CLASSIFYING CONDITIONALS

1. INTRODUCTION

In natural language there exists a type of sentence called *conditionals*. They are characterized in English by the use of connectors. These connectors can be different but fulfil, in most cases, at least two conditions: they link two sentences and they contain the word "if"[1]. The semantic developed to treat this type of sentences is generally a non-classical one and its most famous representative is intensional logic.

The purpose of this paper is to explore the different types of conditional sentences by viewing syntactical and philosophical considerations. The existence of several kinds of conditional sentences is recognized in the current literature but the consequences are not sufficiently shown. We will investigate here the criteria which allow such a division and illustrate these differences by numerous examples from natural language.

This work is not properly technical. It belongs rather to the field of *philosophical logic*, like the first papers published about conditional logic. To develop it in a real mathematical form would need a lot of work. The logical and semantic framework is at best outlined, because we based our analysis primarily on what corresponds to the facts regarding the logic of natural language: the examples of conditional sentences that we employ in everyday life. Nevertheless, a theory is judged by its correspondence to the facts and our approach tries to fulfil this request first. A more technical development will be possible afterwards.

This thesis is organised as follows:

The first part concerns the classification of conditionals into different types. Initially we will introduce briefly the field of conditional logic by a review of its history. Next, we will study the principal problem of conditional logic: different types of conditional sentences exist but sometimes the type is not marked by the connectors used in the sentence (section 2). This study will thus be divided in two branches. The first one is an examination of the sentences that contain sufficient syntactical markers to differentiate the type. It will be shown that each type exemplifies a particular connection between the antecedent and the consequent (section 3). The second one is the study of non-marked cases. We will show that each of them can be

[1] Sentences like "Tell him a joke and he laughs" carry a conditional meaning. But we will restrict our study to the conditional sentences which contain the connector "if".

reduced to one of the marked types that we saw before. These examples contain the trickiest conditional sentences. Some of them are even not always considered as real conditionals, but by our classification, we can integrate all of them in the field of conditional sentences (section 4). Then, we try to characterize inside our approach the usual distinction made between conditionals in the current literature: the difference between the indicative and subjunctive moods (section 5).

The second part of this thesis is concerned with the problems arising from groups of conditional sentences. In section 6, we study the different schemas obtained for the first type of conditional sentences. Also, some consequences of the current representation for mathematical reasoning are exposed. Then, in section 7, the second type of conditional sentence is exposed in a more detailed manner and the schemas used in natural language presented and discussed. Finally, in section 8, two possible developments of this project are highlighted. The first one concerns automatic translation from natural language into formal language and the second one concerns the difficulties with respect to semantics which could formalize our ideas.

2. HISTORY AND PROBLEM

2.1 Historical presentation

This study belongs to the field of conditional logic. Defining Logic as the theory of reasoning, conditional logic is an attempt to give an account of how we use conditional sentences to reason. This problem is very old and can at least to be dated back to stoics. At this time, different conceptions were already in competition and none managed to gain uniform agreement. We can say that more than 2000 years later, the conclusions are the same: no theory actually exists concerning the treatment of conditional sentences that receives general agreement.

Nevertheless, the history of conditional logic during the modern area of logic, which started with the publication of Frege's *Begriffsschrift*, is of great interest. In fact, it shows the development of the field as an autonomous one and the complexity of the problem are exemplified by the various attempts to resolve it. We will present here the most important steps in this history. We can say that the first merit of Frege in logic is to propose a new theory of quantification, for first and higher levels. The modern distinction of the propositional level appears really only in the *Principia Mathematica* by Russell and Whitehead. But the first semantic analysis of the conditional connective under what we now name truth table**,** is usually associated with Peirce and Wittgenstein. At this time, the conception adopted was the material implication, even if some criticisms already appeared, especially for the cases where the antecedent is false.

The first real attempt to give a credible alternative to the material implication is proposed by C.I. Lewis[2]. Roughly, his conception is that the relation postulated between the antecedent and the consequent, with the help of the material implication, has to be necessary. Then, Lewis proposes several systems to represent the different intuitions that we can have concerning the notions of necessity and possibility. For the problem of conditional logic, the results obtained are not very conclusive but they constitute a starting point to the further flourishing research in modal logic, at this time, practiced only from a syntactic point of view. Another conception emerges also at this period and its importance will be revealed only

[2] C.I. Lewis and C.H. Langford, *Symbolic Logic*

decades later. In fact, Ramsey[3] proposes a thought experiment to judge the truth-value of a conditional. Technically, the use of the Bayesian conception of probability enables us to give a formal apparatus to this conception.

Just after the Second World War, the field became autonomous with Goodman's seminal article, "The problem of counterfactual conditionals", which highlights specific problems. Furthermore, the central task is to give an account of the use of conditionals in natural language and not in mathematics. But Goodman's goal is also to show the difficulties of the program of the empiricist logicians (especially Carnap): reducing all scientific language to a formal one. The problem is that the notion of disposition necessitates an analysis of counterfactual conditionals that cannot be performed within classical logic. But the real growth of the field started at the end of the 1960s and the beginning of the 1970s. By importing and adapting the techniques inherited from modal logic (especially what we now name Kripke's models), Stalnaker[4], David Lewis[5] and associates manage to give the first serious analysis of the conditional. Their work contains at the same time philosophy, but more importantly, rigorous syntax and semantics considerations which end up in completeness and decidability proofs for the systems that they present. But for each of these systems, counterexamples were presented. Moreover, for the indicative conditional (which is roughly the matching piece to the counterfactual conditional, usually signalled by the subjunctive mood) some writers prefer to conserve the material implication clothed in pragmatic suits *à la Grice* (for instance D. Lewis and Jackson[6] are representatives of this position). At this time you see also the appearance of a more solid conception of conditionals based on probability. Even if Stalnaker made proposals to interpret his system by the two following semantic tools: possible words and function of probabilities, the principal representative of this approach is Adams, who centres his analysis on the indicative conditional. Nevertheless, Lewis discovered a technical argument which takes away a lot of interest in this approach, because it could be effective only in trivial languages which don't correspond to natural languages, such as English. But it seems that van Fraassen has shown how these problems can be overcome.

From the 1980s, the field lost interest, even if numerous authors continued to recognize the problem: links with other fields are continuously noticed and even established technically. Such fields are for example Belief revision (Gardenfors[7]), Minimal Logic (which contain Nonmonotonic Logic, Deontic Logic and others, see Makinson[8] and Delgrande[9]), Probabilities and Decision Theory (Gibbard and Harper[10]), IA (Morreau[11]), Data Semantics (Veltman[12]) and Linguistics (Haiman, Koning or Iatridou[13]). New insights and points of view are presented in all of these approaches, enriching the available data. But the unity and hope for a general solution which emerged during the 1970s and the growth of conditional logic based on possible worlds, is actually no longer on the agenda of the conditional logicians.

Finally, the consideration of the conditionals in mathematics, even if this issue doesn't belong traditionally to the field of conditional logic, is also linked to this problem. We can distinguish two different usages of the conditional. The first one is the use in mathematical

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[3] F.P. Ramsey, "General Propositions and Causality"
[4] R. Stalnaker, "A theory of Conditionals"
[5] D. Lewis, *Counterfactuals*
[6] F. Jackson, *Conditionals*
[7] Gärdenfors, *Knowledge in Flux*
[8] Makinson, "Five Faces of Minimality"
[9] Delgrande, "An approach to default reasoning based on a first-order conditional logic : revised report*"
[10] Gibbard and Harper, "Counterfactuals and two kinds of expected utility"
[11] Morreau, *Conditionals in Philosophy and Artificial Intelligence*
[12] Veltman, *Logics for Conditionals*
[13] Haiman, , "Constraints on the Form and Meaning of the Protasis"; Koning, , "Conditionals, Concessive Conditionals and Concessives: Areas of Constrat, Overlap and Neutralization"; Iatridou, "If "then", then what?"

reasoning. Even if one conception (the material implication) dominates the others (intuitionist and relevance logic are the principal competitors), the issue is not completely resolved and touches the problem of the foundations of Mathematics. But we will concentrate our study only to the second use of conditionals: in natural language (here English). Traditionally, the field of conditional logic concerns only this second use. But one may notice that the conditional as is it used in mathematics surely has his roots in the conditional used in natural language. It is by applications of practical norms and conventions of the natural conditional that it has been used in mathematical reasoning. So, apart from its own interest, the study of conditionals in natural language could have consequences in the foundations of mathematics, because understanding its use in natural contexts could thereafter help to understand its use in mathematical reasoning.

2.2 The central problem

The final goal of a study in conditional logic is to give a semantic analysis to conditional sentences. But one difficulty immediately appears: several types of conditionals exist. One example will help to see this fact:

(1) If it rains, then I will go to the beach.
(2) Even if it rains, I will go to the beach.

The meaning of (1) is the following: the rain is a sufficient condition for the speaker to go to the beach. The meaning of (2) is different: the rain is not a sufficient condition to discourage the speaker to go to the beach. He will go whatever the weather. The antecedent is here an insufficient[14] but positive condition for the realisation of the negation of the consequent. In more normal circumstances, the antecedent would entail the negation of the consequent. For instance, in (2), the speaker has important reasons to go to the beach. Without these reasons, the rain would discourage him to go.

These two sentences express two different relations between the antecedent and the consequent. In (1), the antecedent is a positive factor for the consequent but in (2), it is a negative factor for the realisation of the consequent, so one speaker cannot assert and believe them both. Someone could think that whatever the exact meaning of (2) is, (2) implies (1). We are opposed to this position. There is a simple argument. We will see later that (1) implies its contraposition "If I don't go to the beach, then it doesn't rain". Obviously, (2) doesn't imply this contraposition and so cannot imply (1). But (2) does imply the following sentence "If it rains, I will go to the beach". We will see later that in this particular case, this sentence is not equivalent with (1).

So, we will have to give different semantics for these two different types of conditionals. But here, we are lucky because there is a syntactic difference between the two conditional connectives in the two sentences. (1) has the following form: "If A, then C" and (2) "Even if A, C". Therefore, we have the easy solution to associate two different formal connectives to these two different natural connectives.

But I will show now that the natural connective "If A, C" can receive the two different preceding interpretations in different contexts of use:

a) <u>If A, C = Even if A, C</u>

In the literature about conditionals, the majority of writers consider that contraposition is not a valid inference. Here, I give Adam's counterexample:

[14] By "insufficient", I mean "not sufficient".

(3) If it rains tomorrow, there will not be a terrific cloudburst.
(4) If there is a terrific cloudburst tomorrow, it will not rain.

One might well accept (3) but would not infer its contrapositive (4). But there is not a difference in terms of meaning between (3) and (5):

(5) Even if it rains tomorrow, there will not be a terrific cloudburst.

Thus, it is clear that conditional sentences of the form "If A, C" will not always permit the passage to the contrapositive. But in all these cases, it is because the conditional has the meaning of an "Even if" conditional.

b) If A, C = If A, then C

These cases are very easy to find because to drop the "then" is very common in English.

(6) If he has some money, he will buy some bread.
(7) If he has some money, then he will buy some bread.
(8) If he doesn't buy some bread, (then) he has no money.

(6) and (7) have the same meaning and both allow the contrapositive (8). We said that a lot of writers consider contraposition as a fallacy. Certainly, this position permits one to avoid to infer the contrapositive in the case where we have an "Even if" conditional. But they cannot explain why in a lot of cases this inference is valid. Furthermore, they didn't stress the fact that a treatment of "even if" conditionals is necessary to hope to understand "If A, C" conditionals.

More tricky examples exist . For instance, conditionals can also be used with rhetorical effects ("If he told the truth, I'm the Pope"). All in all, we can say that the principal task is to classify the "if A, C" conditionals in the different types and uses that we can encounter in natural language. This is the problem that we will treat here.

2.3 Technical Preliminaries

Different components of a conditional sentence help to determine its meaning. Here are some important aspects: the tense of the verbs and their temporal relation, the mood of the verb (indicative versus subjunctive), the presence of modal operators, the different types of conditional connectives. A general theory would have to treat them all. But we will focus just on the components of this study. Because our primary interest is the analysis of the connective, we will not give an account of the tense of the verbs, nor their mood, nor the modal operator. There are two consequences of this position. First, we will not examine the problems concerning the aspects that we don't treat. Second, we will not examine the examples where a difference of meaning arises by one of these factors. For instance, it is well known that the use of the indicative or the subjunctive mood can change the truth-value of the sentence:

(9) If Oswald did not kill Kennedy, someone else did[15].
(10) If Oswald had not killed Kennedy, someone else would have.

[15] Lewis, *Counterfactuals*, p. 3.

Sentence (9) is surely true but people will differ in opinion which truth-value holds for (10). However, you would consider (10) true if you believed in a conspiracy against Kennedy. But you will consider it false if you believe that Oswald was alone. Thus, if there is a difference in meaning with the presence of one of the non-treated aspects, the case will not be considered. As it is a strong tradition in conditional logic to totally separate the indicative mood and the subjunctive mood, we will show that the classification of conditional sentences that we propose appear in the two moods. Secondly, they present the same problems for contraposition, the transitivity, the strengthening of the antecedent, the presence of a false antecedent or a true consequent, the substitution of equivalent antecedents, etc… That means that we don't consider this difference as primordial and that we think that the determination of the type of the conditional connective is the first problem for an analysis of conditional sentences. We will neglect here the differences between these two moods when it comes to tense, because our study is situated at a propositional level, where this aspect is not formalized. In indicative conditionals, it is easy to have an antecedent which precedes the consequent. The translation in the subjunctive mood cannot always be applied. But there are examples of subjunctive conditionals with this employment of the tense for the antecedent and the consequent. They are usually named back-tracking counterfactuals[16].

> (11) If Jim were to ask Jack for help today, there would have to have been no quarrel yesterday.

3. MARKED CASES

In this chapter, we will try to determine what are the basic types of conditionals. We said that a conditional of the form "if A, B" can receive different interpretations in different contexts. But often, a conditional connector can contain a sufficient number of syntactic markers. In that case, the interpretation is fixed. Imagine that we start without knowledge about conditional sentences. Then, many distinctions are important. Here is a list which is not complete but contains at least the most important cases:

(1) If it is sunny, then I will go to the beach.
(2) Even if it is sunny, I will go to the beach.
(3) Only if it is sunny, will I go to the beach.
(4) I will go to the beach if and only if it is sunny.
(5) Necessarily, if it is sunny, I will go to the beach.
(6) A priori, if it is sunny, I will go to the beach.

Only (1)-(3) are basic cases. Case (4) is the biconditional and is only a complex expression formed from the "if, then" and the "only if" type. We will give two reasons to eliminate the last forms. First of all, in both cases, there is a comma between the "if" and the other syntactic marker. So, these markers are not associated with the "if" to form a specific sense of the conditional but are used to qualify the entire conditional sentence. The cases (5)-(6) express a type of judgement. The second reason to eliminate the cases (5)-(6) is to consider what happens when we inverse the order of the antecedent and the consequent. It gives ungrammatical sentences:

> (9) I will go to the beach, necessarily if it is sunny.

[16] Lewis, "Counterfactual Dependence and Time's Arrow" and Kwart, *A theory of Counterfactuals*, p. 250.

(10) I will go to the beach, a priori if it is sunny.

In conclusion, we obtain three irreducible cases: if A, then B; Even if A, B; B only if A. They correspond to three types of connection between the antecedent and the consequent. The idea of connection is introduced historically in the articles of Chisholm[17] and Goodman[18]. Here, I will propose a more elaborated version: a conditional is true if and only if the type of connection indicated by the connective is realised between the antecedent and the consequent. We obtain here three types of connection:

The "if, then" expresses that the connection is sufficient for the realisation of the consequent
The "even if" expresses that the connection is insufficient to avoid the realisation of the consequent or nonexistent considering the link between the antecedent and the consequent
The "only if" expresses that the connection is necessary for the realisation of the consequent

Here we find a classification of the possible connections which correspond, to the classification of conditions. In fact, a condition is often qualified as necessary, sufficient or insufficient. This explanation seems very natural. Why do you think that the sentences that we study are named "conditionals"? But the connection must not be confused with the notion of causality. The latter expresses a link between physical events. It needs also regularities, uniformities or constant conjunctions. The notion of connection is a logical one. It doesn't imply, for instance, that the link is an instance of a causal law; that the antecedent must happen before the consequent, contrary to the priority of the cause with the effect. The link between the antecedent and the consequent can be psychological, logical, etc… For example, we can say, "if Jones is hungry, he will eat the chicken". But Jones' hungriness is an intention and it is difficult to consider it a cause of a physical action. Sometimes, the connection can express a relation of causality, but it is not always the case. Behind this problem, we can find issues in the philosophy of mind and especially the following question: can desires, beliefs or intentions be the cause of our behaviour?

Contraposition is characteristic for conditionals that express a sufficient condition. Contrary to this, when an insufficient condition is used, contraposition is not direct or not even possible. In fact, there are here two possibilities: the antecedent of the conditional and its negation exhaust the universe. In that case, contraposition is not possible. The connection between the antecedent and the consequent is empty. But if the antecedent and its negation do not exhaust the universe, contraposition can take place but by negating the antecedent and its negation. For instance, the negation of the sentence "he drinks a little" is "he doesn't drink a little" which means "he drinks a lot". In that case, the antecedent and its negation leave another possibility: "he doesn't drink".

The conception, defended here, of the "even if" conditional may seem very different from the usual one. The traditional conception is to consider that "even" has a proper semantics and the semantics of the "even if" is given by the application of the semantics of "even" as a modifier on the semantics of "if". So, "even" possesses a semantics which is independent, because this word can be used in non-conditional contexts. For instance, we can say, "Even Attila can solve this problem" which is not so nice a statement about Attila; or "Attila can solve even this problem"[19], which is a compliment. In both cases, the "even" is used to associate with the sentence "Attila can solve this problem" a range of related

[17] Chisholm, "The contrary-to-fact conditional"
[18] Goodman, "The Problem of Counterfactual Conditionals"
[19] Jackson, *Conditionals,* p.46.

sentences. In the first case, the "even" is linked to the name "Attila". So, the associated sentences will be the sentence where another one "Jones", "Mary", etc… replaces the name. But this range of sentences has an orientation: some are more surprising than others. The extreme is the sentence "Attila can solve this problem". "Even" is associated to this name because it is the most informative sentence, as an end-point of the scale. So, among all the individuals, Attila is considered as the one who has got the tiniest chance to solve the problem. It is not far from an insult. In the second case, the "even" is linked to "this problem". Again, the associated sentences start from the sentence "Attila can solve this problem" but this time with a change of "this problem". It means that Attila can solve a whole range of problems and the one touched on in the principal sentence is the most difficult. Here, this sentence is a compliment for Attila.

We can use the same explanation in a conditional context. "Even if it rains, I will go to the beach" is associated with related sentences of the form "If… I will go to the beach". The antecedents of these related sentences express weather conditions like "If the sun shines…", "If there is some wind …", but not like "If it snows …" or "If there is a heavy storm …" which exceed the end-point of the scale. When we said that the antecedent has an insufficient connection with the consequent to be able to change it, it is a parallel explanation. Here, the rain is not a sufficient factor to change my decision to go to the beach. But the insufficiency of the connection is informative if in some cases, this connection is sufficient: for a lot of people, a beach during rain is not something pleasant. So, we can guess that the opposite of the antecedent is also not a problem for the realisation of the consequent. It is perhaps even a positive factor.

But there are objections against a separate semantics for "even". First, at the level of the propositional calculus, "even" is not a correct connector. "It rains" is a proposition but not "Even it rains". So, at this level of treatment, "even if" must be considered as one block. Furthermore, it is not grammatically correct to utter a sentence of the form "even if A, then B". It shows that there is a syntactic difference between the two connectors "if… then…" and "even if…" This is confirmed by our analysis in terms of connection and the different senses obtained if we combine them with the same propositions. Perhaps at a more complex level, a separate semantics, which can combine the use of "even" in conditional and non-conditional contexts, would be preferable, but at our level, the other option seems better. The same argumentation holds also for the case of "only" in " only if".

4. NON-MARKED CASE

We designate the non-marked case the linguistic form of the conditional "if A, B". The difference with the previous forms is that the syntactic marker, which compounds the connective, is inadequate in isolation to determine the type of connection exemplified in the sentence. So, at first sight, the meaning of the conditional may seem ambiguous. In general, the context of utterance of the sentence or the semantic comprehension of the relations of the antecedent and the consequent permit recognition of the connection. Here, we will base our analysis only on the transformation of the conditional sentence in the non-marked case into the same conditional sentence with additional syntactic markers. But we will see that this strategy is not always sufficient. In the most complex cases, we also need to add some piece of information which is presupposed and usually known by the listeners or readers. This information may come from general knowledge of the world or be part of the information learned from the context (for instance by the previous elements of the discussion). However, we will put forward no real technical argument. We address here the comprehension and judgement of the reader who can appreciate by himself if the transformation that we propose changes wrongly or not at all the meaning of the sentence. Some subtleties may be lost, but our goal here is to bring to light and exemplify the type of connection. Sometimes, the resulting sentence is not completely grammatically correct, but at least we can always understand it, so a counter-argument in this vein is not really important. We will first examine the case of the necessary connection, then the sufficient connection and finally the insufficient connection.

4.1 The necessary connection

Examples concerning a necessary connection expressed by an “if A, B” are scarce. But we can give for instance the following example:

We know that the speaker likes to impress people by showing off that he is very rich. This person says:

(1) I will buy this watch, if it takes all the money in my wallet.
(2) I will buy this watch, only if it takes all the money in my wallet.

This possibility seems to exist only when there is an inversion of the antecedent and the consequent. The standard way would be to use an “only if” conditional but when we use a marked-case, the antecedent and consequent can be put in the normal order. So, to exemplify a necessary connection, the non-marked case also needs this inversion. This example is not really convincing to some people but with the right intonation of voice, I think that the listener can understand it in the intended way. At least, in French, listeners generally agree: “J’achèterai cette montre, si cela me coûte tout l’argent dans mon porte-monnaie.” It may not an obligatory interpretation of the sentence, but with the context in mind, some hearers do understand it like a hidden “only if” conditional.

I agree with the conception which says that “A only if B” is equivalent to “if A, then B”. So, we won’t have to investigate separately the logical properties of “only if” in the second part of the thesis.

4.2 The sufficient connection

The examples concerning a sufficient connection represent the majority of the instances of the “if A, B” form. The principal reason of this phenomenon is that we employ this type of connection more often than the others. We have also the tendency, especially in spoken language, to try to economize on the length of the sentence while conveying the same amount of information. So, it is natural to drop the “then” and to use the non-marked case. Here is an example:

(3) If it is sunny, I will go to the beach.
(4) If it is sunny, then I will go to the beach.

But we can find more interesting uses. One is the conditional with a trivially true antecedent. Because the antecedent is sufficient to deduce the consequent, the meaning is that the consequent is also considered as true. So, we employ this type of conditional for a rhetorical effect. Here is one example:

(5) If there is one thing I cannot stand, it is to be caught in traffic jams.[20]
(6) If there is one thing I cannot stand, then it is to be caught in traffic jams.

[20] Veltman, “Data Semantics and the Pragmatics of Indicative Conditionals”

Surely, for everybody, there exists something which is not really appreciated. Here, the real meaning is that the most hated thing is a traffic jam. Among the bad events of the world, the most "true" one for this person is to be blocked in his or her car and not to be able to move forward.

Parallel to this phenomenon, we find conditionals where the consequent is trivially false. The speaker hopes that from this, the listener will be able to deduce that the antecedent is false. There is a simple application of the principle of contraposition:

(7) If he told the truth, I'm the Pope.
(8) If he told the truth, then I'm the Pope.

Contraposition and deduction: I am not the Pope. If I am not the Pope, he doesn't tell the truth. So, he doesn't tell the truth.

This aspect of the sufficient conditional is particularly interesting because it shows that contraposition is one of its logical properties. So, formal systems which don't possess this property, won't be able to give an account of this type of use. But there are well-known examples where contraposition is inappropriate. The explanation is just that we are here dealing with another type of connection.

The last category of sufficient conditionals expressed by the "If A, B" form is what is named "relevant conditionals". These conditionals are the trickiest one because we see here a contraction of a conditional with another piece of information. The result of this contraction is the elimination of the consequent. Always by the knowledge of the world, the listener must be able to reconstruct the argumentation. I will show here two different examples:

(9) If you are hungry, there are biscuits on the table.[21]
(10) If you are hungry, there are biscuits on the table that you can eat.
(11) If you are hungry, then you can eat the biscuits on the table.

(12) If you need me, my name is Marcia.[22]
(13) If you need me, then you can call me. My name is Marcia.

To give an account of this type of examples in formal terms seem really difficult. In fact, we have to guess the presupposed consequent. A purely syntactical solution seems simply impossible. A semantic solution is also really difficult because the missing part of the conditional must be found from the antecedent and the external sentence. Many people could claim that "relevant" conditionals are not really conditionals, because in languages like Dutch and German, the consequent has the word order of a single main clause, while in real conditionals, the verb of the consequent gets second position with respect to the antecedent. Anyway, this construction respects in English the criteria that permit to classify it as a conditional sentence and the case is the same in other languages like in French.

4.3 The insufficient connection

We will study now the forms of the "if A, B" conditionals which correspond to an insufficient connection. The first one is used for rhetorical reasons:

[21] Veltman, "Data Semantics and the Pragmatics of Indicative Conditionals"
[22] Veltman, "Data Semantics and the Pragmatics of Indicative Conditionals"

(14) This is the best book of the month, if not of the year.[23]
(15) Even if it is not the best book of the year, it is the best book of the month.

We can notice here that the inversion of the antecedent with the consequent facilitates this interpretation because very often, the consequent of an "even if" is true. By putting it in the first place, the acceptance by the speaker is reinforced.

The second type is conditionals where the antecedent contains a disjunction. But here, two possibilities exist. The first one is when the antecedent doesn't exhaust the universe. It means that there exists at least a third possibility in complement of those present in the antecedent:

(16) If John is dead or seriously ill, Mary will collect the money.[24]
(17) If John is dead or seriously ill, then Mary will collect the money.

We have here a sufficient connection. We can apply the classical argument to confirm this hypothesis: contraposition is valid. On the other hand, when the disjunction in the antecedent exhausts the universe, the connection is not only insufficient but also moreover inexistent. In that case, we can often replace the conditional by a compound sentence introduced by "whatever":

(18) If John is drunk or not drunk, Bill will vote for him.[25]
(19) Whatever John's consumption of alcohol, Bill will vote for him.

The "whatever" signals a limit case of an insufficient connection because the components of the antecedent don't have any link with the consequent. We have here simply no connection at all. Conversely, a conditional introduced by "even if" signals that the antecedent is a factor which could have changed the consequent in other circumstances.

We can notice also that a conjunction of two conditionals where the antecedent of the second one is the negation of the first one is equivalent to the "whatever" form. From this whatever form can be often deduced an "even if" conditional:

(20) If John is drunk, Bill will vote for him and if John is not drunk, Bill will vote for him.
(21) So, whatever John's consumption of alcohol, Bill will vote for him.
(22) Even if John is drunk, Bill will vote for him.

4.4 Conclusion

As Iatridou has already demonstrated, we can say that the "If A, B" form is not simply equivalent to the "If A, then B" form. It can be equivalent to each of the three basic types of conditionals which correspond to the three possible types of connection. The context of emission permits this determination and without context, the sufficient connection must be preferred. A good test is also the application of contraposition.

[23] Veltman, "Data Semantics and the Pragmatics of Indicative Conditionals"
[24] Iatridou, "If 'then'. Then what?"
[25] Iatridou, "If 'then'. Then what?"

5. THE SUBJUNCTIVE MOOD

In this chapter, we will try to show two things. The first one is that the difference of the indicative and the subjunctive mood is overestimated. The second is that we have also in the subjunctive mood the same problems that we noticed in the indicative mood: a conditional can express different types of connection.

The difference between the indicative and the subjunctive mood employed in a conditional sentence exists. The classical example to show this point is the following one:

(1) If Oswald did not kill Kennedy, then someone else did.[26]
(2) If Oswald had not killed Kennedy, then someone else would have.

The first sentence is surely true, because it is a fact that Kennedy was murdered. So, with the hypothesis that the murder is not Oswald, we are forced to assume that another person shot the president. Conversely, the second sentence could very well be false: Oswald was perhaps the only one to shoot. So, the two moods present two different ways to consider a hypothesis. The first one doesn't erase the consequences of the hypothesis, contrary to the second one.

In my opinion, this difference is considered with a too much importance. In fact, the two moods share a great number of paradoxes.

The paradox of contraposition:

(3) If it were to rain heavily at noon, the farmer would not irrigate his field at noon[27].
(4) If the farmer were to irrigate his field at noon, it would not rain heavily at noon.

(5) If it rains tomorrow, there will not be a terrific cloudburst[28].
(6) If there is a terrific cloudburst tomorrow, it will not rain.

The paradox of the transitivity:

(7) If Carter had not lost the election in 1980, Reagan would not have been president in 1981[29].
(8) If Carter had died in 1979, he would not have lost the election in 1980.
(9) If Carter had died in 1979, Reagan would not have been president in 1981.

(10) If Jones wins the election, Smith will retire[30].
(11) If Smith dies before the election, Jones will win.
(12) If Smith dies before the election, he will retire.

The paradox of strengthening the antecedent:

(13) If the left engine were to fail, the pilot would make an emergency landing[31].
(14) If the left engine were to fail and the right wing were to shear off, the pilot would make an emergency landing.

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26 Lewis, *counterfactuals*
27 Nute, "Conditional Logic"
28 Adams, *The Logic of Conditionals*
29 Nute, "Conditional Logic"
30 Adams, *The Logic of Conditionals*
31 Nute, "Conditional Logic"

(15) If Jones wins the election, then Smith will retire[32].
(16) If Smith dies before the election and Jones wins, then Smith will retire.

The two moods share other properties. In fact there are more similarities than differences, because the mood is not the principal semantic factor, it is the connective. And this connective doesn't change with the mood in English, even if two different formal connectives are often chosen to represent the conditional connective employed with the two different moods (this practice has existed since Lewis' analysis).
The subjunctive mood presents the same classification in types of conditional connectives. For instance, the former example that we gave to contraposition can be reformulated in and "even if" type: "Even if it were to rain heavily at noon, the farmer would not irrigate his field at noon". As usual, the conditionals that don't accept contraposition are of this type.

The subjunctive mood seems often to signal the falsity of the antecedent. That is why they are also called often "counterfactuals". But this name is not totally adequate, because someone could think that the antecedent is false, but his judgement can be erroneous. So, we could think that the subjunctive mood signals that the speaker believes that the antecedent is false. But a counterexample exists to this new definition, given by Anderson:

(17) If Jones had taken arsenic, he would have shown just exactly those symptoms which he does in fact show. [So, it is likely that he took arsenic].

Anyway, this sentence would be typically uttered in a context where the hearers don't think that Jones took arsenic. The speaker uses the subjunctive mood to not violently go against the beliefs of the hearers. By a direct attack, he would take the risk that his audience refuse his proposal. So, he uses a indirect way to show that the antecedent is not so absurd. In fact, the consequent of the conditional is true, the reasoning seems also true, so, even if the subjunctive mood is employed, the antecedent is perhaps true. That's not a demonstration of the truth of the antecedent but it is a way to show that it is not an absurd hypothesis. So, we can say that the subjunctive mood signals that the speaker believes that the antecedent is false, at least as hypothesis. The hypothesis for the current reasoning can be different from the deep beliefs of the speaker. On the contrary, the indicative mood is the non-marked case: no hypothesis is made about the truth-value of the antecedent. Anyway, the speaker can believe that the antecedent is false but by using the indicative mood, he doesn't insert this belief in the hypothesis of his reasoning. So, it is important to distinguish the beliefs of a speaker from the hypothesis that he makes as a reasoning. The hypotheses are not always the beliefs and the beliefs are not always the hypotheses.

6. SCHEMAS FOR THE SUFFICIENT CONNECTION

For the following discussion, we will employ a very simple formal language:
"A", "B", "C" are atomic formulas
"¬", "∧", "∨", "→" are the negation, conjunction, disjunction and conditional
"⇒" , "⇔" are the consequence and the consequence in both directions

6.1 Complex formulas and complex contraposition

[32] Adams, *The Logic of Conditionals*

Previously, we studied simple conditionals, that is conditionals with atomic formulas such as antecedent and consequent. But complex conditionals are more difficult. They contain disjunctions or conjunctions in the antecedent or consequent, or are linked by these connectors. The problem examined here will be how develop or reduce this type of formulas. We will propose eight schemas that are all valid in classical propositional logic and try to see which one are valid or invalid in natural language. All the conditionals considered express a sufficient connection. So, counterexamples to these schemas with another type of connection will not be relevant for our discussion. We will see that our position entails the rejection of the principle RCEA, a basic principle of intensional logic. Our conception of the disjunction will present also some particularities.

The eight schemas are as follows:
D1: $(A \vee B) \rightarrow C \Rightarrow (A \rightarrow C) \wedge (B \rightarrow C)$
D2: $A \rightarrow (B \vee C) \Rightarrow (A \rightarrow B) \vee (A \rightarrow C)$
D3: $(A \rightarrow B) \vee (A \rightarrow C) \Rightarrow A \rightarrow (B \vee C)$
D4: $(A \rightarrow C) \vee (B \rightarrow C) \Rightarrow (A \wedge B) \rightarrow C$
C1: $(A \wedge B) \rightarrow C \Rightarrow (A \rightarrow C) \vee (B \rightarrow C)$
C2: $A \rightarrow (B \wedge C) \Rightarrow (A \rightarrow B) \wedge (A \rightarrow C)$
C3: $(A \rightarrow B) \wedge (A \rightarrow C) \Rightarrow A \rightarrow (B \wedge C)$
C4: $(A \rightarrow C) \wedge (B \rightarrow C) \Rightarrow (A \vee B) \rightarrow C$

They are exposed with the following order, respectively to the left-part of each schema: connector in the antecedent, connector in the consequent, same antecedent for both formulas and same consequent for both formulas. The first four formulas concern the disjunction and the last one concern the conjunction.

We consider six schemas as valid in natural language:
D1: If I'm hungry or tired, I will go home.
⇒ If I'm hungry, I will go home and if I'm tired, I will go home.
D2: If it is sunny, I will go to the beach or to the park.
⇒ If it' sunny, I will go to the beach or if it is sunny, I will go to the park.
D3: If it' sunny, I will go to the beach or if it is sunny, I will go to the park.
⇒ If it is sunny, I will go to the beach or to the park.
C2: If I have money, I will buy some milk and some bread.
⇒ If I have money, I will buy some milk and if I have money, I will buy some bread.
C3: If I have money, I will buy some milk and if I have money, I will buy some bread.
⇒ If I have money, I will buy some milk and some bread.
C4: If I'm hungry, I will go home and if I'm tired, I will go home.
⇒ If I'm hungry or tired, I will go home.

Schema D2 is usually considered as invalid[33]. At first sight, D2 could imply $(A \rightarrow B) \vee (A \rightarrow \neg B)$, whatever A and B are, at least in classical propositional logic:

$B \vee \neg B$ (tautology)
$A \rightarrow (B \vee \neg B)$ (because the consequent is always true)
$(A \rightarrow B) \vee (A \rightarrow \neg B)$ (from D2)

[33] Lewis, *Counterfactuals*

Lewis gives the following counterexample:

(1) If Bizet and Verdi were compatriots, Bizet would be Italian or if Bizet and Verdi were compatriots, Bizet would not be Italian (but Verdi would be French).

We cannot choose between the two options, so no member of the disjunction can be determined as true. So, the schema $(A \rightarrow B) \vee (A \rightarrow \neg B)$ cannot be accepted. We agree with this position. In fact, we said that the antecedent needs a sufficient connection with the antecedent to accept the conditional. Then, we don't consider as valid**,** a conditional with a tautology as its consequent, whatever the antecedent is:

(2) If it is sunny, then my name is John or my name is not John.

So, we cannot derive *simpliciter* the invalid schema. We adopt here a parallel position to the relevance logicians, who say that the antecedent should be relevant to the consequent. It is not because "My name is John or my name is not John" is always true that everything is a sufficient condition for it. We always need a connection between the antecedent and the consequent. The existence of a connection is the same criteria as the relevance. Anyway, our position presents some differences with the one of the relevance logicians. First, their principal interest is mathematical reasoning and not natural language. Secondly, they ask the respect of a second criterion: the valid entailments must be necessarily true[34]. But all the reasoning of natural language cannot be considered as necessary. Finally, our position entails a modification of the consideration of the disjunction (see later). Then, even if our position is close from the one of the relevance logicians, there are some important differences.

Conversely, we have no problems with the following derivation, where the first conditional respects the criteria of connection:

(3) If it is sunny, I will go to the beach.
⇒ If it is sunny, I will go to the beach or I won't go to the beach.
⇒ If it is sunny, I will go to the beach or if it is sunny, I won't go to the beach.

The two last schemas are invalid and we give counterexamples:

D4: If it is sunny, I will run or if I'm injured, I will run. (true because the first conjunct is true)
⇒ If it is sunny and I'm injured, I will run.

C1: If it is sunny and Mary comes, I will go to the beach.
⇒ If it is sunny, I will go to the beach or if Mary comes, I will go to the beach. (false because the both conditions are necessary for the conclusion)

Remark: These schemas can be put two by two to form equivalences:

D1-C4: $(A \vee B) \rightarrow C \Leftrightarrow (A \rightarrow C) \wedge (B \rightarrow C)$
D2-D3: $A \rightarrow (B \vee C) \Leftrightarrow (A \rightarrow B) \vee (A \rightarrow C)$
C2-C3: $A \rightarrow (B \wedge C) \Leftrightarrow (A \rightarrow B) \vee (A \rightarrow C)$
C1-D4: $(A \wedge B) \rightarrow C \Leftrightarrow (A \rightarrow C) \vee (B \rightarrow C)$

We said that contraposition is valid for the simple conditionals which express a sufficient connection. At the same time, we don't want to conserve the schemas C1-D4. But we have

[34] Anderson & Belnap "The Pure Calculus of Entailment"

here a difficulty; these schemas can be obtained with contraposition applied to complex conditionals or groups of formulas, with the help of the schemas D2-D3:

D4: $(A\rightarrow C)\vee(B\rightarrow C) \Rightarrow (\neg C\rightarrow\neg A) \vee (\neg C\rightarrow\neg B)$ [contraposition]
$\Rightarrow \neg C\rightarrow(\neg A\vee\neg B)$ [D3]
$\Rightarrow (A\wedge B)\rightarrow C$ [contraposition]

C1 : $(A\wedge B)\rightarrow C \Rightarrow \neg C\rightarrow(\neg A\vee\neg B)$ [contraposition]
$\Rightarrow (\neg C\rightarrow\neg A) \vee (\neg C\rightarrow\neg B)$ [D2]
$\Rightarrow (A\rightarrow C) \vee (B\rightarrow C)$ [contraposition]

Anyway, we want to conserve the schemas D2-D3. So, one of the possible contraposition must not be valid:

Contraposition 1: $(A\wedge B)\rightarrow C \Leftrightarrow \neg C\rightarrow(\neg A\vee\neg B)$
Contraposition 2: $(A\rightarrow C) \vee (B\rightarrow C) \Leftrightarrow (\neg C\rightarrow\neg A) \vee (\neg C\rightarrow\neg B)$

At first sight, the first contraposition must be the culprit because it uses complex conditionals. But surprisingly, it is the second which is not always valid in natural language:

Counterexample to $(A\rightarrow C) \vee (B\rightarrow C) \Rightarrow (\neg C\rightarrow\neg A) \vee (\neg C\rightarrow\neg B)$

> If you beseech him, he will accept or if you threaten him, he will accept.
> $\Rightarrow$ If he doesn't accept, you didn't beseech him or if he doesn't accept, you didn't threaten him.

We can't accept the conclusion because it is possible that the person refuses because the other adopts both types of behaviour: to beseech and to threaten. This inconstant attitude could be a reason for refusal.

Counterexample to $(\neg C\rightarrow\neg A) \vee (\neg C\rightarrow\neg B) \Rightarrow (A\rightarrow C) \vee (B\rightarrow C)$

> If it is not a mule, its father is not a horse or if it is not a mule, its father is not a donkey.
> $\Rightarrow$ If its father is a horse, it is a mule or if its father is a donkey, it is a mule.

A mule has a horse or a donkey as a father. So, the premises is true. But the conclusion is false. The son of a horse could be a horse and the son of a donkey could be a donkey.

So, we cannot directly apply contraposition to two conditionals linked by a disjunction, at least when they possess the antecedent or the consequent in common.

We resume here the valid and invalid schemas briefly:

Valid schemas: D1-D2-D3-C2-C3-C4
$(A\wedge B)\rightarrow C \Leftrightarrow \neg C\rightarrow(\neg A\vee\neg B)$
$(A\vee B)\rightarrow C \Leftrightarrow \neg C\rightarrow(\neg A\wedge\neg B)$
$(A\rightarrow B) \wedge (A\rightarrow C) \Leftrightarrow (\neg B\rightarrow\neg A) \wedge (\neg C\rightarrow\neg A)$

Invalid schemas: D4-C1
$(A\rightarrow C) \vee (B\rightarrow C) \Leftrightarrow (\neg C\rightarrow\neg A) \vee (\neg C\rightarrow\neg B)$

6.2 Application to mathematical reasoning

Generally, the analysis given by the classical propositional calculus (hereafter CPC) is considered as a good representation of the use of the conditional in mathematical reasoning. Two major critics of this position already exist: the first from the intuitionism logic and the second from the relevant logic. I will not discuss here their arguments, but try to give a new counter-argument against the use of the material implicator as a correct representation of the mathematical conditional.

The formal language used is the following: "(,),[,]" (parenthesis), "¬" (negation), "∧" (conjunction), "∨" (disjunction), "⊃" (material implicator), "R, D, S" (formulas) for the CPC. For the first-order logic we will add "∀" (universal quantification), "∃" (existential quantification), "x, y, z" (variables) and "R, D, S" which stand now for predicates. The inductive definitions of the languages and the semantics associated are the classic ones of the CPC and the FOL.

6.2.1 Propositional Level

First, we must choose a criterion for a correct representation of the mathematical conditional connector in a formal language. We will adopt the following test for the CPC:

(1) "A⊃C" is true iff "If A, then C" is true.

We use here two object languages: the CPC in the first pair of parenthesis and the English language used in mathematical reasoning in the second pair of parentheses. The metalanguage used is the English language with the predicate "is true".

Second, we will use the following semantic equivalence which holds in the CPC:

(2) "A∨B" is true iff "A" is true or "B" is true

Finally, we will use the following tautology of the CPC (schema C1 in section 5.1):

(3) [(A∧B)⊃C] ⊃ [(A⊃C)∨ (B⊃C)]

Counterexample:

Translation keys:
R: the figure ABCD is a rectangle
D: the figure ABCD is a diamond
S: the figure ABCD is a square

In Euclidean geometry, we have the following theorem:
If the figure ABCD is a rectangle and a diamond, then the figure ABCD is a square.
So, we can obtain the following derivation:

1. "If R and D, then S" is true.
2. "(R∧D) ⊃ S" is true (by (1)).
3. "(R⊃S)∨(D⊃S)" is true (by (3)).
4. "R⊃S" is true or "D⊃S" is true (by (2)).
5. "If R, then S" is true or "If D, then S" is true (by (1)).

We obtain here a false conclusion. Neither "If ABCD is a rectangle, then ABCD is a square", nor "If ABCD is a diamond, then ABCD is a square" are theorems of the Euclidean geometry. Intuitively, the problem arises from the tautology:

(3) ([(A∧B)⊃C] ⊃ [(A⊃C)∨ (B⊃C)])

It says that if we need two conditions (A and B) to obtain conclusion C, then only one of the conditions is sufficient to obtain the conclusion. Certainly, by the definition of the material implication with the negation and the disjunction, the formula seems correct:

[(A∧B)⊃C] is equivalent to [¬A∨¬B∨C] is equivalent to [(¬A∨C)∨(¬B∨C)]

But here, we stay in an internal interpretation. We don't try to relate the formal conditional connector with its real use in mathematical reasoning. Our counterexample shows that if this link is examined seriously, we obtain an inadequacy between the inferences allowed by the CPC and the inferences allowed in the current mathematical reasoning. We cannot hope that at the propositional level, all the inferences of mathematical reasoning can be obtained. However, a minimal condition is that we don't obtain already incorrect inferences. The CPC doesn't fulfill this minimal requisite and then is not an adequate representation of mathematics. Finally, we can notice that to give up (3) has as consequence the withdrawal from (1), and inversely.

6.2.2 First Order Level

As is well known, Frege, who is considered as the inventor of the modern logic, proposed a theory in his *Begriffsschrift*, where the distinction between the propositional level and the first-order level is not explicitly made. The principal innovation of his treatment is the introduction of the quantification. So, the issue here will be to evaluate the impact of the quantification for the representation of mathematical reasoning. It is possible that the problem that we noticed on the propositional level disappears on the first-order level by means of the introduction of the quantification. And at first sight, this is the case. If we translate the sentence (3) into FOL, we will use a universal quantification because we refer to figure ABCD as an abstract object that stands for all the possible geometrical figures. Thus, we obtain the following formula which is valid in FOL:

(4) $\forall x[(Rx \wedge Dx) \supset Sx] \supset \forall x\, [(Rx \supset Sx) \vee (Dx \supset Sx)]$

In Euclidean geometry, we have the following theorem :

(5) For all quadrilaterals ABCD, if ABCD is a rectangle and a diamond, then ABCD is a square.

So, we obtain $\forall x[(Rx \wedge Dx) \supset Sx]$ and we can deduce $\forall x\, [(Rx \supset Sx) \vee (Dx \supset Sx)]$ which means:

(6) For all quadrilaterals ABCD, if ABCD is a rectangle, then it is a square or if ABCD is a diamond, then it is a square.

The validation of this formula in FOL is the following:
We can divide the quadrilaterals into four classes: the one which is not rectangular or diamond-shaped, the one which is rectangular but not diamond-shaped, the one which is diamond-shaped but not rectangular and finally the one which is diamond-shaped and rectangular.
For the first one, both conditionals are true because the two antecedents are false.
For the second one and third one, the disjunction is true because in each case, one conditional is true because its antecedent is false.
For the last one, to be a diamond and a rectangle is evidently to be a square so each conditional has the antecedent and the consequent which is true.

But in mathematical reasoning not based on formal logic, we reason differently. If a hypothesis is not fulfilled, we don't consider the conditional as validated or invalidated. We can take the four classes of quadrilaterals again and obtain a different result:
For the first one, the antecedent of the two conditionals is not fulfilled so this is not a counterexample to the formula.
For the second one, the antecedent of the first conditional is fulfilled but not the consequent. So, the conditional is false. The antecedent of the second conditional is not fulfilled, so nothing can be concluded from it. Finally, the only relevant case is the first conditional and it is false. So, we have here a counterexample to the formula.

The third case is the same as the second one. We find again a counterexample to the formula. Finally, the fourth case presents two conditionals that have their true antecedents and consequents, so it is not a counterexample to the formula.
So, the formula cannot be considered as a valid theorem in Euclidean geometry, with the natural way of mathematical reasoning. A quadrilateral has to possess both properties (to be a rectangle and to be a square) to be considered as a square. So, the same problem reappears in the first-order level.

6.2.3 Conclusion

We used for this reasoning the schema that we named C1 (section 5.1). But we could have also start from the schema D4 which is also incorrect. Here is an example of a counterexample to this schema in mathematical reasoning:

D4: $A \rightarrow C \vee B \rightarrow C \Rightarrow (A \wedge B) \rightarrow C$
If x is even, then x is an integer or if x is odd, then x is an integer.
⇒ If x is even and odd, then x is an integer.

Finally, the CPC and the FOL both contain a valid formula that doesn't correspond to a safe rule of reasoning in mathematics (and also in more natural reasoning). The problem arises from the treatment of the conditional reasoning by use of the material implicator. So, we always have the task of finding a formal system that can adequately represent mathematical reasoning.

6.3 SDA

We already used the schema SDA, that we named C1-D4 in section 5.1. Its use seems rather natural in natural language, but in a lot of semantics about conditionals, it is not valid. For instance, Lewis[35] refuses to accept that it represents a good inference for counterfactuals. The reason is that in his system, another principle (named RCEA) forbids the presence of the SDA. Here are the two principles:

SDA: $(A \vee B) \rightarrow C$ is equivalent to $(A \rightarrow C) \wedge (B \rightarrow C)$
RCEA: from $(A \equiv B)$, we can infer $(A \rightarrow C) \equiv (B \rightarrow C)$

These two principles are in conflict, because if we take them both in our system, we can derive the strengthening of the antecedent:

1. $A \equiv ((A \wedge B) \vee (A \wedge \neg B))$ [PCP]
2. $(A \rightarrow C) \equiv (((A \wedge B) \vee (A \wedge \neg B)) \rightarrow C)$ [RCEA]
3. $(A \rightarrow C) \equiv (((A \wedge B) \rightarrow C) \wedge ((A \wedge \neg B) \rightarrow C)))$ [Application of SDA]

So, it is for technical reasons that the SDA is not conserved. In fact, in natural language, this principle seems valid. The only counterexample to the SDA is not very clear:

> "If Spain had fought on either the Allied side or the Nazi side, it would have fought on the Nazi side" has not the following consequence "If Spain had fought on the Allied side, it would have fought on the Nazi side"[36]

But the premise sounds very strange (at least in French). I presented it to English native speakers who had never studied logic and they all said that this sentence is not totally grammatically correct. It stands for "If Spain had fought during the Second World War, it

[35] Lewis, *Counterfactuals*
[36] Mc Kay and Van Inwagen, "Counterfactuals with Disjunctive Antecedents"

would have fought on the Nazi side". But we can imagine that a country fought during the second world war without being on the Allied or Nazi side. For instance, to defend its boundaries, a country can decide to attack all the armies that enter in its territory. So, "Spain had fought on either the Allied side or the Nazi side" is not equivalent to "If Spain had fought during the second world war" and the counter argument is not correct.

Conversely, the RCEA has counterexamples in natural language:

We can't infer from "If this match were struck, it would light" the sentence "If this match were struck and wet or if this match were struck and not wet, it would light".

With the RCEA, "this match were struck" is equivalent to "this match were struck and wet or this match were struck and not wet". But clearly, even if we can accept the premises, the conclusion of the argumentation is false. Furthermore, the argument can take the form of the strengthening of the antecedent:

If this match were struck, it would light

If this match were struck and it were wet or not wet, it would light.

The second antecedent that we add is a tautology ($A \vee \neg A$), but in conjunction with the first, the argument is invalid. So, to accept the RCEA entails the consequence that we accept the strengthening of the antecedent with a tautology.

Anyway, this counterexample to the RCEA is also an argument for the acceptance of the SDA, because the validity of the last one explains it:

If we accept SDA, we can derive from "If this match were struck and wet or if this match were struck and not wet, it would light" the following sentence:

"If this match were struck and wet, it would light; and if this match were struck and not wet, it would light".

In this way, we isolate clearly the piece of reasoning which entails the falsity of the argument. It is the first conjunct:

"If this match were struck and wet, it would light"

So, implicitly, the SDA is used in the counterexample for the RCEA. In conclusion, we can say that from the point of view of the reasoning in natural language, the SDA has numerous advantages confronted to the RCEA. So, the first one has to be conserved and the last one to be rejected.

6.4 CS and Relevance

Another schema is also incompatible with SDA and its name is CS:

CS: $(A \wedge B) \supset (A \rightarrow B)$[37]

We can show this incompatibility by the following deduction:

B	[hypothesis]
$A \vee \neg A$	[tautology]
$(A \vee \neg A) \rightarrow B$	[CS]
$(A \rightarrow B) \wedge (\neg A \rightarrow B)$	[SDA]

It means that if there is a fact B, we can form two conditionals with B and with the first conditional which gets an antecedent A and the second conditional with the negation of the first antecedent. Furthermore, the antecedents are whatever you can choose.

For instance, you know that Mary will come to the party. So, you can say "If Mary is alive, she will come to the party and if Mary is dead, she will come to the party". Obviously, the second conditional is false. We have already said that we have a lot of reasons to accept the SDA as valid: its use is pretty natural, it has no really convincing counterexample, it enables

[37] « ⊃ » will stand for the material implicator and « → » for the intensional connective.

the understanding of the counterexamples to the RCEA. So, from our point of view, we have here a strong argument for the rejection of CS. But more internal reasons exist.

In fact, the schema CS is valid in a lot of semantics (for instance in Stalnaker/Lewis analysis). But we can find obvious counterexamples: "Chirac is the president of France and Tokyo is in Japan" is true but not "if Chirac is the president of France, (then) Tokyo is in Japan". We can notice that the conditional is true if it is interpreted with the "even if" construction but if nothing in the context indicates that this is the intended interpretation, we will interpret the connective as a "if, then" and then obtain a false conclusion. Anyway, we study here conditionals with a sufficient connection and we cannot permit the first interpretation. The default of CS is that if we have two propositions which are true, we can form a conditional with one as its antecedent and the other as its consequent whatever their relation. This is totally contradictory with the use of conditionals in natural language and also with our classification of conditionals. So, we have to reject CS, not only because it is incompatible with SDA but also because it has its own defaults.

At this point, we want to make a general remark about a lot of semantics in conditional logic. The defaults of the material implicator are the major reason for the development of conditional logic. But even if an intensional connective is defined, as in the analysis of Stalnaker/Lewis, they use the material implication as a conditional of "second level". However, they obtain the following bad consequence: imagine that "if the sun shines, then I will go to the beach" and "if Chirac is the president of France, then the RPR won the last presidential election" are true. With the material implication, we obtain that "if I will go to the beach, if the sun shines, then if Chirac is the president of France, then the RPR won the last presidential election" will be true. But there is no connection between the two conditionals. My conclusion is that we can obtain a system which respects the connection only by the total elimination of the use of the material implicator. We cannot keep it as a conditional of "second level". This problem can be seen in the schema CS. From two facts, we cannot derive the consequence that they can form a conditional with one as its antecedent and the other as its consequent.

7. THE INSUFFICIENT CONNECTION

7.1 Even as a pragmatic function

A lot of semantics assume that the treatment of "even if" conditionals must be very close to the one of "if, then" conditionals. They expect that "even if" conditionals should be explained in terms of the interaction of "even" with the connector "if". The first problem of this analysis is that in consequence, a "if, then" conditional should also be explained by the interaction of "if" with "then". We saw that in a lot of samples of use of conditional sentences, the "then" could not be employed to introduce the consequent. So, its use, as the use of the "even" makes a difference for the sense of the sentence. So, we cannot suppose these two views together: "if, then" has not to receive a particular treatment which combines the analysis of "if" and "then", because a "if, then" conditional is equivalent to a "if" conditional; and a "even if" conditional must receive a treatment of "even" applied on "if".

The usual argument for this kind of analysis is that the import of "even" doesn't change the semantic content of the conditional sentence. It means that the truth-conditions of the sentence are the same for a "even if" and a "if, then" conditional. "Even" has only a pragmatic function: it gives indication of the presuppositions or the beliefs of the speaker. We don't accept this analysis precisely because it seems to us that the use of the two different connectors makes a difference for the truth-conditions of the conditional. Let's try to show this by a simple example:

(1) If it rains, then I will go to the beach.
(2) Even if it rains, I will go to the beach.

We can divide the problems in two faces. The first is when the antecedent is true and the second when it is false.

a) The antecedent is true. In that case, the truth-conditions of the two conditionals correspond. If the consequent is true, then the conditional is confirmed by the facts. But if the consequent is false, then the predicted relation is false.

b) The antecedent is false. Contrary to the first configuration, the truth-conditions seem here different. If the consequent is true, then there is no problem for (2) because the rain was not a sufficient condition to avoid going to the beach. So, if it is sunny, the situation is better and so the consequence must really not be surprising: I will go to the beach. But for (1), the analysis is different. The sufficient condition is not fulfilled (the rain), so without more data, we expect that the consequence won't happen. So, we could say that these facts have a tendency to be contradictory with (1).

The situation is inversed when the consequent is false. (1) is confirmed by the facts. The condition is not fulfilled so the consequence is not derived. But (2) is now false. We said in (2) that the consequence will happen with, and moreover without, the rain. So, the facts are here completely against the predicted relation.

Finally, the truth-conditions of the two types of conditionals are really different when the antecedent is false. So, "even" cannot simply import a pragmatic factor to the conditional. It has effect on the truth-conditions themselves. By way of consequence, it justifies that "if, then" and "even, if" must be considered as two different types of conditionals and receive both a particular semantics.

7.2 Does an "even if" conditional entail its consequent?

We said that an "even if" conditional is used to signal that the antecedent is insufficient to avoid the realization of the consequent. So, it seems also that the negation of the consequent is a positive factor for the consequent. Then, neither the antecedent nor its negation can block the consequence. By way of consequence, it seems that this last one must happen, whatever the realization of the antecedent. This thesis is current in conditional logic. It can take several forms, for instance, let's show two:

(3) Even if A, C entails C
(4) Even if A, C is equivalent to $C \wedge (A \rightarrow C)$

But this analysis presents at least two problems. The first one is that if C is an obligatory consequence of the conditional, it would be simpler to assert only the consequent. It would be more economic. It is useless to employ a locution that gives no supplementary information. Furthermore, there is a risk of confusion. A hearer can ask himself why the speaker didn't employ the direct form. Finally, the link between the antecedent and the consequent is doubtful. We could have chosen any other antecedent because whatever is our choice, the realization of the consequent is certain.

The second problem is that counterexamples exist in natural language. Here are two:

(5) Even if he drinks only a little, he will be dismissed.
(6) Even if he is sick, he will come.

In (5), the consequent is not sure because the person can manage to become sober. In (6), we can imagine that the person is not sick but simply dead. In that case, it will be difficult for him to come.

For these two reasons, we cannot accept the thesis that an “even if” conditional always entails its consequent. It establishes a link between the antecedent and the consequent and it is not a redundant way to express a fact.

7.3 The scalarly aspect

An “even if” conditional presents a scalarly aspect. This means that it expresses a scale with two extremes. The first extreme is the antecedent. It is the limiting point up to which the realization of the consequent cannot be avoided. In other circumstances, the antecedent could be a reason to block the situation. But here, even if it is a negative factor, it is insufficient. The other extreme point is the negation of the antecedent. It is a positive factor. Let’s see an example:

(7) Even if it rains, I will go to the beach.

The rain is the lowest expected point for the realization of the consequent. For instance, if the weather is not simple rain, but a terrible hurricane, we can think that the speaker will change his mind. But evidently, if it is sunny, he will go with more pleasure.

Iatridou[38] points out that the scale exhausts the universe. This means that the antecedent and its negation are of the form $A \vee \neg A$. So, the consequent is entailed in any cases. Then, it could be an explanation why an “even if” conditional cannot include the additive connector “then”. In fact, we already saw that it is impossible in English to have the following construction:

(8) Even if it rains, then I will go to the beach.
(9) General form: Even if A, then B.

But the Iatridou’s explanation presents a default. In fact, we saw that “even if” conditionals exist which don’t entail their consequent. In that case, the predicted grammatical construction would be “even if …, then …”. But it is not the case. Nevertheless, perhaps pretending that in the majority of cases and originally, the scale effectively exhausts the universe could save this explanation. So, the grammatical construction would be based on these instances and not on the more particular one which don’t insure that the consequent is true.

7.4 Contraposition

We said that “even if” conditionals cannot admit a direct contraposition. Furthermore, it explains the same phenomena for the “if…, …” conditionals which express an insufficient connection. This fact comes from the scalable aspect of this type of conditionals. Contraposition is obtained in a conditional that expresses a sufficient connection by negating the consequent, putting it as the antecedent of the new conditional and by negating and putting the old antecedent as the consequent in the new conditional:

Contraposition: contraposition of “$A \rightarrow C$” is “$\neg C \rightarrow \neg A$”

[38] Iatridou, *“If “ then”, then what ?”*

But because the "even if" conditional hides a scale, the negation of the antecedent entails also the consequent. So, the negation of the consequent cannot entail the negation of the antecedent. Furthermore, as the negation of the antecedent is a positive factor for the consequent, contraposition is even more mistaken.

The only solution to apply contraposition to an "even if" conditional is to negate the scale before putting it as the consequent of the new conditional:

Even if A, C
Scale: $A \vee \neg A$ that we rename B
Contraposition obtained: If $\neg C$, then $\neg B$

But this solution is not always possible. It depends if the scale exhausts the universe or not. In fact, if the scale exhausts the universe, the negation of the scale is impossible. So, we have here two types of "even if" conditionals that we can illustrate:

(10) Even if Paris is in France, Tokyo is in Japan.
(11) Even if John is sick, he will come.

In (10), the antecedent and its negation exhaust the universe. So, contraposition is impossible. In fact, it means simply that we reach an extreme of the insufficient connection: there is no connection at all between the antecedent (or its negation) with the consequent. In that case, we can reformulate directly the conditional by introducing now the antecedent with "whatever":

(12) Whatever country Paris is in, Tokyo is in Japan.

In (11), the antecedent doesn't exhaust the universe because if John is dead, he will surely not come. So we have here a "real" insufficient connection between the antecedent and the consequent. Then, we cannot say that (11) would be equivalent to the following reformulation:

(13) Whatever John's health, he will come.

But we can apply here contraposition:

(14) If John doesn't come, then he is dead.

So, contraposition of an "even if" conditional depends if there is a "real" insufficient connection and the negation is applied not on the antecedent but on the entire scale.

7.5 Schemas

We will now study complex formulae for the conditionals that express an insufficient connection, as we did before for the conditionals that express a sufficient condition. We will take the same eight schemas again but the symbol for the conditional connective stands now for an "even if" conditional:

D1*: $(A \vee B) \rightarrow C \Rightarrow (A \rightarrow C) \wedge (B \rightarrow C)$
D2*: $A \rightarrow (B \vee C) \Rightarrow (A \rightarrow B) \vee (A \rightarrow C)$
D3*: $(A \rightarrow B) \vee (A \rightarrow C) \Rightarrow A \rightarrow (B \vee C)$
D4*: $(A \rightarrow C) \vee (B \rightarrow C) \Rightarrow (A \wedge B) \rightarrow C$
C1*: $(A \wedge B) \rightarrow C \Rightarrow (A \rightarrow C) \vee (B \rightarrow C)$

C2*: $A\rightarrow(B\wedge C) \Rightarrow (A\rightarrow B) \wedge (A\rightarrow C)$
C3*: $(A\rightarrow B) \wedge (A\rightarrow C) \Rightarrow A\rightarrow(B\wedge C)$
C4*: $(A\rightarrow C) \wedge (B\rightarrow C) \Rightarrow (A\vee B)\rightarrow C$

As for the sufficient conditional, some of them are valid in natural language:

D1*: Even if I'm hungry or tired, I will go home.
⇒ Even if I'm hungry, I will go home and even if I'm tired, I will go home.
D2*: Even if it rains, I will go to the beach or to the park.
⇒ Even if it rains, I will go to the beach or even if it rains, I will go to the park.
D3*: Even if it rains, I will go to the beach or even if it rains, I will go to the park.
⇒ Even if it rains, I will go to the beach or to the park.
C1*: Even if it rains and Mary doesn't come, I will go to the beach.
⇒ Even if it rains, I will go to the beach or even if Mary doesn't come, I will go to the beach.
C2*: Even if I have little money, I will buy some milk and some bread.
⇒ Even if I have little money, I will buy some milk and even if I have little money, I will buy some bread.
C3*: Even if I have little money, I will buy some milk and even if I have little money, I will buy some bread.
⇒ Even if I have little money, I will buy some milk and some bread.
C4*: Even if I'm hungry, I will go home and Even if I'm tired, I will go home.
⇒ Even if I'm hungry or tired, I will go home.

But one of these schemas is clearly not valid:

D4*: Even if it is sunny, I will run or even if I'm injured, I will run. (true because the first conjunct is true)
⇒ Even if it is sunny and I'm injured, I will run.

In comparison with the sufficient conditional, only the status of the schema C1 changes. Invalid for the first type of conditionals, it is now valid. But we don't have here the same problems (derive the invalid schema from valid schemas), because the "even if" conditional doesn't permit contraposition (or only by negating the entire scale). So, the complex formulas for the two types of conditionals are very close but not totally the same.

8. POSSIBLE DEVELOPMENTS

8.1 Automatic translation from Natural Language to Formal Language

Understanding conditionals necessitates at least two steps. The first one is the passage from natural language to formal language. Especially, we must be able, in front of a natural conditional connective, to give its formal counterpart. The second step is to associate a semantic to the formal language. To conclude completely the project, it would be useful to furnish a syntactic system and general metatheorems describing the relation between the level of semantic and the level of syntax. We will discuss here briefly the first problem: the formal translation of conditional sentences belonging to natural language.

For this task, it seems important to use, in the first place, syntactic considerations. But they are not in themselves sufficient. We have also to consider the context of the emission of the conditional. For instance, a previous discourse can give us data which will block one possible interpretation and guide us to a more coherent one. But the syntactic criteria will be considered first, and for the following simple reason: this aspect is always present, contrary to the information of the context of emission which can be irrelevant or unknown.

One recurrent problem in conditional logic is to share what phenomena belong to the semantic and can receive and formal treatment, and which are part of the pragmatic and are thus released from the realm of logic. Especially, in light of an example, one theory which can give an account of it will consider this aspect as a semantic one; when another theory unable to explain it will have the tendency to classify this factor as a pragmatic one. With this attitude, no theory is falsifiable because each counterexample will be put among the pragmatic cases. We propose here another approach. No difference will be put between the semantic and pragmatic aspects. All of them have to receive a formal treatment, even if in spirit, some of them could be seen as closer to the semantic or the pragmatic side. This study could consist of examining a very simple language for conditionals and so we don't pretend here to give a general theory which would be exempt of criticisms.

We hope that, by the examination of the syntactic markers and the previous information, we will be able to determine, in each case, which is exactly the conditional connective at task, to give it a formal counterpart. That means that we take as hypothesis the fact that strong norms exist which govern the use of conditionals by the speakers. We are studying conditional in English. It would be possible that between two different communities of English speakers, some differences appear. In that case, our project would be complicated by the addition of different translation means for the different communities. But we can hope that with the existence of written conventions a sufficient number of grammatical rules exist, which govern the use of conditionals.

Faced with a conditional sentence, it could be possible that some syntactic conventions are not respected. In that case, the only choice will be to declare the conditional as not interpretable. That doesn't mean that a competent hearer will not be able to catch a meaning. For instance, the sentence "You walking with I?" is perhaps understandable but not correct. We will require a strict respect of these conventions to obtain deterministic rules of translation, even if we know that all these rules are not always followed, especially in spoken language.

8.2 Difficulties for semantics

We will resume here the requisites for semantics which can express the ideas that we developed previously. First of all, to give an account of the three types of conditionals that we determine (connection sufficient, insufficient and necessary), we will need two different conditional connectors (the "only if" case can be reduced to the "if, then" case). We already gave the reasons for this choice. This position could be challenged if the level of treatment of our enquiry was not the propositional level. But due to the impossibility of having distinct operator like "even" or "then" which can be applied directly to propositions, we have the only choice of treating them separately. Furthermore, our syntactical considerations show that the "then" can only be present when a sufficient connection is expressed. So, we can treat "if…, then …" and "even if …, …" as separate and distinct syntactical constructions.

But we could give two objections to this analysis. First of all, its is possible to form an "even if" conditional from two "if" conditionals:

(1) If it is sunny, I will go to the beach and if it rains, I will go to the beach.
(2) Even if it rains, I will go to the beach.

We can say that (1) and (2) are equivalent. Now, the question is what is the status of these "if" conditionals that we employed in (1). They cannot be of the type of a sufficient conditional that we define, because put together, they don't admit contraposition. Evidently, they cannot express a necessary connection. Finally, the solution could be to say that both of them express an insufficient connection:

(3) Even if it is sunny, I will go to the beach and even if it rains, I will go to the beach

Even if this sentence has a grammatical construction that seems too heavy, the meaning is not changed. But the same objection can be advanced also against sentence (1). (1) and (3) are both intricate ways of formulating what we say usually in natural language by (2). So, our analysis resists to this objection.

The second problem could be the following one. We said that some "even if" conditionals can admit contraposition by apply it not to the sole antecedent but to the entire scale. It could mean that an "even if" conditional is in fact an "if…, then …" conditionals disguised. But we have reasons to avoid these considerations. Firstly, the complex formulas that we study for the two types are different. The schema C1 is valid for one and invalid for the other. So, the two types are nevertheless not the same. Secondly, some "even if" conditionals do not admit contraposition. These are the ones that express an empty connection. Syntactically, we cannot divide the "even if" conditionals into two categories. In association with the first reason, we can say that "even if" conditionals form a distinct type of conditional sentence.

Finally, the principal problem for a semantics that can express our ideas is the problem of the complex contraposition for the sufficient conditionals. We admit contraposition applied to simple conditionals but not when two conditionals linked by a disjunction possess both a common antecedent or consequent. So, in front of this type of sentence, the semantic analysis must be applied first at the entire sentence and not to its parts. Then, it seems difficult to respect the principle of semantic compositionality, which is the base of the most current theories in logic. Also, the rejection of the principle RCEA entails the fact that we cannot employ intentional semantics. Furthermore, the particular behaviour of the disjunction entails further difficulties to formalize our conceptions.

9. CONCLUSION

The primary purpose of this paper was to provide a classification of conditional sentences. We managed to find three different types that express a sufficient, insufficient and necessary connection between the antecedent and the consequent. They correspond to the syntactically marked constructions which follow: "if…, then …", "even if…, …" and "only if …, …", respectively. The unmarked construction "if …, …" can receive an interpretation in terms of sufficient and insufficient connection; and perhaps even in extreme cases an interpretation in terms of necessary connection. Furthermore, the schemas that are validated in natural language are very close for the two principal types of conditionals but are not totally similar. We saw also that even if the conditionals that express a sufficient connection admit contraposition for simple conditionals, it is not always the case in groups of conditionals.

We did not engage in this work to determine which semantics could render these ideas**,** but this enquiry itself could be extended by the examination of the schemas of the "only if" conditional. That was not done here, because the "if…,…" conditional accepts this interpretation very seldom and the "only if" can be reduced to the "if, then" case. To be complete, an examination of the problems coming from the embedding conditionals could also be carried out. However, we hope that this study contains a sufficient number of philosophical and linguistic insights to inspire a more technical study.

10. BIBLIOGRAPHY

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