Solutions to Sorites

Minithesis

written by

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This essay describes and compares several solutions of the Sorites Paradox, and the problem of vagueness, generally. The essay focuses on Supervaluationism and Degree theories, two non-standard formal semantic theories. Two other solutions are canvassed to show that a non-standard semantics seems the most appealing way of responding to the paradox.

Introduction

A predicate is vague if it could have borderline cases. A predicate has borderline cases if there are objects such that no possible method of enquiry could determine whether the predicate applied to those objects or not. It is not necessary that a predicate have borderline cases to be vague, predicates like 'bald' would be vague even in a world with only hairless people. So a predicate can have a determinate extension and still be vague, so if vagueness is a semantic property of a predicate then it is property of the intension of the predicate. If vagueness is not a property of our semantics, or not only so, then if vagueness is not merely our epistemic inability to decide borderline cases then vagueness is a property of things in the world.

A predicate could have borderline cases of borderline cases and hence the predicate 'vague' is vague. So there is higher-order vagueness.

A seeming further feature of vague predicates is that of tolerance: a vague predicate p is tolerant if there is an irreflexive, transitive relation R relevant to whether p applies such that if xRy then p applies to x if and only if p applies to y. For example, 'tall' (applied to human beings) seems a vague, tolerant predicate as the relation 'taller by 1 millimetre' is such that if x differs in height from y by one millimetre then x is tall if and only if y is tall. Not all vague predicates appear to be tolerant even vague predicates that form comparatives, as 'nice' is vague but there does not seem to be any relevant relation R.

Sorites Paradox

It is when a vague predicate appears to be tolerant that it is subject to a Sorites-type argument (and hence non-tolerant vague predicates appear not to be subject to Sorites). Small differences in relevant respects don't make a difference to the application of the predicate, but small differences add up to large differences that do affect the application of the predicate. The most famous form of the Sorites Paradox is the following:

1. One grain of sand is not a heap.

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0 Another promising solution is Contextualism. The formal accounts I have seen give non-standard semantics. (see Hans Kamp 'The Paradox of the Heap' 1981 Aspects of Philosophical Logic ed. U. Monnich)

2. For all n, if n grains of sand is not a heap then n+1 grains of sand is not a heap.

3. Therefore, 1 million grains of sand is not a heap.

The conclusion follows from 1 and 2 by universal instantiation and repeated application of *modus ponens*. The conclusion contradicts our belief about what 'heap' applies to, but we also accept the premises. The aim of this essay is to evaluate various attempts to defuse this paradox.

There are a number of ways in which the paradox can be defused.

a. Demonstrate that not all the premises are true.

b. Reject the validity of the argument.

c. Admit that the Sorites is sound, and hence accept the conclusion.

d. Claim that the semantics of vague predicates is inconsistent.

There are two immediate non-starters. Firstly, one might make a type (a.) response denying that mathematical induction applies to vague predicates. But the paradox can be generated without the use of induction:

1. One grain of sand is not a heap.

2. If one grain of sand is not a heap then two grains of sand is not a heap.

...

N. If n grains of sand is not a heap then $n+1$ grains of sand is not a heap.

Therefore, one million of grains of sand is not a heap.

The argument would have a million premises, and the conclusion follows by modus ponens and cut.

Secondly, any type (c.) response that is not also a type (d.) response seems hopeless. For if one accepts that nothing is a heap (which follows by induction) then symmetrically one could argue that for any n, n grains of sand is a heap: for with the premise that one million grains of sand is a heap (which is as obvious as that one grain of sand is not a heap) and that if n is more than a million then n grains of sand is a heap and the premise that if n grains of sand is a heap then n-1 grains of sand is a heap (which is a form of tolerance) then for any n, n grains of sand is a heap. Hence, contradiction. But one was assuming that 'heap' has a consistent semantics.

This conclusion can be accepted however, if the type (c.) response is wedded with a type (d.) response; then it follows that vague, tolerant predicates have an inconsistent semantics, this response to the paradox is called Nihilism. We will discuss this shortly.

Constraints on a resolution of the paradox

Any of the responses (a.) to (d.) must meet certain constraints to be a plausible solution to the Sorites Paradox.

(1*) Fidelity: A solution must preserve the semantics and logical relations of sentences that do not contain vague expressions.

(2*) Stability: Any semantics should preserve the truth-values of vague sentences that are obviously true or false.

(2a*) Penumbral connections: As a consequence of Stability any semantics for vague sentences must preserve the logical relations between atomic sentences of the form '*a* is *F*' (call such sentences hard cases) where *F* is a vague, seemingly tolerant predicate and *a* is a borderline case of *F*. For instance, by Tolerance if *a* and *b* are borderline tall men, then if *a* is one millimetre taller than *b* then if *b* is tall then *a* is tall.

More controversial is the question whether all truths of first-order logic hold when the atomic sentences are hard cases. For instance, whether 'a is F or a is not F' is true for hard cases. LEM can be argued for colour predicates thus: if A is a borderline case of an orange and red object, then as every object has a colour, then A is orange or red is definitely true, but if A is red then A is not orange, hence A is orange or not orange.

LEM can be argued for arbitrary vague predicates by generalising the above argument for colours. If a is a borderline case of an F then there is at least one property G such that a is a borderline case of G and such that a cannot be F and G. But there is a class H of predicates (in the case of colours, H is the set of colour predicates) such that F and G are predicates in the class H, and there is a predicate in H such that the predicate applies to a. So a is F or a is G is definitely true, but if a is G then a is not F, hence a is F or a is not F is definitely true.

If a solution to the Sorites Paradox rejects some of the penumbral connections there must be a plausible argument why some connections are respected and others are not.

(3*) An account of what is rejected: As was just said, some of the aforementioned requirements may be rejected by an account of Sorites but to be plausible there must be some explanation of why what was rejected was thought to be obvious, otherwise the theory will seem unmotivated. For instance, the property of Tolerance may be rejected for vague predicates, but as it may be argued this is the convincing feature of vagueness that generates the Sorites, any unmotivated rejection of it will make the account implausible.

(4*) Reflective Equilibrium: Possibly no correct account of the paradox can meet all the requirements aforementioned. So there must be a trade-off where some requirements might be rejected in favour of others, in which case things like the simplicity and economy of an account might be deciding factors on which account seems best. So a correct account should successfully modify the theory to our intuitions (i.e. requirements) and modify our intuitions when economy and simplicity are deciding factors in choosing the correct theory.

Some Common Responses to the Sorites Paradox

The essay will focus on the responses to the paradox that involve degree theories or Supervaluationist semantics. It is necessary to explicate and criticise two other rival proposals to the paradox: Epistemicism and Nihilism. As if Nihilism is not correct then a type (d.) response is ruled out, and so the correct response to the Sorites Paradox is either of type (a.) or type (b.). If this is correct then, if Epistemicism is not correct, the correct solution must involve a non-standard logic or semantics. The most promising of which are Supervaluationism and a degree theory.[0]

It is also helpful in seeing if there are any common, cogent objections to a number of different responses to the paradox. If there are common objections this might help to give an idea of what a correct solution to the paradox and a correct account of vagueness might look like, if none of the discussed responses work.

Nihilism, as mentioned earlier, is the thesis that vague predicates have an inconsistent semantics, as demonstrated by the Sorites paradox. As the semantics of classical first-order semantics is consistent, by the Soundness theorem, then the semantics of vague sentences is not classical. It is difficult to see however how any formal semantic model could lead to an inconsistency . To make this plausible Dummett has suggested that the semantics of vague expressions are rules that determine the use of the expressions. As in certain games, it is possible that rules can prescribe contradictory actions in certain circumstances. So with vague predicates in Sorites-style arguments, the semantics of 'heap' prescribes that the base and inductive premise are true, and hence that the conclusion is true, but also that the conclusion is false.

However, if we discover a contradiction in the rules of a game it usually means we change, or abandon the game, the game loses its point or interest otherwise. Why is this not the case with vague predicates, if Nihilism is correct? One could argue that in normal circumstances of use the semantic rules of vague predicates do not conflict, hence the predicates are still useful. This still does not explain why nearly all people confronted with the Sorites Paradox are more inclined to reject the reasoning or one of the premises than accept the conclusion. As if Nihilism is correct then the semantics of 'heap' licenses the conclusion of the argument as much as the premises. A Nihilist could respond that people are more inclined to reject the conclusion because it is false in all circumstances of use, but each instantiation of the inductive premise is true in normal circumstances. But one would think the inclination to find the conclusion more counter-intuitive should disappear once one is given the Nihilist's explanation, but it seems it does not. So Nihilism seems to fail requirement three, as it cannot explain why the conclusion of the paradox is more counter-intuitive than the premises or the reasoning.

A tempting and common response to the paradox and the phenomena of vagueness generally is Epistemicism. Epistemicism is the thesis that classical logic and classical semantics are correct for vague sentences, hence a vague predicate p has borderline cases only in the sense that there exist objects such that no human enquiry could determine whether the predicate applied to those objects, but for any object x, it is a true that either p applies to x or not p applies to x. So for Epistemicism vagueness is a form of ignorance.

It follows that Epistemicism is a type (a.) response as the inductive premise of the Sorites Paradox is false, as there is an n such that n grains of sand is not a heap and n+1 grains of sand is a heap; but we cannot know what n. It also follows that no non-empty vague predicate is tolerant, as for any vague predicate p, and any irreflexive, transitive relation R relevant to whether p applies there is some world w such that in w there are x and y such that xRy and p applies to x but p does not apply to y. For instance, there may be a tall man x, and a man y such that x is one millimetre taller than y, but y is not a tall man.

In order to meet requirement (3*) an Epistemicist must explain why people believed that some vague predicates are tolerant, and premise two of Sorites. One might argue that in people's minds Tolerance is confused with the property of Epistemic Tolerance: a vague predicate p is epistemically tolerant if there is an irreflexive, transitive relation R relevant to whether p applies such that if xRy then if it is known that x is p then y is p and if it is known that y is p then x is p. For instance, as 'tall' (applied to people) is epistemically tolerant then one cannot know a conjunction of the form 'x is tall and y is not tall' where x differs in height from y by one millimetre because otherwise one must know that x is tall but then by epistemic tolerance (where 'R' is 'taller by one millimetre') y is tall, hence no such conjunction can be known. So people believe that such conjunctions cannot be true (i.e. that premise two of the Sorites cannot be false) as they cannot be known, but given epistemic tolerance, such ignorance is explicable. Hence the attractions of the paradox have an explanation on this account.

A argument for Epistemicism is that if is not correct then, assuming the Sorites argument is not valid or not sound, then either modus ponens is not a valid method of inference or there is an object x and some vague predicate such that in some world w, it is such that 'p applies to x' is neither true or false: hence if epistemicism is not true then either modus ponens is not valid or the Principle of Bivalence is false. If the Principle of Bivalence is false then for some P the following is true:

* Not (T(P) or T(not(P)) (Where T is a truth predicate in the meta-language)

Also, the following is true:

T(P) if and only if P and T(not(P)) if and only if not(P)

Hence by substitution and # and *,

μ Not(P or not(P))

So by De Morgan's laws and μ, assuming the Law of Excluded Middle:

Not(P) and not(not(P))

So it seems PoB cannot be denied (which is not the same thing as saying it cannot be asserted, as in Intuitionistic Logic). So, by Reductio, as neither the invalidity of modus ponens or the rejection of PoB can be correct then Epistemicism must be true.

It is doubtful however that PoB cannot be denied. Firstly, if one can stipulate the meaning of some predicates like 'F': F applies to numbers more than forty and F does not apply to numbers less than 12 then if x is a number between twelve and forty then 'Fx' is without truth value hence it is neither true or false. So one of * and # must be rejected. It seems # is not true as if P is without true-value then T(P) is false, but P is indeterminate, hence on one of the Kleene truth tables for three-valued logic the Tarski-biconditional is indeterminate, so not true. Hence the argument for Epistemicism is inconclusive. It might be objected that F is meaningless and hence any sentence that contains it is meaningless, but PoB only applies to meaningful sentences. This is difficult to sustain as it implies anyone who used F would have the illusion of meaning something by sentences involving F.

A puzzle for Epistemicism is how predicates like 'tall', 'bald' and 'red' get precise extensions given that two people x and y could be physically identical and yet the meaning of 'bald' for x is such that a man with n hairs is bald and a man with n+1 hairs is not bald, but the meaning of 'bald' for y is such that a man with m hairs is bald but a man with m+1 hairs is not bald, where n is not equal to m. That is, the physical properties of the users of 'bald' do not determine where the cut-off point is for 'bald'. A possible answer is that such predicates get their extensions by causal interaction with instances of the properties the predicates express, this is arguably true for water. However, this does not work for non-physical properties like 'small' applied to numbers, as non-physical properties cannot be causally interacted with. So it seems inexplicable how some predicates get precise extensions. However, it might be argued that something so puzzling to our intuitions should be accommodated for the economy and simplicity gained by preserving classical logic and classical semantics. This is in line with Reflective Equilibrium (requirement (5*)) as some intuitions may be rejected in the light of other considerations.

There is perhaps a greater problem for Epistemicism owing to the fact that it posits an unknowable dividing line between things that have a property and things that don't. This can be illustrated by the problem of abortion. It is true that it is wrong to abort people, but it is not wrong to abort foetuses if they are not people. According to Epistemicism, there is an unknowable point in time t at which a foetus becomes a person. But then it is wrong to abort foetuses after time t. If this is correct, then it is a moral principle that one should not abort foetuses after time t. But if p is a moral principle then p can be followed in moral practise. If p can be followed in moral practise then it is possible to know p. But Epistemicism entails that we cannot know when it becomes wrong to abort foetuses therefore we cannot know that one should not abort foetuses after time t.

By the Stability requirement, Epistemicism cannot reject the moral truths involving the vague predicate 'person', so either Epistemicism involves denying that moral statements have truth-values or denying that one can know an important moral principle. The first disjunct would arguably fail the Fidelity requirement, as the semantics of moral statements seems to entail that non-vague moral statements have truth values. The second conjunct could be sustained if a plausible explanation was given of why some important moral principles cannot be known.

The foregoing gives some indication that preserving classical semantics and classical logic for vague statements involves some radical revision of many of our common-sense beliefs unrelated to the more abstruse areas of formal semantics and logic. So it would be hoped a non-classical logic or an alternative semantics for vague sentences may not do as much damage to our everyday beliefs. Two formal accounts of this kind will now be described and analysed.

Supervaluationism

One possible way of looking at vague predicates is to see them as expressing concepts that are ambiguous. A concept is ambiguous if any predicate expressing it can be assigned a different, more precise concept without affecting the truth-value of any sentence that was true or false before the predicate was reinterpreted. A concept F is more precise than a concept G if F is true and false of the same objects as G, but there is at least one object x such that 'Gx' is neither true or false, but 'Fx' is true or false.

Supervaluationism is a non-standard semantic model for vague sentences that formalises the idea that the concepts vague predicates express are multiply ambiguous for more precise concepts. On this model a sentence p with a vague predicate is super-true if and only if it is true on an acceptable way of assigning a more precise concept to the vague predicate in p, and is not false on any acceptable way of assigning a more precise concept to the predicate in p. An acceptable way of assigning a more precise concept to a vague expression is one that does not falsify a penumbral connection. For instance, no way of making 'tall' precise would be acceptable that made y tall, but x short, and x taller than y.

It follows by the above definition of an disambiguation of a vague predicate, that a disambiguation must not change the truth-value of any sentence that had a truth-value prior to disambiguation (this is a version of the Fidelity requirement). Secondly, it seems a requirement on an SV semantics that there must be a disambiguation of all vague terms such that there is no more precise disambiguation: this is the Completability requirement. Thirdly, any such complete disambiguation of all vague expressions in the language must be a classical valuation (this is a version of the Stability requirement).

As will be shown, the Completability and Stability requirements entail that p is super-true if and only if p is true on all complete, classical disambiguations. Supervaluationists claim that a sentence is true if and only if it is true on all classical disambiguations and hence that truth is super-truth.

This is not the same as for the ordinary concept of ambiguity, as usually when someone gives an argument containing an ambiguous term we do not evaluate the argument until the ambiguity has been resolved in one way, the argument is not usually evaluated on all ways of removing the ambiguity, but only on the one that was intended. For instance, if X argues that John is a child of Mary and no child should

cross the road unattended therefore John should not cross the road unattended, we would not say X had given a fallacious argument because 'child' is ambiguous in the first premise, and on one disambiguation John is an adult and 'child of Mary' means John is Mary's son.

It follows by the definition of super-truth that SV preserves classical logic for vague sentences. So where p is a hard case it is true that 'p or not p'. It follows that if a vague term appears in a sentence more than once each disambiguation must uniformly interpret the vague term, otherwise LEM would not be preserved. This differs from ordinary ambiguity where various appearances of an ambiguous term in a sentence can be assigned different meanings in the same disambiguation.

The illustration of the truth of LEM for hard cases also demonstrates that SV is a non-classical semantics as 'p or not p' is true and hard cases are neither true or false, hence the meaning of the logical connective 'or' is not truth-functional.

Supervaluationism's response to the Sorites argument

If SV is the correct semantics for vague sentences then on any complete disambiguation of 'heap' there is an n, such that n grains is not a heap, and n+1 grains is a heap, so the inductive premise of the Sorites argument is super-false. However, it is not super-true that there is an n, such that n grains is not a heap, and n+1 grains is a heap, as any complete disambiguation will draw the boundary between heaps and non -heaps at a different point. If the Sorites argument is in conditional form then a number of the conditionals will not be super-true. As for any complete, classical disambiguation there will be a conditional that is false.

SV, as will be shown, has the advantage of preserving classical logic for vague sentences. So the appeal of SV is that it shows what is wrong with the Sorites argument without rejecting classical logic or penumbral connections. No other such theory of vagueness seems to be able to meet all these requirements. We will now give a formal definition of the SV semantics, and prove that the formal semantics meets the Stability, Completability, and Fidelity requirements. We will also prove that classical logical consequence coincides with SV logical consequence.

Supervalutionist Model Theory[2]

We take as our language L the ordinary language of first-order predicate logic with identity, but without function symbols.

A partial interpretation of the language L is an ordered pair (D,I). D is the non-empty domain. I is a function from the non-logical expressions of L, it is the same as the interpretation function for ordinary model theory, apart from the following exception:

[2] The model theory is based on Shapiro: 'Vagueness: a primer' 2005 May (FoM automated email list for discussion of foundations of mathematics)

If R is n-ary relation of L then I(R) is an ordered pair (p,q) where p and q are subsets of Dn, p is called the extension of R (I(R)+) and q is called the anti-extension of R (I(R)-) in M.

If M is a partial interpretation and R is an n-ary relation of L then if x is a member of Dn and x is not a member of either the extension of R in M or the anti-extension of R in M then x is a borderline case of R in M.

If there are no borderline cases of R in M, then we say 'R' is sharp in M. An interpretation M is sharp if every relation in L is sharp in M.

We now give an 3-valued semantics for partial interpretations on L. The values are T (true), F (false) and I (indeterminate). We ignore variable assignments and assume that every element of the domain is named by one constant. We define the valuation function V from formulas of L to the set {T, F, I} for a partial interpretation M inductively:
Atomic Cases:
If a and b are terms, then V'a=b' is T in M if and only if I(a) = I(b), and is F otherwise.

If R is an n-ary relation, and a1, a2,.....an are terms, then V 'Ra1,a2,...an' is T in M if and only if (I(a1), I(a2),....I(an)) is a member of I(R)+. V 'Ra1,a2...an' is F in M if and only if (I(a1), I(a2),....I(an)) is a member of I(R)-. V 'Ra1,a2...an' is I in M otherwise.

Inductive cases:

~P is T in M if and only if P is F in M. ~ P is F in M if and only P is T in M. ~P is I in M if and only if P is I in M.

'P or Q' is T in M if and only if P or Q is T in M. P or Q is F in M if and only if P is F in M and Q is F in M.

'P and Q' is T in M if and only if P is T in M and Q is T in M. 'P and Q' is F in M if and only if P is F in M or Q is F in M. 'P and Q' is I in M, otherwise.

A universally quantified sentence P is T in M if and only if when each instance of the quantified variable is replaced by a constant in L (remember each element of D is named by a constant) then the resulting sentence is T in M. P is F in M if and only if for some constant c in L when each instance of the quantified variable is replaced by c the resulting sentence is F in M. P is I in M, otherwise. The existential quantifier is defined as usual by means of the universal quantifier.

This initial characterisation of truth for partial interpretations does not preserve classical logic or preserve penumbral connections. So we further define a new concept of super-truth for partial interpretations by means of the following idea:

Let M1 = (D1,I1) and M2 = (D2,I2) be partial interpretations. We say M2 'sharpens' M1 if and only if:

a. D1 = D2

b. I1 and I2 assign the same elements of the domain to the same terms of L.

c. For each n-ary relation R of L, I1(R)+ is a subset of I2(R)+ and I1(R)- is a subset of I2(R)-.

It immediately follows that 'sharpens' is a weak partial ordering on partial interpretations, as it is obviously reflexive, anti-symmetric, and transitive (owing to the transitivity of set-theoretic inclusion).

For any vague predicate R in L there will be a set of sentences with R occurring in each sentence, which we will call the penumbral connections for R. We say M2 is an acceptable sharpening of M1 if and only if M2 sharpens M1 and for no vague predicate R in L is there a penumbral connection p such that p is F in M2.

Let p be a sentence of L and M a partial interpretation. We say p is super-true in M if and only if (1) There is an acceptable sharpening M* of M such that p is T in M* (2) There is no acceptable sharpening M* of M such that p is F in M*. We define super-false similarly, interchanging T and F in the definition of super-truth.

We may now prove Fidelity, Completability and Stability:

(Stability) Theorem 1: If M2 sharpens M1 then if p is T (or F, respectively) in M1 then p is T (or F) in M2.

Proof by induction on the complexity of p:

Atomic cases:

'p' = 'a=b'. If p is T in M1 then I1(a) = I1(b) = I2(b) = I2(a) (by def, of 'sharpens') hence p is T in M2. (The case for F follows)

'p' = 'Ra1,a2,...an'. If p is T in M1 then (I1(a1), I1(a2),...I1(an)) is a member of I1(R)+ hence (by def, of 'sharpens') (I1(a1),...I1(an)) is a member of I2(R)+, but, again (I1(a1),...I1(an)) = (I2(a1), ...I2(an)) so p is T in M2. (The case for F is similar just replacing the extension with the anti-extension)

Inductive cases:

'p' = 'Not(q)'. If p is T in M1 then q is F in M1 hence by induction q is F in M2 so p is T in M2. (The case for F is immediate)

'p' = 'q or r'. If p is T in M1 then q is T in M1 or r is T in M1 hence by induction q is T in M2 or r is T in M2 so p is T in M2.

'p' = 'q and r'. If p is T in M1 then q and r are T in M1 hence by induction q and r are T in M2 so p is T in M2.

If p is a universally quantified sentence then if p is T in M1 then for each constant c the sentence resulting from p by replacing every occurrence of the quantified variable by c is T in M1, hence by induction this is true in M2, hence by the definition of truth in a partial interpretation for universally quantified sentences, p is T in M2 □

(Completability) Theorem 2: Every acceptable partial interpretation M has a maximal acceptable sharpening.

Proof:

If A is a set with a partial-ordering <. We say a subset B of A is a <-chain in A if x,y are members of B then x<y or y<x in A.

We need this definition as we make use of Zorn's Lemma:

ZL: If A is a set with a partial ordering < then if every <-chain in A has an <-upper bound then A has an <-maximal element.
Obviously, 'acceptably sharpens' is a partial-order on partial interpretations.

So let A be the set of all acceptable sharpenings of M.

If B is an acceptably sharpens-chain in A then let M* be the model with the domain of any partial interpretation in B and let M*'s interpretation function I* be the same as any partial interpretation in B except that for any n-ary relation symbol R of L let I*R(+) be the union of the extensions of R for each partial interpretation in B, and let I*(R)- be the union of the anti-extensions of R for each partial interpretation in B. (M* is well-defined because B is a chain, and hence no element of D is in the extension and anti-extension of an R in M*)

Let Mi = (Di,Ii) be a partial interpretation in B. Di = D* by definition of 'sharpens'.
Ii agrees with I* on all terms by definition of M*. So let R be an n-ary relation of L then Ii(R)+ is a subset of I*(R)+ and Ii(R)- is a subset of I*(R)-, hence M* sharpens Mi.

Obviously, as B is a chain then as no member of B falsifies a penumbral connection so neither does M*.

So every acceptably sharpens-chain in A has an upper bound, so by ZL, A has an acceptably-sharpens maximal element: hence M has a maximal acceptable sharpening
□

A classical valuation is an interpretation where each sentence of L is assigned T or F.

(Fidelity) Theorem 3: Every maximally acceptable sharpening is a classical valuation.

It is easy to prove by induction on the complexity of formulas of L that any sharp, acceptable sharpening is a classical valuation.

So, we need only show that every maximally acceptable sharpening is sharp.

Suppose not, then there is a maximally acceptable sharpening M and there is an x in D such that for some R, x is not in the extension or anti-extension of R in M. Let M0 be the partial interpretation just like M except x is in the extension of R in M0. If M0 falsifies a penumbral connection p of R, then by Theorem 1, p was indeterminate at M. So, any false penumbral connection is only false as a result of the truth of 'Rx' in M0, so if 'Rx' is false, then there are no false penumbral connections for R. So, let M1 be the partial interpretation just like M except x is in the anti-extension of R in

M1, then M1 is an acceptable sharpening of M, contrary to the definition of M □

We may now show that super-truth is truth on all complete, acceptable sharpenings:

Theorem 4: p is super-true at M if and only if p is true on every complete, acceptable sharpening of M.
Proof:

If p is super-true at M then p is not false on any acceptable sharpening of M, so a fortiori, p is not false on any complete, acceptable sharpenings M* of M, hence by Theorem 3, p is either true or false at M*, hence p is true at M*, so p is true on every complete, acceptable sharpening of M.

If p is true on every complete, acceptable sharpening of M, then by Completability, M has a complete acceptable sharpening M* so p is true on an acceptable sharpening. Let Mi be an acceptable sharpening of M. Suppose p is false at Mi, by Completability Mi has a complete acceptable sharpening Mi*, as 'acceptably sharpens' is a partial-order, then Mi* is an complete, acceptable sharpening of M. So, by hypothesis, p is true at Mi*. But then p is false at Mi and p is true at Mi*, which contradicts Stability. Hence p is not false on any acceptable sharpening of M □

From Theorem 4, Supervaluationists are inclined to argue that truth is super-truth. An argument is valid only if it preserves truth, so if truth is super-truth, then an argument is valid only if it preserves super-truth. So we get the following notion of validity for SV semantics.

A sentence p is an SV-logical consequence of the set of sentences O if and only if any partial interpretation M that makes every sentence in O super-true at M makes p super -true at M.

Theorem 5: A sentence p is an SV-logical consequence of the set of sentences O if and only if p is a classical-logical consequence of the set of sentences O.

Proof:

If p is a classical-logical consequence of the set of sentences O then any classical valuation that makes all of O true makes p true, hence if O is super-true at M then O is true on each complete acceptable sharpening M* of M. So by Fidelity p is true at M*: hence p is super-true at M. So p is an SV-logical consequence of O.

If p is an SV-logical consequence of the set of sentences O then if all of O are super-true at M then p is super-true at M.

It is easy to prove that any classical valuation M is equivalent to a sharp, acceptable sharpening M*.

So if M makes all of O true then all of O are true at the complete, sharp, acceptable sharpening M*. So as M* is the only acceptable sharpening of M* then all of O are super-true at M*, hence by hypothesis, p is super-true at M*.
So by Theorem 4, p is true at M* so p is true at M. So p is a classical-logical consequence of O □

On the above semantics, the conclusion of the inductive Sorites argument is super-false on the standard, partial interpretation M of 'heap': as one million grains of sand is in the extension of heap. So by Theorem 5, one of the premises is not super-true. In fact, the inductive premise is super-false at M. Any complete, acceptable sharpening M* of M is a classical valuation, so there is an n such that n grains of sand is not a heap, and n+1 grains of sand is a heap, so the inductive premise is false on every complete, acceptable sharpening. With the conditional Sorites, on every complete, acceptable sharpening M* of M, at least one conditional will be false, so at least one conditional premise is not super-true.

A Critique of Supervaluationism

Epistemicism suffered from the problem that it was unclear how vague predicates got precise extensions. Supervaluationism seems to suffer similarly. As was said in the introduction, for many vague predicates like 'tall', the higher-order notion of 'borderline tall' is itself vague. But on the SV semantics just given, the extension of 'borderline tall' is precise (it is the domain minus the union of the extension and anti-extension of tall). Again, as for Epistemicism, two people, x and y, could be physically identical, and yet 'borderline tall' might have a different extension when said by x from the extension of 'borderline tall' when said by y. So, as for Epistemicism, SV seems to fail requirement 3 as it is a mystery as to how vague predicates get the precise boundaries between their extensions, borderline cases, and anti-extensions. SV cannot appeal to reflective equilibrium, as considering only this criticism, SV is as problematic as Epistemicism, but Epistemicism preserves classical semantics.

A way around this problem is to make 'partial interpretation' and 'acceptable sharpening' vague in the meta-language. On this idea, the truth is not evaluated at a particular partial interpretation (in the sense defined before), but on an indeterminate number of different partial interpretations. So as super-truth is defined in terms of partial interpretations it inherits its vagueness.

If a formal account can be given of the semantics of vague sentences it might be thought necessary that the model theory make essential use of vague notions. Otherwise, it seems any assignments to vague predicates will make a precise division between the definite cases, the indefinite cases, and the definitely not cases of a vague predicate. As we shall see, this is true even if one has a continuum of truth-values. So Supervaluationism is not at a disadvantage for having vague notions in the meta-language, if a formal account of vagueness exists. However, different formal accounts will characterise the semantics of vague predicates in terms of different vague concepts in the meta-language. It is yet to be seen why 'partial interpretation' and 'acceptable sharpening' are the right notions by which to account for the vagueness of all other vague predicates. Another semantic account may be able to accommodate all the advantages of SV but differ on the vague notions used in the model theory.

One of the supposed advantages of SV is its preservation of classical logic for sentences containing vague expressions. But, it is not clear that classical logic holds for all vague sentences, as it seems to lead to bizarre conclusions. Imagine the following argument:

Either x is a child or x is not a child. If x is a child then x will not understand the violent film and hence it is alright that x watch it. If x is not a child then it is alright that x watch the violent film. So it is alright that anybody watch the violent film.

The conclusion is false, so at least one of the premises is not true. But it seems to me the second and third premises are true. The problem seems to me that the first premise is not true, for instance when x is a borderline adult (i.e. an adolescent). Presumably, Supervaluationists would think that the second and third premises are not super-true because on some complete sharpening of 'child' at least one of the conditionals would be false.

However, I can imagine someone asserting the second two premises when for instance parents were deciding which of their children could watch the film, and which not. But I cannot imagine someone making the above argument. Either because such reasoning with vague predicates is wrong, or because the first premise is wrong. Perhaps, if x was an impressionable adolescent then I would not assert the first premise, because then the above argument could be made. Possibly this is because where x is a borderline child, if I asserted the first premise I would be committed to asserting either that x was a child or that x was not.

That is, the semantics of 'or' in natural language seems to be truth-functional: an assertion of a disjunction commits one to the truth of one of the disjuncts. Similarly, our understanding of the existential quantifier in natural language entails that a true existential has a true substitution instance, but on the SV semantics, this is not right.

So, ordinary speakers assume that the logical connectives and the existential have a classical interpretation, but SV claims that it does not. So to meet requirement 3, SV must show how people mistakenly believe that the logical expressions in natural language have a classical interpretation. Secondly, as with Epistemicism, SV must give some explanation of how words like 'or' receive an SV meaning, when ordinary speakers do not believe that they do have such a meaning: this is necessary to meet requirement four.

On the SV semantics, if we can grasp the truth of a vague sentence p at M then we must be able to evaluate the truth of p at each complete, acceptable sharpening of M. If we can do that, then we must be able to grasp a complete, acceptable sharpening of M. 'Red' is a vague predicate. However it is not clear that one can grasp a complete, sharpening of 'red'. To understand a colour word one must be able to identify the colour on the basis of perceptible features, but this would not be possible on a complete, sharpening of 'red'. So, it seems, we cannot grasp the truth of a sentence involving 'red' on an SV semantics.

It might be objected the second premise of the above argument is only true on a 'constructivist' understanding of quantification, on which we can only evaluate the truth of an existential statement if we could discover an example that makes the statement true. But this would entail that 'There are objects that we could not possibly know of' is a senseless proposition, whereas it seems plainly true.

But, the difficulty goes deeper. If complete, acceptable sharpenings of 'red' are not identifiable by perceptible features, and hence do not have shades or hues it is difficult to see that it is a sharpening of the concept 'red' at all, and not some

completely different concept altogether, however close a connection it has with the concept red.

Fine[3] has responded that one can abjure the use of complete sharpenings in SV model theory by means of 'generic' partial interpretations. Generic partial interpretations are the limits of a complete sequence of partial interpretations. A sequence of partial interpretations is complete if (a) each member of the sequence is an acceptable sharpening of its predecessor (b) any atomic sentence (and hence any sentence) is true or false on some member of the sequence. On this model, p is super-true at M if and only if it is true on all generic sharpenings of M. So, one can have all the advantages of complete sharpenings without actually quantifying over them or being committed to their existence.

However, this emendation does not go far enough. Complete sequences of partial interpretations entail an infinite number of sharper sharpenings of 'red'. Somewhere in such a sequence there will be a sharpening of 'red' such that no perceptible features can identify it, and the notions of shade and hue for such a sharpening of 'red' are unintelligible. So it seems still that after a finite number of sharpenings of 'red' there is a point at which further sharpening seems senseless. If this is correct, then SV cannot give an account of Sorites for colour words, as sentences with colour words cannot have their truth evaluated on classical valuations.

Degree Theories

Degree theories are semantic models that assign to each sentence a degree of truth. A degree of truth is a real number between zero and one; one is truth, zero is falsity, and a sentence that is neither true or false has a degree between zero and one. A degree theory might be motivated as an account of vagueness on the following grounds: many vague predicates can form comparatives, if F is a vague predicate then there is an irreflexive relation 'more F' so that there are true sentences of the form 'a is more F than b'. One explanation of the truth of these statements is that 'a is F' is more true than 'b is F'. So 'a is F' is true to a greater degree than 'b is F' is true. But, 'a is taller than b' may be true when both a and b are tall (or short), so that they cannot differ in degree of truth. This difficulty can be avoided by degrees of truth only explaining the truth of ' a is more F than b' when a and b are borderline cases of F.

However, this can only be part of the motivation. 'Is true to a greater degree' is a partial order, nothing that has been said entails that it is a total order, or that it has all the order properties of the real numbers, which is what a degree theory commits one to. A more convincing motivation for degree theories may come from the idea of a correspondence theory of truth. Hard cases are not true, but they can correspond more or less to the truth, and there is no limit to how close such cases can correspond to the truth, or how far away they can be. Presumably as well, degrees of correspondence to the truth are always comparable. So, on this motivation, more order properties of the real numbers are needed to explain these degrees, and hence degree theories are better motivated.

[3] In Fine 'Vagueness, Truth, and Logic' 1975 Synthese (30). Also, the model theory must be done differently with generic interpretations.

Different degree theories will distribute degrees to composite sentences differently. The degree of truth of a conjunction can be a function of the degree of truth of its conjuncts in different ways. Alternatively, degrees of truth for propositional connectives needn't be truth functional at all, the degree of truth of a conjunction could be a function of more than merely the degree of truth of its conjuncts. As a result of such differences, degree theories may differ on their definitions of validity (for instance, if a degree theory aims to preserves classical logic). And, hence degree theories may find fault with the Sorites argument for different reasons.
Two different degree theories will be expounded and criticised. The first is a truth-functional degree theory that denies the validity of the Sorites argument. The second is a non-truth-functional degree theory that preserves classical logic, but accounts for the Sorites argument by arguing that completely false conclusions can be inferred from large numbers of premises that are not completely true.

A Truth-Functional Degree Theory[4]

The language L to be interpreted is that of first-order predicate logic without function symbols. A fuzzy interpretation M of L is an ordered pair (D,I), where D is the non-empty domain, and I is the interpretation function on L, as for ordinary model theory. However, I differs from ordinary interpretation functions in one respect:

If R is an n-ary relation symbol of L, and a1,a2,....an are terms of L then I(Ra1,a2,...an) is a real number in the closed subset [0,1].

We assume that every member of D is named by one constant of L in order to avoid having to deal with variable assignments.

Let V denote the valuation function for M, it is defined recursively as follows:

If p is atomic then: $V(p) = I(p)$.

For negation: $V(\text{not}(p)) = 1 - V(p)$

Disjunction: $V(p \text{ or } q) = \max \{V(p), V(q)\}$

Conjunction: $V(p \text{ and } q) = \min \{V(p), V(q)\}$

Implication: $V(p \text{ then } q) = \begin{cases} 1, & \text{if } V(q) \leq V(p) \\ 1 - (V(p) - V(q)), & \text{otherwise} \end{cases}$

These are not the only possible truth-functional definitions of the propositional connectives for fuzzy interpretations. For instance, the value of a conjunction could be defined as the product of the values of the conjuncts (although this has the disadvantage of making 'p and p' differ in value from 'p' when V(p) is strictly between zero and one). One might argue for some of these definitions on the basis of the prior motivation for degrees theories in terms of varying correspondence with the truth. So, a conjunction corresponds to the truth to the degree that its conjuncts do,

[4] The model theory is based on Shapiro 'Vagueness: a primer'

and that is no more than the least conjunct corresponds to the truth, but neither does it seem it could be less.

The value of an existential statement is the least upper bound of the values of each sentence that is got by replacing each occurrence of the quantifier variable by one constant of L. The value of an universal statement is the greatest lower bound of the values of each sentence that is got by replacing each occurrence of the quantifier variable by one constant of L. This is well-defined because the set of real numbers is complete. This might be thought to justify the use of the real numbers for degrees, as this is the most natural partially ordered set that is complete.

The definitions of the quantifiers are the only natural ones given the previous definitions of 'and' and 'or': if the domain is finite then each existential sentence is equivalent to a disjunction and each universal sentence is equivalent to a conjunction.

On these definitions, all tautologies fail to be completely true, for instance if V(p) = 0.5 then V(p or not(p)) = 0.5. However, no tautology can be assigned a value less than 0.5.

There are two plausible definitions of validity for fuzzy interpretations with the above valuation function. We say an argument is strictly valid if and only if no fuzzy interpretation M that gives the truth-value of each premise the value one, assigns the conclusion a truth-value less than one. We say an argument is fuzzy-valid if and only if there is no fuzzy interpretation M such that the truth-value of the conclusion on M is less than the greatest lower bound of the truth-values of the premises on M. Any notion of validity must account for valid reasoning for hard cases (in order to preserve penumbral connections), but hard cases are less than fully true, so there needs be a notion of validity for reasoning with less than true sentences. So, Fuzzy-validity seems a more plausible definition of validity.

On the standard fuzzy interpretation of 'heap' the truth-value of the conclusion of the Sorites argument has a truth value of zero. But each premise of the conditional (or the inductive) Sorites argument has a non-zero truth value, so both arguments are fuzzy-invalid. But both arguments are strictly valid, so an argument can be strictly valid and the conclusion be completely false, while each premise may have a truth-value close to one.

A Critique of the Above Truth-Functional Degree Theory

If a sentence p is assigned a degree of truth x on a fuzzy interpretation then it is completely true that p is true to degree x. As for Epistemicism, how a sentence's degree of truth is determined seems a mystery, at least when that degree of truth is not zero or one. Any number of fuzzy interpretations are compatible with a person's use of 'tall', so what determines the degree of truth of a hard case involving 'tall' for that person is a mystery. Again, it is no use to say that causal interaction with the property 'tall' determines the fuzzy interpretation, as there are non-physical properties this would leave unexplained.

One might be tempted to argue that it is only the particular real number between zero and one that is not determined as the degree of truth, but that the degree of truth is

determined up to a linear transformation. However, this still does not account for the problem, noted to occur for SV, that there is a precise demarcation between the definitely true (one), false (zero), and indefinite (not zero or one) sentences. It will be argued later that this problem can be side-stepped.

An opposite problem is that the degrees of truth prove too much. The real numbers are totally ordered so this entails if a and b are borderline cases of a vague predicate F then either 'a is F' is more true than 'b is F' or visa versa, or they are equal. But, given the 'comparative' motivation of degree theories, this means the same as 'a is more F than b' or 'b is more F than a' or 'a is as F as b' is completely true. But two people can be borderline nice and yet there is no fact of the matter who is nicer or whether both are as nice. It might be argued that any vague predicate F that generates a Sorites argument is comparative (i.e. any borderline cases are comparable in terms of F-ness) so degree theories only apply to those type of predicates. This, again, conflicts with the purported motivation of degree theories which is to explain how borderline cases of a predicate can be more or less true.

One seemingly possible way to avoid this problem is to abjure the use of real numbers and instead use an uncountable partially-ordered set with greatest and least elements (true and false, respectively). This way not all vague predicates need be comparative, and yet the Sorites argument still comes out fuzzy-invalid. However, the valuation function would have to be altered radically, as if two sentences are not comparable in degree of truth then the conjunction of those sentences would not have a maximal element, and hence the valuation function would not be total.

These worries are generated by any degree theory, but a truth-functional degree theory has its own unique problems. As was said, we want to preserve truths that do not involve vague predicates. Logical truth is one such thing we wish to preserve, but logical truths are sentences that are true on any interpretation of the sentence's non-logical terms. Any sentence can receive a value less than one on some fuzzy interpretation on the above truth-functional degree theory, so there are no logical truths. One might object that a logical truth is one that is true on any fuzzy interpretation that only assigns values of zero and one to atomic sentences. A fuzzy interpretation that only assigns values of zero and one is a classical valuation, as can be easily checked with the above valuation function. So logical truth can be preserved on the above degree theory.

Even if this is right, logical relations between hard cases (the penumbral connections) seem to be distorted on this degree theory. For instance, if a is red to degree 0.5 and b is red to degree 0.4 then 'a is not red' is true to degree 0.5 so 'a is red and b is red' is true to degree 0.4 but then so is 'a is not red and b is red'. But, a is redder than b, so it seems it could not be true at all that 'a is not red and b is red', if there are penumbral connections. Even if one allows that contradictions can have a degree of truth, surely 'a is red and b is red' is more true than 'a is not red and b is red'. So, it seems the above valuation for 'and' and 'or' cannot preserve penumbral connections. Edgington's degree theory avoids this problem, and the problem of the existence of logical truths, as we shall now see.

Edgington's Non-Truth-Functional Degree Theory[5]

Edgington's theory has the same notion of fuzzy interpretation as for the above theory, but it has a different valuation function. The valuation function is defined like a probability measure. The key idea is a function 'x given y' from sentences to degrees. The value of the function for x and y is the degree of truth assigned to x on the hypothesis that y has degree of truth one, and all penumbral connections are preserved. The function is necessary in order to preserve penumbral connections between hard cases, which as we saw were violated on the above truth-functional account.

As the atomic cases are the same we only give the other cases:

V(p given q) = the value of q on the hypothesis that the value of p is one.

V(not(p)) = 1 – V(p)

V(p and q) = V(p) x V(q given p)

V(p or q) = V(p) + V(q) – V(p and q)

V(p then q) = V(Not(p and not(q)))

We define the verity of a sentence p to be V(p), and we define the unverity of p to be 1 – V(p). With these notions we define an argument to be E-valid if and only if for any fuzzy interpretation M, the unverity of the conclusion is no greater than the sum of the unverities of the premises on M.

This notion of validity has the advantage of being extensionally equivalent with classical validity, which the previous degree theory could not allow.

Theorem 6: An argument is E-valid if and only if it is classically valid.

Proof:

Right to left:

First, we prove (a) if p entails q then $V(p) \leq V(q)$.

If V(p) = 0, then the conclusion is trivial. If V(p) is more than 0 then V(q given p) is 1 (as p entails q). So V(p and q) = V(p) x V(q given p) = V(p). So as $V(p \text{ and } q) \leq V(q)$ then $V(p) \leq V(q)$.

Second, we prove (b) $V(A1 \text{ or } A2 \ldots .\text{or } An) \leq V(A1) + V(A2) \ldots + V(An)$, when n is 2 or more.

By induction. Base case: When n is 2, the result is immediate by definition of 'or', and the fact that the value is always positive. Inductive case: suppose (b) holds for n.

[5] Edgington 'Vagueness by Degrees' 1997 in 'Vagueness: a reader' ed. Keefe and Smith

So, V(A1 or A2... or An) + V(An+1) ≤ V(A1) + V(A2)...+ V(An) + V(An+1). By the base case, V(A1 or A2... or An or An+1) ≤ V(A1 or A2... or An) + V(An+1), so the result follows.

If the set of sentences O classically entail q then by Compactness, there is a finite set of formulas of p1, p2,...pn of O that classically entail q. So not(q) entails not (p1) or not (p2)... or not (pn).

Hence, by (a) V(not(q)) ≤ V(not(p1) or not(p2)... or not (pn)).
By (b), V(not(p1) or not (p2)... or not (pn)) ≤ V(not(p1)) + V(not(p2)... + V(not(pn)).
So the unverity of q is less than or equal to the sum of the unverity of the premises.

Right to Left: If an argument is invalid, then one can assign degree of truth one to each premise and zero to the conclusion. So the argument is E-invalid □

So both Sorites arguments are E-valid. But the conclusion has zero truth-value on the standard fuzzy interpretation of 'heap'. So the unverities of the premises must sum to at least one. So it follows that the Sorites argument illustrates that one can validly infer a false conclusion from a number of premises whose unverity sums to one.

A number of the premises of the conditional Sorites must receive a degree of truth less than one. Given Edgington's semantics, this is how the conditionals are assigned a degree of truth for a Sorites series involving 'tall': Suppose there are a hundred people: number one is tall, n+1 is taller than n, and number 100 is definitely not tall. Let Pn be the conditional 'if n is tall then n+1 is tall'. Suppose n is a borderline case of tall then 'n is tall' has degree of truth r, so as n+1 is shorter than n then 'n+1 is tall' has degree of truth r – e, for some e. It follows that the degree of truth of Pn is 1 – e. Here's how. V(Pn) = V(not(n is tall and not(n+1 is tall)). So we need only show that V(n is tall and not(n+1 is tall)) = e. Under the hypothetical decision that 'n is tall' has degree of truth one, anyone taller than n is tall to degree one, but any borderline cases of tall in the series shorter than n remain borderline. The hypothesis reduces the borderline cases to the degrees between 0 and r, but leaves the relative values unchanged. So V(n+1 is tall given n is tall) = (r – e)/r; hence V(not(n+1 is tall) given n is tall) = e/r. So V(n is tall and not(n+1 is tall)) = e/r x r = e. So V(Pn) = 1 – e.

It follows that, as the difference in degree of tallness between borderline cases in the series of tall people must add up to at least 1, the average difference of degree of tallness between people in the series is at least 0.01.

A Critique of Edgington's Degree Theory

Edgington's theory preserves classical logic so logical relations hold between hard cases. Penumbral connections that were violated on a truth-functional degree theory are preserved by means of the function 'x given y'. So, for instance if a is redder than b then 'if b is red then a is red' is completely true.

The problem of how degrees of truth are determined is still pressing, however, even with Edgington's modifications. But there is something to be said. It is certainly true that the conclusion of the conditional Sorites argument is false, so Edgington's theory commits one to assigning degrees of truth to the conditional premises in such a way

that the unverities sum to one. This can be done however, in a number of conflicting ways, none of which one is committed to.

Edgington helpfully compares her degree theory with a probability assignment. It is true that people believe things to a certain degree and the structure of their beliefs can be modelled by a probability assignment. But various different probability assignments are possible as long as they correctly model the structure. Similarly, the fact that no particular real number is determined as the degree of truth of a sentence does not entail that a sentence does not have a 'degree of truth', neither does it imply that Edgington's theory is not a correct model of many features of degrees of truth.

If this is correct, then the problem of degrees of truth always being comparable does not arise. If 'a is nice' and 'b is nice' are not comparable in terms of degree of truth, then there may be different, equally acceptable fuzzy interpretations M and M* that model vagueness, such that a is nicer than b on M and b is nicer than a on M*. So neither of 'a is nice' or 'b is nice' are true to a greater degree than the other, and hence by the 'Comparative' motivation for degree theories, neither 'a is nicer than b' or 'b is nicer than a' is true to degree one. The earlier response of using a partially-ordered set instead of the real numbers is not available on Edgington's theory anyhow as the semantics depends crucially on the properties of an ordered field. However, this solution does not sit happily with the purported motivation for degree theories in terms of corresponding with the truth; degrees of correspondence with the truth seem to be always comparable.

It is not clear however that the problem of higher-order vagueness is similarly avoided. There still might be a sharp division in the meta-language between sentences that get degree of truth one, those that get zero, and those sentences that get a value strictly between zero and one. However, given that 'degree of truth' does not determine particular numerical degrees, presumably 'degree of truth' is vague, in which case there may be sentences on the borderline of 'degree of truth one' and 'degree of truth less than one' for instance. As was said, to avoid higher-order vagueness it seems any formal model must have a vague notion in the meta-theory. For Edgington's degree theory, those notions are 'degree of truth' and the function 'x given y'.

Conclusion

Both SV and Edgington's Degree Theory give an account of vagueness that entails that both Sorites arguments have at least one less than true premise. Neither involve any revision of classical logic, so both arguments are valid. In order to preserve classical logic, both theories had to give natural language sentences a non-standard semantics that was not truth-functional.

As we saw, the non-standard semantics that were given to 'or' and the existential quantifier seems to conflict with what people believe is the meaning of those terms. Some explanation must be given of this conflict if the theories are to meet requirements three and four. A good explanation of the conflict is provided by the introduction of a modal operator 'Definitely'. On Edgington's degree theory the operator functions so that V(Dp) = 1 if V(p) =1, and V(Dp) = 0, otherwise. On SV the operator works so that 'Dp' is super-true at a partial interpretation M if p is super-true

at M, 'Dp' is super-false otherwise. With this operator, both theories can claim that people who think that the truth of a disjunction commits one to the truth of a disjunct are committing a modal fallacy by inferring 'Definitely 'p' or definitely 'not p'' from 'definitely 'p or not p''. Similarly, people mistakenly infer 'there is an x such that definitely p' from 'definitely 'there is an x such that p''. Clearly, neither inference is valid on either theory on the above definitions of 'D'.

However, the introduction of the D operator entails that SV no longer preserves central meta-logical results on classical logic. For instance, the inference from 'p' to 'Dp' is SV-valid by definition, but 'p then Dp' can be not super-true when 'p' is not super-true. So the Deduction Theorem fails for SV-validity. But if truth is super-truth then SV-validity is classical validity and hence the Deduction Theorem should hold for SV-validity. So if SV is correct, then one cannot introduce a D-operator. But no such problem attends Edgington's degree theory, so Edgington can give an explanation of why people think natural language semantics is non-standard in terms of the D operator, while SV cannot.

It is not likely that acceptance of SV as a theory of vagueness depends on whether it can provide a plausible explanation of why people are inclined to think natural language has a classical semantics. So, if either SV or a degree theory are true then there must be a more significant difference that makes one theory clearly better than the other.

As was said, in order to avoid the problem of higher-order vagueness both theories had to appeal to vague notions in the model theory by means of which all other vagueness could be explained. The motivations for both theories gave good reasons for believing that those were the correct vague notions to appeal to in the model theory: if a vague predicate expresses a concept that is ambiguous for many precise concepts then 'admissible sharpening' is an essential concept in the model theory, similarly, if vague sentences have a degree of correspondence with the truth then 'degree of truth' is an essential concept in the model theory. So, the correctness of the vague notions utilised in the model theory in either theory depended on the theories' motivation. But, the solution to the problem of comparatives for degree theories did not sit well with the motivation in terms of correspondence with the truth, as was mentioned. So, although both theories can respond to higher-order vagueness in a similar way, the response of SV seems more justified, given that it can consistently appeal to its motivation, while Edgington's degree theory cannot.

However, this advantage for SV is small considering how obscure its motivation is. On this motivation, concepts can be ambiguous for more precise concepts. This differs markedly from ordinary ambiguity, where it is a term or sentence that is ambiguous, and that is because a term or sentence can have several meanings. But the meaning of a predicate is the concept it expresses, so a concept cannot have several meanings, it is one. Also, as was said, several occurrences of an ambiguous term in a sentence can be differently interpreted on one disambiguation, but vague terms must be uniformly interpreted on an SV semantics. So in what sense a concept can be ambiguous is

In conclusion, both theories seem to satisfy requirements one to three. Both theories have small problems none of which were sufficient to decide between them. The only obvious way to decide between them seemed to be by means of what vague notions were utilised to explain all other vagueness, but both theories could only justify the notions they utilised by means of their motivations. However, the motivation for degree theories conflicted with its solution to the problem of comparatives, and the motivation for SV was so obscure as to be unintelligible. So neither theory has a distinct advantage over its rival.