Towards Closed-World Reasoning in Games – Ultimatum Game Revisited

MSc Thesis (*Afstudeerscriptie*)

written by

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Abstract

The Ultimatum Game (UG) is one of the widely studied games in experimental economics. Past data shows a consistent deviation from the classical game theory prediction, which suggests a self-interested money maximizing rational agent would accept any nonzero offer as a responder. However, in reality, people often reject low offers that are less than 30% and above zero. Research from neuro-economics claims that such behavior is mostly emotion driven [40] [48]. However an important cross-cultural study [22] shows that the results in these small-scaled societies deviate drastically from the industrialized societies. In addition, there is a high correlation between the deviations and the social systems. Economists call for a better understanding of how people apply social rules when they make economic decision [17] [16].

We believe that we need a framework that can show the following: 1. Preferences are constructed on demand. 2. Social rules are numerous and complex, and tend to trigger non-monotonic reasoning.

Formally, we developed the *Closed-World Reasoning in Games* framework (CWRiG) adopted from Closed-World Reasoning framework (CWR) by van Lambalgen and Stenning [47]. We defined what can be considered a *monotone game process*. More importantly we show how an agent can make a rational decision change through a non-monotonic reasoning process that brings social rules into the game process.

Experimentally, we ran a modified ultimatum game. The results confirms that people's general, even ranked, attitudes towards some social matters are not necessarily equivalent to their preferences of the same kind of matter during the game. We also discover many instances of non-monotonic reasoning patterns used during the game.

Contents

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Chapter 1

Introduction

In this thesis, we want to investigate if we can apply the framework of closed-world reasoning, which accommodate non-monotonic reasoning, to a game process so that we can understand people's reasoning in games better. More specifically we hope to explain the seemingly irrational behaviors of some responders in the ultimatum game through a different lens.

1.1 Ultimatum Game (UG) and 'Irrationality'

The ultimatum game is one of the most widely experimented games since 1982 [21]. The structure of the game is rather simple. Two players have to decide how to share an initial amount of money, say $10, in two steps. First, the proposer has to decide a division for both players. Second, the responder has to decide whether he or she accepts the offer. If accepted, both players get the portions as the proposer suggests. If rejected, both players get nothing in return. Game theoretical prediction is that the proposer suggests the least amount possible knowing that the responder as a self-interested utility maximizer would accept any offer above zero. However, experimental results have shown that this prediction rarely happens. In western countries, most people offer between 40% to 50%, and offers lower then 30% are often rejected [7]. In smaller and less industrialized societies, there are more varieties in both the offering amount and the level of rejections [22]. Researchers have been debating whether these observed behaviors are rational.

A claim from neuroeconomics, is that the 'irrational' behavior is emotion driven [40] [48] [1]. Another claim is that the behavior is socially driven, and systematic [22]. Therefore, researchers are calling for better economic models that can account for social preference and rules [17] [16].

[1] They use functional magnetic resonance imagine of Ultimatum Game players during the game

1.2 Social Preference and Rationality

In [22], Bowles et al. argue that the concept of *Homo economicus* is outdated. The term was originated [35] in the work of the prominent 19th-century scholar, John Stuart Mill in 1836 [31], "...does not treat the whole of man's nature as modified by the social state, nor of the whole conduct of man in society...". But before that, the founding father of modern economics, Adam Smith, had stated the idea in his famous work 'The Wealth of Nations [44]' in 18th century. He wrote: "It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest". In 1881, Edgeworth even endorsed the idea as "the first principle of Economics" [14].

In 1977, Sen points out the absurdity of such idea in his widely cited paper, "Rational Fools: A Critique of the Behavioral Foundations of Economic Theory" [42]. He gives an amusing example to illustrate the problem of *Homo Economicus* assumption:

"Where is the railway station?" he asks me. "There," I say, pointing at the post office, "and would you please post this letter for me on the way?" "Yes," he says, determined to open the envelope and check whether it contains something valuable.

While Sen agrees with Edgeworth's first principle in a strictly defined context, he also points out that we cannot forgo the complex psychological issues beneath choices and calls for "actual *testing*", which was indeed done by experimental economists years later. Through the flourishing research of behavioral and experimental economics, researchers are able to bring a vast amount of data to the table to show that in many situations people do not just act on self-interest [37] [18]. It lead to the seminal paper of Bowels et al. "In search of homo economicus: behavioral experiments in 15 small-scale societies" [22]. Their results not only show the same kind of deviation from the predictions based on homo economicus assumption, but also show that economic decisions are closely related to the social/group activities. Economists started the journey of defining social preferences [17] [16] and trying to figure out how social rules work when people make decisions.

We have two observations from the literature. First, social concerns tend to be bound with emotion and irrationality. In fact, Nowak, Page and Sigmund have a paper called "Fairness versus Reasoning in Ultimatum Game" [32]. Do social rules have to be emotion driven? Is it possible that we have some kind of social rationality while emotion is the byproduct or the facilitator? Second observation is on the methodology. Economists tend to look at the data from top down at an aggregated level.

We believe that social rules are reasoned as well. Our goal is to study from a bottom up fashion at an individual level. Only then will we know how to parse the data properly at the top, especially given social matters are more complex and demanding.

1.3 Closed-world Reasoning Methodology

The closed-world reasoning framework from van Lambalgen and Stenning [47] brings logic back to actual human reasoning in a refreshing way. The flexible framework logically explains some experimental results that were thought to support the claim that people do not reason logically. In addition, it also brings logic system into the understanding of the rapid and automatic decision making process in system 1.

The two experiments that are elaborately discussed in their book are Wason selection task [50] and the suppression task [6]. In the selection task, subjects have to pick as many as cards needed from a set of four cards, all with a letter on one side and a number on the other, to verify a rule that states "If there is a vowel on the one side, then there is an even number on the other side." In the suppression task, subjects are given a conditional argument such as "If she meets her friend then she will go to a play". Then, they are being tested with their ability to make modus ponens or modus tollens inferences, and a few other inference patterns.

These two studies have driven psychology of reasoning away from logic in different ways. In the selection task, only about 4% choose the right cards according to classical logic, which is considered by Wason the good reasoning and the right interpretation. In the study of the suppression task, people also do not follow the classical rules. Byrne concludes that people suppress the use of formal rules, and argues against the role of logical form in reasoning [47].

Van Lambalgen and Stenning re-explain the data through closed-world reasoning framework, which gives three advantages over the past work. First advantage is to separate the reasoning from and reasoning towards an interpretation. When task is given to subjects, there are different ways of interpreting the task with different logics. After that step, subjects will start to reason from the interpretation they have chosen. Methodologically, van Lambalgen and Stenning and others who adopt this framework use tutorial dialogues that allow the experimenters to gather subjects' interpretation. Second one is that the closed-world reasoning framework work well with non-monotonic reasoning, which happens often in real life situations. When we plan a task, we probably just use the information at hand, and assume what we do not know false. It is this closed-world assumption which is built in their framework. Third advantage is that the system can be applied in situations where reasoning needs to be fast and automatic, such as making an economic decision.

1.4 Towards Closed-world Reasoning in Games

We believe that using this improved understanding of rationality from van Lambalgen and Stenning can give us new insights to the ultimatum game. However, it is a cross-field attempt, so we are making a limited extension which allows us to understand only the reasoning of a responder. We do think it is possible to further extend it to the reasoning of a proposer, and hopefully other games.

In this thesis, we apply closed-world reasoning methodology in two ways: formally and experimentally. We modify the framework into a suitable one for games. It enables non-monotonic reasoning in game reasoning. We have discussed briefly that social rules are numerous and complex. Therefore, we believe people do not apply all social rules at the same time. They use related ones as new information present itself during a game. Experimentally we make two modifications. The first is to survey subjects' general attitude towards some related social situations. The second is that we ask subjects to explain their response in the ultimatum game. Combining these two changes, we are able to have some ideas about whether subjects construct their preference on demand, their interpretation about the task, and some of their reasoning process. The experimental process can certainly be improved. But it has shown some interesting reasoning patterns that were not discovered in the past experiments in the ultimatum game.

We hope to use this result as a starting point for further investigation on the non-monotonic reasoning in social-economic situations. We also want to call for using the kind of simple yet explanatory surveys more often in game theory experiments. In this way, we will be able to have more data on different reasoning patterns in decision making processes that involve social rules. That will further help to build more accurate formal models.

We have structured this thesis in the following way. In chapter 2, we will review the closed-world reasoning framework that is applied in the psychology of reasoning. Then, in chapter 3 we will review a definition of rationality used in economics. In chapter 4, we will review both the theory and the experimental results behind the ultimatum game. Chapter 5 explains a formal framework of closed-world reasoning in games. We will explain the experiment we conducted in chapter 6 and review the results. We will conclude in chapter 7.

Chapter 2

Closed-World Reasoning

The center debate of this thesis is whether the rejection of any positive offer is irrational in the ultimatum game. It is crucial, then, to understand rationality, especially from its original field cognitive psychology rather than economics, which we will also review in a later chapter. We shall clearly define what is rational and review some related historical studies in psychology of reasoning. More importantly we will review the recent development in understanding rationality: the logic of closed-world reasoning [47].

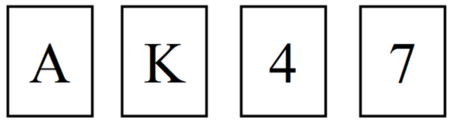
2.1 Rationality in Psychology of Reasoning

Rationality has been an interest of study since the dawn of human history, however, we are far from settling the proper definition of it. When using the current method to look for a definition: searching on Google, the most definitions you get associate the word 'rationality' or 'rational' with 'reasons'. There are two entries under 'rational' on the page of Oxford dictionary. One is 'mathematics (of a number, quantity, or expression) expressible', and the other is 'based on or in accordance with reason or logic'. Sadly, logic and reasoning did not seem to be compatible in psychology of reasoning. Some researchers concluded that people do not generally use logic. We will review two renown studies: Wason Selection Task and Suppression Task, to understand from *their* perspectives why logic is not being used.

2.1.1 Wason Selection Task

Wason published the studies in the late 60's [50] [51], and it was called "mother of all reasoning tasks" [47]. Wason claims, in [50], that this particular task is designed to understand people's reasoning ability about conditional sentences. More specifically, the material implication in the form of "if P then Q". He also conclude that "the subjects did not give evidence of having acquired the characteristics of Piaget's 'formal operational thought' ", which includes the ability

Figure 2.1: 4 cards in the task



to consider the logical relations between given statements only and ignoring the concrete content [10].

Before we get into the detailed results, we will first look at the experiment design itself. Following is the instruction of Wason selection task taken from [47]:

Below is depicted a set of four cards, shown in section 2.1.1 , of which you can see only the exposed face but not the hidden back. On each card, there is a number on one of its sides and a letter on the other.

Also below there is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you must turn in order to decide if the rule is true. Don't turn unnecessary cards. Tick the cards you want to turn.

Rule: If there is a vowel on one side, then there is an even number on the other side.

According to Wason, the key point in this task is that it is not easy for people to reason conditionals in the form of material implications. "If there is a vowel on one side, then there is an even number on the other side." has the form of $p \to q$ with $p =$ "there is a vowel on one side" and $q =$ "there is an even number on the other side", roughly speaking[1]. In this case, the *correct* cards should be A (p) and 7 ($\neg q$). However, the common result [53] of this task is that only about 4% people select p and $\neg q$. 33% people choose A (p) alone, 46% people choose A and 4 (p, q). 7% people choose A, 4 and 7 (p, q, $\neg q$) [2].

Wason concludes that his subjects cannot achieve the formal operations as how Piaget defines. Although, he does say it is possible that the choice of words can be part of the difficulty as well [50]. This has also become the beginning of another line of research that suggests formal logic is not suitable for such tasks [25] [20] [52]. One of the most notable studies is the drinking-age study from Griggs and Cox [20]. The study is basically a version of the selection task with less abstract content. Instead of letters and numbers, the rules are associated with what a person is drinking and the age of the person. It turned out that this version is much "easier" for people to get the "right" answer. Different versions of such less abstract content include postal regulation [25] and production-inspection [52]. All the results show that people can perform better when the context is more familiar. That lead these researchers to believe that formal logic is not suitable for reasoning study.

We do not agree with that, and we will explain why logic does have a role

[1] We will see why in later analysis that the different kinds of more precise interpretation could be the source of 'mistakes'.

[2] Even though, these numbers are from Psychology of Reasoning by Wason and Johnson-Laird in 1972. The recent numbers are comparable, see [47]

in reasoning in the next section. Before that, we will review some results in the suppression task, which has some similarities with the ultimatum game.

2.1.2 Suppression Task

The suppression task from Byrne in 1988 [6] is another important reasoning study. The task is a study about how people sometimes reject *modus ponens* (MP) and *modus tollens* (MT). Byrne argues that people suppress their logical reasoning in certain contexts, even when sentence preserves the basic forms of MP and MT.

Before we explain the task, we will first look at these two rules of inference. In propositional logic, when we have $p \to q$ and p, we can infer q. This is form of argument is called *modus ponens*. Similarly, when we have $p \to q$ and $\neg q$, we can infer $\neg p$. This is *modus tollens*. In natural languages, we use conditionals in the form of "if ... then..." quite often. We will look at one example for each, taken from Byrne's paper in 1988.

modus ponens Statements:
"If she has an essay to write she will study late in the library."[3]
"She has an essay to write."
Conclusion according to MP [6]:
"She will study late in the library."

modus tollens Statements:
"If she has an essay to write, she will study late in the library."
"She will not study late in the library."
Conclusion according to MT:
"She does not have an essay to write."

When given MP form of argument, about 90% [4] of subjects agree with the conclusion stated above. However, only 69% subjects agree with the conclusion in the case of MT shown above. Besides MP and MT, there are also two other forms of arguments that are also studied in suppression task: *affirmation of the consequent* (AC) and *denial of the antecedent* (DA). In AC form presented as above for MT and MP, it combines a statement with its consequent, e.g. Statement 1 and "She will study late in the library". 53% of subjects conclude that "she has an essay to write". An example of DA is Statement 1 and "She hasn't an essay to write.". About 49% conclude that she will not study in the library.

In additional to these four basic forms of argument, an additional premise or an alternative one can be added to all four. Let us see two such examples in MP.

[3] We will refer this as Statement 1 since we will use it multiple times in combination with other statements.

[4] We are using statistics from [13], which has more statistical power than the original study [47].

Alternative :

Statement 1: "If she has an essay to write, she will study late in the library."

Alternative statement: "If she has a textbook to read, she will study late in the library."

Conclusion: "She will study late in the library."

Additional :

Statement 1: "If she has an essay to write, she will study late in the library."

Additional statement: "If she has a textbook to read, she will study late in the library."

Conclusion: "She will study late in the library."

In these two examples, 94% agree with the conclusion with the alternative statement, while only 60% agree with the conclusion with the additional statement.

So how does Byrne explain such experimental results? She categorizes MP and MT inference forms as valid arguments, and AC and DA are fallacies. She raises the fact that ' a major class of theories suggests that reasoning depends on formal rules of inferences that operate in a syntactic way on the abstract logical form of the premises'[5] [6]. Through her experiment, she concludes that people often suppress formal rules. She proposes to improve it by including the interpretation of the input. She believes that the problem is not understanding the logic forms of these sentences. The problems are due to the understanding of our language and the general knowledge. She gives an example in DA, claims that Statement 1 invites its obverse: "If she does not have an essay to write then will not study late in the library.", and concludes that the error does not come from the corresponding rule. Further more, she claims that speakers are *suppose* to give hearer as much information as they need for the current purpose of the exchange.

The reasoning in conditionals is quite relevant to reasoning in games, especially in the games with social perspectives. As Bicchieri emphasizes in her book, The Grammar of Society - The Nature and Dynamics of Social Norms [4], social norms are default rules and we use them in the form of conditionals. In chapter 4, we will explain the ultimatum game and draw the comparison between the game and this suppression task.

2.2 Rationality with Modern Logic

We reviewed two famous reasoning tasks, the Wason selection task and the suppression task. Both reject the logical reasoning to certain degrees through experiments, although from two different angles. Wason seems to suggest that in certain cases, people cannot interpret the correct logical rules. However,

[5] She includes theories till the writing of her paper, which is before 1987.

Byrne believes that people can get the rules right and the problem is with the language and related general knowledge.

Now we will state our own stand on the issue of logical reasoning in real life. We agree with van Lambalgen and Stenning [47] that the understanding of logic in these studies is narrow focused. To be more specifically, they mostly apply logical rules from classical logic. We will argue that using more versatile modern logics will help understand the subjects' reasoning better in both tasks.

In this section, we will first discuss the idea of applying modern logic informally and raise some key concerns in the past studies, and thereby introducing closed-world reasoning. Then, we will discuss some necessary consideration on the formal side. At the end we will review some of the new results in suppression task by applying closed-world reasoning analysis, which is more related to this thesis.

2.2.1 Informally

Before we dive into the discussion about applying modern logic to understand the reasoning in the selection task and suppression task, we shall come back to the definition of rationality first. We have briefly discussed the connection between rationality and logic. Now we will look at the definition of a "rational agency" from the MIT Encyclopedia of Cognitive Science:

"The agent must have a means-end competence to fit its actions or decisions, according to its beliefs or knowledge representations, to its desires or goal-structure."

As Stenning and van Lambalgen point out in [47], the definition has two crucial messages for rationality: logic and planning. The combination of the two imply a more *dynamic* process of reasoning than previously suggested. When facing a goal (or desire), the agent must craft a logical[6] plan and execute it.

In consideration with this adjusted definition of rationality, there are two major problems of analysis in these two studies. The first problem is that reasoning behind them is to assume subjects reading off a fixed set of logical rules, or models. Reasoning is actually a more constructive process, in the sense that subjects have to set a goal, then choose a tool (from different formal systems), and use the tool with the information understood. The latter two steps are also called "reasoning *to* an interpretation" and "reasoning *from* an interpretation" in the book from Stenning and van Lambalgen [47]. Reasoning from an interpretation is actually what Wason and Byrne have done in their studies.

When an agent is reasoning *to* an interpretation, he is defining the domain from which he would reason, and related formal properties. In order to justify the domain, the agent may ask[7] questions such as "what kind of items and concepts I am dealing with?", "how many items will I include in this reasoning process?". He will get different answers in a credulous stance or in a skeptical stance. Below are two examples from [47]:

[6] In [47], authors explain that the term *fit* implies a logical component.

[7] Even though we use the word "ask", we believe that questions that help with defining domain can be raised through a deliberate process or a more automatic process.

- Once upon a time there was a butcher, a baker, and a candlestick maker. One fine morning, a body was discovered on the village green, a dagger protruding from its chest. The murderer's footprints were clearly registered in the mud...
- Some woman is a baker. Some woman is a butcher. Some woman is a candlestick maker. Some person is a murderer. Some person is a corpse. All women are men.

Although theses two examples are both written in a natural language, they tend to activate different reasoning processes. While reading the first one, we think about the number of people and start counting like "a butcher, a baker, and a candlestick maker". So at least three, and we can go deeper and think about which one of the three is the body, or it's from the fourth person. Etc. This is a credulous stance, in which we think about the item in question and its domain locally. The second set of sentences is also about people and their roles, both the professional roles and the roles in crime. However, they are written in such a way, we tend to think about them like solving a puzzle and consider the domain more globally[8]. Another crucial difference between the two stances is that for credulous stance, new information may change the domain. You may ask now: with the new information, does the validity of arguments also change? The choice of a formal language, the semantics and the definition of validity are indeed the crucial elements for consideration when set parameters for reasoning to an interpretation. They are actually related to the second problem of the two tasks we reviewed: focusing only on classical logic.

Both Wason and Byrne ignored the process of reasoning to an interpretation and start their analysis from the given a formal system, namely classical logic[9]. There are a few concerns with using classical logic as the only formal system.

Firstly, it is the truth valuation. A sentence is either true or false. In real life situations, more than often, we answer a yes-no question with neither a "yes", nor a "no". For example, when facing a question like "Do you feel blue?", most people would probably give answer such as "sometimes", "occasionally", etc., and few people would give a definitive yes or no. Even when we describe scientific knowledge, we can face situations in which we cannot give a simple answer. A seemingly simple math question, "Is 811 a prime number?". It is a yes-no question, but probably many people would answer "I am not sure.", especially given some time limit and a no-tool rule [10]. From these two sentences taken from real life, we have exposed ourselves to a few reasons why even with yes-no questions, answers may not comply with the bivalent valuation in classical logic. The most important two, which relate to our examples, are the degree of truth and accessibility of the knowledge.

The second is monotonicity of classical logic. We will leave the formal part to next subsection. But let us first look at the ideas behind it. What is monotonicity (without using mathematical symbols)? Monotonicity is an inference pattern that adding new information to premises should not invalidate a conclusion that we have before. The validation here is defined such that if all premises

[8] Globally might not be the best word, but we mean all possibilities.

[9] Byrne does mention in [6] that the logic rules she uses is also valid in modern logic, but she does not elaborate on it.

[10] That is they cannot use a calculator or search it on the Internet.

are true so is the conclusion. This is compatible with having fallible premises. Non-monotonicity has to do with fallible inference.

So basically when a sentence is true, it is true everywhere. How can that be a problem in daily life reasoning, but not in mathematics? There is a difference in the attitude towards facts between mathematical reasoning and daily life reasoning. In life, problems we face or talk about are generally complex and could involve many different fields. Therefore, we generally accept the limitation of time, our own knowledge and memory. With that acceptance, we talk about things as true if we can verify. We thought "Earth is flat" was true. We also thought "Swans are only in white" was true. These are actually facts about grand knowledge that were accepted by many people. In fact, majority of our truth valuation of facts happens even more locally. These are the planning examples, which are our focus. When we want to find public transportation from A to B, we use ov9292.nl in the Netherlands, or Google map in the US. If we do not find possible transportation between A and B, we *generally* conclude that there is no such public means to travel from A to B. However, in most developed countries, when you search a (young) person on Facebook, you might get "no results". However, we tend to infer that the person probably turned off the function on her account. In this case, we adjust our conclusion with world knowledge. Many sentences, once were true, can be false in the future. We deal with that in real life all the time. This is distinctively different from facts in math, which are basically proven results. They are constructively built, and rigorously verified by many. Any unproven facts are just conjectures, which are considered not true in formal mathematics.

Therefore, we need some kind of logic are can deal with both problems that cannot be solved by applying classical logic. This kind of logic is called closed-world reasoning by Stenning and van Lambalgen [47].

2.2.2 Some formal considerations

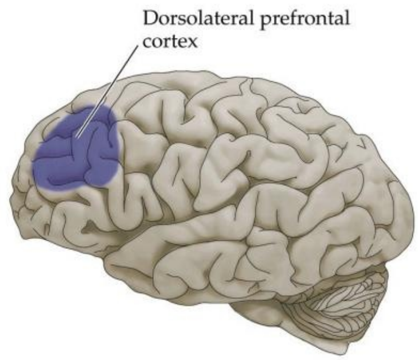
Before we start our discussion about the formal aspects[11] of the closed-world reasoning, we will introduce one more cognitive concept: executive function. It will help understand why particular logic form is picked for the framework.

Completing reasoning tasks, such as the selection task and the suppression task, involves the executive function in the brain. This concept originates from neuro-psychological research, which study the patients who have impairments in that area. This concept seems to include a wide variety of governing skills such as monitoring, controlling and managing other process to achieve a goal, and possibly under adverse circumstances [46]. Some of the tasks that are more specifically related to our purpose, are filtering out unimportant information and holding in mind a plan to carry out [5]. These functions are controlled by the front of the brain. Within that area (see figure 2.2), the dorsolateral prefrontal cortex (DLPFC) is sometimes responsible for such executive tasks[12].

[11] Here we are just raising some required features in closed-world reasoning. Please find the proper formal definitions in chapter 5.

[12] We will come back to discuss more about DLPFC when we present some neuro-economic

Figure 2.2: The dorsolateral prefrontal cortex (DLPFC)



In [46], van Lambalgen and Stenning state that there is a strong connection between logic and executive function: at a logical level, the operation of executive function is a combination of conditional reasoning and abnormality. Even though logic is "traditionally" a costly process, they further explain that closed-world reasoning logic can support a fast and automatic process, which is mostly used by the executive function. When construct a formal framework for this kind of reasoning process, van Lambalgen and Stenning focused on goal maintenance, planning and inhibition. A (finite) sequence of actions can lead to the goal. At the same time, they happen in a state of inertia, in the sense that a property caused to hold by some event will continue to hold unless there is another event that terminates it. Formally:

The use of $\rightarrow$ is only in the form of $p_1 \wedge ... \wedge p_n \rightarrow q$. No iteration of implication is allowed. There is also no negation either in antecedent or consequent[13].

1. Single implication with q as a consequent:
   In $p_1 \wedge ... \wedge p_i ... \wedge p_n \rightarrow q$, if all the p_i are true, then q is true. If there is at least one p_i is not true, then q is false.

2. Multiple implications with q as a consequent:
   For such implications, $j \leq k$, $p_1^j \wedge ... \wedge p_n^j \rightarrow q$, if for every j one p_i^j is false, then q is false.

Next feature needed is to allow a minimal model that includes only the information at hand. If there is no reason for a property to be true, it is assumed to be false. So the framework should be able to handle non-monotonicity. In a monotonic system, $p_1, ..., p_n/q$ means that in every model $\mathcal{M}$ such that $\mathcal{M} \models p_1, ..., p_n$, it also has $\mathcal{M} \models q$. More importantly, when another sentence t is also

results on the ultimatum game.

[13]The semantics of $\vee$ and $\wedge$ are the same as in classical logic. $\rightarrow$ has a different interpretation.

in the model $\mathcal{M}$, we should still have $\mathcal{M} \models q$, regardless what t is. It is not the case with closed-world reasoning.

Last but not the least is a law like feature that can handle exceptions: $p \wedge \neg ab \to q$. Basically if p is the case and “nothing abnormal happens”, $\neg ab$, then we can conclude q.

To combine the formal considerations above, the paper [46] further builds on the format of logic programming by van Lambalgen and Hamm in [28], although without the temporal component. As mentioned earlier, we will define this logic programming perspective formally in the chapter of the model. We will however explain how we handle the negation in the framework for the convenience of discussion in next subsection where we apply closed-world reasoning in the suppression task.

So far the clauses we defined are positive clauses. Before we define the possibility of negation, it is important to make a shift from two-valued semantics to Kleene's three-valued semantics, in which u is undecided besides 0 = false and 1 = true. On an intuitive level, a truth valuation can be undecided at first and latter transform into true or false. Negation here is in the sense that $\neg p$ is true, if we fail to derive p after finite steps in the closed-world reasoning program P. For example, we have a law in the program, $p \wedge \neg ab \to q$. If we fail to show that ab is the case, $\neg ab$ is true. And if we can show p is true, we get q.

At this point, we are ready to explain the basic ideas of a program completion[14]. First of all, for a proposition q in a program, we first look for all clauses that have the form $p_i \to q$, and combine them into the expression $\vee_i p_i \to q$. When there is no such clause, we can add $\perp \to q$. Then, we replace $\to$ with $\leftrightarrow$, which indicates that two sides have the same truth value just as in its classical interpretation [47]. Secondly, a completion can be done with respect to the whole program or a set of atoms that are in P. Lastly, we apply three-valued semantics to a non-monotonic completion.

2.2.3 Closed-World Reasoning in Suppression Task

After proposing the new approach that differs from the classic system, we shall look at how it helps us with understanding the experimental results better in the suppression task. We will review some detailed applications of closed-world reasoning from [45] [47].

Before we start to review the formal analysis, it is important to point out that van Lambalgen and Stenning improved the experiment format a little to accommodate the idea of understanding subjects interpretation from a task. Instead of just asking yes-no questions, the experimenters also ask tutorial questions to understand subject's reasoning patterns.

Byrne is right about subjects using general knowledge in completing the suppression task. However, it does not have to be a problem as she claimed. In fact, a term called abnormality ab can capture large amount of such reasoning with certain general rules. It can be written in the following form: $p \wedge \neg ab \to q$.

[14]It is also properly and formally defined in chapter 5, the model.

Basically it says "given condition p and nothing abnormal happens, we will get q. Such form of arguments happens frequently in our real life situations. For example, if I take subway and given nothing abnormal (like accidents), it will take me 30 minutes to get to the university. Different people could have different interpretations of what is abnormal. Therefore, their domain of abnormality, and even logical relation between the abnormality and known conditions can be different.

Taking all the informal and formal considerations we have raised about past psychology of reasoning studies, it is clear that a logical programming like default reasoning structure is the most suitable system [47]. Now we will look at an example of how the forward inferences (MP and DA) can work[15]. The sentences from the suppression task can be expressed in following formulas.

a. If she has an essay to write, she will study late in the library.
   $p \wedge \neg ab \rightarrow q$

b. If the library is open, she will study late in the library.
   If she has a textbook to read, she will study late in the library.
   Both can be seen as following formula: $r \wedge \neg ab' \rightarrow q$

Then, we use the structure of logical program to process the conditionals. For MP with one conditional, which is "If she has an essay to write, she will study late in the library." + "She has an essay to write.", we can start with following program: $\{p; p \wedge \neg ab \rightarrow q; \bot \rightarrow ab\}$. Because nothing abnormal is indicated, we can have $\bot \leftrightarrow ab$. Because we have both p and $\neg ab$, we can have $p \wedge \neg ab \leftrightarrow q$. Therefore, we will have q, which is "She will study late in the library." Of course, we have illustrated the most common reasoning pattern. It is possible that subjects thinking of some possible disabling factors and use it in the reasoning process. That is also why over 90% subjects conclude with q, but not 100% subjects.

Now let us look at the case of adding an additional premise in the MP case [47] : "If she has an essay to write, she will study late in the library." + "If the library is open, she will study late in the library." + "She has an essay to write". When asked whether she will study late in the library, we will have following program: $\{p; p \wedge \neg ab \rightarrow q; r \wedge \neg ab' \rightarrow q; \bot \rightarrow ab; \bot \rightarrow ab'; \neg r \rightarrow ab; \neg p \rightarrow ab'\}$. After the completion we described above, we get $\{p; (p \wedge \neg ab) \wedge (r \wedge \neg ab') \leftrightarrow q; (\bot \vee \neg r) \leftrightarrow ab; (\bot \vee \neg p) \leftrightarrow ab'\}$. Eventually we can reduce it to $p; p \wedge r \leftrightarrow q$. In this case, since we do not have further information about r, the subject might get stuck.

[15] detailed formal definitions, such as logical programing, completion, etc. will be discussed in chapter 5.

Chapter 3

Rational Choice Theory

In the last chapter, we reviewed rationality in the field of psychology of reasoning. We also briefly explained how closed-world assumption can be used in the framework. Here we want to introduce some traditional views on rationality in economics and game theory. The central point we want to study here is how to explain the experimental derivations from the theoretical predictions. Is there any similarity between the conventional rational choice theory and traditional definition of rationality in psychology[1]? If there is, can we also adopt concepts of rationality from closed-world reasoning?

Answering these questions will help us understand some of the experimental results in the ultimatum game, which is the focus of this thesis. More specifically, we want to know whether the rationality is adequately defined in the ultimatum game. Are people really so irrational as some economists claim?

In this chapter, we will start with basic definitions and some history of rational choice theory such as *preferences*, *utility* and *expected utility* [38] [2]. We will see how uncertainty is being handled in the theory. At the end we will draw some comparison with psychology of reasoning. We will also discuss about how the traditional view of rational choice impact the experimental economics and how closed-world reasoning can play a role in understanding the experimental results better.

3.1 Preferences

In the rational choice theory (RCT), preference is the first layer of the three that are required before a choice is made. We will review the most basic concepts and two properties, namely completeness and transitivity.

[1] Here we mean the definitions that are before closed-world reasoning. More specifically, for a group of researchers classical logic was the only 'correct' formal system to understand reasoning.

[2] In reviewing the concepts, we follow the book Mirco-economics by Rubinstein, with additional references and notes.

3.1.1 Basic Definitions

We define that a variety of options[3] as a finite set X. We then further define a binary relation $\succsim$ that is a collection of ordered pairs of elements from X. For example, when $x, y \in X$, we can have $(x, y) \in \succsim$, which can also be denoted as $x \succsim y$. It means option x is seen at least as good as option y.

This binary relation also includes two additional definitions: one is symmetric, *indifferent*: $\sim$, and the other is asymmetric *strictly better*: $\succ$. For two elements $x, y \in X$, $x \sim y \Leftrightarrow [(x \succsim y)$ and $(y \succsim x)]$. When $x \succ y \Leftrightarrow [(x \succsim y)$ but not $(y \succsim x)]$.

3.1.2 Axioms

***Completeness*:** For any two options $x, y \in X$, $x \succsim y$ or $y \succsim x$.

This axiom says that a decision maker can always choose between two options. Although considering the definition of the binary relation '$\succsim$' and its related definitions, it is clear that completeness axiom entails three situations for any two options. First, when $x \succ y$, it means the decision maker always chooses option x over y. Similarly, the second situation is when $y \succ x$, the decision maker chooses y. The last situation is when $x \sim y$, which means the decision maker chooses x or y at random[4].

***Transitivity*:** For any three options $x, y, z \in X$, if $(x \succsim y$ and $y \succsim z)$, then $x \succsim z$.

This axiom can create cyclical preferences in both individual and group decision making situations. In the latter case, it is the Condorcet paradox.

3.2 Utility

Once we can order different options through preference, we reach the second layer of a rational decision making process: assigning numbers to the preference. We call the functions help us achieve that: the utility functions. We will review some results in both finite and continuous cases.

When we compare two options $x, y \in X$, we often say that we prefer one of them. For example, 'I prefer higher grades'. This can be written in following form:

$$x \succsim y \text{ if } V(x) \geq V(y)$$

Here $V : X \to \mathbb{R}$ is a function that assigns a real number to each element in X. In our example of grades, it has a clear numerical representation. But we also want to represent sets of options that do not have clear numerical evaluation. For example, Jill prefers Thai food over Japanese food because she likes spicy food, i.e. the spicier food has a higher value to her. Ideally we want to define a function $U : X \to \mathbb{R}$ that represents the binary relation $\succsim$ if for any two elements $x, y \in X$, $x \succsim y \Leftrightarrow U(x) \geq U(y)$. We call this function a utility function.

[3] In this paper, we use 'options' interchangeably with 'alternatives'.
[4] As how Gilboa put it in his book 'Rational Choice'.

To show the existence of such utility function, we will look into two situations, namely when X is finite, and when X is continuous [36].

3.2.1 Finite Space

When the set X is finite, a utility function that represents $\succsim$ relationship always exists. To prove that, we can first show that any subset of X has a minimal element[5] through induction on the size of subsets given X is complete and transitive. Then, we can prove following proposition.

Proposition 1. *For a finite set X, the binary relation $\succsim \subseteq X \times X$ that is* complete *and* transitive*, has a utility representation with natural numbers.*

Proof. Given a finite set X, we can define X_1 as a subset of X and contains all the minimal elements of X. Then, we further define X_2 as a subset of $X - X_1$ with all the minimal elements in $X - X_1$. We can keep constructing such minimal subsets till we have $X = X_1 \cup X_2 \cup X_3 \ldots X_k$, and $k \leq |X|$. We then define $U(x) = k$ if $x \in X_k$. Further more, when $a \succsim b$ and $a, b \in X$, we know $U(a) > U(b)$ and $a \notin X_1 \cup X_2 \cup X_3 \ldots X_{U(b)}$. When $a \succ b$, $U(a) = U(b)$. □

3.2.2 Continuous Space

Often in economics the set X is set to be an infinite subset of the Euclidean space, $\mathbb{R}^n$. We want to show that there is a utility representation in that case too. In other words, for two options $a, b \in X$, if a is preferred over b, the "neighboring" elements around a should be preferred over the "neighboring" elements around b. To formalize this, we will start with some definitions.

Definition 2. *Let a be an element in X. We call the set of all points that have distance less than r $(r > 0)$ from a, a **ball** around a, and denote it as **Ball**(a, r).*

Definition 3 (Continuous a)**.** *We call a preference relationship $\succsim$ on X **continuous** when $a \succsim b$ and there are balls B_a and B_b such that $x \succsim y$ for all $x \in B_a$ and $y \in B_b$.*

Definition 4 (Continuous b)**.** *We call a preference relationship $\succsim$ on X **continuous** when the set $(x, y)|x \succsim y \subseteq X \times X$ is a closed set. In other words, for all n and $a_n, b_n \in X$ such that $a_n \to a$ and $b_n \to b$, we have $a \succsim b$*

A preference relation $\succsim$ on X satisfies 'Continuous a' if and only if it satisfies 'Continuous b'.

Debreu [11] proved a famous theorem that pushes a step further.

Theorem 5 (Debreu's Theorem)**.** *For a continuous preference relationship $\succsim$ on X, there exists a continuous utility function $U(x) : X \to \mathbb{R}$.*

[5] Or minimal elements, if they are equivalent.

3.3 Choice

Both utility functions and preference relations are just mental attitudes of a person towards a set of options. However, they do not explain how that person actually makes a choice in a real life situation. A person can think of a preference ordering for different options, but it does not necessary mean that he would make a *choice* accordingly in a real decision problem. Therefore, we need to introduce the definition of a choice function. Before that we will first look at some preliminary definitions.

We will look at the set of possible alternatives X again, in which any non-empty subset of X could be a *choice problem*, such as $A \subseteq X$. Any member $x \in A$ is a *choice*. In some situations, the decision maker considers relevant choice problems. We pair the collection of choice problems $D \subseteq X$ with X and call (X, D) a *context*. When a context is given, for each particular problem $A \subseteq D$, a *choice function* $C(A)$ outputs a unique element from A that is the choice of the problem.

Since this chapter is about *rational* choice theory, we shall discuss what kind of behaviors are considered rational within this theory. Roughly speaking, we consider a decision maker rational when he has a preference relation $\succsim$ on the set of alternatives X, and facing a choice problem A in context D, he chooses the optimal element in A. In other words, we call a choice function C *rationalizable* when $C(A) = C_{\succsim}(A)$ for any A in the domain of C.

Next, we will further review an important condition for rationalizable choice functions: condition α by Sen [41], which is also referred to as Chernoff's condition [9].

3.3.1 Condition α

Given two problems A and B, both in context D, we say that a choice function C satisfies condition α, if $A \subset B$ and $C(B) \in A$, then $C(A) = C(B)$. We say $C_{\succsim}$ satisfies condition α when it always outputs a single most preferred element in X given a preference relation $\succsim$.

Sometimes it is also called "independence of irrelevant alternatives" and was first introduced by Arrow.

3.3.2 Dutch Book Arguments

Dutch Book Argument states that anyone who does not try to maximize a preference relation will not survive. In economics, a decision maker can be Dutch-booked if he or she has intransitive preferences. For example, given three alternatives: A, B, and C, the decision maker, say Tom, has following preference: $A \succsim B$, $B \succsim C$, but $C \succsim A$. Then, someone can take advantage of his by first selling A to Tom for $B + \epsilon$; then selling B to Tom for $C + \epsilon$; then selling C to Tom for $A + \epsilon$. At the end, Tom has paid 3ϵ with nothing in return.

3.3.3 Notes on 'Alternatives'

When we talked about Dutch Book Argument, we revealed an irrational choosing behavior that preserves intransitive preferences on a set of alternatives. In some situations, the violation of rationality is due to inaccurate or changing specification of alternatives. Now we are going to review a famous dinner example from Luce and Raiffa 1957 [29]. In a restaurant, a customer chooses *chicken* from the menu with only *steak tartare* and *chicken*. At the same time, he chooses *steak tartare* from the menu with *steak tartare, chicken* and *frog legs*. It looks like this customer violates the condition α, hence we could consider him irrational with his choices. However, it is possible that he realized the fact that second menu with the frog legs indicates the high level of cooking skills. Making a steak tartare also requires high level of cooking skills. Following such reasoning, you may consider that this customer is actually not that irrational, but actually smart. Rubinstein adds this paragraph in [38] to remind us that sometimes the same set of alternatives can have a different meaning.

We also should realize that the particular reasoning of this customer also implies another condition: he is a new customer. That means even though he has access to his own preferences, he does not have full information about the choices that he could make in this particular restaurant.

3.3.4 Choice Functions

We will continue the definition of choice functions. So far, the functions we discussed have only one solution to every choice problem. It is certainly possible that given a preference relation and a choice problem, there are more than one optimal solutions. Therefore, we will explain a further fine-grained definition: *choice correspondence*.

Given a choice problem A, $C(A)$ is a non-empty subset of A. It is obvious that the decision maker has to select **one** element from $C(A)$. Basically $C(A)$ is the set of equivalent optimal choices he could select from.

3.3.5 The weak Axiom of Revealed Preference [6]

Samuelson originated the revealed preference approach in 1938 [39]. It was also proposed by Houthakker [24] and by von Neumann and Morgenstern [49]. Arrow adapted it to set-valued choice functions in [1]. Sen provides a systematic treatment of the axiomatic structure of the theory of revealed preference in [43].

Definition 6. *For any $x, y \in X$, we say x is **indirectly revealed preferred** to y, denoted as xP^*y, if only if there is a sequence z^i, $i = 0, ..., n$, and $z^1 = x$ and $z^n = y$, such that for all i, $z^{i-1}\tilde{P}z^i$* [7].

[6] Instead of following the discussion from Rubinstein's book, we will review the concepts from the original paper from Sen on 'Choice Functions and Revealed Preference' [43].

[7] xPy is equivalent to our earlier notation: $x \succ y$. Sen defines $x\tilde{P}y$ as x is chosen while y is available but rejected.

Definition 7. *For any $x, y \in X$, we say x is* ***indirectly revealed preferred*** *to y* ***in the wide sense****, denoted as xWy, if only if there is a sequence z^i, $i = 0, ..., n$, and $z^1 = x$ and $z^n = y$, such that for all i, $z^{i-1}Rz^i$* [8].

For all $x, y \in X$, we have following axioms:

1. **Weak Axiom of Revealed Preference (WARP):** If $x\tilde{P}y$, then *not* yRx.

2. **Strong Axiom of Revealed Preference (SARP):** If xP^*y, then *not* yRx.

3. **Strong Congruence Axiom (SCA):** If xWy, then for any non-empty subset B in choice problem A such that $y \in C(B)$ and $x \in B$, x must also belong to $C(B)$.

4. **Weak Congruence Axiom (WCA):** If xRy, then for any B in A such that $y \in C(B)$ and $x \in B$, x must also belong to $C(B)$.

After showing the equivalence of all four axioms mentioned above in [43], Sen raises two important questions: (1) Are the rationality axioms to be used only after establishing them to be true? (2) Are there reasons to expect that some of the rationality axioms will tend to be satisfied in choices over "budget sets" but not for other choices?

3.4 Expected Utility under Uncertainty

So far in our description of rational choice theory, we assumed that the decision maker has the information about available options and the outcome of the choice. However, in reality, we often do not have exact information about the outcome, and face risks or uncertainties. That is the relationship between actions and outcome is not deterministic.

This aspect is especially crucial for this review paper, since ultimately we want to define choices made in a social environment that consists of many people. Then, uncertainty is inevitable. Expected Utility Hypothesis, an idea, which goes as far as 1738 from Daniel Bernoulli [3], provides an important view on how to model uncertainty. In the paper, he wrote:

> "Somehow a very poor fellow obtains a lottery ticket that will yield with equal probability either nothing or twenty thousand ducats[9]. Will this man evaluate his chance of winning at ten thousand ducats? Would he not be ill-advised to sell this lottery ticket for nine thousand ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousand ducats."

[8] xRy is equivalent to our earlier notation: $x \succsim y$.
[9] A ducat was a standard gold coin throughout Europe.

> "...the determination of the value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount."
>
> "If the utility of each possible profit expectation is multiplied by the number of ways in which it can occur, and we then divide the sum of these products by the total number of possible cases, a mean utility [moral expectation] will be obtained, and the profit which corresponds to this utility will equal the value of the risk in question."

Bernoulli pointed out the problem of focusing only on monetary term and suggested to use expected utility instead. He also suggested to use logarithm to calculated the utility.

In 1944, Von Neumann and Morgenstern provide a formalization in their book "Theory of Games and Economic Behavior" [49].

3.4.1 Von Neumann-Morgenstern Utility Theorem

Before we review Von Neumann-Morgenstern Utility Theorem (vNM Theorem), we shall explain some preliminary concepts that extend naturally to the context of uncertainty[10]. In the earlier section, we have defined X as a set of options. Here we will be more precise, and define X to be a set of outcomes. The binary relation $\succsim$ on X is the same as how we introduced preference. $x_1 \succsim x_2$ indicates that x_1 is weakly preferred to x_2, and $\succ$ indicates strict preference while $\sim$ indicates the indifference between the two outcomes. A *lottery* on X is a probability distribution: $[p_1 : x_1; p_2 : x_2; p_3 : x_3; ...; p_n : x_n]$. In addition, we have: $\sum_{i=1}^{n} p_i = 1$ for all $p_i \geq 0$.

In the following paragraphs, we will explain the six axioms that are stated by Von Neumann and Morgenstern [49]: completeness, transitivity, substitutability, monotonicity, continuity, and decomposability.

Axioms

We have introduced the first two axioms (completeness and transitivity) earlier. Therefore, we will just simply state them here.

Completeness: $\forall x_1, x_2 \in X, x_1 \succ x_2$; or $x_2 \succ x_1$; or $x_1 \sim x_2$

Transitivity: $\forall x_1, x_2, x_3 \in X, x_1 \succsim x_2$ and $x_2 \succsim x_3 \Rightarrow x_1 \succsim x_3$

Substitutability: If two outcomes are indifferent for a decision maker, then he is also indifferent between the two lotteries that contain the outcomes separately. To put it formally: Given $x_1 \sim x_2$, $[p : x_1; p_3 : x_3; ...; p_n : x_n] \sim [p :$

[10] For the convenience of readers, we will repeat some basic definition of preference that were introduced earlier.

$x_2; p_3 : x_3; ...; p_n : x_n]$, and $p + \sum_{i=3}^{n} p_i = 1$. In other words, the outcome x_1 can be substituted with x_2.

Monotonicity: For all $x_1, x_2 \in X$, $x_1 \succ x_2$ and $1 \geq p > q \geq 0 \Rightarrow [p : x_1; 1-p : x_2] \succ [q : x_1; 1-q : x_2]$

Continuity: For all $x_1, x_2, x_3 \in X$, $x_1 \succ x_2$ and $x_2 \succ x_3 \Rightarrow \exists p \in [0,1]$ such that $x_2 \sim [p : x_1; 1-p : x_3]$

Decomposability: We denote by $P_{lj}(x_i)$ the probability that x_i is selected by lottery l_j. The axiom states that if we have two lotteries l_1 and l_2 over X, and $P_{l1}(x_i) = P_{l2}(x_i)$ for all $x_i \in X$, then $l_1 \sim l_2$.

An example could be following:
$l_1 = [0.7 : x_1; 0.3 : [0.3 : x_1; 0.7 : x_2]]$
$l_2 = [0.79 : x_1; 0.21 : x_2; 0 : x_3]$
According to decomposability axiom, $l_1 \sim l_2$.

vNM Theorem

Theorem 8. *When a binary preference relation $\succsim$ on a set of outcomes X satisfies completeness, transitivity, substitutability, monotonicity, continuity and decomposability, a utility function u exists and fulfills following two properties:*

$$u(x_1) \geq u(x_2) \textit{ if only if } x_1 \succsim x_2$$

$$u([p_1 : x_1; p_2 : x_2; p_3 : x_3; ...; p_n : x_n]) = \sum_{i=1}^{n} p_i u(x_i)$$

3.5 Comparison with Psychology of Reasoning

So far we have reviewed much of the mathematical aspects and results of rational choice theory. To put them on an intuitive level, a rational agent should be able to order all relevant options according to his preference in a complete and transitive manner, to assign numerical values to these options, and at the end to choose the option that maximize his utility. Mathematical results such as Sen's condition α, the weak axiom of revealed preference, axioms and vNM theorems are properties that are supposed to be obeyed by a rational agent.

Before we start criticizing the problems related to rational choice theory, it is essential for our readers to understand that we are not criticizing the theory itself in this thesis. As Jon Elster put in the very beginning of the book *Rational Choice* [15]:"The theory of rational choice is, before it is anything else, a normative theory. It tell us what we ought to do in order to achieve our aims as well as possible. It does not tell us what our aims ought to be.". Our concern is how the theory is being used in experiments, and how conclusion of actual human reasoning patterns are drawn through these experiments. In that respect, the application of rational choice theory in the laboratory share some drawbacks as in the experiments of psychology of reasoning.

In chapter 2, we discussed the two major problems in applying classical logic directly to the study of reasoning: truth valuation and monotonicity. In the sets of options, there is no truth valuation but ordering. However, there is a relative truth valuation, whether one option is more preferred than the other. Completeness axiom states that a rational agent can always tell the preference between two options. It is true that there is possibility of seeing the two option as indifferent. However, in real life situations, we construct the necessary domain of preference for a decision problem at hand [11]. That means, even if we are reminded of some options available, unless it is in the domain we have defined, we do not position it in the preference set, possibly not even in a set of indifferent options. Using food as an example, we probably all have a few dishes we love and hate. Now someone tells you, there is a delicious Chinese dish called Malandou[12]. Sure, you know it is 'delicious' from that person, but can you place it in your set of preference for dishes? Before you gather more information, such as the looks and taste, you probably cannot even place this dish into a indifferent set either with dishes you love or hate. In real life, there are many such examples that requires a possibility of *undecided*.

Now let us look at the second problem we had raised earlier: monotonicity. In chapter 2, we had talked about updates of knowledge can lead to non-monotonicity in conventional reasoning. Also with the advancement of technology, many un-true facts can be true in the future. In fact, we do not even have to think about such example of future knowledge and technology, because our complex daily life and natural languages provide examples that require non-monotonic preference. When we review the Dutch book argument, we show how agents can be taken advantage of when their preferences are intransitive. However, it is important to look at some possible real reasoning just as in the restaurant example we described later. The customer chooses chicken over steak tartare, when they are the only two options. But he chooses steak tartare, when frog legs are on the menu too. This violets the monotonicity in preference, although it seems reasonable to most of us. We can see that new options may alter new rules and even new set of preference. Therefore, we need a more versatile framework of reasoning in games that supports local and instant constructions.

Besides the theoretical points, which we just discussed above, there are points to be made concerning the experiments. Just as in the past psychology of reasoning experiments, subjects' reasoning is not fully explored. Typical experiments in economics do have more answers to choose from unlike the yes-no style in the reasoning tasks we have reviewed. However, these answers tend to be purely numerical. There are often survey used, but not are related to reasoning, often just their gender, nationality, study, etc. In the book, human reasoning and cognitive science by van Lambalgen and Stenning, they show the great benefit of Socratic tutorials. Through these tutorials, subjects' reasoning patterns are quite clear. We wish to adopt this format. But in order not to de-

[11] By now, we shall state our view on the relationship between rationality in games and rationality in general. We believe that reasoning in games is a subset of reasoning in general, which is a reasonable assumption, especially given van Lambalgen and Stenning have argued that closed-world reasoning is a fast cheap process that can be used in executive function [47] [46].

[12] It is a regular vegetable we eat in the east side of China.

viate too much from the conventional economic experiments, we adopt a survey version of the tutorial, which will be explained in the chapter of experiment.

Another observation we have concerning experiments in economics is that all the subjects are tested for material comprehension before they start the real experiments [22] [23]. For example, they are often asked the amount of reward they would get in different hypothetical situations. That means all the subjects already understand the economic rules, yet the results still sometimes violates the theoretical predictions. It is our belief that other rules and goals are competing with the economic ones, such as social rules and goals.

We understand that building a framework that can accommodate all these concerns is a fairly ambitious goal. We choose to study one of the most well-researched games, the ultimatum game, as our focus for now.

Chapter 4

Ultimatum Game

Ultimatum game(UG) is widely experimented and considered as one of the simplest games in experimental game theory. Güth, Scmittberger and Schwarze first wrote an experimental analysis of this kind of take-it-or-leave-it game in 1982 [21].

The experimental results often deviate from the game theoretical prediction. Game theoretically speaking, the proposer should claim as much as possible for himself, while the responder should not reject any non-zero offers for him/her. However, in reality, people tend to propose much more for the responders, and responders rejects high offers up to 30% of the total share. One stream of publications, especially in neuro-economics, tends to claim that emotional activation is the main reasons for such results [40] [48]. Such a simple claim could not give detailed explanation on cross-cultural results found by Bowles et al. [22]. In [22], the experiments run in 15 small-scaled primitive societies revealed some systematic deviations from the game theoretical results. It indicates a potentially sophisticated mechanism beneath the behavior in the simple game.

In this chapter, we will first explain the game structure and the theoretical predictions. Then, some key experimental results will be reviewed. At the end of the chapter, we will discuss the concept of rationality in this particular game setting.

4.1 Basic Game Structure and The Theory

Ultimatum game has a simple structure. There are two players, who will decide on a distribution of a sum of money. The first player, who is usually called the proposer, suggests a division for both players. Then, the second player, the responder, has two options. She can accept the division, then they each get their parts accordingly. She can also reject it, in which case they both get nothing.

In the original paper from Güth, Scmittberger and Schwarze in 1982 [21], they argue that both players know that the game is finite with two steps: the proposing step and responding step. After second step no one can influence the result. From that the proposer can anticipate which option the responder would take if he knows what is considered a better option by her. Assuming the responder is a rational economic utility maximizer, then the proposer knows that she would accept any non-zero offer, because it would be a better option than rejecting and get zero. As a responder, she just needs to think what is a better option for herself and act on it after the proposal. Therefore, Güth et al. consider this as 1-person game for the responder. To conclude, the proposer should propose the lowest possible amount to the responder.

In this thesis we are only extending the closed-world reasoning into games with a small step, therefore we focus on the reasoning process from the prospective of the responder.

4.2 Experimental Results

In the first experimental study [21], the results are different from the original theoretical predictions. We will start with their results and treat them as the base. Then, we will review other experiments in two other groups. Majority of the Ultimatum Game experiments are conducted at universities in industrialized countries. However, researchers have found differences in both industrialized countries and communities that are less developed. We will first review this group of literature that reviews some cultural differences. Then, we will review a widely cited paper from neuroeconomics.

4.2.1 Results from Güth, Scmittberger and Schwarze in 1982

In the original paper, there are two versions of the Ultimatum Game. First version is basically the same as we described earlier. The other version, which they call "complicated games", includes two different kinds of currency. Here we will only discuss the simple version, which is also the widely experimented version.

There are three phases of the simple game, namely the pilot phase, the naive phase and the experienced phase. In the pilot phase, there were 9 pairs of a proposer and a responder, who played the simple version of the game and were given same amount of money to split. 6 out of 9 proposers demanded between 50% - 60% of the money, and all were accepted. 3 players proposed between 90% - 100%, and were all rejected.

In the naive and experienced phases[1], same group of 42 subjects played the games, and they were expected to have different partners. They were given different amount of money to split. In the experienced phase, the proposers demanded higher shares on average than in the naive phase. But the rejection rate is also higher in the experienced phase. The high rejection rate in this

[1] In the experienced phase, same subjects in the naive phase participated the same experiment after one week.

group offsets the high amount they demand. Therefore, the average payoff in the experienced phase is lower than in the naive one.

Overall, in their experiments, the proposers demanded 63.3% on average, and were rejected when their demands were higher than 70% it was rejected.

4.2.2 Cross-Cultural Differences

Due to the simple game format and surprising deviations from the theoretical prediction s, the ultimatum game was experimented in dozens of countries with a variety of conditions. The general deviation, which is that the proposer tends to give higher offer then expected and the responder tends to reject low offers that seems reasonable for economists, is observed in most of the experiments [33].

However, there are differences in these experimental results. For example, in Henrich 2000 the average offer was 26%, while in the study of Buchan et al. (1999), the average was 51%. Therefore, some claim that culture plays a role. In the paper from Roth et al. (1991), two experiments, the market environment and the ultimatum game, are conducted in four countries, the US, Yugoslavia, Japan and Israel. They observe that there are significant differences in the ultimatum game while the results are similar in the market environment experiment. They argue that the results show that the differences in the ultimatum game could not have been resulted by differences in languages, currencies or the experimenters. At the same time, some argue that there are no cultural differences. A paper by Okada and Riedl shows that the experimental results of the ultimatum game is similar in Vienna and Kyoto.

Oosterbeek, Sloof and van De Kuilen argue in their paper in 2004 [33] that most of the cross cultural studies fail due to two reasons. First of all, majority of the country comparisons were actually done on the level of cities. In the paper we mentioned earlier from Roth et al., the conclusion is about four countries while the experiments were done in one city from each country. This means any difference between two cities within a country is not really considered. Second problem is that before the conclusion is drawn, authors do not specify what kind of cultural traits that could be the potential reason for differences and hence should be the focus of the study. In order to look at the studies more systematically, Oosterbeek, Sloof and van De Kuilen conduct a meta-analysis over 37 papers, which have 75 experimental results in total. Their result is interesting and different than what was claimed earlier. They found less variation in proposers' behavior while there is much more variation in responders' across all these studies. More surprisingly, there is no relation between rejection and the experience of subjects nor their field of study.

The most important result is probably the 15 small-scale society study from Samuel Bowles et al. [22]. It is an unusual collaboration of leading researchers in the field. They intend to challenge the conventional assumption of rationality as we reviewed in chapter 3 that humans are self-interested monetary-maximizing beings. They ask whether the social environments shape the economic behavior too? With that question they conducted experiments in 12 countries on five continents. Three different games are played during the process: the ultimatum game, the public-goods game and the dictator game. For our purpose, we will focus on the results of the ultimatum game. As we can see from table 1 (taken from [22]), the variety of behaviors in the ultimatum game is bigger. For example the mean offers range from 26% to 58%, which is different from the meta analysis we just reviewed above [33]. It is also different from results in industrialized societies, which is usually around 40%. In addition, low offers like 20% are often rejected in industrialized societies, but are rarely rejected in some small-scale societies they studied. For example, only one offer was rejected in Machiguenga, even though 75% of all offers were less than 30%.

Another key result in this study is the strong relationship between the social condition and the behavior in UG. They develop two indexes: *Payoffs to cooperation* (PC) - how important and how large is a group's payoff from cooperation in economic production, and *Market integration* (MI) - how much do people rely on market exchange in their daily lives. The two indexes explain 68 percent of the variance in the mean offers. It shows how social rules can play an important role in economic decisions. However, it is not captured in conventional economic reasoning framework.

4.2.3 Results from Neuro-Economics

Almost all the empirical studies on the ultimatum game relate the rejection behavior[2] to fairness [16]. The concept of fairness does not fit the conventional rational choice theory very well, especially given the self-regarded assumption. More researchers start to get interested in the topic, including psychologists and neuroscientists. Some claim that such rejection behavior is actually irrational and resulted from the emotional process [40] [48].

Here we will review one such claim from a widely cited paper in 2003 by Cohen et al. [40]. They conduct a simple version of the ultimatum game and apply functional neuroimaging techniques that show the active brain areas. During the experiment, two players share \$10. A fair offer is \$5:\$5, unfair offers are \$9:\$1, \$8:\$2, \$7:\$3. The behavioral results were similar to the previous studies, in which unfair offers were mostly rejected. They claim that since the rule of the game is so simple, the experimental deviation is not likely due to the misunderstanding. We disagree on this point. Simple rules can still be interpreted in multiple ways. Even when the rules are interpreted in the same way, if there are multiple goals that are competing in the game, then it is not clear that which goal the subjects are reasoning towards. We have discovered in our experiment that in the case of ultimatum game, subjects can have either social related goals or economic related goals. In [40], they further claim that acceptance of an (positive) offer is cognitively driven while its rejection is emotionally driven.

They find that anterior insula, the area which is generally associated with pain and distress, is more active when subjects reject the offer. Another area

[2]Rejection any amount above 0.

Figure 4.1: UG results from [22]

TABLE 1—THE ULTIMATUM GAME IN SMALL-SCALE SOCIETIES

Group	Country	Mean offer[a]	Modes[b]	Rejection rate[c]	Low-offer rejection rate[d]
Machiguenga	Peru	0.26	0.15/0.25 (72)	0.048 (1/21)	0.10 (1/10)
Hadza (big camp)	Tanzania	0.40	0.50 (28)	0.19 (5/26)	0.80 (4/5)
Hadza (small camp)	Tanzania	0.27 (38)	0.20 (8/29)	0.28 (5/16)	0.31
Tsimané	Bolivia	0.37	0.5/0.3/0.25 (65)	0.00 (0/70)	0.00 (0/5)
Quichua	Ecuador	0.27	0.25 (47)	0.15 (2/13)	0.50 (1/2)
Torguud	Mongolia	0.35	0.25 (30)	0.05 (1/20)	0.00 (0/1)
Khazax	Mongolia	0.36	0.25		
Mapuche	Chile	0.34	0.50/0.33 (46)	0.067 (2/30)	0.2 (2/10)
Au	PNG	0.43	0.3 (33)	0.27 (8/30)	1.00 (1/1)
Gnau	PNG	0.38	0.4 (32)	0.4 (10/25)	0.50 (3/6)
Sangu farmers	Tanzania	0.41	0.50 (35)	0.25 (5/20)	1.00 (1/1)
Sangu herders	Tanzania	0.42	0.50 (40)	0.05 (1/20)	1.00 (1/1)
Unresettled villagers	Zimbabwe	0.41	0.50 (56)	0.1 (3/31)	0.33 (2/5)
Resettled villagers	Zimbabwe	0.45	0.50 (70)	0.07 (12/86)	0.57 (4/7)
Achuar	Ecuador	0.42	0.50 (36)	0.00 (0/16)	0.00 (0/1)
Orma	Kenya	0.44	0.50 (54)	0.04 (2/56)	0.00 (0/0)
Aché	Paraguay	0.51	0.50/0.40 (75)	0.00 (0/51)	0.00 (0/8)
Lamelara[e]	Indonesia	0.58	0.50 (63)	0.00 (3/8)	0.00 (4/20)

Note: PNG = Papua New Guinea.

[a] This column shows the mean offer (as a proportion) in the ultimatum game for each society.

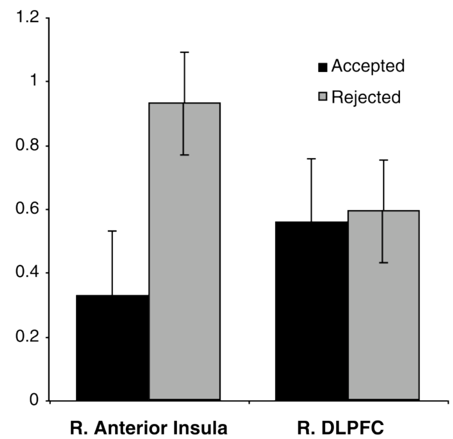
[b] This column shows the modal offer(s), with the percentage of subjects who make modal offers (in parentheses).

[c] The rejection rate (as a proportion), with the actual numbers given in parentheses.

[d] The rejection rate for offers of 20 percent or less, with the actual numbers given in parentheses.

[e] Includes experimenter-generated low offers.

Figure 4.2: Right anterior insula and right DLPFC activation for all unfair offers from Cohen et al [40]



DLPFC, which is considered to be related to goal maintenance and executive controls, is also studied during the experiment. However, it is activated in both acceptance and rejection, as we can see from figure 4.2. Authors do say that the activation of this region alone is not sufficient to predict behavior.

We believe that this neurological result is not convincing enough for their conclusion that a rejection is a result of irrational and emotional reaction. Just as the data shows in their experimental result, DLPFC, which is often linked to executive function activities, is active when accepting and rejecting the unfair offers. So subjects may very well be planning and reaching their goals, which fits the kind of rationality as we are defining in this thesis. Secondly, the activation of anterior insula is an indication of possible pain or stress, but cannot be evidence of absence of the rational process.

4.3 Rationality in Ultimatum Game

It is clear from all the experimental results in UG we have reviewed that people do not always behave as economists expect based on conventional rational choice theory. Therefore, many conclude that the ultimatum game study is a strong support of irrationality among regular people. However, we believe that such a conclusion is hasty. As was discussed in the last chapter, firstly, the concept of rationality in the game calls for better definition. Secondly, the procedure of experiments in games expose similar weaknesses as in psychology of reasoning. Lastly, the formal analysis could be improved by using closed-world reasoning. In the next few subsections, we address them one by one, particularly in the case of the ultimatum game.

4.3.1 Redefining Rationality in UG

In the past chapters, we in fact have introduced two different definitions of rationality. One is an improved definition from cognitive science or psychology of reasoning by van Lambalgen and Stenning [47], which is focusing on the planning. The other is from rational choice theory [38], a theoretical pillar of game theory. In the theoretical prediction of games in general, it is assumed that the set of preferences and utility functions are clear and stable to the agent and he maximizes his own utility, which often meant the monetary gain.

We are calling to redefine the rationality in UG. The deviation from such fixed process is considered irrational. However, as the results shown, there are a lot of regularities in the deviation from the theoretical prediction in UG. The most crucial result is probably from Samuel Bowles et al. [22]. This paper shows that different remote societies have different averages in both proposal and rejection. More importantly, the differences are highly correlated with the local social activities. When the 'irrational' behavior is so highly systematic, it is a good indication of a failure in our models. This is the first reason for redefining rationality in UG. It is essential to provide a system that accommodates the role of social rules.

Second reason is on the theoretical side that just as in the early years in psychology of reasoning, conventional rational choice theory is heavily relying on classic systems as we can see from von Neumann and Morgenstern Theorem. An important objection, which we had mentioned in the earlier chapters, is the monotonicity, in which case a preference relation between two options cannot be switched with additional new information. We shall define a reasonable non-monotonic reasoning process for games like the ultimatum game. Such non-monotonicity can happen especially when people reason with social rules. Bicchieri explains it in her book [4] that social norms are reflected in people's expectations and they often expects others have the same norms as themselves. Clearly it is possible what is considered fair can be slightly different from one person to the other, and across different countries. As the results show, most people behave within the normal range and therefore within the expectation of the others. Occasionally, when the offer is outside of that range, a possible non-monotonic process is activated. In our system described in chapter 5, we will try to accommodate such processes of social rules.

4.3.2 Similar Structure as Suppression Task

We have established the need for redefinition of rationality in UG. But how do we apply it formally? In fact, there are many aspects in UG that are comparable to the suppression task. We will therefore examine the possibility of using the particular closed-world reasoning that is used in re-analyzing suppression task by van Lambalgen and Stenning in [47].

To start off, we will look at the differences and explain why they can be overcome. Indeed at first glance, they may appear quite differently. The ultimatum game has a more complex structure, an interactive game between two players, while the suppression task is a single-person reasoning task. We shall explain this in two steps: first, the matter on the number of subjects; second, how do we account for the complex economic rules?

In the beginning of this thesis, we explained that we intend to extend the closed-world reasoning in games with baby-steps. Therefore, we are only studying the simpler and more reactive reasoning of the responder. In the first experimental study of the game from Güth, Scmittberger and Schwarze in 1982 [21], they state that for the responder in this game, it is basically a one-person game. We agree with this view. In addition, the most controversial behavior is the seemingly irrational rejection of an unfair offer[3]. In addition, there are more cultural differences in the behaviors of the responders [33]. So we reduce our focus to a single-person reasoning process.

In the chapter of rational choice theory and a part of this chapter, we have discussed that reasoning in economic situations is complex. Before we explain how this might be irrelevant for our purpose, we will first clear a misconception about applying reasoning analysis in experimental game theory. Economist tend to argue that making economic decisions is a fast process, which is different from the slow reasoning process that is studied by cognitive psychologists. This would not be a problem in our case. As van Lambalgen and Stenning state in their book [47], the closed-world reasoning system they develop is suitable for both system 1 and 2 since the process is fast and cheap.

So why in the case of the ultimatum game, in fact many other social-related games too, the complex economic rules can be simplified in our reasoning model? In the chapter on rational choice theory, we had briefly mentioned about the comprehension tests [23] used before the real games start. We will elaborate on this, then we will have the answer for our question. In every economic experiments, subjects have to read complicated instructions that described the process. It is important that the subjects understand the instructions. Therefore, there are often a few tests, which are like mini games, to check whether subjects fully understand the economic rules and their implications. An example in a ultimatum game would be a question like: how much you get if the offer to you is $2 out $10 initial amount, and you choose to accept? It happens from time to time that subjects are not allowed to continue participating the game if they fail to pass the tests[4]. Basically, subjects start the games with good understanding of the economic rules and mathematics behind them. However, during the real game, they act as if they do not understand it. From previous results, we suspect social goals and rules are competing with the economic ones[5]. It is more important for us to capture the rational process of dealing with social

[3]Of course, the near-50% offers from the proposers are also puzzling for the economists, but we will not focus them here, as they require slightly more sophisticated reasoning process. We intend to do further research on this in the future.

[4]During the Bsc and Msc study, I have participated in dozens of game theory experiments at Center for Research in Experimental Economics and Political Decision making (CREED). I have observed this, but could not find such a statement in the literature. Papers often just state that they have ensured the comprehension before starting the game.

[5]In fact, we believe if we add a clear goal: maximize your earning, during the game, most subjects would get the 'right'/desired answer.

rules. Therefore, we simplified the economic rules into a single rule, as you will see in the next chapter.

Now we are ready to look at their similarities. In [47] [45], the suppression task results are better understood when a proper logical account of general knowledge is provided. These are the disabling conditions that subjects can think of even though they are not explicitly provided during the experiment. Similarly, in the ultimatum game, the account of general knowledge, more specifically the social conduct of fairness, seems to be the center of the debate. The question is why the behavior of bringing in fairness is considered to be irrational. If we can show using closed-world reasoning that subjects reason logically with such general knowledge, then they can no longer be accused of being irrational.

Secondly, conditionals are the tools used in the suppression task to understand the logical use of rules. In economics, there are multiple attempts to understand people's preferences through rules. Such as in [2], Aumann proposed a rule-based rationality, which states that rather than maximize the utility people follow certain rules. In Bicchieri's book, *The Grammar of Society - The Nature and Dynamics of Social Norms* [4], she explains that social norms are default rules and they are in the form of conditionals.

In our framework, we will try to combine the two points we raised above, that is to provide a conditional rule-like account for both economic and social reasoning in the process of the ultimatum game. More specifically, we assume the focus is on the economic rules, with assumption of no social abnormality in the beginning of the game.

In the next chapter, we will see how it is being implemented formally, and how it can help us understand the results from the experimental studies in the ultimatum game better.

Chapter 5

Modeling

We would like to merge the closed-world reasoning framework to the game settings. The starting point of such merge is to exam the *human* rationality in the process of economic experiments. In that respect, we will formally implement the closed-world assumption, and an account for social rules.

In this chapter, we will first explain the difference between the ultimatum game and the suppression task. That will provide some background on the changes we make in the closed-world reasoning framework of van Lambalgen and Stenning [47] [46]. Then, we will review some necessary formal definitions from their framework. After that, we will introduce the changes we make for reasoning in games. We will also explain the application of this framework in the ultimatum game.

5.1 Closed-World Reasoning in Games

As we explained in the last chapter, the reasoning throughout a game is comparable to that of reasoning tasks, such as suppression tasks. However, there are differences which we have to address before the formalization.

Even though we have explained how we could reduce the complexity of economic reasoning, it is still a 'game'. For a suppression task, a subject read up the instruction and the related sentences to complete it. Even with the tutorial style implemented by [47], the subject still reasons about the same unchanged task. In the situation of games, it is often the case that there are multiple stages of the game. The ultimatum game is one of the simplest games, however it still has two stages for the responder. The initial understanding of the instruction, and the reasoning to respond after receiving the offer. We would like to have multiple reasoning programs throughout the game with minimum changes[1]. In each program we assign a simple time stamp. The entire process of reasoning in a game is a collection of such timed programs.

[1] We will define formally what are the minimum changes allowed in the framework in the next section.

Second difference is built based on the first one. We need to include a definition of rationality that moves from one program to the next. We still apply the closed-world assumption throughout the process. That will enable a non-monotonic yet logical reasoning pattern.

The last difference is a minor point that we are not going to elaborate in this thesis. It is the possibility of multiple players and how they can reason about each other within this framework.

5.2 Formalization

This model of closed-world reasoning in games (CWRiG) is largely adapted from the model in *Human Reasoning and Cognitive Science* [47]. We extend the model into a restricted game situation, in which subjects use a set of programs to process the game task. We aim to show a rational process in completing a game task that allows subjects to logically alter their initial decisions at earlier stages of the game. In addition, the framework does not support arbitrary alternations. It means that not every new fact at a different stage will alert a new *decision*. As we will show formally in this chapter, the CWRiG is monotone in some situations. Formally it is an extension of the least fixed point result from CWR, and we it call *conditional monotone game process*. It basically shows that decisions[2] are changed with new facts that alert adjustment in either old rules or new rules.

5.2.1 Preliminary Definitions from CWR by van Lambalgen and Stenning

Definition 9. A positive clause *has a following form:* $p_1, ...p_n \to q$.
q and p_i are proposition letters. When the antecedent is empty, q is considered as a fact. In addition, q is called the head *and $p_1, ...p_n \to q$ is called the* body *of the clause.*
A positive program is a finite set of positive clauses.

Definition 10. $\mathfrak{N}$ *is the natural language used in game theory experiments. The input of it can be from instructions or opponents' reactions.*

Definition 11. $\mathfrak{L}$ *is the formal translation of* $\mathfrak{N}$ *when a subject needs to process an instruction in a formal manner.* $\mathcal{L}^t \in \mathfrak{L}$ *is a finte set of propositions being processed by subject j at time t.*

Clearly we do not exclude the situation in which two subjects might have two different sets of $\mathcal{L}$ given one identical instruction. In our setting, we just focus on one agent's reasoning pattern in the game. Therefore we will omit the subscript, j from now on.

[2] In our case, it is the output in the logic program, which will be explain in the following sections.

Table 5.1: The truth table for three-valued logic

p	q	$\neg p$	$p \wedge q$	$p \vee q$
1	1	0	1	1
0	0	1	0	0
u	u	u	u	u
1	0	0	0	1
1	u	0	u	1
0	1	1	0	1
0	u	1	0	u
u	1	u	u	1
u	0	u	0	u

Definition 12. *$\mathcal{P}^t$ is a positive program on $\mathcal{L}^t$.*
$\mathcal{M}^t$ is a model of $\mathcal{P}^t$, and assigns truth-values $\{0, 1\}$ to $\mathcal{L}^t$.
Truth conditions on $q \in \mathcal{L}^t$:

- *$\mathcal{M}^t(q) = 1$ if there is a clause $p_1, ..., p_n \to q$ in $\mathcal{P}^t$ such that for all i, $\mathcal{M}^t(p_i) = 1$*
- *$\mathcal{M}^t(q) = 0$ if there is a clause $p_1, ..., p_n \to q$ in $\mathcal{P}^t$ there is some p_i, for which $\mathcal{M}^t(p_i) = 0$*
- *If q is not the head of a clause, $\mathcal{M}^t(p_i) = 0$*

Before we further explain the closed world reasoning in games, we shall first introduce three-valued logic from Kleene to deal with negations in the body of a clause. In three-valued logic, we have an additional truth valuation, u that is undecided. It can later be transformed into true or false as we proceed with the program. The truth valuation of the three-valued connectives is shown in table 1.

Now we can define an important clause in closed world reasoning, the *definite clause* that can have negation in the body.

Definition 13. A definite formula *has the following form:*

$$(\neg)p_1 \wedge ... \wedge (\neg)p_n \to q$$

p_i can be proposition letters, $\top$ or $\bot$, or it can also be in non-negated form.
q is a propositional variable.

Definition 14. *A definite logic program is a finite conjunction of definite clauses.*

Program Completions

Definition 15. *1. Definition of a head q which occurs in a program $\mathcal{P}^t$:*

(a) *Form the expression* $\vee_i \varphi_i \to q$ *with all clauses* $\varphi_i \to q$ *whose head is* q

(b) *if there is no such* φ_i*, then add the expression:* $\bot \to q$

(c) *replace all* $\to$*'s with* $\leftrightarrow$*'s*

2. Program Completion $comp(\mathcal{P}^t)$ *is constructed by taking the conjunction of the definitions of atoms* q*, for all* q *in* $\mathcal{P}^t$

3. *To achieve Partial Program Completion for a set of atoms* S *in* $\mathcal{P}^t$, $comp_S(\mathcal{P}^t)$ *is by taking the conjunction of the definitions of the atoms* q *which are in* S.

4. *If* P *is a logic program, define the non-monotonic consequence relation* $\approx\!\!\!\!|$ *by:*

$$\mathcal{P}^t \mathrel{|\!\!\approx}_3 \varphi \text{ iff } comp(\mathcal{P}^t) \models_3 \varphi$$

Strengthening the Closed-World Assumption: Integrity Constraints

Currently the closed world assumption is only applied to atomic formulas and their negations. For example, in the previous chapter we have mentioned the abnormality, *ab*. When *ab* is not in the database, with closed world assumption we can assume that it is false. We would like to extend it especially when there are multiple clauses that share a same head, such as $\varphi_1 \to q, \ldots, \varphi_n \to q$. In this case, we can only conclude q when one of $\varphi_1, \ldots, \varphi_n$ is the case. However, it is also possible that ψ, which is not in $\varphi_1, \ldots, \varphi_n$, can lead to q, in the form of $\psi \to q$, which we learned later. Van Lambalgen and Stenning call this "closed-world reasoning for rules" [47]. This notion is not yet properly treated in the completion of a program. We will illustrate that with a simple program P that has one clause $\{p \to q\}$. When a fact about q is added, $\top \to q$, the completion will give us $\{(p \vee \top) \leftrightarrow q\}$. We would like to have the database or model updated with p. It is achieved through *integrity constraints* [47] [27]. We will use an example from [27] to introduce some of the technical definitions.

Let us consider a general 'obligation': carrying an umbrella when it is raining. We can formalize it as follows:

$$Holds(rain, t) \to Holds(carry - umbrella, t)$$

Without an integrity constraint, in the ordinary program, it means that when $Holds(rain, t)$ is the case, it would lead to $Holds(carry{-}umbrella, t)$, regardless of whether we actually have an umbrella. We would like to have a machinery in which the consequent is seen as a constraint that has to be satisfied.

Definition 16. *We call a finite (possibly empty) sequence of proposition letters denoted as* $?p_1, \ldots, p_m$ *a query or a* goal. *The empty query denoted by* $\Box$ *is interpreted as* $\bot$*, a contradiction.*

Definition 17. Resolution *is a derivation rule that takes both a clause* $p_1, \ldots, p_n \to q$ *and a query* $?s_1, \ldots, q, \ldots, s_k$ *as inputs, and produces the query* $?s_1, \ldots, p_1, \ldots, p_n, \ldots, s_k$.

Theorem 18 (Theorem 2 of [47]). *A query* $?\varphi$ *succeeds with respect to a program* $\mathcal{P}$ *if* $\mathcal{P}|\approx_3 \varphi$. *I.e. if* φ *is entailed by the completion of* $\mathcal{P}$ *in the sense of* $comp(P) \models_3 \varphi$. *Similarly, a query* $?\varphi$ *fails with respect to a program* $\mathcal{P}$*, if* $comp(P) \models_3 \neg\varphi$. *I.e.* $comp(P) \models_3 \neg\varphi$.

Now we come back to the example of rain and umbrella. We can express the original 'obligation' in the form of a conditional integrity constraint:

$$\text{if } ?Holds(rain, t) \text{ succeeds, then } ?Holds(carry-umbrella, now) \text{ succeeds.}$$

An observation of the fact that it is raining leads to an update with $Holds(rain, now)$. The antecedent of the integrity constraint is satisfied, and the next step is to satisfy the consequent. Then, the agent would connect that goal to the related clause in the program, such as $Initiates(take-umbrella, carry-umbrella, now)$. He will conclude that in order to carry an umbrella, he would have to take the umbrella first.

The integrity constraint is going to be useful when we make any backward inference, for example modeling the reasoning for the proposer in the future.

5.2.2 Further Definitions in CWRiG

Definition 19. $\mathcal{G} = \{\mathcal{P}^1, \mathcal{P}^2, ...\mathcal{P}^t\}$ *is a game reasoning process that contains a set of programs.*

Here is a list of situations where *stage change* happens from $\mathcal{P}^t$ to $\mathcal{P}^{t+1}$:

1. Some new instructions are presented to the subject
2. A response comes from the opponent
3. Or any other new facts that are noticed by the subject. Although we do assume that the understanding and formal translation of the initial instruction are stable.

A stage change is when subjects perform a new reasoning process within a game. When a new instruction or an additional instruction is presented to subject, there is usually a change in $\mathcal{L}^t$. Hence, subjects re-do the reasoning process with a different logical program $\mathcal{P}^{t+1}$ using new set of propositions $\mathcal{L}^{t+1}$.

Generally speaking when a subject realizes some information, a new process is performed. Subjects could be suddenly aware of certain information in a later stage. However, we do not include such situations. When no new instructions or responses from opponents are given, $\mathcal{L}^t$ is constant. In addition, when $\mathcal{L}^t$ is unchanged, there is still a new program $\mathcal{P}^{t+1}$ that is identical to $\mathcal{P}^t$. In this case, the superscript $t+1$ just serves as a time stamp.

An example of such stage change in UG could happen after a responder receives a proposal. In UG, subjects, both responders and proposers, usually read the instruction of the whole game simultaneously before the game starts. It is reasonable to assume that they have formed a reasoning process after reading the instructions. However, the situations interpreted from the instruction by a responder might not include a certain proposal.

In some games, both situations 1 and 2 from definition 7 can be repeated. However, a stage change is different than a game cycle in repeated games. A stage change capture a different reasoning process, which can happen within a game cycle. We also restrict that each stage change can include one new fact in $\mathcal{L}^{t+1}$ comparing with $\mathcal{L}^t$. Formally, the new fact can be presented in three different ways in $\mathcal{P}^{t+1}$:

Definition 20. *When a new fact f_i is added to $\mathcal{L}^{t+1}$:*

1. *f_i does not appear in any of the clauses in $\mathcal{P}^t$. In this case, there are two possible reasoning interpretations for an agent:*
   - *(a) Only the fact, f_i is added to the new program, $\mathcal{P}^{t+1}$.*
   - *(b) Not only f_i is added, a new rule associated with f_i is alerted.*
2. *f_i does appear in one or more of the clauses in $\mathcal{P}^t$*

5.2.3 The Completions in a Game

In this subsection, we define program completions and game completions for a single-shot game.

With the definitions we have so far, we have not included any negations in the body of a positive clause or in a positive program. Before the program completion and game completion are explained, we need to extend our definitions. We will again adapt some formalizations from CWR into CWRiG.

With the extension above, we can further construct the model and the completion of a program.

Game Completions

When the game approach the last moment, i.e. when the subject is ready to make a decision and answer the final question, then the game reasoning process reaches the last program, $\mathcal{P}^t$. The output of the last program is the output of the game.

Definition 21. $comp(\mathcal{G}) = comp(\mathcal{P}^t)$

In the next section, we will first review some formal results in CWR, and then extend them into CWRiG. We hope readers will see that the simple adaption we have made can actually better explain different reasoning patterns in games like the ultimatum game.

5.3 Models

Here we will present how models are constructed in closed world reasoning and then extend it to the reasoning in games. We will also show that the result on the minimal model or least fixed point in CWR can be partially extended to CWRiG.

Again we adjust the definitions in CWR slightly to have a time stamp in the process.

5.3.1 Models in CWR

Definition 22. *The operator T_P transforms a model $\mathcal{M}^t$ in $\mathcal{P}^t$ according to the following rules:*

if $q \in \mathcal{L}^t$:

1. *$T_P(\mathcal{M}^t)(q) = 1$ if there exists a set of proposition letters C such that $\bigwedge C \to q \in \mathcal{P}^t$ and all letters in C are true on $\mathcal{M}^t$*
2. *$T_P(\mathcal{M}^t)(q) = 0$ otherwise.*

Definition 23. *We denote an ordering on two-valued models by $\mathcal{M}^t \subseteq \mathcal{N}^t$ if all the proposition letters that are true in $\mathcal{M}^t$ are also true in $\mathcal{N}^t$.*

Lemma 24 (Lemma 1 of [47]). *If $\mathcal{P}$ is a positive logic program, T_P is monotone in the sense that $\mathcal{M} \subseteq \mathcal{N}$ implies $T_P(\mathcal{M}) \subseteq T_P(\mathcal{N})$.*

This lemma naturally extends to: $\mathcal{M}^t \subseteq \mathcal{N}^t$ implies $T_P(\mathcal{M}^t) \subseteq T_P(\mathcal{N}^t)$.

This kind of monotonicity cannot accommodate negation well in the following sense. When a program P has a clause that contains $\neg q$ and another clause $\neg q \to s$, it is possible to have s true in the beginning and false in a later stage. A more complicated structure is introduced next to deal with negation.

Definition 25. *A fixed point of T_P is a model $T_P(\mathcal{M}^t) = \mathcal{M}^t$.*

Lemma 26 (Lemma 2 of [47]). *If T_P is monotone, it has a least and a greatest fixed point. The least fixed point is the minimal model with respect to the ordering of definition 20.*

As van Lambalgen and Stenning explain in [47], the monotonicity is crucial here to allow incremental computation of the minimal model through iteration of T_P. We start with the empty model $\mathcal{M}_0^t$. All the proposition letters are false in the empty model. From lemma 24, we get $\mathcal{M}_0^t \subseteq T_P(\mathcal{M}_0^t) \subseteq T_P(T_P(\mathcal{M}_0^t)) \subseteq \cdots \subseteq T_p^n(\mathcal{M}_0^t) \subseteq \ldots$. The minimal model is the least fixed point, and can be written as $\cup_n T_P^n(\mathcal{M}_0^t)$.

In the closed-world reasoning framework, $p \wedge \neg ab \to q$ is an important feature. We have mentioned in the earlier chapters that we need to extend the framework into three-valued logic to properly deal with the negation in the body of a clause. Following is how we do that with three-valued logic. For the truth valuation, please refer to the table 5.1. In addition, we assign 1 to $\varphi \leftrightarrow \psi$ when φ and ψ have the same truth value, and 0 otherwise.

Definition 27. *A three-valued model refers to an assignment of truth valuation $\{u, 0, 1\}$ to the set of proposition letters in $\mathcal{L}^t$. When u is not used, we call it a two-valued model.*

Between two models $\mathcal{M}^t$ and $\mathcal{N}^t$, the relation $\mathcal{M}^t \leq \mathcal{N}^t$ means that the truth-value of a proposition letter p in $\mathcal{M}^t$ is less than or equal to the value in $\mathcal{N}^t$, with $u \leq 0, 1$; $0, 1$ are not comparable.

Definition 28. *In a program $\mathcal{P}^t$:*

a. *The operator T_P^3 applies to formulas constructed using only $\neg, \wedge$, and $\vee$ is determined by the three-valued truth table shown above.*

b. *Given a three-valued model $\mathcal{M}^t$, $T_P^3(\mathcal{M}^t)$ is determined by:*

   (a) *$T_P^3(\mathcal{M}^t)(q) = 1$ iff there is a clause $\varphi \to q$ such that $\mathcal{M}^t \models \varphi$. Empty clauses are not allowed.*

   (b) *$T_P^3(\mathcal{M}^t)(q) = 0$ iff there is a clause $\varphi \to q$ in $\mathcal{P}^t$ and for all such clauses $\mathcal{M}^t \models \neg\varphi$.*

   (c) *$T_P^3(\mathcal{M}^t)(q) = u$ otherwise.*

Lemma 29 (Lemma 3 of [47])**.** *If $\mathcal{P}$ is a definite logic program, T_P is monotone when $\mathcal{M}^t \leq \mathcal{N}^t$ implies $T_P^3(\mathcal{M}^t) \leq T_P^3(\mathcal{N}^t)$.*

5.3.2 Monotonic Game Reasoning Process in CWRiG

Corollary 30. *If $\mathcal{P}^t$ and $\mathcal{P}^{t+1}$ are both definite logic programs, T_P is monotone when $\mathcal{M}_n^t \leq \mathcal{M}_n^{t+1}$ implies $T_P^3(\mathcal{M}_n^t) \leq T_P^3(\mathcal{M}_n^{t+1})$. We will call such pair of two consecutive programs a monotone game process.*

Let us look at a simple example. We start with a program $\mathcal{P}^1 : \{p \to q\}$. We add a new fact about p in a stage change and get $\mathcal{P}^2 : \{p \to q, \top \to p\}$. Then, we have following:

- $\mathcal{M}_0^1(p) = \mathcal{M}_0^1(q) = u = T_P^3(\mathcal{M}_0^1(p)) = T_P^3(\mathcal{M}_0^1(q))$.
- In this case, $\mathcal{M}_0^1$ is also the least fixed point of T_P^3.
- $\mathcal{M}_1^1(p) = u = \mathcal{M}_1^1(q)$.
- $\mathcal{M}_1^2(p) = T_P^3(\mathcal{M}_0^2(p)) = 1$.
- $\mathcal{M}_1^2(q) = T_P^3(\mathcal{M}_0^2(q)) = u$.
- etc.

Lemma 31. *A complete monotonic game process is captured by the operator T_P^3 in the following sense: $T_P^3(\mathcal{M}^1) \leq T_P^3(\mathcal{M}^2) \leq \cdots \leq T_P(\mathcal{M}^t)$. It can happen in the following two categories:*

1. *Without any new facts: $\mathcal{L}^1 = \mathcal{L}^2 = \cdots = \mathcal{L}^t$.*
2. *With some new facts:*

(a) are unrelated to any clauses in previous programs and do not alert a new clause

(b) are part of the non-negated body of one or more previous clauses

Proof. • First category is straightforward. When there is no new facts added in any of the programs throughout the whole game, it is essentially one program being repeated.

- 2.(a) When we have a new fact f_i added at time $t+1$ and it is not associated with any of previous clauses, $f_i \in \mathcal{L}^{t+1}$ and $\notin \mathcal{L}^t$. In the new $\mathcal{P}^{t+1}$, we add $\top \to f_i$ or simply f_i. By definition of $\mathcal{M}^t$, we know $\mathcal{M}^t \leq \mathcal{M}^{t+1}$ in this case.

- 2.(b) At time $t+1$, f_i is in *one* of the clauses at time t in the form of $f_i \wedge (\neg)p_1 \wedge \cdots \wedge (\neg)p_n \to q$ in program $\mathcal{P}^t$. We shall note that q is also not a negated part of any clause in both $\mathcal{P}^t$ and $\mathcal{P}^{t+1}$. This kind of fact can affect truth valuation in $\mathcal{M}^{t+1}$ in the following ways:
  - $\mathcal{M}_0^t(f_i) = u$, later it can turn to 0 using closed world completion. In $\mathcal{P}^{t+1}$, $\mathcal{M}_0^t(f_i) = 1$, since we have $\top \to f_i$.
  - Knowing f_i in $\mathcal{P}^{t+1}$ also affects the truth valuation of q.
    * If $\mathcal{M}^t(q) = 1$, given definition of $\mathcal{M}^t$, we know there is at least one clause without f_i that makes q true. In this case, $\top \to f_i$ does not affect q.
    * If $\mathcal{M}^t(q) = 0$, $\top \to f_i$ in time $t+1$ might make q true. For example, $\mathcal{P}^t = \{f_i \wedge \neg p \to q\}$, and $\mathcal{P}^{t+1} = \{f_i \wedge \neg p \to q, \top \to f_i\}$

  Therefore, $\mathcal{M}^t \leq \mathcal{M}^{t+1}$ in this case too.

□

Theorem 32. *In our setting, a rational decision change, i.e. comp($\mathcal{P}^t$) $\neq$ comp($\mathcal{P}^{t+1}$), may happen only when a new (**disabling**) fact affects an old rule, or alert a new rule.*
Note: If $\mathcal{P}^t$ has a clause like $p_i \wedge \neg f_i \to q$, then $\top \to f_i$ is a disabling fact in $\mathcal{P}^{t+1}$. It is also possible that q is a negated body part of a clause.

Proof. Prove by contradiction using corollary 2 and lemma 3. □

Van Lambalgen and Stenning developed the framework of closed reasoning within a program to show how different people can assign different logical structures and domains to the same task. We extend their framework slightly to multiple programs that are used towards one goal in game. In our framework, new facts can be added in the process of a game. Although, it is same kind of non-monotonic logic we use, we are able to extend some of the monotonic aspects in consecutive programs. It shows how people can use pieces of new information constructing new reasoning program towards the goal. A new fact may change the output of a program but only when it is connected to a rule. In this sense, changing a previous decision without alerting any rule is considered irrational, even when new facts are learned.

Now we will see how CWRiG helps us understand the ultimatum game in a richer formal structure.

5.4 CWRiG in UG

CWRiG demonstrates how subjects are reasoning rationally in a strategic situation with information at hand, while still leaving flexibility for future information in the process of a game. So far we have only introduced necessary formal definitions. In this section we are going to draw the big picture by applying these definitions to Ultimatum Game.

We start with assigning some specific propositional letters to particular phases used in Ultimatum Game instructions. We are aware that the definitions of these phases and structures themselves are arguable for each individual. But we want to show that even with such a strong condition, in which we assume every subject understands the instructions in an identical way, we still can show how rejecting a non-zero offer in UG can be a rational reasoning process.

X**:** The proposed percentage of total amount

e_1**:**

$$e_1 = \begin{cases} \text{true} & \text{if } X > 0 \\ \text{false} & \text{if } X \leq 0 \end{cases}$$

Symbolizes an important economic rule: *monetary* utility greater than 0.

unf**:** unf stands for the unfair situations. This assumption of basic trust of cooperation and fairness is encouraged by understanding importance of social contract with trusted cooperation in human evolution. [47] [30] [4]

$$unf = \begin{cases} \text{true} & \text{unfair} \\ \text{false} & \text{fair} \end{cases}$$

unf^***:** There can be different kinds of unfair situations noticed by the subject. But it does not mean all of them will be considered relevant to the reasoning at hand. Therefore, unf^* is an indicator for whether there is relevant unfairness observed that should be used in current reasoning process. U^* is basically a collection of relevant unfair situations. If there is at least one such situation in U^*, unf^* is true.

$$unf^* = \begin{cases} \text{true} & \text{if } U^* \neq \emptyset \\ \text{false} & \text{if } U^* = \emptyset \end{cases}$$

as**:** It is basically a special kind of abnormality. It is true when the subject thinks the other individual is being asocial. When we encounter other people, we usually start with $\neg as$, assuming that they are social people who obey social norms. But when they violet some social rules, such as fairness, we then consider them asocial. This can be written as: $unf^* \wedge \neg ab \rightarrow as$. As we can see from this formula, that there can be other information that can potentially disable this rule (e.g., if ab is true).

A simple economic rule entailed in Ultimatum Game for a responder is:

If the proposed amount is greater than 0, accept the offer.

$e_1 \wedge \neg as \rightarrow q$ Another important feature is $\perp \rightarrow as$, meaning people start with the assumption that others are also social being who do not do asocial things. In this particular case, fairness is the most important social aspect. The responder probably assumes that the proposer would propose an fair offer. This is a typical closed-world assumption that fits the social norms characteristics described in by Bicchieri [4]. When some asocial, i.e. abnormal happens, for instance an unfair offer is presented, the status of $\perp \rightarrow as$ changes.

We will see a few applications of CWRiG in different scenarios in UG.

5.4.1 Fair Offers

This is a rather common situation of UG in industrialized west. In fact, in a fair-offer situation the reasoning process before and after the proposal does not change. People general assume others behaves within the social norms which includes being fair. This is their starting points. Then, the program $\mathcal{P}_j^0$ goes as follow:

1. $\{e_1; e_1 \wedge \neg as \rightarrow q; unf^* \wedge \neg ab \rightarrow as; \perp \rightarrow ab; \perp \rightarrow unf^*\}$
2. $\{e_1; e_1 \wedge \neg as \leftrightarrow q; unf^* \leftrightarrow as; \perp \leftrightarrow unf^*; \perp \leftrightarrow ab\}$
3. $\{\top \leftrightarrow e_1; \neg as; e_1 \wedge \neg as \leftrightarrow q\}$
4. q, accept

5.4.2 Unfair Offers

There are two possible reasoning patterns for an unfair offer and both can be reflected in CWRiG framework. First let us look at the situation in which the subject observes an unfair offer, and think it is relevant to the reasoning task, thus unf^* is true in program $\mathcal{P}_j^1$:

1. $\{e_1; e_1 \wedge \neg as \rightarrow q; unf^* \wedge \neg ab \rightarrow as; unf^*; \perp \rightarrow ab\}$
2. $\{\top \leftrightarrow e_1; e_1 \wedge \neg as \rightarrow q; \top \leftrightarrow unf^*; \top \leftrightarrow as; \}$
3. $\{\perp \leftrightarrow q; \}$
4. $\neg q$, not accept

Formally, a responder j have $\mathcal{P}_j^0$ after reading the instructions. After he receives the information of an unfair offer, the stage change happens and new program $\mathcal{P}_j^1$ takes place. After completing the program $\mathcal{P}_j^1$, we indeed get 'not accept' as the result. In this case, we have a new fact, $\top \rightarrow unf^*$ in $\mathcal{P}^1$ and unf^* is associated to a negated body part of a clause: $\neg as$.

However, it is also possible that the responder considers it as unfair, but does not think it is relevant to the task. Thus unf^* is not true in this case. This possibility is not reflected in the original design of the UG experiment, which we will address in our experiment. We stated that we assumed subjects understand the clauses from the instructions in the same way. Even with that assumption, subjects have decide what is the domain for a particular argument.

Such responders have the following reasoning pattern:

1. $\{e_1; e_1 \wedge \neg as \rightarrow q; unf^* \wedge \neg ab \rightarrow as; \bot \rightarrow unf^*; \bot \rightarrow ab; \top \rightarrow unf\}$
2. $\{\top \leftrightarrow e_1; e_1 \wedge \neg as \rightarrow q; \bot \leftrightarrow as;\}$
3. $\{\top \leftrightarrow q;\}$
4. q, accept

In this case, it is the kind of monotonic game reasoning process we had shown earlier. A new fact unf is learned, but it is un-connected to a clause/rule.

We shall point out that it is certainly possible that a subject may never consider the social rule in the whole process, and any unfair signals do not trigger a social rule. Then, he also accepts. This would be a complete monotonic game process, which is captured by conventional game theory.

From the two situations we discuss above, it is clear that subjects can get two different rational answers due to different settings of the domain for a particular term, even though they both think that the offer is unfair. Such processes cannot be reflected in simple yes-no questions.

5.4.3 Defining Relevant Social Domain

We can see that in the process of completing a game-theory task, there are two kinds of rules that dominates the reasoning, namely the economic rules and social rules, both of which can be represented by conditionals. It is up to the subjects to decide how important each kind of rule is and whether it should be reflected in their reasoning process, especially when they decide the relevant domains. It would be important to know how subjects define this domain. However, it is beyond the scope of this thesis. What we intend to do is to show that given an evidently relevant social rule, we can use CWRiG to explain how people come to a rational decision of rejecting low offers.

During our experiment, we do try to understand the effects of other social rules. Subjects have to decide about a relevant social domain on two levels. The first one is what are the related social norms in this task. In UG, fairness is the most important and is consistently shown in different experimental results. But it is possible that other social norms can be important both in UG and other games. The second level is that given other unfair situations, they have to decide which ones are relevant to the task at hand.

In our pilot experiments, we probed the possible disabling factors for fairness. That is under what other situation would a responder change his rejection to acceptance given an unfair offer. A few such factors are wealth and illness. For example, if the proposer is actually very poor or if the proposer has some congenital illness. Such answers give us an idea to test the domain of fairness when we conduct the real experiment[3].

Here, let us look at the formal implications of these potentially disabling factors.

s_1: Poorness. To be more precise here, it indicates that the *proposer* is very poor.

s_2: Illness. The *proposer* has a serious illness.

If the responder is given an additional information about the proposer that he's very poor, then the additional rule is: $s_1 \wedge \neg ab' \rightarrow ab$. After receiving the poorness information, responder adds this rule and processes it with a new program $\mathcal{P}^2$:

1. $\{e_1; e_1 \wedge \neg as \rightarrow q; unf^* \wedge \neg ab \rightarrow as; unf^*; s_1 \wedge \neg ab' \rightarrow ab; s_1\}$
2. $\{\bot \leftrightarrow ab'; s_1 \wedge \neg ab' \leftrightarrow ab; \bot \leftrightarrow as; e_1 \wedge \neg as \leftrightarrow q\}$
3. q, accept.

Above is the process when the subject treats s_1 as a disabling factor of unf^*. In real life, it is certainly possible that they become some kind of social rules on their own. But in our case, through the way we ask our subjects in the pilot, we guide them into treating them as disabling factors of unf^*.

Comparing $\mathcal{P}^2$ and $\mathcal{P}^0$, even though the results are the same, which is accepting the offer, the reasoning processes are rather different.

As we can see, using a closed-reasoning framework in games allows us to deal with a non-monotonic yet logic reasoning process. Even when the final outcome of the two programs are the same, which is acceptance, the inner processes can be very different. Potentially, if we can use the understanding of such framework into designing better experiments in game theory, we can discover more how people make rational decisions, and how other rules, such as social rules can play a role in the decision process.

5.5 Implication on the Relationship between Social Preference and Individual Preference

The deviation (rejecting an offer lower than 30% of total amount) from the game theoretical prediction is an issue of understanding the operation between

[3]It will be explained in the chapter for experiment.

individual[4] preference and social preference. CWRiG is showing a rational process of a pro-social decision. From the analysis of rejection using CWRiG, it is clear that such decisions reflect a shift in preference domains (from individual to social) with additional information such as an unfair offer or a certain social condition of the proposer.

CWRiG does not deny a potentially consistent and complete preferences set for each individual as conventional rational choice theory suggests. However, it does suggest that people do not reason with the whole set but a partial set.

Rational Choice Theory is being updated frequently with new data and theories as shown in recent works: [12] [34] [19]. In [19], a more subjective focus is raised by Gilboa. In CWRiG, the subjectivity reflects on the inclusion of abnormal terms (as in CWR). Of course, the selection of such abnormal terms is subjective and can be different with different individuals. However, in the particular case with UG, it is a socially concerned term. Therefore, in the formalization it is defined with an abnormal term. Exactly with the inclusion of such a term, it is formally possible to see how subjects' reasoning shifts between the social and individual preferences.

[4] By individual preference we mean self-centered economic preference.

Chapter 6

Experiment

In this chapter, we will explain the experiments we have conducted in order to examine the theoretical idea of applying closed-world reasoning in the ultimatum game. This is key to finding out if there is enough experimental evidence that indicates the clear rationality behind a rejection of none-zero offer.

We ran two separate experiments, one at experimental economics lab at Tilburg University[1], and another at Brooklyn College, the City University of New York. Due to the experimental circumstances, the usable data points at Tilburg University is only 5. We therefore treat the first one as a small pilot, which gave us some inspiration for the second experiment in which 46 students participated.

6.1 Design

This thesis intends to call for a better understanding of rationality in game theory experiments. We have pointed out a few general concerns in conventional rational choice such as the ordering problem and monotonicity. Many experimental results in game theory indeed show such problems. We therefore would like to test our CWRiG framework in an experimental setting in order to check whether it is indeed helpful in understanding reasoning in games.

There are a few key points which we would like to focus on. First of all, it is the relative domain of preferences. As we have argued earlier, we believe that the preference are constructed at the moment they are needed. Options that are irrelevant to the task at hand are not considered in the decision process, and as a result their preference relations can be undecided. In order to test that, we have designed a survey before the ultimatum game to learn about subjects' attitudes towards different unfair situations. In the survey, they have to answer whether a situation is unfair and rank some of the situations. We would like to

[1]Here we want to indicate our sincere appreciation for all the support from Ting Jiang and Prof. Jan Potters.

check if such ranks are preserved in the real ultimatum game, which would be a prediction of conventional rational choice theory.

Secondly, we want to see if our framework captures the reasoning pattern in the UG in general, including both subjects that accept the low offers and those who reject them. More specifically, we would like to see the relationship between social rules and economic rules. To achieve this, we ask our subjects to explain their decisions in the UG.

Last but not least, we are interested in seeing the use of abnormality, which enables non-monotonic reasoning. We examine the answers of the UG carefully to look for such patterns. In addition, we design an extra part of the UG after subjects answer the question whether they will accept or reject an offer. In that question, subjects are given one extra piece of information about the responder. This information is related to one of the situations in the survey. We would like to see if the subjects will use this information because it is made salient.

6.1.1 Pilot

During the pilot experiment, we basically ran the second half of the ultimatum game. That is to give each subject the unfair offer (30% of the total) directly and ask them whether they would accept it. As we have mentioned earlier, unfortunately we have only collected answers from 5 students. 3 out of 5 rejected the offer, which is expected given the results from past experiments. Then, they were asked why they rejected. All three pointed to fairness. We further explored under what condition would they change their rejections. Some conditions mentioned are financial, such as "If the proposer is really poor". Some are other kinds of misfortune, such as illnesses.

During one such interaction with the subject, the responder immediately rejected the unfair offer and gave a very clear threshold of how much is fair in his opinion, which is 40% to 60%. Then, we continued with the following conversation:

- *experimenter:* Is there any situation you would consider to accept the offer?

- *subject:* Yes. There are special situations I would consider to accept. First, if the person is really ill; Second, if the I know the person has very bad economic situation; Or I know he is a very unlucky guy.

Other subjects gave similar answers. However, one subject did say that he would have not considered these unusual situations if he was not asked.

Therefore, we decided to focus on two aspects of CWRiG: the domain and possible disabling factors. We want to understand how the subjects think about fairness, which situations are considered relevant in the task, etc. More importantly, we decided to use the survey format instead of yes-no questions.

6.1.2 Actual Experiment

The actual experiment consists two parts. The first part is mostly designed to show their general attitudes towards fairness. The second part is the second half of the ultimatum game, that is, given an unfair offer the subject has to decide whether to accept or reject.

Part 1 instructions

Name:

This is a small survey for a research. Please answer the questions honestly. There are no tricky questions here. We are just collecting facts.

Please imagine a person X is experiencing following situations.

1. X has to pay a fine which was caused by a person whom he cares. Consider following three amounts separately and answer the questions accordingly.
   (a) The fine is $ 500. Do you think it is fair that he has to pay for it?
   (b) The fine is $ 1,000. Do you think it is fair that he has to pay for it?
   (c) The fine is $ 10,000. Do you think it is fair that he has to pay for it?
2. X has a congenital condition. Consider following three different conditions and answer the questions accordingly.
   (a) He was born blind. Do you think he's in an unfair situation?
   (b) He has a congenital heart disease. Do you think he's in an unfair situation?
   (c) He was born mentally retarded. Do you think he's in an unfair situation?

   Please order three conditions from the most unfair to the least unfair.
3. One of X's classmates and X have an opportunity to share some bonus credits (the total is 10%). The classmate made following proposals
   (a) 5% for the classmate and 5% to X. Is this a fair offer to X?
   (b) 6% for the classmate and 4% to X. Is this a fair offer to X?
   (c) 7% for the classmate and 3% to X. Is this a fair offer to X?

Compare the situation in 1(c) with 2(b) and 3(c), please order three situations from the most unfair to the least unfair for X.

Part 2 instructions

Imagine that you and another student have an opportunity for sharing 10% bonus credits. You both do not have to do anything extra for the credits, but both of you have a decision to make. The other student has to decide how much of 10% each of you will get. He has three options: a) 5% for himself and 5% for you; b) 6% for himself and 4% for you; c) 7% for himself and 3% for you. Once he decides which option to choose, you will get to see the option and make a decision. You have two choices: a) accept whichever option he chooses, and as a result you both get what he proposes; b) you reject what he chooses, and as a result you both get nothing.

1. Now you get the choice he made: c) 7% for himself and 3% for you. Do you accept or reject? Please explain your answer.

2. Now we give you one extra piece of information about the other student: he was born with a heart disease. Do you want to change your offer? Please explain.

3. If you have chosen to reject in both cases, please think of a situation in which you would accept this offer.

6.2 Results

In this section, we will review the results we got from the experiment. In the first part, we give an overview of each part with some basic statistics, such as percentages and trends. In the following parts, we are going to look at some specific statistics and detailed analysis on the three topics:

1. Do students construct related preference?

2. Do people use the reasoning patterns as described in CWRiG?

3. How do people deal with the additional information received about the proposer?

At the end of this section, we will also review some unusual answers from the subjects.

6.2.1 General Results

Experiment Part 1

In this part of the experiment, we ask the subjects three categories of unfair situations, namely finance-concerned situations, diseases, and grading-related. The first two categories are mentioned by the students[2] in our pilot study. The last category is related to the ultimatum game part of the experiment.

[2] All were students from Tilburg University, and mostly European.

Each category has three different instances. For each of them, subjects have to answer whether they consider it as an unfair situation.

We have attached all the descriptive statistics in the appendix. Let us discuss some general results. For the first category, which includes financial situations, the percentages of subjects who think it is unfair increases as the amount of money involved gets larger, from 60.5% to 66.7% to 81.4%. For the second category, which is related to diseases, the percentages of subjects who think it is unfair stay more or less the same, 84.1% to 86.4% to 77.3%, probably due to the fact that it is difficult to rank diseases as some of the subjects wrote on the survey. The last category has an interesting trend, but not surprising. For the first instance, which is to share the bonus credits 50-50, 92% think it is fair. For 60-40, it jumps to 85% who think it is unfair. For 70-30, 87.8% of the subjects think it is unfair. If we count each instance of all three categories as 1 degree of unfairness, we get a minimum 0 and maximum 9 degree to count as their general attitude of unfairness. For this general attitude, 5.93 is the average with 2.370 standard deviation. So on average, our subjects think 6 out the 9 are unfair situations.

In the last question of Part 1, subjects have to rank the unfair situations across the categories. We picked three instance: 1c "paying for $10,000 fine", 2b "having a heart disease" and 3c "giving 7% bonus credits for the classmate, and 3% for X". There are 9 subjects who refused to order these three situations. Majority of the subjects not only ordered them, but also gave an agreement on what is the most unfair and least unfair as we can see from fig. A.5. 78% people thought that 2b is the most unfair. 73% people thought that 1c is the 2nd most unfair. 70% people thought that 3c is the least unfair of the three.

To conclude, this part gives us some idea of how our subjects think about unfairness, and how they would rank different instance if we 'force' them to.

Experiment Part 2

Part 2 of the experiment has three questions as well. The first question puts a subject directly into a role of the responder in a UG. She has to answer if she accept an unfair offer of 30-70. She is also asked to explain the answer. For this question, 26.1% of the subjects rejected the offer, while 73.9% accepted the offer. We will explain their reasoning in a later subsection when we analyze the closed-world reasoning.

Second question is to first give them a piece of information about the proposer, i.e., that he has a congenital heart disease, and then ask the subject if she is going to change her response. For this question, only 1 out 46 students changed the answer. The rest did not. This was surprising for us because illness was mentioned by all our pilot subjects as an abnormal situation in which they would reconsider their response[3].

[3] It is a crucial lesson for us. The abnormal situations should be tested in the same country where the experiment is conducted, especially when we are conducting a study related to social norms.

The last question is basically checking if the subject can think of any situation that would change the rejection. Since over 70% accepted the unfair offer, many did not answer this question. However, there are some interesting answers, which we will review in the later subsections.

6.2.2 Constructed Preference and its Domain

We have argued that it is our belief that people not only use a subset of their preferences, but also have some options un-ordered. We try to capture this by introducing a closed- world reasoning framework with three-valued logic. In addition, due to the multi-stage nature of games, we introduce timed set of propositions $\mathcal{L}^t$ and $\mathcal{P}^t$. Such a construction gives agents the possibility to add new options if necessary and re-compute the program.

Now we will look at the data we gathered and check if there is any evidence supporting this structure.

Let us look at the data from both part 1 and part 2. We have seen that 86% subjects considered a congenital heart disease an unfair situation. Also, 78% people actually ranked it as the most unfair in comparison with credits shared in 70-30. You would think both the case of heart disease and 70-30 credits sharing are in the domain of unfairness and they have an order relation with each other. However, our results show that such domains and relationships did not get carried to the part 2 of the experiment. 45 out 46 subjects refused to change their responses. As high as 91% subjects, voluntarily used the word "irrelevant"[4] to explain why they do not want to change. Following is one actual answer from subject 37: *"I would feel bad for him having such a condition and I wouldn't change my answer. Mainly because heart disease has nothing to do with class assignment or we choose to share the extra credits."* Even though part 1 of the experiment made "having heart disease" salient, and was ranked higher in comparison with sharing credits, our subjects are still very clear about the fact that it is not in the domain for processing.

It is also worth mentioning that the word "unfair" is mentioned by over 60% of the subjects when they answer part 2. Clearly, the word "unfair" has two different meanings with different domain in part 1 and 2.

We can conclude that a framework like CWRiG that allows a constructive process of preference would be useful in general. However, in our framework, the changes from one stage to the next are very limited. For example, in our structure, we cannot accommodate deletion of a proposition in $\mathcal{L}^t$.

6.2.3 CWRiG in Action

We will first look at some individual cases that show clear CWRiG patterns, and then look at some statics for all the cases.

[4]There are a few people used phase equivalent to it, such as "has nothing to do with".

Rejecting

Since the rejection of low offers in the UG is our focus, we will start with these subjects.

Subject 34: *"I reject. Why should the other student get 7% and I get 3%. If my classmate can't be fair then he/she should get nothing along with me."*

This subject understand the economic rule well, i.e. "if I reject, we both get nothing; if I accept he will get 7% and I get 3%". At the same time, she noticed that it is not fair and the social rule is important for her. She even expressed it in a conditional at the end. We can say that this is a *rational decision change* as we defined in chapter 5. Her $\mathcal{P}^1$ could be $\{e_1, e_1 \wedge \neg unf \rightarrow q; \top \rightarrow unf\}$. The unf is the disabling factor here.

Another subject gave an answer very close to the kind of non-monotonic reasoning we want to capture in the framework.

Subject 28: *"No, I don't accept because it's not fair, even though it harms me. But if he has his reasons, I will accept it."*

She clearly understood the economic rule, and mentioning a social rule that is a disabling factor. In addition, she would *rationally change her decision* if there are other disabling factors involved.

Subject 7: *"I reject. Because of the unfair portion/ ratio that student has chosen. I will reject to show that inequality between individuals is injustice."*

Not only we can recognize the non-monotonic reasoning that is not captured by conventional rational choice theory, we can also see how we shall improve our framework formally. In fact, during the experiment, people not only bring in other social rules that require non-monotonic processes, they also have to decide which goal is more important. In most game theory experiments, subjects reply with a yes-no answer or in numbers. But as we can see here, there are different subgoals involved in the process. We will also see later that this did not just happen with a few subjects, but on a larger scale.

Accepting

Some subjects show perfect reasoning according to conventional economic theory without mentioning anything related to social rules:

Subject 12: *"I will accept what the student proposed because if I do not we both end up with nothing. It's better to get a lower percentage than no points at all."*

However, the a more typical answers are following:

Subject 43: *Accept. 3% maybe be unfair due to the circumstances, however 3% is better than zero percent.*

Why did the subject mention unfairness but it did not affect their choice? In order to understand that better, we read through all answers in part 2 for such subjects. We noticed that very often they mention importance of grades or related content. We suspect that they have a subgoal to maximize the grades, even though the word 'goal' is not explicitly mentioned. For example, one subject mention that she would accept it, even though it's unfair, and she would accept without hesitance if she knew she had a good grade already.

Now we are ready to look at some statistics on all subjects, which reflect this kind of goal-related processes. Even though, we did not have thorough conversations with our subjects about their reasoning in the UG, they have provided surprisingly detailed answers. Therefore, we did some basic text analysis shown in fig. 6.1. We will first explain the variables in the table.

- **UGq1** is the result of the 1st question in the part 2 of the experiment.
- **UGQ1 Soc** indicates whether a subject has mentioned a social rule, such "50-50" or "share evenly", etc.
- **UGQ1 Eco** indicates whether a subject has mentioned the economic rules behind the UG. Such as "If I reject I will get nothing." or "Accept means I get 3% and the other student gets 7%", etc.
- **Grade** indicates whether the subject mentions that the grade is important.
- **Soc Emp** is social emphasis, which indicates if a subject mentions unfairness more than once, or strongly indicates the importance of social rules. We do not count for the subjects who just mention unfairness once because our survey in part 1 had made unfairness salient to them. So we want to discount this fact.

This result is quite promising. As we can see that UGQ1 Eco is not highly correlated with UGq1. That is because most of the subjects, both who accepted and rejected offers, mentioned about some form of the economic rule. We have to remark on the 'economic' rule here. Usually it is related to maximizing the monetary return. In our case, it is about maximizing the credits. Secondly, mentioning a social rule explains some rejections, as UGQ1 Soc is negatively correlated with UGq1[5], with Pearson Correlation at -0.535. Thirdly, mentioning importance of grade is correlated with accepting, 0.571. Last, and the highly correlated variable is Soc Emp, -0.814. All three are at the 0.01 significance level.

We believe that the last two variables are some kind of subgoals. Further investigation should be done along this line. In addition, the formal framework should be improved to properly show how to deal with competing subgoals.

6.2.4 Additional Information

We had reviewed some results from the 2nd question in the UG, which is given an extra piece of information about the proposer, in this case a congenital heart disease, whether the subject will change his/her mind. Answers from this question surprised us in two ways. First of all, given many people had accept

[5]Mentioning social rules is counted as 1 in UGQ1 Soc (0 not mentioning), while accept in UGQ1 is 1 (0: reject)

Figure 6.1: Correlation between UG and Soc/Eco Rules

		UGq1	UGQ1 Soc	UGQ1 Eco	Grade	Soc Emp
UGq1	Pearson Correlation	1	-.535**	.293	.571**	-.814**
	Sig. (2-tailed)		.000	.051	.002	.000
	Sum of Squares and Cross-products	8.870	-4.800	1.778	3.571	-5.905
	Covariance	.197	-.123	.040	.132	-.144
	N	46	40	45	28	42

** Correlation is significant at the 0.01 level

Figure 6.2: Correlation between Question 1 and 2 in the UG

Correlations

		UGq1	UGq2
UGq1	Pearson Correlation	1	.944**
	Sig. (2-tailed)		.000
	N	46	46
UGq2	Pearson Correlation	.944**	1
	Sig. (2-tailed)	.000	
	N	46	46

**. Correlation is significant at the 0.01 level (2-tailed).

it already, they wouldn't have to change their answer since the proposer was already 'taking advantage'. However, over 90% people answered the question and elaborated it. Second surprise was from the people who did reject in the first question and also chose not to change their answer for the same reason: irrelevance. Let us look at the correlation between the two questions in fig. 6.2. In UGq1, accepting is 1, rejecting is 0. If a subject accepted in UGq1, and does not change her answer for the second question, then her UGq2 is also 1. We can see from the table that answers in the second question is almost completely predictable from the answers in the 1st question, with Pearson Correlation of 0.944 at 0.01 significance level. The result shows that regardless of whether the subject rejects the offer for the first question, he still stick with his domain of unfairness. It fits the kind of monotone reasoning process we described in the model. When there is a new fact added, as long as it is involving any previous rules or alter a new rule as a disabling factor, the decision should not change. Both rejecting and accepting subjects in our case stick with the original decision and give the same rational reasoning that says heart disease is irrelevant.

6.2.5 Unusual Answers

When answering the first question of the UG, there are subjects that give rather unusual reasoning, such as the one below:

Subject 2: *"I would accept because each person can blame themselves for their* *grade. Meaning we have to assume that the two students tried their best in the class. If he makes a selfish decision, that reflects his work ethics. Further more 30% is better than nothing."*

Subject 2 first gave two arguments: 1. students are responsible for their grades because of the work they put in 2. the other student makes a selfish decision and it reflects on his work ethics. It looks like argument 1 is a general rule for him, and argument 2 is a particular case which he faces here. Eventually, subject 2 did come back to the economic rule here, and stated that 30% is better than nothing.

Following subject's reasoning is also different from most of people who accepted the low offer.

Subject 46: *"I will accept the proposal, because my classmate may really need the bonus credits for his grade."*

Her disabling factor is also social related. But it is rather a social requirement on herself towards others. There are actually quite a few such subjects mentioning such reasoning. This kind of reasoning also provides evidence that supports CWRiG. More importantly, it is hidden in the results from conventional UG experiments, simply because subjects cannot reveal disabling factors that have influenced their decisions.

Chapter 7

Discussion

7.1 Conclusion

In this thesis, we raise our concerns on the issue of rationality in game theory, especially in the ultimatum game. In UG, a rejection of an offer above zero is widely considered irrational. We review the rationality from the latest results in cognitive science and from the conventional rational choice theory in economics. An adapted ultimatum game experiment is designed to reveal the rationality in the process, even with a rejection.

Formally, we have defined a game reasoning process CWRiG, which is a collection of programs from CWR. We have limited the possible changes from one program to the next. Only one fact that may alert one new rule is allowed. Through this structure, we can define what a monotone game process is, and more importantly, what can be considered as a rational decision change through non-monotonic reasoning.

We have also conducted an UG experiment, in which we tested subjects' general attitudes towards different unfair situations before an ultimatum game. During the ultimatum game, every subject was being put into a role of the responder, which is our focus. They were asked to explain their reasoning after the choice was made. With these two modification to the experiment, we were able to gather more information about subjects' reasoning patterns. First of all, we can see how preferences are being constructed and related domain is being defined at the time of the task. Secondly, the answers reveal clear evidence of non-monotonic reasoning that explains the rejection. In addition, we have seen some non-monotonic reasoning for accepting the low offer, which was never discovered nor discussed in previous UG games. Last but not the least is the fact that subjects from both accepting and rejecting groups claim that heart disease was irrelevant for the task at hand. It shows both groups are capable of the kind of monotonic reasoning process as well.

This thesis is a call for better structured experiments in game theory that help understand subjects' diverse reasoning patterns. Simple yes-no questions can hardly show what they are really thinking. Even a small change that gives them a possibility to answer why, helps the experimenter analyze their reasoning better.

We believe that non-monotonic reasoning is especially used in applying numerous and complex social rules. More data needs to be collected properly for further investigation.

7.2 Improvements

Now we will discuss a few improvements concerning the experiment design. In our experiment, we had picked a borderline unfair offer with a ratio of 70-30. We shall expect more rejections if offers with the ratios of 20-80 or 10-90 are used. Hence, we can probably get more data indicating the closed-world reasoning.

Second potential improvement is to add a test that checks the comprehension of economic rules. In our case, many subjects voluntarily mentioned it.

Last but not least is to test what is considered abnormal in the location where the experiment is being conducted. In our case, we did not expect a big gap between European students and American students because of the similar results in all other UG studies in these two regions. However, this is probably a strong evidence that more UG experiments should be done in a CWR methodology to find out different situations considered abnormal across different countries.

7.3 Furture Work

So far we have explicitly focused the reasoning of the responder in the ultimatum game. It is our intention to extend experimental work and formal analysis to the proposer and other games, and to create a more complete closed-world reasoning system in games.

The formal framework is still at very preliminary stage. Three issues need further investigation. First is the order of preferences. We suggest to have two different conditionals for economic rules and social rules. But we have not yet addressed how the concern of ordering coming into the picture. In addition, we do not have a formal framework to address a situation in which two rules are competing with each other. From the experimental results, we can see that subjects do set subgoals such as maximizing grades or social justice. How to design a structure that accommodates such pairs of subgoals is interesting to consider.

Second direction would be to address reasoning about others. Throughout this study, we focused solely on the responder because the reasoning is simpler and more responsive in comparison with the proposer's. This epistemic perspective should be addressed in our future work.

Last concern, which can also be a future research direction, is the involvement of monetary units. We have mentioned earlier that our result would be considered more rational in the eyes of economists because smaller percentage of the students reject an unfair offer. It could be caused by the change of game context. In our version, it is about splitting some extra credits which is more related to these students than abstract monetary unit that is often used by economists.

This is certainly only the beginning of a long journey to bring closed-world reasoning into game theory.

Appendix A

Figures

Figure A.1: Part 1 Question 1

Q1a

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	17	37.0	39.5	39.5
	unfair	26	56.5	60.5	100.0
	Total	43	93.5	100.0	
Missing	System	3	6.5		
Total		46	100.0		

Q1b

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	14	30.4	33.3	33.3
	unfair	28	60.9	66.7	100.0
	Total	42	91.3	100.0	
Missing	System	4	8.7		
Total		46	100.0		

Q1c

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	8	17.4	18.6	18.6
	unfair	35	76.1	81.4	100.0
	Total	43	93.5	100.0	
Missing	System	3	6.5		
Total		46	100.0		

Figure A.2: Part 1 Question 2

Q2a

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	7	15.2	15.9	15.9
	unfair	37	80.4	84.1	100.0
	Total	44	95.7	100.0	
Missing	System	2	4.3		
Total		46	100.0		

Q2b

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	6	13.0	13.6	13.6
	unfair	38	82.6	86.4	100.0
	Total	44	95.7	100.0	
Missing	System	2	4.3		
Total		46	100.0		

Q2c

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	10	21.7	22.7	22.7
	unfair	34	73.9	77.3	100.0
	Total	44	95.7	100.0	
Missing	System	2	4.3		
Total		46	100.0		

Figure A.3: Part 1 Question 3

Q3a

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	39	84.8	92.9	92.9
	unfair	3	6.5	7.1	100.0
	Total	42	91.3	100.0	
Missing	System	4	8.7		
Total		46	100.0		

Q3b

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	6	13.0	15.0	15.0
	unfair	34	73.9	85.0	100.0
	Total	40	87.0	100.0	
Missing	System	6	13.0		
Total		46	100.0		

Q3c

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	fair	5	10.9	12.2	12.2
	unfair	36	78.3	87.8	100.0
	Total	41	89.1	100.0	
Missing	System	5	10.9		
Total		46	100.0		

Figure A.4: Part 1 Degree of Unfairness

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
UnfApt	46	0	9	5.93	2.370
Valid N (listwise)	46				

Figure A.5: The rank of unfair situations

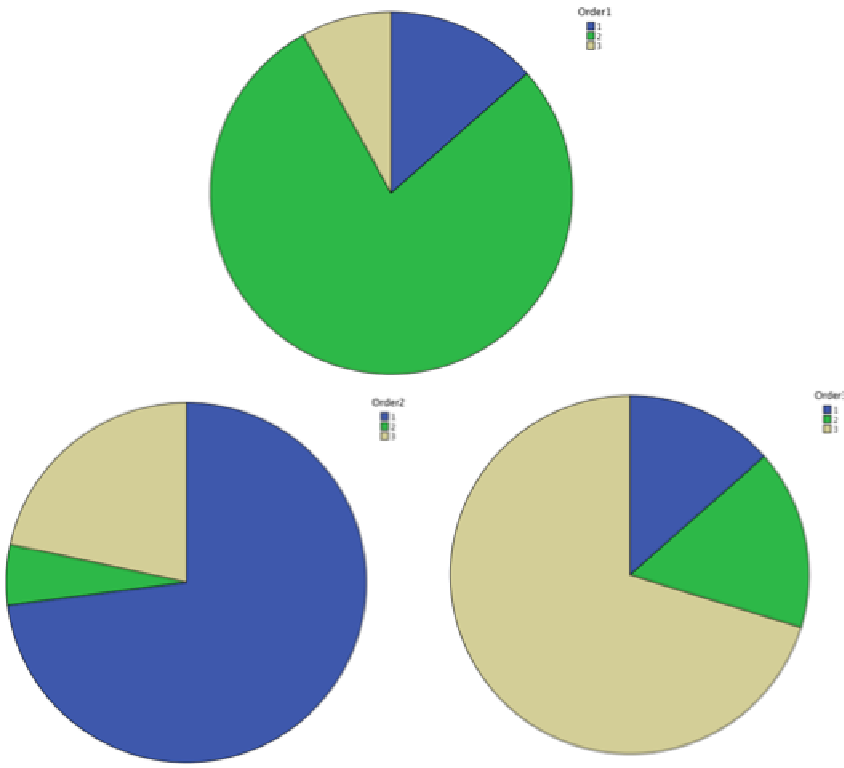


Figure A.6: The rank of unfair situations

UGQ1 Eco

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	not mention	5	10.9	11.1	11.1
	mention	40	87.0	88.9	100.0
	Total	45	97.8	100.0	
Missing	System	1	2.2		
Total		46	100.0		

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