Incompatible Objects and Possible Languages

MSc Thesis *(Afstudeerscriptie)*

written by

Ruiting Hu

under the supervision of **Dr. Thomas Schindler**, and submitted to the Examinations Board in partial fulfillment of the requirements for the degree of

at the *Universiteit van Amsterdam.*

Date of the public defense:	**Members of the Thesis Committee:**
30-08-2024	Dr. Katia Shutova, Dr. Jeroen Smid, Dr. P.J.E. (Paul) Dekker



Institute for Logic, Language and Computation

Abstract

Eklund has put forward maximalism as an ontology for neo-Fregeanism (Eklund, 2006b). He claims this to be a promiscuous ontology that sanctions the existence of any object that is referred to by a singular term in a possible language. Like neo-Fregeanism, maximalism suffers a problem similar to the Bad Company problem. It is called the problem of incompatible objects. This thesis dives into the details of maximalism and seeks a solution for the problem of incompatible objects. Chapter 1 explores the cause for the problem of incompatible objects along with the metaonological principle that motivates maximalism. Chapter 2 considers Linnebo's critical liberalism (Linnebo, 2024) and quantifier variance (Eklund, 2021) as solutions to the problem of incompatible objects and argues that it is inadequate. Chapter 3 puts forward our two solution to the problem with one appealing to the existence of similar but different objects and the other to pragmatic indeterminacy.

Acknowledgements

I would like to express my sincere gratitude to Dr. Thomas Schindler, Tanja Kassenaar and Dr. Benno van den Berg for their continuous support, guidance, and encouragement throughout my study and a difficult period of my life. Special thanks to my family and friends for their unwavering support.

Contents

Acknowledgements **3**

Contents **5**

1 Maximalism and The Problem of Incompatible Objects **7**
1.1 Linguistic-First Metaontologies . . . 7
1.1.1 Basic Assumptions and Key Notions . . . 7
1.1.2 Neo-Qunieanism . . . 11
1.1.3 Neo-Fregeanism . . . 12
1.2 Priority Principle . . . 15
1.2.1 Priority and Syntactic Priority . . . 15
1.2.2 Theories of Truth . . . 17
1.2.3 Rivalries of Priority . . . 19
1.3 Maximalism . . . 21
1.3.1 Maximalism and Language . . . 21
1.3.2 Objections to Maximalism . . . 22
1.3.3 Deflationist Maximalism . . . 23
1.4 Incompatible Objects . . . 24
1.4.1 The Bad Company Problem . . . 24
1.4.2 The Problem of Incompatible Objects . . . 25
1.4.3 Eklund's Solution . . . 25

2 Two Solutions **27**
2.1 Critical Liberalism . . . 27
2.1.1 Grounding . . . 27
2.1.2 From Old to New . . . 28
2.1.3 Problem with All . . . 30
2.1.4 Overkill . . . 31
2.2 Quantifier Variance . . . 31
2.2.1 Warren's Quantifier Variance . . . 32
2.2.2 Objections to Quantifier Variance . . . 33
2.2.3 Quantifier Variance and Maximalism . . . 35

3 Two New Solutions **37**
3.1 The Similarity Solution . . . 37
3.2 Pragmatic Indeterminacy . . . 38

4 Conclusion **41**

References **43**

Chapter 1

Maximalism and The Problem of Incompatible Objects

This chapter is separated into four sections. In the first section we explain the linguistic-first trend in metaontology and introduce two linguistic-first metaontologies and the ontologies that they motivate. We will appeal to the features of these theories in explicating maximalism later on. In the second section we explain the metaontological principle that Eklund puts forward in which maximalism is couched. We will consider interpretations of it and explore its relation with theories of truth and theories of meaning. In the third section we present Eklund's argument for maximalism as an ontology for neo-Fregeanism. In the last section we present the problem of incompatible objects and Eklund's response to the problem.

1.1 Linguistic-First Metaontologies

Ontology is about what there is; metaontology is about the character of ontological claims (Eklund, 2006a). In particular, it is about the nature of key concepts involved in ontological claims such as "object", "property", "existence" (Hale & Wright, 2009). For example, mathematical realism, the view that mathematical objects such as numbers exist, is an ontological claim as it affirms the existence of mathematical objects. Wright's Syntactic Priority Principle, which says that objects are what are referred to by singular terms (Wright, 1983), is a metaontological claim as it is dictates what could be counted as an object.

In the recent decades, a general linguistic-first trend has been gaining popularity in metaontology. Roughly speaking, a linguistic-first metaontology looks at certain features of our language to answer the question of what there is. This stands in contrast to traditional metaontologies such as Aristotelian and neo-Aristotelian metaontologies which does not rely on language to discern what kinds of beings there are (Lowe, 1995). The linguistic-first spirit has great influence in contemporary metaontological and ontological debates.

In this subsection we introduce two major linguistic-first metaontologies and the ontologies they motivate: neo-Quineanism and neo-Fregeanism. The two theories admit book-length detailed explanations. It is neither realistic nor advantageous to our purposes to lay down all the details. Rather, we will introduce important aspects of orthodox versions of these theories and with the help of three notions such as: deflationist ontology, analyticity and realism.

1.1.1 Basic Assumptions and Key Notions

In this part we explain key notions such as analytic truth, the distinction between objects and concepts, metaontological deflationism. We assume the reader is familiar with classical first-order logic and second-order logic. Formally, $t = t$ is true if and only if t refers (Nolt, 2021). This adoption has a complication in that we have to restrict our introduction and elimination rules about the universal quantifier to the following (Linnebo, 2018):

$$\forall x(\phi(x) \wedge P(x)) \rightarrow \phi(t) \qquad (\forall \text{ E})$$

$$((\phi \wedge t = t) \rightarrow \psi(t)) \rightarrow (\phi \rightarrow \forall x \psi(x)) \qquad (\forall \text{ I})$$

This is because universal instantiation and generalizations have to be restricted with respect to referring terms.

Let us distinguish between different two kind of metaphysical realism about X where X is a certain kind of entity:

X exists. (Weak Realism about X)

X exists and its existence is counterfactually independent of intelligent agents, their mind and languages. (Weak Realism about X)

By counterfactually independence we mean that if intelligent agents, their mind and languages had not existed, X would still have existed. Unless otherwise pointed out, we will use "independence" to mean counterfactual independence. We will use "weak (strong) realism" when "X" refers to a general kind of entity such as objects (a notion we specify below) and use "weak (strong) mathematical realism" when "X" refers to mathematical objects.

Object and Concept

There are two popular notions of object in the literature. The first is to take objects to be entities in the broadest sense. Under this notion, properties would also be objects. There is another logical (syntactic) notion of object originating from Frege (Frege, 1951). Frege makes the distinction between concepts and objects. Roughly speaking, concepts are the referents of predicates which are unsatisfied semantic functions such as being red or runs. In this case, "concept" is interchangable with "properties" and saying an object having a property is the same as saying that it falls under a concept. Objects are the referents of proper names which satisfy predicates while concepts are the referents of predicates. Neo-Fregeans such as Wright develop on this notion and take objects to be referents of singular terms whereby a singular term is defined by the syntax of the language and its referential behavior (Wright, 1983).

For the sake of simplicity, throughout this thesis we focus on basic sentences, i.e., sentences consist of only subjects and monadic predicates. A subject is either a proper name such as Aristotle or a definite description such as the person who first landed on the moon. A monadic predicate is a predicate that takes only one subject such as being a man. We define an object to be the referent of a subject in a basic sentence of a possible language where reference is an expression-world relation. We define a concept (property) to be the referent of a predicate in a basic sentence of a possible language.

One immediate worry is that given the similarity of our definition to Frege's definition, our definition might be susceptible to the same problem as Frege's definition. Frege's definition for objects and concepts faces the concept horse problem. Frege subscribes to the claim that no concept can be an object and vice versa (Frege, 1951). Consider the sentence "The concept horse is a concept". The subject of the sentence "the concept horse" refers to the concept of being a horse. However, this is impossible since concepts could only be the referents of predicates and not subjects. Therefore, "the concept horse" can not refer to a concept. But what else should "the concept horse" refers to?

The concept horse problem generates an innumerous amount of discussion and numerous solutions have been proposed. The problem has been analyzed in detail by Hale and Wright (Hale & Wright, 2012), MacBride (MacBride, 2005), Hofweber (Hofweber, 2016) and Trueman (Trueman, 2020). We give a brief presentation of the problem and argue that it should be a concern to our project.

The issue is caused by three jointly contradicting claims

An entity is an object iff (if and only if) it is the referent of a proper name. (O)

O says that proper names refer to objects.

An entity is a concept iff it is the referent of a predicate. (P)

P says that predicates refer to concepts.

If two expressions co-refer, then they are everywhere interchangeable*salvacongruitate*. (RP)

RP (known in the literature as the Reference Principle) says that if two expressions co-refer, then they are interchangeable across all contexts without making syntactic errors. For example, "Aristotle" and "The famous tutor of Alexander the Great" are interchangeable *salvacongruitate* while "Aristotle" and "Being the famous tutor of Alexander the Great" is not because "Being famous tutor of Alexander the Great is Aristotle" is grammatically incorrect. The problem is that "The concept horse" and "Being a horse" intuitively co-refers to the concept of being a horse but they could co-refer given the three claims. This is because they are not interchangeable *salvacongruitate*.

Most of the commentators agree that the problem is that predicate reference could not be the same as proper name reference (Hale & Wright, 2012; MacBride, 2005). There is an unambiguous and clearly understood notion of proper name reference by which proper names refer to objects as first-order reference. It is predicate reference that generates the paradox because concepts seem to be both referred to by proper names and predicates. What kind of expression-world relation does concepts stand to predicates is an interesting but orthogonal problem to our project. Our project focuses on the notion of object and the existence of objects. The concept horse problem forces us to rethink the notion of concept instead of that of object. Thus, so long as the solution to the problem does not modify O, which most solutions do not as they tend to modify P (Trueman, 2020; Hale & Wright, 2012), the concept horse problem has no bearing on our project.

According to Lowe (Lowe, 1989), the notion of a sortal concept (kind), in its contemporary usage, could be traced to Peter Geach (Geach, 1980) who uses the phrase "substantial general term" instead of sortal concept. It might seem that Geach is describing a linguistic notion while we are describing a metaphysical one. But this is not the case because Geach takes substantial general terms to be like predicates which refer to sortal concepts. But today it is more common to say that concepts, instead of terms which refer to concepts, have criteria of identity. According to Geach, a sortal concept, like the concept of horse, has a criterion of identity by which we could tell whether two objects falling under the same concept (of the same kind).

We should be careful not to mistake a criterion of identity with another similar principle, namely Leibniz's Law ($\forall F(Fx = Fy) \leftrightarrow x = y$) which says that two objects are identical if they fall under all the same concepts. Leibniz's law is not a criterion of identity since it is not about the identity between two objects falling under the same concept but rather about any two arbitrary objects whatsoever. In contrast to sortal concepts are adjectival concepts such as the concept of red. There is no criterion by which we could tell whether two red objects are identical. [1] A simple way to distinguish sortal concepts from non-sortal concepts is to see whether we could count the number of its instances. For example, a non-sortal concept like redness does not allow counting of its instances. It makes no sense to ask how many red objects there are. A sortal concept like humans does allow so. It makes good sense to ask how many humans there are.

[1] Leibniz's Law might seem like a candidate but it is not since it is not a criterion of identity.

Analyticity

Most commentators today follow Boghossian in differentiating between metaphysical and epistemological analyticity. According to Boghossian, a sentence is metaphysically analytic if it is true in virtue of its meaning, i.e., does not depend on any facts. By fact, we mean a sui generis entity which that objects fall under concepts (Mulligan & Correia, 2021) unless otherwise noticed.[2] A sentence is epistemologically analytic if we are justified in accepting it as true in virtue of its meaning. This justification consists in an a priori replacement of synonyms for synonyms which results in an a priori knowable sentence (Boghossian, 1996). For example, under the metaphysical notion of analyticity, "Bachelors are unmarried men" is analytic because it is true in virtue of the meaning of "bachelor" and "unmarried men". Under the epistemological notion, we are justified in taking "Bachelors are unmarried" as true because "bachelors" and "unmarried men" are a priori synonyms and the result of replacing "unmarried men" with "bachelors" is "Bachelors are bachelors" which is a logical truth and thus a priori knowable.

Metaontological Deflationism

Following Eklund, let us call a deflationary view on ontology the view that ontological questions are not objective, genuine questions and philosophically significant. Since this is a view about ontology, we will call this view metaontological deflationism (Eklund, 2006a). There is another related view which Thomasson calls existence deflationism, the view that there are no across the board substantive criteria of existence (Thomasson, 2014) or simply put there is no deep nature of existence. By a substantive criterion of existence, Thomasson has in mind such conditions such as mind independence and causally efficacious (which means having causally interactions with other entities). Given that this is a view about the nature of existence, technically it is a metaontological view. To avoid confusion, we will stick with Thomasson's term can call this view existence deflationism throughout this thesis.

Metaontological deflationism was brought into attention in the twentieth century by Carnap (Eklund, 2023). Carnap makes the distinction between internal and external questions about existence to a framework where a framework is taken as language (fragments) (Eklund, 2016a). We introduce this distinction here because it will be helpful to our thesis in Chapter 3. We will follow Eklund's interpretation of Carnap. According to Eklund's interpretation of Carnap, ontological questions can be set into three camps (Eklund, 2016a):

1. Internal questions, which are internal to a framework and ask whether certain objects exist given a framework. Internal questions are legitimate.

2. External factual questions, which are external to a framework and ask whether certain objects exist from a God's-eye perspective outside of any framework. External factual questions are illegitimate.

3. External pragmatic questions, which are external to a framework and ask which framework is most useful to employ. External pragmatic questions are legitimate.

Take the question of whether there are numbers and the framework to be the language of Peano Arithmetic (PA). According to Carnap, this question is only legitimate when asked as an internal question or as an external pragmatic question (Carnap, 1950). When asked as an internal question, the answer is trivially true because the axioms of PA implies the existence of numbers. When asked as an external factual question, which is what mathematical weak realist and mathematical anti-realist tend to ask in their debate as whether numbers "really" exist independent of any framework, the answer is that the question is illegitimate. The only legitimate external question that could be asked about whether numbers exists is the external

[2]We omit objects having relations with other objects since we only consider monadic concepts while relations could be seen as concepts involving more than one object.

pragmatic question of whether it is useful to adopt a weak realist framework in which the internal question of whether numbers exist is trivially true or an anti-realist framework in which the internal question of whether numbers exist is trivially false.

Carnap's original view explains the triviality of internal questions by appealing to the metaphysical analyticity of inference of existence claims from assumptions about the framework. Carnap's justification for his view has fallen out of favor since Quine's famous attack on metaphysical analyticity (W. V. O. Quine, 1951). But the metaontological deflationism that the view entails survives and is picked up by later commentators on ontologies such as Putnam (Putnam, 1991) and Hirsch (Hirsch, 2002). We will introduce their view in Chapter 3.

1.1.2 Neo-Qunieanism

Neo-Quinean Metaontology

In this subsection we focus on the most prominent presentation of neo-Quinean metaontology proposed by van Inwagen (Inwagen, 1998). Van Inwagen lists five theses for the neo-Quinean metaonology: being is not an activity; a criterion of ontological commitment; being is the same as existing; existing is univocal and existence and being is captured by first-order quantification. We consider the last three of these claims as the rest two are not immediately relevant.

Being is the same as existing is addresses a concern that we put off previously. The concern is about reference to non-existent objects. We constantly talk about non-existence objects such as Pegasus. Assume that sentences such as "Pegasus is a flying horse" is meaningful, then it seems that the proper name "Pegasus" is meaningful. The meaning of a proper name is to refer to an object. Thus, "Pegasus" refers to an object (Reicher, 2022). If reference implies existence, then "Pegasus" exists after all. To avoid this complication, people like Meinong distinguish between an object's being and existing (Meinong, 1960). Given this distinction, we could say that reference implies being instead of existence. This view of separating being from existence is no longer prevalent in contemporary debates. It suffices for our purpose to note that all the metaontologies and ontologies we canvass reject such a distinction.

We say that existence is univocal if there is only one meaning of the word "exist" or rather one kind of existence. We will focus on what single notion existence is said to be. According to van Inwagen, the meaning of existence is captured by the first-order existential quantifier (Inwagen, 1998). This is supported by Quine's famous slogan "*To be is to be the value of a bound (first-order) variable.*" (W. V. Quine, 1953).

For example, the Quine-Putnam indispensability argument says that mathematical objects such as numbers exist because mathematical theories such as number theory are indispensable to our best scientific theories of the world (Putnam, 1971). Take the third Peano Axiom which is an axiom of number theory.

$$\forall x(0 + x) = x \tag{Ax3}$$

This implies that

$$\exists x x = 0 \tag{0}$$

Since the interpretation of **0** requires the domain of interpretation to include 0. Thus, according to the neo-Quinean, 0 must exist.

Neo-Quinean Ontology

Given the thesis that existence is captured by first-order existential quantifier, the neo-Quinean ontology answers the question of what objects exist by finding adequate formal theories (formalized in classical first-order logic) that best describe the world and then considers the domain of quantification. Everything in the domain of quantification exists. Traditional Quineans, following Quine himself, take our best scientific theories of the world to be those that best describe

the world (W. V. O. Quine, 1951). Neo-Quineans like van Inwagen tend to allow philosophical theories to play a part alongside scientific theories (Inwagen, 1990). Whatever strand one takes, the neo-Quinean ontology's reliance on our best scientific theory of the world makes it austere because existence hinges upon our best theories of the world.

There is no doubt that the neo-Quinean ontology counts as weak realism. Whether it counts as strong realism is an open matter. Quine himself is not clear on the view as he tends to shift between strong realism (W. V. O. Quine, 1995) and weak realism (W. V. Quine, 1992). Whether the neo-Quinean ontology is deflationary is settled. Neo-Quineans are against metaontological deflationism (van Inwagen, 2019) because the neo-Fregean metaontology affirms an across board substantive criteria of existence, that of being quantified over by the best theories of the world (Thomasson, 2014).

This view has been challenged recently as Marshall and Schindler argue the Quinean metaontology does not entail a substantive criterion of existence. Their argument is that being explanatory for the best scientific theory of the world does not constitute a deep nature of existence (Marschall & Schindler, 2020). This is because objects exist before the existence of theories that quantify over them and would have existed if there were not theories that quantify over them (Marschall & Schindler, 2020). But this assumes that the neo-Quinean ontology is strongly realist as it assumes the objects could exist independent of intelligent agents and their best theories of the world. This entails that there is a substantive criterion of existence, namely that of being mind and theory-independent. Thus, we conclude that the neo-Quinean ontology is not deflationary.

1.1.3 Neo-Fregeanism

Neo-Fregeanism, put forward as an substantial thesis by Wright and Hale (Wright, 1983) is a revival of the Fregean project of logicism (Frege, 1951). Frege aims to reduce mathematics to logic in the sense of finding a proof of mathematical theorems based purely on logic and definitions (Frege, 1951) We need to be bogged down by the details of logicism. It suffices for our purposes to know that a major part of neo-Fregeanism, neo-logicism, is the revival of Frege's project of reducing mathematics to logic. This part won't be the focus of our discussion. What is of our interest is the mathematical strong realism (that neo-Fregeans put forward and its generalization, known as abstractionism which aims to supply on ontology for abstract objects. We will begin with the key notions of implicit definition.

Another assumption is that the neo-Fregeans tend to use negative free logic which is to say that atomic sentences involving non-referring terms are false and thus $t = t$ (where t is a term) is not a trivial truth but depends on t being referring expressions.

Implicit Definition

One notion that is central to the neo-Fregean program is implicit definition. Yet, there are at least two different notion of implicit definition. There is a technical definition within mathematical logic. We assume the reader has basic knowledge of proof theory and model theory. The following definitions are taken from *Routledge's Encyclopedia of Philosophy* (Swijtink, 1998). Let L be a first-order language, P be a predicate and $T(P)$ be a first-order theory in which P occurs, $T(P)$ explicitly defines P if there is $\phi(x)$ in L such that $T(P) \vdash \forall x(P(x) \leftrightarrow \phi(x)$. Let T be the set of first-order consequence of $T(P)$ in which P does not occur, $T(P)$ implicitly defines P if for every model M of T, there is a unique extension of N of M such that N is a model of $T(P)$. Beth's definability theorem for first-order logic says that $T(P)$ explicitly defines P if and only if $T(P)$ implicitly defines P. This technical notion is not the one what is evoked by the neo-Fregeans.

According to Wright and Hale, an explicit definition is a semantically equivalent (synonymous or co-referential) expression of the same syntactic category of the definiendum (Hale & Wright, 2000). For example, an unmarried man is an explicit definition of bachelor. Implicit definition is the complement of explicit definition, i.e., expressions that are not semantically

equivalent or belong to a different syntactic category of the definiendum (Hale & Wright, 2000). While both forms of definition are to fix the meaning of the defined concept, implicit definition is needed when explicit definitions are unavailable. This happens when we define fundamental concepts in logic, mathematics and scientific theories in general. For example, the logical operators are defined by using introduction and elimination laws. There is no explicit definition that tells us what is conjunction or disjunction. Rather, we fix the meaning of these concepts by understanding how to apply them in our logical inferences.

Neo-Fregean Ontology

Compared to neo-Quineans, neo-Fregeans have less agreement on metaontology as commentators offer different accounts of neo-Fregean metaontology (Eklund, 2016b). All the different accounts seek to justify the neo-Fregean ontology which is strong realism. Thus, we will begin with the neo-Fregean ontology also known as abstractionism in the literature. The neo-Fregean ontology (championed by Hale and Wright in its orthodox form) claims that abstract objects, by which we mean pure abstract objects such as pure sets or numbers which are non-spatio-temporal and causally inefficacious, are referred to (and thus exist as the neo-Fregeans also do not distinguish between being and existence) by expressions functioning like singular terms if there are true sentences in which these expressions are introduced by suitable abstraction principles (Hale & Wright, 2009). An abstraction principle (AP) is of the form:

$$\forall\alpha\forall\beta(\S\alpha = \S\beta \leftrightarrow \alpha \sim \beta) \qquad (AP)$$

where α, β are expressions of a certain type such as singular terms or predicates and $\S$ a term-forming operator applicable to α, β resulting in a singular term; $\sim$ is an equivalence relation (reflexive, transitive and anti-symmetric) and $\S\alpha, \S\beta$ refers to abstract objects (Wright, 1983).

The main focus of neo-Fregean ontology has been around a particular version of AP, Hume's Principle (HP):

$$\forall F\forall G(\#F = \#G \leftrightarrow F \cong G) \qquad (HP)$$

where $\#$ is the number-of operator, taking a predicate to the number of instances of the sortal concept referred to by the predicate and $\cong$ the relationship of being equinumerous where by two concepts are equinumerous if there is a one-to-one and onto function from the objects falling under one to those falling under the other. In plain words, HP says that the number of Fs equals the number of Gs if Fs and Gs are equinumerous. HP is a criterion of identity for the sortal concept of numbers.

Frege's logicism (fregeanism) aims to reduce theories of arithmetic to axioms of logic and thus showing that theories of arithmetic are analytic (Ebert & Rossberg, 1964). His project fails but the project is continued by neo-Fregeans. A major victory for the neo-Fregeans is known as Frege's theorem which states that second-order logic with HP proves Peano postulates (Wright, 1999).[3] In plain words, Frege's theorem says that we could derive theories of second-order arithmetic (analysis) from HP within second-order logic. Given this result, if HP is analytic (classically analytic according to Wright (Wright, 1999) which means the same as metaphysically analytic), then we have reduced at least a significant part of mathematical theories to logic. Of course, this depends on second-order logic being logic in the proper sense. Objections have been raised against second-order logic's logical status. Quine famously argues that second-order logic is first-order logic with set theory (W. V. O. Quine, 1970). Following most contemporary commentators of neo-Freganism, we will make no objection to the logicality of second-order logic.

The neo-Fregean ontology is more promiscuous compared to the neo-Quinean ontology because it accepts the existence of abstract objects given that there are adequate abstraction

[3]We do not list the Peano postulates and the proof of Frege's theorem. Readers who wish to see the details could check Boolos' 1996 (Boolos, 1996).

principles that could define the said objects into existence. However, the neo-Fregean ontology does not license the existence of any abstract object whatsoever (Hale & Wright, 2009) as not all abstraction principles are "good". This is known as the Bad Company problem which we will discuss later when it becomes more relevant to our project.

Neo-Fregean Metaontology

As we have seen above, the neo-Fregean metaontology needs to allow the inference from HP to the existence of numbers. This is done in two steps. The first step is the appeal to Wright's Syntactic Priority principle which we mentioned above. In Wright's own words (Wright, 1983):

> *... that a given class of terms are functioning as singular terms, and when it has been verified that certain appropriate sentences containing them are, by ordinary criteria, true, then it follows that those terms do genuinely refer.*

The idea is that a singular term refers if it features in a true sentence of the right kind. According to the neo-Fregeans, the right kind of sentence, in the case of numbers, is the equality on the left hand side of HP. The second step is to justify its truth without making reference to numbers. This is achieved by HP. The truth of the left hand side of HP is entailed by the truth of the right hand side of HP and the truth of HP.

As an example, assume that there are two tables and two chairs. Let F be the concept of being a table and G be the concept of being a chair. According to HP, the number of tables equals the number of chairs if and only if each table is matched to a unique chair and vice versa. The truth of the right hand side of HP, namely that each table is matched to a unique chair and vice versa does not involve reference to numbers at all.

Following in Linnebo, let us call an abstraction principle ontologically symmetric if the sentence on the left side of the biconditional has the same ontological commitment as that on the right (Linnebo, 2018). A sentence is ontologically committed to the referents of singular terms in it. Call an abstraction principle asymmetric if the left side of the biconditional has more ontological commitments compared to the right. Similarly, call an abstraction epistemologically symmetric if recognizing the truth of the sentence on the right side of the biconditional suffices for recognizing of that on the left side and asymmetric if recognizing the truth of the sentence on the right side of the biconditional does not suffice for recognizing of that on the left side.

The core of neo-Fregean metaontology, known as recarving of content in the literature, is the idea that abstraction principles like HP recarves the content on the right hand side of the biconditional into the content of the left hand side. What exactly content means is controversial (Eklund, 2020). Given the controversy regarding content, Wright and Hale appeal to the notion of implicit definition as the one we introduced above. The idea is that HP is an implicit definition for the concept of number (Hale & Wright, 2000). Thus, it is a priori that if we know the truth of the right hand side of HP, we know the truth of the left hand side of HP. Of course this use of implicit definition is not without its problems. The so called Julius Caesar problem objects that HP alone could be an implicit definition of the concept number because it is only a criterion of identity, not a criterion of application[4] which tells whether a given object falls under a concept (MacBride, 2006).The problem has its name from the question that whether Julius Caesar is a number. If HP could not tell us this, then HP can not be an adequate definition for the concept of being a number. We won't dive deep into this problem here as it is not of immediate relevance to our project. It suffices for our purpose that both content recarving and implicit definition aim to justify that abstraction principles like HP are epistemologically symmetric. From now on we will speak of this sort of justification as reconceptualization.

[4]For example, the criterion of application for the concept of being a H2O molecule is for 2 Hydrogen atoms and 1 Oxygen atom to be connected in a certain way.

1.2 Priority Principle

In this section we introduce the Priority Principle (PR) which Eklund motivates as the metaontology for neo-Fregeanism. In subsection 1 we explain what are the possible readings of PR and compare PR with its neo-Fregean cousin, the Syntactic Priority Principle of Wright. In subsection 2 we explore what theories of truth are compatible with PR. In subsection 3 we discuss some rival metaontologies for neo-Fregeanism and PR's advantage over them.

1.2.1 Priority and Syntactic Priority

In this subsection we begin with the three possible readings of the Priority Principle, followed by the exploration of the relationship between the Priority Principle and Wright's Syntactic Priority Principle.

Three Different Readings

Eklund gives a concise definition of what he calls the Priority Principle.

Truth is constitutively prior to reference. (PR)

. It is worth reminding that even though PR says nothing about existence, it is a metaontological principle because Eklund, in line with the neo-Fregeans and neo-Quineans, denies the possibility of reference to non-existent objects.

There are at least three possible different readings of PR: the ontological reading, the explanatory reading and the epistemological reading. To simplify the matter, we will only consider atomic sentences, i.e., sentences of the form $P(a)$ or $a = b$ where P is a monadic predicate and a, b singular terms. On the ontological reading, PR says that if a sentence is true, then the singular term of that sentence refers. On the explanatory reading, PR says that truth is explanatorily prior to reference, which is to say that reference is explained by appealing to truth or rather that it is in virtue of a sentence being true that the singular terms in it refer. The explanatory reading entails the ontological reading. It is clear that if reference of singular terms in a sentence is to be explained by the sentence's being true, then the sentence's truth implies its singular terms having reference. On the epistemological reading, PR says that the recognition of the truth of a sentence implies the recognition that singular terms in that sentence refer.

Since PR is a metaontological thesis, it seems natural to take the ontological reading. Yet, Eklund appears to hold a different view. The ontological reading, according to Eklund, is not so radical (Eklund, 2006b):

> *One claim here is that there is an object to which a given singular term refers if certain sentences in which the singular term occurs in the right way (paradigmatically, identity sentences) are true.This by itself, although it can be denied, is not a terribly radical thesis.*

As we mentioned before, neo-Fregeans tend to take their ontologies to be realist (Linnebo, 2018; Wright, 1983) in the sense that the existence of abstract objects is independent of intelligent agent and their languages and minds. One major reason that Eklund find's the neo-Fregean metaontology inadequate is that it is not very effective against nominalism, the view that abstract objects such as numbers do not exist (Eklund, 2016b). The reason that the ontological reading is not so radical is because a nominalist could agree with the neo-Fregean that truth is ontologically prior to reference but deny that sentences with singular terms seeking to refer to abstract objects are true because the objects these terms refer to do not exist.

The issue is delicate. Let us use the example of "Socrates is a philosopher". Let a realist be the person who claims that Socrates exists and a nominalist the person who denies that Socrates exists. The realist starts by claiming that "Socrates is a philosopher" is true and by

the ontological reading of PR "Socrates" refers in virtue of it featuring in the true sentence "Socrates is a philosopher". This is the same as saying that "Socrates is a philosopher" being true suffices for "Socrates" to refer, which implies that Socrates exists. This could be regimented into propositional logic as:

$$S \rightarrow R \rightarrow E \tag{RR}$$

where "S" is short for the sentence "Socrates is a philosopher", "R" is short for "'Socrates' refers" and E is short for "Socrates" exists.

The nominalist begins by denying that Socrates exists which implies that "Socrates" does not refer. The nominalist goes on to say that 'Socrates is a philosopher" is not true because "Socrates" does not refer. This could be regimented into propositional logic as:

$$\neg E \rightarrow \neg R \rightarrow \neg S \tag{NR}$$

While the realist is using the ontological reading of PR, the nominalist is using its contrapositive which is logically equivalent. Thus, under the ontologically reading there is no argument against the nominalist.

The radical thesis that Eklund puts forward is that truth is "constitutively prior" to reference in the sense that it is in virtue of a sentence being true that the singular term in it refers. The nominalist explains the falsity of S by appealing to the falsity of R while the realist explains the truth of R by appealing to the truth of S. While A is explanatorily prior to B could be formalized as $A \rightarrow B$ which entails $\neg B \rightarrow \neg A$, the last sentence could not be translated back to natural language using the notion of explanatory priority. It is not the case that $\neg B$ is explanatorily prior to A. To illustrate with a simpler example, it is logically equivalent to say that if one eats, then one is not hungry and to say that If one is hungry, then one does not eat. It makes sense to say one's eating explains/is constitutively prior to one's not being hungry one's eating one's being not hungry. However, it does not make sense to say that one's hunger is constitutively prior to one's not eating.

It is noteworthy that explanatory priority is an asymmetric relationship, i.e., it could not be the case that truth is explanatorily prior to reference and reference is explanatorily prior to truth.[5] It is only by holding that truth is explanatorily prior to reference that the neo-Fregean could accuse the nominalist of putting the cart before the horse and thus undercutting the nominalist challenge (Eklund, 2006b). We will follow Eklund, in taking on the explanatory reading of PR throughout this thesis.

Syntactic Priority

PR is very similar to Wright's Syntactic Priority Principle (SP) which we introduced earlier. One reason Eklund gives in support of PR being the new neo-Fregean metaontology is that a similar idea is put forward by Wright in the Syntactic Priority Principle (Eklund, 2006b). Recall that SP says if a singular term appears in a true sentence of the right kind, then it refers. Without the word "constitutively" and further evidence from Wright, it is natural to read SP ontologically. However, we will give a charitable interpretation and take the explanatory reading of SP according to which SP says that It is in virtue of a singular term appearing in a true sentence of the right kind that it has reference.

Be that as it may, we could still draw a significant distance between SP and PR. The problem is what counts as the right kind of sentences. PR does not restrict the kind of true sentences that singular terms appear in. On the other hand, Wright restricts the right kind of sentences to identity statements on the left hand side of abstraction principles (Hale & Wright, 2009). Given our restricted scope on atomic sentences, the difference between SP and PR is that PR allows true sentences of the form $P(a)$ or $a = b$ while SP only allows true sentences

[5]People who accept self-explanatory concepts might object that it explanatory priority could be symmetric and transitive. It follows that truth and reference are both self-explanatory. We rule out this possibility in our thesis as it strikes us to be counter-intuitive about the notion of explanatory priority.

of the form $\S\alpha = \S\beta$. This is due to the fact that the neo-Fregean ontology is tailored to the neo-Fregean ontology which is centered around abstraction principles. The implication of such a centering will be discussed later. For now it suffices to note the difference.

1.2.2 Theories of Truth

If we are asked what does PR mean, a satisfactory answer must explain the meaning of truth or the meaning of reference or both. Under the explanatory reading of PR, we take PR to be a theory of reference that explains the nature of reference in terms of truth, thus we make no assumptions about reference aside from taking reference to be a property of singular terms such that a referring singular term implies existence of its referent. Whether such it is a substantive property or a deflationist one is not of our present concern but will be addressed later. It is not possible in this thesis to survey all theories of truth due to the richness of the area. Instead, we will lay down the conditions that a theory of truth has to meet in order to be compatible with PR. This is followed by one popular theory of truth that fails to be compatible with PR: the correspondence theory of truth where truth is correspondence to reality, one popular theory of truth that is compatible with PR given certain conditions: disquotationalism theory of truth and a theory of truth by truth is correct assertion which Eklund employs.

In contemporary discussion regarding theories of truth, it is common to make the distinction between a substantial theory of truth and a deflationist theory of truth (Glanzberg, 2023). Substantial theories of truth tend to take truth as a substantial property such as a word-world relation or assertibility conditions (which we will explain later) while deflationist theories of truth take truth to be insubstantial such as a tool of disquotation (Horwich, 1990). The correspondence theory of truth and Wright's theory of truth are both substantial theories of truth. We will see shortly that deflationist theories of truth are compatible with PR given certain assumptions. It is substantial theories of truth that tend to incompatible with PR

A helpful way to analyze a theory of truth is to do so in terms of three key notions: truth-bearer, truthmaker and truth condition (Glanzberg, 2023). A truth-bearer is an entity that could be said to be true. Typical candidates for truth-bearer are sentences, propositions (the content expressed by sentences) and beliefs. Truth-bearers are irrelevant as far as compatibility with PR is concerned except that in line with our previous assumptions we take truth-bearers to be sentences instead of propositions. Our focus will be on truthmaker and truth condition. A truthmaker is an entity by virtue of which a truth-bearer is true (MacBride, 2022). In other words, it is an entity that explains the truth of a truth-bearer. Typical candidates for truthmakers are facts, states of affairs and objects. A truth condition determines under what circumstances that a truth-bearer could be true (Glanzberg, 2023). It does so partly by determining when a truth-bearer is meaningful.

Given that explanatory priority is asymmetric, any theory of truth that explains truth in terms of reference or whose definition involves reference would be incompatible with PR. A first minor condition is that truth-bearers should be sentences since PR is defined in terms of truth of sentences. A second major condition is that we should expect reference to play a role in neither of the three notions introduced above. But it is not only reference that is at stake here.

Recall PR, as a metaontological principle, aims to explain the nature of existence by way of reference, i.e, says what makes existence claims true. Although PR stops at reference, there is the implicit assumption that reference explains existence. Thus, any theory of truth that is compatible with PR should not have the existence of objects as part of truthmakers. That is to say, it can not be objects that make truth-bearers true.

The Correspondence Theory of Truth

The most common and intuitive theory of truth is the correspondence theory which says that truth is a correspondence between sentences and reality. It is helpful to distinguish between

an object-based version and a fact-based version of a correspondence theory of truth (David, 2022). They differ in what truthmakers are.

The object-based version states that for a sentence, as a subject-predicate structure, to be true is for there to be an object referred to by the subject falling under the concept[6] exemplified by the predicate. We use the term "exemplified" in the neutral sense as we have seen that the relationship between predicates and concepts is not relevant to our discussion. In this version, truthmakers are objects referred to by singular terms. For example, the sentence "Socrates is a philosopher" is true if only if Socrates exists and Socrates falls under the concept of being a philosopher. It is clear that the object-based version is not congenial to PR because the truth condition of a sentence assumes that the singular terms in the sentence refer. In the example above, "Socrates" has to refer before we can say anything about whether Socrates falls under the concept of being a philosopher.

The fact-based version states that for a sentence to be true is for it to correspond to a fact which is taken to be an entity in its own right (Glanzberg, 2023). This fact-based version might seem innocuous at first but it is still incompatible to PR. This is because we have in the beginning defined facts to be the kind of entities in which objects occur. Thus, the fact-based version violates the rule that existence of objects should not be assumed in truthmakers. But perhaps there are other notions of facts that do not involve objects. There are two other possible notions of facts: facts as true truthbearers and facts as obtaining states of affairs. The first notion is not possible since we are using facts to give a theory of truth. The second notion is also not helpful in that state of affairs typically involves objects as well(Textor, 2021).

The Disquotationalist Theory of Truth

We have briefly introduced deflationist theories of truth in the previous part. Now it is proper to give a more detailed introduction. During the past decades, there has been a rising popularity of deflationist theories of truth (deflationist theories for short). A deflationist theory claims that truth is not a substantial relation such as correspondence between sentences and reality. The predicate "True" is merely a tool of disquotation. For example, without the predicate "True", we will not be able to say that sentences like "Everything John said last night is true" without explicitly mentioning each sentence that John said last night. The most prominent deflationist theory is put forward by Paul Horwich which he calls the minimalist theory of truth. However, Horwich takes truth-bearers to be propositions (Horwich, 1990), this makes his particular version of deflationism, known as minimalism unfit for PR. But there are deflationist theories which take truth-bearers to be sentences. For example, the disquotationalism first proposed by Quine: (W. V. O. Quine, 1970)

$$< s > \text{ is true iff (if and only if) } s \qquad (T)$$

s is any sentence. In the literature, T is called the Tarski scheme as it is a collection of axioms each of which is an instance where s is replaced by a sentence. For example, "Socrates is a philosopher" is true if and only if Socrates is a philosopher.

Since deflationist theories like disquotationalism claim that the nature of truth is exhausted by T, they have a very high hope of being compatible with PR. After all, if a theory says very little about the nature of truth, it is unlikely that it will even mention reference. In particular, unlike substantial theories of truth, deflationist theories of truth says nothing about truthmakers and truth conditions. Therefore, it seems that deflationist theories in which truth-bearers are sentences, like disquotationalism, are compatible with PR.

Although a deflationist theory of truth does not say anything about reference, a deflationist theory of reference does. And for some deflationists such as Horwich, the two go hand in hand (Horwich, 1990). A deflationist theory of reference, like a deflationist theory of truth, says that reference is not a substantial relation between semantic expressions and the external reality.

[6] A more intuitive way is to say "having the property" instead of "falling under the concept".

It can not be reduced to something else. The meaning of "refer" is exhausted by the follow schema

$$\forall x(< n > \text{refers to } x \leftrightarrow n = x) \tag{1.1}$$

where $< n >$ is a token. Whether this contradicts PR depends on whether PR counts as a contribution to the meaning of "refer". If it does, then PR implies that the meaning of "refer" is not exhausted by the disquotation schema and thus contradicts deflationist theories of reference. But if PR does not count as a contribution to the meaning of "refer", then deflationist theories of reference need not conflict with PR and thus neither does deflationist theories of truth in which truth-bearers are sentences such as disquotationalism.

Truth as Correct Assertion

The theory of truth that Eklund uses himself in discussing PR is truth as correct assertion. This notion of truth as correct assertion and truth condition as correctness condition could be traced to Dummett. An assertion is a speech act of saying that s is the case where s is a sentence. According to Dummett, the truth of atomic sentences is the correctness of asserting them (Dummett, 1976) For example, "Socrates is a philosopher" is true because it is correct to assert that Socrates is a philosopher is the case. Correctness conditions are roughly the patterns of use governing when the assertion of a sentence is accepted by a linguistic community and when the assertion of a sentence is rejected by a linguistic community.

Whether the theory of truth as correct assertion is compatible with PR depends on whether the singular terms in a sentence being referring expressions contribute to the acceptance of the sentence's assertion by the native linguistic community in which the sentence is asserted. If there is such a contribution, then reference is involved in explaining the truth of the sentence, which is incompatible with PR. An intuitive argument affirming the contribution of reference is that a major use of singular terms is to refer. After all, the primary use of "Socrates" is to refer to the famous ancient Greek philosopher and it is natural to think that one's accepting it is the case that"Socrates drank the hemlock" depends on the existence of Socrates. It is natural to have a realist picture of use in which the acceptance of assertion of sentences containing proper names depends on/is explained by the proper names being referring to expressions. However, such a case is not the case for proper names that aim to refer to abstract objects. For example, by way of HP, the acceptance of "The number of tables equals to the number of chairs is explained by the acceptance of "Tables are equinumerous with chairs" which does not depend on reference to numbers at all. However, it is doubtful that such arguments could be made for assertions involving singular terms aiming to refer to concrete objects which are spatio-temporal and causally efficacious without being an anti-realist.

1.2.3 Rivalries of Priority

As we have noted when we introduced the neo-Fregean metaontology, PR is not the only neo-Fregean metaontology. In this subsection we consider two rival neo-Fregean metaontologies and present Eklund's arguments of preferring PR over them.

Quietism

A major problem for the neo-Fregean ontology is the nominalist scepticism that there are no abstract objects. The problem begins, according to MacBride, with the seemingly truism that either reality is crystalline or it is plastic (MacBride, 2016). By saying reality is crystalline, MacBride means that the existence of objects such as numbers are independent of the intelligent agents and their languages and minds. If not, then reality is plastic (MacBride, 2016). If reality is crystalline, there is the worry that numbers might not exist and thus HP is not true. This worry is based on the assumptions that HP is not metaphysically analytic but only epistemologically analytic. The rejection of metaphysically analyticity is popularized by

Boghossian (Boghossian, 1996). Without PR, the nominalist could claim that abstract objects such as numbers do not exist. This implies that the equality on the left hand side of HP does not hold.[7] Given the truth of the right hand side of HP, HP is false.

If, on the other hand, reality is plastic, then neo-Fregeanist ontology will not be a realistic one in the sense that the existence of numbers depends upon the languages and minds of intelligent agents. Since neo-Fregeans claim that their ontology is a realist (Wright, 1983; Linnebo, 2018), it seems that neither option is acceptable. The motivation behind this dilemma, according to MacBride, is the need of explaining how language "hooks on" to reality (MacBride, 2016). Thus, MacBride takes the solution to be that such questions are illegitimate because they are asked from a perspective outside of language which he claims to be impossible to take and thus there is not ontological gap that neo-Fregeans need to fill between language and reality (MacBride, 2003).

Eklund's main complaint against quietism is that it moves too quick from the inability to ask a question regarding how language is connected to reality and the lack of ontological gap between language and reality. The former inability, according to EKlund, is an epistemological inability that we are not in a position to know the relationship between language and reality from an perspective above language (Eklund, 2020) There is a strong intuition in support of epistemological thesis because in a sense that language is the bedrock of our thought and thus our knowledge. Yet, this epistemological thesis does not imply the metaphysical thesis that there is no ontological gap between language and reality (Eklund, 2020). Just like one might accept HP as an epistemological thesis about how we could know the truth of numerical identity claims while still in doubt of the metaphysical thesis that numbers exist, we could accept the epistemological thesis that we could not know the relationship between language and reality from an external perspective while accepting the metaphysical thesis that there is a fact of the matter whether language matches reality that is independent of us.

Reconceptualization

As we introduced before, the orthodox neo-Fregean metaontology put forward by Wright and Hale is a combination of the Syntactic Priority Principle and reconceptualization. Reconceptualization, as envisaged by neo-Fregeans, is the epistemological symmetry of abstraction principles. Recall that the neo-Fregean argument for the existence of numbers is that the left hand side of HP is a reconceptualization of the right hand side of HP which is to say that to know the right hand side of HP is true is to know that the left hand side of HP is true. Given the truth of the left hand side of HP, the existence of numbers follows from SP. In this way, HP is ontologically asymmetric.

Eklund's complaint is that it is paradoxical for the neo-Fregeans to hold that HP is epistemologically symmetric while ontologically asymmetric (Eklund, 2006b). For HP to be epistemologically symmetric is for the two sides of HP to have the same content. Here Eklund is working under the assumption that the sameness of content of two sentences requires the two sentences to be ontologically committed to the same objects. This entails that ontological symmetry follows from epistemological asymmetry. Thus, the neo-Fregeans could not hold the ontological asymmetry and the epistemological symmetry of HP at the same time. The ontological asymmetry of HP implies its epistemological asymmetry.

Even if the neo-Fregans come up with a notion of content that allows the two side of HP to be having the same content while being ontologically asymmetric, neo-Fregeans still needs to explain what makes abstraction principles so special that they are necessary for reference to abstract objects. The orthodox neo-Fregean metaontology and PR can agree that abstraction principles like HP are sufficient for explaining reference to abstract objects. To differentiate itself from PR, the orthodox neo-Fregean metaontology needs to offer an argument for the necessity of abstraction principles in securing reference to abstract objects (Eklund, 2016b).

[7] We are of course denying the idea that $a = a$ can be true without 'a' referring.

Yet, the neo-Fregeans give no such argument. There is another complication with the reliance upon abstraction principles.

1.3 Maximalism

Eklund uses PR to motivate what an neo-Fregean ontology which he calls maximalism, a promiscuous ontology. In this section we give a detailed explanation of maximalism, followed by some objections to maximalism and Eklund's response to them. Last but not the least, we consider the two readings of maximalism: a realist one and a deflationist one.

In his 2006, Eklund defines maximalism to be the view that (Eklund, 2006b)

> For any sortal concept F, Fs exist if the hypothesis that Fs exist is consistent and Fs do not fail to exist as a matter of contingent empirical fact. (Max)

The key notions in understanding Max is consistency. Eklund does not specify what kind of consistency is needed. A first guess is logical consistency. For now such a notion is fine.

According to Eklund, maximalism, as an ontology, should give us the existence of concrete objects such as tables, abstract objects such as numbers and counter-intuitive objects such as incars (which is an object like a car but only exists inside a garage) but not fictional objects such as Yetis (a legendary creature living in the Himalayan) (Eklund, 2006a). The hypothesis that Yetis exist is clearly logically consistent. This is where the second condition comes in to rule out the existence of Yetis because they fail to exist as a matter of contingent empirical fact.

However, the problem of this formulation is that it is hard to see how Max is connected to PR. After all, Eklund claims that PR motivates Max. PR is a principle that explains reference in terms of truth but the notion of truth is nowhere to be found in Max. It is helpful to see what other definitions EKlund has for maximalism. Elsewhere, Eklund says that (Eklund, 2006b)

> *... there is the way suggested by maximalism, as motivated by priority: what there is, is a matter of what can be successfully referred to in some possible language or other.*

This definition of maximalism could be formulated as saying that for any sortal concept F, Fs exist if there the hypothesis that Fs exist is true in some possible language and that Fs do not fail to exist as a matter of contingent empirical facts where facts are taken as true truthbearers. Given PR, the truth of the hypothesis that Fs exists in a possible language constitutes F being a successful referring singular term. Adding the claim that what is possible to exist exists, maximalism gives us the promiscuous ontology that Eklund intends. However, what is a possible language is still unexplained. This is what we intend to do next.

1.3.1 Maximalism and Language

A standard formal language L consists a set of symbols, a syntax dictating what are the terms, well-formed formulae and sentences of the language, a logic dictating the validity of inferences and a semantics interpreting the terms of the language and assign truth value to sentences of the language. However, it is clear that we could not take on this concept of language. The main problem is that such an orthodox notion of formal language depends on a theory of truth that's unacceptable to a maximalist. The orthodox stance defines truth in terms of satisfaction. This puts reference before truth. For example, the sentence $Table(t)$ is true because the predicate $Table$ is satisfied by a term t which refers to a table. t must refer to a table before it can satisfy the predicate $Table$ whereby according to PR, the term t refers to a table because $Table(t)$ is true.

For the sake of simplicity, let us define a language to be the combination of a set of symbols, rules for well-formed formulae and sentences and rules for valid inferences. We will assume that the logic is classical first-order logic. Such a language contains a list of sentences that are axioms and the sentences provable from axioms, which are the theorems of that language. Truth condition in this language is taken as correctness conditions of the assertion of sentences. We will also assume that what's provable from a true sentence is true. Thus, the axioms and theorems are all true. What remains is a criterion of identity between languages by which we could tell whether two languages are different. We say that two language are different if they differ in their set of symbols or differ in theorems. A possible language is a language that is used by a linguistic community in a possible world.

Another crucial aspect for a language is to specify what the linguistic expressions mean. This amounts to providing a semantic theory of meaning. We differentiate between semantic theories of meaning and foundational theories of meaning (Speaks, 2021). A semantic theory of meaning tells what are the meanings of particular expressions while a foundation theory of meaning tells how expressions are meaningful. Our focus will be semantic theories of meaning. We will take the inferentialist theory of meaning put forward by Brandom. According to Brandom, for singular terms to be meaningful is for them to be part of sentences governed by acceptable inferential rules (Brandom, 2000). For example, the meaning of "Socrates" is exemplified by how sentences containing "Socrates" such as "Socrates is a philosopher" behave inferentially.

Now that we have specified the notion of a possible language and truth in a possible language, we could reformulate maximalism in the following manner:

$$\begin{gathered}\text{For any sortal concept F, Fs exist if} \\ \text{the hypothesis that Fs exist is true in some possible language} \\ \text{and Fs do not fail to exist as a matter of contingent empirical fact.}\end{gathered} \tag{MM}$$

Call this MM (short for modified maximalism) This is the version of maximalism that we will work with throughout this thesis from now on.

It is clear that MM is entailed by PR. One can see MM as PR under a certain notion of truth. As a simple proof, assume PR and that "There is a philosopher" is true. By the existential elimination law of first-order logic, there is a singular term a such that "a is a philosopher" is true. By PR, a refers to and thus exists. a falls under the concept of being a philosopher. This implies that philosophers exist. If we replace the notion of truth by the correctness condition of assertion in a possible linguistic community that has access to the same empirical facts as the real world, then we have the proof that MM is entailed by PR.

There is no doubt that maximalism is weakly realist. The issue is whether it is possible for it to be strongly realistic. The answer depends on whether there is a notion of truth that is compatible with both PR and strong realism. After all, strong realism demands the mind independence of objects while the substantial theory of truth that is compatible with PR says that truth is correctness condition which seems to depend upon the minds of intelligent agents. This implies that truth is separated from reality in the sense that there is a potential mismatch between what exists and existence claims that we take to be true. This threatens to undermine the character of truth in that truth can be false.

1.3.2 Objections to Maximalism

In this subsection we discuss two objections to maximalism: one argues that the metaontology behind PR is inadequate for concrete objects; another argues that maximalism is not neo-Fregean. Since MM is just PR under a certain definition of truth, any objection to PR would be an objection to MM. It is worth noting here that we are not discussing the adequacy of PR as a neo-Fregean metaontology but rather as the adequacy of PR as a metaontology simpliciter. We will discuss two objections that Eklund considers. The first is that PR is an inadequate meta-ontology. The second is that MM is not in the neo-Fregean spirit.

The Inadequacy of Priority

Eklund considers the following objection to PR as a metaontology. It is counter intuitive to use PR as a metaontology for concrete objects (Eklund, 2006b) It puts the cart before the horse. Take the example of Socrates. It is surely more natural to say that it is because Socrates exists that the term "Socrates" refers and thus sentences such as "Socrates is a philosopher" are true. There is a scientific explanation of Socrates' existence that is more plausible than the explanation given by PR. This is not to deny that in the case of abstract objects such as numbers, PR is a more plausible explanation for their existence. But there is no reason that concrete objects and abstract objects should share the same metaontology and ontology.

Eklund's response to this objection is that given that PR and thus MM is only applicable to abstract objects, this does not block the argument that PR and thus MM should be accepted by the neo-Fregean. As we have mentioned before, the neo-Fregean needs PR to argue against the nominalist's scepticism that uses the contrapositive of the Syntactic Priority Principle.

Maximalism Is Not Neo-Fregean

Another line of argument against MM is that MM goes against the neo-Fregean spirit. MM sanctions the existence of strange objects such as incars (Eklund, 2006b). The neo-Fregeans, even in the loose sense such as Linnebo, are hostile to the existence of such objects (Linnebo, 2018). Wright and Hale reject the existence of incars and accuse MM for being too promiscuous (Hale & Wright, 2009). This goes hand in hand with another objection that MM makes no mention of the abstraction principles that are central to neo-Fregeanism (Eklund, 2006b). The tenet of neo-Fregeanism is existence by abstraction and in the literature neo-Fregeanism is also known as abstractionism because of its reliance on abstraction principles such as HP to back up the existence of abstract objects.

Eklund's response to the latter argument is that abstraction principles such as HP is part of neo-Fregean logicism but not part of neo-Fregean strong realism (Eklund, 2006b). Although PR allows us to derive the existence of abstract objects from abstraction principles like HP, no special attention should be paid to abstract principles in terms of ontological justification. Abstraction principles like HP provide epistemological justification for accepting the truth of atomic sentences with singular terms aiming to refer to abstract objects. Since Wright and Hale only provide negative claims that amount to denying the existence of an ontological gap between language and reality (Wright, 1983; Hale & Wright, 2009), it is not immediately obvious that they have provided justification for neo-Fregean ontology to be a realist one. Of course, there is always the option of taking the neo-Fregean ontology to be an anti-realist one (MacBride, 2016) under which epistemological justification for accepting the truth of HP explains HP's being true. However, Wright and Hale clearly do not have this view in mind (Hale & Wright, 2009).

To understand Eklund's response to the former argument, let A, B be two theories. Say A is hermaneutic of B if A describes B and say A is revolutionary if A says what B ought to be. It is fair to understand Eklund as motivating PR and MM as revolutionary of neo-Fregean metaontology and ontology. Thus, the argument that MM is not neo-Fregean because of its promiscuous ontology need not trouble Eklund. The point that Eklund makes is that neo-Fregeans ought to accept a promiscuous ontology.

1.3.3 Deflationist Maximalism

Eklund claims that there is a reading of maximalism under which it is a deflationary ontology (Eklund, 2008). We have not evaluated whether the neo-Fregean ontology is deflationary but saved it for now. The reason is that how one can read maxiamlism as conditioned deflationary is the same for how one can read neo-Fregean ontology as deflationary. We have proved earlier that MM is entailed by PR plus a certain definition of truth. We have also shown that a deflationist theory of truth is compatible with PR. To read MM as deflationary is to assume

a deflationist theory of truth. The same works for the neo-Fregeans in that neo-Fregean lack a positive account of truth that does not assume the existence of objects (Eklund, 2016b).

It is helpful here to introduce another understanding of the distinction between deflationary and robust ontology that Eklund introduces. Eklund frames the robust ontology as the view that reality can be carved at metaphysical joints in the sense that what exists is objective and the deflationary ontology as the view that reality is an amorphous lump that is without structure and that intelligent agents carve objects into reality using concepts (Eklund, 2008). Understood in this way, it is clear that both the neo-Fregean ontology and maximalism could be seen as a deflationary ontology as they only say how one might carve reality, it does not say whether there is a right way of carving reality.

One might object that given MM appeals to empirical fact, how could MM be a deflationist ontology? In making use of empirical fact, we have appealed to a notion of fact as truth bearers instead of objects having properties or standing in relations or obtaining states of affairs. Thus, at least in the case of mathematical ontology, maximalism is deflationary. In other abstract objects and ordinary objects, it is admittedly difficult to have a notion of empirical fact that does not depend upon the existence of objects being certain ways. But in this case, all is not lost. Eklund anticipates this sort of argument and he admits that maximalism could not be identified with a deflationary ontology, yet it can be a sort of conditioned deflationary ontology (Eklund, 2008). This is because, given Fs do not fail to exist as a matter of empirical fact, Fs criteria of existence is not substantial under a deflationist theory of truth.

1.4 Incompatible Objects

In this section we introduce the problem of incompatible objects, which Eklund takes to be the major objection to maximalism (Eklund, 2006b). To understand the problem of incompatible objects, it is helpful to understand its neo-Fregean analogue, the famous Bad Company problem. Therefore, we will begin with an introduction to the bad company problem followed by the problem of incompatible objects, ending with Eklund's proposed solutions for the problem.

1.4.1 The Bad Company Problem

The Bad Company problem is one of the two major problems facing neo-Fregeanism, the other being the Julius Caesar problem. We have mentioned before in introducing neo-Fregeanism that there needs to be distinction between good and bad abstraction principles. The most infamous abstraction principle is Frege's Basic Law V (Frege, 1893):

$$\forall F \forall G(\{x|F(x)\} = \{x|G(x)\} \leftrightarrow \forall x(F(x) \leftrightarrow G(x))) \qquad (V)$$

V says that for any two concepts F, G, the set of objects falling under F equals the set of objects falling under G if and if every F is a G and every G is a F. V is proven to be inconsistent as it implies Russell's paradox (Zalta, 2024). Therefore, a minimal condition seems to be that a good abstraction principle should itself be consistent.

Yet, this is not enough, as Boolos has shown, there can be two abstraction principles that each is consistently considered on its own but are jointly inconsistent (Boolos, 1990). Boolos introduces the Parity Principle which says that two concepts F, G has the same parity if and only if they differ evenly, which is to say that an even number of objects falls under F but not G (Boolos, 1990). It is proved that the Parity Principle is true only if there are only finitely many objects while HP is true only if there are infinitely many objects (Boolos, 1990). Thus, HP and the Parity Principle can not both be true. One of them has to be given up. The issue is that there is no obvious reason why HP is supposed to be preferred over the Parity Principle without having assumptions about the existence of mathematical objects (such as an infinite set) which HP is supposed to prove.

Most technical solutions for the Bad Company problem develop upon the idea that a good abstraction principle should be conservative (Eklund, 2009a). To say a theory is conservative

is roughly to say that when it is added to another theory, it does not prove the existence of objects that is not alright proven by the theory it is added to (Wright & Hale, 2001). These arguments tend to be technical. Since our project is a philosophical one, we choose to stop here and introduce the problem of incompatible objects. A list of reference for technical arguments could be found in Eklund's 2009 (Eklund, 2009b),

1.4.2 The Problem of Incompatible Objects

The problem of incompatible objects, according to Eklund, is that given two sortal concepts F, G, maximalism is false if both Fs and Gs exist according to maximalism but the hypothesis that Fs exist contradicts the hypothesis that Gs exist (Eklund, 2006b). Under this analysis, numbers and parities are incompatible objects. In this case, Eklund argues that there are two solutions. The first is that we can explore the notion of conservativeness and see whether there is a technical notion that is also philosophically significant that can decide the matter (Eklund, 2006b). This solution is the path the supporters of neo-Fregeans have worked upon. The other solution is to deny abstraction principles in general and find an alternative truth involving singular terms referring to numbers (Eklund, 2006b). This is possible because as we have seen above maximalism, unlike neo-Fregeanism, makes no reliance upon abstraction principles.

Yet, neither solution could make the problem go away for maximalism. Assuming that by some technical notion we are justified in accepting the truth of HP over the truth of the Parity Principle, there are still other incompatible objects possible that are not even in principle possible to develop a technical notion that could allow us to arbitrate their existence hypotheses. Eklund takes the example of xlivers and xhearts. Let xheart be the concept that is exactly like the concept of heart except that xhearts exist only if xlivers do not exist. Let xliver be the concept that is exactly like the concept liver except that xlivers exist only if xhearts do not exist (Eklund, 2006b).

Since they are exactly like livers and hearts except that one's existence implies the none existence of the other, they do not fail to exist as a matter of empirical fact. Each's hypothesis of existence is also consistent which implies that there is a possible language in which it is true. Yet, they could not coexist because this would lead to a contradiction. Assume that xlivers exist and xhearts exist. Since xlivers exist, by definition xhearts do not exist, which contradicts xhearts' existence. This example also invalidates the second solution for the parity example in that the definition of xlivers and xhearts are not by abstraction principles.

1.4.3 Eklund's Solution

In this section we introduce The issue of the problem, as Eklund correctly points out, is that maximalism is too promiscuous as an ontology (Eklund, 2006b). One immediate solution is to restrict what kind of hypothesis of existence is allowed. Neo-Fregeanism could be seen as a solution along this way, namely restricting hypotheses of existence to good abstraction principles. The seeming culprit in the case of xlivers/xhearts is that there is a circularity in definition. Therefore, a first intuitive solution is to outlook circular definition. But as Eklund points out, there are instances of good circular definition (Eklund, 2006b). For example, it is common among logicians to define the concept of number as an object is a number if it is either 0 or the successor of a number (Yablo, 1993).

The second solution is to allow different notions of truth, existence or reference language relative such that the hypothesis that xlivers exist and xhearts exists are both true in their own way or existence in their own way in different languages. The requirement of different language is that the hypothesis that xlivers and xhearts exist could not be true in the same language without breaking the bi-valence of classical logic.[8] The above two options will be considered in

[8]It is only by introducing a third value of indeterminate that we ascribe to the hypothesis that xlivers exist and the hypothesis that xhearts exist.

detail in the next chapter. For the remainder of this subsection, let us consider Eklund's own solution of appealing to indeterminacy.

An intuitive way of solving the problem, according to Eklund, is to accept a form of indeterminacy and claim that it is indeterminate whether xlivers exist or xhearts exist. One immediate worry is how to understand this indeterminacy. As Eklund points out, this indeterminacy could not be semantic in the sense that we do not know which objects the term xliver and xheart refer to because this view of semantic indeterminacy assumes that the objects that are to be referred to already exist (Eklund, 2006b). The indeterminacy has to be ontological which Eklund claims to be a poorly understood notion (Eklund, 2006a). Even granted that there is a clear notion of ontological indeterminacy, Eklund presents a revenge problem for indeterminate object. Let dhearts be such that they are exactly like hearts except that dhearts exist only if dlivers determinately do not exist and let delivers be such that they are exactly like livers except that dlivers exist only if dhearts do not exist (Eklund, 2006b). Saying it is indeterminate that dhearts exists implies that dlivers do not exist. Saying it is indeterminate that dlivers exist implies that dhearts do not exist. Take together, they again lead to a contradiction.

The problem of incompatible objects is that, similar to the bad company problem, the ontology is so promiscuous that it does not rule out the case of objects whose existence is independently consistent but jointly inconsistent. Take for example two concepts xlivers and xhearts. Xlivers exist only if xhearts do not exist and xhearts exist only if xlivers do not exist. Each concept on its own is consistent and thus by maximalism exists. Yet taken together, their existence leads to a contradiction.

A first response might be to try to simulate some of the responses to the bad company problem. A popular response to the bad company problem is to requiring that abstraction principles have to be good in the sense of being conservative [9]. Appeal to conservativeness does not solve the xliver/xheart case.

Another possible response is to accept indeterminacy and claim that it is indeterminate which one of xliver and xheart exist. Eklun claims that the indeterminacy has to be an ontological one which is a notion poorly understood (Eklund, 2006b). There is also the possibility of a revenge problem for indeterminacy which relies on dlivers and dherats. Dlivers exists if dhearts determinately do not exist and dhearts exist if dlivers determinately do not exist.

[9] A theory T is conservative over another theory N if for any $\phi, T + N \vdash \phi$ if $N \vdash \phi$.

Chapter 2

Two Solutions

In this chapter we examine two possible solutions to the problem of incompatible objects: critical realism and quantifier variance and argue that there are not good solutions to the problem of incompatible objects.

2.1 Critical Liberalism

In this section we introduce Linnebo's solution to the problem of incompatible objects which he calls critical realism. We will begin by introducing the key notions of grounding. This is followed by an introduction of critical liberalism. Last but not the least, we will consider three objections to critical liberalism as a solution to the problem of incompatible objects.

2.1.1 Grounding

Grounding is a popular notion in contemporary metaphysical debates and thus admits some complication in terms of definition. It is not realistic to discuss all of them. We will only mention the relevant complications when it is relevant to critical realism. The relevant complications are: the nature of grounding, the difference between partial ground and full ground, the difference between strict ground and weak ground.

In mainstream literature regarding the notion of grounding, there are two readings of grounding, one as a relationship between facts and another as a relationship between objects (Bliss & Trogdon, 2024). Linnebo takes grounding to a relationship between facts which can be taken as either sui generis in which objects have properties or stand in relations (deRosset & Linnebo, 2023). It is also common to speak of ϕ grounding ψ where ϕ, ψ are sentences. To say ϕ groundsψ is to say that the fact expressed by ϕ grounds the fact expressed by ψ. Thus, in discussing critical liberalism, we define fact ϕ grounds fact ψ to be that the fact that ϕ explains the fact that ψ. For example, the fact that I did not have dinner grounds the fact that I am hungry.

A specification could be made between full and partial ground. According to Linnebo, a collection of facts fully ground a fact ϕ if taken together there is no need for any additional fact to ground $f\phi$. For example, the fact that a certain triangle a is an equilateral triangle is fully grounded by the fact that it is a triangle and the fact that all the sides are equal in length. In contrast, a collection of facts partially ground a fact ϕ if they are part of a collection of facts that fully ground ϕ (deRosset & Linnebo, 2023). For example, in our previous example the fact that a is a triangle partially grounds the fact that a is an equilateral triangle.

In mainstream literature, grounding is typically assumed to be a strict order (noted by $<$) in the sense of being a transitive, irreflexive and asymmetric relationship (Bliss & Trogdon, 2024). That is to say that for any fact ϕ, it is not the case that $\phi < \phi$; given facts ϕ, ψ, if $\psi < \phi$ then it is not the case that $\phi < \psi$ and given facts ϕ, ψ, λ, if $\phi < \psi$ and $\psi < \lambda$, then $\phi < \lambda$.

Linnebo defines a new notion of weak grounding $\leq$, which is a transitive and reflexive relation, as a response to the problem of auto abstraction which we need not be concerned with.

According to Linnebo, $\phi \leq \psi$ if and only if there is a corresponding explanatory argument containing only truths (deRosset & Linnebo, 2023) where an explanatory argument is an argument such that its inferences trace the order of dependence and determination among the facts expressed by the sentences involved (deRosset & Linnebo, 2023). For example, the fact that I am hungry weakly grounds the fact that I am hungry or I am tired because there is an explanatory argument from "I am hungry" to "I am hungry or I am tired".

2.1.2 From Old to New

Now that we have introduced the key notion of grounding, let us dive into the ontology itself. We will begin with a motivation for this solution and its core condition. This is followed by an explanation of the notion of plurality which is central to how critical liberalism allows us to turn problematic abstraction principles into unproblematic ones. Last but not the least, we will argue that critical liberalism could not be across the board deflationary ontology in the sense that it is either across the board a robust ontology or robust for the ontology of concrete objects.

Sheep and Wolves

The problem of incompatible objects, according to Linnebo, is that maximalism is too liberal in the sense that it allows too little wolves to control the sheep (Linnebo, 2024). Problematic existences, such as those of incompatible objects or counter-intuitive objects are like uneaten diseased sheep while unproblematic existences such as those of mathematical objects are like healthy sheep Using this metaphor, we can say that the problem of incompatible objects is that we went to far with projecting the sheep by ridding wolves that diseased sheep are not taken out and thus affect the healthy sheep. What critical liberalism does is to take out the diseased sheep by introducing more wolves, stricter criteria of existence.

Recall that the neo-Fregean method of defining good sheep is to appeal to the notion of conservativeness. The rough idea is to start with some already accepted old objects and move up by making a justified step to admit new objects into our ontology. In particular, the old objects could be used in stating correctness conditions for sentences with singular names objects (Linnebo, 2024). For example, given 1 as old objects, we can formulate the correctness condition for sentences with singular names referring to $\{1\}$. Here old and new only refers to the order in which objects are admitted into our ontology, not about any relations between the two objects. For example 1 can be newer than 2 because we might accept 1 as the number of number 2 (because there is only one 2). According to Linnebo, what is important is that the newly admitted objects do not restrict how reality might expand with respect to the old and new objects (Linnebo, 2024). For example, the problem with a concept like xheart is that new xhearts put a restriction upon reality in that there could be no xlivers. Thus, Linnebo's solution is by restricting upon the condition that the hypothesis that Fs exist is true in some possible language.

How to describe a new object is the issue. Linnebo appeals to the notion of intrinsicness (Linnebo, 2024). According to Linnebo, a property F is intrinsic to an object a if a's having F is entirely a matter in virtue of how a is and of nothing else; a binary relation R is intrinsic to objects a, b if $R(a, b)$ holds in virtue of a, b and of nothing else. For example, being equinumerous is intrinsic to fingers on my left hand and fingers on right hand. The specific condition for admitting new objects into ontology are the following (Linnebo, 2024):

> *So long as you provide a reductive way to specify (or to ground) all of the intrinsic features of some "new" objects, you may have them. But you get only what you pay for.*

What Linnebo wishes to say here is that new objects exist if we can reductively ground all facts about intrinsic properties and relations of new objects to facts about facts involving

facts about old objects, and we should not be committed to existence of objects other than the new objects or novel intrinsic property/relations between old objects/future objects and new objects. To say an intrinsic relation/property $R(a, b)$ reduces to another $R'(a', b')$ is to say that $R(a, b) \leftrightarrow R'(a', b')$. For example, identity between numbers could be reduced to equinumerousity between concepts (Linnebo, 2018). In plain and tentative words, new objects exist if we can reduce all important features of them to important features of old objects while making no change to the old objects and no restriction about newer objects. By this condition, the existence of NF sets, whose existence is implied by Quine's New Foundation set theory (W. V. Quine, 1937), is ruled out because there is no clear understanding of what are the intrinsic properties and relations of NF sets (Linnebo, 2024). Even HP becomes unacceptable because it puts a restriction upon reality that there must be infinitely many objects. Frege's V is also unacceptable because it does not specify whether there is a self-membered set (Linnebo, 2024).

Pluralities

Linnebo appeals to the notion of pluralities to fix abstraction principles like HP. A rough notion of plurality is that a plurality of objects are just objects taken collectively (Florio & Linnebo, 2021). One way of understanding a plurality is to think of it as a primitive set. Like a set $\{a, b\}$, there is a relationship between a and the plurality of a and b, this is the relationship of being one of which is similar to the membership relation of a set (Florio & Linnebo, 2021). We say a is one of a, b. Formally, there is a critical plural logic, which we will not explain in detail here but those who are interested should consult Florio and Linnebo's 2017 (Florio & Linnebo, 2021), where plural variables xx are introduced.

The notable features of plurality relevant to us is that pluralities are purely extensionally just like sets in the sense that two pluralities are the same if and only if they have the same objects. Pluralities are also rigid in the sense that if x is one of xx (written as $x \prec xx$), then x is one of xx in all possible worlds (Florio & Linnebo, 2021). These two features make plurality preferable to concepts in defining new objects. Concepts, unlike pluralities, are not defined extensionally. Thus, there is the difference that if we have a plurality of old objects, then we have each member of it as an old object but if we have an old (already accepted) concept, there could be new instances of the concept (Linnebo, 2024). This becomes important when we attempt to formalize sentences about intrinsic properties. Compare the following two restricted universal generalization:

- $(\forall x \prec aa)\phi(x)$
- $(\forall x : F(x))\phi(x)$

Given ϕ is a sentence about an intrinsic property, $(\forall x \prec aa)\phi(x)$ is intrinsic to aa while $(\forall x : F(x))\phi(x)$ is not intrinsic to $\forall x : F(x)$ because there might be a new object b that $F(b)$ is true but $\phi(b)$ is false (Linnebo, 2018).

This benefit of plurality allows us to fix abstraction bad principles by replacing concepts with pluralities. For example, the plural HP is (Linnebo, 2024):

$$\#xx = \#yy \leftrightarrow xx \cong yy \qquad (HP_P)$$

HP_P says that the number of xx equals to the number of yy if and only if xx, yy are equinumerous. Assuming that xx, yy are old objects, we have that a fact about an intrinsic relation on old objects grounds a fact about an intrinsic relation on new objects (deRosset & Linnebo, 2023). The difficult part is to show that this fact of identity is about all the intrinsic properties and relations of $\#xx$.

According to Linnebo, HP_P needs to be supplement by another principle defining the predecessor relationship P between $\#xx, \#yy$ in term of the $Prec$ relationship between xx, yy which is defined as removing one of yy makes the resulting equinumerous to xx: (Linnebo, 2024):

$$P(\#xx, \#yy) \leftrightarrow Prec(xx, yy) \qquad (Inherit_P)$$

This is known as the plural inheritance principle. Linnebo claims that along with HP_P, the two exhausts all the intrinsic properties and relations of $\#xx$.

To illustrate with a concrete example, assume that I have all my fingers, then it is intrinsic to the fingers on my left and those on my right hands that they are equinumerous. This grounds the fact that the number of fingers on my left hand equals the number of fingers on my right hand. This is just the self-identity of 5, which is intrinsic to the number. Now assume I found four carrots and put them next to my left hand. By the plural inheritance principle, the number of carrots precedes the number of fingers on my left hand. Taken together, this allows me to justify that 5 exists.

Robust Ontology

Call a fact zero-grounded if it is grounded by the fact that $\emptyset$ (deRosset & Linnebo, 2023). Whether the fact that natural numbers exist could be zero-grounded is crucial to deciding whether critical liberalism counts as an deflationary ontology for numbers. One might think that facts about numbers need to be grounded in empirical facts. Yet, there is the case of 0. Given that the existence of 0 is grounded and one has $Inherit_P$, one might ground the existence of 1 in that of 0 and 2 in that of 0 and 1 and so on (deRosset & Linnebo, 2023). Thus, one can get facts about all natural numbers in a bottom-up fashion.

Thus, the core of the issue becomes whether 0 can be zero-grounded. The answer is yes. Given HP_P, one might appeal to the notion of an empty plurality $\emptyset\emptyset$. If $\emptyset\emptyset$ is allowed, then $\#\emptyset\emptyset = \#\emptyset\emptyset \leftrightarrow \emptyset\emptyset \cong \emptyset\emptyset$ grounds the fact that 0 exist (deRosset & Linnebo, 2023). However, no such zero-ground are possible for facts regarding the existence of concrete objects or even some abstract objects.

For example, the fact that directions exist could not be grounded in the abstraction principle that two lines have the same direction if and only if they are parallel alone. The fact that there are parallel lines has to be there as well which could not be zero-grounded (deRosset & Linnebo, 2023). For concrete objects such as tables, the fact that a table exists is grounded in the fact about particles arranged tablewise has to be grounded in some empirical fact. To make things worse, one might be suspicious about empty pluralities. If one denies that, critical liberalism has appeal to an alternative more technical method of grounding the fact that natural numbers exist which we will not go into but only note that this method requires the fact that at least one object exists in the universe(deRosset & Linnebo, 2023). Thus, critical liberalism could not be an across the board deflationary ontology in that some empirical facts are assumed to hold which implies the existence of objects being certain ways.

One might wonder if critical liberalism could argue for an across the board deflationism by appealing to a notion of fact that does not involve objects or states of affairs. After all, if maximalism could appeal to the notion of fact as true truth bearers, why could not critical liberalism appeal to the same notion. A tentative answer is that doing so would deny any deeper analysis into the structures within facts which might be problematic. But we will not explain further details here as it involves too many technical details which would lead us astray. We simply notice that this is not the option that Linnebo takes (deRosset & Linnebo, 2023).

2.1.3 Problem with All

In this subsection we explore the objection to critical liberalism from the inadequacy of Linnebo's definition of intrinsic properties and relations. We will first argue that Linnebo's definition is too unspecific to be applicable and then argue that there is no possible notion of intrinsic property that does not conflict with the shallowness of ontology without giving any justification (Linnebo, 2024).

The problem with critical realism is that it requires us to specify all the intrinsic properties and relations about the new object (Linnebo, 2024). It is not always possible for us to know that we have exhausted intrinsic properties and relations for a given new object. In the case of numbers, where Linnebo simply claims that $Inherit_P$ and HP_P exhaust the intrinsic properties and relations about numbers. The intuitive idea seems to be that to specify all the PA postulate would be the exhaustion. In Linnebo's defense, given that grounding is understood as weak grounding defined above, there is an explanatory argument from HP_P to PA postulated by way of Frege's Theorem.

There are at least two notions of property (and relation). There is the extensional notion under which a property is just the objects having the property and a relation is just the objects holding the relation. It follows that two properties are the same if only if they have the same extension (number of objects having the property). Given the extensional notion, there is a clear way of finding out whether we have exhausted intrinsic properties and relations of a new object in the sense that we already know the maximum number of intrinsic properties and relations it can have. There is only one intrinsic property for each new object. If there is only one old object and one new object, then there is only 1 intrinsic binary relation to be specified.

Thus, one might opt for a more fine-grained notion of property such as the intensional notion. Under the intensional notion, two properties are the same if and only if they necessarily have the co-extension. One might feel that this is still not enough and opt for a hyper-intensional notion of property such that two properties are the same if they have the same meaning. Even if we could find a notion of property and relation, it only gives us an upper limit of the number of intrinsic properties and relations that we need to specify.

2.1.4 Overkill

We can assume that critical liberalism is without problems on its own but still argues that it fails to be a solution to the problem of incompatible objects. To solve the problem of incompatible objects is not simply to rid the contradiction from our ontology but to do so in a manner that respects maximalism in the sense that it should try to have an ontology that is as close to maximalism as possible. To use the sheep/wolf metaphor, critical liberalism lets in too many wolves that they start to eat not only diseased bad sheep but also healthy good sheep.

As we have seen previously, critical liberalism out rules the existence of NF sets, which is an object that maximalism happily accepts. The serious issue behind this result that critical liberalism is too austere an ontology that it goes against the spirit of maximalism. Maximalism gladly affirms the existence of counter-intuitive objects and objects whose nature we are not so clear of in the sense that we do not know all the intrinsic properties and relations about them. However, such objects are outlawed by critical liberalism. This is not to say that critical liberalism is problematic ontology taken on its own,the point is that it is more of a new ontology inspired by the problem of incompatible objects than a fix to the old ontology that is troubled by the problem of incompatible objects.

2.2 Quantifier Variance

Given the appeal to truth in a possible language in MM, one intuitive way of solving the problem of incompatible objects is to argue that the hypothesis that xlivers exist and the hypothesis that xhearts exist are both true but true in different languages which are not translatable in the sense that we will specify below. In very rough terms, quantifier variance says that "*there are different languages where different ontological sentences come out true.*" (Eklund, 2021). Our focus will be on Warren's formulation of the view . We will then introduce a key notion of expressive richness of a language which will be central to Eklund's objection to quantifier variance. Last but not the least, we discuss the relationship between quantifier variance and maximalism and argue that it could be a solution to the problem of incompatible objects.

2.2.1 Warren's Quantifier Variance

As we mentioned before, "quantifier variance" is an umbrella term for a range of metaontological theories that differ in detail. We follow Eklund in focusing on Warren's view (Warren, 2015):

> *...there are distinct and inequivalent quantifier meanings such that the different parties in various ontological disputes can be interpreted as using different unrestricted existential quantifiers (which we take to be first-order existential quantifier unless otherwise specified).*

We will analyze different versions of quantifier variance by using the framework developed by Eklund (Eklund, 2021).

In order to analyze different versions of quantifier variance, it is helpful to introduce a simple example. Recall by our definition of language, to say a sentence ϕ is true in a language L is to say that there are axioms Σ in the language of L such that $\Sigma \vdash \phi$. Let L_U, L_N be two languages that share the same set of terms but differ in theorems. Let Σ_U be the axioms of L_U and Σ_E be the axioms of L_N. Let $\exists x Table(x)$ be the sentences that a table exists. Roughly speaking, L_U is the language of a liberal ontology that affirms the existence of tables while L_N is the language of a restrictive ontology that denies the existence of tables.

The traditional ontology dispute is that liberal ontologist claims that $\exists x Table(x)$ while the austere ontologiest claims that $\exists x Table(x)$ is false. However, quantifier variance says that the liberal ontologist and the restrictive ontologist are simply speaking in different languages. In the liberal language L_U $\exists x Table(x)$ is true because $\Sigma_U \vdash \exists x Table(x)$. In the restrictive language L_N $\exists x Table(x)$ is false because $\Sigma_N \vdash \neg\exists_N x Table(x)$. There is no language independent notion of truth because we have defined truth to be what is provable from axioms of a language. It is illegitimate to ask the question whether $\exists x Table(x)$ is true or false.

To apply this argument as a solution to the problem of incompatible objects, let L_L be the xliver language and Σ_L its axioms and L_H be the xheart language and Σ_H its axioms. The quantifier variance solution is to say that $\exists x xheart(x)$ is true in L_H and $\exists x xliver(x)$ is false in L_H while $\exists x xliver(x)$ is true in L_L and $\exists x xheart(x)$ is false in L_L.

Weak and Strong Theses

Now that we have got a concrete example of an argument from quantifier variance, let us move to see in what ways we might analyze the view.

According to Eklund, one way of categorizing theories of quantifier variance is the following: (Eklund, 2021)

> The weak thesis says that there are languages like L_U and L_V which are like our ordinary language in that they differ in what characteristic claims to be true.
>
> The strong thesis says that there are languages like L_U and L_V which are like our ordinary language in that they differ in what characteristic claims to be true and they have different unrestricted existential first-order quantifier that are similar to the unrestricted existential first-order quantifier in that they obey the same introduction and elimination laws.

The difference between the weak and strong thesis is that the weak thesis does not claim that there are different meanings of existential quantifiers. Thus, the strong thesis entails the weak thesis. As Eklund points out, there is the issue of what does it mean to say that two existential quantifiers are different given they share the same inferential behaviour (Eklund, 2021). We will save this problem for later. For now, notice that the examples we provided above are formulated according to the weak thesis, now let's reformulate the examples in line with the strong thesis.

What the liberal ontologist says is $\exists_U x Table(x)$ in the language L_U while the restrictive ontologist says is $\neg\exists_N x Table(x)$ in the language of L_N. Each claim is true in her own language

in the sense that $\Sigma_U \vdash \exists_U x Table(x)$ and $\Sigma_N \vdash \exists_N x Table(x)$. There is no language independent notion of the existential quantifier and thus illegitimate to ask the question whether $\exists x Table(x)$ is true or false.

To apply this argument as a solution to the problem of incompatible objects, let L_L be the xliver language and L_H be the xheart language. The quantifier variance solution is to say that $\exists_H x xheart(x)$ is true in L_H and $\exists_H x xliver(x)$ is false in L_H while $\exists_L x xliver(x)$ is true in L_L and $\exists_L x xheart(x)$ is false in L_L.

XYZ

Now another way that versions of quantifier variance might differ is given by Eklund as the following (Eklund, 2021):

> X: Some ontological disputes are verbal in the sense that the sentences that disputants affirm are consistent with each other.
>
> Y: There is no ontologically privileged quantifier. Each language has its own quantifier. No language is ontologically privileged in describing the world.
>
> Z: Our language is one of many languages that use different representational means to describe the world. No language is ontologically privileged in describing the world.

Ontological realism, according to Sider, is the view that there is an ontologicaly privileged sense of quantifier according to which is the right way to employ that quantifier in describing the world (Sider, 2011). Similarly, to say a language is ontologically privileged is to say that it is the correct way to describe the world. Ontological realism is entailed by strong realism, the view that objects exist independently. According to Eklund, the difference between X and Y, Z is that X is compatible with ontological realism while Y, Z are not (Eklund, 2021). This much should be clear from our definitions. Y, Z both count as metaontological deflationism in the sense that there is no across the board concept of existence. The difference between Y and Z is that while Y still assumes that different languages all use expressions with meanings of existential quantifiers, Z does not assume this at all (Eklund, 2021). Compared to Y, Z has a less restriction on what a possible language might be. Unlike Y, Z implies only the weak thesis like X (Eklund, 2021).

2.2.2 Objections to Quantifier Variance

In this section we consider two objections that Eklund makes against Warren's version of quantifier variance. We will begin with Eklund's two arguments, followed by the response that Hirsch and Warren make.

The Collapse Argument

The first objection that Eklund puts forward is the Collapse argument. The argument goes the following:

Assume that we have the liberal and ontologist dispute described above as a strong thesis. In our language, which is a metalanguage to L_U and L_N, we can say that $\exists_L x Table(x)$. By existential introduction, we have that there is an object a such that a is a table. By existential elimination, we have that $\exists_N x Table(x)$ we have just proven that $\exists_N x Table(x)$ is true in the language L_N. This contradicts quantifier variance which says that $\exists_N x Table(x)$ is false in the language L_N (Eklund, 2021).

An immediate solution is to abandon the strong thesis and take refuge in the weak thesis. After all, if there is not requirement that $\exists_U$ and $\exists_N$ share the same inferential rules, then it

could be that the reference from $Table(a)$ to $\exists_N table(x)$ is illegitimate despite the fact that the inference from $Table(a)$ to $\exists_U table(x)$ is.

Warren's response to this objection follows this route. He does accept that we might merge L_U and L_N but adding one's inferences rule to another. He argues that this merging will change the overall inferential behaviors of the disputed existence claim and thus undermine the Collapse argument (Warren, 2015). Warren argues that merging L_U, L_N into one language would mix two inference rules together and could block the Collapse argument (Warren, 2015). Eklund takes Warren's solution inadequate in that he takes Warren's quantifier variance to be a strong thesis while Warren's solution rests on a weak thesis (Eklund, 2021). It is not obvious how Warren's view might be counted as a strong thesis. Warren does not state that different quantifiers share the same inference rules.

What is more problematic and in Warren's favor is that, as we have seen before, the strong thesis is hard to understand. Given existential quantifier is a logical operator whose meaning tends to be defined inferentially by their inferential behaviours. Given inferentialism, $\exists_N$ and $\exists_U$ could only differ in meaning if they differ in inferential behaviour which the strong thesis assumes they do not. The only way to avoid contradiction, as far as we could see, is to rely on a theory of meaning by which the meaning of existential quantifiers are more than their inferential behaviours. One possible theory is to say that the meaning of an existential quantifier is its extension where the extension of an existential quantifier is the domain of its quantification.

Granted Warren's solution might be brushed away, there is another solution that is worth considering. The first solution is a logical solution offered by Turner. Recall that we have been assuming that our background logic is classical, except when discussing neo-Fregeanism. Turner's solution is to adapt a different background logic, a free logic under which it is not possible to directly infer $\exists_N x Table(x)$ from $Table(a)$; one also needs $\exists_N x(x = a)$ (Turner, 2010). Turner's solution is an acceptable one but accepting it undermines quantifier variance as a solution to the problem of incompatible objects. As Eklund notes, one could not infer from the truth of atomic sentences to the existence of the referents of singular terms in the sentences. This clearly undermines PR who says that we can infer from an atomic sentence's truth to the existence of referents of singular terms.

The Eklund-Hawthorne Argument

In his 2008 Eklund give what he calls the Tarskian argument against quantifier variance (Eklund, 2008), the argument is known as the Eklund-Hawthorne Argument in his 2021 (Eklund, 2021). Following Eklund, we will use Sider's formulation (Sider, 2011).

Take the language L_U, L_N as we have stated above, assume that quantifier variance holds. The restrictive ontologist can say in the language L_N that "$Table(a)$" is true in L_U. Yet, for "$Table(a)$ is true in L_U" to be true in L_N, there should be something that a refers in L_U. If so, it follows that a also refers to the same object in L_N.

The argument is stated by Hirsch and Warren as (Hirsch & Warren, 2017):

1. We are speaking L_N

2. We are in the position to acknowledge the possibility of there being a language L_U corresponding to the standpoint of the liberal ontologist.

3. If it is impossible in one language to formulate a Tarskian semantics for another language, then speakers of the first language are in no position to acknowledge the possibility of the second language. A Tarskian semantics "*...generates truth conditions for sentences compositionally in the familiar fashion by mapping items from each syntactic category to objects of the appropriate type – names map (singularly) to objects, monadic predicates map (dividedly) to the objects they are true of, and so on and so forth*"..

4. It is impossible to formulate in L_N a Tarskian semantics for L_U.

5. Therefore, we are not speaking in L_N.

6. The restrictive ontology is false.

7. Therefore, we cannot view another possible language's ontology as being "bigger" than our own, as quantifier variance requires.

Hirsch and Warren's solution is to argue that 4 does not hold by trying to formulate a Tarskian semantics for L_N (Hirsch & Warren, 2017). To use illustrate with a concrete example. Since tables do not exist given the language L_N, one way to provide a semantics for L_U is to let a, the object that refers to a table given L_U to refer to the particles arranged tablewise. Eklund argues that this solution is inadequate because it does not actually allow the restrictive ontologist to view the liberal ontologist's ontology as bigger than her own as the restrictive ontology has not changed even though the restrictive ontologist could provide a semantics to make the sentence $Table(a)$ meaningful (Eklund, 2021).

As a final objection, Eklund merges the Collapse argument with the Eklund-Hawthorne argument. The major problem to the Collapse argument is that how could one justify that there is a merged language in in which the actual argument goes through. With the Eklund-Hawthorne argument at hand, there is no need to have a merged language. We first infer $Table(a)$ from $\exists_U x Table(x)$ in L_U. But since $Table(a)$ holds in L_U, by the EKlund-Hawthorne argument we could say that in L_N $Table(a)$ holds and thus $\exists_N x Table(x)$ is true in L_N. Call this the Ultimate argument.

2.2.3 Quantifier Variance and Maximalism

Upon a first look, quantifier variance is a rival of maximalism (Eklund, 2006a). They are rival conceptions of deflationary metaontology. What the Ultimate argument attempts proves is that we could not talk about another ontological theory who has a bigger ontology compared to our own. Such an ontology is exactly what maximalism implies. Because PR and maximalism say nothing about whether existence is univocal. We might either see maximalism as the view that there is only one existence predicate or the view that there is one privileged existence predicate. The latter would make maximalism an ontological realism in the sense of Sider. iUnder maximalism, what exists are the ontological commitments of any theory that is true in a possible language without contradicting empirical facts. One can see the ultimate argument as proving that contrary to quantifier variance, there is a notion of existence that is privileged int that it is maximal.

Given the Ultimate argument Warren's quantifier variance could not be a solution to the problem of incompatible objects. Assume maximalism, let L_H be the language in which $\exists_H x heart(x)$ is true and L_L be the language in which $\exists_L x liver(x)$ is true. In L_H we can infer $xheart(a)$ and thus by the Eklund-Hawthorne argument we can say that $xheart(a)$ is true in L_H in L_N, this entails that $xheart(a)$ is true in L_L, a contradiction.

Chapter 3

Two New Solutions

In this chapter we consider two solutions for the problem of incompatible objects The first is to argue that there are at least two languages in which the meaning of xlivers and xhearts are similar but different. This implies that xlivers and something close to xhearts exist or vice versa. The second is to replace the notion of ontological indeterminacy with either pragmatic indeterminacy.

3.1 The Similarity Solution

Warren's quantifier variance is the view that there is a sea full of different, similar and equally privileged quantifiers in a sea of different languages. Maximalism is the view that there is only one existential quantifier or one privileged existential quantifier in a sea of different languages. The Ultimate argument shows that a sea of different equally privileged existential quantifiers is too much. The problem of incompatible objects shows that only one privileged existential quantifier is too little. The ideal should be that there is maybe a small pond of different, similar and equally good existential quantifiers. This means we have to somehow change quantifier variance.

We have seen in the end of the previous chapter that under maximalism, Warren's quantifier variance also faces the problem of incompatible objects. Our inspiration is drawn from what Eklund considers a failed response from quantifier variance against the Eklund-Hawthorne argument. In his 2009, Eklund outlines a response from quantifier variance against the Eklund-Hawthorne argument. Let us assume that xliver(a) is true in L_L. According to Eklund, there is a difference between accepting in L_H that "a" refers and accepting "there is a xliver" (Eklund, 2009b). The xheart ontologist might accept that there is an object that a refers to and it falls under the concept "xlivers" in L_L without accepting in L_H that $\exists_H x xliver(x)$). For the latter she needs that the concept of xliver in L_H and that L_L mean the same or are at least coextensive (Eklund, 2006b). But they need not be. If they are not, then the xliver term "a" in L_H is empty and does not refer while that of L_L does. L_H can talk about the referents of the terms of L_L but not directly. It does so indirectly by talking about the referents of expressions in L_L.

Eklund dismisses this solution as a solution for quantifier variance on the ground that this solution entails that L_H, L_L no longer disagree about ontology. They agree what there are. The difference is that the xhiver term "a"" refers to L_L but not in L_H (Eklund, 2009b). Recall that quantifier variance entails that different languages differ over ontology and there is no privileged answer. Eklund's dismissal is fair. But alas, one theory's loss is another's gain. Our aim is not to solve the problem of incompatible objects via quantifier variance but rather solve the problem of incompatible objects simpliciter. Plus, quantifier variance is a rival of maximalism, if our solution does not depend upon but actually argues against quantifier variance, we can kill two birds with one stone. This is because for quantifier variance to run into the problem of incompatible objects, we do not need to assume maximalism. We only need to assume that there is a language in which the hypothesis that xlivers exist is true and another in which the

hypothesis that xhearts exist is true. If Eklund's proposed solution actually solves a problem faced by two rivals by giving up one, then it proves the superiority of the theory that remains.

The only worry is that this solution makes our ontology profligate. This is because this solution implies that for each pair of incompatible objects, there are at least two languages in which their meanings differ slightly but are mostly similar and there are pairs of these objects. For example, our solution to the xliver/xheart problem implies that there are at least four concepts xliver, xheart, xliver', xheart' such that there are two languages L, L' such that $xliver(a)$ and $xhearts'(b)$ are both true and a, b exist. But again this is not a problem for maximalism which is supposed to be a promiscuous ontology. We have started with the hope of reducing a sea to a pond but we have overachieved and regains the univocality of existence.

3.2 Pragmatic Indeterminacy

Another solution is to revisit the notion that Eklund dismisses, which is indeterminacy. In this section we argue that the notion of pragmatic indeterminacy could serve as the solution to the problem of incompatible objects: pragmatic indeterminacy.

To understand the idea of pragmatic indeterminacy, we need Carnap's distinction between three kinds of ontological questions. Recall that according to Eklund's interpretation of Carnap, ontological questions can be set into three camps: internal questions within a framework (language or language fragments), external (factual) questions about what "really" exists from a God-like perspective outside any framework and pragmatic question about which framework to adopt. Set in this Carnapian framework, we can say that there are at least two different possible languages L_L and L_H such that it is part of the axiom of L_L that xlivers exist and it is part of the axiom of L_H that xhearts exist. In L_L, it is true that xlivers exist and it is false that xhearts exist. In L_H, it is true that xhearts exist and it is false that xhearts exist. Whether xlivers or xhearts exist could be asked as internal questions to L_L and L_H or as a external pragmatic question regarding which framework to adopt as there is a framework in which only $xlivers$ is true and a framework in which only $xhearts$ is true but not a framework in which both are true.

There is a good reason to think that EKlund is at least sympathetic to the notion of pragmatic indeterminacy. Eklund admits the lack of intuition about incompatible possible languages, that is to say that the two possible languages taken together implies a contradiction such as L_L and L_H(Eklund, 2007).

> *Suppose that, as earlier outlined, there are two different languages L and L* both of which can consistently exist but which are incompatible. How can we know which of L and L* exists? .... Second, more to the point, when it comes to choices like that between a language like English except for the possible difference that its truth predicate is transparent and a language where untruth can be expressed, is it really plausible that we have any intuitions that speak to the issue of which language it is that exists. ... If so, then presumably what we think is that the entities quantified over in relatively natural mathematical theories like ZF and PA exist but the entities quantified over in seemingly unnatural theories do not, where perhaps Quine's NF is the prime example of an unnatural theory. Of course serious questions can be raised about the reliability of our intuitions about these matters. My point is just that in this case it can at least be reasonably urged that we have the relevant intuitions. By contrast, do we even have intuitions about what type of negation really exists?*

The reason, Eklund admits, is that we lack a clear intuition in such artificial cases. Eklund's lack of intuition is not directly about what is the most pragmatic framework to adopt. However, we can extract a similar argument for our lack of intuition regarding incompatible objects. Clearly it is pragmatic to adopt a framework where numbers instead of anti-numbers exist where anti-numbers are abstract objects such that they exist only if numbers do not exist (Eklund, 2006b).

But this naturalness from intuition disappears when we consider the case of xlivers and xhearts. It is then that indeterminacy creeps in.

Whether existence is univocal is open for this solution unlike in the previous solution in which univocality of existence is entailed. While adopting of pragmatic indeterminacy is compatible with quantifier variance, it is important to note that the success of this solution depends on the failure of the Ultimate argument. As we have seen, unless one have independent reasons to rule out the existence of one or both of xlivers and xhearts, the Ultimate argument implies that both xlivers and xhearts exist. For a more detailed survey of solutions for the Ultimate problem, one could refer to Eklund's 2009 (Eklund, 2009b),

Chapter 4

Conclusion

We have shown in detail the metaontological principle the Priority Principle which motivates maximalism as an ontology for neo-Fregeaism. We have also seen to what extent maximalism could be considered as a deflationary ontology. We have argued that two prominent solutions to the problem of incompatible objects fail to solve the problem for maximalism. Last but not the least, we have introduced two simple yet effective solutions to the problem of incompatible objects.

References

Bliss, R., & Trogdon, K. (2024). Metaphysical Grounding. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Summer 2024 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/sum2024/entries/grounding/`.

Boghossian, P. A. (1996). Analyticity reconsidered. *Noûs*, *30*(3), 360–391. doi: 10.2307/2216275

Boolos, G. (1990). The standard of equality of numbers. In H. Putnam & G. Boolos (Eds.), *Meaning and method: essays in honor of hilary putnam* (pp. 261–77). Cambridge University Press.

Boolos, G. (1996). On the proof of frege's theorem. In A. Morton & S. P. Stich (Eds.), *Benacerraf and his critics* (pp. 143–59). Blackwell.

Brandom, R. (2000). *Articulating reasons: An introduction to inferentialism*. Cambridge, Mass.: Harvard University Press.

Carnap, R. (1950). Empiricism, semantics and ontology. *Revue Internationale de Philosophie*, *4*(11), 20–40.

David, M. (2022). The Correspondence Theory of Truth. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Summer 2022 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/sum2022/entries/truth-correspondence/`.

deRosset, L., & Linnebo, O. (2023). Abstraction and grounding. *Philosophy and Phenomenological Research*, *109*(1), 357–390. doi: 10.1111/phpr.13036

Dummett, M. (1976). What is a theory of meaning? (ii). In G. Evans & J. McDowell (Eds.), *What is a theory of meaning?* Oxford: Clarendon Press.

Ebert, P. A., & Rossberg, M. (Eds.). (1964). *Gottlob frege: Basic laws of arithmetic*. Oxford, UK: Oxford University Press.

Eklund, M. (2006a). Metaontology. *Philosophy Compass*, *1*(3), 317–334. doi: 10.1111/j.1747-9991.2006.00026.x

Eklund, M. (2006b). Neo-fregean ontology. *Philosophical Perspectives*, *20*(1), 95–121. doi: 10.1111/j.1520-8583.2006.00104.x

Eklund, M. (2007). The liar paradox, expressibility, possible languages. In J. C. Beall (Ed.), *Revenge of the liar: New essays on the paradox.* Oxford University Press.

Eklund, M. (2008). The picture of reality as an amorphous lump. In T. Sider, J. Hawthorne, & D. W. Zimmerman (Eds.), *Contemporary debates in metaphysics* (pp. 382–96). Blackwell.

Eklund, M. (2009a). Bad company and neo-fregean philosophy. *Synthese*, *170*(3), 393–414. doi: 10.1007/s11229-007-9262-x

Eklund, M. (2009b). Carnap and ontological pluralism. In R. Wasserman, D. Manley, & D. Chalmers (Eds.), *Metametaphysics: New essays on the foundations of ontology* (pp. 130–56). Oxford University Press.

Eklund, M. (2016a). Carnap's legacy for the contemporary metaontological debate. In S. Blatti & S. Lapointe (Eds.), *Ontology after carnap.*

Eklund, M. (2016b). Hale and wright on the metaontology of neo-fregeanism. In P. A. Ebert & M. Rossberg (Eds.), *Abstractionism: Essays in philosophy of mathematics.* Oxford University Press UK.

Eklund, M. (2020). The metametaphysics of neo-fregeanism. In R. Bliss & J. Miller (Eds.), *The routledge handbook of metametaphysics.* Routledge.

Eklund, M. (2021). Collapse and the varieties of quantifier variance. In J. Miller (Ed.), *The language of ontology.*

Eklund, M. (2023). Carnapian frameworks revisited. In P. Raatikainen (Ed.), Essays in the Philosophy of Language. *acta philosophica fennica vol. 100.* (pp. 91–113). Societas Philosophica Fennica.

Florio, S., & Linnebo, O. (2021). *The many and the one: A philosophical study of plural logic.* Oxford, England: Oxford University Press.

Frege, G. (1893). *Grundgesetze der arithmetik.* Jena,: H. Pohle.

Frege, G. (1951). On concept and object. *Mind*, *60*(238), 168–180. doi: 10.1093/mind/lx.238.168

Geach, P. T. (1980). *Reference and generality: An examination of some medieval and modern theories.* Ithaca, N.Y.: Cornell University Press.

Glanzberg, M. (2023). Truth. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Fall 2023 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/fall2023/entries/truth/`.

Hale, B., & Wright, C. (2000). Implicit definition and the a priori. In P. A. Boghossian & C. Peacocke (Eds.), *New essays on the a priori* (pp. 286–319). Oxford University Press.

Hale, B., & Wright, C. (2009). The metaontology of abstraction. In D. Chalmers, D. Manley, & R. Wasserman (Eds.), *Metametaphysics: New essays on the foundations of ontology* (pp. 178–212). Oxford University Press.

Hale, B., & Wright, C. (2012). Horse sense. *Journal of Philosophy*, *109*(1-2), 85–131. doi: 10.5840/jphil20121091/24

Hirsch, E. (2002). Quantifier variance and realism. *Philosophical Issues*, *12*(1), 51–73. doi: 10.1111/j.1758-2237.2002.tb00061.x

Hirsch, E., & Warren, J. (2017). Quantifier variance and the demand for a semantics. *Philosophy and Phenomenological Research*, *98*(3), 592–605. doi: 10.1111/phpr.12442

Hofweber, T. (2016). *Ontology and the ambitions of metaphysics.* Oxford, England: Oxford University Press UK.

Horwich, P. (1990). *Truth* (F. Jackson & M. Smith, Eds.). Oxford, GB: Clarendon Press.

Inwagen, P. V. (1990). *Material beings.* Ithaca: Cornell University Press.

Inwagen, P. V. (1998). Meta-ontology. *Erkenntnis*, *48*(2-3), 233–50. doi: 10.1023/a:1005323618026

Linnebo, O. (2018). *Thin objects: An abstractionist account*. Oxford: Oxford University Press.

Linnebo, O. (2024). *Eklund, maximalism, and the problem of incompatible objects.*

Lowe, E. J. (1989). What is a criterion of identity? *Philosophical Quarterly*, *39*(154), 1–21. doi: 10.2307/2220347

Lowe, E. J. (1995). The metaphysics of abstract objects. *Journal of Philosophy*, *92*(10), 509–524. doi: 10.2307/2940785

MacBride, F. (2003). Speaking with shadows: A study of neo-logicism. *British Journal for the Philosophy of Science*, *54*(1), 103–163. doi: 10.1093/bjps/54.1.103

MacBride, F. (2005). Predicate reference. In E. Lepore & B. C. Smith (Eds.), *The oxford handbook of philosophy of language* (pp. 422–474). Oxford University Press.

MacBride, F. (2006). The julius caesar objection : More problematic than ever. In F. MacBride (Ed.), *Identity and modality* (p. 174). Oxford University Press.

MacBride, F. (2016). Neofregean metaontology. In P. A. Ebert & M. Rossberg (Eds.), *Abstractionism: Essays in philosophy of mathematics* (pp. 94–112). Oxford University Press UK.

MacBride, F. (2022). Truthmakers. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Fall 2022 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/fall2022/entries/truthmakers/`.

Marschall, B., & Schindler, T. (2020). Does semantic deflationism entail meta-ontological deflationism? *Philosophical Quarterly*, *71*(1), 99–119. doi: 10.1093/pq/pqaa019

Meinong, A. (1960). On the theory of objects (translation of 'Über gegenstandstheorie', 1904). In R. M. Chisholm (Ed.), *Realism and the background of phenomenology* (pp. 76–117). Free Press.

Mulligan, K., & Correia, F. (2021). Facts. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Winter 2021 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/win2021/entries/facts/`.

Nolt, J. (2021). Free Logic. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Fall 2021 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/fall2021/entries/logic-free/`.

Putnam, H. (1971). *Philosophy of logic* (S. Laurence & C. Macdonald, Eds.). London,: Allen & Unwin.

Putnam, H. (1991). The question of realism. In H. Putnam (Ed.), *? iteputnam:wl* (pp. 295–312).

Quine, W. V. (1937). New foundations for mathematical logic. *Journal of Symbolic Logic*, *2*(2), 86–87. doi: 10.2307/2267377

Quine, W. V. (1953). On what there is. In W. V. O. Quine (Ed.), *From a logical point of view* (pp. 1–19). Harvard University Press.

Quine, W. V. (1992). Structure and nature. *Journal of Philosophy*, *89*(1), 5–9. doi: 10.2307/2026889

Quine, W. V. O. (1951). Two dogmas of empiricism. *Philosophical Review*, *60*(1), 20–43. doi: 10.2307/2266637

Quine, W. V. O. (1970). *Philosophy of logic* (S. Blackburn & K. Simmons, Eds.). Cambridge: Harvard University Press.

Quine, W. V. O. (1995). 10 things and their place in theories. In P. K. Moser & J. D. Trout (Eds.), *Contemporary materialism: A reader* (p. 193). Routledge.

Reicher, M. (2022). Nonexistent Objects. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Winter 2022 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/win2022/entries/nonexistent-objects/`.

Sider, T. (2011). *Writing the book of the world*. Oxford, England: Oxford University Press.

Speaks, J. (2021). Theories of Meaning. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Spring 2021 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/spr2021/entries/meaning/`.

Swijtink, Z. G. (1998). Beth's theorem and craig's theorem. In D. Johnson & C. Smith (Eds.), *The handbook of logic* (pp. 45–67). Cambridge University Press.

Textor, M. (2021). States of Affairs. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Summer 2021 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/sum2021/entries/states-of-affairs/`.

Thomasson, A. L. (2014). *Ontology made easy*. New York: Oup Usa.

Trueman, R. (2020). *Properties and propositions: The metaphysics of higher-order logic*. Cambridge: Cambridge University Press.

Turner, J. (2010). Ontological pluralism. *Journal of Philosophy*, *107*(1), 5–34. doi: 10.5840/jphil201010716

van Inwagen, P. (2019). Against analytic existence entailments. In J. Cumpa & B. Brewer (Eds.), *The nature of ordinary objects* (pp. 173–197). Cambridge University Press.

Warren, J. (2015). Quantifier variance and the collapse argument. *Philosophical Quarterly*, *65*(259), 241–253. doi: 10.1093/pq/pqu080

Wright, C. (1983). *Frege's conception of numbers as objects*. [Aberdeen]: Aberdeen University Press.

Wright, C. (1999). Is hume's principle analytic? *Notre Dame Journal of Formal Logic*, *40*(1), 307–333. doi: 10.1305/ndjfl/1039096303

Wright, C., & Hale, B. (2001). *The reason's proper study: Essays towards a neo-fregean philosophy of mathematics* (C. Wright, Ed.). Oxford: Clarendon Press.

Yablo, S. (1993). Definitions, consistent and inconsistent. *Philosophical Studies*, *72*(2-3), 147–175. doi: 10.1007/bf00989672

Zalta, E. N. (2024). Frege's Theorem and Foundations for Arithmetic. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Spring 2024 ed.). Metaphysics Research Lab, Stanford University. `https://plato.stanford.edu/archives/spr2024/entries/frege-theorem/`.