Would You Believe That?
The Prerogative of Assent and Utility of Disagreement

MSc Thesis *(Afstudeerscriptie)*

written by

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Abstract

By basing interpretation on belief, Davidson incurs heavy constraints regarding similarly of belief across speakers. Among these constraints is taking variance of belief to entail error. It is demonstrated that enriching the structure of belief allows a relaxation of normative criteria adopted to ensure similarity. (This enrichment takes the form of assent as defined by de Sousa, with the complementary semantic concept of overspecification.) With the possibility of acceptable variance in belief, and a corresponding augmentation of Davidsonian interpretation, comes the possibility of linguistically-motivated belief change. A positive account of this belief change is given within a Davidsonian framework.

With the capacity to linguistically coordinate beliefs comes the capacity to strategically coordinate beliefs on a systematic level. The asymmetry between semantic access to systematically coordinated beliefs and semantic access to individual beliefs is shown to be epistemically productive, and can consequently serve as a cornerstone in developing an account of formality.

The interplay between belief and interpretation is examined in a case study of mathematical language, furthering the case that both varying speaker belief and strategically coordinated effort give rise to formality. Finally it is argued that account developed is compatible with and suggestive of multiple descriptive features of formality, beyond bearing on presented examples. Thus, at minimum, augmented Davidsonian interpretation suggests a negative test for formality, while suggesting the basis of a positive account and focusing the existing discussion.

Preface

Godsil and Royle, whose 2001 work is a long-term intellectual companion of mine, suggest in their own preface that the purpose of prefacing an academic work is to suggest to the reader why he or she should struggle with the work at hand. I am afraid my purpose is not quite so helpful, at least not directly. I would like to motivate why I myself have struggled with the subject, with the hope that this suggests to a reader why this struggle might be a worthy one.

Why, you might ask, does this work represent struggle beyond that of any academic work? The answer is frustratingly simple, as its source is not only the content. Bringing two schools of thought together is more difficult than either in isolation and often more thankless. But it is the project undertaken here. I will not speculate as to whether mathematicians are less interested in philosophy of language than scholars of Davidson are in formality, or *vice versa*. Anyone who undertakes to examine the development and communication of concepts should be open to all concepts accessible to them. It is this accessibility which I have sought to widen. In bringing this discussion to mathematical ground, the work of Donald Davidson has proved invaluable for its motivation and demonstration of beginning with available evidence alone. There will be no daydreaming here of a Platonic heaven of formality. The fact is that, if I were the sort of hard-line Platonist many philosophers take classical mathematicians to be, I would have stayed where it is possible to be certain.

Both mathematicians and philosophers suggest logic as a reasonable ambassador between the two disciplines, but the most obvious answer is rarely the most satisfying. This, if anything, forms the basis of this thesis, conviction of mine that the ties are deeper, that Benacerraf is correct in expecting that semantics and epistemology do not observe the bounds of academic departments.

So this writing has been a struggle. Trained as a mathematician, I had never written a piece of such length. I had never experienced the conceptual enrichment that accompanies written formulation, a continual epiphany in this process. I have learned the value in imprecision. Still, I hope to be pardoned my first, and thus foundational, mode of reasoning with the conceptual; mathematics, in all its beauty. If such admission renders me an outsider to philosophy and a deserter to mathematics, let me say that it is only from such a position that any relationship between the two can be viewed. I have tried to bring these ideas

across, and wherever they are unclear, I hope this in some degree attributable not to the clarity of the content, but to the fact that true clarity is the purview of mathematics alone. For such a bold claim, the clarity of mathematics entertains little argument. What we should be wondering is why it might be so. The work that follows seeks to build toward an answer.

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This thesis rests on another division too, that between my land of birth and my land of residence. To bridge this gap, and to do so while remaining not only balanced but intellectually productive, has proven a formidable challenge, both academically and personally. Proportionately, there are many people to whom I am grateful for their role in the completion of this work.

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Contents

Introduction

The central aim of the project at hand is two-fold:

1. To save the Davidsonian picture of interpretation from a strategically-motivated and superficially innocuous assumption, that of uniformity across speaker belief.

2. To lay the groundwork for an account of formality through the development of a theory of interpretation for formal languages.

Success will be largely a function of the degree to which these goals can be related, which they will be observed to do through a minor extension of the Davidsonian toolkit. This strategy has the advantage of preserving Davidson's motivating requirement of empirical adequacy and focus on conditions necessary for interpretation. Care will be taken here to preserve both when adding to the theory. The intermediary step in relating these goals is the problem of linguistic belief change. Requiring uniformity of speaker belief, through the principle of coherence, makes belief change through communication impossible by definition. Perhaps this would not pose such a difficulty, but examples of such communication can be found in scientific and mathematical language.

Progress will be made on the first stated goal by enriching the structure of speakers' belief sets with assent, a mode of reasoning with more flexibility, and less certainty, than classical logic. After observing and exploiting the existence of a complementary semantic extension, belief change through language will be given an account. It is time to preempt the objection that the suggestion of a 'semantic extension' smacks of meanings. The insight of Davidsonian interpretation lies in rejecting intermediary theoretical entities such as determinate meanings in favor of empirically-accessible phenomena such as belief, and moreover motivates the present selection of Davidson as starting point. In fact, section 1.4 represents a pause in the exposition of the central argument to consider the pitfalls of overlooking consequences of the Davidsonian insight and implicitly relying a a more standard concept of meaning. At every juncture, including the introduction of the semantic extension mentioned above, success in preserving this insight will be carefully evaluated. Davidsonian interpretation thus emerges relatively unchanged, and once augmented with assent gives a theory accounting for linguistically motivated belief change, and ultimately a good deal more.

Further examination of assent demonstrates that overcoming an acceptable degree of uncertainty with a leap of estimation when drawing conclusions not only allows belief change thorough language, but also belief coordination on a systematic level. The definitude of utterance, coupled with the uncertainty of (a defined sort of) belief, allows speakers who are in agreement about an utterance a corroboration of that belief supported, not by implicative necessity, but by the beliefs of another speaker. The exact mechanism of this corroboration will be spelled out in detail in section 1.7. The extension remains true to Davidson, for the extent to which the utterance is more definite than its utterer's belief is exactly the extent to which other speakers have the chance to linguistically corroborate that belief. Where language commits us to more than our beliefs can support, the opportunity for the assurance of others arises. Wherever that assurance takes linguistic form, it is in turn supported by the beliefs of those speakers and their disposition to draw the same conclusions.

The more speakers involved, perhaps through a coordinated effort, the more highly corroborated the statement becomes. It is toward the end of epistemic productivity that strategies for coordination are developed. This is the first real step toward formality.

The second chapter turns toward the second goal. The success of formality is often used in many philosophical arguments from Plato onwards. However, if its utility is used as evidence, formal language admits no account of its own. If mere descriptive theories are to be avoided in approaching the formal, we must look to the conditions which allow its existence, which is to say we need to approach it not as a sanctified goal, but as just another form of language.

It is to this end that we argue in favor of an approach based upon mathematical practice, and that Benacerraf's two conditions are introduced in section 2.1.2. The first condition asks a semantics for mathematical language in conformity with the semantics given for language as a whole. The case study of mathematical language presented in section 2.1.3 may be analyzed through our work in the first chapter. Particular care is taken to set aside formalization as a satisfactory account for mathematical interpretation, as it is analogous to set meaning in natural language. This leaves free a belief-based approach to mathematical interpretation, in satisfaction of Benacerraf's second condition.

Finally, in section 2.2, the extent to which Davidsonian interpretation augmented with assent might give rise to formality is assessed. Rather than fixing a particular delineation between formal and natural, the factors which might determine placement on this spectrum are given. The discussion includes a field test of the developed approach in both scientific and mathematical communication, so attainment of formality is established, and one endpoint of the spectrum fixed.

Perhaps our goals can be explained much more simply, through Davidson himself. When considering the empirical constraints upon interpretation, he makes a small mathematical misstep.

> There is a principled, and not merely a practical, obstacle to verifying the existence of detailed, general and abstract non-linguistic beliefs and intentions, while being unable to tell what a speaker's words mean. We sense well enough the absurdity in trying to learn without asking him whether someone believes there is a largest prime, or whether he intends, by making certain noises, to get someone to stop smoking by that person's recognition that the noises were made with that intention. The absurdity lies not in the fact that it would be very hard to find out these things without language, but in the fact that we have no good idea how to set about authenticating the existence of such attitudes when communication is not possible. (Davidson, 1974a, 312)

What is interesting about the quote is that Davidson can so accurately pinpoint constraints placed on interpretation by the accessibility of others' beliefs, while so fundamentally misunderstanding how and why mathematical belief is attributed. It can be claimed without hesitation that Carl Friedrich Gauss did not believe there is a largest prime. Terence Tao does not believe there is a largest prime. I do not believe there is a largest prime and if you've got even a basic understanding of number theory, or just primality, nor do you. Nor does anyone meeting that condition. I can make these claims having never lived contemporaneously with Gauss, and having never met Tao. With apologies to any number theorists, we have been thrown in to show this is not a question of simple expertise. This certitude is motivated by what it means to have a mathematical belief. By virtue of what empirically available conditions might this conviction come to be? Any answer must respect the Davidson's larger point about access to belief. To save the insight motivating Davidsonian project of interpretation, that of basing meaning upon belief, we must find an account which is in harmony with the above quote but allows reasoning about beliefs of others, at least in the case of mathematics, but perhaps a wider class of language as well. Such an account is developed in this work.

Chapter 1

Davidson, Relying on Belief and the Possibility to Change One's Mind

> *It seems the human mind has first to construct forms independently before we find them in things...*
> *...Knowledge cannot spring from experience alone but only from the comparison of the inventions of the intellect with observed fact.*
>
> *–Albert Einstein on the work of Johannes Kepler*

In exposing the mutual indeterminacy between belief and meaning, Donald Davidson makes considerable progress on an account which reconciles the variance across individual belief with our ability to successfully interpret one another. Many of his arguments are offered to bound the discordance between beliefs held by speakers. In fact, in his eagerness to demonstrate the possibility of interpretation in the absence of determinate meaning, Davidson is overzealous in the addition of normative conditions, ultimately giving an account stronger than necessary to allow for the functioning of natural language.

What this chapter aims to show is that it is possible to support Davidson's picture of interpretation with loosened requirements about the structure of speaker beliefs, and that, in appropriate situations, speakers may adopt stricter requirements to meet certain communicative goals. Among these communicative goals is that of belief change through linguistic utterance, and other such belief modulation among speakers, none of which are given satisfactory account in Davidsonian interpretation. It is argued that this sort of coordination on a systematic level ultimately leads to the sort of language that is generally termed formal.

1.1 Davidsonian Interpretation

Perhaps it is to keep in step with his own holistic picture of belief that Davidson casts such a vast philosophical net. While Davidson's theories most relevant to the aims of this thesis will be highlighted here, the breadth and interconnectedness of his work yields no easy summary. Yet this is not to suggest that his work does not stand as a whole. On the contrary, rereading a given work by the lights of claims in another often serves to elucidate both. This, however, means at once that Davidson's work must be approached as a whole and that it is a challenge to do so. What is suggested here, in the guise of an objection, is an extension to the Davidsonian picture of interpretation. As such, it necessarily relies on Davidson's own claims. Where apparent, this reliance will be made as explicit as possible, but the extent to which the approach is successful will remain dependent upon Davidson's own success. Still, it is not necessary that the reader be familiar with Davidson's work on an intimate level, and the themes upon which we will build are given brief summary in this section.

From the standpoint of this thesis, the most interesting point of Davidson's picture of interpretation is where he begins. Davidson asks what sort of evidence is available to speakers in order to interpret, and to decide if the interpretations they generate are correct, requiring only that that evidence ultimately have a non-linguistic source (Davidson, 1974a). This demonstrates a problem of access. The sort of non-linguistic evidence must be whatever it was the sentence in question is about, the belief expressed by the speaker. However, if beliefs are to be the evidence available for validating interpretations, another difficulty arises. We have no manner of access to the beliefs of other speakers, *except through language.* We have come full circle. "A speaker who holds a sentence to be true on an occasion does so in part because of what he means, or would mean, by an utterance of that sentence, and in part because of what he believes. If all we have to go on is the fact of honest utterance, we cannot infer the belief without knowing the meaning, and we have no chance of inferring the meaning without knowing the belief"(Davidson, 1974a, 310). Davidson terms these arguments a positive accentuation of Quine.

Where Davidson deviates from Quine is the sort of solution for the indeterminacy of belief and meaning which he offers. Davidson adopts a Tarskian theory of truth as the basis for interpretation. These are the T-sentences which form the basis of Davidson's principle of charity, *"Snow is white" is true iff snow is white.* (Note with Davidson that this is an inversion of Tarski's motivation, to define truth via syntactic translation.) Davidson supposes that attribution of truth is a (reasonably) uniform habit of speakers, and so the relationship of holding true can be used to associate a sentence with the belief it is taken to express. This is the manner in which a theory of interpretation can be based on a theory of truth.

Although priority of emphasis has been tailored to the needs of this paper, this largely represents the ideas for which Davidson is best known. Of course,

rejecting fixed meaning and placing interpretation on the stage of speaker-shared belief leaves Davidson with many unsettled epistemological obligations. That is, having set up the indeterminacy of meaning and belief, he is left to give an account of what in the nature of belief might allow an interpreter to use his or her beliefs as a road-map for the beliefs of others. Davidson introduces the principle of correspondence, suggesting that an interpreter must use her estimate of what she would believe in the place of the speaker in order to interpret. This is how the right hand side of the T-sentence must obtain its content. That we share beliefs to this degree makes sense for sentences directly related to perception, but what about those that aren't? Can we interpret an irrational speaker, given we should share beliefs? Davidson answers by bounding the concept of irrationality. It is only by a standard of shared beliefs that deviation can be understood at all. To this end, he introduces the principle of logical coherence, stipulating that we may assume our interlocutor's beliefs to be structured by classical logic. The irrational speaker may have transgressed logical norms once or twice, but if this occurs on a systematic level, we cannot treat him as having beliefs, or being interpretable in the first place. Moreover, since we all structure our beliefs by means of classical logic, the evident character of perception sentences only serves to make them the most obvious starting point. They cause, but do not justify, the associated belief. The structure of belief is holistic, related to the world by the impressions of the speaker, but not created by sensory data. Belief is a phenomena of individual concept formation, and we share beliefs because we form concepts in similar ways. In particular, it is not uniformity of sensory stimulus which leads to shared quality of belief, and we cannot individuate one particular stimulus with which a belief should be associated. Developing and defending such claims leads Davidson to focus increasingly on the nature of belief in order to justify his picture of interpretation.

These epistemological considerations also mark Davidson's point of departure from what is typically considered the purview philosophy of language. Of course, such a delineation is the mistake against which Davidson argues. We will take this argument one step further, positing that it is in response to dealing with such epistemic considerations that formality develops in the first place, and that consequently examining them is crucial to categorizing it. Toward the development of such a categorization, Davidson's work will suggest our direction. Consult Joseph (2004) for a detailed overview of Davidson's work. For a primary source beyond Davidson's best known *Radical Interpretation* (1973b), essays worth recommendation include Davidson (1974a), Davidson (1997) and in particular, Davidson (1986a), for a reasonable summary in Davidson's own words.

1.2 The Problem

As Davidson seeks to base an account of interpretation on the beliefs of speakers, he owes extensive accounts as to how individual perspective does not impede this strategy (conceptual scheme), how beliefs correspond to relevant interpretations (triangulation), in what mode we collect belief (Tarskian-style theory of truth), what allows us to have sufficiently similar beliefs in the first place (principle of correspondence), what guarantees these beliefs manifest (similar perceptual apparatus), how these beliefs are structured (principle of coherence) and finally how we can use our own beliefs to interpret others (principle of charity). The (parenthetically indicated) solutions to these difficulties compose the bulk of Davidson's work on language and strongly inform his positions on mind and epistemology.[1] Effectively, to base interpretation on belief, Davidson must show the existence of a shared belief set comprised of *equivalence classes* which suggest the alignment across the respective belief sets of individual speakers, and justify this equivocation through the theories listed above. Beginning with a truth-theory, where truth is unanalyzable, yields a common jumping off point. This equivalence establishes the sufficient similarity between speakers on which the principle of charity relies, giving the conditions upon which perspective can be *factored* out in order for interpretation to function. "This method is intended to solve the problem of the interdependence of belief and meaning by holding belief constant as far as possible while solving for meaning...What justifies the procedure is the fact that disagreement and agreement alike are intelligible only against a background of massive agreement."(Davidson, 1973b, 324) Where this agreement is most evident, and most easily equated across speakers, are perceptual beliefs, as it is most clear that these can be mapped onto some shared concept. However, other sorts of belief must be preserved as well if they are to serve for interpretation. Here is where Davidson's holism comes into play. The less evident presence of non-perceptual belief is the result of their less evident nature, and not any theoretically important difference. The equivalence for non-perceptual beliefs is suggested by the holistic closure of each belief set under the mode of human reasoning.

The gain of this factoring-out individual perspective is considerable. It allows Davidson a secure but novel ground on which to found interpretation, without reliance on meaning. What needs to be recognized is the cost. The involved project of equivocation demands so much apparatus that Davidson hardly addresses the potential side-effects of success. In a theory of interpretation, treating all individual belief sets as one, albeit one with carefully stated properties, makes both theory and interpretation blind to any variation of belief across speakers. Of course, this is only really an issue if there are instances of successful interpreta-

[1] These have been relegated to parenthesis to give the reader familiar with Davidson some points of association. However, each of the concepts indicated requires significant elaboration to be fully understood, and the point holds without this background knowledge. We need to know that Davidson aligns speaker belief. The details of how exceed the space available.

tion which require the belief set of the interpreter to be non-identical to those of the speaker. One such important case is that of belief modification through communication. The hearer gleans from a speaker's utterance that their beliefs do not coincide, thereby, in certain cases, suggesting a manner of modifying a previous belief or obtaining a new one. Moreover, this represents a large class of examples problematic for Davidsonian interpretation.

Rather than attempting to define this entire class of examples just yet, we begin with a significant subset. Scientific communication is rife with utterances which serve to change the belief set of its hearers. "The basic unit from which you are composed is a minute object, which has the structure of positive and neutral particles forming a nucleus orbited by even tinier negative particles." "Objects in motion remain in motion unless an external force is exerted upon them." "The speed of light is constant to all observers." Perhaps none of these instigate a belief change in the learned reader, but this is otherwise upon a first hearing.

The above examples are involved ones, which do not stand alone without the accompanying body of scientific theory. (By virtue of what, exactly, they ultimately do stand, within scientific language, we are not yet in a position to answer.) Still, just as many examples offer immediate verification. "Polarized lenses only filter light in one axis." "A flame deprived of oxygen is extinguished." "Increasing the tension on a vibrating string increases the pitch." These are to be analyzed along the lines of perception sentences. In particular, such immediate scientific facts are subject to the same sort of evidence required of perception sentences, and to bury them in a more theory-leaden layer of the holistic belief structure is to overlook the exact simplicity which allotted perception sentences their prominent place in the discussion.

1.3 Outlining a Solution

Ultimately, the counterexample of scientific communication will be seen to lie in a greater class of counterexamples allowing belief change through communication. Cordoning off this class from the whole of language will be the project of the remainder of this chapter. Once made, this distinction will be shown to fall along another division in language, that between natural and formal, and discussion motivating the correlation between belief changing communication and formal language will follow. Meanwhile, establishing this division will save the project of Davidsonian interpretation from our initial objection by making clear in which cases Davidson's original analysis holds, and those where it requires some modification. Of course, it was Davidson's project only to account for natural language, so if a successful line between formal and natural can be drawn, then our original objection may not even obtain upon Davidson in the first place, and the arguments presented here can be read as extending a Davidsonian account of interpretation into a formal setting. Before going into the details of belief-

changing communication, it may be helpful to have a road-map of the argument as it relates to what has been set out so far before moving forward.

- Language we will eventually term formal represents a class of counterexamples for Davidsonian interpretation.
- This is the result of speakers in these settings consciously adopting a strategy to assure a structured relationship between belief and utterance. (Where typical Davidsonian interpretation takes place through relationship not deliberately structured by speakers.) This strategy includes the beliefs of certain speakers over others.
- Language community wide strategizing of this sort is made possible by communally adopting and correlating a particular structure between beliefs, and a standard by which the community accepts new beliefs into this structure. When an individual does not have the appropriate beliefs, the structure is the means of acquiring them. This acquisition is a consciously mediated process, that of assent which will be defined in detail in section 1.5. The desirability of beliefs is determined by their role in this structure.
- Recognizing this saves Davidsonian interpretation by giving it two modes, one for which the original account will suffice (natural language settings) and one which requires some extension (formal language.)

So far, progress has only been made on part of the first claim. A full-fledged theory of interpretation for formal languages is too ambitious a project for this discussion. What can be established is that, in a Davidsonian setting, such a thing might be both desirable, if not necessary, and also plausible.

1.4 Avoiding Stowaway Meanings

There is a crucial mistake contained in many objections presented to radical interpretation and the Davidsonian picture of interpretation from which we must steer clear. This mistake is as simple as a failure to appreciate what it is to reject semantic meaning as an unanalyzed explanatory concept. Such a rejection requires that any objection to Davidson either attempts to 1) rebut Davidson's dismissal of meaning directly or 2) not rely on meaning, implicitly or explicitly. Few sympathetic to Davidson's goals and approach will attempt the former, but a surprising number of responses to Davidson, even contemporary ones, fail to do the latter.

For the purposes of refining the objection to Davidson to those presented here, and to better define and subsequently avoid the pitfall outlined above, the criticism of radical interpretation made by C. G. Goldberg (2004) is treated

in detail. The error of supposing semantic meaning can be seen in Goldberg's arguments against the strategy of radical interpretation. He argues that the understanding to be obtained in the testimonial transmission of knowledge in a radical interpretation setting cannot be sufficiently full to be considered proper understanding, and thus radical interpretation is drawn into question. His sense of proper understanding is a technical one. (Notice, for our purposes, that the two instances of the word understanding in the preceding sentence play very different roles: the former is understood in that it is information apprehended by the hearer, the latter for the information somehow to be universally ascribed to the utterance itself.) The strategy of this argument involves supposing understanding exists in the hearer after some instance of testimonial knowledge has been offered, then arguing that this understanding requires two conditions "1) The utterance has the content the hearer takes it to have and 2) The hearer is warranted in taking the utterance to have the content she takes it to have" (Goldberg, 2004, 389). Since this sort of understanding is intuitively evident in the hearer, the argument proceeds, it cannot have been derived in a radical interpretation setting, as radical interpretation places far too "stringent evidentiary conditions on the warranted interpretation." Therefore, the knowledge obtained by the hearer exceeds that for which radical interpretation will allow, and radical interpretation must give way to a more anti-individualistic semantic strategy.

Of course, this argument relies heavily on the supposition that such a warranted interpretation exists without itself requiring some sort of radical interpretation, a premise which Davidson would not accept. Notice the decidedly stipulative semantic focus in the first condition and the epistemic one in the second. Again, the extrication of these two is what we value in the Davidsonian picture. Goldberg is unabashed in expressing his simple picture of interpretation in service of understanding, saying "In particular, a good theory of epistemology of understanding should not require too much in the way of substantive positive evidence regarding the truth of one's interpretation of the source testimony"(Goldberg, 2004, 392). This apparently follows from the fact that access to conversant's speech histories is so systematically lacking that any conditions upon interpretation will disallow any hearer from obtaining a warranted interpretation *a la* condition two.

Goldberg, then, gives a very clear example of how easy it can be to smuggle assumptions about meanings into an argument. The goal here is to pose an objection to Davidson on his own terms, with an eye towards being able to suggest an equally compatible fix. If the Goldberg example falls outside the bounds, might the greater field testimonial knowledge inform our inquiry?

Testimonial knowledge is considered to be epistemologically similar to memory, in that its source is beyond first-hand verification and that it is not a generative source of knowledge. It is supposed that the speakers possess the knowledge considered, and that the hearers take the utterance of the speakers as the source

of this knowledge. See Lackey (1999) for further details. However, the problem of testimonial knowledge transmission is typically approached from an epistemological standpoint. Discovering which conditions of utterance are requisite to qualify that utterance as knowledge is the primary concern. Any analysis of such conditions seems to presuppose a reliable means of interpretation, which quickly gives way to talk of meaning. This is a problem when beginning with Davidson, as we do not suppose the absence of additional factors in the communication of the testimony. As always with Davidsion, any theory in which the 'semantic content of an utterance' enters into play as an unanalyzable prime must be rejected, alongside meaning. Because belief itself is the basis of interpretation, even in a solely epistemological analysis, testimony must be treated as more than mere replication of information. Certainly, approaching testimonial knowledge via Davidsonian interpretation may prove an interesting avenue for further investigation, but as the bulk of the literature leaves interpretation unanalyzed, it sheds little light on our problem.[2]

The only insight to be gleaned from all of this, then, is that it would be very easy to make an objection to Davidson similar to the one presented here, thus making a mistake similar to Goldberg's. In fact, should we momentarily tolerate semantic meanings, then it would be very easy to demonstrate belief change as the result of an utterance. If the semantic content of the utterance and the hearer's belief did not match, the hearer could adopt the former as a new belief. Of course, Davidson sees rejection of meaning as necessary to saving philosophy of language as a serious subject (Davidson, 1986a), and no mistake along these lines is being suggested here. What is being suggested is that utterances can sometimes inform belief formation just as sense data does. Nothing external to a mind is the cause of a belief, just the thing the belief is about. All we are suggesting is that there is a mechanism where speech can play a role when an individual is evaluating the relationship between their beliefs and the things their beliefs are about.

1.5 The Structure of Belief

1.5.1 Resituating the Objection

It is a script we could all recite if pressed, even in the absence of familiar prompts "Do you have our discount card? Would you like a receipt? Have a nice day."

[2]In fact, all of this leads to a wider observation. Although Davidsion is typically of interest for those interested in language, the mutual indeterminacy between meaning and belief is just as pertinent to epistemological discussions. This means that, if there is to be no meaning without a concept of belief, so too can there be no mutual establishment of belief without reliance on meaning. At the very least, treating testimony as if it can perfectly duplicate knowledge across parties, even in ideal circumstances, is too simplistic. Further analysis of testimonial transmission of knowledge must be concerned not only with legitimizing interpreted utterances, but with the unavoidable role of interpretation itself.

Most of us do not think twice about it. Moreover, anyone who has been in a foreign language situation can manage to give the right answers at the right moments. Still, it is unlikely they do it without thinking. In a sense, they are parroting the responses of fluent speakers. Worse, we wouldn't want to have to attribute them knowledge of the meaning of any of the terms involved. When they use the right word in a situation, it is done just as a parrot does, as a conditioned response, without reference to a concept held by the speaker. If we suppose the bird cannot possess language (and we will politely follow Davidson here,) through what failing has our expatriate friend been denied competence, given that he or she might have been born to another tongue?

The Davidsonain answer is that such competence originates in possessing a supporting belief, of being aware of a variety of contexts in which those same words could be appropriately employed. If our grocery shopper not is using the words outside the checkout, as for example, a means of distinguishing between instances where he or she wishes another a good day or does not, we can suppose no such concept is to be associated with the vocalizations.[3]

This coherence of structure in the belief set of each speaker is one of two principles Davidson introduces to preserve the possibility of interpretation in the absence of set meanings. Its companion principle, the principle of correspondence, which allows an interpreter to suppose other speakers share very many of her beliefs, typically garners more discussion because of its controversial epistemological stance. Moreover, as a minimal normative requirement goes, it looks innocuous, particularly to philosophers. However, the effect of this shift away from observation sentences as prime to concept formation, isolates the understanding of the speaker. Beliefs are central as they occupy the only role. The total reliance upon the belief of a single individual is central to the principle of charity and to Davidson's entire philosophy, and equally important to his central goal, an empirically defensible account of interpretation.

This being said, placing such strongly normative conditions on speakers as total deference to the rules of classical logic seems mildly incongruent with a rejection of abstract meaning. Conscious of this potential weakness, Davidson gives a long explanation as to why a departure from classical logic in structuring beliefs is on par with irrationality. He goes so far as to claim that for an interpreter to attribute belief in a contradiction, say $p\&\neg p$ to another must be a mistake on the part of the interpreter, for there is always a logical structure on the part of the speaker. This structure may not be totally apparent, but motivated properly it can be understood without the need for contradiction.

Basically, Davidson has supposed a logical structure to all human disposition in order to secure a foothold for interpretation. The position argued here will be

[3]An example more similar to Davidson's is a three-year-old who has been taught to say "Look, a bunch of atoms!" Even if the utterance is amenable to the application of convention T, its correct use in every case cannot be supposed to indicate any understanding of atomic theory. But we're skirting such examples for the moment to avoid question-begging.

that this is both A) too strong to model even basic beliefs and B) too focused on establishing incorrectness on the part of either the speaker or interpreter to display a very real advantage of abandoning meanings as abstract entities. Different concepts can be at play for different speakers, so underlying beliefs may be not only reconciled but compared.

After arguing in favor of A) and B), a solution which substitutes a slightly more involved account of the structure of belief will be introduced. By making this substitution, it becomes possible in the Davidsonian account to allow for variations in belief to be communicated. That is, we will replace the claim that disagreement between speakers implies incorrectness on the part of one, with a description of how such a disagreement might occur, given interpretation's requirement of shared belief. In fact, with this account in hand, it will turn out that it is possible, in the case of disparate beliefs, for speakers to improve their belief sets through communication.

1.5.2 The Principle of Logical Coherence and Why it May be Too Lofty an Aspiration for Language Users

" 'I am inconsistent: I believe both p and ¬p.' This is "indefensible": but is it insight or nonsense?" (de Sousa, 1971, 53)

It is first crucial to recognize what motivates Davidson to adopt his principle of logical cohesion in the realm of belief. Superficially, it is a normative constraint placed on speakers in order to ensure the potential for communication. Of course, for Davidson, language is a phenomenon which extends beyond communication, both indicative and demonstrative of belief. Some intrinsic rationality is requisite for any account of mental phenomena. (Davidson, 1985, 347) Irrationality can only be understood as the total absence of this structure.

Davidson's motivations aside, there are many reasons to reject logical cohesion of belief. The position taken here is that it is simply too stringent a requirement to place on the cognitive activities of most people in their daily lives, that many of us can have some awareness of a proposition without feeling the need to assign it a truth value unless pressed. Moreover, even when speakers have assessed a proposition, holding them to the logical closure of its implications among other propositions held true and false, plus a check for any internal contradiction, is to require a great deal. Most of us simply do not consider the relationships between our beliefs or the totality of implications they may entail. This is even if we suppose that it is possible to survey one's set of beliefs consciously to suit these demands, or in the alternative suppose that there is something fundamental to our cognitive apparatus that plays by the rules of classical logic. Neither option looks appealing, but the second must be set out of hand immediately. Accepting Davidson's methodology means abandoning any privileged access to belief. To claim a property is fundamental to belief is to claim that it is requisite

for communication. This argument fails if a weaker condition may be substituted, thus claiming beliefs naively follow classical logic, while certainly sufficient, may be overzealous. Such a weaker condition will be outlined in a coming section.

Fortunately, Davidson does not take the principle of logical cohesion to such indefensible extremes and he recognizes there is some difficulty in requiring speakers to evaluate themselves for non-contradiction.

> "Someone can believe p and at the same time believe $\neg p$; he cannot believe $p\&\neg p$. In the possible case, of simultaneously, and in some sense actively believing contradictory propositions, the thinker fails to put two and two (or one and one) together, even though this failure is a failure by his own (and our) standards. " (Davidson, 1985, 353)

Davidson is aware, then, of the difficulty of supposing constant self-examination in search of logical consistency, but the presence of a contradiction, p and at the same time $\neg p$ represents a deviation from standards. These standards are absolute. Should an interpreter suppose that a speaker believes p and at the same time $\neg p$, this represents a failure on the part of the interpreter. Davidson goes so far as to explain simultaneously believing a proposition and its contradiction through a compartmentalization of the mind, where presumably each component is logically coherent. Thus such contradictions take the role of errors, which are never apparent to an interpreter and which are sufficiently rare and negligible so as not to upset the project of interpretation.

Davidson elsewhere attributes certain instances of irrationality, such as 'forgetting' the day of the week, to this sort of compartmentalization. (Davidson, 1986c, 146) The agent has failed to be aware of all that was going on in his mind, being in one compartment and not another. To reserve the title of rationality for an un-compartmentalized mind, Davidson directly dismisses this case in which a one might be normally functioning and not aware of potential clashes between propositions held.

At the very least, what is being ignored is the magnitude of the requirement placed on an agent by taking the position that "there is no logical difficulty in supposing that the agent knows what is going on" (Davidson, 1986c). This entails conjuring the relevant beliefs at every appropriate moment, followed by analysis for consistency and implication. For every belief held by a speaker, this is simply too extreme a condition upon everyday. Failure at such constant analysis is more than forgetfulness, but is common enough that it shouldn't relegate the unfortunate agent to a category of globally irrational split-minds. In fact, Davidson posits a total state of synchronous inconsistency would require "All the beliefs, desires, intentions, and principles of the agent that create the inconsistency are present at once....are live psychic forces." (Davidson, 1985, 353) In other words, to make the inconsistency apparent, the agent would have to be aware of the inconsistent propositions simultaneously, and such near-omniscience of introspection would approach insanity. Of course, this is the natural conclusion where one

is looking to find an inconsistency. Making a global check of an individual's belief for consistency leads to the same requirement of simultaneity, and thus the same contradiction.

Finally, there are cases where a contradiction of beliefs is immediately apparent to the agent, and is still embraced. An indecisive agent might be painfully aware he wants p and does not want p, might turn over the lines of reasoning which lead him to each, and yield the contradiction repeatedly. Speakers will consciously enter into a state of denying that p to themselves, while knowing that p is the case. An agent who wants to stay dry and knows it is raining still might simply be too lazy to put on a coat. It is not clear that such states, while perhaps not enviable, constitute irrationality. They too deserve a place in a theory of belief.

There is a reading of Davidson upon which we have straw-manned him into more rigid standards of rationality than intended. Immediately after affirming *modus ponens* in belief structure by claiming that an agent who believes 'if it rains it pours' and 'it is raining,' must believe it is pouring, Davidson does cede that there is no list of the 'basic principles of rationality' (Davidson, 1985, 352) Again, this strategic non-specificity conceals the tension between requiring fundamental similarity of reasoning and prevalence of shared belief, and at the same time allowing for natural deviation in conclusion. A reading in which his conclusions are too strong has been presented, but it does not negatively effect the following arguments if a more sympathetic reading is taken. The posited structure of belief may simply be considered to be an equally non-enumerative, but more interesting, basic principle of rationality than *modus ponens*.

1.5.3 An Alternate Direction

We briefly turn to our second claim, B), that this commitment to a logical structure of belief leads to a blinding focus on false (or incoherent) beliefs and thereby prevents a potential strength of abandoning meanings. This is the possibility of testimonial belief change, but before detailing how such a change might be possible, it must be emphasized that this is in line with the Davidsonian project.

Although Davidson has built an account where interpretation is possible, he has overshot the mark in estimating human capabilities. Accuracy in estimating the beliefs of others is central, and this native competence is much stronger than average competence in classical logic. Culling extraneous assumptions is true to the goal of attaining minimum empirical adequacy, so even if the replacement we find for logic requires lengthy description, we have lightened the burden on speakers and so improved the theory. Should we be able to deal with the challenge of belief change in the same stroke, all the better. We have argued at length against the use of logic as a structure of belief, now it is time to tailor the objection towards a replacement. Davidson's criteria for possessing a concept will serve as a basic measure. After arguing that possessing a concept is distinct from

having the sort of sensory experiences that have the potential to engender that concept, Davidson offers the following criteria: "To have a concept is to classify objects or properties or events or situations while understanding what has been classified may not belong in the assigned class...{correct use} does not prove conceptualization has taken place, even on a primitive level, unless a mistake would be recognized as a mistake." (Davidson, 1997, 22) Thus, concept formation belongs in the realm of beliefs, not in the realm of the sensory input it categorizes. Our grocery shopper would not independently realize a mistake, so even in correct use he or she does not qualify as having the proper concepts. It is here that Davidson dethrones perception sentences, those tied directly to their content by immediate perception, from their position of definitional privilege granted by Quine among others, and embraces an unanchored self-supporting holism of belief.

So beliefs categorize sensory experience, but no individual sensory experience, paired even with the appropriate perception sentence, can engender a belief. Beliefs are the product of generalization from many sensory experiences, and a single instance can never allow for the recognition of an error. It must be remembered that Davidson cannot require that two speakers come to the same belief. The principle of correspondence asserts that beliefs are the same enough for interpretation, that our modes of generalization are similar. Of course, even if they were identical, the point would not be discoverable as the only access to belief is language.

1.5.4 Dennett's Over-Specification of Desire

We now turn to an observation of Daniel Dennett's about the relationship between language and the expression of desires. Consider the following order placed at a cafe. "Could I please have a 1987 French Chardonnay at cellar temperature?"[4] Dennett asks "How could one begin to attribute a desire for anything so specific in the absence of such a verbal declaration? How, indeed could a creature come to contract such a specific desire without the aid of language? Language enables us to formulate highly specific desires, but it also forces us on occasion to commit ourselves to desires altogether more stringent that anything we would otherwise had any reason to endeavor to satisfy." (Dennett, 1981, 63) To square this with Davidson, observe that the implied negative answers to the first two questions are shared.[5] Intentional attitudes offer no access except through language, and we develop these attitudes in parallel with linguistic behaviors. It is the final claim, that sometimes the communication of a desire requires a specificity beyond the native impulse itself which offers no immediate parallel.

[4]This is not Dennett's example, but will pair equally well with his analysis.

[5]...if we allow that Davidson accounts only humans the ability to contract desires, the very contention that Dennett is seeking to dismiss.

To reconcile this, look to the desire underlying the statement. We can easily suppose a desire for chilled white wine, perhaps even for one which is expensive or aged. The diner may even have a more detailed knowledge of wines, that is to say, an unusually rich set of concepts associated with wines. Still, the patron commits to details which cannot pertain directly to his own desire for a (perhaps very specific) bottle of wine. He ultimately imparts a few details which are at minimum necessary for ordering *any wine whatsoever*. And this holds for those of us possessed of a less refined palate. When we ask for something "white and not too sweet" as a wine order, we realize we have deferred the choice of specific detail to the waiter, whose task it becomes to pair our loosely stated desire with one of many bottles in the fridge. In most cases though, it is left to the speaker to add the specificity, even if it goes beyond the demands of their immediate impulse. Even our waiter, so helpful with the wine selection, will be of little help to someone who enters and says merely "I want to eat." even if his hunger is so extreme it earns only this expression.

What it is that necessitates this occasional over-specification of language Dennett doesn't speculate. Nor does he note the possibility of its extension to other intentional attitudes. While desire is a particularly clear example, as it must be related to sensory perception immediately in the relationship of fulfillment, there is reason to suspect it can be extended to belief as well. That is, we sometimes may need to commit ourselves verbally to more specific beliefs than we actually hold in order to commit to any belief at all. Consider someone who attempts to guess the time of day by looking at the position of the sun. They have some notion of the season and the passage of time, some inner sense of how much time must elapse before sundown. Even for one experienced in using the sun's motion to mark time, with a trained inner sense and tailored awareness, placing a number in hours and minutes goes beyond that understanding. Still, in many cases they will be willing to make the leap of associating a number in order to convey the product of that experience to someone else. This is in part because he recognizes that his conversant may lack the same skill.

Having established through example the possibility of over-specification of belief, it would be ideal to know why over-specification happens at all. Without a full answer here, it is at least worth noting that both our wine enthusiast and our solar observer will fail Davidson's test of recognizing error; neither, having made a mistake, is likely to recognize it. The wine drinker will not, upon tasting, announce his dissatisfaction that the wine tastes like a 1980 vintage, nor can the estimator be availed of the accuracy of his guess without the aid of a clock. To the degree that these speakers have overstepped the specificity most befitting their intentional states, they have also overstepped the requirements of what it is to have a concept. And this in the hope of communication. They have each made a deliberate over-saturation of their own understanding.

For now, we merely want to note the possibility of this disparity. It does not

exist in all instances of language use, but that it does at all leads to a complication for radical interpretation.

1.5.5 Elaborating on Dennett

Let's go back to our wine-drinker, who has had the unfortunate luck of ordering a bottle not to his tastes. Now, there are plenty of reasons that he might give for disliking the wine, perhaps the cork has degraded, the temperature is incorrect or the wine turned to vinegar. However, we accept equally that he might be completely unable to give a reason for disliking it, beyond that it is not to his taste. Moreover, there is no contradiction in his ordering this very particular bottle and then, upon tasting it, declaring that it isn't any good.

At first, it looks like the easiest explanation is the *de dicto*/*de re* belief distinction. The wine that our restaurant patron believed *de dicto* he wanted was characterized by being made from Chardonnay grapes, being a certain degree cooled, and of a certain vintage. What prevents him from sipping in satisfaction and declaring of the liquid in his glass "This was the wine I wanted!" is a failure of correct association between the wine-beliefs he had *de dicto* and those he had *de re* . This would lead to many familiar puzzles where he looks at his glass says something to the tune of "I don't like this wine, but I love chardonnay."

It is doubtful, though, that the division between beliefs *de dicto* and *de re* fits into the Davidsonian picture without some explanation as to how an interpreter might attribute misaligned *de dicto* beliefs to a speaker when interpreting charitably, which is to say, when guessing at the belief of the speaker which lead to the utterance. Dennett has a more interesting take on the situation. (Dennett, 1978) In his analysis, the state of wanting a wine with certain properties, white, chilled, etc, is not logically tied to the verdict of liking it or not. Whereas if you believe that if it rains it pours and that it is raining, one might guess you think it is pouring, there is no similar injunction in this case. Even supposing all sorts of rational intents, a careful selection, a knowledge of wine, even the hope that the patron would enjoy his drink, none of this makes it seem illogical, irrational, or even particularly unusual that the wine failed to please. Dennett's observation, that there is no logical tie between the desire for an x with properties $F, G, H, I, J \ldots$ and the judgment of dislike (or of like), points to something more profound than *de dicto* / *de re* belief attribution.

If it is not to be a logical bridge between the desire and the judgment, then by what mechanism does the leap occur? Despite drinking the wine being a result of desiring chardonnay, despite the tasting being best explained by that desire, the judgment of enjoyment is separate. (We are assuming here that the wine has the beverage's characteristic complex flavor, worthy of some examination and assessment beyond visceral reaction.) There is some reflection on the relevant available sensory details, in this case, primarily taste and smell, but perhaps also consistency and color. The relationship of this information to existing concepts

also bears reflection. After all, no connoisseur is in a rush. Then a decision is offered, in which all of these considerations are weighed, but in which the role of any one consideration is not logically clear. Most notably, this lack of logical clarity is not restricted to observers. Even the privileged view of introspection, which may reveal emphasis on particular factors, does not reveal the mechanism of judgment in a logically apparent way. "What is important is just that it is a choice point that terminates a process of deliberation or consideration that is not apparently algorithmic, but rather at best heuristic, At some point, we just stop deliberating. We take a look at the pros and cons, and leap." (Dennett, 1978, 303) The ultimate judgment has more in common with a choice.

At this point in his exposition, Dennett borrows heavily but strategically from the work of Ronald De Sousa. Like Dennett, De Sousa focuses upon the nature of belief, and mechanism of attributing it to others, without central consideration with language. However, De Sousa shares Davidson's commitment to consistency in a model of belief, and is motivated by objections similar to those presented in this chapter in proposing his model of belief. In the paper *How to Give a Piece of Your Mind: Or, the Logic of Belief and Assent*, (1971) De Sousa argues for a reclassification of belief, subjective dispositional beliefs and beliefs to which one consciously assents, the sensations elicited in the wine and the verdict, in our example. The latter form is instantiated in an act, taking into account underlying dispositional beliefs, weighing them and drawing a conclusion. It is the leap for which we could draw no logical bridge, making it is the prerogative of the agent in question. The landing of this leap can be modeled as a classical proposition, with truth value given by the agent. Dennett correctly points out that De Sousa has a terminological difficulty here, as he calls the act of deciding assent, but gives no title to the sort of attitudes that result from assent. It is not immediately clear that the products of assent fit alongside other sorts of belief. Nor is Dennett's suggestion, that the products of assent can be immediately dubbed 'opinions', fully viable without more argument than he gives. For now, we will refer to the products as ascensions, because De Sousa's distinction will offer a good perspective on the problem at hand.

This leaves dispositional beliefs, which are to form the launch pad for assent. As for De Sousa assent as an act, he invokes decision theory. He adopts the perspective of Bayesian decision theory, in which a decision is determined by varying degrees of confidence about circumstance, which can be given a subjective probability. If one accepts a proposition P with probability p, one accepts $\neg P$ with probability $1 - p$. De Sousa argues for modeling dispositional beliefs in this Bayesian fashion, but gives them little further analysis. Beliefs, after all, are prime in decision theory[6], and play an explanatory role only. This presents a major difficulty for De Sousa, for what is to count as a prime dispositional belief and what is to count as an ascension is not totally clear, and he only obliquely

[6]This is interestingly related to where Davidson begins

hints at a distinction. "In reporting someone's beliefs in indirect discourse, our latitude is limited by the canons of acceptable paraphrase; but in the case of animals or infants, it is limited only the the explanatory or descriptive purposes for which the ascription is made. And this is not due to limitations on our access to the beliefs of dumb creatures, but to the fact that they do not have specific belief." (de Sousa, 1971, 61) This is a bit confused, for it contrasts a linguistic report of another linguistic event (about belief) with a linguistic report of an estimation of belief. As we retain Davidson's claim of inaccessibility of belief through language, the former case cannot establish much. Accepting that De Sousa offered it for contrast alone, left safe for examination is the latter case, a situation where there is no danger of assent occurring. [7] Presumably, then, the beliefs possessed by a toddler or a dumb creature minimally belong to the realm De Sousa allots to dispositional belief.

De Sousa makes many insightful arguments in favor of his division, not least of which that it offers a theory of belief which at least suggests how one might (dispositionally) believe p and at the same time $\neg p$, so long as they did not become aware of both propositions at the same time, at which time we expect them to assent one or the other, or at the very least to having a contradiction. Ascension offers a perspective from which we can allow for some of the logically undesirable habits of beliefs, while still maintaining logical coherence. In fact, De Sousa makes the same argument as Davidson against the possibility of total irrationality. "If you ascribe to me inconsistent beliefs you must explain this by assuming that I have make mistakes in manipulating the set of my accepted sentences. If you cannot imagine *what* mistakes might have led me to take something to be true, you are not meeting that requirement." (de Sousa, 1971, 78) Here, De Sousa is making two assumptions, possibility of access to his beliefs, and the existence of similar modes of reasoning, both included in the normative conditions placed on speakers that Davidson establishes. The upshot is that De Sousa requires no compartmentalization of the mind to account for an inconsistency, for mere absent-mindedness will do. Moreover, assent offers a hint as to how we can strengthen the case for maintaining the requirement that speakers share many of the beliefs requisite for interpretation, while still allowing for the many cases in which beliefs diverge.

1.5.6 Incorporating the Ascention

We have seen that incorporating assent might allow a setting in which we can account for incompatible beliefs held by a single speaker and for variance of some conclusions across speakers. What is not entirely clear is that it would be at

[7] The example also implies that language is requisite for assent, and De Sousa readily agrees. Since it is in fact the relationship between belief and language we are most interested in, we will leave aside arguments restricted to the scope of De Sousa's paper.

home in a Davidsonian account. A division between beliefs and ascensions must preserve Davidson's holism. Lower-level beliefs must not have a privileged status in the theory when it comes to concept formation. Finally, and crucially, even if a theory is to account for belief variation across speakers, it must in some degree retain the principle of correspondence, otherwise there will be no grounds for interpretation at all. In the following section, these apparent difficulties will be provided answers which ultimately display the synergy of the two approaches.

If any classification of those beliefs requisite for interpretation of others was forthcoming, the present problem would have been much simpler, for these are never candidates for alteration through communication. The difficulty rests in the fact that we are at once very capable at estimating others through our own beliefs and not consciously availed of all the constraints we apply to these estimations, being subject to them ourselves. Even if we began afresh, with no beliefs at all, followed by a potentially small set of beliefs intimately related to perceptions, and sought to track the relationship of each to every other in our network of beliefs as it developed, the effort of record keeping would be greater than that of belief development. Even if successful, this process would be only a single data point, because speakers are not exposed to the same sensations, even if they do have a similar mechanism of concept formation. We mostly don't know ourselves which beliefs we draw upon in interpretation, because we mostly don't know what beliefs of our own inform a particular concept, and we must assume it is the same for others. In this way, no such categorization should bear on the requisite factors for interpretation. We, having been born blind, are particularly good at navigating dark rooms. If there is another way to interpret, beginning with a minimal set of beliefs for interpretation and no more, it is and as useless as a light switch; its use is external to an already functional operation.

Moreover, uniformity of stimulus doesn't suggest a classification. "Beyond the skin there is mindless causality, but what gets bombarded is a thinking animal with a thoroughly conditioned apparatus. There is no simple relation between the stimulus and the resulting thought." (Davidson, 1997, 21) It is by virtue of our similar perceptual capacities and modes of generalization that beliefs come to be held in common, and not by virtue of their development following identical paths. Considering the many beliefs we have, the sheer number of possible relationships between them is combinatorially impressive. A belief may be an easy leap from perception for someone, and ensue from many others for someone else. It is said that Paul Erdös, who is the most published mathematician ever to live, was unable to prepare a grapefruit, even late in his life. One might imagine an attempted introduction might involve an instruction to, with a knife, bisect the sphere in a plane orthogonal to the line described by two small antipodal indentations in the fruit's skin. Such examples leave little hope for making any sort of belief primary above others. Any belief can be just as theory-infected as any other.

All this is to say that, if perceptual beliefs get any elevated status at all, it is

only because we begin life without any beliefs and must acquire them somehow. To put any further emphasis on all perceptual beliefs is to see a network of beliefs without appreciating its complexity. In particular, perceptual beliefs are not necessarily prior to some other belief in the same manner that De Sousa's basic beliefs are. Notice also that they pass Davidson's test for mistake recognition. If you are startled by a streak in the night and proclaim, "I'm 90% sure that was a cat, but I've never seen a cat run that fast!" it is not your concept of 'cat' you are drawing into question. You are interested in the relationship of your concept of 'cat' to the streak. Even if you aren't fully sure what to call the creature you saw, your notion of cats has not suddenly gotten fuzzy around the edges. Should you check under a car and see a raccoon staring back, you do not suddenly reevaluate your concept of cat and its place in your network of belief to make room for the raccoon. You say "I was mistaken, that was a raccoon." and leave it at that. Again, this is our test of mistake recognition; your possession of the cat concept is confirmed by realizing it has been misapplied.

Of course, should you find a cat after all, then you may modify your concept of cat to include elevated speed. This reveals an interesting asymmetry. Even if your initial uncertainty is a product of the animal's capacity for velocity, the velocity is not sufficient to cause doubt when the animal corresponds in every other apparent way to your concept of cat. Any instance where available evidence does not correspond exactly to one's existing belief, there is the chance for either total negation, in the case of a mistake, or slight revision the existing concept. The very possibility of concept revision exists only in an imperfect match between the concept and the stimulus. Beliefs admit of degrees. Naturally, above a certain threshold of mismatch, the concept is not modified, but the categorization is drawn into question. If neither of these is viable, then you cannot be said to have a cat-belief at all.

It is less a poverty of sensory evidence in this case, and more a recognition that many beliefs upon which we are happy to rely deal with a poverty of information in some dimension or another. This does not undermine our ability to form concepts, it simply recognizes that, by the same token that we are each cognitive apparatus which is tuned to a particular structure of belief acquisition, that apparatus is constantly modifying and improving its network of beliefs. For this modification to take place, each concept must be squared with each situation in which it must apply and the difference between the two measured. Now, rather than simply finding difficulties with requiring logical cohesion, we have a positive motivation for adopting De Sousa's Bayesian beliefs. They offer a mechanism through which a belief may be a product of each instance to which it is relevant, by the possibility of modification. No instance of perception is privileged, and all perceptual beliefs and perceptual sentences are satisfyingly infected by the network of an individual's belief.

There is one special case, that where you require absolute certainty that the

animal is a cat. Naturally, for you, absolute certainty can only be measured relative to your own cat-beliefs. For all normal purposes, even modest concept modification, a glance at the animal was sufficient. Suppose you are recounting the tale of seeing a streak in the night to some unusually inquisitive friends. Suspensfully, you do not reveal that the streak was a cat until the end. When a skeptical friend asks if you are absolutely certain it was a cat you saw, you recognize that he is not questioning your use of the word 'cat' to refer to what you saw, he is inquiring if you have made an evaluation of your sensory recollection, and compared other potentially relevant categorizations, and made a conscious decision to abandon them and commit fully to the term cat. (Perhaps he knows a civet has escaped from your local zoo.) Notice that, even after being sufficiently sure of the relevant beliefs to make a cat concept modification, this question requires more than no thought to answer. In a model of belief based on classical logic, either the answer is trivial or the belief that you saw a cat was a mistaken one. After incorporating assent, the question spurs not only reflection but novel evaluation. You weigh relevant factors, then decide how you will respond, requiring considerably more thought than you are likely to have given the event as it occurred. You realize he is asking about your own personal certainty, rather than about the viability of your sense-data, and this is your subjective certainty to give or withhold at will for whatever reason. It is awareness of the potential non-equality between your best-fit concept and personal certainty concept is what causes your friend to ask in the first place. Assent is a separate phenomenon from basic belief. Even if you voice doubt when pressed, if you feel unwilling to assent to the streak being a cat, your use of the word 'cat' in the story still remains understood as the best reasonable fit and retaining some degree of viability.

1.5.7 Tempered Belief

We've seen some potential advantages of adopting an articulated concept of belief, with a pivot point between basic belief which can be held to some degree between 0 and 1, and beliefs which are the product of assent, which get a binary truth value. While extensive support has been given to the idea that assent is compatible with Davidson's basic goals, what remains to be seen is its compatibility1 with the theory itself. At first glance, a Bayesian model of belief seems incompatible with the T-sentences upon which radical interpretation is built. Finally, the if the distinction is to be incorporated into a Davidsonian theory of interpretation, it must preserve the principle of correspondence, otherwise our newly enriched picture of belief may only serve to make speakers incomprehensible to each other. I have framed these potential difficulties as objections. However, since we are looking to establish and improve the requisite conditions for interpretation, once answered they will serve merely as signposts directing us toward a theory in accordance with Davidson's original insights.

Let us begin with the role of T-sentences, because, being specifically and

technically defined, they present the most obvious and fundamental challenge. At first glance, it does indeed appear as if we are proposing slackening the requirements placed upon us by basing our theory of interpretation on a theory of truth, by introducing degrees of uncertainty. This objection holds insofar as the original motivation for introducing a structure of variegated belief was to relieve interpreters of the cumbersome burden of full and constant accord with classical logic. However, there is nothing to suggest that the manner in which belief is obtained, expressed and otherwise brought to bear on the world is identical with the manner in which one person's network of belief is structured, nor to suggest that the two must be modeled identically. Basic beliefs are exactly those with no component of assent, those which require no conscious consent on the part of the believer. They are absolute precisely because they are non-elective.

This non-elective quality is exactly what is captured by a T-sentence. The very motivation for adopting a theory of truth in pursuit of understanding interpretation was to restrict the resulting theory to faculties possessed by speakers. Davidson is interested in *only the conditions under which speakers hold sentences true.* (Davidson, 1973b, 323) A theory will be more powerful, then, if it is able to reflect both the instances in which speakers find sentences true and the possibility for that ascription of truth to be modified, improved or revoked. T-sentences play the former role, Bayesian beliefs the latter. "The methodological advice to interpret in a way that optimizes agreement should not be conceived as resting on a charitable assumption about human intelligence that might turn out to be false." (Davidson, 1973b) All we've done by incorporating Bayesian belief is account for the real possibility of epistemic error, and awareness of the possibility thereof, by individuals or large communities of speakers. Their role is not the preservation of uncertainty on the part of the believer, but the acknowledgment that many beliefs are not – and should not be – held as absolute. This remains even if we end up relying on them and implementing them for interpreting others.

Having set aside the most pressing objection, it is time to begin to sketch what role De Sousa's division of belief is to play. Of course, De Sousa preserves the type-classification problem of referring to belief, an intention, and assent, an act. While this drives home the participatory quality of the products of assent, this point is the proper juncture to abandon the terminological difficulty so that we might discuss how assent effects an individual's network of belief. To do this, a question left by De Sousa must be resolved: are the products of assent beliefs, or a different species of intentional content, like judgments? The answer in a Davidsonian context, certainly dissatisfying for De Sousa's ends, is: It doesn't really matter which is which, at least most of the time. Because all speakers form concepts much the same way, and use their own beliefs to interpret others regardless of how formation occurs, it makes little difference what the underlying mechanism is, so long as it remains shared. The earlier episode of your uncertainty about cats did nothing to undermine your ability to be understood about cats.

So long as speakers retain sufficiently shared belief structures, the Davidsonian is free to remain agnostic as to what those structures might look like.

There's another reason that the distinction seems less fundamental from a Davidsonian point of view, and that is just how easily we can take the step of assenting from basic beliefs to form another disposition. There is a certain strain in explaining someone as *choosing* to like a certain wine, no matter how drawn out and postured their deliberation. This is not because enjoyment is anything other than a disposition, subjectively-determined by a series of impressions, but because the formation of that disposition is so fundamental and automatic that it goes unnoticed. Still, we can envision the wine drinker requiring several tastes to 'decide' if the wine is enjoyable. As De Sousa defines it, an action is an event caused by certain wants and beliefs, and it is voluntary if we could have acted otherwise. (de Sousa, 1971, p. 60) The wine-drinker had no real alternatives beyond like and dislike, so perhaps his act of assenting to dislike was not uninhibited in the sense required for real freedom. He is at liberty to make a non-logical jump, but only to a limited number of landing places. He is restricted to the options available to his mind, as are we all. Assent can be at once a choice and the product of practiced mental reflex.

Of course, to a given individual, the logically arbitrary choice made in assenting looks well-motivated, perhaps even necessary. This is why you say "I like this wine" more often than "I decided that I like this wine." Considering restocking the same vintage, it is unlikely that you attempt to recall all the beliefs you had, but whether or not you enjoyed it. This brings up an important point: *introspection does not necessarily distinguish between a basic belief and a product of assent, even where assent occurred.* For an individual, there is no necessary awareness of which dispositions are the products of assent. Even where the mechanism of assent is at work, it is easily hidden, even at the level of introspection. This of course transfers directly to being left with only guesses as to instances of assent when interpreting others. This inability to sort bears the important consequence that the products of assent must be considered a species of belief, for they blend so naturally into belief sets as to be indistinguishable. Even a disposition tempered by the evaluation of assent looks to speakers just like a belief, and the resources available to speakers are our only concern.

With products of assent secured into the realm of belief, there is finally ground to settle the terminological difficulty. Beliefs remain beliefs wholesale, and in the specific instances where assent is introspectively apparent to a speaker in the formation of a particular belief, we will refer to as *tempered* belief, as they have been cast, perhaps repeatedly, through the act of assent. Similarly, a tempered utterance is one drawing on tempered belief. Again, many such tempered beliefs will fall under speakers' radar and receive no special qualification either in introspection or interpretation. Even when classification is apparent, it is not often useful, for tempered beliefs are just some of the many shared beliefs which we

are free to assume one another to posses. However, as it will be later be argued, there are cases in which speakers consciously preserve their awareness as to which beliefs are tempered and an ends to which they do so.

This is not to say that speakers generally cannot reconstruct the beliefs associated with a particular tempered belief. As we have seen, the original motivation for the principle of logical coherence is to ensure that there is some relationship between the various instances in which a belief informs language use, a concept playing a distinguishing role. With the addition of assent, this requirement is still met, the relationship binding an individual's belief set is merely no longer that of classical logic. Just as you are fully able to posses the concept of a train locomotive without active recollection of every sensory perception informing your belief, but would still be able to associate any given locomotive-memory with the appropriate concept, it is possible, presented with a supporting belief, to pair it with a tempered belief which it engendered. Even if belief structure is enriched with assent, appropriate navigation and association between concepts remains intact.

Some reasons that tempered belief isn't the dire threat to the principle of correspondence have already been suggested. It is equally familiar to all speakers, and more often we forget that a tempered belief is a product of assent as soon as the act is over. Moreover, it doesn't often matter for interpretation if a belief is tempered or not. Whereas Davidson's speakers must all bend to classical logic, Davidsonian speakers who are also assenters may use logic, but may have (basic) beliefs outside its structure, all the while interpreting other speakers as doing the same. In admitting an individual's beliefs may be logically imperfect, we say the same of all others. No similarity in belief is lost, and the theory is strengthened by abandoning an unrealistically strict requirement. Where assent does not naturally lead us to the same conclusions, there is now a good explanation as to why those conclusions differ. This requires that we all have at least a good idea about which sort of conclusions we won't share, and we do. Taste in wine is just one many preferences no one assumes to be shared. (That is, before a certain threshold. We also know when assent is unlikely to be necessary, as we may all find a spoiled wine unappetizing.) Incorporating assent into our interpretive toolbox allows us to interpret others when a tempered belief is not shared, so long as we share the supporting basic beliefs and understand the mechanism of assent. Again, the theory is stronger, for we no longer need to see one party as having made an error. A final observation is that, even if assent is a logically arbitrary act, never has logical arbitrariness precluded human universality. The idiosyncrasies of our kind facilitate, rather than prevent, us communicating with each other.

1.5.8 Overspecification Explained

With assent incorporated into the Davidsonain picture, grounds are sufficiently developed to examine the Dennett's desire sentences mentioned in section 1.5.4.

Recall that a man ordered a bottle of wine, and this required specification beyond that inherent to his dispositions. Moreover, the distance between the exactingly precise order and the desires which motivated it caused it to fail the Davidson's negatability of error test for possessing a concept. With the addition of assent, the analysis is simple. The individual desires for a certain wine attribute combined supported the wine-drinkers assenting to the belief that he wanted a 1987 French Chardonnay at cellar temperature. That the man could not detect any potential discrepancy between his order and the bottle he was served is precisely because of the arbitrary element of assent. He would notice if it violated the supporting desires which caused him to make his selection, white, dry, expensive, chilled, etc, but would have nothing to contradict if the discrepancy lay elsewhere. Dennett has observed the prevalence of over-specification in the realm of desires because desires directly call for some act of assent to specify whatever is to satisfy them.

A more general claim is at hand. That is, wherever language requires specification beyond the supporting beliefs which seek expression, in precisely those cases where failure of mistake-recognition is at stake, the belief expressed is obtained through assent.[8] This is deeper than a claim about language alone, for giving voice to a disposition is not essential for having one. (Although, at least for Davidson, the ability to use language is.) Assenting to a novel belief may be the result of introspection alone, but we may assent to over-specified propositions that we might not have natively generated in order to say anything at all. That is, our supporting beliefs may be many and varied and difficult to express, so we take the leap of assenting to a more concise more specific proposition. Estimating the time of day from the position of the sun requires observation of shadows, familiarity with the latitude, awareness of the season and an instinct trained by many cases of trial and error. Giving that estimate takes four digits. Over specification occurs when we make a motivated choice to communicate several beliefs together and assent to more than perhaps we would have otherwise in order to do so. And, although this is done for the purpose of communication, it is quite possible that the tempered belief expressed is adopted into the speaker's belief set by virtue of this analysis.

1.5.9 What is Gained: A view from the top

Of course, the principle of logical cohesion was not our original objection. The motivating goal was to gain ground on the possibility of belief change while preserving Davidson's methodologies and insight. In situations of mismatched belief

[8]It is worth noting that Dennett, having circled all the factors incorporated here to make this claim, does not himself reach it. His motivation for introducing over-specified beliefs is to suggest what might mislead philosophers to accept a theory positing a language of thought (a theory which Davidson also does not accept). He is barred from relating assent and over-specification by his hasty conclusion that assent leads directly to opinion, which skirts many of the subtleties of De Sousa's exposition.

between speakers, Davidson was left to account for some error, classical implications giving singular results. For Davidson, differing beliefs must be the result of flaw. With the addition of assent, the resolute bridge of logical necessity is burned and so is the need of characterizing multiple perspectives as error. Of course, we cannot attribute to another speaker an act of assent which is inconceivable to us, but an extended charity dictates that we understand that their tempered belief may be different than our own. We need not accept their subjective bridge so much as recognize it as a possibility. Should a speaker's base beliefs seem to bear no relationship to the tempered belief they instantiate, that speaker runs the risk of being misinterpreted. As they should, for they have violated our dictum of similarity. Thus we are not opening the floodgates of interpretation to arbitrary unmotivated beliefs, merely allowing some multiplicity of conclusion. Every tempered belief, if it is to be used in interpretation, should stand the test of bearing a motivated, if subjective, relationship to the beliefs on which it is based. We hold other speakers to this criteria even in the most subjective realms. You may dislike a lovely chardonnay, but not if you haven't tasted it. You may wear a tinfoil hat, but not without at least the motivation of insulating your brainwaves. Of course, you may offer another equally implausible explanation, but other speakers expect it to be comprehensibly related. If you have dawned cooking wear and give the reason that fish breathe water, you make yourself a serious candidate for a total state of incoherence, rather than just a case of eccentricity.

An interesting corollary of our ability to understand others' capacity for holding different tempered beliefs is that we can be aware of incompatible beliefs simultaneously, and select the appropriate ones for each instance of interpretation. When you tell a child at Disneyland to go talk to Mickey Mouse and ask him about a favorite movie, knowing that the child accepts the presence of Mickey does nothing to impair your awareness that it is a man in a suit, that cartoons aren't video images, and so forth. We interpret others not just on beliefs we have, but beliefs would could imagine having assented to. We are still relying heavily on our own belief set, for it still is our only ground from which to guess at the tempered belief of others. The only real difference is that we are no longer forced to suppose identity, even in the face of having no privileged access to the beliefs of others. Just as our beliefs are separate from the perceptions which are associated with them, so too can we see each cohesive set of tempered beliefs as separate and operating above our basic, everyday beliefs.

Even where we don't consider our own discordant beliefs to be tempered, we aren't prevented from developing a separate picture for the purposes of interpretation. If I interpret an utterance I hear based on what I would mean if I used it, I can imagine a belief underlying it might be a tempered one, even if my own is not. If none of my own beliefs allow successful interpretation of a statement s, and I can predict no tempered belief the utterer of s might have assented to on the basis of our shared beliefs, and if I am going to continue to view the

speaker as rational, I can still try and envision some beliefs which would engender a tempered belief allowing me to interpret s.[9] However, as we shall see, this process is not immune to failure for it is limited by my ability to identify which beliefs led the speaker to assenting, a far less reliable process than going in the opposite direction, from supporting belief to assent. We are better at guessing where someone will go next than at guessing where they are and how they got there. This process of nominating a set of supporting beliefs for the purpose of interpretation we will term descent, which is less reliable and thus markedly not the inverse of assent.

With the resanctification of multiple valid beliefs about a single topic comes the possibility of correction through speech. For Davidson, if two speakers held different beliefs, one of them must have been incorrect. Armed with assent, and the consequent possibility of accurately attributing differing beliefs to others, comes the chance that belief not held by a speaker could be first attributed to another, and then adopted. Mostly, this will come in the form of drawing a different conclusion in an act of assent. If you don't play along, the child is much more likely realize it is a man in a suit. This realization is not sparked by new information about Mickey, but by noticing that you make a different subjective evaluation. In a set few other cases, it can come through suspecting assent in another, in the absence of a supporting belief, and seeking out a basic belief to lead to that assent. If a convincing enough supporting belief is available, it can be accepted as native.[10] Because of the difficulty retracing another's path from many beliefs to just one tempered belief in reverse, this strategy is rarely successful. Later we will examine a special class of cases where it is more reliable.

[9]Finding examples which are neither trivial nor question begging here is challenging, as we near our goal. For the purposes of understanding, an example might go something like this. Someone tells you "If you feel like you are getting a cold, you should take echinacea." To assent requires an idea of what echinacea is. In the absence of beliefs about echinacea, you can guess that the speaker believes that it reduces cold symptoms. With a posited base belief in hand, you can now imagine the speaker assenting to the belief expressed. However, there is nothing to guarantee that you would ever develop the belief that echinacea is good for colds, or in fact that if you were exposed to echinacea that you should have any cause to add 'good for colds' to your echinacea concept.

[10]Again, finding an example of this kind of change in belief, especially a successful one, is challenging, but it does occur. In my adoptive city of residence, on the first Monday of the month, at noon, they test the emergency alert siren. Being out among people on the street when it goes off, and watching them act as if there isn't a iterative blaring scream rattling everyone's eardrums is unsettling, particularly if you don't know that it is a test. The first time I heard it, I quickly concluded that there must be some tempered belief held by the other pedestrians. It was not linguistically expressed, but the natural reaction to such stimulus is to take cover, so they had to have some other motivation. I was at least able to deduce that there was no danger, on the basis that others couldn't have believed there was.

1.5.10 Wrapping Up Belief Structures

Introducing assent into a Davidsonian account resolves the tension of requiring at once logical cohesion in individuals, shared beliefs in speakers, and the possibility of differing beliefs between individuals. Accepting variation of beliefs across individuals is an enhancement of the principle of correspondence. Davidson adopts the principle of correspondence for we have no means of access to the attitudes of others without language and yet require belief to interpret. In a sense, where Davidson claims that interpreters consider what they would hold true from a speaker's perspective, the introduction of assent does nothing more than specify different modes by which this consideration could occur. In fact, it is stronger. Davidson's arguments for correspondence can now be read as supporting agreement between basic beliefs across speakers. Defining assent only suggests which sorts of belief need not correlate, so long as interpreters account for assent. The only real change in the principle of correspondence is added detail.

The principle of logical cohesion has suffered more damage. In defining assent, the requirement of a tight logical connection between a tempered belief and the beliefs which give rise to it is explicitly abandoned. Where tempered beliefs fail to retain any special markers of their origin, they are indistinguishable from any other beliefs a speaker might hold, so no speaker remains certain of the relationships between his own beliefs. Logical cohesion is destroyed, but we have argued that this is no loss. In seeking the requisite empirical conditions for language, Davidson overshot the mark by requiring logical coherence. Certainly it is sufficient to support conjectures about another's reasoning process, but a more minimal consideration will suffice. Beliefs must be internally structured, but demands of classical logic are too rigid to reflect that structure. To enrich the picture of belief structure and relax the requirements placed on speakers, logical coherence may be replaced by something like the *principle of logical collusion: an interpreter's beliefs have a structure which may sometimes follow the rules of classical logic, but is supplemented with other strategies, including assenting; however, since these strategies are common to human concept formation, speakers are no worse at guessing the beliefs of others than if it they modeled on classic logic, and are very often better.* This principle will not be further elaborated, as our focus is extending rather than modifying Davidson's picture of interpretation, but it is Davidsonian in character. "Before there can be learning, there must be unlearned modes of generalization. Before there can be language there must be shared modes of generalization."(Davidson, 1997, p. 24) All that has been proposed is to diversify the sorts of shared modes available; before there can be belief coordination through language, there must be shared modes of assent. The Davidsonian goal of abandoning accepted dicta in pursuit of empirical adequacy is better served by the principle of logical collusion than the principle of logical coherence. This goal entails approaching the problem of language from the resources available to a speaker, and the ability to reason with classical logic is not

among these for all speakers. The majority of Davidson's own arguments can be read as supporting either principle, even when he is talking about relationships between beliefs.

A full logical model of assent will not been suggested here, although De Sousa begins on a few basic implications. What is required after incorporating assent is the possibility of anticipating the attitudes of others in a shared structure of belief, but this was already an explicit condition of language use. If between the two we have introduced the slacker principle, it has only been to suit the somewhat sloppy goal of accounting for minor inconsistencies in individual beliefs, or variance or beliefs across speakers. No longer are we committed to a theory of compartmentalization of the mind if someone believes p and other times $\neg p$, or oscillates between the two. It is unlikely that many assent to $(p\&\neg p)$, but there is a chance interpreters will accept even this, given a proper background of supporting beliefs. Even better is relief from considering one of two possessors of differing beliefs to be incorrect, for it is only abandoning this position which will allow others to help you change your mind.

1.6 Details of Belief Change

After incorporating assent into Davidsonian interpretation as elaborated in section 1.5, there is room for speaker beliefs to differ. Not as evident is the possibility of communicating that difference. The previous examples of belief modification after observing assent in another all amounted to lucky guesses. While some guesswork is unavoidable whenever considering the beliefs of others, any reliable belief change must be based on something less arbitrary than luck or intuition. Here is where boarders in language must be drawn in earnest. The only justification for such an extreme step is that it is one taken by speakers themselves.

It should be clear that basic beliefs are not candidates for belief change, being non-evaluatively governed by attribution of truth or falsehood. Therefore, any utterance which has the potential to instigate belief change must express a tempered belief. This affords the interpreter, the steward of the potential belief change, the opportunity to reason backwards to the speaker's basic beliefs which support the tempered belief expressed. However, this has already been established to be at best an unstable process. There are few resources left to the interpreter. There remains one avenue left to facilitate belief change, a proactive role for the speaker. As assent is a consciously evaluated process, the speaker has some control over the tempered beliefs entertained and subsequently expressed. If the speaker only assents in a predictable, mutually-established manner, then the interpreter has some suggestion as to how to reason backward from the tempered belief expressed.

This is a considerable weight to place on the speaker. Knowledge of which beliefs are tempered must be actively preserved if it is to be preserved at all, and

the subjective leap of assent may no longer be a fully arbitrary one. The speaker is in fact limiting the sort of tempered beliefs they can express. Here, at long last, is the motivation for speakers to draw divisions in language themselves; if the speaker is to be held to all these criteria, it must be in a restricted section of language so as not to become impractical. If a specific strategy for tempering beliefs is to be adopted, it must be adopted in a subset of language. While all this discussion has been about some fixed speaker, the burden naturally extends to everyone employing the subset of language.

So, necessary for belief change is an established strategy by which speakers assent. This strategy need not be fixed across language subsets. All that would seem to be required is that the strategy suggests some means of retrieving the basic beliefs associated with the tempered belief, and that this strategy is shared. What is crucial is establishing a set of rules for assent among speakers and speakers adhering to these rules more or less uniformly. Perhaps this condition sounds normative, but it isn't normativity of belief or normativity of meaning. The norms are selected, dictated and modified by users of the language. Rather than arguing through an attempt to characterize something so elusive and dynamic as belief, this account is based on criteria established by speakers.

There is a final, hidden condition which was hinted at in the previous section. This is the what remains of the principle of correspondence after modification with assent. There remains a set of beliefs which must be shared among speakers, now a subset of the whole, the basic beliefs. It should make sense that they adhere to Davidson's original principle, as they are exactly those beliefs uneffected by the advent of assent. It is to the supporting basic beliefs that an interpreter reasons backward upon hearing an utterance expressing a tempered belief. There is no other criteria upon which to evaluate tempered belief except its source. Only with the relevant basic beliefs at hand can the interpreter evaluate whether or not to assent to the tempered utterance and take it as belief.

So far, an example is missing, but hopefully at this juncture non-formal examples are not forthcoming. Scientific language, which has so far been our testing ground, takes as its strategy the requirement that the experimental data which supports its theories is both measurable and repeatable, and requires its theories to be predictive. If told that deprivation of oxygen will cause a flame to be extinguished, I can descend to my perceptually-established belief "the candle in my study dies whenever placed under a jar," and reason that, given the required repeatability, my basic beliefs support assent to the statement. Still, assent with respect to the scientific strategy may not be so flippant as assent wholesale, and I may require additional evidence before committing. Knowing which supporting beliefs are acceptable and the role they play in strategy-specific assent is a matter of proficiency in the strategy. A flame brought into a mine tends expire whenever the canaries do, suggesting a mutual requirement of oxygen, but a draft can equally extinguish the candle. A speaker's capacity to assent scientifically in the

face of such varied evidence must result from knowledge of and experience with the requirements placed on the scientific strategy of assent. Moreover, should the assent be with respect to any lesser conditions, the resulting belief would not qualify as a scientific one.

There is one more possible avenue for belief modification so far left aside, except perhaps in footnote 10 on page 30. This is the case where the interpreter lacks supporting belief for a tempered utterance upon which to descend. In the vast majority of occurrences lead to no belief change whatsoever. If I say "Pitaya fruit as more interesting in appearance than in taste," and you have never encountered a pitaya and thus have pitaya beliefs or even beliefs somehow associated with pitayas, there is candidate for belief for modification. It is conceivable that you do glean some implications from any such statement, presuming it is a legitimate one. *There is a pitaya concept which our shared cognitive and perceptual apparatus would allow me to acquire in the same fashion you did.* As such evidence can be supposed to exist for any interpretable utterance, it also isn't really an interesting property on its own. If asked about pitayas, you are of course capable of repeating my statement, but this is analogous to the performance of the grocery shopper at the beginning of section 1.5. Correct use is not sufficient evidence for having possession of beliefs.

The case in which this is interesting is when we have established a strategy of assent, which in turn gives a hint to exactly what sorts of supporting beliefs would be required to support the tempered utterance in question. If the relationship dictated by the strategy is tight enough, the interpreter may be able to determine exactly which sort of supporting belief would be required and how to go about adopting such a belief. Suppose we are engaged in botanical research on cacti belonging to the genus *Echinocerus*. You tell me that there are some *Echinocerus* cacti which can survive temperatures well below freezing in dry conditions. Note that our shared method of assent is far more detailed than that which might apply to science as a whole. For instance, through descent I can reason that you do not mean certain individuals scattered within the genus, but that such a subset of individuals will be constituted by one or more species. This is because species adapt specializations such as cold resistance if they improve survival in their environment, and these adaptations are spread across the breeding population through natural selection. The belief that you refer to all the members of one or more species, then, can be obtained through descent exactly because I can be certain you wouldn't have assented without it. Knowing how species are delineated, I can then seek out the characterizing attributes for each species which is a candidate for meeting the requirement of surviving below freezing. This can be broken down further, for what is conveyed by 'survival,' 'dry,' and even potentially 'freezing' depends on what our botanical conditions for assent are, and I can test each species to see if they are met. When I succeed in locating the species, as I eventually should if your original statement was legitimate, I have

somehow been led through the steps to gather the sensory evidence necessary to acquire a belief which can support assent to your statement.

Of course, it is a lot less work for me if you just mention that you have *Echinocereus viridiflorus* in mind, as the concept *Echinocereus viridiflorus* is presumably a basic belief, but the point is that you don't have to. Our mutual means of assent yields enough structure for me to discover this from your tempered utterance alone. What we have accomplished through the communication is my adding the property of 'capacity to endure extreme cold' to my Echinocereus viridiflorus concept. Here there is a danger of the quotation marks becoming misleading. As always, belief formation may occur on the basis of observational evidence, but it is not caused by it. The property may have been deducible from your original utterance, but I have come to believe it only through the process seeking out the appropriate evidence and then forming my own beliefs about it. As a check on this process, having brought about the right belief, I can verify that I assent to your original statement via our botany strategy. Of course, here there is room for error. I may come up with another belief which plays the same role in assent via our strategy. I pursue a belief through descent because it is what I take you to believe in order to make your stated assent. Success in this sort of coordination is really the best sort of success we can hope for. We are still in the position of estimating what beliefs we can suppose others to have, the tools for doing so have just been improved by the incorporation of a mutual strategy. At least the case of *Echnocereus viridiflorus* offers proof positive of communicative belief change employing only the kinds of evidence Davidson allows for belief formation.

1.7 What form might a reasonable strategy take?

So, we have asked for a strategy through which a particular linguistic community can assent to particular propositions. Since assent via this strategy is the manner in which beliefs are given acceptance, the strategy can be considered a sort of belief collector's guide, suggesting the manner in which tempered beliefs are to be vetted. A little of what might be required of the particular strategy adopted for scientific communication has been suggested. However, if an overarching characterization of language which allows coordination and modification of belief between speakers is to be reached, an examination of what exactly might be required of such a strategy is in order

The largest requirement of any such strategy is that it allows assent to occur in a uniform manner, that there is wide agreement about which statements can be assented to via the strategy. This is an extreme restriction on the beliefs available to a speaker. What is to be gained through such coordinative efforts? The goal of speech is typically conceptualized as expression of individual thoughts. This seems in direct opposition to sacrificing one's own patterns of assent in favor

of some strategy. What is the payoff for restricting the sorts of belief held and communicated? Through adopting the structure, a given individual gains access to more beliefs than they would have had the opportunity to collect on their own. As proficiency with the strategy increases, so does access to the beliefs of other speakers communicated through it. Through this structure, a speaker is privy to acquisition strategies for more beliefs than would have been possible without. Any speaker with the patience and skill to become adept in the strategy may too stand on the shoulders of giants.

The requirement of uniformity of assent suggests an immediate account for the 'objectivity' of science. It is less a case of absence of perspective, and more a communal adoption of tightened standards of acceptance, where the goal of the standards is exactly that their results are mutually acceptable to all speakers. The same can be said of predictivity in science. If I state "setup A causes result B," it must be in accord with the strategic requirement that you are able to reproduce the evidence which lead to my tempered utterance. If you setup A, you must get B. I must also be able to do this for myself. The result is predictive ability, but the goal is communicative coordination.

Hopefully, the argument thus far has not diminished what an amazing claim this is, even beyond its obvious incompatibility with unaugmented Davidsonian interpretation. It is possible to acquire through linguistically-directed means vastly more beliefs than otherwise accessible to a single mind in a single lifetime. Supposing language can direct avenues of fruitful belief formation is too fantastic without being accompanied at the same moment by proof positive in the form of science. Moreover, given that typical Davidsonian interpretation is a successful account of most language, what allows for this fantastic extra communication? Uncovering the answer takes another query: by virtue of what do we steer the leap made when making to statements over-specified in Dennett's sense? The answer has to be, none, or rather, none motivated by our own basic belief. How often is there a 'truth of the matter' in a individual's act of assent? The fact is, there isn't one, as assent is non-logical.

Now the utility of including many speakers becomes clear. If an individual act of assent is not logically tied to the associated basic beliefs, it is still motivated by them. Add another individual who assents to the same statement. This second person's supporting beliefs need not match the first's. In total, the over-specified statement now has the support of both belief sets. If we go out for a glass of wine, you may want to order something aged and fruity, and I may want white because red makes my head hurt. We both assent to ordering the same chardonnay. Maybe this example isn't particularly telling, but if you and I are replaced by a group of experienced wine retailers, suddenly the collective selection may be said to have something like taste. Because different individuals use different supporting beliefs to assent to the same statement, the manner in which that statement is over-specified for each is different. Stronger, the higher

the number of assenters, the less over-specified the statement becomes.

Of course, wine retailers won't all agree as to what a good wine is. To direct over-specification in this manner, the assent must be uniform across speakers. Fortunately, this is a simple verification and a requirement already made. The over specification of language relative to one belief set may be steered toward becoming motivated specification relative to many. Again, this also suggests the reason descent cannot be a perfect inverse of assent. Were this the case, there would be nothing to be gained through the strategy. Supporting belief sets are not identical, not always consciously available, and not accessible through language.

So what *can* we have access to? There isn't much, but the role of the strategy should be to maximize what access might be available. Davidson supposes similarity of belief. This will be extended to similarity of assent. To actively coordinate a belief set among many, individual speakers need to get there in the same manner. Fortunately, logically arbitrary motivation in assent need not be fully arbitrary. What a strategy can do, is suggest a method of assent by which to associate supporting beliefs to tempered ones in the hopes of attaining uniformity of assent. This is the most a strategy can ask, that speakers attempt to assent in the same fashion, carefully analyzing each act of assent and remembering its supporting beliefs. This is the best hope of attaining agreement or accounting for disagreement. Success is enforced by speakers discussing particular tempered beliefs and the means by which they were adopted. That is, the strategy itself may be linguistically established.

The requirement of uniformity carries the less evident condition that a speaker should not accept a tempered belief if they do not expect others using the strategy would assent to it as well. Competence in the strategy is a check on oneself. More impressively, an individual who can hold themselves to the dictates of a strategy might succeed in modifying their own beliefs. Notice that this extension breeds opportunity for speaker error. There is the sort of usage mistake recognition of which was taken as evidence for possessing a concept in section 1.5.3. Now a speaker may also fail at the implementation of the strategy. Going forward, the test of recognizing a mistaken assent within a strategy will serve to affirm competence.

If all this seems a bit outlandish, consider what a speaker might require to assent to the existence of something like dark matter. It is difficult to conceive of any single individual's belief set independently coming to contain such a concept, even with assent as a strategy for belief acquisition. It can only be the product of the strategically coordinated efforts of many participant speakers. Examples such as dark matter suggest another approach of strategy consideration. The further removed a tempered belief becomes from the belief set Davidsonian speakers share, through iterations of strategic assent, the stronger the uniformity required by the strategy must be. In a sense, the strategy capitalizes on the asymmetry between assent and descent, but, like all interpretation, it relies on similarity

of belief. Without gaining similarity by requiring uniformity, the statements generated by the strategy run the risk of not being correctly interpreted.

It has already been clear that interpretation in the purview of strategized assent is a more complex task than the interpretation with which we began. Davidsonian interpreters need not be consciously aware of the theory of interpretation employed, so long as their interpretations are accurate. However, recalling one's own motivating beliefs and only reasoning forward from them as others would requires active awareness of the strategy involved. Among the requirements of the strategy is a constant heed for one's own beliefs and the mechanism by which they are structured.

This has been a bit involved, but a clearer motivation and picture of strategized assent has begun to form. We began with two simple facts established in section 1.5: of the natural of variance in speaker beliefs, and the necessity of assent in over-specified language.To these we have introduced the ingredients of a strategy allowing uniform assent and speaker willingness to actively restrict strategy-topical beliefs.The outcome of this combination is a powerful one. Each speaker gains a structured understanding of the beliefs relevant to the strategy. Within this structure, others may be held to the same tempered beliefs for purposes of interpretation. The over-specified statements of the strategy can be taken to be are supported by more beliefs than are possible for a single individual to hold outside of the strategy's guidance. The uniformly accepted statements are tempered repeatedly in the assent of each speaker. In turn, this gives individuals a means, through motivated assent and belief-suggesting descent, to gain and improve upon their own native belief sets.

1.8 Back To Davidson

We are now in the position to address an example in which Davidson seems to touch on scientific communication directly, and revise a mistake of priority.

> Perceptual sentences have an emipirical content given by the situation which stir us to accept or reject them, and the same goes for the beliefs expressed by those sentences...Someone with no understanding of physics could easily come to utter the sentence 'There goes an electron' as the streak appears in the cloud chamber, while having little idea what an electron is. Understanding the sentence depends on prior theory, without which the content would be totally unlike what we think of as the meaning. But isn't theory, in a sense that extends theory to cover tacit understanding, isn't theory always needed for the conditioning of sentence to circumstance to yield the right content? Only someone knowledgeable about sailing ships could recognize on sight that he sees a brig and not a brigantine, though he might use the right words in the right situations...(Davidson, 1997, 24)

The quote serves Davidson's purpose in writing, that of emphasizing the importance of belief even in contexts of correct use. It doesn't serve, however, to make clear the difference between having an electron concept and having a brig concept. In fact, recognizing the difference in acquiring the two is actually of crucial importance when discussing belief formation. While the presence of an electron can be associated with direct observation of cloud chambers, there is no item of sense data which can reliably suggest to individual perception the existence of electrons. Embedding atomic theory in a nest of belief holism isn't really a satisfactory account, particularly when cloud chamber streaks are taken to be perceptual evidence associated with a belief for, well, *something*, and most likely something a bit more similar to beliefs about sailboats.

Davidson wants to make the point language understanding does not exist without speaker belief, and relies on the weight of atomic theory to make this distinction. The analogy made is useful as this isn't clear to his readers in the case of brigs and brigantines than it it is for electrons, exactly because of the theoretical weight involved. We have argued that this theoretical weight is not simply the product of accumulated layers of beliefs in a logically closed holistic belief set, but rather directly reflects the weight of a strategy employed by those who wish to speak about electrons, with standards in accordance to that goal.

For brigs, any mistake takes place with respect to a misalignment of knowledge and application. Davidson points out the extreme case of zero-knowledge, the end of this avenue. Still, given unlimited time and unlimited examples, any speaker could eventually form beliefs about brigs and brigantines sufficient for accurately categorizing the two. For electrons, you can make a mistake relative to the strategy as well, that of unreflective assent. If you don't have a strategically determined method of relating cloud chamber streaks to electrons, then you may know atomic theory and still declare "There goes an electron" in the full absence of understanding. For example, I myself am familiar with the basics of atomic theory. If I were presented with such a cloud chamber in a laboratory setting, even with the knowledge that streaks ought to correlate with the presence of electrons, I might be able to say I saw an electron at the right moments. I could do this even with reasonable knowledge of what an electron is. What I couldn't do is associate that knowledge with my correctly timed declarations of electron presence.

Notice this isn't just a case of my lacking the requisite theory. If I am not in a position to make the leap between what I see and the theory itself, or to consider the sort of evidence I would need to make that leap, then I am not in accordance with the strategy, which requires my individual verification. This is just as damaging as Davidson's correct use without supporting concept example, except supporting concepts and assent are required. If you wouldn't yourself be capable of assenting to the statement, or reconsidering the conditions under which you would assent to the statement, then you fail the criteria of recognizing

a mistake as a mistake. Therefore, having the appropriate beliefs and having competency in the topical strategy are both requisites for a competent speaker.

Chapter 2

Assent in Application, Pursuing Strategies and Attainment of Formality

In the spirit of empirical adequacy and viability, this chapter begins with an application of assent to the formal realm of mathematics. Unfortunately, while a simple application might be hoped for after all the developed legwork, mathematics does not yield to any simple approach to its statements or beliefs. Two conditions for a theory of mathematical truth suggested by Benacerraf will guide the discussion, which will conclude by addressing the non-smoking believer in primes from the introduction. Having established success at least in the case of mathematics, some correlations with formality as a whole will be drawn.

2.1 Mathematical Language, A Style of Understanding and Discourse

With the tool of Davidsonian interpretation augmented with assent in hand, examination of a more extended example is in order. As no language boasts better agreement than mathematical language, mathematics will be our testing ground. It must certainly land on the formal side of any defensible characterization of formality, and so will support our end goal of developing such a characterization, but mathematics itself yields no obvious analysis. The advantage gained from shedding the complexities and unreliability of sensory perception should be clear, but the opportunities to accumulate confusion become more frequent in an abstract setting.

What philosophy is to take from the existence and success of mathematics is a debate perhaps as old as the discipline itself. Fortunately, as we are interested in what makes interpretation possible and the question of belief change through communication, we can focus on evidence from mathematical practice and sidestep

a good portion of the debate. This chapter seeks to establish that belief change can occur through mathematical language, thereby categorizing mathematics as formal by the lights of the previously outlined criteria. What is required to attain this is some discussion of belief and interpretation in a mathematical setting.

2.1.1 Formality, Formalization, and Understanding

It is a story which has long remained puzzling. A mathematics student said of another student "He's so smart that he doesn't even understand what a group is, but after just reading over the (group) axioms[1], he could solve all the exercises." What is most striking about this statement, I suspect, is the difference between the most likely first reaction of a logician to this statement in comparison to what is intended, which is to say the reaction of a mathematician. Of course, the reaction of a logician or a philosopher trained in logic is to see this statement as a trivial one. Presumably the student in question has a grasp on implication rules via classical logic, which should be all that is necessary to derive any theorem from the axioms, and being able to do so makes him no more competent than any other mathematics student. In fact, perhaps the resulting proof would be better, as it suggests directly, through logical connectives, an incontrovertible path from axioms to results, and most likely clearly suggesting full logical formalization.

What remains of the statement outside of this familiar analysis? The implication seems to be that there is something else by virtue of which a mathematician reasons, the suggested *understanding* of what a group is. What might that include beyond knowledge of axioms? This is not a question which logical approaches to mathematical proof are prepared to address. What makes the student in the quote worth notice is his capacity for reasoning in the manner of logical deduction alone, in the absence of some other generally-possessed understanding. This reveals that if logical adequacy is measurably present in mathematical practice, there are other factors contributing to mathematical reasoning.

That fact alone should not be a surprising one. Rather, it is the ability of classical mathematics to be at once indifferent to all but the most basic logical considerations, while remaining in complete harmony, and sometimes even the token example of harmony, with logical dictates. Auzzouni lays out the problem in his *Why Do Informal Proofs Conform to Formal Norms?*. Note that his use of formal is a logical one, much more strict than the use in this paper, and similarly that his use of informal should be taken to mean 'in the common vernacular of mathematical practice'. "The striking point about the *Principia* program *[of Whitehead and Russell]* is this: implementing it didn't expose a crisis in ordinary mathematical practice (i.e., the revelation of numerous informal mathematical

[1]It should be noted that the group axioms are only three, requiring for a set with a binary operation, the existence of an identity element, closure under the operation, and an inverse element for each group element. The value in group theory is seen to lay in the range of what can be done from these axioms.

proofs failing to actually show what they were purported to show.) The recent computerization of formal analogues of informal mathematical proofs, and the checking of such reveals exactly the same thing...Ordinary mathematical practice already is in accord with formalization norms, and apparently has been for its very long history."(Azzouni, 2009, 15) Azzouni goes on to offer his own account of how this harmony comes about. However, his argument ultimately seeks to bring the notion of mathematical truth in the terms of logical/formal deduction.

The student's admiration and the question examined by Auzzouni straddle a divide proposed by Benacerraf (1973) in his essay *Mathematical Truth.* He outlines a basic tension in theories concerning mathematical truth: they end up making mathematics a special case in either its semantics or in its epistemology. That is, there is a fundamental tension between having the exclusive access we do to mathematical facts, and giving a semantics to mathematical language in parallel with the language at large, which generally refers to more apparent sorts of facts. Theories of mathematical truth end up upon one side or another of this division, at the expense of running afoul of whichever of the other. The project of logical formalization of what Azzouni terms 'informal' proofs favors the semantics side, interested only in mathematical propositions for their role in a deduction. An extreme example in the other direction is a hard form of mathematical realism. Positing the reality of mathematical objects makes challenging the project of explaining how all our talk about mathematics uncovers facts about these objects.

What should be apparent through the quote at the beginning of the section is that, while both factors are live considerations for mathematical practice, clearly the apparent tension has no negative effect on the practice of mathematics. The student in question was able to revert to the logical/semantic considerations alone in order to solve exercises, while typically a stronger form of epistemic familiarity is supposed. The ultimate goal of this section will to be to employ Davidsonian interpretation augmented with assent to sketch a picture of mathematical language which allows for both, and the apparent possibility for mathematicians to switch between the two priorities when reasoning.

2.1.2 Benacerraf's Conditions

To examine the theoretical tension Benacerraf describes, a more in-depth look at his two conditions is in order. Notable is that, while the later conclusions of the paper may be dated, the divide remains. The tension between epistemic and semantic considerations has already been briefly explained. Here, each of Benacerraf's conditions is treated in more detail.

2.1.2.1 The First Condition: Meaning in Mathematics

The first of these conditions is that an account "should imply truth conditions for mathematical propositions that are evidently conditions of their truth (and not

simply, say, of their theoremhood in some formal system.)" (Benacerraf, 1973, 666) Here, theoremhood may serve as a truth condition, but not absent an explanation of the link between the theoremhood as defined and truth at large. This condition is to demand some conformity with the theory of truth given for mathematics and that given for non-mathematical language. The semantic account given for natural and mathematical language must not be different in kind.

This condition serves to divorce the notion of truth from systematic and deductive correctness. This serves the intuition common among mathematicians that mathematical truth is best accessed through the action of individual proof, and perhaps through multiple proofs for the same proposition, rather than mere awareness of the existence of a correct proof. This is Rav's dreaded PYTHIAGORA machine (Rav, 1999), which simply spits out a truth value when offered a (potential) theorem. (Of course, this is a hypothetical. Rav sets aside decidability to make the point that, even if it could exist, we wouldn't want such a thing.) His argument is for the preservation of mathematical innovation. Whatever the reasoning, the point to be drawn is that mathematicians care about something beyond the truth or falsehood of a theorem (and nor are they much concerned with the logician's results about when it might be decidable.)

We will spend a bit of attention on this point, as it is tempting for philosophers, and particularly for logicians, to view classical mathematics as simply a system of axioms and the deductions that may be formalized in a logical model as described by Azzouni. This is such a well-established viewpoint, held by philosophers with experience in only basic logic all the way through those writing in philosophy of mathematics, that we will take some time to extract ourselves from it. In this viewpoint, the most interesting point of discussion is under which conditions axioms should be accepted, perhaps alongside a discussion of the implications derivable from a particular set of axioms. This motivates much discussion, particularly in the avenue of set theory and mathematical foundations (Maddy, 1997).

To see placing these debates aside is not detrimental to the project at hand, we need look no further than the reason they are central in the philosophy of mathematics in the first place. Viewing mathematics as a series of classically uninteresting deductions leaves only the axioms as talking points. However, as those with logical training should be equally quick to point out, what is interesting from a modeling perspective is often irrelevant to use. Auzzouni offers the lovely observation that, when presented with a detailed formal proof, beyond the 'informal' proof meeting the standards of mathematical practice, mathematicians fail to respond with any sort of 'eureka!' The original proof is seen as sufficient on its own. Even if basing analysis of mathematics in set theory, foundations and formalizations may be philosophically sound. We do our argument no harm by setting it aside, since we are interested in what information is relevant to interpretation.

The risk of treating mathematics from too structured a perspective is losing any idea of what motivates the structure. As Jamie Tappenden (2008, 256) points out in his essay *Mathematical Concepts and Definitions* "generalizing is much more than just picking constants and replacing them with variables. Articulating the right structures, which then can be generalized, is an incredibly involved process and it is hard to get it right.". Although this quote seems to invite a flippantly basic reading at first, it is embedded in an elevated discussion of the role of the Legendre symbol in the development of algebraic number theory. The technical details speak to the challenge and importance of generalization and definition alongside theoretical development, but will not be treated here[2]. The main goal of the essay is to suggest criteria by which desirable generalizations might be recognized. While it is possible to derive an infinity of trivial inferences from a given set of axioms, finding the valuable definitions leading to far-reaching theorems is the precisely the challenge of mathematical research. Tappenden seeks to categorize these 'natural' definitions (This is to be distinguished from the problem of axiom selection as it is argued definitions are apt in part by virtue of finding application or fruitful relation with other areas of mathematics, a property at best trivially enjoyed by axioms.). Our problem is a bit easier, for the details of such a categorization are not required, so long as there is wide agreement about which definitions are natural. Efforts at categorization of natural definitions in Tappenden's technical mathematical setting provide strong evidence at least for their existence.

Here, a much simpler picture of definition will suffice.[3] We need not know how useful definitions are obtained, so long as there is wide agreement about this utility. It matters not if axioms are fundamental truths or incidental stipulations, so long as the mathematics in question is based on a fixed set accepted by all participants.

Here, we find a second motivation for deviating from mathematics as portrayed in a first-order, or any other, model. While a logic sets up IS TRUE as the ultimate status for a theorem, mathematics values proofs of why the theorem holds over those that merely establish it. It is only when the particulars of a theorem's justification are brought into play that we can begin to discuss mathematical belief and its role in communication. Bertrand Russell emphasizes exactly this for beginning students of mathematics in *Mysticism, Logic and Other Essays*.

[2]For those with a background in number theory and/or algebra, the parallel exposition of a line of conceptual development in higher mathematics and the philosophical issues arising from it presented in the first two-thirds of the paper is compelling.

[3]Notice that 'axiom' and 'definition' are being used as loose approximates of one another here. The intent is to go no deeper into the distinction than a beginning student of analysis might, but this appearance of sloppiness hearkens back to Hilbert's concept of axiom as implicit definition (Friederich, 2011, 6). While appreciating the perspective of a prolific mathematician, we will avoid his mistake of attempting to be overly precise about axioms.

> In geometry, instead of the tedious apparatus of fallacious proofs for obvious truisms which constitutes the beginning of Euclid, the learner should be allowed at first to assume the truth of everything obvious, and should be instructed in the demonstrations of theorems which are at once startling and easily verifiable by actual drawing, such as those in which it is shown that three or more lines meet in a point. In this way belief is generated; it is seen that reasoning may lead to startling conclusions, which nevertheless the facts will verify...
> (Russell, 1918, 60)

A similar issue arises within the bounds of mathematics itself. If logical derivations fail to be sufficiently telling, there are many classical mathematics proofs which are undesirable by the same token.

Proofs that demonstrate *why* a proposition holds are valued over those which show only *that* it must hold. Aided by this consensus, proofs of the latter type are a focus for improvement. While beauty in mathematics is a vague concept, ugliness offers more consensus. The sort of proof that establishes truth but fails to elucidate it, is accepted only where there is no alternative. The use of multiple cases, or excess symbols (particularly sub- and super-scripts) or the invocation theorems do not suggest a connection to the proof statement, or the allowance of unnecessary technicality or length are all considered unaesthetic, precisely because these qualities detract from the clarity of the proof concept. Such proofs find utility when no alternative is forthcoming, but G. H. Hardy requires no definite criteria to issue his dismissal: "there is no permanent place in the world for ugly mathematics."(Hardy, 1967, 14) This is important here, because noting the difference between truth and theoremhood motivates to the role of individual mathematical belief in creating an ideal of the mathematical community.

Motivating this requirement in a Davidsonian context makes a weaker version of an argument often given by mathematical intuitionists. Intuitionists shun reductio proofs because they prefer to have constructions for mathematical objects, and while modern mathematicians are content in the setting of classical logic, proofs which establish why a proposition must hold are valued over those which merely establish that it does. This is to aid in enriching mathematical beliefs. Where proofs of the latter sort dictate propositions to which a mathematician is bound, proofs of the former enrich the mathematician's understanding of how the properties established in proof relate to the object in question.

An example of a proposition alongside both a desirable proof and an undesirable one is in order. We use the cancellation identity of binomial coefficients, both proofs of which require knowledge only of binomial coefficients.[4] We prove twice the following theorem.

[4]see Appendix A

Proposition. *Let n, r and a be non-negative integers. Then*

$$\binom{n}{r+a}\binom{r+a}{r} = \binom{n}{r}\binom{n-r}{a}$$

Our first proof:

Proof.

$$\begin{aligned}
\binom{n}{r+a}\binom{r+a}{r} &= \frac{n!}{(n-r-a)!(r+a)!}\frac{(r+a)!}{(r+a-r)!r!} \\
&= \frac{n!}{(n-r-a)!r!a!} \\
&= \frac{(n-r)!}{(n-r)!}\frac{n!}{(n-r-a)!r!a!} \\
&= \frac{n!}{(n-r)!r!}\frac{(n-r)!}{(n-r-a)!a!} \\
&= \binom{n}{r}\binom{n-r}{a}
\end{aligned}$$

□

This proof establishes the identity using the algebraic definition of a binomial coefficient.
Our second proof:

Proof. We count in two ways in which two disjoint committees, one consisting of r people and one of a, may be selected from a group of n people. The first manner of forming these committees is to select r people from the n total, and then select a people from those who were not chosen for the first committee, of which there are $n-r$. Thus we have $\binom{n}{r}$ ways to make our first selection, and $\binom{n-r}{a}$ ways to make the second. Multiplying, we obtain the right-hand side of the equality. In our second manner of forming these committees, we will first select the group of people who will certainly be on some committee, of which there are $r+a$. Of these selected people, r will form the first committee, leaving a for the second. There are $\binom{n}{r+a}$ ways to choose members for both committees, and $\binom{r+a}{r}$ ways to choose r from these. Multiplying, we obtain the left-hand side of the equation. □

This proof also suffices to establish the validity of the identity, but it also gives its reader, or writer, a notion of precisely why it is necessary, without appealing to a sequence of deductive steps which do not themselves necessarily pertain to the statement to be proved. It can be made do with the deductive syntactic correctness and algebraic manipulation of the first proof, but the approach of the

second proof is evidently to prove the theorem at every juncture, never appealing to seemingly arbitrary manipulations such as multiplication by one (Proof 1, line 3). While the first proof certainly secures the theoremhood of the proposition, the second associates the proposition with the conditions which necessitate its truth. This both secures truth formally and illuminates why it must be the case, offering proposition-focused argumentation to engender belief. This illuminates the utility of Benacerraf's first condition in justifying a belief-based account of mathematical meaning.

2.1.2.2 The Second Condition: Knowledge in Mathematics

Benacerraf's second condition is one about the nature of mathematical knowledge. The epistemology associated with an account of mathematical truth must position mathematical knowledge on par with other sorts of knowledge. Mathematics cannot serve as a mere abstract utility in the nominal sense, nor may its conditions of truth be inaccessible to the knowledge that they have been met. The account must explain why it is possible to posses the mathematical knowledge that we do. "In mathematics, it must be possible to link up what it is for p to be true with my belief that p."(Benacerraf, 1973, p 667) Once again, the tension between the two conditions arises because we must have detailed semantic knowledge of mathematical statements which create definitive information. It must be demonstrated that we are not creating fictitious entities, nor are we expounding groundless properties of objects inaccessible.

Once again, we will turn to a Davidsonian-style framework to heal this divide between semantic meaning and epistemological access. With this goal in mind, we entertain a few constraints upon candidate theories of mathematical knowledge. One characteristic feature is the surprising fixedness of mathematical content across contexts of use. A theorem, once proven, will be accepted by all mathematicians alike. If a mathematical definition exists in one cultural context, so too in any other. Beyond the expressive power being equal, the relevance of such distinctions and categorizations is given equal consideration as well. (In Tappendem's terms, this is the supposition that a definition originating in anther culture is natural.) While these claims require systematic investigation, mathematicians are likely to assume their correctness. A philosophical account of this fixedness is not forthcoming. Prediger to investigates mathematics' status as an "epistemic exception, a culture-independent discipline without any contingency."(Prediger, 2003, 1) While this yields a good description of the standard viewpoint, Prediger's formulation of the opposing position is flawed. "To understand mathematics as a cultural product means to acknowledge the human influence on mathematics. From this perspective there is no principal difference in the epistemic status between mathematics and the arts or natural sciences"(Prediger, 2003, 3). What Prediger has missed is the fact that evident human influence is not the opposite of cultural independence. Presuming we have access only to those cultures and

communications which are human, it is still entirely possible to have a culturally-independent theory which cannot be determined to be free of human influence. This is exacerbated by Prediger's further supposing that human influence, should it be established, places mathematics in the same epistemic bracket as science and the arts, presumably also cultural products. Without expending the space to dismantle this claim, it is clear that sufficient detail is lacking in Prediger's characterization of mathematics-as-cultural to offer insight into the reasons for mathematical fixedness.

Still, the possibility of wide agreement about what is held true in various mathematical settings requires some account. This problem appears particularly pressing when we recall that in radical interpretation of natural language, it is supposed that any uniformity of use arises *in spite* of differences in individual's conception, owing to some large but unspecified set of shared beliefs. Because we cannot suppose that belief is shared in a context of belief change, nor can we assume that interpretation of mathematical utterances is uniform despite strong agreement about truth, some discussion of individual mathematical belief is in order. The preceding discussion of Prediger may be applied in the case where a given culture of interpretation is a single individual. There must be some division between the communal agreement about mathematical theorems and the beliefs of the individual, otherwise individual members of the community would never have new discoveries to offer. Considering this division between the mathematical community and its members, we will, again, restrict any possible contingencies in mathematics to those which take place within standard practice.

How are we to reconcile the possibility of wide-spread agreement with the varying beliefs of individuals? Understanding this relationship will be key to analyzing mathematical interpretation, since interpretation takes place for one speaker relative to the utterance of another. We also wish to satisfy the criteria suggested by Benacerraf for a satisfactory epistemology. It must be possible for a mathematician to associate the truth of their proposition with their belief that it holds. In order to treat this issue, we make a distinction between two types of belief which are relevant to the practice of mathematics.

The first sort of belief relevant for mathematical practice is that of the community, that which may be established and transferred through proof. This type of belief, which we shall call communal belief in the setting of mathematical communication, is the most immediate sort which might be suggested in a naive theory of belief in mathematics. An agent has communal belief in a proposition when she can either generate a proof of that proposition or fully comprehend and agree with the proof of another. Proof is the definitive currency of mathematical information, provers and readers of proofs being subject to its conclusions. Communal belief is the most common sort of information exchanged in mathematics, being the focus in classrooms, journals and research gatherings. An individual faced with a communal belief in the form of a proof must accept it.

Communal belief is to be distinguished from the totality of correct mathematical proofs because it refers not to proofs themselves, but to the beliefs they instantiate in individuals. For this reason, the communal belief of each mathematician need not be identical. However, should two mathematicians be exposed to the same proof of a proposition and both accept it, then they have the same communal belief with respect to that proposition. This implies, for instance, that although Wiles' proof of Fermat's last theorem[5] is widely accepted to be correct, those who have communal belief in its truth are those few mathematicians who fully understand it. (Here we have the beginning of a divorce between mathematical truth and belief, even in the setting of practice.) Moreover, Wiles' initial proof was not the subject of communal belief because it failed to convince all its referees. Thus we see a mathematician may be a quite good judge of when communal belief will apply to a proposition, but not an infallible one. There are a few proofs, such as that of the four-color theorem, which are accepted by some sections of the mathematical community and not others. These are not considered to be the subject of communal belief, because they fail the criteria of certainty of convincing another mathematician. Should such proofs be common, they may undermine the notion of a communal belief, but their relative scarcity, anomalous origin, and recent advent will save us from this objection.

Because communal belief is evident to those external to the practice of mathematics, it will initially appear more prominent than our second type of belief. This type we shall call prover's belief, and it consists of those hunches, suspicions and intuitions about mathematical objects harbored by individual mathematicians. They are those beliefs which aspire to proof but do not (yet) have such a verification. While it is not necessary of such beliefs that a proof always exist, we suppose their source lies with the individual's conception of the objects in question, rather than faith in another's claims. That is, we do not count belief in testimonial accounts of a theorem's accuracy. These are the sorts of belief which precede an agent's self-generated proof or comprehension of another's proof, and may persist once that proof is introduced. Incidentally, this is the sort of belief which is most often romanticized in portrayals of mathematicians, that crazed genius who *understood something* making better drama than accurate portrayal. Bombast aside, it is perhaps more telling of the experience of the mathematician, being the sort of belief sensitive to insight, drive, and idiosyncrasy. Still, it is the prerogative of every mathematician harboring prover's belief to turn it into proof. Without the possibility of transformation into communal belief, prover's beliefs are of no value to mathematics.

Multiple mathematicians may have the impression that they share the same

[5]This example deviates from our rule of shunning theorems with philosophical familiarity, but the only relevant property in this context is the sheer complexity of the proof. Any mathematical theorem requiring the association of disparate mathematical branches in its proof, as Fermat's last required Wiles to prove the TaniyamaShimura conjecture uniting modular forms and elliptic curves, may be considered sufficiently obscure.

prover's belief, but since they are by definition precluded from statement in the language of proof, this impression is in principle unjustifiable. Should the mathematicians harboring it produce a proof, even in collaboration, the belief will no longer be prover's belief, it will have become communal belief. In this sense, this sort of mathematical belief bears much stronger resemblance to Davidson's characterizations of belief and the role it plays in natural language interpretation. Strong similarity based upon commonality of exposure to theories and mathematical objects is the best case option. Supposing the accuracy of the notion of communal belief, the case of multiple mathematicians supposing they share prover's belief can be fruitfully distinguished from a basic Davidsonian explanation, which would suggest that the abstract discussion is similarly theory-leaden for each mathematician. In that picture, the language shared by the mathematicians still finds its roots first in a radical-interpretation scenario based in suppositions about one another's beliefs, and this sort of communication is iteratively layered to successive heights of abstraction, increasingly further away from the base of shared belief upon which the communication is based. This relatively simplistic characterization of the process of abstraction does not allow for the notion of communal belief, in which multiple agents may be assured they have some abstract belief in common. Given the existence of communal belief, we proceed with a set of beliefs which are commonly held even in the absence of an elaborate supporting epistemological account such as that offered by Davidson. We need only adapt the appropriate epistemological criteria to ensure the possibility and existence of both kinds of belief.

There are many documented aspects of mathematical activity to which we might appeal in order to justify this distinction, but we appeal to the context-sensitivity of proof established by Löwe and Müller (Löwe and Müller, 2008)[6]. They expand upon a 'standard view' of knowledge in mathematics, that an agent knows a proposition if and only if the agent has available a proof of the proposition. In the analysis, both the notions of proof and availability emit subtleties. Adequacy of proof within mathematics varies between textbooks, presentations and research papers, even without considering those who require full-scale deductions. Availability of a proof ranges from access to a physical printed copy to capacity to generate a proof meeting certain criteria in a given time frame. They settle upon an agent having knowledge of a proposition if and only if the agent's current mathematical skills are sufficient to produce the form of proof or justification for the proposition required by the actual context.

[6]Löwe and Müller also note that it is possible to have mathematical knowledge without proof, such as the mathematical knowledge held by a general member of the public, or testimonials among researchers. We will set aside the latter case because it occurs out of a wish to build upon a result rather than understand it, so it is not a case in which a full interpretation of the proposition in question is sought–its use is only deductively motivated, a case which we have dismissed in the previous section. The former we will also discard, as we are concerned, for now, for cases of communication of mathematical theory.

We can employ Löwe and Müller's criterion for possessing mathematical knowledge to better establish the concept of communal belief. Suppose we have an undergraduate student, A, who studies mathematics and deduces by repeated observation paired with intuition that a proposition P must hold. Although P is a true proposition, any of its known proofs require tools beyond the ability of A. After repeated failed attempts to prove P, A looks it up and discovers that a proof exists, despite being unable to understand it. Where communal belief for the proposition exists for someone, for the proof is valid in some context, A is stuck with prover's belief alone. It is not a lack of comprehension of P itself, or awareness of its truth, causing the A difficulty. No existing proof can establish knowledge of P for A in Löwe and Müller's sense, and thus no proof can establish communal belief, despite her awareness of its truth. Any epistemology must allow for the existence of this sort of belief, alongside the more conventional notion of belief through proof.

2.1.3 Applying Assent in Mathematics

Perhaps it is apparent where this is leading, given the first chapter. Davidsonian interpretation gives a clear picture of how those things a speaker figures she knows, with the meaning of what she says, are linked. How can precisely that which is inextricable for Davidson be the point of division for Benacerraf? In fact, this is somewhat a false tension, for the exact tension Benacerraf addresses in theories of mathematical truth is the product of treating mathematical meaning and mathematical belief as separate forces. The Davidsonian insight is to observe that they cannot be separated from one another. It is just quite a lot easier to make the mistake of supposing they can in a mathematical setting, as the structure of the communal belief set is quite apparent. Still, mistaking the accessibility of communal belief and the ease of modeling as grounds for creating a semantics for mathematical language is the exact same mistake as introducing meanings as explanatory entities in natural language. This is not to say that formalization of mathematical proofs cannot play some modeling role, but it can't play any explanatory role when it comes to mathematical efficacy or meaning, problems for which we more typically desire semantical theories. To approach such inquiries from the right direction, we need only see that it is quite natural that a logic-based semantics for mathematical language shuts the door on a reasonable epistemology, as it must also always shut the door on the role of mathematician's belief.

Fortunately, recognizing that the problem of giving a formalized logical-semantics for mathematical language is the same mistake as giving a meaning-based semantics for natural language also immediately suggests a solution. Namely, exactly the solution we used for natural language, Davidsonian interpretation. If Davidson's requirements for belief structure were too heavy for natural language speakers, there is the consolation that mathematicians will accept more stringent

structural demands. In the first chapter, the set of tools available to a believer was enriched with assent, which allows us, or more specifically, speakers of the language, to place more specific requirements upon that structure.

In the earlier discussion of perceptual beliefs, it was noted that the most evident sorts of beliefs tend to occupy a disproportionate part of the discussion owing to their accessibility. This is particularly a factor in a mathematical setting, as abstract objects and beliefs about them are infamously inaccessible. Two primary strategies for approaching mathematical beliefs have been developed here. The first is the basic and tempered belief of the first chapter. The second is the prover's belief and the communal belief outlined in section 2.1.2.2. Not many mathematicians are likely to be cognizant of any difference between these two strategies, as the process of verification of one's own belief and the process of gaining mathematical acceptability are the same process in correct mathematical practice. Here 'correct' should of course be taken to mean 'in accordance with established mathematical strategy for abstraction.' This demonstrates the comparability of the two strategies for our purpose as well, as assent within a strategy is the conscious acquisition of a desirable belief. Aligning prover's beliefs and basic beliefs is simpler, if only because each concept is more free-form than its counterpart. Note, however, that mathematical language cannot express beliefs which are not tempered nor beliefs which are not communal. That leaves the medium in which basic beliefs are to be expressed in question. At very least, whatever we call the language which expresses basic beliefs about mathematics, it is not mathematical language proper.

Of course, when analyzing mathematical beliefs in the developed framework, we have failed to escape a difficulty earlier seen to be inherent to tempered belief, the fact that we quite often forget whether held beliefs are the product of assent or not. Quickly: $3 \times 5 =$? If the answer came to mind automatically, there was a time when it didn't (and if it didn't, there is an accessible level of familiarity through which it should.) The fact that we have all been through a phase where this required actual consideration serves again to illustrate the impossibility of disentangling basic and tempered belief. The only case when the distinction is accessible to the speaker is when the assent is fresh and the belief is novel. Fortunately, this is also the most interesting case, as it is exactly this transition which is manifest through proof.

Finally we are in a position to examine belief change through mathematical language. While there may be other possible venues, current discussion will focus on proof. Indeed, although we have not argued our claims in precisely this form, the arguments could be generalized to the claim that belief change through mathematical language occurs whenever a speaker encounters and assents to a previously unknown proof. Motivated by focus on mathematical practice, our example presented will be quoted from a graduate mathematics text. It is a lemma in graph theory, first proved by Bill Tutte, whose best known work outside

combinatorics was done at Bletchley Park, on the Lorentz cipher (Grey, 2013). As proving lemmas is not our current purpose, and not all readers can be held to a familiarity with algebraic combinatorics, the proof of the lemma will not be given in full.[7] Interested readers can find the proof in its entirety in appendix B.

Lemma. *If X is an s-arc transitive graph with girth $2s-2$, it is bipartite and has diameter $s-1$.*

Proof excerpt. We first observe that if X has girth $2s-2$, then any s-arc lies in at most one cycle of length $2s-2$, and so if X is s-arc transitive, it follows that every s-arc lies in a unique cycle of length $2s-2$. Clearly, X has a diameter of at least $s-1$, because opposite vertices in a cycle of length $2s-2$ are at this distance...(Godsil and Royle, 2001, 61)

Let's begin with the second sentence. Notice that the primary clause of the sentence contains the proposition required by the proof, a bound for the diameter. The explanation following the 'because' could be seen as justification for the proposition. Logically, it does serve this purpose, but there is not nearly enough detail to comprise a formalized proof (although this would be possible). A better reading, one that motivates the exact amount of detail given, is that everything in the secondary clause is meant to suggest to the reader a manner of convincing themselves that the proposition holds. The 'clearly' is to suggest a level of ease at which this should occur. If this is one of my first exposures to graph theory, I will sketch the graph, an exercise which the reader is welcome to join. Supposing s is 4, here are six dots in a circle connected by edges. There can be no more edges, as this is assumed to be the minimum cycle. I can count for every pair of vertices if I like, three will be the farthest apart any two of my dots can be. They are, as the proof suggests, opposite each other. If immediate generalization to accepting the claim is not apparent to me, I can repeat the exercise for a different value of s. If I posses no (or only vague) beliefs about what the diameter of a graph meeting the assumptions of the lemma might be, this sentence suggests exactly how to acquire them. If I posses misled beliefs about such a graph, this sentence shows me how to find my way.

Similarly, the opening sentence of the proof offers a proposition necessary to reach the result in the second half of the sentence, supported by the first half of the sentence. Again, the information offered before the conjunction is a minimal road map for the reader to assent to everything after the conjunction, although the concepts necessary to make this inference are too involved to be presented here.

[7]For readers completely unfamiliar with graph theory, a graph is a set of vertices and edges, where one edge connects two vertices. (This is not the technical definition but will suit the current purpose.) The *diameter* of a graph is the maximum distance between two vertices (a path of length n has n edges and $n+1$ vertices), and the *girth* of a graph is the length of its shortest *cycle*, where a cycle is a chain of vertices connected by edges which begins and ends at the same vertex.

Most interesting is the beginning, "we first observe." This language, directing the reader to a fact, is common. Establishing the claim is important for the success of the proof, but focusing the reader on the facts needed to get there takes priority.

An observant reading of this section betrays that nearly all the examples provided are combinatorial ones. This is due in part to the fact that graph theory, and sometimes combinatorics at large, suggests a very immediate point of access. Graphs can be drawn out. As with the example, accepting the reasoning in a proof in graph theory often simply requires sketching a picture and considering it until convinced. Although proof by picture is not an acceptable technique, using some visualization (of a subgraph) of the graph under consideration to reason through or produce a proof is. Why should this be so? Regardless of the footnote on page 54, the definition of a graph uses sets and does not reference pictures. Particular embeddings of a graph into the Cartesian plane (or other surface) can be approached through geometry or topology, but this sort of representation and approach is external to the graph's combinatorial properties. Reference to the definition of a graph is absent in the overwhelming majority of graph theory proofs. Baring an explanation based in definition, the purpose of reference to picture must be to *form the right idea in the reader, or, better, to suggest to the reader what evidence should be sought out to form the idea for themselves.* Even if no picture is or can be drawn, this priority remains[8]. This is not a matter of rhetoric. It is necessitated by the mathematical strategy for assent. If the proof writer cannot convince the audience that every step was worthy of assent via the communal standard, then the proof is not considered valid. If, reasoning via the strategy, even a single member of the proof's audience can draw into question even a single step, the proof is not considered valid. The burden to ensure this will not occur is on the prover. Not only should a good proof suggest how to think along in order to assent to its conclusions, it often does so as a simple consequence of being written to meet the selfsame standard for its writer.

We have shifted emphasis from the role of beliefs supporting mathematical assent to the role of a community-accepted standard for accepting a line of reasoning. To see that the shift between these two is a natural one, we need only remember that the standard is upheld through speakers and believers who must all referee which of their beliefs belong on the side of community sanctification and which must be tempered through proof. Rather than requisitioning similarity

[8]Combinatorics is a particularly interesting testing ground for extending this and related discussions. Not only does it admit a wide class of similar examples where proof steps are argued by asking the reader to simply look at the relevant object to see that a pattern holds, it also is increasingly subject to computational techniques fueled by increasing computational power. The controversy around acceptance of the four-color theorem represents a beginning, but depth is to be gained after the recognition of the accessibility of combinatorial examples in understanding mathematical communication. Of course, by the lights of assent, the four-color theorem is unacceptable as a theorem, its proof suggesting no, or rather incomprehensibly much, evidence through which the reader is to assent.

of belief, as Davidson does for natural language, we can simply point to the fact that users of mathematical language strive for such similarity, an effort which requires both an awareness as to how individual beliefs may differ, and an agreement as to how that difference can be reconciled. Moreover, by allowing belief to take an explanatory role, we have addressed Benecerraf's motivating concern, that both the semantics and the epistemology for mathematical language should mesh comfortably with the semantics and epistemology for the language at large. This divide is dissolved with the realization that our interpretive access to any language, mathematical or otherwise, is based in shared belief.

2.1.3.1 Failure to Qualify: Filtering Non-Mathematical Utterances

Of course, the strategy of assent is interesting not just for the positive analysis it can give to successful mathematical communication such as the proof above, but also for the negative result of excluding certain sorts of suspect mathematical communication. For Davidson, ability for correct use is not sufficient for having a belief. If you have taught a toddler to say 'please' and 'thank you' at the right moments, what you have accomplished is more like fixing a leaky bathtub to release water at appropriate moments than it is like unearthing a demonstration of gratitude. A concept must be gained. We can think of similar examples in a mathematical setting: a student who learns multiplication tables without understanding what multiplication is, or a student who can recite a proof from memory. There is plenty of opportunity to recite mathematical platitudes with the concept-less competence of a parrot. Still, this would have already failed Davidson's test of mistake recognition.

With assent, we have gained a second criteria. Holding a mathematical belief is no longer sufficient, it must also be validated by the mathematical means of assent. There are reports of certain born savants who simply sense whether or not a number is prime. While they do not rely on calculation or proof of any kind, their reports are accurate. In fact, these reports are a great deal faster than those of mathematicians, who must computationally verify the result. When asked how they are able to discover primality or lack thereof, they refer to something like the color or personality of the number. As interesting as such cases are from a psychological point of view, they do not meet our criteria for possessing the tempered belief that the numbers are prime, because doing so requires assenting in the same format as the rest of the mathematical community, or being able to motivate the same assent for others. Failing this is failing to have a mathematical belief at all. Certainly, these primality hunches may have some utility, even for mathematicians, but this more like the utility of a police tracking dog. They have a strong perceptive capacity to pick up on the scent of a suspect, but that hardly entails understanding the alleged crime. If they cannot relate the hunches about primality to the communal idea of what primality is through the standard of proof, no mathematical ground has been gained. Prover's belief, often called intuition

by mathematicians, can play the same role, suggesting, but not validating, a belief. For some mathematicians intuition is crucial, for others, not at all. The point is that this range is unproblematic and can be expected, for this sort of belief falls outside the communal standard of acceptance.

Now mathematical belief has been approached from opposite sides, as we have seen cases where absence of belief causes failure, but also cases where un-tempered belief causes failure as well. We are bound on one side by the requirement of concept formation, and on the other by standards for acceptable concepts, by requiring belief and demanding assent. To see that this is sufficient, we turn to the fellow who Davidson would not assume to believe things about primes or to dislike smoking on page 1. The contention was that we could not learn without asking whether he believed there was a largest prime without some sort of linguistic access to his beliefs. Strictly speaking, this contention will hold, as without some communication it cannot be verified that he employs mathematical assent. However, if this structure is established, and the fellow is a mathematician in the sense of being able to interpret tempered mathematical statements, it may be assumed that he does not believe there is a largest prime if he has any beliefs about the number of primes at all. How can such an assumption be justified? If he has assented to any statement about the cardinality of the primes, it will be the same one I or any other mathematician has. Namely, that they infinite (countably) in number, and that, since they are all positive integers, there cannot be an upper bound. Moreover, if he lacks such a belief and seeks for himself to form a belief about the largest prime through mathematical assent, he will eventually conclude it does not exist. Knowing this, I can in good faith suppose without asking him that he does not believe there to be a largest prime. If this isn't a fully non-linguistic access to belief, it does demonstrate the ability of a structure of tempered beliefs to suggest the beliefs held by an individual speaker.

It is starting to seem as if seeking an account of the fixedness of mathematical content across contexts in section 2.1.2.2 was pursuing the wrong question in the first place. The fixedness is a direct product of the uniformity demanded by every strategy of assent. Rather than focusing on the fact of the fixedness itself, we could better focus on the development of such a rich and deep theory, given that fixedness was to be required by speakers. Mathematics is not an epistemic exception in the sense suggested by Prediger, but it is a bit of an epistemic marvel for supporting such a demanding strategy of assent and thereby creating such strong agreement. If this is because mathematics is the product of the human mind, or somehow externally necessitated, is not relevant for the purpose at hand. Notice Davidson doesn't approach this or similar questions for natural language, either. In fact, claims more involved about mathematical epistemology are pretty much moot. Like Davidson, we are interested exactly to the degree that beliefs can be shared. In seeking to find an empirically adequate theory, it is the measuring devices, the speakers, which have taken priority, rather than the

world and the abstract subjects of belief, or any go-between explanatory entities. This seems a reasonable result for believers who collect sensory impressions but never justify a belief with anything other than another belief. With the addition of assent, they have simply gained a strategy to suggest beliefs to one another, and better justify beliefs to themselves.

2.2 Belief Within, and Through, Formality

So far, specific assent strategies have been examined only for scientific and mathematical language. There remains the possibility that assent, even when calibrated through a fitting strategy, is present in much larger portions of language than just the formal ones. Moreover, proceeding an example by example will bring us no closer to understanding what formal language actually is. (The dual characterization of natural language is similarly absent, but here assumptions are apparently more acceptable.) The goal of this section is to theoretically align languages arising from assent strategies and formal language, and simultaneously align non-systematic assent and basic belief with natural language. We will begin in section 2.2.1 by suggesting how strategized assent can lay the foundation for formality. Then, in section 2.2.2, the potential of assent-augmented Davidsonian interpretation in the project of characterizing formality will be briefly explored.

2.2.1 Formality Arising from Strategies of Assent

This argument must suffer from the problem which plagues any (non-formal) attempt at erecting a partition: there are always to be cases of uncertain alliance upon which must be decided in one direction or another. Such boarders are inevitably the battleground when comparing alternative partitions. Our pre-theoretical notions of where certain examples ought to land can consequently end up informing our theoretical leanings. Here, we have opted to begin with the conditions necessary for formality to develop, drawing from the (hopefully) uncontroversial examples of science and mathematics to do so. It is an emphatic aim of this chapter to continue this focus, and not to engage in any rhetorical attempts at realigning anyone's basic intuitions. Examples can serve to clarify arguments, but not as their basis; they may be causes, but not reasons.

The argument presented so far may be unassumingly compared to Davidson for taking the strategy of exposing theoretical relationships over focusing on developing a theory centered around a single concept. Davidson's stomping grounds range over not only language but decision, belief, mind, and action. We have focused on the structure of belief and its effect on interpretation. Still, the topic has been how each of these determines the other, not setting a fixed point in this balance. This has as a consequence that the resulting delineation is not a particularly strict one, and the position can be tuned within the balance in accordance

with the results desired.

2.2.2 Ground Gained on a Characterization

Still, the literature offers few abstract characterizations by which to set this balance. One is made by Dutilh Novaes (2012), as extended from (Dutilh Novaes, 2011) which separates her encyclopedic chronicle of historically-important notions of formality. Much of the discussion in the latter work can be addressed via this separation, and her focus on developing a cognitive account. Dutilh Novaes sorts formality into the formal as *de-sematification* and the formal as computable. We treat the types in that order, the former being the thornier criteria. De-semantification is defined as "an abstraction from *all meaning whatsoever*"(Dutilh Novaes, 2012, 12). Under this notion is to fall at least Hilbert's formal program and certain readings of Kant. It is difficult at first to see what, exactly this is meant to suggest. The details offered all seem to pertain to mathematical abstraction. De-sematification offers metaphysical freedom, allowing indifference to considerations of existence or actuality. Are infinitesimals 'actual' numbers? As they are de-semantified, we need not ask. De-semantification is also to offer epistemic freedom, allowing indifference to interpretation. The latter sort of proffered freedom we can reject out of hand. Without ability to interpret, there is no ability to state, as both rely on the underlying belief. As for the other freedom on offer, freedom of metaphysical indifference, one might inquire as to the extent to which it is generally restricted. Relevant metaphysical constraints in this analysis (if any) are those imposed by belief. That should not be much of a burden, considering it is our best and only intermediary for perception.

There is a bigger point if de-semantification is to be concieved as a generalization of the process of mathematical abstraction, and we have already done the legwork to get there. The de-semanticized conception of mathematical abstraction represents a whittling away of the unwanted, shed as 'meaning.' What remains is only a shell to used as a placeholder in reasoning. Leaving aside a Davidsonian-style objection, we can instead offer strategized assent as a positive contrast. It too has the result of shedding excess concepts for its terms, those supporting beliefs which one or more individuals may deem relevant for assent but not accepted by the whole. The difference is that the remaining concepts endure due to preference. A profitable mathematical abstraction has the utility of suggesting relevant beliefs to its audience. Contrast this with a de-semantified conception of mathematical abstraction. The abstraction suddenly consists in only what was not shed, an apparently arbitrary leftover. Its large degree of freedom leaves no impetus for its use, and the rules governing its function are unmotivated. We hardly need ask which account better serves the fact of mathematical reasoning when choosing between abstraction as the motivated conferral of preference and abstraction as delousing tank of the actual. Moreover, as mathematical communication follows assent, as does all language, we can extend the

result as Benacerraf would hope to. The strategized assent approach to formality provides grounds for dismissing de-semantification as a worthy characterization.

Dutilh Novaes' other sense of formality is that of the computable. This is computable in roughly the sense of playing well by rules, of being externalizable to machine reasoning, in the basically familiar sense. More interesting is the notion of "computable as in 'not requiring any insight or ingenuity' " (Dutilh Novaes, 2012, 19), which may be a consequence of the more standard definition but does shed a different light. The requirement that formality adheres to rules is a relatively straightforward, and therefore unobjectionable, one. What the presented account sought to expose is the motivation for the implementation of these rules in the first place. Remember that in order to do this, assent is defined as at least a bit arbitrary, if not always insightful. It is a case similar to the one made above. The interesting factor is not the presence of rules, it is what motivates the selection of the rules in force.

Having analyzed the only forthcoming characterization, our balance is not even so much re-calibrated, and the work of solidifying the characterization of the formal must be left as grounds for further research, to which we offer the tools developed here.

Conclusion

An asset of the Davidson's approach is his ability to elucidate the relationships between the theories in a wide range of philosophical debate. We have argued for one such relationship between the strictness of a strategy for assent and an increased level of formality. Following Davidson, we hope to have added a useful tool for discovering balance in an otherwise unforgiving playing field. We have argued that one extreme of that balance is formal language, coupled with a particularly uniform strategy. The other extreme is natural language, where we, and Davidson, began, in the absence of strategizing. In fact, it is not exactly the same beginning, as assent provides the enrichment of allowing disagreement of belief while preserving interpretation. Thus even in the absence of questions about formality and linguistic belief change, we have made a small contribution.

However, more interesting are the new directions in which the developed concepts may take us. At the very least, the relationship between a strategy for assent and formality suggests that the natural is not reducible to the formal. Alternatively, rather hoping to fix a point in that balance, maybe it is better to conclude the boundary between the formal and the natural is a fuzzy one. This, at least, suggests the absence of success in the past. Moreover, it would seem to be supported the arbitrary manner in which speakers sometimes forget which beliefs are tempered and sometimes don't.

More progress has been actually been made on formality than just the notion of a strategy for assent. Wide-spread agreement between speakers is not new to the formality discussion, or particularly interesting. Much more interesting is that any possibility for capitalizing on overspecification requires speaker disagreement and the uncertain leap of assent. That this asymmetry leads to epistemic productivity in a group of speakers with a range of beliefs, particularly when they assent in slightly different manners, is perhaps the best jumping off point for further work. In fact, this thesis can be seen as a blue-print for a much more detailed theory along the same lines. Deeper examination of basic concepts will certainly enrich the results.

Finally, perhaps enough ground has been gained to invert the perspective and carve out a new role for the examination of mathematical practice. If we have enough understanding of what formality entails, mathematics will not be relegated to the unreachable pedestal of best case, but can be treated as a func-

tioning end in its own right. This will be, in the end, the only way to truly benefit from the strength and structure of mathematical reasoning. Of course, the manner in which a sufficient set of beliefs to operate within mathematical practice might be obtained is a different question.

Appendices

Appendix A

Binomial Coefficients

A binomial coefficient, $\binom{n}{k}$, gives the number of possible selections of k element subsets of sets containing n distinct elements, where $k \leq n$. It is read "n choose k." For example, $\binom{3}{2} = 3$, where the three elements may be interpreted as people, only two of which fit in a vehicle.

To derive the algebraic formula, we consider how many ways n distinct objects may be ordered. For the first position, there are n possibilities. For the second position, there are only $n-1$ possibilities, as one object already occupies the first place. Similarly, there are $n-2$ possibilities to occupy the third position and so forth. Notice that there is only one possibility for the final position, as only one object will be remaining. Thus, the number of possible orderings of n objects is

$$n \cdot (n-1) \cdot (n-2) \ldots 3 \cdot 2 \cdot 1$$

This formula is abberviated, for arbitrary integer n as

$$:= n!$$

and is read as "n factorial."

Of course, our goal is not to find the number of possible orderings for n objects, but to find the number of possible different k subsets. That is, we are interested only in the first k objects in the ordering. Thus, the ordering of n objects may be considered as the first k objects, relevant for our purposes, and the next $n-k$, about which we do not care. Since the order of these final $n-k$ objects must not count toward our total, we divide it from the number of possible orderings of the original n objects. Fortunately, we know the way to order $n-k$ items is given by $n!$. The formula given by dividing $\frac{n!}{(n-k)!}$ gives the number of possible ordered k subsets of n items. This should make sense; we have taken a list of n ordered items and unordered the last $n-k$ of them.

Still, the initial k items remain ordered. In fact, we also do not care in which order these items appear. Thus, we must also divide by the number of possible orderings of k objects, $k!$. This gives the formula $\frac{n!}{k!(n-k)!}$, the number of possible ways to choose k elements from a set of n distinct elements.

Appendix B

Tutte's Lemma

Lemma. *(Tutte) If X is an s-arc transitive graph with girth $2s-2$, it is bipartite and has diameter $s-1$.*

Proof. We first observe that if X has girth $2s-2$, then any s-arc lies in at most one cycle of length $2s-2$, and so if X is s-arc transitive, it follows that every s-arc lies in a unique cycle of length $2s-2$. Clearly, X has a diameter of at least $s-1$, because opposite vertices in a cycle of length $2s-2$ are at this distance. Now, let u be a vertex of X and suppose for a contradiction that v is a vertex at distance s from it. Then there is an s-arc joining u to v, which must lie in a cycle of length $2s-2$. Since a cycle of this length has diameter $s-1$, it follows that v cannot be of distance s from u. Therefore, the diameter of X is at most $s-1$ and hence equal to $s-1$.

If X is not bipartite, then it contains an odd cycle.; suppose C is an odd cycle of minimal length. Because the diameter of X is $s-1$, the cycle must have length $2s-1$. Let u be a vertex of C, and let v and v' be the two adjacent vertices in C at distance $s-1$ in C at distance $s-1$ from u. Then we can form an s-arc $(u,\ldots,v,v')$. This s-arc lies in a cycle of C' of length $2s-2$. The vertices of C and C' not internal to the s-arc form a cycle of length less than $2s-2$, which is a contradiction. □

Relevant definitions are too many to be given here, but all are included in Godsil and Royle (2001). The lemma is ultimately used to show that an s-arc transitive, connected graph with girth $2s-2$ is distance transitive, a considerably stronger result. It will not be proved here, despite our theoretical aversion to simply stating results.

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