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MODELING THE KINEMATICS OF MEANING

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MODELING THE KINEMATICS OF MEANING

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1 Meaning and Cognition as Activities

There is a pervasive duality in cognition between human activities and their static forms or products, which is reflected in a certain benign ambiguity in natural language. Thus, the word "judgment" is ambiguous between the act and its content, and so are, say, "reasoning", "dance" or "motion". In most logical and philosophical theories of cognition, however, there is a bias towards the finished product, in that judgment revolves around static propositions (or sentences), and reasoning around static proofs (or texts). But even on a formal computational model of cognition, this view seems impoverished, in that computation always involves an interdependence between more static 'representation' and more dynamic 'procedure'. And indeed, in recent years, there has been a trend toward taking the latter aspect seriously in its own right, not just as a matter of 'implementation' of static essences, but as something on a par with them. Illustrations of this phenomenon are easy to find. In the foundations of mathematics, intuitionism and constructivism in general were always concerned with the creative mathematical activity, but their formalization was directed toward its products (definitions, proofs), without explicitly describing its underlying action structure. Now, modern logical approaches have started investigating the latter too, with their various forms of cognitive 'updating', 'retraction' and 'revision'. Similar movements may be observed in the semantics of natural language (where 'meanings' are now often described in terms of their 'information change potential' rather than their static 'truth conditions') and indeed general epistemology, where 'knowledge' as a focus is giving way to 'cognition'. The present, rather programmatic paper aims at providing some 'strategic depth' for this dynamic research program, from a logical point of view. In principle, modern logic has always been committed to the interplay of proofs, rules and algorithms in both static and dynamic senses, even when this potential has not always been fully realized.

The movement described is not unique to this period in time. There is a long history of attempts at putting cognitive activities at centre stage, of which we mention a few: not just because of historical justice, but also for their contemporary inspirational power. Indeed, more static and more dynamic faces of proof occur as early as Euclid's "Elements", with their distinction between 'theorems' and 'constructions'. In modern times, the procedural aspect of human reasoning was emphasized eloquently in the critical study Toulmin 1958, with its insistence on juridical 'formalities' rather than mathematical 'form' as a guiding model for logic. Procedures of learning by trial and error and the correlated activities of updating and revision have always been prominent in the philosophy of science, with Popper's work as a clear example, especially in the so-called 'probabilistic' (as distinct from the usual 'deductive') tradition. And finally, the time is not long past when the philosophical study of language was dominated, not by concepts of logical and grammatical 'form', but by Wittgenstein's 'language games' as a focus of linguistic activity. (And even before, games were a fashionable unifying paradigm, witness Huizinga 1938 and the references given there.) This perspective has found its logical continuation in various logical game theories, which have been applied to a variety of epistemic concerns (cf. Lorenzen 1959, Hintikka 1973). The difference between modern attempts and these predecessors lies perhaps in a somewhat greater recent sophistication, inspired by various new techniques from computer science. It might actually be of interest to re-examine the tradition in this new light. For instance, traditional key issues like the status of "analytic" versus "synthetic" judgment assume a whole new shape when relocated to the level of activities (which may also be more congenial to the spirit of Kant's writing, whose philosophical terms still seem to breathe the above-mentioned benign ambiguity).

Now, the more specific question to which this paper addresses itself is the following. Which formal models can best bring out this dynamic character of human cognition on a par with its static structure? We shall have no new break-through to report here. Our suggestion is to explore the range of available options in its full extent, and bring out their potential 'fine-structure' when used as an instrument for analyzing cognitive mechanisms. The outcome of the presentation is not one final unified formalism, but rather an attempt at raising some central logical issues in the above movement.

2 Three Models for Cognitive Dynamics

The first model is that of *proofs*. Even though logic is mainly concerned with meaning and truth on some influential modern accounts, there has always been an independent interest in the more precise structure of the argumentation by which one arrives at insights about how the truth lies. Proofs are constellations of inferential steps reflecting (one hopes) certain basic steps and rules in human cognitive competence and performance. In particular, the very richness of existing proof formats (whether axiomatic, via natural deduction trees or otherwise) is another asset in this respect, since it shows that we have quite a variety of ways for bringing out relevant structure. This array of notions has led to the development of a discipline of Proof Theory, which studies, amongst others, when two proof systems may be counted 'equivalent', or how proofs within one format can be effectively transformed into another. This emphasis on equivalence may have to be qualified from our viewpoint, in that we are less interested in 'extensional equality' of input-output behaviour for derivable premise-conclusion sequents than in 'intensional equality' of concrete proof formats for various styles of reasoning. The main cognitive import lies often in the 'little things'. Proof Theory as a more general model for meaning has been proposed at various places in the literature, witness Kneale&Kneale 1962 (using 'Hilbert-style proofs'), Dummett 1976 (relying on intuitionistic proof formats) and recently in the semantics of natural language, Ranta 1991 (advocating Martin-Löf style type theories), Gabbay & Kempson 1992 (using 'labeled deductive systems'). Interestingly, the latter approaches blend in with our next kind of model, in that they 'decorate' proofs with algorithmic information.

The second model, not surprisingly in a computational age, is that of *programs*. Nowadays, many people think of cognitive processes as some form of mental computation triggered by pieces of text. On this view, natural language is partly a programming language for cognition, and the art is to analyze cognitive activities (discourses, arguments, plans) in terms of navigation across information states. This leads to a study of available basic moves as well as the more global programming structure of cognitive activities, in particular, investigating the latter's expressive power and computational complexity. The programming view has its technical logical background too, partly in Recursion Theory, and even more in various parts of computer science. (For instance, the semantics of programming is relevant here, including such areas as 'Dynamic Logic'.) This second model underlies many recent accounts of meaning and cognition, both in linguistics and in general cognitive science, witness Heim 1982, Kamp 1984, Gärdenfors 1988, van Benthem 1991, Groenendijk & Stokhof 1991, Veltman 1991.

The third model, more marginal in the contemporary literature so far, is that of *games*. These too have been proposed as convenient ways of bringing out essential features of human cognitive activities, especially since we often seem to have very vivid intuitions about them. For instance, there are logical games for evaluating statements, comparing model structures, or carrying on debates, with suitably assigned roles for players and winning conventions (an extensive survey of the relevant literature is found in van Benthem 1988). Winning strategies in evaluation or debating games have provided analyses for notions of 'truth' and 'consequence' in the work of Lorenzen 1959, Hintikka 1973 (cf. also Lewis 1979). Further formal background is provided here, partly by mathematical Game Theory, and partly also by certain branches of logic itself (witness the accounts of various back-and-forth games in Model Theory). Of course, there is also a more general academic literature, outside of the logical tradition, studying the role of games both in culture and in nature (cf. again Huizinga 1938, or Berne 1977, Eigen & Winkler 1975).

What is common to all three models is a certain movement through a space of relevant states, be they partial stages of deduction or argumentation, information states, or configurations of a game. This movement is made possible through a repertoire of atomic moves, that can be combined into higher programs through the use of certain 'logical constructions'. What are the natural options here may depend on the model. For instance, proofs involve the basic combinatorics for tree-like argument patterns: 'combination' via conjunction, 'arguing by cases' via disjunction and 'hypothesizing' via implication, creating a dynamic block structure of nested reasoning. With programs, one rather thinks of the usual constructions for instructions or plans such as 'sequential composition', 'indeterministic choice' or 'iteration', possibly guided by 'control assertions'. Finally, games again carry a natural structure of their own, reflecting the involvement of different players, such as various 'choices of continuation' for different players (signalled by conjunctions or disjunctions indicating their rights of choice and duties of response) as well as the notion of 'role change' (signalled by negation). Nevertheless, there also seems to be a certain unity to this logical repertoire, that might come out at a suitably abstract level of analysis.

One useful unifying framework here is that of a General Dynamic Logic, as proposed in van Benthem 1992, which describes various kinds of back-and-forth movement through the 'epistemic time' of a universe of information states ordered by inclusion. Cognitive actions then denote sets of state transitions, i.e., binary transition relations over states. Or more generally, they may be taken to be sets of state transition sequences ('traces of the activity'), so that the proper formalism would become a 'branching time logic' combining evaluation at states with that on trajectories representing 'epistemic histories'. 'Logical operations' may be identified here quite generally as those satisfying certain mathematical invariance properties reflecting their 'topic-neutrality' (cf. van Benthem 1989, Sher 1991). One quite noticeable feature of such a framework is its explicit interplay between two levels: that of activities changing states, and that of standard declarative statements about the states traversed by the former. This design makes sense, since programming involves the interaction of static tests and dynamic actions, whose results may be described using static 'preconditions' and 'postconditions'. Likewise, playing a game often involves the interaction of moves with tests concerning their intermediate results (most games will depend periodically on checking "whether the ball was out".) There is a good deal of mathematical theory about this logical framework, so that we can obtain complete axiomatizations or analyses of complexity and expressive power (cf. de Rijke 1992). Thus, at some level of description, our three models for cognition are indeed similar. But of course, this does not tell us very much yet. Interesting differences and insights will come to light only when we consider further details of the structure of both 'states' and 'moves' on these different ways of thinking, which will emerge when we put them to work.

3 Working with Proofs, Programs and Games

Besides analogies, there are also suggestive differences between the three models. For instance, at a more specific level, not all *logical constants* are equally at home in their associated activities, even when they can be introduced in some technical way. Notably, the proof-theoretic perspective is not really hospitable towards *negation*, unless one treats 'refutation' on a par with 'proof' (cf. Wansing 1991). Likewise, negation as 'complement' of programs is at best a marginal operation ("avoidance") – but by contrast, negation as 'role switching' is a crucial element in games. The proper framework for thinking about 'logical structure' with proofs may be a theory of construction for dynamic data structures (such as possibly nested trees), for programs it may be some form of relational algebra, whereas for games, no general principled analysis of possible operations seems available so far. Analyzing traditional logical notions in these various models may also lead at least to unconventional outcomes. For instance, in all three cases, quantifiers are not really on a par with propositional connectives, even though the two are usually lumped together in standard expositions. Quantifiers rather seem to signal atomic moves of establishing some binding or drawing some object, plus (in some cases) some further propositional superstructure over these. On this view, the syntax of the usual formal languages is even misleading, in that it does not allow us to regard, say, a prefix "$\exists x$" as an independent instruction by itself, stating something like this: 'pick an object ("$\exists$"), and assign it a temporary name x '. A much more sensitive account of this kind of quantificational activity, involving changing data structures and bindings, may be found in Visser 1992.

The different flavours of the three approaches may also be sampled by looking at their respective accounts of the crucial notion of *inference*. The proof-theoretic perspective justifies an inference by composing it as the result of a number of basic moves. This is brought out clearly in the simple example of " A∨B, ¬A / B " , which amounts to a combination of argument by cases and one basic negation step:

$$\frac{A\vee B \qquad \dfrac{A \qquad \neg A}{B} \qquad B}{B}$$

In the program-oriented perspective, however, this inference will rather be viewed as an updating instruction:

> Updating any information state by A∨B and then by ¬A (using some suitable decomposition of these compound instructions) leads to a new information state which may be tested to validate B .

Finally, in the context of games, the story is different again. For instance, in the 'agonistic' Lorenzen style, one would say that

> Defending the claim B in a dialogue against an opponent who has granted the 'concessions' A∨B , ¬A admits of a winning strategy.

One locus where differences may come out here lies in the so-called *structural rules* governing inference, such as the admissibility, without loss of previous conclusions, of shuffling premises by the rule of Permutation, or of adding premises by Monotonicity. The preceding three accounts do not necessarily support the same structural rules. For instance, Permutation seems reasonable on both the proof-theoretic and game-theoretic views, whereas it might be unreasonable on the programming view, since the sequential order of instructions is usually crucial to their total intended effect. Likewise, Monotonicity might be a reasonable principle in games (the more concessions from one's opponent the better), but not on the other two accounts. But actually, even these outcomes will depend on further specification. For instance, if one takes the ordering of premises in a game to convey the priority of commitments incurred, then Permutation will lose its appeal in this model too.

We conclude this brief exploration with a more concrete example where the three models may be put to work. In fact, standard *first-order predicate logic* may still serve as a concrete example for experimentation with such newer ideas. (Although it seems well- and perhaps even over-explored, little twists in interpretation and inference will make it fresh and mysterious again.) A good point of departure here are Beth's 'semantic tableaus', which have always focussed a good deal of semantic imagination. Tableau rules may be viewed in various ways. Read upside down, they are instructions for the stepwise construction of (counter-)models, and as such they fit very well into the programming paradigm, witness a rule like 'Left Quantifier Decomposition':

$\exists x\phi, \Sigma \bullet \Delta$: *introduce a new object* d *and continue with the new construction task* $\phi(d), \Sigma \bullet \Delta$

(We might also read these rules as instructions for evaluation, by interpreting the 'introduction' of d as the result of sampling from some independently given domain.) When read bottom up, these same rules will become left- and right-introduction rules in a Gentzen-style proof-theoretic 'calculus of sequents'. But, as was already pointed out by Lorenzen himself, one can also read semantic tableaus as a record of possible moves in a game. For instance (there are options here), one might have a zero-sum game with player I defending the existence of a counter-example, and player II denying this. Now, the rules represent various obvious game conventions, such as:

$\Sigma \bullet \Delta, \phi\wedge\psi$: *continue with a choice for player* I *as to which component is to be taken:* $\Sigma \bullet \Delta, \phi$ or $\Sigma \bullet \Delta, \psi$

Winning positions for player II will be those atomic end sequents where some formula occurs on both sides. Then, the tableau will close in the traditional sense if and only if player II has a winning strategy in this game.

The three analyses of first-order tableaus can be engineered so as to produce the same valid sequents, and this convergence is even made into a norm in logical text books. But the interesting point from our perspective is rather that there are different natural outcomes, reflecting various decisions concerning the argumentation that we are using, or the kind of game being played. The following illustration is from van Benthem 1990 (cf. also Mey 1992, which presents a rediscovery of the dialogical game tradition in the framework of Linear Logic). Let us look more closely at tableau games. A negation rule

" from $\neg\phi, \Sigma \bullet \Delta$ to $\Sigma \bullet \Delta, \phi$ "

expresses a role change, from defending $\neg\phi$ to attacking ϕ, and so does its converse. The rules for conjunction and disjunction will introduce either multiple commitments or choices (as we have seen). A conjunction on the left or a disjunction on the right just signals the presence of both tasks, witness the tableau rule

" from $\phi\wedge\psi, \Sigma \bullet \Delta$ to $\phi, \psi, \Sigma \bullet \Delta$ ",

while disjunction on the left signals a choice for player I again. Finally, consider the rules for existentially quantified statements $\exists x\phi$. Here, the natural convention on the left is to let player I choose a 'witness' d such that the game continues with $\phi(d)$. Symmetrically, on the right, the choice of a 'challenger' d is the privilege of player II. Now, this plausible semantic game will not lead to standard predicate-logical validity. The reason is this. The corresponding proof rules for the game rules will increase complexity in each step, so that the reverse processs of proof search will terminate in a finite number of steps. Thus, this notion of inference is *decidable*, unlike general first-order consequence. More precisely, the difference with the standard case is that, once one's duty with respect to a formula has been fulfilled, the formula disappears from the list of tasks. But many predicate-logical validities require more than one try on the same false existential statement (or true universal one), or alternatively, a number of single tries on various copies of that statement. (This may be seen, e.g., by constructing a semantic tableau for 'Plato's Law' $\exists x\,(\exists yAy \rightarrow Ax)$.) Put differently, full classical logic requires the structural rule of Contraction for identical formulas, which is invalid in this game. Nevertheless, it would be entirely plausible in a game-theoretic setting to make the following systematic distinction. The primary case is that where an obligation has been incurred and can be discharged in one single act. But in addition, there may also be 'standing obligations' which involve multiple discharges. Typical examples in ordinary argumentation involve commitment to universal quantifiers or conditional statements ("whenever you challenge me with the antecedent, I will respond with the consequent") . But then, one needs some explicit way of distinguishing one from the other, say, by introducing a new logical operator ! encoding standing commitment. Then, a formula $!\phi$ will be treated via the earlier rule for ϕ but with the difference that $!\phi$ itself is also inherited in the process. The corresponding proof rules are these:

$$\frac{!\phi, \Sigma \bullet \Delta}{!\phi, \phi, \Sigma \bullet \Delta} \qquad\qquad \frac{\Sigma \bullet \Delta, !\phi}{\Sigma \bullet \Delta, !\phi, \phi}$$

Note, for instance, that the valid predicate-logical formula $\exists x\,((Pa \vee Pb) \rightarrow Px)$ is unprovable here, since it requires 'two tries' (cf. Mey 1992), but one can prove its 'repetitive version' $!\,\exists x\,((Pa \vee Pb) \rightarrow Px)$. More generally, full predicate logic can be embedded in the game system with the new operator. In earlier Lorenzen-style dialogical analyses, various rather arbitrary conventions on repetition of attacks and defenses had to be chosen so as to arrive at standard predicate logic (cf. Lorenzen & Lorenz 1978). What we advocate instead is a choice of logical apparatus for games which explicitly reflects such distinctions, enabling us to manipulate these in practical inference systems, and making them an object of logical study in their own right. Similar points could be made for the other two models of cognition as well.

4 General Logical Themes

Several general themes may be discerned across all three models for cognition. The first of these may be called *fine-structure*. What is happening nowadays is that many classical logical systems are being reconsidered as to the 'choice points', often implicit, that went into their original formulation, but which may be varied so as to 'parametrize' the system into a family of options, that can be fine-tuned for various applications. Some of these options emerge at first within some richer framework. For instance, the move toward programs in dynamic logic shows that there are many different aspects lumped together under the single classical operation of "conjunction": viz., sequential composition of actions, but also various kinds of parallel execution or mere unordered listing of tasks. Likewise, a single notion of 'implication' will branch out into such dynamic variants as "if condition P holds, then perform action A" or "if action A has been performed, then condition P will hold", etcetera. But then, one can often see where corresponding options would also have been available inside standard systems. For instance, ordinary proof theory turns out to support different 'directed implications' in Categorial Logic, and different 'conjunctions' in Relevant Logic or Linear Logic (cf. van Benthem 1991).

A prominent aspect of fine-structure in these various systems is the notion of inferential *resources*. When cognitive activity comes to the fore, there are no unlimited supplies of information and 'deductive energy', and the task of logical analysis may be extended to bring out precisely which mechanisms are adopted, and which cost in resources is incurred. Very concretely, such considerations will come out in the management of *occurrences* of assertions or instructions in proofs, programs and games. Repeating the same formula twice in a proof means two calls to its evidence, repeating the same instruction in a program calls for two executions and repeating it in the course of a game may signal a new obligation as to its defense or attack. If one wants to record unlimited energy or unfailing commitment or 'standing obligation', this will have to be encoded as a matter of logic (witness the explicit commitment operator ! introduced in the preceding Section). Not surprisingly, therefore, many recent systems of logic have moved to occurrences, thereby operating at a much finer level of detail than classical or intuitionistic calculi of inference. Another broad aspect of fine-structure which is coming to light more systematically these days is that of *dependence*. In the usual systems of inference, it is assumed that the individuals that we are discussing can be freely introduced into our considerations. But in practice, there may be natural constraints. For instance, certain objects may 'depend' on others (as in the theory of 'arbitrary objects' by Kit Fine, or that of 'sampling' by Hintikka and Rantala, and such dependencies will also arise naturally in the course of 'dynamic interpretation'). Thus, one now has to provide explicit conventions as to what can be introduced when. Bringing out this degree of freedom turns out to have very interesting consequences. For instance, on the usual proof-theoretic account of meaning, pervasive non-standard quantifiers such as "most" or "many" always turned out to be difficult to analyze in the proper format advocated by philosophers. But as van Lambalgen 1992 shows, a natural Gentzen-style deduction format introducing 'accessibility management' for individuals in its quantifier rules can provide complete descriptions of inference with various non-standard quantifiers too, where the classical ones become the limiting case when 'unlimited access' is assumed.

There is another virtue to viewing these various models in conjunction, precisely because they also have their separate histories. For, this allows us to pursue some intriguing analogies of a more general nature. Our second general theme concerns the *cognitive claims* embodied in our three models. The programming model has been very prominent in modern approaches to cognition, not just because of its technological glamour, but also because it possesses two solid intellectual assets. One is the existence of a well-developed mathematical theory of computation behind it, the other the challenging philosophical programme enshrined in Church's Thesis which claims that *Any form of effective computation can be programmed on a Turing Machine,* or some equivalent device in Recursion Theory. In its broader version, the Thesis has been expanded to cover all 'rational activities' (which may presumably be encoded numerically inside Turing machines and their ilk). The emphasis on 'machines' rather than 'programs' is somewhat unfortunate here (the hardware of Turing machines or other computing devices is completely irrelevant, the crucial feature is the software); but with that correction, Church's Thesis gives the programming model its bite and appeal. It is interesting to speculate about similar claims concerning proofs or games. Certainly, Jaakko Hintikka's continuing work on logical game theory may be interpreted as implicitly posing a parallel claim (say, 'Hintikka's Thesis') stating that *Any rational human activity can be played via Logical Games,* using their roles and winning strategies. But the problem so far has been that this has not been backed up by any equally powerful logical notion of 'game structure'. Nevertheless, the obvious challenge here is to provide such a framework, and it is much too early to adjudicate the case between the two rival approaches. Likewise, David Israel (private communication) has suggested a 'Gentzen Thesis' making a similar claim about human inference (rather than computation) to the effect that: *All rational inference admits of a Natural Deduction formalization,* (perhaps even one which admits of 'Cut Elimination'). There is still some unclarity here as to the proper construal of these notions in a suitably general Abstract Proof Theory, but the appeal is again unmistakeable.

Given the above style of analysis, further clarification is needed with all three claims. Church's Thesis may be interpreted as the *extensional* statement that the input-output behaviour of every effective function can be adequately programmed on a Turing Machine producing the same results. But it might also have a much stronger *intensional* version, stating that the nature of any algorithm can be reflected faithfully in terms of the programming repertoire provided by Turing machines. In fact, there are many intensional similarities between computation on Turing machines, register machines and even (when all is said and done) logic programs, so that the usual 'evidence' for the Thesis by showing the convergence of different approaches can probably be reworked so as to get some evidence for the stronger version too. But there are also obvious intensional differences in, say, the structure of recursion in Turing Machines with their opaque "goto" loops and that found in higher programming languages for specifying algorithms, that might cast serious doubts on the full intensional version. (For a recent attempt at characterizing the intensional notion of 'algorithm', cf. Moschovakis 1991.) Again, this possible strengthening of Church's Thesis returns for the proof-theoretic and game-theoretic approaches. What are their 'natural repertoires' of logical constructions that should suffice, in principle, for faithful modelling of any rational form of inference or cognitive play? Stated in this form, the concerns behind Church's Thesis become related to various philosophical analyses trying to establish 'Functional Completeness' of operator repertoires in Proof Theory proposed by Prawitz or Schroeder-Heister (cf. Sundholm 1986 – and also van Benthem 1992 on the issue of complete logical repertoires of procedural constructions in the programming model).

As we have seen, the three models of cognition show both clear analogies and differences. But what should one be aiming for in the long run: some kind of Grand Unification, or rather Peaceful Coexistence? There are actually a number of points in favour of the latter course. One practical consideration is that fruitful coexistence is relatively easy to achieve, witness the interaction between the proof-theoretic and algorithmic models in the formalism of current 'type theories' based on statements of the form " function τ provides a method for establishing statement A " . A more empirical argument might be that diversity seems a hall-mark of human cognition: having a varied non-uniform cognitive repertoire may even be a winning strategy in the survival game of Evolution. (Less optimistically, this inherent diversity may also be interpreted as a sign that all our models of cognition will be of the mark, since we are describing what is essentially a physical process without inherent logical structure, driven by neural nets or pattern matching in vast memory banks.) Be this as it may, opting for diversity and coexistence is not a very cheap way-out, as there are costs involved even then. If we want to analyze cognition via a number of these models at once, taking all of them seriously, then a new and interesting aspect arises, that may be dubbed *logical architecture* of communication between the various components of the total system. What are the systematic mechanisms that are available for switching between proof-theoretic and other cognitive modes of activity, and how do we 'transport' information from one to the other? There is quite some material on this issue hidden in various detailed logical studies (such as completeness proofs for dialogue games in terms of proof calculi), but it would have to be put to work more generally, turning theoretical connections into practical channels of communication.

Do the general logical themes developed here have any consequences for 'genuine' philosophical issues, say, in the Theory of Meaning? Or are we just talking about 'epi-cycles' to the definitive analysis of our founding fathers, like Dummett or Prawitz, arising out of (perhaps undue) attention to computational details of implementation? Our answer will be clear. As was stated in the introduction to this paper, dynamic imperative aspects of cognitive activities seem worthy of treatment on a par with the usual declarative ones. This need not result in a break with earlier theories of meaning, but it does imply two things. 'Fine-structure' shows the need for careful re-analysis, trying to see how much of existing accounts of meaning is based on general insights, and how much is based on accidental features of the logical formalisms that served as their inspiration. The other is that there are several new questions that should move higher up on the philosophical agenda. In particular, these concern the explicit design or choice of a repertoire of logical constants as something which is not given once and for all, but which needs periodic reappraisal. For instance, current accounts focussing on always the same set of connectives and quantifiers seem light-years removed from the practice of natural reasoning, where 'logicality' is located in many variants of these expressions, as well as completely different grammatical categories, such as modifiers ("almost", "very") or adverbials ("self"). Also, more conscious attention would be required for higher architectonic aspects of the coexistence of many different cognitive styles, such as their desired communication. How does one manage, for instance, the interaction between static 'truth conditions' and 'dynamic 'update conditions' in some civilized and fruitful manner? It would be good if philosophers of meaning were to contribute some interesting 'regulative ideas' in these matters.

Finally, we want to mention two challenges to any further account of cognition, that lie right around the corner. First, even the simplest realistic situation of language use involves what may be called *global structures*. Current logical theories give us an account of cognition at relatively low levels of aggregation. But what we also need is a better grasp of the more global structure of texts or theories, and of the hierarchy of rules that govern our activities at these various levels. As usual, various interesting bits and pieces in the existing literature may serve as points of departure here, such as the earlier work on the logical structure of scientific theories in the Philosophy of Science (cf. the survey in Suppe 1977), the analysis of more global structures of definition, proof and refutation in Lakatos 1976, or recent computational work on structured data bases (cf. Ryan 1992). But these have not yet been integrated with mainstream logic in an effective manner, so as to broaden the scope of the enterprise. The other inevitable challenge concerns the undeniable fact that cognitive activity is usually a social process with more than one participant. The role of *multiple agents* has been taken seriously in the game-theoretic approach, but not in the other two. What we might need eventually are many-person versions of our whole theory, replacing programs by 'protocols' for a group of distributed agents, and proofs by more interactive formats of reasoning.

5 By Way of Conclusion

This is a paper without a conclusion, but with a recommendation. What we hope to have shown is the utility and inspirational value of putting a number of logical models for cognition side by side, exploring their analogies and differences in a non-dogmatic fashion, and trying to see what they can tell us about the nature of cognition, and also, how well they will stand up to the challenges ahead.

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