Voter Response to Iterated Poll Information

MSc Thesis (*Afstudeerscriptie*)

written by

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Abstract

We develop a formal model of opinion polls in elections and study how they influence voting behaviour, and thereby elections outcomes. We analyse two settings. In the first, we study a voter's incentives to misrepresent her preferences after receiving poll information. We vary the amount of information a poll provides and examine, for different voting procedures, when a voter starts and stops having these incentives. In the second setting, voters repeatedly update their ballot in view of a sequence of polls, and we analyse the effect of this process on the election outcome using both analytical and experimental methods. We consider several ways in which a voter may respond to poll information, and for different combinations of these response policies we study how opinion polls affect the properties of different voting procedures. Together, our results clarify under which circumstances opinion polls can improve the quality of election outcomes and under which circumstances they can have negative effects, due to the increased opportunities for strategic voting they provide.

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Contents

Chapter 1

Introduction

Some countries ban the publication of opinion polls in the days prior to an election because of their presumed effect on voting behaviour. In this thesis, we develop a formal model of opinion polls to study how they may influence voting behaviour, and thereby election outcomes. Our findings help to justify or criticise a ban on opinion polls.

A much-cited example of an election in which polls could have been decisive (but in the end were not) is the 2000 U.S. presidential election. The main candidates running for president then were George W. Bush and Al Gore. Bush won, but only by a tiny margin. A difference that would have been settled in favour of Gore if all supporters of a third, losing candidate, Ralph Nader, had voted for their second choice.

Like the U.S. presidential elections, most political elections are based on the *plurality rule*, under which voters vote for a single candidate and the candidate with the most votes wins. The plurality rule, however, often does not elect the most representative candidate. Other voting procedures do much better in this respect. For example, the *Copeland procedure* asks voters to rank all candidates. The score of a candidate is then computed as the difference between the number of opponents he will beat in a one-to-one majority contest and the number of opponents he will lose to in such a contest. The candidate with the highest score wins. That way, the Copeland procedure also takes voters' second choices into account.

All democratic voting procedures, however, are susceptible to tactical voting behaviour when voters' sole concern is getting the best outcome possible. A classical result in voting theory, the Gibbard-Satterthwaite Theorem, states that if there are three or more candidates, then for any nondictatorial voting procedure there are situations in which voters are better off by not reporting their true preferences (Gibbard, 1973; Satterthwaite, 1975). To recognise these situations, a voter needs to know exactly what everybody else is voting. Clearly, this is not a realistic assumption for actual elections with many voters, but also then voters often have some idea about the voting intentions of others. Opinion polls play an important role in the formation of these beliefs (Faas et al., 2008; Irwin & van Holsteyn, 2002). How much information may a poll provide before voters start knowing when they can benefit from voting tactically? In other words, does the Gibbard-Satterthwaite Theorem generalise to settings where voters only have partial information about other voters' ballots?

In plurality elections, the most natural way to communicate poll results is by publishing the number of votes each candidate would currently receive. Since we are interested in the whole range of possible voting procedures, we also consider other ways of communicating poll results. In Copeland elections, for example, we could publish the Copeland score of each candidate, or we could record how many copies of each possible ballot were received. Alternatively, we could publish the majority graph (a directed graph on the set of candidates that contains an edge from x to y if a majority of voters prefer x over y) or the weighted majority graph (in which each edge is labelled with the strength of the corresponding majority).

The aim of this thesis is twofold. We study how much information a poll may provide before it gives rise to tactical voting behaviour, and we study the effects of tactical voting behaviour on election outcomes. Ultimately, we would like to know under which circumstances opinion polls lead to less representative winners, and under which circumstances they lead to more representative winners.

1.1 Approach

We study opinion polls from the theoretical perspective of *social choice theory*, the formal study of methods for collective decision making (Arrow et al., 2002). To its machinery we add the concept of *poll information function*, a function mapping ballot information obtained via an opinion poll to a communicable format (e.g., a majority graph or a list of scores). From the poll information they receive, voters can infer certain things about the voting intentions of other voters.

We shall analyse two scenarios. In the first, we study a voter's incentives to misrepresent her preferences after receiving poll information. We vary the amount of information a poll provides and examine, for different voting procedures, when a voter starts and stops having these incentives.

In the second scenario, voters repeatedly update their ballot in view of a sequence of polls, and we analyse the effect of this process on the election outcome. We consider several types of responses to poll information: a *strategist* will submit a best response to what she knows about other voters' ballots; a *pragmatist* will support her favourite candidate from a small set of, say, two front-runners; and a *truth-teller* will always vote truthfully. For different combinations of these *response policies*, we study how polls affect the properties of a voting procedure using both analytical methods and simulations. An example of such a property is the frequency of electing a Condorcet winner, i.e., a candidate that would beat any other candidate in a one-to-one majority contest.

Our model of opinion polls is particularly applicable to small elections and straw polls.

1.2 Outline

This thesis is laid out as follows. In Chapter 2 we review related work on opinion polls and tactical voting behaviour. This is also where we discuss some exemplary, experimental results on the influence of opinion polls in real-world political elections. Chapter 3 introduces the basic framework of voting theory which is part of social choice theory. On top of this framework we will build our model. We then focus on the strategic response of a single voter to a single poll. We present the central notion of *poll information function* in Chapter 4, along with our results on manipulation by a single voter under (partial) poll information. Chapter 5 is concerned with voter response to sequences of polls. Here we discuss how polls may affect election outcomes. For this purpose, we also ran numerous simulations of elections. The uncut results of all experiments are listed in Appendix A. Finally, Chapter 6 concludes and gives some directions for future research.

Chapter 2

Influence of Opinion Polls

This chapter discusses related work. Sections 2.1 and 2.2 are concerned with research on opinion polls done in the field of political science. We review the main lines of work: how polls affect a voter's perception of other voters' preferences, and how that in turn may affect her voting intentions. These sections provide some empirical motivation for our work. In Section 2.3, we give an overview of related theoretical research on opinion polls. Finally, Section 2.4 ends with a summary.

2.1 On Perceptions of Collective Opinion

Generally, opinion polls are assumed to influence a voter's expectations regarding the election outcome. Political scientists study whether this is actually the case. Other factors are also believed to play a role. For example, a voter may base her expectations on the opinions of her friends and family, or on the opinions that are most prominent in the media. A voter may also rely on her own preferences in predicting the preferences of others ("wishful thinking"). Or a voter may presume that a candidate's performance in past elections for the same legislative body is illustrative for his performance in future elections.

Irwin & van Holsteyn (2002) studied in how far opinion polls controlled voters' perceptions of collective opinion in comparison to other factors in the 1994 Dutch parliamentary election. They found that poll information best predicted voters' perceptions. In particular, it did so better than past elections, wishful thinking, and general interest in (discussing) politics. In a similar study on the 2005 German parliamentary election it was also found that opinion polls strongly influenced voters' expectations of the election outcome, and that this effect persisted when the influences of wishful thinking and interest in the campaign were cancelled out (Faas et al., 2008). Both studies also found a significant effect of wishful thinking.

Clearly, these studies alone do not provide enough evidence to draw any general conclusions about the effect of opinion polls on voters' expectations. Research in this field is seriously limited by the lack of appropriate data for such analyses. Future research should therefore focus on the gathering and analysis of appropriate data to come to more general conclusions about the relation between opinion polls and voters' perceptions. Additionally, while most work in this area pertains to political elections with very large numbers of voters, it would also be interesting to analyse the effect of polls in elections with few voters.

2.2 On Voting Behaviour

The hypothesised effects of opinion polls on voting behaviour can be categorised into faithful, tactical, and emotional responses. A *faithful* voter does not change her ballot upon receiving poll information. Tactical and emotional voters, on the other hand, do. According to Fisher (2004), a *tactical* voter is "someone who votes for a candidate she believes is more likely to win than her preferred candidate, to best influence who wins in the constituency". Tactical voting behaviour leads to *momentum effects* in which candidates that are winning support win even more support, and candidates that are losing support lose even more support. Formally, a tactical voter can follow different strategies, depending on the voting procedure. The best-known strategy is *compromising;* ranking a candidate higher to get him elected. Other strategies are: *burying* – ranking a candidate lower to get him defeated, and *push-over* – ranking a candidate higher to get some other candidate elected. We refer to Saari (2003) for an overview of strategies for different voting procedures. A voter may also respond *emotionally* to poll information. She may, for example, change her vote to the candidate who is already winning in the polls, because of her intrinsic desire to be part of the winning team. On the contrary, she may also change her vote to the candidate who is losing in the polls, because she feels pity for him. Responses of the first type trigger a *bandwagon effect* in which the winning candidate gains support, and responses of the second type trigger an *underdog effect* in which the losing candidate gains support (terminology from Simon (1954)). Another much-cited emotional reaction to poll information is disillusioned voting. A supporter of a very popular winning candidate then decides to vote for her second most preferred candidate, because she does not feel her vote is needed anymore, or because she does not sympathise with some of the other supporters and therefore no longer feels represented (Riker, 1976).

According to Duverger (1954), tactical voting (compromising) is so common in plurality elections that many such voting systems eventually result in two-candidate systems. Take for example the U.S. presidential elections that completely revolve around the Democrats versus the Republicans. In the Indian general elections, however, votes are often split between three major parties. Hence, *Duverger's law* may not be as law-like as suggested (see Riker (1976) for an analysis).

Typically, in laboratory experiments on opinion polls large numbers of tactical voting are found. In such experiments, each subject is assigned a payoff vector which specifies her reward for each possible election outcome. Voters' preferences can be completely controlled that way, and emotional responses to opinion polls do not need to be taken into account. Following this methodology, Forsythe et al. (1993), for example, found that opinion polls significantly reduced the frequency with which the Condorcet loser won in plurality elections, due to increased rates of tactical voting behaviour.[1]

[1] A Condorcet loser is a candidate that would lose to any other candidate in a one-to-one majority contest.

In real-world political elections, the influence of opinion polls on voting behaviour seems to be much smaller (Faas et al., 2008). Many studies did not find a significant effect of opinion polls on the election outcome. There are several reasons that could explain this apparent discrepancy between laboratory experiments and actual elections. First, subjects in a laboratory experiment simply want to end up with as much reward (money) as possible. They are not confronted with ideological considerations, and the winner of the election is not going to rule their country. Moreover, in many laboratory experiments situations are created that are particularly vulnerable to tactical voting. Additionally, in actual elections emotional responses may cancel out tactical responses to opinion polls. But above all, many other factors may influence a voter's voting intentions in the actual world, and there is not much data available that allows for a direct analysis of the influence of opinion polls (see Section 2.1). More work is needed to come to any definite conclusion on the influence of opinion polls on voting behaviour and election outcomes.

Interestingly, Mutz (1997) argues that opinion polls may not only change the voting intentions of a voter, but also her preferences. According to Mutz (1997), opinion polls trigger a cognitive process in which pros and cons of candidates are mentally rehearsed, thereby possibly causing a voter's own preferences to shift. This would better account for momentum effects than tactical voting alone. Either way, in this thesis we assume that a voter's own preferences are fixed.

2.3 Formal Models of Opinion Polls

Formal models of opinion polls can provide insight into how opinion polls may affect voting behaviour, and thereby election outcomes. They can roughly be divided into what we shall call here 'mathematical' approaches and 'logical' approaches. In the mathematical approach, each voter is assigned a payoff vector which specifies her payoff (or utility) for each possible election winner. All voters are so-called expected utility-maximisers, i.e., they always submit a ballot that maximises their expected payoff given what they know about other voters' ballots. Opinion polls provide voters with information about other voters' ballots with some (unknown) uncertainty. Typically, polls communicate the approximate scores that each candidate would currently receive. In each poll round, all voters may change their ballot.

Myerson & Weber (1993) focused on voting equilibria in such models. A voting equilibrium is a point from which no voter wishes to deviate, i.e., another poll round would not change any voter's ballot. Myerson & Weber (1993) prove that there exists at least one such voting equilibrium for any allocation of payoff vectors under any positional scoring rule (e.g., plurality, veto, and Borda). Myatt (2007) also developed a mathematical model of opinion polls. Contrary to Myerson & Weber (1993), who assume that all voters hold the same beliefs about collective opinion, Myatt (2007) assumes that a voter's perception of other voters' preferences depends not only on opinion polls, but also on her own preferences (wishful thinking) and on the preferences of her friends and family. He proves that under these conditions, tactical voting is limited in equilibria of three-candidate plurality elections. Applied to the 1997 U.K. general election, his model correctly predicted the impact of tactical voting and the reported accuracy of voters' perceptions of collective opinion.

In the logical approach to opinion polls, there is no uncertainty regarding the accuracy of the communicated poll information, although polls may provide partial information about other voters' preferences. Moreover, voters are not necessarily expected utility-maximisers, i.e., voters do not necessarily always play a best response to what they know about other voters' ballots. For example, Brams & Fishburn (1983) proposed a model of opinion polls in which voters always vote for their favourite candidate from a small set of front-runners as identified by the previous poll. They give several examples that show, for both the plurality rule and another system known as *approval voting*, that opinion polls can have both positive and negative effects on the election of the Condorcet winner. Chopra et al. (2004) and Meir et al. (2010), on the other hand, do assume that voters always play a best response to poll information. Chopra et al. (2004) give further examples, showing that a sequence of polls may or may not reach an equilibrium. Meir et al. (2010) identify conditions under which termination can be guaranteed in plurality elections in which exactly one voter changes her ballot in each poll round.

Brams & Fishburn (1983) assume that voters have complete knowledge regarding the current electoral situation, that is, voters know the scores of all candidates and from this information they can derive for each possible way of voting themselves who would win in the next round if all other voters keep their vote. Chopra et al. (2004) and Meir et al. (2010) make the same assumption. Conitzer et al. (2011), however, do consider scenarios in which voters only have partial information about the current electoral situation. Their work on the problem of strategic manipulation under partial information is closely related to the first scenario we study in which a voter may or may not decide to vote strategically on the basis of a single opinion poll: a poll is one way to model the partial information available to a manipulator-to-be.

In this thesis, we take the logical approach. We study the influence of opinion polls on voting behaviour and election outcomes for varying poll information levels and voters' responses, and under different voting procedures.

2.4 Summary

Opinion polls may affect a voter's perception of other voters' preferences, and this may in turn affect her voting behaviour. Ultimately, opinion polls may lead to different election outcomes that way. Experimental research seems to support these claims, but more work needs to be done to come to any definite conclusion (Faas et al., 2008; Forsythe et al., 1993; Irwin & van Holsteyn, 2002). Mathematical models of opinion polls provide insight into how poll information may affect voting behaviour, and thereby election outcomes (Myatt, 2007; Myerson & Weber, 1993). We will take a logical approach and prove general theorems on the relation between opinion polls, voting procedures, voting behaviour, and election outcomes. In addition, we will simulate various elections to provide further insights. The work in this thesis is most closely related to the work of Brams & Fishburn (1983), Chopra et al. (2004), Meir et al. (2010), and Conitzer et al. (2011).

Chapter 3

Voting Theory

In this chapter we describe relevant concepts from voting theory (Taylor, 2005).

3.1 Basic Framework

Let $\mathcal{N} = \{1, 2, \ldots, n\}$ be a finite set of *voters*, and let $\mathcal{X} = \{x_1, x_2, \ldots, x_m\}$ be a finite set of *candidates* (or *alternatives*). To vote, each voter i submits a *ballot* b_i. If not stated otherwise, we adopt the standard assumption that ballots are strict linear orders on $\mathcal{X}$. Let $\mathcal{L}(\mathcal{X})$ be the set of all such orders. A *profile* $\boldsymbol{b} = (b_1, \ldots, b_n) \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$ is a vector of ballots, one for each voter. A *voting procedure* F is a function from ballot profiles to nonempty sets of candidates, the election winners:

$$F : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to 2^{\mathcal{X}} \backslash \{\emptyset\}$$

A voting procedure may give multiple initial winners. A *tie-breaking rule* then picks a unique winner from this set of initial winners. We assume that tie-breaking rules are choice functions: $T : 2^{\mathcal{X}} \backslash \{\emptyset\} \to \mathcal{X}$. An example of a tie-breaking rule that is not a choice function is the random tie-breaking rule which breaks ties randomly. Sometimes we further restrict attention to *rationalisable* tie-breaking rules, i.e., tie-breaking rules under which ties are broken according to some fixed but arbitrary order over the candidates (Definition 1).

Definition 1. *A tie-breaking choice function T is* ***rationalisable*** *if there is a strict linear order over the candidates $\rhd \in \mathcal{L}(\mathcal{X})$ such that for any $C \in 2^{\mathcal{X}} \backslash \{\emptyset\}$:*

$$T(C) = x \quad \textit{where } x \rhd y \textit{ for all } y \in C$$

The following are examples for common voting procedures (Brams & Fishburn, 1983; Taylor, 2005):

- *Positional scoring rules:* A PSR is defined by a scoring vector $(s_1, \ldots, s_m)$ with $s_1 \geqslant \ldots \geqslant s_m$ and $s_1 > s_m$. A candidate receives s_j points for each voter who ranks him at the jth position. The candidate(s) with the most points win(s) the election. Important PSRs are *plurality* with scoring vector $(1, 0, \ldots, 0)$, *antiplurality* (or *veto*) with scoring vector $(1, \ldots, 1, 0)$, and *Borda* with scoring vector $(m{-}1, m{-}2, \ldots, 0)$.

- *Copeland:* A candidate's score is the number of pairwise majority contests he wins minus the number he loses. The candidate(s) with the highest score win(s). A *pairwise majority contest* between candidates x and y is won by x if a majority of voters rank x above y.
- *Maximin* (or *Simpson*): A candidate's score is the lowest number of voters preferring him in any pairwise contest. The candidate(s) with the highest score win(s).
- *Bucklin:* A candidate's score is the smallest k such that a majority of voters rank him in their top k. The candidate(s) with the lowest score win(s).
- *Single transferable vote:* An STV election proceeds in rounds. In each round the candidate(s) ranked first by the fewest voters get(s) eliminated. This process is repeated until only one candidate remains (or until all remaining candidates are ranked first equally often).
- *Approval:* Each voter approves of as many candidates as she wishes. The candidate(s) with the most approvals win(s). Under approval voting ballots are *not* strict linear orders over candidates, but instead they are (not necessarily strict) subsets of candidates.

Example 1 illustrates the working of some of these procedures.

Example 1. *Suppose there are 3 candidates (a, b, c) and 11 voters who submit the following ballots (where underlining indicates approval):*

$$\begin{array}{ll} \textit{5 voters:} & \underline{c} \succ a \succ b \\ \textit{4 voters:} & \underline{a} \succ \underline{b} \succ c \\ \textit{2 voters:} & \underline{b} \succ c \succ a \end{array}$$

Who wins this election? Under the plurality rule, candidate c wins. He receives 5 plurality points against 4 points for candidate a and 2 points for candidate b. Under the Borda rule, candidate a wins. He receives 13 Borda points against 12 points for candidate c and 8 points for candidate b. The Copeland procedure, on the other hand, does not elect a unique winner. All candidates win and lose exactly one pairwise majority contest. Therefore, all candidates are tied under this procedure. Finally, approval voting elects candidate b. He receives 6 approval points against 5 points for candidate c and 4 points for candidate a.

Voting procedures can be categorised by their formal properties, often referred to as *axioms* (Taylor, 2005). Resolute voting procedures always elect a unique winner (Definition 2).

Definition 2. *A voting procedure F is* ***resolute*** *if $|F(\boldsymbol{b})| = 1$ for any ballot profile $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$.*

To simplify notation, we will sometimes think of a resolute voting procedure as a function from ballot profiles to candidates, i.e., $F : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathcal{X}$. A resolute voting procedure is surjective if each candidate wins under at least one ballot profile (Definition 3).

Definition 3. *A resolute voting procedure F is* ***surjective*** *if for any candidate $x \in \mathcal{X}$ there is a ballot profile $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$ such that $F(\boldsymbol{b}) = x$.*

Anonymous voting procedures treat all voters equally (Definition 4). And constant voting procedures always elect the same, unique winner (Definition 5).

Definition 4. *A voting procedure F is* ***anonymous*** *if* $F(b_1, \ldots, b_n) = F(b_{\tau(1)}, \ldots, b_{\tau(n)})$ *for any ballot profile* $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$ *and any permutation* $\tau : \mathcal{N} \to \mathcal{N}$.

Definition 5. *A voting procedure F is* ***constant*** *if there is a candidate* $x \in \mathcal{X}$ *such that* $F(\boldsymbol{b}) = \{x\}$ *for any ballot profile* $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$.

If there is a voter such that her top-ranked candidate is always the unique winner, then the voting procedure is dictatorial (Definition 6). Otherwise it is *nondictatorial.* We call a voter powerful if there is a ballot profile in which her vote matters (Definition 7). Thus a dictatorial voting procedure yields exactly one powerful voter. Note that a powerful voter is the opposite of a dummy voter, as defined in the field of cooperative games.

Below, $\boldsymbol{b}(x \succ y)$ denotes the set of voters ranking x above y in ballot profile $\boldsymbol{b}$.

Definition 6. *A voting procedure F is* ***dictatorial*** *if there is a voter* $i \in \mathcal{N}$ *such that for any ballot profile* $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$:

$$F(\boldsymbol{b}) = \{x\} \quad \text{whenever } i \in \boldsymbol{b}(x \succ y) \text{ for all } y \in \mathcal{X}\backslash\{x\}$$

Definition 7. *A voter i is* ***powerful*** *with respect to a voting procedure F if there are a ballot profile* $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$ *and a ballot* $b'_i \in \mathcal{L}(\mathcal{X})$ *such that* $F(b_i, \boldsymbol{b}_{-i}) \neq F(b'_i, \boldsymbol{b}_{-i})$.

A voting procedure is unanimous if it elects candidate x whenever x is ranked first by all voters (Definition 8). And a voting procedure satisfies the Pareto condition if it does not return a candidate that is ranked below some other candidate by all voters (Definition 9). Note that any Pareto-efficient voting procedure is unanimous (but not vice versa).

Definition 8. *A voting procedure F is* ***unanimous*** *if for any ballot profile* $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$:

$$F(\boldsymbol{b}) = \{x\} \quad \text{whenever } \boldsymbol{b}(x \succ y) = \mathcal{N} \text{ for all } y \in \mathcal{X}\backslash\{x\}$$

Definition 9. *A voting procedure F is* ***Pareto-efficient*** *if for any ballot profile* $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$:

$$y \notin F(\boldsymbol{b}) \quad \text{whenever } \boldsymbol{b}(x \succ y) = \mathcal{N} \text{ for some } x \in \mathcal{X}$$

Finally, a voting procedure is Condorcet-consistent if it elects (only) the Condorcet winner whenever he exists (Definition 10), and it is strongly Condorcet-consistent if it elects (only) the full set of weak Condorcet winners whenever that set is nonempty (Definition 11). A weak Condorcet winner is a candidate that does not lose any pairwise majority contest, although he may tie some. A Condorcet winner wins any pairwise majority contest. Note that (weak) Condorcet winners only exist for some profiles. If a Condorcet winner exists, then he must be unique, while there can be several weak Condorcet winners. A related notion is that of a *Condorcet loser:* a candidate who loses any pairwise majority contest.

Definition 10. *A voting procedure F is **Condorcet-consistent** if for any ballot profile $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$:*

$$F(\boldsymbol{b}) = \{x\} \quad \textit{whenever } |\boldsymbol{b}(x \succ y)| > |\boldsymbol{b}(y \succ x)| \textit{ for all } y \in \mathcal{X}\backslash\{x\}$$

Definition 11. *A voting procedure F is **strongly Condorcet-consistent** if for any ballot profile $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$:*

$$F(\boldsymbol{b}) = W^{\boldsymbol{b}} \quad \textit{whenever } W^{\boldsymbol{b}} \neq \emptyset$$

where $W^{\boldsymbol{b}}$ is the set of all weak Condorcet winners of $\boldsymbol{b}$, i.e.:

$$W^{\boldsymbol{b}} = \{x \in \mathcal{X} | \ |\boldsymbol{b}(x \succ y)| \geqslant |\boldsymbol{b}(y \succ x)| \textit{ for all } y \in \mathcal{X}\backslash\{x\}\}$$

3.2 Manipulation

Each voter i is endowed with a *preference order* $\succ_i$ on $\mathcal{X}$. A voter i votes *truthfully* if she votes $\succ_i$ and untruthfully otherwise. In classical voting theory, a voter i is said to have an incentive to manipulate if she can improve the election outcome with respect to $\succ_i$ by voting untruthfully (Definition 12). A resolute voting procedure is *susceptible to manipulation* if there is a profile in which some voter has an incentive to manipulate (Definition 13). If a resolute voting procedure is not susceptible to manipulation, then it is *immune to manipulation.*

Definition 12. *Given a resolute voting procedure F and a profile $\boldsymbol{b}$, a voter i has an **incentive to manipulate** if there is a ballot $c_i^\star \in \mathcal{L}(\mathcal{X})$ such that $F(c_i^\star, \boldsymbol{b}_{-i}) \succ_i F(\succ_i, \boldsymbol{b}_{-i})$.*

In above definition, $F(\succ_i, \boldsymbol{b}_{-i})$ denotes the election winner under F when everyone votes as in profile $\boldsymbol{b}$, while voter i votes according to $\succ_i$, et cetera.

Definition 13. *A resolute voting procedure F is **susceptible to manipulation** if there are a profile $\boldsymbol{b}$ and a voter i such that i has an incentive to manipulate.*

Impossibility theorems play an important role in voting theory: they describe which combinations of axioms cannot be satisfied by any voting procedure. An influential impossibility result is that of Gibbard (1973) and Satterthwaite (1975):

Theorem 1. (Gibbard-Satterthwaite) *When $m \geqslant 3$, any resolute voting procedure that is surjective and nondictatorial is susceptible to manipulation.*

Proof. See Gibbard (1973) or Satterthwaite (1975). For a simple proof see Barberà (1983) or Benoît (2000). □

In other words, the Gibbard-Satterthwaite Theorem states that if there are three or more candidates, then for any 'democratic' voting procedure there are situations in which voters are better off by not reporting their true preferences. This is problematic for two reasons. In the actual world voting procedures are designed to elect the most representative candidate assuming that all voters vote truthfully, and they may elect a less representative candidate if some vote untruthfully. Additionally, when voting untruthfully can be beneficial, a voter has to strategise over how to vote, which asks a lot of her cognitive abilities.

The Gibbard-Satterthwaite Theorem may, however, be less general than implied. Four of its underlying assumptions can be questioned and provide escapes from its major consequences.

First, the Gibbard-Satterthwaite Theorem makes the *universal domain* assumption: any ballot and preference order is possible. If we restrict the domain of a resolute voting procedure that is surjective and nondictatorial, then it might get immune to manipulation. Moulin (1980) gives a characterisation of such a class of voting procedures for single-peaked domains, a rather natural restriction on domains.

Second, even if it is possible for a voter to manipulate, it may be difficult to do so. Unfortunately, Bartholdi III et al. (1989) find that many common voting procedures are easy to manipulate, among which all positional scoring rules, Copeland and maximin. On the other hand, Conitzer & Sandholm (2003) define a qualifying round that makes common voting procedures hard to manipulate, including plurality, Borda, and maximin. In this round all candidates are paired, and the winners of the corresponding pairwise majority contests qualify for the final round in which the original procedure decides. We refer to Faliszewski & Procaccia (2010) for a review of work on hardness of manipulation.

Third, the Gibbard-Satterthwaite Theorem only applies to voting procedures that take strict linear orders as their input. Thus, voting procedures that are defined on, for example, subsets of candidates might be immune to manipulation. In fact, we will prove that under approval voting no voter ever has an incentive to misrepresent her preferences (see Section 4.3).

Finally, the Gibbard-Satterthwaite Theorem presupposes that a manipulator-to-be knows exactly how all other voters are voting. Limiting information about other voters' ballots, may make voting procedures less susceptible to manipulation, or even immune. In the actual world, voters will often obtain this information from *opinion polls*. In the next chapter, we analyse how much information a voter needs to be able to manipulate successfully. Conitzer et al. (2011) studied a similar scenario, but focused on the computational difficulty of manipulation.

Chapter 4

Response to a Single Poll

In this chapter, we study the scenario in which a single voter strategises in view of a single poll. We vary the amount of information that a poll provides and examine, for different voting procedures, when a voter starts and stops having an incentive to manipulate the election.

4.1 Polling Perspective

In this section we extend the basic framework of voting theory as described in Chapter 3, and define the central notion of *poll information function*.

4.1.1 Poll Information Functions

In an opinion poll, all voters are asked for their ballot. We call the resulting ballot profile a *poll profile*. Often we would not want to communicate the whole poll profile to the electorate, e.g., to respect the privacy of voters, or because it is computationally too expensive to do so. Let $\mathcal{I}$ be the set of all possible pieces of poll information that we might want to communicate to the electorate in view of a given poll profile. A *poll information function* (PIF) is a function $\pi : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathcal{I}$ mapping poll profiles to elements of $\mathcal{I}$. Here are some natural choices for $\mathcal{I}$ and the corresponding PIF π:

- *Profile:* The profile-PIF simply returns the full input profile: $\pi(\boldsymbol{b}) = \boldsymbol{b}$.
- *Ballot:* The ballot-PIF returns a vector recording how often each ballot occurs in the input profile. Formally, $\pi(\boldsymbol{b}) = (c(d_1, \boldsymbol{b}), \ldots, c(d_{m!}, \boldsymbol{b}))$, where $c : \mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathbb{N}$ counts the number of occurrences of a ballot in a ballot profile, and all ballots $d \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$ are ordered lexicographically with ballot d_1 being the lexicographic first and ballot $d_{m!}$ being the lexicographic last.
- *Weighted Majority Graph:* The WMG-PIF returns the weighted majority graph of the input profile. A *weighted majority graph* is a directed graph in which each node represents a candidate. There is an edge (x, y) from x to y if x wins their pairwise majority contest. Each edge (x, y) is labelled with the difference in number between voters ranking x above y and voters

ranking y above x. Let $\textsc{wmg}(\boldsymbol{b})$ be the weighted majority graph of ballot profile $\boldsymbol{b}$. Then $\pi(\boldsymbol{b}) = \textsc{wmg}(\boldsymbol{b})$.

- *Majority Graph:* The MG-PIF returns the majority graph of the input profile. A *majority graph* is a weighted majority graph without weights. Let $\textsc{mg}(\boldsymbol{b})$ be the majority graph of ballot profile $\boldsymbol{b}$. Then $\pi(\boldsymbol{b}) = \textsc{mg}(\boldsymbol{b})$.

- *Score:* Given a voting procedure F, the corresponding score-PIF returns for each candidate its score under the input profile according to F. F should assign points to each candidate for this PIF to be well-defined. Formally, $\pi(\boldsymbol{b}) = (s_F(x_1, \boldsymbol{b}), \ldots, s_F(x_m, \boldsymbol{b}))$, where $s_F : \mathcal{X} \times \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathbb{N}$ computes the score of a candidate under a ballot profile according to F.

- *Rank:* Given a voting procedure F, the corresponding rank-PIF returns the rank of each candidate under the input profile according to F. F should rank all candidates for this PIF to be well-defined. Formally, $\pi(\boldsymbol{b}) = (r_F(x_1, \boldsymbol{b}), \ldots, r_F(x_m, \boldsymbol{b}))$, where $r_F : \mathcal{X} \times \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathbb{N}$ computes the rank of a candidate under a ballot profile according to F. If F is paired with a tie-breaking rule T, then we assume that T is also used to break ties for second place, third place, et cetera.

- *Winner:* Given a voting procedure F, the corresponding winner-PIF returns the winning candidate(s) under the input profile according to F: $\pi(\boldsymbol{b}) = F(\boldsymbol{b})$.

- *Zero:* The zero-PIF does not provide any information, i.e., it simply returns a constant value: $\pi(\boldsymbol{b}) = 0$.

Upon receiving poll information $\pi(\boldsymbol{b})$, and assuming she knows how π is defined, what can voter i infer about the poll profile $\boldsymbol{b}$? Of course, she knows her own ballot b_i with certainty. So, what can she infer about the remainder of the profile, $\boldsymbol{b}_{-i}$? We call the set of (partial) profiles that voter i must consider possible in view of the information she holds after receiving $\pi(\boldsymbol{b})$ her *information set*. It is defined as follows:

$$\mathcal{W}_i^{\pi(\boldsymbol{b})} := \{\boldsymbol{c}_{-i} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}\setminus\{i\}} \mid \pi(b_i, \boldsymbol{c}_{-i}) = \pi(\boldsymbol{b})\}$$

Epistemologically speaking, we may think of poll profile $\boldsymbol{b}$ as the actual world and of $\{(b_i, \boldsymbol{c}_{-i}) \mid \boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\boldsymbol{b})}\}$ as the set of possible worlds that are consistent with i's knowledge in world $\boldsymbol{b}$. It is not difficult to see that $\mathcal{W}$ satisfies all properties of an **S5**-operator (see Blackburn et al. (2001) for an introduction in modal logic). For any PIF π, any voter i, and any ballot a_i, the following holds:

- (REF) $\boldsymbol{b}_{-i} \in \mathcal{W}_i^{\pi(a_i, \boldsymbol{b}_{-i})}$ for any profile $\boldsymbol{b}_{-i} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}\setminus\{i\}}$.

- (SYM) if $\boldsymbol{b}_{-i} \in \mathcal{W}_i^{\pi(a_i, \boldsymbol{c}_{-i})}$, then $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(a_i, \boldsymbol{b}_{-i})}$ for any profiles $\boldsymbol{b}_{-i}$, $\boldsymbol{c}_{-i} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}\setminus\{i\}}$.

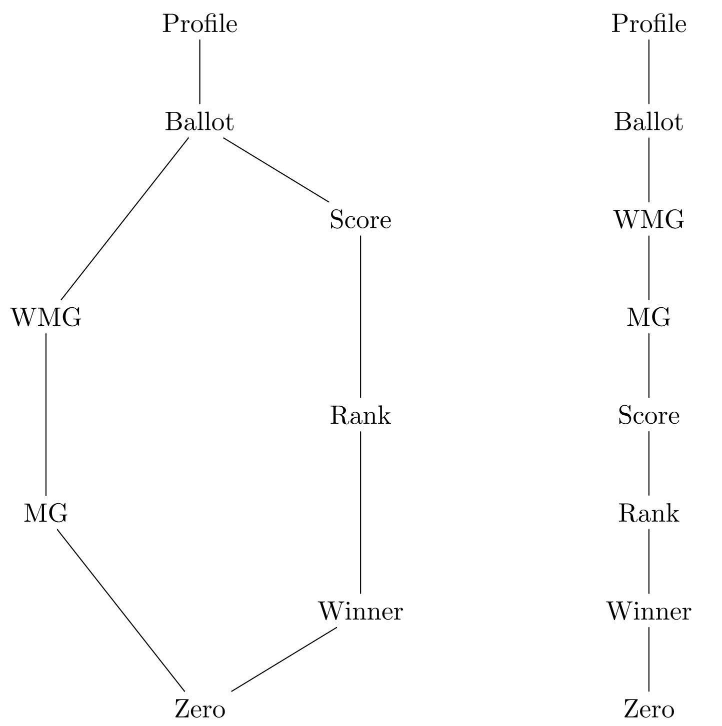
- (TRA) if $\boldsymbol{b}_{-i} \in \mathcal{W}_i^{\pi(a_i, \boldsymbol{c}_{-i})}$ and $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(a_i, \boldsymbol{d}_{-i})}$, then $\boldsymbol{b}_{-i} \in \mathcal{W}_i^{\pi(a_i, \boldsymbol{d}_{-i})}$ for any profiles $\boldsymbol{b}_{-i}$, $\boldsymbol{c}_{-i}$, $\boldsymbol{d}_{-i} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}\setminus\{i\}}$.

Axiom (REF) simply states that the actual poll profile is always part of every voter's information set. And Axioms (SYM) and (TRA) together express that whenever a voter considers some ballot profile possible, then that profile would also induce her current information set. For a discussion of the knowledge-theoretic properties of polls in view of strategic voting we refer to the work of Chopra et al. (2004).

We define the degree of 'informativeness' of a PIF in terms of the information sets it induces:

Definition 14. *A PIF π is said to be* ***at least as informative*** *as another PIF σ, if $\mathcal{W}_i^{\pi(\boldsymbol{b})} \subseteq \mathcal{W}_i^{\sigma(\boldsymbol{b})}$ for any poll profile $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$ and any voter $i \in \mathcal{N}$.*

Definition 14 places a hierarchy on poll information functions, in which a PIF is ranked above all PIFs that are less informative, and below all PIFs that are more informative. Figures 4.1(a) and 4.1(b) show this hierarchy for the above defined PIFs for Borda and Copeland, respectively.



(a) Borda (b) Copeland

Figure 4.1: Information hierarchies of selected poll information functions for Borda (a) and Copeland (b).

We note that Conitzer et al. (2011) work with the same notion of information set as we do here, except that they do not require an information set to be induced by poll information, but rather permit any set of conceivable profiles to form the information set of a given voter. There are also interesting connections to the work of Chevaleyre et al. (2009) on the compilation complexity of voting procedures: their *compilation functions* are the same types of functions as our PIFs.

4.1.2 Manipulation with respect to Poll Information

Now that we extended the basic framework of voting theory to encompass opinion polls and how they affect the information voters have regarding the voting intentions of others, the classical definition of manipulation (Definition 12) does not suffice anymore. We say that a voter has an incentive to π-manipulate if from what she knows about the voting intentions of other voters, by voting untruthfully she has a chance of improving the election outcome according to her true preferences and no chance of worsening it (Definition 15).

In below definition, $\succeq_i$ is the reflexive closure of $\succ_i$.[1]

Definition 15. *Let π be a PIF. Given a resolute voting procedure F, a voter i, and a poll profile $\boldsymbol{b}$ with $b_i = \succ_i$, voter i has an **incentive to π-manipulate** if there is a ballot $c_i^\star \in \mathcal{L}(\mathcal{X})$ such that:*

$$\begin{aligned} & F(c_i^\star, \boldsymbol{c}_{-i}) \succ_i F(\succ_i, \boldsymbol{c}_{-i}) \quad \text{for some profile } \boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\boldsymbol{b})} \\ \text{and} \quad & F(c_i^\star, \boldsymbol{c}_{-i}) \succeq_i F(\succ_i, \boldsymbol{c}_{-i}) \quad \text{for all other profiles } \boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\boldsymbol{b})} \end{aligned}$$

Susceptibility to manipulation of voting procedures is defined in the classical way (Definition 16). Likewise, a resolute voting procedure that is not susceptible to π-manipulation is *immune to π-manipulation.*

Definition 16. *A resolute voting procedure F is **susceptible to π-manipulation** if there are a profile $\boldsymbol{b}$ and a voter i such that i has an incentive to π-manipulate.*

Note that when π is the profile-PIF, returning the full poll profile, then our notion of π-manipulation reduces to the standard notion of manipulability as defined in Chapter 3.

Lemma 1 below relates the degree of informativeness of a PIF to the susceptibility results it brings about.

Lemma 1. *If a PIF π is at least as informative as another PIF σ, then any resolute voting procedure that is susceptible to σ-manipulation is also susceptible to π-manipulation.*

Proof. Let F be a resolute voting procedure that is susceptible to σ-manipulation and let π be a PIF that is at least as informative as σ. By assumption, there are a voter i, a poll profile $\boldsymbol{b}$ with $b_i = \succ_i$, and a ballot $c_i^\star$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \succ_i F(\succ_i, \boldsymbol{c}_{-i})$ for some $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\sigma(\boldsymbol{b})}$ and $F(c_i^\star, \boldsymbol{c}_{-i}) \succeq_i F(\succ_i, \boldsymbol{c}_{-i})$ for all other profiles $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\sigma(\boldsymbol{b})}$. Fix any $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\sigma(\boldsymbol{b})}$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \succ_i F(\succ_i, \boldsymbol{c}_{-i})$. By $\mathcal{W}$-properties (SYM) and (TRA), we get $\mathcal{W}_i^{\sigma(b_i, \boldsymbol{c}_{-i})} = \mathcal{W}_i^{\sigma(\boldsymbol{b})}$. Since PIF π is at least as informative as σ, we have that $\mathcal{W}_i^{\pi(b_i, \boldsymbol{c}_{-i})} \subseteq \mathcal{W}_i^{\sigma(b_i, \boldsymbol{c}_{-i})}$. By $\mathcal{W}$-property (REF), we get $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(b_i, \boldsymbol{c}_{-i})}$. It follows that voter i has an

[1] The reflexive closure of a relation R on a set X is the smallest relation on X that is reflexive and contains R.

incentive to π-manipulate when the poll profile is $(b_i, \boldsymbol{c}_{-i}) = (\succ_i, \boldsymbol{c}_{-i})$. Hence, F is susceptible to π-manipulation. $\square$

Corollary 1.1. *If a PIF π is at least as informative as another PIF σ, then any resolute voting procedure that is immune to π-manipulation is also immune to σ-manipulation.*

In the following two sections, we prove several susceptibility and immunity results for specific PIFs. Lemma 1 and Corollary 1.1 show how such results can be generalised to other PIFs.

4.2 Susceptibility Results

In our poll framework, the Gibbard-Satterthwaite Theorem (Theorem 1) can be restated as follows.

Theorem 2. *When $m \geqslant 3$, any resolute voting procedure that is surjective and nondictatorial is susceptible to profile-manipulation.*

Not every voting procedure requires all information a ballot profile supplies to compute the winner(s). For the plurality rule, for example, it suffices to give for each candidate the number of ballots in which it is ranked first. We would therefore expect that the Gibbard-Satterthwaite Theorem generalises to PIFs that are less informative than the profile-PIF for voting procedures that require less information than full ballot profiles to compute the election winner(s).

For a given PIF $\pi : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathcal{I}$, we say that a voting procedure F is *computable from π-images* if there exists a function $H : \mathcal{I} \to 2^{\mathcal{X}} \backslash \{\emptyset\}$ such that $F = H \circ \pi$. We furthermore say that F is *strongly computable from π-images* if it is computable from π-images and $\pi(\boldsymbol{b}) = \pi(b_i, \boldsymbol{c}_{-i})$ entails $F(c_i, \boldsymbol{b}_{-i}) = F(\boldsymbol{c})$ for any two profiles $\boldsymbol{b}$ and $\boldsymbol{c}$, i.e., upon learning $\pi(\boldsymbol{b})$ a voter i can compute the winners for *any* way of voting herself (rather than just for b_i). For example, the Copeland procedure is computable but not strongly computable from MG-information (i.e., from images under the MG-PIF), while it is strongly computable from WMG-information. Furthermore, any anonymous voting procedure is strongly computable from ballot-information, and any positional scoring rule is strongly computable from score-information.

Theorem 3. *Let π be a PIF. When $m \geqslant 3$, any resolute voting procedure that is surjective, nondictatorial, and strongly computable from π-images is susceptible to π-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $m \geqslant 3$. Let π be a PIF with range $\mathcal{I}$ and let F be any resolute voting procedure that is surjective, nondictatorial, and strongly computable from π-images. From Theorem 2 it follows that F is susceptible to profile-manipulation, i.e., there exist a profile $\boldsymbol{b}$, a voter i, and a ballot $c_i^\star$ such that $F(c_i^\star, \boldsymbol{b}_{-i}) \succ_i F(\succ_i, \boldsymbol{b}_{-i})$. Since F cannot differentiate between profiles that produce the same $\mathcal{I}$-structure, we get $F(\succ_i, \boldsymbol{c}_{-i}) = F(\succ_i, \boldsymbol{b}_{-i})$ for any $\boldsymbol{c}_{-i}$ with $\pi(\succ_i, \boldsymbol{c}_{-i}) = \pi(\succ_i, \boldsymbol{b}_{-i})$. As F is *strongly* computable from π-images, this entails $F(c_i^\star, \boldsymbol{c}_{-i}) = F(c_i^\star, \boldsymbol{b}_{-i})$ for any $\boldsymbol{c}_{-i}$ with $\pi(\succ_i, \boldsymbol{c}_{-i}) = \pi(\succ_i, \boldsymbol{b}_{-i})$. It follows that voter i has an incentive to π-manipulate when the poll profile is $(\succ_i, \boldsymbol{b}_{-i})$. Hence, F is susceptible to π-manipulation. $\square$

The conditions of Theorem 3 are not *necessary* for susceptibility. There are resolute voting procedures that are surjective, nondictatorial, and susceptible to π-manipulation, yet not (strongly) computable from π-images, as our next two results show.

Theorem 4. *When $m \geqslant 3$ and n is even, any strongly Condorcet-consistent voting procedure, paired with a tie-breaking choice function, is susceptible to MG-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $m \geqslant 3$ and n is even, and fix any tie-breaking choice function T. Let F be any strongly Condorcet-consistent voting procedure paired with T, and let π be the MG-PIF. We construct a ballot profile with three weak Condorcet winners such that voter i's second favourite candidate wins if she votes truthfully, and her first favourite wins if she votes untruthfully.

Fix $a, b, c \in \mathcal{X}$ with $a \neq b \neq c$. Without loss of generality, we shall assume that $T(\{a,b,c\}) = a$ and $T(\{b,c\}) = b$. Let $\succ_i = b \succ a \succ c \succ \mathcal{X}\backslash\{a,b,c\}$, where candidates $\mathcal{X}\backslash\{a,b,c\}$ are ranked in any order. And let $c_i^\star = b \succ c \succ a \succ \mathcal{X}\backslash\{a,b,c\}$. Let $\boldsymbol{b}_{-i}$ be a profile in which $\frac{n-2}{2}$ voters submit $b \succ a \succ c \succ \mathcal{X}\backslash\{a,b,c\}$, and $\frac{n-2}{2}+1$ voters submit $c \succ a \succ b \succ \mathcal{X}\backslash\{a,b,c\}$. Then $F(\succ_i, \boldsymbol{b}_{-i}) = a$ and $F(c_i^\star, \boldsymbol{b}_{-i}) = b$. It is not difficult to check that there is no profile $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\succ_i, \boldsymbol{b}_{-i})}$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \prec_i F(\succ_i, \boldsymbol{c}_{-i})$. It follows that voter i has an incentive to π-manipulate when the poll profile is $(\succ_i, \boldsymbol{b}_{-i})$. Hence, F is susceptible to MG-manipulation. □

Examples for voting procedures that are strongly Condorcet-consistent include the maximin procedure, but not, for instance, the (Condorcet-consistent) Copeland procedure. From Lemma 1 it follows that if there are three or more candidates and an even number of voters, then any strongly Condorcet-consistent voting procedure, paired with a tie-breaking choice function, is susceptible to WMG-manipulation, as well as to ballot-manipulation and profile-manipulation (the latter also holds for an odd number of voters by Theorem 2).

Our final π-susceptibility result concerns positional scoring rules. Observe that a positional scoring rule is unanimous if and only if $s_1 > s_2$ holds for the scoring vector defining it.

Theorem 5. *When $m \geqslant 3$ and $n \geqslant 4$, any unanimous positional scoring rule, paired with a tie-breaking choice function, is susceptible to winner-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $m \geqslant 3$ and $n \geqslant 4$, and fix any tie-breaking choice function T. Let F be any unanimous positional scoring rule paired with T, and let π be the winner-PIF with respect to F. We construct a profile where voter i's third favourite candidate wins if she votes truthfully and her second favourite wins if she votes untruthfully.

Fix $a, b, c \in \mathcal{X}$ with $a \neq b \neq c$. Without loss of generality, we shall assume that $T(\{a,b\}) = a$ and $T(\{b,c\}) = b$. Let $\succ_i = c \succ a \succ b \succ \mathcal{X}\backslash\{a,b,c\}$, where candidates $\mathcal{X}\backslash\{a,b,c\}$ are ranked in any order. And let $c_i^\star = a \succ c \succ b \succ \mathcal{X}\backslash\{a,b,c\}$. If n is odd, let $\boldsymbol{b}_{-i}$ be a profile in which $\frac{n-3}{2}$ voters submit $a \succ b \succ \mathcal{X}\backslash\{a,b\}$, $\frac{n-3}{2}$ voters submit $b \succ a \succ \mathcal{X}\backslash\{a,b\}$, and the remaining two voters submit $c \succ b \succ a \succ \mathcal{X}\backslash\{a,b,c\}$ and $b \succ a \succ c \succ \mathcal{X}\backslash\{a,b,c\}$. If n is even, let $\boldsymbol{b}_{-i}$ be a profile in which $\frac{n-2}{2}$ voters submit $a \succ b \succ \mathcal{X}\backslash\{a,b\}$, and $\frac{n-2}{2}$ voters submit $b \succ a \succ \mathcal{X}\backslash\{a,b\}$, and the remaining voter submits $b \succ c \succ$

$a \succ \mathcal{X} \backslash \{a, b, c\}$. Since F is unanimous, i.e., $s_1 > s_2$, we get $F(\succ_i, \boldsymbol{b}_{-i}) = b$ and $F(c_i^\star, \boldsymbol{b}_{-i}) = a$.

It is not difficult to check that there is no profile $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\succ_i, \boldsymbol{b}_{-i})}$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \prec_i F(\succ_i, \boldsymbol{c}_{-i})$. It follows that voter i has an incentive to π-manipulate when the poll profile is $(\succ_i, \boldsymbol{b}_{-i})$. Hence, F is susceptible to winner-manipulation. □

From Lemma 1 it follows that if there are three or more candidates and four or more voters, then any unanimous positional scoring rule, paired with a tie-breaking choice function, is susceptible to rank-manipulation, as well as to score-manipulation, ballot-manipulation, and profile-manipulation. The restriction on the number of voters can be dropped for PIFs that are at least as informative as the score-PIF by Theorem 3.

4.3 Immunity Results

We now turn our attention to voting procedures that are immune to certain types of manipulation. First, it is not difficult to verify that any dictatorial voting procedure is immune to profile-manipulation (Theorem 6). This is in fact the opposite direction of the Gibbard-Satterthwaite Theorem. Clearly, any constant voting procedure is immune to profile-manipulation as well (Theorem 7). From Lemma 1 it follows that these voting procedures are also immune to any other form of manipulation.

Theorem 6. *Any dictatorial voting procedure is immune to profile-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$. Let F be any dictatorial voting procedure and let π be the profile-PIF. Fix any voter i and any poll profile $\boldsymbol{b}$ with $b_i = \succ_i$. If voter i is the dictator, then $F(\succ_i, \boldsymbol{b}_{-i})$ is her favourite candidate and therefore she does not have an incentive to π-manipulate. If voter i is not the dictator, then she does not have any influence on the election outcome and therefore she does not have an incentive to π-manipulate. Hence, F is immune to profile-manipulation. □

Theorem 7. *Any constant voting procedure is immune to profile-manipulation.*

Proof. Immediate from the fact that no voter has any influence on the election outcome. □

At the other extreme, as we shall see next, we can obtain immunity results for two large classes of voting procedures with respect to the weakest form of immunity considered here, namely *zero-manipulation.* The next theorem is inspired by (and corrects two minor mistakes in) a result due to Conitzer et al. (2011).

Theorem 8. *When $n \geqslant 3$, any strongly Condorcet-consistent voting procedure, paired with a tie-breaking choice function, is immune to zero-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $n \geqslant 3$, and fix any tie-breaking choice function T. Let F be any strongly Condorcet-consistent voting procedure paired with T, and let π be the zero-PIF. Fix any voter i, any poll profile $\boldsymbol{b}$ with $b_i = \succ_i$, and any untruthful ballot $c_i^\star$. Since $c_i^\star \neq \succ_i$, there is a pair of candidates such that $x \succ_i y$ and $y \succ_{c_i^\star} x$. Claim: there exists a profile $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\boldsymbol{b})}$ (i.e., any $\boldsymbol{c}_{-i}$) such that $F(c_i^\star, \boldsymbol{c}_{-i}) \prec_i F(\succ_i, \boldsymbol{c}_{-i})$.

We construct a profile $\boldsymbol{c}_{-i}$ such that x and y are the only possible winners. If n is odd, let $\boldsymbol{c}_{-i}$ be a profile in which $\frac{n-1}{2}$ voters submit $x \succ y \succ \mathcal{X}\backslash\{x, y\}$, and $\frac{n-1}{2}$ voters submit $y \succ x \succ \mathcal{X}\backslash\{x, y\}$, where candidates $\mathcal{X}\backslash\{x, y\}$ are ranked in any order. If n is even, let $\boldsymbol{c}_{-i}$ be a profile in which $\frac{n-2}{2}$ voters submit $x \succ y \succ \mathcal{X}\backslash\{x, y\}$, and $\frac{n-2}{2}$ voters submit $y \succ x \succ \mathcal{X}\backslash\{x, y\}$, and the remaining voter submits $y \succ x \succ \mathcal{X}\backslash\{x, y\}$ in case $T(\{x, y\}) = x$ and $x \succ y \succ \mathcal{X}\backslash\{x, y\}$ otherwise. Then $F(\succ_i, \boldsymbol{c}_{-i}) = x$ and $F(c_i^\star, \boldsymbol{c}_{-i}) = y$. It follows that for any untruthful ballot $c_i^\star$ there is a situation where voter i will do strictly better by voting truthfully. Hence, F is immune to zero-manipulation. □

Theorem 9. *When n is odd, any resolute voting procedure that is Condorcet-consistent is immune to zero-manipulation.*

Proof. Immediate from the first part of the proof of Theorem 8. □

Conitzer et al. (2011) state a slightly stronger variant of Theorem 8: any resolute voting procedure that is (not necessarily strongly) Condorcet-consistent is immune to zero-manipulation (or "immune to dominating manipulation when the manipulator has no information", using their terminology). This is true for an odd number of voters, as Theorem 9 shows. For an *even* number of voters, however, Condorcet consistency is not sufficient, as demonstrated by the following example.

Example 2. *Consider a scenario with 4 voters and 3 candidates (a, b, c). Suppose that voting procedure F elects the Condorcet winner if one exists, and otherwise the bottom choice of voter 1. Let $\succ_1 = a \succ b \succ c$, and consider ballot $c_1^\star = a \succ c \succ b$. Now there is a profile $\boldsymbol{c}_{-1}$ such that voter 1 benefits from voting untruthfully, namely when the others vote $a \succ b \succ c$, $b \succ a \succ c$, and $b \succ a \succ c$. Then $F(\succ_1, \boldsymbol{c}_{-1}) = c$ and $F(c_1^\star, \boldsymbol{c}_{-1}) = b$.*

Claim: there is no profile $\boldsymbol{c}_{-1}$ such that $F(c_1^\star, \boldsymbol{c}_{-1}) \prec_1 F(\succ_1, \boldsymbol{c}_{-1})$. Fix any profile $\boldsymbol{c}_{-1}$. If $F(\succ_1, \boldsymbol{c}_{-1}) = a$, then $F(c_1^\star, \boldsymbol{c}_{-1}) = a$, because candidate a can only win $(\succ_1, \boldsymbol{c}_{-1})$ if he is the Condorcet winner, in which case he is also the Condorcet winner of $(c_1^\star, \boldsymbol{c}_{-1})$. Furthermore, if $F(\succ_1, \boldsymbol{c}_{-1}) = b$, then $F(c_1^\star, \boldsymbol{c}_{-1}) = b$, because candidate b can only win $(\succ_1, \boldsymbol{c}_{-1})$ if he is the Condorcet winner, in which case either he also is the Condorcet winner of $(c_1^\star, \boldsymbol{c}_{-1})$ or voter 1's bottom candidate b wins.

It follows that voter 1 has an incentive to zero-manipulate. Hence, F is a resolute voting procedure for $n \geqslant 3$ that is Condorcet-consistent and susceptible to zero-manipulation.

We stress that Theorem 8 also cannot be simplified to stating that any resolute voting procedure that always elects *some* weak Condorcet winner whenever one exists is immune to zero-manipulation. Example 3 presents such a voting procedure that is susceptible to zero-manipulation. This example makes use of a tie-breaking rule that is not a choice function, but instead depends on a specific voter's ballot. This tie-breaking rule is then paired with a strongly Condorcet-consistent voting procedure to get our desired procedure.

Note that Example 3, like Example 2, is also an example of a resolute voting procedure for three or more voters that is Condorcet-consistent and susceptible to zero-manipulation. However, Example 2 is more general and simpler than Example 3 and may therefore be more convincing for this purpose.

Example 3. *Consider a scenario with 4 voters and 3 candidates* (a, b, c). *Suppose that voting procedure* H *always elects the set of all weak Condorcet winners. This set is never empty, because there are 4 voters and 3 candidates and therefore there always is a candidate that is ranked first by at least two voters. Suppose further that tie-breaking rule* T *breaks ties according to* $c \succ b \succ a$ *if voter* 1 *submits* $a \succ b \succ c$, *and according to* $a \succ b \succ c$ *otherwise. Let* $F = T \circ H$. *Let* $\succ_1 = a \succ b \succ c$, *and consider ballot* $c_1^\star = a \succ c \succ b$. *Now there is a profile* $\boldsymbol{c}_{-1}$ *such that voter 1 benefits from voting untruthfully, namely when the others vote* $a \succ b \succ c$, $b \succ a \succ c$, *and* $b \succ a \succ c$. *Then* $F(\succ_1, \boldsymbol{c}_{-1}) = b$ *and* $F(c_1^\star, \boldsymbol{c}_{-1}) = a$.

Claim: there is no profile $\boldsymbol{c}_{-1}$ *such that* $F(c_1^\star, \boldsymbol{c}_{-1}) \prec_1 F(\succ_1, \boldsymbol{c}_{-1})$. *Fix any profile* $\boldsymbol{c}_{-1}$. *We have that* $H(c_1^\star, \boldsymbol{c}_{-1}) \cap H(\succ_1, \boldsymbol{c}_{-1}) \neq \emptyset$, *because* $c_1^\star$ *and* $\succ_1$ *have the same top-ranked candidate. Since* T *breaks ties according to* $c \succ b \succ a$ *if voter* 1 *submits* $\succ_1$, *and according to* $a \succ b \succ c$ *if she submits* $c_1^\star$, *it follows that our claim holds.*

We get that voter 1 has an incentive to zero-manipulate. Hence, F *is a resolute voting procedure for* $n \geqslant 3$ *that always elects some weak Condorcet winner whenever one exists and is susceptible to zero-manipulation.*

In their formulation of Theorem 8, Conitzer et al. (2011) do not require the number of voters to be greater or equal than 3. Example 4 demonstrates why this restriction cannot be dropped.

Example 4. *Consider a scenario with 2 voters and 3 candidates* (a, b, c). *Suppose that voting procedure* H *always elects the set of all weak Condorcet winners. This set is never empty, because there are only 2 voters. Suppose further that* T *is the lexicographic tie-breaking rule, i.e.,* T *breaks ties according to* $a \succ b \succ c$. *Let* $F = T \circ H$. *Let* $\succ_1 = b \succ a \succ c$, *and consider ballot* $c_1^\star = b \succ c \succ a$. *Now there is a ballot* c_2 *of voter 2 such that voter 1 benefits from voting untruthfully, namely when* $c_2 = c \succ a \succ b$. *Then* $F(\succ_1, c_2) = a$ *and* $F(c_1^\star, c_2) = b$. *For all other ballots* c_2 *of voter 2, we have that* $F(\succ_1, c_2) = F(c_1^\star, c_2)$.

It follows that voter 1 has an incentive to zero-manipulate. Hence, F *is a (strongly) Condorcet-consistent voting procedure for* $n = 2$ *that is susceptible to zero-manipulation.*

The following theorem strengthens another result by Conitzer et al. (2011), who use a bound of $n \geqslant 6m - 12$.

Theorem 10. *When* $n \geqslant 2m - 2$, *any positional scoring rule, paired with a tie-breaking choice function, is immune to zero-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $n \geqslant 2m - 2$, and fix any tie-breaking choice function T. Let F be any positional scoring rule paired with T, and let π be the zero-PIF. Fix any voter i and any poll profile $\boldsymbol{b}$ with $b_i = \succ_i$. And fix any untruthful ballot $c_i^\star$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \neq F(\succ_i, \boldsymbol{c}_{-i})$ for some profile $\boldsymbol{c}_{-i}$. Then there exists a pair of candidates such that $x \succ_i y$ and $y \succ_{c_i^\star} x$, and x's score differs from y's in $\succ_i$ and $c_i^\star$. Claim: there exists a profile $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\boldsymbol{b})}$ (i.e., any $\boldsymbol{c}_{-i}$) such that $F(c_i^\star, \boldsymbol{c}_{-i}) \prec_i F(\succ_i, \boldsymbol{c}_{-i})$.

We construct a profile $\boldsymbol{c}_{-i}$ such that x and y are the only possible winners. If n is odd, let $\boldsymbol{c}_{-i}$ be a profile in which $\frac{n-1}{2}$ voters submit $x \succ y \succ \mathcal{X} \backslash \{x, y\}$, and $\frac{n-1}{2}$ voters submit $y \succ x \succ \mathcal{X}\backslash\{x,y\}$, where every candidate $z \in \mathcal{X}\backslash\{x,y\}$ is ranked last by at least one voter. If n is even, let $\boldsymbol{c}_{-i}$ be a profile in which $\frac{n-2}{2}$ voters submit $x \succ y \succ \mathcal{X}\backslash\{x,y\}$, and $\frac{n-2}{2}$ voters submit $y \succ x \succ \mathcal{X}\backslash\{x,y\}$, where every candidate $z \in \mathcal{X}\backslash\{x,y\}$ is ranked last by at least two voters, and the remaining voter submits the ballot that is like $\succ_i$ but with x and y swapped in case $T(\{x,y\}) = x$ and $c_i^\star$ with x and y swapped otherwise. Then $F(\succ_i, \boldsymbol{c}_{-i}) = x$ and $F(c_i^\star, \boldsymbol{c}_{-i}) = y$. Hence, F is immune to zero-manipulation. Observe that the bound of $n \geqslant 2m-2$ follows from our requirements on the number of voters ranking each $z \in \mathcal{X}\backslash\{x,y\}$ last. The case with 4 voters and 3 candidates must be checked separately. □

From the first part of the proof of Theorem 10 it follows that for an odd number of voters, the bound can be improved to $n \geqslant m-1$.

Together, Theorem 8 and Theorem 10 cover a broad range of voting procedures. In particular, as is well known, the classes of Condorcet-consistent procedures and positional scoring rules do not overlap (Taylor, 2005).

So far, all our immunity results involved either trivial voting procedures (dictatorships and constant procedures) or the trivial information set (for zero-manipulation). While we should not expect many positive results between these two extremes, they are not impossible to obtain either:

Theorem 11. *When $n \geqslant 10$, the plurality rule, paired with a tie-breaking choice function, is immune to MG-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $n \geqslant 10$, and fix any tie-breaking choice function T. Let F be the plurality rule paired with T, and let π be the MG-PIF. Fix any voter i and any poll profile $\boldsymbol{b}$ with $b_i = \succ_i$. And fix any untruthful ballot $c_i^\star$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \neq F(\succ_i, \boldsymbol{c}_{-i})$ for some profile $\boldsymbol{c}_{-i} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}\backslash\{i\}}$. Let x be the top-ranked candidate of $\succ_i$, and let y be the top-ranked candidate of $c_i^\star$. It follows that $x \neq y$. Claim: there exists a profile $\boldsymbol{c}_{-i} \in \mathcal{W}_i^{\pi(\boldsymbol{b})}$ such that $F(c_i^\star, \boldsymbol{c}_{-i}) \prec_i F(\succ_i, \boldsymbol{c}_{-i})$.

We construct a profile $\boldsymbol{c}_{-i}$ such that x and y are the only possible winners. If n is odd, let $\boldsymbol{c}_{-i}$ be a profile in which $\frac{n-1}{2}-1$ voters rank x on top, and $\frac{n-1}{2}-1$ voters rank y on top, and the remaining two voters rank some candidate $z \in \mathcal{X}\backslash\{x,y\}$ on top. The remaining part of the profile is filled so that the majority graphs of $(\succ_i, \boldsymbol{c}_{-i})$ and $(\succ_i, \boldsymbol{b}_{-i})$ are the same. Note that this is possible, because each candidate can still win and lose each pairwise contest. If n is even, let $\boldsymbol{c}_{-i}$ be a profile in which $\frac{n}{2}-2$ voters rank x on top, and $\frac{n}{2}-2$ voters rank y on top, and the remaining three voters rank some candidate $z \in \mathcal{X}\backslash\{x,y\}$ on top. Again, the remaining part of the profile is filled so that the majority graphs of $(\succ_i, \boldsymbol{c}_{-i})$ and $(\succ_i, \boldsymbol{b}_{-i})$ are the same. Then $F(\succ_i, \boldsymbol{c}_{-i}) = x$ and $F(c_i^\star, \boldsymbol{c}_{-i}) = y$. Hence, F is immune to MG-manipulation. Observe that the bound of $n \geqslant 10$ ensures that each candidate can still win, lose and tie each pairwise contest in our construction. The case with 2 candidates must be checked separately. □

Our next π-immunity result concerns the antiplurality rule.

Theorem 12. *When $n \geqslant 2m - 2$, the antiplurality rule, paired with a tie-breaking choice function, is immune to winner-manipulation.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$ such that $n \geqslant 2m - 2$, and fix any tie-breaking choice function T. Let F be the antiplurality rule paired with T, and let π be the winner-PIF with respect to F. Without loss of generality, we may assume that voters only submit a candidate they wish to veto. Fix any voter i, any poll profile $\boldsymbol{b}$ with b_i is voter i's true least favourite candidate, and any ballot $c_i^\star \neq b_i$. By definition, $\mathcal{W}_i^{\pi(\boldsymbol{b})} = \{\boldsymbol{c}_{-i} \mid F(b_i, \boldsymbol{c}_{-i}) = F(\boldsymbol{b})\}$. Claim: voter i never has an incentive to π-manipulate.

Suppose $m \geqslant 3$ and $F(\boldsymbol{b}) = w$. If $w = b_i$, then i cannot change the outcome. If $w = c_i^\star$, let $\boldsymbol{c}_{-i}$ be a profile in which $n-1$ voters veto some candidate $x \in \mathcal{X}\backslash\{b_i, w\}$, and all candidates $x \in \mathcal{X}\backslash\{b_i, w\}$ are vetoed by at least one voter. If $w \in \mathcal{X}\backslash\{b_i, c_i^\star\}$, let $\boldsymbol{c}_{-i}$ be a profile in which $n-2$ voters veto some candidate $x \in \mathcal{X}\backslash\{b_i, w\}$, and all candidates $x \in \mathcal{X}\backslash\{b_i, w\}$ are vetoed by at least two voters, and the remaining voter vetoes w in case $T(\{b_i, w\}) = w$ and some candidate $x \in \mathcal{X}\backslash\{b_i, w\}$ otherwise. Then $F(b_i, \boldsymbol{c}_{-i}) = w$ and $F(c_i^\star, \boldsymbol{c}_{-i}) = b_i$. Hence, F is immune to winner-manipulation. Observe that the bound of $n \geqslant 2m-2$ follows from our requirements on the number of voters ranking each $x \in \mathcal{X}\backslash\{b_i, w\}$ last. The case with 2 candidates must be checked separately. □

In its original formulation, the Gibbard-Satterthwaite Theorem only applies to voting procedures that are defined on strict linear orders, as we discussed in Section 3.2. This raises the question whether for example approval voting, defined on subsets of candidates, is susceptible to manipulation. To answer this question we need to revise our definition of a truthful vote: there is no way of fully representing a voter's true preference order in a subset of candidates. A voter i is said to vote *sincerely* if she truly prefers each approved candidate to each disapproved candidate, i.e., if $x \succ_i y$ for any $x \in b_i$ and any $y \in \mathcal{X}\backslash b_i$. Note that a voter has multiple sincere ballots.

Endriss (2009) gives a thorough analysis of manipulation under approval voting. He does not pair approval voting with a tie-breaking rule like we do, but instead lifts a voter's preference order over individual candidates to one over sets of candidates. He proves for different sets of axioms on this lifting that any insincere ballot of a voter is weakly dominated by some sincere ballot, i.e., that no voter will ever have an incentive to vote insincerely when given profile-information. Theorem 13 below is a corollary of his Theorem 7 (Endriss, 2009).

Theorem 13. *Under approval voting, paired with a rationalisable tie-breaking choice function, no voter will ever have an incentive to vote insincerely when given profile-information.*

Proof. Fix any $\mathcal{X}$ and $\mathcal{N}$, and fix any rationalisable tie-breaking choice function T. Let F be approval voting paired with T, and let π be the profile-PIF. Proof by contradiction. Suppose that there are a voter i, a poll profile $\boldsymbol{b}$ and an insincere ballot $c_i^\star$ such that $F(c_i^\star, \boldsymbol{b}_{-i}) \succ_i F(b_i, \boldsymbol{b}_{-i})$ for any sincere ballot b_i. Since $c_i^\star$ is insincere, we have that $c_i^\star$ does not contain candidate x while it does contain candidate y for some x and y with $x \succ_i y$. Two cases. If $F(c_i^\star, \boldsymbol{b}_{-i}) = z$ for some $z \preceq_i x$, then voter i should add candidate x to her ballot (outcome can only improve). And if $F(c_i^\star, \boldsymbol{b}_{-i}) = z$ for some $z \succ_i x$, then voter i should remove all candidates y' with $y' \prec_i x$ from her ballot (outcome is not affected). Repeat the above until voter i has a ballot without any 'gaps'. But then she has a sincere alternative b_i to $c_i^\star$ such that $F(b_i, \boldsymbol{b}_{-i}) \succeq_i F(c_i^\star, \boldsymbol{b}_{-i})$. Contradiction! It follows that under F no voter will ever have an incentive to vote insincerely when given profile-information. □

The above result ceases to hold when we allow approval voting to pair with *any* tie-breaking choice function, as Example 5 shows.

Example 5. *Consider a scenario with 2 voters and 3 candidates (a, b, c). Suppose that voting procedure H is approval voting. Suppose further that T is a (non-rationalisable) tie-breaking choice function that breaks ties as follows: $T(\{a,b,c\}) = a$, $T(\{b,c\}) = b$, $T(\{a,c\}) = a$, and $T(\{a,b\}) = b$. Let $F = T \circ H$. Let $\succ_1 = c \succ b \succ a$, and consider (insincere) ballot $c_1^\star = \{b\}$. Now there is a ballot c_2 of voter 2 such that voter 1 benefits from voting insincerely, namely when $c_2 = \{a\}$. Then $F(b_1, c_2) = a$ for any sincere ballot b_1 of voter 1, and $F(c_1^\star, c_2) = b$.*

It follows that voter 1 has an incentive to vote insincerely when given profile-information. Hence, under approval voting, paired with a non-rationalisable tie-breaking choice function, a voter can have an incentive to vote insincerely when given profile-information.

4.4 Discussion

Does the Gibbard-Satterthwaite Theorem continue to hold in a setting of partial information? In this chapter, we studied the susceptibility to manipulation of different voting procedures while limiting the information about other voters' ballots available to voters. We proved a straightforward generalisation of the Gibbard-Satterthwaite Theorem: any resolute voting procedure for three or more candidates that is surjective, nondictatorial, and strongly computable from π-images is susceptible to π-manipulation. As is well known, at least three different candidates must be able to win for a voting procedure to be susceptible to manipulation. Nondictatorship is also a necessary condition for susceptibility: any dictatorial voting procedure is immune to manipulation. (Strong) Computability from π-images, however, is not. We showed that for an even number of voters, any strongly Condorcet-consistent voting procedure (which may not be computable from (W)MG-information) is susceptible to (W)MG-manipulation.[2] Additionally, any unanimous positional scoring rule (which is not strongly computable from rank- nor winner-information) is susceptible to rank- and winner-manipulation. The unanimity constraint in the latter proposition cannot be dropped. In particular, we proved that the antiplurality rule is immune to winner-manipulation.

The only voting procedures that are (strongly) computable from zero-information are constant procedures. These, however, always elect the same candidate regardless of voters' ballots, and therefore they are immune to any form of π-manipulation. We would expect most 'reasonable' voting procedures to be immune to zero-manipulation. And indeed, we proved this for two large

[2]An example of a voting procedure that is strongly Condorcet-consistent, yet not computable from (W)MG-information is the procedure that elects the set of weak Condorcet winners whenever that set is nonempty, and the Borda winner(s) otherwise.

classes of voting procedures: strongly Condorcet-consistent procedures and positional scoring rules. This is not to suggest that every (resolute) voting procedure is immune to zero-manipulation. We can think of wild procedures that are susceptible to zero-manipulation, for example, the voting procedure that always elects the candidate that is ranked second by voter 1.

What remains open is whether the Gibbard-Satterthwaite Theorem extrapolates to polls that provide ballot-information. That is, is there a resolute voting procedure for three or more candidates that is surjective, nondictatorial and immune to ballot-manipulation? The same question may be asked for WMG-information. From our π-susceptibility result on strong computability from π-images, it follows that the voting procedures we are looking for are not strongly computable from ballot-information and WMG-information, respectively. Note that any voting procedure that is not strongly computable from ballot-information is not anonymous.

We do know that the Gibbard-Satterthwaite Theorem does not extrapolate to polls that provide MG-information, winner-information, or zero-information. More specifically, we found that the plurality rule is immune to MG-manipulation, and that the antiplurality rule is immune to winner-manipulation. Since the score- and rank-PIFs are ill-defined for many voting procedures, we did not investigate the extrapolation of the Gibbard-Satterthwaite Theorem to these information functions. However, it is not difficult to come up with a voting procedure that is immune to score- and rank-manipulation. Take, for example, a procedure that is immune to winner-manipulation and let it assign the same score (different from the winner's score) to all losing candidates.

Finally, in line with the work done by Conitzer et al. (2011), it would also be worthwhile to analyse how hard it is for a voter to π-manipulate under different voting procedures that are susceptible to π-manipulation.

Chapter 5

Repeated Response to Polls

In this chapter, we study the scenario in which voters repeatedly update their ballot in view of a sequence of polls. This *voting game* proceeds in rounds. In each round an opinion poll is held and one of the voters may change her ballot accordingly. We consider three types of responses to poll information: a *strategist* will choose a ballot such that no other ballot provides a strictly better outcome for some and at least as good an outcome for all ballot profiles she considers possible; a *pragmatist* will support her favourite candidate from a small set of, say, two front-runners; and a *truth-teller* will always vote truthfully. For different voting procedures and for different combinations of these *response policies*, we analyse whether the corresponding voting game will always reach a point from which no voter wishes to deviate. We then observe that a voting game can itself be considered a voting procedure if we fix a number of rounds to be played, a voting procedure, a poll information function, and for each voter a response policy: for any input ballot profile it will return a nonempty set of winning candidates. We study how the properties of the original voting procedure relate to the properties of this induced procedure.

This process in which a poll is held after each changed ballot, is common among internet polls and elections. See for example the poll gadgets for Google Wave: `http://sites.google.com/site/polloforwave`. Our approach is also relevant to the study of collective decision making by means of voting in multi-agent systems. In these systems we can precisely model the amount of information available to agents (voters), and agents can be expected to follow relatively simple rules when adjusting their behaviour in response to polls.

5.1 Extended Polling Perspective

In this section, we extend our framework of Chapter 4 to deal with sequences of polls.

5.1.1 Voting Games and Induced Voting Procedures

A voting game is defined by a resolute voting procedure, a poll information function, and for each voter a response policy (Definition 17).

Definition 17. *A **voting game** is a tuple $G = \langle F, \pi, \boldsymbol{\delta} \rangle$, where F is a resolute voting procedure, π is a PIF, and $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)$ is a vector of response policies.*

A voting game G proceeds in rounds. In the first round, each voter i submits a ballot b_i, and the obtained poll information $\pi(\boldsymbol{b})$ is communicated to all voters. In each subsequent round, exactly one voter changes her ballot, and the updated poll information is again communicated to all voters. Whether or not a voter wishes to change her ballot depends on the response policy she is following. A *response policy* determines for each voter i a function $\delta_i : \mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X}) \times 2^{\mathcal{L}(\mathcal{X})^{\mathcal{N}\setminus\{i\}}} \to \mathcal{L}(\mathcal{X})$, mapping voter i's true preference order, her previously submitted ballot, and her current information set to a new ballot. In Section 5.1.2 we give some examples of such response policies. A voting game is said to *terminate* in round t if no voter wishes to change her ballot according to her response policy at the end of t. A game is played until termination (or until a given maximum number of rounds has been reached). We then apply voting procedure F to this final profile to obtain the *winner of the voting game*.

Observe that a voter's response policy does not specify how she should vote in the first round. We shall assume that all voters vote truthfully then. This is not unreasonable, given the immunity results to zero-manipulation in Section 4.3. Furthermore, for a voting game to be uniquely defined, we have to fix the order in which voters may change their vote. Any such order is allowed, even one that only offers voter 1 a chance to update her ballot. We will not specify exactly how a voting game selects the next voter to change her ballot, but simply assume that it has a way of doing so.

A voting game G *induces* a new voting procedure F^t when the number of rounds to be played is t (Definition 18). If G always terminates after at most t rounds (for any truthful, initial profile $\boldsymbol{b}$), then we write $F^\star$ instead of F^t.

Definition 18. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be a voting game, and let $t \in \mathbb{N}$ be the number of rounds to be played. Then a voting procedure F^t is **induced** by G if for any ballot profile $\boldsymbol{b} \in \mathcal{L}(\mathcal{X})^{\mathcal{N}}$:*

$$F^t(\boldsymbol{b}) := \left\{ x \in \mathcal{X} \;\middle|\; \begin{array}{l} x \text{ is a winner of } G \text{ after } t \text{ rounds} \\ \text{when } \boldsymbol{b} \text{ is the truthful, initial profile} \end{array} \right\}$$

As the framework is set up now, voters only take into account information from the latest poll round. However, the framework could be extended to include previous rounds by adding the information sets induced by those rounds to the input arguments of response policies.

5.1.2 Response Policies

In each round of the voting game, a voter receives poll information and has to decide whether or not she wants to update her ballot accordingly. We assume that a voter uses a response policy to determine what to vote next, and that she uses the same policy throughout the election. Recall that a response policy determines for each voter i a function $\delta_i : \mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X}) \times 2^{\mathcal{L}(\mathcal{X})^{\mathcal{N}\setminus\{i\}}} \to \mathcal{L}(\mathcal{X})$, mapping voter i's true preference order $\succ_i$, her previously submitted ballot b_i, and her current information set $\mathcal{W}_i$ to a new ballot c_i. We shall work with the following policies:

- *Truth-teller:* A truth-teller always votes truthfully: $\delta_i(\succ_i, b_i, \mathcal{W}_i) = \succ_i$.
- *Strategist:* A strategist computes her best responses to a poll and submits (any) one of them if her previously submitted ballot b_i is not amongst them, and b_i otherwise. We say that a ballot c_i is a *better response* to voter i's information set than a ballot c'_i if c_i yields a strictly better outcome for some and at least as good an outcome for all profiles considered possible by voter i.[1] Then a ballot c_i is a best response to voter i's information set if there is no ballot c'_i such that c'_i is a better response:

  Definition 19. *Given a resolute voting procedure F, a voter i, an information set $\mathcal{W}_i$, and a ballot c_i, ballot c_i is a* ***best response*** *to $\mathcal{W}_i$ if there is no ballot $c'_i \in \mathcal{L}(\mathcal{X})$ such that:*

  $$\begin{array}{rll} & F(c'_i, \boldsymbol{c}_{-i}) \succ_i F(c_i, \boldsymbol{c}_{-i}) & \textit{for some profile } \boldsymbol{c}_{-i} \in \mathcal{W}_i \\ \textit{and} & F(c'_i, \boldsymbol{c}_{-i}) \succeq_i F(c_i, \boldsymbol{c}_{-i}) & \textit{for all other profiles } \boldsymbol{c}_{-i} \in \mathcal{W}_i \end{array}$$

  Observe that a voter may have multiple best responses to her information set. A specific strategist response policy always picks a unique one. Such a policy may, for example, favour compromising, burying, or push-over strategies (see Section 2.2). All of the results in this chapter hold for any strategist response policy.

  If we restrict attention to the plurality rule and assume that polls give score-information, then the strategist response policy is similar to the one used by Meir et al. (2010).

- *Pragmatist:* A pragmatist cannot or does not want to compute her best response to a poll, e.g., because this takes too much effort. A pragmatist can also be optimistic about other voters following her example and therefore be willing to vote for a candidate that has a better chance of winning than her favourite, even though she does not believe that her vote will change the election outcome at that moment.

  A k-pragmatist always moves her favourite amongst the k currently highest ranked candidates to the first position in her ballot, without changing the relative ranking of the others. For a voter to be a pragmatist, the voting procedure and polls should give enough information to deduce who are the k currently highest ranked candidates.

  The pragmatist response policy is also described by Brams & Fishburn (1983) for the plurality rule.

5.2 Termination Results

Since we do not want the number of poll rounds to have a great influence on the winner of a voting game, we prefer voting games to terminate. In this section, we study which voting games are guaranteed to terminate, and which are not. All of our termination results hold for any way of selecting the next voter to change her ballot.

[1] Note that a voter i has an incentive to π-manipulate in some poll profile $\boldsymbol{b}$ with $b_i = \succ_i$ if there is a ballot $c^\star_i$ such that $c^\star_i$ is a better response to $\mathcal{W}_i^{\pi(\boldsymbol{b})}$ than $\succ_i$.

Obviously, if all voters are truth-tellers, then any voting game terminates after 0 rounds (Theorem 14). Moreover, if F is immune to π-manipulation and all voters are strategists, then G terminates after 0 rounds as well (Theorem 15).

Theorem 14. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that $\boldsymbol{\delta}$ is a vector of truth-tellers. Then G terminates after 0 rounds.*

Proof. Immediate. □

Theorem 15. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is immune to π-manipulation and $\boldsymbol{\delta}$ is a vector of strategists. Then G terminates after 0 rounds.*

Proof. Immediate. □

Corollary 15.1. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is immune to π-manipulation and $\boldsymbol{\delta}$ is a vector of strategists and truth-tellers. Then G terminates after 0 rounds.*

Note that if all voters are strategists or truth-tellers and a voting game G does not always terminate after 0 rounds, then its voting procedure F is susceptible to π-manipulation. It is not the case, however, that if F is susceptible to π-manipulation and all voters are strategists, then there is a (possibly untruthful) initial profile for G such that G never terminates. Consider, for example, the voting procedure that always elects the candidate that is ranked second by voter 1; this procedure can be manipulated, but the voting game will always terminate after at most one round.

Meir et al. (2010) show that for any voting game G with F being the plurality rule, paired with a rationalisable tie-breaking choice function, and π being the score-PIF with respect to F, if p voters are strategists and $n-p$ voters are truth-tellers, then G terminates after at most $m \cdot p$ rounds. To be precise, these authors show this for a specific kind of strategist response policy, namely one in which a voter, whenever her previously submitted ballot is not amongst the best responses, wishes to change her ballot to the best response that matches the next candidate to win (since Meir et al. (2010) focus on the plurality rule, we may assume that voters only submit their top-ranked candidate). Their result ceases to hold when we allow plurality to pair with *any* tie-breaking choice function, as Example 6 shows.

Example 6. *Consider a scenario with 3 voters and 4 candidates (a, b, c, d). Suppose that voting procedure H is the plurality rule. Suppose further that T is a (non-rationalisable) tie-breaking choice function that breaks ties according to $a \succ b \succ c \succ d$ except for: $T(\{a, b, c\}) = b$, $T(\{a, c, d\}) = c$, and $T(\{b, c, d\}) = d$. Let $F = T \circ H$ and let π be the score-PIF with respect to F. Suppose that all voters are strategists according to the definition of Meir et al. (2010). Let their true preference orders be as follows:*

$$\begin{aligned} \succ_1 = \quad & b \succ c \succ d \succ a \\ \succ_2 = \quad & c \succ a \succ b \succ d \\ \succ_3 = \quad & d \succ b \succ a \succ c \end{aligned}$$

Then: $F(b, c, d) = d \xrightarrow{2} F(b, a, d) = a \xrightarrow{1} F(c, a, d) = c \xrightarrow{3} F(c, a, b) = b \xrightarrow{2} F(c, c, b) = c \xrightarrow{1} F(b, c, b) = b \xrightarrow{3} F(b, c, d) = d \xrightarrow{2} \ldots$. *It follows that G does* *not terminate. Hence, a voting game G with F being the plurality rule, paired with a non-rationalisable tie-breaking choice function, and π being the score-PIF with respect to F, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always terminate.*

On the other hand, the result of Meir et al. (2010) does generalise to any strategist response policy (Theorem 16), provided that all voters vote truthfully in the first round.

Theorem 16. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is the plurality rule, paired with a rationalisable tie-breaking choice function, π is the score-PIF with respect to F, and $\boldsymbol{\delta}$ is a vector of p strategists and $n-p$ truth-tellers. Then G terminates after at most $m \cdot p$ rounds.*

Proof. Let G, F, π, and $\boldsymbol{\delta}$ satisfy above conditions. Without loss of generality, we may assume that voters only submit their top-ranked candidate. Fix any truthful, initial profile $\boldsymbol{b}$. Suppose that voter i wishes to change her ballot and may do so next. There are 4 types of strategist moves. Below, $b_{i,t}$ denotes voter i's ballot at round t.

type 0: from $b_{i,t} \neq F^t(\boldsymbol{b})$ to $b_{i,t+1} \neq F^{t+1}(\boldsymbol{b})$

type 1: from $b_{i,t} \neq F^t(\boldsymbol{b})$ to $b_{i,t+1} = F^{t+1}(\boldsymbol{b})$

type 2: from $b_{i,t} = F^t(\boldsymbol{b})$ to $b_{i,t+1} \neq F^{t+1}(\boldsymbol{b})$

type 3: from $b_{i,t} = F^t(\boldsymbol{b})$ to $b_{i,t+1} = F^{t+1}(\boldsymbol{b})$

Meir et al. (2010) only consider moves of type 1 and 3. They prove that there are at most $m \cdot p$ moves of type 1, and no moves of type 3. In addition, we will prove that there are no moves of type 0 nor 2.

Since voter i is a strategist who receives score-information from π, we have that $F^{t+1}(\boldsymbol{b}) \succ_i F^t(\boldsymbol{b})$. Now, suppose that voter i makes a move of type 0. Then old winner $F^t(\boldsymbol{b})$ does not lose any points and new winner $F^{t+1}(\boldsymbol{b})$ does not win any. It follows that $F^t(\boldsymbol{b}) = F^{t+1}(\boldsymbol{b})$, because the applied tie-breaking choice function is rationalisable. Contradiction! Hence, voter i does not have any moves of type 0.

Claim: voter i does not have any moves of type 2. Proof by induction. Since initial profile $\boldsymbol{b}$ is truthful, we have that no strategist wishes to make a move of type 2 at round 1. Now, suppose that no strategist has made a move of type 2 at round t. It follows that only moves of type 1 have been made. Let voter i be the next to change her ballot. Suppose, by contradiction, that i wishes to make a move of type 2. Then $b_{i,t} = F^t(\boldsymbol{b})$ and $F^{t+1}(\boldsymbol{b}) \succ_i F^t(\boldsymbol{b})$. Since i voted truthfully initially, we have that she has made a move of type 1 at some round $t' < t$. It follows that i could not make $F^{t+1}(\boldsymbol{b})$ win at t'. From the induction hypothesis, we get that no voter could make $F^{t+1}(\boldsymbol{b})$ win at any round t'' with $t' < t'' < t$. It follows that at round t the scores of $F^{t+1}(\boldsymbol{b})$ and $F^t(\boldsymbol{b})$ differ at least 1 point if ties are broken in favour of $F^t(\boldsymbol{b})$, and at least 2 points otherwise. Thus, voter i cannot make a move of type 2 at round t. Contradiction! Our claim holds. Hence, strategists only make moves of type 1 in G, and therefore G terminates after at most $m \cdot p$ rounds. □

We obtain a similar result for an electorate composed of truth-tellers and pragmatists (as opposed to strategists). In fact, under these assumptions the result can be generalised to arbitrary positional scoring rules:

Theorem 17. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is a PSR, paired with a rationalisable tie-breaking choice function, π is the rank-PIF with respect to F, and $\boldsymbol{\delta}$ is a vector of p k-pragmatists and $n-p$ truth-tellers. Then G terminates after at most p rounds.*

Proof. Let G, F, π, and $\boldsymbol{\delta}$ satisfy above conditions. Fix any truthful, initial profile $\boldsymbol{b}$. Let $H_k^t(\boldsymbol{b})$ be the set of k highest ranked candidates in $\boldsymbol{b}$ according to F^t. Claim: $H_k^t(\boldsymbol{b}) = H_k^{t+1}(\boldsymbol{b})$ for any number of rounds $t \in \mathbb{N}$. Suppose that voter i changes her ballot at round t. Since i is a k-pragmatist, and F is a PSR, we have that no candidate $x \in H_k^t(\boldsymbol{b})$ loses points and no candidate $y \in \mathcal{X} \setminus H_k^t(\boldsymbol{b})$ wins any. It follows that our claim holds, because the applied tie-breaking choice function is rationalisable. Hence, each k-pragmatist will update her ballot at most once, and therefore G terminates after at most p rounds. □

A similar argument can be used to prove that if F is the Copeland procedure, paired with a rationalisable tie-breaking choice function, π is the rank-PIF with respect to F, and p voters are k-pragmatists and $n-p$ voters are truth-tellers, then G terminates after at most p rounds. This also holds in case F is the maximin procedure or the Bucklin procedure. On the other hand, games defined in terms of other voting procedures or other response policies need not always terminate. The following example illustrates this for the Copeland procedure.

Example 7. *Consider a scenario with 2 voters and 3 candidates (a, b, c). Suppose that voting procedure H is the Copeland procedure. Suppose further that T is the lexicographic tie-breaking rule, i.e., T breaks ties according to $a \succ b \succ c$. Let $F = T \circ H$ and let π be the score-PIF with respect to F. Suppose that all voters are strategists. Consider the following truthful, initial profile: $\boldsymbol{b} = (a \succ b \succ c,\ c \succ b \succ a)$. Then: $F^0(\boldsymbol{b}) = a \xrightarrow{2} F^1(\boldsymbol{b}) = b \xrightarrow{1} F^2(\boldsymbol{b}) = a \xrightarrow{2} F^3(\boldsymbol{b}) = c \xrightarrow{1} F^4(\boldsymbol{b}) = a \xrightarrow{2} \dots$ (voters 1 and 2 alternately move candidate b up and down in their ballot). It follows that G does not terminate. Hence, a voting game G with F being the Copeland procedure, paired with a tie-breaking choice function, and π being the score-PIF with respect to F, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always terminate.*

The Borda voting game with all voters being strategists (or truth-tellers) also does not guarantee termination, as Example 8 shows.

Example 8. *Consider a scenario with 4 voters and 4 candidates (a, b, c, d). Suppose that voting procedure H is the Borda rule. Suppose further that T is the lexicographic tie-breaking rule. Let $F = T \circ H$ and let π be the score-PIF with respect to F. Suppose that all voters are strategists. Consider the following truthful, initial profile:*

$$\begin{aligned} b_1 &= a \succ b \succ c \succ d \\ b_2 &= c \succ b \succ a \succ d \\ b_3 &= a \succ c \succ d \succ b \\ b_4 &= b \succ c \succ d \succ a \end{aligned}$$

Then: $F^0(\boldsymbol{b}) = c \xrightarrow{1} F^1(\boldsymbol{b}) = b \xrightarrow{2} F^2(\boldsymbol{b}) = c \xrightarrow{1} F^3(\boldsymbol{b}) = a \xrightarrow{2} F^4(\boldsymbol{b}) = c \xrightarrow{1} \dots$ (voters 1 and 2 alternately move candidate b up and down in their *ballot). It follows that G does not terminate. Hence, a voting game G with F being the Borda rule, paired with a tie-breaking choice function, and π being the score-PIF with respect to F, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always terminate.*

5.3 Properties of Induced Voting Procedures

In this thesis, we study the effects of polls on election outcomes. In order to judge these effects as positive or negative, we have to be able to identify the candidate who best represents the electorate. For some preference profiles, voting theory provides us with a clear-cut identification method, e.g., Condorcet winners should always win, and Condorcet losers and Pareto-dominated candidates should always lose. For other preference profiles, however, it is not directly clear who should or should not win. For these cases, voting theorists came up with numerous desirable properties that voting procedures should possess. In this section, we investigate the properties of voting games through the voting procedures they induce. More specifically, we are interested in how these properties relate to the properties of the underlying voting procedures.

Let us begin with a simple observation: If G always terminates after 0 rounds, then any property of F is transferred to F^t. This observation leads to the following two transfer results for dictatorship and constancy:

Theorem 18. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is dictatorial, π is a PIF, and $\boldsymbol{\delta}$ is a vector of pragmatists, strategists and truth-tellers. Then F^t is dictatorial for any $t \in \mathbb{N}$.*

Proof. Immediate. □

Theorem 19. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is constant, π is a PIF, and $\boldsymbol{\delta}$ is a vector of response policies. Then F^t is constant for any $t \in \mathbb{N}$.*

Proof. Immediate. □

Our next result is closely related to Theorem 18. It states that any induced voting procedure is dictatorial if its underlying voting procedure is surjective and yields exactly one powerful voter who is a strategist. From Theorem 20 it follows, for example, that any voting game based on the voting procedure that always elects the candidate that is ranked second by voter 1 induces a dictatorial voting procedure if voter 1 is a strategist. Thus, non-dictatorships do not always transfer.

Theorem 20. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is surjective and yields one powerful voter, π is a PIF, and $\boldsymbol{\delta}$ is a vector of response policies such that the powerful voter is a strategist. Then F^t is dictatorial for any $t \geqslant 1$.*

Proof. Immediate. □

Although voting procedures F and F^t in Theorem 20 both yield exactly one powerful voter, we do not have that the set of powerful voters transfers in general. More specifically, the set of powerful voters of F^t can be a strict subset of the set of powerful voters of F. Take, for example, the voting procedure that elects voter 1's top-ranked candidate unless she ranks some candidate a last, in which case voter 2's top-ranked candidate is elected. Then voters 1 and 2 are both powerful in F, but induced voting procedure F^t yields only one powerful voter, namely voter 1 (for $m \geqslant 3$, and all powerful voters being strategists). It is not difficult to see that the set of powerful voters of F^t must always be a (not necessarily strict) subset of the set of powerful voters of F.

Less obvious is the transfer result that we obtain for unanimity:

Theorem 21. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is unanimous, π is a PIF, and $\boldsymbol{\delta}$ is a vector of pragmatists, strategists and truth-tellers. Then F^t is unanimous for any $t \in \mathbb{N}$.*

Proof. Let G, F, π, and $\boldsymbol{\delta}$ satisfy above conditions. Fix any truthful, initial profile $\boldsymbol{b}$ such that there is a candidate w who is ranked first by all voters. Claim: $F^t(\boldsymbol{b}) = w$ for any number of rounds $t \in \mathbb{N}$. Proof by induction. Since F is unanimous, we have that $F^0(\boldsymbol{b}) = w$. Now, suppose that $F^t(\boldsymbol{b}) = w$, and that voter i wishes to change her ballot and may do so next. As no truth-teller or pragmatist who already has her favourite candidate winning will ever change her ballot, we only need to consider the case where i is a strategist. Since strategists always switch to a ballot that is at least as good as their previous ballot for all profiles in their information set (and strictly better for some), we have that $F^{t+1}(\boldsymbol{b}) = w$. Hence, F^t is unanimous for any $t \in \mathbb{N}$. □

However, the Pareto condition, a slightly stronger condition than unanimity, does not always transfer, as we shall see next.

Example 9 (Pareto efficiency)**.** *Consider a scenario with 2 voters and 3 candidates (a, b, c). Suppose that voting procedure H elects all candidates who are not Pareto-dominated. Suppose further that T is the lexicographic tie-breaking rule. Let $F = T \circ H$ and let π be the winner-PIF with respect to F. Suppose that all voters are strategists and suppose that $t \geqslant 1$. Consider the following truthful, initial profile: $\boldsymbol{b} = (b \succ c \succ a,\ c \succ a \succ b)$. Then: $F^0(\boldsymbol{b}) = b \xrightarrow{2} F^1(\boldsymbol{b}) = F^\star(\boldsymbol{b}) = a$, because the second voter will rank candidate a on top, given that she has no chance to make c win, after which no other voter wishes to change her ballot. However, candidate a is Pareto-dominated by candidate c in profile $\boldsymbol{b}$. It follows that F^t is not Pareto-efficient, whereas F is. Hence, a voting game G with F being Pareto-efficient, and π being the winner-PIF with respect to F, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always induce a Pareto-efficient voting procedure.*

We also cannot guarantee the transfer of the Pareto condition for voting games in which all voters are pragmatists. Consider, for this purpose, voting procedure F of Example 9 and pair it with a procedure that assigns 1 point to the winning candidate according to F, and 0 points to all other candidates, and let tie-breaking rule T break ties to obtain a ranking. Then F is Pareto-efficient, whereas F^t is not if all voters are 2-pragmatists. In general, these examples in which all voters are pragmatists are more artificial (and therefore less interesting) than our examples in which all voters are strategists, because all properties under investigation are requirements on the winning candidate and do not pose any restrictions on the ranking of losing candidates, whereas the pragmatist response policy needs voting procedures to produce a ranking of all candidates.

In this section, most of our counterexamples therefore focus on the strategist response policy.

Surprisingly, the transfer of surjectivity, one of our weakest properties defined, is also not guaranteed, as Example 10 shows.

Example 10 (Surjectivity)**.** *Consider a scenario with 4 voters and 3 candidates (a, b, c). Suppose that voting procedure F almost always elects candidate a, unless some other candidate is ranked last by all voters, then F elects that candidate. Let π be the ballot-PIF. Suppose that all voters are strategists and suppose that $t \geqslant 2$. Then $F^t = F^\star$ is a constant voting procedure, always electing candidate a. It follows that F^t is not surjective, whereas F is. Hence, a voting game G with F being surjective, and π being the ballot-PIF, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always induce a surjective voting procedure.*

Again, we can think of an example to show that surjectivity is also not guaranteed to transfer if all voters are pragmatists. Consider a voting procedure F that almost always elects candidate a, unless a is ranked first by all voters except for voter 1, then F elects the top-ranked candidate of voter 1. Now, suppose that F assigns 1 point to the winning candidate, $\frac{1}{2}$ point to some candidate other than a ranked first by some voter other than 1 if such a candidate exists, -1 points to candidate a if a loses, and 0 points to all other candidates. Let some tie-breaking choice function T break ties to obtain a ranking. Then F is surjective, whereas F^t is not if all voters are 2-pragmatists.

Another axiom that is satisfied by many voting procedures, but does not always transfer, is anonymity:

Example 11 (Anonymity)**.** *Consider a scenario with 3 voters and 3 candidates (a, b, c). Let F be the plurality rule paired with the lexicographic tie-breaking rule, and let π be the ballot-PIF. Suppose that all voters are strategists and suppose that $t \geqslant 1$. Suppose further that the next voter to change her ballot is selected from the set of voters who wish to change, and that the voter with the lowest index may do so next. Consider the following truthful, initial profile:*

$$\begin{aligned} b_1 = &\quad a \succ b \succ c \\ b_2 = &\quad b \succ c \succ a \\ b_3 = &\quad c \succ b \succ a \end{aligned}$$

Then: $F^0(\boldsymbol{b}) = a \xrightarrow{2} F^1(\boldsymbol{b}) = F^\star(\boldsymbol{b}) = c$, because the second voter will rank candidate c on top, given that she has no chance to make b win, after which no other voter wishes to change her ballot. However, if we define permutation $\tau : \mathcal{N} \to \mathcal{N}$ as follows: $\tau(1) = 1$, $\tau(2) = 3$, and $\tau(3) = 2$, then $F^0(\boldsymbol{b}) = a \xrightarrow{2} F^1(\boldsymbol{b}) = F^\star(\boldsymbol{b}) = b$, because the second voter is now endowed with true preference order $c \succ b \succ a$. It follows that F^t is not anonymous, whereas F is. Hence, a voting game G with F being anonymous, and π being the ballot-PIF, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always induce an anonymous voting procedure.

Observe that the above example works, because the order in which voters may change their vote is not permuted. We can construct a similar example to show that anonymity is also not guaranteed to transfer if all voters are pragmatists.

Consider, for this purpose, voting procedure F of Example 11 and pair it with a procedure that assigns $m-1$ points to the winning candidate according to F, $m-2$ points to the remaining candidate that is ranked highest by voter 2, $m-3$ points to the remaining candidate that is ranked second highest by voter 2, et cetera. Then F is anonymous, whereas F^t is not if all voters are 2-pragmatists.

Furthermore, Condorcet consistency does not always transfer either:

Example 12 (Condorcet consistency)**.** *Consider a scenario with 4 voters and 4 candidates (a, b, c, d). Suppose that voting procedure F elects the Condorcet winner if one exists, and candidate a otherwise. Let π be the ballot-PIF. Suppose that all voters are strategists and suppose that $t \geqslant 1$. Consider the following truthful, initial profile:*

$$\begin{aligned} b_1 = &\quad a \succ b \succ c \succ d \\ b_2 = &\quad b \succ a \succ c \succ d \\ b_3 = &\quad c \succ b \succ a \succ d \\ b_4 = &\quad d \succ b \succ a \succ c \end{aligned}$$

Then: $F^0(\boldsymbol{b}) = b \xrightarrow{1} F^1(\boldsymbol{b}) = F^\star(\boldsymbol{b}) = a$, because the first voter will rank candidate b on the bottom of her ballot, given that a is the election winner by default, after which no other voter wishes to change her ballot. It follows that F^t is not Condorcet-consistent, whereas F is. Hence, a voting game G with F being Condorcet-consistent, and π being the ballot-PIF, and $\boldsymbol{\delta}$ being a vector of strategists (and truth-tellers) does not always induce a Condorcet-consistent voting procedure.

On the other hand, if all voters are pragmatists (or truth-tellers), then Condorcet consistency does transfer:

Theorem 22. *Let $G = \langle F, \pi, \boldsymbol{\delta} \rangle$ be any voting game such that F is Condorcet-consistent, π is the rank-PIF, and $\boldsymbol{\delta}$ is a vector of pragmatists and truth-tellers. Then F^t is Condorcet-consistent for any $t \in \mathbb{N}$.*

Proof. Let G, F, π, and $\boldsymbol{\delta}$ satisfy above conditions. Fix any truthful, initial profile $\boldsymbol{b}$ with a Condorcet winner w. Claim: $F^t(\boldsymbol{b}) = w$ for any number of rounds $t \in \mathbb{N}$. Proof by induction. Since F is Condorcet-consistent, we have that $F^0(\boldsymbol{b}) = w$. Now, suppose that $F^t(\boldsymbol{b}) = w$, and that voter i wishes to change her ballot and may do so next. Since i is a k-pragmatist for some $k \in \mathbb{N}$, and w is among the k currently highest ranked candidates, we have that w cannot lose support in any pairwise contest with respect to its original pairwise scores. It follows that $F^{t+1}(\boldsymbol{b}) = w$. Hence, F^t is Condorcet-consistent for any $t \in \mathbb{N}$. □

A similar argument as in Example 12 can be used to show that if F never elects the Condorcet loser, then F^t might still elect the Condorcet loser if all voters are strategists.

Example 13 (Condorcet loser)**.** *Consider a scenario with 4 voters and 4 candidates (a, b, c, d). Suppose that voting procedure F almost always elects candidate a, unless he is the Condorcet loser in which case F elects candidate b. Let π be the ballot-PIF. Suppose that all voters are strategists and suppose that $t \geqslant 1$.*

Consider the following truthful, initial profile:

$$\begin{aligned} b_1 = &\quad c \succ d \succ a \succ b \\ b_2 = &\quad b \succ c \succ d \succ a \\ b_3 = &\quad c \succ b \succ a \succ d \\ b_4 = &\quad d \succ b \succ a \succ c \end{aligned}$$

Then: $F^0(\boldsymbol{b}) = b \xrightarrow{1} F^1(\boldsymbol{b}) = F^\star(\boldsymbol{b}) = a$, *because the first voter will rank candidate* a *on top of her ballot, given that* a *and* b *are the only possible winners, after which no other voter wishes to change her ballot. It follows that* F^t *elects the Condorcet loser, whereas* F *does not. Hence, a voting game* G *with* F *being a voting procedure that never elects the Condorcet loser, and* π *being the ballot-PIF, and* $\boldsymbol{\delta}$ *being a vector of strategists (and truth-tellers) does not always induce a voting procedure that never elects the Condorcet loser.*

Unlike Condorcet consistency, this example also works if all voters are 2-pragmatists. In that case, assume that candidates a and b are always the two highest ranked candidates.

5.4 Condorcet Efficiency: Simulations

It is widely acknowledged that Condorcet consistency is a highly desirable property, but many important voting procedures do not satisfy it (Taylor, 2005). The *Condorcet winner efficiency* of a voting procedure is its tendency to elect the Condorcet winner. Theorem 22 identifies conditions under which Condorcet consistency transfers from F to F^t, but it does not say anything about the transfer of Condorcet winner efficiency. Brams & Fishburn (1983) give several examples that show that polls can have both a positive and a negative effect on the Condorcet winner efficiency of plurality and approval voting. We would like to know *how* positive or negative this effect exactly is.

As we prefer Condorcet winners to win, we prefer Condorcet losers to lose. However, under several important voting procedures the Condorcet loser can win sometimes. The *Condorcet loser efficiency* of a voting procedure is its tendency to *not* elect the Condorcet loser. Example 13 shows that the property of never electing the Condorcet loser does not always transfer, but it does not say anything about the transfer of Condorcet loser efficiency. We would like to know how polls influence the Condorcet loser efficiency of a voting procedure.

In this section, we study the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency of six common voting procedures by means of simulations. These voting procedures are: plurality, Borda, Copeland, STV, Bucklin, and approval voting. In our main comparative experiment, we assume that all voters are pragmatists. The pragmatist response policy can be seen as a formalisation of what political scientists call 'tactical voting', a way of voting that is not very common in large elections, like national presidential elections, but presumably occurs much more often in small elections, like the ones we consider here (Fisher, 2004). In a follow-up experiment, we analyse the influence of poll information on election outcomes, and we consider two types of information that polls may provide: each candidate's score or each candidate's rank. The pragmatist response policy does not differentiate between these types of information. The strategist response policy does, but this policy is much more demanding. It is conceivable, however, that voters would use a strategist response policy for the plurality rule, which is computationally particularly easy to reason about for voters. We therefore focus on the plurality rule in this experiment.

Note that the pragmatist response policy is not well-defined for approval voting under which voters submit a subset of candidates rather than a linear order. A *k-pragmatist for approval voting* wishes to approve of exactly one candidate from the k currently highest ranked candidates. If she does not approve of any top k candidate or if she approves of more than one top k candidate, she will approve of her favourite and all candidates preferred to him. Thus, a pragmatist for approval voting may contract and expand her ballot, but will always vote sincerely. This response policy is also proposed by Brams & Fishburn (1983).

In each of our experiments we simulate a large number of elections. That means that we need a large number of truthful, initial ballot profiles.[2] Since it is extremely cumbersome if not impossible to obtain this data from actual elections, we will automatically generate it. How to do this, is a hotly debated topic in social choice theory. One of the most prominent solutions is to assume that any permutation of candidates is equally likely to occur as a voter's preference order; the *impartial culture* (IC) assumption. The limitations of the IC assumption are well known (Regenwetter et al., 2006). In particular, we should not expect the preferences in a real-world electorate to be distributed uniformly. More realistic solutions have been proposed, like using a *spatial model* to generate voters' preferences (Downs, 1957). A spatial model is a multidimensional space in which each dimension corresponds to an election issue. Both voters and candidates are allocated a position in this space according to their stance on these issues. A voter's preference order is then obtained from the distances between her and the candidates; the closer, the higher up in her preference order.

Nevertheless, the IC assumption is still the *de facto* standard used in social choice theory; results based on it provide an important base line and allow for direct comparison with a large number of findings documented in the literature. We shall therefore use the IC assumption to generate preference profiles.

For approval voting, we also need to generate a voter's initial approval set. As for the other voting procedures, we shall assume that all voters vote sincerely initially. Recall that a voter votes sincerely if she truly prefers each approved candidate to each disapproved candidate (see Section 4.3 for more on this). Thus, it suffices to generate for each voter a cutoff point in her preference order such that she approves of all candidates before the cutoff point and disapproves of all other candidates. Since approving of zero or all candidates does not affect the election outcome, we shall assume that cutoff points are placed somewhere after the first and before the last candidate in voters' preference orders. We will generate cutoff points randomly to facilitate the comparison between approval voting and other voting procedures.

5.4.1 Design

The design of our simulation program is depicted in Figure 5.1. We implemented this simple program in JAVA 1.6.0.

[2] Note that if we did not require voters to vote truthfully initially, we would need twice as

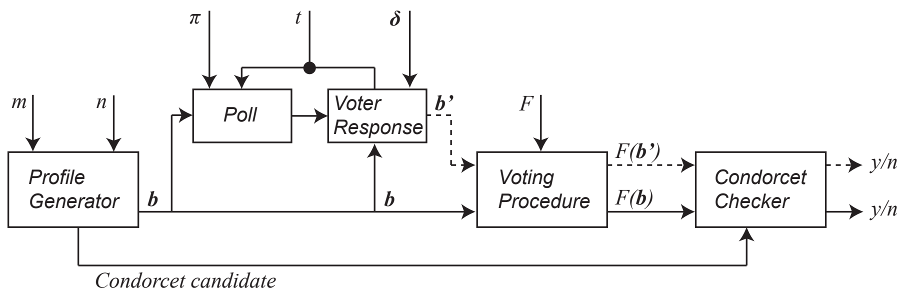


Figure 5.1: Program design.

The profile generator generates preference profiles with a Condorcet candidate (a Condorcet winner of loser) under the impartial culture assumption. We varied the number of candidates (m) from 3 to 15 in steps of 1 while keeping the number of voters (n) fixed at 50, and we varied the number of voters from 10 to 100 in steps of 5 while keeping the number of candidates fixed at 5. Using the IC assumption, the profile generator generated for each of these combinations 10,000 profiles with a Condorcet winner, and 10,000 profiles with a Condorcet loser.

Each generated ballot profile $\boldsymbol{b}$ is then sent through the poll loop t times. In this loop, the poll unit computes poll information $\pi(\boldsymbol{b})$ and communicates it to all voters in the voter response unit. There, a voter i is selected who wishes to change her ballot according to her response policy δ_i, and her updated ballot together with the other voters' ballots is passed to the poll unit again. We fixed the order in which voters may change their ballot to be the *ascending order:* voters are offered a chance to update their ballot according to their index in $\mathcal{N}$, beginning with the successor of the voter who was the last to change (and with voter 1 in the first round).

Voting procedure F calculates the winner of the resulting ballot profile $\boldsymbol{b'}$ and of the original ballot profile $\boldsymbol{b}$. Both winners are compared to the Condorcet candidate of $\boldsymbol{b}$, and the program outputs 'yes' in case these are equal and 'no' otherwise.

Finally, the *Condorcet winner efficiency* of a voting procedure F (or F^t) is computed as the percentage of elections in which the Condorcet winner wins according to F (or F^t) provided one exists, and the *Condorcet loser efficiency* is computed as the percentage of elections in which the Condorcet loser *loses* according to F (or F^t) provided one exists.

We conducted 3 experiments in our simulation program, varying voting procedure F, response policies $\boldsymbol{\delta}$, and poll information function π. All experiments were performed on all generated election data with m ranging from 3 to 15, and n ranging from 10 to 100.

- *Experiment 1:* The aim of this experiment was to investigate the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency of plurality, Borda, Copeland, STV, and Bucklin. We varied F between plurality, Borda, Copeland, STV, and Bucklin, and $\boldsymbol{\delta}$ between a

many ballot profiles, namely a truthful one and an initial one.

vector of all 2-pragmatists and a vector of all 3-pragmatists. PIF π was set to be the rank-PIF with respect to F, and all ties were broken using the lexicographic tie-breaking rule (also ties for second and third place). We ran all voting games until termination.

- *Experiment 2:* The aim of this experiment was to investigate the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency of approval voting. We set F to be approval voting, and we varied $\boldsymbol{\delta}$ between a vector of all 2-pragmatists and a vector of all 3-pragmatists for approval voting. PIF π was set to be the rank-PIF with respect to F, and all ties were broken using the lexicographic tie-breaking rule (also ties for second and third place). We ran all voting games for n rounds and $100n$ rounds.

- *Experiment 3:* The aim of this experiment was to investigate the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency for different poll information levels. We set F to be plurality, and $\boldsymbol{\delta}$ to be a vector of all strategists. PIF π was varied between the score-PIF and the rank-PIF with respect to F, and all ties were broken using the lexicographic tie-breaking rule (also ties for second place, third place, et cetera). We ran all voting games using the score-PIF until termination, and all voting games using the rank-PIF for n rounds and $100n$ rounds.

We used R for the statistical analysis of our data (R Development Core Team, 2011), and we performed McNemar's test on all data pairs to determine whether the poll effect was significant. McNemar's test takes election pairs as input and tests whether the number of elections in which the Condorcet candidate (the Condorcet winner of loser) wins significantly changed before and after the polls.[3]

5.4.2 Results

The results of all our experiments are listed in Appendix A. In this section we present and discuss some exemplary results.

In general, what can we say about the influence of the number of voters and candidates on our results? For all but one of the studied (induced) voting procedures, the Condorcet winner efficiency decreased and the Condorcet loser efficiency increased (or was not affected) as the number of candidates increased. This is not surprising, given that candidates in elections with many other candidates have a smaller chance of winning to start with than candidates with few opponents. Only the (induced) Bucklin procedures displayed a different, irregular pattern. Additionally, for all but one of the studied (induced) voting procedures, the Condorcet winner efficiency was higher when the number of voters was even than when the number of voters was odd. This difference diminished as the number of voters increased. The odd-even effect was also visible in our Condorcet loser efficiency experiments, though to a much smaller extent. Again, this is what we would expect, because for a Condorcet winner (loser) to win (lose) a pairwise majority contest against some candidate x, he must win

[3] To be more precise, let YN denote the number of elections where the Condorcet candidate wins in the no-poll condition, but loses in the poll condition, and let NY denote the number of elections where the Condorcet candidate loses in the no-poll condition, but wins in the poll condition. McNemar's test then checks whether the ratio YN/NY significantly differs from 1.

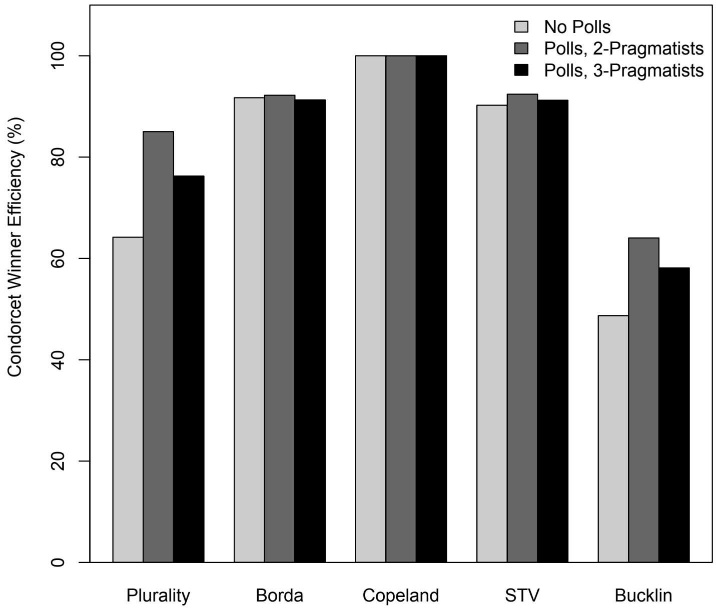


Figure 5.2: Average probability of electing the Condorcet winner for elections with 50 voters and 5 candidates over 10,000 trials. Poll effect on Condorcet efficiency is significant ($p < 0.05$) for plurality, STV and Bucklin.

(lose) with a difference of 2 when the number of voters is even, but only with a difference of 1 when the number of voters is odd. This contrast is relatively large for small numbers of voters. The (induced) Bucklin procedures form an exception. The Condorcet winner efficiency and the Condorcet loser efficiency of these procedures simply decreased as the number of voters increased.

Figure 5.2 shows the Condorcet winner efficiency results of Experiment 1 for elections with 50 voters and 5 candidates. Small variations in the number of voters or candidates did not change this pattern, except for the plurality rule, to which we will come back later. Note that we included the Copeland procedure for comparison, but we already know that the procedure itself is Condorcet-consistent, and thus its induced voting procedure will be as well (cf. Theorem 22).

Polls had a significant positive effect on the Condorcet winner efficiency of plurality, STV and Bucklin, and no significant effect for Borda. Intuitively, one can think of the pragmatist response policy as offering a Condorcet winner another chance to win if he ended up among the k highest ranked candidates in the first round. For plurality and Bucklin we can state this intuition as a general rule: if all voters are 2-pragmatists and the Condorcet winner is among the two highest ranked candidates in the first round, then he will always win under induced voting procedure $F^{\star}$. This follows from the fact that the top 2 candidates do not change throughout these voting games (see also the proof of Theorem 17) and from the voting procedures themselves, and can also be

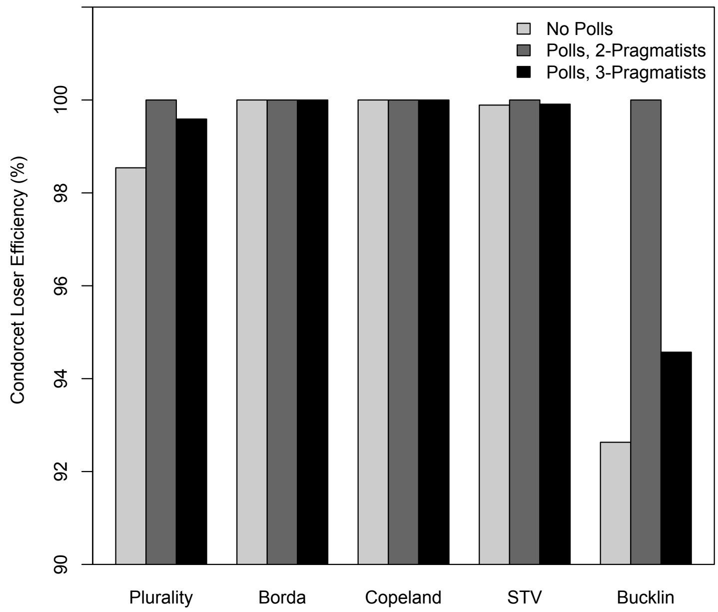


Figure 5.3: Average probability of not electing the Condorcet loser for elections with 50 voters and 5 candidates over 10,000 trials. Poll effect on Condorcet efficiency is significant ($p < 0.05$) for plurality and Bucklin.

seen from our simulation data in Appendix A.1.1. However, if all voters are 3-pragmatists, then this no longer holds. Generally, a candidate in a runoff between 2 candidates has a greater chance of winning to start with than a candidate in a runoff between 3 candidates. We would therefore expect the 2-pragmatist response policy to have a greater positive effect on the Condorcet winner efficiency than the 3-pragmatist response policy. Indeed, our data reflect this expectation. On the other hand, as the number of candidates increases, it becomes less likely that the Condorcet winner ends up amongst the two highest ranked candidates in the first round. This would explain that for large numbers of candidates, the 3-pragmatist response policy had a greater positive effect on the Condorcet winner efficiency of the plurality rule than the 2-pragmatist response policy (see Appendix A.1.1).

Our results also show that polls had a greater effect on plurality and Bucklin than on STV and Borda. This might be due to the substantially lower Condorcet winner efficiencies of plurality and Bucklin compared to STV and Borda, leaving more room for improvement. More specifically, plurality and Bucklin do not respect the relative ranking of candidates within a ballot as much as STV and Borda, whereas this information is crucial for identifying the Condorcet winner. Plurality and Bucklin can therefore profit more from the pragmatist response policy which brings back some of this respect for relative rankings of candidates.

Figure 5.3 shows the Condorcet loser efficiency results of Experiment 1 for elections with 50 voters and 5 candidates. Small variations in the number of voters or candidates did not change this pattern. Note that we included the Copeland procedure for comparison, but we already know that the procedure itself always ranks the Condorcet loser last, and thus its induced voting procedures will never elect the Condorcet loser either.

The Borda rule is also known for satisfying the Condorcet loser criterion, and so is the STV procedure if it breaks ties between plurality losers during execution. In our implementation, however, these ties were not broken and all candidates that were ranked first by the fewest voters got eliminated. In that case the Condorcet loser can win sometimes in an STV election; see our results in Appendix A.1.2.

Polls had a significant positive effect on the Condorcet loser efficiency of plurality and Bucklin, and no significant effect for Borda and STV. Let us now think of the pragmatist response policy as offering a Condorcet loser another chance to lose. For plurality, STV, and Bucklin we can state this intuition as a general rule: if all voters are 2-pragmatists, then the Condorcet loser will always lose under induced voting procedure $F^{\star}$. This follows directly from the definitions of these procedures, and can also be seen from our simulation data in Appendix A.1.2. However, if all voters are 3-pragmatists, then this no longer holds. This would explain that the 2-pragmatist response policy had a greater positive effect on the Condorcet loser efficiency than the 3-pragmatist response policy.

We now turn to the results of Experiment 2. In this experiment we used a modified version of the pragmatist response policy, because approval voting does not fit in our standard framework. Figures 5.4 and 5.5 show the results concerning, respectively, the Condorcet winner efficiency and the Condorcet loser efficiency of approval voting for elections with 50 voters and 5 candidates after 5000 poll rounds. Almost all voting games had terminated by then. Small variations in the number of voters or candidates did not change these patterns (see Appendix A.2). We found that polls had a significant positive effect on the Condorcet winner efficiency of approval voting. Polls also had a significant positive effect on the Condorcet loser efficiency of approval voting in elections with only 2-pragmatists, but in elections with only 3-pragmatists we did not find any significant effect.

Recall that the pragmatist response policy for approval voting and the standard pragmatist response policy have one important thing in common: according to both policies a voter always differentiates between her favourite top k candidate and the other top k candidates. It is therefore not surprising that the results of our second experiment closely resembled the results of our first experiment.

Finally, Figure 5.6 shows the Condorcet winner efficiency results of Experiment 3 for elections with 50 voters and 5 candidates after 5000 poll rounds. All voting games using the score-PIF had terminated by then, but only 37.8% of the voting games using the rank-PIF had. We found that polls had a significant positive effect on the Condorcet winner efficiency of plurality when voters received score-information. On the other hand, when polls only gave rank-information, then we observed no significant effect on the Condorcet winner efficiency of plurality. The latter effect, however, turned significantly positive for large m/n ratios, and significantly negative for small m/n ratios (except for $m = 3$, for which it was significantly positive). For very large m/n ratios the effect of the rank-PIF on the Condorcet winner efficiency of plurality even outweighed the

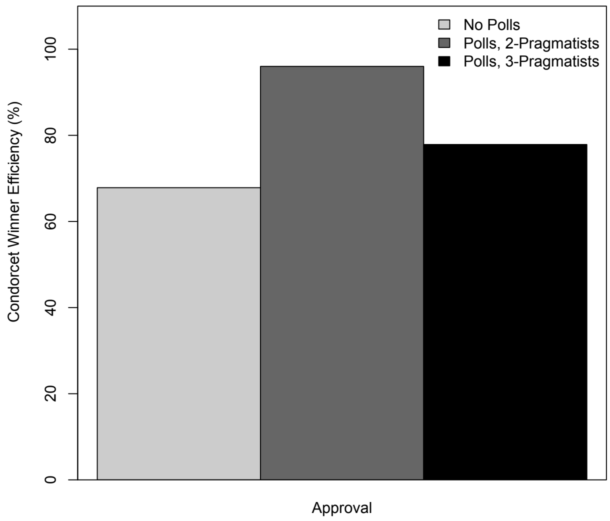


Figure 5.4: Average probability of electing the Condorcet winner for elections with 50 voters and 5 candidates over 10,000 trials. Poll effect on Condorcet efficiency is significant ($p < 0.05$).

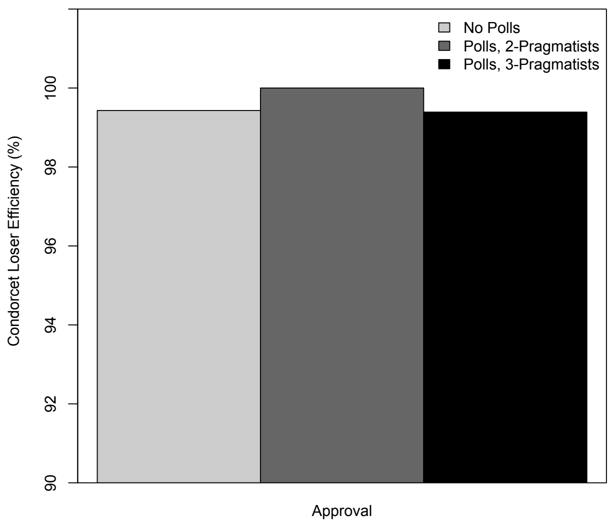


Figure 5.5: Average probability of not electing the Condorcet loser for elections with 50 voters and 5 candidates over 10,000 trials. Poll effect on Condorcet efficiency is significant ($p < 0.05$) if all voters are 2-pragmatists for approval voting.

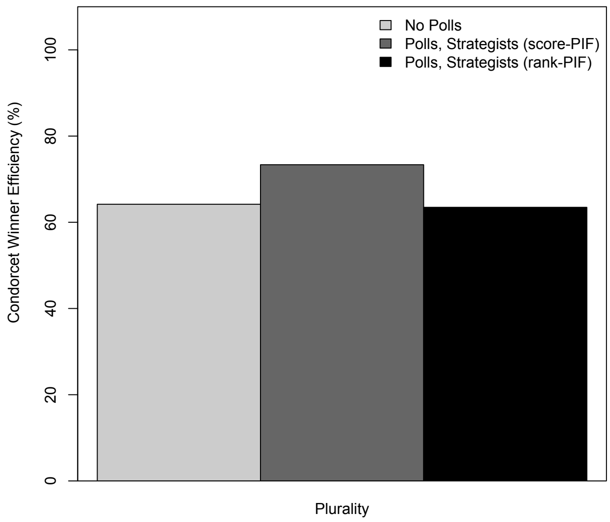


Figure 5.6: Average probability of electing the Condorcet winner for elections with 50 voters and 5 candidates over 10,000 trials. Poll effect on Condorcet efficiency is significant ($p < 0.05$) if polls give score-information.

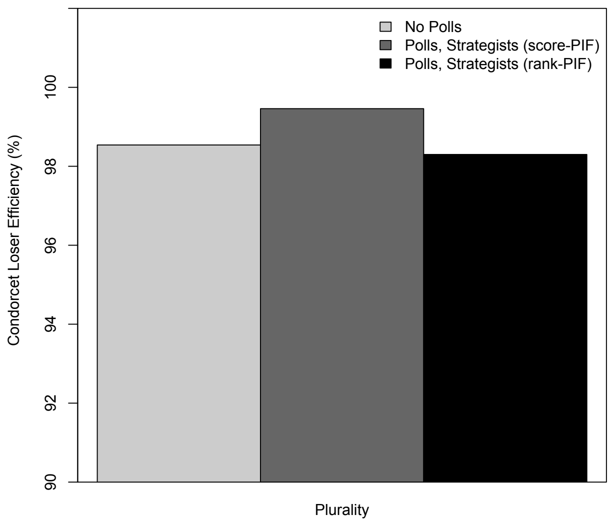


Figure 5.7: Average probability of not electing the Condorcet loser for elections with 50 voters and 5 candidates over 10,000 trials. Poll effect on Condorcet efficiency is significant ($p < 0.05$) if polls give score-information.

effect of the score-PIF.

How can we explain this pattern? First, the less information we provide to voters, the more likely a strategist will consider it possible to benefit from updating her ballot. That this is indeed the case is clear from our experimental data: termination took much longer (or did not occur at all) when we switched from score- to rank-information. Second, strategists are 'selfish' and are not *per se* interested in getting the Condorcet winner elected. Poorly informed strategists may therefore cause elections to drift away from the Condorcet winner. This effect was especially prominent in elections with many voters, possibly because then relatively many voters think they can benefit from updating their ballot. On the other hand, as the number of candidates increases, a strategist will consider it more likely to harm herself by updating her ballot. More specifically, in elections with many candidates, a strategist who is not satisfied with the current outcome is more likely to change to a front-runner, than to a candidate that is currently ranked midway. As the Condorcet winner often is amongst the two or three highest ranked candidates in a plurality election (see the results of Experiment 1 for plurality in Appendix A.1.1), this could account for the increasingly positive effect of the rank-PIF on the Condorcet winner efficiency of plurality for large numbers of candidates (and also for $m = 3$).

Well-informed strategists only change their ballot if they are sure that they can change the election outcome. That is, they only change their ballot if the front-runners' scores differ by at most one vote. Since the Condorcet winner often is one of the front-runners, this would explain that the score-PIF had a positive effect on the Condorcet winner efficiency of plurality, but also that this effect was outweighed by the effect of the rank-PIF for large numbers of candidates, for then the front-runners' scores often differ by more than one vote.

Figure 5.7 shows the Condorcet loser efficiency results of Experiment 3 for elections with 50 voters and 5 candidates after 5000 poll rounds. All voting games using the score-PIF had terminated by then, but only 32.3% of the voting games using the rank-PIF had. The results mirrored the results of the first part of Experiment 3 concerning the Condorcet winner efficiency of plurality, although the (poll) effect size was much smaller. And we may put forward similar reasons to explain the positive effect of score-PIFs and the varying effect of rank-PIFs on the Condorcet loser efficiency of plurality.

5.5 Discussion

In this chapter, we studied the effects of sequences of polls on election outcomes. Under which circumstances is the electorate better represented by a candidate who wins *after* a series of polls than by a candidate who wins *before*, and under which circumstances worse? Often one cannot appoint a candidate who represents the voters' preferences best, but we focused on the preference profiles for which one can. We developed a formal model of elections that are preceded by a series of polls, and we considered three types of responses to poll information: a strategist submits a best response to what she knows about other voters' ballots; a pragmatist supports her favourite candidate amongst the front-runners; and a truth-teller always votes truthfully.

We found that when all voters are pragmatists or truth-tellers, many voting procedures that incorporate multiple poll rounds reach a point from which no voter wishes to deviate, even if each voter is offered only one chance to update her ballot. Such termination is advantageous, for then the number of poll rounds barely influences who ultimately wins an election. However, when all voters are strategists or truth-tellers, termination often cannot be guaranteed. The widely used plurality rule forms an interesting exception.

We also briefly looked into the question which voting games always terminate after 0 poll rounds, i.e., which voting procedures are *immune to polls*? We stated two obvious results: if all voters are truth-tellers, then any voting procedure is immune to polls, and if all voters are strategists, then any voting procedure that is immune to π-manipulation is immune to polls that give π-information. Another example is plurality with runoff, which is immune to polls when all voters are 2-pragmatists. Plurality with runoff is a two-round voting procedure. If there is a candidate that is ranked first by a majority of voters in the first round, then that candidate wins. Otherwise, the two candidates that are ranked first most often in the first round continue to the second round, and the winner of their pairwise majority contest wins the election. It is not difficult to see that plurality with runoff always produces the same winner as the plurality voting game with only 2-pragmatists after termination (Brams & Fishburn (1983), cf. Theorem 17). It then follows that plurality with runoff is immune to polls when all voters are 2-pragmatists.

Returning to our original question about the influence of polls on election outcomes, we studied how polls affect the properties of different voting procedures. Obviously, polls do not affect the properties of voting procedures that are immune to polls, like dictatorial and constant voting procedures. More interestingly, we found that unanimity transfers if all voters are pragmatists or strategists. That is, any unanimous voting procedure remains unanimous if it incorporates multiple poll rounds. Many properties, however, do not always transfer, among which Pareto efficiency, surjectivity, anonymity, and Condorcet consistency, although the latter property does transfer if all voters are pragmatists.

What remains open is whether our (non-)termination and (non-)transfer results continue to hold in a setting where polls are less (or more) informative than we assumed in the theorems and examples of this chapter. In general the answer is no. While the plurality voting game with only strategists always terminates if polls give score-information (Theorem 16), it does not always terminate if polls give rank-information (see our simulation results in Appendix A.3), and it does always terminate if polls give zero-information (Theorem 10). The intuition behind this observation is that when polls give enough information to exclude some candidates as possible winners, but not all, then a voter may update her ballot because she will be better off in situation $\boldsymbol{b}$, whereas another voter may update her ballot because she will be better off in situation $\boldsymbol{c}$, both situations being consistent with the received poll information and possibly none of them being the actual poll profile. For this same reason, (non-)transfer results probably also do not always generalise to PIFs that are less (or more) informative. All of our non-transfer examples, however, do generalise to PIFs that are more informative than we assumed in those examples. We also know that if F is a voting procedure that is immune to zero-manipulation (as most reasonable voting procedures are) and all voters are strategists, then any property of F is transferred to F^t, in which polls give zero-information. It would be interesting to analyse exactly how much information a poll may provide before a property stops being transferred (if such a point is ever reached).

Moreover, it would be worthwhile to investigate whether combinations of properties of voting procedures induce particular properties of elections with polls, like Theorem 20 which states that surjective voting procedures that yield only one powerful voter induce dictatorial voting games.

For properties that do not persist in general, further work on simulations, similar to our study of Condorcet efficiency, will be required. Our simulations showed that polls may strengthen the Condorcet winner efficiency and the Condorcet loser efficiency of a voting procedure when all voters respond pragmatically, and that this also holds when all voters respond strategically provided that polls bound the strategising opportunities of voters. Other suitable properties to study via simulations are: Pareto efficiency and top cycle efficiency, i.e., a voting procedure's tendency to elect a candidate that is in the top cycle (the smallest non-empty subset of candidates such that every candidate in that subset wins every pairwise majority contest against any candidate outside that subset).

In this chapter, we assumed that voters respond strategically, pragmatically, or truthfully to poll information, and that they use the same response policy throughout the voting game. This is, of course, only a very rough approximation of voting behaviour in the actual world. For example, under the pragmatist response policy for approval voting, voters may contract and expand their ballot, and it can happen that a voter ends up approving of candidates that she actually does not approve of. We would like to know whether this is a realistic assumption. It would also be interesting to consider additional and perhaps more realistic types of response policies and to investigate their influence on election outcomes. However, before we can judge a response policy to be more realistic than some other policy, more experimental research on real-world opinion polls needs to be done.

Another way to get an idea of how realistic a response policy is, is to study its complexity. How hard is it for a voter to compute her next (tactical) move? If that turns out to be extremely difficult, then we may wonder whether voters actually follow such a policy.

Finally, in our framework only one voter updates her ballot after each poll round. This is not unusual for internet elections or elections with few voters, but far from usual for, e.g., national presidential elections. A setting in which multiple voters may change their ballot at once is more complex, and most of our results will cease to hold in such a setting. However, if all voters are k-pragmatists (or truth-tellers) and the set of k front-runners does not change throughout the voting game, then a voting game elects the same candidate as a game in which multiple voters may change their ballot at once. It would be worthwhile to study the latter setting in its own right.

Chapter 6

Conclusion

In this chapter we briefly summarise our results and give some directions for future work.

6.1 Results

We have developed a framework to study the effects of opinion polls on voting behaviour and election outcomes. First, we focused on the strategising opportunities of a single voter in view of a single poll. Upon receiving poll information, the voter reconsiders her truthful ballot and changes to an untruthful ballot if she has a chance of improving the election outcome according to her true preferences, and no chance of worsening it. When a poll communicates exactly who is voting what, then we know from the Gibbard-Satterthwaite Theorem that for any reasonable voting procedure there are situations in which a voter has an incentive to vote untruthfully. On the other hand, we found that when a poll does not communicate anything, then for many voting procedures voters never have an incentive to vote untruthfully. This incentive, however, starts coming as soon as voters know who is currently winning, according to the poll.

Thus, polls provoke strategic voting behaviour rather quickly. Does this mean that polls generally have a negative effect on the election outcome? No. In the second part of this thesis, we analysed the scenario in which multiple voters repeatedly update their ballot in view of a sequence of polls. We considered three types of responses to poll information: a strategist submits a best response to what she knows about other voters' ballots; a pragmatist supports her favourite candidate amongst the front-runners; and a truth-teller always votes truthfully. We found that some desirable properties of voting procedures persist, and may even be strengthened, when a voting procedure is preceded by a series of polls to which voters can respond in a strategic, pragmatic, or truthful way. Other desirable properties, however, do not always persist. Among the properties that may be strengthened by polls is the tendency of a voting procedure to elect the Condorcet winner, i.e., a candidate that would beat any other candidate in a one-to-one majority contest. This is an important positive result, because whenever there exists a Condorcet winner, then he is seen as the candidate who best represents the electorate.

Hence, polls can both improve and reduce the quality of election outcomes, but on average they seem to have a positive effect.

6.2 Future Research

Throughout this thesis, we made several recommendations for future research. Here we list the most important ones:

- *Gibbard-Satterthwaite Theorem under partial information:* The Gibbard-Satterthwaite Theorem states that for any 'democratic' voting procedure there are situations in which voters are better off by not reporting their true preferences, provided that they know exactly how all other voters are voting. We showed that this theorem ceases to hold when polls only communicate who is currently winning, or which candidates beat which other candidates in a one-to-one majority contest. However, we did not determine precisely how much information a poll may hide before the Gibbard-Satterthwaite Theorem stops being generalizable. Mapping out this dividing line between the possible and the impossible presents an important challenge for future research.

- *Response policies:* We defined three possible ways in which voters may respond to poll information. These policies are loosely based on the hypothesised influence of polls on voting behaviour as suggested by political science literature. We can think of more such response policies. For example, *pity-feelers* who support the candidate that is currently losing, or *followers* who support the candidate that is currently gaining the most votes. Note that pity-feelers bring about an underdog effect, and that followers bring about a momentum effect. It would be interesting to study the influence of these and other (combinations of) response policies on the election outcome. In addition, it would be worthwhile to investigate whether real voters in real elections actually behave in a way that can be accounted for by the proposed response policies.

- *Properties of elections with polls:* We studied how polls affect the properties of different voting procedures for different response policies and poll information levels. Many desirable properties do not always persist. To find out how bad this is, further work on simulations, similar to our study of Condorcet efficiency, will be required. Beyond this, while we have focussed on the persistence of *single* properties of voting procedures, it would also be interesting to investigate whether *combinations* of such properties induce particular properties of elections with polls.

- *Complexity:* In this thesis, we did not focus on complexity issues. However, it would be worthwhile to analyse for different voting procedures, different response policies, and different poll information functions how hard it is for a voter to compute her response to the received poll information.

Appendix A

Simulation Results

This appendix lists the results of the experiments we conducted (see Section 5.4). The experiments were set up to test the effect of polls on the Condorcet efficiency of different voting procedures while varying the number of voters and candidates, the amount of information polls provide, and the response policies of voters. For all conditions, we ran 10,000 trials and checked how often the Condorcet candidate (the Condorcet winner of loser) was elected compared to the condition without polls. Let YN denote the number of elections where the Condorcet candidate wins in the no-poll condition, but loses in the poll condition, and let NY denote the number of elections where the Condorcet candidate loses in the no-poll condition, but wins in the poll condition. Below, the increase in Condorcet efficiency when elections are complemented with polls is given by $\Delta := \frac{\text{NY}-\text{YN}}{10{,}000} \cdot 100\%$,[1] and the *fluidity* between the no-poll and poll condition is given by $\text{Fl} := \frac{\text{NY}+\text{YN}}{10{,}000} \cdot 100\%$. Note that YN and NY can be computed from this information. To enhance the readability of the tables below, we write $\rightarrowtail$ for the fluidity if $\text{YN} = 0$ and $\text{NY} > 0$, and $\leftarrowtail$ if $\text{YN} > 0$ and $\text{NY} = 0$, and $-$ if $\text{YN} = 0$ and $\text{NY} = 0$.

We used McNemar's test to determine whether the poll effect was significant. Significance is denoted by asterisks: ($\star$) if $p < 0.05$, ($\star\star$) if $p < 0.01$, and ($\star\star\star$) if $p < 0.001$. McNemar's test can only be applied if the fluidity is large enough; we required $\text{NY} + \text{YN} \geqslant 25$. All our data was analysed in R (R Development Core Team, 2011).

A.1 Experiment 1

In this section, we present the results of our first experiment. In this experiment, we examined the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency of plurality, Borda, Copeland, STV, and Bucklin under the assumption that polls provide (at least) rank-information, and that all voters are 2-pragmatists or all voters are 3-pragmatists. Note that we left out the Condorcet winner efficiency results for Copeland, and the Condorcet loser efficiency results for Borda and Copeland. Regardless of the number of candidates, the number of voters, and voters' response policies, those efficiencies

[1] More precisely, Δ gives the increase in Condorcet *winner* efficiency, but the decrease in Condorcet *loser* efficiency.

are known to be 100% with or without polls, except for the Condorcet loser efficiency of Borda with polls for 50 voters and 3 2-pragmatic candidates, which was 99.97%.

A.1.1 Condorcet Winner Efficiency

	Number of Candidates												
	3	4	5	6	7	8	9	10	11	12	13	14	15
Plurality	83.23	72.52	64.18	58.64	54.24	50.10	47.08	44.02	39.86	37.68	35.93	34.37	32.39
Borda	95.64	93.13	91.72	91.33	91.21	91.21	91.11	91.11	90.95	91.24	90.54	91.28	91.00
STV	97.01	93.56	90.23	88.62	86.02	84.03	82.63	80.19	78.12	76.22	74.58	74.31	72.05
Bucklin	36.17	61.82	48.71	44.55	64.37	49.53	60.55	59.78	57.88	63.08	62.40	63.75	65.42

Figure A.1: Condorcet winner efficiency for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials.

		Number of Candidates												
		3	4	5	6	7	8	9	10	11	12	13	14	15
Plurality	Δ	14.45***	18.87***	20.85***	21.64***	20.56***	19.73***	19.83***	19.20***	19.09***	18.36***	17.95***	16.28***	15.72***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Borda	Δ	-1.11***	-0.33	0.48	-0.14	0.23	0.13	0.23	-0.48	0.05	-0.01	0.55	0.59	-0.25
	Fl	6.13	8.67	9.66	10.62	9.97	10.69	11.17	11.06	10.79	10.27	10.73	10.05	10.77
STV	Δ	0.67***	1.35***	2.18***	2.47***	2.75***	3.62***	3.57***	4.56***	4.57***	5.08***	5.49***	5.21***	5.87***
	Fl	$\rightarrowtail$	1.55	2.60	3.35	3.89	4.76	5.35	5.94	6.17	7.10	7.45	7.31	7.99
Bucklin	Δ	32.64***	32.98***	15.33***	34.68***	15.07***	22.31***	24.78***	16.21***	24.54***	19.33***	19.27***	20.49***	17.93***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$

Figure A.2: Poll effects on Condorcet winner efficiency for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *2-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Candidates					
		4	5	6	7	8	9
Plurality	Δ	7.64***	12.08***	14.73***	16.64***	17.77***	18.76***
	Fl	17.52	23.64	26.57	27.40	27.73	28.66
Borda	Δ	-0.28	-0.43	-0.34	-0.45	-0.37	-0.73*
	Fl	4.84	7.55	7.96	8.03	8.13	9.07
STV	Δ	0.35***	0.99***	1.05***	1.53***	1.67***	1.85***
	Fl	0.67	1.59	2.63	3.49	4.15	4.97
Bucklin	Δ	0.27***	9.42***	9.38***	1.36***	16.30***	6.67***
	Fl	$\rightarrowtail$	11.86	13.70	13.96	22.18	19.25

		Number of Candidates					
		10	11	12	13	14	15
Plurality	Δ	18.66***	19.32***	19.96***	19.42***	18.56***	18.97***
	Fl	27.98	28.38	27.40	26.34	25.60	25.53
Borda	Δ	-0.61*	-0.36	-0.45	0.13	-0.51	-0.66*
	Fl	8.53	9.16	8.95	9.21	9.19	9.40
STV	Δ	2.67***	2.71***	3.45***	4.32***	3.53***	3.78***
	Fl	5.35	5.83	6.67	7.54	7.21	7.90
Bucklin	Δ	6.93***	12.68***	5.16***	9.00***	8.00***	7.30***
	Fl	18.81	23.30	20.56	21.72	21.02	21.14

Figure A.3: Poll effects on Condorcet winner efficiency for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *3-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

	Number of Voters						
	10	15	20	25	30	35	40
Plurality	77.45	63.33	71.15	62.12	67.30	60.34	67.07
Borda	97.19	86.33	94.81	85.95	93.34	86.12	92.62
STV	92.11	85.83	92.38	84.99	91.11	87.13	90.35
Bucklin	81.20	66.33	70.98	61.42	62.78	58.02	54.72

	Number of Voters					
	45	50	55	60	65	70
Plurality	59.56	64.34	59.51	64.21	59.27	63.38
Borda	85.87	92.09	85.63	91.56	85.49	91.20
STV	86.78	90.58	87.08	90.56	87.52	90.16
Bucklin	51.29	49.68	45.45	43.47	41.04	39.40

	Number of Voters					
	75	80	85	90	95	100
Plurality	58.62	63.20	58.91	62.20	57.80	62.19
Borda	85.91	90.58	85.71	90.13	85.39	89.99
STV	87.90	90.35	87.69	90.30	87.82	90.08
Bucklin	37.40	34.97	34.29	32.76	32.21	30.27

Figure A.4: Condorcet winner efficiency for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials.

		Number of Voters						
		10	15	20	25	30	35	40
Plurality	Δ	14.05***	21.22***	17.66***	21.27***	19.38***	22.13***	19.25***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Borda	Δ	-1.02***	-0.31	-0.52	0.06	-0.21	0.68	-0.02
	Fl	4.34	12.93	6.98	13.88	8.51	13.78	9.32
STV	Δ	2.37***	2.85***	1.73***	4.07***	2.27***	2.44***	2.28***
	Fl	2.61	3.55	1.91	4.67	2.83	3.14	2.74
Bucklin	Δ	13.03***	19.51***	13.65***	16.17***	13.92***	14.98***	14.46***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$

		Number of Voters					
		45	50	55	60	65	70
Plurality	Δ	22.55***	21.28***	23.19***	21.25***	22.78***	21.45***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Borda	Δ	0.47	0.52	0.98**	0.04	0.69	0.46
	Fl	13.45	9.52	13.72	10.54	14.59	9.98
STV	Δ	2.71***	2.04***	2.98***	2.15***	1.99***	2.29***
	Fl	3.29	2.46	3.90	2.57	2.89	2.71
Bucklin	Δ	14.64***	14.84***	15.63***	16.24***	16.92***	16.83***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$

		Number of Voters					
		75	80	85	90	95	100
Plurality	Δ	23.51***	21.41***	23.15***	21.89***	24.01***	21.73***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Borda	Δ	0.13	0.44	0.83*	0.33	1.37***	0.61
	Fl	14.23	10.94	14.45	11.13	14.67	11.15
STV	Δ	1.93***	1.86***	2.52***	1.83***	2.10***	2.06***
	Fl	2.89	2.34	3.16	2.31	2.72	2.58
Bucklin	Δ	16.89***	17.53***	16.99***	17.96***	17.57***	17.46***
	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$

Figure A.5: Poll effects on Condorcet winner efficiency for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *2-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Voters						
		10	15	20	25	30	35	40
Plurality	Δ	8.61***	8.53***	10.12***	8.79***	11.83***	9.97***	10.79***
	Fl	14.93	20.13	19.12	22.83	22.03	24.17	22.37
Borda	Δ	-0.37*	-0.91**	-0.36	-0.23	-0.13	-0.07	-0.33
	Fl	2.45	9.07	4.30	9.51	5.73	9.51	6.31
STV	Δ	0.58***	0.34	0.68***	0.87***	0.91***	0.79***	0.64***
	Fl	2.12	2.94	1.50	2.37	1.93	1.89	1.56
Bucklin	Δ	4.04***	4.23***	4.89***	4.14***	6.19***	5.13***	7.81***
	Fl	5.82	9.19	8.25	9.84	9.51	10.87	10.71

		Number of Voters					
		45	50	55	60	65	70
Plurality	Δ	10.81***	12.11***	11.10***	11.77***	10.83***	12.46***
	Fl	24.39	23.41	24.98	24.03	24.91	23.46
Borda	Δ	-0.36	-0.13	-0.32	-0.18	0.01	0.18
	Fl	9.68	6.97	9.80	6.96	10.41	6.90
STV	Δ	1.13***	1.06***	0.58***	0.60***	0.70***	0.56***
	Fl	2.11	1.76	2.08	1.52	1.86	1.48
Bucklin	Δ	6.96***	9.04***	8.96***	10.71***	9.85***	11.12***
	Fl	11.54	11.68	12.10	13.03	12.51	12.64

		Number of Voters					
		75	80	85	90	95	100
Plurality	Δ	11.68***	11.47***	11.26***	12.13***	11.82***	12.03***
	Fl	25.50	24.43	25.52	25.55	26.14	25.01
Borda	Δ	-0.70*	-0.44	-0.41	-0.26	0.16	-0.21
	Fl	9.90	7.40	10.15	8.12	9.96	7.65
STV	Δ	0.48***	0.45***	0.70***	0.87***	0.53***	0.46***
	Fl	1.84	1.35	1.84	1.39	1.39	1.26
Bucklin	Δ	10.65***	11.18***	10.61***	10.95***	10.96***	10.55***
	Fl	12.37	12.38	12.43	12.09	12.00	11.15

Figure A.6: Poll effects on Condorcet winner efficiency for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *3-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

A.1.2 Condorcet Loser Efficiency

	Number of Candidates						
	3	4	5	6	7	8	9
Plurality	97.39	98.35	98.54	98.93	98.90	99.15	99.33
STV	99.72	99.85	99.89	99.97	99.97	100.00	99.99
Bucklin	69.07	99.39	92.63	99.87	99.21	99.96	99.94

	Number of Candidates					
	10	11	12	13	14	15
Plurality	99.33	99.44	99.33	99.48	99.50	99.50
STV	99.98	99.99	99.98	99.99	100.00	100.00
Bucklin	99.99	100.00	99.99	100.00	100.00	100.00

Figure A.7: Condorcet loser efficiency for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials.

		Number of Candidates						
		3	4	5	6	7	8	9
Plurality	-Δ	2.61***	1.65***	1.46***	1.07***	1.10***	0.85***	0.67***
	Fl	↣	↣	↣	↣	↣	↣	↣
STV	-Δ	0.28***	0.15	0.11	0.03	0.03	0.00	0.01
	Fl	↣	↣	↣	↣	↣	–	↣
Bucklin	-Δ	30.93***	0.61***	7.37***	0.13	0.79***	0.04	0.06
	Fl	↣	↣	↣	↣	↣	↣	↣

		Number of Candidates					
		10	11	12	13	14	15
Plurality	-Δ	0.67***	0.56***	0.67***	0.52***	0.50***	0.50***
	Fl	↣	↣	↣	↣	↣	↣
STV	-Δ	0.02	0.01	0.02	0.01	0.00	0.00
	Fl	↣	↣	↣	↣	–	–
Bucklin	-Δ	0.01	0.00	0.01	0.00	0.00	0.00
	Fl	↣	–	↣	–	–	–

Figure A.8: Poll effects on Condorcet loser efficiency for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *2-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Candidates					
		4	5	6	7	8	9
Plurality	-Δ	0.60***	1.05***	0.77***	0.90***	0.68***	0.53***
	Fl	1.74	1.41	1.25	1.12	0.94	0.77
STV	-Δ	0.03	0.02	0.01	0.03	0.00	0.00
	Fl	0.07	0.04	$\rightarrowtail$	$\rightarrowtail$	–	0.02
Bucklin	-Δ	0.00	1.94***	-0.14	0.48***	0.00	0.06
	Fl	–	$\rightarrowtail$	0.16	$\rightarrowtail$	–	$\rightarrowtail$

		Number of Candidates					
		10	11	12	13	14	15
Plurality	-Δ	0.59***	0.55***	0.65***	0.47***	0.49***	0.48***
	Fl	0.71	$\rightarrowtail$	0.69	0.53	0.51	0.52
STV	-Δ	0.02	0.00	0.02	0.01	0.00	0.00
	Fl	$\rightarrowtail$	–	$\rightarrowtail$	$\rightarrowtail$	–	–
Bucklin	-Δ	0.01	0.00	0.00	0.00	0.00	0.00
	Fl	$\rightarrowtail$	–	–	–	–	–

Figure A.9: Poll effects on Condorcet loser efficiency for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *3-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

	Number of Voters						
	10	15	20	25	30	35	40
Plurality	99.22	98.25	98.82	97.9	98.72	98.19	98.82
STV	99.81	99.49	99.85	99.47	99.84	99.9	99.81
Bucklin	99.82	98.95	98.76	97.17	97.18	94.89	95.28

	Number of Voters					
	45	50	55	60	65	70
Plurality	98.27	98.49	98.1	98.74	98.39	98.58
STV	99.73	99.89	99.74	99.86	99.89	99.69
Bucklin	92.86	92.75	89.98	90.78	89.14	88.43

	Number of Voters					
	75	80	85	90	95	100
Plurality	98.23	98.31	97.99	98.63	98.17	98.45
STV	99.9	99.92	99.79	99.89	99.93	99.86
Bucklin	87.49	86.97	85.96	86.19	85.23	85.03

Figure A.10: Condorcet loser efficiency for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials.

		Number of Voters						
		10	15	20	25	30	35	40
Plurality	-Δ	0.78***	1.75***	1.18***	2.10***	1.28***	1.81***	1.18***
	Fl	↣	↣	↣	↣	↣	↣	↣
STV	-Δ	0.19	0.51***	0.15	0.53***	0.16	0.10	0.19
	Fl	↣	↣	↣	↣	↣	↣	↣
Bucklin	-Δ	0.18	1.05***	1.24***	2.83***	2.82***	5.11***	4.72***
	Fl	↣	↣	↣	↣	↣	↣	↣

		Number of Voters					
		45	50	55	60	65	70
Plurality	-Δ	1.73***	1.51***	1.90***	1.26***	1.61***	1.42***
	Fl	↣	↣	↣	↣	↣	↣
STV	-Δ	0.27***	0.11	0.26***	0.14	0.11	0.31***
	Fl	↣	↣	↣	↣	↣	↣
Bucklin	-Δ	7.14***	7.25***	10.02***	9.22***	10.86***	11.57***
	Fl	↣	↣	↣	↣	↣	↣

		Number of Voters					
		75	80	85	90	95	100
Plurality	-Δ	1.77***	1.69***	2.01***	1.37***	1.83***	1.55***
	Fl	↣	↣	↣	↣	↣	↣
STV	-Δ	0.10	0.08	0.21	0.11	0.07	0.14
	Fl	↣	↣	↣	↣	↣	↣
Bucklin	-Δ	12.51***	13.03***	14.04***	13.81***	14.77***	14.97***
	Fl	↣	↣	↣	↣	↣	↣

Figure A.11: Poll effects on Condorcet loser efficiency for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *2-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Voters						
		10	15	20	25	30	35	40
Plurality	-Δ	0.56***	0.78***	0.68***	1.10***	0.86***	0.75***	0.71***
	Fl	0.84	1.96	1.22	2.38	1.36	2.17	1.31
STV	-Δ	0.03	-0.08	0.13	0.17**	-0.01	0.05	0.01
	Fl	0.21	0.26	0.15	0.31	0.11	↣	0.09
Bucklin	-Δ	0.07	0.37***	0.43***	1.10***	0.89***	1.84***	1.63***
	Fl	↣	0.49	↣	1.14	↣	1.90	↣

		Number of Voters					
		45	50	55	60	65	70
Plurality	-Δ	0.92***	0.83***	0.90***	0.73***	1.03***	0.79***
	Fl	1.94	1.83	2.10	1.39	1.75	1.71
STV	-Δ	0.11	0.05	0.01	0.03	0.03	0.03
	Fl	↣	↣	0.13	0.09	0.05	0.15
Bucklin	-Δ	2.27***	2.02***	3.47***	2.51***	3.09***	2.58***
	Fl	2.31	↣	3.53	2.53	↣	↣

		Number of Voters					
		75	80	85	90	95	100
Plurality	-Δ	0.95***	1.01***	1.11***	0.76***	0.98***	0.96***
	Fl	2.09	1.89	2.27	1.50	2.08	1.68
STV	-Δ	-0.03	0.01	0.00	0.02	-0.01	0.02
	Fl	0.07	0.05	0.10	0.04	↢	0.08
Bucklin	-Δ	3.35***	2.89***	3.36***	2.70***	3.22***	2.57***
	Fl	↣	↣	3.38	↣	↣	↣

Figure A.12: Poll effects on Condorcet loser efficiency for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were *3-pragmatists*. And all voting games were run until termination. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

A.2 Experiment 2

This section presents the results of our second experiment. In this experiment, we examined the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency of approval voting under the assumption that polls provide (at least) rank-information, and that all voters are 2-pragmatists or all voters are 3-pragmatists for approval voting.

A.2.1 Condorcet Winner Efficiency

		Number of Candidates					
		3	4	5	6	7	8
No Polls		81.06	72.70	67.84	64.45	61.76	59.94
Polls,	Δ	17.62***	24.50***	28.17***	29.99***	31.73***	33.27***
2-pragmatists	Fl	18.14	25.28	29.15	30.75	32.85	34.41
Polls (n rounds),	Δ		7.61***	10.05***	12.85***	14.59***	15.70***
3-pragmatists	Fl		31.67	35.79	37.33	39.83	40.22
Polls (100n rounds),	Δ		7.42***	10.03***	12.00***	13.35***	14.73***
3-pragmatists	Fl		31.64	36.19	37.86	40.45	41.15

		Number of Candidates				
		9	10	11	12	13
No Polls		56.69	56.16	56.41	55.06	55.27
Polls,	Δ	35.31***	35.42***	34.40***	35.76***	35.48***
2-pragmatists	Fl	36.59	36.98	35.70	37.08	36.94
Polls (n rounds),	Δ	18.18***	18.98***	17.48***	18.87***	18.70***
3-pragmatists	Fl	42.22	42.28	42.16	43.29	42.48
Polls (100n rounds),	Δ	16.66***	16.79***	15.69***	17.02***	15.98***
3-pragmatists	Fl	43.44	43.75	43.15	44.48	43.88

		Number of Candidates	
		14	15
No Polls		53.18	53.16
Polls,	Δ	36.84***	36.88***
2-pragmatists	Fl	38.18	38.40
Polls (n rounds),	Δ	20.05***	20.21***
3-pragmatists	Fl	43.55	42.81
Polls (100n rounds),	Δ	17.64***	17.55***
3-pragmatists	Fl	45.26	45.39

Figure A.13: Poll effects on Condorcet winner efficiency of approval voting for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were 2-pragmatists or all voters were 3-pragmatists for approval voting. And all voting games were run twice: once for n rounds and once for 100n rounds. All voting games with only 2-pragmatists had terminated by 100n rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Voters					
		10	15	20	25	30	35
No Polls		72.83	62.08	69.76	61.76	68.79	61.41
Polls,	Δ	25.19***	32.34***	27.39***	32.14***	28.09***	32.45***
2-pragmatists	Fl	26.01	33.9	28.21	33.76	29.01	34.05
Polls (n rounds),	Δ	14.90***	10.71***	12.76***	10.26***	11.29***	9.98***
3-pragmatists	Fl	29.80	38.59	32.98	38.58	34.71	38.78
Polls (100n rounds),	Δ	14.33***	10.10***	11.99***	9.71***	10.95***	9.67***
3-pragmatists	Fl	30.07	38.72	33.27	39.09	34.71	39.05

		Number of Voters					
		40	45	50	55	60	65
No Polls		68.03	62.61	67.25	62.24	67.17	61.54
Polls,	Δ	27.95***	31.09***	28.84***	31.46***	27.91***	31.72***
2-pragmatists	Fl	28.73	32.61	29.4	32.94	29.01	32.94
Polls (n rounds),	Δ	11.13***	9.61***	11.15***	10.17***	10.27***	10.38***
3-pragmatists	Fl	35.19	38.07	35.51	39.33	35.93	39.00
Polls (100n rounds),	Δ	10.60***	9.02***	10.85***	10.18***	10.13***	9.85***
3-pragmatists	Fl	35.48	38.42	35.91	39.66	36.29	39.23

		Number of Voters				
		70	75	80	85	90
No Polls		67.03	62.74	66.59	61.64	65.48
Polls,	Δ	27.82***	30.29***	27.87***	31.25***	28.84***
2-pragmatists	Fl	28.8	31.27	28.97	32.37	29.68
Polls (n rounds),	Δ	10.60***	8.57***	9.46***	11.21***	10.75***
3-pragmatists	Fl	35.62	38.61	35.98	38.69	36.71
Polls (100n rounds),	Δ	10.36***	8.49***	9.37***	10.78***	10.59***
3-pragmatists	Fl	36.26	38.75	36.15	38.74	37.15

		Number of Voters	
		95	100
No Polls		62.07	66.44
Polls,	Δ	30.60***	27.78***
2-pragmatists	Fl	31.58	28.60
Polls (n rounds),	Δ	9.52***	9.26***
3-pragmatists	Fl	38.56	36.24
Polls (100n rounds),	Δ	9.25***	9.21***
3-pragmatists	Fl	38.67	35.87

Figure A.14: Poll effects on Condorcet winner efficiency of approval voting for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were 2-pragmatists or all voters were 3-pragmatists for approval voting. And all voting games were run twice: once for n rounds and once for 100n rounds. All voting games with only 2-pragmatists had terminated by 100n rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

A.2.2 Condorcet Loser Efficiency

		Number of Candidates					
		3	4	5	6	7	8
No Polls		97.71	98.79	99.43	99.71	99.74	99.92
Polls ($100n$ rounds),	-Δ	2.29***	1.21***	0.57***	0.29***	0.26***	0.08
2-pragmatists	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls ($100n$ rounds),	-Δ		0.19	-0.04	-0.04	0.03	-0.05
3-pragmatists	Fl		2.11	1.10	0.62	0.49	0.19

		Number of Candidates				
		9	10	11	12	13
No Polls		99.91	99.98	99.95	99.97	99.99
Polls ($100n$ rounds),	-Δ	0.09	0.02	0.05	0.03	0.01
2-pragmatists	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls ($100n$ rounds),	-Δ	0.01	-0.04	0.03	-0.02	-0.02
3-pragmatists	Fl	0.17	0.08	0.07	0.08	0.04

		Number of Candidates	
		14	15
No Polls		100.00	100.00
Polls ($100n$ rounds),	-Δ	0.00	0.00
2-pragmatists	Fl	–	–
Polls ($100n$ rounds),	-Δ	-0.02	-0.02
3-pragmatists	Fl	$\leftarrowtail$	$\leftarrowtail$

Figure A.15: Poll effects on Condorcet loser efficiency of approval voting for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were 2-pragmatists or all voters were 3-pragmatists for approval voting. And all voting games were run twice: once for n rounds and once for $100n$ rounds. We do not show the results for n rounds, because these did not differ significantly from the results for $100n$ rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Voters					
		10	15	20	25	30	35
No Polls		99.59	99.03	99.41	98.96	99.45	99.01
Polls (100n rounds),	-Δ	0.41***	0.97***	0.59***	1.04***	0.55***	0.99***
2-pragmatists	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (100n rounds),	-Δ	0.19*	-0.15	0.09	0.06	0.06	-0.09
3-pragmatists	Fl	0.63	1.95	1.05	1.90	1.04	2.03

		Number of Voters					
		40	45	50	55	60	65
No Polls		99.44	99.18	99.44	99.05	99.35	99.04
Polls (100n rounds),	-Δ	0.56***	0.82***	0.56***	0.95***	0.65***	0.96***
2-pragmatists	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (100n rounds),	-Δ	0.07	-0.25	-0.10	0.00	0.06	0.28*
3-pragmatists	Fl	1.03	1.81	1.18	1.82	1.24	1.58

		Number of Voters				
		70	75	80	85	90
No Polls		99.19	98.90	99.45	99.05	99.34
Polls (100n rounds),	-Δ	0.81***	1.10***	0.55***	0.95***	0.66***
2-pragmatists	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (100n rounds),	-Δ	0.10	0.18	-0.17	0.03	0.08
3-pragmatists	Fl	1.48	1.96	1.23	1.83	1.24

		Number of Voters	
		95	100
No Polls		99.14	99.34
Polls (100n rounds),	-Δ	0.86***	0.66***
2-pragmatists	Fl	$\rightarrowtail$	$\rightarrowtail$
Polls (100n rounds),	-Δ	0.05	0.14
3-pragmatists	Fl	1.53	1.16

Figure A.16: Poll effects on Condorcet loser efficiency of approval voting for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with rank-information. All voters were 2-pragmatists or all voters were 3-pragmatists for approval voting. And all voting games were run twice: once for n rounds and once for 100n rounds. We do not show the results for n rounds, because these did not differ significantly from the results for 100n rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

A.3 Experiment 3

In this section, we present the results of our third experiment. In this experiment, we examined the effect of polls on the Condorcet winner efficiency as well as on the Condorcet loser efficiency of plurality under the assumption that polls provide score- or rank-information, and that all voters are strategists.

A.3.1 Condorcet Winner Efficiency

		Number of Candidates					
		3	4	5	6	7	8
No Polls		83.23	72.52	64.18	58.64	54.24	50.10
Polls,	Δ	6.31***	8.16***	9.17***	9.76***	9.86***	10.35***
score-PIF	Fl	$\rightarrowtail$	8.20	9.41	10.24	10.46	10.95
Polls (n rounds),	Δ	5.92***	-23.80***	-25.87***	-24.05***	-21.93***	-18.51***
rank-PIF	Fl	18.58	46.42	50.13	49.31	48.19	45.15
Polls ($100n$ rounds),	Δ	6.21***	-6.21***	-0.71	2.95***	5.03***	6.67***
rank-PIF	Fl	18.41	37.95	40.95	41.99	41.75	40.91

		Number of Candidates				
		9	10	11	12	13
No Polls		47.08	44.02	39.86	37.68	35.93
Polls,	Δ	10.79***	10.69***	10.66***	10.70***	10.93***
score-PIF	Fl	11.79	11.53	11.96	11.88	12.01
Polls (n rounds),	Δ	-17.53***	-15.22***	-13.01***	-11.40***	-10.34***
rank-PIF	Fl	43.03	41.98	39.03	37.26	36.64
Polls ($100n$ rounds),	Δ	8.95***	10.54***	11.82***	13.67***	12.97***
rank-PIF	Fl	40.25	39.94	38.10	37.75	35.81

		Number of Candidates	
		14	15
No Polls		34.37	32.39
Polls,	Δ	10.31***	10.42***
score-PIF	Fl	11.85	11.88
Polls (n rounds),	Δ	-10.79***	-9.39***
rank-PIF	Fl	34.61	32.31
Polls ($100n$ rounds),	Δ	12.06***	13.21***
rank-PIF	Fl	35.18	32.75

Figure A.17: Poll effects on Condorcet winner efficiency of plurality for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with score- or rank-information. All voters were strategists. And all voting games using the score-PIF were run until termination, and all voting games using the rank-PIF were run twice: once for n rounds and once for $100n$ rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Voters					
		10	15	20	25	30	35
No Polls		77.45	63.33	71.15	62.12	67.30	60.34
Polls,	Δ	11.52***	13.29***	11.20***	12.17***	10.55***	10.49***
score-PIF	Fl	12.74	14.39	12.14	12.63	10.99	11.01
Polls (n rounds),	Δ	9.17***	0.79	-4.96***	-11.69***	-15.36***	-17.27***
rank-PIF	Fl	19.41	33.95	33.28	41.19	41.88	45.15
Polls (100n rounds),	Δ	13.10***	10.36***	9.75***	6.16***	6.58***	3.76***
rank-PIF	Fl	19.34	33.14	29.17	37.60	35.00	39.70

		Number of Voters					
		40	45	50	55	60	65
No Polls		67.07	59.56	64.34	59.51	64.21	59.27
Polls,	Δ	9.19***	10.14***	9.33***	10.13***	8.64***	8.47***
score-PIF	Fl	9.67	10.48	9.59	10.45	8.84	8.71
Polls (n rounds),	Δ	-23.73***	-21.98***	-26.30***	-25.09***	-29.36***	-28.33***
rank-PIF	Fl	48.17	48.04	49.24	51.07	52.66	53.11
Polls (100n rounds),	Δ	0.69	0.38	-1.19	-3.34***	-5.36***	-6.95***
rank-PIF	Fl	38.83	41.66	39.21	43.00	42.26	44.81

		Number of Voters				
		70	75	80	85	90
No Polls		63.38	58.62	63.20	58.91	62.20
Polls,	Δ	7.59***	8.51***	7.78***	7.41***	6.95***
score-PIF	Fl	7.83	8.77	7.94	7.65	7.13
Polls (n rounds),	Δ	-31.82***	-28.91***	-33.62***	-31.47***	-33.75***
rank-PIF	Fl	54.46	54.07	55.92	55.51	56.03
Polls (100n rounds),	Δ	-8.37***	-9.46***	-12.40***	-12.42***	-14.38***
rank-PIF	Fl	43.85	46.60	45.56	46.94	47.42

		Number of Voters	
		95	100
No Polls		57.80	62.19
Polls,	Δ	8.08***	7.27***
score-PIF	Fl	8.26	7.47
Polls (n rounds),	Δ	-30.31***	-34.11***
rank-PIF	Fl	55.13	57.37
Polls (100n rounds),	Δ	-12.98***	-16.65***
rank-PIF	Fl	47.04	48.39

Figure A.18: Poll effects on Condorcet winner efficiency of plurality for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with score- or rank-information. All voters were strategists. And all voting games using the score-PIF were run until termination, and all voting games using the rank-PIF were run twice: once for n rounds and once for $100n$ rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

A.3.2 Condorcet Loser Efficiency

		Number of Candidates					
		3	4	5	6	7	8
No Polls		97.39	98.35	98.54	98.93	98.90	99.15
Polls,	-Δ	1.55***	0.96***	0.92***	0.77***	0.72***	0.55***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	2.46***	-6.90***	-4.53***	-2.90***	-1.43***	-1.12***
rank-PIF	Fl	2.74	9.72	7.05	4.74	3.27	2.62
Polls (100n rounds),	-Δ	2.53***	-2.17***	-0.24	0.17	0.54***	0.56***
rank-PIF	Fl	2.67	5.37	3.12	1.87	1.64	1.14

		Number of Candidates				
		9	10	11	12	13
No Polls		99.33	99.33	99.44	99.33	99.48
Polls,	-Δ	0.49***	0.44***	0.37***	0.48***	0.44***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	-0.93***	-0.56***	-0.47***	-0.24	-0.09
rank-PIF	Fl	2.01	1.74	1.45	1.40	1.13
Polls (100n rounds),	-Δ	0.38***	0.50***	0.29**	0.60***	0.39***
rank-PIF	Fl	0.96	0.82	0.79	0.74	0.65

		Number of Candidates	
		14	15
No Polls		99.50	99.50
Polls,	-Δ	0.38***	0.39***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	0.00	-0.15
rank-PIF	Fl	0.94	0.97
Polls (100n rounds),	-Δ	0.40***	0.40***
rank-PIF	Fl	0.60	0.56

Figure A.19: Poll effects on Condorcet loser efficiency of plurality for elections with 50 voters and 3 to 15 candidates. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with score- or rank-information. All voters were strategists. And all voting games using the score-PIF were run until termination, and all voting games using the rank-PIF were run twice: once for n rounds and once for 100n rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

		Number of Voters					
		10	15	20	25	30	35
No Polls		99.22	98.25	98.82	97.90	98.72	98.19
Polls,	-Δ	0.71***	1.46***	0.99***	1.51***	0.96***	1.20***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	-0.19	-1.89***	-2.19***	-3.06***	-3.13***	-4.83***
rank-PIF	Fl	1.71	4.95	4.25	6.80	5.41	7.95
Polls (100n rounds),	-Δ	0.53***	0.99***	0.73***	1.01***	0.46**	0.21
rank-PIF	Fl	1.03	2.43	1.59	3.07	2.04	3.29

		Number of Voters					
		40	45	50	55	60	65
No Polls		98.82	98.27	98.49	98.10	98.74	98.39
Polls,	-Δ	0.81***	1.08***	1.03***	0.93***	0.64***	0.91***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	-4.17***	-5.10***	-4.70***	-4.41***	-5.11***	-5.59***
rank-PIF	Fl	6.21	8.00	7.20	7.75	7.31	8.35
Polls (100n rounds),	-Δ	-0.07	-0.14	-0.32	-0.57**	-0.72***	-1.04***
rank-PIF	Fl	2.39	3.38	3.20	4.15	3.10	4.18

		Number of Voters				
		70	75	80	85	90
No Polls		98.58	98.23	98.31	97.99	98.63
Polls,	-Δ	0.74***	0.86***	0.93***	0.94***	0.66***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	-4.87***	-5.89***	-4.80***	-5.65***	-5.46***
rank-PIF	Fl	7.35	8.71	7.66	9.23	7.80
Polls (100n rounds),	-Δ	-1.08***	-1.39***	-1.22***	-1.71***	-2.17***
rank-PIF	Fl	3.86	4.71	4.28	5.55	4.75

		Number of Voters	
		95	100
No Polls		98.17	98.45
Polls,	-Δ	0.83***	0.81***
score-PIF	Fl	$\rightarrowtail$	$\rightarrowtail$
Polls (n rounds),	-Δ	-5.38***	-5.47***
rank-PIF	Fl	8.44	8.13
Polls (100n rounds),	-Δ	-2.17***	-2.04***
rank-PIF	Fl	5.55	4.98

Figure A.20: Poll effects on Condorcet loser efficiency of plurality for elections with 5 candidates and 10 to 100 voters. Average over 10,000 trials. All elections were preceded by a series of polls which provided voters with score- or rank-information. All voters were strategists. And all voting games using the score-PIF were run until termination, and all voting games using the rank-PIF were run twice: once for n rounds and once for 100n rounds. For explanation of the symbols used in this table, we refer to the introduction of Appendix A.

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