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A Case Study on the Paradox of Free Choice Permission

— Master Thesis —

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Contents

1 Introduction

Certainly one the most influential pragmatic theories of the 20^{th} century is the theory of conversational implicatures proposed by Grice [1989]. It has been applied to various semantical problems, inspired many developments in pragmatics, and also received attention in other areas like philosophy and social sciences.

Grice developed the theory as a reaction to the growing bulk of criticism brought forward against the fundamental idea of formal semantics: to describe natural language semantics by mapping natural language to some formal language for which a precise and well-studied semantics is given. The wide range of problems which classical truth-conditional semantics had to face led an increasing number of scholars to question this logicistic program.[1] Grice proposed to rescue the logical stance towards semantics by explaining the critical data as due to additional, pragmatic interpretation mechanisms (this enterprise will be called in the following *the Gricean Program*).

Making pragmatics responsible for the problems truth-conditional semantics had to face asks, of course, for a powerful theory of pragmatics that can account for the critical observations. This role Grice intended his theory of conversational implicatures to play. He claimed that besides semantical conventions there are also rationally motivated rules that govern our use of language. According to him these rules, the *maxims of conversation*, are not arbitrary but determined by the common interest of the participants to maximize the gains of communication. The inferences that can be derived on the assumption that the speaker behaves according to the maxims are called *conversational implicatures*.

Irrespective of its popularity, the theory of conversational implicatures has also been under constant attack. A central objection is that the account lacks precision. The resulting vagueness in its predictions makes it impossible to evaluate the question whether Grice's proposal *does* rescue truth conditional semantics from its threads. One of the main goals of pragmatics has been to overcome this defect and to propose a more detailed description of the class of conversational implicatures (e.g. Horn [1972], Gazdar [1979], Hirschberg [1985]). However, a completely satisfying proposal in this direction is still missing. This raises the question whether a precise formulation of the theory is possible at all[2]. Perhaps Grice's attempt to save classical truth-conditional semantics may have only shifted the problem to the realm of pragmatics. Now, it is this part of interpretation that lacks a logic.

However, there are good reasons to believe that the mentioned attempts to improve on the clarity of Grice's theory did not exhaust their possibilities. When looking at the proposals made it catches the eye that a rather limited set of technical tools are used to formalize Grice's theory. The main role is still played by classical deductive logic; the logic of Frege and Tarski. But also logic has had its revolutions since their times, among them the development of non-monotonic reasoning.

Non-monotonic logics were invented in particular to describe common sense reasoning,

[1] A similar development can be observed in cognitive science half a century later. There a lot of evidence seems to show that not only language but human reasoning in general is not driven by exact logic; see, for instance, the argumentation of van Lambalgen & Stenning [2001] contra such a conclusion.

[2] See the very sophisticated discussion by Davis [1998] who indeed argues that Grice's theory of conversational implicatures cannot be made precise.

especially to account for the strange fact (logical speaking) that in every day life people jump to conclusions even if they are not absolutely reliable. This has the consequence that when later on evidence occurs that shows the conclusions drawn to be false, then, of course, these 'rash' conclusions have to be abandoned. This property, that one loses inferences if they stand in conflict with more reliable information, also gave those logics their name: *non-monotonic*. In this respect they clearly differ from classical deductive logic where inferences once obtained will always be valid no matter how much information is added.

Non-monotonicity has always been considered to be a central feature of conversational implicatures.[3] This suggests that techniques developed in non-monotonic logic may be of use to formalize the theory of Grice. However, this possibility has been widely ignored until now in the research on conversational implicatures.[4] This is even more surprising given that there are development in this area that bear a close connection to the theory of Grice. For instance, the notion of *only knowing* developed by Halpern & Moses (1984) and recently generalized by van der Hoek et al. (2000) shows a strong relation to one of the maxims postulated by Grice: the first submaxim of Quantity.

The goal of the research presented in this thesis was to investigate in how far tools of non-monotonic logic can help to make the theory of conversational implicature precise and the Gricean Program successful. However, to pursue this aim with respect to Grice's theory and its adequateness in general would not have been achievable within the limits of a master thesis. Therefore, we will concentrate on one particular phenomenon that challenged classical truth-conditional semantics and study the questions whether the notion of conversational implicature can be made precise insofar as to account for this concrete observation.

The phenomenon investigated is *free choice permission*. As has often been observed (e.g von Wright [1969] and Ross [1941]) that an utterance of (1a) intuitively entails (1b) and (1c): on hearing (1a) we will conclude that the addressee is allowed to take an apple and she is allowed to take a pear (although not both at the same time). She has *free choice* between both fruits.[5]

(1) a. You may take an apple or a pear.

b. You may take an apple.

c. You may take a pear.

The discussion that arose around the treatment of this observation mirrors to a great extend the general problems truth-conditional semantics got into and from which Grice intended to rescue it: It appeared to be impossible to give formally precise meanings to the involved expressions in a way that can account for the free choice reading - at least, not without making unacceptable predictions in other respects. Therefore, von Wright decided

[3]However, in the linguistic literature they do not refer to this property as *non-monotonicity* but call it instead the *cancellability* of conversational implicatures. This terminus has been also used by Grice himself.

[4]An exception is the dissertation of J. Wainer [1991], who tries to formalize scalar implicatures using McCarthy's [1986] notion of circumscription. In Levinson [2000], it is pointed out that using such techniques may be very useful for the problem of formalizing these inferences but this idea is not worked out further.

[5]Even though free choice permission is a phenomenon that does apply to a wide range of constructions we will concentrate here on English sentences containing the connective 'or'.

to speak of the *paradox* of free choice permission, and the question was raised whether one should not conclude that the way English speakers (and the same observation is made in many other languages too) understand (1a) simply has no logic.

In this thesis we will take a Gricean stance towards the problem. Hence, we will not try to unmask the paradox as illusion by proposing an adequate semantical description of the sentence, but instead look for a Gricean, pragmatic explanation for free choice permission. We will additionally follow Grice insofar as we try to keep the semantics of (1a) simple and close to classical truth-conditional semantics.

In sum, the thesis pursuits two different but strongly connected aims. First, it tries to account for a notorious problematic observation concerning the interpretation of certain sentences of English: the paradox of free choice permission. More particularly, the intention is to account for this observation in terms of Grice's theory of conversational implicatures. Secondly, before this theory can be used first it has to be made precise. Here, the intention is to provide a (part-wise) formalization using results from non-monotonic logic.

The thesis is organized as follows. In the next section we will start with a more extended discussion of the phenomenon of free choice permission. The aim is to get a good understanding of the phenomenon that has to be described. Also two classical accounts to the problem will be discussed that do not only constitute central pillars in the literature on this topic but also build the fundament of the approach developed here: the extensive discussion of free choice permission by Hans Kamp and the proposal of Ede Zimmermann. Afterwards, in section 3, a part-wise formalization of Grice's theory of conversational implicatures is proposed. Building on the work of Halpern & Moses [1984] a pragmatic notion of entailment is introduced that is intended to describe the conversational implicatures due to the first submaxim of Quantity and parts of the maxim of Quality out of Grice's inventory of conversational maxims. The inferences obtained this way essentially come down to Gazdar's [1979] *clausal implicatures*. We will show that such a notion of entailment together with an assumption of *competence* of the speaker - and here we build strongly on Zimmermann [2000] - allows the interpreter to derive the free choice permission. In section 4 we will discuss the proposal of the previous section. As it will turn out, the approach has to fight with one central shortcoming: it over-generates. Some ideas where to look for a solution of this problem will be introduced but a detailed investigation has to wait for another location. We finishes with a section on conclusions and outlines of further research.

2 Classical Approaches to the Free Choice Paradox

As stated in the introduction we will concentrate in this thesis on the question how to account for the phenomenon of free choice permission. This section is dedicated to a closer investigation of the problem we have to solve. Furthermore, a selection of approaches to the paradox of free choice permission will be discussed that were of great influence to the account presented here.

The reader might remember from the introduction the example used to clarify the notion of free choice permission. The critical observation was that an utterance of (2a) intuitively entails (2b) and (2c): on hearing (2a) we will conclude that the addressee has the permission to take an apple and she has the permission to take a pear.

(2) a. You may take an apple or a pear.

b. You may take an apple.

c. You may take a pear.

The challenge for a semanticist who faces this observation is now to assign meanings to the expressions involved in (2a), describe how these meanings are to be combined and perhaps other processes involved in interpreting this utterance such that finally she can correctly predict the way English speakers understand sentences involving these expressions. In particular, a notion of entailment $|\equiv$ between sentences of English has to be definable in the theory of interpretation (probably depending on the context) such that the following relation holds:

(FCP) (2a) $|\equiv$ (2b) $\wedge$ (2c)

However, this seems to be not at all a simple task. Traditionally, '*or*' receives the same semantics (the same truth conditions) as the corresponding logical operator $\vee$ in classical logic: for two sentences '*A*' and '*B*', '*A or B*' is true in case at least one of the both sentences holds. This truth function is called an interpretation as *inclusive disjunction*.[6] But if one assumes '*or*' to take wide scope over '*may*' (and, hence, that the surface structure of (2a) is due to ellipsis), then the inference (FCP) will not be valid. It would hold, however, if in this wide scope analysis '*or*' were interpreted as conjunction. That led some scholars to suggest that in this context '*or*' has indeed a conjunctive interpretation (e.g. Jennings [1994]). But notice, that there is a difference between the interpretation of (2a) and the one of (3).

(3) You may take an apple and a pear.

While the latter allows the addressee to take both (at the same time), (2a) certainly excludes this possibility. Hence, '*or*' has to make its own contribution to the interpretation

[6] This is, thus, the semantic analysis Grice would like to defend with his theory of conversational implicatures.

of (2a) and cannot be simply taken to be semantically the conjunction in this context (as Jennings [1994] does propose).

Let us do a step backward. We observed that assuming '*or*' to take wide scope over '*may*' in (2a) and interpreting it as inclusive disjunction does not allow us to account for the inference pattern (FCP). Still, a classical approach to '*or*' could be rescued by claiming that '*may*' has wide scope over '*or*' in (2a) and then proposing a logic of '*may*' that is operating on the disjunction in its scope in a way that the free choice inferences becomes valid. Unfortunately, there are some problems with such a strategy. First, it has been observed by different authors (e.g. Zimmermann [2000]) that a parallel sentence giving '*or*' explicitly wide scope (see (4)) does also allow a free choice permitting reading.

(4) You may take an apple or you may take a pear.

Second, as we will see in this section, several attempts to make the meaning of '*may*' precise were not able to give an explanation of the free choice effect in a semantical way and, even more, the question has been raised whether such a way of proceeding can be successful at all. A very cautious approach to the meaning of '*may*' is to try to axiomatize it by a list of intuitively valid statements and rules of derivation and then look for a sound and complete semantic. However, it seems to be very difficult to find a system of logic that captures the observed inference. The system either has no theorem $may(p \vee q) \rightarrow may(p)$ or, if we add the inference as an axiom, produces unintuitive consequences. For instance, it seems natural to assume a theorem $may(p) \leftrightarrow \neg\ must\ (\neg p)$, but together with the free choice axiom we can now derive by some simple assumptions about the behavior of $\rightarrow$ and the axiom $\neg\neg p \leftrightarrow p$ that $must(p) \rightarrow must(p \wedge q)$, which is absurd. Because of apparently fundamental misfit between how the logic of '*may*' is supposed to look like and the way we understand and reason with sentences such as (2a) von Wright [1969] spoke of a *paradox of free choice permission.*

Although we will restrict us in this work to free choice readings of utterances containing the sentence connector '*or*', it is important to notice that '*or*' does not seem to stand alone with many of its strange interpretation properties, among them the one discussed here. It shares a pattern of variance in interpretation with a whole class of expressions of English and this holds even cross-linguistically. This class can be roughly described as expressions standardly analyzed as existence quantifiers. In (5) some examples are given.

(5) Du darfst dir einen meiner Stifte nehmen.
Du darfst dir irgendeinen Stift aus der blauen Tasche nehmen.
Du kannst irgendwann morgen vorbeischauen.

The conclusion we have to draw from this observation is that, on the one hand, and fortunately, if we have a solution for the puzzle of '*or*' this answer may be extended immediately to a whole class of problems. On the other hand, it also means that if our solution cannot be generalized, we have to motivate why we need an inhomogen analysis.

2.1 Performative Approaches

An extensive discussion of the problem of free choice permission can be found in two papers ([1973] and [1979]) of Hans Kamp. In his first paper on this topic, Kamp claimed that the paradox of free choice is due to the illegitimate step of reasoning with a descriptive logic on performative utterances. Thus, the mistake made by earlier attempts to find a semantics for '*may*' that can account for the problematic inference was to use the wrong kind of semantic theory to describe the meaning of permissions as (2a). In turn, the thesis is that if one takes the right speech act - sensitive semantics the free choice inference will fall out as semantic consequence. Given this position, the program of Kamp was to develop an extension of recursive truth-conditional semantics to a semantics of permissions and to show that the latter accounts for the free choice reading.

Kamp starts with reviewing the work of Lewis [1970] who has given a model-theoretic analysis of the illocutionary force of permission and obligation sentences. The utterance of such sentences in a context w is interpreted as an action changing the deontic options left to the hearer (slave) by the speaker (master) in w. The deontic options are modeled by the set of worlds in which the hearer fulfills all the restrictions previously imposed on him by the speaker. Obligations, $obl\ \psi$, are now interpreted as imposing additional restrictions on this set: the options shrink to those worlds where ψ holds (i.e. to those where its truth-conditions are fulfilled).[7, 8] Permissions, on the other hand, extend the deontic options of the hearer. By uttering a permission $per\,\psi$ some of the worlds where ψ holds become deontic options of the hearer. This adds a non-monotonic element to the logic of permission: earlier valid obligations will be lifted and become invalid in turn.

In this setting Kamp sketches the following solution of the choice paradox. Sentences of the form '*NP may VP*' are treated as the sentence $per([NP\ VP])$ in the formal language of Lewis.[9] Kamp proposes that the notion of inference also changes with the speech act. He defines a new notion of entailment $\rightarrow_P$ for permissions: a permission sentence $per\,\phi$ entails a permission sentence $per\,\psi$ if in all contexts the latter is redundant after executing the former. $per\,\psi$ is redundant after $per\,\phi$ if all options that $per\,\psi$ would add are enclosed in the options added by $per\,\phi$. Let $[per\,\phi]^p_w$ be the set of worlds added to the options by uttering the permission $per\,\phi$ in context w, then $per\,\phi$ permission-entails $per\,\psi$ ($per\,\phi \rightarrow_P per\,\psi$) iff$_{def}$ $[per\,\phi]^p_w \supseteq [per\,\psi]^p_w$. This notion of entailment is now proposed by Kamp to be responsible for the free choice inferences. Hence, what he has to show is that according to the proposed account for the meaning of permissions the options added by the permission of a disjunction $per(\phi \vee \psi)$ have to enclose the options added by the permission of $per\,\phi$ and $per\,\psi$.

It is clear that whether the idea works depends on how the set of options contributed by a new permission is defined. But, as the title of his article signals, Lewis observes that this is no trivial question to solve. Suppose the options are restricted by the orders to clean the house and not to drink any beer from the fridge. Now the speaker permits the hearer to drink a beer from the fridge. We can in reaction not add all worlds where the hearer drinks

[7]... in some nearby future. We slightly simplify he sophisticated model theory used by Lewis and Kamp. They take deontic options to be variants of the actual world (they have the same history) where all obligations are fulfilled in some nearby future.

[8]This is the same kind of operation that an assertion executes in dynamic semantics on the common belief state in the utterance context w.

[9]However, no connection is made by Kamp between '*may*' and per.

a beer from the fridge to the option sets. This is so, because they also include worlds where she does not clean the house, for instance. So, how is the extension operation restricted?

The idea on which Lewis and Kamp base some of the answers they consider is that we have to add only worlds that are maximally close to the previous option set. The induced change on the deontic alternatives has to be in some sense minimal. Different approaches were discussed by Lewis, Stalnaker and Kamp to make this idea more concrete but they generally fit in one formal pattern: the update is modeled as contraction with respect to an absolute, transitive and irreflexive order $<_P$ on the set of worlds.[10] This order is intended to closeness to the deontic options of the hearer. The deontic options, in consequence, constitute the minimal elements of the order. A permission $per\,\phi$ is interpreted as adding the closest worlds making ϕ true to the set of deontic options: $[per\,\phi]^p_w = \{w_1 \in [\phi]_w | \neg\exists w_2 : w_2 <_P w_1\}$.[11]

The central question such an approach raises is how to specify the order, which, in turn, determines the properties of the function $[\cdot]^p_w$. Stalnaker suggested to define it as *reprehensibility*: how much the hearer deviates from the optimal conformistic behavior. This would mean that a permission $per\,\phi$ adds the set of least reprehensible ϕ-worlds to the deontic options. Unfortunately, the notion of '*reprehensibility*' is still too vague to make concrete predictions. One may interpret it as counting how many previously imposed obligations the hearer breaks in a world - an idea behind database accounts independently developed by Lewis [1970] and Kamp [1973] and generalized in the work of Harper [1976].[12] Kamp gives a different utility oriented interpretation of reprehensibility: according to him, permissions can have different weights depending on their costs for the speaker. But no matter which elaboration is chosen, the question is: can it account for the free choice inferences?

It is easy to see that a minimal contraction account based on any transitive, irreflexive and total order $per(\phi \vee \psi) \rightarrow_P per(\phi) \wedge per(\psi)$ gives us exactly in case the minimal worlds of each disjunct are all of equal distance from the optimum.[13,14] Hence, the free choice permission is not generally valid for minimal contraction of the option set. However, it may be the case that the specific order chosen to model change on deontic options allows to derive the choice principle in those cases where it actually occurs. But neither of the ideas on what reprehensibility may mean we discussed above will have this property. For instance, Kamp claims that there are contexts where it is intuitively more reprehensible to take a pear than to take an apple but where utterances of (2a) would still allow for free choice

[10]The terminus *contraction* is taken over from the literature of belief revision. A *contraction* is an operation defined on a preference structure: a class of models $\mathcal{M}$ together with an order. Applying the operation to a sentence ϕ gives you the minimal elements among the subclass of $\mathcal{M}$ where ϕ is true.

[11]That this idea was so present in the discussion is not surprising: just a few years before Lewis had published his very influential book on the semantics of counterfactuals in which he developed exactly this kind of preferential interpretation mechanism.

[12]Harper himself did not apply his system to permission sentences. Only many years after the publication of Harper's work van Rooy [2000] made an attempt in this direction.

[13]That means, no worlds where ϕ is true is more minimal than any world where ψ is true and the other way around.

[14]We restrict our considerations here to the narrow scope analysis, which is considered by Kamp. Of course, one could also think about the behavior of a wide scope disjunctions $per(\phi) \vee per(\psi)$ in such a setting, which would come with its own problems. For instance, what should it principally mean to utter a disjunction of two performatives?

permission. Hence, his utility based perspective on the order will make false predictions. And, as van Rooy [2000] points out, neither the order introduced by Harper [1976] makes the right predictions. We have to conclude that these theories about the meaning of (2a) cannot account for the free choice inference.

Facing the problems of his proposal, Kamp introduces a new kind of explanation for free choice permission: maybe it is not due to language conventions of how to interpret permissions, maybe it is due to reasoning on rational behavior in conversational contexts - maybe it is a Gricean conversational implicature! Hence, Kamp adopts the strategy of the Gricean program: rescue semantics by letting pragmatics solve the problems. The difference is that it is Kamp's performative permission semantics that has to be saved now.

Kamp discusses a derivation via the maxim of Brevity, one of the rules that, according to Grice, is obeyed by rational and cooperative speakers. It asks the speaker, roughly, to keep his contributions short. The derivation of free choice permission Kamp proposes based on Brevity runs as follows: if it was the intention of the speaker to allow only one of the disjuncts (as Kamp's semantics would predict in case the wrong results are obtained) there would have been a shorter expression the speaker could and, following brevity, should have used, namely (2b) or (2c). However, she did not. Hence, this cannot be the intended interpretation of the utterance. But so far, the reasoning brings us only half the way to the free choice inferences. Now, Kamp continues, we have to rationally motivate why a free choice permitting reading should be chosen as the intended interpretation, instead.[15] Kamp does not provide an answer to this question, but one may think of the following continuation of the argument:[16] the hearer considers the possibility that she based her interpretation on the wrong ordering of worlds and tries to figure out the right one. But she cannot take the order to prefer the other disjunct instead because then the same argument would apply again. Hence, she concludes that the speaker based her utterance on an order where both disjuncts are equally reprehensible.

But besides the objection that it is difficult to motivate why the interpreter should think that the mistake sits precisely in her idea of the order, there is something strange with Kamp's explanation in general. As Kamp [1973] notices himself and as was also pointed out in the introduction, the surprising properties of the interpretation of '*or*' are displayed by other quantificational expressions as well. A sentence like "*You may pick any flower.*" also receives a free choice interpretation. But here Kamp's complexity argument will obviously fail: listing the kinds of flowers that may be picked will raise the length of utterance. We conclude that also this Gricean attempt to derive the free choice inferences based on a performative semantics is not very convincing.

If this were not bad enough already, the performative approach to sentences containing '*may*' has to face other problems as well. Merin [1992] discusses extensively the perspectives of an account based on a contraction function and presents a whole list of objections. Among other things he raises a question that often goes unnoticed in discussing the behav-

[15] This is the big problem any derivation of Brevity implicatures has to face: they easily exclude certain interpretations but have problems in singling out the interpretation obtained instead. Already Grice notices that and suggests that brevity implicatures are actually more conventional than conversational.

[16] An alternative has been suggested and criticized by Merin [1992].

ior of '*or*': do the approaches brought forward also cope with other coordinators, especially with '*and*' ? As he points out, there is a general problem with the minimal contraction approach when it comes to interpret conjunctive permissions. According to it, a permission $per(A \vee B)$ adds to the options of the addressee the closest worlds where $A \wedge B$ is true. Hence, all the options added this way will be such that both conjuncts are true. The *package deal* reading is obtained: no matter what the order is the hearer is not permitted to do one thing without the other. While there is a reading of (6) that can be paraphrased as '*You may take an apple provided that you also take a pear, and vice versa.*' this interpretation only seldom occurs.

(6) You may take an apple and a pear.

The dominating interpretation of sentences as (6) seems to be that the speaker allows the addressee to choose between four options (e.g. van Rooy [2000]): to take both, an apple and a pear, to take either or to take neither of them. No account using contraction will get rid of the package deal prediction.[17]

To the difficulties for the contraction approach to permission sentences we discussed above, others have to be added that concern performative approaches in general.

First, permission sentences, especially the examples under consideration, are in the indicative mood. Given that this mood normally corresponds to assertion one should give the constative approach a serious try before the dominant speech act - sentence mood parallelism is broken. Furthermore, Kamp observes that the sentence (2a) has also a reportive, and thereby assertive use, and even then a free choice reading can occur. Thus, we have to come up with an additional explanation for free choice readings of assertive utterances.

Another problem is that the free choice reading is not restricted to sentences containing *'may'* but seems to generalize to other types of disjunctions as well. Kamp's examples are (Kamp [1979], p. 281)

(7) a. We may go to France or stay put next summer. (with the epistemic reading of '*may*')

b. I can drop you at the next corner or drive you to the bus stop.

c. If John had come to the party with Alice or with Joan we might have had some fun.

[17]Notice, that this is above all a problem for a narrow scope analysis: when the performative update works on the permission $per(\phi \wedge \psi)$. One may think about solving the problem by proposing (i) that in case the reading without package deal occurs the conjunction has wide scope over the permission (but remember that Kamp does not make any connection between *per* and '*may*'), and (ii) that a conjunction of performatives is interpreted as successive execution of the subordinated permission sentences. Of course, this can only be implemented in a system where permission/obligation sentences do not only update the deontic options but the whole order on which the contraction operation for permission sentences is defined (as, for instance, the approach of Harper [1976]). However, while we would get rid of the general prediction of the package deal reading this way, whether we get the dominant four - choices - reading described above would still depend on the specific order we chose.

hence, the free choice inference is not bound to a specific speech act. It may even occur with sentences that seem to have primary assertive use. This is no principal argument against a performative approach, but it teaches us that the explanation given for choice inferences has to fit in a more general frame.

A stronger point against performative accounts is the following. Reportive uses of permission sentences do not always allow for free choice inferences, especially not when the utterance is continued as in (8) or if for other reasons it is known that the knowledge of the speaker on the topic of interest is limited.

(8) You may take an apple or a pear - but I don't know which.

In consequence, if we try to describe free choice in reportive sentences we have to make sure that the inference is not always derived and its occurrence is bound in some sense to what is known about the *competence* of the speaker. This could be ignored by an account that only tries to deal with performative permission sentences. However, Kamp claims that there are also permissions that have no free choice inferences, for instance (9), used by a father addressing his son.[18] (Kamp [1979], p. 271)

(9) You may go to Shoal Creek or go to Shingle Creek. But stay away from the dangerous one.

Kamp suggests the following explanation for the difference in reading. In case we observe free choice permission the act in each disjunct is permitted. In (9), on the contrary, the disjunction as a whole is permitted and extends the deontic options. He considers the possibility that a scope ambiguity serves as a basis for this difference in interpretation. Again, Kamp is not satisfied with this analysis, among other points for the following reason. Utterances of (4) repeated here as (10), for which an underlying narrow scope logical form would be difficult to motivate, prominently get the incompetence and not the free choice reading while Kamp's proposal takes this logical form to underly the free choice interpretation.

(10) You may take an apple or you may take a pear.

Finally, also a serious gap in every performative proposal Kamp discusses speaks against his way of approaching the problem: there is no interpretational contribution of '*may*'. One may think that '*may*' is the part of the sentence that makes it a permission, but Kamp never brings the performative back to the occurrence of this verb, nor does he derive the use of the performative interpretation function compositionally from this part of the structure.[19] So, we are faced with the very strange situation that a systematic ingredient of the utterance form of permissions does not contribute to the interpretation.

[18]Merin [1994] supports the analysis of (9) as performative by pointing out that (9) can be put in the imperative mood, while '*may*' sentences continued with '*... but I don't know which.*' resist such a transformation.

[19]Perhaps this is so for the following reason. If '*may*' would contribute compositionally the imperative mood then the performative interpretation function would only play a role in the scope of the verb. But what is then the interpretation function applied on top of the whole sentence?

2.2 Constative Approaches

Facing all these difficulties, Kamp finally gives up his initial position to account for the paradox of free choice permission by a performative interpretation of sentences as (2a). Instead, he tries to arrive at the choice inferences by taking the involved utterances to be assertions coming with truth-conditional meaning. '*May*' is now interpreted as an existential modal operator claiming that among the deontic options of the hearer in a world w there is an option where ϕ holds.[20]

Also for this constative and modal approach Kamp has to observe that the free choice principle does not hold simply by the semantics outlined above: based on the classical notion of entailment for assertions it is still not valid that (2a) entails (2b), for instance, no matter whether a wide or narrow scope analysis of the disjunction with respect to the modal operator is taken. Again, Kamp considers the possibility to get the inference as a conversational implicature using the maxim of Brevity. The situation is slightly different from the one we considered in the last section when he tried to rescue the performative account by a Gricean argument - simply because different meanings are assigned to (2a). With respect to the performative proposal, the interpreter had to face a maxim clash: the meaning predicted by semantics was not compatible with the maxim of Brevity. Considering the constative interpretation, there is no obvious shorter expression to express the assigned truth conditions. This time Kamp claims, that, in a first step, by brevity all possibilities can be excluded where the speaker knows what is the single disjunct that is permitted. The reason Kamp gives is that if the speaker had this knowledge she should have used a shorter expression. Hence, Kamp here uses brevity not as demanding the speaker to code the meaning she wants to convey the shortest way possible, but to do so with her knowledge. The interpreter learns that way that the actual world has to be such that the speaker takes every disjunct to be possibly permitted. Kamp now claims there are only two situations where this can be the case: (i) the speaker is incompetent, or (ii) the speaker is indifferent.[21] If it is known by the interpreter that the former is not the case, then she infers that the speaker is indifferent and hence that every disjunct permitted.

The free choice inference here becomes a context dependent phenomenon bound to what is known by the interpreter on the competence of the speaker. This picture fits the observations with which we finished the last section much better. But again, the proposal lacks details. Kamp notices this but does not give any elaboration of his ideas, instead he immediately presents arguments that make the correctness of such a Gricean account questionable. He observes that the choice inference can also occur if the permission sentence is embedded in a complex construction as in (11) and can even contribute to the semantic meaning of this construction.

[20]We learn not much about the character of the accessibility relation Kamp assumes besides that it is reflexive: the actual world is among the deontic options. Hence, he only allows consistent sets of obligations to be imposed on the hearer.

[21]Given Kamp's view on assertive permissions (they report that the performative act has occurred) one may question his decision to contrast incompetence of the speaker with indifference instead of with competence. It is here much less clear why the indifference of the speaker will warrant that the permission has actually been given. This is more obvious in case reporting permissions is not about previously executed acts of permitting but for instance, about reporting preferences of the speaker. The force of permissions may then be due to a sharing of preferences between speaker and hearer that determines an authority relation.

(11) Usually you may only take an apple. So if you may take an apple or take a pear, you should bloody well be pleased.

How can a pragmatic process as the concept of conversational implicature be that deeply involved in the calculation of the semantic meaning of (11)? Other counter arguments can be brought forward. Remember the observation made in the last section: the free choice inference also occurs with utterances as *'You may take any apple you want'*. Again, Kamp's argumentation using Brevity would not generalize to these cases. Furthermore, Brevity cannot do the job Kamp assigns to it. According to Kamp's derivation the utterance has to be a maximally short expression of the *knowledge* of the speaker. But Brevity, as formulated by Grice, only asks the speaker to efficiently code the meaning she *wants* to convey. This maxim can, hence, not be the reason why the speaker should intend to communicate all she knows.[22]

To summarize, Kamp left us with a broad and sophisticated discussion of what does not work when one tries to account for the paradox of free choice permission and provides a much more general picture of the phenomenon we have to describe. However, he also left us the problem.

Several years later another very interesting constative approach to the problem has been brought forward by Ede Zimmermann [2000]. A central feature of his account is a more general perspective on the phenomenon. Zimmermann aimed at an account of the free choice inferences that can also deal with (7a). He even added to the list of phenomena that have to been explained the observation that a simple factive sentence involving '*or*' as in (12) allows to infer (in an honest conversation) that the speaker takes both conjuncts as epistemic possibilities.

(12) Peter or Marie took the beer from the fridge.

Zimmermann's approach to the free choice inferences is based on two fundamental claims.

- Semantically, *'or'* denotes a conjunction of epistemic possibilities which he speaker sees[23], (hence (12) is interpreted as stating that *the speaker thinks it possible that Peter took the beer from the fridge* <u>*and*</u> *the speaker thinks it possible that Marie took the beer from the fridge*).
- The choice principle is a pragmatic inference based on the assumption that the speaker is competent.

These two assumptions together allow him to account for the free choice inferences based on a logical form where '*or*' has wide scope over the modal operators. Let's see how this works.

[22] But for this demand other conversational maxims can be made responsible. Actually, this will be the fundament of the approach presented in the third section of this thesis.

[23] Zimmermann does not make the restriction to the belief state of the speaker, but we can ignore this aspect for our purposes.

'*May*', shortened as $\triangle$, and '*might*', shortened as $\diamond$, are analyzed, as in Kamp [1979], as unary modal operators. Hence, the formal system in which the proposal is situated is modal (predicate) logic. The operators are interpreted with respect to accessibility relations[24] $R_\triangle$ and $R_\diamond$, respectively. $R_\diamond$ connects a world w of a model M with all words the speaker takes as epistemically possible in w. Zimmermann does not model belief but knowledge, hence, in every model $M = \langle W, R_\triangle, R_\diamond, V\rangle$ every world is among its epistemic alternatives: $\forall w \in W : \langle w, w\rangle \in R_\diamond$. $R_\triangle$ connects w will all deontic alternative states of affair. Remember, that truth is defined in modal logic with respect to a state: a tuple of a model M and a world w in the set of worlds W^M in M. For a sentence ϕ the truth conditions of $\triangle\phi$ are defined as follows: $M, w \models \triangle\phi$ iff $\exists v \in W_M : \langle w, v\rangle \in R_\triangle \wedge M, v \models \phi$. Truth of $\diamond\phi$ is defined analogously: $M, w \models \diamond\phi$ iff $\exists v \in W_M : \langle w, v\rangle \in R_\diamond \wedge M, v \models \phi$.

Now, we can formalize Zimmermann's semantic analysis of '*or*' in the following way:

$$M, w \models \phi \textit{ or } \varphi \Leftrightarrow_{def} M, w \models \diamond\phi \wedge \diamond\varphi.$$

Zimmermann formalizes competence using Groenendijk & Stokhof's [1984] definition of *exhaustive knowledge*:

> A speaker is *competent* with respect to a predicate P (with domain D) in a state $\langle M, w\rangle$ iff$_{def}$ $\forall v \in W : wR_\diamond v \rightarrow (\forall x \in D : v \in P(x) \leftrightarrow w \in P(x))$.

From the definition of competence Zimmermann derives the following AUTHORITY PRINCIPLE: *If a speaker is competent with respect to predicate P in state $\langle M, w\rangle$ then it holds:* $\forall x \in D : (\exists v \in W : wR_\diamond v \wedge v \in P(x)) \rightarrow (\forall v \in W : wR_\diamond v \rightarrow v \in P(x))$. That means, if a speaker competent with respect to P takes it as possible that some object has property P then she knows that the object has the property.[25] For the derivation of the choice principle Zimmermann instantiates P with the set of deontic options of the hearer in w, the set $\{v \in W | \langle w, v\rangle \in R_\triangle\}$. For a speaker competent with respect to this set, the authority principle says that if she takes a certain world to be possibly a deontic option then she knows that it is a deontic option.

Now, we have everything in place to face the choice inferences again. We start with the plain disjunction A *or* B. By the semantic interpretation rule the sentence is equivalent to $\diamond A \wedge \diamond B$ - what, given the introduced semantics, immediately accounts for the interpretation of (12). Next, we apply the account to the example (2a) containing a deontic modality. $\triangle A$ *or* $\triangle B$ is semantically interpreted as $\diamond\triangle A \wedge \diamond\triangle B$. If the speaker is competent with respect to the deontic alternatives of the hearer in state $\langle M, w\rangle$, and, hence, the authority principle holds we can infer $M, w \models \neg\diamond\neg\triangle A \wedge \neg\diamond\neg\triangle B$. Now, Zimmermann uses his assumption that the beliefs of the speaker are correct (reflexivity of $R_\diamond$) to make the step to $M, w \models \triangle A \wedge \triangle B$, the free choice inference.[26] Finally the epistemic case: (7a). $\diamond A$ *or*

[24] Binary relations on the set of worlds W_M of a model M.

[25] The derivation of the authority principle uses both directions of the equivalence in the definition of competence.

[26] The author does not understand why Zimmermann takes the detour via knowledge. The inference $\diamond\triangle\phi \rightarrow \triangle\phi$ is simply the "$\rightarrow$" direction of the definition of competence that he already uses in deriving the authority principle. Hence, reference to reflexivity is not necessary to account for the free choice interpretation.

$\Diamond B$ is semantically equivalent to $\Diamond\Diamond A \wedge \Diamond\Diamond B$. To allow a derivation from $w \models \Diamond\Diamond\phi$ to $w \models \Diamond\phi$ Zimmermann introduces the SELF-REFLECTION PRINCIPLE: *Any world w satisfies* $\forall v \in W : wR_{\Diamond}v \rightarrow (\forall v' : vR_{\Diamond}v' \leftrightarrow wR_{\Diamond}v')$. This frame condition legitimates the desired derivation and therefore we can state that also for these disjunctions Zimmermann is able to derive the choice inference: $w \models \Diamond A \wedge \Diamond B$.

There have to be didactic reasons why Zimmermann introduces the self-reflection principle additionally to his notion of competence, because the former principle is nothing more than an instantiation of his concept of competence: now, P is taken to be the set of epistemic options of the speaker. Thus, Zimmermann's proposal is more uniform than it may look at first sight. Furthermore, assuming a speaker to be competent with respect to $R_{\triangle}$, her own epistemic state, is equivalent with taking the accessibility relation $R_{\Diamond}$ to be transitive and euclidic. These conditions are very popular if it comes to characterize the way we reason about our own beliefs. They assign an agent positive[27] and negative[28] introspective power with respect to her belief state. This connection adds to the plausibility of the proposed formalization of competence.

The derivation of the free choice inference via a competence axiom has another attractive consequence. As Zimmermann noticed himself, he can straightforwardly account for the observation that free choice permission is not always observed and that it is omitted especially in contexts where the speaker explicitly states his incompetence: the derivation of free choice permission simply relies on taking the speaker to be competent. On the other hand, if the disjunctive sentence is about the epistemic state of the speaker, one observes that the choice principle seems to be always present (if the speaker is taken to be honest). This can now be explained by pointing out that we take every healthy speaker to be a priori competent on his own belief state.

As these results show: Zimmermann's proposal is much more successful in accounting for the free choice inference than any previously discussed alternative. But it has to face its limitations. We have seen above how the approach generates the correct predictions in case '*or*' has wide scope over the modal operators. However, the interpretation mechanism can also be run on the syntactic analysis giving '*or*' narrow scope - unfortunately producing very unsatisfying results. Zimmermann noticed this himself and proposes to exclude narrow scope readings for independent reasons, but this part of his proposal seems a bit ad hoc.

Zimmermann makes also wrong predictions in case '*must*' occurs instead of '*may*'. It seems straightforward to analyze deontic '*must*' as the dual of '*may*': $\nabla =_{def} \neg\triangle\neg$, and epistemic '*must*' as the dual of 'might': $\Box =_{def} \neg\Diamond\neg$. But then, his semantic analysis of a sentence like '*You must ϕ or φ*' is: $\Diamond\nabla\phi \wedge \Diamond\nabla\varphi$. Under the assumption, that the speaker is competent you can now derive that $\nabla\phi \wedge \nabla\varphi$, hence, *You must ϕ <u>and</u> you must φ* - a result that is obviously too strong.

Such sentences, which contain necessity modalities ((13) gives another example) are rarely discussed in the free choice literature. The simple reason is that they seem to be unproblematic.

(13) Mr. X must take a taxi or a boat.

[27]If the agent knows ϕ then he knows that he knows ϕ.

[28]If an agent does not know ϕ then he knows that he does not know ϕ.

But, actually, they play their tricks with us, too. As Zimmermann noticed, also for imperatives there exists an incompetence reading made especially strong by continuation with *'... but I don't know which'*. But this is not the only possible interpretation. Zimmermann observes *'... there appears to be a more straightforward construal according to which Mr. X's obligation are unspecific as to the exact means of transport.'* (Zimmermann [2000], p. 283). Hence, there is a reading of (13) from which you can conclude (14a) and (14b), the obligation (13) still allows Mr. X to chose which disjunct of the obligatory disjunction he is going to fulfill.

(14) a. Mr. X may take a taxi.

b. Mr. X may take a boat.

The similarity between these inferences of (13) and the choice inference of (2a) plus the parallel absence of the inferences if the speaker is taken to be incompetent strongly suggest to speak also for (13) of a free choice inference and to look for one general explanation.[29] Further support for this point comes from the fact that the observation extends to epistemic '*must*':

(15) That must be Tom or Bill.
INFERENCE: *That must be Tom. & That must be Bill.*

But, much stronger than for '*may*', people have difficulties to get the free choice inference for (16), where '*or*' has explicitly wide scope over '*must*', and even found it questionable whether it can get such a reading at all.

(16) You must clean the kitchen or you must go shopping.

All this cannot be explained by the proposal of Zimmermann.

In our opinion, these mispredictions are mainly due to his semantic analysis of '*or*'. This part of his proposal can even be criticized independently from the free choice topic. For instance, if his analysis would be correct, how could it be that sentences like (17) are so useful to communicate the speaker's strong belief in the first disjunct - something obvious if one assume inclusive truth-conditions for '*or*'?

(17) Peter is in love or I'm a monkey's uncle.

And one should be clear about the fact that it is the idea to get the epistemic possibilities for each disjunct from the semantic meaning of '*or*' that bound his approach to a wide scope analysis for this coordination in free choice sentences. This critic does not affect the role competence plays in Zimmermann's approach. On the contrary, this aspect of his theory is very taking.[30]

[29] In view of this it becomes clear that the heading *conjunctive reading* or *conjunctive inference*, that is often found in the literature, is clearly not the right peg for the free choice reading. The parallel sentence of (13) with wide scope conjunction: '*Mr. X must take a taxi and Mr. X must take a boat.*' does not describe the observations. Furthermore, Kamp suggests in his papers that the free choice inference has to be seen as essentially connected with constructions that bring possibilities to the attention of the hearer - this is what all the sentences around (7a) have in common. The observation above makes that questionable or trivial.

[30] Notice the parallelism between Zimmermann's approach and the involvement of competence in the Gricean argument of Kamp's constative proposal.

3 The Proposal

3.1 Introduction

We come now to the main part of the thesis. In the following, a new constative approach to the free choice problem is developed. Because it is a constative approach, we will only discuss the reportive use of permission and obligation sentences.[31] And given the general aim of the paper to defend the Gricean program, a central feature of the approach to be presented will be that it fulfills the restrictions imposed by Grice on a theory on interpretation. In particular we will assume a simple and classical semantics: '*or*' will be interpreted as inclusive disjunction and the modalities are analyzed as unary modal operators. We will see that we can, nevertheless, account for the free choice inferences - if we treat them as a pragmatical phenomenon.

The new proposal will display the same general structure that characterized the account of Zimmermann [2000] and that is also present in the sketched Gricean and constative approach of Kamp [1979]. Both establish the free choice inferences on two premisses. The first ingredient was that the speaker considers certain states of affairs as epistemic possible.[32] For our core-example (18a) it is the assumption that the speaker takes both, (18b) and (18c) as possibly true. Sometimes, this information already accounts for the free choice observation, for instance, for examples like (12): *'Mary or Peter took the beer from the fridge'*. But for the free choice reading of (18a) a second premise was necessary and this was the assumption that the speaker is competent on the subject she is talking about.

(18) a. You may take an apple or a pear.

b. You may take an apple.

c. You may take a pear.

It is obvious that a semantic theory as sketched above will not be able to provide us with these requisites: given an analysis of '*may*' as a modal operator on deontic possibilities, (18a) does neither convey that (18b) and (18c) are possibly permitted according to the speaker, nor that the latter is competent on the deontic options. And because neither Zimmermann's idea to put the first premise in the semantics of '*or*' nor Kamp's suggestion to derive it as conversational implicature via Brevity is convincing we have to come up with something new with respect to this part. However, we will follow Zimmermann in taking free choice permission to depend on the information that the speaker is competent on what is permitted.

The core problem of the new approach is thus to give a account of the first ingredient: the derivation of information about the epistemic alternatives the speaker considers. We will describe them as conversational implicatures derived from the assumption that the

[31] We will not argue for any position with respect to the questions whether there is a performative use and how it should be treated.

[32] Remember, however, that both approaches differ in which level of interpretation they take to be responsible for this information.

speaker obeys the maxim of Quality ($\mathcal{T}$) and the first submaxim of Quantity ($\mathcal{Q}_1$) as introduced by Grice [1989]. We will give a precise description of a class of implicatures that can be derived from these maxims and show that the needed inferences on the epistemic state of the speaker are among them. The formal part is an application of work of Halpern & Moses [1984], that has been recently generalized by van der Hoek et al. [1999, 2000]. This formalization serves at the same time as an argument against critics on the Gricean program we started with: the notorious and justifiable complaint of Kamp and many other scholars that all Gricean approaches lack (a convincing) formally precise implementation.

3.1.1 Free Choice as Clausal Implicature: The Proposal of Gazdar

Actually, this is not the first time that the relevant information about the epistemic possibilities of the speaker has been analyzed as due to the Gricean maxims $\mathcal{T}$ and $\mathcal{Q}_1$. At least part-wise already Gazdar, in his dissertation from [1979], took them to be such conversational implicatures.[33] The account presented here is strongly related to his proposal. To study Gazdar's approach is therefore a good starting point to introduce some general ideas.

Gazdar distinguishes two classes of inferences which he analyzed as generalized $\mathcal{Q}_1$ implicatures. The first class can be described as inferences that some kind of strengthening of the claim made does not hold (as far as the speaker knows). These are the so-called *scalar implicatures*. The second class consists of claims that the speaker does not know for some strengthening of the given utterance whether it holds or not. Gazdar called this group of inferences *clausal implicatures*. The following examples of the latter notion are given by Gazdar (Gazdar [1979], p.50).

(19) a. If John sees me, then he will tell Margaret.
Implicature: I don't know that John will see her.

b. My sister is either in the bathroom or in the kitchen.
Implicature: I don't know that my sister is in the bathroom and I don't know that my sister is in the kitchen.

The reader will have noticed that the clausal implicatures of the second sentence give exactly the kind of information that Zimmermann [2000] analyzed as the semantic contribution of '*or*', the information on the epistemic state of the speaker we are looking for. Here, they appear as pragmatic inferences, as clausal implicatures of a simple[34] disjunction. Of course, this observation raises the question whether we can account for the free choice inferences simply by combining Gazdar's concept of clausal implicatures with Zimmermann's formalization of competence. This will now be investigated.

Gazdar describes the following procedure to calculate clausal implicatures. First, he defines the set of *potential* clausal implicatures (pcis) of a compound sentence ψ (Gazdar

[33] However, he did not consider an application to the paradox of free choice permission.

[34] A sentence is called a simple disjunction, if the only sentential operator it contains is '*or*' and this operator occurs only once.

[1979], p.59, $\Diamond$ is defined as the dual of $\Box$, and $\Box\phi$ stands for *the speaker knows that* ϕ).[35]

$$\left\{\chi \middle| \begin{array}{l} \chi \in \{\Diamond\phi, \Diamond\neg\phi\} \text{ and } \phi \text{ is a subsentence of } \psi \\ \text{such that } \psi \text{ neither entails } \phi \text{ nor its negation} \neg\phi \end{array}\right\}$$

As the reader can see, clausal implicatures are essential inferences about the limitations of the knowledge of the speaker. For instance, for example (19b) the definition predicts the following set of pcis: { *the speaker does not know that her sister is in the bathroom, the speaker does not know that her sister is not in the bathroom, the speaker does not know that her sister is in the kitchen, the speaker does not know that her sister is not in the kitchen*}.

But not all potential clausal implicatures are predicted by Gazdar to become part of the interpretation of an utterance. First, they have to pass a strict consistency check: *A pci becomes part of the interpretation of an utterance if it is a consistent extension of the common ground together with the assumption that the speaker knows her utterance to be true*[36] *and all other potential clausal implicatures.* Because of this filter-condition, clausal implicatures become context dependent and, for instance, the misprediction of Zimmermann concerning example (17) is avoided. However, if nothing special is commonly known, uttering 'ϕ *or* ψ' where ϕ and ψ are logically independent propositions will clausal-implicate $\Diamond\phi$ and $\Diamond\psi$, Zimmermann's semantic meaning of 'ϕ *or* ψ' . Hence, Gazdar can account for the free choice inference which a simple disjunction as (12) or (19b) normally comes with.

But Gazdar cannot only account for these observations. If we take sentences containing epistemic operators, his approach also predicts the free choice inferences we observed. Let's see how. For a sentence $\Diamond(\phi \vee \psi)$ Gazdar predicts the following set of pcis: $\{\Diamond\phi, \Diamond\psi, \Diamond\neg\phi, \Diamond\neg\psi, \Diamond(\phi \vee \psi), \Diamond\neg(\phi \vee \psi)\}$ - among them the free choice inferences $\Diamond\phi$ and $\Diamond\psi$. Whether the pcis are actually generated depends on the context. If the logic of the knowledge operator is $S4$, as Gazdar assumes, and nothing else is commonly known on ϕ and ψ, then all pcis in this set are predicted to be actually derivable from the utterance. Hence, Gazdar can also account for a free choice reading of $\Diamond(\phi \vee \psi)$.[37] Let us consider another case: $\Box(\phi \vee \psi)$. We arrive at exactly the same set of pci's. The element $\Diamond\neg(\phi \vee \psi)$ is not consistent with the assumption that the speaker believed what she said and will therefore be not predicted as actual implicature. However, the free choice implicatures $\Diamond\phi$ and $\Diamond\psi$ survive the consistency check and, hence, Gazdar can account for the free choice interpretation of examples like (15)![38]

Unfortunately, Gazdar's proposal makes generally incorrect predictions if applied to disjunctions containing other modal operators $\triangle$ with $\triangle \neq \Diamond$. In consequence, he cannot account for free choice in deontic contexts. With respect to a narrow scope disjunction $\triangle(\phi \vee \psi)$ there is no hope at all to obtain the needed implicatures, because in this case there will not be any information about the modality $\triangle$ in the set of pcis of this formula:

[35] Gazdar gives one further condition, but this one can be ignored for our purposes. Furthermore, he assumes $S4$ to be the logic of the modal operator $\Diamond$.

[36] This is Gazdar's formalization of the $\mathcal{T}$-implicatures an utterance comes with.

[37] The wide scope analysis gets also a free choice reading in $S4$. However, this is not the case, if the logic of $\Box$ is assumed to be $KD45$. In this case the relevant potential clausal implicatures are generated but no potential implicature will survive the consistency check (see the discussion below for $\triangle\phi \vee \triangle\psi$). Thus, even semantically equivalent sentences may generate different sets of implicatures under Gazdar's treatment.

[38] The pcis of a sentence $\Box\phi \vee \Box\psi$ are different, but still the free choice interpretation is derived. For the logic KD45 we obtain the same results.

$\{\Diamond\phi, \Diamond\neg\phi, \Diamond\psi, \Diamond\neg\psi, \Diamond(\phi \vee \psi), \Diamond\neg(\phi \vee \psi)\}$. For the sentence giving the disjunction wide scope, $\triangle\phi \vee \triangle\psi$, the situation looks a little bit better. Gazdar predicts the following set of pcis: $\{\Diamond\phi, \Diamond\psi, \Diamond\triangle\phi, \Diamond\triangle\psi$ and the respective negations$\}$. A first thing to notice is that the first two members of this list together with their negative counterparts should not be predicted as conversational implicatures. Though in case $\triangle$ in $\triangle\phi \vee \triangle\psi$ is interpreted as deontic *'may'* one (normally) infers from an utterance of this sentence that the speaker takes the asserted deontic options also to be epistemically possible, this inference should rather be analyzed as part of the appropriateness conditions (presuppositions) of permissions (and obligations). To support this standpoint, notice that this inference accompanies also sentences like (20), and, thus, projects trough negation.

(20) You must not take an apple or a pear.

That $\Diamond\phi$ and $\Diamond\psi$ should not be predicted to be conversational implicatures of an utterance of $\triangle(\phi \vee \psi)$ is even more obvious if one takes $\triangle$ to stand for other modalities like, for instance, the belief state of other agents. While also in this case an utterance of (21) does have a free choice reading (Peter thinks it possible that Mr. X is in Berlin and he thinks it possible that Mr. X is in Amsterdam), one does not infer, of course, that also the speaker thinks it possible that Mr. X is in Berlin.

(21) Peter believes that Mr. X is in Berlin or in Amsterdam.

But let us consider the other pcis Gazdar predicts for the sentence $\triangle(\phi \vee \psi)$. Again, Zimmermann's semantic interpretation of the sentence is part of the pcis. Therefore, one may think, if we add an assumption of competence to the context with respect to which $\triangle(\phi \wedge \psi)$ is interpreted, then we will be as successful in deriving the free choice inferences as was Zimmermann. But his formalization of competence will allow us not only to derive from the potential clausal implicature $\Diamond\triangle\phi$ the fact $\triangle\phi$ but also to conclude from its negative counterpart $\Diamond\neg\triangle\phi$ that $\neg\triangle\phi$ holds.[39] Hence, we cannot add the pcis we need to such a context without generating contradiction with other elements of this set. They, therefore, do not pass Gazdar's consistency test and are predicted not to occur. The results for sentences containing deontic '*must*' are equally bad.[40]

After such a successful beginning the question arises: what went wrong? And can we repair Gazdar's approach without major changes? The discussion above revealed two different problems of his approach. For one thing, there is something essentially wrong with the generation process of the pcis, as, for instance, the unintuitive pci $\Diamond p$ for a sentence $\triangle(p \vee q)$ shows. This part of his theory demands serious revision. In particular, the syntactic aspect of Gazdar's notion that allows you to take subsentences out of their modal contexts has to be removed. The problem concerning the incompetence that prevents Gazdar from predicting the free choice inference for $\triangle\phi \vee \triangle\psi$, on the other hand, seems to be less severe. To avoid it one could change the generation process so as to not include the negative pcis or weaken the notion of competence, for instance.

[39]To see this, the axiomatization of Zimmermann's definition of competence in section 3.2.7 may be of use.

[40]There is, of course, no difference in the results if we would assume as epistemic logic KD45.

An objection against the proposal of Gazdar of a totally different character is that it lacks a precise motivation as a formalization of the theory of Grice. And again, it is above all the syntactic aspect of his description of pcis that does not seem to fit. Gazdar sketches a derivation of the implicatures from the maxims $\mathcal{Q}$ and $\mathcal{T}$ that is in its structure as old as the theory of Grice itself: *if the (rational and cooperative) speaker had known the truth-value of a subsentence of her utterance, then she should, in order to obey $\mathcal{Q}_1$ and $\mathcal{T}$, have used a sentence that semantically conveys this information. Hence, the speaker cannot know whether the sub-sentence holds or not.* Given the formulation of the maxims it is absolutely unclear, why this reasoning should apply to sub-sentences of the utterance, and to those only.

To develop our approach and overcome the deficits discussed above we will again start from Grice and motivate a new description of the conversational implicatures due to $\mathcal{T}$ and $\mathcal{Q}_1$. In the end we will see that the inferences we describe can be best understood as, again, Gazdar's clausal implicatures. In particular, all the appealing inferences he predicts are maintained.

3.1.2 Technical Preliminaries

But before we can really start introducing the new approach, the basics of the formalization of natural language we will use have to be laid out. This will be done here: We will make our model-language precise, define its semantics and introduce some technical machinery we will need later on.

Language Our language $\mathcal{L}$ is generated from a finite set of propositional atoms $\mathcal{P} = \{\top, \bot, p, q, r, ...\}$, the logical connectives $\neg, \wedge, \vee$, and $\rightarrow$ and finitely many unary modal operators $\{\triangle_1, \triangle_2 ..., \triangle_n\}$. We will use ∇_i to shorten $\neg\triangle_i\neg$ for all $i \in \{1, ...n\}$. $\Diamond$ is the special modal operator $\triangle_i$ for some $i \in \{1, ..., n\}$ that refers to the belief state of the speaker and $\Box\phi \equiv \neg\Diamond\neg\phi$ expresses in turn that *the speaker believes* ϕ. $\triangle$ and ∇ are used as meta-variables over the set of modalities of $\mathcal{L}$.

$|\cdot| : \mathcal{L} \longrightarrow \mathbb{N}$ denotes the modal depth of formulas, defined recursively: for $p \in \mathcal{P}$ and $\phi, \psi \in \mathcal{L}$, $|p| = |\top| = |\bot| = 0$, $|\neg\phi| = |\phi \vee \psi| = |\phi \wedge \psi| = max\{|\phi|, |\psi|\}, |\triangle\phi| = |\nabla\phi| = |\phi| + 1$.

We will frequently make use of sublanguages $\mathcal{L}' \subseteq \mathcal{L}$. For instance, the languages $\mathcal{L}'_{(n)} = \{\phi \in \mathcal{L}' |\ |\phi| \leq n,\ n \in \mathbb{N}\}$, the set of $\mathcal{L}'$-sentences with modal depth smaller or equal to n, will play a central role. We call $\mathcal{L}_{(0)}$, the language that contains the modal-free part of $\mathcal{L}$, the *basic language*. Another important construct we will use is $\Box\mathcal{L}' = \{\Box\phi | \phi \in \mathcal{L}'\}$, the language that shifts $\mathcal{L}'$ into the belief state of the speaker.

We introduce the following abbreviations for certain $\mathcal{L}$ formulas:

(K)	$\nabla(\phi \rightarrow \psi) \rightarrow (\nabla\phi \rightarrow \nabla\psi)$,	(4) $\Box\phi \rightarrow \Box\Box\phi$,
(Dual)	$\nabla\phi \leftrightarrow \neg\triangle\neg\phi$,	(5) $\neg\Box\phi \rightarrow \Box\neg\Box\phi$,
		(D) $\Box\phi \rightarrow \Diamond\phi$.

Semantics A frame for $\mathcal{L}$ is an (n+1)-tuple of a set of worlds W and for every modality $\triangle$ of $\mathcal{L}$ a binary relation $R_\triangle$ over W. A model for $\mathcal{L}$ is a tuple consisting of a frame for $\mathcal{L}$ and an interpretation function V for the non-logical vocabulary of $\mathcal{L}$; a function from $p \in \mathcal{P}$ to characteristic functions over W. Let $F = \langle W, R_{\triangle_1}, ..., R_{\triangle_n}\rangle$ be a frame for $\mathcal{L}$ and $M = \langle F, V\rangle$ a model. For $w \in W$, $R_{\triangle_i}[w]$ denotes the set $\{v \in W | \langle w, v\rangle \in R_{\triangle_i}\}$. We call the tuple $s = \langle M, w\rangle$, where $w \in W$, a state. Truth of a sentence of $\mathcal{L}$ with respect to a state is defined along standard lines. We will give here only the definition of truth for a modal formula $\triangle\phi$: $M, w \models \triangle\phi$ iff$_{def}$ there is a $v \in W$ such that $v \in R_\triangle[w]$ and $M, v \models \phi$. A sentence ϕ is valid on a class of states S ($\models_S \psi$) iff$_{def}$ for all $s \in S$: $s \models \phi$. A set of formulas Γ entails a formula ϕ relative to a class of states S ($\Gamma \models_S \psi$) iff$_{def}$ for all $s \in S$: $s \models \Gamma$ implies $s \models \psi$. If $\Gamma = \{\phi\}$, we write $\phi \models_S \psi$. $F, w \models \phi$ holds iff$_{def}$ for all valuations V: $\langle F, V\rangle, w \models \phi$. $F \models \phi$ iff$_{def}$ for all $w \in W$: $F, w \models \phi$.

Without going into much details here, models for $\mathcal{L}$ can also be treated as models for the first-order language $\mathcal{L}^1$ corresponding to $\mathcal{L}$ (see Blackburn et al. (2001), definition 2.44). For $\alpha \in \mathcal{L}^1$ we write $M\| - \alpha$ iff$_{def}$ α is true in the model $M = \langle F, V\rangle$. $F\| - \alpha$ iff$_{def}$ for all valuations V: $\langle F, V\rangle\| - \alpha$.

For $A_1, ...A_n \in \mathcal{L}$, $\textsc{State}_{KA_1...A_n}$ is the set of states $s = \langle M, w\rangle$ with $M = \langle F, V\rangle$ such that for all $i \in \{1, ..., n\}$: $F, w \models A_i$.[41] We shorten $\models_{\textsc{State}_{KA_1...A_n}}$ with $\models_{KA_1...A_n}$. In this thesis we will work with (subsets of) $\textsc{State}_{KD45}$. The sentences (D), (4), and (5) locally correspond to well known first order formulas, what allows us to get a good grasp of the structure of the states in $\textsc{State}_{KD45}$.

Fact 1 [42] *Let $F = \langle W, R_{\triangle_1}, ..., R_{\triangle_n}\rangle$ be a frame for $\mathcal{L}$ and $w \in W$. Then the following correspondences hold:*

$F, w \models (D)$ *iff* $F\| - \exists x : R_\diamond(w, x)$
$F, w \models (4)$ *iff* $F\| - \forall x, y : R_\diamond(w, x) \wedge R_\diamond(x, y) \rightarrow R_\diamond(w, y)$
$F, w \models (5)$ *iff* $F\| - \forall x, y : R_\diamond(w, x) \wedge R_\diamond(w, y) \rightarrow R_\diamond(x, y)$

We know, hence, that the underlying frames of the states in $\textsc{State}_{KD45}$ are with respect to $R_\diamond$ locally (in w) transitive, euclidical and non-blind[43]. Conceptually, that means that we adopt a particular perspective on how the belief state of a speaker looks like: the speaker has positive and negative introspective power and the absurd belief state is excluded. This restriction has been necessary because of the limited resources of the study at hand.

The reader may be surprised by the choice to ask only for the local validity of the formulas (D), (4), and (5). In consequence, we allow models where, for instance, the speaker thinks it possible that some other agent takes him to be in a non-transitive, non-euclidian, or absurd belief-state. To realize the idea that KD45 is a general restriction of what belief states can possibly look like, one should impose the axioms globally on the underlying frames. Then one should, of course, also single out the class of modal operators that describe the belief state of other agents and impose parallel conditions on them. One

[41] Hence, A_i is valid in w on the frame underlying M.

[42] Proof: Because (D), (4), and (5) are Sahlqvist-formulas we know immediately that they have first-order correspondents (see Blackburn et al. (2001), theorem 3.54). Applying the Sahlqvist - van Benthem algorithm gives us the claimed equivalencies.

[43] A state $s\langle M, w\rangle$ is non-blind in w with respect to $R_\diamond$ of M iff$_{def}$ $R_\diamond[w] \neq \emptyset$.

reason why this is not done here is that in this paper we will never come in a situation where we will talk about such deeply embedded belief. Hence, the restriction would never show up. Furthermore, later on we will consider restrictions on states that are only plausible when imposed locally.[44]

Another concept that will - in different disguises - play an important role in the theory we are going to develop is n-bisimulation. Let $\langle M, w\rangle, \langle M', w'\rangle \in S$ be two states in S. We say that $\langle M, w\rangle, \langle M', w'\rangle$ are n-bisimilar ($\langle M, w\rangle \cong_n \langle M', w'\rangle$) if there exists a sequence of binary relations $Z_n \subseteq \cdots \subseteq Z_0$ with the following properties (for $i + 1 \leq n$):

(i) wZ_nw'
(ii) If vZ_0v' then $\forall p \in \mathcal{P} : V(p)(v) = V'(p)(v')$
(v and v' agree on all proposition letters)
(iii) If $vZ_{i+1}v'$ and $u \in R_\triangle[v]$
then there exists u' with $u' \in R'_\triangle[v]$ and uZ_iu' (forth)
(iv) If $vZ_{i+1}v'$ and $u' \in R'_\triangle[v']$
then there exists u with $u \in R_\triangle[v]$ and uZ_iu' (back).

We will frequently make use of the following well-known theorem:[45]

Theorem 1 *Let $\mathcal{L}$ be a modal language with a finite set of atomic propositions: $|\mathcal{P}| = m \in \mathbb{N}$. For two states $s, s' \in S$ where S is a set of states for $\mathcal{L}$ the following are equivalent.*

(i) $s \cong_n s'$
(ii) *s and s' agree on all $\varphi \in \mathcal{L}$ with modal depth at most n.*

Finally, we define for $A_1, ..., A_n \in \mathcal{L}$ the $KA_1....A_n$-closure ($CL_{KA_1....A_n}(M)$) of a model $M = \langle W, R_{\triangle_1}, ..., R_{\triangle_n}, V\rangle$ as a model M' with $M' = \langle W, R'_{\triangle_1}, ..., R'_{\triangle_n}, V\rangle \in \textsc{State}_{KA_1...A_n}$, for all $\triangle_i$, $R_{\triangle_i} \subseteq R'_{\triangle_i}$, and there is no $M'' = \langle W, R''_{\triangle_1}, ..., R''_{\triangle_n}, V\rangle \in \textsc{State}_{KA_1...A_n}$ such that for some $\triangle_i$ $R_{\triangle_i} \subseteq R''_{\triangle_i} \subset R'_{\triangle_i}$.

3.1.3 The Data in Overview

Our aim is to describe the diverse free choice inferences we met in section 2. Because we have to account for inferences, the core of the proposal should be to define a notion of entailment, lets call it $|\!\equiv$, such that the relevant inferences become valid with respect to it. Given our formal model-language $\mathcal{L}$ as described above this means that the following entailment statements have to be valid. Let $p, q \in \mathcal{L}_{(0)}$ be two logically independent propositions ($\{A|B\}$ has to be read as '*A is the premise or B is the premise*'):

(D1) $p \vee q \;|\!\equiv \Diamond p \wedge \Diamond q$

[44] The choice to work with local restrictions has some consequences for the logic of $\models_S$: the normal modal logic with additional axioms (D), (4), and (5) is not sound with respect to $\textsc{State}_{KD45}$. However, it is not difficult to find a sound and strongly complete proof system for this class of states: the only thing one has to do is to restrict in the normal modal logic with additional axioms (D), (4), and (5) the rule of generalization to formulas that are also derivable in the minimal normal modal logic **K**. Because syntactic considerations play no role in the rest of the paper we abstain from discussing this topic in more detail.

[45] For a discussion see Blackburn et al. [2001], p. 74.

(D2) $\{\Diamond(p \vee q) | \Diamond p \vee \Diamond q\} \mid\equiv \Diamond p \wedge \Diamond q$

(D3) $\{\triangle(p \vee q) | \triangle p \vee \triangle q\} \mid\equiv \triangle p \wedge \triangle q$
with competence of the speaker.

(D4) $\{\triangle(p \vee q) | \triangle p \vee \triangle q\} \mid\equiv \Diamond(\triangle p \wedge \neg\triangle q) \wedge \Diamond(\triangle q \wedge \neg\triangle p)$
without competence of the speaker, $\triangle \neq \Diamond$

(D5) $\{\Box(p \vee q) | \Box p \vee \Box q^?\} \mid\equiv \Diamond p \wedge \Diamond q$

(D6) $\{\nabla(p \vee q) | \nabla p \vee \nabla q^?\} \mid\equiv \triangle p \wedge \triangle q$
with competence of the speaker.

(D7) $\{\nabla(p \vee q) | \nabla p \vee \nabla q\} \mid\equiv \Diamond(\nabla p \wedge \neg\triangle q) \wedge \Diamond(\nabla q \wedge \neg\triangle p)$
without competence of the speaker, $\nabla \neq \Box$

(D8) $\Diamond(p \wedge q) \mid\equiv \Diamond(p \wedge \neg q) \wedge \Diamond(\neg p \wedge q) \wedge \Diamond(\neg p \wedge \neg q)$

(D9) $\triangle(p \wedge q) \mid\equiv \triangle(p \wedge \neg q) \wedge \triangle(\neg p \wedge q) \wedge \triangle(\neg p \wedge \neg q)$
with competence of the speaker.

The restriction to logically independent propositions reflects the observation that we want the inferences only to be generated in case they are consistent with the context. For instance, we don't want (D1) to be valid if the antecedent is the logical form of (17), here repeated as (22).

(22) Peter is in love or I'm a monkey's uncle.

(D2), (D3), (D5) and (D6) are the core observations: the free choice inferences for epistemic and deontic modalities. Zimmermann has added (D1) to the free choice inferences: the intuition that simple occurrences of '*or*' (no additional sentence operator is involved) allow the inference that both disjuncts are considered to be possible by the speaker. (D4) and (D7) describe the alternative free choice *free* interpretation for the deontic examples. We observed that these readings are particularly forced if the speaker states explicitly her incompetence and that they are not possible for epistemic modalities. (D8) and (D9), finally, refer to one of the points of criticism Merin brought forward against the minimal contraction approach: it cannot handle conjunction properly. As we will show, our approach will predict exactly the reading van Rooy [2000] described as the normal interpretation of such sentences, namely (D8) and (D9).

No claim will be made in advance with respect to the choice of scope on which each inference is based. As we have noticed above, free choice as well as the incompetence inference can come with the surface narrow scope form - but there may be a deletion process working on deep structure, hence, this does not allow us to conclude that both inferences have to be derivable from a narrow scope logical form. For surface wide scope we noticed that free choice readings are more difficult to get and are reported to be absent for all-quantifying modalities. However, because this was not tested properly, we will not exclude this analysis for free choice. We will just see what our approach predicts.

3.2 Developing the Framework

3.2.1 A Gricean Notion of Entailment

Our starting point is Grice's theory of conversational implicatures, especially the first submaxim of QUANTITY ($\mathcal{Q}_1$): *Make you contribution as informative as required (for the current purpose of exchange).*[46] Because of the contrast with the second submaxim and the applications Grice discusses, this maxim is generally understood as an imperative to maximize the quantity of relevant information that is transmitted.[47] But the speaker is not allowed to provide *any* bit of relevant information she can think of. The maxim of QUALITY ($\mathcal{T}$) restricts her contributions to facts she believes to hold and for whose truth she has sufficient evidence. Both maxims can be brought together in the following interpretation rule for a hearer:[48]

> *The contribution ϕ of a rational and cooperative speaker encodes all of relevant information the speaker has; she knows only ϕ.*

We now introduce a pragmatic notion of entailment, $|\!\equiv$, building on this rule of interpretation. $\phi \mid\!\equiv \psi$ is defined to hold in case ψ is true on all models where ϕ represents all the speaker knows. In other words, $|\!\equiv$ describes the inferences an interpreter can draw from an utterance if she assumes the speaker to behave rational and cooperative; in particular, to obey $\mathcal{T}$ and $\mathcal{Q}_1$.

Fortunately, the question how to formalize that a certain formula expresses all an agent knows has gotten some attention from logicians. Our account for the free choice inferences highly relies on this work. More specifically we are going to use an approach introduced by Halpern & Moses [1984] and generalized by van der Hoek et al. [1999, 2000].

The fundamental idea behind their formalization is strikingly simple. The states where a sentence ϕ is all a speaker knows are taken to be those states where she is in the minimal belief-state, given that she believes ϕ. Hence, we impose an order $\preceq$ on all the states s where the speaker believes her utterance ϕ, i.e. $s \models \Box\phi$, that compares how much the speaker believes in these states. And then we define as the class of pragmatic inferences those sentences that hold on the minimal elements of this order. It is obvious that such a formulation of a notion of entailment constitutes nothing more or less than an instance of interpretation in preference structures. We summarize our considerations in the following

[46] We will not discuss why a rational cooperative speaker should obey the maxims of conversation. We simply try to capture what an interpreter can infer if she assumes that the speaker does so.

[47] Although the restriction to *relevant* information has often been neglected in the literature.

[48] This paraphrase does not exactly capture what Grice says in his maxim $\mathcal{Q}_2$. According to his formulation it may be the case that two possible utterances of the speaker are both maximally informative and relevant but nevertheless convey different information. Our formulation implicitly assumes that there exists exactly one maximum. We can make this claim because we will order sentences only with respect to the semantic notion of entailment $\models$. Then it is the case that if two sentences are logical independent their conjunction is more informative than both of them and therefore we can assume the existence of a unique maximum. However, $\models$ may not be an appropriate description of the order (how does it deal with the modification of Grice: 'as informative *as required*', for instance). Nevertheless, if adaptations are necessary they can take the work presented here as a starting point.

definition of a notion of pragmatic entailment.

Definition 1 *(The Inference Relation $|\equiv$)*
Let $\preceq$ be a partial order on some class of states S. We define for sentences $\phi, \psi \in \mathcal{L}$:

$$\phi \mid\equiv_S \psi \text{ iff}_{def}\ \forall s \in S : [s \models \Box\phi \wedge \forall s' \in S : s' \models \Box\phi \Rightarrow s \preceq s'] \Rightarrow s \models \psi.$$

This relation of entailment is the hart of the approach developed here.[49]

3.2.2 Information Orders. Introduction

The crucial question for an approach along these lines is whether we can find a convincing definition of the order $\preceq$, hence, of what it should mean that in one state the speaker knows more than in another. Until now we have said nothing on this point. This will be our next topic.

Van der Hoek et al. [1999] introduce an appealing general characterization of such orders. The underlying idea is very simple: we use our language to define the order. A speaker is said to believe less in a state $\langle M, w\rangle$ than in a state $\langle M', w'\rangle$ with respect to a certain sublanguage $\mathcal{L}' \subseteq \mathcal{L}$, if the set of sentences of $\mathcal{L}'$ she believes in the first state is a subset of the set of sentences of $\mathcal{L}'$ she believes in the second. To make this more precise we define the notion of an information order.

Definition 2 *(Information Orders)*
A sublanguage $\mathcal{L}' \subseteq \mathcal{L}$ defines with respect to a class of states S the relation $\preceq$ iff$_{def}$ $\forall s_1, s_2 \in S : s_1 \preceq s_2 \Leftrightarrow \forall \varphi \in \mathcal{L}' : s_1 \models \varphi \Rightarrow s_2 \models \varphi$. An order that is defined by a sublanguage $\mathcal{L}' \subseteq \mathcal{L}$ is called an information order.

The restriction to orders that have defining languages has some consequences for the notions of entailment that can be defined in terms of them. For instance, these orders are partial but not necessarily total orders. Hence, it may be the case that for a sentence ϕ there exists more that one minimal state. Given the application we have in mind the orders we are interested in compare states with respect to the beliefs of the speaker. Hence, we will use information orders defined by languages $\Box\mathcal{L}^*$ for $\mathcal{L}^* \subseteq \mathcal{L}$. If for a sentence ϕ there exist with respect to the order defined by such a belief-language more than one minimal state this means that for the interpreter a speaker of ϕ can be in (with respect to $\Box\mathcal{L}^*$) different minimal belief states - that she believes in the minimal belief states for ϕ different things. But in such a situation the speaker has to have $\mathcal{L}^*$-beliefs she did not communicate. Thus, it is obvious for the interpreter that she did not obey the maxims $\mathcal{Q}_1$ and $\mathcal{T}$ (as they are interpreted by this account). Such sentences should then be pragmatically not well-formed. The notion of entailment that can be defined on basis of an information order using definition 1 reflects this consideration: if there are different minimal states,

[49]Some reader may miss semantics in this definition: the truth conditions of the antecedent ϕ play no role. This is true and the notion of pragmatic entailment defined above is not conservative with respect to the semantic entailments of an utterance - hence, does not truly strengthen the semantic meaning of an utterance as one might expect. However, this can be easily achieved by defining $\phi \mid\equiv_s \psi$ by $\forall s \in S : s \models (\phi \wedge \Box\phi)$ & ... . Also with respect to this notion, the results we are going to discuss will hold. (This is not trivial: by extending the selection conditions, the set of minimal models may change.)

then $\phi \ |\!\!\equiv \bot$, because there will be no state that fulfills the antecedence in definition 1. Halpern & Moses called such sentences, sentences that allow for different with respect to $\Box\mathcal{L}^*$ incomparable minimal belief state, *dishonest*.

Definition 3 *(Honesty)*
A sentence $\phi \in \mathcal{L}$ is called honest with respect to a language $\Box\mathcal{L}^* \subseteq \mathcal{L}$ and a set of states S, iff$_{def}$ $\{s \in S | s \models \Box\phi \wedge \forall s' \in S : s' \models \Box\phi \Rightarrow s \preceq^* s'\} \neq \emptyset$, where $\preceq^*$ is the order defined by $\Box\mathcal{L}^*$.

There is an interesting and useful connection between being honest and another property of formulas. Let's say that a formula $\phi \in \mathcal{L}$ has the disjunction property with respect to a language $\Box\mathcal{L}^* \subseteq \mathcal{L}$ and a class of states S iff$_{def}$ ϕ is satisfiable in S and for every finite set of sentences $\psi_1, \psi_2, ..., \psi_n \in \Box\mathcal{L}^*$: if $\phi \models_S \psi_1 \vee ... \vee \psi_n$ then for some $i \in \{1, ..., n\}$: $\phi \models_S \psi_i$.[50] Now, assume that ϕ is honest with respect to $\Box\mathcal{L}^*$ and S. This means that there exists a state $s \in S$ such that $s \models \Box\phi$ and $\forall s' \in S : s' \models \Box\phi \Rightarrow s \preceq^* s'$. Assume at the same time that $\Box\phi$ does not have the disjunction property with respect to $\Box\mathcal{L}^*$ and S. Then, there is some sequence of sentences $\psi_1, ..., \psi_n \in \Box\mathcal{L}^*$, such that $\Box\phi \models_S \psi_1 \vee ... \vee \psi_n$ but for no $i \in \{1, ..., n\}$: $\Box\phi \models_S \psi_i$. Of course, if $\Box\phi \models_S \psi_1 \vee ... \vee \psi_n$ then also $s \models \psi_1 \vee ... \vee \psi_n$. Because of the way the truth function of $\vee$ is defined, from this it follows that there is some $k \in \{1, ..., n\}$ such that $s \models \psi_k$. However, by the choice of the $\psi_1, ..., \psi_n$ it cannot be the case that $\Box\phi \models \psi_k$. Hence, there has to be an $s' \in S$ such that $s' \models \Box\phi$ and $s \not\models \psi$. But this impossible: we have chosen s such that $\forall s' \in S : s \preceq^* s'$ and, hence, $\forall \psi \in \Box\mathcal{L}^* : s \models \psi \Rightarrow s' \models \psi$! This proves the following fact:

Fact 2 *If ϕ is honest with respect to $\Box\mathcal{L}^*$ ($\mathcal{L}^* \subseteq \mathcal{L}$) and a class of states S, then $\Box\phi$ has the disjunction property with respect to $\Box\mathcal{L}^*$ and S.*

This result will prove to be quite useful for establishing that certain formulas are not honest.

3.2.3 How to establish Properties of Minimal States

In the last two sections we introduced the central concepts of the new approach. We defined a pragmatic notion of entailment - the notion by means of which we want to account for the free choice inferences - by strengthening the inferences one can get from the premise that the speaker believes what she said with taking the speaker to be in the minimal belief state possible given that she believes what she said. And we characterized a class of orders that can be used to compare the beliefs of the speaker. What else do we have to do before we can see whether the approach gives us the free choice inferences?

Of course, we still have to choose a particular information order. A general characterization is not enough. But before we are going to discuss this question, we will address another problem: given a class of states and an information order, how can we establish that certain inferences are valid with respect to the notion of entailment defined by this order when applied to this class of states? How can we establish properties of minimal states? This question is the topic of the present section.

[50] The definition can be easily extended to arbitrary languages.

We will use a state-construction technique to show that certain sentences hold on the minimal states for a sentence ϕ. It will turn out that the same technique can also be used to prove that a sentence ϕ is honest. Let S be a set of states and $\preceq^*$ an information order on S with defining language $\Box\mathcal{L}^*$. Assume that we could define a mapping $\bigvee$ from the set of subsets of S to S such that for arbitrary $T \subseteq S$ (i) $\bigvee(T) \in S$ and (ii) $\forall s \in T : \bigvee(T) \preceq^* s$, hence $\bigvee$ would map subsets T of S on a state that is $\preceq^*$-smaller that every $s \in T$. Then, we could in terms of this mapping characterize a class of honest formulas for the order-defining language $\Box\mathcal{L}^*$.

Fact 3 *If $\Box\phi$ is $\bigvee$-persistent, i.e. $\forall T \subseteq S : (\forall s \in T : s \models_S \Box\phi) \Rightarrow \bigvee(T) \models \Box\phi)$, then ϕ is honest with respect to S and $\Box\mathcal{L}^*$.*

To see this take $T = \{s \in S | s \models \Box\phi\}$. According to the assumptions we have made about $\bigvee$, we have $\bigvee(T) \in S$ and $\forall s \in T : \bigvee(T) \preceq^* s$. Furthermore, because $\Box\phi$ is $\bigvee$-persistent: $\bigvee(T) \models \Box\phi$. But this means that with $\bigvee(T)$ we have found some state $s \in S$ such that $s \models \Box\phi$ and $\forall s' \in S : s' \models \Box\phi \Rightarrow s \preceq^* s'$. Hence, ϕ is honest with respect to S and $\Box\mathcal{L}^*$.

More relevant for our applications is the following fact.

Fact 4 *If $\forall s \in S : s \models \Box\phi \wedge \neg\psi \Rightarrow [\exists s' \in S : s' \models \Box\phi \;\&\; s' \prec s]$, then $\phi \mid\equiv_S \psi$.*

Assume that $\phi \mid\not\equiv_S \psi$. By definition, it follows that $\exists s \in S : s \models \Box\phi \;\&\; [\forall s' \in S : s' \models \Box\phi \Rightarrow s \preceq^* s'] \;\&\; s \models \neg\psi$. But by the antecedent of fact 4, we know that $\exists s'' \in S : s'' \models \Box\phi \wedge s'' \prec^* s$ - hence, contradiction.

We will see later on that an operation $\bigvee$ that has the properties we have stated above will also allow us to easily show the validity of the antecedent of fact 4. In sum, the main work for establishing the free choice inferences will lay in finding such a mapping $\bigvee$. But it will turn out that this is not a particularly difficult enterprise either. An adapted version of the *disjoint union* of modal models will do the job.

3.2.4 The Basic Information Order

Now, it's time to let all the techniques run on an example. We will, therefore, use this section to study a simple but nevertheless for the free choice inferences very useful instance of an information order: the *basic information order*.

The basic information order is the order defined by the language $\Box\mathcal{L}_{(0)}$.[51]

Definition 4 *(The Basic Information Order $\preceq^0$)*
For $s, s' \in \textsc{State}_{KD45}$[52] we define

$$s \preceq^0 s' \; \textit{iff}_{def} \; \forall\varphi \in \Box\mathcal{L}_{(0)} : s \models_S \varphi \Rightarrow s' \models_S \varphi.$$

[51] This is one of the descriptions Halpern & Moses [1984] use for the definition of their notion of minimal knowledge.

[52] As stated earlier, when discussing applications we will restrict our considerations to $\textsc{State}_{KD45}$.

$\preceq^0$ compares how much the speaker knows on sentences that contain no modal operators. Of course, this is a strong limitation of order-relevant information - and we will have to extend the defining language to get all the intended inferences. But for the moment we will stick to this simplification.

It is easy to see that the following simple fact holds.

Fact 5 *For* $\langle M, w\rangle, \langle M'w'\rangle \in \textsc{State}_{KD45}$*:* $\langle M, w\rangle \preceq^0 \langle M', w'\rangle \Leftrightarrow$
$\forall v' \in R'_\diamond[w']\exists v \in R_\diamond[w]\forall p \in \mathcal{P} : V(p)(v) = V'(p)(v')$.

Thus, we could have equivalently used the stated model-condition to define the order. To give a rough reformulation of the statement, it says that a speaker knows in a state $\langle M, w\rangle$ less than in a state $\langle M', w'\rangle$ if for every epistemic possibility she sees in $\langle M', w'\rangle$ there will be an epistemic possibility of the speaker in $\langle M, w\rangle$ that gives exactly the same interpretation to propositional atoms.

With the use of the basic information order we already can account for the inferences (D1), (D2), (D5) and (D8) from section 3.1.3.

By inserting $\preceq^0$ in Definition 1 we get the first concrete instance of our pragmatic entailment relation: $\phi \mid\equiv^0_{KD45} \psi$ holds iff$_{def}$ on the $\preceq^0$-minimal set of $\textsc{State}_{KD45}$ where the speaker believes ϕ, ψ is valid.

How can we now, for instance for (D1), establish that the inference is valid for $\mid\equiv^0_S$? Given the considerations of the last section, we want to use fact 4 for this purpose. But to be able to do so we first have to show that its antecedent holds: we have to show that a state s_1 where the speaker believes the antecedent of (D1) but where the consequence $\diamond p \wedge \diamond q$ is not true is for $\Box(p \vee q)$ not minimal with respect to $\preceq^0$. Assume, without loss of generality, that the consequence fails because $s_1 \models \neg\diamond p$. An idea how to find for s_1 a strictly smaller s could be to define it by adding to the epistemic possibilities the speaker distinguishes in s_1 one where p holds. In the resulting s, $\neg\diamond p$ would be false and, hence, because $\neg\diamond p \equiv \Box\neg p$ and $\Box\neg p$ is in our order defining language $\Box\mathcal{L}_{(0)}$ it follows that $\neg(s_1 \preceq^0 s)$. On the other hand, because we only added an epistemic possibility where p holds, $\Box(p \vee q)$ is still true in s, and in general the speaker should only have less $\mathcal{L}_{(0)}$-beliefs in s than in s_1. In sum, s would be a state such that $s \models \Box(p \vee q)$ and $s \prec^0 s_1$, and the antecedent of fact 4 for (D1) would be established.

In the following the argumentation sketched above is made more precise. For this purpose we define the s used above as the result of a merge of s_1 with a $KD45$-state where $\Box p$ holds. This operation will be a function with the properties described in the last section. For the basic information order the merge comes down to a simple variation of the disjoint union of modal models.

Definition 5 *(The Merge Function $\bigvee_1$)*
Let T be a subset of $\textsc{State}_{KD45}$ indexed with the set I. Assume that the set of worlds of all states in T are disjoint and w is a world not occurring in any domain. $\bigvee_1(T)$ is the state $\langle CL_{KD45}(M'), w\rangle$, where M' is defined as follows.
$W' = \bigcup_{i\in I} W_i \cup \{w\}$,
$R'_\diamond = \bigcup_{i\in I} R_{i,\diamond} \cup \{\langle w, v\rangle | v \in R_{i,\diamond}[w_i], i \in I\}$,

$R'_\triangle = \bigcup_{i\in I} R_{i,\triangle}$ *for* $\triangle \neq \diamond$,
$V'(p) = \bigcup_{i\in I} V_i(p)$.

The picture below shows how the operator $\bigvee_1$ merges two states $s_1 = \langle M_1, w_1\rangle$ and $s_2 = \langle M_2, w_2\rangle$ to a state $s = \langle M, w\rangle =_{def} \bigvee_1(\{s_1, s_2\})$. As we can see, there are two ways in which arrows are added to the epistemic accessibility relation of s: arrows from the new point w to epistemically accessible points of s_1 and s_2 come by the definition of M'. Arrows connecting the epistemic possibilities of different states with each other are added by the closure operation that is applied to the model M' in the definition.

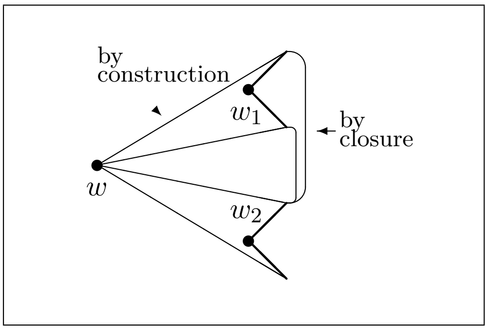


Figure 1:

The presentation of the states is highly simplified. Only some essential $R_\diamond$-connections are depicted. Thick lines represent $R_\diamond$-arrows of the merged states s_1 and s_2, thin lines stand for $R_\diamond$- arrows inserted by the merge operator. The arrow-direction is always from left to right (connections represented by curved lines hold in both directions).

By definition, $\bigvee_1(T) \in \textsc{State}_{KD45}$. Its set of $R_\diamond$-accessible worlds is the union of the epistemic alternatives of the speaker in every $s \in T$ with their respective valuation for atomic propositions. But that means for $\langle M, w\rangle =_{def} \bigvee_1(T)$ that for every $\langle M_i, w_i\rangle \in T$ and for every $v_i \in R_{i,\diamond}[w_i]$ there is an epistemic alternative $v \in R_\diamond[w]$ that assigns in $\langle M, w\rangle$ exactly the same valuation to atomic propositions as v_i in $\langle M_i, w_i\rangle$: $\forall v_i \in R_{i,\diamond}[w_i]\exists v \in R_\diamond[w]\forall p \in \mathcal{P} : V(p)(v) = V_i(p)(v_i)$. But because this is exactly what defines the order $\preceq^0$ (see definition 2), we obtain: $\forall s \in T : \bigvee_1(T) \preceq^0 s$.

Fact 6 *For all* $T \subseteq \textsc{State}_{KD45}$ *the following holds:*
(i) $\bigvee_1(T) \in \textsc{State}_{KD45}$,
(ii) $\forall s \in T : \bigvee_1(T) \preceq^0 s$.

Hence, $\bigvee_1$ has all the properties to apply for the role of $\bigvee$ in fact 3. Thus, we can use the function to characterize a class of honest formulas for $\textsc{State}_{KD45}$ and $\Box\mathcal{L}_{(0)}$. And a short consideration reveals that the antecedents of the inferences in (D1), (D2), (D5) using the first antecedent, and (D8) are persistent under $\bigvee_1$. Hence, they are all honest with respect to $\Box\mathcal{L}_{(0)}$ and $\textsc{State}_{KD45}$. Take, for instance, the sentence $\phi =_{def} \Box(p \vee q)$ and assume $\langle M_1, w_1\rangle, \langle M_2, w_2\rangle \in \textsc{State}_{KD45}$ make this sentence true. Hence, $\forall v_1 \in R_\diamond[w_1] : M, v_1 \models p \vee q$ and the same for $\langle M_2, w_2\rangle$. $\bigvee_1$ maps those states on a new state $\langle M, w\rangle$ such that $R_\diamond[w]$ is simply the union of $R_{1,\diamond}[w_1]$ and $R_{2,\diamond}[w_2]$ - whose members all make $p \vee q$ true. Of course, it will also hold that $\forall v \in R_\diamond[w] : M, v \models p \vee q$. Hence, ϕ is persistent under $\bigvee_1$.

We will now again show that (D1) holds with respect to $\textsc{State}_{KD45}$ and $|\!\equiv^0$ but now formally more precisely, using $\bigvee_1$. By assumption, p and q are logical independent propositions. Assume furthermore that there is a state $s_1 \in \textsc{State}_{KD45}$, such that $s_1 \models \Box(p \vee q)$

but $s_1 \not\models \Diamond p \wedge \Diamond q$. Without loss of generality, we assume $s_1 \models \Box\neg p$. Notice again that $\Box\neg p \in \Box\mathcal{L}_{(0)}$, whereby $\Box\mathcal{L}_{(0)}$ is the defining language of the information order $\preceq^0$. Because $p \not\equiv \bot$, we can find a state $s_2 \in \textsc{State}_{KD45}$ such that $s_2 \models \Box p$. It follows that $s_2 \models \Box(p \vee q)$. Let $s =_{def} \bigvee_1(\{s_1, s_2\})$. Because $\Box(p \vee q)$ is $\bigvee_1$-persistent, $s \models \Box(p \vee q)$. By construction, we additionally have $s \models \Diamond p$ and hence $s \not\models \Box\neg p$. Thus, we found a formula in the defining language that holds at s_1 but not at s. Together with fact (6.ii), this allows us to conclude $s \prec^0 s_1$. To summarize, we have found for every KD45-state verifying $\Box(p \vee q)$ that does not make the free choice inference true a strictly smaller KD45 state. Hence, we verified the antecedent of fact 4 and can conclude that (D1) is valid with respect to $\textsc{State}_{KD45}$ and $|\!\equiv^0$.

Notice, that (D1) is not necessarily valid for logical dependent $p, q \in \mathcal{L}_{(0)}$.[53] Gazdar's inconsistency check on pcis now is a natural consequence of the non-monotonicity of the notion of entailment we defined.

Let's take as second example the inference (D5) with the first antecedent $\Box(p \vee q)$. In this case a state s_1 that verifies the antecedent $\Box\Box(p \vee q)$ ($\equiv_{KD45} \Box(p \vee q)$) but falsifies the conclusion $\Diamond p \wedge \Diamond q$ of (D5) will again (without loss of generality) imply the order-relevant sentence $\Box\neg p$. To show that such a state s_1 cannot be minimal with respect to the order $\preceq^0$, this time, we have to merge s_1 with a witness for $\Diamond p$ such that the stronger claim $\Box\Box(p \vee q)$ still holds in the merge. But the s_2 we used above does already fulfill this requirement: $s_2 \models \Box\Box(p \vee q)$ and hence $s =_{def} \bigvee_1(\{s_1, s_2\}) \models \Box\Box(p \vee q)$ because the sentence is $\bigvee_1$ persistent. And again we can conclude from $s \not\models \Box\neg p$ and fact (6.ii) that $s \prec^0 s_1$. Hence, s_1 is for $\Box\Box(p \vee q)$ not minimal with respect to $\preceq^0$ and (D5) follows by fact 4.

The validity of the inferences (D2) and (D8) can be proven by the same means. However, the argument does not extend to (D5) using the second antecedent $\Box p \vee \Box q$. The reason is simply that $\Box p \vee \Box q$ is not $\bigvee_1$-persistent. But the inference (D5) does hold! Actually we obtain $\Box p \vee \Box q \;|\!\equiv^0_{KD45} \bot$, because the formula $\Box p \vee \Box q$ is not honest with respect to $\Box\mathcal{L}_{(0)}$ and $\textsc{State}_{KD45}$, i.e. there is no state $s \in \textsc{State}_{KD45}$ such that $s \models \Box(\Box p \vee \Box q)$ and $\forall s' \in \textsc{State}_{KD45} : s' \models \Box(\Box p \vee \Box q) \Rightarrow s \preceq^0 s'$. How to see that $\Box p \vee \Box q$ is dishonest? If a speaker believes $\Box p \vee \Box q$ she may, for instance, believe that p but not believe that q. Let s_1 be a state where this is the case. On the other hand, she may also believe that q but not believe that p. Assume that this holds in s_2. Because $\Box p, \Box\neg p, \Box q, \Box\neg q$ are in the order-defining language $\Box\mathcal{L}_{(0)}$ we have $s_1 \not\preceq^0 s_2$ and $s_2 \not\preceq^0 s_1$. Hence, if $\Box p \vee \Box q$ were honest, then there would exists an $s \in \textsc{State}_{KD45}$ such that $s \models \Box(\Box p \vee \Box q)$ and s is $\preceq^0$-smaller than both, s_1 and s_2. However, if s is smaller than s_1, then $s \models_{KD45} \neg\Box p$ because the speaker can believe only less in s that she believes in s_1. For the same reason $s \models_{KD45} \neg\Box q$. But then $s \not\models_{KD45} \Box(\Box p \vee \Box q)$ what contradicts our first assumption about s. Hence, $\Box p \vee \Box q$ has to be dishonest with respect to $\Box\mathcal{L}_{(0)}$ and $\textsc{State}_{KD45}$. Or, in other words, a speaker who utters $\Box p \vee \Box q$ cannot at the same time obey the maxims $\mathcal{T}$ and $\mathcal{Q}_1$: she cannot at the same time believe in her utterance and say all she believes. This sentence is, given our formalization of Grice's maxims, pragmatically not well-formed.[54]

[53] If, for instance, $p = \bot$, then we would not be able to find such an s_2 as we used in the proof.

[54] The same point can be made much easier but less intuitively using the disjunction property. In $\textsc{State}_{KD45}$ we have obviously: $\Box(\Box p \vee \Box q) \models_{KD45} \Box p \vee \Box q$. Both, $\Box p$ and $\Box q$, are in the order-defining language $\Box\mathcal{L}_{(0)}$. In order to have the disjunction property, $\Box(\Box p \vee \Box q)$ should entail one of these sentences.

But this is not such a bad result! Sentences like (23a) and (23b) are indeed reported to be odd. In particular, they do not allow a free choice reading.

(23) a. *Mr. X must be in Amsterdam or Mr. X must be in Frankfurt.

b. *I believe that A or I believe that B.

The approach at hand can explain these intuitions by referring to the notion of pragmatic well-formedness that it allows to define.

This argument nicely illustrates the power of the notion of *honesty*. Normally, we are not able to recognize whether a speaker is really obeying the maxims of Grice: whether she does tell us the truth and gives us all the information she has. The interpreter has to assume that the speaker is trustable. Dishonest sentences, however, directly compromise the speaker. They can never be uttered in accordance with the Gricean maxims. They are therefore also a very powerful testing device for the account presented here.

The information order $\preceq^0$ does not allow us to derive the other inferences in section 3.1.3 we want to account for. In particular, we cannot account for free choice permission. The reason is easily found: the basic information language does not contain any sentence that refers to belief the speaker has about deontic modalities (or any other modality). Thus, if asked to compare two states, the order is not able to see differences in belief about modalities. So how can we expect any minimization effect on such kind of information? The strategy to overcome the limitation we face here should be to extend the order defining language. This will be done in the following sections.

However, a small modification of what we have said above is necessary. Perhaps the reader has recognized that in using the basic information order we *do* have a certain minimization effect on what the speaker beliefs on at least one modality: her own belief state. But this is due to the particular logic of belief used here: KD45. This logic demands a close relation between what the speaker believes and her beliefs on her beliefs: the speaker is assumed to be competent on her beliefs. Therefore, minimizing the $\mathcal{L}_{(0)}$-beliefs of the speaker will also minimize her beliefs on these beliefs. To make this a little bit more precise, consider the language $\mathcal{L}^*$ defined by the following BNF: $\varphi_* ::= p(p \in \mathcal{L}_{(0)})|\varphi_* \wedge \varphi_*|\varphi_* \vee \varphi_*|\Box\varphi_*$. This language allows to express beliefs of various depths but prohibits the occurrence of $\Box$ under negation. Now, it can be shown that $\forall\varphi \in \Box\mathcal{L}^* \exists\varphi' \in \Box\mathcal{L}_{(0)} : \varphi \equiv_{KD45} \varphi'$. Hence, we could have defined $\preceq^0$ equivalently using $\Box\mathcal{L}^*$. However, while $|\equiv^o$ also minimizes beliefs the speaker has on what she does believe, it is easy to see that her beliefs on what she does not believe do not count for the order.[55]

3.2.5 Information Orders with Modal Information

Of course, extending the defining language will in turn widen the class of properties of the states that the information order is sensible to - this is actually our intention. But, to

Of course, this is not true. Hence, the formula cannot have the disjunction property and, therefore, using fact 2, $\Box p \vee \Box q$ cannot be honest with respect to $\Box\mathcal{L}_{(0)}$ and $\textsc{State}_{KD45}$.

[55] Because of the introspective power of the agent minimizing beliefs on beliefs will actually result in maximizing beliefs on what the speaker does not believe.

prove validity on minimal states with respect to such more complex orders we need to know how the minimal states look like. We need a model-theoretic definition of the order. But how to establish such results? Van der Hoek et al. [1999] provide an interesting technique that simplifies this problem for certain order defining languages. They use the notion of *Ehrenfeucht - Fraïssé - Orders* to this purpose:

Definition 6 *(Ehrenfeucht - Fraïssé - Orders)*
Let $\preceq_n, n \in \mathbb{N}$, be an enumerable set of partial orders. We define the Ehrenfeucht-Fraïssé-order *$\preceq$ as follows.*

$$\langle M, w\rangle \preceq \langle M', w'\rangle \Leftrightarrow_{def} \forall n \in \mathbb{N} \forall v' \in R'_\Diamond[w'] \exists v \in R_\Diamond[w] : \langle M, v\rangle \preceq_n \langle M', v'\rangle.$$

For this notion they prove the following result.[56]

Theorem 2 *(Collecting)*
Let $\mathcal{L}^*$ and $\{\mathcal{L}^*_{(n)} | n \in \mathbb{N}\}$ be a system of sublanguages such that for all $n \in \mathbb{N}$: $\mathcal{L}^*_{(n)} = \mathcal{L}^* \cap \mathcal{L}_{(n)}$. If for all $n \in \mathbb{N}$ the language $\mathcal{L}^*_{(n)}$ defines the order $\preceq_n$ and $\mathcal{L}^*$ is closed under $\vee$, then $\Box\mathcal{L}^*$ defines $\preceq$, i.e.

$$s \preceq s' \Leftrightarrow \forall \phi \in \Box\mathcal{L}^* : s \models \phi \Rightarrow s' \models \phi.$$

Given this result, if we know the model conditions of the orders defined by the finite sublanguages $\{\mathcal{L}^*_{(n)} | n \in \mathbb{N}\}$, then we also know that the order defined by the language $\Box\mathcal{L}^*$ is the Ehrenfeucht- Fraïssé order of these 'finite' orders. And to establish model-conditions for a finite language with finite modal depth is much less demanding.

3.2.6 The Objective Information Order

In this section we will study an appealing candidate for an information order that respects the knowledge an agent has on other modalities. However, at the end we will see that the notion of entailment defined by this order will not be able to give us in $\textsc{State}_{KD45}$ all the inferences we are looking for.

To find a modality-sensible defining language we use the simple idea to extend the basic language $\mathcal{L}_{(0)}$ with all sentences of $\mathcal{L}$ that say something on modalities other than the belief state of the speaker.

Definition 7 *(The Objective Information Order $\preceq^o$)*
Let $\mathcal{L}^o \subseteq \mathcal{L}$ be the language defined by the BNF $\phi_o ::= p(p \in \mathcal{L}_{(0)}) | \neg\phi_o | \phi_o \wedge \phi_o | \nabla\phi, \nabla \neq \Box(\phi \in \mathcal{L})$. We call $\mathcal{L}^o$, following Halpern[57], the objective information language *and define the* objective information order, *$\preceq^o$, for $s, s' \in \textsc{State}_{KD45}$ as follows:*

$$s \preceq^o s' \Leftrightarrow \forall \varphi \in \Box\mathcal{L}^o : s \models \varphi \Rightarrow s' \models \varphi.$$

[56] For a proof see van der Hoek et al. [1999]. The result relies on the finiteness property of the set of atomic propositions.

[57] According to van der Hoek et al. [2000] the objective information order has been introduced by Halpern in a paper where he extends the approach in Halpern & Moses [1984] to the multi-agent case.

Building on the result of the last section, van der Hoek et al. [1999] show that the following equivalent model-theoretic definition of $\preceq^o$ can be given.

Fact 7 *We define on KD45 for $n \in \mathbb{N}$ the relations $\cong_n^{-\Diamond}$ as follows:*

$$\langle M, w\rangle \cong_0^{-\Diamond} \langle M', w'\rangle \Leftrightarrow_{def} \forall p \in \mathcal{P} : V(p)(w) = V'(p)(w')$$

$$\langle M, w\rangle \cong_{n+1}^{-\Diamond} \langle M', w'\rangle \Leftrightarrow_{def} \begin{cases} \langle M, w\rangle \cong_n^{-\Diamond} \langle M', w'\rangle \ \& \\ \forall \triangle \neq \Diamond \forall v \in R_\triangle[w] \exists v' \in R'_\triangle[w'] : \langle M, v\rangle \cong_n \langle M', v'\rangle \text{ (forth) } \& \\ \forall \triangle \neq \Diamond \forall v' \in R'_\triangle[w] \exists v \in R_\triangle[w] : \langle M, v\rangle \cong_n \langle M', v'\rangle \text{ (back)} \end{cases}$$

Then $\preceq^o$ is the Ehrenfeucht- Fraïssé order defined by the orders $\cong_n^{-\Diamond}$ for $n \in \mathbb{N}$:

$$\langle M, w\rangle \preceq^o \langle M'w'\rangle \Leftrightarrow \forall n \in \mathbb{N} \forall v' \in R'_\Diamond[w'] \exists v \in R_\Diamond[w] : \langle M, v\rangle \cong_n^{-\Diamond} \langle M', v'\rangle$$

The proof works by induction on the maximal modal depth n of the sublanguages $\mathcal{L}^o_{(n)} = \mathcal{L}^o \cap \mathcal{L}_{(n)}$. For every $n \in \mathbb{N}$ one has to show that $\mathcal{L}^o_{(n)}$ define the respective relation $\cong_n^{-\Diamond}$.[58] Then one applies theorem 3 from section 3.2.5. Remember from section 3.1.2 that $\cong_n$ stands for n-bisimulation between two states. The relation $\cong_n^{-\Diamond}$ is a weakening of $\cong_n$: $R_\Diamond$/ $R'_\Diamond$ -branches starting at the root of two states that are compared are not considered. In other words, the relation $\cong_n^{-\Diamond}$ ignores, the beliefs of the speaker on her own beliefs.

The extended order $\preceq^o$ defines, based on definition 1, a new notion of pragmatic entailment: $|\equiv^o_{KD45}$. The question we now have to address is: can this entailment relation account for the inferences (D1) - (D9)? To investigate this problem we would like to use the same techniques as in section 3.2.4 when we studied this question for the basic information order. In particular, we would like to make use of fact 4. However, to establish the antecedent of this fact, we cannot rely on the merge function $\bigvee_1$ any longer. Remember that we need our operation $\bigvee$ to fulfill two conditions: (i) $\forall T \subseteq \text{STATE}_{KD45} : \bigvee(T) \in \text{STATE}_{KD45}$, and (ii) $\forall s \in T : \bigvee(T) \preceq^o s$. But for $\bigvee_1$ we only showed the claim (ii) for the basic information order $\preceq^0$. In this case the only thing we needed was that for every $s' = \langle M', w'\rangle \in T$ and every $v' \in R'_\Diamond[w']$ there exists for $\langle M, w\rangle =_{def} \bigvee(T)$ a $v \in R_\Diamond[w]$ that agrees with v' in the valuation of propositional atoms. For the objective information order, as fact 7 tells us, we need much more. v and v' have to agree on a great part of their generated submodels: $\langle M', v'\rangle \cong_n^{-\Diamond} \langle M, v\rangle$ for every $n \in \mathbb{N}$! $\bigvee_1$ cannot warrant this. For suppose that in the generated submodel of v' in M' there is a kind of circle leading back to a world $v'' \in R'_\Diamond[w]$, hence, there exists a sequence of accessibility relations $\langle R'_{1,\triangle_i}, ..., R'_{n,\triangle_l}\rangle$ of M', $\triangle_1 \neq \Diamond$ and a sequence of worlds $\langle v'_1, ..., v'_{n+1}\rangle$, such that $\forall i (1 \leq i \leq n) : v'_{i+1} \in R_{\triangle_i}[v'_i]$ and $v'_1 = v'$ and $v'_{n+1} = v''$. If you now merge $\langle M', w'\rangle$ with other states using $\bigvee_1$, then in the resulting state $\langle M, w\rangle$ $R_\Diamond[v'']$ will be much bigger than $R'_\Diamond[v'']$. Hence, in M v'' generates a different submodel than in M'. But therefore, a relevant part of the submodel generated by v' in the new model M will change too and, even though $\cong^{-\Diamond}$ ignores $\Diamond$-branches starting from v', $\langle M, v\rangle \not\cong^{-\Diamond} \langle M', v'\rangle$. To make sure that we do not get in any trouble of this kind, we need a new merge operation that does not pool the set of epistemical possible worlds directly. Instead, we let it work on copies of them. We define:

[58]Essentially for the proof is the well-known result of modal logic that to decide whether a sentence of a modal language with modal depth $n \in \mathbb{N}$ is true it is sufficient to look n-deep into the model.

Definition 8 *(The Merge Function $\bigvee_2$)*
Let T be a subset of $State_{KD45}$ indexed with I. Assume that the domains W of all states $s \in T$ are disjunct and w is a world not occurring in any domain. Let $\tilde{\cdot}$ be a copy-function, an injection from $\bigcup_{i \in I} W_i$ to a distinct set of worlds D. $\bigvee_2(T)$ is the state $\langle CL_{KD45}(M'), w\rangle$, where M' is defined as follows.
$W' = \bigcup_{i\in I} W_i \cup \{\tilde{v_i} | v_i \in R_{i,\diamond}[w_i], i \in I\} \cup \{w\}$,
$R'_\diamond = \bigcup_{i\in I} R_{i,\diamond} \cup \{\langle w, \tilde{v}\rangle | v \in R_{i,\diamond}[w_i], i \in I\}$,
$R'_\triangle = \bigcup_{i\in I} R_{i,\triangle} \cup \{\langle \tilde{v_i}, u_i\rangle | v_i \in R_{i,\diamond}[w_i] \wedge u_i \in R_{i,\triangle}[v_i] \wedge i \in I\}$ *for* $\triangle \neq \diamond$,
$V'(p) = \bigcup_{i\in I} V_i(p) \cup \{\tilde{v_i} | v_i \in V_i(p), i \in I\}$.

The picture below demonstrates the working of the operator $\bigvee_2$ for the merge of two states $s_1 = \langle M_1, w_1\rangle$ and $s_2 = \langle M_2, w_2\rangle$ to a state $s = \langle M, w\rangle$.

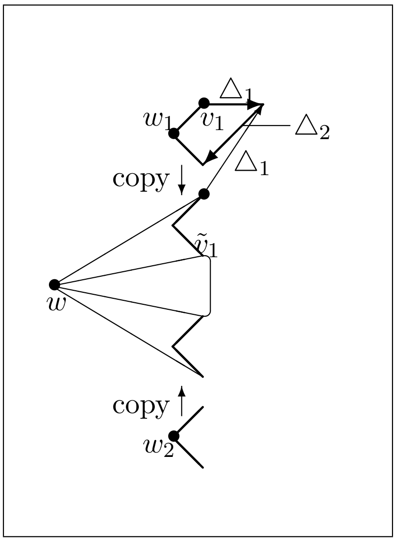


Figure 2:

Now, modalities different from $\diamond$ do occur in the picture. They are represented by arrows annotated with the modality to which they refer. Otherwise the same conventions as for the last picture are in force. The graphic illustrates how the use of copies for the merged states breaks the circles that caused the problem we discussed above. If we had merged s_1 and s_2 directly using $\bigvee_1$, then $\langle M_1, v_1\rangle \not\cong_3^{-\diamond} \langle M, v_1\rangle$, because we would have obtained that in $\langle M, w\rangle =_{def} \bigvee_1(\{s_1, s_2\})$ from v_1 one can reach by $\triangle_1$, $\triangle_2$ and $\diamond$ all elements of $R_{2,\diamond}[w_2]$. However, this is not possible in M_1.

For $\tilde{v_1}$ this problem does not occur. Notice that even though $\langle M, v_1\rangle \cong_n^{-\diamond} \langle M, \tilde{v_1}\rangle$ for $n \in \mathbb{N}$ - hence, they make exactly the same sentences in $\mathcal{L}^o$ true, $\tilde{v_1}$ differs from v_1 by not seeing in M under any modality $\triangle \neq \diamond$ any other point of $R_\diamond[\tilde{v_1}]$.

Fact 8 *For all $T \subseteq$ STATE_{KD45} the following holds:*
(i) $\bigvee_2(T) \in \text{STATE}_{KD45}$,
(ii) $\forall s \in T : \bigvee_2(T) \preceq^o s$.

The proof is quite simple given what we have said above. Take an arbitrary $T \subseteq \text{STATE}_{KD45}$, $\langle M_i, w_i\rangle \in T$ and let $\langle M, w\rangle =_{def} \bigvee_2(T)$. We have to show that $\forall n \in \mathbb{N} \forall v_i \in R_{i,\diamond}[w_i] \exists v \in R_\diamond[w] : \langle M, v\rangle \cong_n^{-\diamond} \langle M_i, v_i\rangle$. We know that by construction $\forall v_i \in R_{i,\diamond}[w_i] : \tilde{v_i} \in R_\diamond[w]$. But this copy $\tilde{v_i}$ of v_i is a $v \in R_\diamond[w]$ such that $\langle M, v\rangle \preceq_n^{-\diamond} \langle M_i, v_i\rangle$ for all $n \in \mathbb{N}$ by the way the construction is set up.

Together with fact 3 this result allows us to establish a general class of $\Box\mathcal{L}^o$-honest formulas: *If $\Box_s\phi$ is $\bigvee_2$-persistent, then ϕ is $\Box\mathcal{L}^o$-honest.* Actually, all antecedents of the inferences in the data-section, beside the part of (D5) we already disqualified in the last section, are honest with respect to $\Box\mathcal{L}^o$.

Parallel to the argumentation in section 3.2.4, we can establish the validity of the free choice inferences using fact 4. However, not all inferences from section 3.1.3 will come out as valid. While we obtain (D1), (D2), (D4), (D5), (D7), and (D8), the inferences (D3), (D6), and (D9) do not hold.

Let's start by discussing one inference we already had as valid given the basic information order: (D2). Let $p, q \in \mathcal{L}_{(0)}$ be two logically independent propositions. Let $s_1 \in \textsc{State}_{KD45}$ be a model of the formula that verifies $\Box\Diamond(p \vee q)$ but fails the conclusion of (D2), hence, without loss of generality, $s_1 \models \Box\neg p$, where $\Box\neg p \in \Box\mathcal{L}^o$, our order defining language. Because p and q are logically independent, p is a contingent fact and, hence, there will be a state $s_2 \in \textsc{States}_{KD45}$ such that $s_2 \models \Diamond p$. It follows that $s_2 \models \Diamond(p \vee q)$ and, by the validity of axiom (5) in $\textsc{State}_{KD45}$, $s_2 \models \Box\Diamond(p \vee q)$. Let $s =_{def} \bigvee_2(\{s_1, s_2\})$. Because $\Box\Diamond(p \vee q)$ is $\bigvee_2$-persistent, we obtain that $s \models \Box\Diamond(p \vee q)$. Additionally, $s \models \Diamond p$ by construction, and hence, $s \prec^o s_1$. By fact 4, $\Diamond(p \vee q) \mid\equiv^o_{KD45} \Diamond p \wedge \Diamond q$.

However, we will not be able to conclude by a parallel argument that in the set of $\preceq^o$-minimal states for $\Box\triangle(p \vee q)$, $\triangle p$ holds and hence to account for (D3). The reasoning fails the moment we want to conclude that on a state s_2 where $\triangle p$ is valid, $\Box\triangle p$ is valid too. In the derivation above we could do the step from $\Diamond p$ to $\Box\Diamond p$ because of axiom (5): because we assumed the speaker to have negative introspective power. We have nothing similar for $\triangle$.

Instead of (D3) we *do* get (D4), the incompetence inference. Let $p, q \in \mathcal{L}_{(0)}$ be two logical independent propositions and let s_1 be a model of the formula that verifies $\Box\triangle(p \vee q)$ but fails the conclusion of (D4). Without loss of generality, $s_1 \models \Box(\neg\triangle p \vee \triangle q)$. The formula $\Box(\neg\triangle p \vee \triangle q)$ is an element of the order defining language $\Box\mathcal{L}^o$. $p \wedge \neg q$ is consistent and hence, there will be a state $s_2 \in \textsc{States}_{KD45}$ such that $s_2 \models \Box\nabla(p \wedge \neg q)$. It follows that $s_2 \models \Box\triangle(p \vee q)$. Let $s = \bigvee_2(\{s_1, s_2\})$. Then, by persistence of the antecedent we have $s \models \Box\triangle(p \vee q)$. Additionally, $s \models \Diamond(\triangle p \wedge \neg\triangle q)$ by construction and hence, $s \not\models \Box(\neg\triangle p \vee \triangle q)$. This allows us to conclude that $s \prec^o s_1$. Hence, by fact 4, (D4) is valid.

To summarize, on $\textsc{State}_{KD45}$ the incompetence inference (D4) is valid with respect to $\mid\equiv^o$ while the free choice permission (D3) is not. Actually this is not such a bad result. As observed in the literature, the incompetence reading does exist. Additionally, the free choice reading seem to be bound to the assumption that the speaker is competent. We did not assume any connection between the deontic facts and the knowledge of the speaker here - hence, we should not obtain free choice permission. On the other hand, for the epistemic modality we build competence into the system - by assuming the axioms (4) and (5) to be valid. As Zimmermann did, we can explain the observation that epistemic free choice seems always valid by claiming that interpreters always take a speaker to have full introspective power.

The aim of the next section has to be to generalize the concept of competence behind introspection to other modalities - here we will rely on the work of Zimmermann [2000] - and then investigate whether we can make the inferences (D3) and (D6) valid this way.

3.2.7 Competence and Multi-Modal Belief

As we have observed in the last section we will be only able to derive the intended inferences in the given setting if we assume additional restrictions on the class of models on which utterances are interpreted. There has to be some kind of connection between what is valid on a certain modality and the speaker's beliefs with respect to this modality: the speaker has to be assumed to be in some sense *competent* on the relevant modality.

In how we make this idea precise we rely on Zimmermann's [2000] formalization of competence. As stated in section 2.2, Zimmermann, building on a proposal of Groenendijk & Stokhof [1984], defines competence by the following first-order model condition.[59]

Definition 9 *(Competence)*
A speaker is strongly competent in a state $\langle M, w\rangle \in \textsc{State}_{KD45}$ *(where* $M = \langle F, V\rangle$ *and* $F = \langle W, R_{\triangle_i}, ..., R_{\triangle_n}\rangle$*) with respect to a modality* $\triangle$ *iff*$_{def}$

$$\forall v \in W : v \in R_\diamond[w] \Rightarrow (R_\triangle[v] = R_\triangle[w]).$$

It is easy to prove that this condition is characterized in modal propositional logic by the two axioms ($C1_\triangle$) and ($C2_\triangle$) described below, i.e. a speaker is strongly competent in some state $s = \langle M, w\rangle$ if the underlying frame locally (hence, in w) satisfies ($C1_\triangle$) and ($C2_\triangle$).

$(C1_\triangle)\ \nabla\phi \rightarrow \Box\nabla\phi$

($C1_\triangle$) will be called the axiom of *positive competence*. Intuitively it expresses that if a sentence holds in the $R_\triangle$-admissible worlds then the speaker knows this. If one substitutes $\Box$ for ∇ one obtains the positive introspection axiom (4). Using the same strategy as in the proof of fact 1 one can show that the following holds for any frame $F = \langle M, R_{\triangle_1}, ..., R_{\triangle_n}\rangle$ for the language $\mathcal{L}$ and any point w in this frame.

$F, w \models C1_\triangle$ iff $F\| - \forall v \in W : v \in R_\diamond[w] \Rightarrow (R_\triangle[v] \subseteq R_\triangle[w])$

We turn to the second axiom.

$(C2_\triangle)\ \neg\nabla\phi \rightarrow \Box\neg\nabla\phi$

($C2_\triangle$) will be called the axiom of *negative competence*. Intuitively it expresses that the speaker knows also for every sentence that is not valid on the $R_\triangle$ accessibility relation that this is the case. It comes as no surprise that this is a multi-modality generalization of negative introspection or euclidicity:

$F, w \models C2_\triangle$ iff $\forall v \in W : v \in R_\diamond[w] \Rightarrow (R_\triangle[v] \supseteq R_\triangle[w])$

[59]The (intensional) predicate $\lambda w \lambda x.P(w)(x)$ in his definition is instantiated here with the characteristic function of the $R_\triangle$-accessibility relation: $\lambda w \lambda v.wR_\triangle v$.

As these considerations show, the definition of competence used by Zimmermann [2000] is a generalization of the concept of full introspective power to arbitrary modalities. Now we have to investigate whether imposing this generalized model condition on the class of states that is considered (hence, to work on $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ instead of on $\textsc{State}_{KD45}$) allows us to account for the free choice *permission* as much as assuming introspective power of speaker allowed us to account for the *epistemic* free choice inferences. Thus, the question is whether (D3) and (D4) are valid conditional on assuming the speaker to be competent in the sense defined above.

Unfortunately, if we use the notion of entailment defined by the objective information order $\preceq^o$ on the set $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ we obtain a system that is much too strong: if the speaker is assumed to be competent on $\triangle$, then every consistent sentence ϕ is dishonest with respect to $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ and $\preceq^o$. Or, in other words, given the way $|\equiv^o$ interprets the maxims $\mathcal{T}$ and $\mathcal{Q}_1$ a speaker competent on $\triangle$ as formalized in $C1_\triangle$ and $C2_\triangle$ cannot utter any consistent sentence and be obeying these maxims. The reason for this is that $|\equiv^o$ demands a strongly competent speaker to provide so much information that there is no finite sentence to encode it all. In some more detail: for every $\varphi \in \mathcal{L}$ both, $\Box\nabla\varphi$ and $\Box\neg\nabla\varphi$ are in the order defining language $\Box\mathcal{L}^o$. Recall that this means that the order $\preceq^o$ prefers states where these sentences are false. But if a speaker is strongly competent with respect to $\triangle$ then for every $\varphi \in \mathcal{L}$: $\Box\nabla\varphi$ and $\Box\neg\nabla\varphi$, cannot be false at the same time (given the formalization we have chosen a speaker strongly competent on $\triangle$ knows for every $\varphi \in \mathcal{L}$ which of $\nabla\varphi$ and $\neg\nabla\varphi$ is true; hence, either $\Box\neg\nabla\varphi$ or $\Box\nabla\varphi$ holds). That means that there will be at least two incompatible classes of minimal states: one where $\Box\nabla\varphi$ is true, and one where $\Box\neg\nabla\varphi$ holds. But this would unmask the speaker to be dishonest. Hence, if a competent speaker does not want to be trapped by not obeying the maxims $\mathcal{T}$ and $\mathcal{Q}_1$ (as formalized in $|\equiv^o$) then she already has to exclude one of the possibilities with her utterance of ϕ: $\Box\phi \models_{KD45C1_\triangle C2_\triangle} \Box\nabla\varphi$ or $\Box\phi \models_{KD45C1_\triangle C2_\triangle} \Box\neg\nabla\varphi$. But the same argument applies for every $\varphi \in \mathcal{L}$! Thus, for every sentence φ her utterance has to convey whether she believes $\nabla\varphi$ or $\neg\nabla\varphi$. However, there can be no finite and consistent sentence that is that strong: that decides every sentence valid on the $R_\triangle$ accessible worlds. Hence, the general dishonesty result.[60]

Given these results we have to accept that the combination of strong (positive and negative) competence with respect to some modality $\triangle$ with a defining language that takes every fact on $\triangle$ to be relevant for the order does not result in an appropriate notion of pragmatic entailment. Two ways to improve the situation can be considered: (1) we can restrict the order defining language $\Box\mathcal{L}^o$ and, hence, weaken the pragmatic entailment, or (2) we can weaken our notion of competence. As we will see, both choices can be turned into a successful approach to the free choice inferences.

[60] Exactly the same point can also be made by using the disjunction property of honest formulas. Parallel to the situation in $\textsc{State}_{KD45}$, $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ has theorems (T1) $\Box\nabla\phi \leftrightarrow \nabla\phi$ (Proof: $(\Rightarrow)\Box\nabla\phi \Rightarrow_D \Diamond\nabla\phi \Leftrightarrow_{Def} \neg\Box\neg\nabla\phi \Rightarrow_{C2_\triangle} \nabla\phi$,$(\Leftarrow)$ by $C1_\triangle$) and (T2) $\Box\neg\nabla\phi \leftrightarrow \neg\nabla\phi$ (Proof: $(\Rightarrow)$ $\Box\neg\nabla\phi \leftrightarrow_{Def} \neg\Diamond\nabla\phi \rightarrow_D \neg\Box\nabla\phi \rightarrow_{C1_\triangle} \neg\nabla\phi$, $(\Leftarrow)$ by $C2_\triangle$) for the modality $\triangle$. Hence, we have $\models_{KD45C1_\triangle C2_\triangle} \Box\nabla\phi \vee \Box\neg\nabla\phi$ (Proof: $\models_{KD45C1_\triangle C2_\triangle} \alpha \vee \neg\alpha$. Instantiate α: $\models_{KD45C1_\triangle C2_\triangle} \Box\nabla\varphi \vee \neg\Box\nabla\varphi$. Using T1 and T2 : $\models_{KD45C1_\triangle C2_\triangle} \Box\nabla\varphi \vee \Box\neg\nabla\varphi$), without the disjuncts being theorems, too. Because formulas that are honest with respect to $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ and $\preceq^o$ have to fulfill the disjunction property it follows that if ϕ is $\preceq^o$-honest in $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ then for all $\psi \in \mathcal{L}$: $\phi \models_{KD45C1_\triangle C2_\triangle} \Box\nabla\psi$ or $\phi \models_{KD45C1_\triangle C2_\triangle} \Box\neg\nabla\psi$, which is impossible to fulfill with a consistent and finite sentence.

3.3 Applying the Framework

3.3.1 Solution 1: Weakening the Order

In this section we will introduce a first combination of a logic and a notion of entailment that gives us the free choice inferences. It will be derived from the framework of the previous section by weakening the information order $\preceq^o$ on which the notion of entailment $|\equiv^o$ was based. However, the notion of competence as used by Zimmermann [2000] will be kept. For the moment, we will assume that the language $\mathcal{L}$ has only one modality $\nabla \neq \Box$. Furthermore, we assume that the speaker is strongly competent with respect to $\triangle$. Hence, we are working in $\text{STATE}_{KD45C1_\triangle C2_\triangle}$. We will come back to the multi-modal case at the end of this section.

One way to look at the problem we ended up with in the last section is that we chose a too strict formalization of the conversational maxims $\mathcal{T}$ and $\mathcal{Q}_1$. As they are described by $|\equiv^o$ a speaker who wants to obey the maxim has to give nearly every information she has. The only thing we excluded from $\mathcal{L}^o$ were the beliefs the speaker has about she does not believes. But all other sentences counted as relevant for the information order. Perhaps we can obtain a more natural notion of pragmatic entailment when we allow the speaker to withhold more information. The problem, then, becomes to find the right restriction that fits our intuitions about when utterances give sufficient information. A promising approach is to generalize the restrictions we have already working for $\Box$-sentences in $\mathcal{L}^o$ to every modality the speaker is competent on. Hence, we systematically remove all sentences from $\mathcal{L}^o$ where ∇ is the modality with widest scope and occurs under negation. We call the new information order that is defined by this language the *positive information order*: [61]

Definition 10 *(The Positive Information Order $\preceq^+$)*
Let $\mathcal{L}^+ \subseteq \mathcal{L}$ be language defined by the BNF $\varphi_+ ::= p(p \in \mathcal{L}_{(0)})|\varphi_+ \wedge \varphi_+|\varphi_+ \vee \varphi_+|\nabla\varphi, \nabla \neq \Box(\varphi \in \mathcal{L})$. We call $\mathcal{L}^+$, the positive information language and define the positive information order, $\preceq^+$, for $s, s' \in \text{STATE}_{KD45C1_\triangle C2_\triangle}$ as follows:

$$s \preceq^+ s' \Leftrightarrow \forall\varphi \in \Box\mathcal{L}^+ : s \models \varphi \Rightarrow s' \models \varphi.$$

This order asks the speaker to give all information she has on what is necessary with respect to $R_\triangle$, but she may know more about what is possible in $R_\triangle$ than she says. Also for the positive information order an equivalent model-theoretic definition can be given.

Fact 9 *We define the relations $\lesssim_n^\diamond$ for $n \in \mathbb{N}$ on $\text{STATE}_{KD45C1_\triangle C2_\triangle}$ as follows:*

$$\langle M, w\rangle \lesssim_0^{-\diamond} \langle M', w'\rangle \Leftrightarrow_{def} \forall p \in \mathcal{P} : V(p)(w) = V'(p)(w')$$

$$\langle M, w\rangle \lesssim_{n+1}^{-\diamond} \langle M', w'\rangle \Leftrightarrow_{def} \begin{cases} \langle M, w\rangle \lesssim_n^{-\diamond} \langle M', w'\rangle \ \& \\ \forall v' \in R'_\triangle[w]\exists v \in R_\triangle[w] : \langle M, v\rangle \cong_n \langle M', v'\rangle \text{ (back)} \end{cases}$$

Then $\preceq^+$ is the Ehrenfeucht- Fraïssé order defined by the orders $\lesssim_n^{-\diamond}$ for $n \in \mathbb{N}$:

$$\langle M, w\rangle \preceq^+ \langle M'w'\rangle \Leftrightarrow \forall n \in \mathbb{N}\forall v' \in R'_\diamond[w']\exists v \in R_\diamond[w] : \langle M, v\rangle \lesssim_n^{-\diamond} \langle M', v'\rangle$$

[61] Notice, that this notion differs from what van der Hoek et al. [2000] call the positive information order.

It does not come not as a surprise that the order $\lesssim_n^{-\diamond}$ differs from the relation $\cong_n^{-\diamond}$ of fact 7 only in the absence of the *forth* condition - which is responsible for perseverance of sentences of the form $\Box\neg\nabla\phi$, $\phi \in \mathcal{L}$. Because the order $\preceq^+$ has not been discussed by van der Hoek et al. the proof of fact 9 is given here. We show by induction on $n \in \mathbb{N}$ that the languages $\mathcal{L}^+_{(n)} = \mathcal{L}_{(n)} \cap \mathcal{L}^+$ define the relation $\lesssim_n^{-\diamond}$. Then, the claim follows immediately by theorem 3.

For $n = 0$, $\mathcal{L}^+_{(0)} = \mathcal{L}_{(0)}$, $s \lesssim_0^{-\diamond} s' \Leftrightarrow s \cong_0 s'$, and the claim follows from theorem 1. Assume $\mathcal{L}^+_{(n)}$ would define $\lesssim_n^{-\diamond}$. We show that $\mathcal{L}^+_{(n+1)}$ defines $\lesssim_{n+1}^{-\diamond}$. Thus:

$$\langle M, w\rangle \preceq_{n+1}^{-\diamond} \langle M', w'\rangle \Leftrightarrow \forall\varphi \in \mathcal{L}^+_{(n+1)} : \langle M, w\rangle \models \varphi \Rightarrow \langle M', w'\rangle \models \varphi \text{ (*)}.$$

("⇒") Notice first that for all $\phi \in \mathcal{L}^+_{(n+1)}$ we can find a sentence $\bigwedge_{i=1}^{l} \bigvee_{j=1}^{k} \varphi_{i,j} \in \mathcal{L}^+_{(n+1)}$, where for all i, j either $\varphi_{i,j} \in \mathcal{L}_{(0)}$ or $\varphi_{i,j} = \nabla\pi_{i,j}$ for $\pi_{i,j} \in \mathcal{L}_{(n)}$, such that $\phi \equiv \bigwedge_{i=1}^{l} \bigvee_{j=1}^{k} \varphi_{i,j}$.[62] Now, assume that for some $\phi \in \mathcal{L}^+_{(n+1)} : M', w' \not\models \phi$. We have to show that then also $M, w \not\models \phi$, given that $\langle M, w\rangle \lesssim_{(n+1)}^{-\diamond} \langle M, w'\rangle$. $M', w' \not\models \phi \Leftrightarrow \exists i(1 \geq i \geq l)\forall j(1 \geq j \geq k) : M', w' \not\models \varphi_{i,j}$. Firstly, assume $\varphi_{i,j} \in \mathcal{L}_{(0)}$. Then, because $\langle M, w\rangle \lesssim_{(n+1)}^{-\diamond} \langle M, w'\rangle$ entails $\langle M, w\rangle \lesssim_0^{-\diamond} \langle M, w'\rangle$, $M, w \not\models \varphi_{i,j}$. On the other hand, if $\varphi_{i,j} = \nabla\pi_{i,j}$, then $\exists v' \in R'_\triangle[w'] : M', v' \not\models \phi$. But $\langle M, w\rangle \lesssim_{(n+1)}^{-\diamond} \langle M', w'\rangle$ entails also, that $\forall v' \in R'_\triangle[w']\exists v \in R_\triangle[w] : \langle M, v\rangle \cong_n \langle M', v'\rangle$. By theorem 1, we can conclude that $\exists v \in R_\triangle[w] : M, v \not\models \pi_{i,j}$ and, hence, $M, w \not\models \nabla\pi_{i,j}$. In sum we have shown that $\exists i(1 \geq i \geq l)\forall j(1 \geq j \geq k) : M, w \not\models \varphi_{i,j}$ and, thus, $M, w \not\models \phi$.

("⇐") Assume, that the left-side of the condition (*) does not hold. Hence, for some $\langle M, w\rangle, \langle M', w'\rangle \in \textsc{State}_{KD45C1_\triangle C2_\triangle}$ either (a) $\langle M, w\rangle \not\lesssim_n^{-\diamond} \langle M', w'\rangle$, or (b) $\exists v' \in R'_\triangle[w']$ $\forall v \in R_\triangle[w] : \langle M, v\rangle \not\cong_n \langle M', v'\rangle$. We show that in both cases the right side of (*) does not hold, either. In case (a) the induction assumption allows us to conclude that there exists $\varphi \in \mathcal{L}^+_n \subseteq \mathcal{L}^+_{n+1}$ such that $M, w \models \varphi$, while $M', w' \not\models \varphi$. Hence, the claim. In case (b) by theorem 1 it follows that there exists a $v' \in R'_\diamond[w']$ such that for every $v \in R_\triangle[w]$ there exists a $\varphi \in \mathcal{L}_{(n)}$ such that $M, v \models \varphi$ but $M', v' \not\models \varphi$. Because $\mathcal{L}_{(n)}$ is finite, we can find a finite sequence $\langle\varphi_1, ..., \varphi_m\rangle$ with $\varphi_1, ..., \varphi_m \in \mathcal{L}_{(n)}$ such that $\forall v \in R_\triangle[w] : M, v \models \bigvee_{i=1}^{m} \varphi_i$ and $M', v' \not\models \bigvee_{i=1}^{m} \varphi_i$. Hence, $M, w \models \nabla \bigvee_{i=1}^{m} \varphi_i$, but $M', w' \not\models \nabla \bigvee_{i=1}^{m} \varphi_i$. However, $\nabla \bigvee_{i=1}^{m} \varphi_i$ is an element of $\mathcal{L}^+_{n+1}$. Thus, we conclude that $\langle M, w\rangle \not\lesssim_{n+1}^{-\diamond} \langle M', w'\rangle$. q.e.d.

In terms of the information order $\preceq^+$ again a notion of entailment $|\!\equiv^+$ can be defined using definition 1. Now, we want to study which of the inferences of section 3.1.3 are valid with respect to this notion. A first thing that catches the eye is that on $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ the formula $\nabla p \vee \nabla q$ is not honest with respect to $\Box\mathcal{L}^+$. The sentence has incomparable minimal states: one, where ∇p but not ∇q holds and the speaker is informed about this, and one, where ∇q but not ∇p holds - again known by the speaker.[63]

[62] This is a straightforward consequence of how $\mathcal{L}^+$ is defined.

[63] Using the disjunction property of honest formulas one argues as follows: the problem is - as one might expect - that by (T1) and (T2) (see footnote 56) $\Box(\nabla p \vee \nabla q) \models_{KD45C1_\triangle C2_\triangle} \Box\nabla p \vee \Box\nabla q$ (Proof: $\Box(\nabla p \vee \nabla q) \equiv \Box(\neg\nabla p \rightarrow \nabla q) \Rightarrow_K \Box\neg\nabla p \rightarrow \Box\nabla q \Rightarrow_{T2} \neg\nabla p \rightarrow \Box\nabla q \equiv \nabla p \vee \Box\nabla q \Rightarrow_{T1} \Box\nabla p \vee \Box\nabla q$) but none of the disjuncts is entailed. Hence, the formula fails the disjunction property of the language $\Box\mathcal{L}^+$ and $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ and cannot, given fact 2, be honest.

Next, we want to establish the inferences (D3) and (D6) with respect to $|\equiv^+$. If we want to use the same kind of argument we employed earlier we need again a proper merge-operation $\bigvee_3$ on $\textsc{State}_{KD45C1_\triangle C2_\triangle}$. Thus, we need (i) $\forall T \subseteq \textsc{State}_{KD45C1_\triangle C2_\triangle} : \bigvee(T) \in \textsc{State}_{KD45C1_\triangle C2_\triangle}$, and (ii) $\forall s \in T : \bigvee_3(T) \preceq^+ s$. Notice that if we would drop the competence axioms and apply $|\equiv^+$ to $\textsc{State}_{KD45}$ we could again use $\bigvee_2$ as merge operation. Condition (ii) would still be fulfilled because $\preceq^+$ is a weakening of the order $\preceq^o$. Hence, $s \preceq^o s \Rightarrow s \preceq^+ s'$. However, if we take the axioms $C1_\triangle$ and $C2_\triangle$ to be valid, then we have to face the fact that $\bigvee_2(T)$ is normally not an element of $\textsc{State}_{KD45C1_\triangle C2_\triangle}$. Therefore, we have to define a new merge function $\bigvee_3$ - but notice that $\bigvee_3$ differs from $\bigvee_2$ only in the stronger closure applied to M'.

Definition 11 *(The Merge Function $\bigvee_3$)*
Let T a subset of $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ indexed with I. Assume that the domains W^s of all states $s \in T$ are disjunct and w is a world not occurring in any domain. $\bigvee_3(T)$ is defined as the state $\langle CL_{KD45C1_\triangle C2_\triangle}(M'), w\rangle$ where M' is defined as in definition 8.

Now property (i) is immediately warranted for $\bigvee_3$, but we might have lost (ii): $\forall s \in T : \bigvee_3(T) \preceq^+ s$. Fortunately, it turns out that we did not.

Fact 10 *For all $T \subseteq \textsc{State}_{KD45C1_\triangle C2_\triangle}$ the following holds:*
(i) $\bigvee_3(T) \in \textsc{State}_{KD45C1_\triangle C2_\triangle}$,
(ii) $\forall s \in T : \bigvee_3(T) \preceq^+ s$.

Intuitively, this can be seen by the observation that the only difference between $\bigvee_3$ and $\bigvee_2$ is that for $\bigvee_3$ extra arrows are added to $R_\triangle[w]$ and $R_\triangle[v]$ for $v \in R_\Diamond[w]$ in $\bigvee_2(T)$. But about these extra arrows $\preceq^+$ does not care - exactly because we dismissed the *forth* condition when going from $\cong_n^{-\Diamond}$ to $\lesssim_n^{-\Diamond}$. Hence, we get $\bigvee_3(T) \preceq^+ \bigvee_2(T) \preceq^+ s$ for all $s \in T$.

Based on $\bigvee_3$ we can use fact 3 to establish: *If $\Box\phi$ is persistent under taking $\bigvee_3$ then ϕ is $\preceq^+$-honest with respect to* $\textsc{State}_{KD45C1_\triangle C2_\triangle}$. The antecedents of (D3) and (D6) are persistent under union $\bigvee_3$ and we can continue to the question central to the whole discussion: do they $|\equiv^+$-entail their consequences? The answer is affirmative. We will only discuss (D6), (D3) works along the same lines. Let s_1 be a state for the honest formula $\Box\nabla(p\vee q)$ that does not satisfy the consequence $\triangle p \wedge \triangle q$ of (D6). Without loss of generality, we assume that $s_1 \models \nabla\neg p$. By $C1_\triangle$, we can conclude that $s_1 \models \Box\nabla\neg p$, whereby $\Box\nabla\neg p \in \Box\mathcal{L}^+$. Because $p \not\equiv \bot$ there will be a state $s_2 \in \textsc{State}_{KD45C1_\triangle C2_\triangle}$ such that $s_2 \models \Box\nabla p$ and, in consequence, $s_2 \models \Box\nabla(p \vee q)$. Let $s =_{def} \bigvee_3(\{s_1, s_2\})$. By the $\bigvee_3$-persistence of $\Box\nabla(p \vee q)$ we have $s \models \Box\nabla(p \vee q)$ and by construction we know $s \models \Diamond\triangle p$. Hence, $s \not\models \Box\nabla\neg p$, which allows us to conclude, together with fact 10.ii, that $s \prec^+ s_1$. Now, we can apply fact 4 and arrive at (D6).

As an aside: in this system (D4) and (D7) will not be valid, of course. According to $KD45C1_\triangle C2_\triangle$, incompetence of the speaker is not possible and hence the conclusions of (D4) and (D7) are not consistent with $C1_\triangle$ and $C2_\triangle$, let alone valid on minimal models.

In this section we restricted our considerations to a language that contained only one additional modal operator: ∇. As we have seen, in this case if the hearer thinks the speaker to be competent on $\triangle$ in the sense of $C1_\triangle$ and $C2_\triangle$, then $|\equiv^+$ predicts the free choice inferences (D3) and (D6) to be valid. Hence, if we interpret $\triangle$ as '*may*' we do obtain free choice permission. Of course, it would be preferable if we could extend the account to a richer language with more modalities - including, for instance, the belief states of other speakers or different kinds of deontic options. But then, one would also like to consider the possibility that the speaker is also competent with respect to these modalities. Such an extension is easily possible. And it is also not difficult to imagine how a language $\Box\mathcal{L}^*$ should be defined that allows, for instance, free choice inferences for a set of modalities Π_1 on which the speaker is competent and no free choice for the modalities: $\mathcal{L}^*$ should look like $\mathcal{L}^o$, except that we exclude for $\nabla \in \Pi_1$ sentences with negative occurrences of ∇ not embedded under another modality. The respective model-conditions are also straightforward, as is the validity of the free choice inferences (D3) and (D6) for $\nabla \in \Pi_1$ and the incompetence inferences (D4) and (D7) for $\nabla \in \Pi_2$.

It is even possible to have a notion of entailment that is independent of the choice of Π_1 and nevertheless provides the free choice inferences for modalities the speaker is competent on. This is the entailment defined by a language that allows *no* modality with widest scope to occur under negation: the BNF is $\varphi ::= p(p \in \mathcal{P})|\varphi \vee \varphi|\varphi \wedge \varphi|\nabla\phi, \nabla \neq \Box(\phi \in \mathcal{L})$. Let us call it $|\equiv^{++}$. One important difference to $|\equiv^+$ is that with respect to $|\equiv^{++}$ (D4) and (D7) are not valid for any modality on which the speaker is not taken to be competent (as described by the axioms $C1_\triangle, C2_\triangle$).[64]

3.3.2 Solution 2: Weakening the Logic

Recall the discussion at the end of section 3.2.7: applying $|\equiv^o$ on $\textsc{State}_{KD45C1_\triangle C2_\triangle}$ will predict dishonesty for all consistent sentences of the language $\mathcal{L}$. The problem was that, on the one hand, a speaker who is competent on a modality ∇ knows everything that can be expressed on this modality. On the other hand, the information order $\preceq^o$ also demands her to say everything she knows on ∇. Both obligations together can never be met by uttering a consistent sentence. In the last section we discussed a way out of this unpleasant situation by weakening the notion of entailment $|\equiv^o$. Another possibility is, of course, not to weaken what the speaker has to communicate, but to take him to be less competent. Parallel to the last section, where we removed sentences from the language that concerned knowledge of the speaker on what is not necessary with respect to the modality ∇, we now consider the case of an interpreter that does not take the speaker to be competent on these sentences: we weaken $KD45C1_\triangle C2_\triangle$ to $KD45C1_\triangle$. In this case, the speaker knows of excluded $\triangle$-options that they are excluded, but she may take it possible that more options are excluded than actually are. For instance, applied to deontic options, she may take it as possible that more obligations exist than actually do.

On $\textsc{State}_{KD45C1_\triangle}$ $|\equiv^o$ will not disqualify every consistent sentence as dishonest. But do we obtain the free choice inferences? Once more we run our machinery. Again, we have

[64] Given the problem that brought us to consider $\preceq^+$ one may also think about using an order that instead of skipping the forth condition of $\preceq^o$ skips the back condition. This would result in minimizing what the speaker takes to be possible and hence maximizing her belief. This approach does not lead to an appeling concept of honesty. Because the speaker's belief can be maximized in many different ways, honesty becomes a very rare thing again.

to adapt our merge-operator[65]: we define $\bigvee_4$ simply by weakening the closure conditions in definition 11 to $KD45C1_{\triangle}$. We can establish again the following fact:

Fact 11 *For all $T \subseteq$ $\text{STATE}_{KD45C1_{\triangle}}$ the following holds:*
(i) $\bigvee_4(T) \in \text{STATE}_{KD45C1_{\triangle}}$,
(ii) $\forall s \in T : \bigvee_4(T) \preceq^o s$.

In consequence, we have again our result about the $\mathcal{L}^o$-honesty of $\bigvee_4$-persistent formulas. It turns out that with the exception of $\Box p \vee \Box q$ all antecedents of the inferences in section 3.1.3 are honest. Particularly, $\nabla p \vee \nabla q$ is honest in $\text{STATE}_{KD45C1_{\triangle}}$ with respect to $\Box\mathcal{L}^+$. This may be a bit surprising on first glance, but take a look at the states sketched in figure 3. While there is no state in $\text{STATE}_{KD45C1_{\triangle}C2_{\triangle}}$ that is $\preceq^o$-smaller than s_1 and s_2, $s_3 \in \text{STATE}_{KD45C1_{\triangle}}$ is $\preceq^o$- smaller than both. Actually, s_3 is the result of merging s_1 and s_2 using $\bigvee_4$. s_3, however, while in $\text{STATE}_{KD45C1_{\triangle}}$ is not an element of $\text{STATE}_{KD45C1_{\triangle}C2_{\triangle}}$. Thus, the sentence $\nabla p \vee \nabla q$ becomes honest because in $\text{STATE}_{KD45C1_{\triangle}}$ less restrictions are imposed on the competence of the speaker.

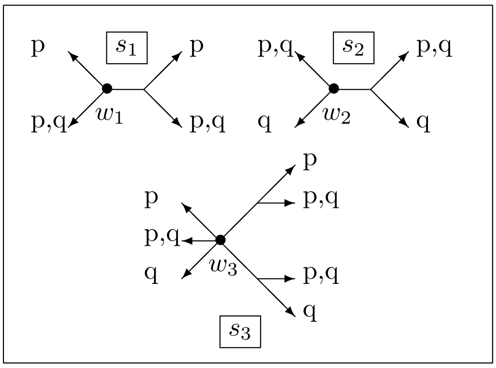


Figure 3:

In this picture, arrows present $\triangle$- admissible worlds, lines $\Diamond$-admissible worlds. Again, only $R_{\Diamond}$-arrows from the root of the state are drawn. Furthermore, only information on the deontic possibility of p and q is specified. Not mentioning a proposition letter at a world means that the proposition is not true there.

Also in this system we can account for the free choice inferences (D3) and (D6) (with respect to the latter now for both antecedents). To show that a state s_1 in $\text{STATE}_{KD45C1_{\triangle}}$ that entails, for instance, $\Box\triangle(p \vee q)$ but not the conclusion $\triangle p \wedge \triangle q$ cannot be minimal with respect to $\preceq^o$ and $\text{STATE}_{KD45C1_{\triangle}}$ one can use the $\bigvee_4$ merge with the same kind of s_2 as used in the last section to prove the validity of (D6).

But actually, the inferences (D4) and (D7) are valid too! This could have been expected, because in both cases the sentences you obtain by negating their conclusions are part of the order defining language $\Box\mathcal{L}^o$. Hence, the interpreter tries to make them false when interpreting the speaker using $|\!\equiv^o$, and, hence the consequences true. On the other hand, the consequences are also consistent with the logic and the assumption that the speaker believes the respective antecedents. Therefore, in sum, the free choice inferences occur together with the inference that the speaker is not competent on the modality. In particular, it is predicted the interpreter will learn more about what is valid on $R_{\triangle}$ than she thinks the speaker knows.

[65] $\bigvee_3$ is not appropriate because it fails condition (ii) in fact 11. $\bigvee_2$, on the other hand, fails condition (i).

We conclude this section with two final remarks. First, in the discussion above we assumed that the speaker is taken to be competent on one modality ∇. The discussed results all extend to the case that the logic is enriched with $C1_{\triangle'}$ for other modalities $\triangle'$ of the language.

Furthermore, it is interesting to look at the second possibility to weaken the logic: by giving up $C1_{\triangle}$ and keeping strong negative competence $C2_{\triangle}$. There seems to be nothing that speaks generally against such a concept of competence. However, it will not allow us to account for the data we want to capture. The order $\preceq^{+}$ minimizes also what the speaker knows about what is possible on $\triangle$-accessible worlds. Hence, in the minimal states as much as possible statements of the form $\Diamond\nabla\phi$ will be valid. Following $C2_{\triangle}$ the interpreter thinks that all the speaker takes possibly to be valid on $R_{\triangle}$ is actually valid (because if not, the speaker would know so). Hence, from $\Diamond\nabla\phi$ the interpreter infers $\nabla\phi$. This leads to results that are again much too strong. For instance, $\triangle p \vee \triangle q$ will be dishonest, because the sentences are consistent with $\nabla p \wedge \nabla\neg q$ as well as with $\nabla q \wedge \nabla\neg p$.

3.3.3 Solution 3: The Combination

Finally, it is interesting to note, that also the combination of $|\!\!\equiv^{+}$ with $\textsc{State}_{KD45C1_{\triangle}}$, hence, the combination of weakening the order and weakening the notion of competence allows us to derive the free choice inferences. The results from the last sections can be transfered nearly directly. One point that may need some discussion is the fact that also with respect to $\preceq^{+}$ we can use the merge $\bigvee_4$ without losing fact (11.ii) for the positive information order. But this follows immediately from the fact that $s \preceq^{o} s' \Rightarrow s \preceq^{+} s'$. The only other point that may be problematic by going from $\preceq^{o}$ to $\preceq^{+}$ is that the sentences we used to show that states that do not verify the conclusion of our inferences are not minimal are no longer in the order defining language. It appears that this is no problem for the free choice inferences (D1), (D2), (D3), (D5) with the first antecedent, (D6) for both antecedents and also for the inferences concerning the conjunction (D8) and (D9). The relevant sentences $\Box\neg p$, $\Box\nabla\neg p$ and $\Box\nabla\neg(p \wedge \neg q)$ are all in $\Box\mathcal{L}^{+}$. Again, this combination of a concept of competence and a notion of entailment differs from the one in section 3.2.6 in not disqualifying the second antecedent of (D6) as dishonest. From the solution studied in the last section this approach differs by not making the incompetence inferences (D4) and (D7) valid. Assume, for instance, that (D7) does not hold in some $s_1 \in \textsc{State}_{KD45C1_{\triangle}}$. This means in particular that $s_1 \not\models \Diamond(\nabla p \wedge \neg\nabla q)$. Hence, $s_1 \models \Box(\neg\nabla p \vee \nabla q)$. This is not a sentence of $\Box\mathcal{L}^{+}$ and hence a state that simply differs from s_1 in not entailing $\Box(\neg\nabla p \vee \nabla q)$ will not be strictly smaller than s_1 with respect to $\preceq^{+}$.

Just for the sake of completeness, we will discuss the inference (D9). Assume, there exists $s_1 \in \textsc{State}_{KD45C1_{\triangle}}$ such that $s_1 \models \Box\triangle(p \wedge q)$ and, without loss of generality, $s_1 \not\models \triangle(p \wedge \neg q)$. By $C1_{\triangle}$ it follows that $s_1 \models \Box\nabla(\neg p \vee q)$. The latter sentence is in the order defining language $\Box\mathcal{L}^{+}$. Because p and q are logically independent from each other, there are states such that $p \wedge q$ as well as $p \wedge \neg q$ is true. Take a $s_2 \in \textsc{State}_{KD45C1_{\triangle}}$ such that $s_2 \models \Box(\nabla p \wedge \triangle q \wedge \triangle\neg q)$. It follows that $s_2 \models \Box\triangle(p \wedge q)$. Hence, for $s =_{def} \bigvee_4(\{s_1, s_2\})$ it hold that $s \models \Box\triangle(p \wedge q)$. Furthermore, by construction, $s \not\models \Box\nabla(\neg p \vee q)$ and together with fact 11(ii) ($\forall s \in T : \bigvee_4(T) \preceq^{+} s$) we can conclude that $s \prec^{+} s_1$. In combination with fact 4 this proves the claim.

4 Discussion of the Proposal

The topic of this last chapter is an evaluation of the account developed in the thesis. We start with comparing the three combinations of a notion of pragmatic entailment and a class of states on which the entailment is evaluated that turned out to predict the free choice inferences. After that we will evaluate the approach in general.

4.1 Comparing the three proposed Approaches

At the end of the third section we discussed three combinations of a notion of entailment (capturing conversational implicatures due to the maxims $\mathcal{Q}_1$ and $\mathcal{T}$) and a class of states on which the notion of entailment is applied (modeling assumptions about the competence of the speaker) that all predicted the free choice inferences. In the following we will broaden our view and evaluate the plausibility of the accounts with respect to other questions. As we will see they do not always perform equally good. In the end we will come up with a ranking of the three approaches concerning their general adequacy and choose one of them as the most convincing account for free choice inferences.

4.1.1 Strong Competence and Weak Order

The first successful system, discussed in section 3.3.1, took a very strong perspective on competence: both, the positive ($C1_\triangle$) as well as the negative ($C2_\triangle$) competence axiom were assumed to be valid in the context of interpretation. This is what we called strong competence of the speaker with respect to $\triangle$. To avoid general dishonesty we had, in turn, to weaken what counts as the relevant information the speaker has to convey and, hence, to restrict the language that defined the pragmatic entailment. We discussed two possible adaptations of the objective information language $\Box\mathcal{L}^o$. In variant 1 we removed for modalities $\triangle$ the speaker is assumed to be competent on sentences from $\mathcal{L}^o$ where a negative occurrence of ∇ is not embedded under another modal operator; this led to the notion $|\!\equiv^+$. In a second variant *all* sentences that contain a negative occurrence of a modal operator in wide scope position are removed from $\mathcal{L}^o$ - *irrespective* of competence; the resulting notion of entailment was $|\!\equiv^{++}$. Both system are, despite the weaker information orders, quite strong in their predictions. For instance, we obtain that an utterance of the sentence $\nabla p \vee \nabla q$[66] made by a competent speaker is pragmatically not well-formed. In particular, no free choice inferences can be derived. The reason is simple: the sentence is not honest with respect to the order defining language $\Box\mathcal{L}^+/\ \Box\mathcal{L}^{++}$ and $\textsc{State}_{KD45C1_\triangle C2_\triangle}$. Or, to put it in other words, given the way $|\!\equiv^+$ and $|\!\equiv^{++}$ formalize the conversational maxims $\mathcal{T}$ and $\mathcal{Q}_1$ a competent speaker uttering $\nabla p \vee \nabla q$ cannot be obeying these maxims.[67]

Another pleasant property of the combination of a weak information order with a strong notion of competence is that it suggests a straightforward way how to account for the possible cancellation of free choice inferences. As we observed in section 2 free choice readings do not occur in case the speaker explicitly mentions her incompetence (cf. the examples (8) and (9) here repeated as (24) and (25)).

[66]This is the second antecedent in the inference schema (D6), which we flagged with a question mark.

[67]Recall that all three accounts we discussed predict the sentence $\Box p \vee \Box q$ to be pragmatically not well-formed for exactly the same reason.

(24) You may take an apple or a pear - but I don't know which.

(25) You may go to Shoal Creek or go to Shingle Creek. But stay away from the dangerous one.

What does this approach predict in such a situation? Of course, any statement of the speaker by which she claims to be (partly) incompetent will be inconsistent with taking her to be strongly competent. Hence, an interpreter that takes the speaker to be strongly competent on the deontic options will not be able to directly incorporate the information of the examples (24) and (25) in her belief state. She has, presumably, first to weaken her belief about the competence of the speaker and to give up strong competence. In this thesis we cannot discuss the topic of belief change, so we can only suggest that this adaption of the belief state of the interpreter leads to a new state on which $|\equiv^{+}$ and $|\equiv^{++}$ will not make the free choice inference valid.[68,69]

Already in section 3.3.1 we hinted at a difference between the two notions of weak entailment, $|\equiv^{+}$ and $|\equiv^{++}$. The inference schemas (D4) and (D7), which we wanted to obtain for modalities the speaker is not taken to be competent on, are not valid for $|\equiv^{++}$. To give a concrete example, it is predicted that a possibly incompetent speaker of (25) takes it as possible that the addressee may go to Shoal creek, but it is not predicted that the speaker takes it as possible that the addressee may go to Shoal Creek and may not go to Shingle Creek. This seems contra intuitions, according to which the sentence is understood as implying that the addressee may go only to one of the creeks and the speaker does not know which one. The reason why this inference is missing is that sentences like $\Box(\triangle p \wedge \neg\triangle q))$ are not elements of the order defining language $\mathcal{L}^{++}$ and, hence, not subject to minimization. Therefore, we cannot derive that the speaker takes $\triangle p \wedge \neg\triangle q$ to be an epistemic possibility and, in turn, establish the consequence of (D4). Even though this may be no serious defect of $|\equiv^{++}$ we have an alternative notion, $|\equiv^{+}$, that *can* account for (D4) and (D7).

One might argue that also $|\equiv^{+}$ has its weak point: it depends on what the hearer believes about the competence of the speaker. One may ask why an interpreter should adapt the completeness expectations she has for the utterance of the speaker this way. The following observation hints at an explanation. If the speaker is believed to be strongly competent

[68]Notice that whether we obtain this result indeed depends on the way the belief change of the interpreter is modeled. If, for instance, in reaction to (24) or (25) the interpreter adopts a belief state where not $C2_{\triangle}$ but still $C1_{\triangle}$ is valid, then $|\equiv^{+}$ and $|\equiv^{++}$ would still derive the free choice inferences. However, this belief adaption is arguably to specific. Why should the interpreter choose to give up only $C2_{\triangle}$?

[69]Of course, this outline of an explanation of the cancellation observation builds on the assumption that the interpreter first takes the speaker to be strongly competent. Only in these situations can we assume that belief change is necessary. But if competence is assumed to be something the hearer learns from incoming information it may be the case that she has only partial knowledge about the competence of the speaker but still enough to derive the free choice inferences. Then the speaker's announcement of incompetence may not be strong enough to contradict the belief of the interpreter and the free choice inferences may still go trough. An example for such a situation is again the case where the hearer thinks $C1_{\triangle}$ to be valid but does not know about $C2_{\triangle}$. A sentence like (24) does not stand in conflict with such a belief state. And, as we know from section 3.3.3, application of $|\equiv^{+}$, $|\equiv^{++}$ on this belief state would still render the free choice inferences valid. An interesting option how to exclude such a situation (and they seem to be contra intuition) is to take also competence to be an assumption the hearer makes about the speaker. We will come back to this perspective in section 4.2.

with respect to some modal operator $\triangle$, then $|\equiv^+$ together with the assumption of strong competence are already enough to infer all the speaker's beliefs on $\mathcal{L}^o - \mathcal{L}^+$, the part left out of the language. Therefore, in case strong competence of the speaker is commonly known a speaker does not need to mention these beliefs and a hearer will not expect her to do so. That the speaker will not give this information seems even more reasonable given conversational maxims as the second submaxim of Quantity. It asks the speaker only to communicate what is necessary. There is no time to go into more details on this point. It has to be left for further research. But so far there seems to be no true objection against $|\equiv^+$.

4.1.2 Weak Competence and Weak Order.

We come now to another approach to the free choice inferences, which was discussed in section 3.3.3. As we observed there, also the application of the weak notions of entailment $|\equiv^+/\ |\equiv^{++}$ to a class of states where the speaker is only taken to be positive competent on some modality $\triangle$ allows to account for the free choice inferences.

Because the notion of entailment used remains the same, what was said above on this topic applies here as well. But there are some negative consequences of dropping $C2_\triangle$ that have to mentioned. First of all, we lose the suggested account for cancellation in case the speaker mentions her incompetence. The reason is that the logic $KD45C1_\triangle$ leaves the speaker elbow-room to be in some sense incompetent on $\triangle$ and this space is not occupied by the information order. This has the effect that in $\preceq^+/\ \preceq^{++}$-minimal states the speaker does not have to have complete beliefs on what does and does not hold on $R_\triangle$. Therefore, if the speaker claims to be incompetent on the modality $\triangle$ this does not necessarily conflict with what the hearer has to think of the beliefs of the speaker in order to derive the free choice inferences. Hence, no inconsistency does occur that forces the interpreter to give up $C1_\triangle$ and, in consequence, the free choice inferences may still go through.[70]

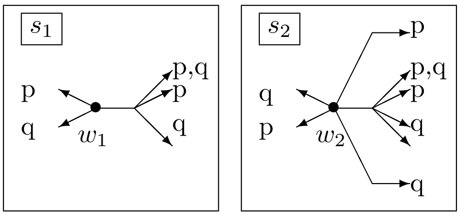
We will not dwell on possible solutions for this problem but instantly turn to another suspicious property of this approach that is strongly related. Without $C2_\triangle$ we are no longer able to derive by $|\equiv^+$, $|\equiv^{++}$ from $\triangle(p \vee q)$ that $\Box(\triangle p \wedge \triangle q)$, hence, that the speaker knows the free choice inference to be valid. (To illustrate this point, figure 4 sketches two states for $\triangle(p \vee q)$ that are minimal with respect to $\preceq^+$ but where $C2_\triangle$ does not hold. In s_2 $\Box(\triangle p \wedge \triangle q)$ is not true.) This result does not seem to be correct. Intuitively, we take a speaker of $\triangle(p \vee q)$ to know that the addressee has free choice.[71]

Finally, this variant of the approach loses, compared with the one discussed above, the possibility to disqualify sentences like $\phi \equiv_{def} \nabla p \vee \nabla q$ as pragmatically not well-formed. However, there is a straightforward way to repair this defect. In a minimal state for such a sentence it will be the case that the utterance itself, ϕ, is not true.[72] This is certainly not a

[70]Notice, that this is particularly true for the two examples of cancellation discussed here : (24) and (25).

[71]Notice that the loss of this particular information seems also to be responsible for the difficulties to account for cancellation. If the speaker is assumed to be aware of the information she implies, then continuation with *" ... but I don't know which"* would be inconsistent with uttering $\triangle(p \vee q)$ (something along the same lines can be said about (25)). Hence, in this case one may again be able to account for the cancellation effects using belief change.

[72]In the minimal belief state of this sentence the speaker distinguishes an epistemic possibility where $\nabla p \wedge \neg\nabla q$ holds and one where $\nabla q \wedge \neg\nabla p$ is true. Hence, $\neg\Box\nabla p \wedge \neg\Box\nabla q$. By $C1_\triangle$ (which we assume to be



The conventions for this figure are the same as for figure 3: arrows present $\triangle$-admissible worlds, lines $\diamond$-admissible worlds. Only $\diamond$-connections from the root of the state are plotted. Furthermore, only information on the deontic possibility of p and q is given. Absence of a proposition letter at a world means that the proposition does not hold there.

Figure 4:

plausible model for the utterance. Already at the beginning of section 3.2.1 we discussed the possibility to strengthen our notion of pragmatic entailment by incorporating the semantic meaning of the utterance: to take the truth of the utterance to be part of the requirements minimal states have to fulfill. It turns out that with respect to such an extended notion of entailment the sentence $\nabla p \vee \nabla q$ would again become dishonest. This can be seen as follows. The first thing to notice is that for two states s_1, s_2, such that $s_1, s_2 \models \nabla p \vee \nabla q$ they either entail ∇p or ∇q. Assume now that s_1 is minimal for $\nabla p \vee \nabla q$ with respect to $\preceq^+$. Without loss of generality, $s_1 \models \nabla p$. We obtain $s_1 \preceq^+ s_2 \Rightarrow s_2 \models \Box\nabla p$ (this follows straightforward from $C1_\triangle$ and the fact that ∇p is in the order defining language). However, there are states s_3 making $\nabla p \vee \nabla q$ true that do not entail $\Box\nabla p$: take, for instance, the s_3 sketched in figure 5. Hence, there are states s_3 such that for a minimal state s_1: $s_1 \not\preceq^+ s_3$. In turn, there cannot be an unique minimal belief state. This implies that with respect to the set of states where $\triangle p \vee \triangle q$ is true the sentence is dishonest with respect to $\mathcal{L}^+$ and $\text{STATE}_{KD45C1_\triangle}$.

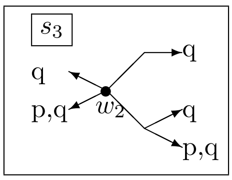


Figure 5:

4.1.3 Weak Competence and Strong Order

Finally, in section 3.3.2 we discussed the possibility to account for the free choice inferences by combining positive competence with the notion of entailment $|\equiv^o$ based on the strong objective information order. This approach faces similar problems as does the last one and actually even in increasing strength. Because the order minimizes not only belief on $R_\triangle$-necessities but now also on $R_\triangle$-possibilities and again positive competence does not lay restrictions on the speaker's belief about what is possible on $R_\triangle$-admissible worlds, a hearer infers using $|\equiv^o$ that the speaker is as far as possible incompetent on $R_\triangle$-possibilities. For instance, remember the states s_1 and s_2 sketched in figure 3. According to the objective

valid in this variant of the approach) it follows that $\neg\nabla p \wedge \neg\nabla q$. Thus, $\neg(\nabla p \vee \nabla q)$.

information order, $s_2 \prec^o s_1$ and even s_2 is not minimal for $\triangle(p \vee q)$ with respect to $\preceq^o$. Particularly, it turns out that using $|\!\equiv^o$ a speaker of $\triangle(p \vee q)$ while implying free choice permission is inferred not to know that this inference holds. These predictions are clearly not in accordance with intuitions.

Given this problem it is not surprising that also the difficulties to predict cancellation of the free choice inferences in case incompetence is stated transfers from the system discussed in the section above. And, as much as for this approach, the pleasant dishonesty properties of the first system are not valid.[73]

Finally, this system predicts that a strongly competent speaker is always dishonestly withholding information. Again, this goes against intuitions.

4.1.4 Conclusions

Let us summarize the discussion so far. While all three accounts we ended up with in section 3 predict the free choice inferences, some differences occur in other respects. Only the first system is able to disqualify the marked inferences in (D5) and (D6) because of dishonesty. However, we have seen that a straightforward modification of the definition of the notion of pragmatic entailment may extend the results also to the other two systems. Furthermore, the intuitions on the well-formedness of sentences like $\nabla p \vee \nabla q$ were not without some doubts. Therefore, the force of this argument in favor of the first account is not particularly strong. However, both other accounts give unintuitive results for the hearers picture of the belief state of the speaker after minimization: while intuitive in case free choice permission obtains the speaker is taken to know this, the system combining $|\!\equiv^+$ with positive competence is not able to make this prediction, and $|\!\equiv^o$ even allows to infer the opposite. These problems lead also to difficulties with the suggested account for cancellation if incompetence is stated. We conclude that therefore the first approach should be preferred. This still allows us to choose between $|\!\equiv^+$ and $|\!\equiv^{++}$. As we have seen, $|\!\equiv^{++}$ fails some of the inference schemes we want to be valid. Therefore, $|\!\equiv^+$ has to be selected as the notion of pragmatic entailment that gives the best results. For the rest of the paper we will always refer to this notion and evaluate it using the strong version of competence.

4.2 Evaluating the Approach

While in the last section we discussed the three successful ways in which the proposed pragmatic account to the free choice inferences can be spelled out, now we turn to a general evaluation of the plausibility of this pragmatic approach.

If one considers the list of pragmatic inferences we wanted to obtain (see section 3.1.3) the approach defended in this thesis is very successful: we were able to come up with a formally precise Gricean notion of entailment that, indeed, makes all the inferences on this list valid. But there are also some problems, or at least, open questions connected to the way free choice inferences are approached here. One point that needs further discussion is

[73]However, also the proposed solution discussed above extends to the present system. The argumentation can be copied directly. Because the defining language of the objective information order is an extension of the order defining language for the positive information order, the latter order is an extension of the former one.

the status of the speaker's competence. The way it is formalized here (as part of the belief of an interpreter) has difficulties to account for some further observations. First, when we look at the data it catches the eye that taking the speaker to be competent seems to be rather the rule than the exception. Free choice is the *normal* interpretation disjunctions undergo; that this reading is not intended has to be signaled explicitly by mentioning incompetence. Because in the proposed account the free choice inferences depend on competence it is predicted that the hearer normally believes the speaker to be competent. On the other hand, one glance on the formalization makes it quite clear that competence is a strong restriction on the belief of the interpreter. One might question whether in every situation where a free choice interpretation occurs indeed the hearer has the factual evidence that the speaker is competent on the matter of discourse. Against this background, it seems more appropriate to understand also the speaker's competence as a default assumption of the interpreter. And remember that also the suggested explanation for cancellation of free choice inferences works much better with this perspective on competence.

An adaption of the presented interpretation function that reflects such a competence assumption can be easily given. But this change in the theory provokes some troublesome questions. For instance, how to motivate such a competence default? None of the Gricean maxims seems to contain a competence assumption. Is it nevertheless something that is reasonable to assume in certain situations? Or is it a linguistic convention?

We will leave these questions unanswered and come to another, more serious problem of the proposed account. As the reader may have recognized, the approach presented here - at least in the way it is spelled out in section 3 - strongly over-generates. What we mean with over-generate in this context is that besides free choice much more pragmatical inferences are predicted and many of them are not in accordance with intuitions. The concrete problem can be formulated particularly clearly using the order defining languages. The reader will remember that in case $\Box\mathcal{L}^*$ is used to define an information order $\preceq^*$, everything that is taken to be in $\mathcal{L}^*$ is subject to belief-minimization and hence, if possible, predicted not to be decided in the speaker's belief state. It seems that we took too much of $\mathcal{L}$ to be relevant for the order, hence, we chose an $\mathcal{L}^*$ that is too big. The language fragment $\mathcal{L}^+$ contains, for instance, all atomic propositions $\mathcal{P}$. We obtain, therefore, for arbitrary $p, q, r \in \mathcal{P}$: $\triangle(p \vee q) \mid\equiv^+ \Diamond r \wedge \Diamond\neg r \wedge \Diamond\triangle r \wedge \Diamond\triangle\neg r$. And under the additional assumption that the speaker is competent on $\triangle$, $\triangle(p \vee q) \mid\equiv^+ \triangle r \wedge \triangle\neg r$.[74] Or, to use more natural examples, given that the sentences *Aunt Hetty is making apple pie*, *Mr. X is in Berlin* and *You take a banana* can be expressed in our formal language, we obtain, for instance, that (26a) $\mid\equiv^+$-entails (26b), (26c) $\mid\equiv^+$-entails (26d), and (26e) $\mid\equiv^+$-entails, assuming $C1_{may}$ and $C2_{may}$ to be valid, (26f). These predictions are certainly wrong.

(26) a. Tomorrow we have better wether or we have to stop.

b. The speaker doesn't know whether aunt Hetty is making apple pie.

c. Mr. X might be in Amsterdam or in Frankfurt.

d. Mr. X might be in Berlin.

[74] The same holds for $\mid\equiv^o$ and $\mid\equiv^{++}$.

e. You may take an apple or a pear.

f. You may take a banana.

What we, thus, need to make the approach to the free choice inference defended here work is a general description of that part of $\mathcal{L}^+$ with respect to which the belief of the speaker really should be minimized.

But even if we had such an description of the order defining language, we still might be in serious trouble. The point is that because of the specific way in which we motivated our approach not any description will do. The defined notion of entailment was set out as formalizing the concept of conversational implicature; as description of what it means to interpret a speaker as obeying certain Gricean maxims. Hence, whatever language restrictions we impose, they have to have a Gricean motivation.

In the remainder of this section we will discuss some possible ways to approach this problem and connected difficulties. However, no final solution will be given. Further investigations have to show whether the challenge of adequate language restrictions can be answered or whether the problem of over-generation reveals that, in the end, we chose the wrong approach.

How could a Gricean description of the order defining languages look like? Let's start with a negative example: with a kind of description that can not be motivated this way. Intuitively, there seems to be a close connection between the inferences we want to obtain (and, hence, the sentences that should define the order) and what has been actually said by the speaker. Given this observation, a first idea one might have is to determine the order defining language using the logical form of the utterance. Gazdar's calculation of potential clausal implicatures comes close to such a way of proceeding. But, as already mentioned when discussing his approach, if it is really the form of the uttered sentence that matters how should a Gricean motivation for such a dependency look like?[75] Is it not the case that then by definition the generated inferences belong - as form-properties - to the sphere of semantics? Therefore, we have to conclude that an approach along these lines would not be in accordance with the general understanding of free choice inferences advanced here.

To find an appropriate description of the order defining language a promising strategy might be to start directly from Grice and asks oneself what kind of order defining languages can get a motivation from his theory of conversational implicatures. Or, to reformulate the question, what part of the belief of the speaker would this theory predict to be exhausted by her contribution? In the formulation of $\mathcal{Q}_1$ Grice explicitly marks that the speaker only has to be complete with respect to *relevant* information. In the maxim of Relevance (and also in the second submaxim of Quantity) he even asks the speaker not to give irrelevant information. Hence, a restriction of the order defining language to sentences that are relevant would fit into a Gricean setting.

There can be two different ways of how to incorporate relevance in our description of the free choice inferences. First, what the speaker thinks to be relevant may be taken by the interpreter to be a subjective variable that she can access only indirectly - by the utterance

[75] We would clearly need more information about the form than pure complexity. Hence, a motivation via Grice's maxim of Brevity would not be an option.

made. In this case the order defining language could be defined as those set of propositions the utterance made is in some sense relevant to, the facts the speaker is giving information about. Second, relevance may also be an inter-subjective contextual factor that can be manipulated by the discourse participants, for instance, by asking questions. In this case the order defining language is defined as those sentences that serve the commonly known goal of discourse at the moment when the speaker makes her contribution.

For both perspectives on relevance there exists a wide range of proposals how to make them precise. To give a simple example how to formalize the first perspective on relevance: one could, for instance, require that the utterance of the speaker gives information about a sentence in case the logical form of the utterance and this sentence are logically dependent on each other. This would, of course be a very coarse-grained notion of relevance. For the second kind of approach we can make use of the well-developed theories of relevance determined in terms of contextual given questions or decision problems (e.g. Groenendijk & Stokhof [1984], van Rooy [2003], van Rooy & Schulz [submitted]). We will use here a rather simple concept of relevance proposed in this area to illustrate some aspects of relevance-based approaches to the order defining language. Let us assume that the meaning of a question is a partition Q of the logical space, hence, a set of sets of states (cf. Groenendijk & Stokhof [1984]). We define a sentence φ to be *about* Q ($\phi \in \textsc{About}(Q)$) iff$_{def}$ $\forall q \in Q$: $(\models_q \varphi)$ *or* $(\models_q \neg\varphi)$.[76] Our notion of entailment $|\equiv^*$ is then defined for answers to the question Q by the language $\Box\mathcal{L}^*$ where $\mathcal{L}^*$ contains those and only those sentences that are about Q, ($\mathcal{L}^* =_{def} \textsc{About}(Q)$).

What would such an approach predict, for instance, for an utterance of (26e)? It seems quite natural to assume in context of the utterance: *'You may take an apple or a pear'* a question of the form *'Which fruits may I take?'*. The set of sentences that are about this question are then $\mathcal{L}^* = \{$ *you make take an apple* ($\triangle apple$), *you may not take an apple* ($\neg\triangle apple$), *you make take a banana* ($\triangle banana$), ...$\}$.

A first thing to notice is that this language - and this would also have been the case if we had used most other notions of relevance - is closed under negation. As we have already seen when discussing the combination of the objective information language and strong competence this results in an unintuitively strong notion of dishonesty. To formulate the problem in some generality:

Fact 12 *If (i) the speaker is assumed to be strongly competence with respect to two sentences* π_1 *and* π_2 *(hence,* $\pi_i \rightarrow \Box\pi_i$ *and* $\neg\pi_i \rightarrow \Box\neg\pi_i$ *are valid for* $i = 1, 2$*), and* π_1 *and* π_2 *and (ii) their respective negations are elements of* $\mathcal{L}^*$*, then the disjunction* $\pi_1 \vee \pi_2$ *is dishonest with respect to* $\Box\mathcal{L}^*$*.*

There are different ways to see this.[77] Intuitively, the problem is simply that the sentence $\Box(\pi_1 \vee \pi_2)$ has three different and incomparable minimal models in this setting: one where $\pi_1 \wedge \neg\pi_2$ holds, one where $\pi_2 \wedge \neg\pi_1$ is true, and, finally, one where $\pi_1 \wedge \pi_2$ is true. To give a concrete example and also to illustrate how serious this problem is for our matters, take $\pi_1 = \triangle p$ and $\pi_2 = \triangle q$. If we assume the speaker to be strongly competent on $\triangle/\nabla$

[76] This notion of aboutness has been independently introduced by Lewis [1998] and Groenendijk [1999].

[77] You can, for instance, use the disjunction property for $\Box\mathcal{L}^*$. Using strong competence one can derive $\Box(\pi_1 \vee \pi_2) \vdash \Box\pi_1 \vee \Box\pi_2$ while the antecedent does not entail one of the disjuncts. Hence, it cannot be honest with respect to $\Box\mathcal{L}^*$.

and $\triangle p, \triangle q, \neg\triangle p, \neg\triangle q \in \mathcal{L}^*$ (notice that $\triangle p, \triangle q, \neg\triangle p, \neg\triangle q \in$ ABOUT(Q)) then $\triangle p \vee \triangle q$ ($\equiv \triangle(p \vee q)$) would turn out to be dishonest.

As with the more general problem of combining strong competence with the objective information language we have two perspectives now: (i) we can weaken the competence assumption, or (ii) we can weaken the order defining language. In the first part of section 4 we have argued that an account that keeps strong competence but describes the order defining language to be (a subpart of) $\Box\mathcal{L}^+$ should be preferred. Of course, we can now postulate, for instance, that the order defining language is the set $\Box$NEG, where NEG= *About*(Q)$\cap\mathcal{L}^+$ and, hence, for the example we have discussed: NEG $= \{\neg\triangle apple, \neg\triangle banana, ...\}$. Indeed, in terms of $\Box$NEG we can account for the free choice inferences. But again: what is the (Gricean) motivation for taking this intersection? Actually, one may argue that POS $=$ ABOUT$(Q)-$ NEG $= \{\triangle apple, \triangle banana, ...\}$ is a much more natural candidate in this respect. For instance, the sentences in POS are prototypical optimal answers to the question *'What fruits may I take?'*, what leads Hamblin to propose this set to be the meaning of the question. Of course, minimizing belief with respect to POS does not give the desired inferences.[78] But a simple adaption brings us back on the right track. We only have to change how a language defines an information order. First, it is easy to prove that the following fact holds.

Fact 13 . *Let $\mathcal{L}^-$ be the language defined by the BNF $\varphi_- ::= p(p \in \mathcal{L}_{(0)})|\varphi_- \wedge \varphi_-|\varphi_- \vee \varphi_-|\triangle\varphi, \triangle \neq \Diamond(\varphi \in \mathcal{L})$. We define further*

$$s \sqsubseteq^o s' \; iff_{def} \forall\varphi \in \mathcal{L}^o : s' \models \Diamond\varphi \Rightarrow s \models \Diamond\varphi,$$
$$s \sqsubseteq^- s' \; iff_{def} \forall\varphi \in \mathcal{L}^- : s' \models \Diamond\varphi \Rightarrow s \models \Diamond\varphi.$$

Then, it holds $\forall s \in$ STATE$_{KD45}$: $s \preceq^o s' \Leftrightarrow s \sqsubseteq^o s'$ and $s \preceq^+ s' \Leftrightarrow s \sqsubseteq^- s'$.

In consequence, the notions of entailment that can be defined in terms of the interchangeable orders (using definition 1) are equivalent too. Hence, minimizing the beliefs of the speaker with respect to $\Box\mathcal{L}^o$/ $\Box\mathcal{L}^+$ provides exactly the same results as maximizing her epistemic possibilities with respect to the languages $\Box\mathcal{L}^o$/ $\Box\mathcal{L}^-$. In the same way it can be shown that $\sqsubseteq^{pos}$ defined by POS is equivalent to $\preceq^{neg}$ defined by NEG. And therefore, also from POS the free choice inferences can be obtained if we take the perspective of maximizing the epistemic possibilities of the speaker.[79]

So far, things are going quite well: we have seen that some problems relevance based accounts face on the first view can easily be solved. A next question one might want to ask is which of the two ways to incorporate relevance in the proposed account for the free choice inferences should actually be adopted: taking relevance to be a contextually given factor

[78] The order defined by POS using definition 2 (section 3.2.2) produces inferences that are inconsistent with a free choice interpretation: having $\triangle apple$ in the order defining language will lead to an order that prefers belief states where the speaker does not believe that $\triangle apple$ is true. In case the speaker is $C2_{may}$-competent we even obtain $\triangle(apple \vee pear) \mid\equiv^{pos} \nabla\neg apple \wedge \nabla\neg pear$.

[79] As a final observation on this topic, the Gricean argument sketched in section 4.1.1 to motivate the language restriction from $\mathcal{L}^o$ to $\mathcal{L}^+$ is independent of the choice of minimizing belief with respect to NEG or maximizing epistemic possibilities with respect to POS and, hence, still an option to explain these necessary language restrictions.

or to be determined by the utterance of the speaker. It is important to notice that there are subtile differences between both perspectives. They make, for instance, different predictions in case the speaker gives information on contextually irrelevant sentences. In this case, the latter notion would give rise to implicatures that are not predicted by the former one. A different situation occurs when the speaker gives no information on sentences that are contextually relevant. Then, using the first concept of relevance as basis would give rise to implicatures that do not occur with a language founded on what the speaker is talking about. We can use these situations where both approaches make different predictions to test them against each other.

The dialogue fragment (27) is set up as example for a speaker giving contextually irrelevant information.

(27) A: Was it raining this morning?
B: Yes, and aunt Hetty is making apple pie or muffins.

Even though the response of B is quite strange, especially in those contexts that interests us here: where aunt Hetty's activities are absolutely irrelevant for answering A's question, intuitively, B is taken not to know what aunt Hetty is preparing for tea. Hence, it seems as if a formulation of the order defining languages in terms of what the speaker is talking about is more appropriately describing our intuitions. One may counter with the suggestion that the speaker's utterance of the disjunction may change what counts as relevant in the discourse. Hence, that what the speaker is talking about can be used to repair situations where the interpreter's picture of what is relevant and the contribution of the speaker are not in accordance. One may furthermore suggest that the speaker's strategy in such a situation is to add the disjuncts to the set of relevant sentences and, hence, the order defining language. 'But why?' one might ask. In particular, why adding the disjuncts and not the whole disjunction?

We leave this question unanswered and consider the second test condition. Examples where the speaker provides (given the semantics assumed here) no information about contextually relevant sentences can be easily found. Take a situation where (26e) is uttered in the context of (28).

(28) A: Which fruits may I take?
B: You may take an apple or a pear.

In this situation standard notions of contextual determined relevance (in particular, the one described above) would predict *'You may take a banana'* to be a relevant sentence. However, the speaker B does not provide any information about its truth. What are the intuitions about this example? Do we infer that the speaker does not know whether it is true that the addressee A may take a banana or is nothing concluded about the speaker's beliefs on the permissibility of taking bananas? The dominant opinion in the literature on comparable examples (e.g. Groenendijk & Stokhof [1984], van Rooy & Schulz [submitted1] and [submitted2]) is that while there are contexts in which the answer of B is taken to represent all the speaker knows (and, hence, the approach using contextually determined relevance would make the correct predictions), in most situations the interpreter of B's utterance will conclude that an apple and a pear is all the addressee may take (hence, taking a banana is prohibited) and this is known to the speaker. Thus, it seems that neither of the relevance approaches gives in general the right result: while using the utterance to determine what counts as relevant is unable to account for those cases where indeed the speaker is inferred to be incompetent about the banana, determining the order defining language with contextual relevance gives in other contexts a language that is still not restricted enough.

In the latter case the predictions made seem to interfere with the so called *exhaustive interpretation* of utterances. The following dialogue fragment gives a standard example for this notion.

(29) Paul: Who knows the answer?
Paula: John and Mary.

In most contexts Paula's answer in (29) is interpreted as conveying that John and Mary are the only people who know the answer. Although the sentence *'John and Mary know the answer'* does not bear this information if occurring in isolation (hence, one could argue, it is not conveyed by its semantic meaning in the strict sense of the word), used as answer it is understood as giving all those and only those who know the answer. In the same sense (26c) on page 65 is understood as giving all whereabouts of Mr. X the speaker thinks possible and (26e) to give a full list of what the addressee may take from the fruit bowl.

When considered more closely, exhaustive interpretation seems to be simply a stronger variant of the non-monotonic reasoning we modeled in chapter 3. Again, the absence of information is made meaningful. But instead of inferring from the fact that the speaker did not claim that some sentence p holds that she does not know whether it does, here the stronger inference is derived that p does not hold - negation as failure. With respect to which sentences this inference is made seems to be strongly dependent on what is relevant in the context of utterance (cf. van Rooy & Schulz [submitted1]).[80] Actually, taking exhaustivity to minimize the set $\Box$Pos described above while at the same time assuming strong competence on the extension of the question predicate yields in many cases a correct description. So, let's adopt for the moment the thesis that exhaustivity can be described in this way.

We have the following options now. We can first adopt the position that what the speaker is talking about determines the order defining languages. Then we have to come up with something else to account for the incompetence readings this approach cannot account for. Independent of the fact that until now we have said nothing about how to make *what the speaker is talking about* precise there are some questions that such a way of proceeding raises. As may have become clear in section 3 the class of inferences described by our notion of pragmatic entailment can be seen as generalization of Gazdar's clausal implicatures. But there are also strong connections between exhaustive interpretation and Gazdar's notion of scalar implicatures (see van Rooy & Schulz [submitted1]). Gazdar took both classes of conversational implicatures to be due to the first submaxim of Quantity (and the maxim

[80] Perhaps the context dependence is even stronger here than with free choice inferences. Consider (30).

(30) A: Who is coming for tea?
B: Mary. And aunt Hetty is making [apple pie]$_F$.

For this example an exhaustive interpretation of the second sentence of B seems hardly possible.

of Quality). We added in this section a third maxim to the derivation: the maxim of Relevance. One question that has to be answered now is why we need two different notions of relevance for both classes of implicatures.

A second option is to take for both: free choice inferences and exhaustification, the same notion of relevance: contextual determined relevance. But this would mean, given the preliminary description of exhaustive interpretation we have adopted, that in both cases $\Box$Pos is the order-defining language. It is difficult to see how then the mispredictions we discussed in connection with example (28) can be excluded. This can, for instance, not be done simply by executing one inference-generating process after the other (as Gazdar proposed for his description of clausal and scalar implicatures). As we have described both processes they work in completely opposite ways on $\Box$Pos: while to obtain the free choice inferences we have to select the maximal elements of the resulting order, we described exhaustivity as doing exactly the opposite: looking for its minimal states. It is obvious that, therefore, no matter which order is adopted, there will be no contribution of the process running as second.

Of course, this whole discussion depends strongly on the provisional description of the exhaustive interpretation we have adopted. Therefore, we conclude that in order to find a solution for the problem we started with: the problem of how to restrict the order defining languages, first, we have to closely study the phenomenon of exhaustive interpretation and its interaction with the free choice inferences. But this would go beyond the scope of this thesis and has, therefore, to be left for another occasion.

5 Conclusions

This thesis proposed a formalization for parts of Grice's theory of conversational implicatures. The proposal builds on work of Halpern & Moses [1984] recently generalized by van der Hoek et al. [1999, 2000]. The core of the approach consists of a notion of pragmatic entailment that is intended to capture the conversational implicatures due to the first submaxim of Quantity and parts of the maxim of Quality introduced by Grice. Technically, the defined notion of entailment represents an instance of reasoning in preferential structures: a sentence ϕ is said to pragmatically entail a sentence ψ if in all states where the speaker is in the unique minimal belief state given that she believes ϕ also ψ is valid.

The proposed formalization was then tested by trying to account in terms of it for the free choice inferences. In particular, we tried to describe free choice permission, a well-known problem for many semantic and pragmatic theories. It was shown that given contextual information about the competence of the speaker - and here we build strongly on work of Zimmermann [2000] - the approach allows us to derive the free choice inferences. Hence, the introduced formalization of the theory of Grice allows us to describe free choice inferences as conversational implicatures certain utterances of competent speakers come with.

Actually, three accounts for the free choice inferences were obtained, differing in how the central notions of competence and pragmatic entailment are spelled out. One of them appears to be particularly promising. It takes as relation comparing the belief state of the speaker the positive information order, $\preceq^+$, which demands the speaker to give for a modality $\triangle$ she is competent on only the information she has on what according to her belief is necessary the case on the set of $R_\triangle$-admissible words. She does not have, however, to convey also her beliefs on what is possible on this set of worlds. The resulting notion of entailment was then combined with the strong version of the competence assumption. This system allows us to account for all inferences that have been worked out in section 2 to be characteristic for the free choice phenomenon.

As has been mentioned earlier the proposed notion of pragmatic entailment is closely related to Gazdar's [1979] description of clausal implicatures; not only with respect to the inferences predicted but also in the way they are thought to be linked to the theory of Grice. A question that this observation immediately provokes is which account should be preferred. When discussing Gazdar's approach we found some of its properties problematic. First, he was not able to account for free choice inferences of modalities different from the one which refers to the speaker's belief state. Furthermore, Gazdar predicts some inferences that are not reasonable. For instance, he predicts for the sentence $\triangle(p \vee q)$ the potential clausal implicature $\Diamond p$. Finally, we complained that the syntactic moment in his description of potential clausal implicatures (he restricted them to subsentences of the sentence uttered) has not been motivated with Grice's theory and that it is also questionable whether such a motivation can be given at all. While the first type of critics does not apply to the proposal made here, the second one does. Also the approach developed in section 3 has to fight with mispredictions. Actually, exactly the same inference we complained about when discussing Gazdar is also predicted here. And furthermore, in some sense the account is also subject to the third type of critic on Gazdar: when introducing the approach in section 3 we did not give a Gricean motivation for the choice of the order defining languages. In section 4.2

some attempts for an improvement on this point were made. We discussed the possibility to determine the order defining languages in terms of relevance. But to be able to see whether such a way of proceeding is successful it appeared to be necessary first to discuss the phenomenon of exhaustive interpretation. However, this could not be achieved in the scope of this thesis and has to be left for another occasion.

To summarize, even though we improve on Gazdar's approach with respect to some points the proposal made here is still not fully satisfying. Further research has to reveal whether it is possible to overcome the remaining shortcomings.

Another topic that needs attention in further work is to what extend the presented formalization of the theory of Grice (if correct) can and should be extended to inferences due to other conversational maxims or other phenomena analyzed as conversational implicatures. Particularly interesting in this respect is an extension to the class of scalar implicatures. As the reader might remember, Gazdar took these inferences to be consequences of the first submaxim of Quantity ($\mathcal{Q}_1$) too. If he is right in this point one would expect that a correct formalization of the implicatures due to this maxim should also account for the scalar implicatures. We already noticed, when discussing the approach of Gazdar, that clausal and scalar implicatures are clearly distinguished by a different grade of strength: while the former only convey that certain statements are not believed by the speaker, scalar implicatures claim that the speaker believes other sentences not to hold.[81] How can inferences so different in character be due to the same maxim?

Interestingly, some linguists working on conversational implicatures (e.g. Soames [1982]) proposed that the strength of $\mathcal{Q}_1$ implicatures depends on whether the speaker is taken to be competent on the subject of conversation. This would suggest that the approach defended here can also account for scalar implicatures: we already can describe clausal implicatures, so the only thing we have to do is to apply our pragmatic notion of entailment in contexts where the speaker is assumed to be strongly competent on the subject of discourse. The approach to exhaustivity we adopted in section 4.2 realizes this program. We have seen there that such a way of proceeding leads to certain problems with respect to the free choice inferences. In van Rooy & Schulz [submitted2], it has been argued that this simple account for exhaustivity will in general not work. The central problem is that typical scalar implicatures are also allowed for utterances that are obviously not made by a strongly competent speaker (if competence is understood as described in section 3.2.7). For more discussion on the question of how to extend the present approach to scalar implicatures see van Rooy & Schulz [submitted2].

We close the thesis by going back to the examples that led von Wright [1969] to speak of a *paradox* of free choice permission and discuss the explanation given by the account presented here for von Wright's paradox.

The problem von Wright faced was that he saw no way to describe a logic of the expressions involved in permission and obligation sentences that can account for the way we

[81] The reader may notice the parallels between the description of scalar implicatures given here and the characterization of the inferences due to exhaustive interpretation, that can be found in section 4.2. One of the central claims made in van Rooy & Schulz [submitted1] is that both types of inferences are to a wide extend identical.

understand these sentences. In particular, for him there seemed to exist no possibility to have free choice permission valid together with other necessary inferences without that the obtained logical system gives rise to absurd predictions. Let us take as an example for such mispredictions the following chain of reasoning.[82]

(31) (1) Mr. X may take a taxi.
(2) Mr. X takes a taxi implies that Mr. X takes a taxi or a boat.
(3) Hence, Mr. X may take a taxi or a boat.
(4) Hence, Mr. X may take a boat.

Intuitively, it should not be possible to conclude from (1) that (4) is valid. However, in a deontic logic (like a logic was supposed to look like at the times of von Wright) that allows free choice permission as a valid inference (31) can be turned into an argument: (2) is valid as a tautology of propositional logic. From the premise (1) we conclude by (2) and a rule of substitution truth conditional weaker sentences in modal contexts that (3) holds. Then, the free choice inference allows us to conclude from (3) to (4).

Let us first recapitulate the position of Zimmermann [2000] regarding this problem. First, in contrast to the early position of Kamp [1973], he does not detect the problem in using a descriptive logic to reason over deontic sentences, but he objects that the wrong translation of natural language sentences in a descriptive logic is used. More particularly, Zimmermann claims that the interpretation rule for '*or*' is wrong. If this is corrected in the way he proposes (A *or* B is interpreted as $\Diamond A \wedge \Diamond B$) (3) will no longer be derivable from premises as (1) and (2).[83] On the other hand, Zimmermann predicts that the step from (3) to (4) is context dependent: it can be made only in case it is known that the speaker is competent with respect to the deontic options '*may*' refers to.

What has the account presented in this thesis to say about the paradox? First, together with Zimmermann, the roots of the paradox are not located in the special features of the semantics of non-assertive speech acts. But in contrast to him we did not blame the assumptions von Wright made on the semantic meaning of the utterance involved. In particular, we adopted an interpretation of '*or*' as inclusive disjunction. With respect to semantics, in turn, (2) and the inference to (3) are valid, but the step from (3) to (4) is not permitted. In this thesis free choice permission is analyzed as a pragmatic inference that is based on reasoning about the beliefs of a competent speaker that observes the maxims of conversation in uttering (3). Only on the level of pragmatics an interpreter may reason from observing an utterance of (3) to the truth of (4). Hence, there is no paradox of deontic logic but confusion about semantic and pragmatic inferences. Notice finally, that (3) as an utterance of a speaker who also uttered (1) or concluded (3) from (1) does not allow us to derive (4). In both cases, given that the speaker of (3) believes (1) to be true, (3) cannot be all she believes. Therefore the speaker was dishonest with her utterance and an interpretation according to the maxims of conversation is not possible. Hence, given an utterance of (31) (1)-(3) from the same speaker (4) is not a valid pragmatic inference.

[82] This example differs from the one we discussed in section 2. It is a weaker argument for the paradox than the earlier one because it involves stronger assumptions on the logic of permission/obligation sentences. Nevertheless, it is more popular in the literature.

[83] It may even be the case that (2) is not valid anymore. This depends on the modal concept of a belief state that is used. In S5 and KD45, at least, $p \rightarrow (\Diamond p \wedge \Diamond q)$ does not hold.

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