‘Limning the True and Ultimate Structure of Reality’

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Considerations on the (In)adequacy of Quine’s Criterion of Ontological Commitment

MSc Thesis *(Afstudeerscriptie)*

written by

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Abstract

This thesis is a close examination of the role and structure of ontology as conceived of by Quine with a particular emphasis on his (in)famous *criterion for the ontological commitment of theories*. In the first part, we investigate the genesis and role of the *criterion* within the overall structure of Quine's ontology and briefly address some challenges that have been advanced, doubting the adequacy of the criterion for capturing what it intends to capture. We conclude that these challenges are only able to call the criterion into doubt if one also abandons other fundamental constraints that Quine imposes on ontology and that limit what it is that one can reasonably hope to achieve in the study of ontology, given its underlying naturalistic framework.

In the second part of the thesis we consider the objection that the language of regimentation required for the proper workings of the criterion, Quine's canonical notation, is inadequate for the task it was designed to do. We reject all previous attempts at demonstrating this claim and offering an emendation of canonical notation to then present our own vindication of it by showing how two principles active in Quine's ontology conflict and render a modification of canonical notation necessary. We will suggest such a modification and show that it respects all constraints that were imposed by the Quinean framework.

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Contents

Chapter 1

Introduction

Asking *what there is* is asking the *ontological* question. Asking what we are asking when asking the ontological question is asking the *meta-ontological* question.[1] Asking the meta-ontological question is asking how one should understand and go about answering the ontological question. Involving oneself in meta-ontology means asking *what it is* that we are asking when we are asking the ontological question, an answer to it specifies what we do when we do ontology.[2]

Disagreeing about an answer to the meta-ontological question entails giving different interpretations to the ontological question. This not merely means answering the ontological question differently, it means seizing to understand it the same way; it means talking about different things when talking about ontology. A necessary precondition for successful discourse about ontology is therefore a shared answer to the meta-ontological question, the question of how one should go about answering the ontological question.

An answer to the meta-ontological question is provided by the formulation of a *criterion of ontological commitment*. Such criterion yields a guideline of how to talk about things when concerned with ontology. It tests the conformity of discourse to a "prior ontological standard" (Quine 1964a, 15), thereby ensuring that what is talked about is understood the same way, making meaningful discourse about ontology possible: "No discussion of an ontological question [...] can be regarded as intelligible unless it has a definite criterion of ontological commitment" (Church 1958, 1012).

Virtually every contemporary debate concerning meta-ontology originates from, defines itself in opposition to, or deals at least in some way with Quine's notorious criterion for the ontological commitment of theories "to be is to be the value of a variable" (Quine 1964a).[3] It was, in fact, Quine's approach to the meta-ontological question and the answer he gave to it in the formulation of the criterion of ontological commitment that 'single-handedly' restored respectability to the ontological enterprise after the 'destruction' of classical metaphysics and ontology by declaring their questions and problems empty and misconstrued through the logical positivists:[4]

> "If we ask when Ontology became a respectable subject for an analytic philosopher to pursue, the mystery disappears. It became respectable in 1948, when Quine published a famous paper titled "On What There Is." It was Quine who single-handedly made Ontology a respectable subject." (Putnam 2004, 78/79)

[1] Cf. (Inwagen 1998).

[2] Cf. (Inwagen 2009; Inwagen 1998) and (Stokes 2005).

[3] The importance of Quine's criterion for the meta-ontological debate can hardly be overrated. Almost every contemporary discussion about meta-ontology orients itself on the Quinean criterion in some way, witness the essays in (Chalmers, Manley, and Wasserman 2009).

[4] See, e.g., (Carnap 1950).

Despite, or because of, its singular status the criterion has been the subject of much criticism and scrutiny over the last 65 years, although its general tendency has, until very recently, been accepted as correct.[5] The topic of this thesis is the investigation of four related objections that have, in one or another form, been advanced against the *formal correctness* of the criterion of ontological commitment, where *formal* should be understood as roughly meaning 'pertaining to its formulation' rather than to its actual content. I am not concerned with whether the criterion provides us with an adequate answer to the ontological question or even allows us to make sense of the ontological enterprise. Neither am I concerned with whether the result it yields are correct or even plausible, though all these concerns are of course related to the question of whether its formulation is adequate. What I am, however, primarily concerned with in this thesis is the question whether, *based on the assumptions made by Quine*, i.e. within the Quinean framework itself, the formulation of the criterion is adequate for the job it was intended to do. Whether, in other words, assuming the correctness of Quine's remaining philosophical framework, it manages to capture what it intended to capture appropriately, based on the presuppositions made about language, theory and the place of ontology therein.

This way of framing the topic of this thesis motivates two remarks. The theses presented in the next section and which provide the occasion for and structure of the thesis have all been brought forward in the literature. However, they have, for the most part, been motivated from different metaphysical assumptions than the ones present in Quine's system. That means their justification derives from very different assumptions than the Quinean. As such, their refutation in the Quinean system itself does not speak against their overall force, they might be legitimate criticisms of a Quinean criterion in a system based on different fundamental assumptions or a criticism of Quine's overall system. My interpretation of them in the next section should therefore best not be seen as a faithful exegesis of the objections from their original sources, but as an intentional misconstruction, intended to guide the progression of this thesis and to test their validity within the Quinean system itself.

This brings me to my second point. The reader who is likely to, or already does, disagree with any part of the Quinean system pertaining to ontology will most likely find my treatment of the challenges inadequate and unsatisfying, leaving much needed things unsaid. While I agree that, ultimately, the Quinean system as a whole will have to be confronted with questions 'from the outside' and will have to justify itself in the face of whatever motivated these criticisms based on rival systems of the world, such project extends the scope of this thesis. Its goal is much more moderate. It is an investigation of the internal consistency of Quine's system with respect to its treatment of ontology, i.e. by looking at its meta-ontological stance. The reader should therefore keep in mind that we are operating from within the Quinean framework, willfully ignoring objections against this or that aspect of it and taken for granted what Quine accepts to be the case and puts forward as true.

I believe such a project merits investigation for two reasons: on the one hand, it demonstrates the robustness, consistency, immense intertwinedness and internal coherence of the Quinean system, which is often ignored and neglected in discussions about one or other of its aspects. This is a mainly exegetical reason. On the other hand, it points us towards inherent limitations and boundaries of the system itself, ultimately putting us into a position to decide on the basis of what such a system can and cannot do whether we accept it as adequate to our philosophical and scientific practice. Only in this way can we clearly articulate why a framework which accepts, say, possible worlds as real objects should be preferred over a purely naturalistic framework. The discovery of such internal

[5]Fine and Schaffner, for example, can be identified as authors who believe that Quine approached the ontological question from a completely wrong angle.

boundaries allows us to see systematic deficiencies which are only in an obscure way alluded to in various objections to Quine's position.

1.1 The Challenge: Four Objections to the Criterion

In a recent paper[6], Jonathan Schaffer outlines qualms that proponents of a so-called *truth-maker* view about ontological commitment, the view that a theory is committed to whatever has to exist in the world for the theory to be made true,[7] have with the traditional Quinean criterion of ontological commitment.[8] One of these qualms consists in the worry that the Quinean criterion "takes too much of a detour through language" (Schaffer 2008, 8). The uneasiness with such detour is caused by the fact that it makes it seem like "serious ontological questions are being decided by linguistic facts" (Cameron 2008, 5). After all, why should the existence of mathematical objects be decided on the basis of whether we can paraphrase talk of numbers away, rather than based on whether there have to be numbers in order to render mathematical statements true? This concern gives rise to the first objection to be considered in this thesis, namely

(Lang) The criterion of ontological commitment makes ontology dependent (too dependent) on language.

A second objection from the same camp involves concern over the way the criterion of ontological commitment is formulated. Its insistence on first-order quantification and first-order variables as primary bearers of ontological commitment, it is said, biases ontological decisions by construction. It biases them, so the criticism continues, in particular against everything which does not have subject position in a sentence whose ontological commitment is being analyzed:

> "Why should we desert Quine's procedure for some other method? The great advantage, as I see it, of the search for truthmakers is that it focuses us not merely on the metaphysical implications of the subject terms of propositions but also on their predicates." (Armstrong 2004, 23)

This leads to the second thesis considered here, viz.

(Bias) The criterion of ontological commitment builds in certain ontological biases by construction.

Putting matters this way is, I believe, too vague to allow for a satisfying and appropriate response. (Bias) is ambiguous between two readings: in order for the criterion of ontological commitment to be applicable at all, the theories to which it is applied need to exhibit a quantificational structure such that the criterion can gain traction. If this is not the case for any theory it needs to be *regimented* into a notation possessing such quantificational structure in order for us to evaluate its ontological commitment. There are then three points (not jointly exclusive) which (Bias) criticizes: (i) the criterion itself could be biased in that its formulation binds the ontological commitment of a theory only to its use of first-order quantifiers and variables; (ii) the criterion could be biased in that the regimentation of a theory into the quantificational notation prefers certain regimentations over

[6] Cf. (Schaffer 2008).

[7] Such view differs in important points and comes apart from the Quinean 'quantifier-view' in that it crucially depends on what kind of theory of truth-making one has.

[8] He names Armstrong, Heil and Cameron as representatives of such view. In the following, whenever we refer to either of these three authors the passages quoted are quoted from (Schaffer 2008).

others; or (iii) the language of the quantificational notation is biased in that it only allows certain and forbids other kinds of constructions, devices and expressions. The problem of the (process of) regimentation into any notation is a problem that has to be considered separately from the problem of whether it is something inherent in the criterion or its underlying conditions that is inadequate, as there is nothing necessary about any particular way of regimentation.[9] (i) and (iii), however, give rise to two very different claims:

(STRONG.Q) The formulation of the criterion of ontological commitment in terms of first-order quantification is inadequate.

and

(WEAK.Q) The language of regimentation upon which the criterion of ontological commitment is based is inadequate.

Although frequently mixed up and not kept separate in the literature, I believe it is of utmost importance to distinguish between these two ways of understanding (BIAS), since they both afford very different replies. In fact, in the context of Quine's philosophical framework I will argue that (STRONG.Q) is false, whereas (WEAK.Q) is true.

This concludes our outline of the guiding considerations of this thesis. It is the above four claims that inform the further structure and progress of this essay.

1.2 The Setting: Quine's Naturalism

In Chapter 2 below we will outline the framework that determines the nature and status of ontology and ontological debates within Quine's overall system. To understand the reason why ontology arises thusly and can only arise this way, it is important to keep in mind the overall driving force of Quine's philosophizing: its commitment to *naturalism*. Naturalism is relevant to ontology because it dictates the conditions under which we can be said to be doing serious ontology, ontology as a scientific enterprise rather than an unfounded game of make-belief and determines the acceptable methods we can use when going about ontology. Naturalism as the overall guiding principle of Quine's philosophy provides an answer as to how, where, why ontology arises and what constraints it is subject to, as well as delimits the very methodology we can use when evaluating what there is. Its emergence, nature and constraints are subject of the next chapter. Here we wish to very roughly sketch the underlying doctrine of Quine's philosophy, *naturalism*.

In a nutshell, naturalism is the view that there is no 'first philosophy' that has a privileged, vantaged viewpoint which allows us to provide a stable foundation for our system of knowledge, outside of that system of knowledge itself. Accepting that science is the best model of knowledge we possess it is the "recognition that it is within science itself, and not in some prior philosophy, that reality is to be identified and described" (Quine 1981e, 21). There is no way of stepping out of 'our theory', our current view of the world, Neurath's boat, to consider the system of knowledge as a whole and place it on a secure foundation. We need to repair the boat while staying afloat and cannot securely dock in a harbor.

The naturalist seeks "no firmer basis for science than science itself" (Quine 1995a, 16), realizing and accepting that "reasoning within the inherited world theory as a going concern" (Quine 1981c, 72) is the only way to go about reasoning, science and knowledge-acquisition. One cannot leave

[9] We will have to say more about this below, see Sections 3.2.2 and 4.1.2.

one's conceptual scheme, 'our theory', step out of it, so to say, in order to test its correspondence and conformity with some extra-theoretical reality. Just as there is no extra-theoretical knowledge since the only way knowledge generation can proceed is from within a specific theory, there is no extra-theoretical reality. Theory is our best (and only) guide in recognizing reality by providing a way of determining what there is *according to theory.* Without it there would neither be a way of telling nor a way of going about answering the question of what there is.

Science itself did not simply emerge as a way of acquiring knowledge, but developed and is developing – it is a continuation of common sense,[10] permanently evolving and improving. What we pursue in our quest for knowledge is the "scientific study of science itself" (Maddy 2007, 439); we can attempt to obtain "a scientific understanding of the scientific enterprise" (Quine 1966c, 253), always from within science and by means of methods devised by it.

Ontology and ontological questions are, on this picture "on a par with questions of natural science" (Quine 1964c, 45), they are of the same kind and should be answered in the same way and by the same methods:

> "Our ontology is determined once we have fixed upon our over-all conceptual scheme which is to accommodate science in the broadest sense [...]. [...] [T]he considerations which determine a reasonable construction of any part of that conceptual scheme, for example, the biological or the physical part, are not different in kind from those considerations which determine a reasonable construction of the whole." (Quine 1964a, 16/17)

This has consequences for ontology and our conception of it. On the one hand, ontology underlies the same constraints as any other scientific theory; its objects, as well as its methods and methodology might develop and change, grow with the development of science which yields new insights. On the other hand, it is subject to the same norms of confirmation and theory construction as other theories, viz. pragmatic norms of simplicity, economy, familiarity, generality, refutability, modesty and confirmation by observation.[11] How and in what way ontology can and does arise in this naturalistic setting, which we were only able to characterize very sketchily here, is the topic of the next chapter.

1.3 The Structure of this Thesis

This thesis is structured as follows: in Chapter 2 we outline the nature of ontology and its different aspects within the Quinean naturalistic framework. Readers sufficiently familiar with Quine and his philosophy should feel free to skip this chapter. Chapter 3 presents my analysis and its basis in Quine's philosophy of the criterion of ontological commitment as the natural result of the interaction between theory and reference. It concludes with some remarks on why I take (LANG), (BIAS) and (STRONG.Q) to not present any problem for the Quinean and are, in fact, unmotivated and unfounded if one takes the Quinean presuppositions for granted.

Chapters 4 and 5 are concerned with a vindication of (WEAK.Q). To this end, Chapter 4 scrutinizes the development of the canonical notation underlying the application of the criterion of ontological commitment, provides an extensive account of its development and on this basis shows where previous attempts to defend (WEAK.Q) went wrong. To my knowledge and in my opinion, none of the attempts present in the current literature succeed in establishing (WEAK.Q)

[10] Cf. (Hylton 2007).
[11] Cf. (Maddy 2007, 441).

because they all misunderstand the interaction between the criterion of ontological commitment and the considerations that led to the development of its underlying notation.

Chapter 5, then, presents my own vindication of (WEAK.Q) and, I believe, ultimately proving it a legitimate criticism of Quine's account of canonical notation even from within its own system, due to the problematic interaction of principles governing ontological reduction and formal properties of canonical notation. Finally, Chapter 6 concludes.

It should be noted at this point that my main concern in this thesis (and the main original contribution of it) lies in the vindication of (WEAK.Q). This claim and investigation of the feature of Quine's system it mentions have therefore received the greatest amount of attention. The other three claims outlined above, on the other hand, have only received a comparatively superficial treatment. The reasons for this are threefold. For one, space and time constraints did not allow an equal treatment of all claims. Moreover, the defense of (WEAK.Q) as a claim successfully challenging a part of the internal consistency of Quine's system appeared most important to me. Lastly, I believe that the inadequacy of (LANG), (BIAS) and (STRONG.Q) as challenges to Quine's conception of ontological commitment can readily be seen after outlining the main features of Quine's take on matters ontological. We will sketch how his system avoids and answers these objections, but will not spent much time developing them and be content with treating their main thrust and aspects, rather than carefully examining and developing each claim in the same way we treat and develop (WEAK.Q).

Chapter 2

A Framework for Ontology

In W.V.O. Quine's philosophical framework, ontology proceeds by way of language-acquisition and theory-formation from an almost presuppositionless starting point to an elaborate framework of existence assumptions and reductions. In the process of learning a language and conceptualizing the world[1] "[e]ntification begins at arm's length" (Quine 1960b, 1), i.e. proper command of a language is acquired by means of reification, the positing of entities in order to better predict and react appropriately to verbal and sensory stimuli. From there onward we strive toward simplicity and economy in our ontology, 'doing science' and developing theories that systematize, classify and simplify the kind of entities we (need to) assume in order to predict sensory stimulation by regimenting scientific discourse.

The ontology we end up with, on this account, is a consequence of our individual history of acquiring a language[2] and a societal effort[3] to explain past and predict future stimuli in the most reliable and simplest way. Ontology is embedded in this process starting from the learning of language and proceeding via a background conceptualization in order to master the linguistic code, toward a reflection upon the role of that background conceptualization in predicting sensory stimuli leading to its explication and the formulation of constantly improving theories.

The following four sections aim to present a rough overview of the setting in which ontological discourse takes place within the Quinean system and against which the discussions and arguments of the following chapters will be set[4].[5]

[1] Cf. (Quine 1960b, 3): "Conceptualization on any considerable scale is inseparable from language [...]."

[2] In talking about the *individual* history of language acquisition the individuality lies in the way the language is acquired, not however in the language thus acquired. Language itself is intersubjective and thereby an intrinsically social phenomenon: "Language is a social art. In acquiring it we have to depend entirely on intersubjectively available cues as to what to say and when" (Quine 1960b, ix).

[3] For the social facet of language and the 'linguistic division of labour', see (Quine 1960b, Ch. 1) and (Putnam 1973).

[4] The exposition in this section follows in no way the historical development of the topic of ontology in Quine's philosophy. It only aims at providing the systematic background against which to see the discussion of later chapters. For a historical perspective on Quine's views of ontology see (Chateaubriand 2010) and (Decock 2004).

[5] The aim of the following four sections is expository in nature in order to provide the systematic backdrop against what is to follow. Luckily, Quine's writings tend to be very clear and concise, which makes a recourse to secondary literature for the purpose of exposition often unnecessary. Nevertheless, this should not be perceived as implying that the things talked about in this section are universally accepted, quite the opposite, most of them are highly contested. Space constraints forbid to discuss but very few and immediately relevant criticisms to Quine's position, however, the most influential objections over the last 60 years have been collected in various essay collections. The reader is referred to (Grandy, Davidson, and Hintikka 1973), (Shahan and Swoyer 1979), (Quine, Barrett, and Gibson 1990), (Leonardi and Santambrogio 1995), (Orenstein and Kotatko 2000), as well as the collections (Follesdal 2001) and (Hahn and Shilpp 1999).

2.1 From Language to Being: Reference

Ontology, according to Quine, deals with questions about *what there is* (Quine 1964a). Factoring in the human perspective these questions are more properly about *what we assume there to be.* A good place to start enquiring into ontology therefore appears to be to ask what the assuming of objects consist in. However, "[t]o ask what the *assuming* of an object consists in is to ask what *referring* to the object consists in" (Quine 1981e, 2) – we ascent to a semantical plane and talk about words and their functions, rather than about what those words appear to talk about directly[6], as this bears the methodological advantage of approaching the problem from a less 'loaded' angle and does not cause any serious distortion; after all, we assume there to be black holes if and only if the term 'black hole' refers.

Having shifted the weight of ontology thus onto the question of what *referring to objects* consists in, i.e. *how* reference works, a good starting point for approaching the question lies in the *genesis of reference*, i.e. in the question when and how reference comes into being. Since it is primarily linguistic expressions (or agent's using linguistic expressions) which we take to refer[7] the genesis of reference is to be located in the process of language-acquisition.

The pre-linguistic infant's initiation into language cannot be afforded on the basis of linguistic means alone, for there is no language to build on – the starting point has to be something common to infant and initiator. The infant's, as well as everyone else's sensory surfaces, are permanently impinged by energy, subjected to constant stimulation: light, sound, touch, they all trigger sensory receptors.[8] The "babbling period of late infancy" in which the infant displays "random vocal behavior" (Quine 1960b, 80) affords the experienced language user in the vicinity of the infant the opportunity to reinforce certain vocalizations in the presence of appropriate stimulatory patterns.[9] The child is conditioned to respond to certain patterns of stimulation with particular noises. These initial vocalizations could include "Mama" upon the stimulation of the infant's retina in the (visual) presence of her mother, or even "It's raining" upon stimulation of her tactile sensory surfaces by something wet. Utterances of this kind Quine calls *observation sentences.* An *observation sentence* is a linguistic unit that can be considered true on some and false on other occasions (Quine 1960b; Quine 1992b), on which speakers of the language in which the sentence was uttered can agree outright upon witnessing the circumstances under which it was uttered (Quine 1992b, 3), i.e. having experienced the situation in which the sentence was uttered they will (uniformly) assent or dissent to the sentence uttered.[10]

Observation sentences are uttered, assented to or dissented from on the basis of stimulation of sensory receptors, they can be "conditioned outright to distinctive ranges of sensory intake" (Quine 1993, 410).[11] "Observation sentences in this pristine purity are the child's port of entry to cognitive

[6]Cf. (Quine 1964a) and (Quine 1960b, 270): "It is the shift from talk of miles to talk of 'mile'."

[7]Ignoring cases of natural meaning a la Grice (Grice 1957), where smoke could be said to refer to fire.

[8]Cf. (Quine 1995a) and (Quine 1993, 410): "Our channel of continuing information about the world is the impact of molecules and light rays on our sensory receptors; just this and some kinesthetic incidentals", as well as (Quine 1992b, 2): "immediate input from the external world, which is [...] the triggering of our sensory receptors."

[9]Cf. (Quine 1969a, 81): "A child learns his first words and sentences by hearing and using them in the presence of appropriate stimuli."

[10]There are 'degrees of observationality' to observation-sentences: language-users with a broader theoretical and linguistic background might dissent from certain observation sentences under circumstances under which other speakers of the same language would assent. This problematizes the notion that observation sentences are mere responses to stimuli. Nevertheless, all the essential properties of the concept of an observation sentence can be salvaged, see (Hylton 2007, pp. 136), (Quine 1992b) and (Quine 2000). See also (Quine 1969b) and (Quine 1981e) for another problematization of observation-sentences and stimulus-similarity.

[11]The range of stimulations associated with an observation sentence Quine calls its *stimulus meaning* (Quine 1992b; Quine 1960b).

language, for it is just these which he can acquire without the aid of previously acquired language" (Quine 1993, 411). Having identified the entering wedge into language as linguistic expressions directly keyed to sensory stimulation the question where in these rudimentary beginnings of language we encounter reference arises. The answer is 'nowhere': on the picture outlined above the way language comes into contact with the extra-linguistic reality is not by means of *referring to that reality or parts thereof*, it is rather *by relating (through conditioning and reinforcement) a series of noises with the stimulatory circumstances under which a community of speakers (in some sense) approves the utterance of these noises and reinforces them.*[12] The fundamental contact between language and reality is thus not referential, it consists in the relation between an utterance and circumstances to which it is conditioned.[13]

Another way of making the same point is the following: so far we have been talking about observation *sentences*. These linguistic utterances are clearly not sentences in any syntactical or grammatical sense (witness 'Mama', 'Red', 'Hungry'), nevertheless, a clear criterion for their 'sentencehood', capturing the essence of any aspired syntactical or grammatical criterion, can be given: a sentence is any linguistic expression, any unit of language, in fact any series of noises (or signs) whatsoever, that is *significant* by itself, without any other linguistic context, i.e. independently of other expressions capable of being assessed as correct or incorrect in terms of being assented to or dissented from. Even a one-word observation sentence such as 'Mama' is, on this account, a *sentence* rather than a *term*, as it can be assessed as true or false on its own, without other linguistic context. It is *terms*, not sentences, that refer and require linguistic context in order to say something true or false, affirmable or deniable, about the objects they refer to. However, at the present linguistic stage of the infant we can make good sense of the concept of a sentence – a linguistic unit that, by itself, can be assented to or dissented from, – not, however, of the concept of a term: essential of a term is that it is subsentential, i.e. that it can feature in different sentences and linguistic contexts in a structured way, by contributing something, its reference in our case, to the meaning of the whole.[14]

However, the observation sentences the infant has learned to master at this stage are "expression[s] that he has learned to associate *directly* (my emphasis) with some range of stimulation" (Quine 1994a, 450). They are not learned piece by piece and composed part by part, but as unstructured wholes, units without significant parts:[15] "they are associated as wholes to appropriate ranges of stimulation" (Quine 1992b, 7); "[a]s a response to neural intake, the sentence is *holophrastic*: the neural intake is keyed to the sentence as a *monolithic whole* (my emphasis)" (Quine 1993, 411). The point here is that no explanatory value can be found in ascribing the learning of terms to the infant, there simply is no ground for doing so, whereas good sense can be made of treating her initial utterances as sentences. It is because of this that "[i]t is occasion sentences, not terms, that are to be seen as conditioned to stimulation" (Quine 1981e, 20).

The 'genetic priority' of sentences over terms in the process of language acquisition renders them the primary vehicles of meaning: "[s]entences come to be seen as the primary repository of meaning, and words are seen as imbibing their meaning through their use in sentences" (Quine 1981e, 3). Terms are to be found by extrapolating from sentences, terms are dependent on sentences for their

[12] Cf. (Hylton 2006, pp. 119).

[13] Ibid. p. 120. See also (Hylton 2007, pp. 147). This implies that the notion of *reference* is derivative and that therefore, rather than just having to accept it as primitive, an informative account and explanation of it can be given.

[14] The fact that 'Mama' cannot refer to an object in the case of the infant can easily be seen by realizing that the utterance of 'Mama' is on par with 'It's raining', after all, they are both learnt the same way. But then what does the latter sentence refer to? The better explanation consists in admitting that the concept of reference simply makes no sense at this stage.

[15] Cf. (Hylton 2007, 166).

meaning.[16] This reverses the traditional picture: terms are learned by breaking down sentences, not sentences by composing terms. Similarity in sound or sign patterns justifies the positing of subsentential structure, identifying terms therein. The necessity of breaking down sentences into parts is dictated by the complexity of language: it would be forever impossible to master the linguistic code of a society, to generate and understand infinitely meaningful sentences, on the basis of conditioned linguistic responses, i.e. observation sentences, alone – adopting the sentence-to-sentence links so essential to language beyond the merely observational level, let alone constructing theory, would be unattainable. We are therefore "bound to attribute significance to constituent parts of sentences because it is only in that way that we can see the patterns and analogies in language that make it possible for us to use it," as "it will in fact be impossible for us to use the theoretical sentences correctly without an analysis of those sentences into significant parts" (Hylton 2006, 137).

How sentences are to be analyzed into parts is, however, a different question and issue. The guiding principle in breaking down sentences into parts is methodological, rather than substantial: based on the available fund of observation sentences learnt by means of stimulus conditioning sentences should be broken down into subsentential parts in such a way as to maximize simplicity, economy and predictive power. Sentences need not be broken down as far as possible into, e.g., phonemes or letters and 'Bachelor or Arts' can be treated as one term, given that predictions of future and explanations of past verbal behavior do not improve through this breakdown. At the same time they need to be broken down until a maximum of predictive power for verbal behavior is reached; it is, e.g., not enough to stop at the term 'horses' since further predictive power could be achieved by analyzing it into the singular 'horse' and plural ending '-s'. The theory underlying the analysis of sentences into part is subject to the same criteria as any other theory (see next section).

What becomes obvious by putting things this way is that theory intervenes when sentences are broken down into parts – here mere stimulation and conditioning are not sufficient any more, what is needed is systematic superstructure without any immediate contact to reality (after all, breaking down the sentences into terms is dictated by methodological criteria rather than any directly observable features of reality).[17] However, theory is underdetermined by evidence:[18] there might be multiple, mutually incompatible, yet equally 'good' ways of analyzing sentences into parts, agreeing on and accounting for all past, present and future evidence; different and irreconcilable ways to attribute structure to them and significance to their parts.[19] Moreover, since the decomposition of sentences into parts occurred on the basis of theoretical, pragmatic virtues alone, without any recourse to an objective reality, there is no way of deciding which analysis is correct: "there is no fact of the matter" (Quine 1969d; Quine 1978).[20] However, insofar as several ways of analysis are equally good and justified and due to the fact that the referential apparatus of language is part of the theoretical superstructure rather than present during the immediate contact with reality

[16] Underlying the conviction that it is possible to break down sentences in order to recover meaningful terms is of course the view that the meaning of a term is exhausted once we have specified the contexts in which it can appear.

[17] Mediately it is connected through the observation sentences, of course (see next section).

[18] Cf. (Quine 1964c), (Quine 1975). For what qualifies as *evidence* see the next section.

[19] For examples, see (Quine 1969d, pp. 35). See also (Quine 2008j, 340): "Moreover, all empirical support for the theory will remain undisturbed. For, consider the nature of empirical support. It resides ultimately in sensory stimulation that has been associated with appropriate sentences, appropriate strings of sounds; and those strings of sounds remain unchanged, down to the last phoneme [...]."

[20] This point is frequently put in terms of *radical translation* and illustrated by means of a field-linguist encountering an unknown tribe speaking the unknown language Jungle and observing members of that tribe uttering "Gavagai" in presence of a rabbit. The analyses of sentences into parts become manuals of translation in this thought-experiment. However, there might be incompatible, yet equally good translation-manuals accounting for all the linguistic evidence. Cf. (Quine 1960b) and (Quine 2008g). See also (Quine 2008m) and (Quine 2008h).

through observation sentences (see above), reference becomes *inscrutable*[21] and ontology relative.[22] This topic will be taken up in Section 2.3 again.

It is important to note at this point that the inscrutability of reference does not constitute a problem for the determination at what stage and how reference intrudes into language: the different ways of analyzing and breaking down sentences into terms disallow a clear answer as to which parts exactly of a language constitute the referential apparatus, but we can still answer the question what the referential apparatus consists in, i.e. through which mechanisms it works. We are now at the following point in our exposition: while "the development leading from sensory stimulation to objective reference is to be seen as beginning with the flat conditioning of simple occasion sentences to stimulatory events" (Quine 1981e, 3) this direct association with stimulation does not constitute objective reference, all we have at this point is conditioned responses, and equally conditioned assent and dissent. Reference enters with the breaking down of observation sentences into parts, the ascription of termhood to subsentential units. This is also the place where theory and indeterminacy enter: "[s]een holophrastically, as conditioned to stimulatory situations, the sentence is theory-free; seen analytically, word by word, it is theory-laden" (Quine 1992b, 7); "[t]erms and their objects are adjuncts of the elaborate superstructure" (Quine 2008d, 361) – reference only emerges after super-imposing a, in some sense arbitrary, structure on sentences.

How does the learning of language proceed after the conditioning of observation sentences? While the extension of the infant's linguistic capabilities and knowledge is based on the meagre stock of observation sentences and the contained observational vocabulary to which she was conditioned, it does not proceed thusly, she would never be able to reach an elaborate mastery of language. "The superstructure is cantilevered outward from that foundation by imitation and analogy, by trial and error" (Quine 1978, 274). A possible progression to an elaborate grammatical superstructure is illustrated by Quine. It is important to notice that nothing hinges on this particular account, it does not matter whether it is correct or not and whether this is the actual order in which grammar is learned. *Nothing depends on the empirical adequateness of this account!* In fact, it is most likely inaccurate or even in plain disagreement with empirical findings, but this does not diminish its purpose of illustrating how a stage of reification and objective reference is reached that cannot be accounted for in any other way, but to accept reference and the positing of objects. Due to this it is instructive to briefly recapitulate it.

The first stage on the long road to objective reference was the one outlined above: through reinforcement verbal units are conditioned to stimulatory input. As it is possible to master this basic stage of language without the need to invoke objective reference in its learning and explanation, the mastery of observation sentences does not present a sufficient condition for having learned to refer, "for this learning consisted simply in learning the circumstances in which to assent or to dissent from the sentences" (Quine 1974, 65).

Central to the second stage is the occurrence of the notion of object.[23] While direct reference might not yet be trackable, it becomes impossible to explain the linguistic phenomena without recourse to a notion of (physical) object. This phase is marked by the advent of *individuative terms*, such as 'apple' or 'dog'. Particular to these terms is that they possess an inbuilt mode of *divided reference*; to fully master a word like 'apple' "it is not sufficient to learn how much of what goes on counts as apple; we must learn how much of what goes on counts as *an* apple, and how much as another. Such terms possess built-in modes [...] of dividing their reference" (Quine 1960b, 91).

In other words, the proper command of individuative terms cannot be learned on the basis of

[21] Cf. (Quine 1969d, 35): "Reference itself proves behaviourally inscrutable."

[22] For the different kinds of indeterminacies playing an important role in Quine's philosophy and their interaction see (Quine 2008m).

[23] "The second phase [...] is where a proper notion of object emerges." (Quine 1969e, 12)

sensory stimulation and conditioning alone, other means are necessary. Based on the vocabulary occurring in the reinforced observation sentences, e.g. 'Dog', 'Fido', expressions are combined in what will later be considered as *predication*, the joining of singular and general terms to form meaningful wholes, e.g. 'Fido (is a) dog'. While utterances of this type can still fully be accounted for by stimulation-reinforcement mechanisms (when both stimulatory circumstances are present the utterance of both conditioned responses is rewarded and thereby reinforced (Quine 1974, pp. 63)), the proper usage of the one term as general presupposes more. In order to master the linguistic code and to evolve a "coherent pattern of usage [...] matching that of society" (Quine 1960b, 93), the child needs to be able to involve in discourse about 'that dog', 'not that dog', 'a dog', 'the same dog', 'a different dog', etc. Mastery of these devices of individuation prove command of the concept of divided reference.[24]

However, the child can never "fully master 'apple' in its divisive use, except as he gets on with the scheme of enduring and recurrent physical objects" (Quine 1960b, 92), i.e. to fully grasp and command a term such as 'apple' in accordance with society, the child needs to be able to apply concepts such as recurrence, identity, discernability. In other words, it needs the notion of an object: "Once the child has mastered the divided reference of general terms, he has mastered the scheme of enduring and recurring physical objects" (Quine 1960b, 94-95). It is worth noting the connection between the notion of an object and identity here. Identity is fundamental to the concept of object, the latter means knowing when something begins and ends, i.e. how long it is identical to itself, when it is different from other things, i.e. when it is not identical to another object, etc. In fact, the two notions are so closely intertwined that it is impossible to have one without the other; knowing what an object is means knowing when/how long one thing is identical to itself and not to another thing, knowing when something is identical (to something) means knowing when something is an object. Entification (proper) means determining identity conditions for what was thus reified and stating conditions under which things are identical or differ amounts to positing objects. Given then that mastering divided reference is grounded in the notion of object, "[i]dentity is intimately bound up with the dividing of reference. For the dividing of reference consists in settling conditions of identity: how far you have the same apple and when you are getting onto another" (Quine 1960b, 115).[25] Hence, "[i]t was only after getting the knack of identity and kindred devices that our own child could reasonably be said to be talking in terms and speaking of objects" (Quine 1969e, 18). The importance and fundamentality of identity in the genesis of reference can thus hardly be underestimated.[26] The previous amounts to saying that we can only be taken to have posited an object once we have brought it into interplay with the *apparatus of individuation*, the grammatical particles of identity, plurals, etc. (Quine 1960b, 236).

Having thus mastered the concept of an object, there is now something to make reference to. The third stage[27] of the ontogenesis of reference does just that by utilizing demonstrative singular terms, such as 'this apple' picking out (or attempting to pick out, as reference can fail at this stage) an object (Quine 1969e, 12). Note that reference is still rudimentary at this stage in that all that can be referred to are things 'given' by or occurring in the observation vocabulary.

In a fourth phase this hurdle is still not quite overcome: the attributive conjoining of general

[24]Cf. (Quine 1960b, pp. 92).

[25]Cf. also (Hylton 2007, 172) and (Quine 1969e, 19): "the very use of terms and the very positing of objects are unrecognizable to begin with except as keyed in with idioms of sameness and difference."

[26]It is precisely this basicness that justifies the demand to be able to provide sensible identity conditions for any entity one intends to posit, sloganized in "No entity without identity" (Quine 1969e; Quine 1995a). See also the next section.

[27]According to (Quine 1969e). This stage was subsumed under other stages in (Quine 1960b). As previously mentioned, nothing depends of the correct or incorrect order and occurrences of these stages, they are more a heuristic device than a description of fact.

terms, creating, for the first time, general terms that might fail to be true of anything, such as, e.g., 'blue apple' or 'round square',"brings mass production of general terms, far outrunning the objects of reference" (Quine 1960b, 109), does not, however, provide reference to any new sort of object. Their denotata, if they have any, are just the same old objects already denoted by either of the compound terms: a blue horse is still a horse and blue.[28]

This shortcoming is remedied in the fifth (fourth in *Word and Object*) phase. Here, *relative terms*, general terms holding of multiple objects, e.g. 'smaller than', 'part of' or 'brother of', are conjoined to other singular or general terms, allowing for terms such as 'smaller than that dog' that, by means of analogy and extrapolation, allow to transcend the domain of merely observable objects without being confronted by the charge of incoherence.[29]

There is a crucial step in the fifth phase in that the realm of reference is extended beyond the observable. Ostension is superseded by the positing of entities.[30] The applying of relative terms to singular terms is a crude method of forming terms that purport to refer to unobservable objects. A much more flexible and direct method is provided through *relative clauses* and *description* (Quine 1969e, 13). A relative clause has the form of a sentence in which a pronoun occupies the spot where a singular term would be needed to complete that sentence. The extraordinary flexibility and openness of the relative clause allows for the positing of entities that differ not only in degree ('smaller than x', 'brother of y') but also in kind from previous objects. In fact, any object that can be described by a description D can be posited, witness the relative clause 'which is D'. It is due to the fact that the relative clause stands in no direct correlation with sensory stimulation that affords it such generality and flexibility. This neutrality towards stimulatory patterns[31] is evidence for the fact that "reference is at hand, full blown and unmistakably, only with mastery of the relative clause" (Quine 2008d, 360); it is here that the full explanatory force of the apparatus of reference is needed in order to account for the linguistic phenomenon of the relative clause.

The status of the relative clause, the pronominal construction,[32] as "the root of objective reference" (Quine 1994a, 451) is underpinned by considering sentences which feature an *essential pronoun*, rather than a *pronoun of laziness*:[33], a pronoun that is not replaceable by its grammatical antecedent (see next section). What is needed to master this linguistic construction is the positing of a carrier. This entity is what is referred to by the pronoun. Reference, and for that matter, reference in its fullest and most unrestricted form, is needed to explain and master the full scope of relative clause constructions. It is here that reference is present in its most complete and fullest manifestation.[34]

However, while reference thus develops naturally to its most full-blown form in a community of speakers this does not yet bring us closer to ontological considerations, for "a fenced ontology is just not implicit in ordinary language." This is because "[o]ntological concern is not a correction of lay thought and practice; it is foreign to the lay culture, though and outgrowth of it" (Quine

[28] Cf. (Quine 1960b, pp. 108), (Quine 1969e, pp. 12).

[29] The non-existence of observable blue horses is tantamount to the non-existence to blue horses, however, the non-existence of observable things smaller than that apple not necessarily to the non-existence of things smaller than that apple (Quine 1969e, 13). Constructions as the one outlined above therefore allow us to legitimately extend our ontology to the realm of unobservable things. See also (Quine 1960b, 109).

[30] Cf. (Decock 2002b, 181).

[31] The words that feature in it are of course still ultimately grounded in stimulation (as everything is ultimately grounded in stimulation), but the objects one is able to posit by means of the relative clause stand in no direct or indirect contact to the objects immediately grounded in sensory stimulation any more.

[32] In the relative clause the channel of reference is the relative pronoun 'that' or 'which', together with its recurrences in the guise of 'it', 'he' or 'her"' (Quine 1981e, 8).

[33] Cf. (Geach 1962).

[34] From here on we can continue and begin referring to abstract objects (6th phase (Quine 1969e)).

1981e, 9). “Putting our house in ontological order is not a matter of making an already implicit ontology explicit by sorting and dusting up ordinary language. [...] It is in deliberately ontological studies that the idea of objective reference gains full force and explicitness. The idea is alien to large parts of our ordinary language” (Quine 1974, pp. 89). We therefore have to turn to reflected, theoretical considerations on ontology, considerations in which the impact and import of different terms is carefully weighed and made explicit: theory.

2.2 From Stimulation to Science: Theory

We located the appearance of reference in the emergence of the relative clause – it was there that it fully and undeniably manifested itself. Having thus determined *when* reference occurs, it remains to be seen *where* it occurs, or, in other words, we have seen what referring to objects, and thereby the assuming of them, consists in, now we have to settle what objects are actually referred to and thereby assumed. We can not simply collapse the where and when in this case unless we wish to trivialize ontological enquiries: we cannot find ontological doctrines in ‘lay thought and practise’, i.e. ordinary language, as indicated at the end of the last section because grammar is a bad guide in devising an inventory of the world: “Should we regard grammar as decisive? Does every noun demand some array of denotata? Surely not; the nominalizing of verbs is often a mere stylistic variation” (Quine 1981e, 9).

So where to draw the line? When to consider the occurrence of a relative clause as genuinely referring and when to regard it as mere stylistic device? With respect to ordinary language “[i]t is a wrong question; there is no line to draw,” for there is only “a succession of dwindling analogies. Various expressions come to be used in ways more or less parallel to the use of the terms for bodies [...]; but there is no purpose in trying to mark an ontological limit to the dwindling parallelism” (Quine 1981e, 9). Grammatical considerations provide a good indicator of how to look for genuine reference – namely, in the use of relative clauses, – they do not yet yield the place where to look.

Successful agency relies on the ability to anticipate and predict. Linguistic agency concerns mastering a language, anticipating and predicting proper verbal responses to verbal and non-verbal stimuli. Successful prediction has to rely on more than random chance and the conditioning of observation sentences – we already saw that total dependence on nothing but observation sentences could only bring us so far in the explanation of how the linguistic code of a society is mastered. Reacting properly to verbal and non-verbal stimuli is related to drawing the correct inferences from the stimulatory input. Conditioning to observation sentences can still fully account for mastery of the implication ‘Whenever there is a raven, there is a black raven’, after all, just because a visual stimulation triggers assent to the sentence ‘There is a white raven’ this does not conflict with assenting to the statement ‘There is a black raven’ – nothing in the verbal behavior is evidence that the conditional was not understood. However, a shift to ‘whenever there is a raven, *it* is a black raven’ changes the situation. Mastery of this implication cannot be explained by reference to dispositions to assent and dissent to certain observation sentences alone anymore: I might be prepared to assent to ‘There is a black raven’ whenever I see a raven, no matter whether it is black or not, I will not be prepared to assent to ‘It is a black raven’ upon seeing a white specimen (Quine 1995a, 27). The “crucial leap to reification” and the justification to speak of *objective reference* depends on the pronominal construction, the pronoun provides an additional link between the component sentences that can only be accounted for by positing an object: in order to fully account for its comprehension we need to accept the positing of a common carrier of different traits, in this case ‘ravenhood’ and blackness (Quine 1995a, pp. 28). Once again we see that mastery of the relative construction requires reference and reification.

Above described 'leap to reification' proceeds by way of transition from *free* to *focal observation categoricals* (Quine 1995a; Quine 1992b). A free categorical is one in which there occurs no pronoun, or a *pronoun of laziness*, in a focal categorical the occurrence of the pronoun is *essential*:[35] in 'I bumped my head and it hurts', the pronoun is one of laziness, it could be replaced by its grammatical antecedent 'my head' without altering the truth-conditions of the sentence.[36] However, in 'There is a raven and it is black' the pronoun is essential and cannot be supplanted by 'a raven' without significantly weakening the message of the sentence (Quine 1995a, 29). An *observation categorical* on the other hand is a generality compounded of observation sentences, "[i]t is a generality to the effect that the circumstances described in the one observation sentence are invariably accompanied by those described in the other" (Quine 1992b, 10), "a generalization built onto observation sentences, to say that fulfillment of the one observation sentence is invariably attended with fulfillment of the other" (Quine 2008l, 330)[37].[38]

The importance of observation categoricals derives from two facts: on the one hand they maintain almost immediate contact with reality due to being composed of observation sentences which are directly keyed to sensory stimulation, our only access to reality. On the other hand, they are no mere occasion sentences, assented to on some and dissented from on other occasions. Due to their generality and universal force they are *standing* or *eternal* sentences, true or false once and for all. This, in turn, has two important consequences: as depending on observation sentences their falsity can be tested and as standing sentences they can be implied by theories, sets of standing sentences. While an observation categorical can never be conclusively verified, as it relies on inductive evidence, it can be refuted and is directly testable against reality: one only has to induce the stimulatory conditions corresponding to the antecedent observation sentence and see whether the sensory input connected with the consequent observation sentence obtains. If not, the observation categorical does not hold and can be considered refuted. If it does, this provides further evidence for its possible truth, although it will never conclusively verify it.[39] Moreover, as a standing sentence observation categoricals are capable of being implied by theory.[40] This makes them the "ultimate empirical checkpoints of science generally" (Quine 1995a, 44) and renders them the arbiter between high-level theoretical generalities and reality – they are the point of connection between world and theory and therefore present the basis and evidence for any theory.

Observation categoricals constitute a first stage in facilitating successful agency in that they help predicting sensory impressions following other sensory impressions and thereby enable appropriate reaction upon sensory stimulation in form of the antecedent condition. However, the strength of observation categoricals in their immediate connection to stimulation is also their greatest weakness – their lack of generality makes it impossible to ever act successfully in a community of evolved agents on the basis of these predictatory devices alone, it would be forever impossible to learn enough observation categoricals in order to act appropriately. Further ascent and generalization is required. By induction and hypothesis, habit-formation and simple conditioning *theories*, sets of standing sentences expressing generalities, establish themselves as guide to human behavior –

[35]Cf. (Geach 1962), as well as (Quine 1995a) and (Quine 1994a).

[36]In such case "[t]he observation categorical just asserts concomitance or close succession of separately specified phenomena." (Quine 1994a, 451).

[37]Note that mastery of an observation categorical does not presuppose knowledge or understanding of logical implication. All that is needed are conditioned expectations.

[38]For the development of the notion of an *observation categorical* (Quine 1995a; Quine 1992b) over the notions of *pegged observation sentences* (Quine 1975) and observation sentences (Quine 1969a), see (Hylton 2007, pp. 179).

[39]Quine described the observation categorical as a miniature scientific theory (Quine 1995a, 43).

[40]A theory does not imply occasion sentences because these are true on some and false on other occasions, whereas the sentences a true theory implies are true simpliciter.

we are doing science.[41] The great generality of theories enables far-reaching predictive power and therefore adequate agency.

Theories are obtained by means of induction and hypothesizing, processes that are everything but infallible. They are connected *upward* to reality through the sharing of vocabulary with observation sentences (Quine 1981b, 26) and *downwards* through the implication of testable observation-categoricals. If some of the implied observation categoricals turn out to be false, the theory needs to be revised and modified. Moreover, there is the possibility that multiple theories can be constructed on the basis of the data available (this is the so-called thesis of the *underdetermination of theory by evidence*, we will come back to it in the next section). On what basis, then, does one construct, revise and choose between theories? As a general principle, simplicity in its many aspects serves as guide: a new hypothesis introduced should be *conservative*, i.e. 'sticking with what is already assumed', as *general* as possible, as *simple* as possible, *modest*, i.e. stick to 'small' explanatory steps, and *refutable* (what would be the point of introducing it otherwise?).[42] Retraction of hypotheses should obey a *maxim of minimum mutilation*, i.e. that hypothesis should be retracted which influences the other sentences in the theory least and blocks the inference of the critical observation categorical.[43]

As for the construction of theories "[s]implicity [...] is what guides [...] extrapolation" (Quine 1966c, 247): a theory should be *simple*, "empirical laws concerning seemingly dissimilar phenomena are integrated into a compact and unitary theory", its principles should be *familiar*, i.e. respecting theoretical principles already in place and accepted, it should be *wide in scope*, "the resulting unitary theory implies a wider array of testable consequences than any likely accumulation of separate laws would have implied", it should be *fecund* in that "successful further extensions of theory are expendited" and, of course, it should be *successful* in that "such testable consequences of the theory as have been tested have turned out well" (Quine 1966c, 247). This also provides us with a heuristic of how to choose between competing theories: choose the one that does better with respect to the criteria outlined above (in case they do equally well or better on different accounts, see the next section). Of course, sometimes faithfulness to one aspect can only be reached at the expense of another aspect. In that case one has to balance advantages and disadvantages against each other.

Science and its theories enable more and more successful prediction of stimulatory events and therefore prove indispensable in human agency and interaction. After having become the focus of reflected enquiry in science, common-sense theories – such as the one about ordinary middle-sized objects, – become refined, more general, comprehensive and a more powerful guide to reality: science is common sense gone self-conscious (Quine 1960b, 3). Nevertheless, their origin in the humble observation-categorical as point of contact with reality necessitates some observations, crucial for the further undertaking of this thesis.

Successful agency within the world is only possible if the predictions of the theory match the ongoings of reality, if and only if the theory in some sense says something true about reality (that this and this stimulatory event will be followed by this and this stimulatory event): "science is a linguistic structure that is keyed to observation sentences here and there" (Quine 2008i, 268). The *empirical content* of a theory, the things it says that establish the contact with reality, is the set of

[41] Cf. (Quine 1981b, 28). Note that we are mainly concerned with *why*, not *how* we adopted theories. The why is increased predictive power and thus an increased ability to orient oneself in the world: "The utility of science [...] lies in fulfilled expectation: true prediction" (Quine 2008i, 258). The how is given the vague answer in terms of evolution, the exact details are lost in the shrouded history of the race.

[42] Cf. (Quine 1992b, 20). For a more worked out account of these criteria, as well as theory testing and construction in general see (Quine and Ullian 1970).

[43] Cf. (Quine 1992b, 14): "[S]o the choice of which of the beliefs to reject is indifferent only insofar as the failed observation categorical is concerned, and not on other counts."

observation-categoricals it implies (Quine 1981b, 30).[44] A theory is a set of sentences.[45] So the net-empirical import of a theory is the sum of the empirical content of the sentences that comprise it? Emphatically, no! For consider the chemist who, upon removing a green colored litmus paper from a liquid, exclaims "There was copper in it".[46] In a community of chemists this statement qualifies as an observation sentence.[47] It is part of the observation-categorical "Whenever litmus paper turns green in liquid, there is copper in the liquid", which in turn is implied by chemical theory. However, there is no single sentence in that theory implying this particular categorical. Instead, it is implied through a complex interaction of sentences from various areas of human knowledge, including, in this case, chemistry and logic: "[w]hat comes of the association of sentences with sentences is a vast verbal structure which, primarily as a whole, is multifariously linked to non-verbal stimulation" (Quine 1960b, 12). Sentences alone and in isolation do not imply categoricals: "[t]he observation conditionals cannot be distributed as consequences of the several sentences of the theory. A single sentence of the theory is apt not to imply any of the overvation conditionals" (Quine 1981c, pp. 70). We therefore shift from sentences to sets of sentences as the primary repository of empirical content, "[w]e come to recognize that in a scientific theory even a whole sentence is ordinarily too short a text to serve as an independent vehicle of empirical meaning[, as i]t will not have its separable bundle of observable or testable consequences" (Quine 1981c, 69). In fact, Quine propounds a holistic doctrine of knowledge:

> The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges. Or, to change the figure, total science is like a field of force whose boundary conditions are experience. (Quine 1964c, 42)

Our system of knowledge and science forms a web, with its sentences multifariously interconnected and sensory stimulation impinging along the edges where they stand in contact with observation sentences; "[i]n an obvious way this structure of interconnected sentences is a single connected fabric including all sciences, and indeed everything we ever say about the world" (Quine 1960b, 12). The fact that the "theory as a whole is a fabric of sentences variously associated to one another and to non-verbal stimuli" (Quine 1960b, 11) has a profound impact on the testing of hypotheses. Due to the fact that "a stimulation will trigger our verdict on a statement only because the statement is a strand in the verbal network of some elaborate theory, other strands of which are more directly conditioned to that stimulation" (Quine 1969e, 16) a refuted observation categorical does not imply the falsity of any single hypothesis or sentence of the theory, but of the theory as a whole. As no sentence of a theory by itself carries empirical content and is connected moreover to the other sentences of the theory in multifarious ways one can modify any of the various hypothesis that implied the refuted observation categorical. Its falsification rendered the entire theory wrong, where the modifications are made within the theory to block the categorical from being implied is

[44] In fact, only the set of *synthetic* observation-categoricals, as opposed to hollow analytic ones, such as $\varphi \to \varphi$, (Quine 1992b; Quine 1995a).

[45] For more on the notion of a theory, see below.

[46] See (Quine 1960b, 11).

[47] This is an example of an observation sentence that is learned indirectly, by subsequent construction from learned vocabulary, rather than through primitive conditioning (Quine 1992b, 5): "here [...] the verbal network of an articulate theory has intervened to link the stimulus with the response" (Quine 1960b, 11). Observation can be learned in two ways, through being keyed to sensory stimulation and later analyzed into parts or as composed from previously learned parts, but directly responsible to stimulation. Observation sentences as viewed in the former way are theory-free and theory-laden in the latter (Quine 1992b, 7): "Seen holophrastically, as conditioned to stimulatory situations, the sentence is theory-free; seen analytically, word by word, it is theory-laden."

inessential. The modifications to the theory should be made in accordance with the above mentioned principles to maintain overall simplicity and leave as much as possible of the theory undisturbed. "It is well [...] not to rock the boat more than it need be" (Quine 1992b, 14).

We moved from terms to sentences as the primary units of meaning and anchored contact with the world in observation sentences – they provided "the link between language [...] and the real world that language is about" (Quine 1992b, 5), not through reference, but through conditioning to stimulation. We now moved from sentences to sets of sentences, even to the whole of science, to tell us about the structure of the world and help us anticipate and predict stimulatory events. By thus divorcing language from what it was supposed to be about and making pragmatic principles the basis on which to construct theory and thereby structure reality, what remains of the objects that we supposed there to be 'out in the world'?

What there is, on the account outlined above, is what suits the need of the scientist. "Everything to which we concede existence is a posit from the standpoint of a description of the theory building process"[48] (Quine 1960b, 22), "[o]ur talk of external things, our very notion of things, is just a conceptual apparatus that helps us foresee and control the triggering of our sensory receptors in the light of previous triggering of our sensory receptors" (Quine 1981e, 1). All objects are theoretical, posits of theory, in this sense. From the perspective of theory and science, there is no difference between molecules, desks and numbers,[49] "[t]he positing of either body is good science insofar merely as it helps us formulate our laws – laws whose ultimate evidence lies in the sense data of the past, and whose ultimate vindication lies in anticipation of sense data of the future" (Quine 1966c, 250). Ultimately, all we have to go on is the triggering of our sensory receptors and chairs are no more found among these surface irritations than molecules and numbers.

The ultimate justification of objects lies in simplification of theory: "objects [...] help us in developing systematic connections between our sensory stimulations" (Quine 1981e, 2), they are posited for the sake of simplifying theory[50] and helping man in "connecting loose ends of experience to produce a structured system of the world" (Quine 1994a, 454). Common man and scientist alike posit objects in order to "simplify his laws by positing an esoteric supplement to the exoteric universe" (Quine 1966c, pp. 249). "[R]eification and reference contribute to the elaborate structure that relates science to its sensory evidence" (Quine 1992b, 31), in fact, reification proved indispensable in connecting loose ends of raw experience through that structure in the simplest possible way. Nevertheless, "[t]he scientific system, ontology and all, is a conceptual bridge of our own making, linking sensory stimulation to sensory stimulation" (Quine 1981e, 20); there is no more to objects than the theory that posits them. Indeed, "our coming to understand what the objects are is for the most part just our mastery of what the theory says about them" (Quine 1960b, 16), for "[t]he very sentences which seem to propound them and treat of them are gibberish by themselves and indirectly significant only as contributory clauses of an inclusive system" (Quine 1966c, 249).

To repeat: all there is to objects is theory, and to any particular object only what the theory which posited it says about it. Objects only make sense within a theory, it is here that their purpose and justification lies, they are instruments in simplifying experience. They *exist only within theory*, as devices simplifying the laws of that theory, all there is outside is the triggering of sensory receptors and it is therefore only within a theory that one can look for objects. Ontology thus becomes a completely artificial enterprise, asking questions that are only answerable to convenience, simplicity

[48]"... and simultaneously real from the standpoint of the theory that is being built" (Quine 1960b, 22). There is no point in looking for a reality beyond the best of our theories, they provide the only access to reality we have.

[49]"Considered relative to our surface irritations, which exhaust our cue to an external world, the molecules and their extraordinary ilk are thus much on a par with the most ordinary physical objects" (Quine 1960b, 22).

[50]"It is the quest of system and simplicity that has kept driving the scientist to posit further entities as values of his variables" (Quine 1966b, 262).

and economy[51] and, more importantly, *only in the context of theory.*

We conclude this section on a note of terminology: usually, when Quine speaks of a theory what he has in mind is an *interpreted theory*, i.e. a set of sentences together with a model of these sentences:[52] "[a] theory [...] is a set of fully interpreted sentences. (More particularly, it is a deductively closed set: it includes all its own logical consequences [...])" (Quine 1969d, 51). When speaking of a theory T we therefore speak of an ordered pair $\langle T', \mathcal{M} \rangle$, where T' is a set of sentences and $\mathcal{M}$ a model of these sentences (in case T' is inconsistent, $\mathcal{M} = \emptyset$). The first coordinate of the pair, T', is called the *theory-form* or *theory-formulation* and strictly speaking not a set of sentences, but an equivalence class of sets of sentences where two theory formulations belong to the same equivalence class if they are *empirically equivalent*, i.e. if they imply the same set of observation-categoricals and can be rendered logically equivalent by means of a reconstrual of predicates.[53]

2.3 Indispensability and Irrelevance

In what follows, I wish to provide a rough sketch of some of the consequences of the views outlined in the previous two sections.[54] The first is a 'positive result' telling us a minimal set of things we must accept as existing if we are serious about what was said before, the remaining are all negative in scope, highlighting how little it actually is that we can say about 'what there is'.

2.3.1 Indispensability

Given the interconnectedness of science in the *web of belief* and the economy, simplicity, clarity and predictive power afforded by the application of mathematical techniques and notions in the special sciences, acknowledging the pragmatic standard of theory leaves one no choice but to accept mathematics as an integral part and clarifying element of every science. This makes mathematics, its notions and objects, an essential component in the simplification and clarification of theories. Put differently, mathematics is *indispensable* for the scientific enterprise if the goal is to strive for the greatest amount of clarity, simplicity and generality possible. Combined with the assumption that we should posit those objects as existing which benefit the simplification of our overall theory about the world (an account which will be made precise below in the formulation of the *criterion of ontological commitment*) and provided that the notions and objects of mathematics are indeed indispensable for that undertaking, no matter what the discipline, we reach the conclusion that the objects of mathematics (or at least sets, as most other parts of mathematics can be reconstructed in terms of sets) are indispensable for our understanding of the world and should therefore be accepted as existing.[55] That means, due to the indispensability for the best possible theory about nature,

[51] Cf. (Hylton 2007, pp. 249).

[52] It has been objected that if one requires a theory to come with its own interpretation Quine's criterion of ontological commitment of a theory becomes superfluous, since what a theory is committed to must be specified before the criterion is applied (Chihara 1973, 37). For replies to this charge see (Resnik 1974) and (Quine 1969d).

[53] See (Quine 1992b), (Quine 1981b) and (Quine 1975). It has been suggested that Quine's notion of theory equivalence amounts to identifying theories which express the same *Ramsey-sentence* (Ramsey 1960), cf. (Quine 1975, n. 4).

[54] Almost all of the points sketched in this section are highly controversial and frequently discussed in the literature. However, space constraints forbid to go deeper into the various arguments for or against them and we will settle for sketching their rough outlines here. The interested reader is referred to the various essay collections mentioned before.

[55] This so-called *indispensability-argument* for realism about mathematical entities is usually considered a consequence of Quine's naturalism, confirmational holism and criterium for the ontological commitment of theory. We have provided a very simplified and more intuitive account of the essence of the argument here.

we should assume the existence of mathematical objects, or at least sets.[56]

2.3.2 Underdetermination

The thesis of the *underdetermination of theory by evidence*[57] states that "scientific theory is underdetermined by all possible data" (Quine 1984, 320), that "physical theory is underdetermined even by all *possible* observations. [...] Physical theories can be at odds with each other and yet compatible with all possible data even in the broadest sense" (Quine 1970, 209), i.e. that "science is empirically underdetermined: there is slack" (Quine 1966c, 254). The evidence of a theory lies in its empirical content, the set of observation categoricals it implies. The thesis of the underdetermination of theory then, in other words, expounds that there can be multiple theories, empirically equivalent, i.e. agreeing on all their empirical content and yet incompatible with each other.[58] They account for all the evidence but are irreconcilable with each other.

In order for the thesis to develop full force it must not only be possible that there be multiple empirically equivalent yet incompatible theories, but also that some of these theories be equally 'good', i.e. possess a comparable amount of simplicity and thus qualify equally well to explain the respective phenomena on a pragmatic count, for otherwise there would be independent criteria on which to decide which theory was the better one: "[u]nder-determination lurks where there are two irreconcilable formulations each of which implies exactly the desired set of observation conditionals plus extraneous theoretical matter, and where no formulation affords *a tighter fit* [my emphasis]" (Quine 1975, 239). It is important to note that the under-determination of theory is *a highly plausible possibility* rather than a necessity:[59] "If all observable events can be accounted for in one comprehensive theory [...] then we may expect that they can all be accounted for equally in another, conflicting system of the world" (Quine 1975, 228).[60] This plausibility is mainly due to two factors; on the one hand, while theories imply observation categoricals they are not determined by the set of categoricals they imply, there might be more than one theory that is compatible with the evidence: "observable consequences of the hypotheses do not, conversely, imply the hypotheses. Surely there are alternative hypothetical substructures that would surface in the same observable ways" (Quine 1975, 228). On the other hand, underdetermination is suggested by confirmational holism: since "our statements about the external world face the tribunal of sense experience not individually, but

[56] Quine never states this argument in its entirety and draws the conclusion in its full force. Considerations to the same effect can be found in (Quine 1960a), (Quine 1964a), (Quine 1964c), (Quine 1981e) and (Quine 1981d). For a full and canonical statement of the indispensability-argument, see (Putnam 1975) and (Putnam 1972). For an interpretation and reconstruction of the argument in Quine see (Decock 2002a). For critique of the argument see (Field 1980), (Maddy 1992), (Maddy 1995), (Maddy 1997), (Sober 1993) and (Azzouni 1997).

[57] Often called the 'Duhem-Quine Thesis'.

[58] When we talk about theory in this section what we usually mean is the theory-form, i.e. the first coordinate of the tuple that constitutes a theory. Note that due to our formulation of theory (form) as equivalence classes (see Section 2.2) this incompatibility is non-trivial: they would have to be empirically equivalent, yet can be neither logically equivalent nor be inter-translatable by means of a reconstrual of predicates: "under-determination says that for any one theory formulation there is another that is empirically equivalent to it, but logically incompatible with it, and cannot be rendered logically equivalent to it by any reconstrual of predicates" (Quine 1975, 237).

[59] Cf. (Quine 1975, 232): "a last-ditch version of the thesis of under-determination would assert merely that our system of the world is bound to have empirically equivalent alternatives which, if we were to discover them, we would see no way of reconciling by by reconstrual of predicates." In fact, there are cases for which the thesis clearly fails, e.g., theories implying only finitely many observation categoricals (as we can in that case just take their conjunction to be our theory) or where all infinitely many implied categoricals can be subsumed under one universally quantified conditional. What is needed is a theory that implies observation categoricals "infinite in number and too ill-assorted to be exactly encompassed by any finite formulation [...] without theoretical foreign matter" (Quine 1975, 239).

[60] Quine here speaks of a 'system of the world' because aptness of a theory for inclusion into an overall system of the world can also weigh in favor of it against any other theory due to the simplicity added to the overall theory.

only as a corporate body" (Quine 1964c, 41) – "scientific statements are not separately vulnerable to adverse observations, because it is only jointly as a theory that they imply their observable consequences" (Quine 1975, 228), – "[a]ny one of the statements can be adhered to in the face of adverse observations, by revising others of the statements" (Quine 1975, 228). Thus, since a theory can be revised in multiple ways upon refutation of an implied observation categorical, there are multiple theories accounting for the remaining empirical content in an equitable way.[61] It is the "looseness of fit between evidence and theory" (Hylton 2007, 193) thus demonstrated, which lends credibility to the thesis of underdetermination.[62]

2.3.3 Inscrutability

As outlined in Section 2.1 the contact between language and world is not direct – there is no immediate connection between terms and objects, – but mediated through stimulation of sensory receptors and conditioning of sentences.[63] Terms, and therewith reference and reification, only come into play once the sentences are analyzed into parts. Given then that the relation of reference is not direct and fixed, but learned and hypothesized, there is room for variance: whether 'gavagai' refers to rabbits, temporal stages of rabbits, instantiations of rabbithood or undetached rabbit-parts is not settled by behavioral evidence. Either of the previous posits fully accounts for the verbal behavior of someone uttering 'gavagai' whenever a rabbit appears in her field of vision.[64]

This lends credence to the thesis of the *inscrutability of reference*, the thesis that "all references can be revised and reshuffled at will without rewriting any of our science, falsifying any of its sentences, or disturbing any of its observational evidence" (Quine 2008d, 361).[65] The thesis of the inscrutability of reference is often conflated with the (much more controversial) thesis of the *indeterminacy of translation*[66] and while they are closely related, the former relies on much weaker assumptions, in particular, it does not necessitate a reinterpretation of the apparatus of individuation and leaves the evidential and logical relations between sentences completely intact,[67], yet still succeeds in establishing that "[r]eference itself proves behaviorally inscrutable" (Quine 1969d, 35). While the indeterminacy of translation is a phenomenon reconcilable only on the level of an entire language, the inscrutability of reference can be reconciled much earlier, on the level of sentences: "It [the indeterminacy of translation] declares for divergences that remain unreconciled even at the

[61] Why neither of these reasons necessitates underdetermination see (Hylton 2007, pp. 192).

[62] For a more thorough treatment of the thesis of the underdetermination of theory by evidence, its implicit assumptions and preconditions as well as possible refinements we refer the reader to (Hylton 2007, pp. 189), (Bergström 2004), (Bergström 1990), (Stegmüller and Gähde 1999), as well as (Vuillemin 1979). Cf. also (Quine 2008m).

[63] Cf. (Hylton 2006).

[64] Ostension does not settle the matter, as one is always pointing at an undetached rabbit-part, temporal rabbit stage, etc. when pointing at a rabbit. Moreover, in situations where this is not the case, the ostension could always be interpreted as *deferred ostension*, e.g., when pointing at the petrol pump in order to refer to the petrol or when pointing at grass while explaining 'green' (Quine 1969d, 40). Furthermore, the dilemma cannot be resolved through observing the interaction of the problematic referential terms with other particles of the language as varying interpretations for these particles can be given, offsetting the thus achieved clarification.

[65] "Quine's thesis of the inscrutability of reference is that there is no way to tell what the singular terms of a language refer to, or what its predicates are true of, at least no way to tell from the totality of behavioral evidence, actual and potential, and such evidence is all that matters to questions of meaning and communication." (Davidson 1979, 227)

[66] In fact, the "inscrutability of reference appears as an accidental corollary of the indeterminacy of translation" (Decock 2002b, 148) and is treated as such in the earlier writings of Quine (see, e.g., (Quine 1960b; Quine 1969d)). They are separated in his later works, cf. (Quine 2008m; Quine 1994b): "The indeterminacy [of reference] depends none on translation, though contributing a dimension to the indeterminacy of translation" (Quine 1994b, 495). See also (Decock 2002b).

[67] Cf. (Glock 2003, pp. 220).

level of the whole sentence, and are compensated for only by divergences in the translations of other whole sentences. Unlike indeterminacy of reference, which is so readily illustrated by mutually compensatory adjustments within the limits of a single sentence, the full or holophrastic indeterminacy of translation draws too broadly on a language to admit of factual illustration" (Quine 1992b, 50). Thus while the indeterminacy of translation only sets in when moving between languages (although this is no real restriction given that "radical translation begins at home" (Quine 1969d, 46)) inscrutability of reference intrudes into our very own idiom.[68]

Now, "reference goes inscrutable if [...] we contemplate a permutational mapping of our language on itself" (Quine 1981e, 20) and is best illustrated and demonstrated by means of a *proxy-function*, a one-to-one reinterpretation of objective reference (Quine 1994a, 458). A *proxy-function* is a one-to-one transformation f between objects of two universes of discourse. It is a rule that assigns to every object x a 'new' object $f(x)$. Given such a proxy-function we can reinterpret every predicate P true of the old objects x in such a way as to now be true of the correlates $f(x)$ of x, i.e. instead of saying that x is a P we now say that x is the f of a P (mutatis mutandis for relations).[69]

Consider, for example, the proxy function given by 'cosmic complement of' (Quine 1992b, 33). 'Francis is a cat' then becomes 'The cosmic complement of Francis is the cosmic complement of a cat'. Pointing at Francis means at the same time pointing at Francis' cosmic complement (be it by deferred ostension); every stimulatory situation that elicited the statement 'Francis is a cat' also elicits 'The cosmic complement of Francis is the cosmic complement of a cat' – the same observations causing assent to the former motivate assent to the latter and identically for dissent. The point is that all observation sentences remain associated with the same ranges of sensory stimulations and the interconnections between sentences remain intact. The reconstruals of objects and predicates cancel each other out. Thus, in terms of verbal behavior, nothing really changes, yet "the objects of the theory have been supplanted as drastically as you please" (Quine 1992b, 32); instead of talking about physical bodies and 'ordinary' objects, our ontology now only contains the cosmic complements of these. "The point is that if we transform the range of objects of our science in any one-to-one fashion, by reinterpreting our terms and predicates as applying to the new objects instead of the old ones, the entire evidential support of our science will remain undisturbed" (Quine 1992a, 404):

> "The original objects have been supplanted and the general terms reinterpreted. There has been a revision of ontology on the one hand and of ideology, so to say, on the other; they go together. Yet verbal behavior proceeds undisturbed, warranted by the same observations as before and elicited by the same observations. Nothing really has changed." (Quine 1981e, 19)

The interchangeability of ontologies and the irrelevance of the nature of the objects to verbal behavior does not reflect inscrutability of fact, or of the world – due to the fact that contact with the world is not referential but can only be found in a relation between sentences taken holophrastically and the stimulatory conditions eliciting their utterance mediated through conditioning and reinforcement there is no fact of the matter as to *what* reference is made to. The decomposing of sentences and positing of objects is carried out on solely pragmatic considerations, i.e. simplicity of theory. It is not that the objects that are being referred to are inscrutable, it is the entire process of referring itself that does not and cannot require there to be any particular objects – reference, just as reification and objects in general, are *mere instruments* in our enterprise to devise the best

[68] For exposition of and comment on the thesis of the inscrutability of reference, often connected with the indeterminacy of translation, see (Davidson 1979), (Aune 1975), (Leeds 1973) and (Cornman 1976).

[69] See (Quine 1969d), (Quine 1981e), (Quine 1992b), (Quine 1995a).

overall theory of the world that systematizes our experience. There is nothing more to them than their contribution to that theory: "Reification is human handiwork, or human mind work. Its utility for science lies in forging logical bonds throughout our manmade theory of the world. This service is structural and hence invariant under one-to-one transformations" (Quine 1994a, 460). "The lesson of proxy functions is that all the technical service rendered by our ontology could be rendered equally well by any alternative ontology provided that we can express a one-to-one correspondence between the two" (Quine 2008m, 376).

Moreover, as there is no fact of the matter as to what reference is made to "the inscrutability of reference can be brought even closer to home than the neighbor's case; we can apply it to ourselves" (Quine 1969d, 47).[70] Even we ourselves when using language (as a society) cannot ultimately pin down what the objects are that we are talking about[71] – and we do not have to: reference is not required to give meaning to language.[72] What gives meaning to linguistic utterances is their use in a community of speakers.[73] As long as these speakers can act and react appropriately to verbal stimuli, the expressions thus uttered have meaning – which objects they needed to posit in order to master the correct usage of the respective expressions has no bearing on their meaningfulness.[74] "The conclusion is that there can be no evidence for one ontology as over against another, so long anyway as we can express a one-to-one correlation between them" (Quine 1992a, 405). Ontological questions "are to be answered within our evolving scientific system, and what proxy functions show is just that the answer must be in large part arbitrary" (Quine 2008d, 362).

2.3.4 Relativity

The inscrutability of reference as applied to ourselves appears to lead to the "absurd position that there is no difference on any terms, interlinguistic or intralinguistic, objective or subjective, between referring to rabbits and referring to rabbit parts or stages“ (Quine 1969d, 47), for nothing in the linguistic behavior changes, whether we posit the former or the latter. But "we cannot rest with a conclusion that allows us to accept both that 'Wilt' refers to Wilt and that 'Wilt' refers to the shadow of Wilt. We can without contradiction accept both only if both can be true, and clearly this is not the case" (Davidson 1979, 231); "it [the inscrutability of reference] made nonsense of reference" (Quine 1969d, 48). If there is no difference between referring to Francis and his cosmic complement, reference itself would become nonsense, for it implies "that there is no difference between the rabbit and each of its parts or stages" (Quine 1969d, pp. 47), we would be identifying the alleged referents.

The thesis of the inscrutability of reference leads us to recognize that one needs to specify in some way *what it is* that one refers to when referring if there is to be any hope of preventing the concept

[70]Cf. also (Quine 1969d, 47): "we can reproduce the inscrutability of reference at home."

[71]In (Quine 1992b, pp. 51) Quine appears to have changed his opinion that the inscrutability of reference also infiltrates the home-language: "And does the indeterminacy or relativity extend also somehow to the home language? In 'Ontological Relativity' I said it did, for the home language can be translated into itself by permutations that depart materially from the mere identity transformation, as proxy functions bear out. But if we choose as our manual of translation the identity transformation, thus taking the home language at face value, the relativity is resolved. Reference is then explicated in disquotational paradigms analogous to Tarski's truth paradigm [...]; thus 'rabbit' denotes rabbits, whatever *they* are, and 'Boston' designates Boston." See also (Quine 1999b, 460) and (Quine 1981e, 20). Nevertheless, the inscrutability is still present in the sense that we ultimately cannot pin down what we are talking about. Doing so would require having a private language in which the references are fixed, but that is not transferable to other speakers (due to the inscrutability of reference). This, however, is absurd as language is an essentially social phenomenon; there is nothing to language, but what is intersubjectively available (see (Hylton 2006, pp. 140)).

[72]This is precisely the point at which Priest (Priest 2002, pp. 214) misconstrues Quine.

[73]Cf. (Decock 2002b, 151): "It is more important to know how the word 'rabbit' functions. [...] It is of no use to know what rabbits really are."

[74]Cf. (Hylton 2006).

of reference from collapsing. It also forces us to realize that any such specification will be arbitrary, after all, there is no fact of the matter: "the relation of reference between objects and words (or their utterances) is relative to an arbitrary choice of a scheme of reference" (Davidson 1979, 227). What we need is a 'coordinate system', a 'frame of reference' against which to use terms to successfully refer without the relation of reference collapsing onto itself. Relative to such a frame of reference "we can and do talk meaningfully and distinctively of rabbits and parts" (Quine 1969d, 48): "reference *is* nonsense except relative to a coordinate system" and it is precisely in this principle of relativity in which "the resolution of our quandary" lies (Quine 1969d, 48). This is because a frame of reference allows us to fix the relation of reference, thereby avoiding the damaging identification of possible referents. A helpful analogy is given by considering the question what the location of an object is: multiple answers can be given and multiple answers are correct, depending on the coordinate system in which we determine the position of the object. By disregarding a frame of reference, our coordinate system, we collapse different answers and locate the object at every possible position – the notion of location looses all content. However, once we specify a coordinate system in which we determine the object's position, we can give an informative answer of the location of the object *relative to that coordinate system.*[75]

In the case of ontology the frame of reference is provided by a *background language* or *background theory*[76] into which the speaker of a language can regress when asked to fix the relation of reference. Here, we have the doctrine of *ontological relativity*, the principle that ontology is relative to a language. In a sense, ontology is doubly relative:[77] on the one hand it is relative to a 'translation-manual',[78] i.e. a way of understanding what the terms of a language correspond to in our own idiom[79] (whether one takes 'gavagai' to correspond to 'rabbit' or 'undetached rabbit part'; note that this includes pondering a permutation of our own idiom onto itself, although this degenerate case is most often resolved through choosing the homophonic translation) and, on the other hand, to the choice of a background language or theory, i.e. a way of understanding *what that referent is*, i.e. what it is that one refers to when using the respective terms.[80]

The background theory fixes the relation of reference allowing one to say *what it is that one is referring to*, be it only relative to that theory:

> "It is meaningless to ask whether, in general, our terms 'rabbit', 'rabbit part', 'number', etc., really refer respectively to rabbits, rabbit parts, numbers, etc., rather than to some ingeniously permutated denotations. It is meaningless to ask this absolutely; we can meaningfully ask it only relative to some background language. When we ask, "Does 'rabbit' really refer to rabbits?" someone can counter with the question: "Refer to rabbits in what sense of 'rabbit'?" thus launching a regress; and we need the background language to regress into. *The background language gives the query sense, if only relative sense; sense relative in turn to it, this background language* [my emphasis]." (Quine 1969d, pp. 48)

The frame of reference provided by the background language gives our query sense by enabling us to *specify the objects one is referring to*, be it only relative to that language. Thus, e.g., if the

[75] Cf. (Quine 1969d).

[76] In (Quine 1969d) Quine uses these terms interchangeably.

[77] Cf. (Quine 1969d, pp. 54).

[78] A translation manual because it has to do with how one understands the terms of a language, how one translates them into one's own idiom.

[79] Cf. (Field 1974) for problems of relativizing ontology to a translation manual.

[80] While Davidson accepts the inscrutability of reference, he disagrees with Quine on the doubly relative character of ontology: "It is ontological relativity that I do not understand" (Davidson 1979, 232).

background language is the same as the object language the fixing of reference will take the form of a disquotational scheme: “thus ‘rabbit’ denotes rabbits, whatever *they* are, and ‘Boston’ designates Boston” (Quine 2008m; Quine 1992b). If the background theory is a chapter of physics it might say that ‘rabbit’ refers to a collection of molecules of such-and-such bonding type standing in such-and-such connection. *Relative to this background language or theory we can say what our terms are about, what they refer to, but only relative to it.* The terms of the background language are of course as much in need of having their reference fixed as the terms of the object language were, for the inscrutability of reference also intrudes here. Its reference, once again, can only be fixed against a further background language or theory. And so on ad infinitum – we are involved in an infinite regress of theories and languages[81]. What this demonstrates is not that reference is impossible, but that it is “elusive” and “empty” (Quine 1969d, 68) and that therefore there is no sense in saying what the objects of a theory are (Quine 1969d, 51), no ultimate sense in which the universe of a theory can have been specified (Quine 1969d, 50).[82]

Thus “[w]hat makes sense is to say not what the objects of a theory are, absolutely speaking, but how one theory of objects is interpretable or reinterpretable in another” (Quine 1969d, 50):

> “The relativistic thesis to which we have come is this [...]: it makes no sense to say what the objects of a theory are, beyond saying how to interpret or reinterpret that theory into another.” (Quine 1969d, pp. 50)

We have repeatedly mentioned that all there is to the existence of objects is their contribution to theory, all there is to their constitution is what the theory that posits them says about them, i.e. how they feature in the laws, axioms and statements of that theory. In short, all there is to objects is structure: “Save the structure and you save all” (Quine 1992a, 405). The significance of ontology for theory building about the world is just its “contribution of neutral nodes to the structure of theory” (Quine 1992b, 33): “what matters for science is not ontology, but structure, together with sensory stimulation. The purported objects serve merely *as neutral nodes in the structure* [my emphasis]” (Quine 2008j, 346).[83]

We do not add a particular object to scientific theory when referring, it is mere *sameness of reference* that ontology contributes to science (Quine 1995a, 72). “Reference and ontology recede thus to the status of mere auxiliaries” (Quine 1992b, 31), instruments in the construction of theory, as “all that matters by way of evidence for the theory is the stimulatory basis of the observation sentence plus the structure that the neutral nodes serve to implement. The stimulation remains rabbity as ever, but the corresponding node or object goes neutral and is up for grabs” (Quine 1992b, 34). It is “[s]entences in their truth and falsity [that] run deep; *ontology is by the way* [my emphasis]” (Quine 1978): “Structure is what matters to a theory, and not the choice of its objects. [...] The objects serve merely as indices along the way, and we may permute or supplant them as we please, as long as the sentence-to-sentence structure is preserved” (Quine 1981e, 20).

The relativity of ontology is a consequence of the inscrutability of reference which in turn follows from the nature of language acquisition. Against the traditional preconception of ontology as the discipline studying the nature of ‘what there is’, ontology on a Quinean understanding has become “the study of structures in which individuation is possible” (Decock 2002b, 156).

[81] Ultimately acquiescing in our mother-tongue; (Quine 1969d).

[82] This appears to contradict what we said above about a theory consisting of an equivalence class of sets of sentences and a model, i.e. a pre-specified universe. The answer lies in what was said above: there is no ultimate way in which the objects of a theory can have been specified, the theory cannot have been fully interpreted, but only relative to our “overall home theory” within which the respective theory was formulated (Quine 1969d, 51).

[83] Cf. also (Quine 1995a, 74/75) and (Quine 1994a, 459/460): “what matters for any objects, concrete or abstract, is not what they are, but what they contribute to our overall theory of the world as neutral nodes in its logical structure.”

2.4 Reduction and Paraphrase

One can distinguish roughly between two kinds of ontological 'reform' in the Quinean framework: *paraphrase* and *reduction*. As a rough-and-ready characterization, paraphrase helps us in determining what there is, whereas reduction guides us with regards to the question of what we can do without. Thus when presented with a theory or piece of discourse whose ontological commitment we wish to determine we first regiment it into canonical notation, i.e. we paraphrase it into a form which makes it susceptible to the criterion of ontological commitment.[84] In the process of this paraphrase or in general in the process of any paraphrase, expressing the discourse at hand in different words, we might realize that we could have said some of the things differently paraphrasing away certain expressions and talking about what they were talking about in different terms. Such paraphrase need not aim at synonymy or fulfill the exact same role as the paraphrased terms did, it is "good insofar as it tends to meet needs for which the original might be wanted" (Quine 1960b, 182). That means, as long as the paraphrase does what we intended to say with the original formulation it can be regarded as a good paraphrase.

Paraphrase is wanted if the original talk is unclear, misleading, obscure and/or ambiguous, when it is not quite clear what it is that we are saying. The paraphrase then helps to flesh out its intended meaning more clearly. In doing so it might, however, stop talking about some of the things the original discourse was considered to talk about. Entities that are paraphrased away in this fashion can then be considered eliminated: we showed that we did not need them in order to express what we wanted to express. We might have posited new entities to fill their role but do so in a more effective and clearer, a less problematic way. By paraphrasing discourse about objects of kind A into discourse about objects of kind B we have eliminated A's in favor of B's and shown that all that we can do with A's we can equally well do in terms of B's. If we prefer B's over A's for any reason whatsoever (they are better behaved, more familiar, already in our ontology) progress has been achieved. Moreover, any problems pertaining to entities of kind A have in an important sense been solved by ridding ourselves of these entities.

Paraphrase has therefore genuine explanatory potential; in paraphrasing problematic entities away we solve the problems that made them problematic: "problems are dissolved in the important sense of being shown to be purely verbal, and purely verbal in the important sense of arising from usages that can be avoided" (Quine 1960b, 261) and "[e]ach elimination of obscure constructions or notions that we manage to achieve, by paraphrase into more lucid elements, is a clarification of the conceptual scheme of science" (Quine 1960b, 161). Moreover, we offered substitutes that were able to do their job and that we could talk about instead. Objects that fulfill the entire purpose for which the originals 'might have been wanted for'. We do not loose anything, but gain much in clarity. We have shown that the reference to problematic objects proved vacuous and that we instead do better of assuming their less problematic 'surrogates' in our ontology. This treatment of eliminating entities by paraphrasing them into acceptable substitutes is the process Quine applies to, among other things, propositions, meanings, properties, propositional attitudes and other intensional or non set-theoretical objects. Talk of these is replaced by naturalistically acceptable talk about expressions themselves, sets and other set-theoretic entities, physical bodies and anything that we have a firmer grasp of. Another consists in the replacing of physical bodies with sets of space-time coordinates. We will demonstrate multiple procedures of paraphrase below.[85]

Reduction, on the other hand, is the interpretation of one theory into another: the elements of

[84] If it was not formulated by us we also have to first translate it into our idiom bringing in separate issues about the adequacy of translation.

[85] (Quine 1960b) is full of examples and repeatedly uses this method to regiment language into clearer scientific discourse.

these theories (sentences or objects equally) are connected according to a specific method allowing an identification of the objects of different theories or revealing a way how to paraphrase talk of the objects of one theory into talk of objects of the other theory, thereby *reducing* the objects to one another. In a process of reduction both theories are already sufficiently clear and unproblematic, but we are guided by considerations of simplicity and economy. Ontological reduction will play a prominent role in Chapter 5 and we will come back to it there and expand on what was said above. As an example of this kind one can consider the reduction of numbers to sets. Both notions are sufficiently clear, but sets will feature in our ontology anyways so we might save on ontology by reducing numbers to them. It is important to note that none of the information of the reduced theory is lost – we merely re-interpret it in another theory, which then takes over the work of that reduced theory according to the interpretation. We merely showed that we could do without certain things, there was no problematic need to paraphrase them away, just pragmatic convenience.

Chapter 3

Ontological Commitment

3.1 Purpose of a *Criterion of Ontological Commitment*

The *criterion of ontological commitment* expresses Quine's meta-ontological stance; it is a guideline of how to talk about things – it is talk about talking about things (Quine 1994a, 449) – testing the conformity of discourse "to a prior ontological standard" (Quine 1964a, 15) and thereby enabling meaningful communication about ontology.

The criterion of ontological commitment provides a standard by which to determine what objects a theory refers to, what objects exist according to it. It talks about reference and sentences, and thus concerns second-order discourse about words (Quine 1960b, 270). Underlying the formulation of the criterion and expressed in it is the crucial assumption that *existence is univocal* (Inwagen 2009), i.e. that there is only 'one mode of being'; objects do not exist in different ways. That means that, supposing there are numbers and wombats, both of them exist in the very same way.[1] There is no difference between abstract and concrete, spatio-temporally located and eternal, observable and unobservable objects with respect to existence: "the distinction between there being one sense of 'there are' for concrete objects and another for abstract ones, and there being just one sense of 'there are' for both, makes no sense" (Quine 1960b, 242). It is "philosophical double talk", thriving on vagaries of ordinary language and intending to "repudiate an ontology while enjoying its benefits" (Quine 1960b, 242).

The reason for Quine not distinguishing different senses of being is not hard to find. From the perspective of theory, the only point of view from which it makes sense to ask questions about the existence of objects (see Sections 2.2 and 2.3), *all objects are posits*. They are all 'convenient fiction'[2], posited for the purpose of simplifying theory and systematizing the 'flux of experience'. In short, all objects are posits, they are simplificatory devices and only exist as such; there is no difference between talking about chairs or atoms or numbers, they all exist because their positing afforded a simplification of theory and thus in the same way, as neutral nodes in the theory: "Unobservable individuals come to be posited too when it is found that our system of statements about the world can thereby be tightened and simplified without detriment to its empirical content. [...] I see no difference in kind between these artful posits and the common sense posits of sticks and stones, the

[1] One could object and say that while wombats exists in space and time, numbers do not, and so clearly those two objects have a different 'mode of being'. However, the only thing one is saying when one insists that wombats exist in space and time is that they possess the property of spatio-temporality; they differ in their properties, not in the way they exist, after all, they are both posits existing for the same purpose of simplifying theory.

[2] Which does not means that we should treat them as such, mere fiction. After all, our best theory of the world is our best (in virtue of only) way of knowing about the world. We should believe what that theory tells us.

macroscopic ontology that stems [...] from the dawn of humankind"[3] (Quine 2008n, 321).

Given that all objects exists in the same way, i.e. that there is only one way of existing[4] it is natural that Quine identifies 'being' and 'existence' (Quine 1964a; Quine 1969c) and makes no distinction between a broader notion of being/subsistence or existence/actuality. Now, while the criterion of ontological commitment provides a way of determining what *kinds of things* exist according to a given theory (Quine 1964a, 1), one should distinguish this *ontological commitment step* from an *ontological specification step* (Stokes 2005). The former tells us what kind of objects there are according to a given theory, i.e. what *roles* have to be filled in the structure of that theory, while the latter determines what objects are fit to play these roles – it is especially in the latter part where ontological reduction occurs.

3.1.1 Talking about Ontology

Without a standard for talking about ontology, discourse about it faces the constant threat of breaking down. Imagine the following dialogue: 'Are there miles?' - 'Of course there are, wherever there are 1760 yards there is a mile.' – 'But there are no yards, only bodies of various lengths.' – 'So earth and moon are separated by bodies of various length?' (Quine 1960b, 272). The entire defective discourse could have been avoided by shifting from talk of mile to talk of 'mile' and asking in what contexts it makes sense to use the word, i.e. what it means to say that places a and b are m miles apart, rather than jumbling on and begging the question. Shifting from talk in certain terms to talking about them is a necessary condition for meaningful ontological talk (Quine 1960b, pp. 271).

Again, consider two scientists, both positing a particle. The one a particle that possesses rest mass, and the other one that does not. Both of them call the particle thus posited 'neutrino' and exclude the possibility of the existence of any other particle. When they now talk about the newly posited particles to each other, they appear to disagree about the properties of neutrinos, for the one scientist claims them to have rest mass and the other one denies this. This is how they would describe their disagreement. In general, every disagreement of such or similar kind could be described as that party A holds that objects x have property P while party B holds that objects x do not have property P. The disagreement is clear, it is about whether an object has or does not have a particular property.

This account becomes problematic, however, in case one proponent of the debate takes something to exist that the other one does not, say, for example, Pegasus. Here they cannot describe their disagreement along the lines of x has P, x does not have P; at least the proponent of the latter position cannot without contradicting herself by saying that there are objects x, such that A recognizes their existence, but I do not – i.e. there *are* objects, such that I claim they do not exists, this is where I disagree with A. Here the proponent denying the existence of such creature as Pegasus appears to be at a serious disadvantage by being unable to state the disagreement without contradicting herself on pain of admitting that there is something whose existence she denies, and it seems "that in any ontological dispute the proponent of the negative side suffers the disadvantage of not being able to admit that his opponent disagrees with him" (Quine 1964a, 1).[5] The proponent of the negative side is thus unable to continue the discourse due to being unable to formulate the

[3] As the empirical content of a theory consists in the observation categoricals it implies and no object will ever feature in observation sentences that are irreferentially keyed to stimulus and make up the categoricals, no posited object will ever be part of the empirical content of a theory.

[4] "I mean 'exists' to cover all there is" (Quine 1969c, 100).

[5] This is of course a reconstruction of Quine's puzzle of 'Plato's beard', the view that one cannot deny the existence of an object without contradicting oneself. See (Quine 1964a) for the most famous exposition of this point.

disagreement properly – if she tries to formulate it any other way, her opponent will charge her with ascribing to him a misconstrual of their disagreement, after all, he can countenance the existence of the object she denies, they won't be able to agree that they disagree (or better, where they disagree). A criterion of ontological commitment ensures that no such breakdown in communication occurs by providing a standard against which both parties involved in an ontological dispute can measure what it is that they are saying. Determining what it means to say that something exists or does not exist (a question that can only be answered from a meta-ontological point of view, in Quine's case that means in accordance with the criterion of ontological commitment) enables a clear statement of the disagreement instead of a paradoxically seeming deadlock.

It does so by shifting the debate upwards onto a semantical plane, translating a "basic controversy over ontology [...] upward into a semantical controversy about words and what to do with them" (Quine 1964a, 16), thereby circumventing a collapse of the controversy into question-begging. It does so by clarifying what saying that something exists amounts to, translating the question 'Does Pegasus exist?' into the question of whether Pegasus is among the values of the variables of the relevant theory T, thereby asking whether the term 'Pegasus' must be taken as referring, and can feature as grammatical antecedent in the context of a relative clause (see next section). This higher-level retreat to deal with ontological issues succeeds because "it carries the discussion into a domain where both parties are better agreed on the objects (viz. words) and on the main terms concerning them. [...] The strategy is one of ascending to a common part of two fundamentally disparate conceptual schemes, the better to discuss the disparate foundations" (Quine 1960b, 272). *Semantic ascent*, withdrawing to a semantical plane of talking about words, how and what properties they have, rather than about the alleged 'meanings' of these words directly, allows to find common ground on which to argue. Mutual understanding is possible on this level because of sufficient convergence of the conceptual schemes of the proponents of the disagreement in their intermediate and upper ramifications (Quine 1964a, 16). While we might disagree about Pegasus and unicorns, it is more likely that we have a shared understanding of words as syntactic and semantic objects. Agreeing therefore on linguistic utterances and their function and properties, ontological disagreement can without contradiction be described in terms of the statements that the proponents of the debate are prepared to affirm and reject: it is possible to formulate the disagreement. This way it is possible to realize that "'unicorn' and 'Pegasus' can be perfectly good terms, well understood in that their contexts are well enough linked to sensory stimulation or to intervening theory, without there being unicorns or Pegasus" (Quine 1960b, 246).

It is worth mentioning that just because disagreement about ontology is translatable upwards into disagreement about words, this does mean that the disagreement is linguistic: "translatability of a question into semantical terms is no indication that the question is linguistic" (Quine 1964a, 17). The acceptance of objects is not a matter of linguistics and "distinct somehow from serious views about reality" (Quine 1960b, 275), but getting clear about the linguistic behavior of terms allegedly referring to objects and agents making such reference can help to resolve ontological issues, or at least formulate the problem properly.

3.1.2 Comparing Theories

An offshoot of enabling discourse about ontology in a uniform and controlled manner is the ability to compare theories according to their 'ontological import', the things which exist according to them. We already mentioned that one of the theoretical virtues of a theory consists in it being simple and therefore, "[w]hen two theories are equally defensible on other counts, certainly the simpler of the two is to be preferred on the score of both beauty and convenience" (Quine 1963a, 255).[6] Given that

[6] For reasons for preferring simple theories see (Quine 1963a).

ontology is one part of a theory, simplicity considerations take hold here as everywhere else, such that, according to Occam's razor (*entia non sunt multiplicanda praeter necessitatem*), economy is the guiding principle in accepting an ontology:[7] "Simplicity is the thing, and ontological economy is one aspect of it" (Quine 1966b, 264).

Thus, given two theories of identical explanatory power, a criterion of ontological commitment provides a measure by which to judge which of the two should be preferred on grounds of ontological economy, on the basis of evaluating their ontological import by the same standard. It is a comprehensive standard, applicable to any theory indiscriminately and thereby constitutes a methodological principle upon which to base at least one aspect of our simplicity considerations. It is not the only aspect however. While economy is the goal, it is economy of theory and not just economy of objects (Quine 1960b, 243).[8] Nevertheless, it provides a clear way of assessing in a uniform and therefore comparable way and evaluating economy and simplicity considerations on the ontological level.

3.2 The *Criterion of Ontological Commitment*

In the previous section we saw that a criterion of ontological commitment allows steering clear of conceptual and linguistic confusions and therefore enables coherent and sensible discourse about 'what there is', providing a way of assessing and comparing the ontological import of theories. It does so by reducing the question of ontology to the question of when "a given pattern of linguistic behavior construes a word [...] as having a designation" (Quine 1939, 706). The criterion of ontological commitment acts as a standard to determine when this is actually the case and this section will attempt to show how and why it does it in the way it does and thereby where it derives its ultimate justification from.

3.2.1 Quine's *Criterion*: A Grammatical Derivation

Epitomized in the slogan "to be is to be the value of a variable" (Quine 1939; Quine 1964a; Quine 1966a; Quine 1992b) the *criterion of ontological commitment* or *criterion of the ontological commitment of theories* can be found in various formulations throughout Quine's writings:[9]

> "To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable." (Quine 1964a, 13)
>
> "We are convicted of a particular ontological presupposition if, and only if, the alleged presuppositum has to be reckoned among the entities over which our variables range in order to render one of our affirmations true." (Quine 1964a, 13)
>
> "[A] theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true." (Quine 1964a, 13/14)

[7] Cf. (Gochet 2007).

[8] Moreover, simplicity is relative to a *conceptual scheme*, a way of constructing the most basic categories in which one takes to perceive reality (Quine 1963a, 255).

[9] While the criterion occurs frequently in his writings, landmark papers treating explicitly of it include (Quine 1966a), (Quine 1939), (Quine 1964a), (Quine 1969c), (Quine 1961). Quine's writings explicitly concerned with ontology began with (Quine 1934a) and arguably reached maturity in (Quine 1964a).

> "To show that some given object is required in a theory, what we have to show is no more nor less than that that object is required, for the truth of the theory, to be among the values over which the bound variables range." (Quine 1969c, 94)

> "What objects does a theory require? [...] [T]hose objects that have to be values of variables for the theory to be true." (Quine 1969c, 96)

> "The ontology to which an (interpreted) theory is committed comprises all and only the objects over which the bound variables of the theory have to be construed as ranging in order that the statements affirmed in the theory be true." (Quine 1951a, 11)

> "[A]n entity is assumed by a theory if and only if it must be counted among the values of the variables in order that the statements affirmed in the theory be true." (Quine 1961, 103)

> "[T]he objects we are understood to admit are precisely the objects which we reckon to the universe of values over which the bound variables of quantification are to be considered to range." (Quine 1960b, 242)

The criterion intends to capture the almost trivial intuition that theories take to exist what they say there is, what they refer to, or, in other words, what must exist if they are to be true.[10] As outlined in Section 3.1.1, it aims at providing a standard for ontological discourse, streamlining and illuminating ordinary discourse about ontology from which it ultimately derives, but which in itself is not sufficiently clear enough in order to enable us to see clearly what is assessed by it. According to the criterion, then, in order to determine when a theory imputes existence one should look at the behavior of the quantified variables. In formulating a standard thusly it therefore crucially relies on three notions: *theory*, *variables* and *quantification*. The adequacy of the criterion to constitute a standard for ontological talk rests on the derivation and interpretation of these notions and this is what we will be concerned with in the following paragraphs.

The role of theory in the formulation of a criterion of ontological commitment becomes clear when considering the function of theory in ontology: there simply is no ontology outside of theory; an objects' existence is inextricably linked with a theory positing it for the sake of simplifying its assumptions. Over and above its structural purpose within that theory there is nothing to it. When talking about ontology, we therefore must talk from within a specific theory. It only makes sense to determine the commitments of discourse about ontology from within a theory, as otherwise the notion of object itself and therewith ontology would loose all content and become meaningless.[11]

The role variables and quantification play is more involved and (thus) more contestable, derives directly, however, from the way reference is incorporated in the Quinean framework. In order to

[10] Cf. (Glock 2003, 56). See also (Rayo 2007) for a helpful way of thinking about the criterion in terms of a sentences demands on the world, i.e. its truth-conditions.

[11] The relativization of ontological commitment to *theories* rather than single sentences (unless these are considered as theories) immediately refutes criticism along the lines of (Rayo 2007, 432) that Quine's criterion undergenerates when dealing with predicates expressing *extrinsic properties*, given that the meaning of these predicates is not settled before and independent of the theory they feature in, but determined by the role they play in the laws of the theory itself; cf. (Quine 1969d, 50): "predicates differ from one another purely in the roles they play in the laws of the theory". Another criticism leveled against Quine's conception of the criterion against which he is immune due to his understanding of theory as a tuple consisting of an equivalence class of sets of sentences together with a model, is the one brought forward by (Martin 1962). He claims that Quine has no means of determining the range of the quantifiers and therefore no way of pinning down any workable notion of quantifier. This objection fails as every theory is, by default so to say, endowed with one or another carrier set.

show that a given theory assumes an object we have to show that the theory refers in some way to that object. Recall that we identified the 'root of objective reference' in the relative clause and that all reference ultimately derives from its role and use in ordinary language (see Section ??!!). Moreover, part of the role of a standard is to clarify and streamline discourse about the things it is a standard for. One of the ubiquitous inaccuracies that befall ordinary language is that of *scope ambiguity*. Consider the following sentence:

(*) Tarski believes that some logician is not Gödel.

This sentence is ambiguous between two readings: on the one hand, it could be understood as Tarski simply believing that not every logician is Gödel, i.e. that there are other logicians besides Gödel. On the other hand, Tarski could believe of a particular person that is not Gödel that she is a logician (e.g. Ruth Barcan-Marcus). The ambiguity of scope here can be described as uncertainty over whether 'some logician' should take wide or narrow scope, i.e. whether its scope contains merely 'some logician is not Gödel' or rather the entire sentence.[12] In case it takes narrow scope the first reading would be appropriate, in case it takes wide scope the second.

There is an idiom of ordinary language, frequently used in mathematical formulations, which circumvents the possibility of such scope ambiguity from the outset. "This basic and neglected idiom is the relative clause, mathematically regimented as the 'such that' idiom" (Quine 1966d, 275). The 'such that' idiom is nothing more than "mathematical pidgin English for our indigenous relative clause" (Quine 1995a, 31), but has the distinct advantage of avoiding above mentioned scope ambiguity and bringing out the sentence structure underlying a particular expression more clearly. In order to construct a 'such that' clause from a sentence all that needs to be done is to prefix the 'such that' to the sentence and to substitute a pronoun for the noun from which one is abstracting, i.e. which is the grammatical subject of the sentence or is responsible for the ambiguity of scope.[13] Completion to a sentence of the thereby obtained 'such that' clause is achieved by prefixing the noun dropped (possibly with the copula 'is') to the respective 'such that' construct. Thus 'I bought Fido from a man who found him' becomes 'Fido (is) such that I bought him from a man who found him' (Quine 1974, pp. 91) and the ambiguity of (*) is resolved by either rendering it 'Some logician (is) such that Tarski believes she is not Gödel' or 'Tarski is such that he believes that some logician is not Gödel' depending on the intended scope. In case of wide scope readings (readings where the 'such that' idiom becomes essential) the way of showing scope "is essentially a matter of getting the indefinite singular term into the position of grammatical subject of a predication which is its scope, and so reducing the question of scope to the question of spotting a subject's predicate" (Quine 1960b, 140); it is a clarification of sentence structure by means of holding (indefinite) singular terms to subject position (Quine 1960b, 161).

Another reason for and advantage of the 'such that' construction is that it preserves the word order of the sentence from which it is abstracted. We already noted above that every relative clause gives rise to a general term, however, making clear what is predicated of what often requires bringing out the relative pronoun to the beginning of the clause and thereby calls for a modification of word order in order to remain grammatical (Quine 1980, 166). This task can be exacting and complex in cases and is largely simplified or completely avoided by falling back on the 'such that' idiom, as it simply divides the responsibility of the relative pronoun: the beginning of the general term given by the relative clause is indicated by the expression 'such that' and the role of standing for a singular term within the relative clause is relegated to a pronoun (Quine 1974, 92/92), e.g. from 'I bought ...' we obtain 'such that I bough it', instead of the more cumbersome 'which I bought'

[12] Cf. (Quine 1960b, 139).
[13] Cf. (Quine 1987b, 238).

(Quine 1960b, 112). The 'such that' construction then immediately enables us (just as the relative clause) to convert every sentence '... c ...' with singular term *c* into a single predicate 'such that ... he/she/it ...' predicable of *c* (Quine 1960b, 141). This, in turn, the predicate abstraction by means of 'such that' (just as what was tentatively claimed about the relative clause without guarantee or algorithm of transforming every relative clause into a grammatically correct phrase beginning with the relative pronoun) confers *predicational completeness*: "whatever we can say about a thing can be said about it by predicating a predicate of it" (Quine 1995a, 32).

It was the power and flexibility of the relative clause (Quine 1980, 165) that forced the assumption of objects upon us in order to explain its linguistic mastery. It was here that reference could be unmistakably and properly identified: "The positing of objects of any sort, from sticks or stones on up or down, is a sophisticated move that makes sense only *after* the mastery of the relative clause" (Quine 2008n, 321, my emphasis). The relative clause could be seen as and transformed into a general term, a predicate, applicable to an object: "the peculiar genius of the relative clause is that it creates from a sentence '... x ...' a complex adjective summing up what the sentence says about x" (Quine 1960b, 110). It could be predicated of an object through its use of a *relative pronoun* – which, who, whom, that, he, she, it, – the pronoun referred to the object the respective relative clause was to be predicated of. It is in the pronoun that we encounter pure and unvarnished reference – there is no other purpose and use to the pronoun than to establish reference, it does not qualify the object it refers to in any way,[14] but merely enables the predication of the relative clause, it supplies *something* that the relative clause can be predicated of: "the pronoun stands by for pure and simple reference. The pronoun is the tenable linguistic counterpart of the untenable old metaphysical notion of a bare particular" (Quine 1980, 165).

Another way of realizing that it is the pronouns to which we must turn when locating the "primary responsibility for reference" (Quine 1995a, 29) is the following: consider a basic declarative sentence, a predication such as, e.g., 'Swans are white'. Here whiteness is predicated of swans, the noun 'swans' denotes objects, swans, of which it is said that they are white. However, the noun 'swan' does much more and much less than merely establishing reference to objects of which the predicate 'are white' is predicated of. It does more because it already qualifies the objects talked about, i.e. it predicates 'swanhood' of the objects, such that they are white. The noun 'swans' in itself is already a predication, predicating 'are swans' of some objects: "the pronoun or variable is the vehicle of pure reference, while names, like predicates, serve to characterize the thing referred to" (Quine 1980, 172).

It does less because it need not refer to anything in order to contribute to a meaningful sentence, viz. 'Unicorns are horned horses' – no matter whether there are objects such that they are unicorns or not, the sentence remains meaningful.[15] The function of reference is therefore neither necessary nor sufficient for successful application of a noun[16] and nouns are thus not a reliable guide for locating reference. However, in abstracting relative clauses from sentences the pronoun takes over the (apparent, in case of non-denoting nouns) essential function of reference from the noun while dropping all other non-referential contributions of the noun thus replaced.[17]

[14] Assuming the pronouns 'which', 'who/whom', 'that', 'it'. 'He', and 'she' qualify the object of reference insofar as they provide information about its gender. We avoid these subtle complication by treating the latter as secondary, 'derived' pronouns that are w.l.o.g reducible to the former, 'purer' pronouns.

[15] Cf. (Quine 1964a).

[16] For the objection that this argument unfairly ignores names or other singular terms see the next section.

[17] It even creates referential function where there was none before, viz. the abstraction of 'which are horned horses' from 'Unicorns are horned horses', which, when completed to a full sentence refers to whatever the completing expression is true of. While certainly controversial the sentence 'Unicorns which are horned horses', or better, 'Unicorns (are) such that *they* are horned horses' appears to possess more referential force than the sentence 'Unicorns are horned horses'.

In any case, while nouns might have additional function over and above reference, the only contribution of a pronoun to a sentence consists in referring and it is therefore pronouns that are "the basic media of reference" (Quine 1964a, 13).[18] They are basic because there is nothing more to a pronoun than the function of reference. It is here one should look for pure, unencumbered, unobstructed and unqualified reference: "the most decisive general marks of reification in our language and kindred ones are the pronouns" (Quine 1992b, 26) and "[w]ere it not for pronouns, or other devices to similar effect, I could make no sense of objective reference" (Quine 1980, 165).[19]

However, disambiguation does not stop with the standardization of the 'such that' idiom and the emphasis on the fundamental importance of the pronoun in reference, for consider the following sentence:

(**) Satan trembles when he sees, the weakest Saint upon his knees.[20]

with the associated such-that paraphrase

(***) Satan (which) is such that he trembles when he sees the weakest Saint upon his knees.

Whose knees are we talking about here? Satan's or the weakest Saint's?[21] One might be able to tell from context, but not from sentence structure. (***) is again structurally ambiguous between a reading on which we talk about the devils knee and another on which the weakest Saint kneels. However, an easy, albeit somewhat artificial, way of disambiguation is readily at hand:

(+) Satan$_1$ is such that he$_1$ trembles when he$_1$ sees the weakest Saint$_2$ upon his$_2$ knees.

By indexing the pronouns we can exactly fix and determine which object, or which noun, they are supposed to refer back to. In case of (+) it is the Saint on his knees (the far more likely situation). However, indexed pronouns are basically *variables*: "Here, I say, is the birth of the variable: in the disambiguation of nested 'such that' clauses, which is to say nested relative clauses" (Quine 1987b, 238). The relative pronoun was the 'prototype', the 'psychogenetical embryo' (Quine 1966d, 281) of the variable which emerged as a disambiguating refurbishment of the traditional and limited relative pronoun, allowing for greater flexibility and reference-tracking, yet, in essence, remaining a pronoun, a device harking back to something denoting an object: "The variable of the 'such that construction' [...] is in effect the relative pronoun" (Quine 1974, 99), it constitutes its regimentation in a clearer and less ambiguous language.

Just as the pronoun, the variable is strictly for reference, "unencumbered with descriptive or identificatory offices" (Quine 1980, 168). Moreover, as heir to the pronoun, the basic function of the variable, its fundamental role is the tracking of reference, resolving structural, syntactic ambiguity and enabling clear and undisguised reference back to its grammatical antecedent. The role of the variable consists in marking and collecting 'scattered reference to the objects', coordinating of cross-reference in sentences.[22] The 'such that' construction combined with variables then becomes the "rectified relative clause, rid of crotchets that could only complicate" (Quine 1974, 93).

[18]Given that pronouns contribute nothing to the object they refer to in the sense of qualifying it, Quine speculates that nouns should have better been called 'propronouns', given that they add something to what the pronouns do, they built on top of the reference established by them (Quine 1964a, 13).

[19]Cf. also (Quine 1980, 167): "Were it not for the irreducibly referential pronoun, or some idiom to the same effect, any distinction between designative words and others would would be idle and arbitrary [...], a notion of object would have no place" and (Quine 1995a, 27): "I see this pronominal construction as achieving objective reference."

[20]Cf. (Quine 1960b, 135).

[21]The situation becomes even more tracked when we are dealing with nested such-that clauses.

[22]Cf. (Quine 1995b, 282): "The basic job of the bound variable is cross-reference to various places in a sentence where objective reference occurs."

Variable and relative clause complement each other: "the variable is best seen as an abstract pronoun; a device for marking positions in a sentence, with a view to abstracting the rest of the sentence as predicate" (Quine 1995b, 228). Due to the predicational completeness conferred by the relative clause this is possible for anything that can be said about any object whatsoever. Moreover, the predicational completeness together with the variables syntactic freedom and ability to track arbitrarily complex cross-reference lets the variable emerge as an "instrument purely of extrication: of rearranging a sentence around some chosen component, so as to segregate it from something it was about" (Quine 2008e, 310); "The variable is a device for marking and linking up various positions in a sentence so as to encapsulate, in an adjective phrase, what a sentence says about something. Its business is linking and permuting" (Quine 1987b, 238).

This freedom of the variable and its ability together with the relative clause to abstract from anything but purely referential position without loss of information or meaning, enables us to see that the variable is "the essence of the ontological idiom, the essence of the referential idiom" (Quine 1966d, 272). It becomes the "distilled essence of ontological discourse" (Quine 1974, 100): "Such, then, is the cosmic burden borne by the humble variable. It is the locus of reification, hence of all ontology" (Quine 1995a, 33).[23]

Having thus isolated *where* reference can uncontroversially and undoubtedly be found, and to which locution it can ultimately be reduced to, it remains to ask *when* we have a case of reference and thus grounds to posit the object referred in our ontology. The easy answer to the question of when would consist in a simple 'when we say we do', but unfortunately it is not always so clear when we really do refer, viz. the uttering of 'Mama' by the infant. We saw above (Section 2.2) that we were forced to posit objects and refer to them when dealing with an *essential pronoun* as opposed to a *pronoun of laziness*, where the latter could, without modifying the meaning of the sentence, be replaced by its grammatical antecedent, but the former could not. This, however, occurred in the case of observation categoricals when the antecedent contained an *indefinite singular term*, such as, e.g., 'a raven'. The presence of an indefinite singular term in the antecedent of an observation categorical forced the pronoun in the consequent to be essential and thus the indefinite term had to be taken as genuinely referring to an object. This as the case because (the pronoun being essential) it contributed something to the sentence (it is essential and therefore needed for whatever the sentence expresses) and the only thing it can contribute is reference. It is thus "with the advent of indefinite singular terms that we find pure affirmations of existence" (Quine 1960b, 112), due to the fact that pronouns, the locus of reification, pure reference and thereby ontology, become essential and contribute reference in case they are combined with an indefinite singular term. It is multiple presentation (of, in some sense, uniform things) that calls for indefinite singular terms, which, in turn, call for essential pronouns in order to keep track and distinguish between these multiple presentations (Quine 1995a, 28).

Indefinite singular terms come in many varieties and are not merely restricted to 'a', viz. 'an', 'any', 'each', 'every', 'some', etc. all qualify.[24] However, the entire class of indefinite singular terms can, through a series of fairly straightforward and uncontroversial conversions (Quine 1960b, pp. 162), be reduced to only the two 'every F' and 'some F' where 'F' represents any general term in substantival form. These are, in turn, reducible to 'everything' and 'something' (for how to do so see Section 4.1.2). All indefinite singular terms can therefore, w.l.o.g., be cut down to 'everything'

[23]There are, in fact, other uses of the variable such as in, e.g., descriptions and class abstraction. These uses can, however, all be repatriated to the basic use of the variable in its referential function as substitute of a pronoun. All other uses can be reduced to that of the variable in reference; cf. (Quine 1980, 168/169), (Quine 1995b, 234), (Quine 1966d, pp. 274), (Quine 1960b, 163).

[24]Cf. (Quine 1960b, 114): "There is no one thing, neither a lion nor a class nor anything else, that is named by 'every lion' any more than by 'a lion'." Cf. also (Quine 1995a, 28).

and 'something', which in turn are prone to a condensed symbolical representation by means of $\forall$ and $\exists$. This artificial notation can be explained as mere symbolic rendering of the locutions 'for all x such that' and 'there is something x such that', which are the intended readings of the well-known universal and existential quantifiers: "Such is simply the intended sense of quantifiers '(x)' and '$(\exists x)$': 'every object x is such that', 'there is an object x such that'. [...] The quantifiers are encapsulations of these specially selected unequivocally referential idioms of ordinary language." (Quine 1960b, 242).

Having thereby reduced all indefinite singular terms to the two quantificational expressions 'every object x is such that', 'there is an object x such that'[25] and the root of objective reference located in the variable, moreover, having linked reification to the essential pronoun and thereby its counterpart, the variable, we see that reference occurs in the binding of a variable as we here connect a pronoun with an indefinite singular term, thereby requiring such pronoun to be essential and demanding reference of it, forcing us to posit the object the indefinite singular term was thought to denote. Quantification thereby becomes the encapsulation of the referential apparatus, constituting a condensed version of an indefinite singular term and an essential pronoun referring back to it. Quantification encapsulates genuine reference and thus becomes the thing we have to look to when doing ontology.

What is referred to in a quantification, however, is what the quantified variables range over: "Hence, down the years, I have identified the objects of a theory with the values of its variables of quantification" (Quine 1980, 169). Moreover, given the fact that the universal quantifier is definable in terms of the existential quantifier[26], as $\forall \equiv \neg\exists\neg$, we can say that it is "the existential quantifier [...] that carries existential import" (Quine 1969c, 94): "The bound variable "x" ranges over the universe, and the existential quantifier says that at least one of the objects in the universe satisfies the appended condition" (Quine 1969c, 94). *Existence is what existential quantification expresses* – there are $F's$ if and only if the theory in questions affirms the existentially quantified identity $\exists x(Fx)$ (Quine 1969c, 97).[27] In terms of traditional grammar this amounts to saying that "to be is to be in the range of reference of a pronoun" (Quine 1964a, 13), for in ordinary English "what one takes there to be are what one takes one's relative pronouns to refer to. [...] The notation of quantification is what is most usual and familiar [...] where one is expressly concerned with ontological niceties" (Quine 1992b, 27). The latter is the case because natural language is unclear, ambiguous and at times misleading, which is why we settled for a regimented notation cleared of all potential obscurities. However, we know for certain that a pronoun refers when it occurs essentially, i.e. when harking back to an indefinite singular term, these indefinite singular terms having been reduced to existential quantification. An existential quantification therefore guarantees us reference and thereby the assumption of an ontology, this is not always so clear in ordinary discourse.

> "Seeing the referential apparatus as epitomized in quantification, we see it as consisting essentially of two sorts of device: there are the quantitative particles 'every' and 'some',

[25]"The indefinite singular terms were built upon general terms. They have now disappeared into quantification" (Quine 1960b, 163).

[26]Existential quantification of course merely is a logically regimented rendering of our standard 'there is' idiom (Quine 1969c, 97).

[27]This is of course not the only way one could think about how one could formulate the conclusion of the development above: "Taking other lines you may say that the objects of a theory are what the general terms are true of; or, again, what the pronouns can refer to. These versions do amount pretty much to saying that the objects are the values of the quantified variables; but quantification is conveniently graphic and explicit" (Quine 1974, 100). Quantification was the excision of a pre-existing idiom, but one could of course also consider developing a formalism for ontology in terms of predicate abstraction as Quine did with his predicate-functor logic (Quine 1971). A criterion for the ontological commitment in such a setting might read "To be is to be denoted by a one-place predicate" and would be equivalent to the criterion stated above.

> as applied to general terms in the categorical constructions, and there are the variables or pronouns as used in abstracting new general terms in the form of relative clauses. The relative clause and the categorical thus stand forth as the roots of reference. The objectual variable is an outgrowth of these two roots, not of one alone; for the variable of the relative clause begins as substitutional." (Quine 1974, 101)[28]

In light of the preceding the identification of ontology with values of variables now appears "less strangely arbitrary than before. There is more to it than a fancy for a nice logical calculus. It lies at the hidden heart of ordinary language" (Quine 1980, 170).[29]

3.2.2 Regimentation

Now, "[o]nce a theory is formulated in quantificational style its objects of reference can be said simply to be the values of its quantified variables" (Quine 1966d, 100). However, in order to apply the criterion of ontological commitment at all a theory *must* be formulated in quantificational style, for otherwise it gains no traction: "the question of the ontological commitment of theories does not properly arise except as that theory is expressed in classical quantificational form or we at least have in mind how to translate it into that form" (Quine 1969c, 106). Natural language is vague and complex. It is not always clear if and what objects we assume in such discourse. For this purpose the criterion was devised as a standard allowing us to judge and measure the ontological import of a particular theory or piece of discourse. In oder to make sure we can always apply it then, and to make sure its application always underlies identical constraints so as to not unfairly skew the result of the criterion we devise a notation, *canonical notation*, exhibiting quantificational structure into which we regiment the obscure natural language or scientific discourse before we assess its ontological import. Note that the purpose of such a notation is not a language reform, it does not urge us to only talk in the idioms of canonical notation and give up natural language completely, but it helps to clarify and 'clean up' the obscure elements of natural language discourse so as to give us a clearer picture to what we are ontologically committed to: "the scientist can enhance objectivity and diminish the interference of language, by his very choice of language" (Quine 1960b, 235). Moreover, it provides a common basis for all theories whose ontological commitment is to be assessed, so as to retain comparability and rule out the possibility of 'cheating' by modifying the quantificational language. Our language of choice that diminishes interference of language is what constitutes the so-called canonical notation and we will have to say more about what it is and how it came to be first-order logic in the next chapter. For now we want to focus on the process of translating or regimenting discourse into this notation, as it is a non-trivial process.

It follows from what was said above that determining the ontological commitment of a theory or piece of discourse divides into two tasks: paraphrasing that discourse into a notation allowing us to apply the criterion and then applying that criterion. Canonical notation itself is supposed to provide a notation that is so explicit on matters quantificational and thus ontological that it simply

[28] This is not uncontroversial. Geach, for example, takes quantification as primitive and obtains relative clauses from it; cf. (Geach 1962) and (Geach 1968). From the point of view of a genetically motivated theory of language acquisition this way is, however, not open.

[29] Note, at this point that the referential role of the variable has nothing to do with the quantificational force of the quantifiers: "The variable comes to appear in its true light as purely a means of identifying and distinguishing the referential places in a sentence, and has nothing to do with 'all' or 'some', which are the business of '$\forall$' and '$\exists$"' (Quine 1995a, 32), as well as (Quine 1980, 275): "The point I want to make is that the quantitative force of the quantifier the 'all' and 'some' is irrelevant to the distinctive work of the bound variable and irrelevant to its referential function." This will become important when we propose to introduce a new quantifier binding the same sort of variables in Chapter 5.

allows us to read off the the commitments of a theory from its canonical regimentation without having any room for obscurity or doubt as to whether something is quantificational or not:[30] "To paraphrase an sentence into the canonical notation of quantification is, first and foremost, to make its ontic content explicit, quantification being a device for talking in general about objects" (Quine 1960b, 242). Paraphrasing a theory into canonical notation constitutes the process of *regimentation*. Regimentation into canonical notation is inevitable for understanding what a given piece of discourse is about, after all "[i]t is scarcely cause for wonder that we should be at a loss to say what objects a given discourse presupposes that there are, failing all notion of how to translate that discourse into the sort of language to which 'there is' belongs" (Quine 1961, 105).

The problem of how to paraphrase discourse into canonical notation presents itself in multiple dimensions. On the one hand, if the discourse we translate is formulated in a different idiom than ours, we first need to translate it into our language to make sense of it, for *reference is parochial* and without identifying the referential elements of discourse there is no way to appropriately regiment it into canonical notation. Translation, however, is *indeterminate*.[31] On the one hand, reference is inscrutable (as mentioned in the previous chapter) allowing multiple and equally good ways to fix the reference of the foreign discourse. On the other hand, it might be holophrastically indeterminate in that there might be multiple equally good and mutually incompatible ways to fix the apparatus of reference, given strong enough adjustments in our translations of the remaining expressions of the language.[32] In any case, the problems do not stop once we have settled on what we take to be an appropriate translation of the interlocuters discourse, for the paraphrase into canonical notation, the regimentation of a piece of discourse is guided as any paraphrase by considerations of simplicity, systematicity, and avoidance of obscurities and problems. There might then be no unique paraphrase, multiple possible options will be open for us. Thus, while we may ultimately be able answer what exists according to a theory once we have paraphrased in into canonical notation, this questions gives way to the question of how to regiment it:

> "The moot or controversial part of the question of the ontic import of a sentence may of course survive in a new guise, as the question how to paraphrase the sentence into canonical notation. But the change of guise conveniently shifts the burden of claims and disavowals. Futile caviling over ontic implications gives way to an invitation to reformulate one's point in canonical notation. We cannot paraphrase our opponents sentences into canonical notation for him and convict him of the consequences [...]; rather we must ask him what canonical sentences he is prepared to offer, consonantly with his own inadequately expressed purposes. [...] To decline to explain oneself in terms of quantification, or in terms of those special idioms of ordinary language by which quantification is directly explained, is simply to decline to disclose one's referential intent. [...] We remain free as always to project analytical hypotheses [...] and translate his sentences into canonical notation as seems most reasonable; but he is no more bound by our conclusions than the native by the field linguist's." (Quine 1960b, 242/243)

There might be multiple equally good ways, respecting simplicity, familiarity and other theoretical values to regiment a piece of discourse and no way of knowing which one to choose. The reason why there will not be one definite correct paraphrase into canonical notation, no unique correct

[30]Cf. (Quine 1960b, 242): "In our canonical notation of quantification, then, we find the restoration of law and order."

[31]Cf. (Quine 1960b) and (Quine 1987a).

[32]The *indeterminacy of translation* is a substantial and controversial thesis. We refer the reader to (Kirk 2006), (Hylton 2007), (Quine 1960b) and (Quine 1987a) for discussion and will ignore the issues pertaining to translation for a large part in the following.

analysis, is because there is *no presupposition of synonymy* between the original theory and its canonical paraphrase.[33] The goal of the regimentation, the paraphrase into canonical notation, was to clarify the original, more obscure discourse. However, if we "paraphrase a sentence to resolve ambiguity, what we seek is not a synonymous sentence, but one that is more informative by dint of resisting some alternative interpretation" (Quine 1960b, 159); such a paraphrase can and will result in 'substantial divergences' such as, e.g., the closing of truth-value gaps or the supplying of explicit reference (Quine 1960b, 159). The relation of a sentence S of the original theory to its regimented paraphrase S' "is just that the particular business that the speaker was on that occasion trying to get on with, with help of S among other things, can be managed well enough to suit him by using S' instead of S" (Quine 1960b, 160). However, it is only the speaker who can judge whether a particular paraphrase is appropriate whether it meets the purpose which he used the original sentence for.

Synonymy between original and paraphrase is not required in order to satisfy the function that the original sentence was used to fulfill, in fact requiring absolute synonymy actually obscures and confuses the goal of paraphrase whose aim is to clarify discourse by focusing on its essential functions in that particular situation and ignoring the rest. Thus, the paraphrase of ordered pairs into certain set-theoretic constructions is, for example, good because "[t]he paraphrases were such as to meet most or all the likely purposes for which the originals might be used, except insofar as such needs included brevity or familiarity" (Quine 1960b, 191). A paraphrase into canonical notation is good "insofar as it tends to meet needs for which the original might be wanted" (Quine 1960b, 182), it is not required, however, that the resulting paraphrase is synonymous with the old expression. "It is only required that, to our satisfaction whatever we hoped to achieve by way of our original sentences can be achieved closely enough by way of their paraphrase" (Rayo 2002, 2), we did not simply translate the old discourse by means of paraphrase, we surrendered it in favor of the new (Rayo 2002).

Regimentation, then, is explication. And explication is elimination: for a troublesome expression we find another, much clearer and better one that satisfies all the originals essential functions. We then simply replace the old expression with the new and forget the old problematic one. We have eliminated it and at the same time explicated what we took to be its essential contribution.[34] Now,

> "[t]his construction is paradigmatic of what we are most typically up to when in a philosophical spirit we offer an "analysis" or "explication" of some hitherto inadequately formulated "idea" or expression. We do not claim synonymy. We do not claim to make clear and explicit what the users of the unclear expression had unconsciously in mind all along. We do not expose hidden meanings, as the words 'analysis' and 'explication' would suggest; we supply lacks. We fix on the particular functions of the unclear expressions that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking, that fills those functions. Beyond those conditions of partial agreement, dictated by our interests and purposes, any traits of the explicans come under the head of "don't cares" [...]." (Quine 1960b, 258/259)

The process of regimentation is thus no trivial matter and should be kept in mind in our further discussion. Neither is, however, the choice of an appropriate canonical notation, the topic of the next two chapters, for while we are able to choose how to regiment a given piece of discourse, "[o]nce we have settled upon a language of regimentation and accepted a theory couched in that language, ontological commitments are forced upon us" (Rayo 2002, 3). Once we have chosen the language

[33]Cf. (Quine 1960b, 159): "In neither case is synonymy to be claimed for the paraphrase."
[34]Cf. (Quine 1960b, pp. 260).

of regimentation and how to paraphrase a particular theory into it, ontological issues are settled determinately and cannot be changed or paraphrased away. We have to live with them. There is thus a lot riding on the choice of a good and sensible canonical notation.

3.3 Remarks on (Lang) and (strong.Q)

The list of objections to Quine's criterion of ontological commitment and where it goes wrong is long and varied.[35] One could doubt the adequacy of the criterion on the basis of its conception of quantification being too narrow, its focus on pronouns rather than names being misguided, its ignorance of 'there is/are' locutions that do not apply to objects, etc., etc. The list is too extensive to offer a substantial defense of the criterion against all of them here, especially given that they attack very different parts of it. What I do want to do is to provide at least a cursory glance at how the criterion resists the challenges as put forward by (Lang), (Bias) and (strong.Q). They are no more than cursory remarks, however, for I will neither develop said theses fully, nor consider all the different directions into which they could be expanded. The main reasons for this are, on the one hand, that the main concern of this thesis lies with (weak.Q) and its vindication and, on the other, that it is especially the fundamental status of canonical notation for the criterion which has forced ontological commitments onto various theories and expressions that several authors have taken issue with, albeit sometimes not clearly seeing that it was (weak.Q) rather than (strong.Q) that they were advancing in their writing.

What, then, could the Quinean respond to the various challenges brought forward against her? With respect to (Lang) her response comes easy given what was said before: our basic contact with reality is not referential (Section 2.1), reference and the positing of objects, whose only function consists in serving as 'neutral nodes' in theory to facilitate predictions, was necessitated by the complexity of theory that helped it explain past and predict future stimuli. Those theories are, however, in an important sense linguistic entities: they are expressed[36] (at least partly) by a set of sentences. Even more importantly, language itself is inextricably connected with theory, for which expressions we take as referring, which as particles, etc. depends on which theory about language delivers the best results in predicting future linguistic stimuli in a linguistic community. Our ontology depended on what we chose to take as referring in our language, a choice dictated by considerations about theoretical virtues. Reference, as a property of linguistic expressions, was in the Quinean setting the only access we possess to ontology. We need reference to make sense of ontology. In short, we need language to make sense of theory and theory to make sense of language, which, in turn, allows us to make sense of reference, which we need to make sense of ontology. Given, then, that the only way we can make sense of ontology is via language, it comes as no surprise that ontology depends on language and is necessitated and facilitated by considerations on language. Moreover, it not only depends on it in some way or other; ontological relativity showed us that it is relative to our choice of language. Other than pragmatic constraints on theory construction the sky is the limit to what we take there to be "out there". The criterion of ontological commitment exploits this connection and makes by its reference to language ontological discourse possible at all. *There just is no prior and extra-theoretical and thereby extra-lingual reality*; what there is, is what we take to be a useful posit to constitute a reference foe expressions in our language, so as to systematize that very language. We cannot take a standpoint outside our language to see what

[35] Cf. (Alston 1958), (Jackson 1980), (Parsons 1982), (Routley 1982), (Hodes 1990), (Lewis 1990), (Melia 1995), (Geach, Ayer, and Quine 1951), (Azzouni 1998), (Yablo 1998), (Rayo 2007) (first part of the list based on references in (Rayo 2007)), to only name a few.

[36] Whether a theory actually is a set of sentences or a set of dispositional states or something else we leave open at this point.

there is and then revise language accordingly. What there is depends on how we speak about it in the sense that it is the only way we can make sense of ontology at all – as bound to language and dependent on language. Our conceptual scheme out of which we cannot step and the language we grow up in are so intimately connected – in fact, the latter is constitutive of the former, – that we cannot reach reality other than via language. And this seems to be something a proponent of (Lang) appears to doubt – he appears to be claiming that there can be a language-independent approach to ontology. However, there is no room for such an approach in the Quinean framework.

Turning to (Bias) we first note that as it stands it is very hollow in that every criterion of ontological commitment whatsoever will build in certain ontological biases by construction by virtue of being a criterion of ontological commitment at all. Such a criterion was supposed to tell us where to look for ontology – it is supposed to be biased towards something, otherwise it would tell us nothing. Thus in considering (Bias) we split it into the claims (weak.Q) and (strong.Q). The former is the topic of the next chapters; we will thus turn here to the latter. (strong.Q) claims that the formulation of the criterion of ontological commitment in terms of first-order quantification is inadequate. This might mean that it undergenerates or overgenerates with respect to the entities that it forces us to assume. Note, however, that what it forces us to assume depends in a crucial way on the language into which we regiment a theory for assessment by the criterion. The language of regimentation, however, is the topic of (weak.Q). Thus what we might take (strong.Q) to say is that it either left important classes of referring expressions out in its derivation or included some which should not have counted as referring expressions. The latter claim appears absurd: the criterion of ontological commitment was founded on an absolutely minimal and uncontroversial basis, that point where we *have to* assume reference in order to explain what it going on – reference became an undeniable necessity in the connection of indefinite singular terms and pronouns. Quine presented the minimal presuppositions for reference and derived the criterion based on these presuppositions. It is doubtful that anyone could have assumed less and still made a strong case for requiring reference. The former claim thus appears much more plausible: that it left out important classes of referring expressions in its derivation and is therefore inadequate by focusing only on first-order quantification.

Here, however, one would have to say that this does not matter at all. The criterion was supposed to deliver a scientific standard by which to compare certain aspects of theories (viz. their ontology), according to their systematicity. In doing so it had to provide a standard on which everyone could agree upon, such that this standard would constitute a recognized means of settling ontological disputes without either party being able to disagree about the standard if they shared the same picture of reference and language. The criterion was built on such minimal basis because it needed maximum confirmation that whatever falls under the first-order quantifiers must exists if we believe in the picture of reference as outlined above.[37] Whenever I connect an indefinite singular term with an essential pronoun I need to refer to an object to make my locution meaningful, this much everyone can agree upon on the picture outlined above. Whether I also posit something when using a name is much more controversial and thus does not offer itself for the basis of a commonly accepted criterion. The assumptions need to be minimal in order to gain maximal acceptance. However, this is all there is to a criterion of ontological commitment as developed here: it need not do justice to our intuitions about ontology, it only needs to constitute a general and efficient standard of comparison allowing us to state our differences. After all, there simply is no sense to speak about ontology absolutely, there is no ontology other than the one posited for systematizing theory. What matters is not what any particular theory says there is, but how I can interpret one

[37]Note that we are still ignoring the issue of canonical notation which might force us to posit certain entities by limiting our ways of paraphrase.

theory in another and this requires me knowing what there is according to the respective theory, not what there is absolutely.

If here is no such independent ontology, however, what is the standard by which to measure the adequacy of the criterion? It can be measured by how well it does in telling me when I should reduce a theory to another. And that, it certainly does, as it delivers a way of comparison of one aspects of a theory's virtues, i.e. with regard to its ontological assumptions. In what way then can (STRONG.Q) be claimed without truly meaning (WEAK.Q)?

Moreover, note that as soon as we change the ontological commitment and take it that there is something other than first-order quantification telling us what there is out there, we also change our account of what qualifies as sufficient evidence for reference. After all, our derivation shows that the criterion was based on the minimal conditions that have to be fulfilled for us to be able to identify a genuine case of reference. This might mean changing our concept of reference, however, and, given the grounding of our concept of existence in that of reference therewith our concept of existence. Thus, when confronted with two different candidates for a criterion of ontological commitment, it is best not to think about them as disagreeing, but of advancing a different account of existence and the grounding of that notion. After all, "philosophical interest, ontological interest, attaches to deviations in quantification theory" for a deviant concept of quantification "carries with it a deviant notion of existence (if existence is still a good word for it)" (Quine 1986, 89).

These remarks remain very unsatisfactory for anyone agreeing with either of the claims allegedly refuted. I do not intend my account here to be a full-fledged defense of the Quinean system against these particular objections, but rather meant to point out how they are better seen as extra-systematic objections, refuting one or another element of the Quinean framework, rather than pointing to an internal inconsistency. This is, I believe, different in the case of (WEAK.Q) to which I now wish to turn.

Chapter 4

Canonical Notation

Whenever Quine's views on ontology, particularly his criterion of ontological commitment, are discussed, it is casually mentioned how this criterion relies on a prior regimentation of any particular theory whose ontological import is to be assessed into a language in which the criterion can gain traction. It is then further parenthetically stated that this language of regimentation in the Quinean framework happens to be standard first-order logic with identity (FOL) and if any justification of that choice is given, it, more often than not, consist in the simple and simplistic claim that first-order logic just happens to be very special to Quine because this is just what he believes logic to be. While it is certainly correct that FOL plays a privileged role in Quine's system, vindications of this kind are not only inaccurate, they misrepresent the function and thereby the true justification and genesis of a canonical notation.[1]

The fact that more than just considerations on logic went into Quine's construction of a canonical notation is not merely an exegetical claim about a correct and more comprehensive Quine interpretation, but constitutes an important feature of Quinean discourse about ontology that has largely gone unappreciated. Its under-appreciation has, on the one hand, led to a confusion of (STRONG.Q) and (WEAK.Q) and, on the other, to misguided arguments against and criticisms of the criterion. The confusion between (STRONG.Q) and (WEAK.Q), facilitated by the misunderstanding of the true nature and origin of the first-order canonical notation in Quine's system is exemplified in the attempt of a formulation of criteria of ontological commitment for *modal languages* by means of utilizing, for example, free modal logic as canonical notation.[2] While on the face of it being an instance of (WEAK.Q) (after all, the criterion is maintained, it is only the underlying language/notation that is changed), it is in fact an instance of (STRONG.Q) given that the meaning of the quantifiers has changed (there can now be non-denoting singular terms).[3]

Furthermore, a misunderstanding of the nature and genesis of a canonical notation in the Quinean system has led to many criticisms of FOL as a "partial notation[...] for discourse on all subjects" (Quine 1960b, 160) on the basis of it forcing ontological commitment onto seemingly harmless and ontologically innocent natural language expressions.[4] While it might very well be correct that the criterion of ontological commitment as it stands (see Section 3.2.1) imposes ontological obligations on seemingly perfectly harmless natural language devices, such as plurals, nominalizations and independence phenomena, this alone cannot be taken as a sufficient argument for their inclusion into

[1] In fact, a caricatural justification of the kind above is no genuine justification in any way; after all, why should the status of any particular logic determine a language suitable for ontological investigations?

[2] Cf. (Rayo 2007, 438)

[3] Never mind the fact that Quine's animadversions to modality (Quine 1953b; Quine 1960b) would disallow the utilization of a modal language as a model language for science anyways.

[4] Cf., e.g., (Rayo 2007; Rayo 2002), (Boolos 1984), (Hintikka 1973), (Hand 1993) and (Patton 1991).

canonical notation. The purpose of a canonical notation is very well-defined and the considerations going into its design are multifarious. Arguments to the effect that Quine's first-order canonical notation should be extended to include such-and-such expressions (plural/branching/higher-order quantification) on the basis of them being ontologically innocent or yielding greater expressive power are fundamentally misguided in that they do not consider the motivations that rendered canonical notation first-order.

The criterion works because there is a subtle interaction between it and its underlying canonical notation and it is important to understand this interaction in order to present a sensible and directed criticism of it. Thus, while I ultimately believe (WEAK.Q) to be correct, i.e. I believe Quine's canonical notation to be too weak to 'do the job' it was designed to do, namely to provide a effective and clear way of structuring ontological discourse, I do not find any arguments in the literature to convincingly argue for this claim – they all take too much for granted or merely construct a Quinean straw-man when attacking his canonical notation on account of its first-order weakness.

I will present my vindication of (WEAK.Q) in the next chapter, before doing so it is, however, important to understand how canonical notation came to be FOL in Quine's system and what the different moments are that influenced it. This will enable a clear view on what must be established in order to launch a justified criticism of Quine's canonical notation and, by extension, where and how previous criticisms failed to demonstrate what they intended to and oversimplified the issue. This is what the present chapter intends to do: here, I will provide a presentation of how and why canonical notation is first-order and what the different moments are that came to play a role in its design.

The design of Quine's canonical notation is based on the two interrelated principles of *clarity* and *simplicity*. Applied to the language of canonical notation and the logic of the theories formulated in it[5] this yields the properties of *extensionality* and *classicality* for the language and logic respectively. Moreover, this is combined with an account of an *objectual interpretation* of quantification and the shunning of all but first-order quantifiers, due to an extra constraint which I will tentatively call *minimality*: we only need to make a notation as complex as necessary for the purpose at hand; as long as it does what it is supposed to do there is no need to introduce extra gadgets in order to make it more expressive.[6]

4.1 The Language of Canonical Notation

4.1.1 The Extensionalist Program

According to Quine, his work relies critically on two fundamental presumptions, *naturalism* and *extensionalism*.[7] We briefly outlined naturalism above (see Section 1.2) and while extensionalism

[5]It is necessary that we specify a logic, for Quine always talks about the *deductive closure* of a theory when talking about a theory.

[6]A proponent of a higher-order canonical notation could object to this by saying that the restriction to a first-order language violates the maxim of simplicity, after all, the greater expressive power yielded by higher-order notation greatly simplifies notation and the objects that need assuming. A Quinean reply could consist in pointing out that all that we gain in notational simplicity is lost in conceptual complexity and that minimization of the entities in the universe of discourse is not the job of a *notation* that is supposed to enable ontological discourse. Nevertheless, what this objection shows is that the requirement of minimality has to be taken with a grain of salt: there are trade-offs between notational simplicity and conceptual clarity. However, I do not believe that this plays a significant role in the argument presented in the next section.

[7]This is the answer he provided in an interview with Fara when asked about the main tenets of his philosophy (as quoted in (Decock 2002b, 47)).

is, of the two, certainly the more neglected aspect, it is nevertheless of utmost importance within Quine's work and a presumption he has maintained throughout his entire career:[8] "I am a confirmed extensionalist. Extensionalism is a policy I have clung to through thick, thin, and nearly seven decades of logicizing and philosophizing" (Quine 2008a, 498).[9]

Extensionality, so Quine, grounds the clarity of discourse and is so essential to it that, in fact, "[n]o theory is fully clear unless extensional" (Quine 2008k, 172)[10]. One of the reasons FOL is so suited to the role of a canonical notation is its clarity, and *the source of this clarity lies in its extensionality* (Quine 2008k; Quine 2008b). A precise definition of what extensionality amounts to can be found via the definition of *co-extensive*:

Definition (co-extensive): We say that

(i) two names/singular terms a and b are *co-extensive* iff they designate the same object.

(ii) two sentences p and q are *co-extensive* iff they have the same truth-value.

(iii) two n-place predicates/relations/general terms/open sentences φ and ψ are *co-extensive* iff they are true of the same ordered n-tuple of objects.[11]

This then allows us to state a concise definition of *extensionality*:

Definition (extensional): An expression/context is *extensional* iff "replacement of its component expressions by co-extensive expressions always yields a co-extensive whole" (Quine 2008a, 498).[12]

Extensionalism is thus "a predilection for extensional theories" (Quine 2008a, 498).[13] Moreover, a language is said to be extensional iff it contains no expressions that give rise to non-extensional

[8]In fact, it might very well be argued that it was the doctrine of extensionalism that motivated and started off his work in mathematical logic (Quine 1934b) (his dissertation was an attempt to rid Principia Mathematica of its intensionality (Quine 2008a, 503)) and later provided the connecting element with his work in philosophy (Decock 2002b, Chapter 3). In any case, it is a doctrine which has motivated much of Quine's work and which he never deviated from.

[9]A strong case can be made that there is a tension between Quine's emphasis on extensionality and the criterion of ontological commitment, for it has been argued that said criterion relies *crucially* on intensional notions ("must"). Quine and others have subsequently tried to provide extensional formulations of it, but it has been argued that these formulations are inadequate and that the insistence on the criterion therefore entails countenancing intensional objects and notions and therefore constitutes a violation of the doctrine of extensionality; cf. (Cartwright 1954) and (Scheffler and Chomsky 1958). This is a serious challenge to Quine's conception, for if the criterion is essentially intensional its own underlying notation will not suffice to properly express the criterion's commitment. However, in the context of this thesis we choose to ignore the issues pertaining to Cartwright's challenge and treat extensionality as one of the fundamental constraints of Quine's system, to be respected by whatever modification is proposed to the criterion or its underlying notation, no matter whether the criterion itself relies essentially on intensional notions or not.

[10]Cf. (Quine 2008a, 500): "I doubt that I have ever fully understood anything that I could not explain in extensional language" and (Quine 1995a, 90-91): "I find extensionality necessary, indeed, though not sufficient, for my full understanding of a theory."

[11]Definition taken from (Quine 2008a, 498).

[12]The clearest and most obvious extensional objects to date are sets, yet Quine's definition ignores them completely and only talks about expressions. This is no limitation or omission, however, for by considering the class abstract "$\{x : Fx\}$" for any set {x: Fx}, a singular term, it is easy to see that if Fx and Gx are co-extensional, "$\{x : Fx\}$" and "$\{x : Gx\}$" will name the same set and we can therefore, w.l.o.g., transfer the notion of extensionality from class abstracts to sets themselves. (Quine 2008a, 500).

[13]Cf. also (Hylton 2007, 288-289).

contexts (Hylton 2007, 289).[14] Given this definition and understanding of extensionalism we see that the clarity conferred by it lies in the fact that co-extensional expressions are interchangeable *salva veritate*, i.e. without modifying the extension/truth-value of the whole they are part of. Why this kind of clarity and transparency is desirable for science is also fairly easy to see: scientific theories are (deductively closed) sets of (declarative) sentences. Declarative sentences themselves are predications, something is predicated of objects in them. Science aims at saying something true about these objects. However, given that we want to be as clear as possible about what we say when doing science, to avoid all kinds of misunderstandings and unnecessary confusions, the truth of a sentence which is about an object, or a number of them, should only depend on that object, but not on the way it is specified or described.[15] Extensionality allows us to remove, so to speak, linguistic/speaker-relevant components (and confusions) from scientific theories and enables us to focus on the factual components of these theories;[16] after all, this is what one is interested in when doing science: facts.[17]

Recalling that objects are nothing but posits of theory to optimize prediction and that really all there is to any particular object is what the theory positing it says about it, moreover, that predication is intimately linked with identity (see Section 5.3.1) "the positing of [...] objects makes no sense except as keyed to identity" (Quine 1969e, 23) and "[r]eification incurs the responsibility to individuate the reified entities, for *there is no entity without identity* (my emphasis)" (Quine 2008a, 501). The fact that identity is inseparable from predication and thereby theory, combined with the demand for an extensional theory formulation calls for clear criteria to individuate and refer to those entities, for how can a theory be extensional if it is not clear whether two expressions are referring to the same object or not?[18] In other words, the reification of entities by a theory and for the purpose of theory, such that predications about these entities can be made, necessitates the possibility of a clear identification of these entities and determines a standard for the admissibility of objects: *no entity without identity.* Unless we can clearly say when two objects are identical or distinct there is no ground for admitting these objects:[19]

[14] Given that above definition of *co-extensive* enumerates three conditions it might appear as if there could be three different degrees of extensionality within a language and that, in order for a language to be fully extensional in our sense, it needs to satisfy three distinct conditions. However, it can be shown that as long as one of the requirements is satisfied, so are the others (Hylton 2007, Appendix), such that "extensionality is not a group of three separate requirements lumped together under a single name, but is rather a single condition on a language" (Hylton 2007, 291).

[15] Cf. (Hylton 2007, 290).

[16] Quine would, of course, despise such talk of factual vs linguistic components since we should, according to him, reject such a distinction ((Quine 1964c) and see as well (Sher 1999b)), and we only mean what we say in a heuristic way here, not to be taken literally or as intending to introduce such a distinction.

[17] Whether extensionalism is the only or even the best way of doing so is a different question that we will not deal with here.

[18] While there is an unquestionable link between identity and predication and the need for clear identity conditions is reinforced by extensionality, the notion of identity itself is not unproblematic: despite the facts that it appears as one of the most natural and simplest concepts that we possess (every objects bears it to precisely one entity, namely itself) questions about its exact nature have caused many puzzling problems. It is a well-known fact that the addition of identity as logical constant to the other axioms of quantification theory still affords a complete proof-procedure, yet it is not a first-order relation, i.e. it is not definable in a first-order language (Hodges 1993, 64-65) (see also (Quine 1950)). Some authors maintain that an absolute notion of identity makes no sense and advocate a notion of *relative identity* (Geach 1967; Geach 1962), identity relative to a kind, whereas others shun any such relation from the outset: "Identity of the object I express by identity of the sign and not by means of a sign of identity" (Wittgenstein 1922, 5.53). Questions surrounding identity and criteria of identity have been discussed extensively in the history of theory, and have featured prominently in the Christian discussion about the trinity; cf. (Cartwright 1987). We will return to some of the problems connected with the notion of identity in Section 5.3.1. For the role identity plays in Quine's philosophy, see the excellent discussion in (Decock 2002b).

[19] In (Quine 1976a) Quine elaborates on some of the ways in which objects can be distinguished from one another

> "Take, for instance, the possible fat man in that doorway; and, again, the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are no *two* possible things alike? Is this the same as saying that it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibles? But what sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct from one another?" (Quine 1964a, 23-24).

The demand that a canonical notation be extensional reinforces and supplements the need for *criteria of identity*,[20] criteria that provide a means of deciding when two entities are identical and when they are not. This is the case because for a theory or language to be extensional the identity conditions for the objects talked about have to be settled, otherwise intensional contexts might arise because of a lack of ground on which to decide whether two terms refer to the same object or different objects, given that it might not be clear whether the respective objects are identical. Criteria of identity, then, provide a way of settling the distinctness or identity of objects, they tell us when we should regard objects as identical or distinct and they do so by specifying a condition the objects have to fulfill in order to qualify as identical. In general, they will be of the form: *for two P's, x and y, x and y are identical iff they satisfy some condition* C_P.[21] An example of such a criterion can be found in the axiom of set extensionality ((a) below: two sets are identical iff they contain the same members) and Frege's criterion for the identity of directions (Frege 1953, §64) ((b) below: the direction of two lines are identical iff they are parallel to each other):

(a) $\forall x \forall y(x = y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y))$

(b) $\forall x \forall y(direction(x) = direction(y) \leftrightarrow x//y)$

We can therefore draw two lessons from the extensionality-constraint imposed on canonical notation: (1) any language that enables intensional contexts, i.e. contexts in which the principle of the intersubstitutability of identicals (co-extensive expressions) fails cannot serve as a proper notation for science; (2) only entities that can be clearly individuated, whose identity conditions are settled, are admissible in an ontology. While (1) is a purely meta-ontological constraint (a constraint telling us how to go about when doing ontology), (2) is ontological as well as meta-ontological: it is ontological in that it bans certain kinds of entities (namely those with indeterminate identity conditions, ontologically vague identities if you will), yet it is meta-ontological in that it formulates necessary and sufficient conditions for an entity to be accepted in an ontology, it tells us conditions under which we can be taken to properly do ontology.

Quine himself has stuck very closely to these constraints throughout his philosophy. In his 'flight from intensions' he banishes all intensional notions from a proper canonical notation (see (Quine 1960b) and below) and he sticks to a purely extensional ontology: the privileged status of physical objects and sets in it is grounded in the fact that clear criteria of identity for these can be given; the axiom of extensionality (a) provides a clear criterion for the latter and an identification of physical

intra-theoretically.

[20] Cf. (Geach 1962, 8): "That in accordance with which we thus judge as to the identity, I call a criterion of identity." See also (Gottlieb 1976).

[21] Cf. (Lowe 1989a). For a more extensive overview, classification and discussion of criteria of identity, we refer the reader to (Noonan 1980; Noonan 2009), (Lowe 1989b; Lowe 1997) and (Williamson 1990).

objects with quadruples of numbers (objects that can be given in terms of sets), sets of space-time coordinates, legitimizes the former:[22]

> A physicalist ontology that is apparently adequate to all reality consists of just the physical objects, plus all classes of them, plus all classes of any of the foregoing, plus all classes of any of the foregoing, plus all classes of any of this whole accumulation, and so on up. (Quine 1995a, 40)[23]

Propositions vs Sentences

The requirement of clear criteria of identity in order for objects to be admissible into an ontology, coupled with the strive for economy and the shunning of unnecessary entities, leads Quine to the rejection of propositions and the acceptance of just the sentences which were thought to express those propositions instead. Propositions were considered to be the meaning of sentences, necessary in order to capture the relationships that hold between different sentences: "Meanings of sentences are exalted as abstract entities in their own right under the name of *propositions*" (Quine 1986, 2). Positing propositions as entities in their own right immediately leads to the question when and under what circumstances two propositions are identical, after all, there is "no entity without identity". "'When are two propositions identical?' or better 'When do two sentences denote the same proposition?"' (Quine 1934a, 472).

Given that we have a better grasp on sentences than on propositions (them being allegedly abstract objects) and the fact that "[i]f there were propositions, they would induce a certain relation of synonymy or equivalence between sentences [...]: those sentences would be equivalent that expressed the same proposition" (Quine 1986, 3) it is easiest to turn towards the sentences themselves, somewhat more concrete objects, when searching for a criterion of identity for propositions. One does not have to look far to find a suitable candidate; exploiting the equivalence relation induced by synonymy one arrives at a criterion such as the following: Two propositions are identical iff the sentences expressing them are synonymous. However, as Quine has repeatedly and forcefully argued, we cannot make sufficient sense of a notion of synonymy given that all we have is behavioral evidence[24] Given, then, that there is no such synonymy between sentences as would be required in order to state appropriate identity conditions for propositions there are not such identity conditions and thus, due to their lack, no propositions.[25]

This is, however, not a big loss and we should have on grounds of Occam's razor done without propositions anyways: "the whole notion of sentences as names is superfluous and figures only as a source of illusory problems" (Quine 1934a, 473). Everything we could have done with propositions we can do with sentences, instead of talking about meaning as abstract objects we can talk about meaning as linguistic speech dispositions, recasting the entire debate in behavioristic terms. Since everything that could have been done with propositions can just as well be done in terms of sentences,[26] we might as well do without propositions, and should, given that their assumption was unnecessary in the first place.

[22] Cf. (Hylton 2007, 300ff) for a discussion of Quine's own preferred ontology.

[23] As quoted in (Decock 2002b, Chapter 3).

[24] Cf. (Quine 1964c) and (Quine 1960b).

[25] This argument strongly relies on the fact that the indentity-criterion in terms of synonymy of sentences is the only feasible one. While this need not be the case, it does not severely weaken Quine's argument, after all, if there were propositions we should be able to make sense of the notion of synonymy between sentences. But we are not. Cf. also (Leeds 1978, 80).

[26] This is a far-reaching claim, but not immediately relevant to what follows. We refer the reader to (Quine 1934a), (Quine 1986) and (Quine 1960b).

Universals Extensionalised

Similar considerations lead to the 'extensionalisation' of universals such as, e.g., properties, attributes and relations. When pressed to provide informative criteria of identity for these alleged entities, we are likely to fall into talk of 'meaning' and 'synonymy' along the lines of saying that two properties/attributes/relations are identical if the units of speech expressing them mean the same, are synonymous. However, talk of synonymy leads to above hinted at problems and unclarities such that this way of stating there identity conditions is problematic at best and impossible at worst if one ultimately gives up any notion of abstract meaning and/or synonymy. However, instead of talking about such problematic intensional notions,[27] i.e. notions having to do with the meaning of expressions, we can also talk about the extensions of properties, attributes and relations, i.e. the class of objects they are true of/ apply to/ hold between.[28]

This then allows us to state their identity conditions as saying that two universals are identical iff their extensions coincide. While this is not completely unproblematic – 'creature with heart' (cordate) and 'creature with kidney' (renade) happen to have identical extensions since every creature with heart happens to have kidneys and vice versa, but are still distinct notions and would be identified on the criterion just expounded – it makes us aware of the fact that the identity conditions thus stated are identical to the ones for sets. This, in turn, identifying universals if they possess the same extension, points us to the fact that we can identify those universals with their extensions, thereby reducing them to sets. This has the advantage of consolidating their stable identity conditions in terms of extensional equivalence on the one hand and removing somewhat dubious intensional entities from our ontology, replacing them with very acceptable and solid surrogates, i.e. sets, on the other. Such a maneuver can clearly only succeed if everything that we were able to do in terms of those universals we can do in terms of sets, but Quine presents a solid case for it being so:[29]

> "we shall find that those further purposes of attributes are well served by classes, which, after all, are like attributes except for their identity conditions. Classes raise no perplexity over identity, being identical if and only if their members are identical" (Quine 1960b, 209)

While only short and superficial, the last two sections thus demonstrate how the interaction of Quine's demand for identity conditions and the general constraint of ontological parsimony lead to a structuring of our ontology.

Modality and Propositional Attitudes

There is much to be said about Quine's treatment and manner towards modality and the propositional attitudes. He himself has gone through great lengths of trying to accommodate these intensional locutions into his extensional framework,[30] with mixed success. However, in the context of this thesis we operate within the constraints of Quine's philosophical framework which mentions, as we saw above, the doctrine of extensionalism as one of its fundamental assumptions. Any position going against this tenet thus has to be considered as breaking with the Quinean position and

[27] Cf. (Quine 1960b).

[28] Cf. (Quine 2008a, 500): "We have no clear basis in general for saying what coextensive properties qualify as identical and what ones do not. In a word, properties lack a clear principle of *individuation*. Groping for such a basis one settles for obscure talk of essence and necessity. Anything that can be described in terms of properties and not equally directly in terms of classes is unclear to my mind."

[29] Cf. especially (Quine 1960b), but also (Quine 1961).

[30] Cf. (Quine 1960b).

advancing an entirely different point of view. The issues pertaining to the propositional attitudes in particular will thus only receive a cursory glance at this point and not the attention that they deserve and that Quine has granted them, simply for the reason that while it is true that they might and do form the basis of a substantial criticism of Quine's criterion of ontological commitment and the extensional canonical notation it is based upon, they fall outside of the scope of (WEAK.Q) because such a proposal would always face the immediate criticism of not being Quinean enough to do its ignoring the constraint of extensionality. While their treatment might therefore form the basis of an objection to the adequacy of Quine's canonical notation because it does not do justice to propositional attitudes, this would be a criticism 'from the outside' of the system, ignoring constraints internal and constitutive of the framework, rejecting one of Quine's fundamental assumptions.

Quine's animadversions to modality are well-known.[31] They stem from the fact that modality gives rise to non-extensional contexts, for consider the statement

(i) $\Box(9 > 7)$

which is arguably true. However, '9' is co-extensional with 'the number of planets'[32], but

(i) $\Box(\text{the number of planets} > 7)$

is nevertheless false. The context created by the modal operator $\Box$ is not extensional. We take this violation of extensionality to be sufficient for rejecting any attempt at trying to supplement canonical notation with model operators in the context of this thesis as inappropriate vindication of (WEAK.Q) because, as mentioned above, any such attempt does not fully buy into the fundamental assumptions of Quine's system itself. This leaves one of course free to disagree with those basic assumptions themselves, but, as mentioned in the introduction, this does not concern us here.

Just as modality, so do propositional attitudes give rise to non-extensional contexts: the sentence "James believed that Cicero was a great orator" or "James was looking for the manager of the store" might be true and yet "James believed that Tully was a great orator" or "James was looking for Michael" might be false despite Tully = Cicero and Michael being co-extensional with 'the store manager'. However, due to their universal presence propositional attitudes play an important role in our conceptual scheme and cannot be dismissed easily. Quine has to offer an ersatz construction allowing us to represent propositional attitudes in his extensional framework. He does so by considering attitudes such as believing, looking for, hunting, etc. as relations between a subject and an object, thereby extensionalising them. This is not unproblematic, for what is that second relatum? It cannot be a proposition as these were dismissed as unnecessary above. Quine here opts to reduce propositional attitudes to relations between a subject and a *sentence*, considered as a concrete object consisting of sets of inscriptions or equivalence classes thereof, claiming thereby to accommodate everything that is important about propositional attitudes in his framework.[33]

We conclude this brief section by emphasizing once more that we do not consider anything that involves the introduction of non-extensional elements into Quine's canonical notation as an appropriate take on (WEAK.Q) under Quinean assumptions, given the fundamental importance of extensionality.

[31] Cf. (Quine 1953b).

[32] Or at least used to be until we lost Pluto.

[33] While this is of course not unproblematic we will end our brief rough-and-ready treatment of this topic greatly deserving of more attention here and refer the reader to (Quine 1960b) and (Hylton 2007).

4.1.2 The Construction of Canonical Notation

Ordinary, everyday language is full of vagueness, ambiguity and unclarity. Science, which strives for precision, objectivity and exactness needs to utilize this compromising language in order to make headway at all, since, ultimately, ordinary language is all we have to go on. "To some degree, nevertheless, the scientist can enhance objectivity and diminish the interference of language, *by his very choice of language* (my emphasis)" (Quine 1957, 7). This is the goal canonical notation attempts to achieve; to reduce the 'noise' of ordinary language, with all its imperfections, unclarities and vague elements.[34] A couple of qualifying remarks are in order: the purpose of devising a canonical notation in order to eliminate obscurity from scientific discourse has as its goal not a "practical language reform" (Quine 1957, 8), i.e. it is not supposed to replace ordinary language. Moreover, as was already hinted at in Section 3.2.2, canonical notation is neither supposed to constitute an analysis of ordinary discourse nor is it even demanded that it be synonymous with it.[35] The only connection there need be between a piece of ordinary discourse and its canonical regimentation is that "[w]e fix on the particular functions of the unclear expression that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking, that fills these functions" (Quine 1960b; Hylton 2007, 258/259).

Canonical notation is supposed to present an idealized form of scientific language, a "philosophical schematism" (Quine 1957, 9), such that, in principle, all science could be done in this way, but does not have to be: "Where the objective of a canonical notation is economy and clarity of elements, we need only show how the notation *could* be made to do the work of all the idioms to which we claim it to be adequate; we do not have to use it" (Quine 1960b, 161). In fact, intermediate notations of varying clarity and austerity might be easier and less laborious to use. Thus reassured "that we are in no way compromising our freedom, we can be uncompromising in our reductions" (Quine 1960b, 161).

Nevertheless, it should not be forgotten that canonical notation is deeply rooted in ordinary language, for "[s]cientific language is in any event a splinter of ordinary language, not a substitute" (Quine 1957, 9). In fact, we cannot do without ordinary language in the construction of canonical constructions, since those constructions are themselves explained in ordinary language. All of the vocabulary and grammatical constructions of a canonical notation will be ordinary (Quine 1960b, 159) and explaining a sentence in canonical notation amounts to saying how it can be expanded in ordinary or semi-ordinary language (Quine 1960b, 159): "Hence to paraphrase a sentence of ordinary language into logical symbols is virtually to paraphrase it into a special part still of ordinary or semi-ordinary language" (Quine 1960b, 159).[36]

However, canonical notation is merely to serve as a means of economic theory construction, not as a linguistic analysis or systematization of ordinary language[37]. Ordinary language is the starting, not, however, the end-point (Hylton 2007, 239), it is almost unavoidable that the canonical regimentation of a piece of ordinary discourse will at some point diverge from it: "We do not ask whether our reformed idiom constitutes a genuine semantical analysis, somehow, of the old idiom; we simply find ourselves ceasing to depend on the old idiom in our technical work" (Quine 1957, 11) and "[i]f the form of paraphrase happens incidentally to produce sense where the original suffered a truth-value gap and so was wanted for no purpose, we may just let the added cases turn out as they will" (Quine 1960b, 182). The purpose of a canonical notation is clarification and simplification of

[34]Cf. (Quine 1985, 170): "I mean predicate logic not as the initial or inevitable pattern of human thought [...], but as the adapted form, for better or for worse, of scientific language."

[35]Cf. (Hylton 2007, 242).

[36]Cf. also (Quine 1960b, 205): "our own language and [...] the canonical part of it".

[37]Synonymy is *not* required, see Section 3.2.2. Cf. also (Hylton 2007, 239ff)

theory, if that requires departures from ordinary language, accidental or substantial, so be it; as long as these "sweeping" artificialities and departures simplify theory they are to be accepted on grounds of economy.[38]

The extensive procedure of how to devise a canonical notation, abstracting from and simplifying ordinary discourse wherever possible, even breaking with ordinary grammar and overstretching the naturalness of certain ordinary language expressions can be found in Chapters 3 - 5 of *Word and Object*. We will here focus on some of the essential steps and provide but a sketch of the original procedure to elucidate how the result became to be the language of FOL.[39] Before going into the details a general remark is in order; the process of devising a canonical notation cuts in two ways: an upward-movement and a downward-movement, so to say. Quine starts, on the one hand, with ordinary language vocabulary and constructions and abstracts away from these, eliminating vaguenesses and bringing stylistically different constructions into a common canonical form. This process is upwards, as it starts with ordinary language and abstracts away from it. Sometimes, however, these canonical constructions happen to encompass more than just the cases they were abstracted from; they might turn out to be able to do the work of other natural language constructions which were not in the first place thought to have much in common with others, subsumed under the same canonical form. This is a welcome side effect and the downward direction of canonical notation: canonical constructions are reapplied to ordinary language constructions, unifying them with other, dissimilar constructions in a canonical form.[40]

Sentences of science are supposed to be true eternally, independent of occasion of utterance (Quine 1960b, 227), so in a first step *indicator words*, such as 'I', 'you', 'this', 'that', 'here', 'there', 'now', 'then', etc. are banned from a canonical notation, thereby rendering truth invariant with respect to speaker and occasion. Their function need not be lost, but can be emulated through mention of concrete persons, places and times in canonical notation (Quine 1957, 8/9). Furthermore, tense is dropped from words of ordinary language once they are incorporated into a canonical regimentation, we might keep their grammatical forms, but treat all terms as temporally neutral.[41] Again, nothing is lost by this move – time and tense can be emulated by adopting a four-dimensional view of time through explicit mention of space-time coordinates or ordering relations (saying that something was before of after something else), – but much is gained in terms of simplicity and economy: logical theory applying to sentences of canonical notation now need not take into account different temporal dimensions of sentences, thereby greatly simplifying its rules of inference.[42] Plurals in ordinary language also introduce unnecessary complications by modifying terms of language through plural endings and can easily be dispensed with. Instead of 'I hear lions' we now say 'I hear a lion other than a lion that I hear' (Quine 1960b, 118), a construction, however unnatural, that bears the great advantage of eliminating plural endings in favor of singular terms, identity and

[38]Cf. (Quine 1960b, 158): "Simplification of theory is a central motive of the sweeping artificialities of notation in modern logic."

[39]For a more extensive treatment and a minute justification and elaboration of the process and steps leading to this result the reader is referred to the relevant chapters of (Quine 1960b).

[40]An example of this downward movement would be the streamlining of all expressions into the 'such-that' idiom, which hopelessly violates grammatical and stylistic rules of many ordinary language expressions, but greatly simplifies canonical forms (Quine 1960b, Chapter 5).

[41]Cf. (Quine 1960b, 170): "ordinary language shows a tiresome bias in its treatment of time. [...] This bias is of itself an inelegance, or break of theoretical simplicity."

[42]We can now say that from p and $p \to q$ we can infer q rather than having to formulate this rule in a way that it takes the temporal relations between p and q into account: "The letter 'p' standing for any sentence, turns up twice in each of these rules; and clearly the rules are unsound if the sentence which we put for 'p' is capable of being true in one of its occurrences and false in the other. But to formulate logical laws in such a way as not to depend thus upon the assumption of fixed truth and falsity would be decidedly awkward and complicated, and wholly unrewarding" (Quine 1957, 9).

cardinality statements[43], all of which will become standard components of a canonical notation, eliminating the need for extra devices to emulate plurals (minimality).

We saw in Section 3.2.1 (sentence (*)) that disambiguation is facilitated by holding singular terms (definite and indefinite) to subject position and reformulating the sentence containing them by means of the 'such-that' paraphrase. Given the clarifying and harmless nature of such linguistic move, it should be adopted as standard in a canonical notation in order to eliminate scope ambiguities. Moreover, the introduction of indexed pronouns, or even better *variables*, allowed for more reliable and accurate reference tracking, greatly simplifying and sharpening grammatical constructions and should therefore also be implemented in a canonical notation. All of these modifications have, so far, been upward-constructions: we began with natural language phenomena and found means of streamlining, standardizing and simplifying their grammar and construction. The last move, the introduction of variables, has, however, a downward effect: it broadens our notion of sentence as we can now distinguish between two types of sentences, *open* and *closed* sentences, i.e. those with free pronouns and those whose variables are all bound, in such a way that they all refer back to an object denoted by an indefinite or definite term at the beginning of the sentence (Quine 1957, 11). While closed sentences behave as before (they have a definite truth-value), open sentences share much more commonalities with predicates in that they have an extension, a set of objects of which they are true, rather than a determinate truth-value. This unintended side effect of introducing variables for pronouns and singular terms in canonical notation is not at all undesirable, in fact it proves the predicational completeness that we located in the relative clause of natural language: everything that can be said about an object can be said by predicating a predicate (= an open sentence) of it.

At this point we see that a sentence in canonical notation will have the form of a predication: there is an object referred to by singular, definite or indefinite terms of which something is predicated via the remainder of the (open) sentence. This general structure affords another downward movement in that no limitations are imposed on the internal complexity that unanalyzable predicates featuring in the open sentences that are predicated of an object possess: "Where canonical notation is cut off, leaving unanalyzed components, will usually vary with one's purpose; what remains unanalyzed has the form of a general term in predicative position" (Quine 1960b, 174). The level of fine-grainedness of analysis and the primitive predicates to be used in the construction of canonical sentences do not need to be natural kind terms, metaphysical primitives or fundamental relations, they completely depend on what the purpose of the paraphrase is. What canonical notation institutes is not which predicates are to denote the basic properties and relations or reality, but that "the only canonical position of a general term is predicative position, whatever the terms uncanonical substructure" (Quine 1960b, 175). It draws no distinction which terms to treat as complex and which as simple; all it does is to unify the position in which a term can occur within a canonical construction.[44]

A final (and crucial) further simplification involves the reduction of all terms that stand in subject position of the pre-canonical constructions outlined above to *universal* and *existential* quantifications $\forall x$ and $\exists x$.[45] The reduction proceeds through different stages: names/singular terms are

[43]Cf. (Quine 1960b, 118) and (Quine 1982, 231ff).

[44]Cf. (Quine 1960b, 160): "There are regimented notations for constructions and for certain of the component terms, but no inventory of allowable terms, nor even a distinction between terms to regard as simple and terms whose structure is to be exhibited in canonical constructions. Embedded in canonical notation in the role of logically simple components there may be terms of ordinary language without limit of verbal complexity. *A maxim of shallow analysis* prevails: *expose no more logical structure than seems useful* for the deduction or other inquiry at hand."

[45]Strictly speaking only one of them is required, as each can be defined in terms of the other, viz. $\exists \equiv \neg\forall\neg$ and $\forall \equiv \neg\exists\neg$. However, such saving would only be notational and not conceptual in nature and for convenience we will therefore retain both.

reduced to definite descriptions (see next section), definite descriptions are reduced in the familiar Russellian fashion and indefinite (singular) terms are captured by means of "every" and "some", which, in turn, can be expressed through "everything" and "something" respectively, expressions that are symbolized by the universal and existential quantifiers (cf. Section 3.2.1). We will turn toward the issue of paraphrasing names in the next section and neglect it here. Definite terms of the form 'the F', where 'F' is any description, can be disposed with in favor of existential quantifications in sentences containing them, thereby lifting the denotational burden from these expressions and the existential import from sentences containing them, as is well known since Russell (Russell 2005). On Russell's analysis, a sentence such as, e.g., "The present king of France is bald" becomes "Someone is such that he$_1$ is king of France and he$_1$ is bald and for anyone such that he$_2$ is king of France he$_2$ is identical to him$_1$". This way definite descriptions give way to existential quantification, predication and uniqueness claims and we can contextually define definite descriptions $\imath(x)(\psi(x))$ (the x, such that $\psi(x)$, i.e. 'the ψ') in sentences in which they occur (φ) the following way:

$$\varphi(\imath(x)(\psi(x)))$$

becomes

$$\exists x(\psi(x) \wedge \forall y(\psi(y) \rightarrow y = x) \wedge \varphi(x))$$[46]

without the need to assume a separate operator for definite terms.

Moreover, the such that idiom already allows us to reduce all indefinite singular terms to locutions of the form 'every F' and 'some F' (possibly plus negation) (Quine 1960b, 162) with 'F' standing for some general term, however complex. These, in turn, give way to 'everything' and 'something' through the following transformations:[47]

(a) Every F is ϕ.

(b) Some F is ϕ.

become

(a') Every F is an object x, such that $\phi(x)$.

(b') Some F is an object x, such that $\phi(x)$.

which then become

(a") Everything is an object x, such that (if $F(x)$ then $\phi(x)$).

(b") Something is an object x, such that ($F(x)$ and $\phi(x)$).

Expressions of the form 'Everything/Something (is an object x, such that)' are, however, precisely what is symbolized by the universal and existential quantifier $\forall x/\exists x$ (see Section 3.2.1) and we have therefore disposed with the entire class of indefinite (and definite) terms in favor of the two quantificational expressions of universal and existential quantifications (of which we, strictly speaking, only require one).[48]

[46] Cf. (Forster 2003, 17).
[47] Cf. (Quine 1960b, 162).
[48] For lack of space and to avoid unnecessary complications we here ignore class-abstraction and attribute-abstraction operators, which, if needed, might require the introduction of additional primitive operators or, depending on the situation, allow reduction within the context of our regimentation (Quine 1960b, pp. 164).

The proposals for paraphrase into canonical notation outlined above have the purpose of simplifying and clarifying theory – resolving issues and eliminating misunderstandings that might pertain to the formulation of such theory in a non-canonical idiom. It has the capability of dissolving problems by showing that they are purely a matter of the way *we speak about things*:

> "Philosophy is in large part concerned [...] with what science could get along with, could be reconstructed by means of, as distinct from what science has historically made use of. If certain problems of ontology, say, or modality, or causality, or contrary-to-fact conditionals, which arise in ordinary language, turn out not to arise in science as reconstituted with the help of formal logic, then *those problems have in an important sense been solved*: they have been shown not to be implicated in any necessary foundation of science." (Quine 1953a, 151, my emphasis)[49]

This does not mean that canonical notation by itself offers solutions to long-standing problems, it merely helps us clarify and simplify theory, thereby showing us that some disputes (such as the one about Pegasus' existence) might have been merely verbal. The task of regimenting a theory into canonical notation and thereby resolving such problems, however, still remains and is not unproblematic: "By developing our logical theory strictly for sentences in a convenient canonical form we achieve the best division of labor: on the one hand there is theoretical deduction, and on the other there is the work of paraphrasing ordinary language into the theory" (Quine 1960b, 159).

Singular Terms Eliminated

One inessential item of canonical notation whose paraphrase enables an attractive solution to the problem of non-existent entities is the proper name. The problems pertaining to non-referring names are well-known. Consider, for example, the sentence "Pegasus does not exist". Here one first appears to be referring to an object (Pegasus) to then deny its existence – a surely contradictory circumstance. Quine is able to circumvent these issues by treating names as singular descriptions. Every name a is associated with a (unique) predicate P_a (e.g. 'pegasizes') and a sentence such as "Pegasus flies" is then paraphrased as $F(\imath(x)(P_a(x)))$, i.e. as $\exists x(P_a x \wedge \forall y(P_a y \rightarrow y = x) \wedge Fx)$, treating Pegasus as a definite description rather than a standard a singular term which would have received the standard paraphrase of Fa. As Kaplan remarked, names are first Quinized into definite descriptions and definite descriptions are then Russelled away.[50]

A similar procedure can be applied to functions, treating them as relations of special kind, namely those whose first-coordinate only occurs once. All singular terms then are paraphrasable in this fashion and therefore unnecessary 'notational ballast' in our most austere scheme of canonical notation. Everything we can do with them we can achieve without them, their function can be emulated by the devices already present and they are therefore dispensable. Moreover, this treatment of names or singular terms in general has two nice side-effects. It closes truth-value gaps in that it assigns a sentence such as "Pegasus flies" the truth-value false, rather than leaving it meaning- or truth-valueless due to the failure of reference connected with the singular term 'Pegasus' (Quine 1964a), thereby solving the issues and puzzles associated with negative existential statements.

Furthermore, it provides us with a robust criterion enabling us to tell when a name refers. A name a refers precisely then when it is able to feature in an affirmed existentially quantified identity $\exists x(x = a)$. This latter way of confirming that a name refers also lets us see that predicates uniquely applying to the object we wish to refer to are readily at hand, simply treat '$= a$' ($= P_a$)

[49] As quoted in (Hylton 2007, 243/244).

[50] For the exact procedure see any of (Quine 1982), (Quine 1964a), (Quine 1960b) and others.

as general term. The abstraction thus becomes a little less artificial. This method of paraphrase is well-known and has almost become canonical. What it shows here is that we can dispense with the class of singular terms and need not recognize them as primitive elements in our canonical notation. Moreover, this notational move enables us to determine when a name refers and solve problematic cases of singular denied existential statements.

4.1.3 A First-Order Notation for All Science

What, then, does canonical notation in its most austere form comprise? Given that names are eliminable (see above) its basic sentences, those that are not decomposable into other sentences, will be of the form '$Rx_1, \ldots, x_n$' for variables $x_1, \ldots, x_n$ and a (polyadic) general term R without recognizable internal structure that is predicated of the objects for which the variables are placeholders.[51] Compound and more complex sentences are then built up from these basic sentences by means of quantification and truth-functions, ultimately reducible to one quantifier and a single truth-function (the Sheffer-stroke).[52] The quantifiers form closed sentences by binding the free variables of the atomic sentences of canonical notation and the truth-functions are precisely those operations that preserve the extensionality of the contexts in which they occur:[53]

> "It is well known, and easily seen, that the conspicuously limited means which we have lately allowed ourselves for compounding sentences – viz., 'and', 'not' and quantifiers – are capable of generating only extensional contexts. It turns out, on the other hand, that they confine us no more than that; the *only* ways of imbedding sentences within sentences which ever obtrude themselves, and resist analysis by 'and', 'not', and quantifiers, prove to be contexts of other than extensional kind."[54] (Quine 1957, 12)

We are thus left with a notation whose only constituents are (polyadic) predicates/relations, variables, quantifiers and truth-functions and whose basic operations consist in predication, quantification and compounding of sentences by means of truth-functions.[55] "What thus confronts us as a scheme for systems of the world is that structure so well understood by present-day logicians, the logic of quantification of calculus of predicates" (Quine 1960b, 228). Now, "[t]his pattern for a scientific language is evidentially rather confining. There are no names of objects. Further, no sentences within sentences safe in contexts of conjunction, negation and quantification. Yet it suffices very generally as a medium for scientific theory. Most or all of what is likely to be wanted in a science can be fitted into this form" (Quine 1957, 11). This notation is not fixed for all eternity: if the overall advantage in bringing about simplicity and economy of theory of, say, intensional constructions becomes undeniable, canonical notation should accommodate such facts. As science advances certain traits neglected by the current version of canonical notation might require integration, after all, its design rested on characteristics of present-day science (Quine 1957, 16).

However, all this shows is that canonical notation is not something that is prior to and a condition of science, but rather continuous and intertwined with it. As part of the scientific enterprise the

[51] Cf. (Quine 1960b, 186) and (Quine 1957, 9). "The ultimate components are the variables and general terms; and these combine in predication to form the atomic open sentences" (Quine 1960b, 228).

[52] A set containing only the Sheffer-stroke (NAND) is *functionally complete*. However, since the savings are only notational we will, for convenience, stick with the traditional boolean connectives $\wedge, \vee, \neg, \rightarrow$.

[53] Cf. (Quine 1957, 12): "In case of closed sentences [...] extensional contexts are what are commonly known as truth functions."

[54] They could be modal contexts, contexts of intensional abstraction or of propositional attitudes, all of which were dropped in favor of ersatz-constructions.

[55] Cf. (Quine 1960b, 203): "At the height of regimentation, sentences were constructed only by adjoining general terms [...] predicatively to variables and applying quantification, truth-functions, and other operations to sentences."

"quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality" and it is "the same motives that impel scientists to seek ever simpler and clearer theories adequate to the subject matter of their special sciences [that] are motives for simplification and clarification of the broader framework shared by all sciences" (Quine 1960b, 161).

Canonical notation provides this framework for the special sciences, it is best seen "not as complete notation[...] for discourse on special subjects, but as partial notation[...] for discourse on all subjects" (Quine 1960b, 160).[56] It does not, however, dictate sciences its assumptions, objects or concepts; what the predicates and general terms of a science are and what objects any particular science wants its quantifiers to range over are its to decide.[57] The doctrine is not that canonical notation decides questions of science. There are no limits as to the unanalyzed terms occurring in any scientific theory or the nature of the objects such theory posits. There are only limits to the ways these things can be talked about and to the way such a theory can be formulated:

> "The doctrine is only that such a canonical idiom can be abstracted and then adhered to in the statements of one's scientific theory. The doctrine is that all traits of reality worthy of the name can be set down in an idiom of this austere form if in any idiom. [...] It [canonical notation] delimits what counts as scientifically admissible construction, and declares that whatever is not thus constructible from given terms must either be conceded the status of one more irreducible given term or eschewed." (Quine 1960b, 228/229)[58]

4.2 The Logic of Canonical Notation

Having settled on a suitable language in which to formulate scientific theories in their most austere form it remains to determine the inferential relations holding between sentences of such theory in order to do justice to the definition of a theory as a deductively closed set of sentences. While the arguments for logic and language of a canonical notation are often run together, even by Quine himself as the citations above demonstrate, we do well to keep to keep these considerations separate. After all, they provide different angles of attack on any adjustment proposed to the Quinean canonical notation as well as different possibilities of modification.[59]

4.2.1 Revisability

According to Quine the totality of our beliefs and knowledge about the world forms a web of interconnected sentences which faces the 'tribunal of experience' only as whole, and not separately piece by piece (see Section 2.3.2). This was the thesis of *confirmational holism.* This model of knowledge entails that no part of the web is privileged above any other in terms of constituting an unchangeable, constant and permanent element of our overall theory of reality and thus *no part is immune to revision in light of new experiences*[60]. This means, in particular, that not even logic is

[56] There are alternative ways of formulating a canonical notation, such as, e.g. type theory. However, there is a case to be made for the simplicity afforded by only having a single universe of values and admitting all general terms on the same footing (Quine 1960b, 229/230).

[57] Cf. (Quine 1957, 14) and (Quine 1960b, 232).

[58] Cf. also (Quine 2008a, 504): "I think of logic in this narrow sense as the grammar of strictly scientific theory."

[59] For an overview of Quine's work in logic see (Ullian 2006). For its connection with ontology and semantics see (Decock 2002b).

[60] Cf. (Quine 1964c, pp. 42): "Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system. [...] by the same toke, no statements is immune to revision."

exempted from the possibility of revision:

> "Logic is in principle no less open to revision than quantum mechanics or the theory of relativity." (Quine 1986, 100)

> "Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?" (Quine 1964c, 43)

This relativistic outlook on science in general and logic in particular, integrates very well with Quine's holistic attempt to circumvent the traditional problems associated with the provision of a foundation for knowledge and if this was all Quine would have said about the status of logic in our overall theory of the world, that its nature depends on the various experiences we face, there would be no problem. Many passages in his work suggest, however, the contrary, i.e. there are deliberations to the effect that logic *does* occupy a privileged position in our web of beliefs which has led to numerous accusations of an internal inconsistency[61].[62]

The tension arises from constraints governing translation: *our* logic – i.e. classical logic on Quine's view – appears to be build into the very notion of translation. It is hardwired into our very ability to translate and make sense of discourse. The upshot of this is that in every attempt to revise the laws of logic, i.e. to deny some of the logical truths we countenance, the deviant logician's predicament lies in the fact that "when he tries to deny the doctrine he only changes the subject" (Quine 1986, 81): change of logic, change of subject. While limits of space disallow a detailed analysis and resolution of the objections outlined above, I do wish to at least sketch their rationale and point out a way in which the Quinean can maintain consistently that logic is, in principle, as revisable as Newtonian mechanics.

Logic is a 'theory of the obvious'. Its truths are either self-evident or can be made evident by deriving them from obvious truths through obvious steps. This obviousness is the result of logic being (partly[63]) grounded in grammar (Quine 1986, 101/102): a logical truth is a sentence "whose grammatical structure is such that all sentences with that structure are true" (Quine 1986, 58), or, in other words, a sentence that cannot be turned false by substituting 'for lexicon'[64].[65] Grammar, however, is obvious: in order to have mastered a language one already needs to have gained an understanding of the grammar of that language – there is no other way to learn a language, but to do so grammatically.

Given the obviousness of grammar and the supervenience of logic's obviousness upon it, as well as the fact that "there can be no stronger evidence for a change of usage than repudiation of what had been obvious [...] one's interlocutor's silliness, beyond a certain point, is less likely than bad translation" (Quine 1960a, pp. 112), dissent from a logical law entails a change of meaning of the logical expressions, for a translation should not impute a denial of the obvious. It is for this reason that the "canon 'save the obvious' bans any manual of translation that would represent the foreigner

[61]Cf. (Dummett 1975) and (Haack 1974). See also (Priest 2006).

[62]I would like to acknowledge my debt to Asgeir Matthiasson for pointers to the literature on this topic and for allowing me to read his essay on problems concerning the revisability of logic in Quine's system. Much of this section is inspired by it and the conclusions reached are essentially the same, albeit for slightly different reasons and despite a somewhat different reconstruction of the argument.

[63]Partly because logical truth does not (solely) depend on language, see below.

[64]We will return to and elaborate on the distinction between lexicon and particles below; see Section 4.2.2.

[65]On Quine's broad understanding of grammar; cf. (Quine 1986) for elaboration on this point. The definition of logical truth in terms of grammar coincides for first-order languages with the usual definition in terms of proof and/or truth in all models.

as contradicting our logic" (Quine 1986, 83) – any translation deviating from our logic would be labelled a mistranslation. This does not mean that every sentence of a 'foreign' language will obey our laws of logic, some sentences might resist translation altogether, but it does mean that the "logics of two cultures will be [...] incommensurable at worst and *never in conflict*, since conflict would simply discredit our translation" (Quine 1986, 96, my emphasis).

We therefore "impute our orthodox logic to him, or impose it on him, by translating his language to suit. We build the logic into our manual of translation" (Quine 1986, 82). Thus, Quine asks us to imagine someone who uses conjunction like we would use disjunction and vice versa. When translating him we would "regard his deviation merely as notational and phonetic" (Quine 1986, 81) and associate his disjunction with our conjunction and vice versa. The question then becomes

> "Could we be wrong in so doing? Could he really be meaning and thinking genuine conjunction in his use of 'and' after all, just as we do, and genuine alternation in his use of 'or', and merely disagreeing with us on points of logical doctrine respecting the laws of conjunction and alternation? Clearly this is nonsense." (Quine 1986, 81)

Similarly if someone were to reject the law of non-contradiction $\neg(p \wedge \neg p)$ and prevent trivialization of his theory by means of compensatory adjustments in other sentences, most notably his rules of inference. It is not that the proponent of such change truly rejected *our* law of non-contradiction; what he did was to change the meaning of negation. Him and I are not talking about the same thing any more when talking about '$\neg$' – after all, the meaning of this particle was dependent on the sentences and rules in which it features, rejecting the law of non-contradiction means changing that class of sentences, and therewith its meaning.[66] Given, then, that any attempt to revise our logic merely amounts to a 'change of topic', how can Quine maintain the universal revisability of every component of the web?

Levin (Levin 1978) points out that even if it is the case that it is always less likely to have denied a logical law than that the translation attributing such denial to a speaker is faulty, this does not entail the impossibility of dissent, merely its unknowability (Levin 1978, 52). Evidence for such a case could be found in the incommensurability of languages, without ever being able to capitalize on this by imputing a deviant logic. Such response, however, falls short of rebutting the objection for the reason that while it enables the deviant logician to deny the classical doctrine, it is still to weak to permit an intra-systematic revision of logic – we are still stuck with the 'classical doctrine'.[67]

A first thing to notice when attempting a response on Quine's behalf is that classical logic is not build into translation as a methodological pre-condition, but, under the slogan 'Save the Obvious', as an inductive generalization; logic is inseparable from translation *because* it is obvious. There is, however, no further significance in the inseparability of logic and translation than the one bestowed through its obviousness[68]: "Logical truths are tied to translation in no deeper sense [...] than other obvious truths, e.g., utterances of 'It is raining' in the rain" (Quine 1986, 96/97) and "Logic is inseparable from translation only as anything obvious is inseparable from translation" (Quine 1986, 97).

In other words, the inseparability of logic from translation is a mere pragmatic inseparability, as opposed to logic being constitutive of, or a condition for translation: just as it would be uncharitable and most likely make for a bad translation to ascribe to a proponent of a foreign language the belief

[66] It is, for example, possible to translate the intuitionist classically; cf. (Levin 1978, 59): "The intuitionist, who seems by his denial of $p \vee \neg p$ to have abandoned [classical] negation, can be interpreted normally in classical logic supplemented by talk of time." See also (Kleene 1945) (as quoted in (Levin 1978)).

[67] Levin has other reasons for rejecting this response; cf. (Levin 1978, pp. 52).

[68] Cf. (Quine 1986, 96): "My point is just that there is no *added* significance in the inseparability of logic and translation. Obviousness, whatever its cause, would already make for that inseparability."

"It is not raining" while witnessing monsoon-season, so it would be when ascribing to him anything but our logic, which we take to be just as obvious. The imputation of our logic is, in this sense, a very strong heuristic for translation, but it remains just that, a heuristic.

"Logic is built into translation more fully than other systematic departments of science. It is in the incidence of obviousness that this difference lies" (Quine 1986, 82). The obviousness Quine places emphasis on here is mere behavioral obviousness "with no epistemological overtones" (Quine 1986, 82).[69] That means, however, that if the behavior of a community changes radically enough – e.g., by accepting the paradoxes of (naive) set theory – so will the logic they regard as obvious. We can then summarize the state of the objection thus: while it might be impossible to ever ascribe a foreign logic in translation, this impossibility is not grounded in the nature of translation, but rooted in our upbringing – translation is not bound to any particular logic. This does not preclude, however, the possibility of genuine disagreement with classical logic, given that the 'change-of-meaning' objection only gains traction from the perspective of the translating individual. Moreover, radically enough change in the behavior of a speaker community might lead to the acceptance of a new logic; no translation has taken place in this process and thus the 'change-of-meaning' objection does not apply.

Classicality

Having established then that logic is, at least in principle, revisable and therefore 'up for grasps' one needs to offer substantial arguments in favor of classical logic in order to demonstrate how it is "better than arbitrary" (Quine 1969c, 112). Not surprisingly Quine's arguments for why logic should be classical rely on considerations of *simplicity*. Deviant logics, so Quine, do not only involve a change in the method of generating the class of logical truths, such as the choice between, say, an axiomatic system and a natural deduction system in the presentation of a logic would, they encompass a change of that class itself (Quine 1986, 82). Given, then, that the truths of logic were supposed to be obvious (we will return to this point below) the price of a deviant logic is a serious loss of simplicity and familiarity; "[t]he price is perhaps not quite prohibitive, but the returns had better be good" (Quine 1986, 86) and unless the returns are substantial deviating from classical logic "runs counter to a generally sound strategy [...] the maxim of minimum mutilation" (Quine 1986, 85). After all, according to Quine, the "classical logic of truth functions and quantification [...] is a paragon of clarity, elegance and efficiency" (Quine 1986, 85); "Classical quantification theory enjoys an extraordinary combination of depth and simplicity, beauty and utility. It is bright within and bold in its boundaries" (Quine 1969c, 112/113). So far this amounts to a mere polemic for FOL, but there appears to be ample reason to assume the familiarity of classical logic if one looks at the scientific practice of non-logicians. As to its simplicity we will have to say something here and in the next sections.

Logic, just like any other good scientific theory, "is under tension from two opposing forces: the drive for evidence and the drive for system." These need to be in balance, for the extremes in either direction would constitute "in the one case a mere record of observations, and in the other a myth without foundation" (Quine 1981a, 31). This tension induces a trade-off between the two forces; "systematicity, achieved through abstraction, is bought at the expense of plurality of evidence and vice versa[70] – [s]cientists of different philosophical temper will differ in how much dilution of

[69]Cf. (Quine 1986, 82): "When I call $1 + 1 = 2$ obvious to a community I mean only that everyone, nearly enough, will unhesitatingly assent to it, for whatever reason; and when I call 'It is raining' obvious in particular circumstances I mean that everyone will assent to it in those circumstances."

[70]Cf. (Quine 1981a, 31): "We gain simplicity of theory, within reason, by recourse to terms that relate only indirectly, intermittently, and rather tenuously to observation. The values that we thus trade off one against the

evidence they are prepared to accept for a given systematic benefit and vice versa" (Quine 1981a, 31). One very contentious disagreement surrounding logic and pertaining to such trade-offs concerns the disagreement between intuitionists and proponents of classical logic that is encapsulated in the issue about the acceptance or rejection of the *law of excluded middle*, $p \vee \neg p$.[71]

In the *Philosophy of Logic* Quine puts forward the 'change-of-logic' objection as a major obstacle for intuitionism or anyone proposing to give up the law of excluded middle: "whoever denies the law of excluded middle changes the subject. [...] In repudiating '$p \vee \neg p$' he is indeed giving up classical negation, or perhaps alternation, or both" (Quine 1986, 83). Rejecting the law of excluded middle amounts to a *rejection of classical negation* (and/or disjunction).[72] The intuitionist "should be viewed [...] as opposing our negation and alteration as unscientific ideas and propounding certain other ideas, somewhat analogous, of his own" (Quine 1986, 87). Intuitionist logic, however, "lacks the familiarity, the convenience, the simplicity and the beauty of our logic" (Quine 1986, 87) which presents ground to reject it as a superior alternative to the classical framework of science.[73]

Examining this last claim a bit closer, one sees that it has two aspects, a positive and a negative. The positive aspect concerns the loss of simplicity and familiarity that intuitionistic logic carries with it as compared to its classical counterpart. Quine does not elaborate much on it, but it is not a far reach to point to the more complicated and rather unintuitive semantics for intuitionistic logic (either in terms of Heyting algebras or via Kripke models) as compared to the very natural Tarskian truth-conditions for classical logic to find one justification behind Quine's claim.

Moreover, and this is quite possibly the more serious objection from Quine, giving up the concept of classical negation amounts to ceding a great amount of systematic simplicity in devising proof-procedures: the true-false dichotomy afforded by *bivalence*, the claim that all sentences are either true or false,[74] and embodied in the law of excluded middle enables a great simplification when considering proof-procedures for a formal calculus. The same procedure used to proof a quantificational schema inconsistent can equally be used to prove the validity of a schema through proving its negation inconsistent.[75] What is needed to demonstrate inconsistency and validity is a single proof-procedure, not two independent ones. By giving up classical negation, however, this advantage is lost and with it a great amount of systematicity, economy and simplicity.[76] Thus

other – evidential value and systematic value – are incommensurable."

[71] While rejection of the law of excluded middle is by no means the only major revision that has been proposed to classical logic, it is of particular importance due to its impetus from mathematics and science. Intuitionism was originally proposed as a reform of mathematics and its central tenet of rejecting the law of excluded middle was later lend additional support by certain phenomena in quantum mechanics (Quine 1986, pp. 83). Other reforms of logic, such as, e.g., paraconsistent or substructural approaches, are motivated by "the pursuit of analogy and generalization" and concern uninterpreted theory, abstract mathematical structure rather than constitute serious attempts to reformulate the framework for science (Quine 1986, 84).

[72] Cf. (Quine 1986, 84): "it is a rejection of the classical true-false dichotomy, of classical negation." Terminology is salvaged through partial analogy between the classical and the intuitionist concept, which, in a certain sense, means that failure of the law of excluded middle is nominal (Quine 1986, 84).

[73] Cf. also (Quine 1999a, 648), "crystalline bivalent logic" vs the "fog of intuitionism".

[74] Cf. (Quine 1986, 83) for three different ways of stating this claim and the proof of their equivalence. Cf. also (Quine 1981a, 36): "Bivalence is a basic trait of our classical theories of nature. It has us positing a true-false dichotomy across all the statements that we can express in our theoretical vocabulary, irrespective of our knowing how to decide them."

[75] Cf. (Quine 1955).

[76] Cf. (Quine 1969c, 111): "The classical logic of quantification has a complete proof procedure for validity and a complete proof procedure for inconsistency; indeed each procedure serves both purposes, since a formula is valid if and only if its negation is inconsistent. [...] Thus classical, unsupplemented quantification theory is on this score maximal: it is as far out as you can go and still have complete coverage of validity and inconsistency by the Skolem proof procedure."

Quine's adherence to bivalence.[77]

Nevertheless, there is also the negative aspect to consider: rejection of the law of excluded middle was not proposed haphazardly, but in order to deal with problems classical logic did not adequately address. In order to be considered superior classical logic should, at least in some way, be able to contribute a solution to these issues in addition of its superior systematicity in order to be considered the better framework for science. There are several motivations for rejecting a classical interpretation of negation to consider:

(i) *Set theoretic paradoxes*: Going along with the rejection of the law of excluded middle is the acceptance of more than two truth-values. Allowing a third truth-value this way offers a solution to the paradoxes of set theory (Russell Class) and semantics (Liar Paradox), by assigning the problematic statements neither the truth value `true` nor `false`. Quine's response consists in pointing out that it is a rather drastic move to change the logic for problems occurring outside of its domain, i.e. in set theory and semantics. It appears more reasonable to solve them within the domains they occur in and that they affect (Quine 1986, 85).[78]

(ii) *Quantum Mechanics*: The same reasoning applies to the proposal of a three-valued logic to deal with certain phenomena in quantum mechanics. Moreover, there are doubts as to whether this change of logic actually achieves what it intends to and whether it is reasonable to impose such a fundamental adjustment (Quine 1986, pp. 85). Moreover, as Levin points out (Levin 1978, 54), the alleged failure of distributivity within quantum mechanics grounding the rejection of the law of excluded middle[79] actually depends on a particular interpretation of logical notions within an algebraic setting.

(iii) *Vagueness*: Three-valued logics have been proposed to deal with problems pertaining to vagueness. Vagueness causes problems in that it gives rise to paradox when common sense, pre-scientific terms (baldness, heap) are combined with scientific principles (induction) (Quine 1981a).[80] Instead of accepting a drastic change of logic, however, Quine suggests replacing the common sense terms with scientific idioms doing the same job as the original terms, whose 'semantic boundaries' have, however, been adequately specified – be it through arbitrary stipulation or precisification, – in order to avoid vagueness.[81]

    There might, however, be common sense terms that resist such precisification without loosing their point.[82] In this case one should "neglect common sense classification in favor of scientifically precise notions" (Quine 1981a, 34), i.e. abandon the problematic notions altogether and ban them from scientific theories.[83]

[77]Cf. (Quine 1981a, 32): "My inclination is to adhere to it [bivalence] for the simplicity of theory that it affords."

[78]Furthermore, if the doubts about classical logic only pertain to constructivist scruples, there are other ways to reconcile them, not 'mutilating' classical logic (Quine 1986, 88).

[79]Cf. (Putnam 1980) for an argument to this effect.

[80]The most well-known of these paradoxes are the so-called *sorites-paradoxes*.

[81]Cf. (Quine 1981a, 33/34): "It is in this spirit that what had been learned as observation terms may be redefined, on pain of paradox, as theoretical terms whose application may depend in marginal cases on protracted tests and indirect inferences. The sorites paradox is one imperative reason for precision in science, along with more familiar reasons. [...] Some terms are adopted from observation language and incorporated into scientific language with their edges refined, others are incorporated as if refined; and sufficient until the day is the evil thereof. We are thus enabled to make do with our bivalent logic and our smooth and simple arithmetic, including mathematical induction."

[82]Quine envisions the common sense term 'table' and attempts to delimit the number of atoms that need to be present in it for it to constitute such (Quine 1981a).

[83]One could, for example, replace the problematic notion of table with the less problematic one of physical body for the purpose of theory.

Whether one deems these modifications sufficient to meet the problems proposed or not, one will have to consider the trade-off between the simplicity of the framework in which the theories are constructed and the global advantage brought by so modifying it in order to avoid problems local to certain theories. Quine's stand, in any case, is clear: bivalence is an essential component of canonical notation.

4.2.2 Demarcating Logic

In order to demarcate a logic, it suffices to delineate the class of logical truths: "demarcate the totality of logical truths, in whatever terms, and you have in these terms specified the logic" (Quine 1986, 80). Where we draw the boundary between logic and something else, however, is, in a sense, arbitrary and relative; nothing hinges on what we call logic and what we call something else.[84] What matters, however, is what we should take to be the notation in which we are able to sensibly debate ontological issues – this notation is to be as clear, precise, succinct and neutral with regards to ontological issues as possible and shares these demands with what might be considered essential traits of logic. It is therefore a reasonable question to ask "where, within the totality of science that we accept, the reasonable boundary falls between what we may best call logic and what we may best call something else" (Quine 1986, 80) and how we best delineate this boundary.

Logicality

Despite the common conception that Quine is a staunch defender of the *first-order thesis*, the thesis that all there is to logic is FOL,[85] Quine nowhere explicitly provides a single, stringent argument as to why and where to draw the boundaries of logic. There are frequent remarks, however, to the effect that certain things should be excluded and other things ought to be taken for granted when dealing with logic. I will here attempt to provide a comprehensive reconstruction of a possible argument one could give on Quine's behalf, which unites the different moments and remarks he makes on logic in a coherent whole and offers, I believe, an adequate account of his view.

The most general definition of logical truth we can give is the one in terms of grammar mentioned above, to repeat: "A logical truth is [...] a sentence whose grammatical structure is such that all sentences with that structure are true" (Quine 1986, 58). It is the most general, because it does not, unlike definitions of logical truth in terms of models, proof, satisfaction, substitution, etc.,[86] make essential mention of the logical particles of the language in question. In giving the recursive truth-definition for truth-in-a-model or by specifying an adequate proof-system and procedure, these devices are already explicitly mentioned. Nevertheless, constraining ourselves to the 'standard grammar', the language of FOL, the definition of logical truth in terms of grammar coincides with the various alternative definitions.[87] We will have to say more about grammar and grammatical structure in the next section. Note at this point that the equivalence of the definitions of logical truth in terms of models and in terms of proof depends on soundness and completeness results – a first clue to the importance of completeness for logicality due to its assurance of the equivalence of two common and fundamental definitions.[88]

[84]Cf. (Hylton 2007, 264).

[85]Cf. (Barwise and Feferman 1985).

[86]Cf. (Quine 1986, pp. 47) or any introductory text on logic.

[87]Adopting the 'standard grammar' logically true sentences are those whose truth only depends on quantification, truth-functions and predication. Within this grammar logical structure and predicates is all there is to any sentence and logical truth is that truth which is independent of predicates (Quine 1986, pp. 47).

[88]Cf. (Quine 1986, 57).

Grammatical structure is obvious to anyone possessing a proper command of the language, given that to have command of a language means to have comprehended its grammar: "[f]or consider, to begin with, the place of grammar in language. Anyone who is said to have learned a language [...] will have learned its grammar. Whoever deviates from the grammar is to be classed either as a foreigner who has not mastered the language or as a native whose dialect is different" (Quine 1986, 101). Due to the supervenience of logical truths on grammatical facts[89] and the obviousness of these grammatical facts

> "the logical truths, being tied to the grammar and not to the lexicon, will be among the truths on which all speakers are likeliest to agree [...]. For it is only lexicon, not grammar, that registers differences in background from speaker to speaker; and the logical truths stay true under all lexical substitutions. Naturally the habit of accepting these truths will be acquired hand in hand with grammatical habits. Naturally therefore the logical truths, or the simple ones, will go without saying; everyone will unhesitatingly assent to them if asked. Logical truths will qualify as obvious [...]" (Quine 1986, 102)

Every truth that is a truth of logic should therefore be, at least potentially if the grammatical construction is very complex, obvious.[90] "This means that every logical truth is either obvious as it stands or can be reached from obvious truths by a sequence of individually obvious steps" (Quine 1986, 82/83) and puts forward the demand for the *completeness* of any system that aims at capturing what we mean by logic. A completeness result will ensure us that every logically valid formula, every logical truth, can be reached from obvious premises by means of obvious steps, thereby demonstrating its obviousness.[91] This, then, places a very concrete demand on any adequate theory of logic, namely that it be *complete* for otherwise there would be no guarantee that logical truths are, in fact, obvious.[92]

Putting matters slightly different: the completeness of logic is demanded because of its supervenience on grammar and the obviousness of grammar during language-acquisition. The obviousness of logic is ensured through completeness because it guarantees us that every logically true sentence can be made obvious through a finite process involving only obvious elements[93].[94]

Another common argument establishing the boundaries of logic rely on its *generality*: logic, it is said, is the most general theory we possess – everything is subject to the laws of logic, but logic

[89] While logical truths can be delineated in terms of grammar, this does not mean that logical truths are true *because* of language; Quine does not put forward a linguistic theory of logical truth: "This much can be said for the linguistic theory of logical truth: we learn logic in learning language. But this circumstance does not distinguish logic from vast tracks of common-sense knowledge that would generally be called empirical. There is no clear way of separating our knowledge into one part that consists merely in knowing the language and another part that goes beyond." (Quine 1986, 100)

[90] On the obviousness of logic see also the introduction in (Sher 1999a).

[91] For proofs of completeness results for FOL, see Quine' own (Quine 1982) or (Quine 1951b).

[92] Tharp (Tharp 1975) reaches the same conclusion in a different way and takes completeness thus to be a minimal condition for logicality.

[93] Cf. (Quine 1960a, pp. 266).

[94] Authors who advocate higher-order systems (e.g. second-order logic) as belonging to logic proper often criticize Quine's 'arbitrary' focus on completeness. After all, the argument goes, decidability is just as important a property as completeness, yet FOL is undecidable (see, e.g., (Boolos, Burgess, P., and Jeffrey 2007, pp. 126)) and Quine still regards is as logic; cf. (Boolos 1975). How, then, to justify the focus on completeness, rather than decidability? The proper answer to this line of reasoning consists in pointing out that Quine is trying to determine a *minimal condition* that anything that claims to be logic has to fulfill. This minimal condition demands that all logical truths be, at least potentially, obvious. Whether completeness is sufficient is a different question, in any case it is necessary. However, there are strong reasons to suppose that Quine also regards it as sufficient, cf. (Quine 2008c, 150): "For logic as a whole then, [...] the most we can require is a complete theory of *proof*, such that every logically valid statement *has* a proof."

is not subject to the laws of, say, physics or of any other of the special sciences. The use of logic is ubiquitous, it "is a handmaiden of all the sciences" (Quine 1986, 98); its characteristics are universal applicability and "impartial participation in all the sciences" (Quine 1986, 102). Such generality can only be maintained by means of *topic-neutrality*, logic is applicable to any topic indiscriminately, without regard to its subject matter. This implies logics' independence from any proper subject matter: "logic favors no distinctive portion of the lexicon, and neither does it favor one subdomain of values of variable over another" (Quine 1986, 98); logic "has no objects to call its own; its variables admit all values indiscriminately" (Quine 1995a, 52).

Any theory requiring the assumption of particular entities or particular predicates in the lexicon – such as, e.g., set theory, requiring the existence of *sets* and the predicate '$\in$'[95] – should therefore be disqualified as logic on account of its lack of topic-neutrality and therefore generality. This then presents sufficient reason to draw a boundary between logic proper and set-theory (Quine 2008c, pp. 150), for set theory requires the existence of certain 'abstract' entities/ universals, namely sets.[96] This "line that can be drawn across modern logic" with the assumption of universals (Quine 2008f, 145) receives additional support from the previous argument: Gödels celebrated incompleteness theorem proves that any theory of sufficient strength is essentially *incomplete*. In particular, any theory sufficiently strong to encompass the theory of the natural numbers, such as set-theory, does not admit of a complete proof-procedure.[97] The assumption of abstract mathematical entities thus leads to incompleteness and any theory positing them cannot be considered logical on account of loosing logic's obviousness. The boundary drawn by the supposition of logics generality and neutrality therefore appears to coincide[98] with the boundary demarcated by its obviousness: "Logic, in this narrower sense, has a complete theory; and logic in this narrower sense is free from ontological commitments" (Quine 2008c, 151). Completeness therefore emerges as a central property delineating logicality, due to its assurance of definitional equivalence of two independent and highly plausible definitions of logical truth (proof- and model-theoretically), its guarantee of the obviousness of logical truths and its conservation of the generality and neutrality of logic.

Logical Constants

Logical truth as truth in virtue of logical form raises the question which elements in a logic qualify as *logical constants*, i.e. as those elements that determine logical form and by means of which one can talk about sameness or distinctness of logical form. The traditional way of determining the logical constants of a language has been by means of *enumeration*. Those elements listed in the recursive definition of truth/satisfaction for a language that are given their own clause are the logical constants of that language. However, sheer enumeration without justification of why it is those and no other elements that qualify as logical is highly unsatisfactory; after all we draw a lot of inferences from the logical form of a sentence so there ought to be some justification for why any particular logical form was chosen.[99] What is needed and wanted is a *criterion of logicality* that enables a (complete and exhaustive) determination of the logical expressions of a language and provides a justification of why these and not others were demarcated as logical.

Given Quine's emphasis on logical truth as truth in virtue of *grammatical* form[100] it comes as

[95]We will deal with the particular status of *identity-theory*, FOL plus the predicate '=', in Section5.3.1.

[96]Cf. also (Quine 2008f, 145).

[97]Cf., e.g, (Boolos, Burgess, P., and Jeffrey 2007, pp. 220).

[98]At least under the plausible assumptions that all candidates deserving of the name that qualify as an extension of logic fall within the realm of mathematics.

[99]This is essentially the criticism Sher (Sher 1991) raises against the traditional way of specifying the logical constants; we will briefly return to this point in Section 5.1.2.

[100]With the qualification that the grammatical form pertains to the language suitable to the enquiry at hand,

no surprise that the criterion he provides delineates the logical expressions in terms of grammar. Quine has a lot to say on the topic of grammar[101] and we will only scratch the surface of these elaborations in order to outline how he devises such a *grammatical criterion of logical particles*.

Grammar, on Quine's account, is an attempt at systematizing the linguistic expressions of a community. The field-grammarian is confronted with utterances of a community which he can break down into phonemes – the basic units of speech that can be combined in order to form meaningful expressions. Grammar then serves to account for *linguistic creativity*, the ability to form an infinite number of meaningful expression from a finitude of resources: in order to capture and delineate the legitimate linguistic constructions of a language, the grammarian will devise a *lexicon*, a list of meaningful simples of the language in question, as well as various *constructions* that allow one to form compound expressions from these simples:[102]

> "the grammarian has recourse to recursion: he specifies a *lexicon*, or list of words, together with various grammatical *constructions*, or steps that lead to compound expressions from constituent ones. His job is to devise his lexicon and his constructions in such a way as to demarcate the desired class: the class of all the strings of phonemes that could be uttered in normal speech. The string of phonemes obtainable from the lexicon by continued use of the construction should all be capable of occurring in normal speech; and, conversely, every string capable of occurring in normal speech should be obtainable from the lexicon by the constructions [...]" (Quine 1986, 16)

In order to economize the grammarian will divide the lexicon into different *categories* – such as, e.g., singular terms, adverbs, copulas, adjectives, etc. – and classify each expression that can be obtained by means of the constructions into one of these categories (Quine 1986, 17). Generally, two expressions will belong to the same category if they are grammatically interchangeable, i.e. if "you put one member for another in a proper sentence of the language you may change the sentence from true to false, but you will not change it to ungrammatical gibberish" (Quine 1986, 18). The division into categories and constructions enables a classification of all linguistic expressions into *lexicon* and *particles*: the lexicon is comprised by all items that occur in the categories, whereas the particles are those elements of speech "that are not thus classified but are handled only as parts of specific constructions" (Quine 1986, 27).

The question then becomes on what grounds certain expressions should be counted among the particles rather than the lexicon; after all why should one count $\neg$ in the sentence $\neg p$ as particle yielding a compound expression from the lexical item p by applying negation, rather than treating it lexically and taking p to be a particle that yields a sentence $\neg p$ by being applied to negation?[103] For Quine it is considerations of quantity that are decisive here: words are classed as belonging to the same category on the basis of being interchangeable *salva congruitate*, i.e. if exchanging the one for the other will not render a formerly grammatical sentence ungrammatical. Those expressions which have few expressions with which they are interchangeable *salva congruitate*, i.e that form a small category, are on grounds of economy counted as integral part of a construction itself, rather than having all the constructions applicable to them listed. These then are the particles (Quine 1986, 29):[104]

otherwise problems about negative existentials would re-arise.

[101]See Chapter 2 in (Quine 1986).

[102]Cf. (Quine 1986, pp. 15).

[103]Cf. (Quine 1986, pp. 28) and (Quine 2008e, 304).

[104]Predicates thus tend to be counted among the lexicon, not necessarily because of their infinitude (for each language the stock of predicates is to be definite and finite) but because of their indefiniteness from language to language and

> "Such, then, is the quantitative criterion: a morpheme [or combination thereof] is a particle or a lexical element according as there are few or many expressions in its grammatical category." (Quine 2008e, 304)

A quantitative criterion of this kind will render predicates such as 'green', 'tall', etc. lexical items and yield as particles, among others, 'and', 'or', 'not'[105]. In the same vein 'something' will have 'little company in its grammatical category', thanks to contexts such as 'There was something new on the Rialto' (Quine 2008e, 305) and therefore qualify as grammatical particle. The logical particles, or logical constants, as those items that occur in the constructions of a logical grammar (recursive definition of satisfaction/ own proof-clause) are those grammatical particles that occur within logical grammar.

The business of the logician consists in the reformulation of grammar in order to resolve structural ambiguities and economize on constructions (Quine 2008e, 307); after all "logical form is what grammatical form becomes when grammar is revised so as to make for efficient general methods of exploring the interdependence of sentences in respect of their truth-values" (Quine 2008e, 308).[106] The language we outlined in Section 4.1 "is the blueprint for a general scientific language. In such a language the grammatical form of any sentence is precisely its logical form"[107] (Quine 2008e, 312). Note that this criterion for logical constant-hood – the logical particles are those grammatical particles that emerge in our logical grammar – provides a complete and exhaustive result in a justified way, because it independently justifies the choice of language and then locates the constants in it by means of a very general criterion.

4.3 Quantification

The upshot of Section 3.2.1 was that "[e]xistence is what existential quantification expresses" (Quine 1969c, 97). The fundamental status of quantification for enquiries into ontology thereby renders the issue of how to actually understand these quantifiers pivotal for devising a proper notation in which to talk about existence. This section considers some of the alternative versions of quantification and their interpretation that have been proposed as supplementing or replacing Quine's favored candidate of *first-order objectual quantification* and the replies he gave (or could have given) to his critics for choosing the latter.

4.3.1 Interpretation of Quantifiers

Quine advances an *objectual interpretation* of quantification for the purpose of canonical notation. On an objectual understanding an existential quantification is true iff *there exists some object* that satisfies the condition which the quantifier in question governs, i.e.

their interchangeability *salva congruitate* when moving from language to language. It also allows for the possibility of predicate-yielding constructions (see, e.g., Quine's *predicate-functor logic* (Quine 1971)) (Quine 1986, pp. 31).

[105] For consider the interchangeability *salva congruitate* of 'not' in 'not only' (Quine 2008e, 305).

[106] Cf. (Quine 2008e, 307): "His interest in grammatical structure is one-sided: he is interested in how it channels truth conditions. If a grammatical reform makes for a more copious channeling of truth conditions and causes no complications in other quarters, he is happy to adopt it. He adopts it not as a reform to be imposed on society, but as a technical by-language to expedited scientific inference."

[107] "... plus predication at the atomic level [...]. Such predication is an additional grammatical construction, for we saw that predication had no place in the grammatical structure that the logician imposes in paraphrasing inward from outside. In recognizing predication thus at the atomic level we round out a self-contained grammar, but we add nothing to the logical truths or implications. The grammatical structure that we have projected into the atomic sentences proves to be logically inert." (Quine 2008e, 313)

> $\exists x\varphi(x)$ is true (in a model $\mathcal{M}$) iff there exists some individual d (from the domain $\mathcal{D}$ of the model $\mathcal{M}$) such that d satisfies condition φ

The case for an objectual understanding of quantification is very strong: the concept of existence is intimately bound up with the concept of reference. Objects were posited as 'neutral nodes in the structure of theory' in order to allow better and more reliable predictions about sensory input by creating stable beacons in the flux of experience that could be referred to repeatedly. In fact, it was this connection between existence and reference that enabled semantic ascent and our ability to formulate a criterion of ontological commitment by talking about linguistic expression in the first place; after all 'Pegasus exists' is true, i.e. Pegasus exists iff 'Pegasus' refers. Objectual quantification exploits and incorporates this moment of reference (by referring to an object of which the condition governed by the quantifier must be true in order for the existential quantification to be true as a whole) thereby lending support to the claim of having successfully captured our natural, everyday concept of existence. Nevertheless, objectual quantification is not the only interpretation of quantification that is 'out there' and alternatives to it merit consideration.

Objectual vs Substitutional

Objectual quantification, so Quine, is *parochial*: "[w]e can locate objectual quantification in our own language because we grow up using those very words" (Quine 1969c, 105). Relative to any other language, however, we can locate these referring expressions only "relative to chosen or inherited codes of translation which are in a sense arbitrary" (Quine 1969c, 105). These 'codes of translation' rely entirely on behaviouristic evidence and criteria for their adoption or rejection. There is, however, an alternative interpretation of quantification that is behaviorally much better determined than objectual quantification, i.e. *substitutional quantification*. On a substitutional understanding an existential quantification $\exists x\varphi(x)$ is true iff *there is a substitution instance* of φ that is true, i.e. iff there is true sentence of the form $\varphi(t)$ for a singular term t. I.a.w.

> $\exists x\varphi(x)$ is true (in a model $\mathcal{M}$) iff there exists some singular term t (in language $\mathcal{L}$) such that $\varphi(t)$ is true.[108]

Substitutional quantification is behaviorally better determined than objectual quantification since once a translator has settled on expressions for quantifiers, singular terms and variables, in order to verify whether a linguistic community would assent to or dissent from a, say, universal quantification $\forall x\varphi(x)$ all we would have to do is to check their assent and dissent with respect to $\varphi(t)$ for each singular term t of the language. Such a method is not available for objectual quantification.

With regards to integrating substitutional quantification into canonical notation, a notation first and foremost designed to allow discourse about ontological issues, there arise substantial difficulties: on the one hand, there is no assurance that every object that we wish to refer to actually has a name. In fact, if we wish to stay within a countable language[109], we are assured that there will be more objects 'out there' than names in the language, one only needs to consider the real numbers and the fact that they are uncountable. Thus, if we assume a 'sufficiently rich' universe

[108] For a formal comparison of objectual and substitutional quantification see (Hand 2007).

[109] It is a justified question to wonder whether we have to require the language to remain countable or might be able to consider anything as a language so long as we have some way to interpret and parse it and thereby also include uncountable vocabularies. Quine would probably dismiss the latter as mathematics and 'uninterpreted algebra' rather than taking them to be a serious attempt of devising an austere notation for science which could, at least in principle, be used by actual scientists. In fact, any theory that is actually devised and devisable will be finite and thus only contain a finite vocabulary.

(Quine 1986, 93) the substitutional characterization of quantifiers will not be co-extensive with their objectual characterization: there are too many objects for all of them to have names and an existential substitutional quantification could therefore turn out false while the same quantification objectually construed might turn out true, due to a lack of name for the relevant witness. Vice versa for universal quantifications; they might turn out true substitutionally, yet false objectually for the mere fact that the counter-instances lack names.[110]

Thus, unless every object has a name objectual and substitutional quantification are in general not equivalent. The objection then goes that "substitutional quantification gives no acceptable notion of existence properly so-called, not if objectual quantification does" (Quine 1986, 106). It is not just that we have already settled on a correct understanding of existence in terms of objectual quantification and then attempt to measure all other attempts to do so by this standard – given that we try to delineate a proper notation in which to talk about existence this would amount to a *petitio principii*, – but rather that substitutional quantification appears to conceal the existence of some objects for lack of names. It appears to not be powerful enough to make reference to all the things that we might find in the ontology of, for example, the real numbers.

Another forceful objection to understanding quantification substitutionally for the purpose of ontology takes up on this latter fact. By utilizing substitutional quantification, the challenge goes, we "turn our backs on questions of reference" (Quine 1969d, 63). However, due to the fact that "referential quantification is the key idiom of ontology" (Quine 1969d, 66) this non-referential orientation of substitutional quantification means that by utilizing it we "turn our backs on ontological questions. Where substitutional quantification serves, ontology lacks point" (Quine 1969c, 107). Now, why is this? Consider the definition of substitutional quantification we provided above and compare it with the definition of objectual quantification. The quantification over objects from the domain of the model in the latter is replaced by *meta-linguistic* quantification over certain expressions of the object language; no reference is made to any objects, we seem to be able to make do with talking about linguistic expressions rather than by talking about the objects they supposedly refer to and talk about – reference disappears from the picture.

The claim that substitutional quantification is due to its shunning of reference unsuited to constitute a proper ontological idiom is, however, premature. It is true that reference does not enter into the specification of the truth-conditions of the quantifiers (and if one insists on the constitutive connection between quantification and reference this might be sufficient to reject substitutional quantification), but it might very well enter on a more basic level, namely in the specification of the truth-conditions for the atomic sentences or clauses governed by the quantifiers and then be inherited 'upwards'. Thus reference can still enters the picture, be it at a more basic level.[111]

However, the account of substitutional quantification given above makes essential use of *singular terms*. Given that they are used in order to provide an account of quantification the method of paraphrasing them away as outlined above (cf. Section 4.1.2), making essential use of quantification is unavailable on pain of circularity; we need to assume names as primitive components of our notation. This in itself does not present a serious problem, however, issues that were solved by quantificationally paraphrasing singular terms re-emerge: when do we know whether a given singular term refers or not, i.e. when do we know whether it ontologically commits us and when not? To make matters even worse, substitutional quantification can be made sense of for any kind of substitution class of expressions "even that whose sole member is the left-hand parenthesis" (Quine 1969c, 106), not only for that of singular terms. This issue is reinforced by the fact that there appears to be no way to restrict substitutional quantification to the class of singular terms, since the means by

[110] Cf. (Quine 1986, 93) and (Quine 1969c, 106).

[111] For a comparison of referential and non-referential base semantics see (Hand 2007). For a comprehensive comparison see Kripke's seminal (Kripke 1976). Cf. also (Haack 1978, pp. 50).

which to distinguish a genuine, referring, singular term from other terms was lost. However, "[t]o conclude that entities are being assumed that trivially, and that far out, is simply to drop ontological questions" (Quine 1969c, 106). Thus, by looking at quantification for ontological commitment we might be forced to attribute substantial ontological import to the left-hand parenthesis and thus the better alternative appears to consist in refraining from attributing referential force to substitutional quantification completely.[112]

> "This does not mean that theories using substitutional quantification [...] can get on *without* objects. I hold rather that the question of the ontological commitment of a theory does not properly arise except as that theory is expressed in classical quantificational form, or insofar as one has in mind how to translate it into that form." (Quine 1969c, 106)

The pointlessness of ontology in the context of substitutional quantification can be seen clearly in the way a theory formulated in terms of substitutional quantification can be translated into a theory using referential, objectual quantification. Such a theory can be construed as only being committed to linguistic expressions: an existential quantification $\exists x\varphi(x)$ is true iff there exists an expression $t \in \mathcal{D}$ (the domain of the model), s.t. $\mathcal{M}, t \models \varphi$; "the theory into which we translate is one that talks about expressions of the original theory, and assumed them among its objects – as values of its variables of objectual quantification" (Quine 1969c, 107). Since the alphabet of the theory in question was countable, we might as well arithmetize the syntax and thus instead of talking about linguistic expressions, talk about natural numbers:

> "Thus we may look upon substitutional quantification not as avoiding all ontological commitment, but as getting by with, if effect, a universe of natural numbers." (Quine 1969c, 107).

Accepting substitutional quantification would thus amount to an acceptance of *Pythagoreanism*, the view that all there is are numbers and that all talk is, ultimately, number talk.

Objectual vs Game-Theoretical

Another, somewhat more recent, semantics for first-order logic that is gaining in popularity is a so-called *Game-Theoretical Semantics* (GTS).[113] In GTS the notion of a game between two players, $\exists$loise and $\forall$belard, serves to extend the concept of truth and satisfaction from atomic to more complex formulas. After having settled the truth-values for atomic sentences of the language the truth for a complex formula ϕ is settled by means of a two-player, zero sum game $G(\mathcal{M}, \phi)$ with respect to a model $\mathcal{M}$ in the following way:[114]

(i) if ϕ is a formula of the form $\psi_1 \vee \psi_2$, $\exists$loise picks a disjunct ψ_i, $i \in \{1, 2\}$, and they play $G(\mathcal{M}, \psi_i)$.

(ii) if ϕ is a formula of the form $\neg\psi$, the roles of the players are switched and the game continues with respect to $G(\mathcal{M}, \psi)$.

[112] Cf. (Quine 1969d, 210): "Ontology is thus meaningless for a theory whose only quantification is substitutionally construed; meaningless, that is, insofar as the theory is considered in and of itself. The question of its ontology makes sense only relative to some translation of the theory into a background theory in which we use referential quantification."

[113] This and the following section draws on and develops aspects and ideas contained in my writing sample submitted with the application for PhD positions in the Netherlands, the UK and the US.

[114] Cf., e.g., (Hintikka 1973) and (Hintikka and Sandu 1998).

(iii) if ϕ is a formula of the form $\exists x\psi(x)$, $\exists$loise chooses a witness for x by picking an individual a from the domain M. They continue the game with respect to an extension $\mathcal{L} \cup \{c_a : a \in M\}$ of $\mathcal{L}$, by playing $G(\mathcal{M}, \psi(c_a))$, where $\psi(c_a)$ is the formula $\psi(x)$ in which all occurrences of the variable x are replaced by the name "c_a" of the object $a \in M$.[115]

In a finite number of moves, i.e. applications of rules (i) – (iii) above, the game yields an atomic sentence ϑ. $\exists$loise wins the game if $\mathcal{M} \models \vartheta$, $\forall$belard wins otherwise. Truth-in-a-model is thus defined as follows:

> A sentence ϕ is true in a structure $\mathcal{M}$, $\mathcal{M} \models \phi$, iff there exists a *winning strategy* for $\exists$loise in the game $G(\mathcal{M}, \phi)$, i.e. "if [she] can always choose [her] moves (depending on what has already happened in the game) so that [she] will win no matter what [$\forall$belard] does." (Hintikka 1973, 157)

A *strategy* for either player instructs them how to play against any possible move of the respective opponent; it tells them which witness, i.e. object from the domain, to pick depending on which individual(s) have been chosen before. Such strategy can be represented by a sequence of functions: the choice of an individual for a variable can be formally captured by a Skolem-function that picks out an individual depending on the objects previously chosen, i.e. that takes as input the values of the variables previously picked and returns a witness for the variable in question. A strategy is then simply a sequence of these functions.

With respect to 'standard grammar', i.e. FOL, these semantics do not tell us anything new or different in that they do not yield any different results than classical Tarskian-semantics[116] and do therefore only propose a harmless alternative to the semantics chosen above. They direct attention to a much neglected phenomenon of FOL, however, which provides the motivation for the impetus to extend FOL by *branching quantifiers* (see the section below). The much overlooked point is that the grammar of FOL, the linear syntax of first-order logic, prohibits certain semantic phenomena. In particular, syntactic scope forces semantic dependence: whenever an existential quantifier stands in the scope of a universal quantifier the value of the former *depends* on the value of the latter. This disallows a straightforward expression of sentences of the form (*) in FOL:

> (*) For every x and for every z there is a y, *dependent only on* x, and a w, *dependent only on* z, such that $P(x, y, z, w)$ holds. (Barwise 1979)

GTS direct attention to this phenomenon because the games mentioned in it are *perfect-information games*; both players always know and remember all previous moves of the game and it seems a natural suggestion to relax this condition and allow games with *imperfect* information.

4.3.2 Deviant Quantification

Having considered some of the alternative ways in which to understand the standard first-order quantifiers, there still remains the question of "how much better than arbitrary [...] our particular quantification theory [is], seen as one in some possible spectrum of quantification theories" (Quine 1969c, 108). This section considers two of the, for present purposes, most important alternative proposals that have been advanced and, especially with respect to the latter, still are advanced as viable alternatives to a first-order canonical notation.[117]

[115] All other connectives and quantifiers are defined in the usual way.

[116] Assuming the Axiom of Choice the two semantics are equivalent; cf. (Hintikka and Sandu 1998).

[117] Commonly, intuitionistic logic is among the first to be mentioned as alternatives for first-order logic, however, we already provided some reason against it replacing classical canonical notation in Section 4.2.1 on the basis of the intuitionists' rejection of the law of excluded middle and thereby classical negation.

First-Order vs Branching

The dependency relations in (*) proved too complex to be expressed in FOL; the only way to respect them entails strong ontological assumptions. Thus

$$(^{**})\ \exists f \exists g \forall x \forall y P(x, f(x), y, g(y))$$

captures the content of (*) at the cost of quantification over functions.[118] The assumption of higher-order objects, however, seems "out of keeping with the elementary character" of (*) (Quine 1969c, 109) and thus an alternative formalization suggests itself:

$$(^{***})\ \begin{matrix} \forall x \exists y \\ \forall z \exists w \end{matrix}\ P(x, y, z, w)$$

In this *branching quantifier* an existentially quantified variable only depends on those other variables that precede it in the same branch. Quantifiers in different branches are independent of each other. Thus (***) captures (*), is equivalent to (**) and is the simplest quantifier that cannot be obtained by linear repetition of the standard first-order quantifiers ∀ and ∃[119].[120] What is more, it appears to be 'perfectly first-order' and not to require abstract objects, such as functions. Branching quantifiers of the form (***) thus demonstrate a way of overcoming the seemingly random and undesired limitations that were induced by the linear syntax of FOL and allow us to capture 'ontologically innocent' content such as the one expressed in (*). They provide a way of accounting for dependence and independence phenomena without appealing to higher-order objects; moreover, an attractive semantics is immediately available: the *GTS* of the previous section can be extended by requiring that a distinct game be played parallel for each branch of the quantifier-prefix. The moves are made simultaneously and without knowledge of the games in the other branches; we are playing games with *imperfect information*.

The argument that branching quantification is not as 'elementary', harmless and ontologically innocuous as it might seem on first view has been made early on: utilization of formulas like (***) allows us to gain the "services of certain abstract objects, namely functions, [...] without recognizing these functions as objects" (Quine 1986, 90); it appears as if (***) hides its true ontological commitment and a theory making essential use of such prefixes is thus *ontologically dishonest*. This argument is supported by the observation that the theory of all *finite partially-ordered quantifiers* (FPO) of the form

$$(+)\qquad \begin{matrix} Qx_1^1 \cdots Qx_1^m \\ \vdots \ddots \vdots \\ Qx_n^1 \cdots Q_n^m \end{matrix}\qquad \phi$$

with $Q \in \{\forall, \exists\}$ and quantifier free formula ϕ is expressively equivalent[121] to existential-second order logic (ESO).[122] It stands to reason that this equivalence justifies the demand that FPO quantification theory should have at least the same ontological commitments as ESO (see next section).

[118] Note that these functions make explicit the notion of a winning-strategy mentioned in the definitions above on the object level and that questions about dependence and independence of quantifiers naturally arise if the semantics are made thus explicit.

[119] Cf. (Barwise 1979).

[120] *Branching quantifiers*, also known as Henkin-quantifiers, were first discussed in (Henkin 1968). The independence phenomena they uncovered later led to the development of *independence-friendly logic* (Hintikka and Sandu 1998) and found application in linguistics; cf. (Barwise 1979), (Fauconnier 1975), (Gierasimczuk and Szymanik 2009), (Hintikka 1973). For a formal discussion of branching quantification see (Enderton 1970), (Walkoe 1970).

[121] I.e. for every ESO formula there exists an FPO formula that is true in the same models.

[122] Cf. (Enderton 1970), (Walkoe 1970).

On this line of argument the skolemization of a formula (***) by (**) just makes explicit what is already implicit in the semantics – the commitment to functions through the notion of a strategy. (**) is the 'honest' version of (***), acknowledging what the semantics imposed ontologically on branching quantification. This essential and irreducible commitment to *strategies* becomes even more obvious when we see that a standard compositional Tarskian semantics cannot be given for branching quantifiers: what is needed is something more to make it work; the question than is whether this something more should go ontologically unpunished.[123]

The discussion surrounding the ontology of FPO quantification theory might be relevant for whether to count it as a logic or not, in the context of (Weak.Q), however, the question is rather whether the ontological assumptions introduced by branching quantifiers should be waived in order to obtain a more accurate notation for ontological issues. Above observations touch only tangentially on these considerations and I will therefore discuss the (supposed) ontological commitment of FPO quantification theory no further.[124]

Much more decisive reasons speaking against the inclusion of branching quantifiers into canonical notation concern the formal properties they import when added to standard FOL. On the one hand, the addition of even only the simplest of branching quantifiers, (***), to FOL causes the set of valid formulas of this system to be non-axiomatizable.[125] Moreover, it is not the case that the negation of a sentence containing a branching quantifier (essentially) is equivalent to a first-order or branching quantifier sentence.[126] In other words, classical negation and thereby the law of excluded middle fails.[127] This then, in addition, renders the existence of a single proof-procedure for both, validity and inconsistency, impossible, introducing severe methodological drawbacks adding to the violations of the desiderata for a canonical notation as outlined in Section 4.2.

Never mind questions of ontology then, formal considerations of the difficulties and drawbacks in terms of simplicity caused by the introduction of branching quantifiers suffice to show that

> "It is at the limits of the classical logic of quantification, then, that I would continue to draw the line between logic and mathematics. Such, also, is the concept of quantification by which I would assess a theory's ontological demands." (Quine 1986, 91)

First-Order vs Higher-Order

Another strong contender for the title of a genuine logic is *second-order logic* (SOL). In addition to quantification over individuals of the domain as in FOL, SOL also allows quantification over predicate variables, such as, for example, in the formulas $\forall P \exists x Px$ or $\exists R \exists x \exists y (xRy)$. How to understand these higher-order quantifications is far from obvious. In general, there are two common ways to interpret quantification in SOL, by means of the so-called *standard semantics* or through *Henkin semantics*. The difference is the following: in standard semantics a predicate variable (or k-ary relation variable) is taken to range over the entire power-set of the domain (of the k-ary Cartesian product of the domain), whereas in Henkin-semantics we specify, for each n, a separate

[123]In general it was not possible to provide a compositional semantics for branching quantifiers along the standard lines. Quantifier-prefixes in essentially branching quantifiers, i.e. those that are not equivalent to a standard FOL formula, cannot be built up quantifier by quantifier, but rather the entire prefix has to be treated en bloc; cf. (Patton 1991). Hodges recently showed how one can obtain a compositional semantics for imperfect information languages through the use of *teams*, i.e. sets of assignments (Hodges 1997).

[124]See however (Patton 1991), (Hand 1993) for further discussion.

[125]Cf. (Henkin 1968). In fact, the set of valid formulas of this system is not recursively enumerable.

[126]Cf. (Burgess 2003).

[127]This is due to the fact that the negation of a functionally-existential sentence will in general be a functionally-universal sentence, falling outside of ESO and therefore not equivalent to any Henkin-sentence.

domain consisting of n-tuples from the domain over which we then take the corresponding relations of arity n to range.[128]

Henkin-semantics determine a much narrower class of logical truths than standard semantics and retain many of the attractive properties of FOL, such as, e.g., compactness and completeness[129]. They do so, however, on cost of being nothing over and above multi-sorted FOL[130] and thereby loosing many of the expressive capabilities the possibility of which justified SOL in the first place. All that can be done with SOL + Henkin-semantics can in principle be done with FOL[131] and thus the systematic advantages offered by it as compared with FOL dwindle. Common with the majority of literature on SOL we therefore take the alternative proposed to FOL by advocates of SOL to consist in SOL under the standard semantics and refer to this interpretation when talking about SOL in the remainder of this section.

Second-order quantification, i.e. quantification over predicates and relations, adds a considerable amount of expressive power to a language: the theories of arithmetic and analysis[132] are *categorically* describable,[133] something that is impossible in FOL. This, of course, makes the resources of SOL very desirable. However, a first challenge the proponent of SOL faces concerns the intelligibility of genuine (as opposed to virtual, cf. (Quine 1986, pp. 68)) second-order quantification. After all, quantification is only allowed into name position (for what can we be said to quantify over other than objects which could, in principle, be named?) and "[p]redicates are not names; predicates are the other parties to predication" (Quine 1986, 27/28).[134] The proponent of SOL thus appears to be forced to at least admit that predicates and relations on his account need to take on some kind of semantic value in order to make quantifying over them intelligible. What objects could those semantic values be? Recourse to properties is not only unhelpful, but appears to immediately rectify Quinean doubts about SOL on account of properties lacking clear identity conditions (see Section 4.1.1 above) and thus violating the principle of extensionality, which was fundamental for canonical notation. However, the property *ersatz* devised by Quine (see above), i.e. sets, is not much more attractive, for this would ascribe substantial ontological commitment to SOL and render it "set theory in sheep's clothing" (Quine 1986, 66). In such case, when using SOL what we would really be doing was set theory in a misleading notation, hiding its true ontological commitments and assumptions.[135]

The question of whether SOL carries commitment to sets or is, as has been argued,[136] ontologically innocent is more relevant to whether SOL should be counted as belonging to logic (given the intuition that logic ought to be topic-neutral and thus ontologically uncomitting) than to the question of whether it should be part of canonical notation. Of course, for Quine these questions

[128]Cf., e.g., (Enderton 2009), (Shapiro 1991) or (Linnebo 2011). Validity of a sentence under Henkin-semantics is then usually defined as truth in all structures that satisfy all so-called *comprehension-axioms*, axioms that in essence guarantee the existence of all definable subsets of the domain in the predicate/relation domains. The validities under Henkin-semantics constitute a much narrower class than the validities under standard semantics and many of the expressive advantages SOL was taken to have are being lost; cf. (Enderton 2009).

[129]SOL under a Henkin semantics is axiomatizable by taking the axioms of FOL plus the comprehension axioms (see previous footnote).

[130]Cf. (Shapiro 1991, Theorem 3.5).

[131]Given the equivalence of SOL with Henkin-semantics to multi-sorted FOL and the possible reduction of the latter to standard FOL through relativization, this fact becomes obvious.

[132]Cf. (Shapiro 1991, 82–84).

[133]A theory is said to be categorical if it has, up to isomorphism, only one model.

[134]Whether quantifiable positions need to be name positions has been doubted, see, e.g., (Boolos 1975).

[135]Cf. (Hylton 2007, 267) and (Quine 1963b, 257): "The existence assumptions, vast though they are, can become strangely inconspicuous; they come to be implicit simply in the old rule of substitution for predicate letters in quantification theory, once we have promoted those letters to the status of genuine quantifiable variables."

[136]Cf., e.g., (Boolos 1984).

are inseparable; after all, logic was to constitute the framework for all of science, to which each science then simply adds its distinctive stock of predicates.[137] Ontology, as the science 'calling no objects its own', but determining the objects assumed by each science is then simply the limiting case of having the empty stock of predicates and thereby coinciding with logic proper. Nevertheless, in the context of (WEAK.Q) making SOL part of canonical notation means giving a 'free-pass' to its alleged ontological import and thus appears to circumvent the question of whether SOL should belong to logic or not on that basis alone. What the above account does, in any case, is to cast justified doubt on the ontological honesty of SOL and thus in the least calls its suitability for a notation that is as unbiased as possible towards ontological issues into question.

There are, however, more substantial reasons to ban SOL from canonical notation. SOL with standard semantics is essentially *incomplete*,[138] i.e. it lacks a complete proof-procedure. We already pointed out above the importance of completeness because of the obviousness it bestows upon the formal system in question; it is the main source of their clarity. This is not only important for logic, but even more so for a canonical notation which is supposed to be as clear as possible about the most basic assumptions of our conceptual scheme. What it does needs to be obvious in order to capture our most basic assumptions about reality, i.e. its ontology. Violating this constraint presents a serious obstacle to inclusion in canonical notation. In addition, it was already pointed out above (see Section 4.2.2) that there appears to exist an intimate connection between the expressive strength of language, completeness and its assumption of abstract entities. The failure of completeness thus lends additional support to the claim about SOL's hidden ontological commitments.

Another objection to the suitability of SOL as canonical notation stems from the observation that it has substantial mathematical content. There exists, for example, a sentence in SOL which is logically true iff the Continuum Hypothesis (CH) holds.[139] The mathematical content of SOL thus appears to exceed the resources of ZFC from the truth of which CH is independent. Moreover, for the demonstration of certain logical facts SOL requires the existence of strongly inaccessible cardinals, objects beyond the reach of ZFC (Linnebo 2011, 123). The possession of such mathematical content is problematic for a candidacy as canonical notation because it calls into question the topic-neutrality and generality that should adhere to canonical notation. On the one hand, canonical notation would not be perfectly general anymore because it was foredoomed to decide substantial mathematical questions (such as the truth or falsity of CH) and thus becomes unsuited for the investigation of questions surrounding these issues (Linnebo 2011, 124) and in particular inapplicable for determining the ontological commitment of theories leaving such questions undecided. On the other hand, it is questionable whether a canonical notation should be given a free-pass on ontological decisions which pertain to substantial scientific theories, such as ZFC, thereby essentially violating its topic-neutrality and calling the universal applicability of the criterion, for purposes of comparing any two theories whatsoever in terms of their ontological commitment, into question.

4.4 Ontological Innocence and Logical Deviance

Surveying the literature for proposals on the emendation of Quine's canonical notation one can, roughly, make out four major serious proposals: (a) Modal languages; (b) Branching Quantifiers; (c) Second- or higher-order logic and (d) Plural Quantification. (b) and (c) were rejected above as good proposals to amend canonical notation because they violated some of the important constraints constitutive of it, such as, e.g., completeness, the law of excluded middle and simple, unified

[137] Such a conception is sometimes called a *generalist* conception of logic.
[138] Cf. (Shapiro 1991), (Linnebo 2011) and (Rossberg 2004).
[139] Cf. (Linnebo 2011, 123).

proof procedures, etc. Proposal (a) transgresses against one of the fundamental tenets, not only of canonical notation, but of Quine's entire philosophy, namely *extensionalism*. Thus when considering (a) a serious option for replacing or amending Quinean canonical notation such suggestion should better be viewed as proposing an alternative treatment of ontology rather than improving Quine's, given the fundamental importance of extensionality in and for Quine's work. In this section we will briefly consider proposal (d) as well as some of the reasons that might lead one to argue for the inclusion of (a) – (d) into canonical notation and provide a rationale for why these considerations are not well-founded in the context of the Quinean ontological enterprise.

4.4.1 Natural Language Constructions and Ontological Innocence

The reasons leading proponents of any of the alternatives (a) – (d) to argue their point fall broadly into three, not necessarily distinct, categories:

(1) The expressive power of canonical notation as conceived by Quine is not adequate for the purpose it intends to achieve.

(2) Logic is broader than Quine thinks and includes more than just the standard first-order constants.

(3) Quine's canonical notation does injustice and misrepresents certain natural language expressions and statements.

I believe that (1) is ultimately correct, however, the reasons for it being so have, to my knowledge, never been adequately pointed out. I will provide an account of why the expressive power of canonical notation is not adequate for its intended purpose in Section 5.4 below, here I will stick to the existing accounts in the literature.

The way the argument for any of (b) – (d) usually goes is something like this: consider a certain sentence S of natural language and try to determine its ontological import. To do so one will have to regiment S into canonical notation. Doing so, however, will either render the ontological commitment of S unintuitive in that a maybe perfectly harmless sentence in ordinary English suddenly carries commitment to abstract mathematical objects – the paraphrase severely misrepresents S (3) –, or the regimented sentence, call it S' does not really capture what we intended to say with S anymore, i.e. the canonical form of S' is unable to capture the meaning of S pointing towards the inadequacy of canonical notation (1). A typical response on behalf of the Quinean then consists in pointing out that this might all very well be true, but that there were solid, well-justified and above all, systematic, reasons for devising canonical notation as it was constructed, even if that means that certain natural language expression receive a treatment that appears unnatural. With an eye to the overall conceptual scheme of science, natural language has to be considered a bad guide anyways, albeit the only one we ultimately have. Thus, unless one can point out serious cases in which the meaning of an expression cannot be captured by canonical notation even if one admits to the unintuitive ontological claim of that expression when paraphrased, those reasons do not suffice to bring about a reconsideration of canonical notation.

This then leads to stage (2) of the argument of the proposals: trying to show that the framework for science as devised by Quine should have been much broader based on the criteria he used to devise it than he made it out to be and that a lot of expressions ought to qualify as logical. The issue with this stage of the argument more often than not happens to be that the arguments used to establish the logicality of certain expressions not adequately recognized by Quine tend to use criteria not themselves recognized by Quine or only respecting them selectively (e.g. ontological innocence, but not completeness, etc.), thereby disregarding his system-internal constraints.

Consider, for example, a typical argument for second- or higher-order quantification. It usually starts by pointing out that there are cases, e.g., *nominalizations*, such as 'Punctuality is a virtue' in which the paraphrase according to Quine's notation forces us to recognize objects 'punctualities' and possibly even sets and reify strange objects from an otherwise perfectly harmless and well-understood expression. Surely canonical notation misrepresents what is going on in natural language here (3) – forcing us to equate predication with set-theoretic inclusion if we wish to express that there is something punctuality is, namely, a virtue. Hence, since canonical notation thereby proves to be inadequate for the purpose for which it was designed, namely to enable us to see what we are committed to in our speech (1) we should extend it by means of higher-order quantification. Against the objection that this covers up substantial set-theoretic commitments the argument then tries to show that higher-order quantification is *really* ontologically innocent and should therefore qualify as logical.[140]

Similarly for branching quantification: sentences such as (x) "Some relative of each villager and some relative of each townsmen hate each other" exhibit such complex dependence and independence relations that make it impossible to symbolize it without the assumption of higher-order objects (functions) in FOL.[141] This, however, appears to be breaking with the innocent character of the sentence whose increased ontological commitment seems to rely on structural features of ordinary discourse which canonical notation cannot adequately represent (3). Since canonical notation thus does not deliver a faithful translation respecting the harmless ontology of (x) (1) we should modify it and allow branching devices. This does not violate any of Quine's constraints; after all, branching quantifiers are perfectly first-order, introducing merely more syntactic and semantic freedom into our language, are ontologically innocent and thus do not violate Quine's views on logic.

The general patterns is clear. Proponents of alternatives (b) – (d) generally argue from (3) to (1) and then justify their choice by alluding to (2). I will have more to say about (3) and the connection between natural language, its assumptions and ontology in the next section, for now notice that in both cases outlined above the proponents allusion to (2) fails because they take ontological innocence to be the only thing that matters for inclusion into logic. But we have shown above that much more than mere considerations on ontology went into the devising of logic and canonical notation (completeness, classicality) and that either of the two violate these constraints. Appeal to (2) therefore fails in their cases. Before I elaborate on why I believe that the arguments from (3) to (1) fail, there is, however, one more alternative to consider which has especially in the last few years received a lot of considerations: *plural logic*. Although it is much less obvious than in cases (b) and (c) I will argue in the last section of this chapter that it as well cannot take the road via (2) as it is in fact, and in a very serious albeit hidden way deviant and thus does not meet all the constraints Quine imposed on logic. However, its argument from (3) to (1) makes a much stronger case than the one's given above a can therefore not as readily be dismissed in the next section as for (b) and (c).

[140] Cf. (Linnebo 2011, 123). An argument for the innocence of second-order logic might, for example, claim that it is unreasonable to say that a consequence of any sentence should have a higher commitment than that very sentence, but that 'There is something that grass is', namely green, is a perfectly valid inference from the sentence 'Grass is green'. There are, however, issues with such a line of argument. Or it might try to interpret it in another logic, such as plural logic, thereby showing that it really only is that other logic in disguise and should not be committed to more than the other logic; (Rayo 2002).

[141] The sentence was proposed by Hintikka (Hintikka 1973) as an example that is not first-orderizable. It has subsequently been doubted, however, whether this sentence really displays the alleged structure (Fauconnier 1975). Nevertheless, convincing examples can be obtained by involving other linguistic expressions such as generalized quantifiers (Barwise 1979).

The language of *plural logic* (PFO)[142] contains, in addition to all the usual symbols of FOL, (i) an additional class of variables, plural variables, written $xx_1, xx_2, \ldots$, (ii) the additional logical predicate $<$ and (iii) an additional class of typed predicates, combining with both, singular and plural variables to form well-formed expressions.[143] New well-formed formulas are of the form, $x < yy$ expressing that it (x) is one of them (yy), $P^{\langle n,m\rangle}(x_1, \ldots, x_m, yy_1, \ldots, y_m)$ expressing a collective predicate[144] and $\exists xx\varphi$ expressing that there exists objects (xx), such that *they* are φ.[145] The language of plural logic is thus a first-order language enriched with plural quantifiers and plural variables.

Plural logic, as the name suggests, tries to account and capture plurals and plural inferences from natural language. Thus it is possible to formulate a sentence such as 'there is an object x and some objects yy, such that it (x) is one of them (yy)' by $\exists x \exists yy(x < yy)$ (Rayo 2002, 9). Rayo proposes it as an alternative language of regimentation due to its superior expressive power:

> "PFO$^+$ languages turn out to be tremendously fruitful. They allow us to give a formal semantics for second-order languages and state important set theoretic propositions; they also provide us with natural formalizations for English plural definite descriptions and generalized quantifiers. I believe this makes a solid case for the use of PFO$^+$ languages as languages of regimentation." (Rayo 2002, 29)[146]

We repeatedly mentioned above that mere superior expressive capabilities do not suffice to constitute a solid argument for inclusion into canonical notation – apart from having to obey additional restrictions on what is allowed, there is a minimality constraint involved to not include anything unnecessary for the purpose at hand. Unless it is needed to achieve the intended goal we do not need to include it, i.e. as long as canonical notation does not suffer any shortcomings due to the lack of expressive power, there is no need and urgency to include devices granting more expressive power, no matter how convenient it would be to have them. It is not convenience but proper function and minimality that counts here. One would thus have to supplement Rayo's argument by the caveat that without these additional resources and expressive power the function of canonical notation is severely hindered, that the additional expressive power is needed, for otherwise canonical notation is not adequate for its desired purpose (1).

Now what are reasons to assume that the expressive resources of FOL are not adequate for ontology? Consider the Geach-Kaplan sentence

(GK) Some critics admire only one another.

Now, unless we allow a complex predicate suppressing some of the quantificational structure of GK, it can be proven[147] that no first-order sentence, and thus in particular no first-order paraphrase

[142] We orient ourselves on the account given in (Rayo 2002).

[143] The latter extension is necessary to account for collective predicates, relating objects to pluralities of objects. Rayo calls the language including these in addition to the standard first-order predicates PFO$^+$ (Rayo 2002, pp. 30). Note, however, that this class of predicates suffices to recover standard first-order predicates, i.e. if we introduce this kind of predicates we can reduce all other kinds to it. Standard first-order predicates are then simply those in which the second type superscript is equal to 0 (the predicate does not take plural variables) and plural predicates are those in which the first type superscript is equal to 0.

[144] $P^{\langle 0,1\rangle}(xx)$ could, for example, express that they (xx) are P; cf. (Rayo 2002, 31).

[145] For a complete and systematic account of PFO, see (Rayo 2002, pp. 30). For the purpose at hand this rather rudimentary account suffices.

[146] We will not show how constructions such as second-order quantification or generalized quantifiers can be interpreted by means of plural quantification, but refer the reader to Rayo's paper (Rayo 2002).

[147] Cf. (Boolos 1984, 432). The proof of this fact is very elegant and relies on the fact that the first-order theory or arithmetic is not categorical.

of GK, is equivalent to its very natural and straightforward second-order rendition $\exists X(\exists x Xx \wedge \forall x \forall y[Xx \wedge Axy \rightarrow x \neq y \wedge Xy])$ with 'A' expressing admiration.[148] However, if one then claims that the second-order sentence adequately captures the meaning of GK, and since it is clear that something is lost when formulating it in FOL, we have here a solid argument that FOL is not adequate to capture the true content of GK. On the other hand, it is possible to restore faith in FOL: assuming sets in the domain we will be able to formulate an adequate paraphrase of GK in FOL albeit at the cost of increasing its silent ontological assumptions, viz. by being committed to sets. Again, this might be taken to count against its adequacy as canonical notation given the seemingly harmless nature of GK which does not make use of set-theoretic talk anywhere, but it means the argument from GK is on the same lines as the ones presented above for branching quantification and second-order logic, and will be addressed in the next section.

There are two more arguments besides the common one that first-order paraphrase appears to burden us with unjustified ontological commitments of harmless ontologically innocent expressions Boolos (Boolos 1984) provides for the claim that FOL is inadequate and severely misconstrues important facts. The first appeals to Occams razor: why should we, he asks, assume a set containing cornflakes in addition to the cornflakes in the bowl in front of me. Entities are not to be multiplied beyond necessity, but everything I can say I can do by reference to conflakes (plural). Nowhere does the need arise for me to assume in addition a set containing all these cornflakes (Boolos 1984, 448). This objection puts the cart before the horse: we can only apply Occams razor to issues ontological once we have determined what there actually is. However, in the process of determining what there is, we discover that when talking about cereals in a bowl we are really also committed to the set of those cereals. Thus after having determined what there is we see that we in fact cannot do without the set of cereals, we assumed it all along, albeit implicitly. Saying that one is able to do without certain things before actually applying the criterion telling one what there is, is 'ontological cheating', I cannot know what I assume before I have actually determined it by applying the criterion. Thus, I cannot exclude certain entities on the basis of economy before I have actually resolved what it is that I truly assume. Boolos objection presupposes that we already have a solid grasp on what it is that we assume when we utter ordinary discourse sentences – the criterion shows that this is not the case. Whether this speaks against the adequacy of the criterion or not, its insufficient overlap with natural language intuitions, is a separate question. Certain is that I cannot apply maxims of economy *before* actually having determined the facts themselves and the criterion of ontological commitment is constitutive of these facts given that it was introduced to allow us to determine what there is in the first place.

The final, and in my opinion strongest, argument against the adequacy of the criterion, brought forward by Boolos concerns the following sentence:

> "There are some sets that are such that no one of them is a member of itself and also such that every set that is not a member of itself is one of them. (Alternatively: There are some sets, no one of which is a member of itself, and of which every set that is not a member of itself is one.)" (Boolos 1984, 440)

[148]There is no first-order sentence that is true in the same models as this second order sentence. This can be seen by substituting $(x = 0) \vee (x = y + 1)$ for Axy and observing that the resulting sentence is true in all non-standard models of arithmetic (which first-order logic allows) and false in the standard model (for which second order logic is categorical) (Boolos 1984, pp. 432).

However, while it at least seems possible that above sentence is true,[149] its most straightforward paraphrase into FOL will be, by necessity, false:[150]

There is a set x, such that, for every set y, $y \in x$ just in case y is a set, such that $y \notin y$. (Rayo 2002, 6)

Here then we seem to have stumbled upon a genuine limitation of canonical notation, a case where it cannot provide a paraphrase doing justice to the original locution. Here we have proven it to be inadequate for the purpose of providing a general framework for all science and a good reason to push for its extension. This extension, it is then argued, should be provided by plural logic, given that plural logic will not have any problems with paraphrasing above sentence contradiction-free and faithfully.

4.4.2 The Artificiality of Ontology

I believe that the general direction taken by the arguments sketched above renders them deficient as arguments against a Quinean conception and role of ontology. This is the case, I content, because the way of arguing from (3) to (1) is just not possible on a Quinean picture of ontological discourse. Inferring from (3) to (1) simply is not an option if one takes serious the role Quine reserved for ontology. That is, we cannot infer from what we think the ontological commitments of a piece of natural language discourse are or should be to the fact that if they do not come out as we anticipated, the entire framework that was devised for ontology is inadequate. This is the case because the ontological enterprise is a fundamentally *scientific enterprise*, thereby underlying the same constraints as any other science and progressing in the same way: by means of systematic considerations of overall simplicity, economy, etc. Natural language does not provide us with privileged access to reality.

The issue here is not that proponents of any of the arguments of the last sections should not complain that canonical notation as devised violates the constraint of familiarity with the subject matter, it is that they completely disregard and ignore any kind of systematic considerations. After all, "[f]undamental metaphysical issues are *not* settled by a priori insight or argument, they are settled by the choice of canonical notation, which in turn is governed *by considerations of systematic simplicity of theory as a whole*" (Hylton 2007, 260, my emphasis). Doubting the adequacy/correctness of paraphrase on account of seemingly harmless natural language constructions suddenly carrying great ontological commitments is not sufficient for establishing the inadequacy of Quine's method of ontology, because there simply is nothing to settle what the ontology of such expressions should be like. There is, to repeat what was said at the very end of Section 1.1, simply no 'fenced' and 'implicit' ontology present in natural language: "[o]ntological concern is not a correction of lay thought and practice; it is foreign to the lay culture, though an outgrowth of it" (Quine 1981e, 9). "Putting our house in ontological order is not a matter of making an already implicit ontology explicit by sorting and dusting up ordinary language. [...] It is in deliberately ontological studies that the idea of objective reference gains full force and explicitness. The idea is alien to large parts of our ordinary language" (Quine 1974, pp. 89).[151]

There simply is no ontology present in natural language as such and thus no way, or at least no point in arguing from intuitions about the ontological commitment of certain expressions *as found in ordinary discourse* to the effect that this intuition is not captured by canonical notation. This

[149] It does not give rise to a Russell-type paradox if one denies that that collection of sets forms itself a set.

[150] Rayo shows that even by generalizing the method of providing sets as 'surrogates' for plural expressions, one does not succeed in avoiding the dilemma. Such divergences will keep reoccurring (Rayo 2002, pp. 5).

[151] Requoting from Section 1.1.

was not what it was designed to do. The criterion of ontological commitment and its underlying canonical notation are supposed to give us a scientific handle on the subject of ontology. While they, just as everything else have to take natural language and ordinary discourse as their starting point, it is neither their goal and target, nor does it impose constraints in such a way as to propose as a criterion of adequacy that whatever the criterion of ontological commitment says must conform to our natural language intuitions.

Ontology is, in a sense, completely artificial – every object is a posit from the standpoint of theory, there is nothing 'natural' or intuitive about these objects, their supposed existence is motivated by systematic considerations of theory. The criterion of ontological commitment then is not supposed to tell us what there 'really is out there', what we refer to in ordinary discourse, it is supposed to give us a scientific handle of how to compare theories with respect to their systematic efficacy in regards to ontology. However, since the enterprise of ontology is in this sense artificial, since all the criterion of ontological commitment tries to do is to provide a standard of comparison by means of which to measure inter-theoretic systematicity and to devise a principle by which to consistently settle ontological disputes, the one thing it does not have to do is to do justice to our intuitions about natural language expressions. But this means that the way of arguing from (3) to (1) is not open and that the proponents of any of the alternative positions above, in order to establish (1), will have to demonstrate the inadequacy of canonical notation with respect to its role and function in science, rather their impoverisation of natural language.

Thus, to do justice as a precise, clear, and motivated by pragmatic scientific principles such as simplicity, etc., standard of comparison, the criterion of ontological commitment and canonical notation disregarded many natural language expression. Plurals were paraphrased away for the sake of systematic simplicity and regarded as dispensable; determiners, such as 'many', 'most' and others were not included in its canon of canonical expressions. In order to make up for the loss and expressive capabilities of these expressions theories utilizing them are ascribed seemingly unjust ontological commitment to sets or other higher-order objects. While this might be a break with natural language intuitions it enables a uniform and impartial comparison between theories utilizing distinct expressive resources. There is no absolute telling what there is anyways, independent of a theory, and we should thus give up the hope for an independent standard allowing us to determine what there is. The one who does best in terms of systematic efficacy is the one we should adopt, whether it is contrary to our intuitions or not.

It might be objected that I am particularly uncharitable to the arguments for the inadequacy of canonical notation here. After all, the criterion of ontological commitment derived its justification from the fact that it based its formulation on structural features of ordinary discourse, it was considered to be adequate because it based its judgement of what there is on whether reference to an object was made. The things it recognized as achieving said reference, however, are devices from natural language, and disrespecting them in the sense of positing reference where there was none therefore undermines its justification and proves its inadequacy. While it is true that the entitiy-positing elements of the criterion are abstracted from the way reference is achieved in natural language, this only alludes to a structural feature of the criterion: only ascribe ontological commitment where a theory could be said to have referred to something. This is the constraint of familiarity, otherwise we would not recognize what the criterion was doing if it was not basing its ascriptions of ontology on reference. However, where a theory could be said to have referred to something translates into where we can construe it as having had reference without violating what it intended to say, what the role of what it tried to express was for present purposes. It does not mean where we take it to have referred to what, it means where it would be best for us, based on overall theoretical considerations to construe it as referring without changing the content of what it said in an unacceptable way. The element of reference from natural language is thus preserved, yet there is no obligation to stick with what we take it to be referring to in any straightforward sense. If it can be reconstrued as referring to something previously hidden and unnoticed and this ascriptions provides us with overall theoretical advantages without changing the meaning of what was said – the meaning of which depends for the most part on its interaction with other sentences, the connection with which thus needs to be maintained in paraphrase (a completely structural constraint), – we should do so, yet only in a way that is consistent with the canonical form of reference that we abstracted from natural language. The criterions' predictions are not correct or incorrect based on some prior way of determining what there is that we are really referring to. They are to be accepted as correct on the basis of systematic and theoretical considerations and on the overall theoretical advantages its application brings about. The foregoing, I believe, suffices to block the argument from (3) to (1).

4.4.3 Logical Deviance of the Proposals

We have already established the logical deviance of (a), (b) and (c) based on their non-extensionality, non-classicality and incompleteness (among other things) above. In this section I wish to discuss the logical deviance of plural logic, i.e. why it cannot argue on the basis of (2) that it should be included in canonical notation and why it, ultimately, fails as a vindication of (WEAK.Q).

We described the business of the logician above as consisting in the reformulation of grammar in order to resolve structural ambiguity and economize on constructions to devise a simple and concise framework for all of science (Quine 2008e, 307). In doing so the logician treated relations, such as "taller than", as a single morpheme, rather than a predicate 'tall' combined with the grammatical particles '-er' and 'than' to form "taller than". By doing so he repudiated the logicality of implications based on the asymmetry and transitivity of the 'taller than'-relation, for treating it as a whole means that whatever we can infer from 'something being taller than something else' depends, to a large part, on the meaning we ascribe to it, rather than the grammatical structure of the sentence itself as provided by the potential particles '-er' and 'than' (Quine 2008e, 307).[152] The choice of analyzing an expression like 'taller than' in this way bore the advantage of simplifying grammar in that we did not have to distinguish between a class or relations to which '-er' and 'than' are applicable and a class to which they are not – it enables a unified treatment of predication in our canonical grammar.

For similar reasons of simplicity and convenience it was chosen to consider all predicates (except for identity which presents a special case which we will consider in Section 5.3.1) as extra-logical. This bore the advantage of providing a nice definition of logical truth, truth in virtue of grammatical structure, i.e. truth under all lexical substitutions (see above) and a good heuristic as to which truths genuinely belong to a particular science and theory and which are true in virtue of logic. Those that are in full generality statable in the object-language itself belong to the respective science and those that require *semantic ascent* for their adequate formulation are logical in nature. In order to state logical truths in their full generality one needed to quantify over sentences and therefore advance to a meta-level, since the only kind of generality statable in the object language is in terms of quantification over objects, not, however, over predicates and relations, something necessary to quantify over when talking about lexical substitutions (Quine 1986, 102).[153] However, with the introduction of *logical predicates* both of these crucial characteristics of logic fail. On the one hand, logical truths become statable in the object language (e.g. (*) $\forall x \exists yy(x < y)$) and the definition of logical truth in terms of lexical substitution fails since sentences like (*) do not remain true under all substitutions of predicates for predicates, i.e. lexical items for lexical items. While this is certainly not sufficient to disqualify $<$ as logical (after all, there is no firm boundary demarcating logic that

[152] Cf. (Quine 2008e, 307): "Relative to grammar as thus revised, the asymmetry and transitivity of 'taller than' cease to count as logical implication; for they are not reflected in the new grammatical structure."

[153] Cf. also Section 5.3.

we need to adhere to) it casts doubt on its status and makes us wonder why one should admit it uncritically into logic, which is considered to be a theory of utmost generality. What qualifies $<$ as belonging to such general theory, i.e. what makes it logical? Similar considerations as the ones pertaining to the logicality of $<$ obviously also pertain to identity, $=$, which we do want to qualify as logical. We will return to this in Section 5.3.1 where we will present an argument as to why identity should nevertheless be counted to logic and will reconsider the status of $<$. For now, however, note that no independent argument as to why $<$ should belong to the realm of logic is given and that this, together with the fact that it appears to violate some of the facts about logic grounding its generality leaves it doubtful whether it genuinely belongs into the category of particles rather than the lexicon. Plural logic, on this count, appears to introduce a non-logical and thereby possibly non topic-neutral element into the language of canonical notation.

The by far more significant issue with plural logic concerns the status of its plural variables xx. Quantification, at least on a strict Quinean understanding, is only possible into name position.[154] But the semantic values of xx are supposed to be pluralities, collections of objects that by themselves are not again treated as objects and are thus not in any straightforward sense nameable. Given then that we are not quantifying into name position in plural logic, i.e. not quantifying over objects in the sense that no unique object needs to be assigned as a value to a plural variable and thus no unique determinable reference can be made out, but only somewhat indeterminate reference to object*s*[155] it is not clear whether such quantification should still qualify as genuine quantification in the same sense as our quantification by means of individual variables or rather as something else that compares to our quantification by means of partial analogy, nothing more.

Even if we were able to make sense of this problematic kind of 'quantification' which appears to be very different from the singular quantification we are used to, we find ourselves confronted by an even more serious problem: in the formal framework set out for plural logic by Rayo and Boolos it is possible to make do completely without individual variables. We can reduce individual variables to plural variables by means of the following definition (Rayo 2002, 22): let $1(xx)$ abbreviate $\forall yy(yy \leq xx \rightarrow xx \leq yy)$, where $xx \leq yy$ stands for 'they (xx) are some of them (yy)'. We can then define quantification over individual variables as follows: $\exists x\varphi \equiv_{def} \exists xx(1(xx) \wedge \varphi)$ and similarly for the other locutions (Rayo 2002, 22). Individual variables are superfluous in plural logic.

But then, given the fundamental status we assigned to the individual variable, what plural logic appears to advocate here is not an emendation of the criterion, but an altogether different understanding of it, not based on reference to individual objects and thereby radically different from our understanding of ontology in terms of reference to to objects. The point becomes clearer when one realizes that, given the reduction of individual variables to plural variables, the restriction of the criterion of ontological commitment to individual variables does not make much sense any more. One can then either modify the criterion to talk about the values of plural variables, by saying that an expression is committed to hose objects that fall under the plural variables/that the plural variables range over,[156] or restrict it to certain sentences in every theory (namely those involving $1(xx)$ above) and say that it is the objects that constitute the pluralities quantified over in the respective sentences which make up the ontology of a theory. In the latter case a proper,

[154] Boolos (Boolos 1975) disagrees with this narrow conception of quantification.

[155] Plural quantification still quantifies over objects, of course. The entire point of plural logic was that it quantifies over objects, but by means of quantifying over pluralities of them, (potentially) multiple objects in the same quantification, at the same time. It might be helpful to think about standard first-order quantification as quantifying over objects in an indefinite sense, indifferent between several objects satisfying the respective condition of the quantification, but nevertheless referring to one of them, no matter which one, whereas plural quantification quantifies over objects in an indeterminate way, not determining its reference as reference to any particular object at all.

[156] This is what Rayo advocates; cf. (Rayo 2007; Rayo 2002).

non-circular justification would have to be given for why we should restrict ourselves to a certain class of *sentences* rather than looking for particular expressions when determining the ontological commitment of a theory (what and why is it that makes particular sentences talk about objects and others not), in the former case we are not dealing with a vindication for (WEAK.Q) anymore, but with a proposal for (STRONG.Q). In either case, however, plural logic appears to advance a concept of existence based on different considerations than our original account, thereby breaking with the Quinean grounding of its legitimacy.

4.5 Conclusion

The goal of this chapter was to demonstrate the multifarious considerations that led canonical notation to coincide with FOL. Attention to these considerations led us to see that previous attempts at vindicating (WEAK.Q) either failed or ignored one or another of these constraints and therefore cannot be said to have succeeded in appropriately demonstrating the inadequacy of Quine's own account, rather than proposing an alternative based on different presumptions. I will try to show in the next chapter that Quine's account of ontology is, however, vulnerable to (WEAK.Q).

Chapter 5

Canonical Notation Revisited

This chapter presents my own attempt at a vindication of (WEAK.Q) after the previous chapter has shown that the other attempts of doing so have to be regarded as unfounded at best and unsuccessful at worst. We will do so in a step-by-step approach: Section 5.1.1 will present the formal machinery and background from which my candidate for inclusion into canonical notation arises. Sections 5.2 and 5.3 will argue that this candidate does not violate any of the constraints Quine imposed on canonical notation as sketched in the previous chapter and Section 5.4 will try to demonstrate the necessity of its inclusion based on an interaction of ontological reduction and ontological commitment which threatens to trivialize the entire ontological enterprise from the outset. This will present a sufficient reason to extend canonical notation, more so than the rather tangential considerations of the candidates treated in the previous chapter.

5.1 Generalized Quantification

Ever since their inception[1] *generalized quantifiers* have received ample attention and generated a constant stream of publications.[2] We will, in his section, not attempt to provide a comprehensive overview of the research on quantifiers (this would be impossible in the context of this thesis), but rather try to provide the minimal background needed to understand how the quantifier "there exists uncountable many", which will constitute our example for the vindication of (WEAK.Q), integrates into logic and language.

5.1.1 Introduction

As the name suggests, *generalized quantifiers* are generalizations of the standard first-order quantifiers $\forall$ and $\exists$ brought about by the following line of reasoning. Consider the subsequent series of equivalences (where M is the domain of the model $\mathcal{M}$):

$\mathcal{M} \models \exists x \varphi(x)$

iff there exists a $d \in M$, such that $\mathcal{M} \models \varphi(d)$

iff (there exists a d, such that) $d \in \{a : \mathcal{M} \models \varphi(a)\}$

[1] Cf. (Mostowski 1957) and (Lindström 1966).

[2] They have received ample treatment in linguistics, as well as in abstract model-theory and mathematical logic. For the former, see, e.g., (Peters and Westerstahl 2006), for the latter (Barwise and Feferman 1985). The research on generalized quantifiers, on their logical, linguistic, computational and cognitive properties, is too vast to allow adequate survey here. We will, in this section, present the bare minimum of background for what is needed for the remainder of the thesis.

iff $\{a : \mathcal{M} \models \varphi(a)\} \neq \emptyset$

iff $\{a : \mathcal{M} \models \varphi(a)\} \in \{A : A \subseteq M \text{ and } A \neq \emptyset\}$[3]

These equivalences allow us to view the standard first-order quantifier $\exists$ as a second-order property, a property true of properties of objects, i.e. a set of sets of individuals from the domain of the respective model. The universal quantifier $\forall$ would, on this account, be the property that is only true of the entire domain of discourse, i.e. $\mathcal{M} \models \forall x\varphi(x)$ iff $\{a : \mathcal{M} \models \varphi(a)\} \in \{A : A = M\} = \{M\}$.

There are, however, a whole range of second-order properties other than $\{A : A \subseteq M$ and $A \neq \emptyset\} = \exists$ and $\{M\} = \forall$, i.e. an entire range of quantifiers between the standard first-order quantifiers, giving rise to the following definition of generalized quantifiers:

Definition (Generalized Quantifer):[4] A generalized quantifier Q is

(i) (syntactically) a variable binding-operator, s.t. whenever x is a first-order variable and $\varphi(x)$ a first-order formula with free variable x, then $Qx\varphi(x)$ is a well-formed formula and Q binds all occurrences of x in $\varphi(x)$.

(ii) (semantically) a set $\mathcal{Q}$ of subsets of the domain. Alternatively, a mapping from arbitrary domains to a set $\mathcal{Q}$ of subsets of these domains.

(iii) (satisfaction) the satisfaction clause for Q is defined as follows: $\mathcal{M} \models Qx\varphi(x)$ iff $\{a : \mathcal{M} \models \varphi(a)\} \in \mathcal{Q}$.[5]

Examples of some such quantifiers include

(i) $Q_{inf} = \{A : A \subseteq M \text{ and } A \text{ is infinite}\}$

(ii) $Q_{\geq 7} = \{A : A \subseteq M \text{ and } card(A) \geq 7\}$

(iii) $Q_{logician} = \{A : A \subseteq M \text{ and } A \text{ contains a logician}\}$

(iv) $Q_{odd} = \{A : A \subseteq M \text{ and } card(A) \text{ is odd}\}$

We will write FOL+Q or FOL$\cup\{Q\}$ for an extension of FOL with the quantifier Q. Extending FOL in such a way can add a substantial amount of expressive power. For example, it is well known that there is no expression in FOL which is true precisely whenever the universe is finite.[6] However, by means of Q_{inf} we can define such an expression by $\neg Q_{inf}x(x = x)$. Generalized quantifiers therefore can add expressive capabilities to the language that far outreach the expressive capabilities of FOL, allowing it to characterize various concepts not characterizable in FOL such as, e.g., finitude, evenness, uncountability, etc. It is immediately obvious from this that some generalized quantifiers are not definable within FOL, for if they were they would add nothing to the expressive strength of the language.[7]

[3] A similar line of reasoning can be pursued for the universal quantifier.

[4] This definition is based on the definition given in (Westerstahl 2011).

[5] What we defined here are generalized quantifiers of type $\langle 1 \rangle$. It is without problems possible to define generalized quantifiers of higher and in multiple types, binding multiple variables in multiple formulas; cf. (Peters and Westerstahl 2006, pp. 65) for a general definition. However, for the purpose at hand this simplified definition suffices. We are not aiming at generality, but at simplicity in our exposition.

[6] For proof see Section 5.4.3 below.

[7] There is a lot more to be said about generalized quantifiers and their formal properties. We have here ignored almost all of their nice properties and formal intricacies. However, for the further purpose of this chapter our short and superficial exposition suffices.

5.1.2 Generalized Quantifiers and Logic

Although originally conceived of as mere mathematical abstractions from the ordinary first-order quantifiers $\forall$ and $\exists$, generalized quantifiers (GQ's) soon established themselves in the domain of logic proper.[8] One guiding consideration behind this assimilation of GQ's to logic concerned the lack of a non-circular and informative criterion for determining the logical constants for first-languages,[9] i.e. languages in which quantifiers only range over individual variables and objects from the domain, but not subsets thereof. While there is a guarantee in propositional languages that all logical constants, taken to be the truth-functions of such language, have been exhausted with the introduction of the standard connectives, no criterion analogous to the one of truth-functionality for propositional languages exist for first-order languages, guaranteeing that we have in fact exhausted all logical constants of the language with $\forall$ and $\exists$ and providing a justification of why it is those and no others. The logical constants for any first-order language are usually given by enumeration and without justification of why it is those and no others.

Addressing this lacunae and based on considerations pertaining to the *generality* and *topic-neutrality* of logic, the so-called *Tarski-Sher criterion for logicality* emerged.[10] Logic is commonly taken to occupy a special place in our conceptual scheme. It is fundamental in the sense that it is the most general theory we possess. Based on the *Erlanger Program* in which Klein showed that theories can be characterized by means of the transformations under which they remain invariant (where a transformation is an injective function of the domain into itself, hence a bijection, also called a permutation of the domain) the late Tarski (Tarski 1986) proposed logic to be that theory which is invariant under the largest class of transformations. Euclidian geometry, for example, is the theory invariant under transformations that preserve distance, affine geometry is the theory invariant under transformations that preserve ratios of distances (a wider class of transformations than for geometry) and logic would thus be the theory invariant under all transformations whatsoever.

A similar result is reached via the route from topic-neutrality. Logic is said to be indifferent to the specific nature and identity of any object, its truths should therefore remain unaffected by any arbitrary switching of objects. A switching of objects is, however, just a permutation of the domain. Logical truths, on this account, are then those truths that are invariant under arbitrary switching of the objects in the domain. Based on this it is not hard to devise a natural criterion for the logicality of expressions in a first-order language: "I suggest that [...] we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself" (Tarski 1986, 146).[11] Accounting for certain undesired and accidental special cases[12] this criterion naturally generalizes to the Tarski-Sher criterion which reads

A notion (operator, expression) is logical iff it is invariant under bijective structures.[13]

Taking a structure to be a tuple $\mathcal{M} = \langle M, P, R, \ldots \rangle$ where M is the domain and $P, R, \ldots$ are the

[8] Much of the material of the present section is based on an essay written for the course "Philosophical Logic" under the supervision of Prof. Frank Veltman (UvA). I would like to express my gratitude for his constructive feedback on the essay.

[9] Cf., e.g., (Sher 1991). Cf. also (Sher 2012).

[10] Cf. (Tarski 1986) and (Sher 1991). Note that this thesis is only interested in devising a criterion for first-order languages and no language of any higher order.

[11] Invariance as mark of the logical has had widespread support throughout the disciplines of mathematics, logic, linguistics and philosophy; cf. (Mostowski 1957), (Lindström 1966), (Barwise and Feferman 1985), (Barwise and Cooper 1981), (Benthem 1989), (Keenan 1999), (Peacocke 1976), (Simons 1988).

[12] Cf. (McGee 1996, 575).

[13] The full criterion as can be found in (Sher 1991) and (Sher 2008) is more complex and more involved, due to also accommodating truth-functions under the first-order criterion. We will ignore the subtleties here. The general statement suffices for the purpose at hand.

extensions of the predicates and relations for the language in questions (subsets of M^n), it can easily be seen that above criterion coincides with the usual isomorphism (ISOM) requirement for generalized quantifiers (Westerstahl 2011).

We can then state the precise conditions that a notion has to fulfill in order to qualify as logical formally as follows:[14] Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be models with domains M_1 and M_2 respectively and $\langle a_1, \ldots, a_n \rangle \in M_1^n$, $\langle b_1, \ldots, b_n \rangle \in M_2^n$, $\langle C_1, \ldots, C_n \rangle \in \mathcal{P}(M_1)^n$ and $\langle D_1, \ldots, D_n \rangle \in \mathcal{P}(M_2)^n$:

(i) For a first-order n-ary relation P[15] (a relation true of/holding between individuals) and bijective structures $\langle M_1, \langle a_1, \ldots, a_n \rangle \rangle$ and $\langle M_2, \langle b_1, \ldots, b_n \rangle \rangle$ where, for the bijection f, $f(a_i) = b_i$, P is invariant under bijective structures if $Pa_1 \ldots a_n$ iff $Pb_1 \ldots b_n$.

(ii) For a second-order n-ary relation Q (a (generalized) quantifier) and bijective structures $\langle M_1, \langle C_1, \ldots, C_n \rangle \rangle$ and $\langle M_2, \langle D_1, \ldots, D_n \rangle \rangle$ where, for the bijection f, $f[C_i] = D_i$, Q is invariant under bijective structures if $\langle C_1, \ldots, C_n \rangle \in \mathcal{Q}(M_1)$ iff $\langle D_1, \ldots, D_n \rangle \in \mathcal{Q}(M_2)$.[16]

Clearly Q_{inf} and $Q_{\geq 7}$ qualify as logical on this criterion as do the standard quantifiers $\forall$, $\exists$ and the predicate of identity.[17] For, all other things being defined as above and $= := \{(a, a) : a \in M\}$, we see that if $\langle M_1, \langle a_1, a_2 \rangle \rangle$ and $\langle M_2, \langle b_1, b_2 \rangle \rangle$ are bijective, s.t. $f(a_i) = b_i$ we have that $a_1 = a_2$ iff $b_1 = b_2$, for otherwise f would not be a bijection. Furthermore, for any sets $A \subseteq M_1$ and $B \subseteq M_2$, s.t. $f : M_1 \mapsto M_2$ is a bijection with $f[A] = B$, we have that $A \neq \emptyset$ iff $B \neq \emptyset$ and thus that $A \in \exists(M_1)$ iff $B \in \exists(M_2)$. Similarly, $A = M_1$ iff $B = M_2$ and thus $A \in \forall(M_1)$ iff $B \in \forall(M_2)$, as otherwise f would not be a bijection.[18] Moreover, a bijection will always preserve cardinality and thus any infinite set will always be mapped onto an infinite set and any set whose cardinality is ≥ 7 never onto any set with cardinality lower than 7. On the other hand, a quantifier such as, e.g. $Q_{logician}$, will not come out as logical under the Tarski-Sher criterion, since not every logician needs to be mapped onto a logician.[19]

It is a curious and welcome fact that if one only takes into account monadic generalized quantifiers, it turns out that a monadic GQ is logical iff it only takes into consideration the sizes of sets and nothing more (Westerstahl 2011). Monadic GQ's deal with sizes of sets, but not with the sets or their content themselves. This also immediately shows that any cardinality quantifier Q_α for any natural or transfinite number or quantity description whatsoever will qualify as logical on the proposed criterion.

5.1.3 Generalized Quantifiers and Language

Generalized Quantifiers have found ample application in natural language semantics, following the pioneering work of Barwise and Cooper (Barwise and Cooper 1981). Not only can they express

[14] The following two conditions are simplifications of the conditions found in (Sher 1991) and (Sher 2008) and appeared first in my essay "Invariance and Overgeneration".

[15] We subsume predicates under relations by considering them to be 1-ary relations.

[16] These conditions exhaust the invariance that needs to be taken into account for first-order languages, due to the fact that individuals are (trivially) not invariant under bijections (cf. (Tarski 1986, 150)) and any relation of order higher than 2 does not denote a quantifier over individuals or relation between them any more, but rather quantifies over sets, sets of sets, etc., or denotes a relation between sets, sets of sets, etc. of individuals and therefore falls outside the scope of a first-order language.

[17] The predicate of identity is in fact the only (first-order) predicate other than n-ary extensions of it and predicates denoting the universal- or null-extension that qualifies as logical.

[18] Proofs first presented in "Invariance and Overgeneration".

[19] The Tarski-Sher criterion has been criticized on the basis of *overgenerating* logical constants and allowing too broad a class of operators to count as logical, among them several that appear to be genuinely mathematical. We will ignore these issues here, but refer the reader to (Feferman 1999), (Feferman 2010), (Bonnay 2008) and (McGee 1996).

all kinds of counting quantifiers like the kind we are interested in here, but they can be used to model *noun phrases*, such as 'many professors', 'most logicians', etc., linguistic structures ubiquitous in almost all natural languages. We will not further elaborate on this as it is not immediately relevant to what follows.[20] However, it is worth mentioning that the wide-ranging utility and the elegant treatment of natural language and mathematical objects afforded by GQ's demonstrates their theoretical utility and emphasizes their important structural status in our overall conceptual scheme.

5.1.4 A Modification of Canonical Notation

The change we propose to canonical notation in this chapter consists in the introduction of the quantifier $Q_{>\aleph_0}$ saying "there exist uncountably many" and given by the following clause:

$\mathcal{M} \models Q_{>\aleph_0} x\varphi(x)$ iff $\{a : \mathcal{M} \models \varphi(a)\}$ is uncountable, i.e. $Q_{>\aleph_0} = \{A : A \subseteq M$ and A is uncountable$\}$

Given that $Q_{>\aleph_0}$ is only concerned with the sizes of sets, it can easily be seen and follows readily from what was said above that it qualifies as logical under the invariance criterion (any uncountable set will be mapped bijectively onto another uncountable set). Moreover, it is a genuine strengthening of FOL as can be seen from the fact that it is not definable within it. For suppose it was definable by a first-order formula χ. Then χ would only be true in an uncountable model. However, by the Löwenheim-Skolem property of first-order logic χ should also have a countable model. Contradiction (supposing that χ has models at all). Thus the introduction of $Q_{>\aleph_0}$ constitutes a genuine extension of FOL.

The remainder of this chapter serves to justify this extension and motivate the introduction of $Q_{>\aleph_0}$ into canonical notation. To this end, Sections 5.2 and 5.3 will show that $Q_{>\aleph_0}$ does not violate any of the constraints imposed on canonical notation by Quine, as outlined in the previous chapter. This alone, however, does not yet suffice to justify the introduction of $Q_{>\aleph_0}$, after all, there are countless expressions which satisfy said constraints. Section 5.4 will then argue that the introduction of $Q_{>\aleph_0}$ is necessary if one wants to save ontological discourse from collapse due to the interaction of principles pertaining to ontological reduction and constraints on the applicability of the criterion of ontological commitment. By demonstrating the necessity of $Q_{>\aleph_0}$ for enabling sensible ontological discourse, we show that $Q_{>\aleph_0}$ does not violate the minimality constraint as it constitutes an essential and necessary element of canonical notation.

5.2 Objections from Language

In order to determine the viability of our proposal to introduce the quantifier $Q_{>\aleph_0}$ into canonical notation we first have to check whether it straight-out violates any of the constrains Quine imposed on the language of canonical notation and thereby disqualifies as an emendation of his program and rather constitutes an alternative approach (as, e.g., in the case of modal logic). The strongest constraint imposed is certainly *extensionality*. Given the fundamental importance of this tenet in Quine's philosophy, anything that violates it will immediately seize to constitute a mere correction and therefore cannot serve to support a claim such as (WEAK.Q).

[20]It should be mentioned, however, that in order to really delineate the class of natural language quantifying expressions that correspond to GQ's as formally defined, one ought to impose other constraints on what qualifies as a quantifier and what does not, such as, e.g. (EXT) and (CONS), for whose exact formulation and meaning we refer the reader to (Peters and Westerstahl 2006).

While not every generalized quantifier is extensional[21], cardinality-quantifiers, i.e. quantifiers of the form "there are x many" where x can stand for any natural, infinite or other number one can think of, clearly are. For if the open sentences $\varphi(x)$ and $\psi(x)$ are true of the same objects, i.e. co-extensional, the classes of objects they determine/are true of will have the same cardinality and thus $Qx\varphi(x)$ will be true whenever $Qx\psi(x)$ for any cardinality quantifier Q. In particular, the quantifier $Q_{>\aleph_0}$ gives only rise to extensional contexts for let the sentence $Q_{>\aleph_0}x\varphi(x)$ be true and $\psi(x)$ be co-extensional with $\varphi(x)$. Then the set of $\{x : \varphi(x)\} = \{x : \psi(x)\}$ and thus they will have the same cardinality. Then, if $Q_{>\aleph_0}x\varphi(x)$ is true, that means $\{x : \varphi(x)\}$ is uncountable and thereby so will $\{x : \psi(x)\}$ be and thus $Q_{>\aleph_0}x\psi(x)$ will be true as well. $Q_{>\aleph_0}$ therefore respects the extensionality constraint.

Moreover, $Q_{>\aleph_0}$ is, just as $\forall$ and $\exists$, perfectly first-order in that it only quantifies over individuals of the domain and not subsets thereof. It does not, therefore, change the referential direction taken to derive the basic quantifiers $\exists$ and $\forall$ by means of abstracting from reference to individual objects. Just as those, $Q_{>\aleph_0}$ only talks about objects of the domain and their number.[22] In terms of the definitions provided above it is easier to talk about the sizes of sets rather than the number of individuals, but it is important to remember that $Q_{>\aleph_0}$ quantifies over individuals, not sets – $Q_{>\aleph_0}$ *only binds individual variables.* Quine nowhere explicitly makes any first-order claim that the only legitimate quantification is quantification over individuals, however, from his skepticism about SOL, and the insistence that it only makes sense to quantify into name position (where every name, at least potentially, denotes an *object*) it is clear that he takes (proper) quantification to be intimately bound up with quantification over individuals. The fact that $Q_{>\aleph_0}$ binds individual rather than higher-order variables then places it in the same framework that Quine delineated for the classical first-order quantifiers, rendering it continuous with them. No foreign concept of quantification is introduced by $Q_{>\aleph_0}$.

One might object and say that the inclusion of a generalized quantifier such as $Q_{>\aleph_0}$ is unnecessary, superfluous and dispensable, thereby violating the minimality constraint, telling us to only assume as much as is necessary for the purpose to be achieved and nothing more. After all, all quantifiers other than $\forall$ and $\exists$ were paraphrased and analysed away in the construction of canonical notation due to us not requiring their function to achieve what we intended to achieve; they were, so to say, linguistic clutter. We already showed above (Section 5.1.4) that $Q_{>\aleph_0}$ is more than mere notational ballast (as it is not definable in standard FOL), but it might still maintained that everything it contributes might be had without it. Some such argument might run as follows: sets are indispensable for our scientific enterprise and will thus form part of any serious ontology anyways. However, everything that can be said in terms of $Q_{>\aleph_0}$ we are able to say in terms of sets (and functions) so that we do not require this additional notational device. We will have to say more about the importance of having $Q_{>\aleph_0}$ in our language of canonical notation despite being able to emulate its function via sets below in Section 5.4. Here and in the following section we only aim to show that $Q_{>\aleph_0}$ does not violate any constraints that Quine imposed when designing his canonical notation. We therefore postpone the issue of the necessity of $Q_{>\aleph_0}$ to Section 5.4.

A further objection could allege that the ontological cost brought about by $Q_{>\aleph_0}$ is too high to warrant its introduction. There is one bad and one good line of reasoning here. The bad line

[21] I am indebted to Prof. Frank Veltman for making me aware of this fact. The quantifier 'many' for example, gives rise to intensional contexts, just consider the two sentences 'Many children died in the fire' and 'Many children do not have breakfast regularly'. If both, 'children who died in the fire' and 'children who do not have breakfast regularly' are co-extensional and apply to, say, three children, truth will not be preserved when replacing the latter expression with the former.

[22] We mentioned in a footnote in Section 3.1.1 that the quantitative force of the quantifier does not impact on the referential force of the variable. We therefore see that our approach appears to be in continuation of Quine's.

alleges that because we provide the semantics of $Q_{>\aleph_0}$ in terms of cardinalities of sets, we thereby commit ourselves to sets from the outset. It is bad because the interpretation of $Q_{>\aleph_0}$ in set-theoretic terms does not commit the object-level theory to the existence of the entities assumed in the meta-linguistic interpretation. Our semantic theory might be committed to sets and all kinds of entities, but these commitments do not translate down into commitments of the object-level theory whose meaning we merely chose to understand in the respective semantic terms. $Q_{>\aleph_0}$ is no more committed to sets because it utilizes set-theoretic talk in the specification of the truth-conditions of formulas involving it then standard FOL is committed to sets because it utilizes talk of sequences (set-theoretic entities) in the specification of truth conditions for atomic sentences.

The good line of reasoning charges us with hiding certain existential commitments that we would have had, had we not introduced $Q_{>\aleph_0}$ into our language. In other words, it complains that $Q_{>\aleph_0}$ hides ontological commitments by allowing us to express things we would otherwise have to assume substantial entities for if we were to express it without its help. It thereby masks the commitment it brings about. I, again, refer the reader to a later section (Section 5.4) where I will formulate an answer to this challenge. It is postponed at this place because I believe we are only in a good position to assess $Q_{>\aleph_0}$ effect on ontology once we have realized its necessity to do ontology. For now let me say that while it does not establish that $Q_{>\aleph_0}$ does not hide certain ontological commitments by making us able to express things we would otherwise have to assume entities for, at least it does not completely hide commitment to entities it itself quantifies over, for it can easily be seen from the semantics that whenever $Q_{>\aleph_0}xPx$ is the case, so will $\exists xPx$, thereby making explicit the commitment to, at least, the entities $Q_{>\aleph_0}$ quantifies over.

A final issue I would like to discuss at this point concerns the naturalness, or better unnaturality, of $Q_{>\aleph_0}$ as compared with $\exists$ and $\forall$. The latter were derived from indefinite singular terms of *natural language*. We abstracted away from particular natural language constructions until the quantifiers crystalized as simplification of these natural expressions. $Q_{>\aleph_0}$, on the other hand, appears to express genuine mathematical content and contrasts therefore rather stark with $\exists$ and $\forall$ as it does not naturally occur in ordinary discourse and thus does not constitute an abstraction from any ordinary language construction. $Q_{>\aleph_0}$ is on this score unnatural and should not serve in a canonical notation which is, after all, a simplification and stylization of natural language discourse. Its content is genuinely foreign to such a conception. This charge, however, oversimplifies. Canonical notation was supposed to be an abstraction from our best scientific language, used to inquire about the world, not just every-day discourse. Given that mathematics is part of that language $Q_{>\aleph_0}$ does occur in it.

Moreover, it is at this point that the upward-downward moment in the development of canonical notation interacts with Quine's naturalism and demonstrates that just because $Q_{>\aleph_0}$ was not directly derivable from natural language constructions does not disqualify it from featuring in canonical notation: ontology is continuous with and not prior to other scientific enterprises. It is part of the same program as all other sciences, namely to achieve knowledge of the world. As such it interacts with the other sciences, including mathematics, and influences and is influenced by these sciences. These mutual influences might lead to the adoption of devices, methods and concepts from other fields of knowledge if they prove advantageous for the purpose at hand. Such is the progression of science and knowledge in general. Ontology is a scientific enterprise, not to be considered separate from it, and it grows and interacts with the developments in the other sciences. Moreover, we already noted in the previous chapter that there is a certain downward-movement in the construction of canonical notation in the sense that certain constructions were applied to objects/expressions they were not necessarily abstracted from because it proved helpful. An example of this could be found in the division of sentences into closed and open sentences and the identification of the latter with predicates, allowing us to confer predicational completeness on canonical notation. For the case at hand we might find the distinctions in cardinality we are able to make in canonical notation (of which we will have to say more in the next sections, as they become particularly pressing with the introduction of the predicate '$=$') to nicely generalize into more abstract mathematical distinctions in cardinality (infinite vs finite; countable vs uncountable) which then prove enormously helpful in organizing our ontological discourse. There thus appears to be a very natural way in which expressions with genuine mathematical content, detached from ordinary natural language communication, might make their way into canonical nation, viz. through the interaction of the field of ontology with other sciences (especially mathematics) and the role of downward movements in the construction of canonical notation (canonical notation was not devised by mere abstraction, it was a much more active process integrating various elements and constructions not found in ordinary discourse into its design).

5.3 Objections from Logic

In this section we will briefly consider the stringency of Quine's conditions of logicality to argue that $Q_{>\aleph_0}$ should qualify as logical constant. After considering the status of identity ($=$) we provide another reason as to why plural quantification is a problematic candidate for inclusion into canonical notation and motivate the idea of integrating $Q_{>\aleph_0}$ instead.

5.3.1 The Case of Identity

In the previous chapter the canonical grammar for our notation consisted in the quantifiers $\forall$ and $\exists$, the usual truth-functional connectives and the device of predication allowing us to form atomic sentences of the form $Rx_1 \dots x_n$ for variables $x_1, \dots, x_n$ and relation symbol R. Logical truths emerged from this conception, according to the grammatical criterion, as those truths which are invariant under substitution for lexicon, i.e. those truths that were true in virtue of grammatical structure alone, whose truth-value did not change if we uniformly substituted for predicates and relations. Characteristic of these truths was the requirement of *semantic ascent*; we were forced to talk about sentences, to quantify over sentences, when articulating logical truths; e.g. $p \vee \neg p$ is a logical truth where p ranges over sentences of the object-language.

> "A second salient trait of the logical truths was seen in our tendency, in generalizing over them, to resort to semantic ascent. This again is explained by the invariance of logical truth under lexical substitutions. The only sort of generality that can be managed by quantifying within the object-language, and thus without semantic ascent, is generality that keeps the predicates fixed and generalizes only over the values of the subject variables. If we are to vary the predicates too, as for logical theory we must, the avenue is semantic ascent." (Quine 1986, 102)

Given our inability to generalize over predicates and relations within the object-language and therefore being forced to resort to semantic ascent in order to state logical truths in their full generality "[t]he contrast between generalities that can be expressed thus by a quantification in the object language, on the one hand, and generalities on the other hand that call for semantic ascent, marks a conspicuous and tempting place at which to draw the line between the other sciences and logic" (Quine 1986, 61). Logic could then be characterized as that science the expression of whose truths requires us to talk about sentences of the object-language, whereas the truths of other sciences are expressible within that very object-language.

This nice and convenient characterization of logic is, however, threatened by the innocent seeming predicate of identity, '$=$'. Identity appears as basic and universal as logic, it is fundamental for all sciences, does not discriminate between objects or imports any conceptual or ontological ballast: "Another respect in which identity theory seems more like logic than mathematics is universality: it treats of all objects impartially" (Quine 1986, 62). Identity theory "knows no preference" (Quine 1986, 62) for any kind of object and is therefore distinctly different from other mathematical predicates such as $\in$ or $<$. Moreover, it is particularly basic and fundamental to our conceptual scheme and intimately bound up with all things referential as repeatedly pointed out in previous sections. Identity is the condition of the possibility of successful cross-reference and reference in general.[23] It appears clear, therefore, that identity shares important traits of logic, implying an important affinity to it.

Nevertheless, it does not qualify as logical on the characterization of logical truth provided above: it violates the structural definition of logical truth because truths of identity theory are falsifiable through substitution of other predicates for $=$. Thus, truths of identity theory do not appear to be logical truths. Moreover, it violates the boundary drawn in terms of semantic ascent because the truths of identity theory, pertaining to objects rather than what is said about them, are statable in the object-language, viz. $\forall x(x = x)$, hence not requiring semantic ascent for their proper articulation. In addition, while identity appears to be as general as the remainder of logic, due to the truth of $\exists x(x = x)$ it virtually renders any sensible treatment of the empty domain impossible; a treatment which is not forthcoming, but at least possible without modification is too complicated.[24]

These are strong reasons why identity should be considered external to logic and as rather belonging to mathematics. There are, however, considerations to the contrary which ultimately convince Quine that identity ought to be counted among the logical constants, despite being an item of the lexicon rather than a particle, a part of a construction. This in turn shows that Quine is open to the possibility of extending the realm of logic if the circumstances consistently support such an extension. In the case of identity these supporting circumstances for integrating identity into the realm of logic stem from two considerations.

On the one hand, extending FOL to FOL+$\{=\}$ by means of adding the following two axiom schemes

(REF) $x = x$

(LL) $(x = y) \rightarrow (\varphi(x) \rightarrow \varphi(y))$ for all sentences φ of the language[25]

still affords a complete proof-procedure. We saw above that the obviousness of logic necessitates the completeness of any system claiming to be (a) logic and, as it turns out, FOL+$\{=\}$ is in fact complete and thereby possesses the essential trait of logic. This completeness result therefore moves identity closer, if not into, the realm of logic. Moreover, "an identity predicate is virtually at hand

[23] See, e.g., (Boolos 1984) for the fundamental importance of identity for cross-reference.

[24] Cf. (Quine 1954).

[25] There are many other equivalent axiom schemes, see (Quine 1986). What is important to note is that (REF) and (LL) in particular, and any first-order axiomatization of identity in general, actually fails to define identity. What they define is rather indiscernability with respect to the language under consideration. It might however always be the case that there are more objects than the resources of the language, its predicates and relations, manage to distinguish. Thus, as Geach (Geach 1967) points out, '$=$' as defined above only gives indiscernability, not, however, identity (he calls a predicate satisfying some such axioms as above an *I-predicable*). In fact, it turns out that identity is not an elementary relation, i.e. it is not first-order definable at all (Hodges 2001). Nevertheless, Quine (Quine 1950) points out a way in which one could always collapse identity and indiscernability according to a language by simply collapsing indiscernible objects.

[...] in any language whose grammar is of the kind that we have called standard" (Quine 1986, 63). For in any language with a finite stock of predicates and relations[26] we can define an identity predicate in the language as follows:[27]

$$x = y \text{ iff } (Px \leftrightarrow Py) \wedge \ldots \wedge \forall z(Rxz \leftrightarrow Ryz) \wedge \ldots \wedge \forall z \forall w(Qxzw \leftrightarrow Qyzw) \wedge \ldots$$

and so on exhausting all predicates, relations and possible combinations.[28] Defining identity in this way allows us to admit "identity to logic without giving up our grammatical theory of logical truth" (Quine 2008e, 314), for identity can then be seen as a schematic predicate symbol, receiving a different interpretation depending on the language in question (with the only restriction that it be formulated in terms of canonical grammar and thus only differ in terms of the lexical items). Statements of identity then require recourse to a meta-language, and thus semantic ascent, given their schematic nature (Quine 2008e). They will qualify as logical truths (truths in virtue of grammatical form) and thus "the identity sign does not qualify as a grammatical particle, but its laws still belong to logic" (Quine 2008e, 314).[29]

Given, then, that "'=' will in effect be present, whether as an unanalyzed general term or in complex paraphrase, at least provided that the vocabulary of unanalyzed terms is finite" we see that "identity thus implicitly accompanies any finite vocabulary of general terms" (Quine 1960b, 230/231). This provides a "kind of justification of one's tendency to view '=', more than other general terms, as a "logical" constant" (Quine 1960b, 231).

The fundamental status of identity in our conceptual scheme and its paramount importance for reference, its universal presence in all reasoning as well as the complete logic it affords, together with its definability in finite contexts, enabling a reconciliation of our structural definition of logical truth and holding on to the grammatical criterion of logical constanthood support the conclusion that "identity theory has stronger affinities with its neighbors in logic than with its neighbors in mathematics. It belongs to logic" (Quine 1986, 64).

'is the same as' (=) vs 'is one of' ($<$)

Given that the account of logic as outlined in the previous chapter excluded the logic of identity "since the identity sign is no mere grammatical particle but a predicate in the lexicon" (Quine 2008e, 314), yet we decided, on second consideration, to nevertheless count identity as a logical constant and can thereby

> "recognize logic in a narrow sense as hinging wholly on grammatical structure, and so as excluding the logic of identity, and logic in a somewhat wider sense as hinging on grammatical structure and the identity predicate" (Quine 2008e, 314)

we might wonder whether the predicate '$<$', 'is one of', of plural logic on which basis we dismissed its logicality in the last chapter, should also be part of logic in this wider sense and therefore warrants a reconsideration.

I believe we should still dismiss the '$<$' of plural logic as extra-logical on the basis of the following considerations: on the one hand, no attractive intra-lingual definition of this predicate as in the case of identity, be it only for finite lexica, appears to exist. On the other hand, and more importantly,

[26] Sometimes even for languages with infinite ideology; cf. (Quine 1963b).

[27] Cf. (Quine 1986, 63), (Quine 1960b, 230), (Quine 1951b) and (Quine 2008e, 314).

[28] Identity defined thusly implies the axioms (REF) and (LL) (Quine 1951b).

[29] Cf. also (Quine 1986, 64): If we understand indentity as defined above "all laws of identity become logical truths of the purely quantificational sort."

it is not clear whether '$<$' is as neutral with regard to topic and objects as it should be if it were logical. The semantics of plural logic do not provide enough information about this fact; while it is true that plural logic can be interpreted in the monadic fragment of SOL and this suggests that '$<$' requires the existence of certain objects, possibly sets, for its proper understanding, this fact cannot serve as conclusive proof, since mere interpretability does not impose the ontological commitments of the language in which another language is interpreted onto the latter.

However, one important way in which the topic-neutrality of logic could be characterized was in terms of invariance (see Section 5.2.1 above). Logic was said to be topic-neutral and the logical notions were characterized as those invariant under permutations of the domain (or better, bijective structures). This sort of invariance was thought to capture the underlying principle of topic-neutrality, since if a notion was invariant under arbitrary reshuffling or replacement of the domain, it thereby proved indifferent to the identity of objects. However, it can be proven that the only level 1 relations and predicates, i.e. predicates and relations having objects, individuals from the domain, as relata that are invariant under bijections are the identity-relation (and n-ary extensions of it) as well as relations/predicates determining the empty set and universal set (Tarski 1986). Arguably '$<$' is none of them. But that means either (i) '$<$' is not a logical notion (as it does not appear among the logical notions of level 1), or (ii) it is not a logical notion of level 1, i.e. it could still be a logical notion of a higher level. In case (i) we can conclude that it in fact does care in some sense about the identity of the objects and should therefore be excluded from logic on Quine's terms and in case (ii) the status of '$<$' as logical becomes even more dubious, given that it appears to be a first-order predicate, but, if taken as logical notion, involves mention of at least subsets of the domain and not just objects thereof. This lack of clarity about its meaning and commitment alone excludes it from logic.

Identity and Cardinality

It is a well-know fact that with the introduction of identity into FOL we 'learn to count' for with identity we can ascertain the existence of exactly/at least/at most n entities for any finite n in the domain. For example:

(i) There are exactly n objects: $\exists x_1 \dots \exists x_n[(\bigwedge_{i,j \leq n} x_i \neq x_j) \wedge \forall y(\bigvee_{i \leq n} y = x_i)]$

(ii) There are at least n objects: $\exists x_1 \dots \exists x_n(\bigwedge_{i,j \leq n} x_i \neq x_j)$

(iii) There are at most n objects: $\exists x_1 \dots \exists x_n(\bigwedge_{i,j \leq n}(x_i = x_j \vee x_i \neq x_j) \wedge \forall y(\bigvee_{i \leq n} y = x_i))$

Moreover, while there famously is no expression saying "there are finitely many" and, equivalently, "there are infinitely many" (for otherwise the following construction together with the compactness theorem yields a contradiction), we are very well able to prevent finitude and force infinity in FOL, be it only implicitly. For the following set of sentences will only have infinite models:

$$\Delta = \{\varphi_n : \varphi_n = \text{"there are at least } n \text{ objects" for all } n \in \mathbb{N}\}$$ [30]

So far this amounts to little more than the statement of trivial facts about FOL+identity.[31] The point is, however, that we already have a (limited) way of counting and distinguishing quantities here. If it should turn out that methods of counting and quantifying become more refined in other

[30] If there were an expression ψ saying "there are finitely many" we could consider the theory $\Delta \cup \{\psi\}$. Every finite subset of this theory clearly has a finite model and thus, by compactness, the entire theory has a model. This model would, in virtue of Δ, have to be infinite and, in virtue of ψ finite. Contradiction.

[31] From hereon FOL will always mean FOL$\cup\{=\}$.

areas of knowledge, it stands to reason that some such methods could (and should if demanded by adequacy constraints) be integrated into our framework of science. The distinctions that come to mind are most notable the one's between countable/uncountable and finite/infinite. However, given that the latter is already expressible in a limited way and the former suffices to make the statements about infinite quantities explicit, minimality demands that we keep the adoption of such new expressions to a minimum and only choose the one explicitly required, not because it would be 'nice' to add the expressive strength, but because it is essential and necessary. The cardinality distinctions we are capable of making due to the introduction of identity, together with the nature of scientific development alluded to in the previous section, motivate the idea of extending canonical notation with other cardinality-quantifiers.

The Logicality of $Q_{>\aleph_0}$

It now remains to consider the logical status of $Q_{>\aleph_0}$ to see whether its inclusion into canonical notation might be denied on grounds of logicality considerations. A serious ground on which to deny inclusion is constituted by the fact that the notion of uncountability incorporated into $Q_{>\aleph_0}$ appears to import genuine mathematical content and that it should therefore be counted as belonging to mathematics, rather than as belonging to logic. It is my contention here, however, that there are considerations to the contrary, so much so, in fact, and so similar to the one's concerning identity (where we similarly wondered whether we should count it as belonging to mathematics or to logic) that $Q_{>\aleph_0}$ should qualify as logical if $=$ does.

It is first worth noting that according to Quine's quantitative criterion of logicality, $Q_{>\aleph_0}$ does not qualify as grammatical particle as it is interchangeable with an infinitude of other quantifier-expressions, such as, e.g. "there are finitely many", "there are infinitely many", "there are $\aleph_{95}$ many", etc. Being that it is thus not a grammatical particle and the logical constants are those grammatical particles that occur in our strictly logical grammar, $Q_{>\aleph_0}$ does not qualify as logical. That is, at least in the narrower sense, for we have seen above that although identity qualifies neither on the narrower understanding of logic there are other considerations suggesting its inclusion on a broad understanding. I believe that considerations similar to those that led to the inclusion of identity, also justify the inclusion of $Q_{>\aleph_0}$, despite it not being a grammatical particle.

On the one hand, different from identity and similar to the other quantifiers, $Q_{>\aleph_0}$ requires semantic ascent and disallows the statement of logical truths within the object language (ignoring the problematic cases involving identity). In fact, it is even weaker than the existential quantifier in the sense that $Q_{>\aleph_0}x(x = x)$ will not be a logical truth of the system (whereas $\exists x(x = x)$ will be). It does, in its interaction with identity, however, allow the statement of a different logical truth on the object level, viz. Axiom (ii) below. The only new logical truths added by it (ignoring identity) therefore require semantic ascent. Moreover, given that it is not a predicate but a quantifier it does not violate the definition of logical truth in terms of substitution for lexicon (as this, in our strictly logical grammar, only includes predicates). A formula involving $Q_{>\aleph_0}$ will be a logical truth iff it will remain true when substituting for predicates and the logical truths of FOL+$Q_{>\aleph_0}$ will therefore be those invariant under substitution for lexicon. Nothing changes here.

Moreover, according to the invariance criterion outlined above, $Q_{>\aleph_0}$ qualifies as logical in the sense that it is invariant under arbitrary permutations of the domain/under bijections.[32] We took this to mean that it is indifferent to the identity of the objects it quantifies over, an essential trait of logic which had 'no objects to call its own'. Most importantly, however, it affords a complete

[32]It is not hard to see this; in fact, every cardinality quantifier is invariant under bijections because whatever the size of the set it quantifies over, under a bijection the size of the set it is mapped to remains steady.

proof-procedure.[33] The basicality of ontology to one's conceptual scheme[34] necessitates the kind of obviousness afforded by completeness. A completeness result is therefore inevitable in order for any notion to qualify as logical and such completeness result is available for $Q_{>\aleph_0}$. The logic in the language of FOL+$Q_{>\aleph_0}$ is complete[35] with respect to the following axiom-*schemes*:[36]

(i) any axiom system for FOL and the universal closures of (ii) – (v)

(ii) $\neg Q_{>\aleph_0}x(x = y \vee x = z)$

(iii) $\forall x(\phi \rightarrow \psi) \rightarrow (Q_{>\aleph_0}x\phi \rightarrow Q_{>\aleph_0}x\psi)$

(iv) $Q_{>\aleph_0}x\phi(x) \leftrightarrow Q_{>\aleph_0}y\phi(y)$

(v) $Q_{>\aleph_0}y\exists x\phi \rightarrow (\exists xQ_{>\aleph_0}y\phi \vee Q_{>\aleph_0}x\exists y\phi)$

together with the inference rules of *modus ponens* and *universal generalization* (which is derivable)[37].[38]

We therefore conclude on above basis, despite the mathematical content apparently imported by $Q_{>\aleph_0}$, that, just as in the case for identity, considerations pertaining to the completeness result, the common definition of logical truth, etc., justify the qualification of $Q_{>\aleph_0}$ as a logical notion (at least in the broad sense in which = is) and as thereby belonging to logic rather than mathematics. In the very least, based on its formal properties, it does not obstruct the ontological enterprise in any way that FOL does not already.

5.4 An Inner Tension

There are two distinct considerations playing into the determination of "what there really is out there", i.e. *which entities actually exists*, one of a *qualitative* kind the other *quantitative* in nature.[39] That is, in order to completely resolve what there is we, on the one hand, need to know what *kinds* of entities exist and, on the other hand, *how many* of those there are.

These considerations of quantity and quality with respect to ontological discourse are intertwined in a non-trivial way. The *criterion of ontological commitment*, for one, only offers an answer to the qualitative question telling us which sets of entities are non-empty, i.e. what kinds of entities exist, but remains silent on the question of how many of those there are. Another very important aspect pertaining to ontology, however, concerns *reduction*, i.e. the process of reducing one kind of entity to another kind, explaining the former, so to say, in terms of the latter. Reduction plays a

[33] (Fuhrken 1964) and (Vaught 1964) initiated the search for an axiom system for $Q_{>\aleph_0}$ and proved that the set of formulas of FOL+$Q_{>\aleph_0}$ is recursively enumerable. Keisler (Keisler 1970) provided an explicit set of axioms (the one listed below) and proved the completeness of FOL+$Q_{>\aleph_0}$ w.r.t. it. See also (Kaufmann 1985).

[34] Cf. (Quine 1964a).

[35] It is even compact.

[36] The fact that they are schemata grounds our claims about $Q_{>\aleph_0}$ requiring semantic ascent for the articulation of logical truths.

[37] Cf. (Keisler 1970, 1) and (Kaufmann 1985, 132) for the very involved proof.

[38] Note that the axioms do not force any universe to be uncountable.

[39] The section "An Inner Tension" is based on and a development of ideas that were first expounded in two essays, "Commitment and Reduction" and "Reduction, Pythagoreanism and Emergence", written under the supervision of Prof. Martin Stokhof (UvA, Netherlands) and Prof. Christian Wüthrich (UCSD, USA) in the context of an individual project on "Quine and Davidson" and a course on "Reduction and Emergence in the Sciences", respectively. Both papers have profited greatly from their feedback and I would like to thank them for their suggestions and comments. The responsibility for all remaining mistakes and unclarities lies of course with the author of this thesis.

fundamental role in the sciences and is often considered an extremely, if not the most, important mode of progression. After all, by reducing one theory to another, possibly better, theory we not only replace one by another, but have explained the reduced theory in terms of the reducing thereby showing that the work it did could just as well be done in a better way; reduction presents one way in which science progresses.

While the reduction of one *theory* to another often brings with it the reduction of one *kind* of object to another, considerations of quantity also enter the picture at the level of reduction. For would it not be deemed progress if one was able to show that all that was needed to do the work of one A was the existence of 5 B's rather than, say, 50? We will have to say more about this below. In any case, it is my contention here that the quantitative moment which enters the 'ontological stage' when we are concerned with ontological reduction introduces a tension into Quine's account of ontology and renders the expressive power of canonical notation as conceived by Quine inadequate for the purposes it intends to achieve. Considerations on ontological reduction and its interaction with the *criterion* therefore justify, I claim, a revision and modification of canonical notation. I will be concerned with developing this line of thought here and suggest a reasonable way to avoid the unwelcome consequences by introducing the quantifier $Q_{>\aleph_0}$ into canonical notation, thereby vindicating (WEAK.Q).

5.4.1 Ontological Reduction

Ontological reduction occurs when we show that *widgets are just gadgets*, when it turns out that entities of kind A are *nothing over and above* entities of kind B, that *all there really is to* an entity X are entities Y that constitute it. In such case we say that widgets have been *reduced* to gadgets, A's have been reduced to B's and X has been reduced to Y's. Examples of such reductions, or at least attempted reductions, exists in abundance: mental states (being in pain, seeing a red spot, etc.) have been said to be nothing over and above, to be identical with the physical state a brain is in when having these perceptions[40], tables are just atoms of a certain kind arranged in a certain way, numbers are just sets and so on and so forth.

There are, in general, two ways to go about ontological reduction: on the one hand, close observation of certain items, such as, e.g., a table, might lead one to discover that it consists of certain parts (atoms) and thereby to the insight that the existence of such object need not be taken as simple and brute, but that it can be explained in terms of (the existence of) its constituent parts (atoms) and their arrangement and that we therefore need not assume the existence of tables in addition to the existence of atoms in our best, most fundamental theory of the world. The latter suffice in order to explain the existence of the former. On the other hand, one might realize that everything that can be said about objects of a certain kind, say numbers, can just as well be said in terms of other objects, say sets. Realizing that talk of numbers can be completely and without loss of cognitive content be replaced by talk of sets we replace numbers/widgets by sets/gadgets through talking about the former in terms of the latter – we have reduced one kind of entity to another by referring only to entities of a certain kind (and possibly their arrangement) when talking about entities of another kind. Given, then, that everything that can be said about the reduced entities in terms of the reducing there is no need to assume the former in our ontology in addition to the latter – a maxim of economy dictates that we should just replace them by the latter. Reduction by means of observation can always be subsumed by reduction through 'theory-comparison'; just replace talk of the reduced objects by talk about the observed constituents in the respective theory. In what follows it therefore suffices to focus on the latter kind of reduction.

[40]Doubts about this kind of reduction arise because it appears to be unable to explain the phenomenal quality these mental events possess.

Thus, by reduction of one *theory* to another we might precipitate ontological reduction as well, by replacing talk about entities of kind A with talk about entities of kind B, thereby rendering the assumption of entities of kind A superfluous. We call the reduction of one theory to another *theoretical*- or *inter-theoretical* reduction. The upshot is, of course, that theoretical reduction is more than sufficient for ontological reduction. If we can show that the explanatory work done by entities of kind A can just as well be done by entities of kind B for whose assumption we have independent reasons, we might as well do without the former. The picture sketched in the last few paragraphs can be summarized in the following two points:[41]

(i) In reducing one theory to another the reducing theory plays the same cognitive role as the reduced theory, i.e. accounts for the same phenomena, and does so while preserving explanatory patterns (talk of widgets can be replaced uniformly by talk of gadgets).

(ii) Inter-theoretical reduction induces ontological reduction: by reducing one theory to another we thereby reduce the entities that were talked about in the reduced theory to entities that form the subject matter of the reducing theory.

The immediate concern about the coherency of such an account of reduction – after all, in what way can entities of kind A be said to have been reduced to entities of kind B if, by the very process of reduction, we explain the existence of the former void, i.e. if there was nothing to be reduced in the first place? – can be met by pointing to how ontological considerations actually come to enter into theory-construction (see Section 3.2.1) and what it actually is that we do when talking about the objects of a theory: "[w]hat makes sense is to say not what the objects of a theory are, absolutely speaking, *but how one theory of objects is interpretable or reinterpretable in another*" (Quine 1969d, 50, my emphasis) and, with an eye to ontological relativity (see Section 2.3) "[t]he relativistic thesis to which we have come is this [...]: it makes no sense to say what the objects of a theory are, beyond saying how to interpret or reinterpret that theory into another" (Quine 1969d, 50). Reduction, on this picture, is then simply one kind of these interpretations, namely one in which we do not need to suppose the existence of the reduced objects in order to make sense of a certain part of discourse.[42]

Any account of ontological reduction needs to be such that it disallows the trivialization (in a sense to be specified later) of ontological discourse. The challenge is to develop an account of inter-theoretical reduction that enables such a non-trivializing way of ontological reduction.[43] A first attempt undertaken by Quine to capture a relation of inter-theoretical reduction enabling a well-defined ontological reduction (Quine 1964b, 211/212) relies on the idea that all that is needed for the conceptual content of one theory to be preserved in another is for every sentence of the reduced theory to be mapped to a sentence of the reducing theory, such that the distribution of truth-values is preserved, i.e. that the sentence φ^* of the reducing theory corresponding to a sen-

[41] Cf. (Kroon 1992, 54).

[42] A lot more remains to be said on this point, but space constraints forbid going deeper into this issue. The point here is that in the process of interpreting one theory into another we step 'outside' of either theory and thus need not regard either as true, but only compare them w.r.t. their structure. The one that fares better in the end, is the one we can choose to adopt as true. In (Quine 1969d), Quine compares this process to a proof by contradiction in mathematics; suppose there was a biggest prime, then... Similar for the case here: suppose there were objects of kind A, then ... we could as well do without them.

[43] In what follows we stick to the view of theory as expounded in Section 2.2. Unless indicated otherwise we always refer to the *theory-form* T' of a theory $T = \langle T', \mathcal{M} \rangle$ when talking about theory T.

tence φ of the reduced theory is true iff φ was true.[44] A bit more formal:[45]

Definition (c-reduction): T_2 is *c-reducible* to T_1, $T_1 c T_2$, iff there exists a mapping $\sigma : T_2 \mapsto T_1$ that sends closed sentences of T_2 to closed sentences of T_1 and preserves truth, i.e. for all $\varphi \in T_2$, $\sigma(\varphi) \in T_1$ and $\mathcal{L}_1 \cup \mathcal{L}_2$-model $\mathcal{M}$[46] we have that $\mathcal{M} \downarrow \mathcal{L}_2 \models \varphi$ iff $\mathcal{M} \downarrow \mathcal{L}_1 \models \sigma(\varphi)$.[47]

Unfortunately, mere distribution of truth-values proves inadequate for the purpose of defining a suitable reduction-relation; *c*-reduction allows for a straightforward trivialization. For, for any theory T consider the theory T' that consists of a predicate Tr and Gödel-numerals n_φ for all sentences φ. For every sentence $\varphi \in T$ let $Tr(n_\varphi) \in T'$. Take Tr to be the truth-predicate for T, i.e. define it such that it is satisfied by all and only the Gödel-numbers of (true) sentences in T. Moreover, define the mapping σ by $\sigma(\varphi) = Tr(n_\varphi)$ (Quine 1964b, 212). Then, clearly, $T'cT$ and thereby all the entities of T are reduced to the entities that T' talks about. However, Gödel-numerals can be taken to refer to natural numbers and it therefore appears as if *c*-reduction provides a blanket method of reducing the ontological commitments of any theory whatsoever to (a subset of) the natural numbers $\mathbb{N}$. This surely trivializes all ontological enterprises from the outset because we know that there is a method for reducing all discourse about anything to discourse about numbers. A very unwelcome conclusion: "a doctrine of blanket reducibility of ontologies to natural numbers surely trivializes most further ontological endeavor" (Quine 1964b, 213). As we will see, arguments of this kind will generalize unless certain quantitative considerations are taken seriously. For now let us call the view that "all talk is number talk", that all there is in an ontology are natural numbers, *Pythagoreanism*.

Another very influential account of inter-theoretical reduction is the deductive account by Nagel (*n*-reduction). Nagel[48] attempts to repatriate the relation of reduction between theories to that of logical deducibility. For that purpose he distinguishes between so-called *homogeneous* and *inhomogeneous* reductions. Homogeneous reductions are those in which the vocabulary of the language of the reduced theory is a subset of the vocabulary of the reducing theory, i.e. both theories make use of the same stock of concepts. In the homogenous case a theory T_2 is reducible to a theory T_1 iff T_2 is deducible from T_1, i.e. iff $T_1 \vdash T_2$.

This method obviously only works if the languages of T_1 and T_2 sufficiently overlap. That, however, need not always be the case; in a situation where the vocabulary of T_1 and T_2 differ we are dealing with a case of *inhomogeneous* reduction. Here we cannot make straightforward use of the deduction relation $\vdash$, but must first find a way of relating the concepts of the reduced theory that are foreign to the reducing theory to concepts known to it. Nagel does so by a recourse to *bridge-laws*, bi-conditionals that relate concepts of the reduced theory to concepts of the reducing theory.[49] With the assistance of the connecting bridge-laws we are then in a position to state a

[44] Cf. (Quine 1964b, 211): "What do we require of a reduction of one theory to another? Here is a complaisant answer: any effective mapping of closed sentences on closed sentences will serve if it preserves truth." In the following definition we do not require the mapping to be effective, thereby liberalizing Quine's original definition. Nothing hinges on this fact.

[45] All definitions for the various forms of inter-theoretical reduction, as well as the theorem comparing n- and o-reduction were originally developed in my essay "Reduction, Pythagoreanism and Emergence" and are only minimally modified for their reproduction here. In what follows we always take a theory to be formulated in a first-order language.

[46] Where $\mathcal{L}_i$ is the language of T_i for $i \in \{1, 2\}$.

[47] Where $\mathcal{M} \downarrow \mathcal{L}_i$ is the reduction of $\mathcal{M}$ to language $\mathcal{L}_i$; cf. (Hodges 1993, pp. 2). We use *model* to mean structure plus signature.

[48] We orient our presentation on (Nickles 1973). For Nagel's original account see (Nagel 1961) and (Nagel 2008).

[49] The status of these bridge-laws and the meaning of the bi-conditional in this context is highly controversial. Are they mere instructions for translation? Do they express a real connection between the two concepts? Are they factual, conventional or conceptual? Do they precisify the meaning of the respective concepts? We will ignore these

deductive definition of reducibility for the inhomogenous case: T_1 reduces T_2 iff T_2 is deducible from T_1 together with a set B of bridge-laws, i.e. iff $T_1 \cup B \vdash T_2$. If we let $voc(T)$ denote the set of non-logical expressions used in a theory T we can state the preceding thus:

Definition (n-reduction): T_2 is n-reducible to T_1, $T_1 n T_2$ iff

(i) $T_1 \vdash T_2$ when $voc(T_2) \subseteq voc(T_1)$ (homogeneous reduction) or

(ii) $T_1 \cup B \vdash T_2$ for some suitable set B of bridge-laws when $voc(T_2) \nsubseteq voc(T_1)$ (inhomogeneous reduction)

n-reduction avoids the trivialization as put forward for the case of c-reduction above, because it implicitly appeals to a connection of sub-sentential parts between the sentences of the different theories.[50] In order for T_2 to be *deducible* from (and thereby reducible to) T_1 the structure of the two theories must be sufficiently similar, similar in a way that extends mere preservation of truth values; the sentences need to be related in such a way as that the sentential structure, the structure of the sentences of the reduced theory themselves is mirrored or preserved in the reducing theory,[51] otherwise they will not be deducible from one another.

Preservation of sub-sentential structure is thus implicit in the Nagelian account of reduction. Quine makes similar observations and locates the failure of c-reduction in the coarse-grainedness of its approach to theoretical structure. He suggests an approach that provides a more fine-grained analysis and makes *explicit* the sub-sentential connections of the sentences with each other and inter-theoretically. Moreover, he weakens the (rather restrictive) requirement that the languages of the theories need to be so similar as to be relatable by the relation of deducibility and relaxes it insofar as that his improved account allows for the possibility of heterophonic relations between sub-sentential parts.[52] The new proposal might read as follows (Quine 1964b, 213):[53]

Definition (o-reduction): T_2 is o-reducible to T_1, $T_1 o T_2$, iff there exists a function $\sigma : \mathcal{L}_2 \mapsto \mathcal{L}_1$ that sends predicates/relations of $\mathcal{L}_2$ which occur in T_2 to open sentences formed from the vocabulary of $\mathcal{L}_1$ and occurring in T_1, s.t. substituting a relation R with an open sentence $\varphi = \sigma(R)$ in a sentence ψ preserves the truth-value of ψ. Equivalently, for an $\mathcal{L}_1 \cup \mathcal{L}_2$-model $\mathcal{M}$ and an n-ary relation $R \in \mathcal{L}_2$ and open sentence $\varphi(\bar{x})$ in n-free variables and formed from the vocabulary of $\mathcal{L}_1$, s.t. $\sigma(R) = \varphi$, $\mathcal{M} \downarrow \mathcal{L}_2 \models R\bar{x}$ iff $\mathcal{M} \downarrow \mathcal{L}_1 \models \varphi(\bar{x})$.

o-reduction preserves predicate structure of sentences thereby also maintaining the sentence to sentence links that are established by common vocabulary and enabling a much more nuanced approach to reduction.[54] Given the two proposals of n-reduction and o-reduction it is an interesting question

questions in what follows, but refer the reader to (Nagel 1961, 354).

[50]Nagel has to defend his account against other criticisms, see (Nickles 1973), (Kuhn 1996), (Feyerabend 1962).

[51]The reduced theory could, for example, include a sentence Pa and the reducing theory the universal generalization of this sentence $\forall x P x$. Sometimes the relation between subsentential parts is made obvious through the bridge-laws themselves.

[52]It remains a conjecture of this thesis that Quine's account of relating the languages in question is substantially more lenient by appealing to truth-in-models rather than deducibility. Evidence is provided by the characterization of the connection between n- and o-reducibility below, as well as the fact that n-reduction forces homophonic relations between theories, whereas o-reducibility does not.

[53]Quine does not propose this account as a serious alternative to c-reduction, but rather as a way of necessitating a detour through the Löwenheim-Skolem theorem to reduce every theory to a pythagorean universe. We choose to treat it as a separate attempt at capturing a sensible reduction-relation here.

[54]Cf. (Quine 1964b, 213): "Instead of accepting just any and every mapping of closed sentences on closed sentences so long as it is effective and truth-preserving, we can insist rather that it preserves predicate structure. That is, instead

to ask how the two are related. The result is not surprising. Given that we already pointed out that o-reduction is a liberalization of n-reduction, the following shows that n-reduction imposes much stronger constraints on reduction than o-reduction does.

THEOREM: $\langle T_1, T_2\rangle \in n \Rightarrow \langle T_1, T_2\rangle \in o$

Proof: Suppose $T_1 n T_2$. That means, w.l.o.g., $T_1 \cup B \vdash T_2$ for some suitable set of bridge-laws (we allow $B = \emptyset$ for the case of homogeneous reductions).

Define σ as follows for any n-ary relation $R \in voc(T_2)$:

If $R \in voc(T_1)$, $\sigma(R) = id$. In this case, replacing R with itself in any sentence clearly preserves truth-value.

If $R \notin voc(T_1)$ we know that there exists a bridge-law $\chi \in B$ of the form $R\bar{x} \leftrightarrow \varphi(\bar{x})$. Set $\sigma(R) = \varphi$. Clearly if $\mathcal{M} \models T_1 \cup B$, $\mathcal{M} \models \chi$, i.e. $\mathcal{M} \models R\bar{x} \leftrightarrow \varphi(\bar{x})$ (under some valuation v). But that means, $\mathcal{M} \models R\bar{x}$ iff $\mathcal{M} \models \varphi(\bar{x})$ (under v). The restrictions of the model to the respective languages are easily seen to apply. Hence $T_1 o T_2$ and therefore $n \subseteq o$.[55]

Thus, every pair of theories that is n-reducible is also o-reducible. The notion of o-reducibility determines a class that is at least as subsuming as that of n-reducibility.[56] o-reduction, and thereby n-reduction, despite its more facetted approach, nevertheless still faces its own trivialization problem in the form of the (downward) *Löwenheim-Skolem Theorem* (LS-Theorem). Although well-known it is worth stating the theorem here:

THEOREM (downward Löwenheim-Skolem): Let T be a countable set of first-order sentences which is satisfiable, i.e has a model $\mathcal{M}$. Then T has a countable model and, moreover, the model can be chosen such that it is a substructure of $\mathcal{M}$.[57]

Although there are different versions of the LS-Theorem[58] it in essence guarantees us the existence of a countable model for any (countable) first-order theory T. But that means that every theory $T_1 = \langle T_1', \mathcal{M}\rangle$ can, simply by means of the homophonic translation $\sigma = id$, be o-reduced to a theory $T_2 = \langle T_2', \mathcal{M}'\rangle$ with $\mathcal{M}'$ being a *countable* sub-structure of $\mathcal{M}$.

Now, why would the enumerability of the domain of a theory trivialize the ontological enterprise? In order to understand why ontology is 'pointless' (Quine 1969c, 107 n.1) at the denumerable level and why, in the end, "[i]ndenumerable and indefinite universes are what [...] give point to objectual quantification and ontology" (Quine 1969c, 107) it is helpful to first consider the finite case. Thus assume we are confronted with a theory T with finite universe $\mathcal{U}$. Given that there are only finitely many objects we are able to name them. This, in turn, enables us to paraphrase 'away'

of mapping just whole sentences of θ on sentences, we can require that each of the erstwhile primitive predicates of θ carry over into a predicate or open sentence about the new objects [...]."

[55]Theorem first proved in "Reduction, Pythagoreanism and Emergence".

[56]The reverse direction, however, need not hold, i.e. we cannot conclude that all o-reducible theory's are also n-reducible. The reason for this can be found in the fact that the translation σ is not strong enough to allow us to conclude the derivability of the reduced theory from the reducing theory.

[57]The original statement of the theorem can be found in (Skolem 1967). The proof of this result was subsequently refined; for a more comprehensive statement see, e.g., (Hodges 1993), (Ebbinghaus 1991) or any introduction to the meta-theory of first-order logic. Conceptual and philosophical problems of this result are nothing new, the Löwenheim-Skolem theorem has many seemingly puzzling consequences; see, e.g. (Putnam 1980) and (Bays 2006).

[58]And it is not always clear which version Quine had in mind when talking about the difficulties it presented; cf. (Quine 1964b) and (Quine 1969d).

all quantified statements of the theory into finite disjunctions and conjunctions:[59] every existential quantification $\exists x Px$ can be paraphrased away in favor of a finite (!) disjunction $Pa \vee Pb \vee \ldots \vee Pn$ for names $a, b, \ldots, n$ of (all) the objects in $\mathcal{U}$; every universal quantification $\forall x Qx$ can be replaced by a finite conjunction $Qa \wedge Qb \wedge \ldots \wedge Qn$ where $a, b, \ldots, n$ name all the objects in $\mathcal{U}$; similarly for all other quantificational expressions involving n-ary relations. These conjunctions and disjunctions are clearly equivalent to the quantificational statements and moreover, since they are finite they are well-formed expressions of the object-language.

This renders quantificational expressions nothing but 'inessential abbreviations' (Quine 1969d, 209) for finite disjunctions and conjunctions. However, with the dissolution of quantificational expressions "ontological considerations lose all [their] force" (Quine 1964b, 213) given that "referential quantification is the key idiom of ontology" (Quine 1969d, 212). The referential apparatus, however, was constituted by variables and quantifiers and with their removal any recognizably referential apparatus disappeared as well, and therewith any point to ontology: "[v]ariables [...] disappear, and with them the question of a universe of values of variables" (Quine 1964b, 209).

A qualifying remark is in order here: names were explained away in Section 4.1.2 as inessential notational luxury and thus one could argue that their reintroduction in the context of finite theories (i.e. theories with a finite universe of objects) should have no bearing on ontological questions – we should only assess the ontological commitment of T once it is brought into appropriate quantificational form, i.e. only when all names are 'quantified away'. However, the problem runs deeper. For one names were made out as names by means of quantification. Something a was to only count as a name if it satisfied the existential quantification $\exists x(x = a)$ (Quine 1969c, 94); the hallmark for a name to import ontology is to occur in variable position of an existentially quantified identity. This criterion for namehood is however only possible if quantification expresses 'something more' than names, for otherwise, if they express the same, the criterion becomes circular and useless. In the context at hand, however, quantifiers are nothing but a notational variant of finite disjunctions and conjunctions, they are nothing more than typographical abbreviations and it thus appears reasonable to ask where they are supposed to derive their existential force from. The question is how much ontological force a notational abbreviation should carry and whether, being just a typographical move, it can introduce referential content. One can then either insist on them still carrying ontological commitment, thereby essentially restoring ontological import to names, at least in this particular context (given that they are equivalent in these contexts), and reintroducing all of the problems that went along with this move (cf. (Quine 1964a)) or take the other horn of the dilemma and agree that "[o]ntology [...] is emphatically meaningless for a finite theory of named objects, considered in and of itself" (Quine 1969d, 209).

An argument of this form generalizes: as soon as we have a way of clearly specifying each entity in our universe of discourse the constraints on ontological reduction are not strong enough to avoid ontological trivialization. Thus consider, in full generality, a theory T with a denumerable universe $\mathcal{U}$. Since $\mathcal{U}$ is denumerable we can enumerate its elements. Let $f : \mathbb{N} \mapsto \mathcal{U}$ be that enumeration. There are then two arguments to demonstrate the pythagorean trivialization. (i) Introduce for every $n \in \mathbb{N}$ a name, the numeral $\mathbf{n}$, into the language of T. Having named all objects in $\mathcal{U}$ and given the enumeration f it is not hard to reconstrue the quantifications substitutionally. Given that every object is named we are assured that the same sentences will come out true no matter whether they are substitutionally or objectually construed. But then, why wouldn't one choose the substitutional over the objectual interpretation given that the former is more ontologically parsimonious and thus, on these grounds, appears to be the better theory to choose. The conclusion seems paradoxical: on grounds of ontological parsimony we abandon and 'turn our back on ontological questions' (see

[59] Cf. (Quine 1964b, 213) and (Quine 1969d, 209).

Section 4.3.1 above).

Even if one is not convinced by this latter argument a stronger case can be made: (ii) Given the enumeration f it is not hard to reinterpret the predicates of the original theory T in such a way that for any relation R we have that $Rn_1 \ldots n_m$ iff $Rf(n_1) \ldots f(n_m)$. Applying the homophonic translation $\sigma = id$ to the language in question we have then o-reduced T to a theory with a universe consisting solely of natural numbers. Pythagoreanism prevails.

The quantitative element that enters with issues pertaining to enumerable universes into ontological reduction, together with the Löwenheim-Skolem Theorem and the fact that if every theory has prima facie the same ontological commitment as any other – given that it is only qualitative in nature it will tell us for all Pythagorean universes that the theory in question is committed to natural numbers, – questions of ontology become pointless, for what is there to settle? All theories have the same ontological commitments, there is no point in comparing them on a qualitative level and thus no purpose for a criterion providing a way of actually deciding what a theory is committed to. More precisely, the following argument appears to render the ontological enterprise moot:

(P1) Questions about the ontological commitment of a theory do not make sense if the universe of that theory is denumerable.

(P2) The ontological commitment of a theory can only be determined once that theory is translated into canonical notation.

(P3) Canonical Notation is standard first-order logic (with identity).

(C1) The ontological commitment of a theory can only be determined once that theory is translated into standard first-order logic (with identity).

(P4) Every first-order theory that has infinite models has a denumerable model (downward Löwenheim-Skolem Theorem).

(P5) When evaluating a theory's ontological commitment we look at the smallest possible universe rendering its assumptions true.

(C2) When evaluating the ontological commitment of a theory with infinite models we consider a denumerable universe.

(C3) There is no point in asking questions about the ontological commitment of theories and thereby no sense in talking about ontology.

Above we established (P1), (P2) and (P3) were argued for in previous sections. (C1) thus straightforwardly follows. (P4) is an (unfortunate) fact of FOL and thus cannot be doubted.[60] (P5), on the other hand, is supposed to be a mere constraint on methodological fairness, although it is not as straightforward as it appears.[61] It is a sound methodological principle that one should not assume more than need be and leave out all unnecessary ontological and conceptual ballast. Thus the force of the dreaded "must" in the formulation of the criterion of ontological commitment. 'Smallest possible universe' thus suggests qualitative parsimony, parsimony in terms of kinds. One should not assume more kinds than a theory demands for its truth when evaluating its ontological commitment.

[60] Although it gives rise to curious phenomena, viz. *Skolems-paradox* (Bays 2006).

[61] I would like to thank Prof. Martin Stokhof for making me aware of this point. In my essay "Commitment and Reduction" I argued for this point by taking quantitative parsimony to be a sound methodological principle, thereby overlooking that ontological parsimony is, in Quine's sense, a fundamentally qualitative notion concerning not the number of objects, but the number of kinds. I would like to offer a slightly different argument for (P5) here.

This, however, says nothing about whether one should consider a denumerable or bigger universe when evaluating a theories' ontological commitment – considerations of quantity have no place here. Nevertheless, it is also a reasonable standard to only evaluate the 'best' version of any theory when assessing its ontological commitment. Given that we take ourselves to be making scientific progress when reducing one theory to another, the best possible theory to be evaluated is, on this account, the 'most reduced' theory, i.e. a theory, in our case, with the most quantitative parsimonious universe. Thus one might wish to reformulate (P5) the following way to bring out its features more clearly:

(P5*) When evaluating a theory's ontological commitment we look at the *best possible version of that theory.*

Here the 'best possible version' of a theory is not only the most ontologically parsimonious (qualitative), but also the quantitatively smallest (given that we consider reduction progress and therefore a reducing theory better than a reduced theory). This establishes (P5)/(P5*). (C2) then follows from (C1), (P4) and (P5) and (C3) is established by (P1) and (C2). Ontology has lost all its point.

An important question to ask at this point is whether this actually is so, whether ontology has in fact lost its point or whether it has merely shifted and this result is something one should have expected from the start. After all

> "[w]e must note [...] that this triumph of hyper-Pythagoreanism has to do with the values of the variables of quantification, and not with what we say about them. It has to do with ontology and not with ideology. The things that a theory deems there to be are the values of the theory's variables, and it is these that have been resolving themselves into numbers and kindred objects – ultimately into pure sets. *The ontology of our system of the world reduces thus to the ontology of set theory, but our system of the world does not reduce to set theory*; for our lexicon of predicates and functors still stands stubbornly apart." (Quine 1976b, 503, my emphasis)

Moreover, it is not surprising that the ontological content of any theory whatsoever might turn out to consist in the set of natural numbers, all there is to objects is the structure they contribute to theory, they are nothing but neutral nodes and their identity can only be settled by describing their structural role within the network of sentences of a theory.[62] However, the formal postulates of a theory are completely indifferent to the actual identity of the object that fills any particular structural role and it might therefore be filled by many objects as long as they behave the same way in the relevant respects. This inscrutability and relativity is what makes reduction possible in the first place, but it is false to ask for what a theory actually contributes empirically in terms of objects – just as everything else, ontology is a purely structural undertaking[63] and talk of 'real empirical content of theories' therefore looses its basis and renders above result harmless. While our ontology dissolves into a universe of natural numbers, the ideology of that theory, its stock of predicates, does not dissolve into arithmetical predicates and relations, but 'stands stubbornly apart'. The 'empirical significance' of a theory is thus preserved. While our talk might be about numbers, it is not number-talk and therefore the theory's empirical content is upheld.

If this was all there was to the issue we could rest at this point. However, there are situations in which we want to deny the possibility that natural numbers could possibly be all there is flat-out, viz. in cases where we assume there to be *more* objects than there are natural numbers. Here it

[62]Cf. (Quine 1969d, 45): "arithmetic is all there is to number."

[63]See Section 3.1 for the role of a criterion of ontological commitment in such a structural undertaking.

appears prima facie impossible to settle for a universe consisting solely of members of $\mathbb{N}$, regardless of the assurances of the LS-Theorem. In such a case the possibility of a reduction of a theory assuming the existence of uncountably many objects to a theory containing only countably many should be ruled out by any adequate account of reduction, regardless of the structural roles to be filled. We are not interested in qualitative constraints, but quantitative.

5.4.2 Avoiding Ontological Trivialization

I believe it is precisely this last point that Quine had in mind when he proposed *proxy-functions* to save his account of ontological reduction and block blanket reductions through the LS-Thm. Just as ontology is "internally indifferent [...] to any theory that is complete and decidable" (Quine 1969d, 209) on account of us being able to settle truth-values mechanically and therefore no need for us to posit objects and refer to them by means of variables and quantifiers in order to predict linguistic behavior, so "a finite and listed ontology is no ontology" (Quine 1981e, 7) and ontology at the denumerable level is 'pointless' (Quine 1969c). In order to avoid a total collapse of ontological enquiry what needs to be averted is the breakdown of any theory whatsoever into a theory with denumerable universe, in short, the inference from (C1), (P4) and (P5)/(P5*) to (C2) must be blocked. However, (C1) and (P4) appear immune to criticism if one wishes to leave the remainder of Quine's system intact.

Quine of course anticipated problems with the LS-Theorem from the outset[64] and attempted to design his final account of ontological reduction accordingly. With the introduction of *proxy-functions*, functions relating the objects of the reduced and reducing domain in a certain way, he effectively tries to block (P5)/(P5*) by showing that the best possible version of a theory need not possess a denumerable universe (despite the progress promised by quantitative reduction), i.e. that the smallest possible universe of a theory need not be countable if the original universe was not.

He does so by imposing the constraint of a *proxy-function* into the definition of reduction. Such function relates the elements of the 'old' and 'new' universe in such a way as to allow one to see which object (token, rather than type) was reduced to which other precise object. The proxy-function is supposed to derive its rationale from the fact that what happens when one theory becomes reduced to another is not 'mere modelling', it does not suffice to simply find a new model of (some translation) of the old theory. The reifications of previous theories are supposed to serve as epistemic 'stepping stones' (Quine 1992b, 33) and should therefore not simply be disregarded in the process of reduction. In other words, ontological reduction should not only take into account the relation between the language and concepts of the old and new theory, but also the relation between the (supposed) old and new entities. Put slightly differently: ontological reduction should have something to do with the original ontology and not merely with the formal framework of the theory.[65]

While this reasoning succeeds in providing *some* intuition for the introduction of proxy-functions it fails to justify why one should be explicitly concerned with the relation between tokens, i.e. singular object-object relations, rather than the type-type, i.e. kind to kind relations already implicit in *o*-reduction. It appears to make sense in some cases, but not in others: in the case of numbers, for example, it appears reasonable to know which precise number is related to which precise set. In the case of, say, tables or mental events, however, it does not really matter which precise mental event is related to which precise physical brain state or which precise table is related to which precise atoms – in fact, it appears difficult to implement some such relation given that every simple object of the old domain is related to at most one other object in the new domain (given that it

[64] Cf. (Quine 1969d) and (Quine 1964b).
[65] Cf. (Decock 2002b, 169).

is a proxy-*function*).[66] In any case, I agree with several interpreters of the relevant passages in Quine that the introduction of a proxy-function appears to be a very *ad hoc* maneuver to block the unwelcome consequences of the LS-Theorem.

Be all this as it may, the final account of reduction that Quine provides, integrating the notion of a *proxy-function*[67], "a function whose values exhaust the old things [...] as their arguments range over the new things" (Quine 1964b, 214), reads as follows:[68]

> The standard of reduction of a theory θ to a theory θ' can now be put as follows. We specify a function, not necessarily in the notation of θ or θ', whose values exhaust the universe of θ for arguments in the universe of θ'. This is the proxy function. Then to each primitive n-place predicate of θ, for each n, we effectively associate an open sentence of θ' in n free variables, in such a way that the predicate is fulfilled by an n-tuple of values of the proxy function always and only when the open sentence is fulfilled by the corresponding n-tuple of arguments. (Quine 1964b, 215)

More formally we define q-reduction as follows:

Definition (q-reduction): A theory T_2 with domain D_2 is q-reducible to theory T_1 with domain D_1, $T_1 q T_2$, iff there exists a pair of mappings $\langle \sigma, \rho \rangle$ with $\sigma : \mathcal{L}_2 \mapsto \mathcal{L}_1$ and $\rho : D_2 \mapsto D_1$, s.t. σ sends every n-ary relation R of $\mathcal{L}_2$ that occurs in T_2 to an open sentence $\varphi(\bar{x}) \in \mathcal{L}_1$ in n free variables, s.t. for all $d_1, \ldots, d_n \in D_2$ and any $\mathcal{L}_1 \cup \mathcal{L}_2$-model $\mathcal{M}$ we have that $\mathcal{M} \downarrow \mathcal{L}_2 \models Rd_1, \ldots, d_n$ iff $\mathcal{M} \downarrow \mathcal{L}_1 \models \sigma(R)\rho(d_1), \ldots, \rho(d_n)$.[69]

q-reduction can, without problems, be extended to guarantee the preservation of truth-values of closed sentences[70] by means of the following recursive extension of the demands on σ:[71]

(i) $\rho(\neg\varphi) = \neg\rho(\varphi)$

(ii) $\rho(\varphi * \psi) = \rho(\varphi) * \rho(\psi)$ for $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$

(iii) $\rho(Qx\varphi) = Qx\rho(\varphi)$ for $Q \in \{\exists, \forall\}$

(iv) for all $d_1, \ldots, d_n \in D_2$ and any $\mathcal{L}_1 \cup \mathcal{L}_2$-model $\mathcal{M}$: $\mathcal{M} \downarrow \mathcal{L}_2 \models \varphi(d_1, \ldots, d_n)$ iff $\mathcal{M} \downarrow \mathcal{L}_1 \models \sigma(\varphi)\rho(d_1), \ldots, \rho(d_n)$.[72]

[66] Those latter cases fall, more likely, under the heading of *ontological destruction*. We will have to say more about this later.

[67] Proxy-functions also later played a significant role in arguments for the inscrutability of reference by enabling arbitrary permutations of the domain, cf., e.g., (Quine 1995a).

[68] It is worth noting that Quine does not require the proxy-function to be definable in the object-language of either theory (Quine 1964b).

[69] This definition makes Quine's account more general in only demanding the existence of such function, not that it needs to be specified.

[70] It is not completely clear whether Quine demands this be the case for any successful reduction, but he appears to suggest it at times: "Reduction of a theory θ to natural number [sic!] – true reduction by our new standard, and not mere modeling – means determining a proxy function that actually enumerates the objects of θ and maps the predicates of θ into open sentences of the numerical model. Where this can be done, *with preservation of truth values of closed sentences*, we may well speak of reduction to natural number." (Quine 1964b, 216, my emphasis)

[71] This account is due to (Iwan 2000, 203).

[72] A further technical demand imposed on the specification of function σ might require the integration of a relativization formula to restrict translated formulas to relevant parts of the domain; cf. (Iwan 2000, pp. 203). Tharp (Tharp 1975, 155) argues to the same effect and points out that the condition of integrating a relativization formula is equivalent to requiring the proxy-function to be surjective. We will ignore these further difficulties here.

What is important to note at this point is that as long as we have a function that is injective, i.e. one-to-one, we also always have a proxy-function allowing ontological reduction. For, in such case and given an injective function g, we define for every predicate/relation R of the old theory, true of objects $x_1, \ldots, x_n$ a new predicate/relation R' true of exactly the correlates $g(x_1), \ldots, g(x_n)$ of the original objects.[73] Thus as soon as we have an injective function from one domain into another, we can, albeit artificially, construe an ontological reduction by taking the function to be the proxy-function called for in the definition of q-reduction.

However, when being confronted with a denumerable universe, the enumeration whose existence we are guaranteed provides such a one-to-one function into the domain of the natural numbers and therefore automatically enables an ontological reduction to a Pythagorean universe.[74] Denumerable universes, therefore, remain as pointless for ontological enquiries as before.

Given, then, that whenever we have a one-to-one function from one domain into another we are able to reduce the two, a good place to start looking for failures of reduction to the Pythagorean universe, $\mathbb{N}$, seem to be domains with cardinality greater than $\mathbb{N}$, for there exists no injective function from sets with cardinality greater than $\mathbb{N}$ into $\mathbb{N}$. This is precisely the point at which the proxy-function requirement blocks a general reduction of any theory to a denumerable universe by means of the LS-Theorem, viz. by *disallowing reductions in cardinality*.[75] For consider theories T_1 and T_2 with domains D_1 and D_2 respectively. We want to show that $T_1 q T_2$. Then, however, for any two distinct elements, i.e. $x \neq y$ for $x, y \in D_2$, we cannot have $\rho(x) = \rho(y)$, for otherwise $\mathcal{M} \models x \neq y$, but $\mathcal{M} \not\models \rho(x) \neq \rho(y)$.[76] This means that objects distinguishable in the original theory need to remain distinguishable in the reducing theory and, since for a reduction in cardinality multiple objects of the old theory need to be identified with one and the same object of the reducing theory, a decrease in cardinality of the domain is thereby ruled out if the original objects are all distinguishable. If the cardinality of the old domain was greater than $\aleph_0$, so will the cardinality of the new domain be and a prima facie reduction to a denumerable domain by means of the LS-Theorem is thereby ruled out. (P5)/(P5*) is blocked and the fatal argument averted.

Adequacy of the Solution I

Quine's solution to the threat of Pythagoreanism through the LS-Theorem has never received as much attention as other parts of his philosophical system, despite the importance proxy-functions later came to play in his attempt to establish the inscrutability of reference.[77] There have, nevertheless, been occasional treatments of the issue.[78] In this section I will briefly present and refute two arguments that have been advanced to the effect that (i) the proxy-function requirement was not necessary to solve the problem caused by LS-reductions and (ii) that it is not sufficient to

[73] Cf. (Quine 1969d, 57).

[74] There might still be the objection that just knowing of the existence of such function is not sufficient for ontological reduction, we need to be able to specify it. This would shift the meaningfulness of ontological discourse in denumerable cases onto the contingent fact of whether something is known or not and therefore appears to be a weak defense.

[75] At least for theories with identity, but this presents no real restriction since canonical notation encompasses $=$ and it could otherwise always be approximated by means of an indistinguishability-predicate definable within any language, see above. Cf. also (Tharp 1971, 155) and (Jubien 1969, 538).

[76] More formally $\mathcal{M} \downarrow \mathcal{L}_2 \models x \neq y$, but $\mathcal{M} \downarrow \mathcal{L}_1 \not\models \neg(\rho(x)\sigma(=)\rho(y))$. Note, however, that identity will always be mapped onto identity by σ, for either (i) it is a logical constant and thus not part of the vocabulary of either theory to be mapped, or (ii) any relation playing the same role as identity and thereby satisfying the identity axioms (REF) $x = x$ and (LL) $x = y \rightarrow (\phi(x) \rightarrow \phi(y))$ is co-extensive with identity; cf. (Quine 1986, 62).

[77] The majority of the content of this section is based on the essay "Reduction, Pythagoreanism and Emergence".

[78] There are some articles and chapters dealing explicitly with his account of reduction, most of them negative in nature and pointing out various shortcomings. Cf. (Bonevac 1982), (Chihara 1973), (Tharp 1971), (Grandy 1978), (Grandy 1969), (Iwan 2000), (Kroon 1992), (Gottlieb 1976).

do so, before, in the next section, elaborating why I believe that the proxy-function requirement overshoots its goal and should be abandoned.

Argument (i), that the proxy-function requirement was not necessary to solve the Pythagorean trivialization relies on a particular understanding of Pythagoreanism:[79] call *weak Pythagoreanism* the claim that every theory can be reduced to a theory about numbers only and *strong Pythagoreanism* the claim that every theory can be reduced to the theory of arithmetic.[80] The difference between the two claims is the following; whereas weak Pythagoreanism allows for the existence of predicates and relations that are not representable/definable in terms of purely arithmetical predicates and relations, strong Pythagoreanism does not.

The argument can then be given as follows: Quine's account of *o*-reduction cannot seriously feel threatened by weak Pythagoreanism, as this would imply that he, on the one hand, considers ontology at the denumerable level pointless from the outset and, more importantly, that the only thing that prevents ontological discourse about the natural world to collapse into talk about natural numbers is the fact that any such theory would also have to talk about mathematical entities, such as real numbers or sets, given that the number of non-mathematical objects is most likely not even infinite. Thus what the proxy-function requirement was really introduced for was to block strong Pythagoreanism. However, the argument might continue, there is in general no reduction of any theory to PA, especially not by the LS-Theorem.[81] Then all that is needed to save *o*-reduction from the threat of Pythagoreanism is an example of a case of reduction in which the reduced theory cannot be interpreted in purely arithmetical terms. And, in fact, examples of such theories abound (Iwan 2000, 213), thereby saving the concept of *o*-reduction from trivialization by strong Pythagoreanism and thus forestalling any threat of the LS-Theorem without even mentioning proxy-functions. They appear to have been unnecessary in the first place and Quine's reason for introducing them was mis-motivated.

One can conceive of a way of strengthening this argument by restricting oneself to finitely axiomatizable theories[82] and appealing to *Bernays-Lemma* which yields (very roughly) a reduction of any axiomatizable theory to PA plus a consistency postulate (Iwan 2000), meaning that the predicates and relations of such finitely axiomatizable theory can be defined in terms of purely arithmetical predicates and relations, thereby reinstating the worry about strong Pythagoreanism.[83]

Both of the arguments nevertheless crucially rely on the claim that what Quine is truly worried about is strong Pythagoreanism. The reasons for this claim are weak and can easily be refuted, for Quine explicitly states that ontology at the denumerable level is pointless and that it is indenumerable universes that give ontology its significance (Quine 1969c, 107), thus suggesting that what he is worried about is, in fact, weak Pythagoreanism. Moreover, at least in the context of reduction, ontology is to be seen as a mostly structural enterprise trying to fill the neutral nodes of each theory as fits best. In finite and countable contexts, however, there are easy methods available to settle

[79]The argument as given here appears in different forms in (Grandy 1978) and (Iwan 2000). All mistakes made in the reconstruction, which does not faithfully reproduce either of their precise claims, lie with the author of the present thesis.

[80]Suppose that this theory consists of the axioms of standard Peano-arithemtic (PA).

[81]Cf. (Quine 1964b, 212): "There is, under the Löwenheim-Skolem theorem, a reinterpretation that carries all these truths into truths about natural numbers; but there may be no such interpretation in arithmetical terms."

[82]A restriction that might be justified on account of us being finite beings and the concept of reduction discussed being such that it can serve in actual scientific practice (at least in principle) and does not merely serve as an abstract thought-experiment.

[83]Grandy (Grandy 1978, pp. 76) points out that in order to reinterpret the predicates and relations of the old theory, we still need to make essential reference to that theory, thereby only enabling a reduction by presupposing what we intended to reduce: "The Bernays lemma gives us a way of mapping discourse about an arbitrary domain into discourse about numbers, but what is said about the numbers is still dependent on reference to the original theory." He takes this to be a sufficient refutation of the challenge.

questions of ontology and it is only within infinite contexts that ontology as a methodological enterprise providing a means to choose between theories becomes important. Contexts of an infinite kind are, however, essentially mathematical and it therefore should come as no surprise that ontological questions only make sense once mathematical entities are taken into account as well. Furthermore, mathematics is so fundamental to our explanations of the world and so central to our conceptual scheme that the need to require the presence of mathematics to make sense of ontological questions even in the most basic enquiries into nature is no serious drawback, as such enquiries without the apparatus of mathematics would hardly be conceivable.

There is also a more positive argument for Quine taking the challenge posed by weak Pythagoreanism very serious: it relies on the fact that ontological reduction is generally conceived to be an important, non-trivial achievement. If all that is talked about are numbers, however, no matter how they are talked about, i.e. in what terms we talk about them, this fact makes ontological reduction inherently uninteresting. The only thing left to do in such a situation would be to exclude certain subsets of natural numbers from our universe, but genuine question of ontological reduction have lost all their appeal and we might consider all ontological issues forever solved. This, however, seems to be an absurd conclusion.

Turning now to the insufficiency objection, it was noted early on by Grandy (Grandy 1969) that Quine's model of reduction disallows certain cases of theory-relations that should be classified as reductions, namely cases in which certain objects turn out to be superfluous and can therefore simply be dropped from the reducing domain. Grandy labelled such a reduction which "consists simply in dropping objects whose absence will not falsify any truths expressible in the notation" (Quine 1969d, 68) *ontological destruction* and Quine later, in an addendum to (Quine 1969d), acknowledged the status of such reductions. Such cases are not captured by q-reduction because every object of the old theory that is distinguished within that theory has to be mapped to some object of the new theory, but sometimes it is impossible to just arbitrarily map superfluous objects somewhere without off-setting the truth-value distribution of the respective theory.[84] Thus there is no way to simply forget the unwanted elements, but there is also no way to map them arbitrarily to some element.

In order to account for such cases Quine added the condition that whenever the universe of a theory is reduced to a sub-universe of that theory we can forget a class of objects and still qualify the process as reduction, as long as the class of the objects dropped is specifiable, i.e. definable. This last requirement blocks a version of the LS-Theorem which enables the reduction of any universe to a countable sub-universe. Given, then, that there is no guarantee that the class of objects be definable the trivialization threatened by the modified version of the LS-theorem is blocked. Grandy then develops a method allowing him to always reduce a universe to a countable sub-universe through above method, by applying a series of inflations and deflations of the universe, thereby intending to show that even if Quine accounts for these extra kinds of reductions his extended criterion q is insufficient to block trivialization. Tharp (Tharp 1971) advances a similar argument intending to show that one is always able to specify the class of objects dropped in order to reduce a theory to a countable universe, so long as the original model is definable.

Neither argument attacking the sufficiency of condition q (plus extension) succeeds, however. Contra Tharp there is no reason to suppose that the original model is definable. Against Grandy's attempt speaks the crucial use he makes of inflations of universes, for there is no guarantee or criterion to decide when an inflation counts as a reduction (it moreover appears to be a contradiction in terms). To make it more explicit Quine might add the condition to his definition of q-reduction

[84] Such cases occur, for example, when one tries do reduce a non-standard model of arithmetic to the standard model; cf. (Grandy 1969, 810).

that the relation q is non-transitive, thereby ruling out multi-step reductions such as the ones proposed by Grandy (but possibly inviting other objections, for transitivity does not appear to be an undesirable property for a reduction relation) or, and more naturally, he could require that reduction between one and the same theory-forms has to always proceed from the universe with the higher cardinality to a universe with equal or lower cardinality. Since Grandy's process does not guarantee that this is always the case his objection is blocked.

Adequacy of the Solution II

It is my contention that the proxy-function requirement in Quine's final account of ontological reduction overshoots its goal and does in fact exclude a significant class of cases which, almost trivially, should fall under any adequate concept of reduction. Whether the reason for this is that q-reduction was a mere *ad hoc* solution to the problems presented by the Löwenheim-Skolem Theorem (as it appears at times) or not, does not change the fact that it fails in preventing the quantitative worries presented by the threat of Pythagorean reduction if it is to be taken seriously as an account of ontological reduction.

The difficulty is the following: in order to enable meaningful ontological discourse under the permanent threat of a Pythagorean collapse, the Quinean was forced to delineate conditions under which it made sense to talk about ontology. She did so by requiring a proxy-function thereby (allegedly) disabling reductions in cardinality and marking the class of those universes with cardinality greater than $\aleph_0$ as the class of cases in which application of the criterion of ontological commitment made sense, as they disallowed reduction to an enumerable universe. Such reduction was prohibited because in case the elements of these universes were intra-theoretically distinguishable the proxy-function had to be one-to-one. If one were to erode this delineation, i.e. if one were able to show that there are (or should be) genuine many-to-one reductions despite intra-theoretical distinguishability of the elements of the respective universes, it would, once again, become questionable whether, in any particular situation, ontological discourse made sense or was mere musing about natural numbers. Moreover, the burden of proof is again shifted onto the Quinean to provide a clear demarcation of those cases in which ontological discourse makes sense, and of those in which it is sophisticated talk about numbers.

Thus, in the absence of a clear boundary where and when we can sensibly talk about ontology and thus apply the criterion of ontological commitment we might just as well remain silent about ontological issues, given that we never know whether the reasoning we apply in such cases holds up, as it always might be discovered later that all we were talking about were natural numbers.

Quine already conceded one exception to his criterion, which we called *ontological destruction* above. There are two more cases he mentions, where many-to-one functions yield acceptable reductions. One such situation is the case where two kinds of objects are "so invariably and extravagantly unlike that the identity question [does] not arise" (Quine 1969d, 56) and it is possible to reduce these unlike kinds of objects to a single kind. For example, if we are in a situation where it applies we should 'cheerfully' reduce both, expressions and ratios to natural numbers. Another example of this kind might include the reduction of physical objects to space-time coordinates, the identity question between physical objects and sets of numbers being unlikely to arise. In short, such many-to-one reductions are possible because 'no capital' was made of the distinction of these kinds of objects within the theory, although they were distinguishable (Quine 1969d).

A second kind of acceptable many-to-one reduction concerns the already hinted at case in which the theory itself does not distinguish between some of its objects. Here it is possible to form equivalence classes of intra-theoretically indistinguishable objects and map all members of one equivalence class to a single representant of that class, or to the class itself without loss of information. "The reason such a reduction is acceptable is that it merges the images of only such individuals as never had been distinguished by the predicates of the original theory. Nothing in the old theory is contravened by the new identities" (Quine 1969d, 56). An example for this kind of legitimate many-to-one reduction can be found in the case of an economic theory whose predicates only establish income-classes, rather than individual earners. A model of this theory could include persons, but would inevitably identify persons with the same income: "[t]he interpersonal relation of equality of income enjoys, within the theory, the substitutivity property of the identity relation itself; the two relations are indistinguishable" (Quine 1969d, 55).[85] Nothing is lost by moving from persons to income-classes because, from the standpoint of the theory in question, this was all that was talked about before anyways.

Except for the somewhat special case of *ontological destruction* mentioned in the previous section the other two cases merely confirm the importance of identity and our intra-theoretical ability to tell objects apart for ontology. This is not surprising, for "[w]e cannot know what something is without knowing how it is marked off from other things. Identity is thus of a piece with ontology" (Quine 1969d, 55). So long as questions of identity do not arise between different (kinds of) objects or coincide with questions of indistinguishability ontological reduction is warranted no matter whether the proxy-function is one-to-one or many-to-one – if identity does not matter there is no harm in collapsing entities for which questions of identity and non-identity never arise. Thus the demarcated boundary of the Quinean: ontological enquiries make sense there, where the cardinality of the set of (relevantly) intra-theoretically distinguishable entities is larger than $\aleph_0$. Failure of ontological reduction to a denumerable universe will occur precisely where there are indenumerably many intra-theoretically distinguishable elements whose identity and distinctness matters (in some underspecified sense) and whose presence is not superfluous. Many-to-one reduction fail in these cases because the identity and distinctness of the objects of the theory mattered in some sense – their positing was not arbitrary but based on certain considerations and any reduction that ignores their distinctness and collapses them ignores these considerations and thereby provides an unfaithful reinterpretation of the reduced theory, such that it can not be regarded as a genuine reduction: "A proxy function that did not preserve the distinctness of the elements of such a theory would fail of its purpose of reinterpretation" (Quine 1969d, 57).

An example of a theory in which a many-to-one reduction to a denumerable universe fails is the first-order theory of the real numbers $\mathbb{R}$ restricted to their usual ordering $<_{\mathbb{R}}$. Here we have indenumerably many intra-theoretically distinguishable – albeit not uniquely specifiable – elements, since for every two distinct numbers $r_1, r_2 \in \mathbb{R}$ we either have $r_1 < r_2$ or $r_2 < r_1$, but never $r_i < r_i$ for $i \in \{1, 2\}$ (Quine 1969d). No two real numbers can be collapsed without disrespecting (relevant) intra-theoretical distinguishability and any reduction will therefore have to respect the cardinality of $\mathbb{R}$. To repeat, the criterion determining the situation in which it makes sense to ask the ontological question could therefore read thus: it makes sense to ask the ontological question and to determine the ontological commitment of a theory if the universe of that theory is indenumerable and the entities contained in that universe are intra-theoretically, i.e. by means of the predicates and relations of that theory, distinguishable and, moreover, their distinctness is relevant, each of them plays a non-superfluous role in the theory and they are sufficiently similar so as to be able to appear in similar contexts.[86]

In the following I will present an example, demonstrating that this criterion is faulty and thereby shift the burden of proof to provide a sensible criterion for the intelligibility of ontological discourse back on the Quinean. Thus consider the following example: let $\mathbb{Q}$ be the set of rational numbers

[85] See also (Quine 1950).

[86] The latter part of this criterion is admittedly vague, but I believe it to be concrete and clear enough to make the intended points.

whose elements q are equivalence classes over $\mathbb{Z} \times \mathbb{Z}/\{0\}$, together with their standard ordering $<_{\mathbb{Q}}$, definable by means of operations on the natural numbers.[87] Defined thus, rational numbers are sets (equivalence classes) of tuples of integers. Moreover, consider the real numbers $\mathbb{R}$ defined as *Dedekind Cuts* together with their usual ordering $<_{\mathbb{R}}$, defined in terms of set-theoretic inclusion. The nature of the exact construction is not of importance at this point, suffices to say that every real number $r \in \mathbb{R}$ is a tuple of sets $(\alpha, \beta) \in \mathcal{P}(\mathbb{Q}) \times \mathcal{P}(\mathbb{Q})$ of rational numbers (s.t. $\alpha \cup \beta = \mathbb{Q}$ and $\alpha \cap \beta = \emptyset$). $\mathbb{Q}$ can be embedded into $\mathbb{R}$ by the mapping $\mathbb{Q} \mapsto \mathbb{R}$, $q \mapsto (\mathbb{Q}/\gamma, \gamma = \{x : x \in \mathbb{Q} \wedge q <_{\mathbb{Q}} x\})$, however, it is important to note that, on this view, it is not the case that $\mathbb{Q} \subseteq \mathbb{R}$, for every $q \in \mathbb{Q}$ is a set of tuples of integers, whereas every $r \in \mathbb{R}$ is a tuple of sets of rationals. Here, the rationals and the reals are *genuinely distinct entities*, for *no* $q \in \mathbb{Q}$ and $r \in \mathbb{R}$ we have $q = r$.

Consider now the theory $T = Th(\mathbb{Q}) \cup Th(\mathbb{R})$, i.e. the (first-order) theory consisting of the theory of the real numbers and the theory of the rational numbers,[88] where $Th(\mathbb{Q})$, as well as $Th(\mathbb{R})$, are *relativized* theories.[89] Clearly, then, $\mathbb{Q} \cup \mathbb{R}$ is a model of T (arguably the intended model), clearly the universe of T is indenumberable (after all, $\mathbb{R}$ is), clearly its elements can occur in the same contexts (are sufficiently similar, given that they are all numbers),[90], clearly they are intra-theoretically distinguishable (since we have, for no $q \in \mathbb{Q}$ and $r \in \mathbb{R}$, $q <_{\mathbb{X}} r$ or $r <_{\mathbb{X}} q$ for $\mathbb{X} \in \{\mathbb{Q}, \mathbb{R}\}$ due to the definitions of $<_{\mathbb{X}}$), yet clearly we should be able to do without $\mathbb{Q}$.

An ontological reduction is plausible and possible here, but the intended and practiced candidate for such a reduction involves a many-to-one proxy-function, mapping every real number to itself and every rational number according to the embedding above[91].[92] Such a plausible reduction does not fall under any of the special cases Quine exempted from his general definition, i.e. we cannot simply 'forget' the rational numbers due to the presence of Q and $<_{\mathbb{Q}}$ and the objects of the theory can occur in the contexts, that is, questions of their identity and distinctness do arise, that is, the theory makes capital of their distinctness. But that means we have here a genuine many-to-one ontological reduction which does not fall under Quine's notion of q-reduction, yet it should clearly qualify as ontological reduction proper.

Note that, according to the criterion as outlined above it should still make sense in this situation to ask ontological questions and apply the criterion of ontological commitment, after all, the elements of the universe are indenumerable, intra-theoretically distinguishable and fall under none of the three

[87]For all mathematical details about this and the following constructions the reader is referred to (Ebbinghaus 1991, pp. 20).

[88]By the theory of the real numbers and the theory of the rational numbers we mean the theory including the order-, as well as the usual mathematical operation axioms.

[89]*Relativization* of a theory is a standard procedure by means of which sub-universes of a theory can be pooled into one standardized universe without the other pooled entities interfering with the statements of the relativized theory. It proceeds by means of introducing predicates, in our case for example Q and R, interpreted as true of all and only the entities of one of the sub-universes. Quantifications are then reformulated the following way: $\exists x\varphi(x)$ becomes $\exists x(Qx \wedge \varphi(x))$ and $\forall x\varphi(x)$ becomes $\forall x(Rx \rightarrow \varphi(x))$; cf. (Henkin 1968; Quine 1956). Thus, $Th(\mathbb{Q})$ is relativized by means of the predicate Q which is interpreted as being true of all $q \in \mathbb{Q}$ and similarly for $\mathbb{R}$.

[90]Even if one doubts that the elements are sufficiently similar in the example at hand to warrant occurrence in the same contexts, it is not hard to devise a more encompassing theory in which this should be the case. Imagine, for example, a theory about measurements. The point here is that in any such theory the identity question between rational and real numbers will arise – in the theory about measurement just envisaged one can imagine providing the length in terms of real and in terms of rational numbers and the asking in what cases the 'real length' and the 'rational length' coincide.

[91]Note that a devious Hilbert's Hotel style construction seems possible here, maintaining the one-to-one claim of q-reduction, but only counter to common mathematical practice and common sense and involving needlessly involved and complicated reconstruction of the operations on $\mathbb{Q}$ and $\mathbb{R}$. Surely this price is too high to pay in order maintain the (rather arbitrary) definition of q-reduction.

[92]Curiously Quine employs a similar move in (Quine 1956), where he reduces the universe of sets and classes of the set-theory NBG to a universe of classes only and fuses the relations of set-membership ϵ and class-membership η.

special cases. And it does make sense due to the lucky fact that the resulting theory still possesses an uncountable universe. But there is no guarantee that this will always be the case. What this example shows is that q-reduction does not capture everything there is to ontological reduction. There are important and valid (for lack of a better word) many-to-one reductions that should qualify as ontological reductions. However, if there are genuine many-to-one ontological reductions nothing speaks, in principle, against the possibility that there are many-to-one ontological reductions to denumerable universes.[93]

That in turn means, however, that a criterion telling us when it makes sense to talk about ontology and to apply the criterion of ontological commitment which relies solely on q-reduction (plus exceptions) in order to determine when sensible ontological discourse is possible – which the criterion provided above does, as it relies on the fact that if the universe of the theory in question is indenumerable and contains intra-theoretically distinguishable objects it is not q-reducible to a denumerable universe – is not strong enough to formulate a criterion which articulates proper constraints and conditions when such enquiries make sense. After all, it might happen that an indenumerable universe is neither q-reducible, nor any of the other three special cases, but still happens to be ontologically reducible, possibly even to a denumerable universe thereby, again, rendering ontological discourse moot. A criterion only relying on q-reduction to delineate the cases in which application of the criterion of ontological criterion is sensible is simply to weak to serve for the intended purpose. The failure of q-reduction to capture all (or even most as the example above is arbitrarily extendable) cases of ontological reduction conditions the need for a new criterion to tell us when we should refrain from asking ontological questions. However, without a proper, more encompassing concept of ontological reduction such criterion appears to be hard to come by. Note that it does not simply suffice to formulate a criterion in such a way as to tell us that ontological enquiries make sense whenever the domain of the theory in question is irreducible to a countable domain, for it does not provide us with a means of telling when this is the case, i.e. it remains completely silent on providing the problematic constraints of when and how that is the case and when not.

5.4.3 A Way Out

One way to counter the objections formulated in above section would consist in the attempt to formulate further constraints on ontological reduction, extend the allowable cases that qualify as such or add more special cases that encompass the situation envisaged in the previous section.[94] However, it is questionable whether one could ever achieve a highly formalized account of what should qualify as ontological reduction without being vulnerable to various counter-examples. Moreover, the attempt to sensibly capture what qualifies when as proper ontological reduction should, in my opinion, be separated from the question when it makes sense to apply the criterion of ontological commitment and when it does not. After all, the connection between the two was only established due to the (unfortunate) LS-Theorem, a theorem which deals primarily with the formal properties of the notation in which the theories are formulated and less with the actual universes.[95] It therefore stands to reason that the problems brought about by it should be tackled at the precise point where

[93]It might turn out, for example, that all one needs to do analysis are the computable reals in which case a reduction of all reals to the computable reals is certainly desired.

[94]Again, simply complementing q-reduction by an extra special case allowing ontological reduction so long as the reduced universe remains indenumerable not only causes new problems, but is also question-begging, for for what reason do we disallow such reductions to lower cardinality? Such exception would be conceptually unjustified and thereby does not provide a conceptually well-motivated way out of the Löwenheim-Skolem dilemma.

[95]The proof of the theorem relies on the fact that in a countable language one can only distinguish countably many different things and therefore need not more.

they arise: in the notation itself.

Given the failure of Quine's own way to deal with and avert the negative consequences for his account of ontology brought about by the LS-Theorem, I believe that it is justified to modify canonical notation in such a way that theories formulated in it are not vulnerable to a theorem which, in the first place, concerns the notation in which we talk about the world, rather than the entities in that world itself (granted, this distinction cannot be drawn like this on Quinean terms, but the rationale behind it still exhibits some force). I therefore propose to make explicit in canonical notation what Quine tried to integrate implicitly with his proxy-function requirement; viz. the irreducibility of indenumerable universes. The proposal unsurprisingly consists in the introduction of the quantifier $Q_{>\aleph_0}$, "there exist uncountably many", into canonical notation. We already showed above (Sections 5.2 – 5.3) that this quantifier does not violate any of the constraints Quine imposed on a language appropriate for ontological discourse and the previous section showed that its introduction is warranted due to problems attaching to ontological discourse formulated in a language without it. In other words, it does not violate any minimality-constraints and is mere notational luxury which is dispensable in an austere notation for ontological discourse, but is necessary to prevent such discourse from collapsing. It is necessary for making sensible ontological discourse possible in the first place.

It is not hard to see how the introduction of this quantifier blocks the problems caused by the LS-Theorem for it is in general not the case that a theory T formulated in language FOL+$\{Q_{>\aleph_0}\}$ has a denumerable universe, for let that theory include the sentence $Q_{>\aleph_0}x\varphi(x)$ and have a model $\mathcal{M}$. Suppose now that T has the Löwenheim-Skolem property, i.e. it has a countable model $\mathcal{M}'$. But then $\mathcal{M}' \models Q_{>\aleph_0}x\varphi(x)$ and $\mathcal{M}' \models Q_{>\aleph_0}x\varphi(x)$ iff $\{x \in \mathcal{D}' : \mathcal{M}' \models \varphi(x)\} \subseteq \mathcal{D}'$ (where $\mathcal{D}'$ is the domain of $\mathcal{M}'$) is uncountable. But $\mathcal{D}'$ is itself only countable. Contradiction. Therefore T does not possess the Löwenheim-Skolem property.[96]

We argued above that Quine's attempt to block (P5)/(P5*) fails because the account of ontological reduction he provided did not do the job it was supposed to do. Strictly speaking he succeeded in blocking the argument though, because all he had to do was to provide an example of a case in which the 'best', i.e. most reduced, universe was not denumerable and this was achieved by q-reduction and a single counterexample already sufficed to block the fatal argument from section 5.4.1. However, the account he provided was very unsatisfying because it left open the question when and under what circumstances ontological enquiries make sense and should therefore, together with his objection to the argument, be rejected. We suggest here to modify premise (P3) to saying

(P3') Canonical notation is FOL+$\{Q_{>\aleph_0}\}$.

thereby automatically blocking (C2) – since (P4), the LS-Theorem, does not apply anymore – and solving the trivialization worry of ontological discourse. For the remainder of this section we will briefly consider two possible objections to our attempt at a solution.

Ontological Objections: Change of Logic, Change of Subject

An immediate objection to our proposed introduction of $Q_{>\aleph_0}$ into canonical reduction concerns its effect on the concept of existence. After all, with a change of logic goes along a change of subject and with a change of the quantifiers goes along a change in our very concept of existence. When I talk about what exists in the canonical notation FOL+$Q_{>\aleph_0}$ and you talk about what exists in FOL we really do not mean the same when we say that some thing x exists or does not exist, our

[96] A more elegant proof would proceed by saying that FOL+$\{Q_{>\aleph_0}\}$ is compact and extends FOL and thereby, by the Lindström-theorem, cannot possess the Löwenheim-Skolem property.

concepts of existence differ, due to the different quantifiers we utilize and them constituting and grounding the very notion of existence:[97]

> "The question of deviance in the logic of quantification is relevant to ontology – to the question what there is. What there are, according to a given theory in standard form, are all and only the objects that the variables of quantification are meant in that theory to take as values. [...] Consequently some philosophical interest, ontological interest, attaches to deviations in quantification theory. They can affect what to count as there being. The intuitionist's deviant quantification (if 'quantification' is still a good word for it) carries with it a deviant notion of existence (if 'existence' is still a good word for it)." (Quine 1986, 89).

Changing the quantification we allow by necessity changes the concept of existence being that existence is precisely what quantification expresses. This approach is, however, not nuanced enough. Existence is what quantification expresses because we settled and determined what we mean by existence through the criterion of ontological commitment. We derived it from ordinary language (see Section 3.2.1) and thus came to the conclusion that what exists is whatever has to be a value of a first-order existential quantifier of a theory in order that the assumption of that theory be true. However, we are defending (WEAK.Q) here: we are modifying canonical notation and not the criterion. What we take to exist are still all and only those objects that the first-order existential quantifiers of our theory have to take as value to render the assumptions of the theory whose ontological commitment we are determining true. *Neither did we change the meaning of the first-order existential quantifier* (as in intuitionism)[98] *nor did we extend the reach of the criterion to include anything other than that very quantifier* (as in cases of (STRONG.Q)). If the criterion is constitutive our notion of existence then that notion of existence remains the same, whether we adopt the new and improved canonical notation or remain in the old.

Nevertheless, one might insist that, just as in the case of second-order quantification, the introduction of $Q_{>\aleph_0}$ causes ontological dishonesty. After all, this quantifier enables us to hide certain existential commitments by permitting us to express certain things without the need to assume objects such as sets or functions that would have otherwise been required for us to assume in order to articulate these same things. The case in point being that we can express "uncountably many" without having to assume functions and sets.[99] There is, I believe, a weak and a strong answer to this challenge.

The weak response points to the fact that $Q_{>\aleph_0}$ is much more ontologically honest than, say, second-order quantification. After all, whenever we have $Q_{>\aleph_0}xPx$ we will also have (by the semantics of the quantifier) $\exists xPx$. Thus, whatever kind of object $Q_{>\aleph_0}$ quantifies over will be assumed among the ontological commitments of the respective theory in question, but, importantly, not because $Q_{>\aleph_0}$ quantifies over these kind of objects, but because a statement involving this quantifier will imply an existential quantification over the same kind of object. Thus, $Q_{>\aleph_0}$ does not hide any

[97] Cf. also (Quine 1969c, 113): "Deviations from it are likely, in contrast, to look rather arbitrary. But insofar as they exist it seems clearest and simplest to say that deviant concepts of existence exist along with them."

[98] One could of course allege that we implicitly did change the meaning of the quantifier since it now interacts differently with the remaining elements of the theory, given that it also interacts with the new quantifier $Q_{>\aleph_0}$. However, in the same vein one would then have to say that the meaning of the quantifier changes as soon as we introduce new predicates into the language as it also interacts with them. Thus no two sciences would use the same notion of existence on behalf of a different ontology, which is certainly an absurd conclusion.

[99] In order to express this in ordinary FOL we would have to deny the existence of a function which is a bijection into the natural numbers (or finite subset thereof), as in the 'solution' to Skolem's paradox (Bays 2006), requiring us to assume functions, sets and possibly even numbers.

kind of object it is quantifying over, which is different in the case of second-order logic, where an object of the kind a second-order quantifier quantifies over need not be assumed among the objects that the first-order quantifier quantify over. This reply remains inadequate to the challenge however, because $Q_{>\aleph_0}$ still makes the assumption of certain objects which would have to be assumed without it in FOL to match its expressive strength unnecessary and therefore 'masks' these.

The strong response then reflects on the discussion of the last few sections: the quantifier $Q_{>\aleph_0}$ was deemed necessary because without it ontological discourse was to lose any point. It is needed in order to make sense of ontological questions at all – without it ontological discourse would not be possible. Given then, that its assumption is necessary for the very possibility of any kind of meaningful ontological discourse it is meaningless to say that it hides the existence of entities that would have to be assumed if it was not part of the language in which to deal with ontology, for without it there simply would be no ontology and thus no sensible way to talk about any entities to be assumed.

Formal Objections: Uncountability, Infinity and a Framework for Science

In the previous chapter we painstakingly delineated a horizon for logic in such a way that it is "bright within and bold in its boundaries" (Quine 1969c, 113). Classical quantification theory, i.e. FOL, emerged as logic proper because it "enjoys an extraordinary combination of depth and simplicity, beauty and utility" and is, on this score "maximal" (Quine 1969c, 113/111). Now, however, we are suggesting to transcend these boundaries and integrate a non-standard element into canonical notation thereby offsetting the beautiful balance achieved by classical quantification theory. A first attempt at an answer to this admittedly vague challenge might try to point out that the boundaries determined were supposed to delineate, in the first place, *logic*. There is, however, nothing that compels us to equate logic with canonical notation – maybe all the argument above shows is that logic and canonical notation come apart and that canonical notation happens to be logic plus something else. Maybe the most austere scheme in which we try to capture reality (logic) is simply not sufficient for doing ontology.

A reply along these lines misunderstands the fundamental place that logic occupies in the Quinean system. We emphasized in the last chapter that it is not just considerations on logic that went into the construction of canonical notation, there is, however, a good reason why the two coincided. Logic was supposed to provide a 'partial notation for all of science' a framework in which all scientific undertaking could, in principle, be conducted.[100] Logic was supposed to supply the framework, every science then added its specific stock of predicates and relations. Canonical notation was the most austere of these scientific schemes, having no specific predicates of its own – its core idiom was the existential quantifier. Our proposed modification, however, *modifies the very framework* in which to do science rather than merely filling out some of the details.

Our response to this challenge merely takes up what we tried to show in Sections 5.2 and 5.3 above: that by all the criteria laid out by Quine for what constraints there must be imposed on something to qualify as logical, $Q_{>\aleph_0}$ qualifies as belonging to logic. It does not illegitimately encroach on the territory of logic, but is an integral part of it. A logic containing this quantifier remains classical in the sense that it respect the law of excluded middle and is complete. Moreover, given that it is specified as part of the constructions of a language it qualifies as logical particle, as logical constant. From the standpoint of logic simpliciter nothing puts it in contrast with the other logical constants in any relevant aspect. We briefly mentioned above that Quine's grammatical criterion for the demarcation of the logical constants guarantees the generation of an exhaustive

[100] This is meant when it is said that Quine was a *generalist* about logic.

list, because the language to which the criterion is applied has been justified by independent means, thereby allowing him to claim that the logical constants are all and only those particles of that particular language satisfying the criterion. However, the argument above shows that the language he devised is too weak and insufficient to do the job it was designed to do, namely to provide a framework for all of science – it trivializes ontological endeavors. This was sufficient reason to extend it by the quantifier $Q_{>\aleph_0}$ and to therefore raise it to the status of a logical constant.

Another objection to the logicality of $Q_{>\aleph_0}$ stems from the similarity of the quantifier "there exist uncountably many" to the quantifier "there exist infinitely many" $Q_{\geq\aleph_0}$.[101] The alleged similarity between these two suggests that if the one was logical, so should the other. However, no logic containing $Q_{\geq\aleph_0}$ is complete, thereby disqualifying it as logical and, so the argument goes, by extension $Q_{>\aleph_0}$. Moreover, the cardinal $\aleph_1$ plays a privileged role in the logic FOL+$Q_{>\aleph_0}$, suggesting that it imports a certain amount of set-theoretical content, "a state of affairs sufficiently unnatural to discredit [$Q_{>\aleph_0}$] as a logical notion" (Tharp 1975, 40).[102]

My attempt at an answer has to be brief at this point. On the one hand, I would point to the fact that just a superficial similarity between the meaning of two quantifiers does not warrant an inference to the claim that both of them should have the same status. Recall the similarity between branching quantifiers which appeared to be 'quite elementary' and continuous with the standard first-order quantifiers and just those. They turned out to be fundamentally different when their logical properties were taken into consideration. Similarly here, $Q_{\geq\aleph_0}$ is not axiomatizable, whereas $Q_{>\aleph_0}$ is. Moreover, the downward *LS*-Theorem holds for a logic with contains $Q_{\geq\aleph_0}$, but, as we showed above, not for a logic containing $Q_{>\aleph_0}$. There are then deeper reasons to not simply rely on the superficial similarity of the two quantifiers and sufficient grounds for considering them different in their nature and status. The objection thereby looses much of its force. Another response might mention the constraint of minimality: we only need to introduce into our language what is necessary to achieve our purpose. $Q_{>\aleph_0}$ is essential in devising a notation doing justice to science, whereas $Q_{\geq\aleph_0}$ is unnecessary and unhelpful ballast.[103] There is therefore no need to include it in a language that is to be assessed by our criterion of logicality and it thereby is not among the candidates for a logical notion in the first place.

As to the set-theoretic content brought about by $Q_{>\aleph_0}$ through its elevation of the cardinal $\aleph_1$ to a privileged position one could respond as follows: on the one hand, the concept of infinity is no stranger to FOL. While there is no single formula forcing the universe of a model to be infinite there are well-known constructions such that the truth of certain theories requires the universe to be infinite. What is lacking is a characterization of this infinity. Although the concept of uncountability has received a fundamentally set-theoretical treatment, it does not seem to me that its content is by necessity set-theoretical. It merely says that there are more objects of a certain kind than there are natural numbers, i.e. than I could distinguish within my language. As such its import seems much less harmful, merely allowing us to express some of the limitations of our language within the language itself. In any case, even if a strong case were to be made to the effect that it in fact does import set-theoretical content, I believe this import to be so minimal as to not deem it worth causing an issue, after all we still need a substantial theory with its own commitments to do

[101] Cf. (Tharp 1975).

[102] Her criticism in fact goes further as she points to the existence of certain uncountability axioms, introducing certain predicate letters into the language that are only satisfied if the universe of the model has a certain cardinality. However, this clearly breaks with the Quinean approach to canonical notation and can therefore not be regarded as a serious alternative; cf. (Tharp 1975, pp. 40).

[103] One could point to the fact that it allows us to express the finitude of objects in a universe, but the inability to do so is no serious shortcoming of FOL+$Q_{>\aleph_0}$ as I can already express for every finite number n that there are at most or exactly n objects.

set-theory – set theory is not given and presupposed as soon as we begin using FOL+$Q_{>\aleph_0}$.

5.5 Conclusion

The conclusion of this chapter is a vindication of the claim (WEAK.Q). In Sections 5.2 and 5.3 we showed that the introduction of the quantifier $Q_{>\aleph_0}$ does not violate any of the constraints Quine imposed on the logic and language of canonical notation as outlined in the previous chapter. Its introduction can therefore be seen as a continuation of the Quinean program in alignment with its goals and not as a break with its principles and purpose. Section 5.4 then demonstrated the necessity of the introduction of such quantifier. It was argued that the interaction of the two principles of ontological commitment and ontological reduction caused a trivialization in Quine's account of ontology, rendering the ontological enterprise desolate. While Quine clearly perceived and attempted to block this danger by means of modifying what qualifies as ontological reduction and what does not through the introduction of a proxy-function requirement we argued that this effort to block the trivialization argument failed on account of being unable to clearly delineate a class of cases in which it made sense to apply the criterion of ontological commitment. In order to block the trivialization argument the Quinean did not only have to present a principle based upon which the argument failed, but also had to demonstrate that there actually are cases which fall under the principle, i.e. counterexamples to the trivialization argument. However, we showed that the characterization of ontological reduction provided by q-reduction is insufficient to clearly delineate such cases as it does not provide us with a reliable way of deciding when and where reduction to a denumerable universe it really blocked and thus when and where ontological enquiries make sense.

After rejecting Quine's solution intended to block the trivialization argument we sketched our own, consisting in modifying canonical notation by means of introducing $Q_{>\aleph_0}$ into it, so as to account for the features which Quine tried to implicitly induce by means of the proxy-function requirement. This introduction is continuous with and not in violation of any of the constraints imposed by Quine on canonical reduction and blocks the problematic argument, thereby avoiding the trivialization of ontological discourse through the unfortunate interaction of ontological commitment and ontological reduction.

It is interesting to reconsider the challenge that it is the fact that $Q_{>\aleph_0}$ imports genuine mathematical content into canonical notation which should disqualify it from inclusion in it. We have treated this objection in three different ways above: first, the charge of importing mathematical content could be understood as introducing concepts into canonical notation which properly belong to another branch of science and should not overlap with a notation for ontology. We countered this charge by pointing to the fact that ontology is not prior to and separate from other scientific, particularly mathematical, enquiries, but continuous with it and that this continuity not only allows but necessitates an overlap of concepts and methods (after all, the entire treatment of canonical notation is mathematical through and through). Second, we could understand the charge as saying that the mathematical content imported by $Q_{>\aleph_0}$ forces it to belong to mathematics, rather than logic – a specific theory, rather than a general framework for all theory. However, considerations similar to those pertaining to Quine's discussion of the logical status of identity, pertaining to completeness and maintenance of the original definition of logical truth showed us that $Q_{>\aleph_0}$ should qualify as logical if $=$ does and that it therefore should belong to logic, rather than mathematics (so that then the necessity of its presence in a general framework for all of science could be justified, which we tried to do in Section 5.4). Third, one could understand the charge as saying that the genuine mathematical content imported by $Q_{>\aleph_0}$ manifests in it hiding serious ontological commitments and thereby not truly admitting what commitment is carried by it. Here we responded that, given that $Q_{>\aleph_0}$ appears to be necessary to make sense of ontological discourse as such, it is constitutive of it and the question of what it imports ontologically does not make sense, because it cannot be addressed without its help. Either way, the challenge pertaining to its apparent mathematical, rather than logical, nature seems to be met and does thereby not constitute a serious objection. And maybe the distinction between countable and uncountable will at some point be as obvious as the distinction between natural and real numbers.

Now, where does this leave us. In a sense, little has been gained. Neither did we propose an alternative, better account of ontological reduction, nor did we completely answer the trivialization fear; after all, analogous Löwenheim-Skolem Theorems can be proven for logics with generalized quantifiers, guaranteeing us that theories which have infinite models also have models of size κ for some infinite cardinal κ[104] and thereby plausibly enabling reducibility to the particular subset of the set-theoretic hierarchy of appropriate size if nothing absolutely major is changed in our concept of ontological reduction. However, (i) it can be argued that Quine might be quite happy with a universe containing only sets (and uncountably many of them); at least at some point in his career he appeared to have held that view, though he later abandoned it in favor of a view positing particulars and sets of these, sets of sets of these, etc. and (ii) our major concern was not to prevent ontology from trivialization. While it might very well be the case that various versions of LS-Theorems push the problem merely upward our goal was merely to vindicate (WEAK.Q) and to show that canonical notation as FOL is too weak for the purpose of ontology and needs to be augmented. This we succeeded in doing. Whether it just pushes the problem to the next level and we could argue for an analogue of (WEAK.Q) alleging that the refined canonical notation is not strong enough for the task at hand, is not our concern here and left for future investigation. The goal was to defend (WEAK.Q), not to devise a notation taking care of all problems that might afflict a new notation, solving the problems of the old one and, in any case, not being plagued by anything more than the old, but rather facing fewer problems.

[104]In fact, the Löwenheim-Skolem number of FOL+$Q_{>\aleph_0}$ is $\aleph_2$, meaning it always has a model of size $\aleph_1$, see (Magidor and Väänänen 2011).

Chapter 6

Conclusion

6.1 A Brief Look Back

What, if anything, have we achieved in this thesis? In an important sense not very much – we are no step closer to assessing how well Quine's system does in achieving what it intended to achieve, namely to provide a model of knowledge superior to those of his predecessors and free of the difficulties that plagued them. In another sense, however, we have shown that, by its own standards, it appears to be doing well: I believe that the preceding Chapters demonstrated the robustness of Quine's system, its strength consisting in the development of an elaborate, extensive and sophisticated system from a very meagre starting point. While the holistic nature of his framework with its countless intertwined aspects threatens by its very nature and complexity to contradict itself in some way, we have, I believe, shown that large parts of his ontology cohere in an elaborate and fruitful interaction.

However, we have also shown that collapse of the ontological enterprise looms large in the formal requirements imposed on the language of theory coupled with principles of ontological reduction and the determination of what there is according to a theory, two fundamental moments of ontology. Moreover, this collapse does not loom large from outside the system, as previous arguments tried to establish, but from within it. We suggested a modification, but it is not clear whether this adjustment holds up under scrutiny or merely constitutes a quick-fix.

6.2 An Outlook

So where does this leave us? I believe that with respect to the Quinean system itself this thesis leaves us with two major tasks. On the one hand, the development of a canonical notation should be re-thought and possibly adjusted, accounting for the vast developments within mathematical logic, linguistics and computer science since the 60's. New insights from these areas might lead us to conclude that if one wants to save Quine's ontological project, one will have to match it with the improvement the formal tools he used to generate it with have experienced. On the other hand, we should rethink the nature of theories and the concept of ontological reduction associated with them and see whether one could integrate a different account, accommodating insights pertaining to the nature, logic and structure of explanations.

This then leads to the more encompassing project, to see how well Quine's system fares with respect to our thinking about knowledge and science in general. Whether he still has something to contribute or whether engagement with him is of mere historical interest. To this end, one would have to do what I excluded from the outset: consider criticisms coming from the outside of the system itself, challenging its very adequacy not on the basis of internal inconsistencies, but on the basis of not doing justice 'to the facts' or not doing justice the best way possible. Be that as it may, I still believe that the sheer systematicity of Quine's philosophy, its clarity and resourcefulness make it an object worth of study.

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