transl.), *Sun Tzu, The art of War*, Oxford University Press, Oxford, UK, 1963
- [18] Luo Guanzhong, *Three Kingdoms*, vol, I, II and III, ed. and transl. Moss Roberts, Foreign Languages press/ Univ. of California press, Beijing, China, 1994

- [19] Halpern, J.Y., Rêgo, L.C., *Extensive games with possibly unaware players*, Mathematical Social Sciences (2012), doi: 10.1016/j.mathsosci.2012.11.002
- [20] Hodges, Wilfrid, "Logic and Games", *The Stanford Encyclopedia of Philosophy (Spring 2013 Edition)*, Edward N. Zalta (ed.), <http://plato.stanford.edu/archives/spr2013/entries/logic-games/>, last accessed 20160125.
- [21] Ian Johnston(transl. and ann.), *The Mozi; a Complete Translation*, Columbia University press, new York, NY, USA, 2010
- [22] David Kahn, *The Codebreakers*, Macmillan, New York, 1967
- [23] Mamoru Kaneko & J. Jude Kline, *Information Protocols and Extensive Games in Inductive Game Theory*, in L.A. Petrosjan & V.V. Mazalov eds., *Game theory and Applications*, vol. XIII, Nova Science publ., 2008
- [24] Mingming Leng & Mahmut Parlar, *Game-theoretic analysis of an ancient Chinese horse race problem*, Computers & Operations Research, 33 (2006), 2033-2055
- [25] Benedikt Löwe, Eric Pacuit & Sanchit Saraf, *Analyzing stories as games with changing and mistaken beliefs*, Rep. ILLC PP-2008-31
- [26] Nicolò Machiavelli, *The Art of War; revised and with an introduction by Neal Wood*, Da Capo Press, second edition, Cambridge Mass, USA, 2001
- [27] Victor H. Mair(ed. and transl.),*Sun Tzu, The art of War*, Columbia University Press, 2007
- [28] G.A Michelson & St. Michalson, *Sun Tzu - The Art of War for Managers: 50 Strategig Rules Updated for Today' Business*, 2nd edition, Adams Media, Avon Mass, 2010
- [29] Yang Min, *The Diversity of the English Translations of The Art of War and the Translator's Subjectivity*, M.A. Thesis, School of foreign Languages, Central China Normal University, May 2007
- [30] John Minford(ed. and transl.),*Sun Tzu, The art of War*, Penguin classics, Penguin group NY. New York, USA, 2002
- [31] *the essential John Nash*, ed. H.W. Kuhn & S. Masar, Princeton, USA, 2002
- [32] Emerson M.S. Niu & Peter C. Ordeshook, *A Game-Theoretic interpretation of Sun Tzu' The Art of War*, J. of Peace Research, 31 (1994), 161-174
- [33] Anatol Rapoport, *Editors introduction*, in Carl von Clausewitz, On War, [10]
- [34] Wu Rusong & Wu Xianlin(eds.) & *Sunzi, The art of War*, & Lin Wusun(transl.), *Sun Bin, The art of War*, , Library of Chinese classics, Foreign Language press, Hunan peoples publishing house, Beijing, China, 1999
- [35] Ralph D. Sawyer(ed. and transl.), *Sun Tzu, The art of War*, Barnes and Noble, New York, USA, 1994

- [36] Ralph D. Sawyer, *The Tao of Spycraft; Intelligence Theory and Practice in Traditional China*, Westview Press, Boulder, CO, USA, 2004
- [37] Ralph D. Sawyer & Mei-Chün Sawyer(ed. and transl.), *The seven Military Classics of Ancient China*, Basic books, NY, New Yourk, USA, 2007
- [38] Ralph D. Sawyer, *Ancient Chinese Warfare*, Basic Books, New York, NY, USA, 2011
- [39] Sebastian Sequoiah-Grayson, review on [3], Bull. ASL. 20(4), 501–503, 2014.
- [40] Sima Qian, *Records of the Grand Historian; Qin Dynasty; transl. Burton Watson*, Renditions - Columbia Press Book, Hong Kong, 1993
- [41] Sima Qian, *Selections from Records of the Historian*; transl. into modern Chinese and ed. by An Pingqiu and transl. into English by Yang Xianyi and Gladys Yang, Foreign Language Press, Beijing, 2008; three volumes
- [42] Sun Tzu, *The art of War*, transl. essays and comm. by the Denma Translation Group, Shambala, London, 2009
- [43] Jiang Taigong *The Six Strategies*, ed. Kong Deqi and transl. Nie Songlai, Library of Chinese classics, Military Science publishing house,Shenzhen Jiaxinda Printing co., Shenzen, China, 2004
- [44] Thucydides, *History of the Peloponesian War*, ed. M.I. Finley and transl. Rex Warner, Penguin books, London, UK, 1954
- [45] J. von Neumann & O. Morgenstern, *Theory of Games and Economic Behavior*, sixtieth-iversary edition, Princeton Univ. press, Princeton, USA, 2004
- [46] Stephan H. Verstappen, *the thirty-six Strategies of Ancient China*, China Books, San Francisco, USA, 1999
- [47] *The Wiles of War; 36 Military Strategies from Ancient China*, Foreign Language Press, beijing, 2009
- [48] Lin Wusun(ed. and transl.), *Sun Tzu, The art of War*, Long River Press, San Francisco, 2003
- [49] Andrew W. Ziegler(transl. and comm.), *Sun Tzu' original Art of War*, Colours Network, Canada 2010.
- [50] http://freemind.sourceforge.net/wiki/index.php/Main_Page, last accessed 20160125
- [51] <http://scienceofstrategy.org>, last accessed 20160125
- [52] Wikipedia, *The Art of War*, http://en.wikipedia.org/wiki/The_Art_of_War, last accessed 20160125
- [53] <http://www.sonshi.com>, last accessed 20160125

- [54] Wikipedia, *Records of the Grand Historian*,
http://en.wikipedia.org/wiki/Records_of_the_Grand_Historian, last accessed 20160125
- [55] Wikipedia *Warring States period*,
http://en.wikipedia.org/wiki/Warring_States_period, last accessed 20160125
- [56] Wikipedia, *End of the Han Dynasty*,
http://en.wikipedia.org/wiki/End_of_han_dynasty, last accessed 20160125
- [57] Wikipedia, *Records of the Three Kingdoms*,
http://en.wikipedia.org/wiki/Records_of_the_Three_Kingdoms, last accessed 20160125
- [58] Wikipedia, *Romance of the Three Kingdoms*,
http://en.wikipedia.org/wiki/Romance_of_the_Three_Kingdoms, last accessed 20160125
- [59] Wikipedia, *Timeline of the Three Kingdoms period*,
http://en.wikipedia.org/wiki/Timeline_of_the_Three_Kingdoms_period, last accessed 20160125
- [60] Wikipedia, *List of people of the Three Kingdoms*,
http://en.wikipedia.org/wiki/List_of_people_of_the_Three_Kingdoms, last accessed 20160125
- [61] Wikipedia, *List of fictional people of the Three Kingdoms*,
http://en.wikipedia.org/wiki/List_of_fictional_people_of_the_Three_Kingdoms, last accessed 20160125
- [62] Wikipedia, *List of fictitious stories in Romance of the Three Kingdoms*,
http://en.wikipedia.org/wiki/List_of_fictitious_stories_in_Romance_of_the_Three_Kingdoms, last accessed 20160125
- [63] Wikipedia, *Military history of the Three Kingdoms*,
http://en.wikipedia.org/wiki/Military_history_of_the_Three_Kingdoms, last accessed 20160125
- [64] Wikipedia, *Battle of Red Cliffs*,
http://en.wikipedia.org/wiki/Battle_of_Red_Cliffs, last accessed 20160125
- [65] Wikipedia, *Red Cliff (film)*,
[http://en.wikipedia.org/wiki/Red_Cliff_\(film\)](http://en.wikipedia.org/wiki/Red_Cliff_(film)), last accessed 20160125
- [66] Wikipedia, *Battle of Tong Pass (211)*,
[http://http://en.wikipedia.org/wiki/Battle_of_Tong_Pass_\(211\)](http://http://en.wikipedia.org/wiki/Battle_of_Tong_Pass_(211)), last accessed 20160125
- [67] The Dao of Strategy, *Zhuge Liang (Kong Ming): The Hidden Dragon of The Three Kingdoms*,
<http://daofstrategy.blogspot.nl/2012/11/zhuge-liang-kong-ming-hidden-dragon-of.html>, last accessed 20160125

- [68] *Dynamic Spectrum Access and Management in Cognitive Radio Networks*, Cambridge, 2009,
<http://ebooks.cambridge.org/chapter.jsf?bid=CBO9780511609909&cid=CBO9780511609909A043>,
last accessed 20160125
- [69] Ian Bogost, *Unit Operations; an Approach to Videogame Criticism*, MIT Press 2006,
http://http://www.skynet.ie/~ogami/notes/year_4/writing/Unit_Operations_-_An_Approach_to_Videogame_Criticism.pdf, last accessed 20160125