Quine's challenge and Logical Pluralism

MSc Thesis (*Afstudeerscriptie*)

written by

Antonio Negro
(born December 24th, 1978 in Poggiardo (LE), Italy)

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Date of the public defense:	**Members of the Thesis Committee:**
July 16th, 2010	Drs. Theodora Achourioti
	Prof. Dr. Dick De Jongh
	Dr. Catarina Dutilh Novaes
	Prof. Dr. Martin Stokhof

Institute for Logic, Language and Computation

Abstract

A number of deviant deductive systems have been proposed which differ in some way from classical logic. An influential philosophical interpretation of plurality in logic has been developed by Quine, who introduced the *meaning-change thesis*, summarized in his "Change of logic, change of subject".

We show that if Quine is right about the consequences of his thesis, then the systems of classical logic can only be *rejected* as unscientific or meaningless, and not *refined* as inadequate. We show that there are good reasons both for and against his thesis.

We argue against Local Pluralism, on the basis that it is incompatible with the universality and normativity of logic. We then assess Beall and Restall's logical pluralism, which is shown to be not sufficiently fine-grained in that it leads to relativism in logic.

We finally introduce Dalla Pozza's Global Pluralism, where deviant logics are viewed as dealing with specific pragmatic meta-concepts, which are distinct from the semantic concept of truth as captured by classical logic, while presupposing it. We show that Dalla Pozza's analysis corroborates the meaning-change thesis without leading to the monistic outcome of Quine.

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Chapter 1

Introduction

But the LORD came down to see the city and the tower that the men were building. The LORD said: «If as one people speaking the same language they have begun to do this, then nothing they plan to do will be impossible for them. Come, let us go down and confuse their language so they will not understand each other»

GENESIS 11: 5-7

The history of sciences surely shows that the quest for knowledge needs not only relevant issues to be raised and important developments of theories to be carried out - but also revolutionary approaches. This has also been the case for logic.

In spite of the clear historical predominance of syllogistic as the quintessential logical system, one should recall that already Stoic "propositional logic", Boethius' notion of *hypothetical syllogism* and the Medieval theories of *consequentia* added more and more details to Aristotle's sophisticated deductive system - viewed by Kant as the final stage of the development of logic. Besides, a number of intricate debates over the epistemological status of the discipline have been taking place for centuries. Frege's and Russell & Whitehead's works succeeded in laying down the foundations for a possible agreement over the subject, granting indeed to what we now call *classical logic* a long-lasting special role among sciences and philosophy.

However, starting with C. I. Lewis, Lukasiewicz and Post, a number of non-standard deductive systems have been proposed, which differ in some way from classical logic. In fact, the current scenario may be described as a "jungle of logics" [7], which represents a challenge to the traditional view of the discipline as the most general and universal instrument for rationality.

Even before addressing the question as to how to make sense of so many different logics, one is curious about the reasons that lead to their development. These include both philosophical criticisms and the «mathematical appeal of the prospect of extensions and modification of classical logic» ([32], p. xii). For example, doubts over the ability of the material conditional to adequately represent the indicative conditional led eventually to the so-called "strict conditional" (and then to modal systems), many-valued logics questioned the Principle of Bivalence and intuitionistic logic has its origins in a refusal of the logicist view of mathematics. Formal innovations, then, inspired others. Epistemic and deontic logics developed from modal calculi, and relevance logic from doubts over the ability of strict conditionals to represent informal entailments. It can happen also that a system of logic is supported for *new* reasons, different from the inspiring ones, as with Dummett's defence of intuitionistic logic, which is based indeed on general philosophical considerations pertaining to a theory of meaning.

The three taxonomies at the end of this introduction are examples of possible classifications of the best-known logical systems. In a sense, the small differences between these classifications suggest that an agreement over the way of making sense of this plurality is still far from being reached. This is nothing but what one should expect. Indeed, an extensively shared view of the situation can be obtained only by addressing some of the most difficult issues about the nature itself of logic, and especially about whatever it is that a system of logic is supposed to capture, if anything at all. As Susan Haack observes,

> «the vital philosophical issues in logic are focussed by consideration of the plurality of logical systems and of the ways in which formal calculi bear on the assessment of informal arguments» ([32], p. xiii).

Thus, on the one hand, the philosophical relevance of the problem must absolutely not be underestimated. On the other, we have little ground for optimism in looking for a complete resolution. In fact, there are too many divergences on essential points and there is a tight but still obscure link between formal aspects and philosophical criticisms. So much, that one would be tempted to give up the project and accept a

status of general impasse.

However, this is actually what happens with the most important philosophical problems, which often can be given only a history - not a solution. Thus, it seems opportune to try to investigate the possibility of a *different approach* to the problem. We want to follow this line in our work, which thus should be viewed as a critical overview of the relevant conceptions on plurality in logic and as an introduction to a possibly *preferable* point of view on it. Not really a complete resolution maybe, rather an interesting way to reach an acceptable agreement over most of the crucial problems.

An important and influential interpretation of plurality in logic has been developed by Willard Van Orman Quine, a philosopher who condensed some of his major ideas using mottos. His *Change of logic, change of subject* has become a mantra for those thinkers who maintain that no *real* conflict is possible between different logics. However, the thesis endorsed by Quine - the *meaning-change* thesis - is not easy to understand. In this work we will give an account of the thesis in relation both to the problem of compatibility between different logics and to *Logical Pluralism*, a philosophical position that accepts the existence of different logical systems as *all correct*, in some sense. The procedure adopted will be thus as follows.

In **Chapter 2**, we introduce what one can call "Quine's challenge".

We first describe Quine's starting problem (§2.1), namely the possible conflict between logics *and* the possible revision of orthodox - classical - logic in favour of some other one.

Then, in §2.2.1 we maintain that in Quine's perspective two general kinds of conflicts between logics are possible (*genuine* and *partial*), which we hold are viewed by Quine as leading to two different kinds of revision of a system of logic: a *refinement* and a *rejection*, respectively. In §2.2.2, we reconstruct Quine's original rationale against the possibility of a *genuine* conflict between logics (hence, of a *refinement* of a logic) and against the specific *rejection* of classical logic (in the form suggested by some deviant logicians).

Section §2.3 is dedicated to some crucial clarifications. In §2.3.1 we explain how our distinction between two kinds of conflicts saves Quine from the charge of inconsistency on the topic of logical revision. In §2.3.2 we stress that the meaning-change thesis is independent from a specific conception of logical systems (viz., qua *calculi* or qua *models of correct deductive reasoning*).

In §2.4 we analyse how relevant is the meaning-change thesis to the problem of compatibility between different logics within the *model*-conception of logic. To this aim, we describe the relations between the two kinds of conflicts and some traditional responses to the question whether there is a uniquely correct logical system (Monism, Local Pluralism and Global Pluralism). We conclude, *pace* Quine, that even if a refinement of a logic is in fact impossible, the meaning-change thesis does not lead necessarily to the *rejection* of a system of logic, the thesis being coherently endorsed also within the *compatibility*-scenario offered by Global Pluralism.

Chapter 3 is dedicated to the assessment of the strength of Quine's analysis.

In §3.1 we show that Quine's defence of classical logic as the unique acceptable logical system cannot be considered conclusive. However, we hold that his analysis is not dependent from such defence, so that the weakness of the latter does not cast doubts over the plausibility of the former.

In §3.2 we analyse the most important responses to Quine's challenge with respect both to his theory-dependence thesis for the meaning of logical operators (§3.2.1) and to his thesis that a genuine conflict between logics is impossible (§3.2.2). We show that none of them can be said either conclusive or preferable. Hence, we conclude that Quine's challenge should be considered as a matter of fact *not yet resolved*.

In §3.3 we offer a summary of the partial results of our study and argue that some form of logical pluralism is a desirable way to deal with the issues concerning plurality in logic.

In **Chapter 4** we introduce two attempts to defend a form of pluralism in logic.

In §4.1 we describe Beall and Restall's Logical Pluralism, which we consider not sufficiently fine-grained and unacceptable for the effects it induces over the notion of "truth".

In §4.2 Dalla Pozza's Global Pluralism is introduced, which is developed endorsing (and corroborating) Quine's thesis in a compatibility-scenario. We argue that it constitutes the most promising analysis proposed so far that is able to reach a *general* way to *understand* and *resolve* conflicts between different logics.

In **Chapter 5** we give an account of the conclusions of our work.

- Classification 1 (Haack [32]):

-*traditional* logic: Aristotelian syllogistic
-*classical* logic: 2-valued sentence calculus; predicate calculus
-*extended* logics: modal, tense, deontic, epistemic, preference, erotetic logics
-*deviant* logics: many-valued, intuitionistic, quantum, free logics
-*inductive* logics

- Classification 2 (Dalla Pozza [21]):

-*standard* logic: classical logic
-*non-standard* logics: **SUPPLEMENTARY** logics:
extended: modal, epistemic, deontic, tense logics
integrative: pragmatic logic
DEVIANT logics:
many-valued: 3-valued, fuzzy logics
non-classical 2-valued: intuitionistic, quantum, paraconsistent logics
SUBSTRUCTURAL logics: non-monotonic, relevance, linear, basic logics

- Classification 3 (Palladino [41]):

-*classical* logic
-*extended* logics: Kripke's, minimal modal, alethic, deontic, epistemic, tense logics
-*alternative* logics: many-valued, relevance, conditional, intuitionistic, paraconsistent logics
-*Artificial Intelligence* logics: non-monotonic, fuzzy, linear logics

Chapter 2

Quine's challenge

2.1 The criticisms against classical logic

The three classifications in the introduction agree at least on one point: alethic, deontic, tense and epistemic logics are considered *extensions* of classical logic (CL), while the latter is considered the standard - official - logic. There is indeed quite a wide consensus that the relation between (classical) modal logics and CL is not problematic. Even if - as we noticed - modal systems developed from doubts over the characterization of the classical material conditional, there has never been a deep dispute between these two systems. The question as to why this happened arises, and the tension between formal aspects of a logic and its philosophical aspirations starts to show its importance. First, from a formal point of view, modal systems are obtained from CL by adding to its logical vocabulary some modal operators, whose properties are then defined in terms of the classical theory and of the new notion of *frame*. In this way, the class of theorems/valid inferences of a modal logic properly includes the class of theorems/valid inferences of CL, and this holds as well for the well-formed formulas (wffs). Secondly, from a philosophical point of view, modal logics were meant to capture those *qualifications* (or modalities) of the truth of a sentence that could not be expressed in CL, but that anyway *presupposed* CL. Apparently, the compatibility between these systems is then easily defended. They do not try to model different notions of validity or incompatible aspects of the same notion of validity; rather, they model different - and yet compatible - aspects of one and the same notion of validity. This may be not a completely acceptable philosophical explanation, but we want now to introduce the problem of the possible *revision* of CL and hence we need to focus on those systems that have been widely

proposed in *conflict* with it, viz., as *rivals*. They are known as *deviant* (or alternative) logics and constitute a *non-conservative critical logical response* to orthodox logic.

According to Haack [31], a deviant logic can in first instance be described as a system such that the class of its wffs coincides with the class of the wffs of CL, but such that the class of its theorems/valid inferences *differ* from the one for CL. Usually, the systems proposed as rivals of CL are deviant in the sense that they apparently do not agree with *all* the classical principles, so that typically they *lack* certain theorems that CL has.[1]

Observation: As Haack herself [31] notices, this idea of "deviance" leads to some difficulties in isolating the class of rival systems. First, deviance seems not to be a clear *necessary* condition for conflict. For example, Van Frassen's presuppositional language [26], which one would say to be in conflict with CL (it allowing truth-value gaps), is not a deviant system in that sense (its theorems being all and only the classical tautologies). Also, deviance seems not to be a *sufficient* condition for conflict. If one takes a specific formulation of CL - which differs from the usual one for employing a different but inter-translatable notation (e.g., $\oplus$ for $\wedge$) - one is intuitively forced to consider such formulation only a *notational variant* of CL, certainly not a system in conflict with it. Indeed, this formulation would certainly lack some theorems of the usual formulation, but only because the notation would be different.

With reference to the last part of our observation, the question arises as to whether deviant logics could be analyzed *as if* they used different - maybe inter-translatable - notations. In this possibility, the mere lack of a theorem on a *typographical* level would certainly not lead to any conflict. This consideration constitutes an important indication that a fruitful analysis of our problem should be mostly based on the *philosophical aspirations* of different logics, rather than on their linguistic formulation. In a sense, thus, one should look *behind* the notation and see what it stands for. In the next section, it will be clear how this is relevant for Quine's rationale. For the moment, let us sketch in more detail the reasons why the most known deviant systems have been proposed in fact as conflictive with CL.

[1] More rarely, the opposite holds: they can allow for some theorems that CL lacks. Also, in principle, the disagreement can be such that the deviant system allows for the contradictory of a classical theorem. For the purposes of this work, we will not take in consideration such cases.

- Intuitionistic logic rejected the Law of Excluded Middle (LEM) on the basis that it entails the resolution of each and every mathematical problem, whereas this was considered in disagreement with the view according to which mathematical results are products of a construction-based activity [12];

Later, intuitionistic logic was defended by Dummett [22, 23, 24], on the basis that it is the only logic in agreement with what he considered the necessary constraints for any theory of meaning, namely the so-called "principle of manifestability";

- Quantum logic, as defended by Putnam [54], rejected the Law of Distributivity on the basis of its being empirically refuted by the experiments;

- Three-valued logic rejected the Principle of Bivalence and was proposed by Lukasiewicz [36] on the grounds that CL was unable to govern future-tense sentences without leading to fatalism;

- Relevance logic [63] was defended by arguing against the classical paradoxes of implication and on the basis that any deduction should be such that the premises are *effectively* and *materially* involved in the inference toward the conclusion (whereas classical implication only depends on the truth-value of the premise and of the conclusion);

- Paraconsistent logics [50] rejected the general validity of the Principle of Non-contradiction, on the basis that some theories can be inconsistent (thus allowing a contradiction) but not trivial (thus preventing to derive every sentence).[2] For example, Bohr's model of the atom was inconsistent with Maxwell's equations, but no trivialization was conceded on the whole;

- Fuzzy logic [77] rejected the Principle of Bivalence on the basis that it does not allow for the treatment of sentences containing vague expressions.

The peculiar character of deviant systems is a bit clearer now. The difference between them and the modal systems with respect to CL is the same between talking about old things in a new way and talking about new things - so to say. Indeed, deviant logics are not proposed as mere reformulations of CL, neither supposed to be a supplement or a conservative extension of CL. Rather, they are essentially based on different - revolutionary - ideas about fundamental logical principles, developed for whatever reason. They constitute a *conscious* challenge to CL and its presuppositions, which are viewed as deeply *mistaken* on a conceptual level. More specifically, one can be a *realist* reformer of logic, if one maintains that CL can be *falsified*; or a *pragmatist*

[2]Whereas, according to CL, a contradiction allows the inference to any sentence and leads thus to trivialization.

reformer, if one maintains that his own logic must be chosen on grounds of convenience, simplicity or economy within applications (see [31], p. 3).

Thus, deviant systems are expected to be employed *instead* of CL, in all or some applications. As Quine put it:

> «[t]he systems of orthodox logic are themselves many and varied [...] It is *one* logic variously expounded and variously serviced by computers or proof procedures. Demarcate the totality of logical truths, in whatever terms, and you have in those terms specified the logic. Which of these truths one chooses to designate as axioms, and what rules he devises for generating the rest of the logical truths from those axioms, is indifferent [...] The kind of deviation now to be considered is of *a more substantial* kind. It is not just a change of methods of generating the class of logical truth, but a change of the class itself. It is not just a change of demarcation, either, between what to call logical truth and what to call extra-logical truth. It is a question rather of outright *rejection* of part of our logic as not true at all» ([58], pp. 80-81; emphasis added).

2.2 Quine on alternative logics

2.2.1 Two kinds of conflicts

As one would expect, the first attempts to argue for a revision of some classical principle or, in Carnap's words, «[to cast] the ship of logic off from the *terra firma* of the classical forms»[3] were mostly considered bold or unreasonable. After all, some of those principles (like LEM) had been supported for centuries in virtue of their purported evident accordance with intuition and rationality (even if Aristotle himself doubted of the Principle of Bivalence, as Lukasiewicz maintains). Not unexpectedly then, Graham Priest considers «the analogy between non-standard logics and non-Euclidean geometries an important and interesting one» ([51], p. 156). Euclid's fifth postulate was indeed considered correct (better, *true*) for centuries, even if there had always been doubts about its self-evidence.

[3][14], p. xv.

However, there is something about logic that one would say is "special": «If sheer logic is not conclusive, what is? What higher tribunal could abrogate the logic of truth functions or of quantification?» ([58], p. 81). According to Quine, two ways of interpreting the deviant critique to a classical logical principle are possible. The first one consists in considering the principle unable to capture the *real* behaviour of some logical operator - whatever that is. In this sense, for example, if a deviant logician maintains that sometimes a conjunction of the form $p \wedge \neg p$ is true, he is thereby claiming that the *real* behaviour of the logical operator "conjunction" allows for that truth-value. According to the second one, instead, when a deviant logician denies the correctness of a classical principle, he is simply talking about *something else* - though using the same notation. The difference is between talking about the same "thing", while disagreeing on its properties, and talking about something else altogether. It seems to us, moreover, that this corresponds in Quine's view to a difference between what we will call a *refinement* and a *rejection* of CL. The latter distinction is *not* expressed explicitly in Quine's. However, we believe that not only it is in real agreement with his analysis, but also a good way to make sense of his apparent incoherency on the topic of logical revision, as we will see later. Let us offer the first insights into this delicate difference.

If two logics were dealing with the same logical operators, then their disagreement over the validity of a logical principle could be surely interpreted as a dispute over which of them is the *unique* correct rendering of a certain operator. This would thus constitute a real conflict. Assume now that CL is challenged in this sense by a deviant logic. Surely, if the conflict could be resolved to the advantage of the latter, then the deviant logic would in fact constitute a *refinement* of CL. That is, CL could be seen as having been sufficiently *refined* to reach the properties of the deviant system. In an important sense, thus, one would have here an improvement of one and the same system of logic. Besides, this improvement would be viewed anyway as a theoretical development in the understanding of the *real properties* of a certain operator, even if the disagreement was inspired by pragmatic considerations. We will call this first case "*genuine* conflict" (Gc, henceforth).

Let us consider instead the case in which two logics deal with different things, namely different operators. *Whatever* this means, the disagreement over the validity of a logical principle could *not* be viewed as a dispute over which is the unique correct rendering of a certain operator, because the characterizations of the operators of the two logics could *not* be proposed in fact as different versions/developments of one and

the same thing. One would be tempted to say that there would be no dispute at all in this case. However, one should notice that - *whatever* these different operators could refer to - both systems would be still proposed in this scenario qua *logics* and, as such, possibly defended on the grounds that their application in some branch of science would be preferable for some reason. Specifically, on the one hand, *if* one maintained that there must be only one system of logic, then those two systems would be in fact in disagreement as to which among them is *the* system of logic, *even if no one could understand what these systems really deal with*. This kind of disagreement would be then mostly based on *pragmatic* issues. On the other hand, the disagreement could be a purely *theoretical* disagreement, if one could eventually understand *what* the two systems deal with. Indeed, *if* one maintained that anyway there must be only one system of logic, then this scenario would inevitably lead to principle-disputes over *what* a logic has to deal with, viz., again, over which of the two systems is the uniquely correct one. In both cases, thus, we would have here two deeply different systems of logics. Assume now that CL is challenged in this sense by a deviant logic. Surely, if the conflict could be resolved to the advantage of the latter, then the deviant logic would in fact *reject* CL as a whole. More specifically, CL would *not* be *modified*, but rather abrogated as not convenient (in the first, pragmatic, case), unscientific (in the second, theoretical, case) or even not understandable (if the deviant logician could not really conceive what CL deals with). We will call this second case "*partial* conflict" (Pc, henceforth).[4]

There is an important difference between the two conflicts with respect to the so-called Monism, a philosophical position on plurality in logic according to which there is (or there must be) only *one* correct system of logic. We can summarize this difference by noticing that in the Gc-case Monism is a *result*, whereas in the Pc-case it is more a philosophical *presupposition*. In other words, the Gc-scenario cannot be really resolved without naturally leading to Monism, for a resolution of this conflict would quite necessarily individuate *the* unique correct system.[5] A resolution of a Pc-scenario, instead, leads necessarily to such an individuation *only* if Monism is already chosen as the best position on plurality of logics. On the contrary, if one accepted the possibility that a number of logics *can* coexist, then one could in principle *accept* the Pc-scenario as not problematic at all. This could happen, for example, if no system could be definitively

[4]In this work, we will use the expression "revision" as a general term. Thus, both "refinement" and "rejection" must be interpreted as particular kinds of revision.

[5]As we will see, a singular exception to this outcome - local pluralism - is also possible.

defended as the most convenient in applications, or if one could show that those different things the two systems deal with are *compatible* in some sense. On the whole, we should say that a Gc-scenario naturally leads to a refinement of a system, whereas a Pc-scenario leads necessarily to a rejection of a system only in some cases (namely, only if Monism is supported).

As a matter of fact, all the logics we mentioned above have been proposed as alternatives to CL in the sense of the Gc-case. Also, the Pc-case seems so obscure and convoluted that one is tempted to consider it as a really improbable scenario. Strangely enough, not only the outcome of Quine's analysis is that the Gc-case is theoretically absurd (and so that only the Pc-case is possible), but all the attempts to show that a Gc-scenario can in fact hold seem to fail in some way - as we will show.

2.2.2 Quine's rationale

All in all, Quine has argued against both **(G)**: the possibility of a genuine conflict, and so against a subsequent refinement of CL - and **(P)**: the specific rejection of CL in favour of some deviant logic induced in a Pc-scenario.

(G) Quine's rationale against the Gc-case is twofold. The first part is essentially a theoretical one; the second is a pragmatic one, based on his insights concerning the translation that an imaginary linguist makes of a native's logic (in a behaviourist framework). They together support Quine's *meaning-change thesis*: deviant logics emerge from a mere change in the usage - meaning - of logical words. Actually, the two arguments are not tightly separated in Quine's exposition, but rather a unique (merged) argument, which is heuristically connected to the pivotal role he assigns to (classical) logic with respect to our logical beliefs. However, one can adequately analyse them separately, since the second argument can be seen essentially as a way to show the consequences that the first one has in an imaginary pragmatic scenario. In this work, we will then focus mainly on clues about the first argument, also because touching upon «the thorny issues related with indeterminacy of translation» ([42], p. 542) would lead us too far from our aims.

G1 (first argument):

- (a) A Gc-scenario involving a deviant and a classical logician would be possible only if they both wanted to capture the *essence* of some logical expression, but disagreed about the laws that rule the expression;

- (b) then, if a Gc-scenario can be possible, there must be a *residual* essence of a logical expression, independent in some sense from those laws;

- (c) this is absurd: there is nothing like a residual essence of a logical expression «in addition to the [...] notations and the laws in conformity with which a man uses those [...] notations» ([58], p. 81);

- (d) hence, there cannot be a Gc-scenario. The disagreement is not about the same thing: "change of logic, change of subject".[6]

G2 (second argument):

For a Gc-scenario to be possible - Quine continues - the deviant and the classical logician should have in mind the same logical operator (likely, as it emerges in the *use* of language, the so-called "vernacular operator"), while disagreeing about its properties. But - he argues - when faced with the new deviant characterization of the operator, the classical logician cannot change his own laws in that he cannot change the meaning of his operator, which is the only one he knows and which he must *impute* to the reformer's one in order to make sense of what he says. The classical logician is simply unable to understand the deviant reasons, for he can only behave according to the canon "Save the obvious!", that is, "Save the meanings of your own (classical) connectives!". From the classical point of view, thus, the deviant logician is talking about something that should be *translated* into a classical framework, *if* possible:

> «If a native is prepared to assent to some compound sentence but not to a constituent, this is a reason not to construe the construction as conjunction [...] We impute our orthodox logic to him, or impose it on him, by translating his language to suit [...] The canon "Save the obvious" bans any manual of translation that would represent the foreigners as contradicting our logic» ([58], pp. 82-83);

[6] Since below we will talk about *domains*, in the sense of "areas of discourse", which one can possibly refer to using "subjects", let us specify that the reader should in the first instance interpret "subject" in Quine's motto as adequately replaced by "meaning". Thus, "change of subject" should *not* be read as "change of domain/area of discourse".

> «We are then clearly free to say that [the deviant logician] is merely using the familiar particles "and", "all", or whatever, in other than the *familiar* senses, and hence that *no real contrariety is present after all* [...] For, there can be no stronger evidence of a change in usage than the repudiation of what had been obvious, and no stronger evidence of *bad translation* than that it translates earnest affirmations into obvious falsehoods» ([57] pp. 351, 355; emphasis added).[7]

Finally, from the reasoning in (G), Quine concludes that the only possible outcome of the dispute is a Pc-scenario, where the deviant logician challenges CL *not* as inadequate (thus leading to a *refinement*), but as completely *unscientific* or *meaningless* or *not convenient* (if so, to be *rejected*):

> «My view of [the] dialogue is that neither party knows what he is talking about. They think they are talking about negation, ["¬"], "not"; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form $[p \wedge \neg p]$ as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: *when he tries to deny the doctrine he only changes the subject*» ([58], p. 81; emphasis added);
>
> «The [deviant logician] should not be viewed as controverting us as to the true laws of certain fixed logical operations, namely, negation and alternation. He should be viewed rather as opposing our negation and alternation as *unscientific ideas* and propounding *certain other ideas, somewhat analogous*, of his own» ([58], p. 87; emphasis added).

(P) However, Quine is aware that, notwithstanding his own arguments in (G), the deviant logician can still maintain that CL can be challenged - for some reason - in a genuine sense. Then, he tries to assess anyway the deviant logician's reasons, which we know should be evaluated in the sense of the Pc-case, but which nevertheless the deviant logician proposes in the sense of the Gc-case:

[7] The second part of the observation at page 8 describes then a trivial kind of partial conflict. In this optimistic case ([58], p. 81), *all* the principles of one logic would have other meanings than the ones of the other, but nevertheless they would be all translatable in *acceptable* principles of the other. As Quine notices, in this case each logic should be considered just a notational variant of the other. See [31], pp. 22-23.

> «[Whoever] denies the law of excluded middle changes the subject. This is not to say that he is wrong in so doing. In repudiating $p \vee \neg p$ he is indeed giving up classical negation or perhaps [disjunction], or both; and *he may have his reasons* [...] If anyone questions the meaningfulness of classical negation, we are tempted to say in defence that negation of any given closed sentence is explained thus: it is true if and only if the given sentence is not true. This, we may feel, meets the charge of meaninglessness by providing [...] a meaning that assures that any closed sentence or its negation is true. [...] However, our defence here begs the question; *let us give the dissident his due*. In explaining the negation as true if and only if the given sentence is not true, *we use the same classical "not" that the dissident is rejecting*. Let us grant, then, that the deviant can coherently challenge our classical true-false dichotomy. But why should he want to?» ([58], pp. 83, 84-85; emphasis added).

Though his rationale against the possible rejection of classical logic is specific for the classical true-false dichotomy (or for LEM), his arguments are methodologically meant to be a general standard in evaluating such a rejection. Quine is looking here at logic as applied and used in a larger context of scientific practices. The core of the argument is indeed based on the *maxim of minimum mutilation*, stating that the theoretical virtues of CL (simplicity, familiarity, transparency, beauty, efficiency and convenience) can be given up only if the adoption of a deviant logic in some branch of science - in place of CL - would lead both to a resolution of a serious oddness and to an overall simplicity of the new scenario so obtained. If, instead, there is no such resolution, or the resulting scenario is overall too convoluted, then the methodological maxim tells us not to prefer the deviant system, even if CL contains what one could consider some unfortunate handicap (which he calls "fat"). Thanks to his maxim, he can then reject the *purported genuine* reasons of a deviant logician, arguing that none of them is anyway acceptable.

For example - Quine remarks - one rejection of LEM on the ground of the peculiarities of Quantum Theory is based on the fact that Heisenberg's principle of indeterminacy implies that certain magnitudes cannot be jointly ascertained; so that, applying CL, one would allow empty questions in that theory.[8] However - he continues - it is far from clear how one can obtain a *preferable* scenario if one needs to complicate his logic to

[8]Thus one would allow - in Quine's words - «an excess of accepted questions over possible answers» in that theory ([58], p. 87).

avoid those questions:

> «Certainly the scientist admits as significant many sentences that are not linked in a distinctive way to any possible observations. He admits them as to round out the theory and make it simpler, just as the arithmetician admits the irrational numbers so as to round out arithmetic and simplify computation; just, also, as the grammarian admits such sentences as Carnap's "This stone is thinking about Vienna" [...] so as to round out and simplify the grammar. Other things being equal, the less such fat the better; but when one begins to consider complicating logic to cut fat from quantum physics, I can believe that other things are far from equal. The fat must have been admirably serving its purpose of rounding out a smooth theory, and it is rather to be excused than excised» ([58], p. 86).

Quine objects in the same way to the suggestion that one should allow a middle truth value in order to avoid paradoxes, e.g., with respect to the sentence asserting that Russell's paradoxical class is a member of itself:

> «The classical logic of truth functions and quantification is free of paradox, and incidentally it is a paragon of clarity, elegance, and efficiency. The paradoxes emerge only with set theory and semantics. Let us then try to resolve them within set theory and semantics, and not lay a *fairer fields waste*» ([58], p. 85; emphasis added).

Last, even recognizing the fact that intuitionism - qua one school of constructivism - is congenial and admirable, Quine underlines how one can practice «a very considerable degree of constructivism without adopting intuitionistic logic [...] On this approach, constructivist scruples can be reconciled with the convenience and beauty of classical logic» ([58], p. 88).

As should be clear, Quine develops all his rationale from a monistic perspective. From his point of view, thus, the connection between the Pc-case and rejection is as natural as the one between the Gc-case and refinement. We can say that he in fact assumes Monism as the only acceptable philosophical position with respect to plurality of logics. Thus, if one completely agrees with Quine's approach, deviant logics cannot be taken too seriously after all. That is, if one assumes that only one system of logic can exist, then his arguments in (P), though not really definitive, can hardly be surpassed by the other side. Besides, most deviant logicians have stressed that only in a Gc-scenario

can their reasons be adequately represented and safeguarded. However, we will show in the last chapter that a Pc-scenario does not lead *necessarily* to the rejection of a system of logic - *pace* Quine, but possibly also to a special form of pluralism, where the reasons behind the deviant systems *can be viewed as* adequately represented - *pace* the deviant logicians.

2.3 Observations on Quine's position

As will be evident throughout the work, almost all the main issues concerning philosophy of logic can be found in analyzing Quine's challenge, which can be then considered "the quick brown fox" of philosophy of logic - so to say. A firm and detailed reconstruction of his view is however not easy to reach and we believe that most of the reactions to his thesis are based somehow on a misunderstanding. Let us then try to fix some crucial points, before analyzing such reactions in the next chapter.

2.3.1 Quine and the radical Quine

Quine's views on the topic of logical revision do not appear stable throughout his texts. Usually, this passage of Quine in *Two Dogmas of Empiricism* (1960) is quoted to express the ideas of the "radical Quine" - as Shapiro [2] calls it:

> «[No sentence] is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a mean of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?» ([55], p. 43).

This passage is then put in tension with the rationale in "Philosophy of Logic" (1970) [55] that we have sketched above, which is supposed to imply that (classical) logic cannot be reformed.[9] However, we believe that there is no contrast between the "radical Quine" and the other one, simply because we just do not assent to this distinction. That is why we did distinguish terminologically between a refinement and a rejection of CL - even if he does not. And we have our reasons.

As we have seen, after arguing *against* a genuine disagreement over one and the same meaning (subject) of an operator, Quine is ready to grant that the deviant logician who

[9] See, e.g., [31].

denies LEM "may have his reasons". For what? For changing the subject, of course, or "[for completely giving up] classical negation, or perhaps alternation, or both", in his words. Therefore, an attack to CL, though not genuine, is still conceived as possible by him, and so is a revision of logic. Indeed, Quine himself in the same text reiterates his "radical" ideas, emphasizing that

> «[l]ogic is in principle no less open to revision than quantum mechanics or the theory of relativity [...] If revisions are seldom proposed that cut so deep as to touch logic, there is a clear enough reason for that: the principle of minimum mutilation. The maxim suffices to explain the air of necessity that attaches to logical and mathematical truth» ([58], p. 100).

Thus, claiming that a revision based on a Gc-scenario, i.e., a refinement, is not possible (for *that* scenario itself is not possible) is one thing. But claiming that no revision at all is possible is another thing. On the whole, what Quine coherently maintains is that a revision is indeed possible which *rejects* the classical principles as unscientific or meaningless, and that no rejection of CL has been yet proposed which is in agreement with his maxim of minimum mutilation. To sum up, Quine is not conservative about (classical) logic in principle; he is instead conservative about logic in practice (through the application of his maxim of minimum mutilation).

2.3.2 Immanent vs. transcendent operators

According to Haack [32], one can say that a formal system is a logic when an isomorphism is shown to hold between the system and a "structured part of reality". For example, she considers many-valued calculi as logics in the sense that they have interpretations in terms of electrical circuits. However, the traditional view about logic is a bit more specific. A logic is supposed to be a formal system able to capture an extra-systematic notion of *validity*, embodied in the so-called "informal arguments". In her words:

> «the claim of a formal systems to be a logic depends [...] upon its having an interpretation according to which it can be seen as aspiring to embody canons of valid arguments» ([32], p. 3).

For such an interpretation to be possible, one usually needs to offer a formal *explication*[10] of the so-called "logical expressions" of the (natural) language in which those arguments are expressed. As Cook put it:

> «[A] formal language is a mathematical model of a natural language, in roughly the same sense as, say, a Turing machine is a model of calculation, a collection of point masses is a model of a system of physical objects, and the Bohr construction is a model of an atom. In other words, a formal language displays certain features of natural languages, or idealizations [...] while simplifying other features» ([17], p. 500).

In this way, the behaviour and the properties of the logical operators in the system constitute a model of the behaviour and the properties of the logical expressions in the natural language. Thus, one can apparently maintain that on this view a logic embodies a *theory* of logical constants able to account for the vernacular logical expressions, viewed as external data captured by the model.

Now, in trying to better understand Quine's ideas, the question arises as to whether the meaning-change thesis *must* be interpreted within such traditional perspective or not. Let us try to address this issue.

When discussing intuitionistic logic, Quine claims:

> «[T]he names and notations of negation and [disjunction] carry over to a deviant logic [...] only by a rough and somewhat arbitrary analogy. [Negation and disjunction] are *immanent* rather than *transcendent*» ([58], p. 87; emphasis added).

Quine is maintaining here that the logical operators should be viewed as defined for a *particular* language, not for languages in general. This is another way to stress that there is *no residual essence* of a logical operator in addition to the laws and the principles that govern its use - *within* a particular language, we should add. Thus, he considers a logic essentially as a particular type of language, conventionally constructed. In the words of Paoli, for Quine «[t]here is nothing [...] that precedes or transcends formalization, no external data to "get right"» ([42], p. 542). Hence, his meaning-change thesis could be so expanded: "change of logic, change of subject, change of

[10] Or "rational reconstruction", in Carnap's words [13]. Carnap introduced this notion to explain how formal methods can be applied to philosophical problems involving informal concepts of ordinary language.

language". With respect to the two kinds of conflicts, we can say that the genuine one would be possible only if different interpretations for a certain fixed logical operator were proposed within the *same* language, which Quine considers absurd: different logics correspond to different languages.

The latter conclusion, together with Quine's awareness that a deviant logician may nevertheless have his reasons to change the subject/language, makes his position *apparently compatible* with the one endorsed by the well-known Carnap's *Principle of Tolerance*:

> «It is not our business to set up prohibitions, but to arrive at *conventions* [...] In logic, there are no morals. Everyone is at liberty to build his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments» ([14], pp. 51, 52; emphasis added).

As Restall put it: «Carnap's aim is not to ban the choice of this or that logic, but to explore the consequences of such choices. [For Carnap], all there is to logic is *language adoption*, and language adoption is radically unconstrained» ([67], pp. 429, 430). The unique accepted notion of validity from this perspective is an internal one - a system-relative validity - so that an argument cannot be merely said to be valid/invalid, but valid/invalid with respect to a specific system. This clearly supports a conception of logical systems more as convention-based useful *calculi* than as logics in a traditional sense, viz., as models of correct deductive reasoning. In fact, the choice of which language is to be preferred must be done for Carnap on a pragmatic level, on the basis of its being more or less useful within applications.

However, Quine's and Carnap's views do not match in all details. It is true that the conventional aspect of a system of logic is relevant to the former as well as to the latter, as is clear from this passage by Quine:

> «[T]he non-Euclidean geometries came of *artificial* deviations from Euclid's postulates, without thought (to begin with) of true interpretation. These departures were doubly conventional [...] For the deviating logician the words "or" and "not" are unfamiliar, or defamiliarized [...] Any such revised usage is conspicuously a matter of *convention*, and can be declared by legislative *postulation*» ([57], pp. 358, 362-363; emphasis added).

Also, it is true that apparently for both of them there are no external data a logic should "get right". Last, it is true that the maxim of minimum mutilation can be said analogous to the Carnapian idea that only pragmatic issues are involved in the use/choice of a particular logic. But there is a *fundamental* difference between Quine and Carnap, namely that Quine thinks one should not be changing logics/languages at will, since in his view logic is after all *more than a pure linguistic instrument* and classical logic has a privileged role in our web of *beliefs*. More specifically, he maintains that the systems of orthodox logic embody in fact the familiar and obvious senses of our logical words, granted though that these senses are conceived as intimately defined through our logical principles (immanent) and not as existing somehow independently from them (transcendent). Thus, although conventionally constructed, those systems are *not arbitrary* in his view. Also, whereas for Quine any effective change of logic leads to the rejection of the precedent system (in a monistic-excluding perspective), in Carnap's perspective a system of logic cannot be really rejected, but only employed or not employed (granted that it is clearly constructed).

Going back to the original issue of this paragraph, we need to conclude as to whether the meaning-change thesis can be maintained only within the traditional conception of logic. It is then interesting to notice that in no stage of Quine's analysis has there been a reference to the ability/adequacy of a logic to capture informal valid arguments. We hold indeed that Quine's rationale simply is *not dependent* on this traditional assumption. Such an independence is evident if one considers that: all the steps in G1 can be adequately read without the traditional assumption; G2 simply does not make any reference to it; the P-rationale against the rejection of CL is so essentially based on methodological/pragmatic principles, that one can conclude that the assumption is, again, not relevant here.

This means that Quine's meaning-change thesis can be conceived *both* from the point of view of logic as based on its ability to capture an informal notion of validity *and* from the point of view of logic as pure language. Or, in other words, both for logics qua *models* and for logics qua *calculi*. We will use this fact in the following.

2.4 Setting the stage: the two kinds of conflicts and logical pluralism

The latter consideration raises an essential issue. As already noticed in introducing the two kinds of conflicts (§2.2.1), one is tempted to say that there is no real dispute in a Pc-case. Indeed, *those who endorse Quine's meaning-change thesis usually stress that different logics are then products of a* MERE *change of meaning*, thus implying that a Pc-scenario is *less problematic* than a Gc-scenario (or even not perturbing at all). Of course, if a complete inter-translation of two logics was possible, then no conflict at all would hold, for they would be mere notational variants of each other. But, as we already noticed, this case is far too optimistic. Hence, the question arises as to whether the Pc-scenario endorsed by the meaning-change thesis is less problematic in a *wider* sense.

In a logic-qua-calculus conception, Carnap's perspective offers a first possible insight about why this could happen. Indeed, in this perspective the existence of different pure mathematical structures, each of them with a peculiar meaning for an operator, does not raise important problems for philosophy of Logic (traditionally meant), but for philosophy of Mathematics, at most. The only relevant problems here would be practical ones, concerning which system one finds more useful within applications.

However, when the meaning-change thesis is interpreted within the traditional framework, logic-qua-model, one needs to offer other reasons as to whether a *mere* change of meaning and the existence of different logics are in fact less problematic. After all, as Quine maintains in his monistic perspective, the resolution of a Pc-case leads to a *rejection* of a system of logic, that is to a clear incompatibility-scenario; and one cannot really see how *mere* a conflict can be whose outcome is an incompatibility-scenario.

Indeed, philosophers such as Dummett [23, 22] and Prawitz [47] have argued - contra Quine, in their view - that even a terminological disagreement can lead to a *real* conflict if two logics disagree on general meaning-theoretical principles. The typical case is the conflict between CL and intuitionistic logic (IL), which they conceive as originated from two different conceptions of the meaning of a sentence, based respectively on its truth-conditions and on its assertibility-conditions. The difference in perspective from Quine's framework is crucial in their view. The clues they offer to understand the supposed conflict are no longer constrained by considerations about the use of a logic in ordinary

speakers, but rather on principle-issues. Whereas Quine puts in high consideration the actual set of our logical beliefs as expressed in CL, these philosophers have argued that CL's principles not only are not obvious, but also in disagreement with the actual practice and with some normative principles that every theory of meaning *must* have (and that IL satisfies, according to them).

However, the difference in perspective from Quine's approach is not substantial to us. For Quine, a conflict is possible even if the disagreement is indeed only terminological, that is not genuine (this is indeed our Pc-case). In fact, Dummett's and Prawitz's views can be rephrased by saying that a conflict can still exist even if two logics disagree not as to which of them gives the best account of an operator (they in fact deal with different operators, hence the terminological disagreement), but for *other reasons*, namely, the meaning-theoretical principles. But this is exactly what Quine's Pc-scenario implies. In other words, Dummett and Prawitz argue *for* the rejection of CL in the *same sense* in which Quine argues *against* such a rejection, that is in the Pc-case's sense. The main difference is that Quine's argument is based on the maxim of minimum mutilation (i.e., on pragmatic principles), whereas their rationale is instead based on the claim that CL is *unscientific*.[11]

The fact that an important conflictive situation can still exists even within the bounds of a "mere" terminological disagreement has been emphasized by Graham Priest, who offers en passant a confirmation that our distinction between Gc and Pc is convincing:

> «[Quine] is absolutely correct to insist that *the views concerning meaning-change do not render rivalry and revision impossible.* One way to see this is to recall that a number of influential writers in the philosophy of science [like Feyerabend and Kuhn] argued for a version of meaning variation for scientific theories. According to them, the meanings of terms in scientific theories

[11] As is clear from the well-known Dummett's rationale: (a) a theory of meaning has to be a theory of the understanding of meaning, capable of determining what is it that a speaker knows when he knows the meaning of the expressions of language; (b) nothing could count as knowledge of meaning which could not be fully manifested in use; (c) a theory of meaning should then be based on the assertibility-conditions of a sentence (d) intuitionistic logic is better than CL in ensuring - through its constructive constraints - that such conditions are respected; (d) so, intuitionistic logic is the only correct logic. We cannot here investigate in details the strength of this position, which of course is more sophisticated than we make it sound. However, it is a fact that Dummett's rationale has been deeply criticized, essentially for the confusion it endorses between a *semantic* theory of meaning and a *pragmatic* theory of understanding (the meaning) - and between a semantic notion (truth) and a pragmatic criterion to establish the truth of a sentence ([32], pp. 86-91; for details on such critiques, see [38, 44, 19]).

> are defined by the scientific principles in the theory, and thus, e.g. "mass" in Newtonian mechanics means something different from "mass" in relativistic mechanics. They concluded from this that the theories are incommensurable, i.e. that no direct comparison of content is possible between them. *But they did not infer that the theories are not rivals.* They obviously are: they give different and incompatible accounts of, e.g., motion, to the point of making inconsistent predictions» ([51], §10.09; emphasis added).

Now, if CL and DL endorse in fact incommensurable logical principles that have nothing in common (no common residual meaning), then the question arises - if one wants to preserve Priest's analogy - as to *what* they give different incompatible accounts of. If, so to say, "mass" corresponds to "or" and "scientific principle" corresponds to "LEM", *what* exactly in logic corresponds to "motion"? An excellent reasonable candidate for this is of course "correct deductive reasoning". Hence, if one wants to claim that a Pc-case is really a less problematic scenario (or, equivalently, that Priest's analogy does not apply completely), one needs to show not only that there are different extra-systematic types of correct deductive reasonings (each uniquely captured by a specific logic), but also that they are in some sense compatible. We will start now to assess this issue, which appears to be a particularly tricky one.

First of all, let us describe the most important responses as to handling a general conflictual situation between logics. As described by Haack [32], when one is faced with the question whether there is a uniquely correct logical system, one can endorse three broad kinds of positions:

- **Monism**: there is just *one* correct system of logic. Accordingly, CL and DL cannot be both correct because they contain rival claims about which formalism correctly and uniquely represents informal valid arguments/logical truths.

- **Pluralism**: there is more than one correct system of logic. Accordingly, CL and DL can be *both* correct, so that the conflict is not problematic. More specifically, there are two ways in which CL and DL can be both correct:

(LP) A pluralist is a *local pluralist* if he adopts a view of logic as *domain-relative*: different logics deal with the *same* subject/meaning (the same general *kind* of informal valid arguments/logical truths), but they are correctly applicable to different and specific areas of discourse (domains), for each area allows for its own peculiar class of valid arguments/logical truths (each area supports a specific extension of the class of arguments/truths). So, for example, CL is best applicable to macroscopic phenomena,

quantum logic to microscopic ones. However, a local pluralist maintains, the wffs of these logics have *one and the same* meaning, though the respective logical truths are *not* true of *any* domain. A *local pluralist* proposes thus the employment of his logic in *some* domains, namely in those applications that are *not compatible* with an application of CL. An extremely important disadvantage of this form of pluralism is that it allows logical truths to change with the field of research or with the theory about such field. In principle, a local pluralist can in fact endow *any* theory with its own specific logic, this logic being *imputed* by the theory itself. In other words, he can find the "correct" logic for a given area of discourse a-posteriori. This means not only that we would be forced to deny the universality of logic in such a perspective, that is its being applicable to *any* area of discourse, but also that we would be left without an important criterion of rational evaluation of theories. In a nutshell, both the universality and the normativity of logic would be definitely lost.

(GP) A pluralist is a *global pluralist* if he adopts a view of logic as «[applicable] irrespective of subject-matter» ([32], p. 223), i.e., all-purpose logic. In this view, different logics talk about *different* meanings (different general *kinds* of informal valid arguments/logical truths), and they are (should be) correctly applicable to *any* area of discourse. Thus, the typographically identical wffs of two logics have *different* meanings, so that they cannot capture the very same informal sentences/arguments. An interesting possible outcome of this scenario is that two different logics could be applied without any incompatibility at all. *But a crucial point for this is to produce an acceptable rational account of* WHAT *those different meanings are.*

- **Instrumentalism**: there is "no" correct logic. The notion of correctness is inappropriate, since there are no informal (i.e., extra-systematic) valid arguments/logical truths to be captured. A logic can only be useful, fruitful or convenient *in* applications. The only necessary condition for accepting a logic is soundness (all the theorems are logically true *in the system*) or non-triviality (the system should not demonstrate every sentence).[12]

Carnap's perspective sketched above can be correctly called an instrumentalist perspective. Thus, let us focus on the non-instrumentalist possibilities, where a logic is viewed as a model of an external notion of correct reasoning.

[12]In the instrumentalist perspective, a logic is conceived in the same way in which different geometries are interpreted in a conventionalist view: they are not purportedly true of the world, they are just meant to be used in scientific applications.

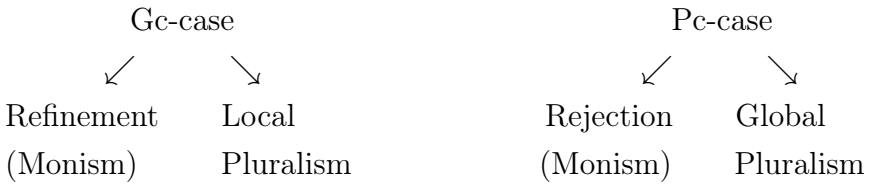
If there was a Gc-scenario, a real disagreement between logics, then these could be said to disagree on *one and the same* principle, e.g., *the* Law of Excluded Middle, conceived as something whose meaning is the same for everybody, a transcendental entity. The reformer, in this case, would *understand* the classical principle in the same sense the classical logician would, but he would then disagree on its general correctness or - in Quine's words - he would deny the classical doctrine. More specifically, a realist reformer of LEM would claim that it can be empirically *falsified* (from facts).[13] The question then would arise as to who is right, as it can happen for disagreements over generally uniformly interpreted sentences like "Rome is the capital of Italy". Let us emphasize that, being the disagreement over *one and the same* principle, the Gc-case naturally endorses a *monistic* view of logic: there must be, or there is, only one correct system of logic, namely the only one which captures the unique extensively correct properties of the operators involved in the principle. That is why thus the "wrong" logic could be then *refined*, namely because it would deals indeed with the same things the reformer's logic deals with, but still not in a sufficiently correct way. A singular exception to such a monistic outcome in a Gc-scenario is obtained if one adoptes a view of logic as domain-relative. In this case, a genuine scenario would lead indeed to *local pluralism*: different logics would deal with the *same* LEM, but the application of such LEM would be adequate only for some domains, not for all.

In the Pc-case, instead, more than a disagreement over the principle, there would be a *misunderstanding* of each other's reasons. Namely, the reformer and the classical logician would just use the *same notation* while referring to completely different meanings, or sufficiently different - more likely. Only by accident, then, could one talk of *the* Law of Excluded Middle, since there would actually be *two* different laws, which would embody *two* different meanings one can express by their means. Here again there is more than one possible outcome, *although from Quine's work only a rejection appears to be possible*. Indeed, one can also be *in principle* a *global pluralist* in this case, by showing that a number of different logics can indeed coexist in a compatibility-scenario. In this view, the correct (or fruitful) application of LEM in *one certain sense* (say, the CL sense) would be in principle trivially compatible with its not being applicable in another, different, sense (say, the DL sense). These logics would embody senses of logical principles so much different that they could not be said to give different incompatible

[13]But even a pragmatist reformer of LEM would nevertheless argue against it assuming that *the* meaning of LEM would lead to unacceptable or inconvenient results.

accounts of one thing, not even of what a "correct deductive reasoning" is.

On the whole, these are the four possibilities:



Now, summing up the differences among the pluralists:

- Local Pluralism "resolves" the problem of incompatibility *within* applications, by individuating the specific domains that are uniquely appropriate for one logic;

- Global Pluralism is able in principle to "dissolve" the problem of incompatibility, *if* an *understanding* and an *acceptance* of each other's reasons are reached.

Thus, our reasoning shows that the Pc-case can be less problematic than the Gc-case - or even not perturbing at all - and that *it can be regarded as a desideratum in that it can lead to a compatibility-scenario* (Global Pluralism). Quine's thesis, hence, is shown able in principle to produce a *general* way to *understand* and *dissolve* conflicts. In order to emphasize this consideration, let us notice that Dummett's approach could hardly lead to a preferable way to *understand* conflicts, let alone to resolve them. Indeed, even assuming that the approach makes good sense of the CL-IL case, it is unclear how one could individuate different *theories of meaning* for other different logics. But even assuming this, the approach would certainly lead only to a monistic resolution of a conflict, namely to a rejection.[14]

One last observation is worth being made now as to whether modal logics are mere extensions/supplements of CL or systems in competition with CL. On the one hand, one could be a pluralist holding that both CL and modal systems are correct systems of logic (for they capture different, and yet compatible, notions). On the other, one could be a monistic by considering both CL and modal systems as fragments of the *one* correct system of logic. As Haack [32] notices, the difference *here* is only verbal. CL and modal systems share the core of one and the same language; by adding *new* vocabulary to CL, modal systems amply the extension of the classical notion of validity.

[14]Of course, no one can be sure that there is indeed a *general* way to understand *any* conflict between logics, for there could be irreducible "varieties of deviance" - in Haack's words [31].

Chapter 3

Reactions to Quine's challenge

We have identified in the first part of Quine's rationale two arguments - G1 and G2 - which together support his meaning-change thesis. Although the rationale is not dependent on the supposed ability of a logic to capture a transcendental notion of validity, his perspective assigns as a matter of fact to classical logic the very privileged role of being «*our* orthodox logic» ([58], p. 82), that is the system in which our logical beliefs are somehow expressed. In the following we will describe some difficulties related to these points and we will assess some relevant critical attempts to attack Quine's rationale.

3.1 The *maxim of minimum mutilation* and the special role of classical logic

On the one hand, the maxim of minimum mutilation is unproblematic insofar it refers to theoretical virtues that are considered desirable in any theory. The case of Einstein's Theory of General Relativity is a typical example: Riemannian geometry - which is undoubtedly more complex than Euclidean geometry - was nevertheless preferred to describe the theory, for it leads to an overall less intricate scenario than the one obtained using the Euclidean one. Thus, the virtues themselves do not constitute a problem. On the other hand, a realist reformer who defends his alternative logic as the only able to *efficiently* capture some real, informal notion, cannot easily accept Quine's maxim, this being based on the supposed efficiency and convenience of CL instead. Even if we agree, as it seems correct, that the reformer's logic leads to a more convoluted scenario, its purported efficiency and convenience is certainly enough for the reformer to question

Quine's maxim. The maxim, in sum, embodies virtues that can be variously satisfied by more than one system or, in other words, that will not necessarily narrow down the options to only one.

The way in which Quine uses his maxim should be seen in fact as intimately connected with the general privileged role that he assigns to CL. Such a role can be appreciated in his argument from translation (G2), whose conclusion is that the imaginary linguist can only impose his own classical logic to the native's one, by rejecting both all the sentences translated by classical contradictions (if accepted by the native) and all the sentences translated by classical tautologies (if dissented by the native). To reach this conclusion, Haack [31] notices, Quine needs not only to maintain his *principle of maximal agreement*, quite a rephrase of his meaning-change thesis: (M) «whenever you can trace back a divergence to a misunderstanding rather than to a mismatch of beliefs, do so» ([42], p. 538). But - she continues - he also needs to defend classical criteria as the only correct ones. However, even if one can easily agree with the methodological principle (M) in a translation-framework (without some agreement in beliefs the translation could hardly even begin), the privileged role of CL is far from being unproblematic according to her. Indeed - she claims - if the linguist is an Intuitionist who accepts (M), then he would impose intuitionistic logic to the native's one (possibly CL). But then, assuming that the classical native invariably assents a two-compound sentence ϕ from his own native language, the linguist could never translate ϕ with $p \vee \neg p$: this translation would hold only for a sentence to which the native does *not* invariably assent. On the whole, as noticed also by Priest [51], there is no reason to believe that Quine's argument cannot be perfectly symmetric, so that the deviant translator imposes her deviant logic to the classical native. Thus, «the maxim "Save the obvious!" preserves classical logic only granted that classical logic *is* obvious» ([31], p. 20), or - going back to our G2 - only granted that "the classical logician can't change his own laws exactly in that he can't change the meaning of his operator, which is the only one he knows".

This part of Quine's argument is indeed not so strong at first glance, because it seems reasonable that one can change his own meaning if someone points out that according to him it is not correct (which is exactly what some reformers of CL maintain). Anyway, we consider the critiques above not fully appropriate. Indeed, Quine seems to be completely aware that the deviant logician considers his own deviant logic as the obvious one. His defence of CL in not as much based on its being obvious as much as on the principle of minimum mutilation. As we will see in the next section, he does not question the deviant beliefs as such, for he is aware that the question as to which beliefs are the

correct ones cannot really be answered, but rather for the consequence they would have with respect to the constraints expressed through the principle of minimum mutilation.

Anyway, even assuming that the above critiques are reasonable, it is important to notice that they cast doubts *only* on Quine's defence of CL, and not also on the adequacy of his meaning-change thesis. The independence of the maxim of minimum mutilation from the thesis is quite evident. As for the translation, not only the "symmetry-fault" in Quine's argument does not make the thesis untenable, but the possibility itself of *any* translation (hence also of the symmetric translation) is *dependent* on the correctness of the thesis in his view. Indeed, Quine would claim, the Gc-case simply makes translations a nonsense, because it would be unclear *what* should be translated: a mismatch of beliefs cannot be compatible with a fruitful translation, only a misunderstanding can.

The latter consideration confirms in a sense that Quine's real argument is G1, the theoretical one; whereas the argument from translation G2 is in fact a way to show those practical consequences of G1 that relate to his fictional/exemplifying translation scenario. To be more clear, G2 is *based* on G1, namely on G1-(c) (p. 14). Thus, the essence of the argument is not the defence of CL (as Haack's and Priest's critiques seem to assume), but rather G1-(c) itself.[1] On the whole, the faulty "virtual" defence of CL developed in G2 does not corrupt the possible integrity of Quine's meaning-change thesis, as a defence of DL along the same line would not.

3.2 The strength of Quine's rationale

As a matter of fact, only the Quinean thesis against the Gc-case is a *positive* - articulated - thesis. The Pc-scenario is derived simply by the confutation of the Gc-scenario, and no independent argument is proposed for its plausibility. On the whole, one can express the core of the Quinean schemata by the disjunctive syllogism: $Gc \vee Pc$, $\neg Gc \vdash Pc$. Quine's essential assumption behind the premise $Gc \vee Pc$ is that a terminological disagreement is not compatible with a genuine scenario (and that in fact Gc e Pc are the only two options). Let us recall in the following schema the ideas involved in his rationale (DL indicates a deviant logic challenging some CL's principle):

[1] Thus, the expression "the classical logician can't change his own laws exactly in that he can't change the meaning of his operator, which is the only he knows" should be better rephrased as "the classical logician *can* change his own laws, and then change the meaning of his operator, but he would then endorse another *wholesale different* operator, not a refinement of the old one".

- Gc: There exists a *real* behaviour of some logical expression → A logic should capture/express such behaviour → CL and DL try to capture one and the same real behaviour → DL captures such behaviour better than CL, i.e., only DL succeeds in the individuation of the unique correct properties of the expression → CL is *inadequate* → CL must be *refined*;

- Pc: CL and DL simply talk about different (maybe incommensurable) things → CL's operators are unscientific, meaningless or not convenient → CL must be *rejected* in favour of DL.

Not surprisingly, Quine's argument against the Gc-case has been largely criticized both for the rationale itself and for its conclusion. The meaning-change thesis is however so hardly defendable or rebuttable that one tends to believe that no definitive solution can be given to the issue. As Resnik put it:

> «Suppose that until now my mathematical proofs used non-constructive principles, but now I announce that I will restrict myself to constructively acceptable proofs. Have I revised my logic, while continuing to mean the same by "not" and "or" or have I decided to use those words with a different meaning? I don't perceive a fact of the matter here» ([66], p. 180).

From the reasoning in the conclusion of the last section, it follows that a relevant critique to Quine's meaning-change thesis should be addressed to G1, more than to G2. Now, G1 is mainly criticisable with respect to two distinct but deeply interconnected aspects one can find in it:

(P1): If there is a disagreement on logical principles, then there is a change of the meaning of logical operators

(P2): If there is a change of the meaning of logical operators, then there is no genuine conflict.

Let us notice that from P1 and P2 one gets - by transitivity - (P3): if there is a disagreement on logical principles then there is no genuine conflict, viz., a genuine conflict is simply impossible. In order to definitively emphasize how odd is Quine's conclusion on the whole (but nevertheless hardly refutable), let us express P3 through its contrapositive rephrase: if there is a genuine conflict, then there is an agreement on logical principles. No comment.

3.2.1 The *theory-dependence* thesis

- P1 is essentially our G1-(c) in disguise (p. 14), in fact Quine's *substantial* supposition on which the disjunctive syllogism above is based. It supports the *theory-dependence* of the meaning of logical operators, according to which when one changes some logical principle (or an axiom/rule of inference), one changes the meaning of the logical operators used in the principle (in the axiom/rule of inference).

Putnam suggested a critique to this conception, introducing the notion of "law-cluster":

> «[T]he logical words "or" and "not" have a certain core meaning which is [...] independent of [LEM]. Thus in a certain sense the meaning does not change if we go over to a three-valued logic or Intuitionistic logic [...] Law-cluster concepts are constituted not by a bundle of properties [...] but by a cluster of laws [and any of them] can be abandoned without destroying the identity of the law-cluster concept involved» ([53], pp. 50-52).

Such an identity would be enough to allow for a Gc-scenario and for the refinement of a logic, which would be thus able to individuate a transcendent core meaning of the operators, within a *minimalist view*. For example, in the case of negation ($\neg$), one could say that the notions of "opposition" or "denial" are constitutive of its *core meaning* and, as such, independent by the specific laws that negation supports.

Of course, no suggestion can be taken seriously which is not adequately developed. Thus, if one notices that Putnam himself considers the «notion of "change of meaning" [not] refined enough to handle [the] issue» ([54], p. 233), one should conclude that no important contribution has been offered through his suggestion, or at least that P1 cannot be easily rejected in this way.

What is particularly odd is that Quine himself [57] considers the notion of "meaning" not sufficiently clear to show the correctness of the theory-dependence thesis. As we said, G1-(c) is a crucial *assumption*, not really a thesis, and is defended only by suggesting that the opposite thesis (the meaning-invariance one) leads to an absurd. It seems thus that one cannot decide between the strong version of P1 (change of meaning as a wholesale change, i.e., Quine's view) and the weak version of P1 (where a change holds, but such that a core-meaning is in fact guaranteed). As Field notices:

> «The question [of meaning change] is clear only to the extent that we know how to divide up such firmly held principles into those that are "meaning constitutive" or "analytic" and those which aren't, and this is notoriously difficult» ([25], p. 176).

Quine has anyway an advantage over the others, for he can tolerate the absence of a clear idea of "meaning" better than the supporter of the weak notion. Quine's reply to Field could be indeed quite simple: *whatever* the meaning is, *every* logical principle gives its own contribution to the meaning of the operators it includes. On the contrary, the weak-supporter must hold that some principles are simply not essential, and he then needs a watertight criterion for distinguishing between essential and non-essential principles. For example, if each of two competing systems shares *one* principle of the other one, the weak-supporter needs to maintain that *nothing* pertaining the meaning is in fact added to the core meaning of the relevant operator through that principle. Also, assume we have a bounded sequence of systems such that the class of principles of one system is properly included in the class of the following one:[2] $S_1 \subset S_2 \subset S_3 \subset ...$. The weak-supporter would need to maintain that it is possible to individuate which of the systems in the sequence embodies only the necessary/constitutive principles for a certain operator; which appears to be quite an improbable enterprise.

On the whole, the notion of "core-meaning" has to face the same objections that the ancient notion of *substance* has to face, as well as any form of essentialism. As Russell put it:

> «[Substance is a] mere imaginary hook, from which the occurrences are supposed to hang [...] But when we take away the properties, and try to imagine the substance by itself, we find that there is nothing left» ([69], p. 193).

A suggestive attempt to individuate the essence of an operator, due to Priest, is worth mentioning:

> «Someone who rejects classical logic, say a paraconsistent logician, need not deny that the (classical) meaning of "$\neg$" is sufficient to guarantee the validity of inference $(p \wedge \neg p) \vdash q$ in classical logic (the pure abstract logic); what they will certainly deny is that this is the meaning of negation, as it occurs

[2] As for IL and CL. This case is actually a bit more convoluted than it could seem, for a result from Gödel shows that an opportune translation of CL into IL is possible which preserves all CL theorems.

> in *vernacular reasoning* [...] According to them, the semantics of their pure logic is the correct semantics for vernacular negation. Seen in this way, a dispute between rival logics is, then, exactly a dispute over meaning» ([51], §10.11; emphasis added).

In other words, he maintains that a residual essence of an operator can be in fact identified with the meaning of that operator in the vernacular reasoning, viz., everyday reasoning. Having this in mind, it is interesting to notice that, according to Priest, Quine's defence of P1 and CL is based on the confusion between a theory of logic and logic itself:

> «Quine [...] complains that someone who denies ex contradictione quodlibet just doesn't know what they are talking about, since changing the laws is changing the subject. [Yet] such a person needs only be suggesting a revision of a *theory of logic*, *not* logic itself. One cannot simply *assume* that classical logic gets it right» ([48], p. 102; italics in the text).[3]

A deviant logician could then genuinely disagree with CL at the level of the theory, namely about which logic best describes a certain underlying phenomenon (the informal behaviour of an operator), and not necessarily at the level of which logical beliefs we do have (or we think we do have). Different *theories* of logic are in this sense about one and the same thing (the vernacular essence), and each of them can be in principle equally defended (as well as, e.g., different physical theories can). In his words, «logics, then—that is, our theories of logic—are fallible theories» ([51], §10.15).

We do not have a definitive opinion as to how convincing can be Priest's suggestion. On the one hand, that logic is a fallible theory seems quite reasonable, if only by recalling that already during the age of Diodorus and Philo there were disputes over the nature of conditionals which were so long-lasting that «Callimachus wrote an epigram saying, 'Even the crows on the roofs crow about the nature of conditionals'» ([35], p. 128). On the other hand, however, there are reasons to be sceptic about the relevance that Priest's distinction has for Quine's challenge. First of all, it seems deeply unclear how a theory of logic could be adequately defended *without* some reference to our logical beliefs. Differently from the case of physics, or at least of macro-physics, the "external data to get right" here are not objective, if even existing. Hence, their characterization would be *dependent* on one's beliefs in an essential way. Thus, if Quine is right about

[3] Resnik makes a similar distinction in [66].

P1 in the second sense described by Priest (the "logic of our beliefs"-case), then it seems he should be right in the first one as well (the "theory of logic"-case). Quine himself, of course, has argued against the *existence itself* of *the* "meaning of a vernacular operator" - in agreement with his idea of logical connectives as *immanent* operators. The whole point is quite clear in the following quote, concerning the relation between the sentences of ordinary language (S) and their paraphrases (S') into logical symbols. Quine says:

> «The speaker can be advised in his paraphrasing, and on occasion he can even be enjoined to accept a proposed paraphrase or substitute another [...] but *his* choice is the only one that binds him. A foggy appreciation of this point is expressed in saying that *there is no dictating another's meaning*; *but the notion of there being a fixed, explicable, and as yet unexplained meaning in the speaker's mind is gratuitous.* The real point is simply that *the speaker is the one to judge* whether the substitution of S' for S in the present context will forward his present or evolving program of activity to his satisfaction» ([56], p. 160; emphasis added).

Differences among different supporters of a logic would not then concern as to how a certain informal operator should be captured. Indeed, each different paraphrase cannot be said to deal with the one and the same thing, the paraphrase itself being *the only thing* that one can deal with. Given "one and the same language" - Quine holds - it is not possible to find the unique correct regimentation (paraphrase) of that language. The enterprise would not be undecidable, rather a nonsense: the *what* to be modelled and the *how* to model it would be quite the same entity.[4] The distinction between immanent and transcendent operators that we have sketched above finds its reason exactly in the consideration that the meaning of an operator is wholly defined *internally* to a language in function of one's own logical *beliefs* (expressed within a paraphrase)[5]. If there is a dispute, thus, it is not originated from a mismatch of beliefs/paraphrases over the same thing, but rather from a possibly incommensurable wider diversity *in* logical beliefs.

Quine's defence of CL then is not meant to be a foundational enterprise. Even when he stresses its obviousness, he is simply emphasizing such a wider diversity. On the whole, the principle of minimum mutilation is the only real defence of CL he develops. A corollary of his approach is that a logical principle cannot be really falsified from a

[4]Let us point out that this thesis concerns with speakers of the *same* language, rather than of different languages (as in Quine's scenario for the Indeterminacy of Translation).

[5]For an analysis of how different logics can emerge from different models of language, see [76].

scientific experiment. At first, the principle can only be accepted or rejected in function of one's (logical) beliefs; then, the correspondent logical system can be accepted or rejected as a whole in function of the principle of minimum mutilation, conceived by Quine as sufficient to decide between different logics (and this constitutes the ultimate reason for his Monism).[6]

3.2.2 *Change of meaning* considered harmless

- P2 (If there is a change of the meaning of logical operators, then there is no genuine conflict) is essentially our G1-(a) in disguise (p. 14). Indeed, from P2 one obtains by contraposition that if there is a genuine conflict then there is no change of meaning of logical operators (and so there must be a residual meaning for a logical operator, absurdly independent from the principles which contain it - Quine continues). Haack [31] has argued against P2 by counterexample.
The first example is based on the following fictional scenario:

-DL and CL both have the same operators (typographically identical). However, the meaning of $\vee$ is different: it is discovered that what the deviant logician means by $\vee$ is by chance what the classical logician means by $\wedge$;

-DL lacks the wff *w:* $(p \vee q) \rightarrow (\neg p \rightarrow q)$ as a theorem, which instead is a theorem in CL;

-CL has the wff *v*: $(p \wedge q) \rightarrow (\neg p \rightarrow q)$ as a theorem.

It follows - Haack argues - that when the deviant logician denies *w*, he is in fact denying *v*. And since the latter is a theorem of CL, then DL is denying the *same* thing that CL accepts. Hence, there is what one would call a Gc-case, despite the change of meaning of $\vee$. End of the argument.

The argument is clever and seems correct, but we think it is not cogent. For it to be an effective critique to Quine's, it should go on showing that this scenario is actually possible, namely that the very assumption that the deviant logician means by $\vee$ *exactly* what the classical means by $\wedge$ is indeed possible. For, without this step, the strength of Haack's rationale reduces to the strength of a petitio principii. As we have seen, according to Quine's *substantial* assumption G1-(c), Haack's scenario is simply absurd: the deviant logician cannot mean by $\vee$ *exactly* what the classical logician means by $\wedge$,

[6] According to Quine, if one thinks that a certain classical paraphrase S' for S will not "forward his present or evolving program of activity to his satisfaction", then one is in principle authorized to suggest a rejection of CL. Again, what one *cannot* do is defending his own - deviant - paraphrase for its superior capacity in capturing the *same* meaning that CL tries to capture.

without then accepting w as a theorem. For Quine, thus, w is part of the very meaning of $\vee$ so much that one gets another *different* operator by denying or not accepting it. Far from being a proof that Quine is right, this shows nevertheless that he can be.

The second example is actually "a suggestive consideration" - in Haack's words:

> «[Gentzen's sequent] formulation of minimal logic differs from classical logic not in respect of the introduction and elimination rules for the connectives, but in respect of the structural rules for deducibility [...] Since the restriction involves no essential reference to any connectives, it is hard to see how it could be explicable as arising from divergence of meaning of connectives» ([31], p. 10).

What Haack is saying here is that if Quine is right about the impossibility of a genuine conflict, then the conflict in the quote would be only apparent; and therefore explicable, through P2, as arising from a change of meaning. However, she notices, in this specific case one simply cannot keep that a change indeed holds, since no reference to the operators seems essential in explaining the divergence between the two systems (which apparently depends only from the restriction on the rules for deducibility). However, this conclusion of Haack would indicate a relevant oddness of P2, only if it could be shown that the operators involved in her example have *in fact* the same meaning, which in a sense she suggests in virtue of the identity of the introduction and elimination rules for the operators. But, as we already noticed, the problem is that no agreement has been reached so far as to when two operators have the same meaning (this is actually *what the whole Quinean problem is about*).[7] Again, far from being a proof that Quine is right, this shows nevertheless that he can be. Indeed, as Haack herself notices:

> «the argument is not wholly conclusive, since it could be suggested that the reason for the restriction on deducibility lies in a desire to avoid certain theorems, e.g., $p \vee \neg p$, and that the desire to avoid these theorems may spring from idiosyncrasy of connectives» ([31], p. 10).[8]

[7] Quine himself gives only a *necessary* condition for the identity of meaning, through the contrapositive of P1, namely the agreement on logical principles. Analogously, Quine does not offer a definition of genuine conflict. Again, he only gives a *necessary* condition, through the contrapositive of P2, namely the identity of meaning.

[8] We think it is opportune here to give an idea of a recent suggestion developed by Francesco Paoli [42]. His idea can be seen as a gloss to Haack's observation. Paoli's view is based on the distinction between two aspects of the meaning of a logical operator. The first one is the "molecular meaning", that is the meaning as defined - in a proof-theoretical approach in theory of meaning - by

The latter quote - we recall - represents the good path that one should take in analyzing our issue, viz., focussing on the motivations/aspirations behind a formal system, rather than on its specific formal characterization. This clue will be of crucial importance for the planned suggestion of our study.

3.3 Intermezzo - So far, so bad

Before going on, it is useful to offer a brief summary.

On the one hand, we have suggested that Quine's meaning-change thesis apparently cannot be really demonstrated, and that it should be considered an assumption whose precise meaning is nevertheless not really clear. Not only did Quine not offer a precise definition of "subject" (or "meaning"), but he also proposed the Pc-scenario endorsed by his thesis only through the exclusion of the genuine case. On the other hand, all the attempts to show that a genuine disagreement in logic can in fact hold seem to fail in some manner, however "meaning" is interpreted.[9]

As summarized by Haack: «The question is tricky because there are reasons both for and against meaning-variance» ([32], p. 230). The correctness of the meaning-change thesis should be then considered still to be decided, even if Quine's approach

the introduction-rules and elimination-rules for an operator. The second one is the "holistic meaning" - meant to be «the meaning of [the operator] as encoded in the relationships between [the operator] itself and the other [operators] in the language» ([43], p. 556). According to Paoli, in the framework of a deductive system, namely a sequent calculi S, one can express both these two aspects. He calls the first one *operational meaning*, defined by the *operational* S-rules for the operator, that is those rules that govern the derivations involving that particular connective. The second one - which he calls *global meaning* - can be said to be represented in a sequent calculi S by the class of all those provable S-sequents involving formulae containing the operator. From this distinction - and from the fact that most sequent calculi for available deviant logics share with CL the same operational rules - he concludes that a core-meaning can be said to persist among different logics, namely the operational meaning. What changes among different logics is the global meaning of the operator, for the class of provable formulae is typically not the same. For example, the sequent calculi for the negation-implication fragments of CL and the one for Relevance Logic have the same negation rules, ($\frac{\Gamma \Rightarrow \triangle, A}{\Gamma, \neg A \Rightarrow \triangle}$ $(\neg L)$ and $\frac{A, \Gamma \Rightarrow \triangle}{\Gamma \Rightarrow \neg A, \triangle}(\neg R)$), but the sequent $A, \neg A \Rightarrow B$ is provable in the former and not in the latter calculus. In addition to the objection arising by the same consideration that Haack made about her own analogous approach above, let us just notice that the operational/structural distinction for a sequent calculi S is not firm, but dependent on the specific axiomatization of S. For a more detailed critique to Paoli, see [34]. Other minimalist attempts are developed in [64] and [67].

[9] *One* of the classical problems here is connected indeed to the notion of meaning defined by the introduction-rules and elimination-rules for an operator. This notion must face the traditional objections that go back to Prior's fictional connective "tonk" [52] (which - by trivializing a deductive system - is indeed a counterexample to the plausibility of such defined meaning). A number of responses have been developed against Prior's - mostly based on the constraint-notion of "Logical Harmony" (see, e.g., [9], [46], [24]). However, one hardly could maintain that a clear agreement has been reached so far.

is able to tolerate the absence of a clear idea of "meaning" better than the minimalist approach. On the other hand, however, Quine needs to maintain a really disturbing and problematic point. That is, that different logics deal with different languages in a deep sense, so that - if he is right - it cannot be taken for granted that different logicians are in fact able to understand each other.

Our comments about Priest's critique to Quine show also that it is not easy - if possible - to defend a logic on the base of its ability to capture the "real" meaning of an operator as used in a language - the vernacular meaning. Indeed, even assuming that there is something like a *stable* objective vernacular meaning, one could not actually *show* that one's logic captures it in the unique correct way, for the only possible arguments seem to be anyway built within the same logic one wants to defend. Thus, no resolution of a conflict can be based on this approach, since we have no final or definitive test able to assess the "winner", so to say.

Because of this apparent general impasse on Quine's challenge, one cannot make sense of the considerable number of logics existing today, so that we still do not know whether they should be considered in absolute competition or in agreement to a certain extent, whether they talk about the same things or rather about completely different things. In more practical terms, we cannot really decide whether, for a given particular argument, a classical and a deviant logician may or may not disagree about its validity. We still lack a plausible story on this.

However, what is quite certain is that the following scenario holds*, whatever* the meaning of an operator is (see p. 28). If Quine is wrong, then Monism is the natural outcome (and Local Pluralism only an unpleasant possibility). If Quine is right, then Global Pluralism is in principle possible (Monism being only one of the possibilities in this case).

Now, it seems that all the attempts to defend a specific logic as the only correct one are doomed to be inconclusive. Be that as it may, such attempts - if successful - would anyway leave some interesting systems out of the scope of logic. We believe thus that one should in fact conclude that a scenario of pluralism is surely preferable, if possible. On the other hand, the instrumentalist position and Carnap's principle of tolerance - though allowing a form of pluralism - do not give sufficient relevance to the traditional view of logic as representing an extra-systematic idea of validity, and reduce logic to a mere calculus. We have shown instead that one can endorse Quine's meaning-change thesis without defending a particular conception of logic, thus also within the traditional view.

Hence, if only a problematic impasse seems possible by trying to reject the meaning-change thesis or by accepting it but for a defence of one specific system, then Global Pluralism - which emerges from *accepting the meaning-change thesis but not to defend one specific system* - seems the most interesting possibility.

Two other considerations support this suggestion. First, the fact that, even accepting the meaning-change thesis, «straightforward and wholesale inter-translation is not [always] possible, [so that] the Deviant logician will have to be taken seriously after all» - in Haack's words ([31], p. 23). Second, the fact that if a *specific* different meaning is shown able to characterize the operators of a certain logic, then - in principle - Global Pluralism could explain the differences among logics in a more extensive way than Dummett's approach sketched above. Besides, in this case one would actually corroborate the meaning-change thesis.

As should be evident by now, the issues raised by the plurality of logics are extremely complicated. Not surprisingly, one tends to think thus that the study of the relations between different systems should be developed - in *some* manner - in a setting different from the one we have followed so far. An influential tendency in a new perspective has being carried out in recent logical studies:

> «The original quarry, the best logic for natural argumentation, has given way to something of higher generality: a structure that integrates the best features of a plurality of logics—an *Erlanger Programm* for logic. The articulation of such a structure as applied to natural argumentation is still in its earliest stages, but much recent work toward the provision of a general account of logical systems may lend itself to the advancement of this program» ([1], p. 698. See, e.g., Sambin's Basic Logic [70]).

This minimalism about logic - not to be confused with the minimalism about the meaning of logical operators indicated above - is developed nevertheless in a perspective that we do not consider particularly fruitful on a philosophical level. These studies can be interesting, and in a relevant sense, on a *mathematical* level, for they can show how the rules and the principles of different logics can emerge in some way from a basic structure. This would certainly be a highly relevant result, for it would offer us a common and explicative framework in an unifying approach. However, the approach would be nevertheless too much *formal*, since what it could explain is how different *calculi* can be seen as originating from a unique source; but still - prima facie - *qua* calculi, not qua models of correct reasoning. In other words, for such minimalism to be

a more interesting analysis of the disagreement in logic, it should be developed having constantly in mind also the philosophical aspirations of each logic. As noted by Haack,

> «[in advance] of some philosophical work, it is sometimes uncertain what formal investigations are likely to be fruitful. This is why [...] serious examination of the philosophical, rather than the purely formal, consequences of adoption of non-standard systems, is [...] overdue» ([31], xii).

Now, in rough terms, an understanding of the philosophical aspirations behind the systems of logic reduces to the understanding of *what* the systems talk about, i.e., of what they are supposed to be a model. In the next chapter, we will introduce two recent approaches in this sense, in the perspective of some *logical pluralism*.

Chapter 4

Logical Pluralism

4.1 Beall and Restall's logical pluralism

Beall and Restall [4, 5, 6] proposed a view about plurality in logic based on the tenet that the central notion in logical theorizing is the notion of "logical consequence" - or "follow from". They hold that this notion is essentially based on some pre-theoretical facts, which however are in some sense incomplete or not fully specifiable, so that they *underdetermine* the notion. Therefore, these facts allow for *different*, but *equally legitimate*, theoretical specifications; that is, for different, equally good, notions of logical consequence. And since the latter is what determines a system of logic, these facts leave room for different - *equally good* - systems of logic:

> «Logic is a matter of truth preservation in all cases. Different logics are given by *different explications of these cases*. This account of the nature of logical consequence sheds light on debates about different logics. Once this realisation is made, apparent disagreements between some formal logics are shown to be just that: merely apparent. A number of different formal logics, in particular, classical logics, relevant logics and intuitionistic logics, have their place in formalising and regulating inference. Each is an elucidation of our pretheoretic, intuitive notion of logical consequence» ([4], p. 491, emphasis added).

The idea that the intuitive notion of logical consequence is in fact not really well-determined is not new in logical studies. Already Tarski [74] was aware that the clarity of this notion is not superior to the clarity of other concepts of everyday-language, so

that every attempt to define it would inevitably include some arbitrary features. In a sense, then, Beall and Restall (B&R) try to transform this weakness of logical theorizing in a useful instrument toward an understanding of logical disagreement. Their flexible notion of "cases" allows in fact for different arbitrary intuitions to take form, while apparently making sense of how each logic succeeds in specifying a certain particular conception of validity.

In order to understand how they conceive their ductile notion, let us recall the usual Tarskian - model-theoretic - definition of logical consequence *(TC)*: *A sentence ϕ is a logical consequence of a set of sentences Γ, if and only if every model that satisfies Γ also satisfies ϕ.* B&R's idea is that TC is only *one* of the possible acceptable definitions of logical consequence, namely the one where their cases are identified with classical (complete and consistent) set-theoretical structures. They introduce instead a wider definition, where the validity of an argument is parameterized, that is relative to the particular notion of cases involved. They call it *Generalised Tarski Thesis (GTT)*: *An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion.* Their crucial idea is that there are different admissible specifications of cases, and then different admissible specifications of validity (GTT), hence of logics, too. For example, if cases are "situations", then one obtains a *relevant* definition of validity; if cases are "constructive stages", then one obtains an *intuitionistic* notion of validity. Since GTT is proposed as a general shared notion of validity of an argument, B&R maintain that any of the specifications of GTT - obtained through a specification of "cases" - can be accepted only if it can be shown to be *necessary*, *formal* and *normative*. That is, only if it has those features of deductive logic that are almost invariably taken as distinctive, even if widely considered problematic and not easily definable. Now that the central idea of B&R has been sketched, let us make a relevant consideration, before going on in the analysis.

There is here an evident important shift from the approaches we have discussed so far. B&R's Logical Pluralism is in fact an attempt to develop a common framework from which different logics can emerge (hence, it constitutes a form of minimalist approach to logic), while giving high relevance also to the philosophical - rather than purely formal - characterization of a certain logic. Also, B&R's Logical Pluralism aims at making sense of the existence of a number of logics not from a conflictive perspective, but giving all of them a peculiar dignity. According to B&R, indeed, the existence itself of different system-based notions of logical consequences does not lead to a Gc-scenario on the level of logical theorizing (only on the level of applications, at most). Analogously to Quine's,

they hold that the very one thing different logics try to capture (i.e., the intuitive notion of logical consequence) is not well determined, so that these logics cannot be said to disagree in a genuine sense. Rather, each of them gives its own elucidating contribution to the determination of the intuitive core-notion: their notion of logical consequence allows for a certain «unsettledness [that] affords different, legitimate precisifications of the core notion» ([6], p. 104). Each logic is thus relative to the specific aspect of the logical-consequence relation one intends to codify.

Two relevant differences emerge between this view and Quine's. First, differently from B&R, Quine argued against a Gc-scenario *while* defending a monistic view on logic (even if, as we noticed, his approach is in principle compatible with a form of pluralism). Second, differently from Quine's (and Carnap's), B&R's view leads to a scenario where different systems of logic share *one and the same language*. Their logical pluralism is indeed a consequence-pluralism, based *not* on different specifications of languages (meanings), but on different specification of cases within one and the same (formal) language. On the whole, they see the plurality of logics as a plurality of pairs $(L, \models_c)$, where L is a language and $\models_c$ is a consequence relation over L, depending on one particular specification of cases - c.

GTT is thus supposed to represent a pre-theoretic *shared* view of consequence, a core notion of consequence based on the idea that «*truth-preservation in cases* does the work required of logical consequence» ([6], p. 29). To understand how such unsettled notion leads to a specific characterization of consequence when a certain specification of cases is offered, one needs to understand how such cases are to be conceived. Here the weakness of B&R's approach starts to unveil. B&R claim indeed that

> «[c]ases [...] are "things" in which claims may be true. By specifying "truth conditions" for claims, you thereby specify cases [...] We do not pretend to have given precise individuation conditions for cases. We are not sure that such conditions can be given» ([6], p. 89).

B&R's writings on Logical Pluralism are not always entirely clear. However, they suggest some important examples of specification of cases (for classical, relevant and intuitionistic logic), from which it seems that cases are a way to express the different philosophical views on *truth* that the supporters of a logic hold.

In this sense, "situations" (the cases for relevant logic) are meant as *partial* representations of the world, rather than completely and consistently structured representations (as in the case of CL). Hence, for example, the disjunctive syllogism $\phi \vee \psi$, $\neg\phi \vdash \psi$ is not

necessarily valid here, for in such cases $\neg\phi$ could be true, ϕ could be also true (hence, $\phi \vee \psi$ could be true), while ψ not being specified. The "constructive stages" (the cases for intuitionistic logic) are instead conceived as successive steps in proof procedures. More specifically, they are ordered through an anti-symmetric relation of inclusion and are such that each stage may verify neither a claim nor its negation (as in Kripke's models for intuitionistic logic). Hence, for example, the inference from $\neg\neg\phi$ to ϕ is not necessarily valid here, for a stage can validate $\neg\neg\phi$ without validating ϕ. In all these examples (as in the cases for CL, i.e., Tarskian models), the three constraints on GTT above are then shown to be all satisfied in some sense.

However, the question arises as to how general the approach they suggest can be, that is how many kinds of "cases" one can specify for GTT to obtain specific logics. Even if their approach seems to work for classical, intuitionistic and relevance logic, one wants to understand whether it can actually work for other cases. Thus, what makes a type of cases admissible? Apparently, the answer B&R offer to the question appeals to the flexibility of the settled notion of consequence expressed in GTT:

> «Whether candidates [to be cases] are admissible turns on whether they agree with the settled parts of language, on whether they exhibit the features required by the (settled) notion of logical consequence. We hold that the notion settles some but not all features of any candidate relation of logical consequence; the unsettled features leave room for plurality» ([6], p. 29).

The idea seems to be that the necessary constraints endorsed by the settled notion are only that it must capture the idea of "truth-preservation in cases" and that it must be necessary, formal and normative. *Any* other feature one may want to add to the settled notion of consequence is acceptable if useful, ensured that the necessary constraints are satisfied: «[t]he question is ultimately one of utility» ([6], p. 29). This is in fact in agreement with the general B&R's perspective, where there is no possible answer to the question as to which account is the *right* account of logical consequence.

The latter point is however one of the most problematic features of B&R's Logical Pluralism, for one tends to conclude that it leads to some form of relativism. They are aware of this consideration:

> «Recall that we are not *relativists* about logical consequence, or about logic as such. We do not take *logical* consequence to be relative to languages, communities of inquiry, contexts, or anything else [...] We are *pluralists*

> about logical consequence because we take there to be a number of different consequence relations, each reflecting different precisifications of the pre-theoretic notion of deductive logical consequence» ([6], p. 88; emphasis in the text).

However, B&R *must* anyway conclude that, for a given particular argument of natural language, their Logical Pluralism does not allow for a unique answer as to its validity. As we have seen, their notion of validity is parameterized, so that a determinate answer is possible only *relatively* to a particular setting of cases. More specifically, in their framework an argument can be considered valid in one setting and not valid in another, but still remaining *one and the same argument* in some sense (since expressed in one and the same language).

Now, as noticed, B&R's idea of one and the same common language among different logics constitutes a departure from Quine's approach and it is purported as a way to give different logics *equal* dignity. However, for their Logical Pluralism to be a really different approach from Quine's, one should show that the characterization of a type of cases does not affect the meaning of the operators involved in the definition of truth for that type of cases, that is that no change of meaning of logical constants does hold (in other words, that there is here one and the same language in a *deep* sense). In this way, one would show that in fact an equal dignity is guaranteed to each logic. But such a result is anyway not easy to reach in B&R's framework, because - as observed by Priest [49] - different types of cases endorse as a matter of fact different *theories* of vernacular connectives, which are based on different truth-conditions (so, different meanings) and *cannot all be right*, if a form of relativism is not endorsed.

These considerations support a conclusion that should not surprise the reader. That is, that one cannot easily reject Quine's meaning-change thesis *without* being a monistic (or a local pluralist, at most). Indeed, if an invariance of meaning for logical operators is in *some* way guaranteed, then different logics should be said to offer different theories for one and the same operator - and as a consequence the question arises as to which is the only correct one. In other words, one should not be a pluralist - by holding that different accounts of cases are acceptable - while maintaining that the different clauses for truth are *equally* acceptable for one and the same operator.

However, B&R - in replying to Priest - offer for the first time a different perspective on the issue, which is connected to their conception of logical consequence as an *unsettled* notion. Analogously to Quine's, they hold indeed that different types of cases

(that is, different logics offering different truth-conditions) do not try to capture one and the same vernacular connective. Rather - they continue - the meaning of a vernacular connective is unsettled as well, and to be specified in different ways. Thus, different types of cases do not offer different *renderings* of one and the same vernacular connective, they rather capture different *features* of it - those features that all give their contribute to the overall meaning of the operator. In a sense, then, there is nothing here like a residual meaning, an essence for an operator, since all the different features isolated by different logics equally participate to that general meaning of the operator that different logics all *specify*. In order to avoid the threat of Monism, however, they need to put a *strong* constraint in this scenario, that is that those features must be *compatible*, i.e., that non-conflicting truth-conditions hold among different logics. As a consequence, only those types of cases which can be shown reciprocally compatible are admissible, and this of course excludes from the range of logics a number of systems. On the other hand, from a monistic point of view, B&R's Logical Pluralism not only counts too many logics as correct, but it also offers a useful framework where the reasons of one certain logic can be defended as the only correct ones. Trivially, a supporter of, e.g., intuitionistic logic can argue in B&R's framework that the only admissible cases are the "constructive stages" (see [63]).

Be that as it may, what seems particularly interesting to us is that - from this new perspective - B&R can still maintain that different logics deal with one and the same language (contra Quine), without necessarily talking of the same vernacular operators (rather, of different *features* of a yet-to-be-settled meaning of vernacular operators):

> «[Different] clauses can [...] be equally accurate in exactly the same way as different claims about a thing can be equally true: they can be equally true of one and the same object simply in virtue of being *incomplete* claims about the object. What is required is that such incomplete claims do not conflict [...] Each clause picks out a different feature of [an operator]» ([6], p. 98; emphasis added).

B&R's Logical Pluralism has been attacked on a number of points, essentially for its not being really able to make «the most sense of contemporary work in logic», as they maintain ([4], p. 476). More specifically, a monistic logician like Priest cannot agree with the treatment they make of the philosophical aspirations of deviant logics. Even if their framework suggests a way to express what it is that characterizes a certain logic, he cannot easily accept the whole approach, for he considers non-classical logics not

only as alternatives to CL, but also candidates to take its place as the only correct one (typically, a monistic argues in a genuine-conflict's perspective; see [62]). Let us emphasize, anyway, that this scenario holds for *any* form of pluralism - not only for B&R's. Roughly said, if one *wants* to be a monistic, then it is difficult to conceive reasons for convincing him that he can be wrong. Monism cannot be easily defended as true or rejected as false, but only preferred as opportune or rejected as inopportune.

There are two other considerations we want to stress for their relevance to the purpose of this work. The trouble with B&R's approach we want to emphasize is essentially the *relativism about truth* it seems to endorse. As suggested by Priest [49] and Read [62], suppose that the argument from α to β is classically but not relevantly valid, and suppose α is (known to be) true. Is β true or not? Clearly, a possible scenario of *incompatibility* would arise, for β would be *certainly* true only in one of the two accounts. We believe that this consideration raises an important issue, for we hold that no real scenario of compatibility between logics is possible which leads to a possible relativism about truth. According to us, different logics cannot be *equally accepted* if the outcome is the unsettledness of the notion of truth itself: how good can be a logic if it cannot be used to definitively establish the truth-value of a sentence? This outcome of B&R's ideas depends not from the privileged role that they assign to logical consequence in logical theorizing, but from their conception of the clauses included in *all* types of cases as *truth*-clauses.[1] Analogously, the outcome is dependent on their idea that the different features of an operator are all *semantic* features. Not surprisingly, in sum, it is not possible to avoid relativism about truth if different logics are conceived as all dealing, though in different ways, with truth itself.

There is another unpleasant feature of B&R's pluralism that is linked to their truth-based framework. Indeed, even if according to them different logics deal with the same language, it is difficult to see how two different logicians can even understand each other if their disagreement is explained as arising by different notions of truth. This is another way to stress that what they purport to be one and the same language is not one language in a *deep* sense (we think that it is reasonable to claim that within the same language no *essential* misunderstanding should be possible). The whole problem lies in the fact that they look at the relation between logics from the perspective of what we called genuine conflict: after all, they treat different logics as all dealing with the same thing, as a matter of fact.

[1] Their cases, after all, are nothing but another guise for the specific recursive definition of truth included in any system of logic.

However, we believe that there is much to be commended in B&R's attempt. Their focus on the philosophical aspirations of different logics (which they try to capture through the notion of "cases") is the right - albeit not sufficiently fine-grained - path to follow. Also, their general idea of different logics as dealing with different *aspects* (features) of *one and the same language* is an interesting one. Their analysis is thus attractive to us because it seems to allow a pluralistic interpretation of logical disagreement able to avoid *both* the use of the difficult and obscure notion of "core-meaning" (as in the minimalist approaches to the meaning of operators) *and* the undesirable Quine's outcome that different logics deal with different languages tout court.

4.2 Dalla Pozza's Global Pluralism

From what we said throughout, one should conclude that a good solution to the problem of plurality in logic should meet the following desiderata, in order to accommodate in an organic manner most of the crucial points we have discussed:

(a) - It should allow a pluralism able to give to the largest number of logics their due *philosophical* dignity. Of course, an issue connected to this is what is to count as logic in the first place;

(b) - It should be in agreement with (even better, corroborate) Quine's meaning-change thesis, for otherwise it would naturally lead to Monism (possibly, to Local Pluralism) and it would probably endorse the obscure notion of *core-meaning*;

(c) - It should ensure that «logical principles [...] apply irrespective of subject-matter», in Haack's words ([32], p. 223) (for the reasons sketched against Local Pluralism);

(d) - It should be developed in the perspective of Global Pluralism, so that it could ensure that a real compatibility between those logics hold (that is that the relative types of different deductive reasonings are in fact compatible);

(e) - It should show that different logics are generated within one and the same language, in such a way that different logicians can presumably understand each other;

(f) - It should consider different logics as dealing with different explicable *aspects* of such a language. Hence, it should show that different logics deal with different *kinds* of operators (within the same language);

(g) - It should *not* lead to any form of relativism, in particular about truth;

(h) - It should *not* treat the different explicable aspects of a language all as *semantic*

features (differently from B&R's approach);

(i) - It should constitute a *general* way to *understand* the relations between different logics and to *resolve* possible conflicts.

We hold that Dalla Pozza's global pluralism (dpGP) [19, 29, 7, 20] meets these desiderata.

The starting point of dpGP is the following tenet: two logics cannot both deal with the notion of truth or with the same logical operators. In order to better appreciate Dalla Pozza's general suggestion, let us get back to what we presented as the disadvantages of Local Pluralism (LP), which - we recall - can be proposed as a way to recover the compatibility between logics (see, e.g., [18]).

According to LP, different logics deal with the same thing (i.e., truth) and use operators having the same meaning (contra Quine). However, for LP to be a reasonable way to make the compatibility between different logics practically possible, it must endorse the thesis of locality of logics, so that the validity of logical principles depends on the subject-matter and is *determined* by a certain specific area of discourse. This makes the correctness of a logic depends on the point of view or on the theoretical context. For example, CL could deal only with macro-phenomena, Quantum logic instead only with quantum ones. The outcome of LP is thus that the validity of logical principles may change with the domains of application. Alternatively, that the extension of the concept of truth/validity itself may change with these domains. For example, one could say in this perspective that the Law of Distributivity is valid when dealing with macro-phenomena, but not valid when dealing with quantum ones.

This position is exposed to a crucial philosophical objection, as well indicated by Garola in defending dpGP ([29]):

> «[LP] implies that the logical apparatus which underlies a given [scientific] theory may depend on the theory itself, since the latter selects [the logical apparatus] by means of which it is expressed; thus, *we have no "rationality principle"* which allows a *preliminary* choice between theories on the basis of external criteria like "consistency" or "coherence", and *any theory can, in principle, justify itself*» ([29], p. 201; emphasis added).

Now, the central idea of dpGP is that a non-classical logic can be characterized by a certain specific linguistic meta-concept that is *different from (and compatible with) the concept of truth*. As will be clear, such concepts are meant as *not determined* by

specific domains (thus safeguarding the normativity and universality of logic), while corresponding to actual deductive practices. For example, intuitionistic logic is viewed as dealing with the concept of *constructive proof*, quantum logic with the concept of *empirical testability*, many-valued logics with the concept of *epistemic probability*. But all these concepts are not conceived as characterizing a particular domain (or employable only in particular domains). Rather, they just characterize a particular use/aspect of one and the same language; analogously, they characterize a particular deductive practice that makes use of that language. Also, these different concepts are viewed as *depending* in an important sense on the concept of (classical) truth (while not being reducible to it).[2] Such dependence is on the whole what produces a real compatibility-scenario in dpGP, as we will see. In fact, this situation is *analogous* to the one involving CL and modal logics, where the modalities of a sentence presuppose the notion of truth as expressed in CL.

Before giving the details of dpGP, we can already make two relevant considerations in connection to B&R's Logical Pluralism. First of all, both Dalla Pozza and B&R want to give to a number of different logics their due philosophical dignity. However, this aim is purportedly obtained in B&R's by the introduction of the notion of "case", which is based on the idea that those logics all deal with (different clauses for) *truth*. This - we noticed - is the reason why their pluralism can lead to relativism about truth, which of course is undesirable. Such an outcome is clearly not possible in dpGP, where each logic is endowed with a characteristic concept that however is different from the concept of truth. Secondly, both B&R and Dalla Pozza consider different logics as dealing with one and the same language. However, differently from B&R's, we will see that dpGP can guarantee that such language is the same in a deep sense, so that different logicians can indeed understand each other and do not equivocate.

We can say that an instance of Global Pluralism is useless if it cannot meet the following *two constraints*. First, it should show how deviant logics can be *adequately* characterized. Second, it should show that the so-characterized deviant logics can indeed coexist with classical logic in a compatibility-scenario. If the first constraint is missing, then a deviant logician would be unsatisfied with the rendering of his own logic. If the second constraint is missing, then obviously there would not be an interesting pluralism at all, for the universality of logic would be impossible and our rationality should be considered defeated. We hold that dpGP is a promising extensive approach

[2]The latter is meant to be exhaustively described by Tarski's truth theory [75] and adequately captured only by CL.

able to satisfy both constraints.

In order to obtain this result, Dalla Pozza developed two kinds of analysis, strictly connected. The first one is philosophical, concerning the specific meta-concept characterizing a specific logic. The second one is instead a semiotic-linguistic analysis, based on the insight (also present in B&R's) that different logics deal with different *features* of the same language. Since we have already sketched the philosophical approach and since the two analyses are tightly connected, we will now focus on the linguistic one, while other relevant philosophical aspects will be clear and emphasized throughout. Also, we will focus on how dpGP treats the supposed conflict between CL and IL, because this is in fact sufficient to give a detailed idea of the whole approach. Let us notice however that dpGP has been already applied also to other logics (e.g., Quantum logic [29] and Substructural logics [7]) and that there is a lot of work in progress on the subject (see [8, 59, 60]).

From what we said throughout, it seems that, when one wants to claim that two systems of logics are compatible, one can only show either that a complete inter-translation is possible which save the theorems of the systems (as in Quine's optimistic case) or that one of the system can be viewed as an extension of the other (as in the case of modal systems and CL). However, Dalla Pozza showed that it is possible to endorse the Quinean meaning-change thesis and to reach a compatibility in a third - *integrative* - way.

To this aim, he developed the idea of a *pragmatic* analysis of sentences that was originally suggested by Frege [27, 28] and then refined by Reichenbach [65]. According to this conception, every sentence can be analyzed in terms of two distinct linguistic components (or features), each of them having different *semiotic* roles: the *pragmatic mood* and the *radical formula*. The latter expresses the descriptive content of the sentence, whereas the former simply *shows* how the proposition expressed by the sentence is used (e.g., assertions, questions and norms are modalities in this pragmatic sense). The pragmatic mood, as a consequence, does not give any contribution to the descriptive content of the sentence. Therefore, whereas the radical has a truth-value, the whole pragmatic sentence (obtained by adding the mood sign to the radical) can only be *justified* or *unjustified*. An example of such *assertive* sentences is $\vdash \phi$, where $\vdash$ is the pragmatic sign for assertions and ϕ the radical formula. Using this approach, Frege proposed in fact his system for CL in terms of assertive sentences. However, in Frege's and Reichenbach's models, no assertive sentence contains more than one pragmatic sign; in other words, their models apply only to *elementary* assertive sentences. Dalla

Pozza's approach goes beyond this limit by introducing a formalized Pragmatic Language (PL) endowed with *pragmatic connectives*, thus allowing also the construction of *complex* assertive sentences.[3]

This step turns out to be crucial in showing that a logic is possible which deals with concepts that are different from the concept of truth. Indeed, in this way, it is possible to develop a *pragmatic logic* by defining the conditions of *justification* of an assertive sentence (in a way analogous to the one used for defining the conditions of truth for CL's sentences), combining within *one and the same* language both the descriptive aspect and the pragmatic/illocutionary aspect of a sentence - in a logical-semiotic perspective already suggested by Morris [40].

Let us describe in more details the pragmatic language introduced by Dalla Pozza to treat the supposed conflict between CL and IL. We will see that Dalla Pozza's crucial philosophical idea - that non-classical logics can be formulated as dealing with *pragmatic concepts* of language - can find a suitable instantiation in PL, whose apparatus is indeed able to express the formal properties of such concepts. On a formal level, this (propositional) pragmatic language is obtained as an extension of the standard language for (classical) propositional logic, by adding to its vocabulary a new class of pragmatic signs: the assertion-sign $\vdash$ and the pragmatic connectives $\sim$, $\cap$, $\cup$, $\supset$, $\equiv$. These connectives can be intuitively viewed as the pragmatic counterpart of the usual connectives. More specifically, $\sim$ can be said to be the pragmatic negation, $\cap$ the pragmatic conjunction, $\cup$ the pragmatic disjunction, $\supset$ the pragmatic implication and $\equiv$ the pragmatic biconditional. Two kinds of formation-rules are then introduced, which recursively define two kinds of wffs: the radical formulas (identical to CL's formulas) and the assertive formulas. If ϕ and ψ are radicals, then examples of the latter are: $\vdash (\phi \wedge \psi)$; $\sim\vdash \phi$; $(\vdash \phi) \cup (\vdash \psi)$; $((\vdash \phi) \cap (\vdash \psi)) \supset (\sim\vdash \phi)$, which one can intuitively read as "$\phi \wedge \psi$ is asserted/assertable", "ϕ is not asserted/assertable",[4] "ϕ is asserted/assertable or ψ is asserted/assertable", et cetera.[5] Any assertive formula contains at least one radical formula as a proper sub-formula.

After this, the semantic and pragmatic rules are introduced. The first ones are

[3]The apparatus of PL is able to treat a number of different mood-signs corresponding to different *illocutionary* acts, e.g., questions, commands, norms, et cetera. In the following we will focus on assertions.

[4]Not to be confused with "$\neg\phi$ is asserted": $\vdash \neg\phi$.

[5]Let us stress that the assertive formulas are conceived by Dalla Pozza as «purely logical entities, without making reference to the speaker's intentions or beliefs» ([19], p. 83). Thus, assertions are considered as "impersonal acts".

identical to the usual classical Tarskian rules and specify the truth-conditions (only for radical formulas) through an assignment function σ, thus regulating the *semantic interpretation* of PL. The second ones regulate the *pragmatic evaluation* π, specifying the justification-conditions for the assertive formulas in function of the σ-assignments of truth-values for their radical sub-formulas (in the following, J stays for "justified" and U for "unjustified"):

JR1- Let ϕ be a radical formula. Then, $\pi(\vdash \phi) = J$ iff a proof exist that ϕ is true, i.e., that σ assigns to ϕ the value "true". Hence, $\pi(\vdash \phi) = U$ iff no proof exists that ϕ is true.

JR2- Let Φ be an assertive formula. Then, $\pi(\sim \Phi) = J$ iff a proof exists that Φ is unjustified, i.e., that $\pi(\Phi) = U$.[6]

JR3- Let Φ and Ψ be assertive formulas. Then:
(i) $\pi(\Phi \cap \Psi) = J$ iff $\pi(\Phi) = J$ and $\pi(\Psi) = J$
(ii) $\pi(\Phi \cup \Psi) = J$ iff $\pi(\Phi) = J$ or $\pi(\Psi) = J$
(iii) $\pi(\Phi \supset \Psi) = J$ iff a proof exists that $\pi(\Psi) = J$ whenever $\pi(\Phi) = J$[7]
(iv) $\pi(\Phi \equiv \Psi) = J$ iff $\pi(\Phi \supset \Psi) = J$ and $\pi(\Psi \supset \Phi) = J$.

As it is evident, the pragmatic evaluation is defined in terms of the intuitive notion of proof, which is defined in terms of the notion of truth (see JR1). It is relevant to emphasize that in PL the latter concepts are precisely differentiated using different syntactic counterparts for them: the radical and the assertive formulas. As summarized by Dalla Pozza:

> «It is important to observe that the notions of truth and justification defined in the semantics and pragmatics of [PL] respectively, also exhibit *a different logical behaviour* [...] In fact, the semantic assignment function [...] satisfies the truth rules [...] which define the meaning of the semantic connectives by assuming that they conform to classical logical laws; the pragmatic assignment function [...] satisfies the justification rules JRI-JR3 [...] which define the meaning of the pragmatic connectives by assuming that they conform to intuitionistic-like logical laws» ([19], p. 103; emphasis added).

[6] That is, by JR1, iff a proof exists that no proof exists that the radical is true (only for Φ *elementary* assertive formulas).

[7] That is, iff a proof exists that if Φ is proved then Ψ is proved.

The crucial tenet behind the linguistic apparatus is then that *a proof is always a proof of the truth of a radical formula* or - in other words - that the general notion of proof presupposes the notion of truth. This idea is not new in philosophical debates on verificationist theories of meaning,[8] but once it is expressed in such a linguistic apparatus, it becomes more interesting in relation to the compatibility between CL and IL. In order to appreciate this, we first need to describe some crucial properties of Dalla Pozza's pragmatic language.

The language just described establishes a *general* framework. The notion of proof in the pragmatic rules is still meant to be undetermined (except for the "correctness criterion": if there is a proof that ϕ is true, then ϕ is true). In other words, what we have described so far is a purely formal pragmatic language useful to rigorously express those general properties of the concept of proof that are *independent* from a particular procedure of proof. More specifically, such general properties are meant to hold both for empirical proofs (verifications) and logical proofs (demonstrations), as well as for actual or potential proofs. In this sense, the pragmatic rules settle the normative constraints that each and every notion of proof must satisfy in relation to the notion of classical truth.

In the same general perspective, Dalla Pozza gives the definition of *pragmatic validity* in PL. In a nutshell, an assertive formula Φ is pragmatically valid in PL iff $\Phi = J$ for every semantic interpretation of the radical formula and every pragmatic evaluation.

Given the general framework just described, he then succeeds in showing that both CL and IL can be adequately expressed in PL, thus making them coexist in one and the same language. To this aim, he first specifies the undetermined notion of proof as exclusively being a *classical logical* proof, so that only classical procedures of proofs are used in the metalanguage for PL. Then, he succeeds in expressing both CL and IL within PL by individuating two different (assertive) formal calculi. The first one, which he calls Assertive Classical Propositional Calculus (ACPC), consists of the set of all *elementary* assertive formulas of PL, endowed with one of the usual sets of axioms for CL in assertive disguise (e.g., the axiom $\vdash (\phi \to (\psi \to \phi))$). The second one, which he calls Assertive Intuitionistic Propositional Calculus (AIPC), consists of the set of all assertive formulas of PL (i.e., both elementary and complex) which contain only *atomic* radicals, endowed with one of the usual sets of axioms for IL in assertive disguise (e.g., the axiom $\vdash \phi \supset ((\sim\vdash \phi) \supset\vdash \psi)$, with ϕ and ψ *atomic*). A suitable rule for the

[8] See, e.g., [68],[16],[45],[11].

pragmatic renderings of modus-ponens is of course included in both calculi ($\frac{\vdash\phi, \vdash(\phi\rightarrow\psi)}{\vdash\psi}$ for ACPC; $\frac{\vdash\phi, \vdash\phi\supset\vdash\psi}{\vdash\psi}$ for AIPC, with ϕ and ψ *atomic*). In this way, ACPC and AIPC are shown to correspond respectively to CL's calculus and IL's calculus.

This being made, he shows the following two crucial results. First, that every theorem of ACPC is a pragmatically valid *elementary* assertive formula of PL and vice versa. Second, that every theorem of AIPC is a pragmatically valid assertive formulas of PL containing only *atomic* formulas and vice versa. PL is then able to adequately express both CL and IL, and a theorem of completeness for the two calculi is also proved. PL in fact constitutes a common neutral formal platform - regulating one and the same abstract notion of proof. Such notion is then used to capture both logics by introducing some constraints on the assertive formulas, these constraints corresponding to *the different properties of the two meta-concepts* underlying the two logics. Indeed, ACPC can be said to state the properties of the concept of "classical proof" (hence, of - classical - truth), whereas AIPC can be said to state the properties of the concept of "constructive proof".

Thus, we can say that Dalla Pozza's treatment of the relation between CL and IL corroborates Quine's meaning-change thesis, for it shows that these two logics can be viewed as dealing with different operators in a threefold sense. Indeed, the operators are semiotically *differentiated*, endowed with *different* meanings and meant to behave in agreement with the properties of *distinct* meta-concepts. This is a truly relevant result, if one recalls that Quine's meaning-change thesis was not defended with positive arguments, but only through the purported absurdity of the opposite thesis (the meaning-invariance thesis).

Now, it is clear that the notion of truth is classically-oriented,[9] while the notion of justification is intuitionistically oriented. However, this does not mean that a classical and an intuitionistic logician cannot be viewed here as understanding each other. Rather, the opposite seems to hold.

The central clue to see this is that both the intuitionistic and the classical logician can possibly agree that a proof is always a proof of the truth of a sentence. For the

[9]As noticed by Dalla Pozza himself [19], this observation - together with the proposed constraint that only *classical* logical procedures of proofs are accepted in the metalanguage for PL - shows that non-classical procedures of proof are *not* required in order to express intuitionistic logic. An analogous point of view is defended by Sundholm in [73], where he maintains that constructivism and intuitionistic logic must not necessarily go hand in hand. Besides, these results strengthen Quine's observation that one can practice «a very considerable degree of constructivism without adopting intuitionistic logic [...] On this approach, constructivist scruples can be reconciled with the convenience and beauty of classical logic» (see §2.2).

latter, this is nothing but his natural habit of mind. For the former, this is something he can accept as far as it allows his own theorems to be captured in PL. Thus, more specifically, the meaning of *any* radical formula can be certainly said to be the same for both of them, since it is defined in one and the same way through the (classical) semantic interpretation of PL. The only disagreement between them reduces as to which *type* of radicals they accept as *possibly assertable*. For the intuitionistic logician, only atomic radicals can work. For the classical, instead, also molecular radicals can be accepted. We can say thus that, from a classical perspective, intuitionistic logic seems explicable and *understandable* as the logic that underlies a particular kind of illocutionary act/logical proof, which one can call "constructive assertion" (obtained through the restriction that the radical there involved is atomic). What happens instead from an intuitionistic perspective? Here classical logic seems explicable and *understandable* simply as the logic that deals with that kind of assertions whose justification does not necessarily need an atomic construction of the proofs. In other words, the intuitionistic logician needs only to accept that the classical logician uses a different notion of *justification* (rather than truth), namely one that allows the latter to assert *kinds* of things that his notion cannot treat, i.e., non-atomic radicals. However, such things are understandable from the intuitionistic logician in the same way as they are understood by the classical one, once the common (classically-oriented) notion of truth is introduced. Consider the case in which both logicians are presented with a non-atomic radical, namely R: $\phi \vee \neg\phi$. Surely, the intuitionistic notion of justification cannot be used to assert R, simply because it is a non-atomic (molecular) radical. The classical notion of course can be used to assert R. Does this mean that then R is true for one and not true for the other? No. The meaning of the radical is the same for both, it is the classical meaning. In other words, the radical *is* true, for both of them. There is no disagreement as to which are the true radicals and which the false ones. The difference is only about which radicals, among the true ones, can be asserted in a justified manner.

What changes thus is only the notion of assertion. Does this lead to some form of relativism about assertions? We believe not. To show this, let us start by recalling the desideratum (c) above, describing the crucial property of any Global Pluralism: ensure a logic to be domain-independent, that is ensure that «logical principles [...] apply irrespective of subject-matter», in Haack's words ([32], p. 223). This point can be said to be satisfied by Dalla Pozza's approach. Once a logic is shown to be characterized by a *specific* meta-concept (compatible with other meta-concepts), then its laws and principles will hold independently from the domains of application. Indeed, those laws

and principles are not viewed as valid *of* a domain, but rather as properties of the meta-concept itself. For example, LEM is viewed here as a property of the meta-concept "truth". Analogously, the fact that LEM does not hold in IL is viewed here as a consequence of the properties of the meta-concept "constructive proof", which is shown sufficient to characterize IL. In this sense, it is obvious why Global Pluralism naturally leads to a compatibility-scenario: as we already noticed, the correct application of LEM in *one certain sense* (say, the CL sense) is in principle trivially compatible with its not being applicable in another - different - sense (say, the IL sense). In other words, these logics so characterized can be *consistently* viewed as talking of one and the same area of discourse from different and compatible *perspectives*, rather than as endorsing different subsets of formulas within one and the same perspective - as in the case of Logical Pluralism. Thus, the different meanings of the logical operators are here *integrated* in a wider logical perspective, which not only makes CL and IL compatible as a matter of fact, but also represented within *one and the same* (pragmatic) language.

Now, both ACPC's and AIPC's formulas are *assertive* formulas.[10] The meaning of these formulas is well defined by the semantic and pragmatic interpretations, in function of the notions of justification and truth. The compatibility between the two systems is obtained neither by translating one calculus into the other nor by showing that the class of theorems of one calculus is a proper subset of the class of theorems of the other one (as in the case of CL and modal logics). The two logics are indeed precisely separated in PL. Therefore, the choice of one specific logic depends only on *how* one *needs* to talk about a certain domain, that is on what he is interested in. If one needs a constructive proof of a sentence for a certain domain (and there are plenty of reasons why one would need this), then he should use IL. If one needs a non-constructive proof (and there are plenty of reasons why one would need this, as well), then he should use CL. As we said above, different logics are viewed here as characterizing particular *uses/aspects* of one and the same language.

Thus, going back to the issue of relativism about assertions, we can say that, *once the common framework is shared*, both the intuitionistic and the classical logician can in principle use the other's logic, if they are interested in assessing a particular domain from the perspective of the other one. *This means that the respective kinds of assertions are not mutually exclusive*, that is they are not different notions of one and the same

[10]Behind such treatment of CL, there is the tenet that (classical) logic deals with the *recognition* of the (classic) truth-value of a sentence, rather than with the truth-value itself. This was the original conception of logic as developed by Frege (see [72]). Recently, it has been endorsed by Martin-Löf [37].

thing. They are different things tout court, but *equally employable by users*, given that they belong to the same language in a deep sense. Choosing one of them is not a metaphysical decision (as in the case of intuitionistic "truth" as opposed to classical truth). Rather, it is a practical choice, motivated by what kind of logical analysis one is interested in. Thus we can say that we have now a way to reply to Resnik's question (introduced above):

> «Suppose that until now my mathematical proofs used non-constructive principles, but now I announce that I will restrict myself to constructively acceptable proofs. Have I revised my logic, while continuing to mean the same by "not" and "or" or have I decided to use those words with a different meaning?» ([66], p. 180).

Dalla Pozza's pragmatic language allows for the second answer.

In the first part of this work, we argued - contra Quine - that endorsing the meaning-change thesis does not lead necessarily to the rejection of a system. Also, we noticed that a deviant logician typically interprets the thesis as a challenge to the inspiring reasons of the system he supports; as if an adequate way to safeguard such reasons was possible only by supporting the meaning-invariance thesis. Hence, we believe that the results of dpGP are particularly interesting in that they allow to satisfy the meaning-change thesis (and there are good reasons for that), while safeguarding the compatibility between logics, the reciprocal understanding of different logicians and the notion of (classical) truth itself. As already suggested, the same result is obtained by Dalla Pozza for other systems, by characterizing a logic in function of a specific pragmatic meta-concept which is different from the concept of truth, but which presupposes it. These characterizations are all formally possible thanks to the flexibility of PL, which can be *specified* to treat a number of different *kinds* of assertions - corresponding to different kinds of meta-concepts. A certain specification of PL can be obtained through syntactical constraints on the assertive formulas and/or through a particular (material) characterization of the notion of proof as it appears in JR1-JR3. In this way, Linear logic has been classified as the logic for *undetermined illocutionary force*, Quantum logic as the logic for *empirical testability*, Many-valued logics as the logic for *epistemic probability*, and - as we have seen - IL as the logic for *constructive assertions*.

One may wonder whether the two constraints we stated above for any acceptable form of Global Pluralism are met in dpGP. The second constraint (compatibility between CL and IL) can be said met through the compatibility between ACPC and AIPC.

As for the first one, the fact that IL'c calculus can be completely translated into PL through AIPC seems enough to claim that an adequate characterization of IL has been reached (Dalla Pozza showed that AIPC has an Heyting algebra underlying). Of course, an intuitionistic logician could still oppose such a treatment of his logic on the grounds that it makes a mockery of his own reasons. In fact, what PL shows is that IL does not deal with a notion of truth different from the classical one (rather with the same notion *and* in a compatible way). Thus, if one still wants to defend a *genuine* challenge to CL (to the extent that CL should be considered completely erroneous), one can certainly do so; not differently from one who wants to be a monistic. More precisely, thus, what Dalla Pozza showed is that *if* the intuitionistic logician agrees in stipulating that his own logical beliefs are not about the notion of truth but about the notion of constructive proof (as based on a classical notion of truth), then there is a way to express both CL and IL in a compatible way.

The relevance of this result must not be underestimated. In the usual approach to the problems concerning CL and IL with respect to mathematics, one is forced to maintain that the set of truths accepted from a classical mathematician properly includes the set of truths accepted from an intuitionistic mathematician. The inevitable outcome of this view is that no agreement on the nature of mathematical truth can be reached and - more importantly - that there are mathematical sentences that are true for someone and not true for others. So that, if one wants to preserve an acceptable idea of rationality, one can only hope that one of the two approaches can be shown erroneous. However, as Quine has argued, one cannot really obtain a definitive foundation for one's own logical beliefs (classical or deviant ones). At most, we continue, one can try and show that a certain set of logical principles can be interpreted in a new and acceptable way while making them still work with the usual logical engine *and* compatible with the principles of other logics. According to us, Dalla Pozza succeeded in showing this.[11] After his treatment of IL, there is a way *not* to be in disagreement as to which are the mathematical truths. One can only *prefer* to use in mathematics the constructive methods, rather than the classical ones, for certain specific aims. But this does not mean that a dispute still exists within the philosophy of logic or mathematics, at most within the philosophy of mathematical *practice*.

We must recall however that dpGP excludes from the scope of logic those systems that are usually described as calculi. Dalla Pozza, thus, endorses Quine's meaning-

[11] Let us notice that such reinterpretation can be in principle developed as based on a deviant notion of truth, rather than on a classical one. We are not aware of any such attempt.

change thesis (which we showed to be *independent* from a particular conception of logic), while considering "logics" only those systems able to capture an acceptable type of deductive practice.[12] Let us stress however that the rational reconstruction (explication) of the properties of a meta-concept in PL must *not* be viewed as *determined* by the corresponding deductive practice (like in the Logical Pluralism view), but only *inspired* from that, in line with the canonical distinction between the "context of justification" and the "context of discovery". The philosophical justification of a logic, of course, does not depend in dpGP on the existence itself of a deductive practice that is in agreement with it. Rather, it is essentially based on *normative* constraints that are *imposed* to the behaviour of a notion (meta-concept). For example, through the rules JR1-JR3, one is *not* trying to capture the intuitionistic deductive practice. Rather one is just isolating, based on what one considers to be acceptable rational criteria, those properties that *any* informal notion of proof *must* satisfy - from the armchair, so to say.[13] It is just *after* this reconstruction that IL is shown expressible in this framework through the *extra*-constraint that the radicals must be atomic. The whole enterprise is thus a *normative* one and the fact that JR1-JR3 are in fact intuitionistic-like rules should be considered, *prima facie,* a coincidence.[14] As indicated by Garola:

> «[Dalla Pozza's] perspective has the advantage of making it easier to find an effective interpretation of a given formal language endowed with a non-standard formal semantical interpretation whenever the metalinguistic concept that *has guided* the choice of axioms and inference rules is specified (the effective interpretation will usually coincide with an intended interpretation underlying the language, which is thus *clarified* and *systematized*» ([29], p. 203; emphasis added).

In a sense, thus, we can rephrase Dalla Pozza's thesis using the following restricted version of Carnap's Principle of Tolerance:

> Everyone is at liberty to build his own logic, as he wishes. All that is required of him is that, if he wishes us to accept it, he must *not only* state

[12] In agreement with his thesis: "Every logic is a calculus, but not all calculi are logics".

[13] In a way that is analogous to the one used by Tarski for the notion of truth. The semantic assignment function σ - through the recursive clauses - isolates the properties that any notion of truth must satisfy.

[14] The rules are only *analogous* to the rules given in Heyting's definition of "truth" for IL [33]. In Dalla Pozza's words, they are *intuitionistic-like* rules (notice that they indeed do not mention the constraint that the radical must be atomic).

> his methods clearly, *but also* show that an interesting intuitive meta-concept and a relative type of deductive reasoning are *possible* which are captured by his logic in agreement with classical logic.

The latter constraint is highly desirable in his view to avoid the threat of the destruction of the *unity of logic*, which he considers necessary to safeguard a strong conception of rationality.[15] This aspect is analogous in fact to the one suggested by B&R about the admissibility of cases, that is that only those types of cases which can be shown reciprocally compatible are admissible. However, it seems to us that B&R's pluralism allows in principle for a smaller number of compatible systems than dpGP, for it is clearly more difficult to obtain a compatibility between systems that all deal with one thing, namely "truth". Let us emphasize how crucial is the difference between B&R's and Dalla Pozza's in this respect:

> «[W]henever one embraces [dpGP], some changes in the terminology seem necessary; to be precise, the noun "semantics" should be reserved to a semantical apparatus based on some standard [classical] formal semantical interpretation, while *similar* apparatuses based on non-standard formal semantical interpretations could be classified as pragmatics (in a broad sense)» ([29], p. 203; emphasis added).

Such a broad sense of "pragmatics" has a counterpart in the flexibility of PL, which is able to treat different kinds of meta-concepts expressible in one and the same language. This, in turn, allows for a classifications of non-standard languages, since in fact these can be classified according to the meta-concepts formalized through their algebraic structures.

As for CL, it clearly has a privileged role in dpGP: Dalla Pozza in fact shares Kneale and Kneale's view [35] that «it is rather difficult to imagine how we can give up classical logic and go on reasoning» ([19], p. 103). However, if this is the price one has to pay to reach a possible *general* way to *understand* and *resolve* conflicts in a compatibility-scenario, we believe it is worth paying it.

[15] Paraconsistent logic - as defended by Priest - seems not easily interpretable as a logic compatible with CL, in this sense. Notoriously, the relation between these two systems is particularly subtle. In dpGP's terms, it is difficult to understand how a meta-concept can be found here, which is really *non semantical* and in agreement with the (classical) meta-concept of truth. In Haack's terms, it seems we have a case of (tricky) variety of deviance.

Chapter 5

Conclusions

This work has been concerned with the difficult questions raised by the existence of different logics. We started out our investigation by noticing that, in spite of the philosophical relevance of the problem, there is in fact little ground for optimism in looking for an acceptable resolution of the issues involved.

We offered a critical overview of the most important conceptions on plurality, starting from Quine's complex and influential *meaning-change* thesis. A considerable part of our work has been dedicated to the interpretation of the thesis, which revealed to be difficult not only to assess, but even to understand. We have shown that Quine's thesis constitutes a challenge for those logicians who hold that a revision of classical logic is possible. According to the thesis, indeed, the systems of classical logic can only be rejected as unscientific or meaningless, and *not* refined as inadequate. From the point of view of deviant logicians, this position makes a mockery of their own reasons. They hold indeed that their logics do not involve a change of meaning of logical operators. Rather, they maintain that the validity of certain classical logical principles is questioned by them in the *same sense* in which it is affirmed by the classical logician. Thus, they purport a *genuine* critique to classical logic.

We suggested that two kinds of conflicts correspond respectively to the meaning-change thesis and to the meaning-invariance thesis: a *partial* conflict and a *genuine* conflict. Differently from Quine, however, we argued that the meaning-change thesis does not necessarily lead to the rejection of a logic. An alternative outcome is indeed Global Pluralism, which seems to be an interesting way to reach a compatibility between different logics while giving them a philosophical dignity.

We assessed the strength of Quine's rationale and the objections raised against

his position, and found that there are indeed good reasons both for and against the meaning-change thesis. However, the meaning-invariance thesis has to face a particularly intricate problem. Indeed, any promising attempt to defend such thesis must try to individuate a core-meaning for a logical operator, viewed as a common ground for the operators of different systems of logic. We have shown that this notion is not only obscure *per se* but also hardly defensible, due to the absence of an acceptable definition of "meaning" in the first place. Quine's view, however, is able to tolerate the absence of this definition in a better way. Our conclusion was that Quine's challenge should be considered unresolved.

We then maintained that some form of logical pluralism would be desirable, in that it would allow for the existence of different logical systems as all correct. However, we argued against Local Pluralism, on the grounds that it leads to the unpleasant conception of logical principles as dependent from specific areas of discourse, thus challenging the normativity and universality of logic. We then analyzed Beall and Restall's logical pluralism, according to which different logics all deal with the notion of truth while being equally good. This form of pluralism is however not sufficiently fine-grained and can lead to a form of relativism about truth, which we consider unacceptable.

We concluded that, if only unsatisfactory outcomes seem possible by trying to reject the meaning-change thesis or by accepting it but for a defence of one specific system, then Global Pluralism - which emerges from *accepting the meaning-change thesis but not to defend one specific system* - is a quite interesting possibility. In Dalla Pozza's Global Pluralism, alternative logics are viewed as dealing with specific pragamatic meta-concepts, which are distinct from the concept of truth as captured by classical logic, while presupposing it. We showed how this idea is practically possible by introducing Dalla Pozza's pragmatic language, where both classical logic and alternative logics can be adequately and rigorously expressed by using distinct semiotic signs in an integrative framework. This solution corroborates the meaning-change thesis, while giving to a number of different logics a philosophical dignity. We thus maintained that Dalla Pozza's suggestion is the most preferable and promising analysis in that it allows a compatibility between different logics, while safeguarding the traditional view of logic as normative and applicable irrespective of subject matter.

Bibliography

[1] Aberdein A. - Read S. (2009), "The Philosophy of Alternative Logics", in The Development of Modern Logic, ed. L. Haaparanta, Oxford UP (2009), pp. 613-723.

[2] Arnold J. - Shapiro S. (2007), "Where in the (world wide) web of belief is the law of non-contradiction?", Nous; 41:276-297.

[3] Barwise J. - Etchemendy J. (1999), "Language, Proof and Logic", CSLI-publications.

[4] Beall J.C. - Restall G. (2000), "Logical pluralism", Australasian Journal of Philosophy 78, pp. 475– 493.

[5] Beall J.C. - Restall G. (2001), "Defending logical pluralism", In J. Woods and B. Brown (eds.), "Logical Consequence: Rival Approaches", Proceedings of the 1999 Conference of the Society of Exact Philosophy. Stanmore: Hermes, pp. 1–22.

[6] Beall J. C. - Restall G. (2006), "Logical Pluralism", Clarendon Press, Oxford.

[7] Bellin G. - Dalla Pozza C. (2002), "A pragmatic interpretation of substructural logics", in "Reflections on the Foundations of Mathematics, Essays in Honor of Solomon Feferman", W.Sieg, R.Sommer and C.Talcott eds. ASL Lecture Notes in Logic ; 15, 2002.

[8] Bellin G. - Biasi C. (2004), "Towards a logic for pragmatics: Assertions and conjectures", In: Journal of Logic and Computation Vol. 14, 4, 2004, pp. 473-506.

[9] Belnap N. D. (1961) "Tonk, Plonk, and Plink", Analysis, 22, pp. 130-134, 1961.

[10] Berto F. (2006), "Teorie dell'Assurdo. I rivali del Principio di Non-Contraddizione", Roma, Carocci.

[11] Bradley R. - Swartz N. (1979), "Possible worlds. An introduction to logic and its philosophy", Blackwell, Oxford.

[12] Brouwer L. E. J. (1923), "On the Significance of the Principle of Excluded Middle in Mathematics, Especially in Function Theory", reprinted in J. van Heigenoort (ed.) "From Frege to Gödel", Harvard University Press, Cambridge, 1967, pp. 334–345.

[13] Carnap R. (1928), "Der Logische Aufbau der Welt", Leipzig, Felix Meiner Verlag (English translation as "The Logical Structure of the World - Pseudoproblems in Philosophy", University of California Press, Berkeley, 1967).

[14] Carnap R. (1934), "Logische Syntax der Sprache", Springer-Verlag, Vienna; Eng. trans. as "The Logical Syntax of Language", Open Court, Chicago, 2002.

[15] Carnap R. (1963), "Charles Morris on Pragmatism and Logical Empiricism", collected in P. A. Schlipp (ed.), "The Philosophy of Rudolf Carnap I", The Library of Living Philosophers Inc., Evanston, Illinois.

[16] Carnap R. (1966), "Philosophical Foundations of Physics", Basic Books Inc., New York.

[17] Cook R. (2010), "Let a thousand flowers bloom", Philosophy Compass 5 (6): 492-504.

[18] Dalla Chiara M. L. (1974), "Logica", ISEDI, Milano.

[19] Dalla Pozza C. - Garola C. (1995), "A Pragmatic Interpretation of Intuitionistic Propositional Logic", Erkenntnis, 43, 1995 (81-109).

[20] Dalla Pozza C. (1997), "Una logica pragmatica per la concezione "espressiva" delle norme", in: "Logica delle Norme" (A. Martino, Ed.), SEU, Pisa, 1997.

[21] Dalla Pozza C. (2002), "Logiche devianti e logiche substrutturali: aspetti filosofici", abstract of a Seminar at the Department of Philosophy, University of Genoa, May 3th 2002. Web-address: http://www.dif.unige.it/epi/con/dallapozza.pdf [accessed on May 11th, 2010].

[22] Dummett M. (1975), "What is a Theory of meaning? 1", collected in S. D. Guttenplan (ed.), "Mind and Language", Oxford University Press, Oxford, pp. 97-138.

[23] Dummett M. (1978), "Truth and other enigmas", Duckworth, London.

[24] Dummett M. (1991), "The Logical Basis of Metaphysics", Harvard University Press, Cambridge, Mass.

[25] Field, H. (2008), "Saving Truth from Paradox", OUP, Oxford.

[26] van Frassen B. C. (1969), "Presuppositions, Supervaluations, and Free Logic", in "The Logical Way of Doing Things", ed. Lambert K. (1969), Yale U.P..

[27] Frege G. (1879), "Begriffschrift", Nebert. Halle.

[28] Frege G. (1893), "Grundgesetze der Arithmetik 1", Pohle, Jene (Olms, Hildesheim, 1962).

[29] Garola C. (1992), "Truth versus Testability in Quantum Logic", Erkenntnis, 37, 1992 (197-222).

[30] Gödel K. (1933, "Eine Interpretation des Intuitionistichen Aussagenkalkulus", Ergebnisse eines Mathematischen Kolloquiums IV, 39-40.

[31] Haack S. (1974), "Deviant Logic", Cambridge University Press, Cambridge.

[32] Haack S. (1978), "Philosophy of Logics", Cambrdige University Press, Cambridge.

[33] Heyting A. (1956), "Intuitionism. An introduction", North Holland, Amsterdam.

[34] Hjortland O. T. (2010), "The structure of logical consequence : proof-theoretic conceptions", PhD-thesis, University of St Andrews. Web-address: http://research-repository.st-andrews.ac.uk/handle/10023/892 [accessed on June 5th, 2010].

[35] Kneale, W. C. - Kneale, M. (1962), "The Development of Logic", Clarendon Press, Oxford.

[36] Lukasiewicz J. (1920), "O logice trójwartościowej" (in Polish), Ruch filozoficzny 5:170–171. English translation: "On three-valued logic", in L. Borkowski (ed.), "Selected works by Jan Łukasiewicz", North–Holland, Amsterdam, 1970, pp. 87–88.

[37] Martin-Löf Per (1996), "On the Meanings of the Logical Constants and the Justifications of the Logical Laws", Nordic Journal of Philosophical Logic, 1(1) (1996), pp. 11–60.

[38] McDowell J. (1981), "Anti-realism and the Epistemology of Understanding", collected in H. Perrett and J. Bouveresse (eds.), "Meaning and Understanding", De Gruyter, Berlin (1981), pp. 225-248.

[39] McGee V. - McLaughlin B. (1994), "Distinctions without a difference", Southern Journal of Philosophy XXXIII (supp) (1994), pp. 203-250.

[40] Morris C. (1963), "Pragmatism and Logical Empiricism", collected in P. A. Schlipp (ed.), "The Philosophy of Rudolf Carnap I", The Library of Living Philosophers Inc., Evanston, Illinois.

[41] Palladino D. - Palladino C. (2007), "Logiche Non Classiche", Carocci, Roma.

[42] Paoli F. (2003), "Quine and Slater on paraconsistency and deviance", Journal of Philosophical Logic, 32, pp. 531-548.

[43] Paoli F. (2007), "Implicational paradoxes and the meaning of logical constants", Australasian Journal of Philosophy, 85, 4, 2007, pp. 553-579.

[44] Peacocke C. (1981), "The theory of meaning in Analytical Philosophy", collected in G. Floistad (ed.), "Contemporary Philosophy I", Martinus Nijhoff Publishers, Dordrecht, pp. 35-36.

[45] Popper K. (1969), "Conjectures and Refutations", Routledge and Kegan Paul, London.

[46] Prawitz D. (1965), "Natural Deduction", Almqvist and Wiksell, Stockholm.

[47] Prawitz D. (1987), "Dummett on a Theory of Meaning and Its Impact on Logic", collected in B. M. Taylor (ed.), "Michael Dummett", Martinus Nijhoff Publishers, Dordrecht, pp. 117-165.

[48] Priest G. (1999), "What not? A defence of dialetheic theory of negation", in D. Gabbay and H.Wansing (eds.), "What is Negation?", Kluwer Pcademic Publishers, Dordrecht, pp. 101–120.

[49] Priest G. (2001), "Logic: One or Many?", in "Logical Consequence: Rival Approaches", ed. J. Woods and. B. Brown, Oxford: Hermes Science, 2001: 23–38.

[50] Priest G. - Beall J. C. - Armour-Garb B. (eds.) (2004), "The Law of Non-Contradiction. New Philosophical Essays", Clarendon Press, Oxford.

[51] Priest G. (2005), "Doubt Truth to be a Liar", Oxford University Press, Oxford; published to Oxford Scholarship Online: May 2006.

[52] Prior A. N. (1960), "The Runabout Inference Ticket", Analysis 21, 38–39.

[53] Putnam H. (1962), "The Analytic and Synthetic", In "Mind, Language and Reality: Philosophical Papers", Cambridge University Press, (1975).

[54] Putnam H. (1968), "Is logic empirical?", in R. Cohen and M. P. Wartofski (eds.), "Boston Studies in the Philosophy of Science" (Volume 5), Dordrecht: D. Reidel; reprinted as "The logic of quantum mechanics" in H. Putnam, Mathematics, Matter and Method, Cambridge University Press, 1976.

[55] Quine W. V. O. (1951), "Two Dogmas of Empiricism", originally published in The Philosophical Review, 60 (1951): 20-43. Reprinted in W.V.O. Quine, "From a Logical Point of View", Harvard University Press, 1953; second, revised, edition 1961.

[56] Quine W. V. O. (1960), "Word and Object", MIT Press, Cambridge, Massachusetts.

[57] Quine W. V. O. (1960b), "Carnap and logical truth", Synthèse, Vol. 12, Number 4 (December, 1960).

[58] Quine W. V. O. (1970), "Philosophy of Logic", Prentice-HaU Inc., Englewood Cliffs, New Jersey.

[59] Ranalter K. - Bellin G. (2003), "A Kripke-style semantics for the intuitionistic logic of pragmatics ILP", In: Journal of Logic and Computation 13(5), pages 755-775, 2003.

[60] Ranalter K. (2008), "A semantic analysis of a logic for pragmatics with assertions, obligations, and causal implication", In: Fundamenta Informaticae 84(3-4), pages 443-470, 2008.

[61] Read S. (2000), "Harmony and autonomy in classical logic", Journal of Philosophical Logic, 29 (2000), no.2, pp.123-154.

[62] Read S. (2006a), Review of "Logical Pluralism" [6], in Notre Dame Philosophical Reviews, 2006.05.16; Web-address: http://ndpr.nd.edu/review.cfm?id=6643 [accessed on May 2th, 2010].

[63] Read S. (2006b), "Monism: the one true logic", in "A Logical Approach to Philosophy: Essays in Honour of Graham Solomon", ed. D. DeVidi and T. Kenyon, Springer 2006, 193-209.

[64] Read S. (2008), "Harmony and Modality", in "Dialogues, Logics and Other Strong Things: Essays in Honour of Shahid Rahman", ed. C. Dégremont, L. Kieff and H. Rückert, College Publications 2008, pp. 285-303.

[65] Reichenbach H. (1947), "Elements of Symbolic Logic", The Free Press, New York.

[66] Resnik M. D. (2004), "Revising Logic", in Priest G. - Beall J. C. - Armour-Garb B. (eds.) (2004), pp. 178-193.

[67] Restall G. (2002), "Carnap's Tolerance, Meaning and Logical Pluralism", Journal of Philosophy 99, 426–443.

[68] Russell B. (1940), "An inquiry into Meaning and Truth", Allen & Unwin, London.

[69] Russell B. (1945), "History of Western Philosophy", G. Allen & Unwin Ltd., London. Reprinted in Routledge Classics (1994).

[70] Sambin G. - Battilotti G. - Faggian C. (2000), "Basic logic: Reflection, symmetry, visibility", Journal of Symbolic Logic 65: 979–1013.

[71] Sambin G. (2002), "Steps towards a dynamical constructivism", In "Proceedings of the XI International Congress of Logic", Methodology and Philosophy of Science, Krakow, August 1999, eds. P. Gärdenfors, K. Kijania-Placek, and J. Wolenski, 261–284. Dordrecht: Kluwer.

[72] Smith, N. J. J. (2009), "Frege's Judgement Stroke and the Conception of Logic as the Study of Inference not Consequence", Philosophy Compass, Volume 4, Issue 4 (2009), pp. 639–665.

[73] Sundholm G. (2004), "The proof-explanation of logical constants is logically neutral", Revue internationale de philosophie n° 230 (4/2004), pp. 401-410.

[74] Tarski A. (1936), "On the concept of logical consequence", in J. Corcoran (ed.), *Logic, Semantics, Meta-mathematics*, Hackett Publishing Company, Indianapolis.

[75] Tarski A. (1956), "The Concept of Truth in Formalized Languages", collected in "Logic, Semantics, Metamathematics", Oxford University Press, Oxford (1956).

[76] Varzi A. (2002), "On Logical Relativity", Philosophical Issues, 12 (2002), pp. 197-219.

[77] Zadeh L. A. (1974), "Fuzzy logic and its application to approximate reasoning", in: Information Processing 74, Proc. IFIP Congr. 1974 (3), pp. 591–594.