institute for logic, language and computation

JOHAN VAN BENTHEM

Two Essays on Semantic Modelling

X-94-01, *received*: Jan. 1994

ILLC Research Report and Technical Notes Series
Series editor: Dick de Jongh

Technical Notes (X) Series

Institute for Logic, Language and Computation (ILLC)
University of Amsterdam
Plantage Muidergracht 24
NL-1018 TV Amsterdam
The Netherlands
e-mail: illc@fwi.uva.nl

Institute for Logic, Language and Computation **ILLC**
University of Amsterdam
20 September 1993

Reflections on Epistemic Logic

to appear in Paul Gochet, ed., special issue of *Logique et Analyse*.

Johan van Benthem

****

Traditionally, 'epistemic logic' has been a specialized area of general modal logic devoted to the epistemic modalities and their interaction with quantification, predication and identity. In this brief Note, we shall step back and see which broader forms of epistemic semantics exist, and discuss how they affect traditional concerns of the field.

****

1 Standard Epistemic Logic

Traditional epistemic logic started largely with the pioneering classic Hintikka 1962, which presented modal logics for the notions of a person knowing and believing a proposition, based upon standard possible worlds semantics with an accessibility relation of 'epistemic indistinguishability'. Roughly speaking, I know a proposition if that proposition holds in all worlds which I cannot distinguish from my current one, that is, if it holds across my whole 'range of uncertainty'. Different epistemic attitudes may then vary in the extent of that semantic range, or in the requirements imposed upon it. In a subsequent series of papers, Hintikka also developed an epistemic predicate logic, dealing with issues of individual identity across epistemic worlds and the interplay of knowledge and ('de re' versus 'de dicto') quantification. No extensive mathematical theory developed around this initial focus – the technical monograph Lenzen 1980 is a respectable exception – but the paradigm did find a number of active areas of application in the eighties, especially in computer science with the work of Joe Halpern and his colleagues at IBM San José, and in Artificial Intelligence by Bob Moore and some colleagues at SRI, Menlo Park. What the computational connection added, in particular, was a more concrete view of epistemic models derived from computational protocols or processes, where agents are well-defined and philosophical questions concerning the intentionality of human cognition can be safely side-stepped.

Notions which then came to the fore were 'autoepistemics', concerning the equilibrium states that an agent can achieve in non-monotonic default reasoning (Moore 1987), and also various forms of collective epistemics expressed in the interplay of operators K_aB ('agent a knows that B ') for various agents a , as well as 'common knowledge' for groups of agents (Halpern & Moses 1990). Moreover, these epistemic concerns were embedded in a larger environment of communication and physical action. As a result, there is now a flourishing community around the so-called TARK Conferences (cf. Halpern, ed., 1986, Vardi, ed., 1988 and subsequent volumes), which brings together philosophers, mathematicians, computer scientists, economists and linguists. In what follows, we shall take the basic core of epistemic logic for granted, while making only occasional reference to its computationally inspired extensions.

2 Implicit versus Explicit Epistemics

The above enterprise of epistemic logic has not been without its critics. In fact, many people have regarded it as a typical form of 'shallow analysis', describing epistemics by merely imposing some modal superstructure on top of an ordinary classical semantics, rather than reanalyzing the latter in depth. To introduce a distinction, the Hintikka-Halpern paradigm is one of 'extrinsic epistemics', which does not affect standard classical semantics, witness its key truth definition stating that, in any model, " K_aB is true at a world w iff B is true at all worlds v that are R_a-related to w ". By contrast, a more radical 'intrinsic epistemics' would not take classical semantics for granted, and reanalyze the whole notion of truth in the light of epistemic considerations. The most prominent historical instance of such a radical enterprise is of course intuitionistic logic, where 'truth' recedes as the central logical notion in favour of 'provability' or 'assertability'. Thus, its guiding philosophy has been epistemically oriented from the start. (Compare the constructive theories of meaning based upon the 'Brouwer-Heyting-Kolmogoroff interpretation' that are surveyed in Troelstra & van Dalen 1988.) To be sure, intuitionistic logic, too, has its Kripke-style possible worlds semantics, but with an entirely different flavour. Worlds stand for information states, accessibility encodes possible informational growth, and truth at a world corresponds intuitively to epistemic 'forcing' by the available evidence there. Thus, the basic logical operations themselves become 'epistemically loaded'. But then, given an epistemic reinterpretation of what logic and semantics is about, where is the need for any separate 'epistemic logic'?

It is not so easy to adjudicate this debate. Certainly, the intuitionistic approach has generated more interesting mathematical theory, while also being more influential in philosophical and computational circles. But there are many different causes for this course of events, not all of them having to do with the relative merits of implicit versus explicit epistemics. Moreover, as a matter of fact, there are also some advantages to the extrinsic approach. For instance, it can incorporate whatever sound analysis is provided in a classical semantics for a certain kind of propositions, without having to worry about philosophical compatibilité d'humeurs. By contrast, more intuitionistic proof-theoretic approaches have had difficulties making sense of logical operators such as generalized quantifiers that have perfectly straightforward model-theoretic explications. (But cf. van Lambalgen 1991 for a fresh start.) Moreover, the very explicitness of Hintikka's approach has encouraged an active search for new epistemic operators, such as the above ones referring to knowledge of multiple agents or groups which have no counterpart in intuitionistic logic. Even though mathematics is a social activity, too, where insights may depend essentially on cooperation of rational agents, the implicit intuitionist stance has not been very conducive so far to bringing this out formally.

Finally, the distinction betweeen intrinsic and extrinsic epistemics is an 'intensional' one, which may be undercut at a more formal level of analysis. For instance, the well-known Gödel translation embeds intuitionistic logic faithfully into S4, the logic of choice for much of epistemic logic, thereby making it a kind of 'forward persistent' part of the latter approach. (Van Benthem 1990 provides further formal detail here. Incidentally, no converse embedding seems to work.) In this line, the explicit 'epistemic mathematics' of Shapiro 1985 may also be viewed as a natural extension of intrinsic intuitionistic mathematics. Conversely, the analyses of 'common knowledge' put forward in Barwise 1988 seem more 'intrinsic' analyses of this phenomenon, changing the underlying classical model theory to a form of information-oriented situation semantics. Generally speaking, then, there seems no problem of principle in combining both the agenda and the technical apparatus of intrinsic and extrinsic approaches to epistemic logic.

3 Information-Based Semantics

In recent years, the 'intrinsic' approach has gained ground in various new guises. Current semantics for natural languages and computation shows a trend away from truth conditions and correspondence with the world outside to explaining propositions in terms of their role in information processing over models that are now viewed as 'information structures'. Here, the classical turn-stile becomes a notion of 'forcing' for statements by the available information. Examples of this trend are data semantics (Veltman 1985), various forms of partial modal logic (Thijsse 1992), constructive semantics in the style of Nelson (Jaspars 1993) or substructural semantics for relevant or categorial logics (van Benthem 1991, Wansing 1993). This development can be seen as epistemic logic in a broader sense, since much of cognition can in fact be subsumed under the heading of information processing. What this new phase adds, however, is an explicit concern with the nature of information states and possible updates over these effected by successive propositions. Much is still unclear at this stage, witness the persistent debate between 'eliminative' accounts of information processing (which proceed via successive elimination of epistemic possibilities) and 'constructive' ones (which build ever larger representation structures). Either way, these concerns do seem a natural addition to the foundations of epistemic logic.

Moreover, one clear trend can be observed which does not depend on the exact nature of epistemic states. Current information-oriented modellings suggest a richer and more systematic design of important epistemic operators. This phenomenon is illustrated by the basic case of Kripke models themselves. Initially, these were designed to model a particular epistemic language, whose operators were given independently. But then, one can also reverse the perspective and ask, given such models for information structure, which epistemic languages would best bring out their semantic potential, thereby redesigning the original language. For instance, as in temporal logic, there are two natural directions in the growth ordering of information states, both backward and forward. Knowledge may be mostly concerned with epistemic 'advance', but it has to do also with epistemic 'retreat', in contraction or revision of our information (cf. the symmetric theory of updates and contractions in Gärdenfors 1988). Thus, a more adequate epistemic logic should also incorporate operators reflecting these additional directions and their interplay, with 'forward knowledge' referring upward to all possible extensions of the current state, and 'backward knowledge' referring to the epistemic past. Their interplay will then generate different routes for epistemic revision ("if A had been found, then ..."). Van Benthem 1990 explores the resulting hierarchy of operators over information models, using a framework of enriched modal logics. In particular, a next natural stage of epistemic expressiveness would involve operators reflecting the addition of information pieces (i.e., suprema in the ordering of possible growth) as well as their downward counterparts (i.e., their infima). For instance, the binary epistemic modality $\phi+\psi$ would hold at those states which are the sum of a state verifying ϕ and a state verifying ψ. These will allow us, e.g., to refer to logical features of composite knowledge arising from the combination of various sources.

Of course, by this time, one will have left the traditional areas of the Theory of Knowledge which inspired Hintikka's initial enterprise. But that might also be considered a virtue, and one could certainly translate many of the newer technicalities back into genuine philosophical issues that might revitalize the somewhat fossilized agenda found in most philosophical textbooks.

4 Adding Justifications

The above type of enrichment may be seen as an extension of the modal viewpoint on epistemics, with a reinterpretation of its models in a Kripke-style intuitionistic spirit. But there is more to be learnt from a confrontation of extrinsic and intrinsic approaches. One conspicuous feature of contemporary intuitionistic logic and mathematics has been the development of type theories whose proof format rests essentially on binary assertions of the form $\pi : A$, meaning that function π is of type A, or that proof π establishes proposition A. Proof rules in the usual constructivist interpretation then typically build up compound conclusions in both components, witness a case like

$$\frac{\pi_1 : A \qquad \pi_2 : B}{(\pi_1, \pi_2) : A\&B} \qquad\qquad \frac{\pi_1 : A \to B \qquad \pi_2 : A}{\pi_1(\pi_2) : B}$$

Note how both assertions and their justifications are affected here. This binary logical format is gaining popularity these days for its greater perspicuity in bringing out combination of linguistic propositions plus their underlying manifestations or justifications, where the latter need not always be explicitly linguistically encoded. This makes sense in mathematics and computation (cf. Barendregt 1993), but also more broadly in linguistics and Artificial Intelligence, witness the new research program of 'labeled deductive systems' put forward in Gabbay 1993.

Now here too, there is a very attractive move for epistemic logic. Much of the classical theory of knowledge and its initial logical formalizations seems hampered by the absence of any systematic way of bringing out the justifications underlying our knowledge as first-class citizens. Put somewhat formally, saying that someone knows a proposition is an existential quantification stating that she has a justification for that proposition. But by keeping those justifications hidden in our logical framework, we create both technical and conceptual difficulties. For instance, much of Hintikka's own work on the Kantian notion of analyticity (cf. Hintikka 1973) has to wrestle with the fact that with Kant, 'analyticity' is a qualification of reasons or justifications as much as of statements, which makes its projection into standard logical systems somewhat problematic. An even more striking example is the so-called 'Problem of Omniscience', where knowledge of a proposition entails knowledge of all its logical consequences. This problem is highlighted in the standard epistemic Distribution Axiom $K(A \to B) \to (KA \to KB)$. By contrast, in the above binary format, this problem would not arise in the first place, because any logical inference will come with an explicit cost record in terms of a more complex justification. To see this, compare the above distribution principle with the corresponding binary type-theoretic inference from the two premises

$\pi_1 : A \to B$ and $\pi_2 : A$ to the conclusion $\pi_1(\pi_2) : B$. Of course, one further question here is what systematic type-theoretic calculus will support explicit epistemic operators. There are various options to this effect, such as the following two introduction rules for epistemic operators:

$$\frac{\pi : A}{- : KA}\ (\text{'forgetful'}) \qquad\qquad \frac{\pi : A}{{}^*(\pi) : KA}\ (\text{'reminiscent'})$$

For a more elaborate system of this kind, cf. the modal type theories in Borghuis 1993. (E.g., deriving the above epistemic distribution axiom will also involve suitable elimination rules for the K-operator.) The conceptual task remains to set up a complete plausible base theory for epistemic logic with an explicit calculus of justifications.

5 Cognitive Action

When model-theoretic semantics is reinterpreted as a theory of information structures, one further move becomes quite natural. Justifications and revisions are really examples of cognitive actions – and it would be quite appropriate then to embed our epistemic logic into an explicit dynamic logic. This move is in fact foreshadowed in the earlier-mentioned computational tradition in epistemic logic (cf. Moore 1984). For instance, the above binary schema $\pi : A$ has one further useful interpretation, stating that "program or action π achieves an effect described by proposition A " . Examples of relevant cognitive actions are the earlier-mentioned 'updates' or 'contractions', but one can also think of a much richer repertoire of 'testing', 'querying', etcetera. Moreover, on top of this basic repertoire, one can describe complex cognitive actions or plans via the usual programming operations, such as sequential or parallel composition and choice. Evidently, human cognitive plans have compound structures not unlike those found in computer programs. The precise extent of this analogy is an interesting issue by itself. For instance, can cognitive plans also display more infinitary structures like recursion?

Stated in this way, we need a two-level system for combining epistemic statements with 'cognitive programs'. One possible logical architecture here is the 'dynamic modal logic' of van Benthem 1993, which has a relational repertoire of actions over information models interacting with a standard modal logic. Typical assertions in such a formalism will be a dynamic modality $[\pi]\, A$, stating that " action π always achieves effect A ", (compare the above binary schema), or modal iterations like $[\pi_1]\langle\pi_2\rangle\, A$ stating that " action π_1 'enables' action π_2 to achieve effect A " . The model theory and proof theory of this system are well-understood (cf. Harel 1984, de Rijke 1992). Moreover, its expressive power subsumes at least the better-known theory of belief revision in

Gärdenfors 1988. But dynamic modal logic also supports more radical deviations from classical logic. For instance, van Benthem 1993 considers 'dynamic styles of inference' from premises $\pi_1, \ldots, \pi_n$ to conclusions C based on the idea that propositions are cognitive actions which are being processed in reasoning. Various options to this effect may be expressed in the above terms. For instance, one plausible dynamic style would state that 'sequential processing of the premises is a way of getting the conclusion', which may be expressed by the modal formula $[\pi_1] \ldots [\pi_n] C$. Another typical dynamic style would rather state that 'processing the premises is a way of doing the conclusion' – which involves a stronger modal apparatus than that of standard dynamic logic (cf. Kanazawa 1993.)

6 Combinations and Conclusions

The above Sections point at various attractive enrichments of standard epistemic logic. Putting all of these ideas together, however, raises some obvious further issues. For instance, which logical architecture would most naturally combine statements, justifications and actions? A type-theoretic approach looks promising here. For instance, one of its key statements might be of the form $\pi : A \to B$, expressing that action π will always lead from states satisfying precondition A to states satisfying postcondition B. In such a format, one might describe the behaviour of compound cognitive actions or plans, mentioned in the above, with rules like the following:

$$\frac{\pi_1 : A \to B \qquad \pi_2 : B \to C}{\pi_1 ; \pi_2 : A \to C} \qquad\qquad \frac{\pi_1 : A \to B \qquad \pi_2 : A \to B}{\pi_1 \cup \pi_2 : A \to B}$$

Nevertheless, even this kind of generalization will still fail to capture some further essentials of cognition. First, we need logical systems that lift all of the above to many-person settings, which allow, amongst others, for updating common knowledge of groups via various acts of communication (cf. Jaspars 1993). But also, we would eventually need a theory of 'synchronization' between internal cognitive actions and external physical actions which change the world.

Even with all these open ends, our conclusions concerning epistemic logic will be clear. We would recommend expansion beyond Hintikka's original modal statement format, thereby effecting a junction with a much larger logical environment of epistemic import. Moreover, we think the effort would be well worth-while to systematically rethink much of traditional philosophical epistemology in this broader light.

Acknowledgment

This paper grew out of notes for two lectures. I wish to thank the organizers of TARK IV (Asilomar 1992), and especially Yoram Moses, for inviting me to their meeting. Moreover, I am grateful to Professor Paul Gochet for inviting me to his workshop on Epistemic Logic (Liege 1993).

References

S. Abramsky, D. Gabbay & T. Maibaum, eds., 1993,
Handbook of Logic in Computer Science, Oxford University Press, to appear.

H. Barendregt, 1993,
'Lambda Calculi With Types', to appear in S. Abramsky et al., eds.

J. Barwise, 1988,
'Three Views of Common Knowledge', in M. Vardi, ed., 365-379.

J. van Benthem, 1990,
'Modal Logic as a Theory of Information', CSLI Report 90-144, Center for the Study of Language and Information, Stanford University.

J. van Benthem, 1991,
Language in Action. Categories, Lambdas and Dynamic Logic, North-Holland, Amsterdam, (Studies in Logic 130).

J. van Benthem, 1992,
'Abstract Proof Theory', manuscript, Institute for Logic, Language and Computation, University of Amsterdam.

J. van Benthem, 1993,
'Logic and the Flow of Information', in D. Prawitz et al., eds.

T. Borghuis, 1993,
'Interpreting Modal Natural Deduction in Type Theory', in M. de Rijke, ed., 67-102.

J. van der Does & J. van Eyck, eds., 1991,
Generalized Quantifier Theory and Applications, Dutch Network for Language, Logic and Information, Amsterdam, (to appear with CSLI Lecture Notes, Chicago University Press).

D. Gabbay, 1993,
Labeled Deductive Systems, Department of Computing, Imperial College, London, (to appear with Oxford University Press).

D. Gabbay & F. Guenthner, eds., 1984,
Handbook of Philosophical Logic, volume II, Reidel, Dordrecht.

P. Gärdenfors, 1988,
Knowledge in Flux. Modeling the Dynamics of Epistemic States, The MIT Press, Cambridge (Mass.).

M. Ginzburg, ed., 1987,
Readings on Non-Monotonic Reasoning, Morgan Kaufmann Publishers, Los Altos.

J. Halpern, ed., 1986,
Proceedings First Conference on Theoretical Aspects of Reasoning About Knowledge, Morgan Kaufmann Publishers, Los Altos.

J. Halpern & Y. Moses, 1990,
'Knowledge and Common Knowledge in a Distributed Environment', *Journal of the Association for Computing Machinery* 37, 549-587.

D. Harel, 1984,
'Dynamic Logic', in D. Gabbay & F. Guenthner, eds., 497-604.

J. Hintikka, 1962,
Knowledge and Belief, Cornell University Press, Ithaca (New York).

J. Hintikka, 1973,
Logic, Language Games and Information, Clarendon Press, Oxford.

W. van der Hoek, 1992,
Modalities for Reasoning About Knowledge and Quantities, dissertation, Department of Computer Science, Free University, Amsterdam.

J. Jaspars, 1993,
Logics for Communicative Action, dissertation, Institute for Language and Knowledge Technology, University of Brabant, Tilburg.

M. Kanazawa, 1993,
'Completeness and Decidability of the Mixed Style of Inference with Composition', Center for the Study of Language and Information, Stanford University.

M. van Lambalgen, 1991,
'Natural Deduction for Generalized Quantifiers', in J. van der Does & J. van Eyck, eds., 143-154.

W. Lenzen, 1980,
Glauben, Wissen und Wahrscheinlichkeit, Springer, Vienna.

R. Moore, 1984,
'A Formal Theory of Knowledge and Action', CSLI Report 31, Center for the Study of Language and Information, Stanford University.

R. Moore, 1987,
'Possible Worlds Semantics for Auto-Epistemic Logic', in M. Ginzburg, ed., 137-142.

D. Prawitz, B. Skyrms & D. Westerståhl, eds., 1993,
Proceedings 9th International Congress of Logic, Methodology and Philosophy of Science. Uppsala 1991, North-Holland, Amsterdam.

M. de Rijke, 1992, 'A System of Dynamic Modal Logic', Report LP-92-08, Institute for Logic, Language and Computation, University of Amsterdam.

M. de Rijke, 1993,
Extending Modal Logic, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

M. de Rijke, ed., 1993,
Diamonds and Defaults, Kluwer, Dordrecht.

S. Shapiro, ed., 1985,
Intensional Mathematics, North-Holland, Amsterdam.

E. Thijsse, 1992,
Partial Logic and Knowledge Representation, dissertation, Institute for Language and Knowledge Technology, University of Brabant, Tilburg.

A. Troelstra & D. van Dalen, 1988,
Constructive Mathematics, North-Holland, Amsterdam, (Studies in Logic 121, 123).

M. Vardi, ed., 1988,
Proceedings Second Conference on Theoretical Aspects of Reasoning About Knowledge, Morgan Kaufmann Publishers, Los Altos.

F. Veltman, 1985,
Logics for Conditionals, dissertation, Department of Philosophy, University of Amsterdam.

H. Wansing, 1993,
The Logic of Information Structures, Springer, Berlin, (Lecture Notes in Artificial Intelligence 681).

Institute for Logic, Language and Computation **ILLC**
University of Amsterdam
December 1993

To appear in M. Marx and L. Polós, eds.,
Logic@Work, Oxford University Press.

THE SOURCES OF COMPLEXITY: CONTENT VERSUS WRAPPING

I Contents and Wrappings

Any description of a subject carries its own price in terms of complexity. To understand what is being described, one has to understand the mechanism of the language or logic employed, adding the complexity of the encoder to the subject matter being encoded. Put more succinctly, "complexity is a package of subject matter plus analytic tools". This price is inevitable, and scientific or common sense insight does result all the same. Nevertheless, there is also a persistent feeling that one should never pay more than is necessary. Aristotle already formulated the necessary intellectual 'lightness' as follows. "It is the hall-mark of a scientific mind to give a subject no more formal structure than it can support" (as paraphrased in Kneale & Kneale 1961). Critics of formal logic would certainly agree with this dictum, and they have pointed at many cases in philosophy, linguistics, computer science and even the foundations of mathematics where general mathematical sophistication or just essayistic common sense is the more appropriate road towards insight than elaborate logical formal systems. But also inside formal logic, this question seems a legitimate concern. Are our standard modelings really appropriate for certain phenomena of reasoning, and are the received conclusions that we draw about complexity of phenomena in our field (using qualifications like 'undecidable' or 'higher-order') warranted, or rather an artefact of those modelings? More disturbingly, could it be that much of the respectable literature in our journals, which aims at proving difficult theorems as a sign of the academic 'worthiness' of a topic, mostly derives its continuity from the fact that one encounters the same issues over and over again, precisely because they derive from the formalisms employed, not from the subject matter at hand? I think these are serious questions, that deserve constant attention. Of course, many logicians do care for these, either implicitly or explicitly – but it will do no harm to keep them high on the agenda by means of some occasional extra advertisement.

But then, of course, the question arises how one can distinguish the two sources of complexity in specific cases. Often, one has a suspicion that received views as to what is difficult in some form of reasoning or computation and what is not, might be challenged, but how to separate the two components? There may not be any systematic method of separation here – but there is certainly a roundabout answer in the practice of our field, which tends to generate scores of alternative formal modellings. Thus, the role of the wrappings becomes visible indirectly, by comparison with alternatives (e.g., think of set-theoretic versus algebraic or category-theoretical formulations of the same problem). In other words, although formal logical modellings may be part of the complexity problem themselves, logicians mitigate this drawback by producing so many of them!

There is room for more detailed analysis here, by considering more concrete examples. Received views on complexity in formal semantics often come in the form of warnings concerning certain dangerous 'thresholds' where complexity is generated or increased. For instance, one well-known danger zone is the transition from finite to infinite structures, and another that from first-order to higher-order objects. These moves often produce undecidability or even non-axiomatizability in the description of computational or linguistic phenomena. For instance, in the semantics of programming languages, undecidability may arise through the introduction of infinitary structures for iteration or recursion (Harel 1984, Goldblatt 1987). Likewise, temporal semantics for concurrency usually employ higher-order logics of temporal branches or histories, which involve quantification over sets (Burgess 1984, Stirling 1990). In the semantics of natural languages, similar thresholds arise. For instance, many quantificational phenomena are generally considered to be essentially second-order – such as linguistic 'branching quantifiers' which involve parallel rather than serial processing, with corresponding non-linear quantifier prefixes going beyond first-order logic (cf. Barwise 1979). Other sources of higher-orderness are traditionally located in the semantics of plural quantification (cf. van der Does 1992). Undecidability has also been claimed for variable binding phenomena in natural language anaphora (cf. Hintikka 1979 on the so-called "Any-Thesis"). Finally, new thresholds of complexity have emerged in Artificial Intelligence, with corresponding forms of 'received wisdom'. Notably, non-monotonic reasoning mechanisms, such as 'circumscription', are usually considered to be essentially higher-order and highly complex in general (McCarthy 1980) – and indeed, the same seems to hold, perhaps disturbingly, for any meaningful human cognitive task (cf. Kugel 1986).

Do these views square with the intuitive expectations that one might have on the basis of a plausible initial estimate of the intrinsic complexity of a subject, prior to its formalization? Not always. And in general, it is hard to make uniform predictions here. Sometimes, formal analysis does confirm our intuitive hopes or suspicions concerning complexity of a phenomenon, witness the complexity theory of logical formalisms (cf. Spaan 1993). But at other times, plausible expectations may turn out to be wrong, and we stand corrected by the formal analysis. A well-known historical instance is the undecidability of predicate logic, understandable eventually via Gödel's and Church's arguments, which clashed with a dominant tradition of searching for decision procedures in the logical literature. Moreover, there do not seem to be infallible rules for keeping complexity down by simply avoiding the above danger zones. For instance, a blanket restriction to finite models has been advocated in natural language semantics as a way of keeping complexity down (cf. van Benthem 1986). But this same restriction may also make many results harder to obtain (if available at all), witness the additional complexities of 'finite model theory' over general model theory found in Gurevich 1985. Likewise, the use of infinitary logics does increase complexity in some ways, but it also decreases complexity in other ways, witness the discovery of smooth infinitary axiomatizations for various programming logics (Goldblatt 1983), which can be more perspicuous than their first-order counterparts. Let us now turn to some specific cases where influential styles of modeling have supported well-known received views on complexity. These will provide more specific points for our subsequent discussion.

II From Higher-Order to First-Order

There are many cases in semantics where higher-order modelling is thought appropriate and inevitable, even though it brings the cost of employing a formal system whose logical validities are non-axiomatizable (and indeed non-arithmetically definable). Pointing at these dangers is indeed one of the well-known social bonding procedures in our community, where collective shudders run through lecture halls at the mention of this Evil One. Nevertheless, upon closer inspection, in all these cases, an important distinction should be made. It is one thing to employ a higher-order *language*, referring to non-first-order individuals such as sets, choice functions or branches – but quite another to insist that this language should have full set-theoretic *standard models*, making it behave in the above-mentioned fashion. For, insisting on the latter expresses an additional commitment: namely that we want to use one particular mathematical implementation of our formalism, whose complexities will tend to 'pollute', naturally, the validities of our logic which was designed to mirror the core phenomenon.

There is an insidious term in the practice of our field, which confuses the issue, namely the often praised "concreteness" of set-theoretic models. What this merely means is that we insist on using one particular well-known abstract mathematical structure as the wrappings of our theory: not that other mathematical modelings would be less concrete. For instance, is a set-theoretic model really 'more concrete' than an algebraic structure or a geometrical picture? Intuitively, in the latter case, the opposite would seem to be true. Moreover, the specificity of a mathematical structure may be the very source of complexity indicated above. Thus, there are reasons for preferring a more neutral stance. Another insidious groove of thought here is the 'separation of concerns' favoured by many authors proposing semantic innovations. They prefer not to raise too many issues at once, and therefore choose "standard set-theoretic modelling" as their working theory. (Cf. the dominant style of presentation in current 'dynamic semantics', cf. Kamp 1984, Groenendijk & Stokhof 1991, Veltman 1991). And of course, this is an excellent research strategy. But in fact, as we shall see below, working with this standard background may not always be an orthogonal decision. It can even be detrimental to the new proposals, as it may involve them in the hereditary sins, complexity-wise, of the old paradigms – precisely when dynamic semantics is partly inspired by the need to provide some cognitive relief from these.

In the case of higher-order logic, a more neutral perspective has been around for a long time, with so-called 'general models' for higher-order languages allowing in general just restricted ranges for set quantification (Henkin 1950). Here, standard models are the limiting case where all mathematically possible sets or predicates have to be present – whether needed or not for the phenomenon under study. This broadening of the model class reinstates the usual properties of first-order logic, but it would be misleading to view it as just an opportunistic tactic. More importantly, on this view, higher-order logic becomes a *many-sorted first-order* logic treating, amongst others, individuals, sets and predicates on a par (cf. Enderton 1972). This move has a good deal of independent philosophical justification, witness the 'property theories' advocated in Bealer 1982, Turner 1989. Indeed, we achieve a moral rarity, being a combination of philosophical virtue with computational advantage, in trading set-theoretic complexity for new sorts of individuals. Even so, this move has always carried a stigma of ad-hoc-ness, and one common complaint is its lack of canonicity. No unique behaviour is specified for the sort of sets, so that the resulting 'logic' is subject to intensive manipulation. In other words, this solution is 'too easy' – again, one of the harmless shibboleths of the field around which we celebrate our professional consensus. But is it really? Considered from another angle, general models do just the right thing. First, they radically block the importation of extraneous set-theoretic truths, letting the subject being described stand out. Next, they force us to replace Platonic complacency by honest work. If we want to add an explicit new sort of objects ('sets', 'predicates' or whatever), then it should indeed be our task to analyze those principles about these objects that are germane to our subject, and formulate them explicitly. Just what about the behaviour of 'sets' is relevant to programming semantics, or to natural language?

By answering questions like this, one often arrives at some compromise. The usual semantic restriction to standard set-theoretic models is too complex, and unilluminating – whereas a liberalization to all general models is too weak, and unilluminating for the opposite reason. But after some hard work, we may arrive at an appropriate model class somewhere in between. Examples of this style of analysis may be found everywhere, once one perceives matters in this light. For instance, second-order semantics with quantification over branches of some sort may always be replaced by many-sorted theories of suitable 'individuals' (points in time, states, etc.) and 'branches' or 'paths', where one now has to study the key principles concerning branches as well as their interaction with the individuals occurring on them. E.g., Stirling 1989 identifies various interesting modal-temporal second-order principles – such as 'fusion closure' stating that, for any state occurring in two histories, its past in the one and its future in the other may be glued together so as to form a new history. Likewise, in branching temporal logic, second-order axioms of choice have been invoked to support 'confluence principles' stating when two points on different histories may come together in some common future by suitable further histories (De Bakker, de Roever & Rozenberg 1989). But, such principles are better viewed as geometrical conditions on the availability of branches in two-sorted first-order models, on a par with standard geometrical axioms concerning points and lines. This is at least as good mathematics as set theory. After all, standard axiomatizations of geometry use 'points', 'lines', 'planes' on a par, rather than construing the latter outlandishly as point sets. For a final example, consider two closely related semantics for intuitionistic logic. 'Kripke models' are first-order, with truth conditions referring only to possible worlds and accessibility, whereas 'Beth models' are second-order, involving also branches (Troelstra & van Dalen 1988). For instance, Beth's truth clause for a disjunction A-or-B says that there exists some barrier of states across all possible future histories such that each state on the barrier verifies either A or B . But again, the latter models can be reformulated by viewing worlds and branches on a par, and analyzing what (little) explicit theory about branches – rather than some Platonic oracle about sets – is needed to explain validity for intuitionistic logic. (The Appendix to Rodenburg 1986 contains a first exploration of the resulting two-sorted model theory, including the interaction of states and branches.)

The above task of localizing key principles on many-sorted models may be made more concrete and systematic by employing the tools of *Correspondence Theory* (cf. van Benthem 1984, 1985). Let us demonstrate this approach for the case of temporal logic. There are two complementary ways of arriving at our desired concrete 'branch theory'. The first involves general reflection on 'branches', sorting out general logical principles governing these from more extravagant mathematical existence claims. The other approach takes the kind of temporal reasoning that is to be analyzed as a guide-line, looking for 'correspondences' between its intended principles and assumptions about branches. This interplay is well-documented for pure temporal logic (cf. van Benthem 1983). Here is a simple example in the branching semantics. Some forms of temporal reasoning allow a quantifier shift between 'future possibility' and 'possible futurity'. This will correspond to a simply computable condition on the pattern of states and branches: namely that time-travel along my current history and then switching to another future history may also be performed by first switching to another history and then traveling into its future. Correspondence Theory has only been investigated very systematically in Modal Logic, but in principle, this style of analysis is available everywhere – witness the powerful extensions presented in Venema 1991, De Rijke 1993. (Admittedly, there is still a subtle issue of methodological consistency here. Correspondence analysis in its usual format itself involves computation in higher-order logic! One line to take here might be to say that we are merely using this technique as a semantic heuristic, and that 'defeating second-order logic from within' is an elegant philosophical stratagem. The more sober line would note that many of the relevant insights would in fact also be forthcoming on suitable general frames.)

Clearly, not every principle that comes up in the above way should be taken for granted. Some correspondences for putative principles of temporal reasoning may turn up clear general desiderata on two-sorted point-branch models, such as linearity for branches, or unicity of their initial points. Others may turn up interesting but only negotiable options. For instance, more complex modal-temporal quantifier shifts in temporal reasoning turn out to correspond to axioms of choice, reflecting a certain 'fullness' of state-branch models. No very clear demarcation line exists between the first case and the second – although there have been some systematic proposals. For instance, if genuine logical principles should be entirely free from existential import (cf. Etchemendy 1990), then a case can be made for admitting just the purely universal first-order requirements arising in a correspondence analysis as genuine core conditions on our semantics, relegating all conditions with existential import to some negotiable mathematical part. (Van Benthem 1983 even defends the further restriction to mere universal Horn clauses, viewing propositional disjunctions as existential too.)

Despite the methodology advocated here, there is no general presumption that any reasonable new semantics will or should be simple, or even first-order. For instance, it might well be that in many computational applications, temporal branches should be *finite* (a non-first-order condition). But also, in the opposite direction, it has been argued with some force that the essential control structures for programming ought to be *infinitary*, and hence non-first-order (Goldblatt 1983). Either way, the emergence of such discussions does not count against our broader framework: it rather speaks for its fruitfulness. Whether computation hinges essentially on finiteness for its 'traces' or its control structures is a substantive issue, which should be on the agenda explicitly, and not prejudged in the use of 'concrete standard modeling'.

All these points may also be demonstrated for the case of natural language. Consider the following linguistic phenomenon mentioned earlier on. From the start (cf. Hintikka 1979), 'branching quantification' has been considered a typical example of a second-order semantic mechanism, requiring quantification over Skolem functions not represented by ordinary first-order linear quantifier prefixes. To be sure, there is still some opposition to this linguistic claim, and first-order guerrilleros are still active. But, even granting the move toward using non-linearly orderable Skolem functions here, all that is shown by its proponents is that we need to consider a certain family of *relevant* 'choice functions' explicitly among our semantic individuals – not necessarily the whole mathematical space (containing lots of linguistically irrelevant items). Moreover, we can even defend this move on the basis of independent linguistic evidence. For instance, so-called 'functional answers' to questions ("Whom does every man love? His mother.") suggest that we need more abstract 'functional objects' as citizens in our semantic world. But even then, the real issue remains how many of these choice functions are required to explain natural language branching patterns, satisfying which combinatorial constraints. More generally, this point applies to more general linguistic 'polyadic quantification' (van Benthem 1989, Keenan and Westerståhl 1993), whose higher-orderness has been taken for granted so far. The complexity of the set-theoretic denotations arising here might also point at constraints on some family of 'available' predicates and functions. Likewise, considerations of this kind apply to several current discussions of complexity in Artificial Intelligence. For instance, the above-mentioned method of 'circumscription' might just employ minimization in general models, performed with respect to some family of 'relevant predicates' – say, those that are explicitly represented in our computational environment. (Morreau 1985 investigates the resulting two-sorted first-order model theory, and shows how it can deal with many of the non-monotonic reasoning patterns that originally motivated circumscription.)

Finally, it should be emphasized that not all broader semantic spaces can or should arise from the above 'general model' strategy. What the latter provides is one systematic way of stepping back and rethinking the semantic issues. Without independent semantic evidence fort its outcomes, however (as in the preceding linguistic example), the resulting many-sorted model might remain an ad-hoc theoretical curiosity. And in fact, eventually, one may come to prefer some alternative modelling altogether. A general illustration of this freedom is the ubiquitous switch found in mathematics between algebraic and geometric viewpoints – and a more specific one, the emergence of some recent analyses of branching quantification in natural language that employ a different circle of ideas concerning 'groups' and collective predication (cf. Landman 1989, Hoeksema 1983, van der Does 1992).

Summing up, changing to broader first-order model classes is not an ad-hoc move of desperation or laziness in taming complexity. It rather represents a more finely-tuned style of analysis, forcing us to do our conceptual homework, and eventually – another potential benefit – suggesting new applications beyond the original field, precisely because of the available new semantic models. Moreover, this is not a risky new-fangled approach of uncertain prospects. For, when all is said and done, this move is precisely what abstract mathematical analysis has always been about.

III From First-Order to Decidable

The preceding move from higher-order to first-order logic brings a clear gain in complexity. But how satisfactory is the latter system as a universal semantic medium? Although effectively axiomatizable, predicate logic is undecidable – and again, this feature may import external complexity into the description of subjects whose 'natural complexity' would be decidable. One factor responsible for this situation, as was observed above, is the lure of 'concrete set-theoretic models'. We all think of standard Tarskian models as the essence of concreteness and simplicity (although there has been some underground opposition from the earlier-mentioned 'property theorists' – cf. also Zalta 1993). But here again, are we perhaps still importing extraneous set theory, which might account for the undecidability of our logic? For instance, working logicians in linguistics or computer science often have a gut feeling that the styles of reasoning they are analyzing are largely decidable (cf. the percentual estimate given in Bacon 1985, or the analysis of 'natural logic' in Sanchez Valencia 1991), but it is hard to give any mathematical underpinning to these working intuitions.

So, could there be well-motivated decidable versions of predicate logic, arising from giving up certain standard semantic prejudices? As it happens, there are even several such roads towards decidability. A traditional one would be to work only with restricted fragments of predicate logic (monadic, universal or otherwise, as is treated in many standard text books), while a very modern route is provided by recent linear logics of occurrences (Girard 1987, van Benthem 1991). But indeed, here too, a very general standard strategy exists for broadening our model class. There is a whole mathematical spectrum running from concrete set-theoretic models for predicate logic to abstract algebraic ones. In fact, this is precisely the domain of *algebraic logic*, which has produced a good deal of information concerning these very issues: cf. Andréka 1991, Németi 1991, 1993, Venema 1991. In principle, this method will work whenever some modest minimal requirements are met by the underlying base logic. But of course, as with the above strategy of introducing general models, the interesting possibilities will lie somewhere in between. Now, just like the general model strategy, the algebraization strategy has been criticized in the literature for its ad-hoc-ness and lack of clear constraints. As an old saying goes, 'algebraic semantics is syntax in disguise'. But this is only true for the bottom level (where syntactic Lindenbaum algebras would do the job), whereas usually, algebras of independent interest emerge in abundance through further semantic considerations. Thus, a key concern in our present setting should be the search for interesting independently motivated 'semantic parameters' that can be set differently from the specific choices made in the standard Tarskian paradigm. But then, this search does not seem hopeless. For instance, on the more traditional ontological side, we have already seen that property theorists want to treat individuals and properties on a par as intensional entities, viewing the usual Tarskian 'set–tuple style' of treating predicate denotations as just one, extensional, option out of many.

Here is a more wide-ranging shift in current attitudes concerning semantic modelling. More 'procedurally', various intriguing new ideas have been put forward recently concerning first-order interpretation. In particular, one may view the 'cylindric modal algebra' of Venema 1991, and indeed cylindric algebra in general (Henkin-Monk-

Tarski 198) as a more fine-structured account of regimented access to successive variable assignments when interpreting quantified formulas, which may lead to natural decidable predicate logics (witness the contributions by Andréka & Németi in this volume). In this case, the more general models will carry some accessibility pattern on assignments, or more generally states for predicate logic, with existentially quantified statements $\exists x \phi$ referring to some new assignment verifying ϕ which should be x-accessible from the present one. Correspondence theory will then identify the procedural import of various predicate-logical principles over these abstract models, such as S5-laws for x-accessibility or more delicate 'path principles' (cf. van Benthem 1993). Judicious combinations of such conditions will then produce attractive decidable predicate logics, witness again the Andréka and Németi papers in this volume. Independent sources of intuitions concerning such transitions between aassignments exist already. One example is the 'dynamic predicate logic' of Groenendijk & Stokhof 1991, which views first-order formulas as explicit programs for effecting transitions between assignments when interpreting predicate logic. But also, in some recent semantics of generalized quantification, individual domains are allowed to carry an abstract 'accessibility' or 'dependence' structure, determining in which order individuals may become available in the course of interpreting an existence statement. Concrete motivations may be found in the theory of 'arbitrary objects' in Fine 1985, or the work on probabilistic independence relations in van Lambalgen 1991, as well as its 'modal first-order' version proposed in Alechina & van Benthem 1993. This gives a much broader space of models for the language of first-order predicate logic, with the original Tarskian ones becoming the special case where one has 'random access' to the full Cartesian space of all mathematically possible assignments of objects to variables.

As before, the advantage of this style of analysis is that one makes predicate logic into a more finely-tuned tool, with a decidable 'core' and a 'periphery' of more demanding principles. This is of purely logical interest, as it allows us to see new distinctions among what used to be one uniform batch of logical truths, that need not correspond to traditional classifications of propositions (say, using prenex forms or predicate arities). It is also of applied interest, in that we may try to see whether the richer picture fits the natural structure of the subject described. For instance, 'accessibility constraints' are natural from many viewpoints, mathematical, philosophical and linguistic. Moreover, the decidable subsystems of first-order logic that emerge in the resulting landscape of logical systems may fit in more naturally with proposed mechanisms of 'natural logic' in human reasoning (cf. Sommers 1982, Sanchez Valencia 1991).

Taking this viewpoint seriously is not a minor philosophical move, as it brings a lot of radical repercussions with it. For instance, in the broader perspective sketched here, what remains the point of Gödel's celebrated Completeness Theorem, stating a natural fit between Tarskian semantics and Fregean axiomatization? From our current stance, one would have to say that Gödel's result ties one particular natural choice of predicate-logical validity, which may be motivated independently from various angles (model-theoretic, proof-theoretic, game-theoretic) to 'standard set-theoretic modelling'. Kreisel 1967 and Etchemendy 1990 have stressed the surprise in this outcome, noting that it somehow manages to ensnare natural intuitive validity by means of exact mathematical notions. We would have to disagree here. From equally natural semantic points of view, other logical equilibria may arise, this time, having a decidable set of validities. What still remains is Gödel's lasting more abstract achievement of having put completeness issues on the map, and made them amenable to mathematical statement and proof.

Actually, we could go one step further here, as well as in the earlier cases. The very multiplicity of modellings for predicate logic suggests that there need not be one unique preferred choice – and that one should rather view all these options as living together. Having many modelings around in the literature for some particular phenomenon is not a nuisance or disease, to be eliminated by natural competition, in favour of one canonical solution. It may even be a definite virtue to be prized. For instance, it is quite possible to think of semantic modelling as an open-ended process, which allows ever finer levels of 'grain size'. As an example, one may think of a recent proposals by Zalta 1993, following Etchemendy, to split first-order semantics into two stages. The first goes from syntactic forms to abstract propositions (this is the domain of 'linguistic change'), and the second goes from these propositions to standard models (this is the domain of 'real change'). The burden of 'predicate logic' could then be divided between these two stages, with a possibly decidable component for the first stage, and an undecidable one for the second. Another example would be the grounding of decidable linear predicate logics in Lorenzen-style or Hintikka-style game-theoretical semantics for first-order logic (Mey 1992), or in intermediate Tarskian evaluation algorithms serving as Fregean senses (Moschovakis 1991). Of course, this does raise new issues of what may be called 'logical architecture' (van Benthem, to appear), as the interplay between the various components now becomes the explicit business of logic, too. Thus, traditional semantics and its rivals might coexist in one broader logical framework, whose task it would be to indicate precisely at which fault lines complexity emerges.

IV Digression: Illustrations in Arrow Logic

A more compact testing site for the above general considerations may be found in the 'Arrow Logic' which is so prominent in this volume. Here are a few illustrations of the above general themes at work in this more limited domain. One guiding motivation in the arrow logic research programme has been to challenge another well-known 'complexity threshold' in logics of computation, namely that undecidability is inevitable once a full relational algebra of programming operations is allowed. For, how much of this undecidability is due to the essential complexity of computation, and how much is merely a by-product of the mathematics of ordered pairs that lies underneath standard modelling in this area? As it happens, the resulting theory nicely demonstrates the above general points (cf. Venema's introductory chapter in this volume). First, a new sort of individual objects has been put on the map, namely transitions or 'arrows', which occur in most computational intuitions, and which arguably should be first-class citizens in any analysis of computation. Moreover, we are forced to reflect on the essential structure among these arrows, such as their composition, conversion or possibly other modes of combination (including parallel constructions of 'sheafs of arrows'). At least in the simplest similarity type, here, various decidable core calculi have been discovered, which can do quite a bit of the basic combinatorics of program combination. In particular, these calculi support all Boolean operators on top of relational composition, without tripping over any purported undecidability threshold. On top of that, correspondence analysis reveals the additional semantic content of further programming principles, distinguishing between purely universal ones (whose totality might form the largest desirable 'core theory') and more demanding existence principles. Outcomes have not been totally routine here, in that, e.g., associativity of composition has been identified as a danger point (cf. Simon et al. 1993), whereas most people would consider the latter an entirely harmless domestic assumption. Finally, this is not an isolated piece of semantic engineering, because relational transitions underly such a vast range of computational processes. Thus, arrow-style analysis is appropriate for relational algebra and propositional dynamic logic, but also for logics of belief revision or general process algebras. Indeed, the whole theory of labeled transition systems in computer science (cf. Stirling 1990, van Benthem & Bergstra 1993, van Benthem, van Eyck & Stebletsova 1993) is probably better viewed as a theory of states and arrows, with appropriate notions of two-sorted bisimulation for process equivalences. (One might even consider the introduction of computation 'paths' or 'branches' as a third kind of independent semantic object, thereby reflecting an earlier strand in our discussion.) Finally, there is no reason for staying inside the realm of computation, once our general points about modeling have been adopted. For instance, concrete geometrical arrows also reflect human thinking about their preferences, both scientific and domestic, and thus switching to arrow semantics might also throw some new light on preference logic (cf. the first exploration made in van Benthem, van Eyck & Frolova 1993) and more generally, on various theories of individual and collective choice in the social sciences.

V General Issues

This paper has raised some general issues concerning semantic modelling, with a special interest in the locus of complexity. We have recommended a systematic search for alternative logical modelings, with the search for 'minimal complexity of wrappings' as a moving force toward new conceptual schemes. Moreover, we have noted some general features and benefits of this enterprise, as well as some of its philosophical repercussions. All this has by no means exhausted the topic of logical complexity. For instance, other important technical approaches exist in our field to high-lighting 'core content'. Among these, one may mention the use of 'oracles' supplying extraneous information to be bracketed out – as happens in Cook-style completeness theorems in programming semantics (Cook 1978) and in various parts of Complexity Theory (Buhrmann 1993). Such strategies remain to be compared with the present proposals. Another well-known way of coping with complexity is that of the 'moderate drinker': 'ingest only small amounts'. For instance, second-order complexity might be avoided by using only small fragments of the full second-order language, and the same holds for first-order logic. (By contrast, we have advocated retaining the full language as the expressive medium, while lowering the over-all complexity of its logic.) Finally, there are various statistical approaches to average-case complexity of logical reasoning that takes the sharpest edges off the usual worst-case problems. In this division of labour, the statistician takes care of the harsh realities of life, so that the logical theorist can live at peace. What these alternative strategies have in common is that they do not require rebuilding basic logical frameworks, but rather offer ways of accommodating ourselves to their complexity. Perhaps, then, our own proposals here have been unduly principled and calvinistic. Indeed, they might also run against the no-nonsense spirit of our times. In particular, they seem to clash with a recent trend in logical modelling which aims at replacing language-oriented semantic analysis by what is called 'direct mathematics'. The latter is illustrated by current developments in the Dutch branch of Process Algebra (cf. the program outlined in Baeten 1992), but also by earlier 'set-theoretical' programs in the Philosophy of Science (Suppes 1960, Sneed 1971), whose proponents want to minimize encounters with the common syntactic idiosyncracies of logical formalisms when describing their phenomena of interest. There are certainly good practical reasons for exploring such a line of investigation. Nevertheless, on the view of this paper, its guiding slogan may also be slightly misleading. 'Direct mathematics' with its seemingly innocent emphasis on concrete 'standard mathematical tools' carries many pre-wired decisions as to complexity, whereas our advice has been to take nothing for granted.

In all, this short essay has a very modest aim. We would be happy if our points were to contribute to a sustained probing discussion of received views in semantic modelling.

VI References

Abramsky, S., D. Gabbay & T. Maibaum, eds.

to appear *Handbook of Logic in Computer Science*, Oxford University Press.

Alechina, N. & J. van Benthem

1993 'Modal Quantification over Structured Domains', Report ML 93-02, Institute for Logic, Language and Computation, University of Amsterdam.

Andréka, H.

1991 *Complexity of Equations Valid in Algebras of Relations*, D. Sc. Thesis, Mathematical Institute, Hungarian Academy of Sciences, Budapest.

Bach, E.

1986 'Natural Language Metaphysics', in R. barcan Marcus et al., eds., 573-595.

Bacon, J.

1985 'The Completeness of a Predicate-Functor Logic', *The Journal of Symbolic Logic* 50, 903-926.

Baeten, J.

1992 'Informatica als Wetenschap (formele specificatie en wiskundige verificatie)', Inaugural Lecture, Department of Computer Science, Technological University, Eindhoven. (*TUE Informatie* 35 (7/8), 493-499, 1993.)

Bakker, J. de, W-P de Roever & G. Rozenberg, eds.

1989 *Linear Time, Branching Time and Partial Order in Logics and Models for Concurrency*, Springer, Berlin.

Barcan Marcus, R., G. Dorn & P. Weingartner, eds.

1986 *Proceedings 7th International Congress of Logic, Methodology and Philosophy of Science. Salzburg 1983*, North-Holland, Amsterdam.

Barwise, J

1979 'On Branching Quantifiers in English', *Journal of Philosophical Logic* 8, 47-80.

Bealer, G.

1982 *Quality and Concept*, Oxford University Press, Oxford.

Benthem, J. van

1983 *The Logic of Time*, Reidel, Dordrecht.

1984 'Correspondence Theory', in Gabbay & Guenthner, eds., 167-247.

1985 *Modal Logic and Classical Logic*, Bibliopolis, Napoli.

1986 *Essays in Logical Semantics*, Reidel, Dordrecht.

1989 'Polyadic Quantifiers', *Linguistics and Philosophy* 12:4, 437-464.

1991 *Language in Action. Categories, Lambdas and Dynamic Logic*, North-Holland, Amsterdam.

1993 'Modal State Semantics', manuscript, Institute for Logic, Language and Computation, University of Amsterdam.

to appear 'Logic and the Flow of Information', Report CSLI-91-160, Center for the Study of Language and Information, Stanford University. (To appear in D. Prawitz et al., eds.)

Benthem, J. van & J. Bergstra

1993 'Logic of Transition Systems', Report CT-93-03, Institute for Logic, Language and Computation, University of Amsterdam.

Benthem, J. van, J. van Eyck & A. Frolova

1993 'Changing Preferences', Report CS-93-10, Centre for Mathematics & Computer Science, Amsterdam.

Benthem, J. van, J. van Eyck & V. Stebletsova

1993 'Modal Logic, Transition Systems and Processes', Report CS-93-21, Centre for Mathematics and Computer Science, Amsterdam.

Benthem, J. van & A. ter Meulen, eds.

to appear *Handbook of Logic and Language*, Elsevier, Amsterdam.

Buhrmann, H.

1993 *Resouce Bounded Reductions*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Burgess, J.

1984 'Basic Tense Logic', in Gabbay & Guenthner, eds., 89-133.

Chierchia, G., B. Partee & R. Turner, eds.

1989 *Properties, Types and Meaning*, Reidel, Dordrecht.

Cook, S.

1978 'Soundness and Completeness of an Axiom System for Program Verification', *SIAM Journal of Computing* 7, 70-90.

Does, J. van der

1992 *Applied Quantifier Logics*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Does, J. van der & J. van Eyck, eds.,

1991 *Generalized Quantifier Theory and Applications*, Dutch Ph. D. Network for Language, Logic and Information, Amsterdam. (To appear: CSLI Lecture Notes, Chicago University Press.)

Enderton, H.

1972 *A Mathematical Introduction to Logic*, Academic Press, New York.

Etchemendy, J.

1990 *The Concept of Logical Consequence*, Harvard University Press, Cambridge (Mass.)

Fine, K.

1985 *Reasoning with Arbitrary Objects*, Blackwell, Oxford.

Gabbay, D. & F. Guenthner, eds.

1984 *Handbook of Philosophical Logic*, vol. II, Reidel, Dordrecht.

Girard, J-Y

1987 'Linear Logic', *Theoretical Computer Science* 50, 1-102.

Goldblatt, R.

1983 *Axiomatizing the Logic of Computer Programming*, Springer, Berlin.

1987 *Logics of Time and Computation*, CSLI Lecture Notes, Chicago University Press.

Groenendijk, J., Th. Janssen & M. Stokhof, eds.

1984 *Truth, Interpretation and Information*, Foris, Dordrecht.

Groenendijk, J. & M. Stokhof

1991 'Dynamic Predicate Logic', *Linguistics and Philosophy* 14, 39-100.

Gurevich, Y.

1985 'Logic and the Challenge of Computer Science', Report CRL-TR-10-85, Computer Research Laboratory, University of Michigan, Ann Arbor.

Harel, D.

1984 'Dynamic Logic', in Gabbay & Guenthner, eds., 497-604.

Henkin, L.

1950 'Completeness in the Theory of Types', *Journal of Symbolic Logic* 15, 81-91.

Henkin, L., D. Monk & A. Tarski

198 *Cylindric Algebra*, Parts I and II, North-Holland, Amsterdam.

Hintikka, J.

1979 'Quantifiers in Natural Languages: Some Logical Problems', in E. Saarinen, ed., 81-118.

Hoeksema, J.

1983 'Plurality and Conjunction', in Ter Meulen, ed., 63-83.

Kamp, H.

1984 'A Theory of Truth and Semantic Representation', in J. Groenendijk et al., eds., 1-41.

Keenan, E. & D. Westerståhl

1993 'Generalized Quantifiers', to appear in van Benthem & ter Meulen, eds.

Kneale, W. & M.

1961 *The Development of Logic*, Clarendon Press, Oxford.

Kreisel, G.

1967 'Informal Rigour and Completeness Proofs', in I. Lakatos, ed., 138-186.

Kugel, P.

1986 'Thinking May be More Than Computing', *Cognition* 22, 137-198.

Lakatos, I., ed.

1967 *Problems in the Philoophy of Mathematics*, North-Holland, Amsterdam.

Lambalgen, M. van

1991 'Natural Deduction for Generalized Quantifiers', in van Eyck & van der Does, eds., 143-154.

Landman, F.

1989 'Groups I & II', *Linguistics and Philosophy* 12, 559-605 and 723-744.

McCarthy, J.

1980 'Circumscription – A Form of Non-Monotonic Reasoning', *Artificial Intelligence* 13, 27-39.

Meulen, A. ter, ed.

1983 *Studies in Modeltheoretic Semantics*, Foris, Dordrecht.

Mey, D.

1992 'Game-Theoretical Interpretation of a Logic Without Contraction', Department of Computer Science, Swiss Federal Institute of Technology, Zürich.

Morreau, M.

1985 'Circumscription: A Sound and Complete Form of Non-Monotonic Reasoning', Technical Report, Mathematical Institute, University of Amsterdam.

1992 *Conditionals in Philosophy and Artificial Intelligence*, dissertation, Philosophical Institute, University of Amsterdam.

Moschovakis, Y.

1991 'Sense and Reference as Algorithm and Value', Department of Mathematics, University of California, Los Angeles.

Németi. I.

1991 'Algebraizations of Quantifier Logics: An introductory overview', Mathematical Institute, Hungarian Academy of Sciences, Budapest.

1993 'Decidability of Weakened Versions of First-Order Logic', Workshop on Algebraization of Logic, Fifth European Summer School in Logic, Language and Information, Lisbon.

Prawitz, D., B. Skyrms & D. Westerståhl, eds.

to appear *Proceedings 9th International Congress of Logic, Methodology and Philosophy of Science. Uppsala 1991*, Elseviers Science Publishers, Amsterdam.

Rodenburg, P.

1986 *Intuitionistic Correspondence Theory*, dissertation, Mathematical Institute, University of Amsterdam.

Rijke, M. de

1993 *Extending Modal Logic*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Saarinen, E., ed.

1979 *Game-Theoretical Semantics*, Reidel, Dordrecht.

Sain, I.

1978 'Observations on Abstract Model Theory', CL&CL 1978 XIII, 195-250.

Sanchez Valencia, V.

1991 *Studies on Natural Logic and Categorial Grammar*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Simon, A. , A. Kurucz, I. Németi & I. Sain

1993 'Undecidability Issues of Some Boolean Algebras with Operators and Logics Related to Lambek Calculus', Workshop on Algebraization of Logic, Fifth European Summer School in Logic, Language and Information, Lisbon.

Sneed, J.

1971 *The Logical Structure of Mathematical Physics*, Reidel, Dordrecht.

Sommers, F.

1982 *The Logic of Natural Language*, Cambridge University Press, Cambridge.

Spaan, E.

1993 *Complexity of Modal Logics*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Stirling, C.

1990 'Modal and Temporal Logics', to appear in Abramsky, Gabbay & Maibaum, eds.

Suppes, P.

1960 'A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences', *Synthese* 12, 287-301.

Troelstra, A. & D. van Dalen

1988 *Constructivism in Mathematics*, North-Holland, Amsterdam.

Turner, R.

1989 'Two Issues in the Foundations of Semantics', in G. Chierchia et al., eds., 63-84.

Veltman, F.

1992 'Defaults in Update Semantics', Report LP-91-02, Institute for Logic, Language and Computation, University of Amsterdam. (*Journal of Philosophical Logic*, to appear.)

Venema, Y.

1992 *Many-Dimensional Modal Logic*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Zalta, E.

1988 *Intensional Logic and the Metaphysics of Intentionality*, MIT Press, Cambridge (Mass.)

1993 'A Philosophical Conception of Propositional Modal Logic', Center for the Study of Language and Information, Stanford University.

ILLC Research Reports and Technical Notes

Coding for Research Reports: *Series-Year-Number*, with LP = Linguistics and Philosophy of Language; ML = Mathematical Logic and Foundations; CL = Compuational Linguistics; CT = Computation and Complexity Theory; X = Technical Notes.
All previous ILLC-publications are available from the ILLC bureau. For prepublications before 1990, contact the bureau.

LP-90-01 *Jaap van der Does, A Generalized Quantifier Logic for Naked Infinitives*
LP-90-02 *Jeroen Groenendijk, Martin Stokhof, Dynamic Montague Grammar*
LP-90-03 *Renate Bartsch, Concept Formation and Concept Composition*
LP-90-04 *Aarne Ranta, Intuitionistic Categorial Grammar*
LP-90-05 *Patrick Blackburn, Nominal Tense Logic*
LP-90-06 *Gennaro Chierchia, The Variablity of Impersonal Subjects*
LP-90-07 *Gennaro Chierchia, Anaphora and Dynamic Logic*
LP-90-08 *Herman Hendriks, Flexible Montague Grammar*
LP-90-09 *Paul Dekker, The Scope of Negation in Discourse, towards a Flexible Dynamic Montague grammar*
LP-90-10 *Theo M.V. Janssen, Models for Discourse Markers*
LP-90-11 *Johan van Benthem, General Dynamics*
LP-90-12 *Serge Lapierre, A Functional Partial Semantics for Intensional Logic*
LP-90-13 *Zhisheng Huang, Logics for Belief Dependence*
LP-90-14 *Jeroen Groenendijk, Martin Stokhof, Two Theories of Dynamic Semantics*
LP-90-15 *Maarten de Rijke, The Modal Logic of Inequality*
LP-90-16 *Zhisheng Huang, Karen Kwast, Awareness, Negation and Logical Omniscience*
LP-90-17 *Paul Dekker, Existential Disclosure, Implicit Arguments in Dynamic Semantics*

ML-90-01 *Harold Schellinx, Isomorphisms and Non-Isomorphisms of Graph Models*
ML-90-02 *Jaap van Oosten, A Semantical Proof of De Jongh's Theorem*
ML-90-03 *Yde Venema, Relational Games*
ML-90-04 *Maarten de Rijke, Unary Interpretability Logic*
ML-90-05 *Domenico Zambella, Sequences with Simple Initial Segments*
ML-90-06 *Jaap van Oosten, Extension of Lifschitz' Realizability to Higher Order Arithmetic, and a Solution to a Problem of F. Richman*
ML-90-07 *Maarten de Rijke, A Note on the Interpretability Logic of Finitely Axiomatized Theories*
ML-90-08 *Harold Schellinx, Some Syntactical Observations on Linear Logic*
ML-90-09 *Dick de Jongh, Duccio Pianigiani, Solution of a Problem of David Guaspari*
ML-90-10 *Michiel van Lambalgen, Randomness in Set Theory*
ML-90-11 *Paul C. Gilmore, The Consistency of an Extended NaDSet*

CT-90-01 *John Tromp, Peter van Emde Boas, Associative Storage Modification Machines*

CT-90-02 *Sieger van Denneheuvel, Gerard R. Renardel de Lavalette, A Normal Form for PCSJ Expressions*

CT-90-03 *Ricard Gavaldà, Leen Torenvliet, Osamu Watanabe, José L. Balcázar, Generalized Kolmogorov Complexity in Relativized Separations*

CT-90-0 *Harry Buhrman, Edith Spaan, Leen Torenvliet, Bounded Reductions*

CT-90-05 *Sieger van Denneheuvel, Karen Kwast, Efficient Normalization of Database and Constraint Expressions*

CT-90-06 *Michiel Smid, Peter van Emde Boas, Dynamic Data Structures on Multiple Storage Media, a Tutorial*

CT-90-07 *Kees Doets, Greatest Fixed Points of Logic Programs*

CT-90-08 *Fred de Geus, Ernest Rotterdam, Sieger van Denneheuvel, Peter van Emde Boas, Physiological Modelling using RL*

CT-90-09 *Roel de Vrijer , Unique Normal Forms for Combinatory Logic with Parallel Conditional, a case study in Conditional Rewriting*

X-90-01 *A.S. Troelstra , Remarks on Intuitionism and the Philosophy of Mathematics, Revised Version*

X-90-02 *Maarten de Rijke, Some Chapters on Interpretability Logic*

X-90-03 *L.D. Beklemishev, On the Complexity of Arithmetical Interpretations of Modal Formulae*

X-90-04 *Annual Report 1989,*

X-90-05 *Valentin Shehtman, Derived Sets in Euclidean Spaces and Modal Logic*

X-90-06 *Valentin Goranko, Solomon Passy, Using the Universal Modality: Gains and Questions*

X-90-07 *V.Yu. Shavrukov, The Lindenbaum Fixed Point Algebra is Undecidable*

X-90-08 *L.D. Beklemishev, Provability Logics for Natural Turing Progressions of Arithmetical Theories*

X-90-09 *V.Yu. Shavrukov, On Rosser's Provability Predicate*

X-90-10 *Sieger van Denneheuvel, Peter van Emde Boas, An Overview of the Rule Language RL/1*

X-90-11 *Alessandra Carbone, Provable Fixed points in ID0+W1, revised version*

X-90-12 *Maarten de Rijke, Bi-Unary Interpretability Logic*

X-90-13 *K.N. Ignatiev , Property, Craig's Property*

X-90-14 *L.A. Chagrova, Undecidable Problems in Correspondence Theory*

X-90-15 *A.S. Troelstra, Lectures on Linear Logic*

LP-91-01 *Wiebe van der Hoek, Maarten de Rijke, Generalized Quantifiers and Modal Logic*

LP-91-02 *Frank Veltman, Defaults in Update Semantics*

LP-91-03 *Willem Groeneveld, Dynamic Semantics and Circular Propositions*

LP-91-04 *Makoto Kanazawa, The Lambek Calculus enriched with Additional Connectives*

LP-91-05 *Zhisheng Huang, Peter van Emde Boas, The Schoenmakers Paradox: Its Solution in a Belief Dependence Framework*

LP-91-06 *Zhisheng Huang, Peter van Emde Boas, Belief Dependence, Revision and Persistence*
LP-91-07 *Henk Verkuyl, Jaap van der Does, The Semantics of Plural Noun Phrases*
LP-91-08 *Víctor Sánchez Valencia, Categorial Grammar and Natural Reasoning*
LP-91-09 *Arthur Nieuwendijk, Semantics and Comparative Logic*
LP-91-10 *Johan van Benthem, Logic and the Flow of Information*

ML-91-01 *Yde Venema , Cylindric Modal Logic*
ML-91-02 *Alessandro Berarducci, Rineke Verbrugge, On the Metamathematics of Weak Theories*
ML-91-03 *Domenico Zambella, On the Proofs of Arithmetical Completeness for Interpretability Logic*
ML-91-04 *Raymond Hoofman, Harold Schellinx, Collapsing Graph Models by Preorders*
ML-91-05 *A.S. Troelstra, History of Constructivism in the Twentieth Century*
ML-91-06 *Inge Bethke, Finite Type Structures within Combinatory Algebras*
ML-91-07 *Yde Venema, Modal Derivation Rules*
ML-91-08 *Inge Bethke, Going Stable in Graph Models*
ML-91-09 *V.Yu. Shavrukov, A Note on the Diagonalizable Algebras of PA and ZF*
ML-91-10 *Maarten de Rijke, Yde Venema, Sahlqvist's Theorem for Boolean Algebras with Operators*
ML-91-11 *Rineke Verbrugge, Feasible Interpretability*
ML-91-12 *Johan van Benthem, Modal Frame Classes, revisited*

CT-91-01 *Ming Li, Paul M.B. Vitányi , Kolmogorov Complexity Arguments in Combinatorics*
CT-91-02 *Ming Li, John Tromp, Paul M.B. Vitányi, How to Share Concurrent Wait-Free Variables*
CT-91-03 *Ming Li, Paul M.B. Vitányi, Average Case Complexity under the Universal Distribution Equals Worst Case Complexity*
CT-91-04 *Sieger van Denneheuvel, Karen Kwast, Weak Equivalence*
CT-91-05 *Sieger van Denneheuvel, Karen Kwast, Weak Equivalence for Constraint Sets*
CT-91-06 *Edith Spaan, Census Techniques on Relativized Space Classes*
CT-91-07 *Karen L. Kwast, The Incomplete Database*
CT-91-08 *Kees Doets, Levationis Laus*
CT-91-09 *Ming Li, Paul M.B. Vitányi, Combinatorial Properties of Finite Sequences with high Kolmogorov Complexity*
CT-91-10 *John Tromp, Paul Vitányi, A Randomized Algorithm for Two-Process Wait-Free Test-and-Set*
CT-91-11 *Lane A. Hemachandra, Edith Spaan, Quasi-Injective Reductions*
CT-91-12 *Krzysztof R. Apt, Dino Pedreschi, Reasoning about Termination of Prolog Programs*
CL-91-01 *J.C. Scholtes, Kohonen Feature Maps in Natural Language Processing*
CL-91-02 *J.C. Scholtes, Neural Nets and their Relevance for Information Retrieval*

CL-91-03 *Hub Prüst, Remko Scha, Martin van den Berg, A Formal Discourse Grammar tackling Verb Phrase Anaphora*

X-91-01 *Alexander Chagrov, Michael Zakharyaschev, The Disjunction Property of Intermediate Propositional Logics*

X-91-02 *Alexander Chagrov, Michael Zakharyaschev , On the Undecidability of the Disjunction Property of Intermediate Propositional Logics*

X-91-03 *V. Yu. Shavrukov, Subalgebras of Diagonalizable Algebras of Theories containing Arithmetic*

X-91-04 *K.N. Ignatiev, Partial Conservativity and Modal Logics*

X-91-05 *Johan van Benthem, Temporal Logic*

X-91-06 *Annual Report 1990,*

X-91-07 *A.S. Troelstra, Lectures on Linear Logic, Errata and Supplement*

X-91-08 *Giorgie Dzhaparidze, Logic of Tolerance*

X-91-09 *L.D. Beklemishev, On Bimodal Provability Logics for P1-axiomatized Extensions of Arithmetical Theories*

X-91-10 *Michiel van Lambalgen, Independence, Randomness and the Axiom of Choice*

X-91-11 *Michael Zakharyaschev, Canonical Formulas for K4. Part I: Basic Results*

X-91-12 *Herman Hendriks, Flexibele Categoriale Syntaxis en Semantiek: de proefschriften van Frans Zwarts en Michael Moortgat*

X-91-13 *Max I. Kanovich, The Multiplicative Fragment of Linear Logic is NP-Complete*

X-91-14 *Max I. Kanovich, The Horn Fragment of Linear Logic is NP-Complete*

X-91-15 *V. Yu. Shavrukov, Subalgebras of Diagonalizable Algebras of Theories containing Arithmetic, revised version*

X-91-16 *V.G. Kanovei, Undecidable Hypotheses in Edward Nelson's Internal Set Theory*

X-91-17 *Michiel van Lambalgen, Independence, Randomness and the Axiom of Choice, Revised Version*

X-91-18 *Giovanna Cepparello, New Semantics for Predicate Modal Logic: an Analysis from a standard point of view*

X-91-19 *, Papers presented at the Provability Interpretability Arithmetic Conference, 24-31 Aug. 1991, Dept. of Phil., Utrecht University*

LP-92-01 *Víctor Sánchez Valencia, Lambek Grammar: an Information-based Categorial Grammar*

LP-92-02 *Patrick Blackburn, Modal Logic and Attribute Value Structures*

LP-92-03 *Szabolcs Mikulás, The Completeness of the Lambek Calculus with respect to Relational Semantics*

LP-92-04 *Paul Dekker, An Update Semantics for Dynamic Predicate Logic*

LP-92-05 *David I. Beaver, The Kinematics of Presupposition*

LP-92-06 *Patrick Blackburn, Edith Spaan, A Modal Perspective on the Computational Complexity of Attribute Value Grammar*

LP-92-07 *Jeroen Groenendijk, Martin Stokhof, A Note on Interrogatives and Adverbs of Quantification*

LP-92-08 *Maarten de Rijke, A System of Dynamic Modal Logic*

LP-92-09 *Johan van Benthem, Quantifiers in the world of Types*

LP-92-10 *Maarten de Rijke , Meeting Some Neighbours (a dynamic modal logic meets theories of change and knowledge representation)*

LP-92-11 *Johan van Benthem, A Note on Dynamic Arrow Logic*

LP-92-12 *Heinrich Wansing, Sequent Caluli for Normal Modal Propositional Logics*

LP-92-13 *Dag Westerstæhl, Iterated Quantifiers*

LP-92-14 *Jeroen Groenendijk, Martin Stokhof, Interrogatives and Adverbs of Quantification*

ML-92-01 *A.S. Troelstra, Comparing the Theory of Representations and Constructive Mathematics*

ML-92-02 *Dmitrij P. Skvortsov, Valentin B. Shehtman, Maximal Kripke-type Semantics for Modal and Superintuitionistic Predicate Logics*

ML-92-03 *Zoran Markovi,c, On the Structure of Kripke Models of Heyting Arithmetic*

ML-92-04 *Dimiter Vakarelov, A Modal Theory of Arrows, Arrow Logics I*

ML-92-05 *Domenico Zambella, Shavrukovs Theorem on the Subalgebras of Diagonalizable Algebras for Theories containing ID0 + EXP*

ML-92-06 *D.M. Gabbay, Valentin B. Shehtman, Undecidability of Modal and Intermediate First-Order Logics with Two Individual Variables*

ML-92-07 *Harold Schellinx, How to Broaden your Horizon*

ML-92-08 *Raymond Hoofman, Information Systems as Coalgebras*

ML-92-09 *A.S. Troelstra, Realizability*

ML-92-10 *V.Yu. Shavrukov, A Smart Child of Peanos*

CT-92-01 *Erik de Haas, Peter van Emde Boas, Object Oriented Application Flow Graphs and their Semantics*

CT-92-02 *Karen L. Kwast, Sieger van Denneheuvel, Weak Equivalence: Theory and Applications*

CT-92-03 *Krzysztof R. Apt, Kees Doets, A new Definition of SLDNF-resolution*

X-92-01 *Heinrich Wansing, The Logic of Information Structures*

X-92-02 *Konstantin N. Ignatiev, The Closed Fragment of Dzhaparidze's Polymodal Logic and the Logic of S1 conservativity*

X-92-03 *Willem Groeneveld, Dynamic Semantics and Circular Propositions, revised version*

X-92-04 *Johan van Benthem, Modeling the Kinematics of Meaning*

X-92-05 *Erik de Haas, Peter van Emde Boas, Object Oriented Application Flow Graphs and their Semantics, revised version*

LP-93-01 *Martijn Spaan, Parallel Quantification*

LP-93-02 *Makoto Kanazawa, Dynamic Generalized Quantifiers and Monotonicity*

LP-93-03 *Nikolai Pankrat'ev, Completeness of the Lambek Calculus with respect to Relativized Relational Semantics*

LP-93-04 *Jacques van Leeuwen, Identity, Quarrelling with an Unproblematic Notion*

LP-93-05 *Jaap van der Does, Sums and Quantifiers*

LP-93-06 *Paul Dekker, Updates in Dynamic Semantics*

LP-93-07 *Wojciech Buszkowski, On the Equivalence of Lambek Categorial Grammars and Basic Categorial Grammars*

LP-93-08 *Zisheng Huang, Peter van Emde Boas, Information Acquisition from Multi-Agent resources; abstract*

LP-93-09 *Makoto Kanazawa, Completeness and Decidability of the Mixed Style of Inference with Composition*

LP-93-10 *Makoto Kanazawa, Weak vs. Strong Readings of Donkey Sentences and Monotonicity Inference in a Dynamic Setting*

LP-93-11 *Friederike Moltmann, Resumptive Quantifiers in Exception Sentences*

LP-93-12 *Jaap van der Does, On Complex Plural Noun Phrases*

LP-93-13 *Natasha Alechina, Binary Quantifiers and Relational Semantics*

LP-93-14 *Mati Pentus, Lambek Calculus is L-complete*

LP-93-15 *David Ian Beaver, What comes first in Dynamic Semantics*

ML-93-01 *Maciej Kandulski, Commutative Lambek Categorial Grammars*

ML-93-02 *Johan van Benthem, Natasha Alechina, Modal Quantification over Structured Domains*

ML-93-03 *Mati Pentus, The Conjoinablity Relation in Lambek Calculus and Linear Logic*

ML-93-04 *Andreja Prijatelj, Bounded Contraction and Many-Valued Semantics*

ML-93-05 *Raymond Hoofman, Harold Schellinx, Models of the Untyped l-calculus in Semi Cartesian Closed Categories*

ML-93-06 *J. Zashev, Categorial Generalization of Algebraic Recursion Theory*

ML-93-07 *A.V. Chagrov, L.A. Chagrova, Algorithmic Problems Concerning First-Order Definability of Modal Formulas on the Class of All Finite Frames*

ML-93-08 *Raymond Hoofman, Ieke Moerdijk, Remarks on the Theory of Semi-Functors*

ML-93-09 *A.S. Troelstra, Natural Deduction for Intuitionistic Linear Logic*

ML-93-10 *Vincent Danos, Jean-Baptiste Joinet, Harold Schellinx, The Structure of Exponentials: Uncovering the Dynamics of Linear Logic Proofs*

ML-93-11 *Lex Hendriks, Inventory of Fragments and Exact Models in Intuitionistic Propositional Logic*

ML-93-12 *V.Yu. Shavrukov, Remarks on Uniformly Finitely Precomplete Positive Equivalences*

ML-93-13 *V.Yu. Shavrukov, Undecidability in Diagonizable Algebras*

ML-93-14 *Dick de Jongh, Albert Visser, Embeddings of Heyting Algebras*

ML-93-15 *G.K. Dzhaparidze, Effective Truth*

ML-93-16 *Maarten de Rijke, Correspondence Theory for Extended Modal Logics*

ML-93-17 *Alexander Chagrov, Michael Zakharyaschev, On the Independent Axiomatizability of Modal and Intermediate Logics*
ML-93-18 *Jaap van Oosten, Extensional Realizability*
ML-93-19 *Raymond Hoofman, Comparing Models of the Non-Extensional Typed l-Calculus*
ML-93-20 *L.A. Chagrova, Dick de Jongh, The Decidability of Dependency in Intuitionistic Propositional Logic*
ML-93-21 *Max I. Kanovich, The Relational Knowledge-Base Interpretation and Feasible Theorem Proving for Intuitionistic Propositional Logic*
ML-93-22 *Andreja Prijatelj, Connectification for n-contraction*

CT-93-01 *Marianne Kalsbeek, The Vanilla Meta-Interpreter for Definite Logic Programs and Ambivalent Syntax*
CT-93-02 *Sophie Fischer, A Note on the Complexity of Local Search Problems*
CT-93-03 *Johan van Benthem, Jan Bergstra, Logic of Transition Systems*
CT-93-04 *Karen L. Kwast, Sieger van Denneheuvel, The Meaning of Duplicates in the Relational Database Model*
CT-93-05 *Erik Aarts, Proving Theorems of the Lambek Calculus of Order 2 in Polynomial Time*
CT-93-06 *Krzysztof R. Apt, Declarative programming in Prolog*
CT-93-07 *Janusz A. Pomykala, Approximation, Similarity and Rough Constructions, Part I. Elementary Introduction*
CL-93-01 *Noor van Leusen, László Kálmán, Computaional Linguistics*
CL-93-02 *Theo M.V. Janssen, An Algebraic View On Rosetta*
CL-93-03 *Patrick Blackburn, Claire Gardent, Wilfried Meyer-Viol, Talking about Trees*

X-93-01 *Paul Dekker, Existential Disclosure, revised version*
X-93-02 *Maarten de Rijke, What is Modal Logic?*
X-93-03 *Michiel Leezenberg, Gorani Influence on Central Kurdish: Substratum or Prestige Borrowing*
X-93-04 *A.S. Troelstra (editor), Metamathematical Investigation of Intuitionistic Arithmetic and Analysis, Corrections to the First Edition*
X-93-05 *A.S. Troelstra (editor), Metamathematical Investigation of Intuitionistic Arithmetic and Analysis, Second, corrected Edition*
X-93-06 *Michael Zakharyashev, Canonical Formulas for K4. Part II: Cofinal Subframe Logics*

ML-94-01 *Domenico Zambella, Notes on polynomially bounded arithmetic*
LP-94-01 *Dimitar Gelev, Introducing Some Classical Elements of Modal Logic to the Propositional Logics of Qualitative Probabilities*
CT-94-01 *Harry Buhrman and Leen Torenvliet, On the Cutting Edge of Relativization: the Resource Bounded Injury Method*
X-94-01 *Johan van Benthem, Two Essays on Semantic Modelling*

Previous titles in the ILLC Dissertation Series:

1993-1 *Transsentential Meditations; Ups and downs in dynamic semantics*, Paul Dekker
1993-2 *Resource Bounded Reductions*, Harry Buhrman
1993-3 *Efficient Metamathematics*, Rineke Verbrugge
1993-4 *Extending Modal Logic*, Maarten de Rijke
1993-5 *Studied Flexibility*, Herman Hendriks
1993-6 *Aspects of Algorithms and Complexity*, John Tromp
1994-1 *The Noble Art of Linear Decorating*, Harold Schellinx
1994-2 *Generating Uniform User-Interfaces for Interactive Programming Environments*, Jan Willem Cornelis Koorn
1994-3 *Process Theory and Equation Solving*, Nicoline Johanna Drost

institute for logic, language and computation

JOHAN VAN BENTHEM

Two Essays on Semantic Modelling

X-94-01, *received*: Jan. 1994

ILLC Research Report and Technical Notes Series
Series editor: Dick de Jongh

Technical Notes (X) Series

Institute for Logic, Language and Computation (ILLC)
University of Amsterdam
Plantage Muidergracht 24
NL-1018 TV Amsterdam
The Netherlands
e-mail: illc@fwi.uva.nl

Institute for Logic, Language and Computation ILLC
University of Amsterdam
20 September 1993

Reflections on Epistemic Logic

to appear in Paul Gochet, ed., special issue of *Logique et Analyse*.

Johan van Benthem

****

Traditionally, 'epistemic logic' has been a specialized area of general modal logic devoted to the epistemic modalities and their interaction with quantification, predication and identity. In this brief Note, we shall step back and see which broader forms of epistemic semantics exist, and discuss how they affect traditional concerns of the field.

****

1 Standard Epistemic Logic

Traditional epistemic logic started largely with the pioneering classic Hintikka 1962, which presented modal logics for the notions of a person knowing and believing a proposition, based upon standard possible worlds semantics with an accessibility relation of 'epistemic indistinguishability'. Roughly speaking, I know a proposition if that proposition holds in all worlds which I cannot distinguish from my current one, that is, if it holds across my whole 'range of uncertainty'. Different epistemic attitudes may then vary in the extent of that semantic range, or in the requirements imposed upon it. In a subsequent series of papers, Hintikka also developed an epistemic predicate logic, dealing with issues of individual identity across epistemic worlds and the interplay of knowledge and ('de re' versus 'de dicto') quantification. No extensive mathematical theory developed around this initial focus – the technical monograph Lenzen 1980 is a respectable exception – but the paradigm did find a number of active areas of application in the eighties, especially in computer science with the work of Joe Halpern and his colleagues at IBM San José, and in Artificial Intelligence by Bob Moore and some colleagues at SRI, Menlo Park. What the computational connection added, in particular, was a more concrete view of epistemic models derived from computational protocols or processes, where agents are well-defined and philosophical questions concerning the intentionality of human cognition can be safely side-stepped.

Notions which then came to the fore were 'autoepistemics', concerning the equilibrium states that an agent can achieve in non-monotonic default reasoning (Moore 1987), and also various forms of collective epistemics expressed in the interplay of operators K_aB ('agent a knows that B ') for various agents a , as well as 'common knowledge' for groups of agents (Halpern & Moses 1990). Moreover, these epistemic concerns were embedded in a larger environment of communication and physical action. As a result, there is now a flourishing community around the so-called TARK Conferences (cf. Halpern, ed., 1986, Vardi, ed., 1988 and subsequent volumes), which brings together philosophers, mathematicians, computer scientists, economists and linguists. In what follows, we shall take the basic core of epistemic logic for granted, while making only occasional reference to its computationally inspired extensions.

2 Implicit versus Explicit Epistemics

The above enterprise of epistemic logic has not been without its critics. In fact, many people have regarded it as a typical form of 'shallow analysis', describing epistemics by merely imposing some modal superstructure on top of an ordinary classical semantics, rather than reanalyzing the latter in depth. To introduce a distinction, the Hintikka-Halpern paradigm is one of 'extrinsic epistemics', which does not affect standard classical semantics, witness its key truth definition stating that, in any model, " K_aB is true at a world w iff B is true at all worlds v that are R_a-related to w ". By contrast, a more radical 'intrinsic epistemics' would not take classical semantics for granted, and reanalyze the whole notion of truth in the light of epistemic considerations. The most prominent historical instance of such a radical enterprise is of course intuitionistic logic, where 'truth' recedes as the central logical notion in favour of 'provability' or 'assertability'. Thus, its guiding philosophy has been epistemically oriented from the start. (Compare the constructive theories of meaning based upon the 'Brouwer-Heyting-Kolmogoroff interpretation' that are surveyed in Troelstra & van Dalen 1988.) To be sure, intuitionistic logic, too, has its Kripke-style possible worlds semantics, but with an entirely different flavour. Worlds stand for information states, accessibility encodes possible informational growth, and truth at a world corresponds intuitively to epistemic 'forcing' by the available evidence there. Thus, the basic logical operations themselves become 'epistemically loaded'. But then, given an epistemic reinterpretation of what logic and semantics is about, where is the need for any separate 'epistemic logic'?

It is not so easy to adjudicate this debate. Certainly, the intuitionistic approach has generated more interesting mathematical theory, while also being more influential in philosophical and computational circles. But there are many different causes for this course of events, not all of them having to do with the relative merits of implicit versus explicit epistemics. Moreover, as a matter of fact, there are also some advantages to the extrinsic approach. For instance, it can incorporate whatever sound analysis is provided in a classical semantics for a certain kind of propositions, without having to worry about philosophical compatibilité d'humeurs. By contrast, more intuitionistic proof-theoretic approaches have had difficulties making sense of logical operators such as generalized quantifiers that have perfectly straightforward model-theoretic explications. (But cf. van Lambalgen 1991 for a fresh start.) Moreover, the very explicitness of Hintikka's approach has encouraged an active search for new epistemic operators, such as the above ones referring to knowledge of multiple agents or groups which have no counterpart in intuitionistic logic. Even though mathematics is a social activity, too, where insights may depend essentially on cooperation of rational agents, the implicit intuitionist stance has not been very conducive so far to bringing this out formally.

Finally, the distinction betweeen intrinsic and extrinsic epistemics is an 'intensional' one, which may be undercut at a more formal level of analysis. For instance, the well-known Gödel translation embeds intuitionistic logic faithfully into S4, the logic of choice for much of epistemic logic, thereby making it a kind of 'forward persistent' part of the latter approach. (Van Benthem 1990 provides further formal detail here. Incidentally, no converse embedding seems to work.) In this line, the explicit 'epistemic mathematics' of Shapiro 1985 may also be viewed as a natural extension of intrinsic intuitionistic mathematics. Conversely, the analyses of 'common knowledge' put forward in Barwise 1988 seem more 'intrinsic' analyses of this phenomenon, changing the underlying classical model theory to a form of information-oriented situation semantics. Generally speaking, then, there seems no problem of principle in combining both the agenda and the technical apparatus of intrinsic and extrinsic approaches to epistemic logic.

3 Information-Based Semantics

In recent years, the 'intrinsic' approach has gained ground in various new guises. Current semantics for natural languages and computation shows a trend away from truth conditions and correspondence with the world outside to explaining propositions in terms of their role in information processing over models that are now viewed as 'information structures'. Here, the classical turn-stile becomes a notion of 'forcing' for statements by the available information. Examples of this trend are data semantics (Veltman 1985), various forms of partial modal logic (Thijsse 1992), constructive semantics in the style of Nelson (Jaspars 1993) or substructural semantics for relevant or categorial logics (van Benthem 1991, Wansing 1993). This development can be seen as epistemic logic in a broader sense, since much of cognition can in fact be subsumed under the heading of information processing. What this new phase adds, however, is an explicit concern with the nature of information states and possible updates over these effected by successive propositions. Much is still unclear at this stage, witness the persistent debate between 'eliminative' accounts of information processing (which proceed via successive elimination of epistemic possibilities) and 'constructive' ones (which build ever larger representation structures). Either way, these concerns do seem a natural addition to the foundations of epistemic logic.

Moreover, one clear trend can be observed which does not depend on the exact nature of epistemic states. Current information-oriented modellings suggest a richer and more systematic design of important epistemic operators. This phenomenon is illustrated by the basic case of Kripke models themselves. Initially, these were designed to model a particular epistemic language, whose operators were given independently. But then, one can also reverse the perspective and ask, given such models for information structure, which epistemic languages would best bring out their semantic potential, thereby redesigning the original language. For instance, as in temporal logic, there are two natural directions in the growth ordering of information states, both backward and forward. Knowledge may be mostly concerned with epistemic 'advance', but it has to do also with epistemic 'retreat', in contraction or revision of our information (cf. the symmetric theory of updates and contractions in Gärdenfors 1988). Thus, a more adequate epistemic logic should also incorporate operators reflecting these additional directions and their interplay, with 'forward knowledge' referring upward to all possible extensions of the current state, and 'backward knowledge' referring to the epistemic past. Their interplay will then generate different routes for epistemic revision ("if A had been found, then ..."). Van Benthem 1990 explores the resulting hierarchy of operators over information models, using a framework of enriched modal logics. In particular, a next natural stage of epistemic expressiveness would involve operators reflecting the addition of information pieces (i.e., suprema in the ordering of possible growth) as well as their downward counterparts (i.e., their infima). For instance, the binary epistemic modality $\phi+\psi$ would hold at those states which are the sum of a state verifying ϕ and a state verifying ψ . These will allow us, e.g., to refer to logical features of composite knowledge arising from the combination of various sources.

Of course, by this time, one will have left the traditional areas of the Theory of Knowledge which inspired Hintikka's initial enterprise. But that might also be considered a virtue, and one could certainly translate many of the newer technicalities back into genuine philosophical issues that might revitalize the somewhat fossilized agenda found in most philosophical textbooks.

4 Adding Justifications

The above type of enrichment may be seen as an extension of the modal viewpoint on epistemics, with a reinterpretation of its models in a Kripke-style intuitionistic spirit. But there is more to be learnt from a confrontation of extrinsic and intrinsic approaches. One conspicuous feature of contemporary intuitionistic logic and mathematics has been the development of type theories whose proof format rests essentially on binary assertions of the form $\pi : A$, meaning that function π is of type A, or that proof π establishes proposition A. Proof rules in the usual constructivist interpretation then typically build up compound conclusions in both components, witness a case like

$$\frac{\pi_1 : A \qquad \pi_2 : B}{(\pi_1, \pi_2) : A\&B} \qquad\qquad \frac{\pi_1 : A \to B \qquad \pi_2 : A}{\pi_1(\pi_2) : B}$$

Note how both assertions and their justifications are affected here. This binary logical format is gaining popularity these days for its greater perspicuity in bringing out combination of linguistic propositions plus their underlying manifestations or justifications, where the latter need not always be explicitly linguistically encoded. This makes sense in mathematics and computation (cf. Barendregt 1993), but also more broadly in linguistics and Artificial Intelligence, witness the new research program of 'labeled deductive systems' put forward in Gabbay 1993.

Now here too, there is a very attractive move for epistemic logic. Much of the classical theory of knowledge and its initial logical formalizations seems hampered by the absence of any systematic way of bringing out the justifications underlying our knowledge as first-class citizens. Put somewhat formally, saying that someone knows a proposition is an existential quantification stating that she has a justification for that proposition. But by keeping those justifications hidden in our logical framework, we create both technical and conceptual difficulties. For instance, much of Hintikka's own work on the Kantian notion of analyticity (cf. Hintikka 1973) has to wrestle with the fact that with Kant, 'analyticity' is a qualification of reasons or justifications as much as of statements, which makes its projection into standard logical systems somewhat problematic. An even more striking example is the so-called 'Problem of Omniscience', where knowledge of a proposition entails knowledge of all its logical consequences. This problem is highlighted in the standard epistemic Distribution Axiom $K(A \to B) \to (KA \to KB)$. By contrast, in the above binary format, this problem would not arise in the first place, because any logical inference will come with an explicit cost record in terms of a more complex justification. To see this, compare the above distribution principle with the corresponding binary type-theoretic inference from the two premises

$\pi_1 : A \rightarrow B$ and $\pi_2 : A$ to the conclusion $\pi_1(\pi_2) : B$. Of course, one further question here is what systematic type-theoretic calculus will support explicit epistemic operators. There are various options to this effect, such as the following two introduction rules for epistemic operators:

$$\frac{\pi : A}{- : KA} \text{ ('forgetful')} \qquad\qquad \frac{\pi : A}{{}^*(\pi) : KA} \text{ ('reminiscent')}$$

For a more elaborate system of this kind, cf. the modal type theories in Borghuis 1993. (E.g., deriving the above epistemic distribution axiom will also involve suitable elimination rules for the K-operator.) The conceptual task remains to set up a complete plausible base theory for epistemic logic with an explicit calculus of justifications.

5 Cognitive Action

When model-theoretic semantics is reinterpreted as a theory of information structures, one further move becomes quite natural. Justifications and revisions are really examples of cognitive actions – and it would be quite appropriate then to embed our epistemic logic into an explicit dynamic logic. This move is in fact foreshadowed in the earlier-mentioned computational tradition in epistemic logic (cf. Moore 1984). For instance, the above binary schema $\pi : A$ has one further useful interpretation, stating that "program or action π achieves an effect described by proposition A " . Examples of relevant cognitive actions are the earlier-mentioned 'updates' or 'contractions', but one can also think of a much richer repertoire of 'testing', 'querying', etcetera. Moreover, on top of this basic repertoire, one can describe complex cognitive actions or plans via the usual programming operations, such as sequential or parallel composition and choice. Evidently, human cognitive plans have compound structures not unlike those found in computer programs. The precise extent of this analogy is an interesting issue by itself. For instance, can cognitive plans also display more infinitary structures like recursion?

Stated in this way, we need a two-level system for combining epistemic statements with 'cognitive programs'. One possible logical architecture here is the 'dynamic modal logic' of van Benthem 1993, which has a relational repertoire of actions over information models interacting with a standard modal logic. Typical assertions in such a formalism will be a dynamic modality $[\pi] A$, stating that " action π always achieves effect A " , (compare the above binary schema), or modal iterations like $[\pi_1]\langle\pi_2\rangle A$ stating that " action π_1 'enables' action π_2 to achieve effect A " . The model theory and proof theory of this system are well-understood (cf. Harel 1984, de Rijke 1992). Moreover, its expressive power subsumes at least the better-known theory of belief revision in

Gärdenfors 1988. But dynamic modal logic also supports more radical deviations from classical logic. For instance, van Benthem 1993 considers 'dynamic styles of inference' from premises $\pi_1, ..., \pi_n$ to conclusions C based on the idea that propositions are cognitive actions which are being processed in reasoning. Various options to this effect may be expressed in the above terms. For instance, one plausible dynamic style would state that 'sequential processing of the premises is a way of getting the conclusion', which may be expressed by the modal formula $[\pi_1] ... [\pi_n] C$. Another typical dynamic style would rather state that 'processing the premises is a way of doing the conclusion' – which involves a stronger modal apparatus than that of standard dynamic logic (cf. Kanazawa 1993.)

6 Combinations and Conclusions

The above Sections point at various attractive enrichments of standard epistemic logic. Putting all of these ideas together, however, raises some obvious further issues. For instance, which logical architecture would most naturally combine statements, justifications and actions? A type-theoretic approach looks promising here. For instance, one of its key statements might be of the form $\pi : A \rightarrow B$, expressing that action π will always lead from states satisfying precondition A to states satisfying postcondition B . In such a format, one might describe the behaviour of compound cognitive actions or plans, mentioned in the above, with rules like the following:

$$\frac{\pi_1 : A \rightarrow B \qquad \pi_2 : B \rightarrow C}{\pi_1 ; \pi_2 : A \rightarrow C} \qquad\qquad \frac{\pi_1 : A \rightarrow B \qquad \pi_2 : A \rightarrow B}{\pi_1 \cup \pi_2 : A \rightarrow B}$$

Nevertheless, even this kind of generalization will still fail to capture some further essentials of cognition. First, we need logical systems that lift all of the above to many-person settings, which allow, amongst others, for updating common knowledge of groups via various acts of communication (cf. Jaspars 1993). But also, we would eventually need a theory of 'synchronization' between internal cognitive actions and external physical actions which change the world.

Even with all these open ends, our conclusions concerning epistemic logic will be clear. We would recommend expansion beyond Hintikka's original modal statement format, thereby effecting a junction with a much larger logical environment of epistemic import. Moreover, we think the effort would be well worth-while to systematically rethink much of traditional philosophical epistemology in this broader light.

Acknowledgment

This paper grew out of notes for two lectures. I wish to thank the organizers of TARK IV (Asilomar 1992), and especially Yoram Moses, for inviting me to their meeting. Moreover, I am grateful to Professor Paul Gochet for inviting me to his workshop on Epistemic Logic (Liege 1993).

References

S. Abramsky, D. Gabbay & T. Maibaum, eds., 1993,
Handbook of Logic in Computer Science, Oxford University Press, to appear.

H. Barendregt, 1993,
'Lambda Calculi With Types', to appear in S. Abramsky et al., eds.

J. Barwise, 1988,
'Three Views of Common Knowledge', in M. Vardi, ed., 365-379.

J. van Benthem, 1990,
'Modal Logic as a Theory of Information', CSLI Report 90-144, Center for the Study of Language and Information, Stanford University.

J. van Benthem, 1991,
Language in Action. Categories, Lambdas and Dynamic Logic, North-Holland, Amsterdam, (Studies in Logic 130).

J. van Benthem, 1992,
'Abstract Proof Theory', manuscript, Institute for Logic, Language and Computation, University of Amsterdam.

J. van Benthem, 1993,
'Logic and the Flow of Information', in D. Prawitz et al., eds.

T. Borghuis, 1993,
'Interpreting Modal Natural Deduction in Type Theory', in M. de Rijke, ed., 67-102.

J. van der Does & J. van Eyck, eds., 1991,
Generalized Quantifier Theory and Applications, Dutch Network for Language, Logic and Information, Amsterdam, (to appear with CSLI Lecture Notes, Chicago University Press).

D. Gabbay, 1993,
Labeled Deductive Systems, Department of Computing, Imperial College, London, (to appear with Oxford University Press).

D. Gabbay & F. Guenthner, eds., 1984,
Handbook of Philosophical Logic, volume II, Reidel, Dordrecht.

P. Gärdenfors, 1988,
Knowledge in Flux. Modeling the Dynamics of Epistemic States, The MIT Press, Cambridge (Mass.).

M. Ginzburg, ed., 1987,
Readings on Non-Monotonic Reasoning, Morgan Kaufmann Publishers, Los Altos.

J. Halpern, ed., 1986,
Proceedings First Conference on Theoretical Aspects of Reasoning About Knowledge, Morgan Kaufmann Publishers, Los Altos.

J. Halpern & Y. Moses, 1990,
'Knowledge and Common Knowledge in a Distributed Environment', *Journal of the Association for Computing Machinery* 37, 549-587.

D. Harel, 1984,
'Dynamic Logic', in D. Gabbay & F. Guenthner, eds., 497-604.

J. Hintikka, 1962,
Knowledge and Belief, Cornell University Press, Ithaca (New York).

J. Hintikka, 1973,
Logic, Language Games and Information, Clarendon Press, Oxford.

W. van der Hoek, 1992,
Modalities for Reasoning About Knowledge and Quantities, dissertation, Department of Computer Science, Free University, Amsterdam.

J. Jaspars, 1993,
Logics for Communicative Action, dissertation, Institute for Language and Knowledge Technology, University of Brabant, Tilburg.

M. Kanazawa, 1993,
'Completeness and Decidability of the Mixed Style of Inference with Composition', Center for the Study of Language and Information, Stanford University.

M. van Lambalgen, 1991,
'Natural Deduction for Generalized Quantifiers', in J. van der Does & J. van Eyck, eds., 143-154.

W. Lenzen, 1980,
Glauben, Wissen und Wahrscheinlichkeit, Springer, Vienna.

R. Moore, 1984,
'A Formal Theory of Knowledge and Action', CSLI Report 31, Center for the Study of Language and Information, Stanford University.

R. Moore, 1987,
'Possible Worlds Semantics for Auto-Epistemic Logic', in M. Ginzburg, ed., 137-142.

D. Prawitz, B. Skyrms & D. Westerståhl, eds., 1993,
Proceedings 9th International Congress of Logic, Methodology and Philosophy of Science. Uppsala 1991, North-Holland, Amsterdam.

M. de Rijke, 1992, 'A System of Dynamic Modal Logic', Report LP-92-08, Institute for Logic, Language and Computation, University of Amsterdam.

M. de Rijke, 1993,
Extending Modal Logic, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

M. de Rijke, ed., 1993,
Diamonds and Defaults, Kluwer, Dordrecht.

S. Shapiro, ed., 1985,
Intensional Mathematics, North-Holland, Amsterdam.

E. Thijsse, 1992,
Partial Logic and Knowledge Representation, dissertation, Institute for Language and Knowledge Technology, University of Brabant, Tilburg.

A. Troelstra & D. van Dalen, 1988,
Constructive Mathematics, North-Holland, Amsterdam, (Studies in Logic 121, 123).

M. Vardi, ed., 1988,
Proceedings Second Conference on Theoretical Aspects of Reasoning About Knowledge, Morgan Kaufmann Publishers, Los Altos.

F. Veltman, 1985,
Logics for Conditionals, dissertation, Department of Philosophy, University of Amsterdam.

H. Wansing, 1993,
The Logic of Information Structures, Springer, Berlin, (Lecture Notes in Artificial Intelligence 681).

Institute for Logic, Language and Computation **ILLC**

University of Amsterdam December 1993

To appear in M. Marx and L. Polós, eds.,
Logic@Work, Oxford University Press.

THE SOURCES OF COMPLEXITY: CONTENT VERSUS WRAPPING

I Contents and Wrappings

Any description of a subject carries its own price in terms of complexity. To understand what is being described, one has to understand the mechanism of the language or logic employed, adding the complexity of the encoder to the subject matter being encoded. Put more succinctly, "complexity is a package of subject matter plus analytic tools". This price is inevitable, and scientific or common sense insight does result all the same. Nevertheless, there is also a persistent feeling that one should never pay more than is necessary. Aristotle already formulated the necessary intellectual 'lightness' as follows. "It is the hall-mark of a scientific mind to give a subject no more formal structure than it can support" (as paraphrased in Kneale & Kneale 1961). Critics of formal logic would certainly agree with this dictum, and they have pointed at many cases in philosophy, linguistics, computer science and even the foundations of mathematics where general mathematical sophistication or just essayistic common sense is the more appropriate road towards insight than elaborate logical formal systems. But also inside formal logic, this question seems a legitimate concern. Are our standard modelings really appropriate for certain phenomena of reasoning, and are the received conclusions that we draw about complexity of phenomena in our field (using qualifications like 'undecidable' or 'higher-order') warranted, or rather an artefact of those modelings? More disturbingly, could it be that much of the respectable literature in our journals, which aims at proving difficult theorems as a sign of the academic 'worthiness' of a topic, mostly derives its continuity from the fact that one encounters the same issues over and over again, precisely because they derive from the formalisms employed, not from the subject matter at hand? I think these are serious questions, that deserve constant attention. Of course, many logicians do care for these, either implicitly or explicitly – but it will do no harm to keep them high on the agenda by means of some occasional extra advertisement.

But then, of course, the question arises how one can distinguish the two sources of complexity in specific cases. Often, one has a suspicion that received views as to what is difficult in some form of reasoning or computation and what is not, might be challenged, but how to separate the two components? There may not be any systematic method of separation here – but there is certainly a roundabout answer in the practice of our field, which tends to generate scores of alternative formal modellings. Thus, the role of the wrappings becomes visible indirectly, by comparison with alternatives (e.g., think of set-theoretic versus algebraic or category-theoretical formulations of the same problem). In other words, although formal logical modellings may be part of the complexity problem themselves, logicians mitigate this drawback by producing so many of them!

There is room for more detailed analysis here, by considering more concrete examples. Received views on complexity in formal semantics often come in the form of warnings concerning certain dangerous 'thresholds' where complexity is generated or increased. For instance, one well-known danger zone is the transition from finite to infinite structures, and another that from first-order to higher-order objects. These moves often produce undecidability or even non-axiomatizability in the description of computational or linguistic phenomena. For instance, in the semantics of programming languages, undecidability may arise through the introduction of infinitary structures for iteration or recursion (Harel 1984, Goldblatt 1987). Likewise, temporal semantics for concurrency usually employ higher-order logics of temporal branches or histories, which involve quantification over sets (Burgess 1984, Stirling 1990). In the semantics of natural languages, similar thresholds arise. For instance, many quantificational phenomena are generally considered to be essentially second-order – such as linguistic 'branching quantifiers' which involve parallel rather than serial processing, with corresponding non-linear quantifier prefixes going beyond first-order logic (cf. Barwise 1979). Other sources of higher-orderness are traditionally located in the semantics of plural quantification (cf. van der Does 1992). Undecidability has also been claimed for variable binding phenomena in natural language anaphora (cf. Hintikka 1979 on the so-called "Any-Thesis"). Finally, new thresholds of complexity have emerged in Artificial Intelligence, with corresponding forms of 'received wisdom'. Notably, non-monotonic reasoning mechanisms, such as 'circumscription', are usually considered to be essentially higher-order and highly complex in general (McCarthy 1980) – and indeed, the same seems to hold, perhaps disturbingly, for any meaningful human cognitive task (cf. Kugel 1986).

Do these views square with the intuitive expectations that one might have on the basis of a plausible initial estimate of the intrinsic complexity of a subject, prior to its formalization? Not always. And in general, it is hard to make uniform predictions here. Sometimes, formal analysis does confirm our intuitive hopes or suspicions concerning complexity of a phenomenon, witness the complexity theory of logical formalisms (cf. Spaan 1993). But at other times, plausible expectations may turn out to be wrong, and we stand corrected by the formal analysis. A well-known historical instance is the undecidability of predicate logic, understandable eventually via Gödel's and Church's arguments, which clashed with a dominant tradition of searching for decision procedures in the logical literature. Moreover, there do not seem to be infallible rules for keeping complexity down by simply avoiding the above danger zones. For instance, a blanket restriction to finite models has been advocated in natural language semantics as a way of keeping complexity down (cf. van Benthem 1986). But this same restriction may also make many results harder to obtain (if available at all), witness the additional complexities of 'finite model theory' over general model theory found in Gurevich 1985. Likewise, the use of infinitary logics does increase complexity in some ways, but it also decreases complexity in other ways, witness the discovery of smooth infinitary axiomatizations for various programming logics (Goldblatt 1983), which can be more perspicuous than their first-order counterparts. Let us now turn to some specific cases where influential styles of modeling have supported well-known received views on complexity. These will provide more specific points for our subsequent discussion.

II From Higher-Order to First-Order

There are many cases in semantics where higher-order modelling is thought appropriate and inevitable, even though it brings the cost of employing a formal system whose logical validities are non-axiomatizable (and indeed non-arithmetically definable). Pointing at these dangers is indeed one of the well-known social bonding procedures in our community, where collective shudders run through lecture halls at the mention of this Evil One. Nevertheless, upon closer inspection, in all these cases, an important distinction should be made. It is one thing to employ a higher-order *language*, referring to non-first-order individuals such as sets, choice functions or branches – but quite another to insist that this language should have full set-theoretic *standard models*, making it behave in the above-mentioned fashion. For, insisting on the latter expresses an additional commitment: namely that we want to use one particular mathematical implementation of our formalism, whose complexities will tend to 'pollute', naturally, the validities of our logic which was designed to mirror the core phenomenon.

There is an insidious term in the practice of our field, which confuses the issue, namely the often praised "concreteness" of set-theoretic models. What this merely means is that we insist on using one particular well-known abstract mathematical structure as the wrappings of our theory: not that other mathematical modelings would be less concrete. For instance, is a set-theoretic model really 'more concrete' than an algebraic structure or a geometrical picture? Intuitively, in the latter case, the opposite would seem to be true. Moreover, the specificity of a mathematical structure may be the very source of complexity indicated above. Thus, there are reasons for preferring a more neutral stance. Another insidious groove of thought here is the 'separation of concerns' favoured by many authors proposing semantic innovations. They prefer not to raise too many issues at once, and therefore choose "standard set-theoretic modelling" as their working theory. (Cf. the dominant style of presentation in current 'dynamic semantics', cf. Kamp 1984, Groenendijk & Stokhof 1991, Veltman 1991). And of course, this is an excellent research strategy. But in fact, as we shall see below, working with this standard background may not always be an orthogonal decision. It can even be detrimental to the new proposals, as it may involve them in the hereditary sins, complexity-wise, of the old paradigms – precisely when dynamic semantics is partly inspired by the need to provide some cognitive relief from these.

In the case of higher-order logic, a more neutral perspective has been around for a long time, with so-called 'general models' for higher-order languages allowing in general just restricted ranges for set quantification (Henkin 1950). Here, standard models are the limiting case where all mathematically possible sets or predicates have to be present – whether needed or not for the phenomenon under study. This broadening of the model class reinstates the usual properties of first-order logic, but it would be misleading to view it as just an opportunistic tactic. More importantly, on this view, higher-order logic becomes a *many-sorted first-order* logic treating, amongst others, individuals, sets and predicates on a par (cf. Enderton 1972). This move has a good deal of independent philosophical justification, witness the 'property theories' advocated in Bealer 1982, Turner 1989. Indeed, we achieve a moral rarity, being a combination of philosophical virtue with computational advantage, in trading set-theoretic complexity for new sorts of individuals. Even so, this move has always carried a stigma of ad-hoc-ness, and one common complaint is its lack of canonicity. No unique behaviour is specified for the sort of sets, so that the resulting 'logic' is subject to intensive manipulation. In other words, this solution is 'too easy' – again, one of the harmless shibboleths of the field around which we celebrate our professional consensus. But is it really? Considered from another angle, general models do just the right thing. First, they radically block the importation of extraneous set-theoretic truths, letting the subject being described stand out. Next, they force us to replace Platonic complacency by honest work. If we want to add an explicit new sort of objects ('sets', 'predicates' or whatever), then it should indeed be our task to analyze those principles about these objects that are germane to our subject, and formulate them explicitly. Just what about the behaviour of 'sets' is relevant to programming semantics, or to natural language?

By answering questions like this, one often arrives at some compromise. The usual semantic restriction to standard set-theoretic models is too complex, and unilluminating – whereas a liberalization to all general models is too weak, and unilluminating for the opposite reason. But after some hard work, we may arrive at an appropriate model class somewhere in between. Examples of this style of analysis may be found everywhere, once one perceives matters in this light. For instance, second-order semantics with quantification over branches of some sort may always be replaced by many-sorted theories of suitable 'individuals' (points in time, states, etc.) and 'branches' or 'paths', where one now has to study the key principles concerning branches as well as their interaction with the individuals occurring on them. E.g., Stirling 1989 identifies various interesting modal-temporal second-order principles – such as 'fusion closure' stating that, for any state occurring in two histories, its past in the one and its future in the other may be glued together so as to form a new history. Likewise, in branching temporal logic, second-order axioms of choice have been invoked to support 'confluence principles' stating when two points on different histories may come together in some common future by suitable further histories (De Bakker, de Roever & Rozenberg 1989). But, such principles are better viewed as geometrical conditions on the availability of branches in two-sorted first-order models, on a par with standard geometrical axioms concerning points and lines. This is at least as good mathematics as set theory. After all, standard axiomatizations of geometry use 'points', 'lines', 'planes' on a par, rather than construing the latter outlandishly as point sets. For a final example, consider two closely related semantics for intuitionistic logic. 'Kripke models' are first-order, with truth conditions referring only to possible worlds and accessibility, whereas 'Beth models' are second-order, involving also branches (Troelstra & van Dalen 1988). For instance, Beth's truth clause for a disjunction A-or-B says that there exists some barrier of states across all possible future histories such that each state on the barrier verifies either A or B . But again, the latter models can be reformulated by viewing worlds and branches on a par, and analyzing what (little) explicit theory about branches – rather than some Platonic oracle about sets – is needed to explain validity for intuitionistic logic. (The Appendix to Rodenburg 1986 contains a first exploration of the resulting two-sorted model theory, including the interaction of states and branches.)

The above task of localizing key principles on many-sorted models may be made more concrete and systematic by employing the tools of *Correspondence Theory* (cf. van Benthem 1984, 1985). Let us demonstrate this approach for the case of temporal logic. There are two complementary ways of arriving at our desired concrete 'branch theory'. The first involves general reflection on 'branches', sorting out general logical principles governing these from more extravagant mathematical existence claims. The other approach takes the kind of temporal reasoning that is to be analyzed as a guide-line, looking for 'correspondences' between its intended principles and assumptions about branches. This interplay is well-documented for pure temporal logic (cf. van Benthem 1983). Here is a simple example in the branching semantics. Some forms of temporal reasoning allow a quantifier shift between 'future possibility' and 'possible futurity'. This will correspond to a simply computable condition on the pattern of states and branches: namely that time-travel along my current history and then switching to another future history may also be performed by first switching to another history and then traveling into its future. Correspondence Theory has only been investigated very systematically in Modal Logic, but in principle, this style of analysis is available everywhere – witness the powerful extensions presented in Venema 1991, De Rijke 1993. (Admittedly, there is still a subtle issue of methodological consistency here. Correspondence analysis in its usual format itself involves computation in higher-order logic! One line to take here might be to say that we are merely using this technique as a semantic heuristic, and that 'defeating second-order logic from within' is an elegant philosophical stratagem. The more sober line would note that many of the relevant insights would in fact also be forthcoming on suitable general frames.)

Clearly, not every principle that comes up in the above way should be taken for granted. Some correspondences for putative principles of temporal reasoning may turn up clear general desiderata on two-sorted point-branch models, such as linearity for branches, or unicity of their initial points. Others may turn up interesting but only negotiable options. For instance, more complex modal-temporal quantifier shifts in temporal reasoning turn out to correspond to axioms of choice, reflecting a certain 'fullness' of state-branch models. No very clear demarcation line exists between the first case and the second – although there have been some systematic proposals. For instance, if genuine logical principles should be entirely free from existential import (cf. Etchemendy 1990), then a case can be made for admitting just the purely universal first-order requirements arising in a correspondence analysis as genuine core conditions on our semantics, relegating all conditions with existential import to some negotiable mathematical part. (Van Benthem 1983 even defends the further restriction to mere universal Horn clauses, viewing propositional disjunctions as existential too.)

Despite the methodology advocated here, there is no general presumption that any reasonable new semantics will or should be simple, or even first-order. For instance, it might well be that in many computational applications, temporal branches should be *finite* (a non-first-order condition). But also, in the opposite direction, it has been argued with some force that the essential control structures for programming ought to be *infinitary*, and hence non-first-order (Goldblatt 1983). Either way, the emergence of such discussions does not count against our broader framework: it rather speaks for its fruitfulness. Whether computation hinges essentially on finiteness for its 'traces' or its control structures is a substantive issue, which should be on the agenda explicitly, and not prejudged in the use of 'concrete standard modeling'.

All these points may also be demonstrated for the case of natural language. Consider the following linguistic phenomenon mentioned earlier on. From the start (cf. Hintikka 1979), 'branching quantification' has been considered a typical example of a second-order semantic mechanism, requiring quantification over Skolem functions not represented by ordinary first-order linear quantifier prefixes. To be sure, there is still some opposition to this linguistic claim, and first-order guerrilleros are still active. But, even granting the move toward using non-linearly orderable Skolem functions here, all that is shown by its proponents is that we need to consider a certain family of *relevant* 'choice functions' explicitly among our semantic individuals – not necessarily the whole mathematical space (containing lots of linguistically irrelevant items). Moreover, we can even defend this move on the basis of independent linguistic evidence. For instance, so-called 'functional answers' to questions ("Whom does every man love? His mother.") suggest that we need more abstract 'functional objects' as citizens in our semantic world. But even then, the real issue remains how many of these choice functions are required to explain natural language branching patterns, satisfying which combinatorial constraints. More generally, this point applies to more general linguistic 'polyadic quantification' (van Benthem 1989, Keenan and Westerståhl 1993), whose higher-orderness has been taken for granted so far. The complexity of the set-theoretic denotations arising here might also point at constraints on some family of 'available' predicates and functions. Likewise, considerations of this kind apply to several current discussions of complexity in Artificial Intelligence. For instance, the above-mentioned method of 'circumscription' might just employ minimization in general models, performed with respect to some family of 'relevant predicates' – say, those that are explicitly represented in our computational environment. (Morreau 1985 investigates the resulting two-sorted first-order model theory, and shows how it can deal with many of the non-monotonic reasoning patterns that originally motivated circumscription.)

Finally, it should be emphasized that not all broader semantic spaces can or should arise from the above 'general model' strategy. What the latter provides is one systematic way of stepping back and rethinking the semantic issues. Without independent semantic evidence fort its outcomes, however (as in the preceding linguistic example), the resulting many-sorted model might remain an ad-hoc theoretical curiosity. And in fact, eventually, one may come to prefer some alternative modelling altogether. A general illustration of this freedom is the ubiquitous switch found in mathematics between algebraic and geometric viewpoints – and a more specific one, the emergence of some recent analyses of branching quantification in natural language that employ a different circle of ideas concerning 'groups' and collective predication (cf. Landman 1989, Hoeksema 1983, van der Does 1992).

Summing up, changing to broader first-order model classes is not an ad-hoc move of desperation or laziness in taming complexity. It rather represents a more finely-tuned style of analysis, forcing us to do our conceptual homework, and eventually – another potential benefit – suggesting new applications beyond the original field, precisely because of the available new semantic models. Moreover, this is not a risky new-fangled approach of uncertain prospects. For, when all is said and done, this move is precisely what abstract mathematical analysis has always been about.

III From First-Order to Decidable

The preceding move from higher-order to first-order logic brings a clear gain in complexity. But how satisfactory is the latter system as a universal semantic medium? Although effectively axiomatizable, predicate logic is undecidable – and again, this feature may import external complexity into the description of subjects whose 'natural complexity' would be decidable. One factor responsible for this situation, as was observed above, is the lure of 'concrete set-theoretic models'. We all think of standard Tarskian models as the essence of concreteness and simplicity (although there has been some underground opposition from the earlier-mentioned 'property theorists' – cf. also Zalta 1993). But here again, are we perhaps still importing extraneous set theory, which might account for the undecidability of our logic? For instance, working logicians in linguistics or computer science often have a gut feeling that the styles of reasoning they are analyzing are largely decidable (cf. the percentual estimate given in Bacon 1985, or the analysis of 'natural logic' in Sanchez Valencia 1991), but it is hard to give any mathematical underpinning to these working intuitions.

So, could there be well-motivated decidable versions of predicate logic, arising from giving up certain standard semantic prejudices? As it happens, there are even several such roads towards decidability. A traditional one would be to work only with restricted fragments of predicate logic (monadic, universal or otherwise, as is treated in many standard text books), while a very modern route is provided by recent linear logics of occurrences (Girard 1987, van Benthem 1991). But indeed, here too, a very general standard strategy exists for broadening our model class. There is a whole mathematical spectrum running from concrete set-theoretic models for predicate logic to abstract algebraic ones. In fact, this is precisely the domain of ***algebraic logic***, which has produced a good deal of information concerning these very issues: cf. Andréka 1991, Németi 1991, 1993, Venema 1991. In principle, this method will work whenever some modest minimal requirements are met by the underlying base logic. But of course, as with the above strategy of introducing general models, the interesting possibilities will lie somewhere in between. Now, just like the general model strategy, the algebraization strategy has been criticized in the literature for its ad-hoc-ness and lack of clear constraints. As an old saying goes, 'algebraic semantics is syntax in disguise'. But this is only true for the bottom level (where syntactic Lindenbaum algebras would do the job), whereas usually, algebras of independent interest emerge in abundance through further semantic considerations. Thus, a key concern in our present setting should be the search for interesting independently motivated 'semantic parameters' that can be set differently from the specific choices made in the standard Tarskian paradigm. But then, this search does not seem hopeless. For instance, on the more traditional ontological side, we have already seen that property theorists want to treat individuals and properties on a par as intensional entities, viewing the usual Tarskian 'set–tuple style' of treating predicate denotations as just one, extensional, option out of many.

Here is a more wide-ranging shift in current attitudes concerning semantic modelling. More 'procedurally', various intriguing new ideas have been put forward recently concerning first-order interpretation. In particular, one may view the 'cylindric modal algebra' of Venema 1991, and indeed cylindric algebra in general (Henkin-Monk-

Tarski 198) as a more fine-structured account of regimented access to successive variable assignments when interpreting quantified formulas, which may lead to natural decidable predicate logics (witness the contributions by Andréka & Németi in this volume). In this case, the more general models will carry some accessibility pattern on assignments, or more generally states for predicate logic, with existentially quantified statements $\exists x\phi$ referring to some new assignment verifying ϕ which should be x-accessible from the present one. Correspondence theory will then identify the procedural import of various predicate-logical principles over these abstract models, such as S5-laws for x-accessibility or more delicate 'path principles' (cf. van Benthem 1993). Judicious combinations of such conditions will then produce attractive decidable predicate logics, witness again the Andréka and Németi papers in this volume. Independent sources of intuitions concerning such transitions between aassignments exist already. One example is the 'dynamic predicate logic' of Groenendijk & Stokhof 1991, which views first-order formulas as explicit programs for effecting transitions between assignments when interpreting predicate logic. But also, in some recent semantics of generalized quantification, individual domains are allowed to carry an abstract 'accessibility' or 'dependence' structure, determining in which order individuals may become available in the course of interpreting an existence statement. Concrete motivations may be found in the theory of 'arbitrary objects' in Fine 1985, or the work on probabilistic independence relations in van Lambalgen 1991, as well as its 'modal first-order' version proposed in Alechina & van Benthem 1993. This gives a much broader space of models for the language of first-order predicate logic, with the original Tarskian ones becoming the special case where one has 'random access' to the full Cartesian space of all mathematically possible assignments of objects to variables.

As before, the advantage of this style of analysis is that one makes predicate logic into a more finely-tuned tool, with a decidable 'core' and a 'periphery' of more demanding principles. This is of purely logical interest, as it allows us to see new distinctions among what used to be one uniform batch of logical truths, that need not correspond to traditional classifications of propositions (say, using prenex forms or predicate arities). It is also of applied interest, in that we may try to see whether the richer picture fits the natural structure of the subject described. For instance, 'accessibility constraints' are natural from many viewpoints, mathematical, philosophical and linguistic. Moreover, the decidable subsystems of first-order logic that emerge in the resulting landscape of logical systems may fit in more naturally with proposed mechanisms of 'natural logic' in human reasoning (cf. Sommers 1982, Sanchez Valencia 1991).

Taking this viewpoint seriously is not a minor philosophical move, as it brings a lot of radical repercussions with it. For instance, in the broader perspective sketched here, what remains the point of Gödel's celebrated Completeness Theorem, stating a natural fit between Tarskian semantics and Fregean axiomatization? From our current stance, one would have to say that Gödel's result ties one particular natural choice of predicate-logical validity, which may be motivated independently from various angles (model-theoretic, proof-theoretic, game-theoretic) to 'standard set-theoretic modelling'. Kreisel 1967 and Etchemendy 1990 have stressed the surprise in this outcome, noting that it somehow manages to ensnare natural intuitive validity by means of exact mathematical notions. We would have to disagree here. From equally natural semantic points of view, other logical equilibria may arise, this time, having a decidable set of validities. What still remains is Gödel's lasting more abstract achievement of having put completeness issues on the map, and made them amenable to mathematical statement and proof.

Actually, we could go one step further here, as well as in the earlier cases. The very multiplicity of modellings for predicate logic suggests that there need not be one unique preferred choice – and that one should rather view all these options as living together. Having many modelings around in the literature for some particular phenomenon is not a nuisance or disease, to be eliminated by natural competition, in favour of one canonical solution. It may even be a definite virtue to be prized. For instance, it is quite possible to think of semantic modelling as an open-ended process, which allows ever finer levels of 'grain size'. As an example, one may think of a recent proposals by Zalta 1993, following Etchemendy, to split first-order semantics into two stages. The first goes from syntactic forms to abstract propositions (this is the domain of 'linguistic change'), and the second goes from these propositions to standard models (this is the domain of 'real change'). The burden of 'predicate logic' could then be divided between these two stages, with a possibly decidable component for the first stage, and an undecidable one for the second. Another example would be the grounding of decidable linear predicate logics in Lorenzen-style or Hintikka-style game-theoretical semantics for first-order logic (Mey 1992), or in intermediate Tarskian evaluation algorithms serving as Fregean senses (Moschovakis 1991). Of course, this does raise new issues of what may be called 'logical architecture' (van Benthem, to appear), as the interplay between the various components now becomes the explicit business of logic, too. Thus, traditional semantics and its rivals might coexist in one broader logical framework, whose task it would be to indicate precisely at which fault lines complexity emerges.

IV Digression: Illustrations in Arrow Logic

A more compact testing site for the above general considerations may be found in the 'Arrow Logic' which is so prominent in this volume. Here are a few illustrations of the above general themes at work in this more limited domain. One guiding motivation in the arrow logic research programme has been to challenge another well-known 'complexity threshold' in logics of computation, namely that undecidability is inevitable once a full relational algebra of programming operations is allowed. For, how much of this undecidability is due to the essential complexity of computation, and how much is merely a by-product of the mathematics of ordered pairs that lies underneath standard modelling in this area? As it happens, the resulting theory nicely demonstrates the above general points (cf. Venema's introductory chapter in this volume). First, a new sort of individual objects has been put on the map, namely transitions or 'arrows', which occur in most computational intuitions, and which arguably should be first-class citizens in any analysis of computation. Moreover, we are forced to reflect on the essential structure among these arrows, such as their composition, conversion or possibly other modes of combination (including parallel constructions of 'sheafs of arrows'). At least in the simplest similarity type, here, various decidable core calculi have been discovered, which can do quite a bit of the basic combinatorics of program combination. In particular, these calculi support all Boolean operators on top of relational composition, without tripping over any purported undecidability threshold. On top of that, correspondence analysis reveals the additional semantic content of further programming principles, distinguishing between purely universal ones (whose totality might form the largest desirable 'core theory') and more demanding existence principles. Outcomes have not been totally routine here, in that, e.g., associativity of composition has been identified as a danger point (cf. Simon et al. 1993), whereas most people would consider the latter an entirely harmless domestic assumption. Finally, this is not an isolated piece of semantic engineering, because relational transitions underly such a vast range of computational processes. Thus, arrow-style analysis is appropriate for relational algebra and propositional dynamic logic, but also for logics of belief revision or general process algebras. Indeed, the whole theory of labeled transition systems in computer science (cf. Stirling 1990, van Benthem & Bergstra 1993, van Benthem, van Eyck & Stebletsova 1993) is probably better viewed as a theory of states and arrows, with appropriate notions of two-sorted bisimulation for process equivalences. (One might even consider the introduction of computation 'paths' or 'branches' as a third kind of independent semantic object, thereby reflecting an earlier strand in our discussion.) Finally, there is no reason for staying inside the realm of computation, once our general points about modeling have been adopted. For instance, concrete geometrical arrows also reflect human thinking about their preferences, both scientific and domestic, and thus switching to arrow semantics might also throw some new light on preference logic (cf. the first exploration made in van Benthem, van Eyck & Frolovą 1993) and more generally, on various theories of individual and collective choice in the social sciences.

V General Issues

This paper has raised some general issues concerning semantic modelling, with a special interest in the locus of complexity. We have recommended a systematic search for alternative logical modelings, with the search for 'minimal complexity of wrappings' as a moving force toward new conceptual schemes. Moreover, we have noted some general features and benefits of this enterprise, as well as some of its philosophical repercussions. All this has by no means exhausted the topic of logical complexity. For instance, other important technical approaches exist in our field to high-lighting 'core content'. Among these, one may mention the use of 'oracles' supplying extraneous information to be bracketed out – as happens in Cook-style completeness theorems in programming semantics (Cook 1978) and in various parts of Complexity Theory (Buhrmann 1993). Such strategies remain to be compared with the present proposals. Another well-known way of coping with complexity is that of the 'moderate drinker': 'ingest only small amounts'. For instance, second-order complexity might be avoided by using only small fragments of the full second-order language, and the same holds for first-order logic. (By contrast, we have advocated retaining the full language as the expressive medium, while lowering the over-all complexity of its logic.) Finally, there are various statistical approaches to average-case complexity of logical reasoning that takes the sharpest edges off the usual worst-case problems. In this division of labour, the statistician takes care of the harsh realities of life, so that the logical theorist can live at peace. What these alternative strategies have in common is that they do not require rebuilding basic logical frameworks, but rather offer ways of accommodating ourselves to their complexity. Perhaps, then, our own proposals here have been unduly principled and calvinistic. Indeed, they might also run against the no-nonsense spirit of our times. In particular, they seem to clash with a recent trend in logical modelling which aims at replacing language-oriented semantic analysis by what is called 'direct mathematics'. The latter is illustrated by current developments in the Dutch branch of Process Algebra (cf. the program outlined in Baeten 1992), but also by earlier 'set-theoretical' programs in the Philosophy of Science (Suppes 1960, Sneed 1971), whose proponents want to minimize encounters with the common syntactic idiosyncracies of logical formalisms when describing their phenomena of interest. There are certainly good practical reasons for exploring such a line of investigation. Nevertheless, on the view of this paper, its guiding slogan may also be slightly misleading. 'Direct mathematics' with its seemingly innocent emphasis on concrete 'standard mathematical tools' carries many pre-wired decisions as to complexity, whereas our advice has been to take nothing for granted.

In all, this short essay has a very modest aim. We would be happy if our points were to contribute to a sustained probing discussion of received views in semantic modelling.

VI References

Abramsky, S., D. Gabbay & T. Maibaum, eds.

to appear *Handbook of Logic in Computer Science*, Oxford University Press.

Alechina, N. & J. van Benthem

1993 'Modal Quantification over Structured Domains', Report ML 93-02, Institute for Logic, Language and Computation, University of Amsterdam.

Andréka, H.

1991 *Complexity of Equations Valid in Algebras of Relations*, D. Sc. Thesis, Mathematical Institute, Hungarian Academy of Sciences, Budapest.

Bach, E.

1986 'Natural Language Metaphysics', in R. barcan Marcus et al., eds., 573-595.

Bacon, J.

1985 'The Completeness of a Predicate-Functor Logic', *The Journal of Symbolic Logic* 50, 903-926.

Baeten, J.

1992 'Informatica als Wetenschap (formele specificatie en wiskundige verificatie)', Inaugural Lecture, Department of Computer Science, Technological University, Eindhoven. (*TUE Informatie* 35 (7/8), 493-499, 1993.)

Bakker, J. de, W-P de Roever & G. Rozenberg, eds.

1989 *Linear Time, Branching Time and Partial Order in Logics and Models for Concurrency*, Springer, Berlin.

Barcan Marcus, R., G. Dorn & P. Weingartner, eds.

1986 *Proceedings 7th International Congress of Logic, Methodology and Philosophy of Science. Salzburg 1983*, North-Holland, Amsterdam.

Barwise, J

1979 'On Branching Quantifiers in English', *Journal of Philosophical Logic* 8, 47-80.

Bealer, G.

1982 *Quality and Concept*, Oxford University Press, Oxford.

Benthem, J. van

1983 *The Logic of Time*, Reidel, Dordrecht.

1984 'Correspondence Theory', in Gabbay & Guenthner, eds., 167-247.

1985 *Modal Logic and Classical Logic*, Bibliopolis, Napoli.

1986 *Essays in Logical Semantics*, Reidel, Dordrecht.

1989 'Polyadic Quantifiers', *Linguistics and Philosophy* 12:4, 437-464.

1991 *Language in Action. Categories, Lambdas and Dynamic Logic*, North-Holland, Amsterdam.

1993 'Modal State Semantics', manuscript, Institute for Logic, Language and Computation, University of Amsterdam.

to appear 'Logic and the Flow of Information', Report CSLI-91-160, Center for the Study of Language and Information, Stanford University. (To appear in D. Prawitz et al., eds.)

Benthem, J. van & J. Bergstra

1993 'Logic of Transition Systems', Report CT-93-03, Institute for Logic, Language and Computation, University of Amsterdam.

Benthem, J. van, J. van Eyck & A. Frolova

1993 'Changing Preferences', Report CS-93-10, Centre for Mathematics & Computer Science, Amsterdam.

Benthem, J. van, J. van Eyck & V. Stebletsova

1993 'Modal Logic, Transition Systems and Processes', Report CS-93-21, Centre for Mathematics and Computer Science, Amsterdam.

Benthem, J. van & A. ter Meulen, eds.

to appear *Handbook of Logic and Language*, Elsevier, Amsterdam.

Buhrmann, H.

1993 *Resouce Bounded Reductions*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Burgess, J.

1984 'Basic Tense Logic', in Gabbay & Guenthner, eds., 89-133.

Chierchia, G., B. Partee & R. Turner, eds.

1989 *Properties, Types and Meaning*, Reidel, Dordrecht.

Cook, S.

1978 'Soundness and Completeness of an Axiom System for Program Verification', *SIAM Journal of Computing* 7, 70-90.

Does, J. van der

1992 *Applied Quantifier Logics*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Does, J. van der & J. van Eyck, eds.,

1991 *Generalized Quantifier Theory and Applications*, Dutch Ph. D. Network for Language, Logic and Information, Amsterdam. (To appear: CSLI Lecture Notes, Chicago University Press.)

Enderton, H.

1972 *A Mathematical Introduction to Logic*, Academic Press, New York.

Etchemendy, J.

1990 *The Concept of Logical Consequence*, Harvard University Press, Cambridge (Mass.)

Fine, K.

1985 *Reasoning with Arbitrary Objects*, Blackwell, Oxford.

Gabbay, D. & F. Guenthner, eds.

1984 *Handbook of Philosophical Logic*, vol. II, Reidel, Dordrecht.

Girard, J-Y

1987 'Linear Logic', *Theoretical Computer Science* 50, 1-102.

Goldblatt, R.

1983 *Axiomatizing the Logic of Computer Programming*, Springer, Berlin.

1987 *Logics of Time and Computation*, CSLI Lecture Notes, Chicago University Press.

Groenendijk, J., Th. Janssen & M. Stokhof, eds.

1984 *Truth, Interpretation and Information*, Foris, Dordrecht.

Groenendijk, J. & M. Stokhof

1991 'Dynamic Predicate Logic', *Linguistics and Philosophy* 14, 39-100.

Gurevich, Y.

1985 'Logic and the Challenge of Computer Science', Report CRL-TR-10-85, Computer Research Laboratory, University of Michigan, Ann Arbor.

Harel, D.

1984 'Dynamic Logic', in Gabbay & Guenthner, eds., 497-604.

Henkin, L.

1950 'Completeness in the Theory of Types', *Journal of Symbolic Logic* 15, 81-91.

Henkin, L., D. Monk & A. Tarski

198 *Cylindric Algebra*, Parts I and II, North-Holland, Amsterdam.

Hintikka, J.

1979 'Quantifiers in Natural Languages: Some Logical Problems', in E. Saarinen, ed., 81-118.

Hoeksema, J.

1983 'Plurality and Conjunction', in Ter Meulen, ed., 63-83.

Kamp, H.

1984 'A Theory of Truth and Semantic Representation', in J. Groenendijk et al., eds., 1-41.

Keenan, E. & D. Westerståhl

1993 'Generalized Quantifiers', to appear in van Benthem & ter Meulen, eds.

Kneale, W. & M.

1961 *The Development of Logic*, Clarendon Press, Oxford.

Kreisel, G.

1967 'Informal Rigour and Completeness Proofs', in I. Lakatos, ed., 138-186.

Kugel, P.

1986 'Thinking May be More Than Computing', *Cognition* 22, 137-198.

Lakatos, I., ed.

1967 *Problems in the Philoophy of Mathematics*, North-Holland, Amsterdam.

Lambalgen, M. van

1991 'Natural Deduction for Generalized Quantifiers', in van Eyck & van der Does, eds., 143-154.

Landman, F.

1989 'Groups I & II', *Linguistics and Philosophy* 12, 559-605 and 723-744.

McCarthy, J.

1980 'Circumscription – A Form of Non-Monotonic Reasoning', *Artificial Intelligence* 13, 27-39.

Meulen, A. ter, ed.

1983 *Studies in Modeltheoretic Semantics*, Foris, Dordrecht.

Mey, D.

1992 'Game-Theoretical Interpretation of a Logic Without Contraction', Department of Computer Science, Swiss Federal Institute of Technology, Zürich.

Morreau, M.

1985 'Circumscription: A Sound and Complete Form of Non-Monotonic Reasoning', Technical Report, Mathematical Institute, University of Amsterdam.

1992 *Conditionals in Philosophy and Artificial Intelligence*, dissertation, Philosophical Institute, University of Amsterdam.

Moschovakis, Y.

1991 'Sense and Reference as Algorithm and Value', Department of Mathematics, University of California, Los Angeles.

Németi, I.

1991 'Algebraizations of Quantifier Logics: An introductory overview', Mathematical Institute, Hungarian Academy of Sciences, Budapest.

1993 'Decidability of Weakened Versions of First-Order Logic', Workshop on Algebraization of Logic, Fifth European Summer School in Logic, Language and Information, Lisbon.

Prawitz, D., B. Skyrms & D. Westerståhl, eds.

to appear *Proceedings 9th International Congress of Logic, Methodology and Philosophy of Science. Uppsala 1991*, Elseviers Science Publishers, Amsterdam.

Rodenburg, P.

1986 *Intuitionistic Correspondence Theory*, dissertation, Mathematical Institute, University of Amsterdam.

Rijke, M. de

1993 *Extending Modal Logic*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Saarinen, E., ed.

1979 *Game-Theoretical Semantics*, Reidel, Dordrecht.

Sain, I.

1978 'Observations on Abstract Model Theory', CL&CL 1978 XIII, 195-250.

Sanchez Valencia, V.

1991 *Studies on Natural Logic and Categorial Grammar*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Simon, A. , A. Kurucz, I. Németi & I. Sain

1993 'Undecidability Issues of Some Boolean Algebras with Operators and Logics Related to Lambek Calculus', Workshop on Algebraization of Logic, Fifth European Summer School in Logic, Language and Information, Lisbon.

Sneed, J.

1971 *The Logical Structure of Mathematical Physics*, Reidel, Dordrecht.

Sommers, F.

1982 *The Logic of Natural Language*, Cambridge University Press, Cambridge.

Spaan, E.

1993 *Complexity of Modal Logics*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Stirling, C.

1990 'Modal and Temporal Logics', to appear in Abramsky, Gabbay & Maibaum, eds.

Suppes, P.

1960 'A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences', *Synthese* 12, 287-301.

Troelstra, A. & D. van Dalen

1988 *Constructivism in Mathematics*, North-Holland, Amsterdam.

Turner, R.

1989 'Two Issues in the Foundations of Semantics', in G. Chierchia et al., eds., 63-84.

Veltman, F.

1992 'Defaults in Update Semantics', Report LP-91-02, Institute for Logic, Language and Computation, University of Amsterdam. (*Journal of Philosophical Logic*, to appear.)

Venema, Y.

1992 *Many-Dimensional Modal Logic*, dissertation, Institute for Logic, Language and Computation, University of Amsterdam.

Zalta, E.

1988 *Intensional Logic and the Metaphysics of Intentionality*, MIT Press, Cambridge (Mass.)

1993 'A Philosophical Conception of Propositional Modal Logic', Center for the Study of Language and Information, Stanford University.

ILLC Research Reports and Technical Notes

Coding for Research Reports: *Series-Year-Number*, with LP = Linguistics and Philosophy of Language; ML = Mathematical Logic and Foundations; CL = Compuational Linguistics; CT = Computation and Complexity Theory; X = Technical Notes.
All previous ILLC-publications are available from the ILLC bureau. For prepublications before 1990, contact the bureau.

LP-90-01 *Jaap van der Does, A Generalized Quantifier Logic for Naked Infinitives*
LP-90-02 *Jeroen Groenendijk, Martin Stokhof, Dynamic Montague Grammar*
LP-90-03 *Renate Bartsch, Concept Formation and Concept Composition*
LP-90-04 *Aarne Ranta, Intuitionistic Categorial Grammar*
LP-90-05 *Patrick Blackburn, Nominal Tense Logic*
LP-90-06 *Gennaro Chierchia, The Variablity of Impersonal Subjects*
LP-90-07 *Gennaro Chierchia, Anaphora and Dynamic Logic*
LP-90-08 *Herman Hendriks, Flexible Montague Grammar*
LP-90-09 *Paul Dekker, The Scope of Negation in Discourse, towards a Flexible Dynamic Montague grammar*
LP-90-10 *Theo M.V. Janssen, Models for Discourse Markers*
LP-90-11 *Johan van Benthem, General Dynamics*
LP-90-12 *Serge Lapierre, A Functional Partial Semantics for Intensional Logic*
LP-90-13 *Zhisheng Huang, Logics for Belief Dependence*
LP-90-14 *Jeroen Groenendijk, Martin Stokhof, Two Theories of Dynamic Semantics*
LP-90-15 *Maarten de Rijke, The Modal Logic of Inequality*
LP-90-16 *Zhisheng Huang, Karen Kwast, Awareness, Negation and Logical Omniscience*
LP-90-17 *Paul Dekker, Existential Disclosure, Implicit Arguments in Dynamic Semantics*

ML-90-01 *Harold Schellinx, Isomorphisms and Non-Isomorphisms of Graph Models*
ML-90-02 *Jaap van Oosten, A Semantical Proof of De Jongh's Theorem*
ML-90-03 *Yde Venema, Relational Games*
ML-90-04 *Maarten de Rijke, Unary Interpretability Logic*
ML-90-05 *Domenico Zambella, Sequences with Simple Initial Segments*
ML-90-06 *Jaap van Oosten, Extension of Lifschitz' Realizability to Higher Order Arithmetic, and a Solution to a Problem of F. Richman*
ML-90-07 *Maarten de Rijke, A Note on the Interpretability Logic of Finitely Axiomatized Theories*
ML-90-08 *Harold Schellinx, Some Syntactical Observations on Linear Logic*
ML-90-09 *Dick de Jongh, Duccio Pianigiani, Solution of a Problem of David Guaspari*
ML-90-10 *Michiel van Lambalgen, Randomness in Set Theory*
ML-90-11 *Paul C. Gilmore, The Consistency of an Extended NaDSet*

CT-90-01 *John Tromp, Peter van Emde Boas, Associative Storage Modification Machines*
CT-90-02 *Sieger van Denneheuvel, Gerard R. Renardel de Lavalette, A Normal Form for PCSJ Expressions*
CT-90-03 *Ricard Gavaldà, Leen Torenvliet, Osamu Watanabe, José L. Balcázar, Generalized Kolmogorov Complexity in Relativized Separations*
CT-90-0 *Harry Buhrman, Edith Spaan, Leen Torenvliet, Bounded Reductions*
CT-90-05 *Sieger van Denneheuvel, Karen Kwast, Efficient Normalization of Database and Constraint Expressions*
CT-90-06 *Michiel Smid, Peter van Emde Boas, Dynamic Data Structures on Multiple Storage Media, a Tutorial*
CT-90-07 *Kees Doets, Greatest Fixed Points of Logic Programs*
CT-90-08 *Fred de Geus, Ernest Rotterdam, Sieger van Denneheuvel, Peter van Emde Boas, Physiological Modelling using RL*
CT-90-09 *Roel de Vrijer , Unique Normal Forms for Combinatory Logic with Parallel Conditional, a case study in Conditional Rewriting*

X-90-01 *A.S. Troelstra , Remarks on Intuitionism and the Philosophy of Mathematics, Revised Version*
X-90-02 *Maarten de Rijke, Some Chapters on Interpretability Logic*
X-90-03 *L.D. Beklemishev, On the Complexity of Arithmetical Interpretations of Modal Formulae*
X-90-04 *Annual Report 1989,*
X-90-05 *Valentin Shehtman, Derived Sets in Euclidean Spaces and Modal Logic*
X-90-06 *Valentin Goranko, Solomon Passy, Using the Universal Modality: Gains and Questions*
X-90-07 *V.Yu. Shavrukov, The Lindenbaum Fixed Point Algebra is Undecidable*
X-90-08 *L.D. Beklemishev, Provability Logics for Natural Turing Progressions of Arithmetical Theories*
X-90-09 *V.Yu. Shavrukov, On Rosser's Provability Predicate*
X-90-10 *Sieger van Denneheuvel, Peter van Emde Boas, An Overview of the Rule Language RL/1*
X-90-11 *Alessandra Carbone, Provable Fixed points in ID0+W1, revised version*
X-90-12 *Maarten de Rijke, Bi-Unary Interpretability Logic*
X-90-13 *K.N. Ignatiev , Property, Craig's Property*
X-90-14 *L.A. Chagrova, Undecidable Problems in Correspondence Theory*
X-90-15 *A.S. Troelstra, Lectures on Linear Logic*

LP-91-01 *Wiebe van der Hoek, Maarten de Rijke, Generalized Quantifiers and Modal Logic*
LP-91-02 *Frank Veltman, Defaults in Update Semantics*
LP-91-03 *Willem Groeneveld, Dynamic Semantics and Circular Propositions*
LP-91-04 *Makoto Kanazawa, The Lambek Calculus enriched with Additional Connectives*
LP-91-05 *Zhisheng Huang, Peter van Emde Boas, The Schoenmakers Paradox: Its Solution in a Belief Dependence Framework*

LP-91-06 *Zhisheng Huang, Peter van Emde Boas, Belief Dependence, Revision and Persistence*
LP-91-07 *Henk Verkuyl, Jaap van der Does, The Semantics of Plural Noun Phrases*
LP-91-08 *Víctor Sánchez Valencia, Categorial Grammar and Natural Reasoning*
LP-91-09 *Arthur Nieuwendijk, Semantics and Comparative Logic*
LP-91-10 *Johan van Benthem, Logic and the Flow of Information*

ML-91-01 *Yde Venema , Cylindric Modal Logic*
ML-91-02 *Alessandro Berarducci, Rineke Verbrugge, On the Metamathematics of Weak Theories*
ML-91-03 *Domenico Zambella, On the Proofs of Arithmetical Completeness for Interpretability Logic*
ML-91-04 *Raymond Hoofman, Harold Schellinx, Collapsing Graph Models by Preorders*
ML-91-05 *A.S. Troelstra, History of Constructivism in the Twentieth Century*
ML-91-06 *Inge Bethke, Finite Type Structures within Combinatory Algebras*
ML-91-07 *Yde Venema, Modal Derivation Rules*
ML-91-08 *Inge Bethke, Going Stable in Graph Models*
ML-91-09 *V.Yu. Shavrukov, A Note on the Diagonalizable Algebras of PA and ZF*
ML-91-10 *Maarten de Rijke, Yde Venema, Sahlqvist's Theorem for Boolean Algebras with Operators*
ML-91-11 *Rineke Verbrugge, Feasible Interpretability*
ML-91-12 *Johan van Benthem, Modal Frame Classes, revisited*

CT-91-01 *Ming Li, Paul M.B. Vitányi , Kolmogorov Complexity Arguments in Combinatorics*
CT-91-02 *Ming Li, John Tromp, Paul M.B. Vitányi, How to Share Concurrent Wait-Free Variables*
CT-91-03 *Ming Li, Paul M.B. Vitányi, Average Case Complexity under the Universal Distribution Equals Worst Case Complexity*
CT-91-04 *Sieger van Denneheuvel, Karen Kwast, Weak Equivalence*
CT-91-05 *Sieger van Denneheuvel, Karen Kwast, Weak Equivalence for Constraint Sets*
CT-91-06 *Edith Spaan, Census Techniques on Relativized Space Classes*
CT-91-07 *Karen L. Kwast, The Incomplete Database*
CT-91-08 *Kees Doets, Levationis Laus*
CT-91-09 *Ming Li, Paul M.B. Vitányi, Combinatorial Properties of Finite Sequences with high Kolmogorov Complexity*
CT-91-10 *John Tromp, Paul Vitányi, A Randomized Algorithm for Two-Process Wait-Free Test-and-Set*
CT-91-11 *Lane A. Hemachandra, Edith Spaan, Quasi-Injective Reductions*
CT-91-12 *Krzysztof R. Apt, Dino Pedreschi, Reasoning about Termination of Prolog Programs*
CL-91-01 *J.C. Scholtes, Kohonen Feature Maps in Natural Language Processing*
CL-91-02 *J.C. Scholtes, Neural Nets and their Relevance for Information Retrieval*

CL-91-03 *Hub Prüst, Remko Scha, Martin van den Berg, A Formal Discourse Grammar tackling Verb Phrase Anaphora*

X-91-01 *Alexander Chagrov, Michael Zakharyaschev, The Disjunction Property of Intermediate Propositional Logics*
X-91-02 *Alexander Chagrov, Michael Zakharyaschev , On the Undecidability of the Disjunction Property of Intermediate Propositional Logics*
X-91-03 *V. Yu. Shavrukov, Subalgebras of Diagonalizable Algebras of Theories containing Arithmetic*
X-91-04 *K.N. Ignatiev, Partial Conservativity and Modal Logics*
X-91-05 *Johan van Benthem, Temporal Logic*
X-91-06 *Annual Report 1990,*
X-91-07 *A.S. Troelstra, Lectures on Linear Logic, Errata and Supplement*
X-91-08 *Giorgie Dzhaparidze, Logic of Tolerance*
X-91-09 *L.D. Beklemishev, On Bimodal Provability Logics for P1-axiomatized Extensions of Arithmetical Theories*
X-91-10 *Michiel van Lambalgen, Independence, Randomness and the Axiom of Choice*
X-91-11 *Michael Zakharyaschev, Canonical Formulas for K4. Part I: Basic Results*
X-91-12 *Herman Hendriks, Flexibele Categoriale Syntaxis en Semantiek: de proefschriften van Frans Zwarts en Michael Moortgat*
X-91-13 *Max I. Kanovich, The Multiplicative Fragment of Linear Logic is NP-Complete*
X-91-14 *Max I. Kanovich, The Horn Fragment of Linear Logic is NP-Complete*
X-91-15 *V. Yu. Shavrukov, Subalgebras of Diagonalizable Algebras of Theories containing Arithmetic, revised version*
X-91-16 *V.G. Kanovei, Undecidable Hypotheses in Edward Nelson's Internal Set Theory*
X-91-17 *Michiel van Lambalgen, Independence, Randomness and the Axiom of Choice, Revised Version*
X-91-18 *Giovanna Cepparello, New Semantics for Predicate Modal Logic: an Analysis from a standard point of view*
X-91-19 *, Papers presented at the Provability Interpretability Arithmetic Conference, 24-31 Aug. 1991, Dept. of Phil., Utrecht University*

LP-92-01 *Víctor Sánchez Valencia, Lambek Grammar: an Information-based Categorial Grammar*
LP-92-02 *Patrick Blackburn, Modal Logic and Attribute Value Structures*
LP-92-03 *Szabolcs Mikulás, The Completeness of the Lambek Calculus with respect to Relational Semantics*
LP-92-04 *Paul Dekker, An Update Semantics for Dynamic Predicate Logic*
LP-92-05 *David I. Beaver, The Kinematics of Presupposition*
LP-92-06 *Patrick Blackburn, Edith Spaan, A Modal Perspective on the Computational Complexity of Attribute Value Grammar*

LP-92-07 *Jeroen Groenendijk, Martin Stokhof, A Note on Interrogatives and Adverbs of Quantification*

LP-92-08 *Maarten de Rijke, A System of Dynamic Modal Logic*

LP-92-09 *Johan van Benthem, Quantifiers in the world of Types*

LP-92-10 *Maarten de Rijke , Meeting Some Neighbours (a dynamic modal logic meets theories of change and knowledge representation)*

LP-92-11 *Johan van Benthem, A Note on Dynamic Arrow Logic*

LP-92-12 *Heinrich Wansing, Sequent Caluli for Normal Modal Propositional Logics*

LP-92-13 *Dag Westerstæhl, Iterated Quantifiers*

LP-92-14 *Jeroen Groenendijk, Martin Stokhof, Interrogatives and Adverbs of Quantification*

ML-92-01 *A.S. Troelstra, Comparing the Theory of Representations and Constructive Mathematics*

ML-92-02 *Dmitrij P. Skvortsov, Valentin B. Shehtman, Maximal Kripke-type Semantics for Modal and Superintuitionistic Predicate Logics*

ML-92-03 *Zoran Markovi,c, On the Structure of Kripke Models of Heyting Arithmetic*

ML-92-04 *Dimiter Vakarelov, A Modal Theory of Arrows, Arrow Logics I*

ML-92-05 *Domenico Zambella, Shavrukovs Theorem on the Subalgebras of Diagonalizable Algebras for Theories containing IDO + EXP*

ML-92-06 *D.M. Gabbay, Valentin B. Shehtman, Undecidability of Modal and Intermediate First-Order Logics with Two Individual Variables*

ML-92-07 *Harold Schellinx, How to Broaden your Horizon*

ML-92-08 *Raymond Hoofman, Information Systems as Coalgebras*

ML-92-09 *A.S. Troelstra, Realizability*

ML-92-10 *V.Yu. Shavrukov, A Smart Child of Peanos*

CT-92-01 *Erik de Haas, Peter van Emde Boas, Object Oriented Application Flow Graphs and their Semantics*

CT-92-02 *Karen L. Kwast, Sieger van Denneheuvel, Weak Equivalence: Theory and Applications*

CT-92-03 *Krzysztof R. Apt, Kees Doets, A new Definition of SLDNF-resolution*

X-92-01 *Heinrich Wansing, The Logic of Information Structures*

X-92-02 *Konstantin N. Ignatiev, The Closed Fragment of Dzhaparidze's Polymodal Logic and the Logic of S1 conservativity*

X-92-03 *Willem Groeneveld, Dynamic Semantics and Circular Propositions, revised version*

X-92-04 *Johan van Benthem, Modeling the Kinematics of Meaning*

X-92-05 *Erik de Haas, Peter van Emde Boas, Object Oriented Application Flow Graphs and their Semantics, revised version*

LP-93-01 *Martijn Spaan, Parallel Quantification*

LP-93-02 *Makoto Kanazawa, Dynamic Generalized Quantifiers and Monotonicity*

LP-93-03 *Nikolai Pankrat'ev, Completeness of the Lambek Calculus with respect to Relativized Relational Semantics*

LP-93-04 *Jacques van Leeuwen, Identity, Quarrelling with an Unproblematic Notion*

LP-93-05 *Jaap van der Does, Sums and Quantifiers*

LP-93-06 *Paul Dekker, Updates in Dynamic Semantics*

LP-93-07 *Wojciech Buszkowski, On the Equivalence of Lambek Categorial Grammars and Basic Categorial Grammars*

LP-93-08 *Zisheng Huang, Peter van Emde Boas, Information Acquisition from Multi-Agent resources; abstract*

LP-93-09 *Makoto Kanazawa, Completeness and Decidability of the Mixed Style of Inference with Composition*

LP-93-10 *Makoto Kanazawa, Weak vs. Strong Readings of Donkey Sentences and Monotonicity Inference in a Dynamic Setting*

LP-93-11 *Friederike Moltmann, Resumptive Quantifiers in Exception Sentences*

LP-93-12 *Jaap van der Does, On Complex Plural Noun Phrases*

LP-93-13 *Natasha Alechina, Binary Quantifiers and Relational Semantics*

LP-93-14 *Mati Pentus, Lambek Calculus is L-complete*

LP-93-15 *David Ian Beaver, What comes first in Dynamic Semantics*

ML-93-01 *Maciej Kandulski, Commutative Lambek Categorial Grammars*

ML-93-02 *Johan van Benthem, Natasha Alechina, Modal Quantification over Structured Domains*

ML-93-03 *Mati Pentus, The Conjoinablity Relation in Lambek Calculus and Linear Logic*

ML-93-04 *Andreja Prijatelj, Bounded Contraction and Many-Valued Semantics*

ML-93-05 *Raymond Hoofman, Harold Schellinx, Models of the Untyped l-calculus in Semi Cartesian Closed Categories*

ML-93-06 *J. Zashev, Categorial Generalization of Algebraic Recursion Theory*

ML-93-07 *A.V. Chagrov, L.A. Chagrova, Algorithmic Problems Concerning First-Order Definability of Modal Formulas on the Class of All Finite Frames*

ML-93-08 *Raymond Hoofman, Ieke Moerdijk, Remarks on the Theory of Semi-Functors*

ML-93-09 *A.S. Troelstra, Natural Deduction for Intuitionistic Linear Logic*

ML-93-10 *Vincent Danos, Jean-Baptiste Joinet, Harold Schellinx, The Structure of Exponentials: Uncovering the Dynamics of Linear Logic Proofs*

ML-93-11 *Lex Hendriks, Inventory of Fragments and Exact Models in Intuitionistic Propositional Logic*

ML-93-12 *V.Yu. Shavrukov, Remarks on Uniformly Finitely Precomplete Positive Equivalences*

ML-93-13 *V.Yu. Shavrukov, Undecidability in Diagonizable Algebras*

ML-93-14 *Dick de Jongh, Albert Visser, Embeddings of Heyting Algebras*

ML-93-15 *G.K. Dzhaparidze, Effective Truth*

ML-93-16 *Maarten de Rijke, Correspondence Theory for Extended Modal Logics*

ML-93-17 *Alexander Chagrov, Michael Zakharyaschev, On the Independent Axiomatizability of Modal and Intermediate Logics*
ML-93-18 *Jaap van Oosten, Extensional Realizability*
ML-93-19 *Raymond Hoofman, Comparing Models of the Non-Extensional Typed l-Calculus*
ML-93-20 *L.A. Chagrova, Dick de Jongh, The Decidability of Dependency in Intuitionistic Propositional Logic*
ML-93-21 *Max I. Kanovich, The Relational Knowledge-Base Interpretation and Feasible Theorem Proving for Intuitionistic Propositional Logic*
ML-93-22 *Andreja Prijatelj, Connectification for n-contraction*

CT-93-01 *Marianne Kalsbeek, The Vanilla Meta-Interpreter for Definite Logic Programs and Ambivalent Syntax*
CT-93-02 *Sophie Fischer, A Note on the Complexity of Local Search Problems*
CT-93-03 *Johan van Benthem, Jan Bergstra, Logic of Transition Systems*
CT-93-04 *Karen L. Kwast, Sieger van Denneheuvel, The Meaning of Duplicates in the Relational Database Model*
CT-93-05 *Erik Aarts, Proving Theorems of the Lambek Calculus of Order 2 in Polynomial Time*
CT-93-06 *Krzysztof R. Apt, Declarative programming in Prolog*
CT-93-07 *Janusz A. Pomykala, Approximation, Similarity and Rough Constructions, Part I. Elementary Introduction*
CL-93-01 *Noor van Leusen, László Kálmán, Computaional Linguistics*
CL-93-02 *Theo M.V. Janssen, An Algebraic View On Rosetta*
CL-93-03 *Patrick Blackburn, Claire Gardent, Wilfried Meyer-Viol, Talking about Trees*

X-93-01 *Paul Dekker, Existential Disclosure, revised version*
X-93-02 *Maarten de Rijke, What is Modal Logic?*
X-93-03 *Michiel Leezenberg, Gorani Influence on Central Kurdish: Substratum or Prestige Borrowing*
X-93-04 *A.S. Troelstra (editor), Metamathematical Investigation of Intuitionistic Arithmetic and Analysis, Corrections to the First Edition*
X-93-05 *A.S. Troelstra (editor), Metamathematical Investigation of Intuitionistic Arithmetic and Analysis, Second, corrected Edition*
X-93-06 *Michael Zakharyashev, Canonical Formulas for K4. Part II: Cofinal Subframe Logics*

ML-94-01 *Domenico Zambella, Notes on polynomially bounded arithmetic*
LP-94-01 *Dimitar Gelev, Introducing Some Classical Elements of Modal Logic to the Propositional Logics of Qualitative Probabilities*
CT-94-01 *Harry Buhrman and Leen Torenvliet, On the Cutting Edge of Relativization: the Resource Bounded Injury Method*
X-94-01 *Johan van Benthem, Two Essays on Semantic Modelling*

Previous titles in the ILLC Dissertation Series:

1993-1 *Transsentential Meditations; Ups and downs in dynamic semantics*, Paul Dekker
1993-2 *Resource Bounded Reductions*, Harry Buhrman
1993-3 *Efficient Metamathematics*, Rineke Verbrugge
1993-4 *Extending Modal Logic*, Maarten de Rijke
1993-5 *Studied Flexibility*, Herman Hendriks
1993-6 *Aspects of Algorithms and Complexity*, John Tromp
1994-1 *The Noble Art of Linear Decorating*, Harold Schellinx
1994-2 *Generating Uniform User-Interfaces for Interactive Programming Environments*, Jan Willem Cornelis Koorn
1994-3 *Process Theory and Equation Solving*, Nicoline Johanna Drost

Universiteit
van
Amsterdam
Nwe Achtergracht 166
1018 WV
Amsterdam
tel.: 5256033

B
O
HUISDRUKKERIJ

OPDRACHTGEVER : v.d. Linden

FACULTEIT : FWI

VAKGROEP : VGII/Ille

FASNUMMER* : 4-15521

TELEFOONNUMMER : 6090

Datum: 7-06-1994

(bon a.u.b. in 3 voud in blokletters invullen)

Omschrijving: RAPPORTEN

Aankruisen wat wordt verlangd

Nr		Aantal orig	Oplage sets	Enkelz	Dubbelz	Omslag (kleur)	Binden ....mm	Nieten 1 - 2	Vouwen sets	Tot.aant druks	Bedrag
1.		33	36		X			X			
2.		15	60		X			X			
3.		33	78		X			X			
4.											

andere wensen: kopie ...

bijgeleverd:

handtekening gemachtigde:

[redacted]

gewenste leverdatum: -- -- . uur

kaft voor ☒ ☒ kaft achter

vergaren: ☐ ja ☐ nee

vertrouwelijk ☐ ja ☐ nee

boren ☐ snijden ☐ lijmen ☐

stempel:

(Dit gedeelte niet beschrijven)

platen	*f*	....
kopieen	*f.*	....
1.2.	*f.*	....
3.4.	*f.*	....
afwerking	*f.*	....
	f.	....
Totaal.	*f.*	....

geboekt:
handtekening voor ontvangst.

U gelieve bovengenoemd bedrag over te maken op girorekening 5032162 t.n.v. Beheersorganisatie Roeterseiland, Centrale Repro, Nieuwe Achtergracht 166. onder vermelding van FACTUUR NR. en t.g.v. 870-721