The Problem of the Perfect Agent: Investigations into Determinism.

MSc Thesis (*Afstudeerscriptie*)

written by

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Contents

1 Introduction

This thesis is a continuation of work that was begun as a short project on Ancient and Medieval Logic, in which I examined medieval approaches to the apparent incompatibility of God's omniscience and future contingents. It is born of an interest in the philosophy and structure of time, together with an interest in the 'big questions' of philosophy which have fascinated scholars from the early Greeks, through the Middle Ages, and still provoke debate today.

This project builds on the earlier work in two ways: firstly by introducing modern formalisations which can be used to analyse ancient and medieval approaches to questions of necessity, possibility, tense and agency; and secondly by considering not only omniscience but additional properties which can be ascribed to a 'perfect agent', namely omnipotence and omnibenevolence. The aim is to determine whether the existence of a perfect agent is compatible with the notion of free will.

Given the wide scope of this topic, it is inevitable that much that is relevant must be omitted. Even outside the boundaries of logic and philosophy there is relevant work to be found in the field of theology. So the scope of this work has been limited to those questions relating to the issue of determinism, and further to those which can be modelled together in a single system without rendering the system so complex as to obscure any results which might be obtained from it. In particular we ask the question, "Which properties of a perfect agent are sufficient conditions for a deterministic system?"

§2 presents the formal system which will be used to analyse this question. Although the system itself is new, it borrows many ideas from existing work on modal and hybrid logics, including temporal, epistemic and deontic logic. The treatment of knowledge is based on that of Pacuit, Parikh and Cogan [PPC06], and the treatment of agency is heavily influenced by Horty [Hor96]. The result is a multi-agent, event-based branching temporal structure with valuations for truth and goodness, and a language incorporating operators for necessity and possibility, tense, knowledge, free choice and obligation. This section is required reading since the system introduced here will be used throughout the rest of the thesis.

One consequence of the historical and philosophical bias of this investigation is that applications of this system to modern computer science problems are not considered. I feel however that fruitful work could be done along these lines, in particular in the areas of game theory and computational social choice. The branching time structure lends itself well to discussion as a game tree and the goodness valuation is, in essence, a utility function. In the study of social procedures, the results around determinism could be explored further in terms of protocol design for obtaining robust outcomes. This paper does not cover any of these ideas but simply notes them as related areas of interest which could provide the basis for further work using the formal system presented here.

§3 takes a detailed look at an argument from antiquity, the so-called "Master Argument" of Diodorus Cronus. This argument brings together modal logic and temporal logic and introduces us to the problem of determinism. We examine this argument in two ways—first from a historical perspective and then by looking at a wide range of modern reconstructions of the argument. Combining the results of these two studies, we present a reconstruction of the argument which we believe to be the most historically plausible and conclude that the purpose of the argument was to define the notion of *necessitas per accidens* against the backdrop of the more generally accepted version of necessity as 'truth at all times'.

§4 moves to the medieval period and considers an argument supporting theological fatalism: the argument that human freedom and divine foreknowledge are incompatible. In addition to temporal and modal considerations we examine the notions of omniscience and immutability, and consider the deterministic consequences of introducing an agent with these attributes. We show using our formalisation that the argument for fatalism is not logically sound, and also discuss the solutions of Thomas Aquinas and William of Ockham. We conclude that the presence of an omniscient agent is neither necessary nor sufficient to ensure a deterministic system.

§5 goes on to define the concepts of omnipotence and benevolence. Here we discuss the problem of evil: the argument that that the existence of an omniscient, omnipotent and omnibenevolent God is inconsistent with the presence of evil in the world. We generalise this argument to consider not just evil, but also conflicting obligations, moral dilemmas and free choice. Finally we show that the existence of an omnipotent and omnibenevolent agent ensures a system can support free choice only if it also contains moral dilemmas.

2 Building the Formal System

2.1 Foundations

The basic model $M = \langle S, V \rangle$ is a set of atomic propositions P, a nonempty set of states S, and a valuation function

$$V : P \to 2^S$$

which for every proposition $p \in P$ yields the set $V(p) \subseteq S$ of states in which the proposition is true. In Figure 1, $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ and $V(p) = \{s_1, s_2, s_4, s_5, s_6\}$.

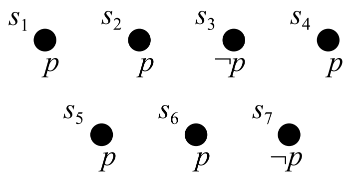

Figure 1: A basic model.

We define the basic logical language in Backus-Naur form:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi).$$

By selecting a given $s \in S$ we get a pointed model which we denote by the pair (M, s). Using the model we define semantics for the language in the usual way:

$$\begin{aligned}
&(M, s) \models p \text{ iff } s \in V(p)\\
&(M, s) \models \neg\phi \text{ iff } (M, s) \not\models \phi\\
&(M, s) \models (\phi \wedge \psi) \text{ iff } (M, s) \models \phi \text{ and } (M, s) \models \psi.
\end{aligned}$$

Additional operators $\vee$ and $\to$ are introduced as abbreviations in the usual way:

$$\begin{aligned}
\phi \vee \psi &:= \neg(\neg\phi \wedge \neg\psi)\\
\phi \to \psi &:= \neg\phi \vee \psi.
\end{aligned}$$

By extending this basic system with the concepts of modality, tense, knowledge, action and obligation we will construct a framework within which the problems relating to future contingents, free will and determinism can be studied.

2.2 Modality

We begin by adding a general concept of modality to the system by introducing a reflexive binary relation R on S, and a unary operator $\Diamond$ with semantics

$$(M, s) \models \Diamond\phi \text{ iff there is some } s' \in S \text{ such that } sRs' \text{ and } (M, s') \models \phi.$$

So with the addition of modality, our model is $M = \langle S, R, V \rangle$ and our language is

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Diamond\phi.$$

The dual operator $\Box$ is defined as an abbreviation

$$\Box\phi := \neg\Diamond\neg\phi$$

and it follows that it has the semantics

$$(M, s) \models \Box\phi \text{ iff for all } s' \in S \text{ such that } sRs', (M, s') \models \phi.$$

The operators $\Diamond$ and $\Box$ are usually taken to represent possibility and necessity, such that $\Diamond\phi$ is read as 'it is possible that ϕ', and $\Box\phi$ is read as 'it is necessary that ϕ'. The condition that R is reflexive ensures as a minimum that everything necessary is true ($\Box\phi \rightarrow \phi$) and that everything true is possible ($\phi \rightarrow \Diamond\phi$).

Note that we can re-use these general definitions to define new modal operators by substituting different relations in place of R. We will use this method to define temporal and epistemic modalities for our system.

2.3 Tense

2.3.1 Temporal Frames

To examine the notion of tense, we need to define an order on our states. We introduce an irreflexive, transitive order $\prec$ on S and define a *temporal frame* as a structure

$$T = \langle S, \prec \rangle.$$

We also introduce the converse relation

$$s \succ s' \text{ iff } s' \prec s$$

and their reflexive counterparts

$$s \preceq s' \text{ iff } (s \prec s') \vee (s = s')$$
$$s \succeq s' \text{ iff } (s \succ s') \vee (s = s').$$

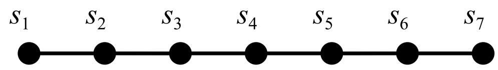

Figure 2: A linear temporal frame.

We say that T is a *linear* temporal frame if $\prec$ is a total order. We adopt the convention that diagrams are ordered left to right, so Figure 2 shows an example of a linear temporal frame in which

$$s_1 \prec s_2 \prec s_3 \prec s_4 \prec s_5 \prec s_6 \prec s_7.$$

Definition 2.1. A relation $\prec$ on a set S is said to be *backward linear* if for any $s, s', s'' \in S$, if $s \prec s''$ and $s' \prec s''$ then either $s = s'$ or $s \prec s'$ or $s' \prec s$.

If $\prec$ is irreflexive, transitive and backward linear then T is a *forest*. If it is also connected it is a *tree*. In either case we may also refer to T as a *branching* temporal frame (see Figure 3). From now on we will consider only branching temporal frames.

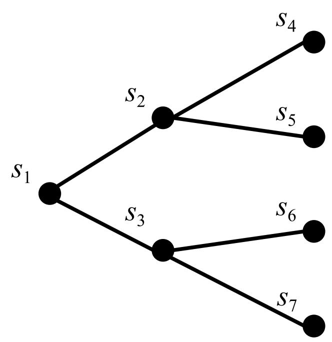

Figure 3: A branching temporal frame.

2.3.2 Nominals and Satisfaction Statements

In our formal system a statement ϕ is generally taken to be *temporally indefinite*, since we evaluate against a pointed model (M, s) where the state s provides the temporal reference, and we write

$$(M, s) \models \phi.$$

These statements can also be regarded as functions from states to truth values, and are sometimes called propositional functions. However we can also consider statements which are *temporally definite*, in which the temporal reference comes from ϕ itself. Temporally definite statements contain an explicit temporal reference, e.g. 'it is the case at s that ϕ'.

To formalise this notion we need to introduce a set N of *nominals* to the system. This is a concept from hybrid logic (see [BRV01, p.435]). Nominals are atomic elements of the language which are used to name states, and so act as temporal references within statements. A state can have any number of nominals associated with it, but each nominal must correspond to a unique state.

Because we are adding extra atoms to the system, we must also extend the valuation function. Previously we had $V : P \to 2^S$, now we extend this so that $V : P \cup N \to 2^S$. Because each nominal denotes a unique state, V takes each nominal $n \in N$ to a singleton subset of S. For ease of use we define a function $g : N \to S$ such that

$$V(n) = \{g(n)\}.$$

We call g the *assignment function*. The assignment function allows us to use the expression $g(n)$ to refer directly to the state denoted by the nominal n.

The simplest statement including a nominal is a single nominal. The semantics are the same as for atomic propositions, i.e. $(M, s) \models n$ iff $s \in V(n)$ but using the assignment function g we can write this as

$$(M, s) \models n \text{ iff } s = g(n).$$

This is interpreted as 'the current state is s'.

Nominals can combine with atomic propositions and other formulae using any of the usual operators, e.g. $n \wedge p$, $n \to \phi$. However we will usually want to use nominals to form temporally definite statements. This is a type of statement that we cannot form with our existing language, so we must add a new operator to the system. Following [BRV01, p.435], we introduce the *satisfaction operator* $@_n$, where $n \in N$ is a nominal. A *satisfaction statement* $@_n\phi$ says that ϕ is true at the state named by n, and it has formal semantics as follows:

$$(M, s) \models @_n\phi \text{ iff } (M, g(n)) \models \phi.$$

Note that although we specify a pointed model, the selected state is irrelevant. So in fact if $(M, s) \models \phi$ for some $s \in S$, then $(M, s) \models \phi$ for all $s \in S$ and the semantics could be written instead as

$$M \models @_n\phi \text{ iff } (M, g(n)) \models \phi.$$

With the addition of nominals and the satisfaction operator, our language is

$$\phi ::= p \mid n \mid \neg\phi \mid (\phi \wedge \phi) \mid \Diamond\phi \mid @_n\phi.$$

Now we can formally define what we mean by a temporally definite statement.

Definition 2.2. A statement ϕ is *temporally definite* iff it is a satisfaction statement.

For ease of use, we will abbreviate satisfaction statements of the form $@_n\phi$ to $\phi(s)$ where $s = g(n)$. This is in line with our view of temporally indefinite statements as propositional functions. We can move from temporally definite statements to temporally indefinite ones and vice versa by noting that

$$M \models \phi(s) \text{ iff } (M, s) \models \phi.$$

2.3.3 Temporal Operators

Using the relations $\prec$ and $\succ$, we can define temporal operators F and P with the semantics

$$(M, s) \models F\phi \text{ iff there is some } s' \in S \text{ such that } s \prec s' \text{ and } (M, s') \models \phi$$
$$(M, s) \models P\phi \text{ iff there is some } s' \in S \text{ such that } s \succ s' \text{ and } (M, s') \models \phi$$

and their duals are defined as abbreviations $G := \neg F \neg$ and $H := \neg P \neg$ with the semantics

$$(M, s) \models G\phi \text{ iff for all } s' \in S \text{ such that } s \prec s', (M, s') \models \phi$$
$$(M, s) \models H\phi \text{ iff for all } s' \in S \text{ such that } s \succ s', (M, s') \models \phi.$$

The standard interpretation for $F\phi$ is 'it will be the case that ϕ'. In a linear temporal frame there is no problem with this interpretation. However, in a branching temporal frame we can have $s \prec s'$ and $s \prec s''$ with s', s'' on different branches. If ϕ is true at s' but not at s'', then $F\phi$ is true at s even though there is no certainty that ϕ will become true. For branching frames $F\phi$ can be interpreted as 'it will possibly be the case that ϕ'. To be certain that ϕ will become true we need some s' on each branch such that $s \prec s'$ and $(M, s') \models \phi$. We will formalise this notion in §2.3.5. For a discussion of why 'it will be' on branching frames must correspond to either 'it will possibly be' or 'it will certainly be', see [McA74].

The standard interpretations for $G\phi$ ('it will always be the case that ϕ') and $H\phi$ ('it was always the case that ϕ') are not affected by the branching structure because G and H are universally quantified, so the condition is satisfied for all branches. $P\phi$ ('it was the case that ϕ') is also not affected because although it is only existentially quantified, the temporal frame is backward linear so there is only one branch to be considered.

By combining the basic temporal operators it is possible to build *compound tenses* e.g. FPF, PFP, $PPPFFF$. However with compound tenses it can become difficult or even impossible to identify whether the subject matter of a statement concerns the past, present or future. To make this easier, we will generally want to reduce a compound tense statement to its *simple tense* equivalent where possible.

Definition 2.3. A statement ϕ is called a *simple tense* statement iff it contains at most one temporal operator.

To find the simple tense of a statement, we need to compare the temporal reference of the subject matter with the temporal reference of evaluation.

For example, consider a model with $s_1 \prec s_2 \prec s_3$ and where $(M, s_2) \models PF\phi$. Then $PF\phi$ can be true if 'it was the case at s_1 that it would be the case at s_2 that ϕ', which at s_2 is a statement about the present; or it can be true if 'it was the case at s_1 that it would be the case at s_3 that ϕ', which at s_2 is a statement about the future. So although $PF\phi$ at s_2 has the form of a past tense statement, in its simple tense it is really about the present or the future. To determine the simple tense of $PF\phi$ at s_2 we need more information about ϕ. If $M \models \phi(s_2)$, then $(M, s_2) \models PF\phi$ reduces to $(M, s_2) \models \phi$ which is present tense. If $M \models \phi(s_3)$ then $(M, s_2) \models PF\phi$ reduces to $(M, s_2) \models F\phi$ which is future tense.

Temporally indefinite statements are always tensed because they are evaluated against a pointed model. They are taken to be in the present tense unless modified by a temporal operator. Temporally definite statements should be considered untensed when evaluated against an unpointed model, and tensed when evaluated against a pointed model. The tense of a temporally definite statement does not affect the truth value.

2.3.4 Histories in Branching Frames

In order to express certainty about the future we need to be able to quantify over sets of branches. To formalise this we introduce the notion of a *history*, which is standard in the branching time literature (see e.g. [BPX01, p.181], [Hor96, p.271]).

Definition 2.4. In a temporal frame $T = \langle S, \prec \rangle$, a *history* h is a maximal subset of S which is totally ordered by $\prec$.

Note that in a linear temporal frame, there is only one history, S. In a branching temporal frame, there will be multiple histories. In the frame in

Figure 3 (p.5) there were four histories:

$$
\begin{aligned}
h_1 &= \{s_1, s_2, s_4\} \\
h_2 &= \{s_1, s_2, s_5\} \\
h_3 &= \{s_1, s_3, s_6\} \\
h_4 &= \{s_1, s_3, s_7\}.
\end{aligned}
$$

Let $\mathcal{H}$ be the set of histories in $T = \langle S, \prec \rangle$. For a given state $s \in S$, define

$$H(s) = \{h \in \mathcal{H} \mid s \in h\}$$

as the set of histories running through s. So in Figure 3, we have

$$
\begin{aligned}
H(s_1) &= \{h_1, h_2, h_3, h_4\} \\
H(s_2) &= \{h_1, h_2\} \\
H(s_3) &= \{h_3, h_4\} \\
H(s_4) &= \{h_1\} \\
H(s_5) &= \{h_2\} \\
H(s_6) &= \{h_3\} \\
H(s_7) &= \{h_4\}.
\end{aligned}
$$

Note that for any $s, s' \in S$, $s \prec s'$ implies $H(s) \supseteq H(s')$.

2.3.5 Another Future Operator

Using this notion of histories, we can now define semantics for a new operator F', where $F'\phi$ is taken to mean 'it will certainly be the case that ϕ'.

$$
\begin{aligned}
(M, s) \models F'\phi \text{ iff for all } h \in H(s) \\
\text{there is some } s' \in h \text{ such that } s \prec s' \text{ and } (M, s') \models \phi.
\end{aligned}
$$

This is sometimes known as the *Peircean future tense* (see e.g. [BPX01, p.159]). It is possible to analogously define an operator P' but it is unnecessary since our definition of P is equivalent to P', due to backward linearity.

Let us add to our branching frame of Figure 3 a valuation V such that $V(p) = \{s_1, s_2, s_4, s_5, s_6\}$ to get the model in Figure 4.

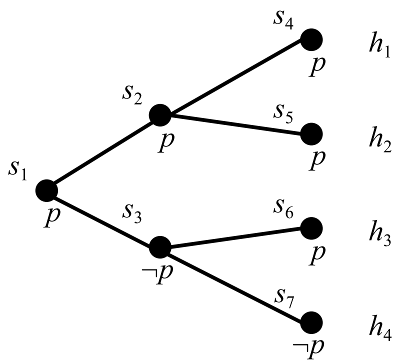

Figure 4: A branching model.

In our model we can see that

$$
\begin{aligned}
&(M, s_1) \models Fp \\
&(M, s_2) \models Pp \wedge Hp \wedge F'p \wedge Gp \\
&(M, s_3) \models Pp \wedge Hp \wedge Fp \\
&(M, s_4) \models Hp \\
&(M, s_5) \models Hp \\
&(M, s_6) \models Pp \\
&(M, s_7) \models Pp.
\end{aligned}
$$

2.3.6 Combining Tense and Modality

To be in line with our intuitions, necessity and possibility should be defined in such a way that if something is necessary, it will always be necessary; and if something is possible, it was always possible. That is, they should satisfy the postulates

$$
\begin{aligned}
\Box\phi &\rightarrow G\Box\phi \\
\Diamond\phi &\rightarrow H\Diamond\phi.
\end{aligned}
$$

The branching time structure naturally leads us to define the ideas of (temporal) necessity and possibility in the following way:[1]

$$
\begin{aligned}
&(M, s) \models \Diamond\phi \text{ iff for some } h' \in H(s) \text{ and some } s' \in h', (M, s') \models \phi \\
&(M, s) \models \Box\phi \text{ iff for all } h' \in H(s) \text{ and all } s' \in h', (M, s') \models \phi.
\end{aligned}
$$

[1] This definition is not the Diodorean notion of modality, which (on a linear frame at least) is given by $\Diamond\phi := \phi \vee F\phi$ and $\Box\phi := \phi \wedge G\phi$, or by defining R as sRs' iff $s \preceq s'$. The Diodorean definition also satisfies our postulates. We will consider this definition further in §3.2.3 during our discussion of the Master Argument.

This corresponds to defining R as

$$sRs' \text{ iff } s \prec s' \text{ or } s \succ s' \text{ or } s = s'.$$

Just as R can be defined in terms of the temporal order, so can $\Diamond$ be defined in terms of the temporal operators. In this case

$$\Diamond\phi := P\phi \vee \phi \vee F\phi$$

and consequently

$$\Box\phi := H\phi \wedge \phi \wedge G\phi.$$

So with the addition of tense, our model becomes $M = \langle T, V\rangle$ where T is a branching temporal frame, and our language becomes

$$\phi ::= p \mid n \mid \neg\phi \mid (\phi \wedge \phi) \mid @_n\phi \mid P\phi \mid F\phi \mid F'\phi.$$

In the model given by Figure 4 we can see that

$$\begin{aligned}
(M, s_1) &\models \Diamond p \wedge \Diamond\neg p \\
(M, s_2) &\models \Box p \\
(M, s_3) &\models \Diamond p \wedge \Diamond\neg p \\
(M, s_4) &\models \Box p \\
(M, s_5) &\models \Box p \\
(M, s_6) &\models \Diamond p \wedge \Diamond\neg p \\
(M, s_7) &\models \Diamond p \wedge \Diamond\neg p
\end{aligned}$$

which satisfies our postulates.

2.4 Knowledge

We base our formalisation of knowledge on that developed by Pacuit, Parikh and Cogan in [PPC06] and add to our system a set $A = \{1, \ldots, n\}$ of agents, a set E of events, an equivalence relation $\sim_i$ on S where $s \sim_i s'$ is taken to mean that agent i cannot distinguish between states s and s', and an operator K_i with $K_i\phi$ meaning 'agent i knows that ϕ'. We define semantics for K_i as

$$(M, s) \models K_i\phi \text{ iff for all } s' \in S \text{ such that } s \sim_i s', (M, s') \models \phi.$$

From now on, we always assume that a state is a set of events; more precisely: $s \in S$ implies $s \subseteq E$ and $s \prec s'$ iff $s \subset s'$.[2] Events should be interpreted as 'things that happen' and which can affect the truth value of propositions.

[2] So a state corresponds to what Pacuit, Parikh and Cogan refer to as a finite prefix of a history.

For each agent $i \in A$, we fix sets E_i and P_i such that $P_i \subseteq E_i \subseteq E$. We call E_i the set of events which are *witnessed* by agent i and P_i the set of events which are *performed* by agent i. Events performed by agents are called *actions*.[3]

For example, consider a model with two agents i and j, and two states s_1 and s_2. Let $E = \{e_i, e_j\}$, where e_i represents agent i sitting down and e_j represents agent j sitting down. Then $P_i = \{e_i\}$ and $P_j = \{e_j\}$. Suppose that in s_1, both agents are standing up. Now suppose that $s_2 = s_1 \cup e_i$. So in s_2, agent i is sitting down and agent j is standing up. If we further stipulate that $E_i = E_j = E$, then both agents witness the event of agent i sitting down. So both agents know that the proposition 'agent i is sitting down' is now true.

To properly formalise this notion we need to introduce some extra machinery. For any event e, we define a corresponding *witnessed event* for agent i as

$$w_i(e) = \begin{cases} e & \text{if } e \in E_i \\ \mathrm{t} & \text{otherwise} \end{cases}$$

where $\mathrm{t} \in E$ is an uninformative 'time passes' event. Now given a state s, we define the *local state* of agent i to be

$$L_i(s) = \{w_i(e) \mid e \in s\}.$$

We can now define our relation $\sim_i$ in terms of the agent's local state, giving

$$s \sim_i s' \text{ iff } L_i(s) = L_i(s').$$

In our model of Figure 5, let the set of events be $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. Let $s_1 = \{e_1\}, s_2 = \{e_1, e_2\}, s_3 = \{e_1, e_3\}, s_4 = \{e_1, e_2, e_4\}, s_5 = \{e_1, e_2, e_5\}, s_6 = \{e_1, e_3, e_6\}, s_7 = \{e_1, e_3, e_7\}$.

For agent i, let $E_i = \{e_1, e_2, e_3\}$. Then the local states are

$$\begin{aligned}
L_i(s_1) &= \{e_1\} \\
L_i(s_2) &= \{e_1, e_2\} \\
L_i(s_3) &= \{e_1, e_3\} \\
L_i(s_4) &= \{e_1, e_2, \mathrm{t}\} \\
L_i(s_5) &= \{e_1, e_2, \mathrm{t}\} \\
L_i(s_6) &= \{e_1, e_3, \mathrm{t}\} \\
L_i(s_7) &= \{e_1, e_3, \mathrm{t}\}
\end{aligned}$$

[3] We add P_i to the system to accommodate agency as well as knowledge and will consider it further in §2.5. Following [PPC06], we do not require E_i, E_j or P_i, P_j to be disjoint for $i \neq j$.

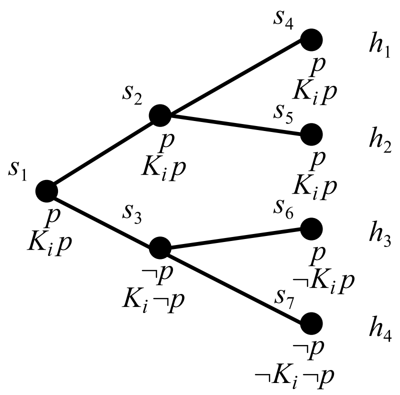

Figure 5: Knowledge of agent i.

and $\sim_i$ generates us the equivalence classes $\{s_1\}, \{s_2\}, \{s_3\}, \{s_4, s_5\}, \{s_6, s_7\}$. So we have

$$\begin{aligned}
(M, s_1) &\models K_i p \\
(M, s_2) &\models K_i p \\
(M, s_3) &\models K_i \neg p \\
(M, s_4) &\models K_i p \\
(M, s_5) &\models K_i p \\
(M, s_6) &\models \neg K_i p \\
(M, s_7) &\models \neg K_i \neg p.
\end{aligned}$$

2.5 Agency

In order to examine the concept of agency we need to focus our attention on the actions *performed* by an agent, represented by the set $P_i \subseteq E_i$. Whenever an agent performs an action, the state changes. To look at the actions available to an agent at a given state, we need to consider the possible successor states.

Definition 2.5. A state s' is an *immediate successor* of s iff $s \prec s'$ and there is no s'' such that $s \prec s'' \prec s'$.

Note that this definition only permits the existence of immediate successors if $\prec$ on S is not dense. But by defining S and $\prec$ in terms of E and $\subset$ we have ensured that this condition is met. Similarly,

Definition 2.6. A state s' is an *immediate predecessor* of s iff $s' \prec s$ and there is no s'' such that $s' \prec s'' \prec s$.

Note that because of backward linearity, there can be at most one immediate predecessor for any given state.

An individual agent may not have access to the full range of successors. In order for a successor to be available to agent i at s, we require that agent i can perform some action which determines the next state in the history. Thus for an agent to choose to move from s to s', there must be some action e performed by agent i which is in s' but not in s, i.e., $e \in P_i \cap s'$ but $e \notin s$. So the set $C(s,i)$ of *choices* available to agent i at s is given by

$$C(s,i) = \{s' \in S \mid s' \text{ is an immediate successor of } s \text{ and } (s' - s) \cap P_i \neq \emptyset\}.$$

In our discussion of free will and determinism we will want to talk about possible actions, but in our language the modal operators apply to propositions and not to events. So rather than talking about actions directly, instead we will refer to actions by way of their consequences, i.e., statements which are true after the action has been performed.

Definition 2.7. A statement ϕ is a *consequence* of an action e iff for all $s \in S$ such that $e \in s$, $(M,s) \models \phi$.

Next we introduce $\Diamond_i \phi$ meaning 'agent i can choose such that ϕ' with the semantics

$$(M,s) \models \Diamond_i \phi \text{ iff there exists } s' \in C(s,i) \text{ such that } (M,s') \models \phi.$$

Just as $\Diamond \phi$ can be true only if there is some state in which ϕ is true, $\Diamond_i \phi$ can be true only if there is some *successor* state in which ϕ is true. It follows that $\Diamond_i \phi \to \Diamond \phi$.

Now we introduce a new operator to enable us to talk about the choices that the agent actually makes. We will consider the agent to have made a choice only if it is a free choice, i.e. that the agent could have chosen otherwise. We write $\triangle_i \phi$ to mean 'agent i (freely) chose such that ϕ' and give the semantics as

$$\begin{gathered}(M,s) \models \triangle_i \phi \text{ iff } (M,s) \models \phi \text{ and there exists an immediate predecessor } s' \text{ of } s \\ \text{such that } (M,s') \models \Diamond_i \phi \wedge \Diamond_i \neg \phi.\end{gathered}$$

This type of operator is similar to the *achievement stit*[4] described by Belnap et al. in [BPX01, p.36], but is much simpler due to the simplified notion of choices and the elimination of the history parameter.

[4]The *stit* operator is so-called because the agent "sees to it that" the outcome obtains.

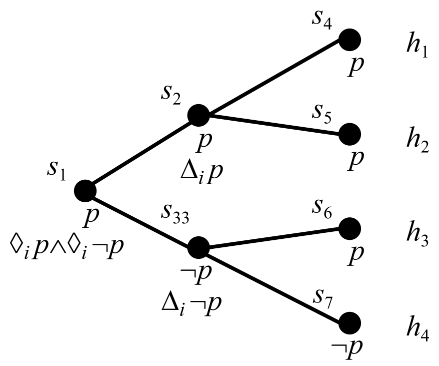

Figure 6: Choices of agent i.

In Figure 6, let the actions performed by i be given by $P_i = \{e_2, e_3\}$. Then

$$C(s_1, i) = \{s_2, s_3\}$$

and

$$\text{for } 2 \leq n \leq 7, C(s_n, i) = \emptyset.$$

In this model, agent i only has a choice at s_1. So we have

$$\begin{aligned}(M, s_1) &\models \Diamond_i p \wedge \Diamond_i \neg p \\ (M, s_2) &\models \triangle_i p \\ (M, s_3) &\models \triangle_i \neg p\end{aligned}$$

and these are the only states at which a choice formula is true.

2.6 Obligation

We will keep our notion of obligation very general. First we introduce a *goodness valuation* on actions

$$u : E \to \mathbb{R}.$$

We say that e is a *better* action than e' if $u(e) > u(e')$. No other restrictions are placed on u. Just as states are composed from events and histories are composed from states, so we can compose goodness valuations for states and histories based on the goodness valuation for actions. We place no restrictions on *how* these valuations should be defined, and simply introduce $\mu : S \to \mathbb{R}$

as a goodness valuation for states, and $U : \mathcal{H} \rightarrow \mathbb{R}$ as a goodness valuation for histories.[5]

For a set $C(s,i)$ of choices available to agent i at a state s, we define the set $B(s,i)$ of *best choices* as

$$B(s,i) = \{s' \in C(s,i) \mid \mu(s') \geq \mu(s'') \text{ for all } s'' \in C(s,i)\}.$$

So the best choices are those choices which maximise the value of the immediate successor. Note that the best choice is not necessarily the state obtained by performing the best action.

Now we introduce a new operator O_i with the intention that $O_i\phi$ means 'agent i ought to choose such that ϕ'.

$$(M,s) \models O_i\phi \text{ iff there is some } s' \in B(s,i) \text{ such that } (M,s') \models \phi.$$

This says that an agent ought to choose such that ϕ if there is a best choice in which ϕ is true. Note that this definition does not rule out the possibility of conflicting obligations in the case of multiple best choice states being available.

Definition 2.8. We say that ϕ and ψ are *conflicting obligations* for i at s iff $(M,s) \models O_i\phi$ and $(M,s) \models O_i\psi$ and $(M,s) \models \phi \rightarrow \neg\psi$.

We do not consider this a deficiency of the O_i operator, since it is not uncommon for agents to be caught between conflicting obligations. Any conflict is generated by the inability of the μ-function to distinguish between two states, and it is at the level of the μ-function that the conflict should be resolved. We can stipulate that the μ-function generate no conflicts by insisting that two successor states have the same μ-value if and only if they are the same state.

Definition 2.9. We say that a model $M = \langle T, V, \mu \rangle$ contains *no moral dilemmas* iff for any states $s, s', s'' \in S$, if s' and s'' are immediate successors of s and $\mu(s') = \mu(s'')$, then $s' = s''$.

[5]The simplest interpretation of μ is as a summation of the u-values of the events which make up the state, i.e., $\mu(s) = \sum_{e \in s} u(e)$. However, this simplistic interpretation of the value of states could lead to unexpected results, since it requires actions to be strictly independent but intuitively the moral value of an action depends on its context. So either the context must be ignored, or it must be included in the notion of an action, so that an action is no longer a primitive object. In the latter case, an action is better modelled as a pair (e,s). Similarly, the simplest interpretation of U is as a summation of the μ-values of the states which make up the history, i.e., $U(h) = \sum_{s \in h} \mu(s)$ but this definition is also likely to lead to counter-intuitive results, since the length of the history has an enormous impact on its value. This interpretation of U is really only useful for comparing histories of equal length.

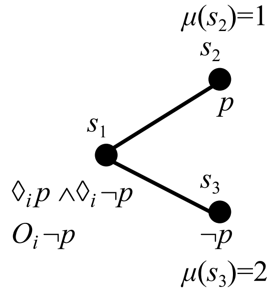

Figure 7: Obligations of agent i.

In our model of Figure 7, let $s_1 = \emptyset$, $s_2 = \{e\}$ and $s_3 = \{e'\}$. Let p be a consequence of e and $\neg p$ a consequence of e'. Now let $u(e) = 1$ and $u(e') = 2$, so e' is a better action than e, and define $\mu(s) = \sum_{e \in s} u(e)$. This means that $\mu(s_3) > \mu(s_2)$, so we have $B(s_1, i) = \{s_3\}$, and since $(M, s_3) \models \neg p$, this gives us

$$(M, s_1) \models O_i \neg p$$

which is the only obligation in this small model.

3 The Master Argument of Diodorus Cronus

3.1 Historical Background

Epictetus (c55–c135) tells us in Discourses, Book II, Chapter 19 that the argument known as the κυριεύων λογος[6] appears to be based on the principle that the following three propositions form an inconsistent triad:[7]

(D1) τῷ πᾶν παρεληλυθὸς ἀληθὲς ἀναγκαῖον εἶναι

(D2) τῷ δυνατῷ ἀδύνατον μὴ ἀκολουθεῖν

(D3) τῷ δυνατὸν εἶναι ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται

So if the argument is valid, then only two of the propositions can be maintained. Epictetus tells us that Diodorus Cronus (c340–280 B.C.) used the first two propositions to show that (D3) must be false i.e. every possibility is realised either now or in the future.[8]

Epictetus claims not to have his own opinion on which of the three should be false but refers us instead to a list of authors whose writings are related to the issue, in particular Cleanthes,[9] Chrysippus,[10] and Antipater.[11]

To Cleanthes he attributes the position of maintaining (D2) and (D3), and denying (D1), a position which was later defended by Antipater.

The third position of maintaining (D1) and (D3) and denying (D2), he attributes to Chrysippus. It should be noted that Chrysippus interpreted (D2) as referring to logical consequence and not temporal succession [Mat61, p.39] and he attempted to refute it by way of counterexample [Mat61, p.30]. This evidence suggests that Chrysippus interpreted (D2) as a statement of *reductio ad impossibile* [Mic76, p.234].

Unfortunately Epictetus does not include the reasoning behind the argument, only the propositions themselves and the position held by each philosopher regarding which should be maintained and which denied. But despite

[6]Usually translated as "Master Argument" [Old26, p.359] although "Ruling Argument" would be more accurate [Sed77, p.99].

[7][D1]"Everything true as an event in the past is necessary", [D2]"An impossible does not follow a possible", and [D3]"What is not true now and never will be, is nevertheless possible" [Old26, p.359].

[8]This is a restriction of the Megarian-Aristotelian definition of possibility, which allows (D3) by including past events. This will be discussed further in §3.2.3.

[9]c301–252 B.C., successor of Zeno and second head of the Stoic school [Sha91, p.14].

[10]c280–207 B.C., pupil of Cleanthes and third head of the Stoic school [Sha91, p.14].

[11]Antipater of Tarsus (c200–c129 B.C.), author of *On the Master Argument* and *On the Possible* [Bob98, p.3] and head of the Stoic school between c150 and c130 B.C. [Mat61, p.85].

their conflicting positions, there was no challenge to the result that the three propositions form an inconsistent triad, so we conclude that in the opinion of the ancients, the argument was logically valid (see [McK79, p.224]).

Although Epictetus gives the fullest account of the Master Argument, there are references to it in other literature. In *De Fato*, Cicero (106–43 B.C.) discusses the conflicting positions of Diodorus, Chrysippus and Cleanthes. He says of Diodorus:

> *ille enim id solum fieri posse dicit, quod aut sit verum aut futurum sit verum, et quicquid futurum sit, id dicit fieri necesse esse, et quicquid non sit futurum, id negat fieri posse.*[12] (De Fato, VII, 13)

To Chrysippus he says:

> *tu et quae non sint futura posse fieri dicis, ut frangi hanc gemmam etiam si id numquam futurum sit, neque necesse fuisse Cypselum regnare Corinthi, quamquam id millesimo ante anno Appolinis oraculo editum esset.*[13] (De Fato, VII, 13)

Thus we see that according to Cicero, Chrysippus maintains proposition (D3). There is also more evidence to show that Chrysippus maintained proposition (D1), and that this was denied by Cleanthes:

> *omnia enim vera in praeteritis necessaria sunt, ut Chrysippo placet dissentienti a magistro Cleanthe, quia sunt inmutablia nec in falsum e vero praeterita possunt convertere*[14] (De Fato, VII, 14)

So Cicero is in agreement with Epictetus.

Diodorus' Master Argument was intended to justify his definitions of 'possible' and 'necessary' (see [Mat61, p.38] and [Sed77, p.99]). These are given to us by Boethius (c480–c525) in his second commentary on Chapter 9 of Aristotle's *On Interpretation* [Mei80, p.234], although he makes no mention of the Master Argument itself:

[12] "For he [Diodorus] says that only what either is true or will be true can happen, and he says that whatever is going to happen must necessarily happen, and that whatever will not happen cannot happen." [Sha91, p.65].

[13] "You [Chrysippus] say that things that will not happen, too, can happen, for example that this precious stone should be broken can happen, even if this is never going to happen, and that it was not necessary for Cypselus to rule in Corinth although this had been declared by the oracle of Apollo a thousand years before." [Sha91, p.65].

[14] "[F]or all true statements about past things are necessary, in the view of Chrysippus who disagrees with his teacher Cleanthes, because they are unchangeable and cannot be turned from true to false[.]" [Sha91, p.65].

> *Diodorus possibile esse determinat, quod est aut erit; impossibile, quod cum falsum sit, non erit verum; necessarium, quod cum verum sit, non erit falsum; non necessarium, quod aut jam est aut erit falsum.*[15]

The Master Argument came to the attention of twentieth-century logicians primarily through the work of Arthur Prior (1914–1969), although Martha Kneale wrote about it in 1938 [Kne38] and tells us that previous commentators included the nineteenth century German philosophers Carl Prantl [Pra55] and Eduard Zeller [Zel75, Zel82]. Having read of Diodorus in Mates' *Stoic Logic* [Mat61], Prior decided to investigate Diodorus' notions of possibility and necessity using modern modal logic techniques. The result of this investigation was his 1955 paper "Diodoran Modalities" [Pri55] in which he developed a formalised tense logic and used it to construct a proof of the Master Argument.[16] Since Prior's work there has been much discussion of the Master Argument. We will look at several modern reconstructions in the following sections.

3.2 Modern Reconstructions

There have been many modern attempts to reconstruct the argument. These reconstructions differ primarily on how to interpret the premises, namely over whether (D1) applies to temporally definite or indefinite statements, and whether 'follow' in (D2) should be read as referring to temporal order (i.e. $\succ$) or some sort of implication (e.g. $\rightarrow$, $\vdash$).

As part of this investigation into the Master Argument, we have examined and classified a wide range of reconstructions.[17] Table 1 summarises each author's position with respect to the type of basic statement under discussion and the meaning of 'follow'. As we can see, all positions are represented, although the majority of authors favour an interpretation with temporally indefinite statements and 'follow' representing some notion of implication. In the following sections we examine some of these reconstructions in more detail, formalising the premises using the system developed in §2. This allows

[15] "Diodorus delimits the possible as that which either is or will be; the impossible as that which when it is false will not be true; the necessary as that which when it is true will not be false; the non-necessary as that which either now is or will be false." [Kre98, p.176].

[16] For more information on Prior and the origins of tense logic, see [Cop07].

[17] Readers proficient in French, German and Italian are also referred to the following authors who have written on the Master Argument but are not included in the above survey: Becker [Bec61], Blanché [Bla65], Celluprica [Cel77], Giannantoni [Gia81], von Kutschera [Kut86], Mingucci [Min66], Schuhl [Sch60], Stahl [Sta79], Wiedemann [Wie99].

	Type of basic statement	
	Temporally indefinite	Temporally definite
'Follow' means implication	Denyer [Den99] Gaskin [Gas99] Gundersen [Gun97] Hintikka [Hin64] Kneale [Kne38] McKirahan [McK79] Øhrstrøm & Hasle [ØH95] Prior [Pri67] Purtill [Pur73] Sedley [Sed77] Trzęsicki [Trz87] White [Whi80]	Mates [Mat61] Michael [Mic76]
'Follow' means temporal succession	von Wright [Wri79]	Rescher [Res66] Zeller [Zel82]

Table 1: Classification of Master Argument reconstructions.

the different positions to be compared from within a single system so as to better understand their similarities and differences.

3.2.1 The First Premise

(D1) is translated by Oldfather as "Everything true as an event in the past is necessary" [Old26, p.359], by Mates as "Every proposition true about the past is necessary" [Mat61, p.38], and by Carter as simply "Everything past is necessarily true" [Car57, p.110], so there is clearly some ambiguity regarding what, precisely, is taken to be necessary.

Although our formal language does not enable us to talk directly about the necessity of events, it is clear that in our system an event that has already occurred (i.e. a past event), cannot 'un-occur'. And in §2.3.2 we defined both temporally definite statements (i.e. those with a fixed temporal reference) and temporally indefinite statements (i.e. functions from states to truth values). How then are we to interpret this premise formally?

(D1) is a combination of past tense and necessity, so we would expect any formalisation to contain the operators P and $\Box$, or if expressed as a restriction on the class of frames, to include both relations R and $\prec$.

Rescher [Res66, p.440] interprets (D1) as 'Everything that is past and true is (now) necessary' which he formalises in terms of temporalised modalities as

$$(\forall t)\{[T_t(p) \ \& \ t < n] \to N_n(p)\}$$

which reads as: 'for all times t, if p is true at time t and t is earlier than now (i.e. t is in the past), then p is necessary now'. We do not have temporalised modal operators in our system, so we must express this concept in terms of our model semantics

$$\text{for all } s, s' \in S, \text{ if } (M, s) \models p \text{ and } s \prec s' \text{ then } (M, s') \models \Box p$$

but since $s \prec s'$ we could also write

$$\text{for all } s' \in S, \text{ if } (M, s') \models Pp \text{ then } (M, s') \models \Box p$$

and then this is equivalent to

$$\models Pp \to \Box p.$$

However this only really makes sense if p is an untensed temporally definite proposition, since the truth value of a tensed proposition is dependent on the time of evaluation. Rescher and Urquhart explicitly exclude assertions which are not temporally definite [RU71, p.189]:

> We rule out such assertions as "It rained in London yesterday" which may be true at some times and false at others, because they involve a shifting time-indicator (today, yesterday) rather than a definite date. The propositions at issue must thus be taken as temporally definite.

Michael [Mic76, p.231] offers a similar formalisation of the first premise:

$$T_{n-d}(p) \to N(p)$$

where d is a variable ranging over increments of time greater than zero. Michael differs from Rescher in that he does not use temporalised modalities, but he shows that his argument can be applied equally well to temporalised modalities with only the notation varying [Mic76, p.232].

In our formal system, if p is temporally definite, then it includes an explicit temporal reference, say s, and can be written $p(s)$. Now if $p(s)$ is true anywhere, it must be true at the state s. But then it must be true at every state on all histories containing s. In other words, $p(s)$ is true at all states on all histories in $H(s)$. This is precisely the definition we gave for (temporal) necessity in §2.3.6. So by making only the assumption that p is temporally definite, we obtain not only

$$Pp \to \Box p$$

but also the "unpalatably deterministic thesis" [RU71, p.195]

$$p \to \Box p.$$

This is temporalised necessity in the sense of the principle "*unumquodque, quando est, oportet esse*",[18] which is precisely how Mates interprets (D1) [Mat61, p.39, fn.57]. Mates considers all propositions to be temporally definite, and refers to our notion of temporally indefinite propositions as "propositional functions". He gives explicit support for the temporally definite interpretation of (D1) by saying [Mat61, p.39],

> Although Diodorus usually predicates necessity of what are in effect propositional functions, it seems that in the first of his three incompatibles, necessity is predicated of a proposition.

McKirahan [McK79, p.227] also uses temporalised modalities, but in contrast with Rescher, considers the propositions under discussion by Diodorus to be temporally indefinite. As a result the reading he gives to (D1) is rendered in his notation as

$$(\exists t)[t < n \ \& \ T_t(p)] \supset \sim\Diamond_n\sim(\exists t)[t < n \ \& \ T_t(p)]$$

which translates in our system to

$$Pp \to \Box Pp$$

since $\sim$ and $\supset$ are just notational variants of $\neg$ and $\to$ respectively.

Prior [Pri67, p.32] formalises (D1) in Polish notation as *CPpNMNPp*, which also translates in our system to $Pp \to \Box Pp$, so Prior's formalisation of (D1) is equivalent to that of McKirahan. Purtill [Pur73, p.34] takes a slightly different approach notationally, and formalises (D1) simply as $\Box a$ where a is some statement or set of statements about the past. If we assume that this is equivalent to saying that a can be expressed in the form Pp, then this can also be translated in our system to $Pp \to \Box Pp$.

Denyer [Den99, p.245] and White [Whi99, p.233] both ensure that this reading of (D1) holds by way of a first-order frame restriction, which in our system is written as:

$$\forall s \forall s' \forall s''(sRs' \to ((s'' \prec s) \to (s'' \prec s')))$$

i.e. 'everything that has already happened, must have already happened in all possible worlds too'.

[18] Cited by Leibniz in the "Theodicy" [Far51, p.152], translation given by Rescher [Res66, p.440] as "whatever is, when it is, is necessarily".

One way of satisfying this restriction is to use the relation $\preceq$ for R. It is then saying that if $s \preceq s'$ and $s'' \prec s$ then $s'' \prec s'$, and this is clearly true because of the transitivity of $\prec$. This approach, if taken together with the restriction that $\prec$ is linear, directly leads to a notion of the possible as being that which either is or will be true. This is precisely the Diodorean notion of possibility as given by Boethius' second commentary on Chapter 9 of Aristotle's *On Interpretation* [Mei80, p.234]. This interpretation of Diodorus' first premise can be expressed using only temporal operators as

$$Pp \rightarrow GPp.$$

So to summarise, we have three formalisations of (D1), which is a statement of the principle of the necessity of the past. $Pp \rightarrow \Box p$ applies only to temporally definite statements and has as a consequence that truth and necessity are equivalent. $Pp \rightarrow \Box Pp$ is applicable to all statements and is the form used by most modern reconstructions. $Pp \rightarrow GPp$ is a special case of this form using the Diodorean notion of necessity.

3.2.2 The Second Premise

The translation of ἀκολουθεῖν as 'follow' in (D2) is ambiguous between a temporal and logical interpretation, but the word is usually used by Diodorus to mean "is a consequent of" [Mat61, p.39], and indeed this is how Carter translates it.[19]

Nevertheless, Rescher [Res66, p.440], along with Zeller [Zel82], chooses to interpret it in a temporal way rendering what he calls the possibility-conservation principle that 'the once possible is always possible thereafter'. In our system, this reading of (D2) can be expressed as

$$\Diamond p \rightarrow G\Diamond p.$$

Viewed in this way, it is clearly another restriction on the way R and $\prec$ interact. Comparing to (D1), we can see that it is in the same form as $Pp \rightarrow \Box Pp$, but with $\Diamond$ replacing P and using G for $\Box$. The corresponding frame restriction is

$$\forall s \forall s' \forall s''[s \prec s' \rightarrow (s''Rs \rightarrow s''Rs')].$$

Again this is satisfied if R is the relation $\prec$ or $\preceq$. But taking this approach then means that (D1) and (D2) essentially represent the same condition,

[19] While Oldfather translates (D2) as "An impossible does not follow a possible" [Old26, p.359], Carter translates it as "An impossibility is not the consequence of a possibility" [Car57, p.110].

captured by the restriction

$$\forall s \forall s' \forall s''[s \prec s' \to (s'' \prec s \to s'' \prec s')]$$

i.e. 'everything that has already happened, has already happened in all future worlds too', which is self-evident because of the transitivity of $\prec$. So using the Diodorean notion of possibility, this reading of (D2) adds nothing to the argument, and we will not consider it further.

Even amongst those authors who reject the temporal reading in favour of some notion of implication, there are differences of opinion regarding how to interpret (D2). For example, Kneale & Kneale consider it "a recognized thesis of modal logic" [KK62, p.119], while Hintikka says it is "the assumption hardest to understand" [Hin64, p.103].

Denyer [Den99, p.245] takes the line that 'follow' refers to logical consequence. In particular, he argues that (D2) is already implicit in his formalism, if it is taken to mean that formula ϕ follows from a set of formulae Φ iff ϕ is true in every model in which every member of Φ is true. This is just the standard model-theoretic definition of entailment, with ϕ being a logical consequence of the theory Φ [Hod97, p.37]. So this interpretation is implicit in our system too, and using $\vdash$ for entailment we can write 'q follows from p' as $p \vdash q$.

Michael [Mic76, p.234] also takes this line and interprets (D2) as a general statement of *reductio ad impossibile* (i.e. proof by contradiction), formalising it as

$$\text{if } \sim P(\Gamma_1), \text{ and if } \Gamma_2 \vdash \Gamma_1, \text{ then } \sim P(\Gamma_2)$$

where P is here taken to mean possible rather than past. Thus in our system this interpretation of (D3) is written as

$$\text{if } p \vdash q \text{ then } \neg\Diamond q \to \neg\Diamond p.$$

White [Whi99, p.226] also interprets (D2) as the principle of *reductio ad impossibile* but chooses to make a distinction between logical necessity and relative or conditional necessity (*necessitas per accidens*) which he denotes by Nec_r. This type of necessity he says can be thought of as "the necessity of unalterability, unavoidability, or irrevocability" and allows that something may be nonnecessary at one state/time and necessary at another. This is in keeping with our semantics for $\Box$, although we also stipulate that a necessary proposition must be invariable along the whole length of any history it belongs to, and this does not seem to be a requirement of White's notion of relative necessity. He then uses this notion of relative necessity in his interpretation of 'follow', representing (D2) as a rule of inference by means of which a premise of the form $\mathrm{Nec}_r(\sim p \supset \sim q)$ together with a premise of the form $\sim \mathrm{Pos}_r \sim q$ yield a conclusion of the form $\sim \mathrm{Pos}_r \sim p$ [Whi99, p.229]. This interpretation of (D2) can be formalised in our system as

$$(\Box(\neg p \to \neg q) \wedge \neg\Diamond\neg q) \to \neg\Diamond\neg p.$$

Prior [Pri55, p.211] formalises (D2) as *CNMqCLCpqNMp*, which is equivalent in our system to

$$\Box(p \to q) \to (\neg\Diamond q \to \neg\Diamond p)$$

and hence represents the same sense of 'follow' as White's formalisation. Prior's formalisation is intended to capture the Diodorean sense of implication [Pri55, p.206] i.e. that the implication must be valid at all times, but this then presupposes a definition of $\Box$ which is not in accordance with White's interpretation, or with Diodorus' notion of necessity. So we cannot say that this interpretation of 'follow' represents true Diodorean implication, which is the same as the medieval notion of *consequentia simplex* [Pri57, p.1], [ØH95, p.67] and which we know today as strict implication. Instead it is a *necessitas per accidens* version of the medieval notion of *consequentia ut nunc*, which we know today as material implication.

A more accurate approach to formalising Diodorean implication, or *consequentia simplex*, without presupposing definitions of possible and necessary, is that taken by Øhrstrøm & Hasle [ØH95, p.24] who introduce the notation $\Rightarrow$ and define it as

$$p \Rightarrow q \text{ iff } (\forall t)(T(t,p) \supset T(t,q))$$

to give a formalisation of (D2) as

$$((p \Rightarrow q) \wedge Mp) \supset Mq.$$

This is equivalent in our system to

$$\begin{gathered}\text{if for all } s \in S, (M,s) \models p \to q \\ \text{then for all } s \in S, (M,s) \models \neg\Diamond q \to \neg\Diamond p.\end{gathered}$$

So to summarise, we have four different readings of ἀκολουθεῖν, giving four different interpretations of (D2), of which one uses straightforward temporal succession ($\Diamond p \to G\Diamond p$), and three use different notions of implication, namely

- entailment (if $p \vdash q$ then $\neg\Diamond q \to \neg\Diamond p$),
- *consequentia simplex* (if for all $s \in S, (M,s) \models p \to q$ then for all $s \in S, (M,s) \models \neg\Diamond q \to \neg\Diamond p$), and

- a *necessitas per accidens* version of material implication $(\Box(p \to q) \to (\neg\Diamond q \to \neg\Diamond p)$, where $\Box$ represents *necessitas per accidens*).

with the *consequentia simplex* formalisation being the closest to Diodorus' own sense of implication. Both historical and modern analysis indicates that (D2) is most likely a statement of *reductio ad impossibile.*

3.2.3 The Conclusion

(D3) can be formalised in our system as the existential statement

$$\text{there is a } p \text{ such that } \Diamond p \wedge \neg p \wedge \neg F p.$$

The conclusion of Diodorus' argument is therefore a negation of this statement, essentially stating that all possibilities must be actualised either now or in the future. Sedley suggests [Sed77, p.116, fn.140] that the Master Argument was turned into a determinist argument by converting all propositions into temporally definite ones. However he rejects the common view that Diodorus himself was a hardline determinist [Sed77, p.99] and notes that a deterministic interpretation is not consistent with the distinction that Diodorus maintained between the possible and the necessary. Recall that Diodorus defined these modalities as

- The possible is that which either is or will be true.
- The necessary is that which, being true, will not be false.

These definitions can be formalised in our system as

$$\Diamond_{\mathrm{D}}\phi := \phi \vee F\phi$$
$$\Box_{\mathrm{D}}\phi := \phi \wedge G\phi$$

which is consistent with Prior's definitions of the Diodorean modalities [Pri55, p.206]. On a linear temporal frame this captures precisely the Diodorean notions. On a branching frame it could be argued that the operator F is not strong enough to capture the sense of 'will be', and that F' should be used instead. But this does not preserve the interdefinability $\Box_{\mathrm{D}} \equiv \neg\Diamond_{\mathrm{D}}\neg$ which we assume, along with Øhrstrøm & Hasle [ØH95, p.25], that Diodorus accepted. So we will retain the use of F, and argue that Diodorus' wording is due to an assumption that time is linear.

The Diodorean notion of necessity, where a proposition is necessary if it is true now and at all future (later) times (i.e. irrevocability), can be contrasted with the Megarian-Aristotelian notion of necessity (see Byrd [Byr78, p.463]) where a proposition is necessary if it is true now and at all past and future (earlier and later) times, i.e. it is true at all times. Note that the Megarian-Aristotelian notion does not result in a system where necessity is dependent on what time is now, which is an undesirable aspect of the Diodorean system, since it does not really fit with our intuitions of what 'necessary' should mean. The definition of necessity which we gave in §2.3.6 conforms to the Megarian-Aristotelian notion. Both the Diodorean and the Megarian-Aristotelian notion of necessity differ from the standard notion of logical necessity in that they do not include *all* possible states, only those which are in some way temporally related, and as such are limited in scope to states which lie on the same history.

Note that the Diodorean definition of possibility does not allow for possibilities having being actualised in the past, but being no longer true. It is in effect a restriction of the Megarian-Aristotelian notion. So something that was true in the past but is no longer true and will never be true again is neither possible nor necessary. This in itself seems to be a contradiction of the necessity of the past. It can therefore be argued that the Master Argument was meant to distinguish the Diodorean notion of possibility from the Megarian-Aristotelian notion in such a way that preserves the necessity of the past. Thus the Diodorean definition of necessity can be taken as a definition of *necessitas per accidens*. It is with this interpretation in mind that we propose a formal reconstruction of the argument itself.

3.2.4 The Argument

We will now apply all these lessons to build a reconstruction of the argument in our formal system.

For temporally definite propositions, the reconstruction is a simple one. We aim to show that the Megarian-Aristotelian definition of possibility with (D1) and (D3) together produce a contradiction. Then by using (D2) as the principle of *reductio ad impossibile*, we conclude that either (D1) or (D3) must be false. Rejecting (D1) rejects the principle of the necessity of the past, and rejecting (D3) generates the Diodorean definition of possibility.

$$
\begin{array}{lrr}
Pp \rightarrow \Box p & \text{(premise D1)} & (1) \\
\Diamond p \wedge \neg p \wedge \neg Fp & \text{(premise D3)} & (2) \\
\Diamond p & \text{(from 2)} & (3) \\
Pp \vee p \vee Fp & \text{(from 3 and definition of } \Diamond) & (4) \\
Pp & \text{(from 2 and 4)} & (5) \\
\Box p & \text{(from 1 and 5)} & (6) \\
\neg \Diamond \neg p & \text{(from 6 and definition of } \Box) & (7) \\
\neg p & \text{(from 2, contradicts 7).} & (8)
\end{array}
$$

This reconstruction makes use of the interdefinability of $\Diamond$ and $\Box$, although the weaker principle $\Box p \rightarrow \neg\Diamond\neg p$ will suffice. Diodorus needed to state the principle of *reductio ad impossibile*, because as we have shown, there is evidence that Chrysippus rejected this principle. This, I believe, is the most historically plausible reconstruction of the argument, as it uses the premises in accordance with the views of Mates.[20] It is essentially the same as the reconstruction proposed by Michael [Mic76].

Nevertheless, this is not the most popular form of reconstruction. The majority of authors prefer to interpret (D1) as referring to temporally indefinite statements. The most rigorous reconstruction of this form is that of Prior [Pri67]. For a restatement of Prior's reconstruction in more modern notation, see Øhrstrøm & Hasle [ØH95, pp.20–23].

[20] We take Mates to be our best historical authority on the Master Argument of Diodorus. Of previous commentators, he remarks that "it is apparent that Prantl and Zeller did not understand Stoic logic" [Mat61, p.86]. Martha Kneale also chose to disagree with Prantl and Zeller's interpretations on historical grounds [Kne38, p.254].

4 Freedom and Foreknowledge

In this section we want to use our formalisation to define the concept of omniscience and use this to examine whether the existence of an omniscient agent is compatible with the ability of agents to make free choices. We will outline a version of the argument supporting fatalism, and show the approaches used by Thomas Aquinas and William of Ockham in their attempts to refute it.

4.1 Omniscience

There are numerous ways to formalise the concept of omniscience, many of which were examined by Prior in his 1962 paper, "The Formalities of Omniscience" [Pri62]. In this section we will follow the line of Zagzebski [Zag91, p.6] and define two ways in which omniscience can be interpreted, although while Zagzebski talks about belief, we will talk directly about knowledge. The first is the notion of *infallible knowledge*.

Definition 4.1. We say that an agent i has *infallible knowledge* in a model M iff for all $s \in S$, $(M, s) \models K_i\phi \to \phi$.

This means that if agent i knows that ϕ, then ϕ must be true. But this is implicit in our definition of knowledge. Recall from §2.4:

$$(M, s) \models K_i\phi \text{ iff for all } s' \in S \text{ such that } s \sim_i s', (M, s') \models \phi.$$

But $\sim_i$ is an equivalence relation, and hence reflexive. So it is a trivial result that for any agent, if that agent has any knowledge, then it is infallible knowledge.

So we must define a stronger notion of omniscience, which Zagzebski calls *essential omniscience*.

Definition 4.2. We say that an agent i is *essentially omniscient* in a model M iff for all $s \in S$, $(M, s) \models K_i\phi \leftrightarrow \phi$.

Now we introduce a new agent into our system, which we will call God, and denote by G. Thus if agent G knows ϕ, we write $K_{\mathrm{G}}\phi$. Thomas Aquinas (c1225–1274) is unequivocal in his assertion that God is omniscient, stating

> *Deus scit omnia quaecumque sunt quocumque modo.*[21]

[21] Summa Theologica, I, q.14, a.9: 'God knows all things whatsoever that in any way are.' [EDP20].

We stipulate then that agent G is essentially omniscient, so that

$$\forall s \in S, (M, s) \models K_{\mathrm{G}}\phi \leftrightarrow \phi.$$

This can be satisfied by defining the similarity relation to be the identity relation:

$$s \sim_{\mathrm{G}} s' \text{ iff } s = s'.$$

It is easy to see that as a result, we obtain

$$(M, s) \models K_{\mathrm{G}}\phi \text{ iff } (M, s) \models \phi$$

i.e. $K_{\mathrm{G}}\phi \leftrightarrow \phi$, which is what we want for essential omniscience: God's knowledge is identical with truth. So this formalisation captures the meaning of 'God knows all true propositions'.

This definition of $\sim_{\mathrm{G}}$ is compatible with our general definition of $\sim_i$ which was given in §2.4 as

$$s \sim_i s' \text{ iff } L_i(s) = L_i(s').$$

Recall that

$$L_i(s) = \{w_i(e) \mid e \in s\}$$

and

$$w_i(e) = \begin{cases} e & \text{if } e \in E_i \\ \mathrm{t} & \text{otherwise} \end{cases}$$

This means that for our system we can give a definition of omniscience based on events.

Definition 4.3. We say that an agent i is *omniscient* in a model M iff $E_i = E$.

So for agent G, we set $E_{\mathrm{G}} = E$, which gives us $w_{\mathrm{G}}(e) = e$, from which it follows that $L_{\mathrm{G}}(s) = s$, and hence $s \sim_{\mathrm{G}} s'$ iff $s = s'$, which is what we want.

4.2 Knowledge of Contingent Propositions

Aquinas also clearly asserts that God knows future contingents.

> *Cum supra ostensum sit quod Deus sciat omnia non solum quae actu sunt, sed etiam quae sunt in potentia sua vel creaturae; horum autem quaedam sunt contingentia nobis futura; sequitur quod Deus contingentia futura cognoscat.*[22]

[22]Summa Theologica, I, q.14, a.13: 'Since as was shown above, God knows all things; not only things actual but also things possible to Him and creature; and since some of these are future contingent to us, it follows that God knows future contingent things.' [EDP20].

We will consider future contingents as a special case of a more general notion of contingent propositions.

Definition 4.4. For a proposition p, we say that p is:

- a *contingent proposition* iff there is some $s \in S$ such that $(M, s) \models \Diamond p \wedge \Diamond \neg p$,
- a *contingent proposition at s* iff $(M, s) \models \Diamond p \wedge \Diamond \neg p$, and
- a *future contingent proposition at s* iff $(M, s) \models Fp \wedge F\neg p$.

Aquinas gives the argument against God's knowledge of contingent propositions as follows:

> *omne scitum a Deo necesse est esse: quia etiam omne scitum a nobis necesse est esse, cum tamen scientia Dei certior sit quam scientia nostra. Sed nullum contingens futurum necesse est esse. Ergo nullum contingens futurum est scitum a Deo.*[23]

To show why the argument fails, first we rephrase this informal argument in terms of our system: Let p be a contingent proposition. If God knows the truth of all propositions, he must also know the truth of contingent propositions. Suppose God knows that p is true. Then since $K_{\mathrm{G}}p \leftrightarrow p$, it follows that p is true. But then $\neg p$ cannot be true, i.e. $\neg\Diamond\neg p$. But this is equivalent to $\Box p$, which contradicts p being contingent. Similarly, if God knows that p is false, then we get $\Box\neg p$, which also contradicts p being contingent. So either God does not know the truth value of p, or p is not contingent.

The argument hinges on the definitions of possibility and necessity. There is an underlying assumption here that $p \to \neg\Diamond\neg p$, and hence $p \to \Box p$. So this is trying to apply the principle "*unumquodque, quando est, oportet esse*", which we first met in §3.2.1. For our formalisation, it is clear that this argument does not hold up, since possibility is defined in section §2.3.6 as

$$(M, s) \models \Diamond\phi \text{ iff for some } h' \in H(s) \text{ and some } s' \in h', (M, s') \models \phi$$

and so the inference $p \to \neg\Diamond\neg p$ is not valid. Similarly for future contingents, it does not follow from Fp that $\neg F\neg p$. So it is easy to show that the argument

[23] Summa Theologica, I, q.14, a.13, obj.3: 'everything known by God must necessarily be, because even what we ourselves know, must necessarily be; and, of course, the knowledge of God is much more certain than ours. But no future contingent things must necessarily be. Therefore no contingent future thing is known by God.' [EDP20].

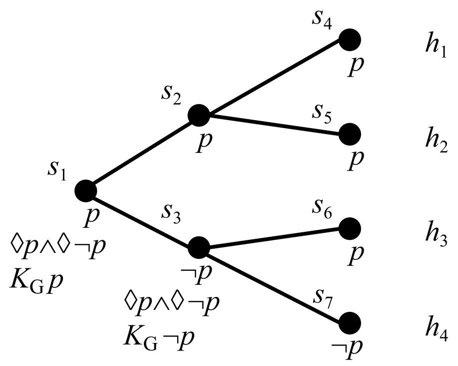

Figure 8: God's knowledge of contingent propositions

is invalid by way of a counterexample. For instance in Figure 8, we have

$$
\begin{aligned}
&(M, s_1) \models p \\
&(M, s_1) \models K_{\mathrm{G}} p \\
&(M, s_1) \models \Diamond p \wedge \Diamond \neg p \\
&(M, s_3) \models \neg p \\
&(M, s_3) \models K_{\mathrm{G}} \neg p \\
&(M, s_3) \models \Diamond p \wedge \Diamond \neg p.
\end{aligned}
$$

So in our model, at s_1 agent G knows that p, and at s_3, agent G knows that $\neg p$. But all along the histories h_3 and h_4, p is a contingent proposition.

4.3 Immutability and Eternity

Aquinas also argues that God's knowledge is immutable.

> *Cum scientia Dei sit eius substantia, ut ex dictis patet; sicut substantia eius est omnino immutabilis, ut supra ostensum est, ita oportet scientiam eius omnino invariabilem esse.*[24]

Definition 4.5. We say that agent i has *immutable knowledge* in a model M, iff whenever $(M, s) \models K_i \phi$ for some $s \in S$, then for all $h \in H(s)$, we have that $(M, s) \models K_i \phi$ for all $s \in h$. In other words, $M \models K_i \phi \to \Box K_i \phi$.

[24] Summa Theologica, I, q.14, a.15: 'Since the knowledge of God is His substance, as is clear from the foregoing, just as His substance is altogether immutable, as shown above, so His knowledge likewise must be altogether invariable.' [EDP20].

This is saying that if an agent has immutable knowledge, then that knowledge does not change along a history, and so it is necessary. In [Kre83, p.642], Kretzmann poses the following question:

> Are the beginninglessness and immutability of God's knowledge of all particulars compatible with the fact that some of them are contingent?

For our definition of immutable, it seems that they are not compatible. To see this we need look only at Figure 8, where we can see that

$$(M, s_1) \models K_{\mathrm{G}} p$$
$$(M, s_3) \models \neg K_{\mathrm{G}} p$$

and both s_1 and s_3 lie on both h_3 and h_4. Note however that this definition of immutable relies on the known proposition having an immutable truth value, so the proposition itself must be necessary. So combining immutable knowledge with essential omniscience has deterministic consequences for the model, as shown by the following theorem:

Theorem 4.6. If a model M contains an essentially omniscient agent with immutable knowledge, then $M \models \phi \to \Box\phi$.

Proof. Let M contain an essentially omniscient agent G with immutable knowledge. Then

$$M \models K_{\mathrm{G}}\phi \leftrightarrow \phi \quad \text{(from Definition 4.2)} \tag{9}$$
$$M \models K_{\mathrm{G}}\phi \to \Box K_{\mathrm{G}}\phi \quad \text{(from Definition 4.5)} \tag{10}$$
$$M \models \phi \to \Box\phi \quad \text{(use 9 to substitute } \phi \text{ for } K_{\mathrm{G}}\phi \text{ in 10).} \tag{11}$$

□

So a model containing an essentially omniscient agent with immutable knowledge cannot contain any contingent propositions. To define God's knowledge in this way would be to restrict God's knowledge to necessary propositions, and we have already seen in §4.2 that this was not what Aquinas intended. We can note, however, that this category of proposition does include all temporally definite propositions, so God can be said to have immutable knowledge of all temporally definite propositions. But his knowledge of contingent propositions must be defined in a different way.

Aquinas' solution was that God's immutability meant that he was an 'eternal' being, outside of time. Consider

> *Ratio aeternitatis consequitur immutabilitatem, sicut ratio temporis consequitur motum, ut ex dictis patet. Unde, cum Deus sit maxime immutabilis, sibi maxime competit esse aeternum.*[25]

and

> *Aeternitas est tota simul: in tempore autem est prius et posterius.*[26]

So according to Aquinas, the terms 'before' and 'after' cannot apply to God, because he is outside of time. We capture this notion in our formal system by placing the eternal agent's knowledge outside of the scope of the temporal order.

Just as the terms 'before' and 'after' cannot apply to God, neither can they apply to God's knowledge. This means that statements about God's knowledge cannot be tensed. So we cannot say that in the past, God *knew* that p, or in the future, God *will know* that $\neg p$. We can incorporate this view into our formal system by defining the notion of *atemporal knowledge*.

Definition 4.7. An agent i has *atemporal knowledge* iff for all $s \in S$,

$$(M, s) \models K_i\phi \text{ iff } M \models K_i\phi(s).$$

This is saying that if an agent has atemporal knowledge, then if at s he knows that 'ϕ is true', then at every state in the model he knows that 'ϕ is true at s', and vice versa. Atemporal knowledge follows from essential omniscience, as we see in the following theorem.

Theorem 4.8. If an agent is essentially omniscient, that agent has atemporal knowledge.

In the proof we will make use of the equivalence

$$M \models \phi(s) \text{ iff } (M, s) \models \phi \tag{12}$$

from §2.3.2.

[25]Summa Theologica, I, q.10, a.2: 'The idea of eternity follows immutability, as the idea of time follows movement, as appears from the preceding article. Hence, as God is supremely immutable, it supremely belongs to Him to be eternal.' [EDP20].

[26]Summa Theologica, I, q.10, a.4: 'Eternity is simultaneously whole. But time has a "before" and an "after".' [EDP20].

Proof. Let G be an essentially omniscient agent. Then

$$\begin{aligned}
M &\models K_{\mathrm{G}}\phi \leftrightarrow \phi && \text{(from Definition 4.2)} \quad (13)\\
(M,s) &\models K_{\mathrm{G}}\phi \leftrightarrow \phi && \text{(from 13)} \quad (14)\\
(M,s) &\models K_{\mathrm{G}}\phi \text{ iff } (M,s) \models \phi && \text{(equivalent to 14)} \quad (15)\\
(M,s) &\models K_{\mathrm{G}}\phi \text{ iff } M \models \phi(s) && \text{(from 15 and 12)} \quad (16)\\
(M,s) &\models K_{\mathrm{G}}\phi \text{ iff } M \models K_{\mathrm{G}}\phi(s) && \text{(from 13 and 16)} \quad (17)
\end{aligned}$$

so G has atemporal knowledge. □

So adding atemporal knowledge to essential omniscience has no effect on the model.

If an agent i has atemporal knowledge, then the temporal order $\prec$ cannot be used to evaluate statements about the agent's knowledge, and so the K_i operator must always have scope over any temporal operators.

Definition 4.9. If agent i has atemporal knowledge, we say that statements of the form $F\phi$ and $P\phi$ are only well formed if ϕ contains no occurrence of K_i.

This prohibits statements of the form $FK_{\mathrm{G}}\phi$ and $PK_{\mathrm{G}}\phi$, but it does not prohibit us from saying that *at* s_1, $K_{\mathrm{G}}p$ and *at* s_3, $K_{\mathrm{G}}\neg p$, which still seems to imply a change in God's knowledge. But by telling us that eternity is simultaneously whole, Aquinas is telling us that s_1 and s_3 are simultaneous for God, so the fact that he knows p at one state and $\neg p$ at another does not result in a change in God. So we can have

$$\begin{aligned}
(M,s_1) &\models K_{\mathrm{G}}p\\
(M,s_3) &\models \neg K_{\mathrm{G}}p
\end{aligned}$$

but we cannot have

$$\begin{aligned}
(M,s_1) &\models K_{\mathrm{G}}p \wedge F\neg K_{\mathrm{G}}p\\
(M,s_3) &\models \neg K_{\mathrm{G}}p \wedge PK_{\mathrm{G}}p.
\end{aligned}$$

This formal restriction has no effect on the objects of God's knowledge, since the equivalence $K_{\mathrm{G}}\phi \leftrightarrow \phi$ means that we can always eliminate the K_{G} operator to form a well-formed statement.

4.4 The Fatalism Argument

The fatalism argument can be constructed in a similar (informal) form to the argument against God's knowledge of contingent propositions: Let e be an action that I can perform freely. If God knows that in the future I will perform e, then in the future I will perform e. Suppose God knows that in the future I will perform e. Then it is true that in the future I will perform e. Then it is not possible that in the future I do not perform e. So it is necessary that in the future I perform e. So I do not perform e freely.

Suppose that ϕ is a consequence of e and let s_1, s_2, s_3 be states, with $s_1 \prec s_2$ and s_3 any immediate successor of s_2. Then we can formalise the argument as follows:

$$\begin{array}{lrr}
(M, s_1) \models K_{\mathrm{G}} F \triangle_i \phi & \text{(premise)} & (18) \\
(M, s_1) \models F \triangle_i \phi & \text{(from 18 and God's omniscience)} & (19) \\
(M, s_1) \models F \phi & \text{(from 19 and definition of } \triangle_i\text{)} & (20) \\
(M, s_2) \models P F \phi & \text{(from 20 and definition of } P\text{)} & (21) \\
(M, s_2) \models \Box F \phi & \text{(from 21 and necessity of the past)} & (22) \\
(M, s_2) \models \neg F \neg \phi & \text{(from 22 and definition of } \Box\text{)} & (23) \\
(M, s_2) \models \neg \Diamond_i \neg \phi & \text{(from 23 and definition of } \Diamond_i\text{)} & (24) \\
(M, s_3) \models \neg \triangle_i \phi & \text{(from 24 and definition of } \triangle_i\text{).} & (25)
\end{array}$$

The crucial step in the argument is to infer $\Box F\phi$ from $PF\phi$. This inference is based on the principle of the necessity of the past. Recall from §3 that this is premise (D1) from the Master Argument. In §3.2.1 we discussed two formulations of this principle; the form used here ($P\phi \rightarrow \Box\phi$) is applicable only to temporally definite statements. We will look at this form first and then consider the other form ($P\phi \rightarrow \Box P\phi$).

Let ϕ be a temporally definite statement e.g. 'agent i does e at s_2', so we could write it instead as $M \models \phi(s_2)$. Then (25) says that 'at s_3, i has not freely chosen to do e at s_2'. Since s_3 can be any successor of s_2, it follows that there is no state in which agent i has freely chosen to do e at s_2. So the fatalism argument holds if we accept $P\phi \rightarrow \Box\phi$, and in fact it is not possible to deny this for temporally definite propositions, since the truth or falsity of the untensed statement 'agent i does e at s_2' is a property of the whole model. So if it is true anywhere, then it is necessarily true. It follows that (23) is valid for temporally definite propositions, even though it does not follow directly from (22), but rather from the principle "*unumquodque, quando est, oportet esse*" ($\phi \rightarrow \Box\phi$) which we discussed in §3.2.1. So the argument contains the mistaken assumption $\Box F\phi \equiv \neg F \neg\phi$, in other words it is confusing $\Box F$ with G, but this mistaken assumption has no bearing on the outcome, and the conclusion of the fatalism argument seems to hold for temporally definite statements. This is not surprising since the principle "*unumquodque, quando est, oportet esse*" is in itself explicitly deterministic.

Now we consider $P\phi \to \Box P\phi$, the second form of the necessity of the past, which is applicable to temporally indefinite statements. To do this we must reformulate the argument, so consider ϕ to be a temporally indefinite statement e.g. 'agent i does e'. We pick up the argument after (21) to give us

$$(M, s_2) \models \Box PF\phi \qquad \text{(from 21 and necessity of the past)} \qquad (26)$$
$$(M, s_2) \models \neg PF\neg\phi \qquad \text{(from 26)} \qquad (27)$$
$$(M, s_2) \models \neg P\Diamond_i\neg\phi \qquad \text{(from 27 and definition of } \Diamond_i\text{)} \qquad (28)$$
$$(M, s_3) \models \neg P\triangle_i\phi \qquad \text{(from 28 and definition of } \triangle_i\text{).} \qquad (29)$$

This argument fails on a number of levels. Firstly, (29) tells us that 'before s_3, agent i has not freely chosen to do e.' But this is not a contradiction, since at s_3 or later we can still have that i has freely chosen to do e. Secondly, the argument contains a mistaken assumption similar to that used in the previous version of the argument. Where previously $\Box F$ was confused with G, in this version of the argument $\Box PF$ is confused with $\neg PF\neg$ in the move from (26) to (27). And finally, the premise $P\phi \to \Box P\phi$ is not valid in our system, as shown by the counterexample in Figure 9. To see this, consider h_4. At s_7, $P\neg p$ is true, but at s_3, $P\neg p$ is not true, and $h_4 \in H(s_7)$, so $\Box P\neg p$ is not true at s_7, and so the principle of the necessity of the past does not hold. So the fatalism argument does not hold for temporally indefinite statements, and so does not hold in general for our system.

4.5 Foreknowledge

Now we look at one proposed way of refuting the argument in its entirety. Kretzmann, in accepting the eternal God as an explanation for the apparent incompatibility between immutability and knowledge of contingent propositions, argues that this rules out any possibility of foreknowledge [Kre83, p.643]:

> When the beginningless and immutability of God's knowledge are understood as essential aspects of his atemporal mode of existence rather than as special features of his omniscience, they do not entail foreknowledge, they rule it out. It is impossible that any event occur later than an eternal being's atemporally

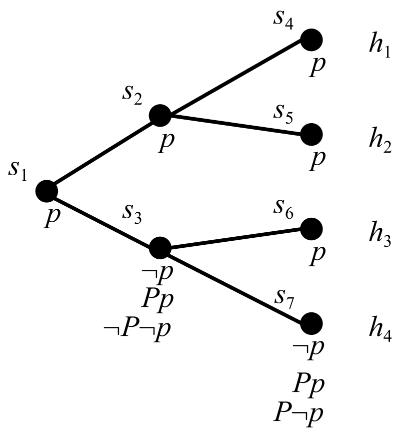

Figure 9: A counterexample to $P\phi \to \Box P\phi$.

> present state of awareness, since every temporal event is atemporally simultaneous with that state; and so an eternal being *cannot* foreknow anything.

This is also the solution of Boethius and Aquinas.[27] If true, then this renders the fatalism argument invalid by rejecting premise (18). To examine this claim, we first give a definition of foreknowledge.

Definition 4.10. We say that agent i has *foreknowledge of ϕ at s* iff

$$(M, s) \models K_i F\phi$$

Since the K_i operator occurs outside the scope of the F operator, there is no incompatibility between foreknowledge and atemporal knowledge in our system. This may seem strange but it follows from the definition of omniscience, $K_{\mathrm{G}}\phi \leftrightarrow \phi$. To evaluate whether God knows something, we need only evaluate whether it is true. The atemporal nature of God's knowledge does not prevent us from using the temporal order $\prec$ to evaluate $F\phi$ provided ϕ does not itself include K_{G} as a modality. God does not need to perform this evaluation step—his essential omniscience ensures that he knows it simply because it is true—and so it is not in conflict with his eternality. Aquinas himself offers support for this position by asserting:

> *In scientia enim nostra duplex est discursus. Unus secundum successionem tantum: sicut cum, postquam intelligimus aliquid in actu, convertimus nos ad intelligendum aliud. Alius discursus*

[27] See Zagzebski's discussion of 'The Boethian Solution' [Zag91, Ch.2, pp.36–42].

est secundum causalitatem: sicut cum per principia pervenimus in cognitionem conclusionum. Primus autem discursus Deo convenire non potest. Multa enim, quae successive intelligimus si unumquodque eorum in seipso consideretur, omnia simul intelligimus si in aliquo uno ea intelligamus: puta si partes intelligamus in toto, vel si diversas res videamus in speculo. Deus autem omnia videt in uno, quod est ipse, ut habitum est. Unde simul, et non successive omnia videt. Similiter etiam et secundus discursus Deo competere non potest. Primo quidem, quia secundus discursus praesupponit primum: procedentes enim a principiis ad conclusiones, non simul utrumque considerant. Deinde, quia discursus talis est procedentis de noto ad ignotum. Unde manifestum est quod, quando cognoscitur primum, adhuc ignoratur secundum. Et sic secundum non cognoscitur in primo, sed ex primo. Terminus vero discursus est, quando secundum videtur in primo, resolutis effectibus in causas: et tunc cessat discursus. Unde, cum Deus effectus suos in seipso videat sicut in causa, eius cognitio non est discursiva.[28]

So in our system we have no reason to reject premise (18), and we must therefore identify other grounds on which to reject the argument.

4.6 Necessity of the Past

William of Ockham (c1287–1347) rejected the fatalism argument by attacking its use of the principle of the necessity of the past. So far we have examined

[28]Summa Theologica, I, q.14, a.7: 'In our knowledge there is a twofold discursion: one is according to succession only, as when we have actually understood anything, we turn ourselves to understand something else; while the other mode of discursion is according to causality, as when through principles we arrive at the knowledge of conclusions. The first kind of discursion cannot belong to God. For many things, which we understand in succession if each is considered in itself, we understand simultaneously if we see them in some one thing; if, for instance, we understand the parts in the whole, or see different things in a mirror. Now God sees all things in one (thing), which is Himself. Therefore God sees all things together, and not successively. Likewise the second mode of discursion cannot be applied to God. First, because this second mode of discursion presupposes the first mode; for whosoever proceeds from principles to conclusions does not consider both at once; secondly, because to discourse thus is to proceed from the known to the unknown. Hence it is manifest that when the first is known, the second is still unknown; and thus the second is known not in the first, but from the first. Now the term discursive reasoning is attained when the second is seen in the first, by resolving the effects into their causes; and then the discursion ceases. Hence as God sees His effects in Himself as their cause, His knowledge is not discursive.' [EDP20].

two formalisations of the necessity of the past using the definition of necessity given in §2.3.6. We have shown that only when applied to temporally definite propositions is this a valid principle in our system, and then only as a trivial consequence of "*unumquodque, quando est, oportet esse*". However in §3 we also mentioned a third formalisation ($Pp \rightarrow GPp$) using the Diodorean notion of necessity, which we took to be a formalisation of *necessitas per accidens*. $Pp \rightarrow GPp$ holds on Figure 9, and it is with this notion of necessity in mind that we consider Ockham's argument.

Recall that the fatalism argument uses the necessity of the past to infer $\Box F\phi$ from $PF\phi$. Ockham would argue that this is an invalid use of the principle of the necessity of the past. In $PF\phi$, the past tense operator is applied not to a statement about the present, but to a statement about the future, $F\phi$. Thus the resulting statement $PF\phi$ is past tense in form only, and is not really about the past in terms of subject matter. Ockham explains his viewpoint in Assumption 3 of Question I of the *Tractatus de praedestinatione et de praescentia dei et de futuris contingentibus* [Boe45, pp.12–13]:

> *Quod aliquae sunt propositiones de praesenti secundum vocem et secundum rem, et in talibus est universaliter verum, quod omnis propositio de praesenti vera habet aliquam de praeterito necessariam, sicut tales: Sortes sedet, Sortes ambulat, Sortes est iustus, et huiusmodi. Aliquae sunt propositiones de praesenti tantum secundum vocem et sunt aequivalenter de futuro, quia earum veritas dependet ex veritate propositionum de futuro; et in talibus non est ista regula vera, quod omnis propositio vera de praesenti habet aliquam de praeterito necessariam. Et hoc non est mirabile, quia sunt propositiones verae de praeterito et de futuro, quae nullam habet veram de praesenti, sicut istae: Album fuit nigrum, Album erit nigrum, quae sunt verae, et sua de praesenti et falsa: scilicet ista: Album est nigrum.*[29]

[29] "Some propositions are about the present as regards both their wording and their subject matter. Where such propositions are concerned, it is universally true that every true proposition about the present has corresponding to it a necessary one about the past—e.g.,'Socrates is seated,' 'Socrates is walking,' 'Socrates is just,' and the like. Other propositions are about the present as regards their wording only and are equivalently about the future, since their truth depends on the truth of propositions about the future. Where such propositions are concerned, the rule that every true proposition about the present has corresponding to it a necessary one about the past is not true. And this is not remarkable, since there are true propositions about the past and about the future that have no true proposition about the present corresponding to them. For example, 'what is white was black' and 'what is white will be black' are true while their corresponding proposition about the present—'what is white is black'—is false." Translated by Adams & Kretzmann [AK83, pp.46–47].

In the fatalism argument, we said that ϕ represented the statement 'agent i does e at s_2', so we can write $M \models \phi(s_2)$. So, as we saw in §2.3.3, $(M, s_2) \models PF\phi$ is actually a present tense statement when reduced to its simple tense. So (21) becomes

$$(M, s_2) \models \phi \qquad \text{(converting 21 to simple tense)} \tag{30}$$

and the principle of the necessity of the past does not apply.

5 Goodness and Omnipotence

In this final section we use our formalisation to define the concepts of omnipotence and benevolence, and then use these definitions to examine whether the existence of an omniscient, omnipotent, omnibenevolent agent is consistent with the existence of free will.

5.1 Omnipotence

We can propose formalisations for omnipotence along the same lines as our formalisations of omniscience. Parallel to infallible knowledge we can define the concept of infallible agency.

Definition 5.1. An agent i has *infallible agency* in a model M iff for all $s \in S$, $(M, s) \models \triangle_i \phi \rightarrow \phi$.

This follows directly from our definition of $\triangle_i$. Parallel to essential omniscience we can define the concept of essential agency.

Definition 5.2. An agent i has *essential agency* in a model M iff for all $s \in S$, $(M, s) \models \triangle_i \phi \leftrightarrow \phi$.

This notion is analogous to the notion of essential omniscience, but since it only applies to actions that have already been performed, it is not a definition of essential *potency*. Aquinas acknowledges that there are different ways of interpreting 'omnipotence', and himself proposes a definition based on possibility.

> *Communiter confitentur omnes Deum esse omnipotentem. Sed rationem omnipotentiae assignare videtur difficile. Dubium enim potest esse quid compehendatur sub ista distributione, cum dicitur omnia posse Deum. Sed si quis recte consideret, cum potentia dicatur ad possibilia, cum Deus omnia posse dicitur, nihil rectius intelligitur quam quod possit omnia possibilia, et ob hoc omnipotens dicatur.*[30]

We can capture this notion in our formal language with the following definition of essential omnipotence.

[30] Summa Theologica, I, q.25, a.3: 'All confess that God is omnipotent; but it seems difficult to explain in what His omnipotence precisely consists: for there may be doubt as to the precise meaning of the word 'all' when we say that God can do all things. If, however, we consider the matter aright, since power is said in reference to possible things, this phrase, "God can do all things," is rightly understood to mean that God can do all things that are possible; and for this reason He is said to be omnipotent.' [EDP20].

Definition 5.3. An agent i is *essentially omnipotent* in a model M iff for all $s \in S$, $(M, s) \models \Diamond\Diamond_i\phi \leftrightarrow \Diamond\phi$.

This definition uses the modality $\Diamond_i$ which was introduced in §2.5 and which means 'agent i can choose such that'. So we are saying that for an essentially omnipotent agent, something is possible if and only if it is possible that the agent can choose to make it so.

With this in mind, we propose the following definition of omnipotence for our system.

Definition 5.4. An agent i is *omnipotent* in a model M iff $P_i = E$.

Note that this definition seems to imply not only that an omnipotent agent *can* perform every action, but also that he *does* perform every action, i.e. that omnipotence is equivalent to essential agency. This is counterintuitive in two ways. Firstly, it means that there are no unperformed actions; and secondly, every performed action is performed by the omnipotent agent. The first is essentially the principle of no unactualised possibilities, which we discussed in connection with the Master Argument. This does not lead to any difficulties, since just as our notion of a possible proposition is truth in some possible world, so our notion of a possible action is performance in some possible world.

Slightly more problematic is the concept of the omnipotent agent performing every action. This goes to the heart of what we mean by *agency* and it is difficult to get a grasp of this concept using a single natural language term. But it too can be explained by means of the principle of no unactualised possibilities. For in order to say that it is *possible* for agent i to perform an action e, then there must be some possible world in which he *does* perform action e. So it follows that for every $e \in E$, if it is possible for agent i to perform e, then $e \in P_i$, so for an omnipotent agent i it must be true that $P_i = E$.

This still leaves us with the problem of joint agency. In this context it helps to think of agency as responsibility. Take the example of agent i sitting down, which we previously represented as $e_i \in P_i$. Then $e_i \in E$. Then suppose there is an agent j who is omnipotent. Then $e_i \in P_j$. So when agent i sits down, both agents i and j are responsible. This initially seems counterintuitive, since we started by specifically attributing agency to agent i, but when we consider that agent j could have prevented the action e_i, it seems reasonable that agent j should also be held responsible. This leads to the conclusion that in a system containing an omnipotent agent, that agent is responsible for *every* action that takes place, even though that responsibility may be shared with another agent.

5.2 Benevolence and the Problem of Evil

The problem of evil runs (informally) like this: God is omnipotent, omniscient and omnibenevolent. If God is omnipotent he can prevent evil. If God is omnibenevolent he wants to prevent evil. If God is omniscient he knows when evil is occurring. But there is evil in the world. So God cannot be omnipotent, omniscient and benevolent.

If we are to analyse this argument further we need a formal definition of what evil is. In our formalisation we have attributed a valuation, the u-value, to actions as a measure of 'goodness'. Now we use that to define what we mean by an evil action.

Definition 5.5. We call an action e an *evil action* iff $u(e) < 0$. Similarly we call e a *neutral action* iff $u(e) = 0$ and a *good action* if $u(e) > 0$.

Mackie argues in his 1955 paper 'Evil and Omnipotence' [Mac55] that belief in the existence of an omnipotent, wholly good God and the existence of evil is logically inconsistent—he does not consider omniscience essential to the argument. Our formalisation supports this argument, if we assume that a wholly good God would perform no evil actions.

Proposition 5.6. If evil actions exist, and God is an omnipotent agent, then God performs evil actions.

Proof. If evil actions exist, then there is some $e \in E$ such that $u(e) < 0$. But if God is omnipotent, then $P_{\mathrm{G}} = E$, and so there is some $e \in P_{\mathrm{G}}$ such that $u(e) < 0$. So God performs the evil action e. $\square$

Intuitively we think that if God is wholly good, he should prevent evil actions. However it is possible that allowing an evil action can produce an overall increase in the value of the history. Plantinga [Pla75] holds that omnipotence, omnibenevolence and evil are consistent because God may have a good reason for allowing evil. Our analysis suggests that the 'good reason' for allowing evil may be that any negative effect from an evil action can be offset by the positive effect from good actions, thus raising the goodness valuation of the state overall; or evil actions might make additional future good actions possible, and so increase the goodness valuation of the history. So the problematic premise in the argument is 'If God is omnibenevolent he wants to prevent evil', which should be replaced by 'If God is omnibenevolent he wants to maximise goodness.'

Gottfried Wilhelm Leibniz (1646–1716) maintains that the actual world is the best possible world, even though it may not seem that way to us:

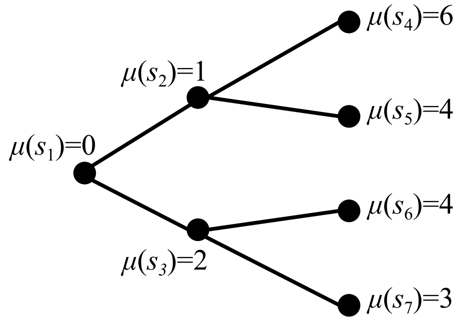

Figure 10: Choosing the best possible world.

> It is true that one may imagine possible worlds without sin and without unhappiness, and one could make some like Utopian or Sevarambian romances: but these same worlds again would be very inferior to ours in goodness. I cannot show you this in detail. For can I know and can I present infinities to you and compare them together? But you must judge with me *ab effectu*, since God has chosen this world as it is. We know, moreover, that often an evil brings forth a good whereto one would not have attained without that evil.[31]

With this in mind we propose the following definition of benevolence, which is based not on the goodness valuation of actions, but on that of states:

Definition 5.7. An agent i is *essentially benevolent* in a model M iff for all $s \in S$, $(M, s) \models \Diamond_i \phi \leftrightarrow O_i \phi$. It follows directly that $C(s,i) = B(s,i)$.

This says that an agent can make a choice if and only if it is a choice that he ought to make, i.e. a best choice. If the agent is also omnipotent, then this means that the successor state is the best possible successor state, in terms of maximised value. In Figure 10, an omnipotent, benevolent agent starting at s_1 would choose s_3 then s_6. But $\mu(s_4) > \mu(s_6)$ so with only two iterations we have already selected a state which is not the best state reachable from s_1. So the existence of an omnipotent, benevolent agent as we have defined it is no guarantee that the 'actual' state is the best state that could have been reached, only that it is the best state that was reachable from the immediate predecessor.

This restriction of an agent's power seems to be at odds with the notions of omnipotence and free choice, and we can show that the assumption of the

[31] Theodicy I, Ch.10: Translation from [Far51].

existence of an essentially benevolent agent does have consequences for the model in terms of free choices.

Lemma 5.8. For an essentially benevolent agent, free choices are equivalent to conflicting obligations.

Proof. If G is essentially benevolent then $C(s, \mathrm{G}) = B(s, \mathrm{G})$. Suppose ϕ is a free choice, then $\neg\phi$ is also a free choice. Then there is some $s \in S$ such that $(M, s) \models \Diamond_{\mathrm{G}}\phi \wedge \Diamond_{\mathrm{G}}\neg\phi$. So there is some $s' \in C(s, \mathrm{G})$ such that $(M, s') \models \phi$ and some $s'' \in C(s, \mathrm{G})$ such that $(M, s'') \models \neg\phi$. But $C(s, \mathrm{G}) = B(s, \mathrm{G})$ so $s' \in B(s, \mathrm{G})$ and $s'' \in B(s, \mathrm{G})$, hence $(M, s) \models O_{\mathrm{G}}\phi \wedge O_{\mathrm{G}}\neg\phi$, and ϕ and $\neg\phi$ are conflicting obligations. Similarly in the opposite direction. $\square$

Theorem 5.9. If a model $M = \langle T, V, \mu \rangle$ has no moral dilemmas, and it contains an essentially omnipotent and essentially benevolent agent G, then M contains no free choices.

Proof. If G is omnipotent then $P_{\mathrm{G}} = E$, and so for any $s \in S$, $C(s, \mathrm{G}) = \{s' \mid s' \text{ an immediate successor of } s\}$. If G is also essentially benevolent then $C(s, \mathrm{G}) = B(s, \mathrm{G})$ and so $B(s, \mathrm{G}) = \{s' \mid s' \text{ an immediate successor of } s\}$. If M has no moral dilemmas then $B(s, \mathrm{G})$ is a singleton set, and hence so is $\{s' \mid s' \text{ an immediate successor of } s\}$. So every state $s \in S$ has a unique successor. So for any agent i and state s, $(M, s) \models \Diamond_i\phi \rightarrow \neg\Diamond_i\neg\phi$, and so M contains no free choices. $\square$

So the existence of an essentially omnipotent and essentially benevolent God means that the world either contains moral dilemmas or does not support free will.

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