What should have been the case.
A temporal update semantics for necessity deontic modals

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Abstract

The thesis develops a formal semantics for present necessity deontic modals, such as *should*, *ought to*, *must*, and past necessity deontic modals, such as *should have*, *ought to have*, *had to*. Contrary to the traditional approaches in deontic logic, we concentrate on the prescriptive use of such modals.

In analyzing the different behavior of present and past necessity deontic modals in everyday discourse practice, we focus on the contrast that arises when the proposition embedded under those modals is eventive and non-progressive. We distinguish between a "counterfactual" and an "open-past" reading of past necessity deontic modals, and argue that the semantic differences between present and (the two readings of) past necessity deontic modals are ultimately due to two factors: (i) when the embedded proposition is evaluated, and (ii) whether the embedded proposition is an open possibility according to the common ground of the participants in the conversation.

We develop a framework from the update semantics' tradition. The approach we propose, called Temporal Deontic Update Semantics (TDUS), permits us to take into account the two factors (i) and (ii), and to model present and past necessity deontic modals by means of a single deontic operator: **Oblige**.

We conclude our thesis by taking into account some more classical topics in the literature of deontic logic, such as conditional obligations and deontic paradoxes. We show that our considerations regarding present and past necessity deontic modals are also relevant to a solution of the paradoxes.

Acknowledgments

Contents

Chapter 1

Introduction

The goal of this thesis is to provide a new formalism which captures some aspects of the semantics of necessity deontic modals. In particular, we aim at laying the foundations for an analysis of present necessity deontic modals (such as *must, ought to, should*) and past necessity deontic modals (such as *ought to have, should have, had to*). Literature in deontic logic, however, has concentrated mostly on problems related to conditional obligations. Different approaches have been proposed, but they tend to focus only on present necessity deontic modals, and to consider the difference between present and past deontic modals as an unrelated issue. We think that that view is misleading. We propose an update semantics for present and past deontic modals, and show how it may also enlighten some of the puzzles which have been traditionally ascribed to conditional obligations.

The starting points of our analysis are Thomason's (1981) work on temporal deontic logic, Condoravdi's (2002) and Arregui's (2010) works on present and past modals, and van der Torre and Tan's (1999) work on dynamic deontic logic. Our framework is intended to be a development of those works. In particular, none of them can account for sentences like *You ought to have sent everybody a remainder by email (but you didn't)*, where the necessity deontic modal *ought to have* is used to express a new obligation, rather than report an existing one. The present paper aims at filling that gap. We propose an update semantics in which obligation sentences are interpreted as inducing updates on *betterness-relations*. Moreover, we take *histories* as basic elements of the framework. Intuitively, a history represents the evolution of the world in time, while betterness-relations establish which histories are deontically ideal. Information states consist in sets of histories and betterness-relations, and represent agents' knowledge of the facts of the world and of its normative configuration. In particular, an information state can be understood as the representation of the conversational context that speaker and listener share in conversation. The framework is dynamic, in the sense that information states can be modified and evolve in time.

The thesis has the following structure. Chapter two is devoted to the analysis of the difference between present and past deontic modals. In particular, it

focuses on the contrast that emerges when the proposition embedded under the modals is eventive and non-progressive. Moreover, two different readings of past necessity deontic modals are discussed: the counterfactual reading and the so-called "open-past" reading.

Chapter three is devoted to our framework, which we call *Temporal Deontic Update Semantics* (TDUS). The framework aims at modeling the aspects of the semantics of present and past deontic modals that have been discussed in Chapter two. We provide the basic definitions of our framework, and focus on updates induced by sentences which contain present necessity deontic modals and past necessity deontic modals.

Chapter four focuses on some classical deontic paradoxes, such as Forrester's Paradox, the Miners' Paradox and Chisholm's Paradox. The paradoxes involve contrary-to-duty obligations, disjunction of obligations and conflicting obligations. Those scenarios are not problematic in our framework, and the paradoxes do not arise in TDUS. Moreover, we show that our previous considerations regarding present and past necessity deontic modals are relevant to a solution of the paradoxes, especially in the case of Chisholm's Paradox.

Chapter five concludes with some final remarks.

Before turning to Chapter two, it is worth introducing some of the preliminaries, and setting the boundaries of our inquire. In particular, we choose to focus our attention on obligation sentences, and provide an analysis of their prescriptive use.

1.1 Obligation sentences

Deontic logic is the formal study of normative concepts, such as obligation, permission and forbidness.[1] Here we concentrate on the concept of "obligatory". In natural language, obligations can be expressed by the use of several modals, which may differ in their strength. Consider, for instance, the following sentences:

1. You must go

2. Sara should return the library book on time

When modals such as *must* and *should* are used to express a sort of duty or obligation, they are called *necessity deontic modals*.[2] In the present paper, we take 1 and 2 to have the following logical form:

3. must [you go]

4. should [Sara returns the library book on time]

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[1] See von Wright (1951), page 1.

[2] It is common to distinguish between strong necessity modals (such as *must*) and weak necessity modals (such as *should* and *ought to*). However, in the present paper, we do not deal with that distinction. We are interested in the general notion of obligation, and we aim at providing a formal analysis which captures the semantic core that all the necessity deontic modals have in common.

where *must* and *should* are necessity deontic modals which range over the domain of propositions. We formalize 1 and 2 as **Oblige**(α), where α is a descriptive sentence and **Oblige** is an impersonal deontic necessity operator – meaning that "It must/should/ought to be that".

In our analysis, necessity deontic modals are, therefore, treated as S-operators, i.e, modals which operate on sentence phrases. There are a few examples in which the reduction of all obligation sentences to the form **Oblige**(α) may be problematic.[3] However, for the purposes of the present thesis, it is sufficient to consider that in the majority of cases (but not all) we can fairly represent obligations in the form **Oblige**(α). By doing so, we are able to give a preliminary and general semantic account of necessity deontic modals.

1.2   Two readings

One of the first topics discussed in the development of deontic logic was the question whether obligation sentences have truth-values. The question was motivated by the fact that obligation sentences like 1 and 2 look like commands, rather than descriptive sentences. Thus arises a puzzle sometimes known as "Jørgensen's Paradox". The puzzle can be reformulated as follows: If obligation sentences do not describe facts of the world, they cannot have truth values; and, since logic concerns itself exclusively with things that have truth values, then how can there be anything as deontic logic? [4]

The paradox, which appeared to challenge the entire enterprise of deontic logic, relies instead on a number of questionable assumptions. Firstly, it should be observed that obligation sentences can be used both to *report* and to *bring about* obligations. To illustrate the difference, consider the following examples:[5]

- Tax lawyer to client:
  You must report this donation as income.

- Lecturer to class:
  The final assignment must be in my mailbox by six pm on Friday.

In the first case, the lawyer reports to the client an already existent obligation (hence, reportative use). In the second case, however, the lecturer enacts a new obligation (hence, prescriptive use). Concerning the first, reportative obligation,

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[3]Consider, for instance, the following scenario. Suppose that Fred ought to dance with Ginger. This is represented, using the impersonal necessity deontic operator, as "It ought to be that Fred dances with Ginger". However, the relation of dancing is symmetric. In any possible world in which Fred dances with Ginger, it is also the case that Ginger dances with Fred. So, it seems that the two statements "Fred dances with Ginger" and "Ginger dances with Fred" are equivalent. It follows that if two statements α and β are equivalent, then the statement **Oblige**(α) cannot be distinguished from **Oblige**(β). Thus, since it ought to be that Fred dances with Ginger, it ought to be also that Ginger dances with Fred, which is an incorrect conclusion. These are examples in which the notion of agency plays a predominant role, hence the use of an impersonal deontic operator is problematic. Cf. Horty (2001), pp. 50-51.

[4]For the original formulation of the paradox, see Jørgensen (1937).

[5]Examples taken from Condoravdi (2010).

it seems perfectly reasonable to admit that it can be true or false *according to a particular norm*: for instance, it can be a true (or false) description of a norm of the tax code.

Secondly, we argue that there can be a *logic* also for prescriptive obligations, even if they lack truth values. In particular, we are interested in the semantics of prescriptive necessity deontic modals, and semantics deals with meaning, which should not be reduced *a priori* to truth conditions.

1.2.1 Reportative Reading

We have said that obligation sentences are used reportatively if they are a description of some norm. Therefore, under the reportative reading, our sentences 1 and 2 are interpreted as:

5. *According to the law/ moral code/ library rules*,
   You must go

6. *According to the law/ moral code/ library rules*,
   Sara should return the library book on time

where the agent who utters those sentences refers to some pre-existing code of norms, and intends to give a description of it. Traditionally, works in the field of deontic logic have focused precisely on the reportative reading of obligation sentences, by providing a truth-conditional analysis based on a primitive ordering among possible worlds (cf. Hansson 1969; Lewis 1973; Spohn 1975; Kratzer 1981). The approach is similar to the one in doxastic logic and conditional logic: obligation sentences are interpreted by making use of an "ideality relation" between possible worlds, and by the notion of maximality. In particular, from Hansson (1969) on, conditional obligations of the type *If α, it is obligatory that β* (where α expresses the condition under which the obligation holds) are formalized by the use of a dyadic deontic operator $\bigcirc(\beta/\alpha)$, and interpreted as follows:

(∗) $M, s \models \bigcirc(\beta/\alpha) \iff Max([\![\alpha]\!]_M) \subseteq [\![\beta]\!]_M$

where M is a model built on a Kripke frame $F = (S, \preceq)$ and $[\![\cdot]\!]_M$ is the truth-set function of M.[6] Given our purposes, it is worth to note that truth-conditional analyses take the ideality relation $\preceq$ to be primitive, that is, what counts as "ideal" is already given in the model.

Thus, according to (∗), obligation sentences express only true or false descriptions of a normative code, which is (semantically) represented by the ideality ordering $\preceq$.[7] In particular, a sentence such as *If α, it is obligatory that β* asserts that the best worlds in which α is the case, i.e., the worlds which are maximal in

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[6]Depending on the proprieties of the ideality relation $\preceq$, different logics can be obtained. For instance Hansson (1969) considers frames in which $\preceq$ is only reflexive, and then moves to frames in which $\preceq$ is a total pre-order. See Hansson (1969) and van Benthem, Grossi and Liu (2010).

[7]Cf. van Benthem, Grossi and Liu (2010).

the ideality ordering $\preceq$, are also worlds in which β is the case. As we said above, that semantic analysis is very similar to the one used in Lewis-Stalnaker's semantics for counterfactual conditionals, and in plausibility models for conditional belief.[8]

However, by interpreting obligations as descriptions, truth-conditional analyses miss the prescriptive effect which is a distinguishing feature of deontic statements. As Austin (1962) noticed, obligation sentences are typically used to *bring about* new duties and obligations. Sentences like 1 and 2 are generally used to *command* something rather than *assert* it.

1.2.2 Prescriptive Reading

Alchourrón explains the difference between reportative and prescriptive readings of obligation sentences in terms of the difference between making a picture of reality and building a part of reality. He uses the following box metaphor: Think of the obligatory set as a box ready to be filled. When an authority uses an obligation sentence prescriptively, his activity belongs to the same category as *putting something into a box*. When that authority, or someone else, uses the obligation sentence reportatively his activity belongs to the same category as *making a picture of the authority putting something into a box*.[9]

In other words, only under the prescriptive reading, obligation sentences are used to *create* and *change* an aspect of reality, i.e., its normative configuration. In that case they have, therefore, a *dynamic* character.

One question should now be addressed: what is the relation between the two readings? In the current literature, there is no agreement about that issue. Some authors argue that deontic modals are ambiguous between the *two meanings* (e.g., Kamp 1973; van Rooij 2000), while others assume that the semantics of deontic modals is constant, and the prescriptive effect pertains to the *pragmatic domain* (e.g., Schwager 2006; Kaufmann and Schwager 2009). According to the latter view, the semantics of *must*, *ought to* and *should* is the same across the two readings, while the prescriptive effect is trigged by certain contextualist settings. For instance, in Schwager (2006) and Kaufmann & Schwager (2009), the contextualist settings responsible for the prescriptive effect are characterized on the basis of a Kratzer-style semantics. This means that obligation sentences are always interpreted in terms of truth conditions, and the prescriptive effect arises just in the presence of particular contextual factors.

In the present thesis, we adopt a different perspective. We take the prescriptive use of obligation sentences to be the primary one, and provide an update semantics which models it. The reportative use, on the other hand, can be derived in the semantics by means of the notion of "support".

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[8]See Lewis (1973), Stalnaker (1968).

[9]See Alchourrón (1993).

Chapter 2

Deontics, Time and Common Ground

The present chapter analyses deontic expressions such as *must*, *ought to*, *should*, *have to* (usually called *present modals*[1]) and deontic expressions such as *ought to have*, *should have*, *had to* (usually called *past modals*[2]). In particular, we focus on the contrast that emerges between present and past necessity deontic modals in the case in which the embedded proposition is eventive and non-progressive.

2.1   Present necessity deontic modals

Present necessity deontic modals are expressions such as *must*, *ought to*, *should*, *have to* which are used to indicate obligations, duties, or requirements.[3] Consider the following example:

7. (a) You should call your parents

   (b) should [you call your parents]

Intuitively, sentence 7a says that a certain event has to obtain. It establishes, indeed, a duty: in uttering 7a, the speaker prescribes that the listener has the duty of calling her parents. If the listener, in turn, accepts 7a, she comes to believe that writing a letter to her parents is better than not doing it.[4]

7b illustrates the logical form of 7a. The deontic modal *should* ranges over the sentence *You call your parents*, whose verb-phrase (VP) is headed by an eventive predicate with a non-progressive aspect. The predicate *call* is eventive

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[1] See Condoravdi (2002), pp. 59-60.

[2] *Ivi.*

[3] By defining deontic modals in terms of their meaning contribution, we can include under that category also expressions like *have to*, which are not, strictly speaking, modal verbs. Cf. Adger (2004), pp. 158-159.

[4] Cf. Condoravdi (2010).

because it denotes a property of events (rather than a property of states of affairs), and is non-progressive because the action of calling is not ongoing (as it would be in the case of *You are calling your parents*).[5] It is worth noticing that even if the eventive predicate is in present tense, the evaluation of the embedded clause takes place after the utterance time.[6] This is reasonable if we consider that a deontic sentence like 7a can be used to direct the addresse to perform the action of calling. In the case of 7a, the eventive predicate refers to an event that is contained in an interval which starts immediately after the utterance time and ends in an indefinite future. Frame adverbials can, however, make explicit the temporal interval in which the event has to be located:

8. You should call to your parents by 5pm/ (later) today/ tomorrow

Present deontic modals with eventive embedded clause establish, therefore, that the corresponding event has to obtain somewhere in the future. The event can take place in the interval which starts immediately after the utterance time, and ends at 5pm or at the end of the current day, or in an interval which starts tomorrow and expands till the end of that day. However, the event cannot take place in the past:

9. You should call to your parents #yesterday

2.1.1 Presuppositions

Present necessity deontic modals appear to be sensitive to the context in which they are used. Consider the following scenario:

> Today you need to call the Municipality Office. The office usually closes at 3pm, but today it is exceptionally open until 5pm.

10. (a) It is 7pm and you forgot to call the Municipality Office. # But you should call the Municipality Office by 5pm.

    (b) It is 7pm and you forgot to call the Municipality Office. But you should have called the Municipality Office by 5pm.

Given the context above, the use of present necessity deontic modals is not really possible. It might be argued that the oddity of 10a is basically a temporal issue. Temporal considerations play indeed a prominent role in the scenario above. At 7pm the addressee's duty of calling the Municipality Office by 5pm cannot obtain; rather, it becomes a duty which refers to an event of the past. After 5pm, it makes no sense to say that the addressee has the duty of calling the Municipality Office by 5pm. At that point, what makes sense is only to say that she *had* that duty.

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[5]For a discussion on eventive *vs.* stative predicates and the various aspectual distinctions, see, for instance, Arregui (2010) and Condoravdi (2002).

[6]On this point see, for instance, Arregui (2010), Condoravdi (2002), Mastop (2005), Schwager (2006).

That line of argument is suggested, for instance, by Feldman (1986). He proposes a time-based analysis of obligation sentences, and argues that the contrast between 10a and 10b is ultimately reducible to a tense contrast. Even if his semantics is truth-functional, i.e., it is concerned only with the evaluation of descriptive obligation sentences, it is worth to discuss here some aspects of his analysis.

Feldman (1986) develops a possible worlds-semantics where statements' truth values are made relative to certain temporal parameters. His main points are that deontic modals have tense, and that, in evaluating an obligation sentence, one has to see what happens in the ideal accessible worlds that are *physically* possible at the relevant temporal parameter. In particular, according to Feldman, *should/ ought to/ must* are anchored in the present, while *should have/ ought to have/ must have* are nothing but the past forms of the corresponding present deontic modals. Sentences like *It should be the case that* α would contain, therefore, a silent temporal parameter corresponding to the time of utterance, while sentences like *It should have been the case that* α indicate that the temporal parameter precedes the time of utterance. This means that simple *should*-statements like 7a require to look at the worlds which are physically possible at the utterance time, while *should have*-statements shift the temporal parameter of the accessibility relation to the past.

Consider 10b. At 7pm, there are no physically possible worlds in which the addressee has called the Municipality Office by 5pm; in fact, she forgot to do it. That is the reason why we need to use the deontic modal *should have* to express an obligation sentence which might be true. The deontic modal *should have* takes us back to a moment of time t' in which it was still possible to call the Municipality Office by 5pm, a moment in the past such that among the physically possible worlds at t' there are worlds in which the addressee calls the Municipality Office by 5pm.

The analysis proposed by Feldman (1986) suggests that deontic modals are dependent on temporal parameters, and are evaluated in light of the physically possible alternatives available at the corresponding time. However, it is possible to find some examples which cannot be captured by Feldman's analysis. Consider the following conversation between a mother and her son:

11. m: Remember that tomorrow you must take your little brother to the Luna Park.

   s: Oh, no! So, all day tomorrow I will be stuck there.

   m: Well, I have already bought the entrance tickets for you both

   s: OK. I will go even though I should have written my report tomorrow!

Feldman's temporal analysis cannot predict the use of *should have* in 11. At the utterance time, it is indeed still *physically* possible for the son to spend the following day writing his report, instead of going to the Luna Park with his brother. Moreover, the duty of spending the day writing the report concerns the future, which casts some doubts on the reduction of *should-have* to the past form of *should*.

It is worth noticing that the use of *should-have* happens in a context in which the son has *agreed* to take the little brother to the Luna Park (agreement expressed by the utterance of *OK*). In *that* context, spending the day writing his report is not taken to be possible anymore.

The choice between *should* and *should-have* seems to have little to do with physical possibilities. Rather, what matters are the possibilities that the speaker assumes to be opened during the conversation. Consider the case in which the son refuses to take his little brother to the Luna Park. In that case, the use of *should* is not problematic:

12.   s: I am sorry, Mom. I prefer not to take him to the Luna Park tomorrow.
           I should write my report tomorrow.

Feldman's temporal analysis fails in distinguishing the context in 11 from the one in 12. In fact, the two contexts do not differ in terms of the physical possibilities. However, the use of the simple deontic modal *should* is acceptable in 12 but not in 11.

Similar criticisms to the purely temporal account of the difference between *should* and *should-have*-statements have been suggested also by Arregui (2010). She argues that the choice between *should* and *should-have* is not conditioned by what has happened in the world so far (i.e., by the physical possibilities that are left open). Rather, she concludes, the choice is determined by what the agents accept as knowledge, or are willing to assume in the conversational context.[7] The example we have discussed above agrees indeed with what Arregui claims.

As Arregui herself points out, the idea of focusing on the role of the conversational context comes from Stalnaker's (1975) analysis of indicative and subjunctive conditionals. While indicative conditionals are concerned with possibilities that are within the common ground, i.e., the common knowledge among the participants in a conversation, subjunctive conditionals regard possibilities that can fall outside it.[8] The subjective mood, Stalnaker argues, indicates that the assumptions made during the conversation are suspended.[9]

Contrary to Feldman (1986), we argue that the difference between *should* and *should-have* is not about tense (where *should-have* would be conceived as the past form of *should*), nor about the physical possibilities open at the corresponding temporal anchoring. Rather, the difference has to do with the embedded clause and the common ground of the participants in the conversation. By combining Stalnaker's and Arregui's analyses with what we have observed in the examples above, we can conclude that felicitous uses of present necessity deontic modals require the propositions embedded under them to be compatible with what has been assumed during the conversation, i.e., with the common ground that speaker and listener share at the utterance time.[10] We call it the *Compatibility*

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[7]See Arregui (2010), p.271.

[8]See Stalnaker (1975), p. 273

[9]*Ibidem*, p. 276

[10]Here we follow Arregui (2010) in understanding *compatibility* in a broad sense, meaning that the common ground does not contain any information which causes the exclusion of the embedded proposition from the possibilities. We will return on this point in the next chapter.

Presupposition:

Compatibility Presupposition: The embedded proposition is compatible with the common ground.

Some further remarks are needed here. First, the Compatibility Presupposition (CP) does not depend on any particular reading of deontic modals. That is, the observations which led to CP apply to the case where deontic modals are interpreted reportatively, and to the case where deontic modals are interpreted prescriptively. Consider, for instance, 12. The deontic modal *should* can have been used by the son to inform the mother about an already existent duty (for example, to describe a duty previously established by his teacher), or to put into existence that duty (for instance, because he has just realized that the report is due the day after). In both cases, CP holds, and the utterance of *should* is felicitous.

Second, we have argued that simple deontic modals *presuppose* compatibility; that is, we have considered compatibility a presupposition, and not an implicature. The reason is that, contrary to implicatures, CP cannot be cancelled. Compatibility constitutes, indeed, a felicity condition of present deontic modals in which the embedded clause is eventive and non-progressive. In the case of an eventive, non-progressive embedded clause, if compatibility fails the use of present deontic modals is not acceptable.[11] Moreover, CP can be considered as an instance of the well known *Ought implies Can Principle* (OiC).[12] It links, indeed, the notion of obligation with the notion of possibility, where the latter is understood as epistemic possibility.

Finally, it is worth to remark that present deontic modals with eventive embedded clauses establish that the corresponding event has to obtain in an interval which ends somewhere in the future. We have already observed that the temporal interval in which the event is located can be specified by the use of frame adverbials. In certain cases, such a specification is needed to see whether CP is violated or not. For the sake of illustration, consider the following example:

13. You ought not to steal

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[11]CP does not hold, however, in the cases where the embedded clause contains a stative predicate, or is in progressive form. Consider the following examples:

13. Your mother isn't here. She should be here.

14. You forgot to wear your school uniform. Your teacher sees you when you enter in class and utters: you should be wearing your school uniform!

We will not consider those cases in the present thesis. However, we think that the difference may lie on the fact that stative and progressive indicate a state of affairs or an action which is continuous, located in a temporal interval of time whose end is not determined. If the state of affairs or action are in progress, then it is not possible to say that they contradict the events assumed in the common ground. We leave this issue to further investigations. For the semantic interpretation of stative predicates, see Arregui (2010), Condoravdi (2002), Mastop (2003).

[12]For a discussion on OiC see, for instance, Sinnott-Armstrong (1984), Thomason (1981), Fischer (2003).

In uttering 13, we might want to forbid that the event of stealing takes place today, tomorrow, or from the utterance-time on. We can make it explicit, by means of frame adverbials:

14. You ought not to steal today/ tomorrow/ the day after tomorrow/...

What CP says is that, if 13 is interpreted as *You ought not to steal today*, then the common ground does not have to contain the information that the listener has already stolen something today. On the other hand, if that information is included in the common ground, CP predicts that *You ought not to steal today* is not felicitous. Of course, if it is known that the addressee has already stolen something today, the speaker can still felicitously utter 13. But it can do it only by shifting the evaluation of the embedded proposition further in the future, e.g., by interpreting it as *You ought not to steal tomorrow*.

We want to discuss now a proposal that has been suggested by Arregui (2010). In her analysis, Arregui considers a further felicity condition for present necessity deontic modals, which can be called *Diversity Presupposition*:

Diversity Presupposition: The embedded proposition is an open possibility within the common ground.

While CP only requires the embedded proposition to be conceived as a possibility at the utterance time, the Diversity Presupposition (DP) goes further, by imposing the condition that the embedded proposition does not trivially hold in the common ground at the utterance time.[13] Intuitively, DP claims that genuine and felicitous obligations do not prescribe actions that are taken to be inevitable. DP could be motivated by appealing to the idea that obligations, like imperatives, are generally uttered with the aim of prescribing something and, therefore, of influencing deontic preferences and addressee's actions.[14] If DP was violated and the embedded proposition was already taken to be the case, there would not be any proper alternatives to prefer nor actions to perform. Even if there are also meta-ethical considerations which support DP,[15] Arregui's motivation for taking the embedded proposition to be an open possibility is mainly technical. In particular, she is interested in avoiding that sentences like ϕ: *If α, then must/ought to/should α* trivially hold. If ϕ is always the case, it follows that the truth of α makes the deontic statement *Must/ought to/should α* trivially acceptable. That is something problematic, indeed.[16] Consider, for

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[13]A similar condition is adopted by Condoravdi (2002) for the semantics of non-root modals. See Condoravdi (2002), p. 83.

[14]Cf. Condoravdi (2010).

[15]As CP, also DP is relevant to the ethical and meta-ethical debate about determinism and moral responsibility. In the ethical tradition, (moral) obligations are analyzed in relation to the notions of blame and praise. In particular, it is traditionally argued that an agent cannot be praised for having fulfilled an obligation if she could not do otherwise. DP seems, therefore, to affirm the same principle. See Eshleman (2009) for an overview and a discussion about moral responsibility and determinism.

[16]However, ϕ is derivable in Standard Deontic Logic and in many other deontic systems (e.g., Hansson 1969; Kratzer 1981; Thomason 1981). Some authors have also tried to argue

instance, the case in which it is known that John robs banks. From that it does not trivially follow that John ought to rob banks.[17] Another example: assume that it is known that today the sun rises at 6 am; if ϕ was the case, we could derive that today the sun is obliged to rise at 6 am. All those unwanted derivations are blocked by DP.

We are not claiming that adopting DP is the only way of blocking the derivation of ϕ. In the literature of deontic logic, other solutions have been suggested. For instance, Frank (1997) and van Rooij (2000) argue that the derivation of ϕ shows that the traditional treatment of conditional obligations is unsatisfactory. The problem would not be due to a violation of a deontic presupposition (like DP); rather it would concern the analysis of sentences of the form *If α, then must/ought to/should β*, where α constitutes the *condition* under which the obligation *Must/ought to/should β* applies. In that perspective, a proper treatment of the interaction between the conditional operator and deontic modals would, therefore, avoid the derivation of ϕ. We will not present here the details of the semantics which invalidate ϕ.[18] What is relevant to our discussion is that there exist at least two different strategies to block ϕ: (i) ruling out ϕ as a violation of DP (as in Arregui 2010) or (ii) providing an interpretation of conditional obligations which falsifies ϕ (as in van Rooij 2000) .

In this paper, we adopt DP and follow strategy (i). The reason behind our choice is that DP is supported by considerations that are independent from ϕ. It enlightens some aspects of the prescriptive meaning of deontic modals, and is related to traditional meta-ethical issues. However, it remains an open question whether DP constitutes a presupposition for every kind of obligation (moral, legal, ...) that can be expressed by present necessity deontic modals. We agree that more research has to be done in that direction, and we leave the issue to future investigations.

2.2   Past necessity deontic modals

This section is devoted to *past necessity deontic modals*, that is, deontic modals like *ought to have*, *should have*, *had to*. Semantically, they are ambiguous between two readings: a counterfactual reading and an open-past reading.[19] We discuss them in what follows.

that ϕ is not problematic. For instance, Thomason (1981) justifies ϕ by saying that ϕ holds in deliberative contexts, i.e., contexts in which deontic statements are used to give "practical advices". If there are no other options than α, then α is inevitable, and *Must/ought to/should α* is harmless. We find that solution unsatisfactory. As we have already argued, if there are no other options than α, then what *Must/ought to/should α* looses is precisely its character of practical advice. Therefore, especially in deliberative contexts, ϕ turns out to be not felicitous.

[17] Example taken from van Rooij (2000), p.10.

[18] On that point, see van Rooij (2000).

[19] The distinction draws back to Condoravdi's (2002) work on the temporal interpretation of non-root modals. She uses a different terminology, calling the two readings metaphysical and epistemic, respectively. Since, however, we are focusing only on deontic modals and referring to metaphysical and epistemic readings might have led to confusion, we decided to adopt a more neutral terminology.

2.2.1 Counterfactual Reading

Under the counterfactual reading, past necessity deontic modals are used to prescribe events that might not be realizable anymore. This is possible only for modals that are in subjective form in English, such as *ought to have* and *should have*.[20] Consider the following example:

15. You ought to have sent everybody a reminder by email (but you didn't)

When the speaker utters 15, she communicates that we are now located in a world whose past included the (unactualized) possibility of the addressee sending everybody a remainder by email.[21] Sentence 15 establishes, indeed, what would have been ideal. It requires to go back to a certain point of the past in which the addressee had the possibility of sending everybody a remainder by email; the worlds in which she did it are deontically more ideal than the one we occupy now.

It is worth noticing that he counterfactual reading is allowed also in cases of obligations about issues that are located in the future. Compare 15 with the last sentence in 11:

11.   m: Remember that tomorrow you must take your little brother to the Luna Park.

      s: Oh, no! So, all day tomorrow I will be stuck there.

      m: Well, I have already bought the entrance tickets for you both

      s: OK. I will go even though I should have written my report tomorrow!

In one case, *ought to have* indicates an obligation about an event in the past; in the other, *should have* refers to an event located in the future. However, what is relevant for the counterfactual reading is that, in both sentences, the obligations refer to issues that are considered already settled: neither the listener sent everybody a remainder via email, nor the son will spend the following day writing his report.

A purely temporal analysis cannot make sense of the counterfactual reading of *ought to have* and *should have* in 15 and 11. First, as we have already noticed above, if we reduce *should have* to the past form of *should* we are not able to predict its use in contexts which regard the future (like in 11). Second, if *ought to have* and *should have* were interpreted simply as the past forms of *ought to* and *should*, then we would expect that all the presuppositions which hold for *ought to* and *should* apply to *ought to have* and *should have*. However, it is clear that neither CP nor DP hold in 15 and 11 at the utterance time.

In fact, the counterfactual reading indicates that the presuppositions CP and DP are suspended. The reading is indeed "counterfactual", in the sense that the propositions embedded under the past necessity deontic modals may be incompatible with what is currently assumed in the common ground. To

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[20]See Condoravdi (2002), p. 75.

[21]*Ibidem*, p. 62.

paraphrase Arregui (2010), under the counterfactual reading past necessity deontic modals do not encode any presupposition regarding the common ground: the worlds which satisfy the embedded proposition may be inside the common ground or not.[22] On the other hand, it seems that the counterfactual reading implies something stronger, that is, that the worlds which satisfy the embedded proposition actually *are* outside the common ground. *Ought to have* and *should have*-statements, like 15 and 11, are generally used by the speaker to convey the information that the relevant events did not/will not in fact take place. In order to account for that, Condoravdi (2002) uses an argument based on conversational maxims: if at the utterance time it were still possible that the event did take place, considering also what falls outside the common ground would not be necessary and would only make the statement weaker. So, for reasons of informativity and economy, it makes sense to exclude that the embedded proposition is verified inside the common ground.[23]

Even if the embedded propositions are not assumed to be possible at the time of utterance, when the speaker utters 15 and 11, she makes a felicitous and meaningful conversational contribution. Even if the embedded propositions are not inside the common ground anymore, *ought to have* and *should have* can be used prescriptively, in the sense that they can bring about new obligations. Actually, as Mastop (2005) notices, those modals induce a vivid sense of commitment in the addressee:

> Sentences starting with "you should have" are not always, perhaps not even normally, responded to by saying "Oh, I did not know that" merely accepting its informational content. Natural reactions, such as "I guess you are right, I should have" or "Yeah, stupid of me", express an attitude of remorse or regret. This remorse or regret indicates the presence of an actual, felt commitment instead of merely the belief that the commitment would have been reasonable under the circumstances [...] "you should have" sentences can be used in a non-negative sense, suggesting how some alternative course of action would also have been nice, perhaps even better than the one actually taken.   (Mastop 2005, p.145)

Hence, the fact that the embedded proposition is not conceived as a realizable possibility does not prevent the speaker to make a prescriptive deontic statement about it. In other words, when a sentence like 15 or 11 is uttered and accepted, it creates a new ideality order among possible worlds, by establishing, for instance, that the worlds in which the listener sent everybody the remainder by email are more ideal than the ones in which she did not. Under the counterfactual reading, *ought to have* and *should have* in 15 and 11 create a new ideality order which is not restricted to the worlds in the common ground.

We have argued that, in order to understand the counterfactual reading of past necessity deontic modals, it is crucial to look at the common ground. The

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[22]See Arregui (2010), pp. 251, 261.
[23]See Condoravdi (2002), p. 86 and Mastop (2005), p. 140.

reading is indeed "counterfactual", because past necessity deontic modals express obligations about issues that contradict (what are assumed to be) the facts. However, it is worth noticing that the temporal dimension plays a role in the counterfactual reading as well. The counterfactual reading, as we have described it, does not consist in simply imagining that things are different than they are presumed to be by the participants in the conversation. It makes reference to alternative possibilities that were open in the past. As observed also by Arregui (2010), Condoravdi (2002) and Mastop (2005), the counterfactual reading of statements such as *It ought to have/should have been the case that* α requires backtracking, i.e., to go back to a moment of time in which *It ought to/should be the case that* α was felicitous. All this suggests that the counterfactual reading can be obtained by means of a **PAST** operator (responsible for the cg-backtracking). In the next chapter, we will argue that it is indeed the case, and propose to analyze deontic statements such as 11 and 15 as follows:

16. (a) I should have spent the day writing my report
    (b) have [should [I spend the day writing my report]]
    (c) **PAST** [should [α]]

17. (a) You ought to have sent everybody a reminder by email
    (b) have [ought to [you send everybody a reminder by email]]
    (c) **PAST** [ought to [α]]

2.2.2 Open-Past Reading

Under the open-past reading, past necessity deontic modals are used to express obligations about facts which happened in the past. Contrary to the counterfactual reading, those facts are consistent with the common ground. Consider the following scenario:

> You work as a teacher, and Sara is one of your students. You told your students that they have time till Friday afternoon to leave their homework in your pigeonhole. On Thursday, everybody but Sara handed in the homework. On Friday you could not check your pigeonhole. Now it is Saturday, and the department is closed.

You meet Sara's parents and you utter:

18. Sara had to hand in her homework yesterday

Sentence 18 expresses, indeed, an obligation about an issue of the past which is supposed to be settled at the utterance time, but such that the speaker does not know which way it was settled.[24] In other words, 18 refers to an issue of the past which is still open according to the knowledge of the speaker. In the example above, it is indeed clear that the issue whether Sara handed in her homework

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[24]Compare with Condoravdi (2002), p. 62.

yesterday or not is actually settled; however, the speaker cannot know if she succeeded in doing it or if she failed.

Given the speaker's state of knowledge, the common ground contains at least two worlds which differ in their past history: one world such that yesterday Sara handed in her homework, the other such that she did not do it. A sentence like 18 establishes that the worlds whose past included the event of Sara handing in her homework are more ideal than the ones in which she did not do it. In that sense, 18 is prescriptive; it induces a modification of the ideality order among the worlds in the common ground.

The open-past reading differs from the counterfactual one because it only requires to look at the worlds which belong to the common ground. Moreover, while the counterfactual reading expresses a contrast between the embedded proposition and the facts, the open-past reading has an epistemic flavor. It is about epistemic uncertainty at the utterance time. A sentence like 18 does not communicate what would have been ideal, but what *was* ideal. It establishes an obligation about an issue of the past which is still an epistemic possibility. Even if, in the present thesis, we are largely abstracting from differences between the various deontic modals, we would like to speculate that the open-past reading is generally excluded in the case of subjective modals. In fact, we have already argued that a sentence like *Sara should have handed in her homework yesterday* conversationally implies that she did not do it. Unless the implicature is canceled, the open-past reading is excluded.

Finally, we can observe that only under the open-past reading, past necessity deontic modals meet the conditions CP and DP. By definition, the open-past reading requires the embedded proposition to be compatible with the common ground, and to be an open possibility at the time of utterance. In fact, under the open-past reading, past necessity deontic modals behave as present necessity deontic modals, with the exception that the embedded clause is evaluated in the past. Then, we propose to represent the the logical form of 18 as follows:

19. (a) Sara had to hand in her homework yesterday

    (b) have to [Sara handed in her homework yesterday]

    (c) have to [**Past** [Sara hand in her homework]]

    (d) have to [**Past** [α]]

where **Past** is a temporal operator which shifts the evaluation of the verb in α to the past ("yesterday").

2.3   Time and common ground, summing up

We have argued that, in order to interpret present and past necessity deontic modals, two factors need to be taken into account: (i) time (with respect to the evaluation of the embedded proposition, and the common ground) and (ii) inside/outside the common ground. Our observations are summarized in the Table 2.1.

Table 2.1: Deontics, time and cg.

	Evaluation embedded clause	Common Ground (cg)
Present	future	inside present cg
Counterfactual	past or future	?outside present cg inside past cg
Open-past	past	inside present cg

Present necessity deontic modals express an obligation about an event located in the future, which is conceived as an open possibility at the utterance time (hence, it is inside the present common ground). Under the open-past reading, past necessity deontic modals are like present necessity deontic modals except for the fact that the embedded proposition concerns a past event. Finally, under the counterfactual reading, past necessity deontic modals express an obligation about an event which was previously conceived as an open possibility (i.e., it may be incompatible with the current common ground).

Chapter 3

Temporal Deontic Update Semantics (TDUS)

The present chapter represents present necessity deontic modals and past necessity deontic modals by making use of a deontic operator **Oblige**. It analyses the different necessity deontic modals in terms of scope interactions between the deontic operator, a past tense operator and an operator responsible for cg-backtracking. Finally, it develops a temporal update semantics which models the prescriptive use of such deontic modals.

3.1 Logical forms

In Chapter two we have considered the semantic differences between present necessity deontic modals, past necessity deontic modals under the counterfactual reading, and past necessity deontic modals under the open-past reading. One question now naturally arises: how many deontic operators do we need to model those differences? The answer, we argue, is: one.

We have seen that time and common ground (cg) should be taken into account for the interpretation of present and past necessity deontic modals. In particular, we can express the differences that those modals display in terms of when the evaluation of the embedded proposition takes place, and whether it belongs to the current common ground or to a past one.[1] We argue that even if those factors may vary, the semantic contribution of the deontic operator is constant.

Let **Oblige** be a sentential deontic operator. Let **PRESENT** and **PAST** be operators which applies to cg: **PRESENT** anchors the deontic operator to the current cg, while **PAST** is responsible for cg-backtracking, and anchors the deontic operator to a past cg. Let **Past** be a past-tense, sentential operator, which moves the evaluation of the embedded clause to a time which precedes the

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[1]Recall that we are limiting our analysis to the cases in which the embedded propositions are eventive and non-progressive.

utterance time. Then, the logical forms of sentences containing present necessity deontic modals, past necessity deontic modals under the counterfactual reading, and past necessity deontic modals under the open-past reading are along the lines of 20b, 21b and 22b, respectively.

20. (a) You should call your parents

    (b) **PRESENT Oblige** (you call your parents)

21. (a) You ought to have sent everybody a remainder by email

    (b) **PAST Oblige** (you send everybody a remainder by email)

22. (a) Sara had to hand in her homework yesterday

    (b) **PRESENT Oblige Past** (Sara hand in her homework)

Since we are dealing with eventive predicates, the default is that the embedded event follows the anchoring time of the deontic operator, but this can be changed by introducing a past tense operator in the scope of **Oblige**.[2] If we define the semantics of **Oblige** such that the presuppositions CP and DP hold at the anchoring time, then we can take care of all the semantic differences between present necessity deontic modals and past necessity deontic modals.

Consider 20a. The proposition *You call your parents* is an open possibility at the time of utterance, and is evaluated sometime after the utterance time. Hence the deontic operator is anchored in the current common ground, CP and DP hold there, and the embedded proposition is evaluated in the future. This is predicted by 20b.

Now, consider past necessity deontic modals under the counterfactual reading. We have said that it does not really matter whether the embedded event takes place at a time that follows or precedes the utterance time. What matters for the counterfactual reading is that the embedded proposition may be outside the current common ground. Take 21a. It establishes what would have been ideal, and requires to go back to a certain point of the past in which the proposition *You send everybody a remainder by email* was an open possibility. This is predicted by 21b. The **PAST** operator shifts the anchoring of the deontic operator to a past cg. At that anchoring time CP and DP hold. Moreover the embedded proposition is evaluated at a time which follows the anchoring time.

Consider 22a. The proposition *Sara hands in her homework* is evaluated at a time which precedes the utterance time ("yesterday"). Moreover, that Sara handed in her homework (yesterday) is an open possibility at the utterance time. This is what 22b says. The deontic operator is anchored in the current common

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[2]As Condoravdi (2002) notices, when eventive predicates appear in the scope of present modals, they shift forward the evaluation of the sentence in which they occur. It is worth to remark that our analysis follows the main results presented by Condoravdi in her decompositional analysis of non-root modals. However, while she analyses scope restrictions between non-root modals and a *perfect* operator, we make use of the past tense operator **Past**. This is due to the fact that we are taking into account also deontic expressions such as *had to*, in which there is no perfect. Cf. Condoravdi (2002), pp. 64-69.

ground, and CP and DP hold there (for **Past**(Sara hand in her homework)). Moreover, **Past** shifts the evaluation of *Sara hands in her homework* to the past.

Finally we propose that, unless it is specified otherwise, the deontic operator **Oblige** anchors to the current cg. This permits us to simplify 20b, 21b and 22b as 23, 24 and 25, respectively:

23. **Oblige** (you call your parents)

24. **PAST Oblige** (you send everybody a remainder by email)

25. **Oblige Past** (Sara hand in her homework)

In the next section we develop a formal semantics for present necessity deontic modals, past necessity deontic modals under the counterfactual reading, and past necessity deontic modals under the open-past reading. We adopt the analysis described above.

3.2 A dynamic setting

As we have already said, the present thesis focuses on the prescriptive reading of obligation sentences. As noticed by Austin (1962), in everyday discourse practice the prescriptive reading takes the priority over the reportative one: obligation sentences are typically used to bring about new duties and obligations, rather than to give a true or false description of the existing ones. They are intrinsically prescriptive, and they are used to *create* and *change* an aspect of reality, i.e., its normative configuration. They have, therefore, a dynamic character. That is what we want to model in our semantics.

We develop here a framework from the dynamic logic tradition, especially Veltman's (1996) work on Update Semantics.[3] Given that we aim at representing the dynamics of discourse practice, especially concerning obligation sentences, the semantic framework interprets "meanings" as the change that every sentence induces in the context in which it is uttered and accepted by the agents involved in the conversation. As it is said, the slogan "You know the meaning of a sentence if you know the conditions under which it is true" is replaced by the following one: "You know the meaning of a sentence if you know the change it brings about in the information state of anyone who accepts the news conveyed by it."[4]

In a well-run conversation, speaker and listener share a certain background information, or conversational context, which gets updated during the dialogue.[5] When a sentence is uttered and accepted by the participants in the conversation, the information the sentence carries (i.e., its meaning) modifies and strengthens that context, by putting aside all possibilities that are incompatible with it. Every sentence affects the conversational context: during a conversation, agents

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[3]For another use of Veltman's update semantics for the analysis of prescriptive obligation sentences, see van der Torre and Tan (1999).

[4]See Veltman (1996), page 221.

[5]Cf. Stalnaker (1978).

can acquire factual knowledge and new moral obligations. In what follows, we consider the conversational exchange between a moral authority (the speaker) and a listener.

3.3   Information states

In the present section, we define the notion of information state. An information state represents the conversational context that speaker and listener share during a conversation. The basic elements of information states are *histories*, which represent the temporal evolution of the world, and *deontic betterness-relations* between histories. Just as it happens during a dialogue, information states can be modified and evolve in time.

Definition 1 (History)**.** Let I be a set of indeces, and $\mathbb{N}$ the set of natural numbers. For $i \in I$, a history h_i is defined as follows: $h_i = \{(i,n) \mid n \in \mathbb{N}\}$.

We take every $n \in \mathbb{N}$ to represent a temporal span, i.e., a certain temporal interval. Then, the pair (i,n) indicates the temporal span n which belongs to a certain evolution i of the world. Let $<$ be the "less than"-relation on natural numbers (i.e., a binary strict total relation on $\mathbb{N}$). If $m < n$, then (i,m) temporally precedes (i,n). Hence, a history is a set of lineally ordered temporal spans. For the sake of simplicity, we assume that all histories have the same temporal structure.

This is a simplistic representation of time. However, it permits us to enlighten the semantic contribution of the different deontic modals, and, as we will see in the next chapter, to take into account some of the main deontic paradoxes. Nevertheless, our simplistic representation has some drawbacks. We discuss some of its limits when we define complex updates, and when we consider Chisholm's Paradox in the next chapter.

We introduce now the language L_0 (which contains only descriptive sentences) and the valuation function V.

Definition 2 (Language L_0)**.** Language L_0 is built from a countable set A of atoms according to the following BFN:

$L_0 : \phi := p \mid \top \mid \neg\alpha \mid \alpha \wedge \beta \mid \alpha \vee \beta \mid \textbf{Past } \alpha$

where $p \in A$.

Definition 3 (Valuation function V)**.** Let A be the set of atoms of L_0, and let v be the interpretation function $v \colon A \longrightarrow \mathcal{P}(I \times \mathbb{N})$. A formula $\phi \in L_0$ is true at (i,n) under the interpretation v (written $(i,n) \in V_v(\phi)$) iff:

- if ϕ=p, then $(i,n) \in V_v(p)$ iff $(i,n) \in v(p)$

- if $\phi = \neg\alpha$, then $(i,n) \in V_v(\neg\alpha)$ iff $(i,n) \notin V_v(\alpha)$

- if $\phi = \alpha \wedge \beta$, then $(i,n) \in V_v(\alpha \wedge \beta)$ iff $(i,n) \in V_v(\alpha)$ and $(i,n) \in V_v(\beta)$

- if $\phi = \alpha \vee \beta$, then $(i,n) \in V_v(\alpha \vee \beta)$ iff $(i,n) \in V_v(\alpha)$ or $(i,n) \in V_v(\beta)$

- if $\phi = \textbf{Past } \alpha$, then $(i,n) \in V_v(\textbf{Past } \alpha)$ iff $\exists (i,m)$, such that $m < n$ and $(i,m) \in V_v(\alpha)$

Given the definition above, every sentence is evaluated with respect to a temporal span and a history. Let ϕ be *You call your parents*. If $(i,n) \in V_v(\phi)$, then we say that the event of you calling your parents obtains inside the temporal span n according to the history h_i. Finally, notice that the operator **Past** shifts the evaluation of the embedded clause to a certain past time.

We can now define the notion of information state.

Definition 4 (Information State). An information state S^n is a tuple $\langle n, H, H*^n, v, Better \rangle$ where:

- $n \in \mathbb{N}$ represents the current temporal collocation

- H is the set of *all physically possible* histories h_i, for $i \in I$

- $H*^n \subseteq H$ is the common ground at n

- v is an interpretation function

- *Better* is a function that assigns to every history $h_i \in H$ and every m$\in \mathbb{N}$ a set of couples (h_k, h_l), such that $h_k, h_l \in H$ and $h_k \preceq_m h_l$ (read as: history h_k is at least as deontically ideal as h_l from the point of view of h_i at m).

The information state S^n represents the conversational context that the speaker and the listener share. It has a temporal collocation, n, which is the current temporal location of the speaker and the listener. H is the set of all possible evolutions of the world which obey to the physical laws. $H*^n$ is a subset of physically possible histories that are considered possible at n. In other words, $H*^n$ represents the current common ground, i.e., the knowledge that the speaker and the listener have in common at time n. Finally, the *Better*-function represents what we have called the normative configuration of reality. Take, for the sake of illustration, $Better(h_i, n)$. It consists of a set of pairs $\{(h_k, h_l), (h_s, h_t), ...\}$ which indicates what are the deontic preferences according to the history h_i at n. Of course, deontic preferences may vary with respect to another history, or change with time. It is worth highlighting that, for every $m \in \mathbb{N}$, we take the relation $\preceq_m$ to be reflexive, but we do not assume transitivity.[6]

We conclude the present section by introducing two special information states: the initial state **0** and the absurd state **1**.

Definition 5 (Initial State). We call initial state **0** every information state $S^n = \langle n, H, H*^n, v, Better \rangle$ such that:

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[6]We agree that talking about a "*Better*-function" might convey the idea that the relation $\preceq$ is transitive. However, we just use that terminology figuratively. We will come back on the failure of transitivity in the next chapter, when we will introduce conditional obligations. See also van der Torre and Tan (1999).

- $H*^n = H$

- For every $h_i \in H$ and $m \in \mathbb{N}$, $Better(h_i, m) = H \times H$

Definition 6 (Absurd State)**.** We call absurd state **1** every information state $S^n = \langle n, H, H*^n, v, Better \rangle$ such that:

- $H*^n = \emptyset$;

  or:

- for some $h_i \in H$ and $m \in \mathbb{N}$, $Better(h_i, m)$ is such that there are $h_k, h_j \in H$: $(h_k, h_j) \notin Better(h_i, m)$ and $(h_j, h_k) \notin Better(h_i, m)$

Intuitively, in the initial state **0** agents have no factual knowledge (hence, all physical possibilities are still open) and no deontic preference is assumed (hence, for every $h_i \in H$ and $m \in \mathbb{N}$ the relation $\preceq_m$ is universal). On the other hand, the absurd state **1** is such that the agents have reached a contradiction either about what they take to be the facts (hence, there are no possibilities left open), or about what counts as deontically ideal.[7]

3.4 Updates with descriptive sentences

We define now various update operations on information states. We develop an eliminative approach, since every update operation results in an elimination of the relevant possibilities. First consider the case of updates with descriptive sentences. When an information state is updated with a sentence which describes an event of the world, the common ground shrinks. The resulting information state has a new temporal collocation, and is such that the new common ground contains only histories in which that event happened. We write $S^n[\phi]$ to denote the result of updating S^n with the sentence ϕ.[8]

Definition 7 (Update with descriptive sentence)**.** If ϕ is a descriptive sentence of L_0, then:

- $S^n[\phi] = S^{n+1} = \langle n+1, H, H*^{n+1} = \{h_i \mid h_i \in H*^n \text{ and } (i, n) \in V_v(\phi)\}, v, Better' = Better \rangle$

In other words, the resulting information state is such that the new common ground includes the information that the event described by ϕ has obtained in n.

Consider, for example, the following information state $S^n = \langle n, H, H*^n, v, Better \rangle$, where:

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[7]The latter condition refers to the so-called deontic dilemmas: if there are two histories such that the first one is better than the second one (according to a certain obligation) and the second one is better than the first one (according to another obligation), then we have a deontic dilemma. The present framework rules out deontic dilemmas, in the sense that they lead to absurd states.

[8]Cf. Veltman (1996).

- $H = H*^n = \{h_i, h_j\}$

- v is such that $(i, n) \in V_v(p)$ and $(j, n) \in V_v(\neg p)$

- $Better$ is such that: $Better(h_i, n-1) = Better(h_j, n-1) = \{(h_i, h_j), (h_i, h_i), (h_j, h_j)\}$, and for all $m \neq n - 1$, $Better(h_i, m) = Better(h_j, m) = H \times H$.

Hence, no factual knowledge is assumed (since $H = H*^n$) and, from the point of view of both h_i and h_j at a past time $n - 1$, h_i is deontically more ideal that h_j. However, no other deontic preferences are assumed (in particular, no current deontic preference is assumed since $Better(h_i, n) = Better(h_j, n) = H \times H$). Then if the speaker utters "p" (i.e., "p holds in n") and then the listener accepts it, the resulting state is $S^{n+1} = \langle n + 1, H, H*^{n+1}, v, Better' \rangle$ such that:

- $H = \{h_i, h_j\}$

- $H*^{n+1} = \{h_i\}$

- $Better'(h_i, n - 1) = Better'(h_j, n - 1) = \{(h_i, h_j), (h_i, h_i), (h_j, h_j)\}$ and for all $m \neq n$, $Better'(h_i, m) = Better'(h_j, m) = H \times H$.

That is, the history h_j where p was false at n is eliminated from the common ground.

Finally, notice that an information state can be updated also with a sentence like **Past** α, i.e., agents can acquire information concerning facts of the past.[9]

3.5 Updates with obligation sentences

In this section, we enrich the language L_0 with the deontic operator **Oblige**, and define the main update operations with obligation sentences.

Definition 8 (Language L_1)**.** $L_1 = L_0 \cup \{\textbf{Oblige}\}$.

3.5.1 Present necessity deontic modals

We have argued that obligation sentences have a certain relation with the common ground. Before introducing the update operations with sentences which contain present and past necessity deontic modals, we define what it means to be an open possibility:

Definition 9 (Open possibility)**.** Let ϕ be a descriptive sentence of L_0. For $m \in \mathbb{N}$, we say that ϕ is an open possibility in $H*^m$ iff:

- $\exists h_i \in H*^m$ such that $(i, m) \in V_v(\phi)$; and

- $\exists h_j \in H*^m$ such that $(j, m) \in V_v(\neg\phi)$

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[9]It is also possible to add a future operator **Fut** to our language L_0, in order to allow agents to acquire information concerning future events. The evaluation clause would be: if $\phi = \textbf{Fut}$ α, then $(i, n) \in V_v(\textbf{Fut}\ \alpha)$ iff $\exists(i, m)$, such that $n < m$ and $(i, m) \in V_v(\alpha)$.

In particular, present necessity modals obey to preconditions CP and DP with respect to the current common ground. Hence the embedded proposition is an open possibility in $H*^n$, where n is the temporal collocation of speaker and listener.

Now we define the update operation trigged by the utterance and the acceptance of an obligation sentence containing a present necessity deontic modal. As we argued above, we represent it as "**Oblige** α":[10]

Definition 10 (Update with present necessity deontic modal). If $\phi =$ **Oblige** α (where $\alpha \in L_0 \setminus \{\mathbf{Past}\}$), then:

- if α is an open possibility in $H*^n$, then $S^n[\phi] = S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$ where:

  - $H*^{n+1} = H*^n$
  - for every $h_i \in H*^n$, $Better'(h_i, n+1) = Better(h_i, n+1) \setminus \{(h_k, h_l) \mid (k, n) \in V_v(\neg\alpha)$ and $(l, n) \in V_v(\alpha)\}$
  - for all the other $h_j \in H$ and $m \in \mathbb{N}$, $Better'(h_j, m) = Better(h_j, m)$

- otherwise $S^n[\phi] = \mathbf{1}$

In the previous chapter, we have argued that the proposition embedded under a present deontic modal has to be an open possibility in the current common ground. If it is not the case, the utterance is not felicitous and the obligation sentence cannot be accepted as such. This is predicted by definition 10: if α is not an open possibility in the current common ground, then $S^n[\mathbf{Oblige}\alpha]$ results in the absurd state. On the other hand, if α is an open possibility, then accepting $\mathbf{Oblige}\alpha$ results in a new state in which the deontic preferences are different from the previous ones. Given that present deontic modals are defined in light of the possibilities in the common ground (i.e, felicitous utterances of sentences containing present deontic modals depend on the physical possibilities that are in the common ground), the update concerns only the deontic preferences of histories which belong to $H*^n$. In particular, the updated information state is such that the histories in which the event described by α obtained are better than the ones in which it did not. Let α be *You call your parents*. Assume that the sentence $\mathbf{Oblige}\alpha$ is uttered by the speaker at a certain moment in the temporal span n. If the listener then accepts that sentence, she comes to have a new deontic preference: she agrees that calling her parents is better than not doing it. With the listener's acceptance, histories in which the event of calling her parents occurred within the temporal span n (after the speaker's utterance) become therefore more ideal than the others.

Consider, for example, the following information state $S^n = \langle n, H, H*^n, v, Better \rangle$, where:

- $H = \{h_i, h_j, h_k\}$

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[10] See 23 in section 3.1.

- $H*^n = \{h_i, h_j\}$

- v is such that $(i, n) \in V_v(\alpha)$, $(j, n) \in V_v(\neg\alpha)$ and $(k, n) \in V_v(\neg\alpha)$

- for every $m \in \mathbb{N}$, $Better(h_i, m) = Better(h_j, m) = Better(h_k, m) = H \times H$

Hence, the histories that are considered still possible at n are h_i and h_j, and no deontic preference is assumed. If the speaker utters "**Oblige**α" and then the listener accepts it, the resulting state is $S^{n+1} = \langle n + 1, H, H*^{n+1}, v, Better'\rangle$ such that:

- $H = \{h_i, h_j, h_k\}$

- $H*^n = \{h_i, h_j\}$

- $Better'(h_i, n+1) = Better'(h_j, n+1) = \{(h_i, h_j), (h_i, h_k), (h_j, h_k), (h_k, h_j)\}^{11}$, and $Better'(h_k, n + 1) = Better(h_k, n + 1) = H \times H$

- for all $m \neq n + 1$, $Better'(h_i, m) = Better'(h_j, m) = Better'(h_k, m) = H \times H$.

Since α is an open possibility in $H*^n$, $S^{n+1} \neq \mathbf{1}$. In particular, S^{n+1} is such that from the point of view of the histories in the common ground at $n + 1$, the histories which led to α are deontically better than the others.

3.5.2 Past necessity deontic modals under the open-past reading

Updates with past necessity modals under the open-past reading are like updates in definition 10. The only difference is that they concern the cases in which $\alpha:=\mathbf{Past}\beta$.

Definition 11 (Update with past necessity deontic modal, open-past). If $\phi=\mathbf{Oblige}\ \mathbf{Past}\beta$ (where $\beta \in L_0$), then:

- if $\mathbf{Past}\ \beta$ is an open possibility in $H*^n$, then $S^n[\phi] = S^{n+1} = \langle n + 1, H, H*^{n+1}, v, Better'\rangle$ where:

  - $H*^{n+1} = H*^n$
  - for every $h_i \in H*^n$, $Better'(h_i, n+1) = Better(h_i, n+1) \setminus \{(h_k, h_l) \mid (k, n) \in V_v(\neg\mathbf{Past}\beta)$ and $(l, n) \in V_v(\mathbf{Past}\beta)\}$
  - for all the other $h_j \in H$ and m$\in \mathbb{N}$, $Better'(h_j, m) = Better(h_j, m)$

- otherwise $S^n[\phi] = \mathbf{1}$

As we have argued above, the difference between past deontic modals under the open-past reading and present deontic modals lies on the fact that the embedded event takes place in the past. That is precisely the difference between updates defined in 11 and in 10.

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[11] We leave the reflexive closure implicit.

3.5.3 Past necessity deontic modals under the counterfactual reading

We consider now updates with past necessity deontic modals under the counterfactual reading. Contrary to updates defined in 10 and 11, the events embedded under past necessity deontic modals under the counterfactual reading may obtain outside the common ground. Updating a state with past necessity deontic modals under the counterfactual reading eventually requires to go back to a past common ground where the embedded proposition was an open possibility. The update, hence, concerns those histories which were part of that past common ground.

Definition 12 (Language L_2). $L_2 = L_1 \cup \{\textbf{PAST Oblige}\}$.

We define now the update operation trigged by the utterance and acceptance of an obligation sentence containing past necessity deontic modals under the counterfactual reading. As we have argued above, we represent it as **PAST Oblige**α:[12]

Definition 13 (Update with past necessity deontic modal, conterfactual). Let $\phi =$ **PAST Oblige** α (where $\alpha \in L_0$). $S^n[\phi]$ is such that:

- if there exists $S^b = \langle b, H, H*^b, v, Better'' \rangle$ such that $b \leq n$ and α is an open possibility in $H*^b$, then $S^n[\phi] = S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$ where:

    - $H*^{n+1} = H*^n$
    - for every $h_i \in H*^b$, $Better'(h_i, b+1) = Better(h_i, b+1) \backslash \{(h_k, h_l) \mid (k, b) \in V_v(\neg\alpha) \text{ and } (l, b) \in V_v(\alpha)\}$
    - for all the other $h_j \in H$ and m$\in \mathbb{N}$, $Better'(h_j, m) = Better(h_j, m)$[13]

- otherwise $S^n[\phi] = \textbf{1}$

In the previous chapter, we have argued that the counterfactual reading of statements such as *It ought to have/should have been the case that* α may require backtracking, i.e., to go back to a moment of time in which *It ought to/should be the case that* α was felicitous. That is what definition 13 establishes. Updating the current information state with **PAST Oblige** α requires to consider a (possibly) past information state in which **Oblige** α was felicitous. In particular, updating the current information state with **PAST Oblige** α corresponds to updating that past information state with **Oblige** α.[14] Finally, notice that, contrary to

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[12]See 24 in section 3.1.

[13]Notice that, given that **PAST** is a function from information states and sentences to information states, while $[\cdot]$ is a function which assigns to each sentence ϕ an operation $[\phi]$ on information states, it is not possible to define [**PAST Oblige** α] compositionally.

[14]If more than one S^b satisfies the condition in definition 13, we establish that the past information state to be updated is the one which is pragmatically salient. In this way we guarantee the operation $[\cdot]$ to be a function. I am grateful to Maria Aloni for pointing out this issue.

the updates we have considered in definitions 10 and 11, no presuppositions about the current common ground are assumed. However, the event described by α has to be a physically possible event, i.e., there must be a (possibly past) common ground according to which α was possible.

Let α be *You call the Municipality Office by 5pm*. Assume that now it is 7pm, and that the current information state is $S^n = \langle n, H, H*^n, v, Better \rangle$, where:

- $H = \{h_i, h_j, h_k\}$

- $H*^n = \{h_i, h_j\}$

- v is such that $(i, n) \in V_v(\neg\alpha)$, $(j, n) \in V_v(\neg\alpha)$

- for every $m \in \mathbb{N}$, $Better(h_i, m) = Better(h_j, m) = Better(h_k, m) = H \times H$

According to the current common ground, *You call the Municipality Office by 5pm* is false (i.e, the event of the listener calling the municipality office by 5pm does not obtain in the temporal span n, i.e., in the interval defined by the hour 7pm). Hence, updating S^n with **Oblige** α leads to the absurd state **1**. However, there has been a time, say at 3pm, in which the event of the listener calling the Municipality Office by 5pm was still an open possibility. Let S^c be the information state at 3pm. Let $S^c = \langle c, H, H*^c, v, Better'' \rangle$, where:

- $H = \{h_i, h_j, h_k\}$

- $H*^c = \{h_i, h_j, h_k\}$

- v is such that $(i, c) \in V_v(\neg\alpha)$, $(j, c) \in V_v(\neg\alpha)$ and $(k, c) \in V_v(\alpha)$

- for every $m \in \mathbb{N}$, $Better''(h_i, m) = Better''(h_j, m) = Better''(h_k, m) = H \times H$

Therefore, *You call the Municipality Office by 5pm* was an open possibility according to the agents' common knowledge at 3pm.

Now update S^n with **PAST Oblige** α. $S^n[\textbf{PAST Oblige } \alpha] = S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$ where:

- $H = \{h_i, h_j, h_k\}$

- $H*^{n+1} = \{h_i, h_j\}$

- $Better'$ is such that $Better'(h_i, c+1) = Better'(h_j, c+1) = Better'(h_k, c+1) = \{(h_i, h_j), (h_j, h_i), (h_k, h_i), (h_k, h_j)\}$[15]

- for all the other $h_j \in H$ and m$\in \mathbb{N}$, $Better'(h_j, m) = Better(h_j, m)$

Updating the current information state with the sentence *You should have called the Municipality Office by 5pm* results in an information state which has "new past preferences". In particular, the agents come to know that at 3pm h_k was more ideal than h_i and h_j. In other words, with the acceptance of **PAST Oblige** α, the agents come to have deontic preferences which they should have had before.

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[15] Again, we left the reflexive closure implicit

3.6 Complex *vs* Sequential Updates, Acceptance and Logical Consequence

In this section we discuss the notion of complex update, and define the notions of acceptance and logical consequence.

For "complex update" we mean an update which involves multiple sentences which refer to the same temporal span. Therefore, a complex update differs from a sequence of updates as defined in 10, 11 and 13.[16]

Let S^n be an information state, and let ϕ and ψ be two sentences of L_2. While $S^n[\phi][\psi]$ results in updating S^n with ϕ and then S^{n+1} with $[\psi]$, a complex update of S^n with ϕ and ψ indicates that both ϕ and ψ refer to n.[17]

Definition 14 (Complex Update)**.** Let $\alpha_1, \alpha_2, ..\alpha_x \in L_2$, and $S^n = \langle n, H, H*^n,$ $v, Better \rangle$. Take $S^n[\alpha_1] = \langle n{+}1, H, H^1*^{n+1}, v, Better^1 \rangle$, $S^n[\alpha_2] = \langle n{+}1, H, H^2*^{n+1},$ $v, Better^2 \rangle$, etc. A complex update with $\alpha_1, \alpha_2, ..\alpha_x$ on S^n (written $S^n[\alpha_1; \alpha_2;$ $...; \alpha_x]$) is defined as follows:

$S^n[\alpha_1; \alpha_2; ...; \alpha_x] = \langle n + 1, H, H*^{n+1}, v, Better' \rangle$ such that:

- $H*^{n+1} = H^1 *^n \cap H^2 *^n \cap ... \cap H^x *^n$

- for every $h_i \in H$ and m$\in \mathbb{N}$, $Better'(h_i, \text{m}) = Better^1(h_i, m) \cap Better^2(h_i, m)$ $\cap ... \cap Better^x(h_i, m)$

In the case of a complex update, the set $H*^{n+1}$ and the $Better'$-function of the new information state are obtained by intersecting those resulting from single updates of the original state. Therefore, according to the above definition 14, complex updates differ from sequential updates.

It is worth noticing that the above definition 14 does not take into account the order in which the single updates of α_i occur. However, in some cases one might want to distinguish between the complex updates $[\phi; \psi]$ and $[\psi; \phi]$ (e.g., when $\phi := \neg p$ and $\psi :=$**Oblige**p). In particular, even if ϕ and ψ refer to the same temporal span, it would be relevant to distinguish between the order in which ϕ and ψ are accepted. Unfortunately, given our definition of update function, we cannot express the above-mentioned distinction. The complex update $[\phi; \psi]$, as we have defined it, communicates only that *there is* an order in which ϕ and ψ can be processed such that both ϕ and ψ hold in the same temporal span.

The notion of complex update as defined in 14 seems to be the best we can get from our framework. Taking care of the distinction in which the simple

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[16]For instance, affirming that both ϕ and ψ are true *now* is different from saying now that ϕ is true, and then saying that ψ is true (things may have changed, and ϕ may have ceased to hold).

[17]In our framework, the notion of Complex Update is essential to model scenarios which involve contrary-to-duty obligations (CTDs). CTDs create a sort of degree of sub-ideality, in which histories that violate the primary obligation **Oblige**α but satisfy the secondary one ($IF\neg\alpha,$ **Oblige**β) are better than histories which violate both obligations. It is necessary, indeed, that the primary obligation and the secondary obligation refer to the same temporal span.

updates of ϕ and ψ happen would require a more refined setting. In particular, it would probably require to modify our definition of history (definition 1) and of valuation function (definition 3).[18] We leave the issue for future developments of the framework.

Finally, we define the notion of acceptance in a state, and the notion of logical consequence:

Definition 15 (Acceptance)**.** Let $\phi \in L_1$, and $S^n = \langle n, H, H*^n, v, Better \rangle$. Take $S^n[\phi] = \langle n+1, H, H*^{n+1}, v, Better' \rangle$. We say that ϕ is *accepted* in S^n, or that S^n *supports* ϕ (written $S^n \models \phi$), iff:

- $H*^{n+1} = H*^n$; and

- for every $h_i \in H$ and $m \in \mathbb{N}$, $Better'(h_i, m+1) \supseteq Better(h_i, m)$

In other words, we require that ϕ is *accepted* in S^n if and only if ϕ conveys *less* information than what is already assumed.[19]

Definition 16 (Logical Consequence)**.** Consider $\phi = \alpha_1, \alpha_2, ...\alpha_x$ and β, with $\alpha_1, \alpha_2, ...\alpha_x, \beta \in L_1$. We say that β is a *logical consequence* of ϕ (written $\phi \models \beta$), iff for every information state S^n: if $S^n \models \alpha_1$ and $S^n \models \alpha_2$,..., and $S^n \models \alpha_x$, then $S^n \models \beta$.

Premises in ϕ and the conclusion β should hold in the same temporal span n. Definition 16 is motivated, again, by the difference between complex update and sequential updates.[20]

3.7 Conclusions

This chapter aimed at providing a formal representation of the semantic differences between present and (the two readings of) past necessity deontic modals. We argued that the main differences rely on temporal considerations and on the common ground. We introduced a dynamic framework to deal with the prescriptive reading of obligation sentences, and we described the updates with present deontic modals, past deontic modals under the counterfactual reading and past deontic modals under the open-past reading.

A final remark on the distinction between reportative and prescriptive uses of obligation sentences. We have shown that, contrary to the traditional frameworks

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[18]One possible modification would consist in defining histories as sets of linearly ordered moments (rather than spans), and evaluate eventive sentences with respect to intervals of moments. Making explicit the moment in which the sentence is accepted, and the temporal interval in which the event takes place would permit one to move from one moment m to $m+1$ in the history while staying in the same temporal interval in which the event happens. Cf. Condoravdi (2002), p. 70.

[19]The clause for the *Better*-function is motivated by the fact that when S^n is updated with a sentence like **Oblige**ϕ, the *Better*-function at time $n+1$ is modified from scratch (the same for **PAST Oblige**ϕ and the *Better*-function at the relevant time $b+1$). See definitions 10, 11 and 13.

[20]Cf. Veltman (1996).

in deontic logic, our dynamic semantics permits us to model the prescriptive use of obligation sentences, by allowing the ideality order *Better* to be updated every time a new obligation is uttered and accepted. However, even if our semantics focuses on the prescriptive reading, it does not follow that the reportative reading is out of the picture. It is worth noticing that, thanks to the notion of support, our semantics permits us to take into account also the reportative use of obligation sentences. Recall that, according to the reportative use, an obligation sentence like *You ought to do* α describes an already existent obligation, and does not trigger any change in the ideality order *Better*. It amounts to saying that the ideality order *Better* already encodes the information that the ideal possible worlds are the ones in which you do α, i.e., that the sentence *You ought to do* α is supported in the current information state.

Chapter 4

Applying TDUS to deontic paradoxes

We conclude our paper by considering some of the most known deontic paradoxes: Forrester's Paradox, the Miners' Paradox and Chisholm's Paradox. We extend our framework with an update rule for conditional obligations, and prove that TDUS blocks the paradoxes. Moreover, we show that our considerations on present and past necessity deontic modals are relevant also in the context of deontic paradoxes.

4.1 Conditional obligations, contrary-to-duty obligations and Forrester's Paradox

Conditional obligations are sentences like *If α, then should/ought to/have to/must β*, where α indicates the condition under which the obligation that β applies. Consider, for instance, the following sentence:[1]

26. If Sara returns the library book late, she should pay a fine

According to 26, the obligation of paying a fine does not hold unconditionally. Rather, Sara has to pay a fine if it is true that she returned the library book late. The conditional operator works, therefore, as a restrictor of possibilities: the obligation in the consequent applies only to the cases in which the antecedent is true.[2] In other words, if we interpret 26 prescriptively, it establishes that the histories in which Sara returned the library book late and paid a fine are better than the ones in which she returned the library book late but did not pay a fine. This suggests that the semantics of sentences like 26 is along the

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[1] Example taken from Arregui (2010), p. 277.

[2] It follows that, in a conditional obligation, the antecedent α cannot be a prescriptive obligation itself. In principle, the listener need to *check* where the antecedent is the case in order to establish if the obligation in the consequent applies. Hence, α must be descriptive.

lines of the semantics of sentences like **Oblige** ϕ, with the restriction that the update concerns only the histories in which the antecedent holds. Therefore, we represent and interpret 26 as follows:

27. **If** (Sara returns the library book late)**, Oblige** (Sara pays a fine)

Where **If ,** is a conditional operator for obligation sentences.

Definition 17 (Language L_3). $L_3 = L_2 \cup \{\textbf{If}, \textbf{Oblige}\}$.

Given that we have interpreted **If** α**, Oblige** β in such a way that the if-clause expresses a *restriction* on **Oblige** β, we need to reconsider also the presuppositions CP and DP. In particular, we require that the proposition β embedded under the deontic operator **Oblige** is an open possibility in the *restriction* of the common ground given by α.[3]

Definition 18 (Update with conditional obligation). If $\phi = $ **If** α**, Oblige** β (where $\alpha, \beta \in L_0$), then:

- if β is an open possibility in $H*^n_{\upharpoonright \alpha} = \{h_i \mid h_i \in H*^n \text{ and } (i,n) \in V_v(\alpha)\}$, then $S^n[\phi] = S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$ where:

  - $H*^{n+1} = H*^n$
  - for every $h_i \in H*^n_{\upharpoonright \alpha}$, $Better'(h_i, n+1) = Better(h_i, n+1) \backslash \{(h_k, h_l) \mid (k,n) \in V_v(\neg\beta) \text{ and } (l,n) \in V_v(\beta)\}$
  - for all the other $h_j \in H$ and $m \in \mathbb{N}$, $Better'(h_j, m) = Better(h_j, m)$

- otherwise $S^n[\phi] = \mathbf{1}$

Notice that, given definition 18, **If** $\top$**, Oblige** β is equal to the unconditional obligation **Oblige** β. Moreover, now that we have provided the update rule for conditional obligations, we can see why sentences of the form **If** α**, Oblige** α are not valid.[4] It is also possible to see why, for every $m \in \mathbb{N}$, the relation $\preceq_m$ is not transitive.[5]

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[3]Cf. definition 9.

[4]Recall the discussion about *If John robs banks, then he ought to rob banks*. See Section 2.1.1.

[5]Counter-example to transitivity. Take the following information state $S^n = \langle n, H, H*^n, v, Better \rangle$, where:

- $H = H*^n = \{h_i, h_j, h_k\}$
- v is such that $(i,n) \in V_v(\neg p), (i,n) \in V_v(q), (j,n) \in V_v(p), (j,n) \in V_v(q)$ and $(k,n) \in V_v(\neg q)$.
- for every $m \in \mathbb{N}$, $Better(h_i, m) = Better(h_j, m) = Better(h_k, m) = H \times H$.

Now consider $S^n[\textbf{If} q, \textbf{Oblige} p]$. It gives the new state $S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$, where:

- $Better'(h_i, n+1) = Better'(h_j, n+1) = \{(h_i, h_k), (h_j, h_i), (h_j, h_k), (h_k, h_i), (h_k, h_j)\}$
- $Better'(h_k, n+1) = H \times H$.
- for every $m \neq n+1$, $Better'(h_i, m) = Better'(h_j, m) = Better'(h_k, m) = H \times H$.

Consider, for instance, $Better'(h_i, n+1)$. According to h_i: $h_i \preceq_{n+1} h_k$, $h_k \preceq_{n+1} h_j$ but $h_i \npreceq_{n+1} h_j$.

Conditional obligations are often used to describe so-called "secondary obligations" (of the form **If** α, **Oblige** β) which come into play when "primary obligations" (of the form **Oblige**$\neg\alpha$) are violated. It is customary to use the term *contrary-to-duty obligations* (CTD's) to refer to those secondary obligations.[6] It is worth noticing that CTDs do *not* establish *exceptions* to the primary obligations. In deontic logic's literature, CTDs are sometimes treated as rules of exceptions, but the two notions are different. Consider the primary obligation **Oblige**$\neg\alpha$. An exception to the primary obligation may have the form **If** β, **Oblige** α. Being an exception, it establishes that if β is the case the primary obligation is canceled.[7] On the contrary, in the case of CTD the primary obligation is still in force. **If** α, **Oblige** β establishes a new duty in the case in which the primary obligation is violated, but that does not prevent the primary obligation **Oblige**$\neg\alpha$ from holding unconditionally.[8]

A typical CTD-scenario is Forrester's Paradox of the gentle murder. Imagine that a moral authority utters:

[1] Smith should not murder Jones

[2] If Smith murders Jones, then Smith should murder Jones gently

The paradox emerges in Standard Deontic Logic (SDL) because [1]+[2] cannot be expressed consistently in that framework. However, the problem does not arise in TDUS.

We formalize [1]+[2] as:

[3] **Oblige** $\neg m$

[4] **If** m, **Oblige** g

where m is *Smith murders Jones*, and g is *Smith murders Jones gently*.

Consider the information state $S^n = \langle n, H, H*^n, v, Better \rangle$, where:

- $H = H*^n = \{h_i, h_j, h_k\}$

- v is such that $(i, n) \in V_v(\neg m), (j, n) \in V_v(m), (j, n) \in V_v(g), (k, n) \in V_v(m)$ and $(k, n) \in V_v(\neg g)$.

- for every $m \in \mathbb{N}$, $Better(h_i, m) = Better(h_j, m) = Better(h_k, m) = H \times H$.

Now let us update S^n with [1]+[2]. That amounts to the complex update $S^n[\textbf{Oblige } \neg m; \textbf{If } m, \textbf{Oblige } g]$.

First consider $S^n[\textbf{Oblige } \neg m] = \langle n + 1, H, H^1*^{n+1}, v, Better^1 \rangle$, where:

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[6] See, for instance Prakken and Sergot (1997).

[7] In the present thesis, we will not consider conditional obligations as exceptions. Modeling conditional obligations as exceptions would probably require the introduction of a normality order among histories, and that goes beyond the purposes of the present work. A formalization of exceptions by means of a normality order among possible worlds has been proposed by van der Torre and Tan. In their perspective, primary obligations indicate what counts as normal, while worlds in which **If** β, **Oblige** α holds are the abnormal ones. See van der Torre and Tan (1999 b).

[8] See van der Torre and Tan (1997).

- $H^1*^{n+1} = H*^n$

- $Better^1(h_i, n+1) = Better^1(h_j, n+1) = Better^1(h_k, n+1) = \{(h_i, h_j), (h_i, h_k), (h_j, h_k), (h_k, h_j)\}$

- for every $m \neq n+1$, $Better^1(h_i, m) = Better^1(h_j, m) = Better^1(h_k, m) = H \times H$.

Now take $S^n[$ **If** m, **Oblige** $g] = \langle n+1, H, H^2*^n, v, Better^2 \rangle$, where:

- $H^2*^{n+1} = H*^n$

- $Better^2(h_i, n+1) = H \times H$

- $Better^2(h_j, n+1) = Better^2(h_k, n+1) = \{(h_i, h_j), (h_i, h_k), (h_j, h_i), (h_j, h_k), (h_k, h_i)\}$

- for every $m \neq n+1$, $Better^2(h_i, m) = Better^2(h_j, m) = Better^2(h_k, m) = H \times H$.

Finally, $S^n[$**Oblige** $\neg m$; **If** m, **Oblige** $g] = S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$, where:

- $H*^{n+1} = H*^n$

- $Better'(h_i, n+1) = \{(h_i, h_j), (h_i, h_k), (h_j, h_k), (h_k, h_j)\}$

- $Better'(h_j, n+1) = Better'(h_k, n+1) = \{(h_i, h_j), (h_i, h_k), (h_j, h_k)\}$

- for every $m \neq n+1$, $Better'(h_i, m) = Better'(h_j, m) = Better'(h_k, m) = H \times H$.

Since updating S^n with [1]+[2] does not result in the absurd state, no paradox arises. Moreover, the state S^{n+1} makes sense of the difference between the primary obligation and the CTD. Not only it is the case that the history h_i, in which Smith does not murder Jones, is better than h_j and h_k; but also the history h_j, in which Smith murders Jones gently, is better than h_k, in which Smith murders Jones and he does not do it gently.

4.2   The Miners' Paradox

Another paradox which is intensively studied in deontic logic and recent metaethics is the Miners' Paradox,[9] an example that runs as follows:

> Ten miners are trapped either in shaft A or in shaft B, but we do not know which one. Water threatens to flood the shafts. We only have enough sandbags to block one shaft but not both. If one shaft is blocked, all of the water will go into the other shaft, killing every miner inside. If we block neither shaft, both will be partially flooded, killing one miner.[10]

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[9]See for instance Kolodny and MacFarlane (2010), Willer (2012).
[10]See Kolodny and MacFarlane (2010), page 1.

The reason this paradox is so prominent is that it incorporates other familiar paradoxes generated by SDL: paradoxes involving conditional obligations, disjunction of obligations, and conflict of obligations. Why is the Miners' Paradox a paradox? Lacking any information about the miners' position, it seems right to that the outcome should be:

[5] We ought to block neither shaft.

However, in deliberating about what to do, we accept:

[6] If the miners are in shaft A, we ought to block shaft A.

[7] If the miners are in shaft B, we ought to block shaft B.

We also accept:

[8] Either the miners are in shaft A or they are in shaft B.

But [6]-[8] seem to entail:

[9] Either we ought to block shaft A or we ought to block shaft B.

And this is incompatible with [5]. Thus we have a paradox.

In order to block the Miners' Paradox, several escape routes have been proposed.[11] In particular, it has been suggested that two different *oughts* are involved in the paradox: a *subjective ought* and an *objective ought*.

Intuitively, subjectivism and objectivism differ with respect to the body of information in light of which the deontic modal is evaluated. Under the objectivist reading, a sentence like *X ought to do α* indicates that α is the best option available to the agent X in light of all facts, known or unknown; while, under the subjectivist reading, *X ought to do α* indicates that α is the best option available to the agent X in light of what X knows. If one adopts a purely objectivist reading of the deontic *ought*, the paradox does not arise, since the very premise [5] is rejected. Under the objectivist perspective, [5] does not hold: in light of all facts, the best option is to block one shaft, the one the ten miners are in. On the other hand, if one adopts a purely subjectivist reading of *ought*, premises [6] and [7] do not seem to be acceptable. For instance, take [6]: *If the miners are in shaft A, we ought to block shaft A*. While the antecedent refers to a fact that might be unknown to the agent *X*, the consequent establishes a moral obligation in light of the knowledge of the agent *X*. Thus the conditional is not acceptable, since it may be the case that the antecedent is true and the consequent is false.

Both objectivism and subjectivism have, however, some difficulties. As Kolodny and MacFarlane (2010) also point out, objectivism seems too strong, since it does not allow deontic reasoning for partially-informed agents. If the objectivist view had to be adopted, only an omniscient agent with a complete knowledge of all facts would be able to determine whether *X ought to do α* is

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[11]See Kolodny and MacFarlane (2010), pp. 117-129.

true. On the other hand, subjectivism seems to be too weak, since it does not validate conditional obligations like *If α, then X ought to do β*.[12] That appears to be a remarkable loss, since those conditional constructions are very common in ordinary deontic reasoning, in particular in deliberation. Subjectivism is, therefore, too weak if it cannot deal with those common constructions.

However, that does not imply that some of the claims made by objectivism and subjectivism are not worthy to be considered. In what follows we show that, in the case of the Miners' Paradox, it is possible to provide a solution which takes into account some aspects of the objectivism's and subjectivism's views, but uses one single deontic operator that remains neutral between the two positions.

The Miners' Paradox is about agents who consider what they ought to do in a context of partial factual information. Again, we can understand the paradox's scenario in terms of a conversational exchange between a moral authority (the speaker) and a listener. Depending on what the moral authority commands, the listener may decide to accept or reject certain obligation sentences. In particular, we can see that even if the listener has accepted all the premises of the Miners' Paradox, the paradoxical conclusion [9] is not supported in the updated information state.

We formalize sentences [6]-[8] as follows:

[10] **If** a, **Oblige** $block_a$

[11] **If** b, **Oblige** $block_b$

[12] $a \vee b$

where a is *The miners are in shaft A*, b is *The miners are in shaft B*, $block_a$ is *We block shaft A* and $block_b$ is *We block shaft B*.

Assume that the moral authority has already uttered *If the miners are in shaft A, we ought to block shaft A* and *If the miners are in shaft B, we ought to block shaft B*, and the listener has already accepted those obligations. Consider the information state $S^n = \langle n, H, H*^n, v, Better \rangle$ such that:

- $H = H*^n = \{h_i, h_j, h_k, h_l\}$

- v is such that:

    - $(i, n) \in V_v(a), (i, n) \in V_v(block_a), (i, n) \in V_v(\neg b), (i, n) \in V_v(\neg block_b)$
    - $(j, n) \in V_v(b), (j, n) \in V_v(block_b), (j, n) \in V_v(\neg a), (j, n) \in V_v(\neg block_a)$
    - $(k, n) \in V_v(a), (k, n) \in V_v(\neg b), (k, n) \in V_v(\neg block_a), (k, n) \in V_v(\neg block_b)$
    - $(l, n) \in V_v(b), (l, n) \in V_v(\neg a), (l, n) \in V_v(\neg block_a), (l, n) \in V_v(\neg block_b)$

- $Better(h_i, n) = Better(h_k, n) = \{(h_i, h_k), (h_i, h_j), (h_i, h_l), (h_k, h_l), (h_j, h_k), (h_j, h_l), (h_l, h_k)\}$

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[12]Subjectivism may validate only weaker conditionals such as *If X knows that α, then X ought to do β*. See Kolodny and MacFarlane (2010) , page 118.

- $Better(h_j, n) = Better(h_l, n) = \{(h_i, h_k), (h_i, h_l), (h_k, h_l), (h_j, h_i), (h_j, h_k),$
  $(h_j, h_l), (h_l, h_k)\}$

- for every $m \neq n$, $Better(h_i, m) = Better(h_j, m) = Better(h_k, m) = Better(h_l, m) = H \times H$.

The common ground consists of four histories: h_i in which the miners are in shaft A and we block shaft A (hence no miner dies), h_j in which the miners are in shaft B and we block shaft B (hence no miner dies), and h_k and h_l where one miner dies. From the point of view of h_i and h_k at n, h_i represents the best course of events; while from the point of view of h_j and h_l at n, h_j is the best one. Therefore, we get that:

- $S^n \models a \vee b$

- $S^n \models$ **If** a, **Oblige** $block_a$

- $S^n \models$ **If** b, **Oblige** $block_b$

In the state S^n, all the premises of the paradox are accepted. However, we get that:

- $S^n \not\models$ **Oblige** $block_a$

- $S^n \not\models$ **Oblige** $block_b$

To see that, it is sufficient to consider the result of updating S^n with **Oblige** $block_a$ and **Oblige** $block_b$. Consider the case of $S^n[\textbf{Oblige } block_a]$.[13] $S^n[\textbf{Oblige } block_a] = S^{n+1} = \langle n+1, H, H*^{n+1}, v, Better' \rangle$ such that:

- $Better'(h_i, n+1) = \{(h_i, h_k), (h_i, h_j), (h_i, h_l), (h_k, h_j), (h_k, h_l), (h_j, h_k),$
  $(h_j, h_l), (h_l, h_k), (h_l, h_j)\}$

- $Better'(h_j, n+1) = \{(h_i, h_k), (h_i, h_j), (h_i, h_l), (h_k, h_j), (h_k, h_l), (h_j, h_k),$
  $(h_j, h_l), (h_l, h_k), (h_l, h_j)\}$

- $Better'(h_k, n+1) = \{(h_i, h_k), (h_i, h_j), (h_i, h_l), (h_k, h_j), (h_k, h_l), (h_j, h_k),$
  $(h_j, h_l), (h_l, h_k), (h_l, h_j)\}$

- $Better'(h_l, n+1) = \{(h_i, h_k), (h_i, h_j), (h_i, h_l), (h_k, h_j), (h_k, h_l), (h_j, h_k),$
  $(h_j, h_l), (h_l, h_k), (h_l, h_j)\}$

Since $Better'(h_j, n+1) \not\supseteq Better(h_j, n)$ and $Better'(h_l, n+1) \not\supseteq Better(h_l, n)$, it follows that $S^n \not\models$ **Oblige** $block_a$.

So S^n does not support **Oblige** $block_a$ nor **Oblige** $block_b$. That is, neither we ought to block shaft A nor we ought to block shaft B. Even if all the premises of the Miners' Paradox hold in the information state S^n, the paradoxical conclusion [9] **Oblige** $block_a \vee$ **Oblige** $block_b$ does not hold. Therefore no paradox arises.

Notice that, however, we get that:

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[13]The case of $S^n[\textbf{Oblige } block_b]$ is analogous.

[13] $S^n \models$**Oblige** $(block_a \lor block_b)$

This is an intuitive result. It means that, *in absolute terms*, the listener knows that we should block one shaft (as no miner killed is better than one miner killed), but given her *partial information* about the world, she cannot conclude which shaft has to be blocked. She rejects, indeed, the obligation of blocking shaft A and the obligation of blocking shaft B. In fact, in our semantics the contrast between what we ought to do in absolute terms and what we ought to do relative to our partial information can be represented in terms of a scope distinction between one single deontic operator and the disjunction. We can, therefore, account for the distinction to which subjectivism and objectivism appeal, without accepting a proliferation of different deontic operators.

4.3 Chisholm's Paradox

We conclude the chapter by discussing Chisholm's Paradox. As Forrester's Paradox, also Chisholm's concerns CTDs and cannot be formalized consistently in SDL. However, contrary to Forrester's, it also involves some factual information:

[14] Smith ought to refrain from robbing Jones

[15] Smith robs Jones

[16] If Smith robs Jones, he ought to be punished for robbery

[17] If Smith refrains from robbing Jones, he ought not be punished for robbery

Let r be *Smith robs Jones*, and p be *Smith is punished for robbery*. We propose the following formalization of the Chisholm's set [14]-[17]:

[18] **Oblige**$\neg r$

[19] r

[20] **If** r, **Oblige** p

[21] **If** $\neg r$, **Oblige** $\neg p$

Consider the state $S^n = \langle n, H, H*^n, v, Better \rangle$, where:

- $H = H*^n = \{h_a, h_b, h_c, h_d\}$

- v is such that $(b, n) \in V_v(p), (c, n) \in V_v(p), (c, n) \in V_v(r)$ and $(d, n) \in V_v(r)$

- for every $h_i \in H$ and m$\in \mathbb{N}$, $Better(h_i, m) = H \times H$

Now update S^n with [18]-[21]. The state $S^n[$**Oblige**$\neg r$; **If** r, **Oblige** p; **If** $\neg r$, **Oblige** $\neg p$; $r] = \langle n+1, H, H*^{n+1}, v, Better' \rangle$, where:

- $H*^{n+1} = \{h_c, h_d\}$

- $Better'(h_c, n+1) = Better'(h_d, n+1) = \{(h_c, h_d)\}$[14]

- for every $h_i \in H$ and $m \neq n+1$, $Better'(h_i, m) = H \times H$.

In presenting sentences [14]-[17], Chisholm argued that an appropriate solution to his paradox should satisfy three constraints: (i) it formalizes [14]-[17] consistently, (ii) it makes sense of the distinction between primary and secondary obligations, and (iii) it formalizes [14]-[17] in such a way that they are all logically independent from each other. The solution we propose satisfies all the three constraints.

We have just shown that it is possible to successfully update an information state with the Chisholm's set. Hence, constraint (i) is met. Moreover, we have formalized the distinction between primary obligations and secondary obligations, as required in (ii). In the resulting state, in which the primary obligation *Smith ought to refrain from robbing Jones* is violated, the history h_c in which Smith is punished is better than h_d in which he is not punished. Finally, all sentences are logically independent from each others, as required in (iii). In particular, we get that $r \not\models \mathbf{Oblige}\neg r$ and $r \not\models \mathbf{If} \neg r, \mathbf{Oblige} \neg p$. For every S^m, if r holds in the common ground at m, then $\mathbf{Oblige}\neg r$ and $\mathbf{If} \neg r, \mathbf{Oblige} \neg p$ violate the open possibility condition. Therefore updating S^m with $\mathbf{Oblige}\neg r$ and $\mathbf{If} \neg r,$ $\mathbf{Oblige} \neg p$ results in the absurd state. As we have argued in Chapter 3, if it is known that Smith has robbed John, the utterance of *Smith ought to refrain from robbing Jones* and *If Smith refrains from robbing Jones, then he ought not be punished for robbery* are infelicitous.[15]

In discussing sentences [14]-[17], Chisholm did not take into account that the presuppositions that a sentence like $\mathbf{Oblige}\neg r$ carries are incompatible with the previous acceptance of r.[16] There exists, therefore, an order in which [14]-[17] are processed such that they all hold in the same temporal span. Our notion of complex update should be modified to take care of that order. One of the first future developments of TDUS will go in that direction.

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[14] In $Better'$ we write only the histories which belong to $H*^{n+1}$. We leave the others implicit.

[15] While the use of past deontic modals as in *Smith ought to have refrained from robbing Jones* and *If Smith had refrained from robbing Jones, then he ought not have been punished for robbery* is felicitous.

[16] On that point, see also Arregui (2010).

Chapter 5

Conclusion

The primary aim of this thesis was to explore the difference between present necessity deontic modals, such as *should*, *ought to*, *must*, and past necessity deontic modals, such as *should have*, *ought to have*, *had to*. Contrary to the traditional approaches in deontic logic, we concentrated on the prescriptive use of such modals. The reason for giving the priority to the prescriptive use came, indeed, from everyday discourse practice, in which obligation sentences are primarily used to bring about new obligations and duties, rather than to describe the already existing ones.

In analyzing the different behavior of present and past necessity deontic modals in everyday discourse practice, we focused on the contrast that arises when the proposition embedded under those modals is eventive and non-progressive. We argued that a purely temporal analysis is not sufficient to take care of that contrast, and that also the common ground of the participants in the conversation should be taken into account.

Felicitous utterances of present deontic modals require, indeed, that the participants in the conversation consider the embedded proposition to be an open possibility at the utterance time. We noticed that such presuppositions not only are reflected in ordinary language practice, but are also connected to classical topics in ethics and meta-ethics, such as the *Ought implies Can* Principle and the notion of moral responsibility.

Past necessity deontic modals, on the other hand, do not always obey to the same presuppositions. In particular, we distinguished between two readings of past deontic modals: the counterfactual reading and the open-past reading. It is when past deontic modals are interpreted under the counterfactual reading that those presuppositions do not hold.

We moved, then, from a theoretical analysis to a formalization of the different semantic contribution of present and past necessity deontics modals. We developed a framework from the dynamic logic tradition, especially Veltman's (1996) work on update semantics. Dynamic logic provides, indeed, a natural setting for the formalization of the prescriptive use of deontic modals.

The advantage of the framework we proposed is that it permits us to take

into account the two factors that are determinant in the interpretation of present and past deontic modals, i.e., time and common ground. The structure of our information states allows us to express when the embedded proposition is evaluated, and whether it belongs to the current common ground or to a past one. We argued that even if those factors may vary, the semantic contribution of the deontic operator is constant. We were indeed able to model present deontic modals and the two readings of past deontic modals by means of a single deontic operator: **Oblige**.

We concluded our paper by taking into account some more classical topics in deontic logic literature, such as conditional obligations and deontic paradoxes. We considered Forrester's Paradox, the Miners' Paradox and Chisholm's Paradox, and showed that our framework blocks those paradoxes. Finally, even if those paradoxes do not concern directly the difference between present and past deontic modals, we observed that that difference is still relevant, especially in the case of Chisholm's Paradox.

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