When an Algorithm Cannot Help You Find a Wife: Modeling Two-Sided Matching Markets Using Stochastic Matching

MSc Thesis *(Afstudeerscriptie)*

written by

Wouter P.J. Kroese
(born August 14th, 1989 in Heemstede, Netherlands)

under the supervision of **Dr Michael Franke**, and submitted to the Board of Examiners in partial fulfillment of the requirements for the degree of

MSc in Logic

at the *Universiteit van Amsterdam.*

Date of the public defense:
August 17, 2015

Members of the Thesis Committee:
Dr Maria Aloni
Dr Ulle Endriss
Dr Michael Franke
Dr Nina Gierasimczuk
Dr Sonja Smets

INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Acknowledgements

Most of all I want to thank my supervisor Michael Franke. You have been a great supervisor and taught me a lot about doing research. I really appreciate the fact that you managed to have me preserve the fun of chasing my own ideas and playing around in my model, but continually helped me to make this all contribute to a decent academic work. I was furthermore impressed on your quick and thorough understanding of my work, with corresponding helpful feedback, although it was outside your field most of the time. Next to my supervisor I want to thank Maria Aloni, Nina Gierasimczuk, Sonja Smets and Ulle Endriss for being part of my committee. I also want to thank Femke Bekius for her thorough and helpful proof reading.

I want to thank Alvin Roth from Stanford University for helping me to pin down the core of my ideas and welcoming me to the weekly Market Design coffees, where he introduced me to many interesting researchers. Amongst them was Peter Biro, who has been extremely helpful, both at Stanford University as through online correspondence. I want to thank Peter for guiding me through the field of Stochastic Matching. I am also flattered by, and thankful for, Peter's invitation to work together on further research. I want to thank Jonathan Bendor, Marcus Feldman, Steve Callander and Daniel McFarland from Stanford University for looking at my work from an Economics, Biology, Business and Sociology perspective respectively. I want to thank Vaclav Petricek for inviting me to the headquarters of datingsite eHarmony in Los Angeles, to discuss how my work could be directly applied to help people find the love of their lifes. I also want to thank Bogdan State from Facebook for investigating opportunities to work with data from Facebook. Furthermore I want to thank Pieter Gautier from the Vrije Universiteit and Matthijs van Veelen from the Universiteit van Amsterdam for discussing the economics of love with me.

Lastly, I want to thank my parents for always supporting me and I want to thank Lara for making the many long nights at the Amsterdam Science Park a great way of spending my time. Ending up in a stable matching with you is the greatest result of my master's.

Abstract

In Stable Marriage Problems two groups of agents have preferences over being paired with one another. The Deferred Acceptance Algorithm is a great and elegant tool for finding stable matchings for these Stable Marriage Problems, where no blocking pair, consisting of two agents that would rather be matched with one another exists. In many real-world two-sided matching markets, like people in search of a romantic partner, it is however not possible to execute a procedure like the Deferred Acceptance Algorithm. We constructed a stochastic matching model in which all agents randomly meet another agent from the other side of the market in consecutive daterounds, with whom they pair up if they form a blocking pair. We first show that our model will converge towards stable matchings. In our model agents have randomly assigned preferences over all agents of the other side of the market. We show that our model obtains stable matchings, which on average yield both higher and more egalitarian payoffs than in the Deferred Acceptance Algorithm or in a randomly selected stable matching. Furthermore we extend our model such that it can be used for simulating and improving real-world two-sided markets.

Table of contents

Chapter 1

Introduction

1.1 Motivation

The greatest value of Game Theory is possibly in supporting people or institutions in making better strategic choices. Game Theory potentially is very valuable by its application to social and economic problems. Game Theory could for example be used to learn how markets can be designed such that more preferred equilibria will be obtained. Many real-world problems however seem to complex and noisy for Game Theory to have value in supporting to make better decisions. This thesis will form a starting point for investigating the power of agent-based models, for better understanding real-world two-sided matching markets. We are convinced that agent-based models can be of great power for explaining dynamics and results in real-world markets from a bottom-up perspective. Therefore, we build a stochastic matching model for two-sided markets, where two groups of agents have preferences over the agents of the other side of the market and have to be matched in pairs. This thesis forms a starting point for using agent-based models to better understand the behaviour of agents in real-world two-sided markets like the job market, people renting a house and people in search of a romantic partner. Our goal is to build a model that could be further extended by researchers and policy makers such that they can use it to understand and improve the outcomes in two-sided markets respectively.

This thesis will be focusing on two-sided matching problems, so called Stable Marriage Problems, consisting of two sets of agents, $M = \{m_1, \dots, m_n\}$, the male agents, and $F = \{f_1, \dots, f_m\}$, the female agents. All agents of M have a complete, strict and transitive preference order over F and the other way around. This means every agent is able to rank being matched with every one of the agents from the other sex, from the most to the least attractive and finds no two agents equally attractive. The goal of the agents is to be paired with an agent they prefer the most. Gale and Shapley (1962) devised the Deferred Acceptance Algorithm which gives us a stable matching for every instance of the Stable Marriage Problem [6]. In stable matchings no two agents would rather be matched with each other, than with the agent they are currently matched with. Furthermore these matchings are optimal in the sense that for one side of the market, all agents are better off than in any other stable matching.

The Deferred Acceptance Algorithm provides us with an elegant tool to find stable matchings for two-sided markets where both sides of the market have preferences over one another. In many real-world cases, however, there is no central authority to execute such an algorithm and agents do not have the resources to have an overview and access to the whole market. This might keep agents from finding a stable matching. Take the job market and people in search of a romantic partner as examples: A man has no overview of every potential job or every potential partner, let alone does he have the time to get the chance to apply for every job or date every woman. Furthermore, job offers will be valid for limited time only and women won't wait forever after offering their love to a man.

Illustrated and inspired by people in search of a romantic partner, we build a model for two-sided markets where all agents have preferences over one another but where there is no central authority. In our model agents furthermore have limited resources such that they have no full overview of, and access to, the whole market.

In our model agents from different sides of the market meet each other randomly in sequence in consecutive daterounds. Agents will also meet new potential partners when they are already in a relationship. Every dateround every agent will be randomly assigned a "date" with an agent from the other side of the market. A male and a female agent can only pair with each other, when they form a blocking pair at the moment they "date", and thus both strictly prefer being paired with their "date" above their current partner. Agents have no overview of the market and do not know which agents they will meet next, nor can they review their decision later on. Agents are seeking to maximize the payoff they assign to being matched with an agent from the other side of the market, at the end of the limited time available. We are interested in investigating the outcomes and dynamics when we run simulations for different parameter settings and compare the outcomes to those obtained in the stable matchings obtained with the Deferred Acceptance Algorithm. We are furthermore interested to find out whether our agent-based model can be extended, such that it can be used to simulate real-world two-sided matching markets, like people in search of a romantic partner or the job market.

1.2 Thesis build-up

Theoretical Framework

In Chapter 2, we cover the theoretical framework consisting of the Secretary Search Problem and the Stable Marriage Problem. The Secretary Search Problem is used for defining our model. The solution to the Stable Marriage Problem, the Deferred Acceptance Algorithm, is an elegant and efficient procedure that always gives a stable matching in two-sided matching markets. We use the results of this algorithm as a benchmark for the results of the simulations of our model. We also discuss how our model differs from the models and problems discussed in this chapter.

Model

In chapter 3, we define the basis of our stochastic model for simulating real-world two-sided matching markets. We describe how payoffs and utility of agents are defined. We then describe how agents get the opportunity to pair up and to which rules they adhere when they decide to do so.

Convergence towards a stable matching

In chapter 4, we prove that our model will ultimately converge with probability 1 towards a stable matching. Which means that our model ends up in matchings between the two sides of the market where no two agents form a blocking pair, which means they had rather been paired with one another than with their partner in the obtained matching. We furthermore show that our model can converge towards stable matchings other than the ones resulting from the Deferred Acceptance Algorithm.

Simulations

In chapter 5, we start by describing how preferences are randomly generated, based on the two parameters p and s that influence the objectivity and symmetry of the preferences respectively.

Thereafter, we will first investigate, by running simulations, what the effects of changing the parameter values on the results of the Deferred Acceptance Algorithm are. In particular, we will investigate what the effects are of making the preferences of agents more or less objective or symmetric on the number of rounds it takes to obtain the stable matching in the Deferred Acceptance Algorithm. In addition we consider the payoffs the agents obtain in these matchings.

Thirdly, we investigate why our model obtains better results than the Deferred Acceptance Algorithm. We do this by listing all stable matchings for 450 instances of the Stable Marriage Problem with uniform random preferences. For every instance we then run 500 simulations in our model and investigate which stable matchings are more often obtained than others.

Extensions

In chapter 6 we make extensions to our model and thereby take a first step towards modeling real-world two-sided markets. First we construct a more advanced procedure to investigate the symmetry and objectivity in preferences. We then extend and adjust our model where the agents and simulations seem to differ critically from real-world two-sided matching markets. Thereafter, we construct a model for instances of the Stable Marriage Problem with relatively objective preferences, in which agents have some awareness of their attractiveness. We furthermore add a cost to being in a relationship by having agents meet less new agents once they are in a relationship. This will make it less attractive for agents to start a relationship when single, since it reduces the chance of finding a better partner when they are in one. Following, we will therefore extend our model with pickiness which makes agents not want to start a relationship with any agent they meet when single.

Discussion

In Chapter 7 we will discuss possible downsides of our research method and possible points of improvement for our model.

Conclusion

In Chapter 8 we will elaborate on what we can conclude from our research and the simulations in our model.

Further research

In Chapter 9 we will do some suggestions for further research, such that our model might become of even greater value for simulating, understanding, improving and clearing two-sided matching markets.

Chapter 2

Theoretical framework

We use two mathematical problems as theoretical framework, the Secretary Search Problem and the Stable Marriage Problem. The Secretary Search Problem is mainly an inspiration for constructing our model. Because of the crucial differences between our model and the Secretary Search Problem, the solution to the Secretary Search Problem seems to be of little value for our model. As opposed to the Secretary Search Problem the solution to the Stable Marriage Problem, the Deferred Acceptance Algorithm, will be of great value for our research. In the first place, the proof that shows that there always exists a stable matching will form an important element in several of our proofs. Furthermore, the resulting matching from the Deferred Acceptance Algorithm will form a benchmark for assessing the results obtained in our model.

2.1 Secretary Search Problem (Martin Gardner 1960)

The Secretary Search Problem was first mentioned by Martin Gardner in a column in the Scientific American in 1960 as explained by Ferguson [5]. The problem is as follows:

A businessman has n applicants for the position of secretary. The potential secretaries are interviewed sequentially in random order. Immediately after the interview the businessman has to decide to reject or accept the applicant. The businessman cannot come back from decisions made. The businessman can rank all the applicants he has interviewed so far but does not know the quality of the applicants to come. The question now is what the optimal stopping rule is, that maximizes the probability of accepting the best possible candidate.

2.1.1 Solution to Secretary Search Problem

There is an elegant solution to this problem, first published by Lindley [9]. The optimal stopping rule is always rejecting the first $\frac{n}{e}$ applicants after the interview (where e is the base of the natural logarithm) and then stopping at the first applicant who is better than every applicant interviewed so far (or continuing to the last applicant if this never occurs). Sometimes this strategy is called the $\frac{1}{e}$ stopping rule, because the probability of stopping at the best applicant with this strategy is about $\frac{1}{e}$, already for moderate values of n.

2.1.2 Importance for this thesis

There are a number aspects about the Secretary Search Problem that inspired our model on two-sided markets. In a lot of real-world two-sided markets it is hard to have an overview of all possible partners. Furthermore, it is hard to know which opportunities will come later on and where you have to decide on the specific moment an opportunity comes along whether to take your chances or let the opportunity go. The Secretary Search Problem elegantly models an agent's consideration on whether the current opportunity is the best that will ever come or that the agents deems it likely that better opportunities will come along later. Furthermore, our model and the Secretary Search Problem involve stochasticity in similar ways.

2.1.3 Differences compared to our model

The most important difference between our model and the Secretary Search Problem is that we will model two-sided markets. In the Secretary Search Problem all applicants want the job, where in two-sided markets, like the search for romantic partners it takes two to tango. To stick with the secretary example: In our model the applicants can apply for different jobs, over which they all have their preferences. On the one hand this makes our model more complex such that it will be harder to find elegant results like the $n \div e$ stopping rule. On the other hand we might find interesting dynamics because of the interaction between the two sides of the market.

In the Secretary Search Problem the businessman will meet all candidates, as long as he does not say “yes" to an applicant and he will meet all candidates only once. Furthermore, the businessman is aware of this fact. In our model, however, agents do not know which or how many potential agents they could pair up with. Because of the stochastic elements it is also possible that they do not get to “date" and thus pair up with every possible partner. On the other hand it is possible that an agent “dates" a potential partner more than once.

The final important difference between the Secretary Search Problem and our model will be that in the Secretary Search Problem the businessman has to employ a “stopping rule". When the businessman accepts a candidate he will meet no other candidates anymore and cannot come back from his decision. In our model agents will however keep meeting new possible partners, also when they are already paired with an agent. A male agent m_a can decide to break up with female agent f_b when male agent m_a meets an agent f_c with whom male agent m_a would rather be paired up with than with his current partner female agent f_b, but can only do so if agent f_c also prefers being paired with male agent m_a above being paired with her current partner.

2.2 Stable Marriage Problem (Shapley and Gale 1962)

The Stable Marriage Problem was first mentioned, and directly solved, by Gale and Shapley [6]:

In a community of n men and n women all people from the one sex rank all people from the other sex according to their preferences. Assumed is there are no ties in the ranking. Gale and Shapley first ask whether there is a way to marry all the men and women such that we obtain a stable matching. Thus, does there exist a matching where male agents are matched with female agents, in which there is no blocking pair, consisting of a man and a woman who would have rather married each other than the partner they are assigned to in this matching. Secondly, Gale and Shapley ask whether we can find a stable matching optimal for either all males or all females in the case multiple stable matchings exist. Here optimal means that every person from this sex is at least as well off in this matching as under any other stable matching [6].

2.2.1 Deferred Acceptance Algorithm

Shapley and Gale devised the Deferred Acceptance Algorithm, which for any Stable Marriage Problem, with any number of male agents n, any number of female agents m and any strict preferences, ensures the existence of a stable matching [6]. The Deferred Acceptance Algorithm works as follows:

Each man proposes to his favorite woman. Each woman who receives more than one proposal rejects those that proposed her, except her favorite of her proposers. These women don't accept their favorite proposers right away but engage with them to allow for the possibility that someone they like more might propose later. In the second stage all the rejected men propose to their next best choices. Each woman that received proposals chooses her favorite from the set consisting of the man she is engaged with (if she was proposed before) and the new proposers. Again she gets engaged with her new favorite and rejects all others. This second stage is iterated until (within $n^2 - 2n + 2$ stages) every woman will have received a proposal. Since as long as not all women have received a proposal there will be rejections and thus new proposals. However, no man can propose to a woman more than once. When the last woman gets her proposal, each woman will have to accept the man she is engaged with. We now have every man matched with a woman in a stable matching.

2.2.2 Deferred Acceptance Algorithm always obtains a stable matching

Theorem 1 (Gale and Shapley, 1962)**.** *The Deferred Acceptance Algorithm always obtains a stable matching.*

Proof. Assume male A and female B form a blocking pair, which means they are not matched with each other, but would rather have been matched with one another than with the agents they are currently matched with. If male A would have preferred female B above his current match, he would however have proposed female B before he would have proposed his current match. There thus must have been a man who female B likes more then male A, since she did not accept male A at that moment. Male A and female B thus not form a blocking pair and since this would hold for any blocking pair we thus have obtained a stable matching [6]. □

Note that there is complete duality for the proposing party and the proposed party, and we could thus switch the roles of the males and the females.

2.2.3 Extensions

The Deferred Acceptance Algorithm also results in a stable matching for instances of the Stable Marriage Problem with unequal numbers of male and female agents. When we have n agents of the proposing party (males) and m agents of the accepting party (females) and $n < m$ then we execute the algorithm until n agents of the accepting party (females) are proposed to. When $n > m$ then the procedure continues until every agent from the proposing party (males) is either engaged with an agent of the accepting party (females) or rejected by everyone of them.

Furthermore, we can extend this solution to instances where some possible partners are unacceptable to some agents. In this case, it could happen that some agents prefer being single over being matched with some of the agents of the other side of the market. This means that it could be the case that the Deferred Acceptance Algorithm obtains a stable matching in which not all agents are matched.

2.2.4 Optimal stable matching for the proposing side of the market

The Deferred Acceptance Algorithm not only gives a stable matching, it also gives the optimal matching for the proposing side of the market (in this case the males). This means that every person from the proposing side of the market is at least as well off under the matching obtained by the Deferred Acceptance Algorithm, as under any other stable matching [6].

Theorem 2 (Gale and Shapley, 1962)**.** *The Deferred Acceptance Algorithm always results in the optimal stable matching for the proposing side of the market.*

Proof. We call a female agent (accepting) “possible” for a male agent (proposing) if there exists a stable matching in which they are matched. We now do a proof by induction (where the empty matching at the beginning of the procedure forms the base case). Assume that somewhere during the Deferred Acceptance Algorithm no male is yet rejected by any of the females with whom he would form a pair in a stable matching. Now assume female B has proposals from male A,B,C and D of which she rejects male D. We now must prove that female B is “impossible” for male D. We know that males A,B and C preferred female B above all other females, except the ones that males A,B and C have proposed to before, but got rejected by and which thus are impossible to males A,B and C. Now consider a matching where female B is paired with male D and where all other males are matched with females that are possible to them. This means at least one of the male agents, say male B, is paired with a female that is less favorable to them than female B. This means that the matching is unstable because female B and this male B would rather be with each other than with their current match. Thus this assumed matching is unstable and thus female B is impossible for male D. We now have shown that the Deferred Acceptance Algorithm gives the optimal stable matching for the proposing side of the market. [6]

□

2.2.5 Pessimal stable matching for the accepting side of the market

The Deferred Acceptance Algorithm thus gives a stable matching that is the optimal stable matching for the proposing side of the market (in this case the males). For the accepting side of the market (females), the Deferred Acceptance Algorithm, however, gives us the least optimal stable matching. Where in the least optimal stable matching every agent from the accepting side of the market is always worse off under the matching obtained by the Deferred Acceptance Algorithm than under any other stable matching.

Corollary 2.1. *The Deferred Acceptance Algorithm always results in the pessimal matching for the accepting side of the market.*

Proof. Suppose the Deferred Acceptance Algorithm with the male agents as proposing party gives us the optimal stable matching μ_s for male agents. We claim this stable matching is the pessimal stable matching for the female agents, meaning that every female agent would be at least as good off in any other stable matching. We now do a proof by contradiction. Suppose female agent f_a is matched with male agent m_a in μ_s, but there exists a stable matching μ_w in which female agent f_a is matched with a male agent m_y, who she finds a worse partner than male agent m_a. Let female agent f_x be the partner of male agent m_a in μ_w. We know, however, that male agent m_a prefers female agent f_a above female agent f_x, since in the optimal stable matching for males μ_s male agent m_a is paired with female agent f_a. We thus have that μ_w is no stable matching, which contradicts our assumption. We thus have shown that μ_s is the pessimal stable matching for the female agents.

□

2.2.6 Complexity class

Theorem 3. *The Deferred Acceptance Algorithm is in complexity class P.*

Proof. Assume we have an instance of the Stable Marriage Problem with n male agents and m female agents. Assume that the male agents are the proposing party in the Deferred Acceptance Algorithm. The Deferred Acceptance Algorithm consists of rounds in which the male agents that are not engaged propose to their favorite female agent that they are not rejected by yet. The female agents then engage with their favorite proposer or stay engaged with the same agent as in the previous round. Every round thus maximally n proposals and rejections take place. The Deferred Acceptance Algorithm finishes when either all male or all female agents are engaged. Since all agents will only propose or reject every agent once, we know that within $m \times n$ stages either all male agents will be engaged or all female agents will be proposed to. We can thus conclude that the Deferred Acceptance Algorithm obtains a stable matching within polynomial time ($\mathcal{O}(q^2)$, where q is the largest of n and m). When $n = m$ we have that it takes at most $n^2 - 2n + 2$ stages to obtain a stable matching [6].

$\square$

2.2.7 Importance of Deferred Acceptance Algorithm for this thesis

The Deferred Acceptance Algorithm will be very important for our research. In the first place we use the fact that there always exists a stable matching for any instance of the Stable Marriage Problem in many of our proofs. Furthermore, the optimal stable matchings that result from the Deferred Acceptance Algorithm form the benchmark that we will compare the outcomes of our simulations to. Although properties like fairness and optimality are hard to define, stable matchings seem good outcomes for two-sided matching problems since we have that every agent will be matched and since there exists no pair of agents that would rather be matched with one another than with their current partner.

2.2.8 Shortcomings of Deferred Acceptance Algorithm

The Deferred Acceptance Algorithm is more than an elegant result for a mathematical problem. In the 1980s Alvin Roth applied the Deferred Acceptance Algorithm to solving many real-world matching problems including the assignment of new doctors to hospitals, students to schools, and organs to transplant recipients [10] [11] [12]. Alvin Roth and Lloyd Shapley were awarded the Nobel prize in economic science “for the theory of stable allocations and the practice of market design” in 2012. Alvin Roth and Lloyd Shapley have shown that by introducing a central authority we can better match children to schools and doctors to hospitals. For many real-world two-sided matching markets it is however not realistic to apply the Deferred Acceptance Algorithm for finding a stable or in any other sense good matching.

Central authority

For many real-world matching problems, like people in search of a romantic partner, it is not feasible and probably even unwanted to introduce a central authority to obtain a stable matching.

Overview of the market

Even if agents have strict preferences over every possible pair they could be part of, in many real-world situations agents would not have a full overview of the whole market. A recent graduate could for example be able to rank every possible job, but she might not be able to be aware of every possible vacant position.

Access to the market

Even if agents are aware of some or all of the possible partners, these partners might not be available at any time. In many real-world two-sided markets opportunities to form a pair come by, for which agents do not have endless consideration time. In the noisy real world there seems to be a lot of randomness and agents will not have an overview of the whole market, let alone have access to it. People for example have no resources to date every possible partner or apply for every available job. Very often in real-world markets chance influences which possibilities for forming pairs arise.

Chapter 3

Model

3.1 Goal of the model

The Deferred Acceptance Algorithm thus provides us with an elegant procedure to find optimal stable matchings for instances of the Stable Marriage Problem. We have observed that in many real-world two-sided matching markets there is no central authority to execute this algorithm and agents do often not have the resources to have an overview of, and access to, the whole market at any time. Besides that, it also seems to be the case that in many real-world two-sided matching markets agents do not behave anything like agents in the Deferred Acceptance Algorithm. In this thesis we want to use agent-based modeling to improve and better understand the matching process in two-sided markets that are too complex or noisy to execute the Deferred Acceptance Algorithm. We do this by constructing a model that takes into account the shortcomings of the Deferred Acceptance Algorithm. In the first place the goal of our model is to provide researchers with a tool to simulate and better understand the behaviour of agents in real-world two-sided matching markets. We however will find that our model can also be directly used to obtain stable matchings in which on average higher payoffs are obtained than in the Deferred Acceptance Algorithm and higher than in an randomly chosen stable matching, without having the need of a central authority. Furthermore, we find that our model obtains stable matchings that on average are both more egalitarian than the stable matchings obtained with the Deferred Acceptance Algorithm and than a randomly selected stable matching in two ways. Firstly, the differences in average payoffs between the male and female agents are on average smaller. Secondly, the difference in payoffs between the best and worst scoring agent of an instance is smaller.

We have built a model for two-sided matching markets where all agents have preferences over all agents from the other side of the market but where there is no central authority and agents have limited time and resources to find their optimal partner. Just as in the Secretary Search Problem agents have no overview of all potential partners. In our model, agents from different sides of the market meet each other in sequence and can only decide to match with an agent from the other side of the market, when at the moment they meet, they both choose to. In our model, randomness influences with which agents of the other side of the market an agent gets the chance to form a relationship with and when this possibility to pair up occurs. This is also the case in the Secretary Search Problem. The agents in our model have no overview of the market and do not know which agents they will meet next, nor can they review their decision later on. As opposed to the Secretary Search Problem agents do not meet all agents serially. This means not every agent of the other side of the market has to be met and it could be the the case that an agent meets a particular agent more than once. Contrary to the Secretary Search Problem, where no new applicants are met after the businessman accepted a candidate, in our model agents keep meeting agents after they started a relationship and our model does allow for break-ups. Thus when male agent m_a is in a relationship with female agent f_a, and male agent m_a meets female agent f_b, then if male agent m_a and female agent f_b, who is matched with male agent m_b, would rather be matched with one another than with their current partners (and thus form a blocking pair), then male agent m_a starts a relationship with female agent f_b and male agent m_b and female agent f_a are left single.

Before we apply our model for simulating real-world matching markets we want to understand the behaviour of agents in our model, and the effect of changing parameter settings like the number of agents, the time available to obtain a matching and two parameters that influence the preferences of the agents over one another. We are interested in running our model for different parameter settings and compare the outcomes to those in the optimal stable matchings resulting from the Deferred Acceptance Algorithm. Better understanding of the agents in our model and the effects of certain parameters can help explain behaviour that is found empirically. Ultimately it could even help people, companies and policy makers to design markets such that matchings in which more agents are matched and which are more stable or yield higher average payoffs are more likely to be obtained. Our model will be inspired by people in search of a romantic partner and we use corresponding terminology. We however show that our model can be applied for better understanding other two-sided matching markets too.

3.2 The model

3.2.1 Agents, relationships and preferences

We have a set of n male agents $M = \{m_1, \dots, m_n\}$ and a set of m female agents $F = \{f_1, \dots, f_m\}$. Our simulations consist of k daterounds. We define the function $P(x,z) = y$ which represents agent x having a relationship with agent y at dateround z and where $P(x,z) = 0$ represents agent x being single at dateround z. If $P(x,z) = y$ then $P(y,z) = x$. Agents cannot be in a relationship with more than one agent. Preferences of all agents of one side of the market over all agents of the other side of the market are strict, transitive and complete, meaning that for every two possible partners female agents f_a and f_b, male agent m_c either prefers being in a relationship with female agent f_a above being in a relationship with agent f_b or the other way around. All male agents assign a preference score to being in a relationship with every one of the female agents and all female agents assign a preference score to being in a relationship with every one of the male agents. These scores are assigned by the preference function $u : M \times F \to \mathbb{R} \times \mathbb{R}$ where $u_m(m_i, f_j)$ is the preference score m_i assigns to being in a relationship with f_j and $u_f(m_i, f_j)$ is the preference score f_j assigns to being in a relationship with m_i with $0 < u_m(m_i, f_j) \leq 10$ and $0 < u_f(m_i, f_j) \leq 10$. Agents can only be in a relationship with maximally one agent at every dateround, who has to be from the other side of the market. Agents prefer being in a relationship above being single and we thus have $u_m(x,0) = 0$ and $u_f(0,x) = 0$ for any agent x.

Although many similar matching problems are based on preferences expressed by ranking agents, we choose to express our preferences in preference scores ranging from zero to ten. There are several reasons why we choose to do so. In the first place it allows us to use the preference agreement factor p and symmetry factor s when generating preferences, that will form important parameters for our research as will be discussed in Chapter 5. Furthermore we choose to express payoffs in preference scores ranging from zero to ten since agents do not have a full overview of the whole market. Since agents do not know which agents they will meet, it is more natural to express payoffs in the form of a score than in the form of a ranking. Lastly we are convinced that in most real world two-sided matching markets it does not necessarily hold that agents have ordinal payoffs. Note that from our preference scores we can of course deduce a ranking of the agents, which would not influence the behaviour of the agents in our model.

3.2.2 Deferred Acceptance Algorithm

When preferences are assigned we start with the Deferred Acceptance Algorithm from Shapley and Gale [6] to obtain the stable matching μ_m with optimal outcomes for all agents in M, and the stable matching μ_f with optimal outcomes for all agents in F. These results will form a benchmark for the results of the simulations in our model.

3.2.3 Simulations

We then run simulations consisting of k daterounds. At round 0 all agents are single ($\forall x \in F \cup M : P(x,0) = 0$). Every dateround l, with $0 \leq l \leq k$ all of the agents are matched with an agent of the other side of the market forming the one-to-one bipartite dateround graph $G_l = (D_m, D_f, E)$, where $D_m = M$ and $D_f = F$ (in Chapter 6 we investigate an extension of our model where $D_m \subseteq M$, $D_f \subseteq F$, and thus not all agents meet an agent from the other side of the market every dateround). Every edge in E from G_l consisting of a male agent $m_i \in M$ and a female agent $f_j \in F$ represents a date at dateround l in which male agent m_i and female agent f_j get the opportunity to pair up, which we will call being in a relationship. Note that it could be the case that an agent meets the same agent as the one he was already in a relationship with before the dateround started.

3.2.4 Update rule

In our model agents are myopic and behave according to simple deterministic update rules, called the "Better response dynamics" by Ackermann et al. [1]. When agents are single, they will start a relationship with any agent they would get the chance to. Suppose a male agent m_a is in a relationship with female agent f_b but gets the chance to start a relationship with a female agent f_c, because they date in a dateround and female agent f_c prefers being matched with male agent m_a above her current situation. Then if that male agent m_a prefers female agent f_c above female agent f_b, then male agent m_a and female agent f_c start a relationship and female agent f_b is left single. Formally we thus have: If $\{m_i, f_j\} \in E$ of G_l then agents m_i and f_j date at dateround l and if $u_m(m_i, f_j) > u_m(m_i, f_p)$ and $u_f(m_i, f_j) > u_f(m_p, f_j)$, where $P(f_j, l-1) = m_p$, $P(m_p, l-1) = f_j$, and $P(m_i, l-1) = f_p$ and $P(f_p, l-1) = m_i$, then $P(m_i, l) = f_j$, $P(f_j, l) = m_i$. Every dateround l all couples in the one-to-one matching $G_l = (D_m, D_f, E)$ all sequentially decide whether they start a new relationship in a randomized order. This means that if an agent is left by its partner at the beginning of a dateround, but has a date later on in this same dateround, this agent would consider itself single and thus accept his date (since all agents prefer every relationship above being single). So only if we have that agents f_p and m_p, that were left by m_i and f_j respectively during dateround l, will not have a date in dateround l that results in pairing with a new partner, we have $P(m_p, l) = 0$, $P(f_p, l) = 0$. After every dateround l we will have obtained a matching μ_l consisting of all pairs (m_i, f_j) such that $P(m_i, l) = f_j$ and $P(f_j, l) = m_i$ (which means it does not have to contain all agents).

3.2.5 Blocking pair counter

One of the great things about the Deferred Acceptance Algorithm is that it not only ensures that a stable matching is obtained that is optimal for the proposing side of the market. It is also clear when this stable matching is obtained. Even though our model will eventually converge towards a stable matching, as we will prove in the next chapter, it is not directly clear when a stable matching is obtained. We therefore extended our model with a blocking pair counter (where a blocking pair consists of a pair of agents that would rather be matched with each other than with their current partners, which could also mean not being matched), that counts the number of blocking pairs in the obtained matching after every dateround. This means that every dateround we have to check for all $n \times m$ (with n male agents and m female agents) possible pairs whether they form a blocking pair. Only if the number of blocking pairs is 0, we have obtained a stable matching. Note that in the model explained above every single male and single female agent form a blocking pair.

3.2.6 Utility

Although our agents behave according to simple rules in which they continually aim to be in a relationship that they assign the highest preference score to, it is important to think about the utility of the agents in our model. This is the case because this utility helps us to assess results obtained in our model. This utility is also important when we investigate in Chapter 6 what other, less myopic, strategies agent could use to obtain higher utilities. In our model we will start with setting the utility of an agent equal to the preference score assigned to its partner at the last dateround. Thus in a simulation with k daterounds, $\pi_{m_i} = u_m(m_i, P(m_i, k))$ and $\pi_{f_j} = u_f(P(f_j, k), f_j)$ for any male agent m_i and any female agent f_j.

Chapter 4

Convergence towards stability

4.1 Introduction

The first question we want to answer is whether our model will always converge towards a stable matching. At first sight this seems trivial since there are only finitely many matchings and we know from the Deferred Acceptance Algorithm that a stable matching exists. This would mean that at some point in our simulation we would randomly have generated a dateround in which all agents of one side of the market date the agent with whom they would be paired in this stable matching. It could however be the case that at that point an agent is in a relationship with an agent he finds more attractive than the agent he dates (and thus is matched with) in this optimal stable matching, and would therefore not start a relationship with this agent. This would imply that the optimal stable matching that resulted from the Deferred Acceptance Algorithm would not be obtained at that moment. It is per definition the case that if we obtained a stable matching μ_l without a blocking pair somewhere during the simulation after dateround l, with $l < k$, that no mutations take place and our matching will stay the same and thus $\mu_l = \mu_k$. When we have a positive number of blocking pairs in matching μ_l then we know we have no stable matching. Suppose m_i and f_j form a blocking pair for μ_l. Since every pair consisting of a male and female agent has a positive probability of dating each other in every dateround p (daterounds consist of random bipartite graphs G_p between M and F) and since we only have a finite number of agents the probability that we will never obtain a dateround p with $\{m_i, f_j\} \in E$ of G_p for some dateround p with $p > l$ goes to zero when we increase the number of daterounds in our simulation. Note however that there might be more blocking pairs that need to be satisfied than the blocking pairs in μ_l since new blocking pairs can form whenever a blocking pair is satisfied. We thus need to show that in our model for any instance of the Stable Marriage Problem for any matching there always exists a finite sequence of blocking pairs that could be satisfied consecutively in our model such that we obtain a stable matching, without any blocking pairs.

4.1.1 Roth-Vande Vate Algorithm

Theorem 4 (Roth-Vande Vate(1990))**.** *For any instance of the Stable Marriage Problem for any matching μ_0 there exists a finite sequence of matchings $\mu_0, \dots, \mu_i, \dots, \mu_k$ such that μ_k is stable and μ_i is obtained from μ_{i-1} by satisfying a blocking pair. [13]*

To find out whether our simulations will eventually converge towards a stable matching we will start with the proof of Roth and Vande Vate, that shows that from any unstable matching we can always obtain a stable matching by subsequently satisfying a finite number of blocking pairs [13]. The proof of Roth and Vande Vate shows that from any unstable matching there exists a finite sequence of matchings $\mu_0, \dots, \mu_i, \dots, \mu_k$ such that μ_k is stable and μ_i is obtained from μ_{i-1} by satisfying a blocking pair. There are only finitely many matchings since F and M are finite. If every blocking pair has at any point a positive probability of being satisfied, it will thus be the case that for every matching there is a positive probability that exactly the finite sequence of blocking pairs needed to obtain a stable matching is generated. This means that when blocking pairs are randomly satisfied there will be convergence towards a stable matching.

Proof. Suppose that we have a set of n male agents M and a set of m female agents F and an unstable matching $\mu_0 = \{(m_1, f_1), \dots, (m_k, f_k)\}$ with $m_i \in M$, $f_j \in F$, $k \leq n$ and $k \leq m$. We shall show that we can reach a stable matching by successively satisfying finitely many blocking pairs. The Roth-Vande Vate algorithm works as follows. We gradually extend a set $S \subseteq (M \cup F)$. Initially let $S(0) = \{\emptyset\}$. Now suppose there exists a blocking pair (m_i, f_j) for μ_0 (if there exists no blocking pair we are done). Now let μ_1 be the next matching in the finite sequence described in the theorem with $(m_i, f_j) \in \mu_1$ by satisfying the blocking pair (m_i, f_j). Now we extend S by having $S(1) = \{m_i, f_j\}$. Note that if (m_i, f_j) forms a blocking pair for μ_1, it would not be contained in $S(1)$. We now construct the needed sequence of matchings ending with a stable matching by extending S such that no blocking pairs for $\mu(k)$ are contained in $S(k)$. We prove this by induction (with the extension of $S(0)$ to $S(1)$ as the base case). Suppose we have a set $S(k)$ such that no two agents are part of it, that would form a blocking pair for μ_{k+1} and such that μ_{k+1} does not match any agent in $S(k)$ with an agent outside $S(k)$. Now if μ_{k+1} is not stable, there exists a blocking pair (m_{k+1}, f_{k+1}) such that at most one member of the blocking pair is contained in $S(k)$. We now consider three cases:

First, suppose that (m_i, f_j) is a blocking pair for μ_k with m_i contained in $S(k)$, such that it is the blocking pair such that among all blocking pairs (m, f_j), m_i is the most preferred partner of f_j in $S(k)$. Now form μ_{k+2} by satisfying this blocking pair and let $S(k+1) = S(k) \cup \{f_j\}$. If m_i was unmatched in μ_{k+1}, then $S(k+1)$ contains no blocking pair for μ_{k+2}. Otherwise there could be a blocking pair (m_g, f_h) for μ_{k+2} such that m_i is matched with f_h in μ_{k+2} and with m_g and f_h both contained in $S(k+1)$. Here we choose (m_g, f_h) such that among all blocking pairs (m, f_h), m_g is the most preferred partner of f_h in $S(k+1)$. The next matching in the sequence μ_{k+3} is now formed by satisfying this blocking pair. We continue this process until we reach a matching μ_t, with $t > k$ such that no blocking pairs for μ_t are contained in the set $S(t) \equiv S(k+1)$. Note that we know that this must eventually happen, since we have finitely many agents and no male agent ever gets matched with a less preferred partner and thus no blocking pair appears twice in the finite sequence of matchings $\mu_k + 2, \ldots, \mu_t$. Now $S(t)$ is the set we need, which strictly contains $S(k)$ and contains no blocking pairs for μ_t.

For the second case we consider blocking pairs (m_i, f_j) for μ_k with f_j contained in $S(k)$, such that it is the blocking pair such that among all blocking pairs (m_i, f), f_j is the most preferred partner of m_i in $S(k)$. We can now simply proceed as in the first case, with the roles of m_i and f_j switched.

For the third case all blocking pairs (m_i, f_j) for μ_k are disjoint from $S(k)$. We can then simply select one of these blocking pairs and satisfy it to obtain the next matching in the sequence and let $S(k+1) \equiv S(k) \cup \{m_i, f_j\}$.

$S(k+1)$ contains $S(k)$ and contains no blocking pairs for μ_{k+2}. Obtaining $S(k+1)$ with this algorithm always takes finitely many steps since $S(k)$ can be strictly increased until a stable matching is obtained, but it can never grow larger than $M \cup W$.

We thus obtain the sequence of blocking pairs that we need to satisfy to reach a stable matching starting from μ_0. Since we never satisfy a pair twice when adding a new agent to S, it follows that the number of steps in the path to stability is at most mn and thus finite. Thus, if we randomly satisfy blocking pairs, than from any unstable matching μ we will eventually obtain a stable matching μ_s, since there always is a positive probability that the needed finite sequence of blocking pairs needed to obtain stable matching μ_s will be satisfied consecutively. [13]

□

If we start with the empty matching and run the Roth-Vande Vate algorithm, then the resulting stable matching will depend on the order in which the agents arrive. This is called the random order mechanism.

4.1.2 Obtaining the right sequence of blocking pairs in our model

From the Roth-Vande Vate proof [13] we know that from any unstable matching we can obtain a stable matching by satisfying a finite number of blocking pairs. In our model all agents randomly meet another agent from the other side of the market every dateround. Agents will only form a relationship when they meet an agent with whom they form a blocking pair. We now need to show that in our model, every blocking pair in the finite sequence of the Roth-Vande Vate algorithm has a positive probability of being the next blocking pair to be satisfied at the right moment during the simulation. In our model it is not the case that we randomly satisfy blocking pairs, since an agent can only start a new relationship once every dateround. Thus, when the Roth-Vande Vate algorithm for example gives us a sequence in which an agent is part of three consecutive blocking pairs it is not trivial that there is a positive probability that this sequence of blocking pairs can be obtained. It could for example be the case that there is no dateround possible in which only the second of the three consecutive blocking pairs can be satisfied, at the moment needed. This could be the case since in every possible dateround in which this blocking pair is satisfied, also other blocking pairs will be satisfied.

We thus cannot conclude from the Roth-Vande Vate proof that eventually we will always obtain a stable matching in our model. We do however know that from any unstable matching, there exists a dateround, in which all single agents meet another single agent and every other agent meets the agent they are already in a relationship with, such that we can obtain a matching μ_c in which every agent is matched. For this matching μ_c there of course also exists a finite sequence of blocking pairs that can be satisfied sequentially to obtain a stable matching. Since all agents are now matched we know there exist daterounds in which all agents meet the agents they were already paired with in μ_c except the two agents in the next blocking pair in the finite sequence, say (m_i, f_j) and their partners in μ_c, say (m_i, f_j).

Knuth [8] asked whether for any instance of the Stable Marriage Problem we can obtain a stable matching from any complete matching (in which all agents are matched) by executing a sequence of interchanges. Here an interchange consists of satisfying a blocking pair, and immediately after the partners they left behind. Tamura [14] and also Tan and Su [15] have shown independently that this is not the case by giving a counterexample. This means we cannot use interchanges to prove convergence towards stability in our model.

4.1.3 Proof of convergence towards a stable matching in our model

To prove that our model always converges towards a stable matching we use a proof of Diamantoudi et al. [4] who made a generalization of the proof of Roth and Vande Vate [13]. Diamantoudi et al. [4] have shown that we can always find a finite sequence of blocking pairs that can be satisfied consecutively to obtain a stable matching for the Stable Roommate Problem with strict preferences. The Stable Roommate Problem is a generalization of the Stable Marriage Problem in which there is only one group of agents. Here all agents have preferences over being paired with the agents in this group, for which a stable matching needs to be obtained.

Theorem 5. *For any instance of the Stable Marriage Problem with n male and m female agents, for any arbitrary matching μ_0 there exists a finite sequence of blocking pairs that has a positive probability of being satisfied consecutively to result in a stable matching μ_s, by iteratively generating random bipartite graphs consisting of n pairs (m_i, f_j) who meet sequentially in random order and will satisfy a blocking pair at that moment if they form one.*

Proof. We know from Shapley and Gale that for any instance of the Stable Marriage Problem there exists a stable matching μ_s [6]. Assume we have an unstable matching μ_c for an arbitrary instance of the Stable Marriage Problem for which μ_s is a stable matching. For any matching μ and a stable matching μ_s for this instance let $n(\mu, \mu_s)$ be the number of pairs (m_i, f_j) that are both part of μ and μ_s. Diamantoudi et al. show that from μ_c there always exists a finite sequence of blocking pairs that can be satisfied to obtain a matching μ_p such that $n(\mu_p, \mu_s) > n(\mu_c, \mu_s)$ [4]. Since we only consider cases with finite number of agents, there thus is a finite sequence of blocking pairs that can be satisfied to obtain a stable matching. We will show that there always is a positive probability of consecutively satisfying this sequence of blocking pairs in our model, and thus that our model will always converge towards a stable matching.

We thus start with an arbitrary unstable matching μ_c and we fix the stable matching μ_s. Now the first step in the strategy of Diamantoudi et al. [4] is to obtain a matching μ_e in which no blocking pair exists that forms a pair in μ_s and in which every agent is matched. First we need to make sure that all agents are matched. We know that there exists a dateround in which all agents meet the agents they are paired with in μ_c and where all single male agents meet a single female agent, who will pair up since in our model all agents prefer being matched above being single. This dateround will give us μ_d. Now if there exists a blocking pair (m_i, f_j) for μ_d such that $(m_i, f_j) \in \mu_s$, then we know that there exists a dateround in which all agents meet the same agent they are paired with in μ_d except for m_i and f_j and their partners, in which m_i and f_j date each other before their partners do (who will start a relationship since they were left by m_i and f_j). After this dateround we obtain a matching μ_e and $n(\mu_e, \mu_s) > n(\mu_c, \mu_s)$ and in which all agents are matched. Which means we are done for the unstable matching μ_c.

Now if there are pairs of agents that form a blocking pair in μ_e that are a part of μ_s, then we repeat the procedure we used to obtain μ_e from μ_d. We repeat this procedure until we obtained a matching μ_f such that no blocking pair (m_a, f_a) exists such that $(m_a, f_a) \in \mu_s$.

Now suppose μ_f is not stable. Diamantoudi et al. have shown that we now know the following [4]:

1. In μ_f all agents are matched.
2. There exists no pair of agents $(m_i, f_j) \in \mu_s$ such that both m_i and f_j prefer each other above their current partner in μ_f.
3. There exists no pair of agents $(m_i, f_j) \in \mu_f$ such that both m_i and f_j prefer each other above their partner in μ_s.

Diamantoudi et al. now define the following function h (where $\mu(i)$ is the agent that agent i is matched with in μ) [4]:

$$h(i) = \begin{cases} \mu_f(i) & \text{if } \mu_f \succ_i \mu_s \\ \mu_s(i) & \text{otherwise.} \end{cases}$$

Every agent thus "points" to the agent of the other side of the market they would prefer to be with if they could choose between their partner in the current matching and the one in the stable matching. Because of 2. and 3. and the fact that we only have finitely many agents, we can partition the set of all agents in cycles $c = (m_1, f_2, m_3, \ldots, f_k)$ consisting of an even number of k agents such that $h(m_i) = f_{i+1}$ and $h(f_i) = m_{i+1}$ for $i < k$ and $h(f_k) = m_1$. We also know that either all male agents or all female agents in a cycle prefer being in μ_f above being in μ_s. We furthermore know that if m_i and f_j form a blocking pair for μ_f, that only one of those two prefer being in μ_s above being in μ_f. If m_i is this agent, then all male agents in his cycle prefer matching μ_s over μ_f and all the female agents prefer μ_f over μ_s. If f_j would be this agent, then all female agents in her cycle prefer matching μ_s over μ_f and all the male agents prefer μ_f over μ_s. Diamantoudi et al. [4] also show that if for any agent i, $\mu_f(i) \neq \mu_s(i)$ then agent i is part of a cycle with at least four members.

Now assume there exists a blocking pair (m_1, f_j) for μ_f (for which 2. and 3. thus holds). Then either one of the two prefers μ_s over μ_f. Let this agent be m_1 without loss of generality. We know that m_1 is part of a cycle $c_1 = (m_1, f_2, m_3, \ldots, f_k)$ with $k \geq 4$. We thus have that all male agents in this cycle prefer μ_s over μ_f and all female agents in this cycle prefer μ_f over μ_s. We now have that $\mu_s = [(m_1, f_2), (m_3, f_4), \ldots, (m_{k-3}, f_{k-2}), (m_{k-1}, f_k), \ldots]$ and $\mu_f = [(m_3, f_2), (m_5, f_4), \ldots, (m_{k-1}, f_{k-2}), (m_1, f_k), \ldots]$ [4]. We now need to show first that there exists a finite sequence of blocking pairs that can be satisfied to obtain a matching μ_k such that $n(\mu_k, \mu_s) > n(\mu_f, \mu_s)$. We then need to show that we can obtain this sequence in our model.

When we would satisfy (m_1, f_j) we would obtain a matching μ_g with $n(\mu_g, \mu_s) = n(\mu_f, \mu_s) - 1$ if $\mu_f(f_j) = \mu_s(f_j)$ (since the pair $(\mu_s(f_j), f_j) \in \mu_s$ would be broken) and with $n(\mu_g, \mu_s) = n(\mu_f, \mu_s)$ if $\mu_f(f_j) \neq \mu_s(f_j)$. We have that f_k is single under μ_g and $h(m_{k-1}) = f_k$. Thus (m_{k-1}, f_k) forms a blocking pair that we could satisfy to obtain μ_h.

If we would have that $\mu_f(f_j) \neq \mu_s(f_j)$ then $n(\mu_h, \mu_s) > n(\mu_f, \mu_s)$. Note that the two blocking pairs we have satisfied now consist of four different agents. This means that there exists a dateround such that if that dateround would be generated in our model at μ_f we would obtain matching μ_h with $n(\mu_h, \mu_s) > n(\mu_f, \mu_s)$. In this dateround every agent meets the same agent as it is matched with in μ_f except (m_1, f_j) and (m_{k-1}, f_k) who meet each other and the partners they left behind who meet each other after being left behind. It does not matter which male agent that is left single meets what other female agent that is left single.

If however $\mu_f(f_j) = \mu_s(f_j)$ then we obtain $\mu_h = [(m_3, f_2), (m_5, f_4), \ldots, (m_{k-1}, f_k), (f_{k-2}), (m_1, f_j), (\mu_s(f_j)), \ldots]$ with $n(\mu_h, \mu_s) = n(\mu_f, \mu_s)$ by first satisfying blocking pair (m_1, f_j) and then blocking pair (m_{k-1}, f_k) for μ_f. We now show that (m_{k-3}, f_{k-2}) is a blocking pair for μ_h which we could satisfy to obtain a matching μ_k such that $n(\mu_k, \mu_s) > n(\mu_f, \mu_s)$.

When $k \leq 6$ this is trivial since f_{k-2} prefers being matched with m_{k-3} above being single and since $h(m_{k-3}) = f_{k-2}$ and all male agents prefer being matched with the agent they are paired with in μ_s above being paired with their match in μ_f [4]. Note that the three blocking pairs we could satisfy consecutively to obtain a matching μ_k such that $n(\mu_k, \mu_s) > n(\mu_f, \mu_s)$ now consist of six different agents. This means that there exists a dateround in which every agent meets the same agent as it is matched with in μ_f except (m_1, f_j), (m_{k-1}, f_k) and (m_{k-3}, f_{k-2}) who meet each other before every on of the male agents that is left single meets by the female agents in these blocking pairs meet one of the female agents that are left single by the male agents in these blocking pairs. When we generate this dateround at μ_f we would obtain matching μ_k with $n(\mu_k, \mu_s) > n(\mu_f, \mu_s)$.

Now for the case where $k = 4$ (which is the only case for $k < 6$ since k must be even and $k \geq 3$) we have $\mu_s = [(m_1, f_2), (m_3, f_4), \ldots, (\mu_s(f_j), f_j), \ldots]$ and $\mu_f = [(m_1, f_4), (m_3, f_2), \ldots, (\mu_s(f_j), f_j), \ldots]$. After satisfying (m_1, f_j), (m_3, f_4) becomes a blocking pair since m_3 prefers f_4 above f_2 and f_4 is left single by m_1. After satisfying (m_1, f_j) and (m_3, f_4) consecutively, we would obtain $\mu_i = [(m_1, f_j), (m_3, f_4), (f_2), \ldots, (\mu_s(f_j)), \ldots]$. Now since (m_1, f_j) forms a blocking pair for μ_f, we know female agent f_j prefers m_1 to $\mu_s(f_j) = \mu_f(f_j)$. We have that (m_1, f_j) does not block μ_s since μ_s is stable. Thus m_1 prefers f_2 to f_j, which means (m_1, f_2) blocks μ_i. [4]

We can however not guarantee that we can obtain a dateround in which blocking pair (m_1, f_2) will be satisfied after satisfying (m_1, f_j) and (m_3, f_4) and thus forming μ_i. This is the case since we cannot satisfy a blocking pair that contains m_1 twice in one dateround. This means that we have to let m_1 and f_2 pair in a second dateround.

Now we need to show that our model could generate the daterounds in which these blocking pairs are satisfied consecutively such that we would obtain a matching μ_k from μ_f with $n(\mu_k, \mu_s) > n(\mu_f, \mu_s)$. We start with dateround $d_1 = [(\mu_s(f_j), f_2), (m_1, f_j), (m_3, f_4), \ldots]$, where all other agents meet the partner they are already matched with in μ_f. Here, $(\mu_s(f_j), f_2)$ is no blocking pair since $(\mu_s(f_j), f_2)$ is no blocking pair for μ_s either (since μ_s is stable) and since f_2 prefers m_3 over m_1 (and thus since f_2 does not prefer m_1 over $\mu_s(f_j)$ it does also not prefer m_3 over $\mu_s(f_j)$). We now obtain the matching $\mu_i = [(m_1, f_j), (m_3, f_4), (f_2), \ldots, (\mu_s(f_j)), \ldots]$. Now consider dateround $d_2 = [(m_1, f_2), (\mu_s(m_j), f_j), \ldots]$, where all other agents meet the partner they are already matched with in μ_i. If our model generates d_2 for μ_i then we would obtain $\mu_k = [(m_1, f_2), (m_3, f_4), \ldots, (\mu_s(f_j)), (f_j), \ldots]$ by satisfying (m_1, f_2). We now have obtained a matching μ_k such that $n(\mu_k, \mu_s) = n(\mu_f, \mu_s) + 1$. Note that by satisfying $(\mu_s(f_j), f_j)$ we would end dateround d_2 with $\mu_l = [(m_1, f_2), (m_3, f_4), \ldots, (\mu_s(f_j), f_j),$

...] for which $n(\mu_l, \mu_s) = n(\mu_f, \mu_s) + 2$.

□

We have now covered all the possible cases. We thus have shown that for any instance of the Stable Marriage problem, for any unstable matching there always exists a finite sequence of blocking pairs, that can be satisfied to obtain a matching which has strictly more pairs in common with a particular stable matching. Furthermore we have shown that this sequence always has a positive probability of being satisfied consecutively in our model. Since there are only finitely many agents this means that our model will eventually converge towards a stable matching. Note however that this does not mean that we can always identify a finite number of simulations after which we know a stable matching will be obtained. There namely always is a positive probability that we have not obtained a stable matching, since for example our model would have generated a dateround in which all agents meet the agents they were already in a relationship with over and over. We do however know that for infinite number of daterounds the probability of obtaining a stable matching in our model goes to 1.

4.2 Optimality of obtained matchings

4.2.1 Existence of suboptimal matchings

The Deferred Acceptance Algorithm always gives us the optimal stable matching for the proposing side of the market. It is however also possible that other stable matchings exist.

Theorem 6. *There exist stable marriage problems for which not every stable matching is an optimal stable matching for one of the two sides of the market.*

We can show this using a simple counterexample (Table 4.1) from the famous paper by Shapley and Gale [6], in which we have three male agents $\{m_a, m_b, m_c\}$ and three female agents $\{f_a, f_b, f_c\}$ and preferences that are expressed in Table 4.1. Here the first number is the rank the male agent (row) assigns to being matched with the female agent (column) and the second number is the rank the female agent assigns to being matched with the male agent (thus for male agent m_a female agent f_a is his most preferred candidate while male agent m_a is the least preferred candidate of female agent f_a).

The matching $\mu_1 = \{(m_a, f_c), (m_b, f_a), (m_c, f_b)\}$ is stable since there exists no blocking pair consisting of two agents who would rather have been matched with one another than with their current match (since one of the two agents will always switch from their second

	m_a	m_b	m_c
f_a	$1,3$	$2,2$	$3,1$
f_b	$3,1$	$1,3$	$2,2$
f_c	$2,2$	$3,1$	$1,3$

Table 4.1 Example of a stable matching problem in which a stable matching exists that is not optimal for either sides of the market

favorite to their least favorite). There exists however a stable matching in which a male agent (even all male agents) are better off, namely the matching $\mu_2 = \{(m_a, f_a), (m_b, f_b), (m_c, f_c)\}$ where all male agents are matched with their most favorite female agent. This is the matching that would result from the Deferred Acceptance Algorithm with the male agents as proposing party. There also exists a stable matching in which all female agents are better off, namely the matching $\mu_3 = \{(m_a, f_b), (m_b, f_c), (m_c, f_a)\}$ where all male agents are matched with their most favorite female agents. This is the matching that would result from the Deferred Acceptance Algorithm with the female agents as proposing party.

4.2.2 Our model obtaining suboptimal matchings

Theorem 7. *Our model does not necessarily converge to one of the optimal stable matchings that is the result of the Deferred Acceptance Algorithm.*

Proof. We can show this using a simple counterexample. Take the same example (Table 4.1) from the Shapley and Gale paper as in the previous theorem with the stable matching $\mu_1 = \{(m_a, f_c), (m_b, f_a), (m_c, f_b)\}$, that is not optimal for either side of the market. Remember we start every simulation with single agents who would start a relationship with any agent they meet the first dateround. If we would thus have that in the first dateround we obtain a bipartite graph G_1 exactly similar to the suboptimal stable matching μ_s then we will immediately obtain the suboptimal stable matching μ_s and since μ_s is stable it will be the result of the simulation in our model.

□

Note that we could easily extend this proof to any number of male and female agents.

Chapter 5

Understanding the model

5.1 Research questions

The goal of our model is to simulate real-world two-sided matching markets. We start by explaining how preferences in the utility function $u : M \times F \rightarrow \mathbb{R} \times \mathbb{R}$ are randomly generated, based on the parameters p and s. Here p is the preference agreement factor that affects the objectivity of the preferences within one side of the market and s is the symmetry factor that affects the symmetry of the preferences between the two sides of the market.

We first investigate, by running simulations, what the effects are of changing the parameter values on the results obtained with the Deferred Acceptance Algorithm of Shapley and Gale [6]. We specifically examine what the effects are of changing the preference agreement factor p, the symmetry factor s and the number of agents in the market. We investigate the effect of changing these parameters on the number of rounds it takes to reach the optimal stable matching in the Deferred Acceptance Algorithms and what the payoffs of the agents in the obtained matchings in this algorithm are. We also investigate what the effects of changing these parameters are on the percentage of the simulations in which the optimal stable matching for male agents is equal to the optimal stable matching for female agents.

We then investigate the effects of changing the same parameters when we run simulations in our model. Next to payoffs of agents and the number of rounds it takes until a stable matching is reached, we investigate how many agents are in a relationship and the number of blocking pairs during the simulations. furthermore, we will examine the number of agents in a relationship that is the same as the one they would end up with in one of the stable matchings resulting from the Deferred Acceptance Algorithm. We also investigate whether a stable matching is obtained, other than the optimal ones resulting from the Deferred Acceptance

Algorithm.

One of the goals of this thesis is to provide researchers with a model that can be extended, such that it simulates real-world two-sided matching markets. It is therefore important to provide the reader with thorough understanding of our model. To be able to extend our model, it is crucial to understand the effects of changing the parameter settings in both our model and in the Deferred Acceptance Algorithm. This chapter contains a relatively detailed investigation of the effects of changing parameter settings in both our model and the Deferred Acceptance Algorithm. To ensure that the reader does not have to miss the most important conclusions, without having to dive into all the details, we end section 5.3, 5.4 and 5.5 with summaries. In these summaries the reader finds the most important results of the simulations in the Deferred Acceptance Algorithm, our model and our exhaustive research of all stable matchings, respectively.

We find that our model will on average converge to stable matchings with average payoffs higher than, or equal to, the average payoffs in the two stable matchings that result from the Deferred Acceptance Algorithm, independent of the parameter settings. The greatest difference we find for subjective preferences in which there is no symmetry ($p = 0$ and $s = 0$). In order to better understand why our model yields higher payoffs we investigated all possible matchings for 450 instances of the Stable Marriage Problem with 10 male and 10 female agents, with $p = 0$ and $s = 0$. For all instances we investigate which matchings are stable and what the average payoffs in these stable matchings are. We then run 500 simulations in our model until a stable matching is reached for every instance, to examine which stable matchings are more often obtained.

5.2 Preferences

The values of the preference function u are assigned randomly. There are however two parameters that affect the values assigned by u to every pair consisting of a male and a female agent. There are many properties we can assign to the strict, transitive and complete preferences of the agents. Two properties, however, seem to be most important when we want to run simulations to model real-world two-sided markets. The first is the degree to which agents of the same side of the market have similar preferences. The second is the degree to which there is symmetry between the preferences of the two sides of the market. We can imagine that it makes an important difference for reaching a stable matching whether everyone wants the same job or partner or whether preferences are totally subjective. When, however, there is some subjectivity in place, it seems to be important whether this matter of taste is symmetric. Thus, whether it is the case that when a male agent m_a highly prefers being in a relationship with female agent f_b, then this female agent f_b also likes agent m_a relatively more. We therefore introduce two parameters that influence the preferences assigned by the preference function u. The first parameter is the preference agreement factor p with $0 \leq p \leq 1$. The second parameter is the symmetry factor s with $0 \leq s \leq 1$. The higher p, the more similar the preferences of agents within one side of the market over the other side of the market are. The higher s, the closer $u_m(m_i, f_j)$ and $u_f(m_i, f_j)$ are.

The values for the function u are generated by first generating four lists: $gm = (a_1, \ldots, a_m)$ (which represents the objective preferences of the male agents over the female agents), $gf = (b_1, \ldots, b_n)$ (which represents the objective preferences of the female agents over the male agents), $sm = ((c_{11}, \ldots, c_{1m})_1, \ldots, (c_{n1}, \ldots, c_{nm})_n)$ (which represents the subjective preferences of the male agents over the female agents) and $sf = ((d_{11}, \ldots, d_{1n})_1, \ldots, (d_{m1}, \ldots, d_{mn})_m)$ (which represents the subjective preferences of the female agents over the male agents) with n the number of male agents and m the number of female agents, and $a_i, b_j, c_{ij}, d_{ji} \in \mathbb{R}$ where $0 < a_i \leq 10$, $0 < b_j \leq 10$, $0 < c_{ij} \leq 10$, $0 < d_{ji} \leq 10$ and where a_i, b_j, c_{ij} and d_{ji} are all randomly generated and $1 \leq i \leq n$ and $1 \leq j \leq m$. With preference agreement factor p and symmetry factor s, we now have that $u_m(m_i, f_j) = pa_j + (1-p)c_{ij}$ and $u_f(m_i, f_j) = s(u_m(m_i, f_j)) + (1-s)(pb_i + (1-p)d_{ji})$. Note that when $p = 0$ the preferences are fully subjective and when $p = 1$ all male agents have exactly the same preferences over the female agents and the other way around. For $s = 1$, $u_m(m_i, f_j) = u_f(m_i, f_j)$ for any two agents m_i and f_j. If we have $p = 0$ and $s = 0$, then the preferences are uniformly random. Note that when $u_m(m_i, f_j) = u_f(m_i, f_j)$, this does not automatically mean that m_i and f_j rank each other the same way over all agents of the other side of the market, since it might for example be the case that male agent m_i and f_j both assign a preference score of 9 to each other, but that there exists no male agent that agent f_j likes more than m_i while there exists some female agent f_k that m_i assigns a score of 9.5 to, who is thus ranked better by him than f_j.

If our model generates preferences for any agent m_i, f_j and f_k where $u_f(m_i, f_j) = u_f(m_i, f_k)$ then we slightly increase $u_f(m_i, f_j)$ with 0.0001 if $j < k$ or $u_f(m_i, f_k)$ with 0.0001 if $k < j$ to ensure we have strict preferences. The same holds for the preferences of the female agents. We iterate this procedure until no agent is indifferent about pairing with any pair of agents of the other side of the market.

Note that although preferences of agents over the other side of the market are complete, agents are not aware of their preferences. Agents are only aware of the preference score they assign to their current partner and the agents they date, at the moment they date them.

Example

An example with three male agents m_1, m_2 and m_3 and three female agents f_1, f_2 and f_3 and with $p = 0.5$ and $s = 0.75$: First we randomly generate $gm = (3.5, 6.7, 4.2)$, $gf = (8.5, 2.1, 4.9)$, $sm = ((4.5, 1.0, 9.0), (2.5, 1.8, 9.2), (7.5, 8.0, 1.0))$ and $sf = ((1.5, 8.0, 9.8), (2.0, 6.0, 8.5), (1.5, 3.0, 7.5))$. We now for example have that $u_m(m_2, f_3) = pa_3 + (1-p)c_{23} = (0.5 \times 4.2) + (0.5 \times 9.2) = 6.7$ and $u_f(m_2, f_3) = s(u_m(m_2, f_3)) + (1-s)(pb_2 + (1-p)d_{32}) = (0.75 \times 6.7) + (0.25 \times ((0.5 \times 2.1) + (0.5 \times 3.0)) = 5.6625$. We can compute the utilities assigned to the eight other possible relationships in the same way.

5.3 Deferred Acceptance Algorithm with different parameter settings

We know that the Deferred Acceptance Algorithm gives us the optimal stable matching for one (the proposing) side of the market for any instance of the Stable Marriage Problem. Our model however has four parameters. We expect the values for these parameters to influence the number of rounds it takes before a stable matching is obtained in the Deferred Acceptance Algorithm, the payoffs of agents obtained in these stable matchings and the percentage of the time the optimal stable matching for male agents is equal to the optimal stable matching for female agents.

The four parameters in our model:

1. Number of male agents $n \in \mathbb{N}$.
2. Number of female agents $m \in \mathbb{N}$.
3. Preference agreement factor $p \in \mathbb{R}, 0 \leq p \leq 1$.
4. Symmetry factor $s \in \mathbb{R}, 0 \leq s \leq 1$.

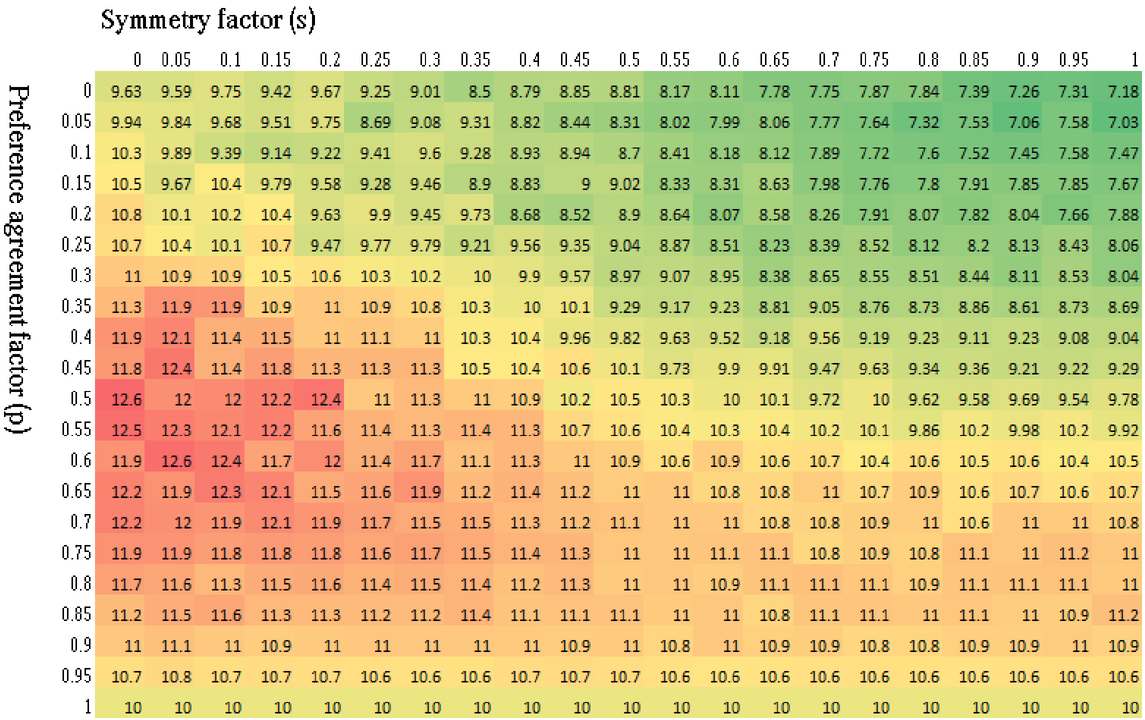
In this thesis we set the number of male agents and female agents equal. We run simulations with different parameter settings for preference agreement factor p and symmetry factor s. We let the possible values for p and s range from 0 to 1 with steps of 0.05 and run simulations for all possible combinations of these values. To reduce noise we run 200 simulations for all parameter settings and take averages as our results. Every simulation we start by generating agents with randomly assigned preferences based on s and p. We then run the Deferred Acceptance Algorithm, once with the male agents as proposing party and once with the female agents as proposing party. This gives us the optimal stable matching for male agents and for female agents respectively. We keep track of the number of rounds it takes in the Deferred Acceptance Algorithm before the optimal stable matching for male agents is obtained. We also investigate whether the optimal stable matching for male agents is equal to the optimal stable matching for female agents. Lastly, we consider the average payoffs of all male agents in the optimal stable matching for male agents and the optimal stable matching for female agents. Note that there is a duality between the male agents in the optimal female matching and the female agents in the optimal stable matching for male agents. We focused our research on simulations for instances with 10 male agents and 10 female agents, but also investigated the effects of increasing the number of agents by doing the same simulations for 20 male and 20 female agents and even 100 male and 100 female agents.

5.3.1 Rounds before stable matching is obtained

As can be seen in Figure 5.1 we find that it takes the least rounds to obtain the optimal male stable matching in the Deferred Acceptance Algorithm when we have a low value for p and a high value for s. It seems to be the case that we reach stability fastest in this case since a lot of different female agents are proposed to per round. This follows from the fact that there is a maximum amount of subjectivity in the preferences of the proposers due to the low value of p and the fact that a lot of proposals are answered positively by the female agents because of the symmetry in preferences due to the high value of s. We also find that for $p = 1$, it takes exactly 10 rounds to obtain the stable matching in the Deferred Acceptance Algorithm. We can prove and extend this to any number of agents.

Theorem 8. *In any instance of the Stable Marriage Problem for which all agents of the one side of the market have the same preferences over all agents of the other side of the market, it takes exactly as many rounds in the Deferred Acceptance Algorithm to obtain the optimal stable matching as the number of agents in the side of the market consisting of the least number of agents.*

Proof. Take any instance of the Stable Marriage Problem with n male agents and m female agents where all male agents have the same preferences over the female agents and all female agents have the same preferences over the male agents. Now in any round in the



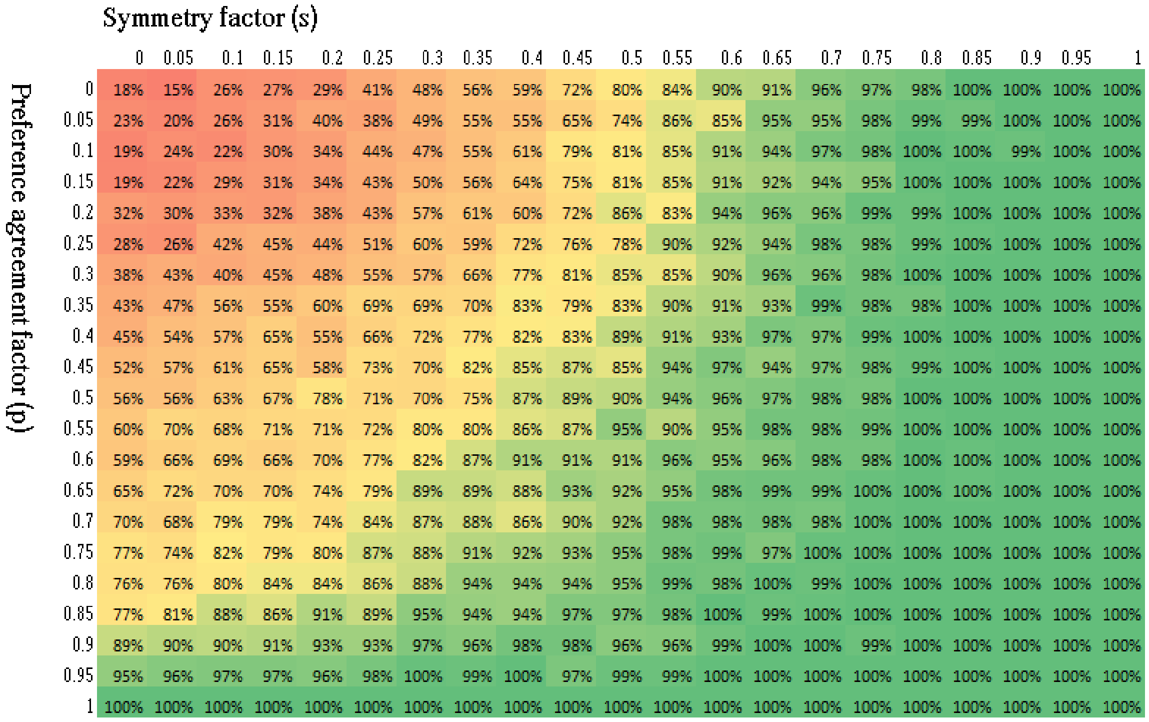
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	9.63	9.59	9.75	9.42	9.67	9.25	9.01	8.5	8.79	8.85	8.81	8.17	8.11	7.78	7.75	7.87	7.84	7.39	7.26	7.31	7.18
0.05	9.94	9.84	9.68	9.51	9.75	8.69	9.08	9.31	8.82	8.44	8.31	8.02	7.99	8.06	7.77	7.64	7.32	7.53	7.06	7.58	7.03
0.1	10.3	9.89	9.39	9.14	9.22	9.41	9.6	9.28	8.93	8.94	8.7	8.41	8.18	8.12	7.89	7.72	7.6	7.52	7.45	7.58	7.47
0.15	10.5	9.67	10.4	9.79	9.58	9.28	9.46	8.9	8.83	9	9.02	8.33	8.31	8.63	7.98	7.76	7.8	7.91	7.85	7.85	7.67
0.2	10.8	10.1	10.2	10.4	9.63	9.9	9.45	9.73	8.68	8.52	8.9	8.64	8.07	8.58	8.26	7.91	8.07	7.82	8.04	7.66	7.88
0.25	10.7	10.4	10.1	10.7	9.47	9.77	9.79	9.21	9.56	9.35	9.04	8.87	8.51	8.23	8.39	8.52	8.12	8.2	8.13	8.43	8.06
0.3	11	10.9	10.9	10.5	10.6	10.3	10.2	10	9.9	9.57	8.97	9.07	8.95	8.38	8.65	8.55	8.51	8.44	8.11	8.53	8.04
0.35	11.3	11.9	11.9	10.9	11	10.9	10.8	10.3	10	10.1	9.29	9.17	9.23	8.81	9.05	8.76	8.73	8.86	8.61	8.73	8.69
0.4	11.9	12.1	11.4	11.5	11	11.1	11	10.3	10.4	9.96	9.82	9.63	9.52	9.18	9.56	9.19	9.23	9.11	9.23	9.08	9.04
0.45	11.8	12.4	11.4	11.8	11.3	11.3	11.3	10.5	10.4	10.6	10.1	9.73	9.9	9.91	9.47	9.63	9.34	9.36	9.21	9.22	9.29
0.5	12.6	12	12	12.2	12.4	11	11.3	11	10.9	10.2	10.5	10.3	10	10.1	9.72	10	9.62	9.58	9.69	9.54	9.78
0.55	12.5	12.3	12.1	12.2	11.6	11.4	11.3	11.4	11.3	10.7	10.6	10.4	10.3	10.4	10.2	10.1	9.86	10.2	9.98	10.2	9.92
0.6	11.9	12.6	12.4	11.7	12	11.4	11.7	11.1	11.3	11	10.9	10.6	10.9	10.6	10.7	10.4	10.6	10.5	10.6	10.4	10.5
0.65	12.2	11.9	12.3	12.1	11.5	11.6	11.9	11.2	11.4	11.2	11	11	10.8	10.8	11	10.7	10.9	10.6	10.7	10.6	10.7
0.7	12.2	12	11.9	12.1	11.9	11.7	11.5	11.5	11.3	11.2	11.1	11	11	10.8	10.8	10.9	11	10.6	11	11	10.8
0.75	11.9	11.9	11.8	11.8	11.8	11.6	11.7	11.5	11.4	11.3	11	11	11.1	11.1	10.8	10.9	10.8	11.1	11	11.2	11
0.8	11.7	11.6	11.3	11.5	11.6	11.4	11.5	11.4	11.2	11.3	11	11	10.9	11.1	11.1	11.1	10.9	11.1	11.1	11.1	11
0.85	11.2	11.5	11.6	11.3	11.3	11.2	11.2	11.4	11.1	11.1	11.1	11	11	10.8	11.1	11.1	11	11.1	11	10.9	11.2
0.9	11	11.1	11	10.9	11	11	11	11	11	10.9	11	10.8	11	10.9	10.9	10.8	10.8	10.9	10.9	11	10.9
0.95	10.7	10.8	10.7	10.7	10.7	10.6	10.6	10.6	10.7	10.7	10.7	10.6	10.6	10.6	10.6	10.6	10.6	10.6	10.6	10.6	10.6
1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Fig. 5.1 The average number of rounds in the Deferred Acceptance Algorithm before the stable matching is obtained for 10 male and 10 female agents over 200 simulations

Deferred Acceptance Algorithm, all male agents that are not engaged with any female agent will propose to the same female agent. Consequently this female agent will accept the one candidate she likes most and will never be proposed to again in a later round. This process continues until either all male or all female agents are engaged. This means that it takes exactly n rounds to obtain the optimal stable matching for the male agents in the Deferred Acceptance Algorithm when $n \leq m$ or that it takes m rounds until all female agents are engaged when $m \leq n$. ☐

Most surprising is, that it takes the most rounds in the Deferred Acceptance Algorithm to reach the optimal stable matching for minimal values for s and moderate values for p. It seems to be the case that for high values for p, we find that the Deferred Acceptance Algorithm proceeds fairly orderly (as described in the case where $p = 1$ there seems to be continuous progress towards stability). For minimal values for p we however have that a lot of male agents propose to different female agents, which also seems to speed up the trajectory towards stability. This is the case since it will take less rounds before all women are proposed to. As expected, higher values for s always benefit the speed of convergence towards stability, although the effect of changing s diminishes when agents agree more. When all agents from the same side of the market have exactly the same preference, changing the value of s has no effect at all.

5.3.2 Equality of the stable matching optimal for male agents and the stable matching optimal for female agents



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	18%	15%	26%	27%	29%	41%	48%	56%	59%	72%	80%	84%	90%	91%	96%	97%	98%	100%	100%	100%	100%
0.05	23%	20%	26%	31%	40%	38%	49%	55%	55%	65%	74%	86%	85%	95%	95%	98%	99%	99%	100%	100%	100%
0.1	19%	24%	22%	30%	34%	44%	47%	55%	61%	79%	81%	85%	91%	94%	97%	98%	100%	100%	99%	100%	100%
0.15	19%	22%	29%	31%	34%	43%	50%	56%	64%	75%	81%	85%	91%	92%	94%	95%	100%	100%	100%	100%	100%
0.2	32%	30%	33%	32%	38%	43%	57%	61%	60%	72%	86%	83%	94%	96%	96%	99%	99%	100%	100%	100%	100%
0.25	28%	26%	42%	45%	44%	51%	60%	59%	72%	76%	78%	90%	92%	94%	98%	98%	99%	100%	100%	100%	100%
0.3	38%	43%	40%	45%	48%	55%	57%	66%	77%	81%	85%	85%	90%	96%	96%	98%	100%	100%	100%	100%	100%
0.35	43%	47%	56%	55%	60%	69%	69%	70%	83%	79%	83%	90%	91%	93%	99%	98%	98%	100%	100%	100%	100%
0.4	45%	54%	57%	65%	55%	66%	72%	77%	82%	83%	89%	91%	93%	97%	97%	99%	100%	100%	100%	100%	100%
0.45	52%	57%	61%	65%	58%	73%	70%	82%	85%	87%	85%	94%	97%	94%	97%	98%	99%	100%	100%	100%	100%
0.5	56%	56%	63%	67%	78%	71%	70%	75%	87%	89%	90%	94%	96%	97%	98%	98%	100%	100%	100%	100%	100%
0.55	60%	70%	68%	71%	71%	72%	80%	80%	86%	87%	95%	90%	95%	98%	98%	99%	100%	100%	100%	100%	100%
0.6	59%	66%	69%	66%	70%	77%	82%	87%	91%	91%	91%	96%	95%	96%	98%	98%	100%	100%	100%	100%	100%
0.65	65%	72%	70%	70%	74%	79%	89%	89%	88%	93%	92%	95%	98%	99%	99%	100%	100%	100%	100%	100%	100%
0.7	70%	68%	79%	79%	74%	84%	87%	88%	86%	90%	92%	98%	98%	98%	98%	100%	100%	100%	100%	100%	100%
0.75	77%	74%	82%	79%	80%	87%	88%	91%	92%	93%	95%	98%	99%	97%	100%	100%	100%	100%	100%	100%	100%
0.8	76%	76%	80%	84%	84%	86%	88%	94%	94%	94%	95%	99%	98%	100%	99%	100%	100%	100%	100%	100%	100%
0.85	77%	81%	88%	86%	91%	89%	95%	94%	94%	97%	97%	98%	100%	99%	100%	100%	100%	100%	100%	100%	100%
0.9	89%	90%	90%	91%	93%	93%	97%	96%	98%	98%	96%	96%	99%	100%	100%	99%	100%	100%	100%	100%	100%
0.95	95%	96%	97%	97%	96%	98%	100%	99%	100%	97%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Fig. 5.2 The average percentage of cases in which the optimal stable matching for male agents is equal to the optimal stable matching for female agents over 200 simulations for 10 male and 10 female agents

When we have the male agents as the proposing party we obtain the optimal stable matching for the male agents in the Deferred acceptance Algorithm. When the female agents are the proposing party, we obtain the optimal stable matching for female agents. It could be the case that the optimal stable matching for male agents is equal to the optimal stable matching for female agents. We investigate the effect of changing the parameter settings on the percentage of the cases in which the optimal stable matching for male agents is equal to the optimal stable matching for female agents. We again ran 200 simulations with 10 male and female agents with the same parameter settings for the symmetry factor s and the preference agreement factor p (both from 0 to 1 with steps of 0.05) and plotted the results in

Figure 5.2.

One stable matching when preference agreement factor $p = 1$

We find that in all cases where the preference agreement factor is 1, the optimal stable matching for male agents is equal to the optimal stable matching for female agents. We can support this with the following proof that shows that when preference agreement factor is $p = 1$, there is only one stable matching. Therefore, the optimal stable matching for male agents is equal to the optimal stable matching for female agents in these cases:

Theorem 9. *For any instance of the Stable Marriage Problem where all agents of the one side of the market have the same preferences over all agents of the other side of the market* ($p = 1$)*, there exists only one stable matching.*

Proof. Take any instance of the Stable Marriage Problem with a set M consisting of n male agents and a set Fconsisting of m female agents where all male agents have the same preferences over the female agents and the other way around ($p = 1$). We know from theorem 8 that it takes as many rounds as there are agents in the smallest side of the market to obtain the optimal stable matching for the proposing side of the market (M or F) in the Deferred Acceptance Algorithm since every round all agents from the proposing side of the market who are not engaged will propose to the same agent, which is the most attractive one of the agents of the other side of the market that they have not been rejected by yet. This proposed agent will always choose the proposer that this agent finds most attractive. Since all proposing agents have the same preferences and since all accepting agents have the same preferences, the first round the most popular male agent will start an engagement with the most popular female agent. The second round the second most popular male agent starts an engagement with the second most popular female agent and so on. Since the procedure will move from the most popular agent to the next less popular agent, until there are either no agents left that have not been proposed to or no agents left that are not engaged, all agents will be matched with the agent they are engaged with. Note that this proof holds for both sides of the market as the proposing party.

We now show inductively that all pairs in this matching must be part of any stable matching in this two-sided matching problem. For the pair consisting of the two most popular agents we know that there is no other (stable) matching in which they would yield a higher payoff, which means they both would not form blocking pairs with any other agent. This matching thus has to be part of any stable matching.

Now assume that for k pairs of agents we know that they have to be part of any stable matching, and who all prefer being matched with their current partner above being matched with any of the agents that are not part of one of the k pairs. From all the agents that are not part of one of the k pairs we now select the pair of agents that are the next, $k+1$-th, most popular agents. Since all agents in the k pairs do not form a blocking pair with these two agents we know that these two agents will also have to be a part of any stable matching. We go on adding pairs that have to be part of the stable matching until either no male agents or no female agents are left.

$\square$

By showing that when $p = 1$, there exists only one stable matching we have shown that when $p = 1$ the optimal stable matching for male agents is equal to the optimal stable matching for female agents.

One stable matching when symmetry factor $s = 1$

For $s = 1$ we also find that, independent of the value of p, for all simulations the optimal stable matching for male agents is equal to the optimal stable matching for female agents. We can support and extend this to any number of agents by proving that when $s = 1$ there exists only one stable matching.

Theorem 10. *For any instance of the Stable Marriage Problem where for all agents m_i and f_j holds that the preference score agent m_i assigns to being a relationship with f_j is equal to the preference score agent f_j assigns to being a relationship with m_i ($s = 1$), there exists only one stable matching.*

Proof. We have a set M of n male agents and a set F of m female agents. The preferences of an agent over all agents of the other sex form a strict total order. Symmetry factor $s = 1$ means that the preference score male agent m_i assigns to female agent f_j is equal to the preference score m_i assigns to f_j. ($\forall m_i \in M \, \forall f_j \in F \; u_m(m_i, f_j) = u_f(m_i, f_j)$). We thus can construct a matrix for the payoffs of both the male and the female agents as seen in (Table 5.1).

Note that it can be the case that male agent m_i and female agent f_j assign the same preferences score to being in a relationship with one another as male agent m_k and female

	m_1	m_2	...	m_n
f_1	7	6	...	3
f_2	2	9	...	8
...	...	...	...	...
f_m	4	7	...	10

Table 5.1 Example of a payoff matrix when symmetry factor $s = 1$

agent f_l would (with $f_j \neq f_l$ and $m_i \neq m_k$). It however cannot be the case that an agent is indifferent between being matched with one of two agents from the other side of the market.

We construct a procedure to obtain matchings, for which we ensure that every pair must be part of every stable matching for that particular instance of the Stable Marriage Problem. We also show that every agent of that instance is part of this matching. This way we show that there exists only one stable matching and thus that the optimal stable matching for male agents is equal to the optimal stable matching for female agents.

In the first round of our procedure we let the pair(s) consisting of the male and female agent(s), that yield the highest payoff of being in a relationship of all possible pairs start a relationship. Then we remove these agents from the market. Of the resulting restricted market we again let the pair(s) consisting of the male and female agent(s), that yield the highest payoff of being in a relationship of all remaining possible pairs in the restricted market start a relationship. Now we remove these agents and proceed until all agents are in a relationship. It is trivial that the resulting stable matching contains every agent, and every agent once.

We now show inductively that all pairs in this matching must be part of any stable matching in this instance of the Stable Marriage Problem. For the pairs that are matched in the first round we know there is no other (stable) matching in which they would yield a higher payoff, which means they both would not form blocking pairs with any other agent. This matching thus has to be part of any stable matching of this instance of the Stable Marriage Problem.

Now assume that for k pairs of agents we know that they have to be part of any stable matching, and who all prefer being matched with their current partner above being matched with one of the agents that are not part of one of the k pairs. From all the agents that are not part of one of the k pairs we now select the pair(s) of the agents that assign the highest payoff to being in a relationship with one of the agents outside the k pairs. Since all agents in the k pairs do not form a blocking pair with any agent in these pair(s) we know that these pair(s) will also have to be a part of the stable matching. We go on adding pairs that have to be part of any stable matching until either no male agents or no female agents are left and we have added all of the agents once. We thus have shown that when $s = 1$ there exists only one stable matching. □

An example with four male agents and four female agents:

	m_1	m_2	m_3	m_4
f_1	4	6	3	7.5
f_2	2	9	8.5	8
f_3	10	3.5	2.5	6.5
f_4	3	7	5	10

Table 5.2 Payoff matrix when symmetry factor $s = 1$ for 4 male agents and 4 female agents

We see (Table 5.2) that agent m_1 and f_3 and agent m_4 and f_4 both assign payoff 10 to being matched with one another, which is the highest payoff in all of the market. We thus know that the pairs (m_1, f_3) and (m_4, f_4) will be part of any stable matching, and thus match them and go on with the restricted market without these agents.

	m_2	m_3
f_1	6	3
f_2	9	8.5

Table 5.3 Restricted payoff matrix when symmetry factor $s = 1$ for 4 male agents and 4 female agents, where f_3, f_4, m_1, m_4 are matched and removed

In the resulting restricted market (Table 5.3) we find that m_2 and f_2 could yield the highest payoff, 9, by starting a relationship. We thus add (m_2, f_2) to our matching and further restrict the market by removing them, leaving just the pair (m_3, f_1) (Table 5.4) who of course will also be matched. This gives us the stable matching $\mu = \{(m_1, f_3), (m_4, f_4), (m_2, f_2), (m_3, f_1)\}$.

	m_3
f_1	3

Table 5.4 Restricted payoff matrix when symmetry factor $s = 1$ for 4 male agents and 4 female agents, where only f_1, m_3 are not matched yet

By showing that there exists only one stable matching for any instance of the Stable Marriage Problem where the payoffs are symmetric ($u_m(m_i, f_j) = u_f(m_i, f_j)$ for any male agent m_i and any female agent f_j), we have shown that the optimal stable matching for male agents is always equal to the optimal stable matching for female agents.

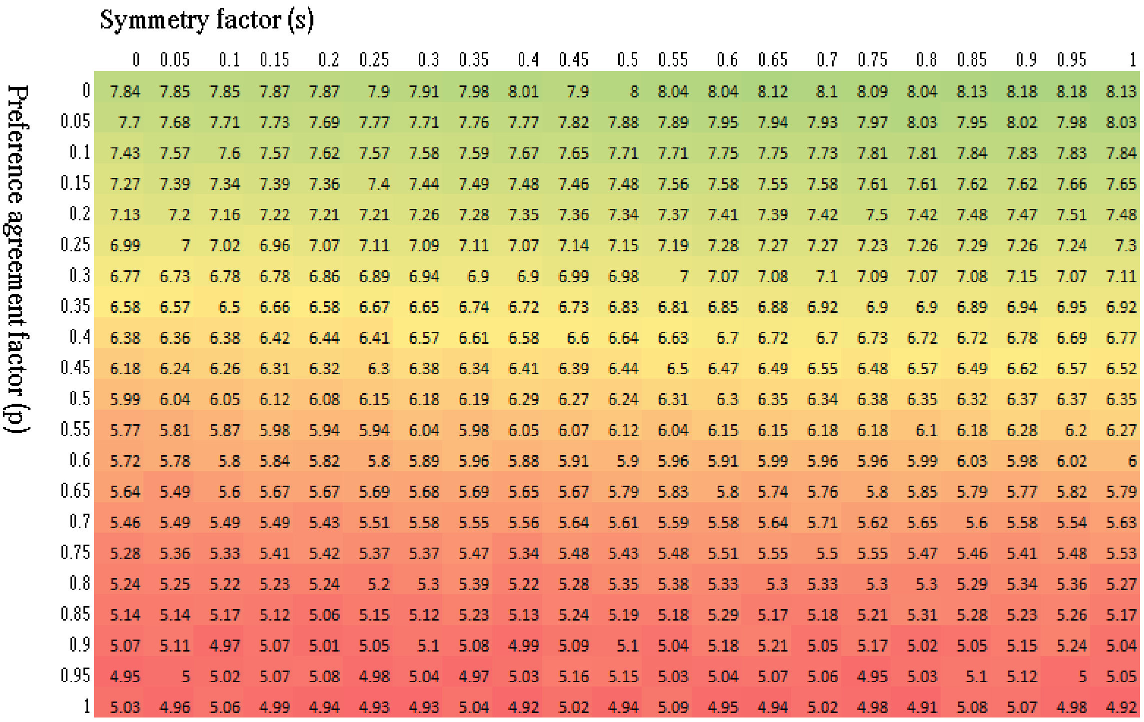
Other results on the equality of optimal stable matching for male and female agents

The optimal male and female matching are equal when preference agreement factor $p = 1$ or when symmetry factor $s = 1$. However, we find that this is not always the case when $p < 1$ and $s < 1$. The lower the symmetry factor s the more $u_m(m_i, f_j)$ and $u_f(m_i, f_j)$ tend to differ for any male agent m_i and any female agent f_j. Therefore it is more likely that the optimal stable matching for male agents is not equal to the optimal stable matching for female agents. The lower the preference agreement factor p, the less it will be the case that there exist agents that are more popular on average. It is thus less likely it is that in any round in the Deferred Acceptance Algorithm, all agents will propose to the same agents, when p is lower. The more disagreement within one side of the market the more likely it is that the optimal stable matching for female agents is different from the optimal stable matching for female agents. We find a faster decrease in the percentage of the simulations where the optimal male stable matching is equal to the optimal female stable matching when lowering p than for lowering s. For 10 male and 10 female agents we find that for $p = 0$ and $s = 0$ we only obtain the same optimal stable matching for male agents as the optimal stable matching for female agents in $18,5\%$ of the 200 cases.

5.3.3 Payoffs in optimal stable matchings

When we investigate the average payoff of male agents in the optimal stable matching for male agents (Figure 5.3), we find that the highest average payoff in the optimal male stable matching is obtained when all agents have independent preferences ($p = 0$) and when it is more likely that an agent is liked by agents that they like ($s = 1$). These payoffs are worst in the opposite case, when all male agents fight for the same female agents ($p = 1$) and when it is not more likely that agents are liked by the agents they like.

When we investigate the average payoff of male agents in the optimal stable matching for female agents (Figure 5.4) we find similar effects of changing the parameter values for s and p as for the average payoff of male agents in the optimal stable matching for male agents. The most important difference between payoffs for male agents in the optimal stable



Preference agreement factor (p)	Symmetry factor (s)																				
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	7.84	7.85	7.85	7.87	7.87	7.9	7.91	7.98	8.01	7.9	8	8.04	8.04	8.12	8.1	8.09	8.04	8.13	8.18	8.18	8.13
0.05	7.7	7.68	7.71	7.73	7.69	7.77	7.71	7.76	7.77	7.82	7.88	7.89	7.95	7.94	7.93	7.97	8.03	7.95	8.02	7.98	8.03
0.1	7.43	7.57	7.6	7.57	7.62	7.57	7.58	7.59	7.67	7.65	7.71	7.71	7.75	7.75	7.73	7.81	7.81	7.84	7.83	7.83	7.84
0.15	7.27	7.39	7.34	7.39	7.36	7.4	7.44	7.49	7.48	7.46	7.48	7.56	7.58	7.55	7.58	7.61	7.61	7.62	7.62	7.66	7.65
0.2	7.13	7.2	7.16	7.22	7.21	7.21	7.26	7.28	7.35	7.36	7.34	7.37	7.41	7.39	7.42	7.5	7.42	7.48	7.47	7.51	7.48
0.25	6.99	7	7.02	6.96	7.07	7.11	7.09	7.11	7.07	7.14	7.15	7.19	7.28	7.27	7.27	7.23	7.26	7.29	7.26	7.24	7.3
0.3	6.77	6.73	6.78	6.78	6.86	6.89	6.94	6.9	6.9	6.99	6.98	7	7.07	7.08	7.1	7.09	7.07	7.08	7.15	7.07	7.11
0.35	6.58	6.57	6.5	6.66	6.58	6.67	6.65	6.74	6.72	6.73	6.83	6.81	6.85	6.88	6.92	6.9	6.9	6.89	6.94	6.95	6.92
0.4	6.38	6.36	6.38	6.42	6.44	6.41	6.57	6.61	6.58	6.6	6.64	6.63	6.7	6.72	6.7	6.73	6.72	6.72	6.78	6.69	6.77
0.45	6.18	6.24	6.26	6.31	6.32	6.3	6.38	6.34	6.41	6.39	6.44	6.5	6.47	6.49	6.55	6.48	6.57	6.49	6.62	6.57	6.52
0.5	5.99	6.04	6.05	6.12	6.08	6.15	6.18	6.19	6.29	6.27	6.24	6.31	6.3	6.35	6.34	6.38	6.35	6.32	6.37	6.37	6.35
0.55	5.77	5.81	5.87	5.98	5.94	5.94	6.04	5.98	6.05	6.07	6.12	6.04	6.15	6.15	6.18	6.18	6.1	6.18	6.28	6.2	6.27
0.6	5.72	5.78	5.8	5.84	5.82	5.8	5.89	5.96	5.88	5.91	5.9	5.96	5.91	5.99	5.96	5.96	5.99	6.03	5.98	6.02	6
0.65	5.64	5.49	5.6	5.67	5.67	5.69	5.68	5.69	5.65	5.67	5.79	5.83	5.8	5.74	5.76	5.8	5.85	5.79	5.77	5.82	5.79
0.7	5.46	5.49	5.49	5.49	5.43	5.51	5.58	5.55	5.56	5.64	5.61	5.59	5.58	5.64	5.71	5.62	5.65	5.6	5.58	5.54	5.63
0.75	5.28	5.36	5.33	5.41	5.42	5.37	5.37	5.47	5.34	5.48	5.43	5.48	5.51	5.55	5.5	5.55	5.47	5.46	5.41	5.48	5.53
0.8	5.24	5.25	5.22	5.23	5.24	5.2	5.3	5.39	5.22	5.28	5.35	5.38	5.33	5.3	5.33	5.3	5.3	5.29	5.34	5.36	5.27
0.85	5.14	5.14	5.17	5.12	5.06	5.15	5.12	5.23	5.13	5.24	5.19	5.18	5.29	5.17	5.18	5.21	5.31	5.28	5.23	5.26	5.17
0.9	5.07	5.11	4.97	5.07	5.01	5.05	5.1	5.08	4.99	5.09	5.1	5.04	5.18	5.21	5.05	5.17	5.02	5.05	5.15	5.24	5.04
0.95	4.95	5	5.02	5.07	5.08	4.98	5.04	4.97	5.03	5.16	5.15	5.03	5.04	5.07	5.06	4.95	5.03	5.1	5.12	5	5.05
1	5.03	4.96	5.06	4.99	4.94	4.93	4.93	5.04	4.92	5.02	4.94	5.09	4.95	4.94	5.02	4.98	4.91	5.08	5.07	4.98	4.92

Fig. 5.3 The average payoff of male agents in the stable matching optimal for male agents resulting from the Deferred Acceptance Algorithm over 200 simulations for 10 male and 10 female agents

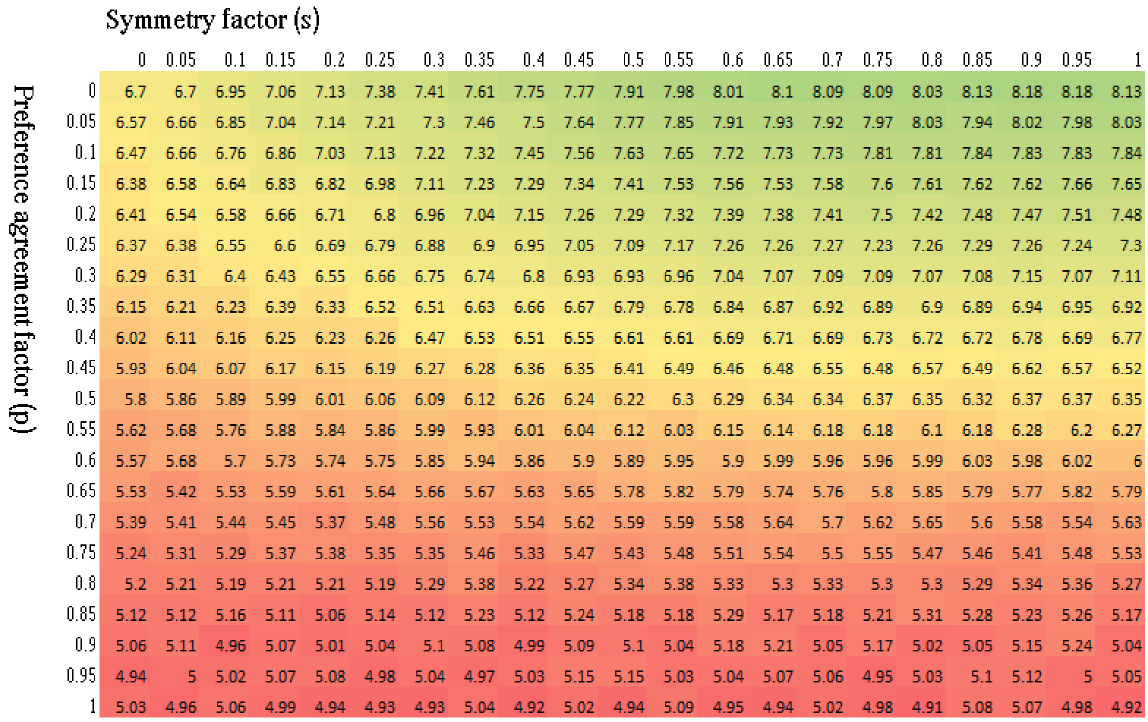
matching for male agents and the one for female agents is that the average payoffs for male agents are substantially higher in the optimal male stable matching for lower values for p and s. For 10 male and 10 female agents we for example find for $p = 0$ and $s = 0$ that the average payoff of male agents in the optimal stable matching for male agents is 7.84 where this is only 6.7 for the optimal stable matching for female agents.

5.3.4 Increasing the number of agents

We ran the same simulations also for 20 male and 20 female agents and for 100 male and 100 female agents, to see whether this effects the results of the outcome of the Deferred Acceptance Algorithm.

Rounds in Deferred Acceptance Algorithm

When we increase the number of agents we again find that the parameter setting for which it takes the most rounds to obtain the optimal stable matching in the Deferred Acceptance



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	6.7	6.7	6.95	7.06	7.13	7.38	7.41	7.61	7.75	7.77	7.91	7.98	8.01	8.1	8.09	8.09	8.03	8.13	8.18	8.18	8.13
0.05	6.57	6.66	6.85	7.04	7.14	7.21	7.3	7.46	7.5	7.64	7.77	7.85	7.91	7.93	7.92	7.97	8.03	7.94	8.02	7.98	8.03
0.1	6.47	6.66	6.76	6.86	7.03	7.13	7.22	7.32	7.45	7.56	7.63	7.65	7.72	7.73	7.73	7.81	7.81	7.84	7.83	7.83	7.84
0.15	6.38	6.58	6.64	6.83	6.82	6.98	7.11	7.23	7.29	7.34	7.41	7.53	7.56	7.53	7.58	7.6	7.61	7.62	7.62	7.66	7.65
0.2	6.41	6.54	6.58	6.66	6.71	6.8	6.96	7.04	7.15	7.26	7.29	7.32	7.39	7.38	7.41	7.5	7.42	7.48	7.47	7.51	7.48
0.25	6.37	6.38	6.55	6.6	6.69	6.79	6.88	6.9	6.95	7.05	7.09	7.17	7.26	7.26	7.27	7.23	7.26	7.29	7.26	7.24	7.3
0.3	6.29	6.31	6.4	6.43	6.55	6.66	6.75	6.74	6.8	6.93	6.93	6.96	7.04	7.07	7.09	7.09	7.07	7.08	7.15	7.07	7.11
0.35	6.15	6.21	6.23	6.39	6.33	6.52	6.51	6.63	6.66	6.67	6.79	6.78	6.84	6.87	6.92	6.89	6.9	6.89	6.94	6.95	6.92
0.4	6.02	6.11	6.16	6.25	6.23	6.26	6.47	6.53	6.51	6.55	6.61	6.61	6.69	6.71	6.69	6.73	6.72	6.72	6.78	6.69	6.77
0.45	5.93	6.04	6.07	6.17	6.15	6.19	6.27	6.28	6.36	6.35	6.41	6.49	6.46	6.48	6.55	6.48	6.57	6.49	6.62	6.57	6.52
0.5	5.8	5.86	5.89	5.99	6.01	6.06	6.09	6.12	6.26	6.24	6.22	6.3	6.29	6.34	6.34	6.37	6.35	6.32	6.37	6.37	6.35
0.55	5.62	5.68	5.76	5.88	5.84	5.86	5.99	5.93	6.01	6.04	6.12	6.03	6.15	6.14	6.18	6.18	6.1	6.18	6.28	6.2	6.27
0.6	5.57	5.68	5.7	5.73	5.74	5.75	5.85	5.94	5.86	5.9	5.89	5.95	5.9	5.99	5.96	5.96	5.99	6.03	5.98	6.02	6
0.65	5.53	5.42	5.53	5.59	5.61	5.64	5.66	5.67	5.63	5.65	5.78	5.82	5.79	5.74	5.76	5.8	5.85	5.79	5.77	5.82	5.79
0.7	5.39	5.41	5.44	5.45	5.37	5.48	5.56	5.53	5.54	5.62	5.59	5.59	5.58	5.64	5.7	5.62	5.65	5.6	5.58	5.54	5.63
0.75	5.24	5.31	5.29	5.37	5.38	5.35	5.35	5.46	5.33	5.47	5.43	5.48	5.51	5.54	5.5	5.55	5.47	5.46	5.41	5.48	5.53
0.8	5.2	5.21	5.19	5.21	5.21	5.19	5.29	5.38	5.22	5.27	5.34	5.38	5.33	5.3	5.33	5.3	5.3	5.29	5.34	5.36	5.27
0.85	5.12	5.12	5.16	5.11	5.06	5.14	5.12	5.23	5.12	5.24	5.18	5.18	5.29	5.17	5.18	5.21	5.31	5.28	5.23	5.26	5.17
0.9	5.06	5.11	4.96	5.07	5.01	5.04	5.1	5.08	4.99	5.09	5.1	5.04	5.18	5.21	5.05	5.17	5.02	5.05	5.15	5.24	5.04
0.95	4.94	5	5.02	5.07	5.08	4.98	5.04	4.97	5.03	5.15	5.15	5.03	5.04	5.07	5.06	4.95	5.03	5.1	5.12	5	5.05
1	5.03	4.96	5.06	4.99	4.94	4.93	4.93	5.04	4.92	5.02	4.94	5.09	4.95	4.94	5.02	4.98	4.91	5.08	5.07	4.98	4.92

Fig. 5.4 The average payoff of male agents in the stable matching optimal for female agents resulting from the Deferred Acceptance Algorithm over 200 simulations for 10 male and 10 female agents

Algorithm stays at around $p = 0.5$ and $s = 0$ and that the the optimal stable matching is obtained in the least rounds for $p = 0$ and $s = 1$. The number of rounds it takes to obtain the optimal stable matching in the Deferred Acceptance Algorithm scales exactly linear for the cases where $p = 1$ since it exactly takes as many rounds in the Deferred Acceptance Algorithm as there are agents. For other parameter settings the number of rounds it takes to obtain the optimal stable matching in the Deferred Acceptance Algorithm scales above linear. For the parameter settings for which it takes the most rounds it scales fastest. For $p = 0.5$ and $s = 0$ for 10 agents it takes 12,6 rounds on average in the Deferred Acceptance Algorithm to obtain the optimal stable matching where this is 29,7 for 20 agents (2.33 times more than for 10) and 207 rounds for 100 agents (17.3 times more than for 10 agents).

Equality of optimal stable matching for male agents and optimal stable matching for female agents

When we increase the number of agents we see a very strong decline in the number of cases in which the optimal stable matching for male agents is equal to the optimal stable matching for female agents. For 100 male and 100 female agents, we find that in none of the 200 simulations for $s = 0$ and $p = 0$ the optimal female stable matching was equal to the optimal male stable matching. The effect of the preference agreement factor p on equality of the optimal male and female stability decreased enormously when we increased to 100 male and 100 female agents. Even for $p = 0.95$ and $s = 0$ the optimal stable matching for male agents and the optimal stable matching for female agents are only equal in 10.0% of the cases.

Payoffs

Increasing the number of agents increases the average payoffs obtained by agents in the stable matchings. The parameter settings $p = 0$ and $s = 1$ are the parameter settings for which the male agents obtain the highest payoffs for the optimal stable matching for male agents (which is the same as the optimal stable matching for female agents) and increases from 8.13 for 10 agents to 8.80 for 20 agents and 9.62 for 100 agents.

5.3.5 Summary on simulations in the Deferred Acceptance Algorithm

In the Deferred Acceptance Algorithm it takes the least rounds to obtain a stable matching for instances of the Stable Marriage Problem with high symmetry between the preferences of the two sides of the market ($s = 1$) and with a lot of disagreement in the preferences within one side of the market ($p = 0$). This is the case, because it is most likely that agents propose to different agents from the other side of the market and since it is likely that proposals are answered positively. We find that it takes the most rounds in the Deferred Acceptance Algorithm for instances with low symmetry between the two sides of the market and mediocre agreement of preferences within the two sides of the market. When preferences are fully objective ($p = 1$) we find that it takes exactly as many rounds to obtain the stable matching in the Deferred Acceptance Algorithm as there are agents in the smallest side of the market. We prove this for any number of agents in Theorem 8.

For uniform random preferences it is the least likely that the stable matching optimal for male agents is equal to the stable matching optimal for female agents. When $p = 1$ or $s = 1$ we have that the stable matching optimal for male agents is always equal to the stable matching for male agents. We prove this in Theorem 9 and Theorem 10 respectively, by showing that there exists only one stable matching in these cases.

Average payoffs are highest when we have symmetric ($s = 1$) and subjective ($p = 1$) preferences. When all agents want to be matched with the same agent from the other side of the market ($p = 1$) we yield the lowest average payoffs.

5.4 Simulations in our model with different parameter settings

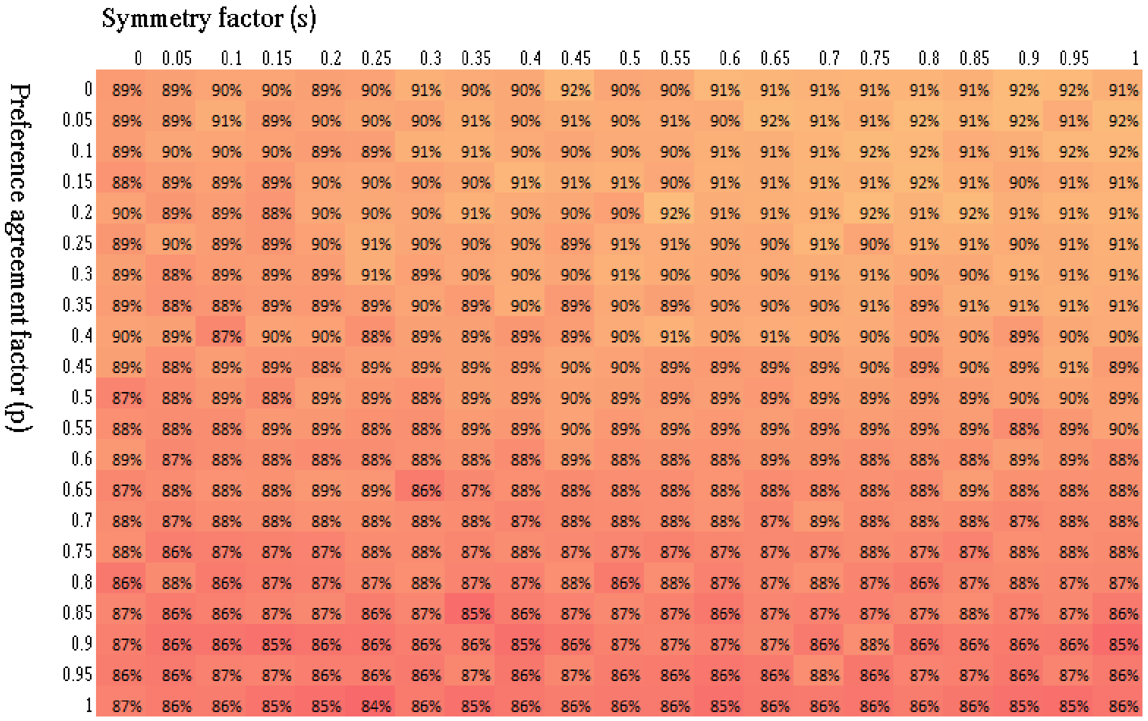
We know that our model will ultimately converge towards a stable matching with a probability that goes to 1 for increasing the number of daterounds. Ackermann et al. [1] give an exponential lower bound for the convergence time of the random better response dynamics in two-sided markets. We are interested to find out how changing the parameter settings influences the time it takes before a stable matching is reached, and whether this is a stable matching that is optimal for the male or the female agents or none of the two sides of the market. We are also interested in the stability, payoffs and the percentage of agents in a relationship when a stable matching is not yet reached.

We started the simulations again with 10 male and 10 female agents. We first ran simulations for 1 dateround (since this gives us a random matching of agents because all agents start a relationship at the first round because they started single and prefer being in a relationship with any agent above being single). We then ran simulations for 20, 60 and 100 daterounds and ended with a simulation in which we would continue until a stable matching was obtained.

5.4.1 Percentage of agents in a relationship

After one dateround we obtain a random matching in which all agents are in a relationship since all agents prefer being matched with the agent they date in the first dateround above being single. After later daterounds it could be the case that agents are single, since it might be the case that an agent m_a is left by their partner agent f_b if agent f_b met an agent m_c with whom this agent f_b could satisfy a blocking pair. Note however that in our model at least 50% of the agents will be in a relationship at any time during the simulation since an agent m_a can only leave an agent f_b by starting a new relationship himself.

After 20 daterounds we find in Figure 5.5 that on average 89% of the agents are in a relationship. This percentage is highest for $s = 1$ and $p = 0$ with 91%. For simulations where $p = 1$ this percentage is lowest with around 86%. After 60 daterounds (Figure 5.6) the average percentage of agents in a relationship has increased towards a maximum of 99% for



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	89%	89%	90%	90%	89%	90%	91%	90%	90%	92%	90%	90%	91%	91%	91%	91%	91%	91%	92%	92%	91%
0.05	89%	89%	91%	89%	90%	90%	90%	91%	90%	91%	90%	91%	90%	92%	91%	91%	92%	91%	92%	91%	92%
0.1	89%	90%	90%	90%	89%	89%	91%	91%	90%	90%	90%	90%	91%	91%	91%	92%	92%	91%	91%	92%	92%
0.15	88%	89%	89%	89%	90%	90%	90%	90%	91%	91%	91%	90%	91%	91%	91%	91%	92%	91%	90%	91%	91%
0.2	90%	89%	89%	88%	90%	90%	90%	91%	90%	90%	90%	92%	91%	91%	91%	92%	91%	92%	91%	91%	91%
0.25	89%	90%	89%	89%	90%	91%	90%	90%	90%	89%	91%	91%	90%	90%	91%	90%	91%	91%	90%	91%	91%
0.3	89%	88%	89%	89%	89%	91%	89%	90%	90%	90%	91%	90%	90%	90%	91%	91%	90%	90%	91%	91%	91%
0.35	89%	88%	88%	89%	89%	89%	90%	89%	90%	89%	90%	89%	90%	90%	90%	91%	89%	91%	91%	91%	91%
0.4	90%	89%	87%	90%	90%	88%	89%	89%	89%	89%	90%	91%	90%	91%	90%	90%	90%	90%	89%	90%	90%
0.45	89%	88%	89%	89%	88%	89%	89%	89%	89%	90%	90%	89%	89%	89%	89%	90%	89%	90%	89%	91%	89%
0.5	87%	88%	89%	88%	89%	89%	88%	89%	89%	90%	89%	89%	89%	89%	89%	89%	89%	89%	90%	90%	89%
0.55	88%	88%	88%	89%	89%	88%	88%	89%	89%	90%	89%	89%	89%	89%	89%	89%	89%	89%	88%	89%	90%
0.6	89%	87%	88%	88%	88%	88%	88%	88%	88%	89%	88%	88%	88%	89%	89%	88%	88%	88%	89%	89%	88%
0.65	87%	88%	88%	88%	89%	89%	86%	87%	88%	88%	88%	88%	88%	88%	88%	88%	88%	89%	88%	88%	88%
0.7	88%	87%	88%	88%	88%	88%	88%	88%	87%	88%	88%	88%	88%	87%	89%	88%	88%	88%	87%	88%	88%
0.75	88%	86%	87%	87%	87%	88%	88%	87%	88%	87%	87%	87%	87%	87%	87%	88%	87%	87%	88%	88%	88%
0.8	86%	88%	86%	87%	87%	87%	88%	87%	87%	88%	86%	88%	87%	87%	88%	87%	86%	87%	88%	87%	87%
0.85	87%	86%	86%	87%	87%	86%	87%	85%	86%	87%	87%	87%	86%	87%	87%	87%	87%	88%	87%	87%	86%
0.9	87%	86%	86%	85%	86%	86%	86%	86%	85%	86%	87%	87%	87%	87%	86%	88%	86%	86%	86%	86%	85%
0.95	86%	86%	87%	87%	86%	86%	86%	87%	86%	87%	86%	86%	86%	86%	88%	86%	87%	87%	86%	87%	86%
1	87%	86%	86%	85%	85%	84%	86%	85%	86%	86%	86%	86%	85%	86%	86%	86%	86%	86%	85%	85%	86%

Fig. 5.5 The average percentage of agents in a relationship over 200 simulations after 20 daterounds for 10 male and 10 female agents

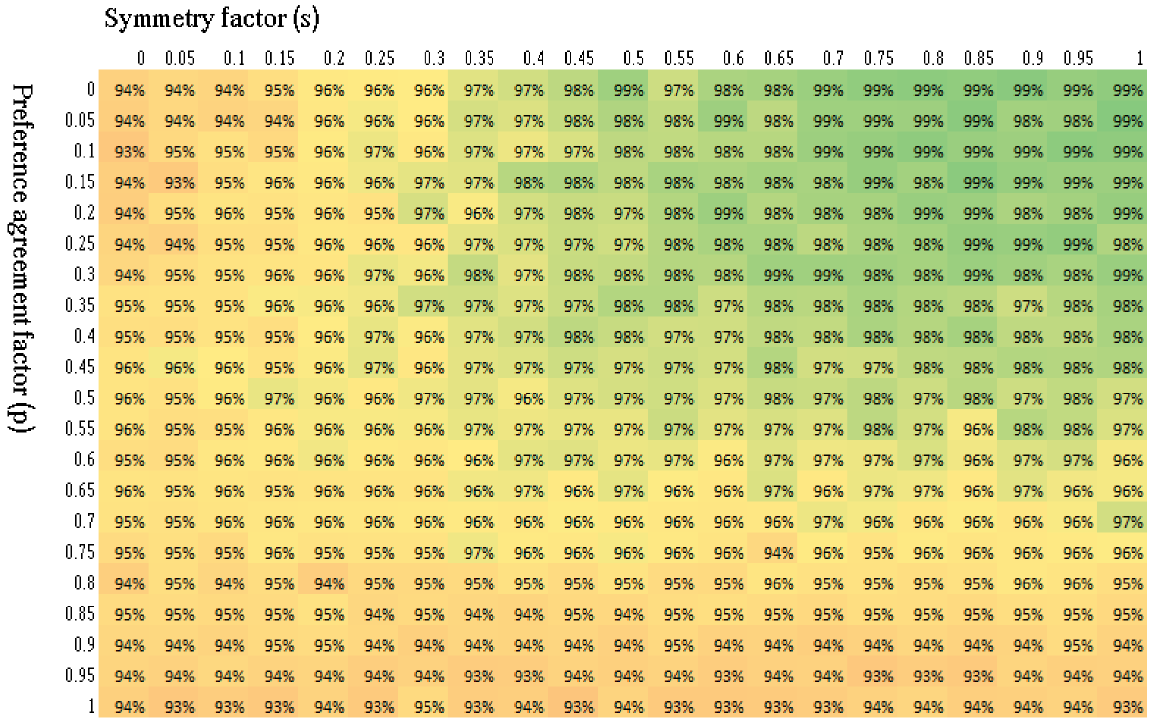
$s = 1$ and $p = 0$ and a minimum for $p = 1$ at around 94%. After 100 daterounds (Figure 5.7) all agents were in a relationship for $s = 1$ and $p = 0$. The average percentage of agents in a relationship is now lowest for $p = 0$ and $s = 0$ with 96%.

5.4.2 Stability

The most important notion of the stability of a matching seems to be the number of blocking pairs. We also investigate the percentage of simulations in which a stable matching is obtained and whether this is a suboptimal stable matching or one of the optimal ones resulting from the Deferred Acceptance Algorithm. Furthermore, we investigate the number of agents that have the same partner as they would have in the optimal stable matching for male agents and for female agents.

Blocking pairs

When we have obtained random matchings after 1 dateround for 10 male and 10 female agents we find that the average number of blocking pairs, thus pairs of agents who would rather have been matched with one another than with their current partner, varies between 22



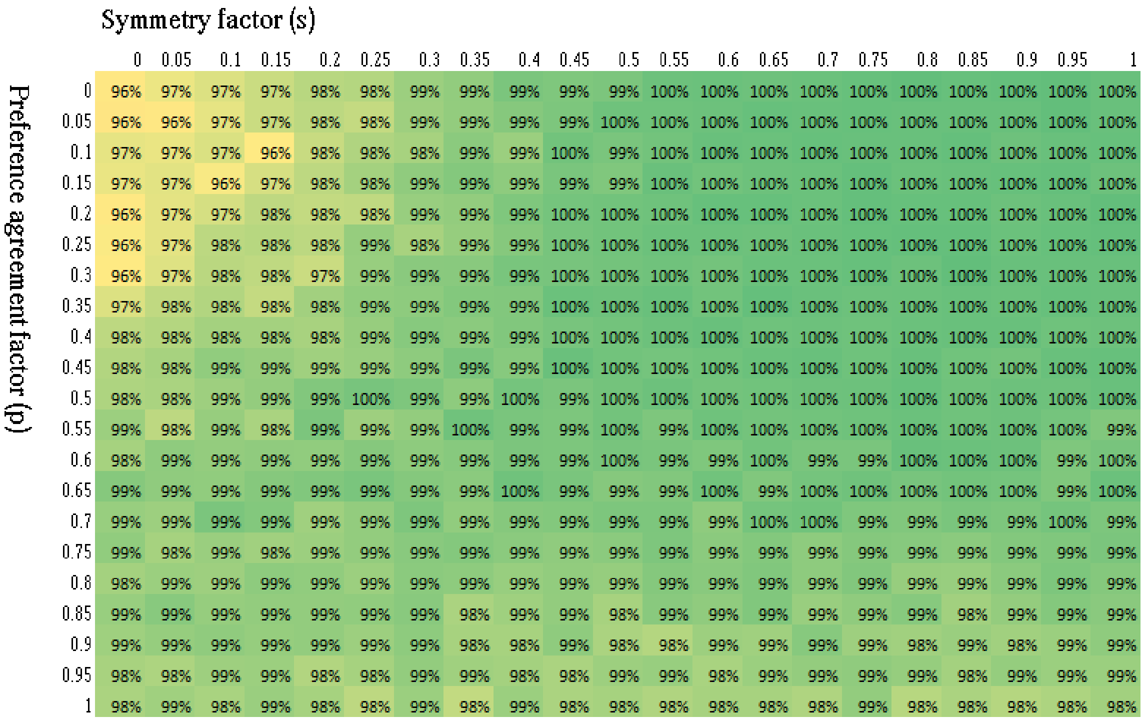
Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	94%	94%	94%	95%	96%	96%	96%	97%	97%	98%	99%	97%	98%	98%	99%	99%	99%	99%	99%	99%	99%
0.05	94%	94%	94%	94%	96%	96%	96%	97%	97%	98%	98%	98%	99%	98%	99%	99%	99%	99%	98%	98%	99%
0.1	93%	95%	95%	95%	96%	97%	96%	97%	97%	97%	98%	98%	98%	98%	99%	99%	99%	99%	99%	99%	99%
0.15	94%	93%	95%	96%	96%	96%	97%	97%	98%	98%	98%	98%	98%	98%	98%	99%	98%	99%	99%	99%	99%
0.2	94%	95%	96%	95%	96%	95%	97%	96%	97%	98%	97%	98%	99%	98%	98%	98%	99%	99%	98%	98%	99%
0.25	94%	94%	95%	95%	96%	96%	96%	97%	97%	97%	97%	98%	98%	98%	98%	98%	98%	99%	99%	99%	98%
0.3	94%	95%	95%	96%	96%	97%	96%	98%	97%	98%	98%	98%	98%	99%	99%	98%	98%	99%	98%	98%	99%
0.35	95%	95%	95%	96%	96%	96%	97%	97%	97%	97%	98%	98%	97%	98%	98%	98%	98%	98%	97%	98%	98%
0.4	95%	95%	95%	95%	96%	97%	96%	97%	97%	98%	98%	97%	97%	98%	98%	98%	98%	98%	98%	98%	98%
0.45	96%	96%	96%	95%	96%	97%	96%	97%	97%	97%	97%	97%	97%	98%	97%	97%	98%	98%	98%	98%	98%
0.5	96%	95%	96%	97%	96%	96%	97%	97%	96%	97%	97%	97%	97%	98%	97%	98%	97%	98%	97%	98%	97%
0.55	96%	95%	95%	96%	96%	96%	96%	97%	97%	97%	97%	97%	97%	97%	97%	98%	97%	96%	98%	98%	97%
0.6	95%	95%	96%	96%	96%	96%	96%	96%	97%	97%	97%	97%	96%	97%	97%	97%	97%	96%	97%	97%	96%
0.65	96%	95%	96%	95%	96%	96%	96%	96%	97%	96%	97%	96%	96%	97%	96%	97%	97%	96%	97%	96%	96%
0.7	95%	95%	96%	96%	96%	96%	96%	96%	96%	96%	96%	96%	96%	96%	97%	96%	96%	96%	96%	96%	97%
0.75	95%	95%	95%	96%	95%	95%	95%	97%	96%	96%	96%	96%	96%	94%	96%	95%	96%	96%	96%	96%	96%
0.8	94%	95%	94%	95%	94%	95%	95%	95%	95%	95%	95%	95%	95%	96%	95%	95%	95%	95%	96%	96%	95%
0.85	95%	95%	95%	95%	95%	94%	95%	94%	94%	95%	94%	95%	95%	95%	95%	95%	95%	95%	95%	95%	95%
0.9	94%	94%	94%	95%	95%	94%	94%	94%	94%	94%	94%	95%	94%	94%	94%	94%	94%	94%	94%	95%	94%
0.95	94%	94%	94%	94%	94%	94%	94%	93%	93%	94%	94%	94%	93%	94%	94%	93%	93%	93%	94%	94%	94%
1	94%	93%	93%	93%	94%	93%	95%	93%	94%	93%	94%	93%	93%	93%	93%	94%	94%	94%	94%	94%	93%

Fig. 5.6 The average percentage of agents in a relationship over 200 simulations after 60 daterounds for 10 male and 10 female agents

and 31 (Figure 5.8). The random matchings that are least stable, and thus have the highest number of blocking pairs, are found for high values of s and low values of p. This is the case since when we have high values for s it is more likely that when an agent a prefers being in a relationship with an agent b that is not the current partner of agent a, it is also more likely that agent b would also want to be in a relationship with agent a. In the same manner, for lower values of p random matchings are less stable, since agent's preferences differ more. Since when agents have more similar preferences, preferences become less symmetric since if for example every male agent likes the same female agent most, it can of course not be the case that all these male agents are all the most preferred male agent by this female agent. Interestingly we find that after 20 daterounds that for $s = 1$ and $p = 0$ we find the most stable matchings with only 3.2 blocking pairs on average (Figure 5.9). The matchings are least stable for $p = 1$, with around 10 blocking pairs on average, independent of the value of s. It seems to be the case that for high values of s and low values of p we converge towards a stable matching fastest. This is the case because agents all have different preferences and it is very likely that when an agent wants to start a new relationship with an agent he or she meets it is also more likely that this is the case the other way around. After 60 (Figure 5.10)



Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	96%	97%	97%	97%	98%	98%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.05	96%	96%	97%	97%	98%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.1	97%	97%	97%	96%	98%	98%	98%	99%	99%	100%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.15	97%	97%	96%	97%	98%	98%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.2	96%	97%	97%	98%	98%	98%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.25	96%	97%	98%	98%	98%	99%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.3	96%	97%	98%	98%	97%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.35	97%	98%	98%	98%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.4	98%	98%	98%	98%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.45	98%	98%	99%	99%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.5	98%	98%	99%	99%	99%	100%	99%	99%	100%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.55	99%	98%	99%	98%	99%	99%	99%	100%	99%	99%	100%	99%	100%	100%	100%	100%	100%	100%	100%	100%	99%
0.6	98%	99%	99%	99%	99%	99%	99%	99%	99%	99%	100%	99%	99%	100%	99%	99%	100%	100%	100%	99%	100%
0.65	99%	99%	99%	99%	99%	99%	99%	99%	100%	99%	99%	99%	100%	99%	100%	100%	100%	100%	100%	99%	100%
0.7	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	100%	100%	99%	99%	99%	99%	100%	99%
0.75	99%	98%	99%	98%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%
0.8	98%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%	99%
0.85	99%	99%	99%	99%	99%	99%	99%	98%	99%	99%	98%	99%	99%	99%	99%	99%	99%	98%	99%	99%	99%
0.9	99%	99%	99%	99%	99%	99%	99%	98%	98%	99%	98%	98%	99%	99%	99%	99%	98%	99%	98%	99%	99%
0.95	98%	98%	99%	99%	98%	98%	99%	99%	98%	98%	99%	99%	98%	99%	99%	99%	99%	98%	99%	99%	99%
1	98%	99%	98%	99%	98%	98%	99%	98%	99%	98%	98%	98%	98%	98%	98%	99%	98%	98%	98%	98%	98%

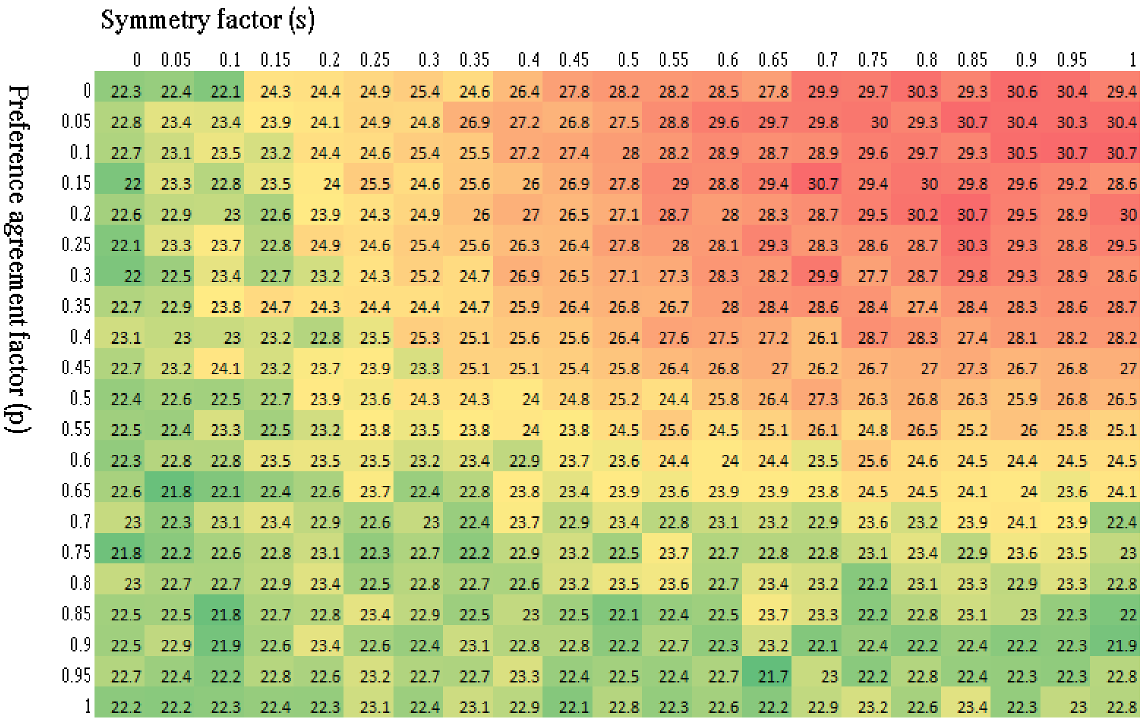
Fig. 5.7 The average percentage of agents in a relationship over 200 simulations after 100 daterounds for 10 male and 10 female agents

and 100 (Figure 5.11) daterounds we find that matchings are least stable for $p = 0$ and $s = 0$ with on average 4.1 and 2.8 blocking pairs respectively.

Percentage of simulations in which a stable matching is obtained

We have investigated how many of the simulations for 10 male and 10 female agents have reached a stable matching after 20 (Figure 5.12), 60 (Figure 5.13) and 100 (Figure 5.14) daterounds and investigated how many of those stable matchings were suboptimal and how many were optimal for either the male or the female agents or both. We found that most stable matchings were obtained for high values for s and low values for p with 7% after 20 daterounds, 34% after 60 daterounds and 99% after 100 daterounds. For $p = 0$ and $s = 0$ the least stable matchings are obtained.

When we run simulations until our model has converged to a stable matching Figure 5.15 we find that a stable matching is obtained fastest for $p = 0$ and $s = 1$, with 44.7 daterounds on average. When there is no symmetry and all agents have different preferences ($p = 0$ and $s = 0$) it takes the most rounds to obtain a stable matching with 161 rounds on average. For simulations where $p = 1$ it takes about 81 rounds, independent of the value of s.



Symmetry factor (s)

Preference agreement factor (p)

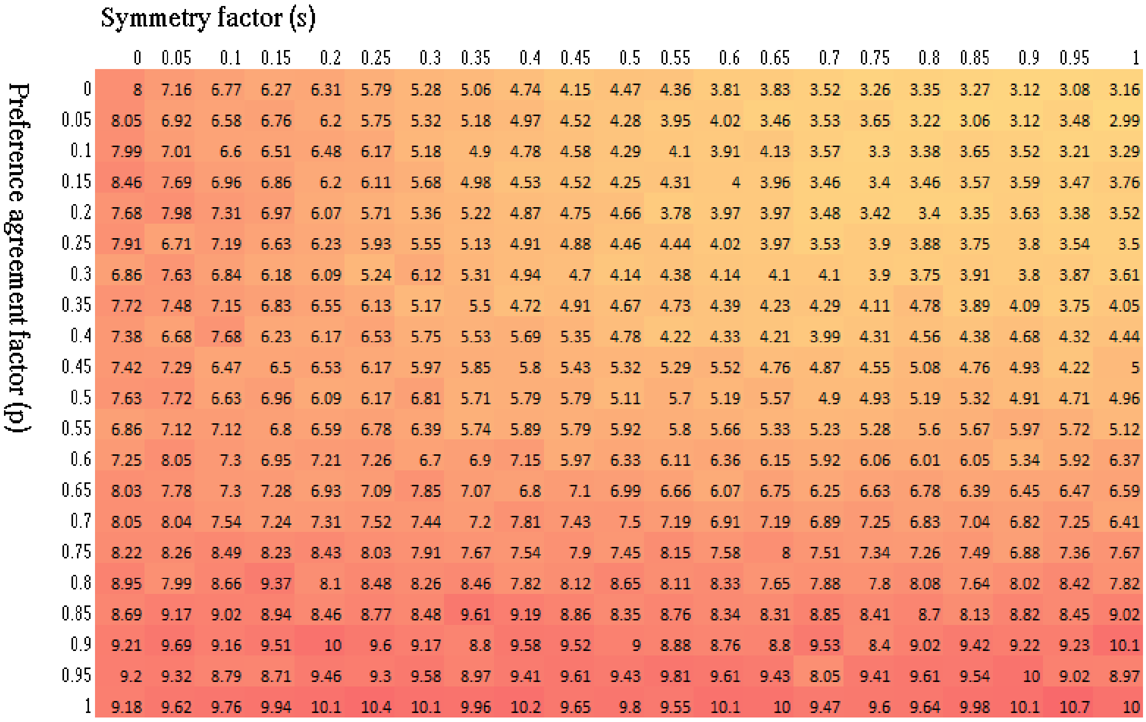
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	22.3	22.4	22.1	24.3	24.4	24.9	25.4	24.6	26.4	27.8	28.2	28.2	28.5	27.8	29.9	29.7	30.3	29.3	30.6	30.4	29.4
0.05	22.8	23.4	23.4	23.9	24.1	24.9	24.8	26.9	27.2	26.8	27.5	28.8	29.6	29.7	29.8	30	29.3	30.7	30.4	30.3	30.4
0.1	22.7	23.1	23.5	23.2	24.4	24.6	25.4	25.5	27.2	27.4	28	28.2	28.9	28.7	28.9	29.6	29.7	29.3	30.5	30.7	30.7
0.15	22	23.3	22.8	23.5	24	25.5	24.6	25.6	26	26.9	27.8	29	28.8	29.4	30.7	29.4	30	29.8	29.6	29.2	28.6
0.2	22.6	22.9	23	22.6	23.9	24.3	24.9	26	27	26.5	27.1	28.7	28	28.3	28.7	29.5	30.2	30.7	29.5	28.9	30
0.25	22.1	23.3	23.7	22.8	24.9	24.6	25.4	25.6	26.3	26.4	27.8	28	28.1	29.3	28.3	28.6	28.7	30.3	29.3	28.8	29.5
0.3	22	22.5	23.4	22.7	23.2	24.3	25.2	24.7	26.9	26.5	27.1	27.3	28.3	28.2	29.9	27.7	28.7	29.8	29.3	28.9	28.6
0.35	22.7	22.9	23.8	24.7	24.3	24.4	24.4	24.7	25.9	26.4	26.8	26.7	28	28.4	28.6	28.4	27.4	28.4	28.3	28.6	28.7
0.4	23.1	23	23	23.2	22.8	23.5	25.3	25.1	25.6	25.6	26.4	27.6	27.5	27.2	26.1	28.7	28.3	27.4	28.1	28.2	28.2
0.45	22.7	23.2	24.1	23.2	23.7	23.9	23.3	25.1	25.1	25.4	25.8	26.4	26.8	27	26.2	26.7	27	27.3	26.7	26.8	27
0.5	22.4	22.6	22.5	22.7	23.9	23.6	24.3	24.3	24	24.8	25.2	24.4	25.8	26.4	27.3	26.3	26.8	26.3	25.9	26.8	26.5
0.55	22.5	22.4	23.3	22.5	23.2	23.8	23.5	23.8	24	23.8	24.5	25.6	24.5	25.1	26.1	24.8	26.5	25.2	26	25.8	25.1
0.6	22.3	22.8	22.8	23.5	23.5	23.5	23.2	23.4	22.9	23.7	23.6	24.4	24	24.4	23.5	25.6	24.6	24.5	24.4	24.5	24.5
0.65	22.6	21.8	22.1	22.4	22.6	23.7	22.4	22.8	23.8	23.4	23.9	23.6	23.9	23.9	23.8	24.5	24.5	24.1	24	23.6	24.1
0.7	23	22.3	23.1	23.4	22.9	22.6	23	22.4	23.7	22.9	23.4	22.8	23.1	23.2	22.9	23.6	23.2	23.9	24.1	23.9	22.4
0.75	21.8	22.2	22.6	22.8	23.1	22.3	22.7	22.2	22.9	23.2	22.5	23.7	22.7	22.8	22.8	23.1	23.4	22.9	23.6	23.5	23
0.8	23	22.7	22.7	22.9	23.4	22.5	22.8	22.7	22.6	23.2	23.5	23.6	22.7	23.4	23.2	22.2	23.1	23.3	22.9	23.3	22.8
0.85	22.5	22.5	21.8	22.7	22.8	23.4	22.9	22.5	23	22.5	22.1	22.4	22.5	23.7	23.3	22.2	22.8	23.1	23	22.3	22
0.9	22.5	22.9	21.9	22.6	23.4	22.6	22.4	23.1	22.8	22.8	22.2	22.7	22.3	23.2	22.1	22.4	22.2	22.4	22.2	22.3	21.9
0.95	22.7	22.4	22.2	22.8	22.6	23.2	22.7	22.7	23.3	22.4	22.5	22.4	22.7	21.7	23	22.2	22.8	22.4	22.3	22.3	22.8
1	22.2	22.2	22.3	22.4	22.3	23.1	22.4	23.1	22.9	22.1	22.8	22.3	22.6	22.2	22.9	23.2	22.6	23.4	22.3	23	22.8

Fig. 5.8 The average number of blocking pairs over 200 simulations in a random matching after 1 dateround for 10 male and 10 female agents

Percentage of simulations in which a suboptimal stable matching is obtained

Not many of the obtained stable matchings for 10 male and 10 female agents are suboptimal, meaning a stable matching other than one of the optimal stable matchings that would result from the Deferred Acceptance Algorithm. We find that for low values for p and s around 1% of the simulations ended up in a suboptimal stable matching after 20 daterounds. After 60 daterounds this percentage has increased, with a maximum of 8% for $p = 0$ and $s = 0$. After 100 daterounds this percentage has increased to 13%. Eventually, after 161 daterounds (Figure 5.16) on average we find that 35% of the 200 simulations for $s = 0$ and $p = 0$ ended in a stable matching that is neither optimal for the male agents nor for the female agents. For $p = 1$, independent of the value for s we never obtain a suboptimal stable matching. This is also the case for $s = 1$ and any value of p.

We have proven Theorem 9 that said that in any Stable Marriage Problem in which all agents of one side of the market have the same preferences over the other side of the market and the other way around ($p = 1$), there exists only one stable matching. From this proof trivially follows that no suboptimal stable matching exists for any instance of Stable Marriage



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	8	7.16	6.77	6.27	6.31	5.79	5.28	5.06	4.74	4.15	4.47	4.36	3.81	3.83	3.52	3.26	3.35	3.27	3.12	3.08	3.16
0.05	8.05	6.92	6.58	6.76	6.2	5.75	5.32	5.18	4.97	4.52	4.28	3.95	4.02	3.46	3.53	3.65	3.22	3.06	3.12	3.48	2.99
0.1	7.99	7.01	6.6	6.51	6.48	6.17	5.18	4.9	4.78	4.58	4.29	4.1	3.91	4.13	3.57	3.3	3.38	3.65	3.52	3.21	3.29
0.15	8.46	7.69	6.96	6.86	6.2	6.11	5.68	4.98	4.53	4.52	4.25	4.31	4	3.96	3.46	3.4	3.46	3.57	3.59	3.47	3.76
0.2	7.68	7.98	7.31	6.97	6.07	5.71	5.36	5.22	4.87	4.75	4.66	3.78	3.97	3.97	3.48	3.42	3.4	3.35	3.63	3.38	3.52
0.25	7.91	6.71	7.19	6.63	6.23	5.93	5.55	5.13	4.91	4.88	4.46	4.44	4.02	3.97	3.53	3.9	3.88	3.75	3.8	3.54	3.5
0.3	6.86	7.63	6.84	6.18	6.09	5.24	6.12	5.31	4.94	4.7	4.14	4.38	4.14	4.1	4.1	3.9	3.75	3.91	3.8	3.87	3.61
0.35	7.72	7.48	7.15	6.83	6.55	6.13	5.17	5.5	4.72	4.91	4.67	4.73	4.39	4.23	4.29	4.11	4.78	3.89	4.09	3.75	4.05
0.4	7.38	6.68	7.68	6.23	6.17	6.53	5.75	5.53	5.69	5.35	4.78	4.22	4.33	4.21	3.99	4.31	4.56	4.38	4.68	4.32	4.44
0.45	7.42	7.29	6.47	6.5	6.53	6.17	5.97	5.85	5.8	5.43	5.32	5.29	5.52	4.76	4.87	4.55	5.08	4.76	4.93	4.22	5
0.5	7.63	7.72	6.63	6.96	6.09	6.17	6.81	5.71	5.79	5.79	5.11	5.7	5.19	5.57	4.9	4.93	5.19	5.32	4.91	4.71	4.96
0.55	6.86	7.12	7.12	6.8	6.59	6.78	6.39	5.74	5.89	5.79	5.92	5.8	5.66	5.33	5.23	5.28	5.6	5.67	5.97	5.72	5.12
0.6	7.25	8.05	7.3	6.95	7.21	7.26	6.7	6.9	7.15	5.97	6.33	6.11	6.36	6.15	5.92	6.06	6.01	6.05	5.34	5.92	6.37
0.65	8.03	7.78	7.3	7.28	6.93	7.09	7.85	7.07	6.8	7.1	6.99	6.66	6.07	6.75	6.25	6.63	6.78	6.39	6.45	6.47	6.59
0.7	8.05	8.04	7.54	7.24	7.31	7.52	7.44	7.2	7.81	7.43	7.5	7.19	6.91	7.19	6.89	7.25	6.83	7.04	6.82	7.25	6.41
0.75	8.22	8.26	8.49	8.23	8.43	8.03	7.91	7.67	7.54	7.9	7.45	8.15	7.58	8	7.51	7.34	7.26	7.49	6.88	7.36	7.67
0.8	8.95	7.99	8.66	9.37	8.1	8.48	8.26	8.46	7.82	8.12	8.65	8.11	8.33	7.65	7.88	7.8	8.08	7.64	8.02	8.42	7.82
0.85	8.69	9.17	9.02	8.94	8.46	8.77	8.48	9.61	9.19	8.86	8.35	8.76	8.34	8.31	8.85	8.41	8.7	8.13	8.82	8.45	9.02
0.9	9.21	9.69	9.16	9.51	10	9.6	9.17	8.8	9.58	9.52	9	8.88	8.76	8.8	9.53	8.4	9.02	9.42	9.22	9.23	10.1
0.95	9.2	9.32	8.79	8.71	9.46	9.3	9.58	8.97	9.41	9.61	9.43	9.81	9.61	9.43	8.05	9.41	9.61	9.54	10	9.02	8.97
1	9.18	9.62	9.76	9.94	10.1	10.4	10.1	9.96	10.2	9.65	9.8	9.55	10.1	10	9.47	9.6	9.64	9.98	10.1	10.7	10

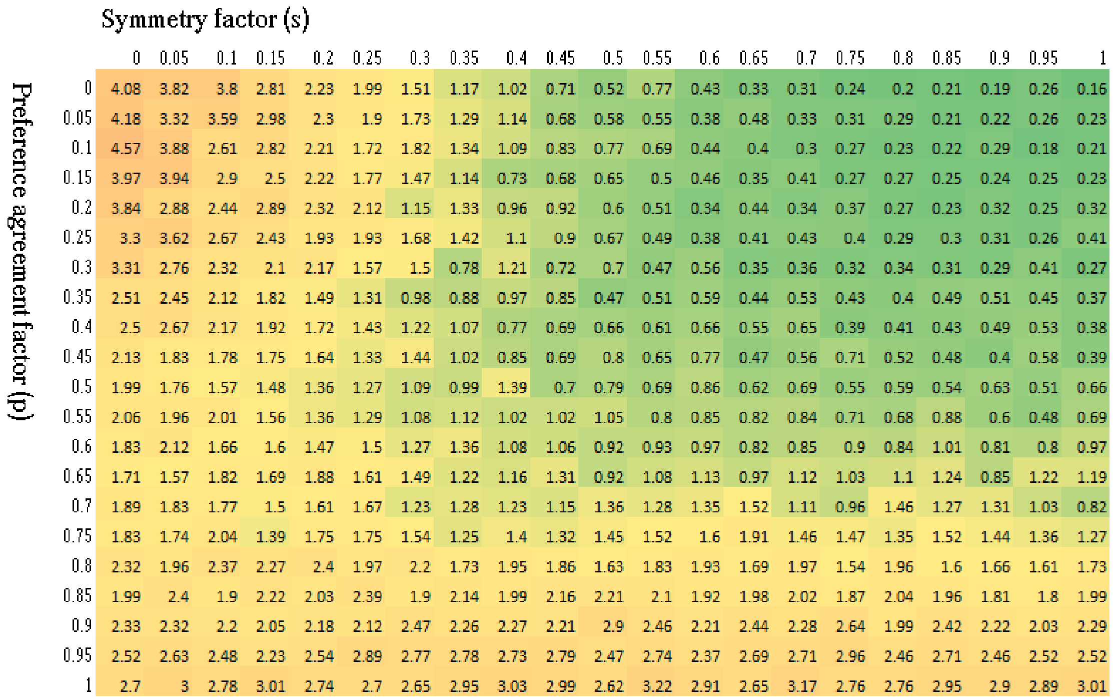
Fig. 5.9 The average number of blocking pairs over 200 simulations after 20 daterounds for 10 male and 10 female agents

Problem in which all agents of one side of the market have the same preferences over the other side of the market and the other way around.

We have proven Theorem 10 that said that in any Stable Marriage Problem with symmetric preferences ($s = 1$), there exists only one stable matching. From this proof trivially follows that for any instance of Stable Marriage Problem with symmetric preferences no stable matching exists that is not optimal for one side of the market.

Percentage of agents in optimal stable matchings

We also kept track of the percentage of the agents that were matched with the same agents as they would be with in the optimal stable matching for male agents and the optimal stable matching for female agents during the simulation. The average percentage of male agents matched with the same partner as they would be with in one of the optimal stable matchings is lowest for $p = 0$ and $s = 0$ with both 44% of the male agents being matched with the same partner as they would be in the optimal stable matching for male agents and 44% of the male agents being matched with the same partner as they would be in the optimal stable



Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	4.08	3.82	3.8	2.81	2.23	1.99	1.51	1.17	1.02	0.71	0.52	0.77	0.43	0.33	0.31	0.24	0.2	0.21	0.19	0.26	0.16
0.05	4.18	3.32	3.59	2.98	2.3	1.9	1.73	1.29	1.14	0.68	0.58	0.55	0.38	0.48	0.33	0.31	0.29	0.21	0.22	0.26	0.23
0.1	4.57	3.88	2.61	2.82	2.21	1.72	1.82	1.34	1.09	0.83	0.77	0.69	0.44	0.4	0.3	0.27	0.23	0.22	0.29	0.18	0.21
0.15	3.97	3.94	2.9	2.5	2.22	1.77	1.47	1.14	0.73	0.68	0.65	0.5	0.46	0.35	0.41	0.27	0.27	0.25	0.24	0.25	0.23
0.2	3.84	2.88	2.44	2.89	2.32	2.12	1.15	1.33	0.96	0.92	0.6	0.51	0.34	0.44	0.34	0.37	0.27	0.23	0.32	0.25	0.32
0.25	3.3	3.62	2.67	2.43	1.93	1.93	1.68	1.42	1.1	0.9	0.67	0.49	0.38	0.41	0.43	0.4	0.29	0.3	0.31	0.26	0.41
0.3	3.31	2.76	2.32	2.1	2.17	1.57	1.5	0.78	1.21	0.72	0.7	0.47	0.56	0.35	0.36	0.32	0.34	0.31	0.29	0.41	0.27
0.35	2.51	2.45	2.12	1.82	1.49	1.31	0.98	0.88	0.97	0.85	0.47	0.51	0.59	0.44	0.53	0.43	0.4	0.49	0.51	0.45	0.37
0.4	2.5	2.67	2.17	1.92	1.72	1.43	1.22	1.07	0.77	0.69	0.66	0.61	0.66	0.55	0.65	0.39	0.41	0.43	0.49	0.53	0.38
0.45	2.13	1.83	1.78	1.75	1.64	1.33	1.44	1.02	0.85	0.69	0.8	0.65	0.77	0.47	0.56	0.71	0.52	0.48	0.4	0.58	0.39
0.5	1.99	1.76	1.57	1.48	1.36	1.27	1.09	0.99	1.39	0.7	0.79	0.69	0.86	0.62	0.69	0.55	0.59	0.54	0.63	0.51	0.66
0.55	2.06	1.96	2.01	1.56	1.36	1.29	1.08	1.12	1.02	1.02	1.05	0.8	0.85	0.82	0.84	0.71	0.68	0.88	0.6	0.48	0.69
0.6	1.83	2.12	1.66	1.6	1.47	1.5	1.27	1.36	1.08	1.06	0.92	0.93	0.97	0.82	0.85	0.9	0.84	1.01	0.81	0.8	0.97
0.65	1.71	1.57	1.82	1.69	1.88	1.61	1.49	1.22	1.16	1.31	0.92	1.08	1.13	0.97	1.12	1.03	1.1	1.24	0.85	1.22	1.19
0.7	1.89	1.83	1.77	1.5	1.61	1.67	1.23	1.28	1.23	1.15	1.36	1.28	1.35	1.52	1.11	0.96	1.46	1.27	1.31	1.03	0.82
0.75	1.83	1.74	2.04	1.39	1.75	1.75	1.54	1.25	1.4	1.32	1.45	1.52	1.6	1.91	1.46	1.47	1.35	1.52	1.44	1.36	1.27
0.8	2.32	1.96	2.37	2.27	2.4	1.97	2.2	1.73	1.95	1.86	1.63	1.83	1.93	1.69	1.97	1.54	1.96	1.6	1.66	1.61	1.73
0.85	1.99	2.4	1.9	2.22	2.03	2.39	1.9	2.14	1.99	2.16	2.21	2.1	1.92	1.98	2.02	1.87	2.04	1.96	1.81	1.8	1.99
0.9	2.33	2.32	2.2	2.05	2.18	2.12	2.47	2.26	2.27	2.21	2.9	2.46	2.21	2.44	2.28	2.64	1.99	2.42	2.22	2.03	2.29
0.95	2.52	2.63	2.48	2.23	2.54	2.89	2.77	2.78	2.73	2.79	2.47	2.74	2.37	2.69	2.71	2.96	2.46	2.71	2.46	2.52	2.52
1	2.7	3	2.78	3.01	2.74	2.7	2.65	2.95	3.03	2.99	2.62	3.22	2.91	2.65	3.17	2.76	2.76	2.95	2.9	2.89	3.01

Fig. 5.10 The average number of blocking pairs over 200 simulations after 60 daterounds for 10 male and 10 female agents

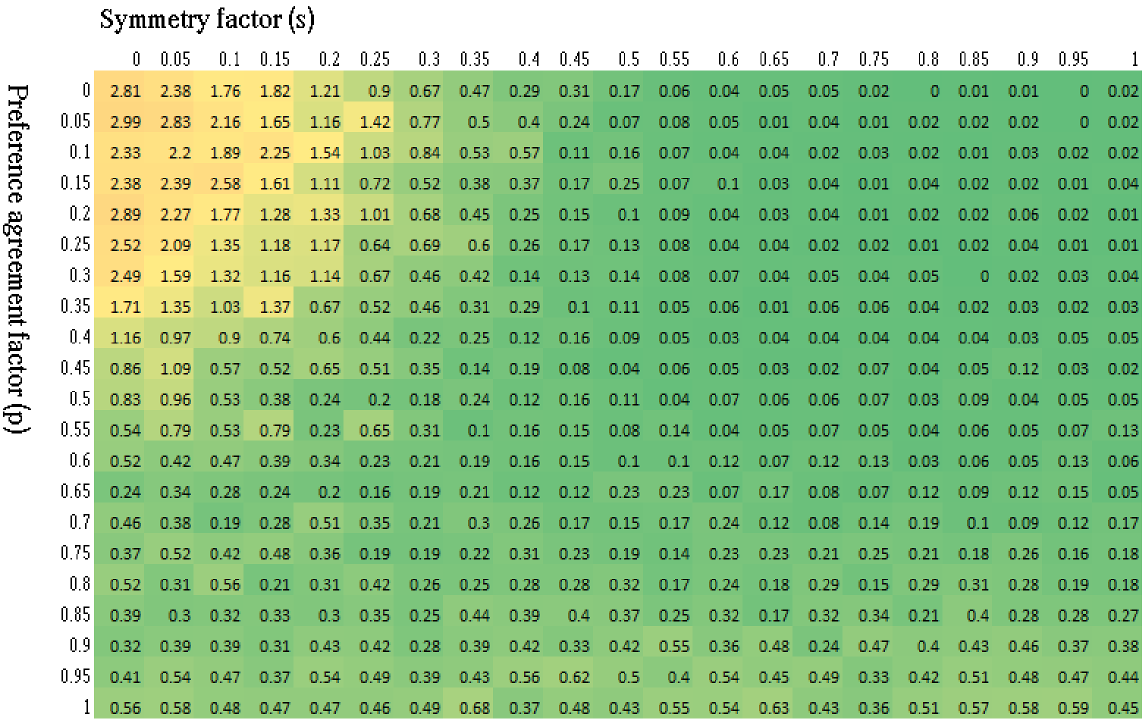
matching for female agents on average after 20 daterounds over 200 simulations. After 60 daterounds these percentages both increased to 63% and after 100 daterounds to 68% respectively. Eventually about 75% of the male agents are being matched with the same partner as they would be in the optimal stable matching for male agents (Figure 5.17) and of course also about 80% of the agents being matched with the same partner as they would be in the optimal stable matching for female agents on average over 200 simulations.

Increasing the number of agents

Although it takes many more daterounds to obtain stable matchings we find similar effects of changing the values for *p* and *s* on the stability in simulations for 20 male agents and 20 female agents.

5.4.3 Payoffs

Next to the different indicators for stability, the payoffs obtained in the matchings resulting from the simulations are also an important measure of the optimality of a matching. In this section we investigate the payoffs obtained by the agents during the simulations and compare



Symmetry factor (s)

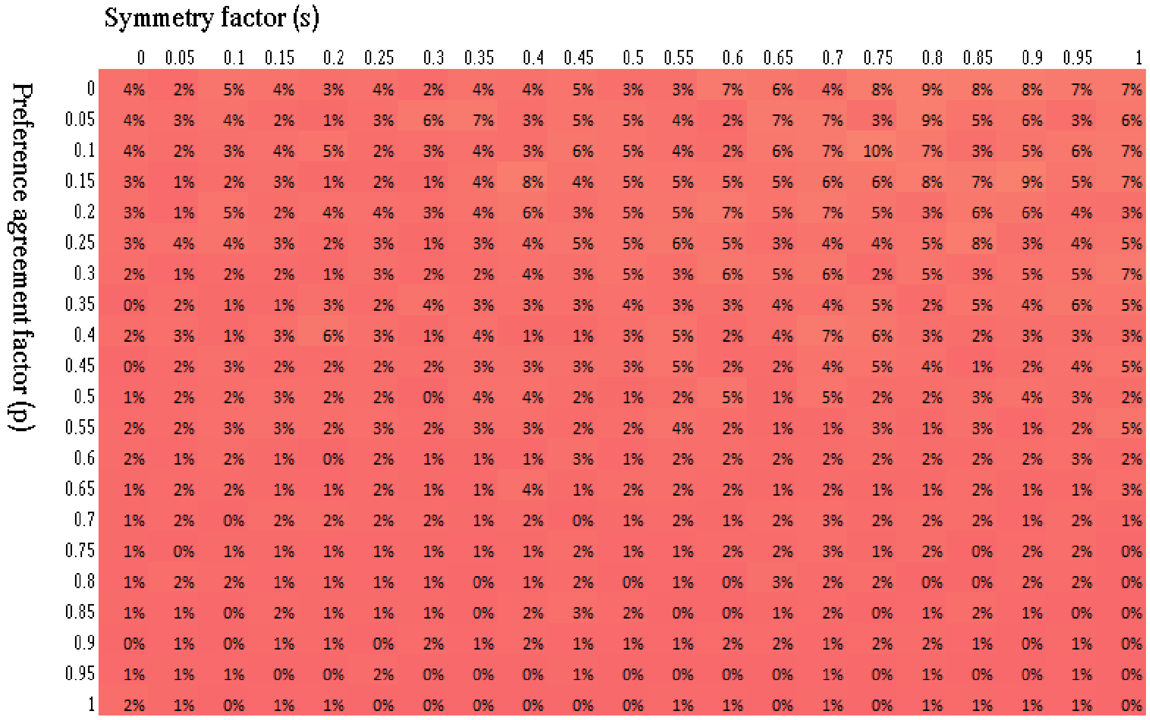
Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	2.81	2.38	1.76	1.82	1.21	0.9	0.67	0.47	0.29	0.31	0.17	0.06	0.04	0.05	0.05	0.02	0	0.01	0.01	0	0.02
0.05	2.99	2.83	2.16	1.65	1.16	1.42	0.77	0.5	0.4	0.24	0.07	0.08	0.05	0.01	0.04	0.01	0.02	0.02	0.02	0	0.02
0.1	2.33	2.2	1.89	2.25	1.54	1.03	0.84	0.53	0.57	0.11	0.16	0.07	0.04	0.04	0.02	0.03	0.02	0.01	0.03	0.02	0.02
0.15	2.38	2.39	2.58	1.61	1.11	0.72	0.52	0.38	0.37	0.17	0.25	0.07	0.1	0.03	0.04	0.01	0.04	0.02	0.02	0.01	0.04
0.2	2.89	2.27	1.77	1.28	1.33	1.01	0.68	0.45	0.25	0.15	0.1	0.09	0.04	0.03	0.04	0.01	0.02	0.02	0.06	0.02	0.01
0.25	2.52	2.09	1.35	1.18	1.17	0.64	0.69	0.6	0.26	0.17	0.13	0.08	0.04	0.04	0.02	0.02	0.01	0.02	0.04	0.01	0.01
0.3	2.49	1.59	1.32	1.16	1.14	0.67	0.46	0.42	0.14	0.13	0.14	0.08	0.07	0.04	0.05	0.04	0.05	0	0.02	0.03	0.04
0.35	1.71	1.35	1.03	1.37	0.67	0.52	0.46	0.31	0.29	0.1	0.11	0.05	0.06	0.01	0.06	0.06	0.04	0.02	0.03	0.02	0.03
0.4	1.16	0.97	0.9	0.74	0.6	0.44	0.22	0.25	0.12	0.16	0.09	0.05	0.03	0.04	0.04	0.04	0.04	0.04	0.03	0.05	0.05
0.45	0.86	1.09	0.57	0.52	0.65	0.51	0.35	0.14	0.19	0.08	0.04	0.06	0.05	0.03	0.02	0.07	0.04	0.05	0.12	0.03	0.02
0.5	0.83	0.96	0.53	0.38	0.24	0.2	0.18	0.24	0.12	0.16	0.11	0.04	0.07	0.06	0.06	0.07	0.03	0.09	0.04	0.05	0.05
0.55	0.54	0.79	0.53	0.79	0.23	0.65	0.31	0.1	0.16	0.15	0.08	0.14	0.04	0.05	0.07	0.05	0.04	0.06	0.05	0.07	0.13
0.6	0.52	0.42	0.47	0.39	0.34	0.23	0.21	0.19	0.16	0.15	0.1	0.1	0.12	0.07	0.12	0.13	0.03	0.06	0.05	0.13	0.06
0.65	0.24	0.34	0.28	0.24	0.2	0.16	0.19	0.21	0.12	0.12	0.23	0.23	0.07	0.17	0.08	0.07	0.12	0.09	0.12	0.15	0.05
0.7	0.46	0.38	0.19	0.28	0.51	0.35	0.21	0.3	0.26	0.17	0.15	0.17	0.24	0.12	0.08	0.14	0.19	0.1	0.09	0.12	0.17
0.75	0.37	0.52	0.42	0.48	0.36	0.19	0.19	0.22	0.31	0.23	0.19	0.14	0.23	0.23	0.21	0.25	0.21	0.18	0.26	0.16	0.18
0.8	0.52	0.31	0.56	0.21	0.31	0.42	0.26	0.25	0.28	0.28	0.32	0.17	0.24	0.18	0.29	0.15	0.29	0.31	0.28	0.19	0.18
0.85	0.39	0.3	0.32	0.33	0.3	0.35	0.25	0.44	0.39	0.4	0.37	0.25	0.32	0.17	0.32	0.34	0.21	0.4	0.28	0.28	0.27
0.9	0.32	0.39	0.39	0.31	0.43	0.42	0.28	0.39	0.42	0.33	0.42	0.55	0.36	0.48	0.24	0.47	0.4	0.43	0.46	0.37	0.38
0.95	0.41	0.54	0.47	0.37	0.54	0.49	0.39	0.43	0.56	0.62	0.5	0.4	0.54	0.45	0.49	0.33	0.42	0.51	0.48	0.47	0.44
1	0.56	0.58	0.48	0.47	0.47	0.46	0.49	0.68	0.37	0.48	0.43	0.55	0.54	0.63	0.43	0.36	0.51	0.57	0.58	0.59	0.45

Fig. 5.11 The average number of blocking pairs over 200 simulations after 100 daterounds for 10 male and 10 female agents

them with the payoffs obtained in the optimal stable matchings. Without loss of generality we only investigated the payoff of the male agents in our simulations because of the duality between the two sides of the market (in our simulations there is no proposing or accepting party, as opposed to the Deferred Acceptance Algorithm) .

In a random matching i.e. after one dateround, for all values for p and s we find average payoffs of 5. During the simulations we find that the highest average male payoff is obtained when all agents have independent preferences ($p = 0$) and when it is more likely that an agent is liked by agents that they like ($s = 1$), with 7.5 on average after 20 daterounds (Figure 5.18), 8.1 after 60 daterounds and 100 daterounds (after 44.7 daterounds on average the optimal stable matching is obtained in these cases). The payoffs of the male agents are lowest when all male agents fight for the same partners ($p = 1$). Since payoffs are based on random generated real numbers between 0 and 10 and since all agents of one side of the market have the same preferences over the agents of the other side of the market, the average payoff obtained by the agents cannot exceed 5. When we continue the simulations until a stable matching is obtained then we find maximal payoffs for $p = 0$ and $s = 1$ around 8.15. For $p = 0$ and $s = 0$ the payoffs are 7.28 and for $p = 1$ they are trivially around 5, independent of the value of s (Figure 5.19).



Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	4%	2%	5%	4%	3%	4%	2%	4%	4%	5%	3%	3%	7%	6%	4%	8%	9%	8%	8%	7%	7%
0.05	4%	3%	4%	2%	1%	3%	6%	7%	3%	5%	5%	4%	2%	7%	7%	3%	9%	5%	6%	3%	6%
0.1	4%	2%	3%	4%	5%	2%	3%	4%	3%	6%	5%	4%	2%	6%	7%	10%	7%	3%	5%	6%	7%
0.15	3%	1%	2%	3%	1%	2%	1%	4%	8%	4%	5%	5%	5%	5%	6%	6%	8%	7%	9%	5%	7%
0.2	3%	1%	5%	2%	4%	4%	3%	4%	6%	3%	5%	5%	7%	5%	7%	5%	3%	6%	6%	4%	3%
0.25	3%	4%	4%	3%	2%	3%	1%	3%	4%	5%	5%	6%	5%	3%	4%	4%	5%	8%	3%	4%	5%
0.3	2%	1%	2%	2%	1%	3%	2%	2%	4%	3%	5%	3%	6%	5%	6%	2%	5%	3%	5%	5%	7%
0.35	0%	2%	1%	1%	3%	2%	4%	3%	3%	3%	4%	3%	3%	4%	4%	5%	2%	5%	4%	6%	5%
0.4	2%	3%	1%	3%	6%	3%	1%	4%	1%	1%	3%	5%	2%	4%	7%	6%	3%	2%	3%	3%	3%
0.45	0%	2%	3%	2%	2%	2%	2%	3%	3%	3%	3%	5%	2%	2%	4%	5%	4%	1%	2%	4%	5%
0.5	1%	2%	2%	3%	2%	2%	0%	4%	4%	2%	1%	2%	5%	1%	5%	2%	2%	3%	4%	3%	2%
0.55	2%	2%	3%	3%	2%	3%	2%	3%	3%	2%	2%	4%	2%	1%	1%	3%	1%	3%	1%	2%	5%
0.6	2%	1%	2%	1%	0%	2%	1%	1%	1%	3%	1%	2%	2%	2%	2%	2%	2%	2%	2%	3%	2%
0.65	1%	2%	2%	1%	1%	2%	1%	1%	4%	1%	2%	2%	2%	1%	2%	1%	1%	2%	1%	1%	3%
0.7	1%	2%	0%	2%	2%	2%	2%	1%	2%	0%	1%	2%	1%	2%	3%	2%	2%	2%	1%	2%	1%
0.75	1%	0%	1%	1%	1%	1%	1%	1%	1%	2%	1%	1%	2%	2%	3%	1%	2%	0%	2%	2%	0%
0.8	1%	2%	2%	1%	1%	1%	1%	0%	1%	2%	0%	1%	0%	3%	2%	2%	0%	0%	2%	2%	0%
0.85	1%	1%	0%	2%	1%	1%	1%	0%	2%	3%	2%	0%	0%	1%	2%	0%	1%	2%	1%	0%	0%
0.9	0%	1%	0%	1%	1%	0%	2%	1%	2%	1%	1%	1%	2%	2%	1%	2%	2%	1%	0%	1%	0%
0.95	1%	1%	1%	0%	0%	2%	0%	0%	0%	1%	0%	0%	0%	0%	1%	0%	1%	0%	0%	1%	0%
1	2%	1%	0%	1%	1%	0%	0%	0%	0%	0%	0%	1%	1%	0%	1%	0%	1%	1%	1%	1%	0%

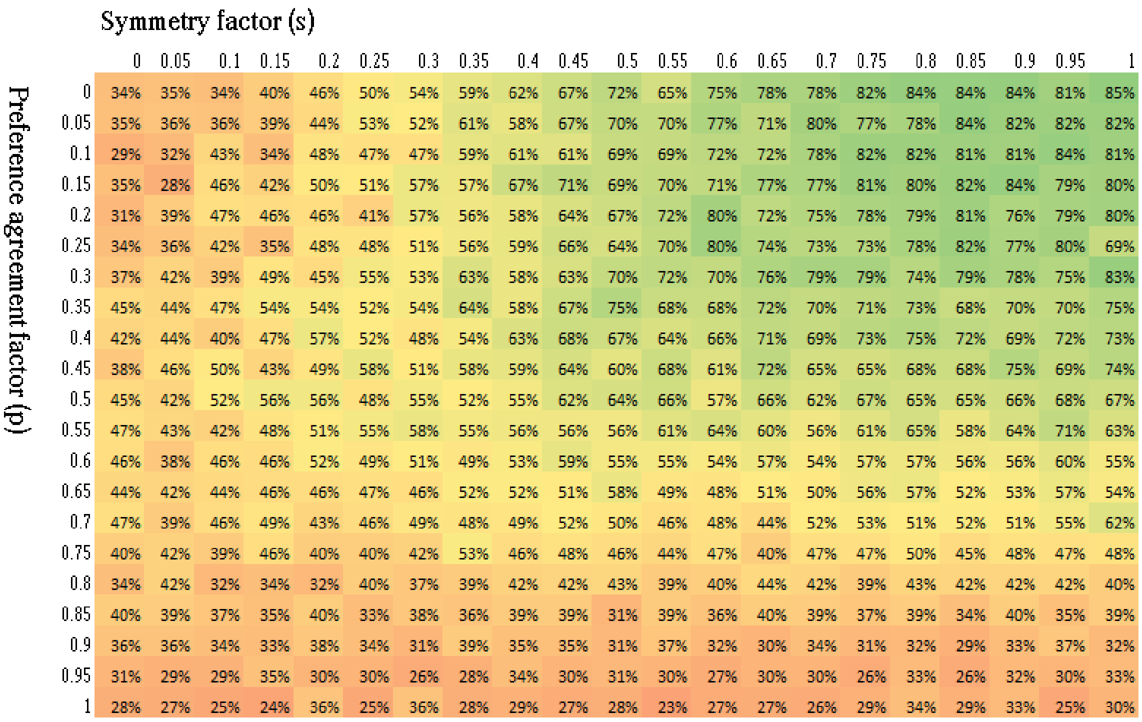
Fig. 5.12 The average percentage of simulations in which a stable matching was obtained within 20 daterounds for 10 male and 10 female agents over 200 simulations

Payoffs of male agents in a relationship

When we only look at payoffs of the male agents who are in a relationship then we find that in the beginning of the simulation, for example at dateround 20, we find average payoffs of around 5.4 for $p = 1$, independent of the value of s. For $p = 0$ and $s = 1$ we find that the average payoffs of all male agents in a relationship are 8.3. We can thus conclude that the pairs in which relatively high payoffs are obtained match earlier in the simulation.

Payoffs of model compared to those obtained in the optimal stable matchings

When we run simulations until the simulations have converged to stable matching we find that for $p = 1$ and any value for s and for $s = 1$ and any value for p that we trivially have that all agents obtained 100% of the payoff obtained in the two stable optimal stable matchings since there exists only one stable matching in these cases. For $p = 0$ and $s = 0$ we have multiple stable matchings, and our simulations do not always converge to one of the optimal stable matchings. For $p = 0$ and $s = 0$ we find that the male agents obtain 94% of the payoffs they would obtain in the optimal stable matching for male agents. For $p = 0$ and $s = 0$



Symmetry factor (s)

Preference agreement factor (p)

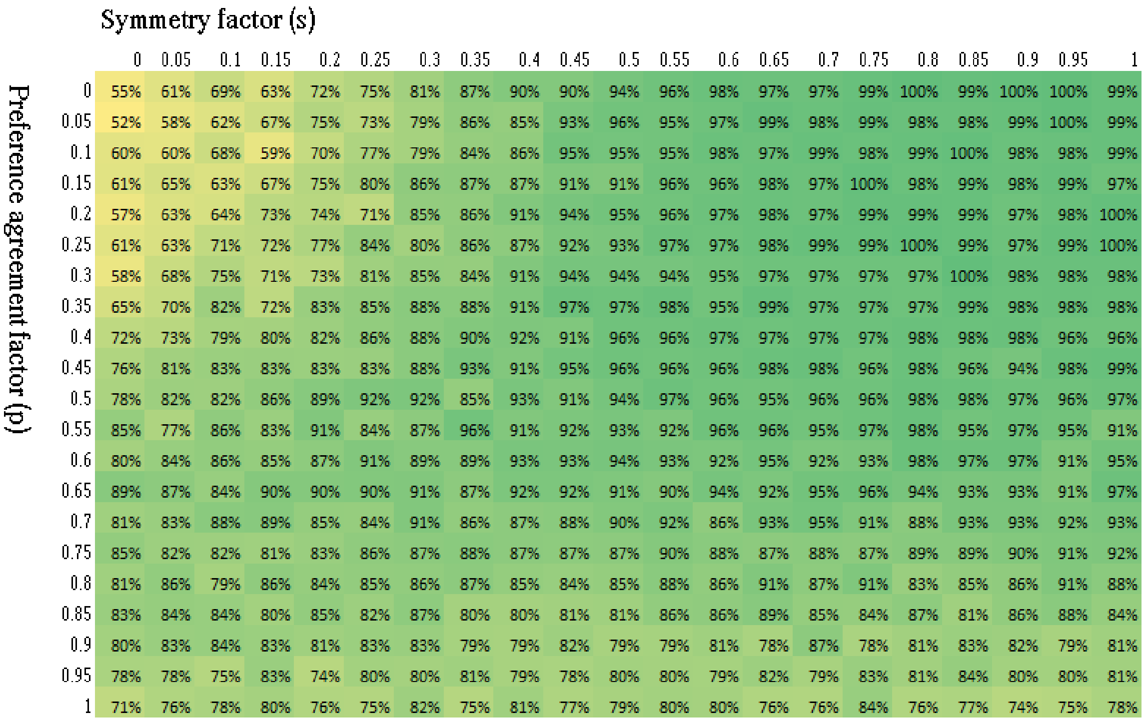
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	34%	35%	34%	40%	46%	50%	54%	59%	62%	67%	72%	65%	75%	78%	78%	82%	84%	84%	84%	81%	85%
0.05	35%	36%	36%	39%	44%	53%	52%	61%	58%	67%	70%	70%	77%	71%	80%	77%	78%	84%	82%	82%	82%
0.1	29%	32%	43%	34%	48%	47%	47%	59%	61%	61%	69%	69%	72%	72%	78%	82%	82%	81%	81%	84%	81%
0.15	35%	28%	46%	42%	50%	51%	57%	57%	67%	71%	69%	70%	71%	77%	77%	81%	80%	82%	84%	79%	80%
0.2	31%	39%	47%	46%	46%	41%	57%	56%	58%	64%	67%	72%	80%	72%	75%	78%	79%	81%	76%	79%	80%
0.25	34%	36%	42%	35%	48%	48%	51%	56%	59%	66%	64%	70%	80%	74%	73%	73%	78%	82%	77%	80%	69%
0.3	37%	42%	39%	49%	45%	55%	53%	63%	58%	63%	70%	72%	70%	76%	79%	79%	74%	79%	78%	75%	83%
0.35	45%	44%	47%	54%	54%	52%	54%	64%	58%	67%	75%	68%	68%	72%	70%	71%	73%	68%	70%	70%	75%
0.4	42%	44%	40%	47%	57%	52%	48%	54%	63%	68%	67%	64%	66%	71%	69%	73%	75%	72%	69%	72%	73%
0.45	38%	46%	50%	43%	49%	58%	51%	58%	59%	64%	60%	68%	61%	72%	65%	65%	68%	68%	75%	69%	74%
0.5	45%	42%	52%	56%	56%	48%	55%	52%	55%	62%	64%	66%	57%	66%	62%	67%	65%	65%	66%	68%	67%
0.55	47%	43%	42%	48%	51%	55%	58%	55%	56%	56%	56%	61%	64%	60%	56%	61%	65%	58%	64%	71%	63%
0.6	46%	38%	46%	46%	52%	49%	51%	49%	53%	59%	55%	55%	54%	57%	54%	57%	57%	56%	56%	60%	55%
0.65	44%	42%	44%	46%	46%	47%	46%	52%	52%	51%	58%	49%	48%	51%	50%	56%	57%	52%	53%	57%	54%
0.7	47%	39%	46%	49%	43%	46%	49%	48%	49%	52%	50%	46%	48%	44%	52%	53%	51%	52%	51%	55%	62%
0.75	40%	42%	39%	46%	40%	40%	42%	53%	46%	48%	46%	44%	47%	40%	47%	47%	50%	45%	48%	47%	48%
0.8	34%	42%	32%	34%	32%	40%	37%	39%	42%	42%	43%	39%	40%	44%	42%	39%	43%	42%	42%	42%	40%
0.85	40%	39%	37%	35%	40%	33%	38%	36%	39%	39%	31%	39%	36%	40%	39%	37%	39%	34%	40%	35%	39%
0.9	36%	36%	34%	33%	38%	34%	31%	39%	35%	35%	31%	37%	32%	30%	34%	31%	32%	29%	33%	37%	32%
0.95	31%	29%	29%	35%	30%	30%	26%	28%	34%	30%	31%	30%	27%	30%	30%	26%	33%	26%	32%	30%	33%
1	28%	27%	25%	24%	36%	25%	36%	28%	29%	27%	28%	23%	27%	27%	26%	29%	34%	29%	33%	25%	30%

Fig. 5.13 The average percentage of simulations in which a stable matching was obtained within 60 daterounds for 10 male and 10 female agents over 200 simulations

we however find that the male agents obtain 111% of the payoffs they would obtain in the optimal stable matching for female agents (Figure 5.20) and (Figure 5.21).

Higher average payoffs in model than in Deferred Acceptance Algorithm

We compare the average payoff obtained in our model, to the average payoff obtain over the two optimal stable matchings resulting from the Deferred Acceptance Algorithm. We do this by expressing the average obtained payoffs in our model as a percentage of the payoff that would have been obtained by the same agents in the stable matchings resulting from the Deferred Acceptance Algorithm.Very interestingly we now find (Figure 5.22), that the matching to which our model eventually converges, yields higher payoffs than the optimal stable matching for male agents and the optimal stable matching for female agents do on average. For $p = 1$ and any value for s and for $s = 1$ and any value for p our model, of course, converges to the same stable matching as the optimal stable matchings that result from the Deferred Acceptance Algorithm with both the male agents and the female agents as proposing party. However, the lower p and the lower s, the relatively higher the obtained payoffs in the stable matchings that result from our model are, with payoffs of around 102%



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	55%	61%	69%	63%	72%	75%	81%	87%	90%	90%	94%	96%	98%	97%	97%	99%	100%	99%	100%	100%	99%
0.05	52%	58%	62%	67%	75%	73%	79%	86%	85%	93%	96%	95%	97%	99%	98%	99%	98%	98%	99%	100%	99%
0.1	60%	60%	68%	59%	70%	77%	79%	84%	86%	95%	95%	95%	98%	97%	99%	98%	99%	100%	98%	98%	99%
0.15	61%	65%	63%	67%	75%	80%	86%	87%	87%	91%	91%	96%	96%	98%	97%	100%	98%	99%	98%	99%	97%
0.2	57%	63%	64%	73%	74%	71%	85%	86%	91%	94%	95%	96%	97%	98%	97%	99%	99%	99%	97%	98%	100%
0.25	61%	63%	71%	72%	77%	84%	80%	86%	87%	92%	93%	97%	97%	98%	99%	99%	100%	99%	97%	99%	100%
0.3	58%	68%	75%	71%	73%	81%	85%	84%	91%	94%	94%	94%	95%	97%	97%	97%	97%	100%	98%	98%	98%
0.35	65%	70%	82%	72%	83%	85%	88%	88%	91%	97%	97%	98%	95%	99%	97%	97%	97%	99%	98%	98%	98%
0.4	72%	73%	79%	80%	82%	86%	88%	90%	92%	91%	96%	96%	97%	97%	97%	97%	98%	98%	98%	96%	96%
0.45	76%	81%	83%	83%	83%	83%	88%	93%	91%	95%	96%	96%	96%	98%	98%	96%	98%	96%	94%	98%	99%
0.5	78%	82%	82%	86%	89%	92%	92%	85%	93%	91%	94%	97%	96%	95%	96%	96%	98%	98%	97%	96%	97%
0.55	85%	77%	86%	83%	91%	84%	87%	96%	91%	92%	93%	92%	96%	96%	95%	97%	98%	95%	97%	95%	91%
0.6	80%	84%	86%	85%	87%	91%	89%	89%	93%	93%	94%	93%	92%	95%	92%	93%	98%	97%	97%	91%	95%
0.65	89%	87%	84%	90%	90%	90%	91%	87%	92%	92%	91%	90%	94%	92%	95%	96%	94%	93%	93%	91%	97%
0.7	81%	83%	88%	89%	85%	84%	91%	86%	87%	88%	90%	92%	86%	93%	95%	91%	88%	93%	93%	92%	93%
0.75	85%	82%	82%	81%	83%	86%	87%	88%	87%	87%	87%	90%	88%	87%	88%	87%	89%	89%	90%	91%	92%
0.8	81%	86%	79%	86%	84%	85%	86%	87%	85%	84%	85%	88%	86%	91%	87%	91%	83%	85%	86%	91%	88%
0.85	83%	84%	84%	80%	85%	82%	87%	80%	80%	81%	81%	86%	86%	89%	85%	84%	87%	81%	86%	88%	84%
0.9	80%	83%	84%	83%	81%	83%	83%	79%	79%	82%	79%	79%	81%	78%	87%	78%	81%	83%	82%	79%	81%
0.95	78%	78%	75%	83%	74%	80%	80%	81%	79%	78%	80%	80%	79%	82%	79%	83%	81%	84%	80%	80%	81%
1	71%	76%	78%	80%	76%	75%	82%	75%	81%	77%	79%	80%	80%	76%	76%	84%	76%	77%	74%	75%	78%

Fig. 5.14 The average percentage of simulations in which a stable matching was obtained within 100 daterounds for 10 male and 10 female agents over 200 simulations

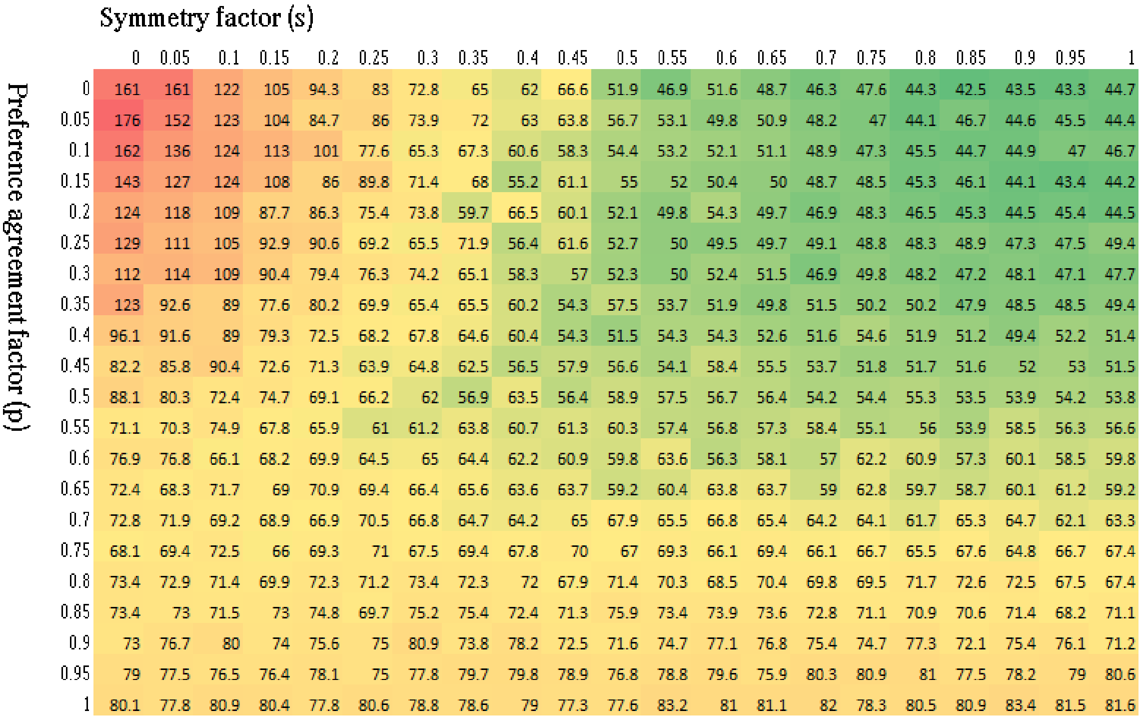
for $p = 0$ and $s = 0$. This means that our model on average gives us a stable matching with higher average payoffs than the Deferred Acceptance Algorithm would.

Increasing the number of agents

Increasing the number of agents has no interesting effects on the obtained payoffs. We do find that for low values for p we eventually obtain higher payoffs when we increase the number of agents. When there are more possible partners, agents have “more to choose from", and when preferences are not totally objective, this results in an increase in the obtained payoff. For example 8.15 is maximal payoff for 10 male and 10 female agents, where this is 8.8 for 20 male and 20 female agents (both for $p = 0$ and $s = 1$). On the other hand it of course takes longer for these relatively higher payoffs to be obtained since it takes longer to obtain a stable matching.

5.4.4 Summary on simulations in our model

In our simulations most agents are in a relationship during the simulations for instances of the Stable Marriage Problem with high symmetry between the preferences of the two sides of the



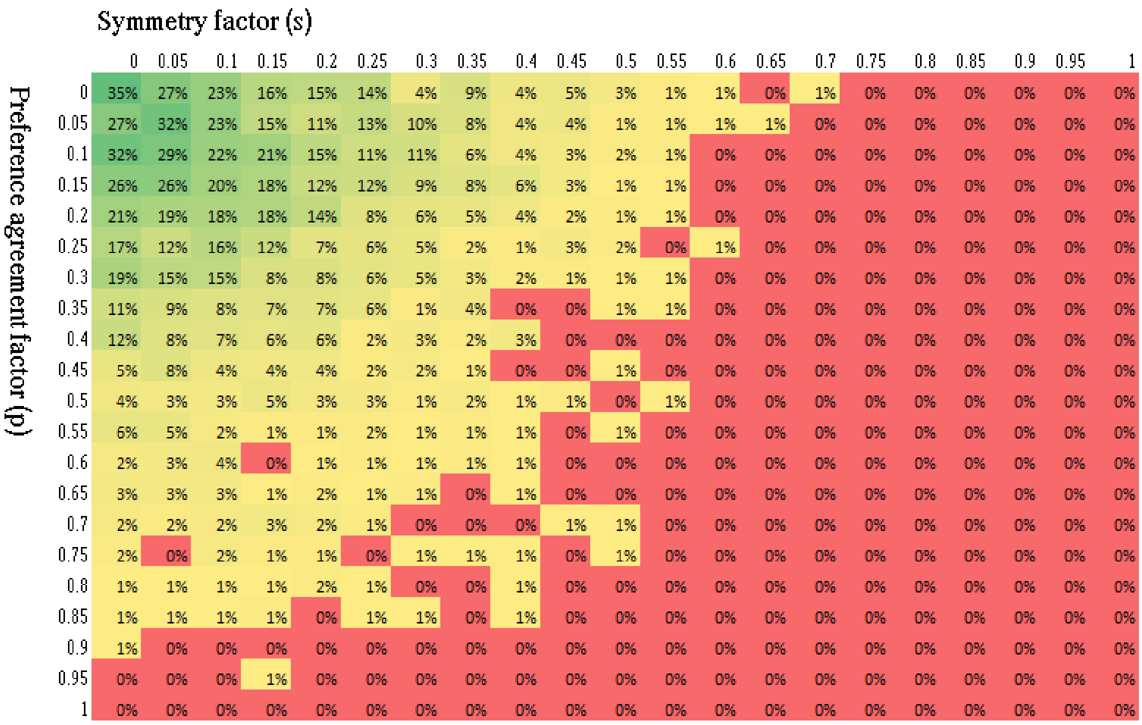
Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	161	161	122	105	94.3	83	72.8	65	62	66.6	51.9	46.9	51.6	48.7	46.3	47.6	44.3	42.5	43.5	43.3	44.7
0.05	176	152	123	104	84.7	86	73.9	72	63	63.8	56.7	53.1	49.8	50.9	48.2	47	44.1	46.7	44.6	45.5	44.4
0.1	162	136	124	113	101	77.6	65.3	67.3	60.6	58.3	54.4	53.2	52.1	51.1	48.9	47.3	45.5	44.7	44.9	47	46.7
0.15	143	127	124	108	86	89.8	71.4	68	55.2	61.1	55	52	50.4	50	48.7	48.5	45.3	46.1	44.1	43.4	44.2
0.2	124	118	109	87.7	86.3	75.4	73.8	59.7	66.5	60.1	52.1	49.8	54.3	49.7	46.9	48.3	46.5	45.3	44.5	45.4	44.5
0.25	129	111	105	92.9	90.6	69.2	65.5	71.9	56.4	61.6	52.7	50	49.5	49.7	49.1	48.8	48.3	48.9	47.3	47.5	49.4
0.3	112	114	109	90.4	79.4	76.3	74.2	65.1	58.3	57	52.3	50	52.4	51.5	46.9	49.8	48.2	47.2	48.1	47.1	47.7
0.35	123	92.6	89	77.6	80.2	69.9	65.4	65.5	60.2	54.3	57.5	53.7	51.9	49.8	51.5	50.2	50.2	47.9	48.5	48.5	49.4
0.4	96.1	91.6	89	79.3	72.5	68.2	67.8	64.6	60.4	54.3	51.5	54.3	54.3	52.6	51.6	54.6	51.9	51.2	49.4	52.2	51.4
0.45	82.2	85.8	90.4	72.6	71.3	63.9	64.8	62.5	56.5	57.9	56.6	54.1	58.4	55.5	53.7	51.8	51.7	51.6	52	53	51.5
0.5	88.1	80.3	72.4	74.7	69.1	66.2	62	56.9	63.5	56.4	58.9	57.5	56.7	56.4	54.2	54.4	55.3	53.5	53.9	54.2	53.8
0.55	71.1	70.3	74.9	67.8	65.9	61	61.2	63.8	60.7	61.3	60.3	57.4	56.8	57.3	58.4	55.1	56	53.9	58.5	56.3	56.6
0.6	76.9	76.8	66.1	68.2	69.9	64.5	65	64.4	62.2	60.9	59.8	63.6	56.3	58.1	57	62.2	60.9	57.3	60.1	58.5	59.8
0.65	72.4	68.3	71.7	69	70.9	69.4	66.4	65.6	63.6	63.7	59.2	60.4	63.8	63.7	59	62.8	59.7	58.7	60.1	61.2	59.2
0.7	72.8	71.9	69.2	68.9	66.9	70.5	66.8	64.7	64.2	65	67.9	65.5	66.8	65.4	64.2	64.1	61.7	65.3	64.7	62.1	63.3
0.75	68.1	69.4	72.5	66	69.3	71	67.5	69.4	67.8	70	67	69.3	66.1	69.4	66.1	66.7	65.5	67.6	64.8	66.7	67.4
0.8	73.4	72.9	71.4	69.9	72.3	71.2	73.4	72.3	72	67.9	71.4	70.3	68.5	70.4	69.8	69.5	71.7	72.6	72.5	67.5	67.4
0.85	73.4	73	71.5	73	74.8	69.7	75.2	75.4	72.4	71.3	75.9	73.4	73.9	73.6	72.8	71.1	70.9	70.6	71.4	68.2	71.1
0.9	73	76.7	80	74	75.6	75	80.9	73.8	78.2	72.5	71.6	74.7	77.1	76.8	75.4	74.7	77.3	72.1	75.4	76.1	71.2
0.95	79	77.5	76.5	76.4	78.1	75	77.8	79.7	79.8	78.9	76.8	78.8	79.6	75.9	80.3	80.9	81	77.5	78.2	79	80.6
1	80.1	77.8	80.9	80.4	77.8	80.6	78.8	78.6	79	77.3	77.6	83.2	81	81.1	82	78.3	80.5	80.9	83.4	81.5	81.6

Fig. 5.15 The average number of daterounds until a stable matching is obtained in our model for 10 male and 10 female agents over 200 simulations

market ($s = 1$) and with a lot of disagreement in the preferences within one side of the market ($p = 0$). For those parameter settings we furthermore find that random complete matchings, that are obtained in the first dateround, contain the most blocking pairs (and are thus least stable). However, we find that for these parameter settings it takes the smallest number of daterounds to obtain a stable matching. For uniform random preferences ($p = 0$ and $s = 0$) it takes the longest before a stable matching is obtained. For these parameter settings it was also most likely that our simulations converged to a stable matching that is different from the stable matchings resulting from the Deferred Acceptance Algorithm, which thus are not optimal for both sides of the markets. The most important result of our simulations is that our model converges to stable matching in which on average higher payoffs are obtained than on average in the stable matchings resulting from the Deferred Acceptance Algorithm. This is most strongly the case for uniform random preferences ($p = 0$ and $s = 0$), since in these cases it is most likely that our model converges to a stable matching other than the ones resulting from the Deferred Acceptance Algorithm.



Symmetry factor (s)

Preference agreement factor (p)

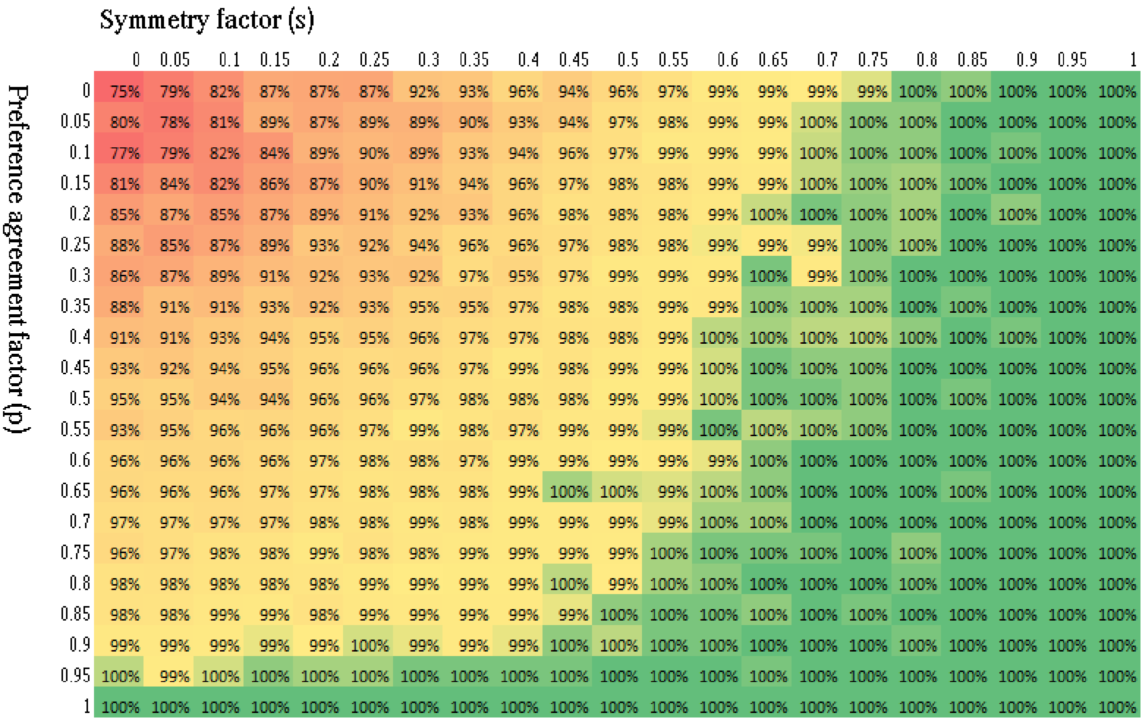
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	35%	27%	23%	16%	15%	14%	4%	9%	4%	5%	3%	1%	1%	0%	1%	0%	0%	0%	0%	0%	0%
0.05	27%	32%	23%	15%	11%	13%	10%	8%	4%	4%	1%	1%	1%	1%	0%	0%	0%	0%	0%	0%	0%
0.1	32%	29%	22%	21%	15%	11%	11%	6%	4%	3%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.15	26%	26%	20%	18%	12%	12%	9%	8%	6%	3%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.2	21%	19%	18%	18%	14%	8%	6%	5%	4%	2%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.25	17%	12%	16%	12%	7%	6%	5%	2%	1%	3%	2%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
0.3	19%	15%	15%	8%	8%	6%	5%	3%	2%	1%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.35	11%	9%	8%	7%	7%	6%	1%	4%	0%	0%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.4	12%	8%	7%	6%	6%	2%	3%	2%	3%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.45	5%	8%	4%	4%	4%	2%	2%	1%	0%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.5	4%	3%	3%	5%	3%	3%	1%	2%	1%	1%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.55	6%	5%	2%	1%	1%	2%	1%	1%	1%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.6	2%	3%	4%	0%	1%	1%	1%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.65	3%	3%	3%	1%	2%	1%	1%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.7	2%	2%	2%	3%	2%	1%	0%	0%	0%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.75	2%	0%	2%	1%	1%	0%	1%	1%	1%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.8	1%	1%	1%	1%	2%	1%	0%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.85	1%	1%	1%	1%	0%	1%	1%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.9	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0.95	0%	0%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
1	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Fig. 5.16 The average percentage of simulations in which a suboptimal stable matching was eventually obtained for 10 male and 10 female agents over 200 simulations

5.5 Investigating why our model obtains higher average payoffs

We have seen that our model on average converges to a stable matching in which the payoffs are on average higher than in the optimal stable matchings obtained with the Deferred Acceptance Algorithm. As shown in Theorem 7 our model can also converge to a stable matching that is not optimal for either side of the market. From this proof we can easily conclude that every stable matching could be obtained by our model (since every matching could be equal to the pairing in the first round where every agent starts a relationship with the agent they date). We now want to understand why our model on average converges to a stable matching with higher average payoffs than the optimal stable matchings resulting from the Deferred Acceptance Algorithm.

We first show that stable matchings can yield both higher and lower payoffs using the following examples:



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	75%	79%	82%	87%	87%	87%	92%	93%	96%	94%	96%	97%	99%	99%	99%	99%	100%	100%	100%	100%	100%
0.05	80%	78%	81%	89%	87%	89%	89%	90%	93%	94%	97%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%
0.1	77%	79%	82%	84%	89%	90%	89%	93%	94%	96%	97%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%
0.15	81%	84%	82%	86%	87%	90%	91%	94%	96%	97%	98%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%
0.2	85%	87%	85%	87%	89%	91%	92%	93%	96%	98%	98%	98%	99%	100%	100%	100%	100%	100%	100%	100%	100%
0.25	88%	85%	87%	89%	93%	92%	94%	96%	96%	97%	98%	98%	99%	99%	99%	100%	100%	100%	100%	100%	100%
0.3	86%	87%	89%	91%	92%	93%	92%	97%	95%	97%	99%	99%	99%	100%	99%	100%	100%	100%	100%	100%	100%
0.35	88%	91%	91%	93%	92%	93%	95%	95%	97%	98%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%
0.4	91%	91%	93%	94%	95%	95%	96%	97%	97%	98%	98%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.45	93%	92%	94%	95%	96%	96%	96%	97%	99%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.5	95%	95%	94%	94%	96%	96%	97%	98%	98%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.55	93%	95%	96%	96%	96%	97%	99%	98%	97%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.6	96%	96%	96%	96%	97%	98%	98%	97%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%
0.65	96%	96%	96%	97%	97%	98%	98%	98%	99%	100%	100%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.7	97%	97%	97%	97%	98%	98%	99%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.75	96%	97%	98%	98%	99%	98%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.8	98%	98%	98%	98%	98%	99%	99%	99%	99%	100%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.85	98%	98%	99%	99%	98%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.9	99%	99%	99%	99%	99%	100%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.95	100%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

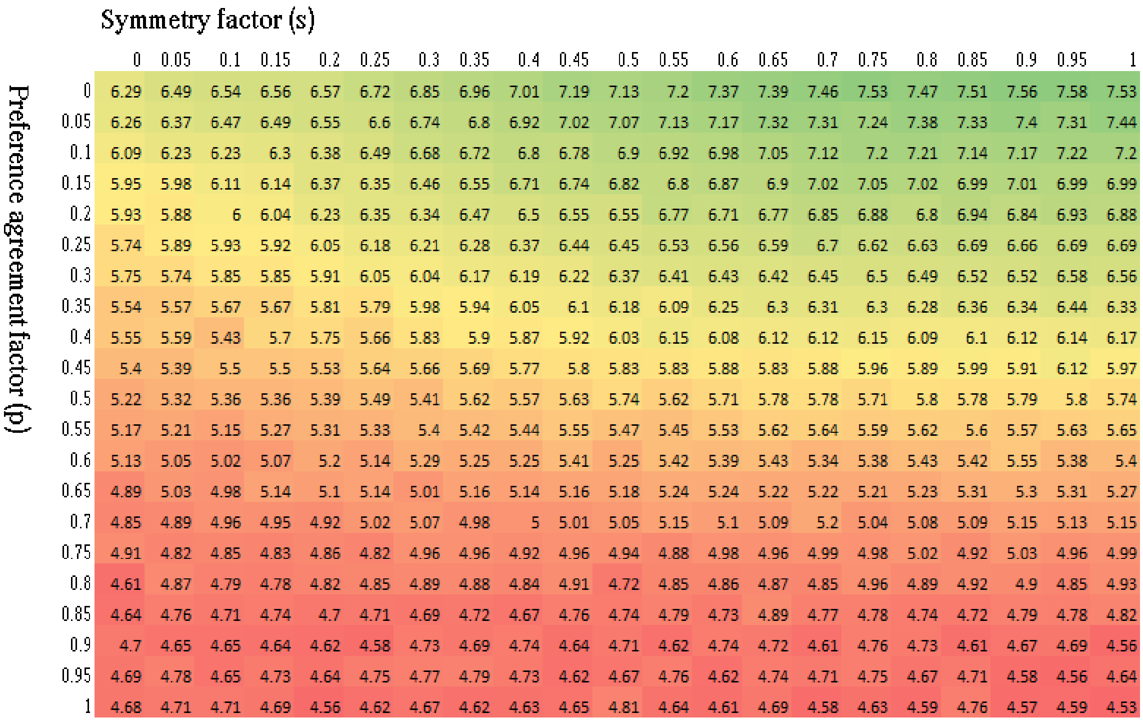
Fig. 5.17 The average percentage of agents that eventually ended up with a partner that is the same as they would be matched with in the optimal stable matching for male agents for 10 male and 10 female agents over 200 simulations

	A	B	C
a	1, 5	2, 2	5, 1
b	5, 1	1, 5	2, 2
c	2, 2	5, 1	1, 5

Table 5.5 Example of a Stable Marriage Problem in which a stable matching exists in which the average payoffs are lower than on average in the optimal stable matchings

	m_1	m_2	m_3
f_1	1, 5	4, 4	5, 1
f_2	5, 1	1, 5	4, 4
f_3	4, 4	5, 1	1, 5

Table 5.6 Example of a Stable Marriage Problem in which a stable matching exists in which the average payoffs are higher than on average in the optimal stable matchings



Symmetry factor (s)

Preference agreement factor (p)

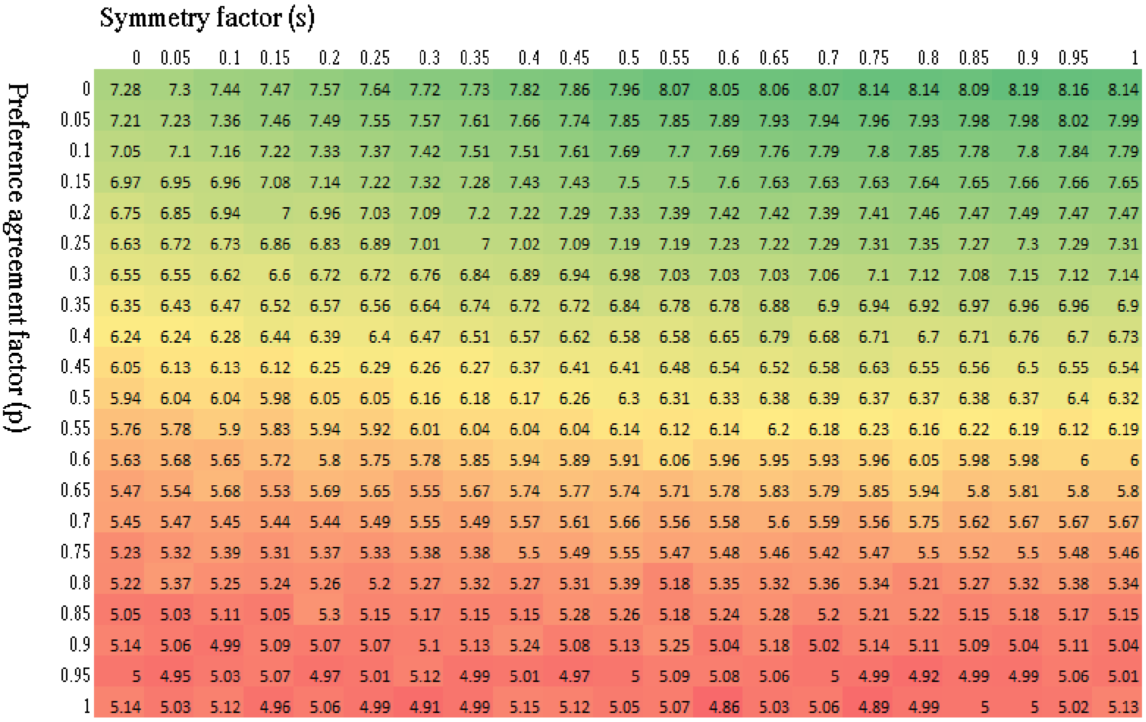
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	6.29	6.49	6.54	6.56	6.57	6.72	6.85	6.96	7.01	7.19	7.13	7.2	7.37	7.39	7.46	7.53	7.47	7.51	7.56	7.58	7.53
0.05	6.26	6.37	6.47	6.49	6.55	6.6	6.74	6.8	6.92	7.02	7.07	7.13	7.17	7.32	7.31	7.24	7.38	7.33	7.4	7.31	7.44
0.1	6.09	6.23	6.23	6.3	6.38	6.49	6.68	6.72	6.8	6.78	6.9	6.92	6.98	7.05	7.12	7.2	7.21	7.14	7.17	7.22	7.2
0.15	5.95	5.98	6.11	6.14	6.37	6.35	6.46	6.55	6.71	6.74	6.82	6.8	6.87	6.9	7.02	7.05	7.02	6.99	7.01	6.99	6.99
0.2	5.93	5.88	6	6.04	6.23	6.35	6.34	6.47	6.5	6.55	6.55	6.77	6.71	6.77	6.85	6.88	6.8	6.94	6.84	6.93	6.88
0.25	5.74	5.89	5.93	5.92	6.05	6.18	6.21	6.28	6.37	6.44	6.45	6.53	6.56	6.59	6.7	6.62	6.63	6.69	6.66	6.69	6.69
0.3	5.75	5.74	5.85	5.85	5.91	6.05	6.04	6.17	6.19	6.22	6.37	6.41	6.43	6.42	6.45	6.5	6.49	6.52	6.52	6.58	6.56
0.35	5.54	5.57	5.67	5.67	5.81	5.79	5.98	5.94	6.05	6.1	6.18	6.09	6.25	6.3	6.31	6.3	6.28	6.36	6.34	6.44	6.33
0.4	5.55	5.59	5.43	5.7	5.75	5.66	5.83	5.9	5.87	5.92	6.03	6.15	6.08	6.12	6.12	6.15	6.09	6.1	6.12	6.14	6.17
0.45	5.4	5.39	5.5	5.5	5.53	5.64	5.66	5.69	5.77	5.8	5.83	5.83	5.88	5.83	5.88	5.96	5.89	5.99	5.91	6.12	5.97
0.5	5.22	5.32	5.36	5.36	5.39	5.49	5.41	5.62	5.57	5.63	5.74	5.62	5.71	5.78	5.78	5.71	5.8	5.78	5.79	5.8	5.74
0.55	5.17	5.21	5.15	5.27	5.31	5.33	5.4	5.42	5.44	5.55	5.47	5.45	5.53	5.62	5.64	5.59	5.62	5.6	5.57	5.63	5.65
0.6	5.13	5.05	5.02	5.07	5.2	5.14	5.29	5.25	5.25	5.41	5.25	5.42	5.39	5.43	5.34	5.38	5.43	5.42	5.55	5.38	5.4
0.65	4.89	5.03	4.98	5.14	5.1	5.14	5.01	5.16	5.14	5.16	5.18	5.24	5.24	5.22	5.22	5.21	5.23	5.31	5.3	5.31	5.27
0.7	4.85	4.89	4.96	4.95	4.92	5.02	5.07	4.98	5	5.01	5.05	5.15	5.1	5.09	5.2	5.04	5.08	5.09	5.15	5.13	5.15
0.75	4.91	4.82	4.85	4.83	4.86	4.82	4.96	4.96	4.92	4.96	4.94	4.88	4.98	4.96	4.99	4.98	5.02	4.92	5.03	4.96	4.99
0.8	4.61	4.87	4.79	4.78	4.82	4.85	4.89	4.88	4.84	4.91	4.72	4.85	4.86	4.87	4.85	4.96	4.89	4.92	4.9	4.85	4.93
0.85	4.64	4.76	4.71	4.74	4.7	4.71	4.69	4.72	4.67	4.76	4.74	4.79	4.73	4.89	4.77	4.78	4.74	4.72	4.79	4.78	4.82
0.9	4.7	4.65	4.65	4.64	4.62	4.58	4.73	4.69	4.74	4.64	4.71	4.62	4.74	4.72	4.61	4.76	4.73	4.61	4.67	4.69	4.56
0.95	4.69	4.78	4.65	4.73	4.64	4.75	4.77	4.79	4.73	4.62	4.67	4.76	4.62	4.74	4.71	4.75	4.67	4.71	4.58	4.56	4.64
1	4.68	4.71	4.71	4.69	4.56	4.62	4.67	4.62	4.63	4.65	4.81	4.64	4.61	4.69	4.58	4.63	4.59	4.76	4.57	4.59	4.53

Fig. 5.18 The average payoff of male agents after 20 daterounds over 200 simulations for 10 male and 10 female agents

In both Table 5.5 and Table 5.6 matching $\{(m_1,f_1),(m_2,f_2),(m_3,f_3)\}$ is the stable matching optimal for female agents and the matching $\{(m_1,f_3),(m_2,f_1),(m_3,f_2)\}$ is the optimal stable matching for male agents with an average payoff of 3 for all agents. The matching $\{(m_1,f_3),(m_2,f_1),(m_3,f_2)\}$ is stable but not optimal for either side of the market in both Table 5.5 and Table 5.6. In Table 5.5 the agents however yield an average payoff of 2, which thus is lower than on average in the optimal stable matchings. In Table 5.6 the agents yield an average payoff of 4, which thus is higher than in the optimal stable matchings.

5.5.1 Exhaustive investigation of stable matchings

In order to understand why our model tends to converge to matchings with higher average payoffs we want to know what the average payoffs of all stable matchings are for an instance of the Stable Marriage Problem, and which of these stable matchings are more likely to be obtained in our model. We focus on instances of the Stable Marriage Problems where preferences are uniformly random, thus with preference agreement factor $p = 0$ and symmetry factor $s = 0$, since for these values of p and s it is most likely that there are multiple stable matchings and for which the difference between the obtained payoffs in simulations and the



Symmetry factor (s)

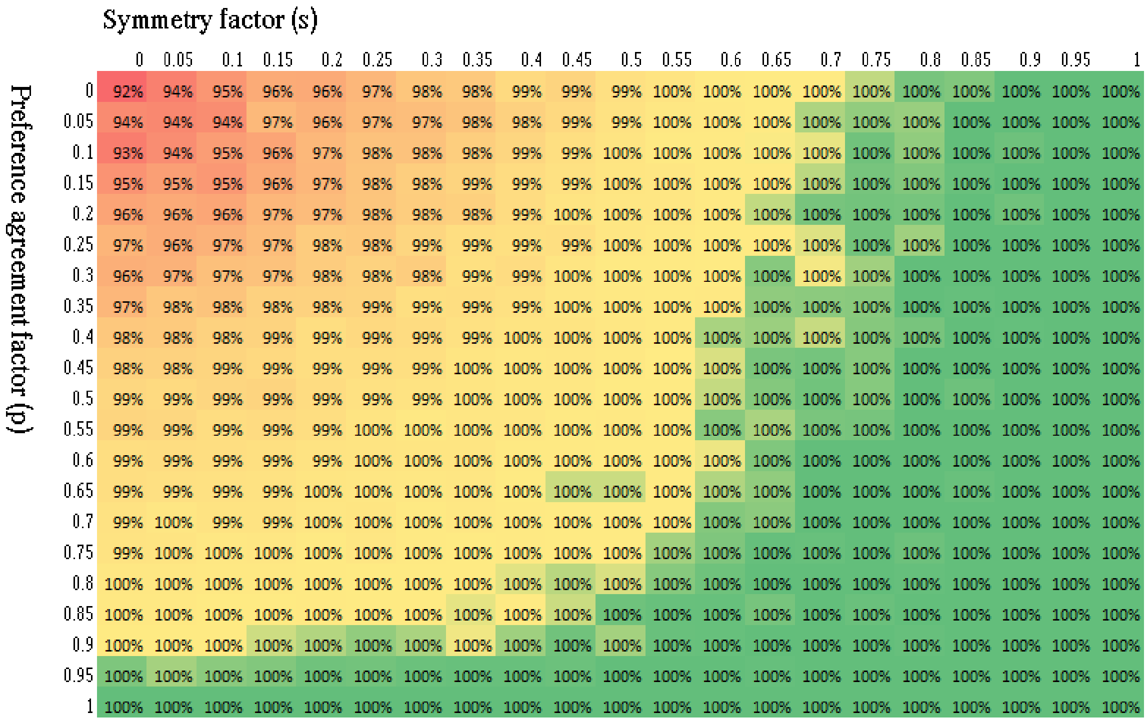
Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	7.28	7.3	7.44	7.47	7.57	7.64	7.72	7.73	7.82	7.86	7.96	8.07	8.05	8.06	8.07	8.14	8.14	8.09	8.19	8.16	8.14
0.05	7.21	7.23	7.36	7.46	7.49	7.55	7.57	7.61	7.66	7.74	7.85	7.85	7.89	7.93	7.94	7.96	7.93	7.98	7.98	8.02	7.99
0.1	7.05	7.1	7.16	7.22	7.33	7.37	7.42	7.51	7.51	7.61	7.69	7.7	7.69	7.76	7.79	7.8	7.85	7.78	7.8	7.84	7.79
0.15	6.97	6.95	6.96	7.08	7.14	7.22	7.32	7.28	7.43	7.43	7.5	7.5	7.6	7.63	7.63	7.63	7.64	7.65	7.66	7.66	7.65
0.2	6.75	6.85	6.94	7	6.96	7.03	7.09	7.2	7.22	7.29	7.33	7.39	7.42	7.42	7.39	7.41	7.46	7.47	7.49	7.47	7.47
0.25	6.63	6.72	6.73	6.86	6.83	6.89	7.01	7	7.02	7.09	7.19	7.19	7.23	7.22	7.29	7.31	7.35	7.27	7.3	7.29	7.31
0.3	6.55	6.55	6.62	6.6	6.72	6.72	6.76	6.84	6.89	6.94	6.98	7.03	7.03	7.03	7.06	7.1	7.12	7.08	7.15	7.12	7.14
0.35	6.35	6.43	6.47	6.52	6.57	6.56	6.64	6.74	6.72	6.72	6.84	6.78	6.78	6.88	6.9	6.94	6.92	6.97	6.96	6.96	6.9
0.4	6.24	6.24	6.28	6.44	6.39	6.4	6.47	6.51	6.57	6.62	6.58	6.58	6.65	6.79	6.68	6.71	6.7	6.71	6.76	6.7	6.73
0.45	6.05	6.13	6.13	6.12	6.25	6.29	6.26	6.27	6.37	6.41	6.41	6.48	6.54	6.52	6.58	6.63	6.55	6.56	6.5	6.55	6.54
0.5	5.94	6.04	6.04	5.98	6.05	6.05	6.16	6.18	6.17	6.26	6.3	6.31	6.33	6.38	6.39	6.37	6.37	6.38	6.37	6.4	6.32
0.55	5.76	5.78	5.9	5.83	5.94	5.92	6.01	6.04	6.04	6.04	6.14	6.12	6.14	6.2	6.18	6.23	6.16	6.22	6.19	6.12	6.19
0.6	5.63	5.68	5.65	5.72	5.8	5.75	5.78	5.85	5.94	5.89	5.91	6.06	5.96	5.95	5.93	5.96	6.05	5.98	5.98	6	6
0.65	5.47	5.54	5.68	5.53	5.69	5.65	5.55	5.67	5.74	5.77	5.74	5.71	5.78	5.83	5.79	5.85	5.94	5.8	5.81	5.8	5.8
0.7	5.45	5.47	5.45	5.44	5.44	5.49	5.55	5.49	5.57	5.61	5.66	5.56	5.58	5.6	5.59	5.56	5.75	5.62	5.67	5.67	5.67
0.75	5.23	5.32	5.39	5.31	5.37	5.33	5.38	5.38	5.5	5.49	5.55	5.47	5.48	5.46	5.42	5.47	5.5	5.52	5.5	5.48	5.46
0.8	5.22	5.37	5.25	5.24	5.26	5.2	5.27	5.32	5.27	5.31	5.39	5.18	5.35	5.32	5.36	5.34	5.21	5.27	5.32	5.38	5.34
0.85	5.05	5.03	5.11	5.05	5.3	5.15	5.17	5.15	5.15	5.28	5.26	5.18	5.24	5.28	5.2	5.21	5.22	5.15	5.18	5.17	5.15
0.9	5.14	5.06	4.99	5.09	5.07	5.07	5.1	5.13	5.24	5.08	5.13	5.25	5.04	5.18	5.02	5.14	5.11	5.09	5.04	5.11	5.04
0.95	5	4.95	5.03	5.07	4.97	5.01	5.12	4.99	5.01	4.97	5	5.09	5.08	5.06	5	4.99	4.92	4.99	4.99	5.06	5.01
1	5.14	5.03	5.12	4.96	5.06	4.99	4.91	4.99	5.15	5.12	5.05	5.07	4.86	5.03	5.06	4.89	4.99	5	5	5.02	5.13

Fig. 5.19 The average payoff of male agents when a stable matching is obtained in our model over 200 simulations with 10 male and 10 female agents

payoffs in the optimal stable matchings thus was highest. We generated 450 instances of the Stable Marriage Problem with 10 male and 10 female agents with $p = 0$ and $s = 0$. We then investigated which of the 3628800 (10!) possible matchings are stable and what the average payoff is in these stable matchings. We then ran 500 simulations for every instance until a stable matching was obtained and investigated which stable matchings were more often obtained than others.

Biro et al. [3] have shown that for some examples of the Stable Marriage Problem, suboptimal stable matchings (called egalitarian matchings in their article) are more likely to be obtained than the optimal stable matchings when we randomly satisfy blocking pairs. They furthermore show that in these cases suboptimal stable matchings are more likely to be obtained by randomly satisfying blocking pairs than would be the case using the algorithm of Roth and Vande Vate [13]. We are interested to find out whether suboptimal stable matchings are also more likely to be obtained in our model and whether this is the reason our model on average obtains higher payoffs than the optimal stable matchings.



Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	92%	94%	95%	96%	96%	97%	98%	98%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.05	94%	94%	94%	97%	96%	97%	97%	98%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.1	93%	94%	95%	96%	97%	98%	98%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.15	95%	95%	95%	96%	97%	98%	98%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.2	96%	96%	96%	97%	97%	98%	98%	98%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.25	97%	96%	97%	97%	98%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.3	96%	97%	97%	97%	98%	98%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.35	97%	98%	98%	98%	98%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.4	98%	98%	98%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.45	98%	98%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.5	99%	99%	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.55	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.6	99%	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.65	99%	99%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.7	99%	100%	99%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.75	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.85	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.9	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.95	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

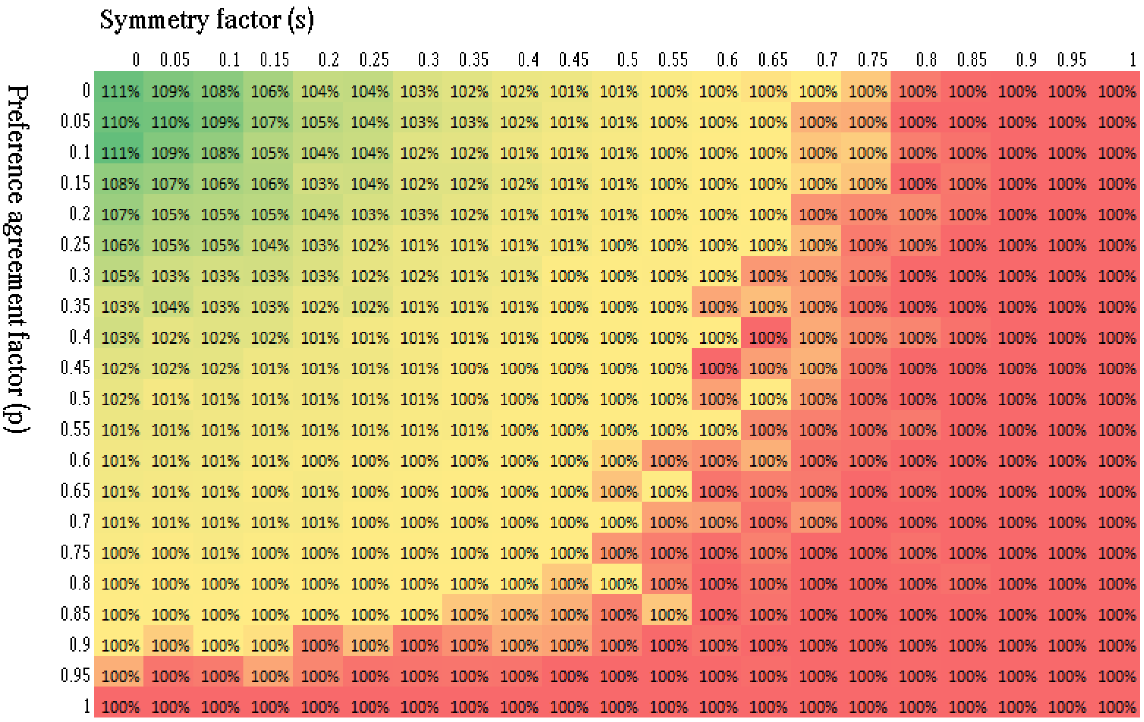
Fig. 5.20 The average payoff of male agents as a percentage of payoff in the optimal stable matching for male agents when a stable matching is eventually reached over 200 simulations with 10 male and 10 female agents

Number of stable matchings

We found that on average over 450 instances that for every instance on average 3.02 out of the 3,628,800 possible matchings are stable, with the number of stable matchings per instance ranging from 1 to 12 (Figure 5.23).

Payoffs of stable matchings and simulations

The 1,358 stable matchings had an average payoff of 7.194, with an average absolute deviation of 0.323. The 825 stable matchings that were either optimal for the male agents, the female agents or both sides of the market, obtained an average payoff of 7.218. It thus not seems to be the case that stable matchings that are not optimal for either side of the market tend to yield higher average payoffs than the stable matchings resulting from the Deferred Acceptance Algorithm. The average payoff obtained on average over the 225,000 simulations in our model was however 7.301 with an average deviation of 0.312. We thus do find that our model obtains both higher payoffs than would be the case in the matchings resulting from the Deferred Acceptance Algorithm and in a random stable matching. If we would select the



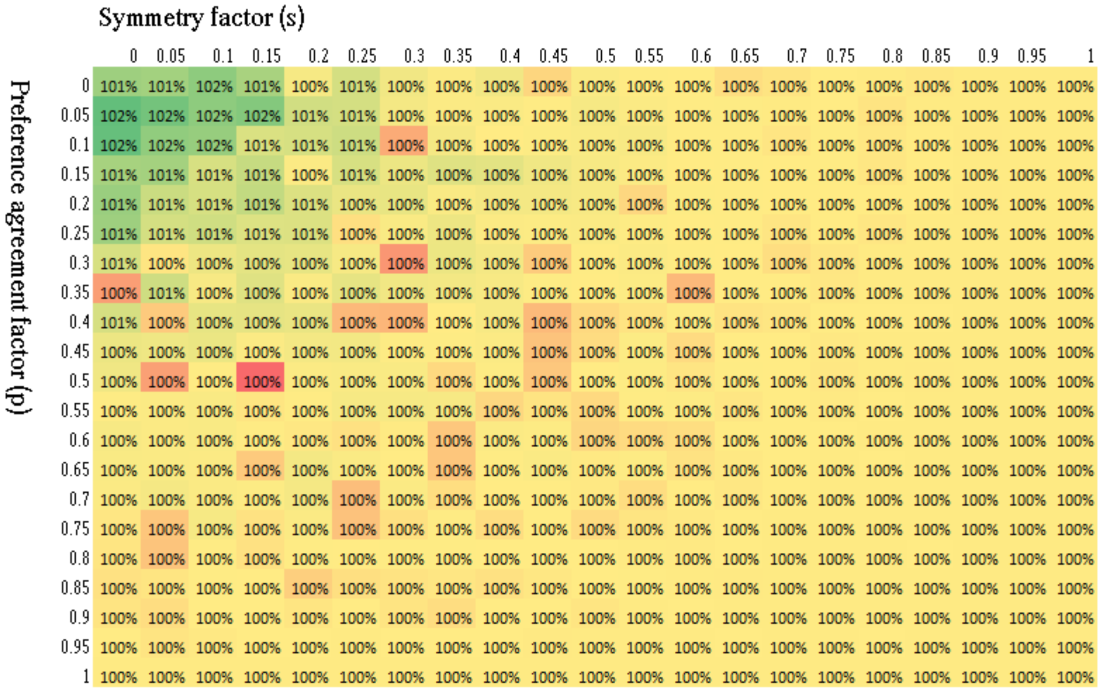
	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	111%	109%	108%	106%	104%	104%	103%	102%	102%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.05	110%	110%	109%	107%	105%	104%	103%	103%	102%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.1	111%	109%	108%	105%	104%	104%	102%	102%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.15	108%	107%	106%	106%	103%	104%	102%	102%	102%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.2	107%	105%	105%	105%	104%	103%	103%	102%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.25	106%	105%	105%	104%	103%	102%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.3	105%	103%	103%	103%	103%	102%	102%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.35	103%	104%	103%	103%	102%	102%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.4	103%	102%	102%	102%	101%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.45	102%	102%	102%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.5	102%	101%	101%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.55	101%	101%	101%	101%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.6	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.65	101%	101%	101%	100%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.7	101%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.75	100%	100%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.85	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.9	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.95	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Fig. 5.21 The average payoff of male agents as a percentage of their payoff in the optimal stable matching for female agents when a stable matching is eventually reached over 200 simulations with 10 male and 10 female agents

stable matchings with the highest average payoff for every one of 450 instances we would have obtained an average payoff of 7.392.

Suboptimal stable matchings

Of the 1,358 stable matchings we found for the 450 instances of the Stable Marriage Problem 533 matchings were neither optimal for the male agents nor for the female agents. These suboptimal stable matching had an average payoff of 7.159, which is lower than the average payoff of the stable matchings that were either optimal for the male or the female agents (7.218). It is thus not the case that suboptimal stable matching yields higher payoffs on average than the stable optimal stable matchings. We have found that the suboptimal stable matchings were 15% more likely to be obtained in our model, than would be the case if every stable matching was equally likely to be obtained.



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	101%	101%	102%	101%	100%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.05	102%	102%	102%	102%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.1	102%	102%	102%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.15	101%	101%	101%	101%	100%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.2	101%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.25	101%	101%	101%	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.3	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.35	100%	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.4	101%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.45	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.55	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.65	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.75	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.85	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.9	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.95	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Fig. 5.22 The average payoff of male agents as a percentage of the average of the payoff obtained by them in the optimal stable matching for male agents and the optimal stable matching for female agents when a stable matching is eventually reached over 200 simulations for 10 male and 10 female agents

Stable matchings optimal for both sides of the market

Of the 1,358 stable matchings we found for the 450 instances of the Stable Marriage Problem 75 matchings were optimal for both the male agents and the female agents. This means that for 75 of the 450 instances the Deferred Acceptance Algorithm would yield the same matching with the male agents as the proposing party as with the female agents as the proposing party. Furthermore, we found that when the optimal stable matching for male agents was equal to the optimal stable matching for female agents, that there never existed another stable matching. The optimal stable matching for both sides of the market had an average payoff of 7.385, which is higher than the average payoff of the stable matchings that were either optimal for the male or the female agents (7.218) and almost as high as the average payoff in the stable matchings with the highest payoff for every instance (7.392).

Suboptimal stable matchings with maximal payoffs

In 124 of the 450 generated instances of the Stable Marriage Problem there existed a stable matching that yielded the highest average payoff of all stable matchings for this instance but

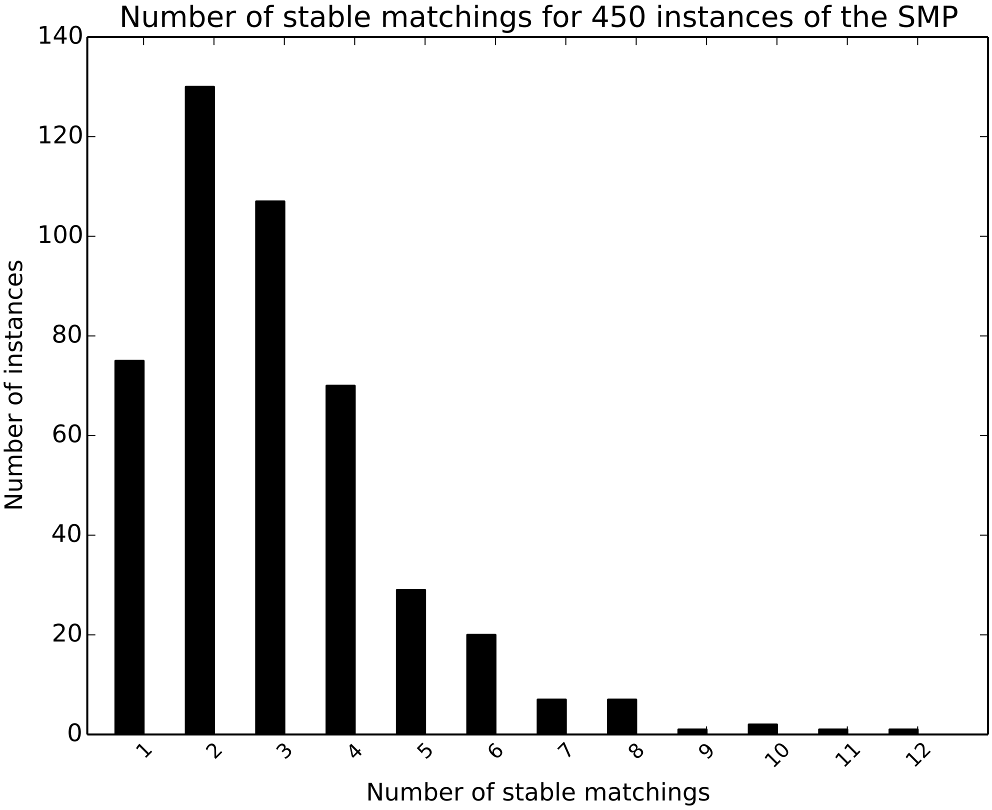


Fig. 5.23 Distribution of the number of stable matchings for 450 instances of the Stable Marriage Problem with uniformly distributed payoffs and 10 male and 10 female agents

which was not equal to one of the stable matchings resulting from the Deferred Acceptance Algorithm, and thus not optimal for either side of the market. If every stable matching would have been equally likely to be obtained, we would obtain a stable matching that yields the highest average payoff of all stable matchings, but which is not a stable matching that results from the Deferred Acceptance Algorithm, in 15,356.8 of the 225,000 simulations. We however have obtained such stable matchings 22,342 times. This means in our model stable matchings that yield the highest payoffs of all stable matchings of that instance and which are not the same as the optimal stable matchings resulting from the Deferred Acceptance Algorithm are 45% more likely to be obtained in our model than when we would randomly select a stable matching.

5.5.2 Obtained average ranks

Although we find that our model obtains higher average payoffs than we would obtain by running the Deferred Acceptance Algorithm or by randomly selecting a stable matching (since suboptimal stable matchings with high average payoffs are more likely to be obtained), we do not necessarily have that this is also the case for the average rank agents assign to their parter. We therefore also investigated the ranking every agent assigns to their partner in the stable matchings and our simulations.

In the 1,358 stable matchings agents on average were matched with an agent they ranked at 3.081 on average (with rank 1 for the most popular agent and rank 10 for the least popular agent), with an average absolute deviation of 0.263. In the stable matchings that were either optimal for the male agents, the female agents or both sides of the market, agents assigned an average rank of 3.066 to the agent they were matched with. It thus not seems to be the case that stable matchings that are not optimal for either side of the market tend to yield better average payoffs rankings than the stable matchings resulting from the Deferred Acceptance Algorithm. The average ranking agents assign to their match on average over the 225,000 simulations was however 3.021 with an average deviation of 0.269. We thus do find that our model obtains not only higher payoffs, but also better average rankings than would be the case in the matchings resulting from the Deferred Acceptance Algorithm and in a random stable matching. The differences in the obtained average ranking are however not as convincing as for the payoffs. The average of the 501 stable matchings with the minimal average ranking for all instances (in same instance multiple stable matchings had the same average ranking) we would have obtained an average ranking of 2.897.

Average ranking in suboptimal stable matchings

The 533 suboptimal stable matchings that were neither optimal for the male agents nor for the female agents had an average ranking of 3.103, which is worse than the average ranking of the stable matchings that were either optimal for the male or the female agents (3.066) and the average ranking of all stable matchings (3.081).

Average ranking in stable matchings optimal for both sides of the market

In the 75 matchings that were optimal for both the male agents and the female agents, the agents on average gave their partners a ranking of 2.873. This is better than the average ranking of the stable matchings that were either optimal for the male or the female agents (3.066) and the average ranking of all stable matchings (3.081).

Suboptimal stable matchings with minimal ranking

In 165 of the 450 generated instances of the Stable Marriage Problem there existed a stable matching that yielded the lowest average ranking of all stable matchings for this instance but which was not equal to one of the stable matchings resulting from the Deferred Acceptance Algorithm, and thus not optimal for either side of the market. If every stable matching would have been equally likely to be obtained, we would obtain such a stable matching in 20,300.4 of the 225,000 simulations. We however have obtained such stable matchings 31,019 times. This means in our model stable matchings that yield the lowest ranking of all stable matchings of that instance and which are not the same as the optimal stable matchings resulting from the Deferred Acceptance Algorithm are 53% more likely to be obtained in our model than when we would randomly select a stable matching. In 353 of the 450 cases the stable matching with the maximal payoff was equal to the stable matching with the minimal ranking. In 88 of the 450 instance there existed a stable matching that had the highest average payoff and the minimal average ranking of that instance, and which was not optimal for neither side of the market. These stable matchings were 58% more likely to be obtained in our model, than if we would have randomly selected stable matchings.

We can thus conclude that both stable matchings that are suboptimal and yield highest payoffs of all stable matchings and stable matching that are suboptimal and have the lowest average rank are more likely to be obtained in our model.

5.5.3 Egalitarity in our model

Next to the fact that our model on average obtains higher average payoffs and better average rankings than we would by randomly selecting a stable matching or by running the Deffered Acceptance Algorithm. It might also obtain more egalitarian results.

Egalitarity between two sides of the market

From Corollary 2.1 we know that the optimal stable matching for male agents resulting from the Deferred Acceptance Algorithm is the pessimal stable matching for female agents, and the other way around. We can therefore conclude that for an instance of the Stable Marriage Problem the stable matchings resulting from the Deferred Acceptance Algorithm the difference between the average ranking of agent's matches of one side of the market and the average ranking of agent's matches of the other side of the market is largest of all stable matchings. For many two-sided matching problems it seems preferable and more fair to have the average ranking of agent's matches of one side of the market and the average ranking of agent's matches of the other side of the market as close to one another as possible. We found that the 1,358 stable matchings for the 450 instances of the Stable Marriage Problem had an average difference between the two sides of the market of the rankings agents assigned to their partner of 1.416 (and an average difference in payoff of 1.376). The stable matchings resulting from the Deferred Acceptance Algorithm had an average difference of the ranking of the partners of the two sides of the market of 1.639 (and average difference in payoff of 1.563). In the 225,000 simulations in our model we however obtained an average difference of the rankings assigned to all agent's partners of 1.024 (and an average difference in payoff of 1.060). When we would select the stable matchings where the rankings of the agents of the two sides of the market is closest to one another we would obtain an average difference of 0.528 (with an average rank of 2.961). This means that our model obtains stable matchings that are more egalitarian (considering the two sides of the market) than picking a random stable matching, or by running the Deferred Acceptance Algorithm. We indeed find that when there exists a stable matching that is not optimal for one of the sides of the market and has the lowest difference in ranking between the two sides of the market, then this stable matching is 36% more likely to be obtained than we would expect if every stable matching was equally likely to result from simulations in our model.

Egalitarity for all agents

Next to the differences between the results for the two sides of the market, we are also interested whether our model obtains more egalitarian stable matchings in the sense that the difference between the ranking and payoff assigned to the partner by the agent that yields the worst results in the resulting stable matching is closer to the ranking and payoff assigned to the partner by the agent that yields the best results. We found that in the 1,358 stable matchings the difference in the ranks of the partners of the best and worst ranking is 7.327 (the difference between the agent's with the highest and the lowest payoff is 7.966). The stable matchings resulting from the Deferred Acceptance Algorithm had an average difference in ranking between the agent that ended up with the lowest ranked partner and the agent that ended up with the highest ranked partner of 7.452 (and average difference between the agent with the highest payoff and the one with the lowest payoff of 8.038). In the 225,000 simulations in our model we however obtained an average difference between the agent that ends up with the lowest ranked partner and the agent that ends up with the highest ranked partner of 7.019 (the difference between the agent's with the highest and the lowest payoff is 7.707). When we would for every instance select one of the stable matchings where the agent with the worst ranked partner and the agent with the best ranked partners rank their partner closest to one another of all stable matchings for this instance (of which there are 728 in total), we would obtain an average difference of 6.524. If we would for every instance choose one of the stable matchings where the payoff of the agent with the highest payoff is closest to the payoff of the agents with the lowest payoff (of which there are 552 in total) we find an average difference between the best and worst scoring agent of 7.209. This means that our model obtains stable matchings that are more egalitarian (considering the difference in payoff and ranking between the two agents with the highest and lowest payoff) than picking a random stable matching, or by running the Deferred Acceptance Algorithm. Out of the 450 instances of the Stable Marriage Problem we found that there existed a stable matching that is both the most egalitarian stable matching for all agents based on ranking and the most egalitarian stable matching based on payoff in 407 cases.

5.5.4 Summary of exhaustive investigation of all stable matchings

We can conclude that our model on average yields both higher payoffs and better average ranks than would be the case when we would either run the Deferred Acceptance Algorithm or when we would randomly pick a stable matching (which could be done by checking random matchings until a stable one is found). Although suboptimal stable matchings are more likely to be obtained in our model, this does not account for the good results in our model, since suboptimal stable matchings do not yield better payoffs than a random stable matching or the stable matchings resulting from the deferred acceptance algorithm. We however do find that suboptimal stable matchings exist that yield better results than the optimal stable matchings, and we find that these stable matchings are relatively more likely to be obtained in our model. Furthermore, we find that our model yields stable matchings that are more egalitarian in the sense that the differences between the two sides of the market are on average lower than in a randomly selected stable matching and lower than in the optimal stable matchings resulting from the Deferred Acceptance Algorithm. We also found that our model on average obtained stable matchings in which the differences between the best and worst scoring agent (considering both ranks and payoffs) were smaller than in a randomly chosen stable matching and in the optimal stable matchings resulting from the Deferred Acceptance Algorithm.

Chapter 6

Extending our model for simulating real-world two-sided matching markets

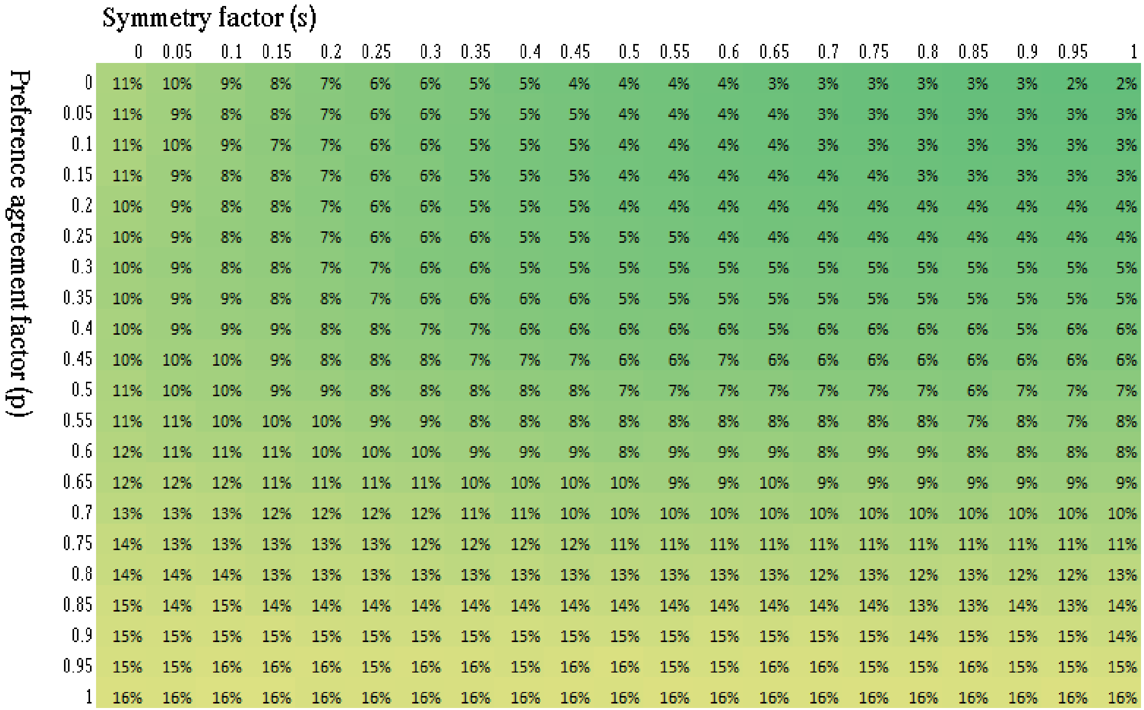
Many real-world two-sided matching markets are too complex to be solved by the Deferred Acceptance Algorithm since agents have no full overview of the market or since there is no central authority to execute the algorithm. Besides that, agents in many real-world two-sided matching markets seem to behave differently than agents are ought to behave in the Deferred Acceptance Algorithm which makes this algorithm inappropriate for modeling behaviour in real world two-sided markets. The ultimate goal of this thesis is to propose a method to model real-world two-sided matching markets. So far, we have built a model that ultimately converges towards a stable matchings that seems to include the randomness needed to model noisy real-world markets and which has no need for a central authority to obtain this stable matching. We furthermore identified two important parameters that affect the distribution of attractiveness; the preference agreement factor p and the symmetry factor s. We investigated the effects of changing these parameters on the stability, payoffs and percentage of agents in a relationship during the simulations.

To use our model to simulate and better understand real-world two-sided markets there are still some steps to take. Firstly, there still seem to be some aspects in our current model that make it not suitable yet for modeling, for example, the search of people for a romantic partner. In this chapter, we will investigate some of these aspects and propose some adjustments to our model to tackle them.

Secondly, for modeling a certain real-world two sided matching market we need to find the parameter settings that resemble the real-world market best. In this chapter we will discuss a more advanced measure of the preference agreement and symmetry in the preferences of the agents, and propose some methods of translating preferences in real-world problems to our model.

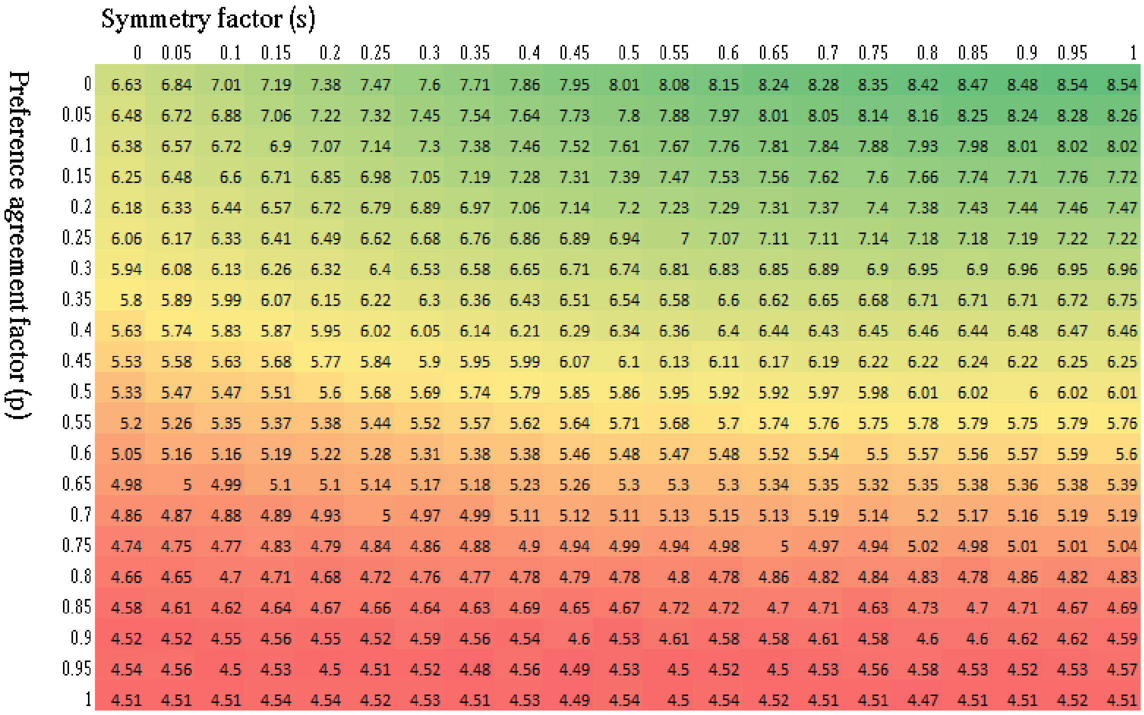
6.1 Guaranteeing limited access to the market

In many real-world two-sided matching markets, especially the ones that are to complex to solve with the Deferred Acceptance Algorithm like the job market or people in search of a romantic relationship, agents do not get to meet all candidate matches. In our model, however, it takes many times more rounds than there are agents in the simulation before a stable matching is obtained. It took for example 161 daterounds on average for instances with completely random preferences over 200 simulations with 10 male and 10 female agents, which means that agents are "dated" many times. For modeling real-world two-sided matching markets we will run simulations with 100 agents, for 80 rounds. This way we will ensure that agents do not get to meet every potential partner.



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	11%	10%	9%	8%	7%	6%	6%	5%	5%	4%	4%	4%	4%	3%	3%	3%	3%	3%	3%	2%	2%
0.05	11%	9%	8%	8%	7%	6%	6%	5%	5%	5%	4%	4%	4%	4%	3%	3%	3%	3%	3%	3%	3%
0.1	11%	10%	9%	7%	7%	6%	6%	5%	5%	5%	4%	4%	4%	4%	3%	3%	3%	3%	3%	3%	3%
0.15	11%	9%	8%	8%	7%	6%	6%	5%	5%	5%	4%	4%	4%	4%	4%	4%	3%	3%	3%	3%	3%
0.2	10%	9%	8%	8%	7%	6%	6%	5%	5%	5%	4%	4%	4%	4%	4%	4%	4%	4%	4%	4%	4%
0.25	10%	9%	8%	8%	7%	6%	6%	6%	5%	5%	5%	5%	4%	4%	4%	4%	4%	4%	4%	4%	4%
0.3	10%	9%	8%	8%	7%	7%	6%	6%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
0.35	10%	9%	9%	8%	8%	7%	6%	6%	6%	6%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
0.4	10%	9%	9%	9%	8%	8%	7%	7%	6%	6%	6%	6%	6%	5%	6%	6%	6%	6%	5%	6%	6%
0.45	10%	10%	10%	9%	8%	8%	8%	7%	7%	7%	6%	6%	7%	6%	6%	6%	6%	6%	6%	6%	6%
0.5	11%	10%	10%	9%	9%	8%	8%	8%	8%	8%	7%	7%	7%	7%	7%	7%	7%	6%	7%	7%	7%
0.55	11%	11%	10%	10%	10%	9%	9%	8%	8%	8%	8%	8%	8%	8%	8%	8%	8%	7%	8%	7%	8%
0.6	12%	11%	11%	11%	10%	10%	10%	9%	9%	9%	8%	9%	9%	9%	8%	9%	9%	8%	8%	8%	8%
0.65	12%	12%	12%	11%	11%	11%	11%	10%	10%	10%	10%	9%	9%	10%	9%	9%	9%	9%	9%	9%	9%
0.7	13%	13%	13%	12%	12%	12%	12%	11%	11%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
0.75	14%	13%	13%	13%	13%	13%	12%	12%	12%	12%	11%	11%	11%	11%	11%	11%	11%	11%	11%	11%	11%
0.8	14%	14%	14%	13%	13%	13%	13%	13%	13%	13%	13%	13%	13%	13%	12%	13%	12%	13%	12%	12%	13%
0.85	15%	14%	15%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	13%	13%	14%	13%	14%
0.9	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	14%	15%	15%	15%	14%
0.95	15%	15%	16%	16%	16%	15%	16%	16%	15%	16%	16%	15%	15%	16%	16%	15%	15%	16%	15%	15%	15%
1	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%	16%

Fig. 6.1 The average percentage of the market agents would form a blocking pair with over 200 simulations after 80 daterounds for 100 male and 100 female agents



Symmetry factor (s)

Preference agreement factor (p)	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	6.63	6.84	7.01	7.19	7.38	7.47	7.6	7.71	7.86	7.95	8.01	8.08	8.15	8.24	8.28	8.35	8.42	8.47	8.48	8.54	8.54
0.05	6.48	6.72	6.88	7.06	7.22	7.32	7.45	7.54	7.64	7.73	7.8	7.88	7.97	8.01	8.05	8.14	8.16	8.25	8.24	8.28	8.26
0.1	6.38	6.57	6.72	6.9	7.07	7.14	7.3	7.38	7.46	7.52	7.61	7.67	7.76	7.81	7.84	7.88	7.93	7.98	8.01	8.02	8.02
0.15	6.25	6.48	6.6	6.71	6.85	6.98	7.05	7.19	7.28	7.31	7.39	7.47	7.53	7.56	7.62	7.6	7.66	7.74	7.71	7.76	7.72
0.2	6.18	6.33	6.44	6.57	6.72	6.79	6.89	6.97	7.06	7.14	7.2	7.23	7.29	7.31	7.37	7.4	7.38	7.43	7.44	7.46	7.47
0.25	6.06	6.17	6.33	6.41	6.49	6.62	6.68	6.76	6.86	6.89	6.94	7	7.07	7.11	7.11	7.14	7.18	7.18	7.19	7.22	7.22
0.3	5.94	6.08	6.13	6.26	6.32	6.4	6.53	6.58	6.65	6.71	6.74	6.81	6.83	6.85	6.89	6.9	6.95	6.9	6.96	6.95	6.96
0.35	5.8	5.89	5.99	6.07	6.15	6.22	6.3	6.36	6.43	6.51	6.54	6.58	6.6	6.62	6.65	6.68	6.71	6.71	6.71	6.72	6.75
0.4	5.63	5.74	5.83	5.87	5.95	6.02	6.05	6.14	6.21	6.29	6.34	6.36	6.4	6.44	6.43	6.45	6.46	6.44	6.48	6.47	6.46
0.45	5.53	5.58	5.63	5.68	5.77	5.84	5.9	5.95	5.99	6.07	6.1	6.13	6.11	6.17	6.19	6.22	6.22	6.24	6.22	6.25	6.25
0.5	5.33	5.47	5.47	5.51	5.6	5.68	5.69	5.74	5.79	5.85	5.86	5.95	5.92	5.92	5.97	5.98	6.01	6.02	6	6.02	6.01
0.55	5.2	5.26	5.35	5.37	5.38	5.44	5.52	5.57	5.62	5.64	5.71	5.68	5.7	5.74	5.76	5.75	5.78	5.79	5.75	5.79	5.76
0.6	5.05	5.16	5.16	5.19	5.22	5.28	5.31	5.38	5.38	5.46	5.48	5.47	5.48	5.52	5.54	5.5	5.57	5.56	5.57	5.59	5.6
0.65	4.98	5	4.99	5.1	5.1	5.14	5.17	5.18	5.23	5.26	5.3	5.3	5.3	5.34	5.35	5.32	5.35	5.38	5.36	5.38	5.39
0.7	4.86	4.87	4.88	4.89	4.93	5	4.97	4.99	5.11	5.12	5.11	5.13	5.15	5.13	5.19	5.14	5.2	5.17	5.16	5.19	5.19
0.75	4.74	4.75	4.77	4.83	4.79	4.84	4.86	4.88	4.9	4.94	4.99	4.94	4.98	5	4.97	4.94	5.02	4.98	5.01	5.01	5.04
0.8	4.66	4.65	4.7	4.71	4.68	4.72	4.76	4.77	4.78	4.79	4.78	4.8	4.78	4.86	4.82	4.84	4.83	4.78	4.86	4.82	4.83
0.85	4.58	4.61	4.62	4.64	4.67	4.66	4.64	4.63	4.69	4.65	4.67	4.72	4.72	4.7	4.71	4.63	4.73	4.7	4.71	4.67	4.69
0.9	4.52	4.52	4.55	4.56	4.55	4.52	4.59	4.56	4.54	4.6	4.53	4.61	4.58	4.58	4.61	4.58	4.6	4.6	4.62	4.62	4.59
0.95	4.54	4.56	4.5	4.53	4.5	4.51	4.52	4.48	4.56	4.49	4.53	4.5	4.52	4.5	4.53	4.56	4.58	4.53	4.52	4.53	4.57
1	4.51	4.51	4.51	4.54	4.54	4.52	4.53	4.51	4.53	4.49	4.54	4.5	4.54	4.52	4.51	4.51	4.47	4.51	4.51	4.52	4.51

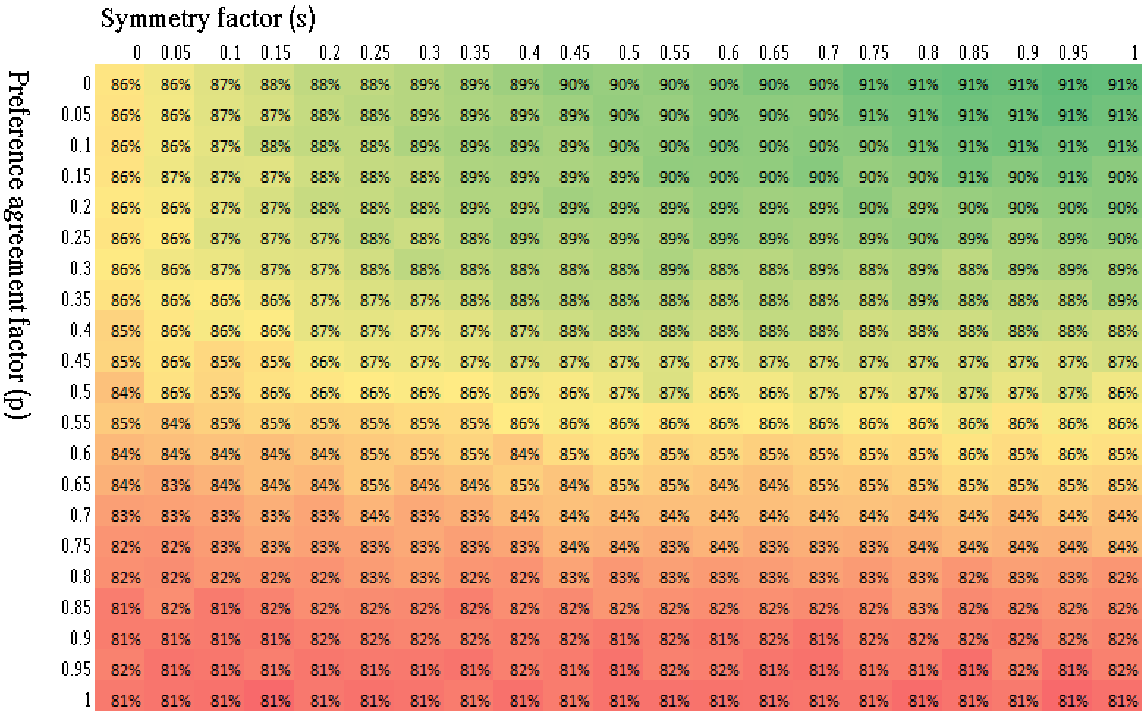
Fig. 6.2 The average payoff of male agents over 200 simulations after 80 daterounds for 100 male and 100 female agents

We find that after running 200 simulations for 80 daterounds with 100 male and 100 female agents that for $p = 1$ we have the least stable matchings with about 80% of agents in a relationship (Figure 6.3) and around 1600 blocking pairs, independent of the value for s. This means that an average agent would form a blocking pair with 16% of the potential partners (Figure 6.1). In these cases male agents obtained an average payoff of 4.5 (Figure 6.2), which is 90% of what would have been obtained in the optimal stable matchings (Figure 6.4 and Figure 6.5). For $p = 0$ and $s = 1$ we find the highest stability with 239 blocking pairs, which means an average agent would form a blocking pair with only 2.39% of the potential partners. Agents obtained the highest average payoff in this case, with 8.54 which is 89% of the payoff obtained in the optimal stable matchings (Figure 6.4 and Figure 6.5).

6.2 Finding the right parameter settings

6.2.1 Advanced method for investigating preference agreement

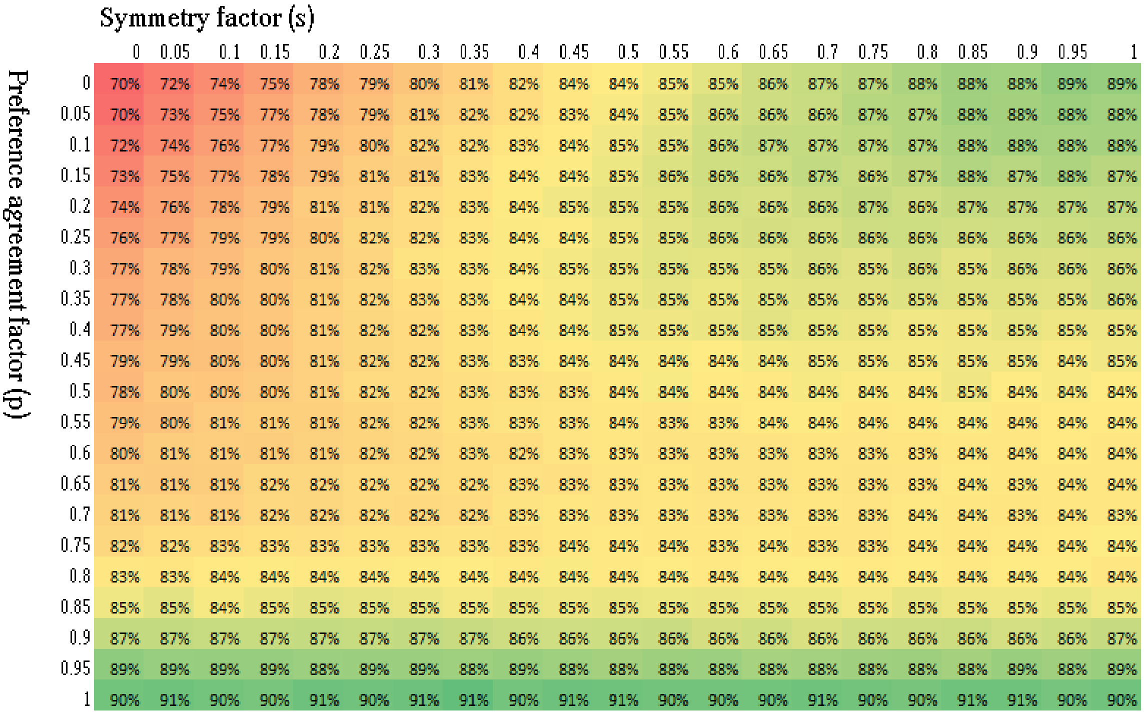
The preference agreement factor p seems to be a very simple and powerful method to influence the subjectivity of agents in our model. However, we could think of more advanced



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	86%	86%	87%	88%	88%	88%	89%	89%	89%	90%	90%	90%	90%	90%	90%	91%	91%	91%	91%	91%	91%
0.05	86%	86%	87%	87%	88%	88%	89%	89%	89%	89%	90%	90%	90%	90%	90%	91%	91%	91%	91%	91%	91%
0.1	86%	86%	87%	88%	88%	88%	89%	89%	89%	89%	90%	90%	90%	90%	90%	90%	91%	91%	91%	91%	91%
0.15	86%	87%	87%	87%	88%	88%	88%	89%	89%	89%	89%	90%	90%	90%	90%	90%	90%	91%	90%	91%	90%
0.2	86%	86%	87%	87%	88%	88%	88%	89%	89%	89%	89%	89%	89%	89%	89%	90%	89%	90%	90%	90%	90%
0.25	86%	86%	87%	87%	87%	88%	88%	88%	89%	89%	89%	89%	89%	89%	89%	89%	90%	89%	89%	89%	90%
0.3	86%	86%	87%	87%	87%	88%	88%	88%	88%	88%	88%	89%	88%	88%	89%	88%	89%	88%	89%	89%	89%
0.35	86%	86%	86%	86%	87%	87%	87%	88%	88%	88%	88%	88%	88%	88%	88%	88%	89%	88%	88%	88%	89%
0.4	85%	86%	86%	86%	87%	87%	87%	87%	87%	88%	88%	88%	88%	88%	88%	88%	88%	88%	88%	88%	88%
0.45	85%	86%	85%	85%	86%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%	87%
0.5	84%	86%	85%	86%	86%	86%	86%	86%	86%	86%	87%	87%	86%	86%	87%	87%	87%	87%	87%	87%	86%
0.55	85%	84%	85%	85%	85%	85%	85%	85%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%
0.6	84%	84%	84%	84%	84%	85%	85%	85%	84%	85%	86%	85%	85%	85%	85%	85%	85%	86%	85%	86%	85%
0.65	84%	83%	84%	84%	84%	85%	84%	84%	85%	84%	85%	85%	84%	84%	85%	85%	85%	85%	85%	85%	85%
0.7	83%	83%	83%	83%	83%	84%	83%	83%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%
0.75	82%	82%	83%	83%	83%	83%	83%	83%	83%	84%	84%	83%	84%	83%	83%	83%	84%	84%	84%	84%	84%
0.8	82%	82%	82%	82%	82%	83%	83%	82%	82%	83%	83%	83%	83%	83%	83%	83%	83%	82%	83%	83%	82%
0.85	81%	82%	81%	82%	82%	82%	82%	82%	82%	82%	82%	82%	82%	82%	82%	82%	83%	82%	82%	82%	82%
0.9	81%	81%	81%	81%	82%	82%	82%	82%	82%	82%	81%	82%	81%	82%	81%	82%	82%	82%	82%	82%	82%
0.95	82%	81%	81%	81%	81%	81%	81%	81%	82%	81%	81%	82%	82%	81%	81%	81%	81%	81%	82%	81%	82%
1	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%	81%

Fig. 6.3 The average percentage of agents in a relationship over 200 simulations after 80 daterounds for 100 male and 100 female agents

ways to express this preference agreement. We did this by for every agent computing the average rank they received in the preferences of all female agents. We then plotted these average ranks, ordered from the most popular male agent to the least popular male agent. The resulting graph expresses the preference agreement in a more advanced way. When this plot forms the diagonal we have full preference agreement where the most popular agent is ranked first by every other agent and the second most popular second by every other agents and so on. The closer this plot to the horizontal line the more subjective the preferences are. This method allows us to make a plot when we have empirical data from a real-world matching market, such that we could search for a way to generate instances in our model with similar preferences. In Figure 6.6 we find the results of generating 200 instances of the stable matching problem with $p = 0, p = 0.25, p = 0.5, p = 0.75, p = 1$ (we found that changing the value for s had no effect on the resulting plot). We indeed find that for decreasing p the plot moves closer to the horizontal line. But this horizontal line will never be obtained since there is randomness involved in generating preferences.

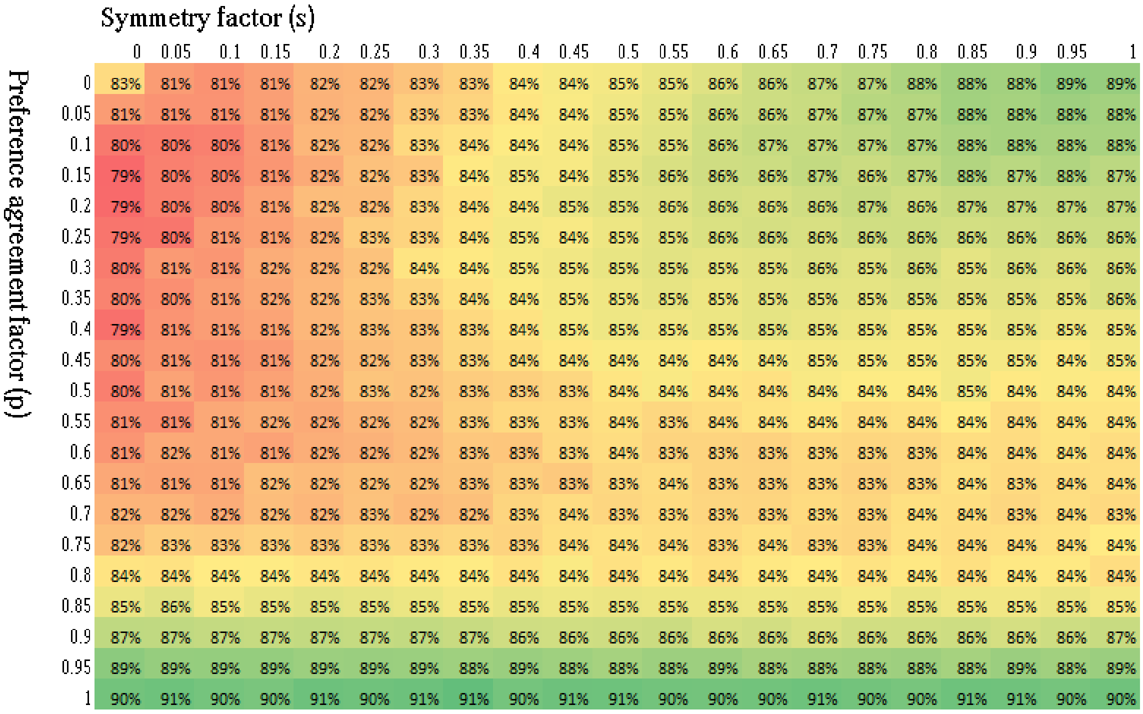


	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	70%	72%	74%	75%	78%	79%	80%	81%	82%	84%	84%	85%	85%	86%	87%	87%	88%	88%	88%	89%	89%
0.05	70%	73%	75%	77%	78%	79%	81%	82%	82%	83%	84%	85%	86%	86%	86%	87%	87%	88%	88%	88%	88%
0.1	72%	74%	76%	77%	79%	80%	82%	82%	83%	84%	85%	85%	86%	87%	87%	87%	87%	88%	88%	88%	88%
0.15	73%	75%	77%	78%	79%	81%	81%	83%	84%	84%	85%	86%	86%	86%	87%	86%	87%	88%	87%	88%	87%
0.2	74%	76%	78%	79%	81%	81%	82%	83%	84%	85%	85%	85%	86%	86%	86%	87%	86%	87%	87%	87%	87%
0.25	76%	77%	79%	79%	80%	82%	82%	83%	84%	84%	85%	85%	86%	86%	86%	86%	86%	86%	86%	86%	86%
0.3	77%	78%	79%	80%	81%	82%	83%	83%	84%	85%	85%	85%	85%	85%	86%	85%	86%	85%	86%	86%	86%
0.35	77%	78%	80%	80%	81%	82%	83%	83%	84%	84%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	86%
0.4	77%	79%	80%	80%	81%	82%	82%	83%	84%	84%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%
0.45	79%	79%	80%	80%	81%	82%	82%	83%	83%	84%	84%	84%	84%	84%	85%	85%	85%	85%	85%	84%	85%
0.5	78%	80%	80%	80%	81%	82%	82%	83%	83%	83%	84%	84%	84%	84%	84%	84%	84%	85%	84%	84%	84%
0.55	79%	80%	81%	81%	81%	82%	82%	83%	83%	83%	84%	83%	83%	84%	84%	84%	84%	84%	84%	84%	84%
0.6	80%	81%	81%	81%	81%	82%	82%	83%	82%	83%	83%	83%	83%	83%	83%	83%	83%	84%	84%	84%	84%
0.65	81%	81%	81%	82%	82%	82%	82%	82%	83%	83%	83%	83%	83%	83%	83%	83%	83%	84%	83%	84%	84%
0.7	81%	81%	81%	82%	82%	82%	82%	82%	83%	83%	83%	83%	83%	83%	83%	83%	84%	84%	83%	84%	83%
0.75	82%	82%	83%	83%	83%	83%	83%	83%	83%	84%	84%	84%	83%	84%	83%	83%	84%	84%	84%	84%	84%
0.8	83%	83%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%
0.85	85%	85%	84%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%
0.9	87%	87%	87%	87%	87%	87%	87%	87%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	87%
0.95	89%	89%	89%	89%	88%	89%	89%	88%	89%	88%	88%	88%	88%	88%	88%	88%	88%	88%	89%	88%	89%
1	90%	91%	90%	90%	91%	90%	91%	91%	90%	91%	91%	90%	90%	90%	91%	90%	90%	91%	91%	90%	90%

Fig. 6.4 The average percentage of the payoff in the optimal stable matching for male agents over 200 simulations after 80 daterounds for 100 male and 100 female agents obtained by the male agents

6.2.2 Advanced method for investigating symmetry

In a similar way as for the preference agreement we can investigate the symmetry more thoroughly. Of all male agents we investigate what rank they receive by their first ranked agent, their second rank agent and so on. For all these ranks we take the averages. We now plot these and in a similar way as for the preference agreement we have that the we find the diagonal when preferences are completely symmetric (if a male agent is the x-th favorite partner of a female agent then this is also the case the other way around). We ran 100 simulations with 100 male and 100 female agents, with values for p and s ranging from 0 to 1 with steps of 0.25. We find that the higher s, the closer the plot is to the diagonal (Figure 6.7). The higher p, the less effect increasing s has on the symmetry, and the closer the plotted line is to the horizontal line (Figure 6.8), (Figure 6.9), (Figure 6.10) and (Figure 6.11).



Symmetry factor (s)

Preference agreement factor (p)

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
0	83%	81%	81%	81%	82%	82%	83%	83%	84%	84%	85%	85%	86%	86%	87%	87%	88%	88%	88%	89%	89%
0.05	81%	81%	81%	81%	82%	82%	83%	83%	84%	84%	85%	85%	86%	86%	87%	87%	87%	88%	88%	88%	88%
0.1	80%	80%	80%	81%	82%	82%	83%	84%	84%	84%	85%	85%	86%	87%	87%	87%	87%	88%	88%	88%	88%
0.15	79%	80%	80%	81%	82%	82%	83%	84%	85%	84%	85%	86%	86%	86%	87%	86%	87%	88%	87%	88%	87%
0.2	79%	80%	80%	81%	82%	82%	83%	84%	84%	85%	85%	86%	86%	86%	86%	87%	86%	87%	87%	87%	87%
0.25	79%	80%	81%	81%	82%	83%	83%	84%	85%	84%	85%	85%	86%	86%	86%	86%	86%	86%	86%	86%	86%
0.3	80%	81%	81%	82%	82%	82%	84%	84%	85%	85%	85%	85%	85%	85%	86%	85%	86%	85%	86%	86%	86%
0.35	80%	80%	81%	82%	82%	83%	83%	84%	84%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	86%
0.4	79%	81%	81%	81%	82%	83%	83%	83%	84%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%
0.45	80%	81%	81%	81%	82%	82%	83%	83%	84%	84%	84%	84%	84%	84%	85%	85%	85%	85%	85%	84%	85%
0.5	80%	81%	81%	81%	82%	83%	82%	83%	83%	83%	84%	84%	84%	84%	84%	84%	84%	85%	84%	84%	84%
0.55	81%	81%	81%	82%	82%	82%	82%	83%	83%	83%	84%	83%	84%	84%	84%	84%	84%	84%	84%	84%	84%
0.6	81%	82%	81%	81%	82%	82%	82%	83%	83%	83%	84%	83%	83%	83%	83%	83%	83%	84%	84%	84%	84%
0.65	81%	81%	81%	82%	82%	82%	82%	83%	83%	83%	83%	84%	83%	83%	83%	83%	83%	84%	83%	84%	84%
0.7	82%	82%	82%	82%	82%	83%	82%	82%	83%	84%	83%	83%	83%	83%	83%	83%	84%	84%	83%	84%	83%
0.75	82%	83%	83%	83%	83%	83%	83%	83%	83%	84%	84%	84%	83%	84%	83%	83%	84%	84%	84%	84%	84%
0.8	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%	84%
0.85	85%	86%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%	85%
0.9	87%	87%	87%	87%	87%	87%	87%	87%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	86%	87%
0.95	89%	89%	89%	89%	89%	89%	89%	88%	89%	88%	88%	88%	89%	88%	88%	88%	88%	88%	89%	88%	89%
1	90%	91%	90%	90%	91%	90%	91%	91%	90%	91%	91%	90%	90%	90%	91%	90%	90%	91%	91%	90%	90%

Fig. 6.5 The average percentage of the payoff of male agents in the optimal stable matching for female agents over 200 simulations after 80 daterounds for 100 male and 100 female agents obtained by the male agents

6.2.3 Symmetry and preference agreement in real-world matching problems

In real-world two-sided matching markets we could obtain a distribution of preferences that could not be generated by our model by varying the preference agreement factor p and the symmetry factor s. It could for example be the case that only very few male agents are really popular, but that there is more disagreement on which male agents are the least preferred partners by the female agents. With the more advanced expression of preference agreement and symmetry we could express all different sorts of distributions of preferences.

If we want to model people's search for a romantic partner we want to know what the distribution of preferences are for people looking for a partner, such that we could generate agents that have similar preferences as the people we want to model. If we would for example have data from a speeddate event, where people meet different potential partners, such that we obtain the preferences of all agents, we could compute how much agents have similar preferences and what the degree of symmetry was. We can plot these results using the

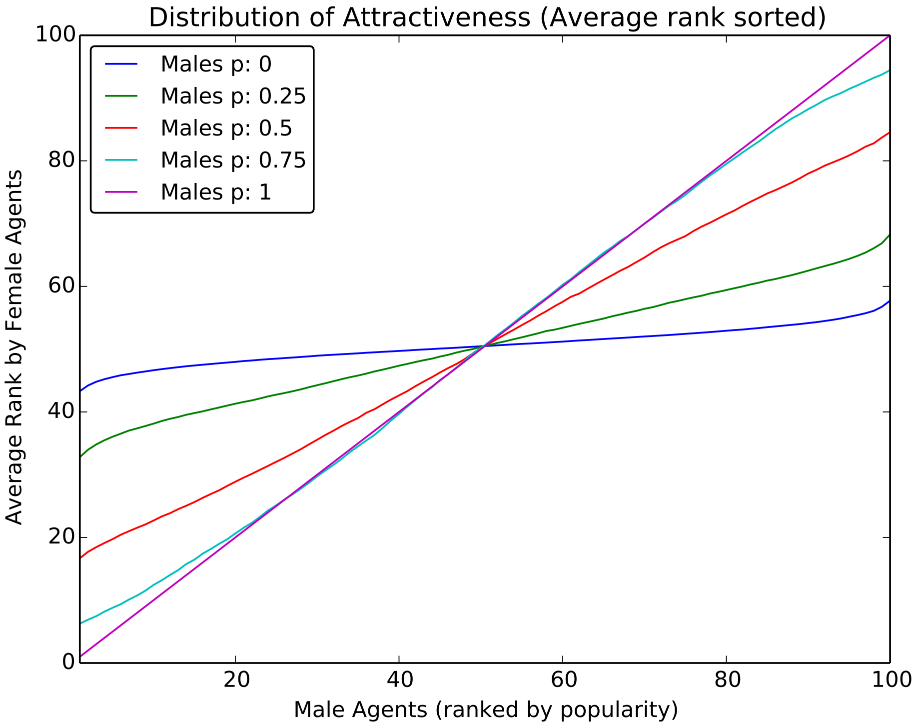


Fig. 6.6 The average ranks of male agents by the female agents per rank, from most popular to least popular over 200 instances for 100 male and 100 female agents for different values for p

advanced method for expressing preference agreement and symmetry.

To adjust our model for simulating people in search of a romantic relationship we have tried to use data from the social network website Facebook, a study from the University of Michigan on Sexually Transmitted Diseases and the American dating site eHarmony to distill the preferences of agents. Alas this data was not rich enough such that it enabled us to draw the plots in the advanced method of expressing preference agreement and symmetry.

However, it seems to be the case that in most interesting real-world two-sided matching markets there is both intermediate preference agreement, and intermediate symmetry. Take for example people in search of romantic partner. To some degree people seem to agree on what makes someone a preferable partner. Some people are more popular than others, and there seem to be certain character traits that are welcomed by many or all. On the other hand it (luckily) is also the case that attractiveness is a matter of taste.

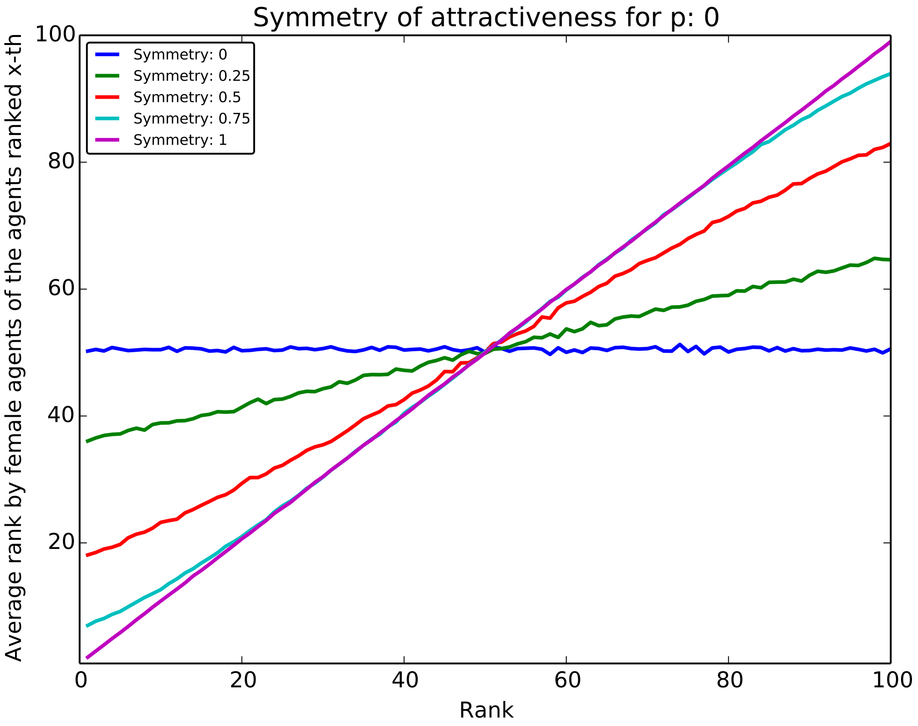


Fig. 6.7 The average rank the female agents assign to the male agents that rank them first, second and so on. Average over 200 instances for 100 male and 100 female agents for different values of s with $p = 0$

6.3 Multi-dimensional weighted preferences

There are many ways we could distribute preferences in ways we could not obtain by varying p and s. We will use an example, that intuitively seems to give preferences that resemble those in real-world situations where there is some agreement on what makes a candidate preferable over another and where agents are liked relatively more by agents they like. Assume there are j dimensions of attractiveness. Every agent x scores a randomly generated real number $0 \leq u_x(d_i) \leq 10$ on every dimension d_i with $i \in \mathscr{N}$ and $1 \leq i \leq j$. For every such dimension d_i agents agree on what is most and least attractive in another agent (they all prefer higher scores). Agents however disagree on the weights $w_x(d_i)$ they assign to each dimension d_i with $\sum_{i=1}^{j} w_x(d_i) = 1$ with $0 \leq w_x(d_i) \leq 1$ and $w_x(d_i) \in \mathscr{R}$ for any agent s. The higher an agent x scores on a dimension, the higher this agents weighs this dimension when meeting other agents. In this multi-dimensional weighted preference model we have $w_x(d_i) = \frac{u_x(d_i)}{x_{tot}}$ where $x_{tot} = \sum_{i=1}^{j} u_x(d_i)$. In our simulations we fixed the number of dimensions of attractiveness at

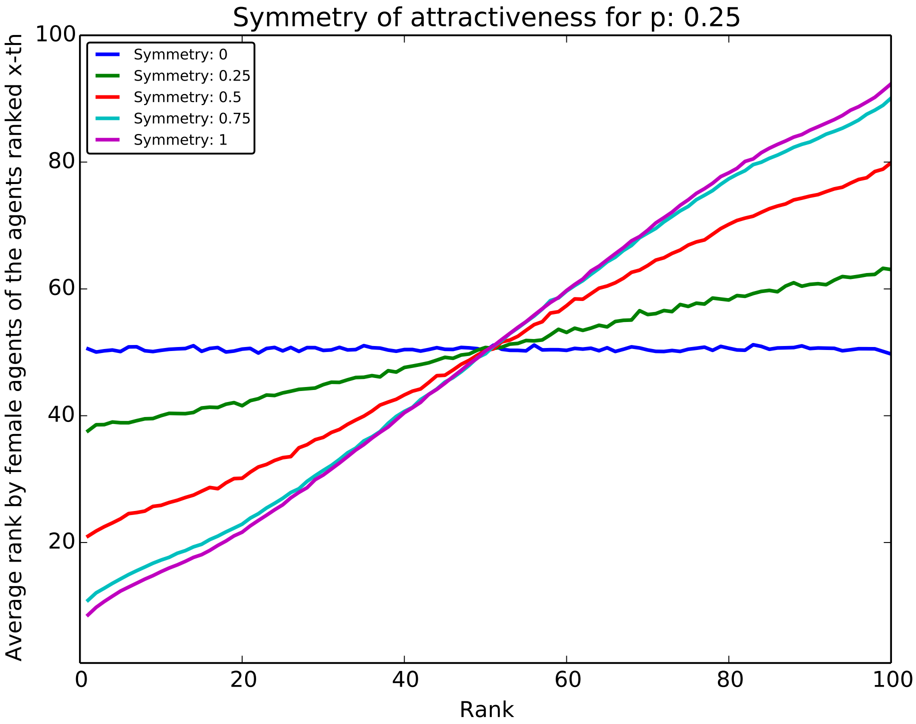


Fig. 6.8 The average rank the female agents assign to the male agents that rank them first, second and so on. Average over 200 instances for 100 male and 100 female agents for different values of s with $p = 0.25$

four. We started our simulations by plotting the preferenceagreement and symmetry for 100 male and 100 female agents using the advanced methods discussed earlier in this chapter.

Symmetry and preference agreement with four-dimensional preferences

With four-dimensional weighted preferences we find that agents very much agree on attractiveness, and that the distribution of attractiveness (Figure 6.12) is thus close to the diagonal. We however do find that there is some subjectivity. The most popular male agent is for example ranked 2.38th on average by the female agents and the 6th most preferred male agent is ranked 10.67th. We also obtain some symmetry (Figure 6.13). We see that agents are more likely to be preferred by agents that they like more and less likely to be preferred by agents they like less.

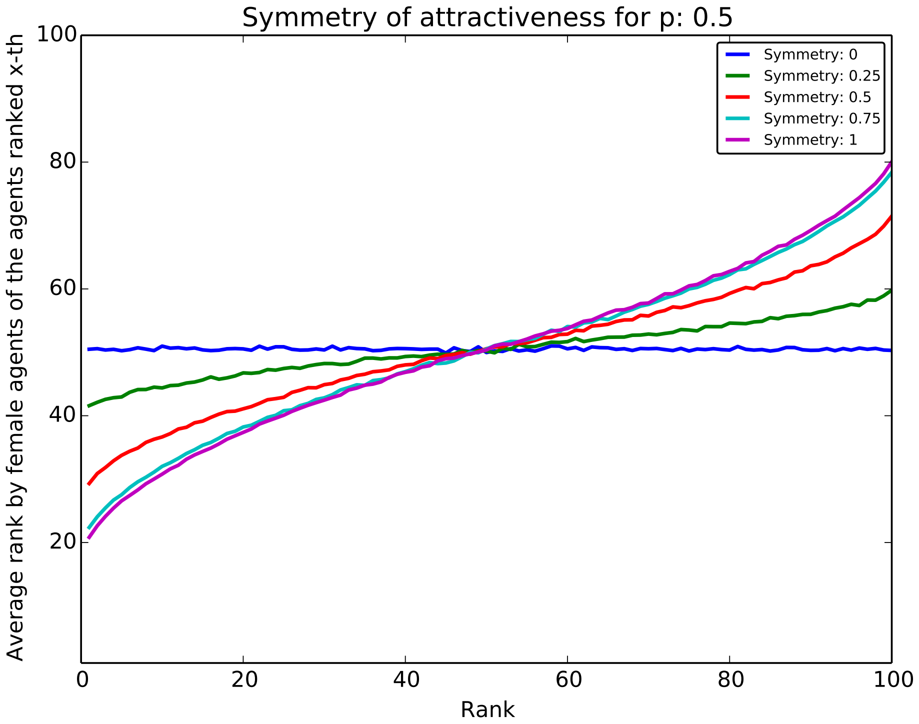


Fig. 6.9 The average rank the female agents assign to the male agents that rank them first, second and so on. Average over 200 instances for 100 male and 100 female agents for different values of s with $p = 0.5$

Simulations with four-dimensional preferences

When we run simulations with four-dimensional weighted preferences for 80 daterounds with 100 male and 100 female agents we find that we obtain matchings with on average 90% of agents in a relationship with an average payoff of 5.2 and in which agents on average form a blocking pair with 9.7% of the agents of the other side of the market.

Attractiveness estimate

With four-dimensional weighted preferences we find that agents have some inherent attractiveness, since it is based on their scores on the four dimensions of attractiveness. Using these scores we can give agents some awareness of their attractiveness. An agent m_k has the attractiveness estimate $e_{m_k} = \frac{d_{tot}}{n_d}$ where $d_{tot} = \sum_{i=1}^{j} m_k(d_i)$ and n_d is the number of dimensions. Note however that e_{m_i} only gives male agent m_k an estimate of its attractiveness, since there is some objectivity in the distribution of preferences. The attractiveness estimate e_{m_k} (and

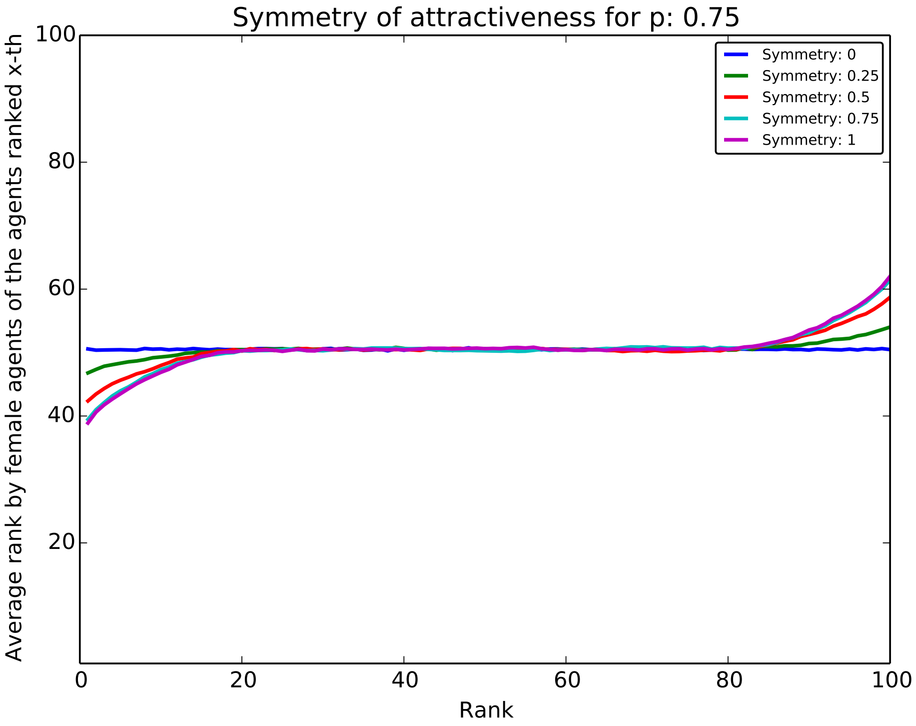


Fig. 6.10 The average rank the female agents assign to the male agents that rank them first, second and so on. Average over 200 instances for 100 male and 100 female agents for different values of s with $p = 0.75$

e_{f_k} for female agent f_k) will be of use when we insert a cost of being in a relationship and a pickiness for starting a relationship in our model.

6.4 Inserting pickiness in accepting potential partners

In our current model agents all adhere to the simple rule of saying “yes” to any partner when single and trading up when a better partner comes along, called the “better response dynamics" by Ackermann et al. [1]. In the type of real-world matching markets we want to model we however find that agents often behave differently than those rules would prescribe. We take people in search of a romantic partner as a matching market we want to model. We now find some results in our model that are not in line with how people seem to behave in the real world. In our mode,l all agents always are willing to start a relationship with any person they meet when single. We therefore find that all agents directly start a relationship in the first round. In real life it is however not the case that a single person would start a relationship with anyone they meet, not even if they are looking for a partner. It seems to be

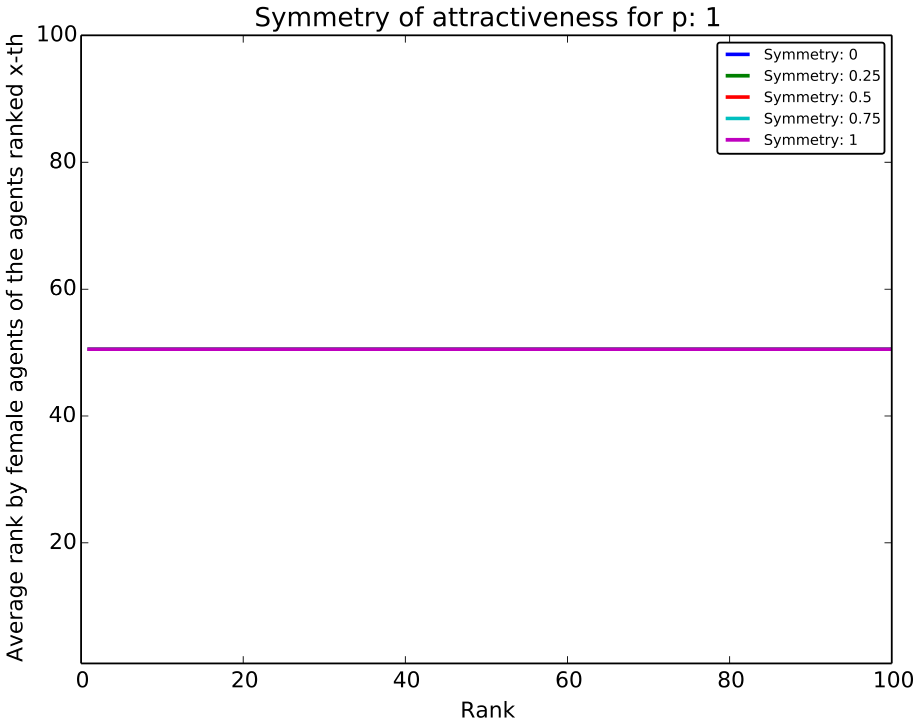


Fig. 6.11 The average rank the female agents assign to the male agents that rank them first, second and so on. Average over 200 instances for 100 male and 100 female agents for different values of s with $p = 1$

the case that there is some pickiness in starting a relationship with a potential partner when single. Pickiness can be seen as the lower bound of $u_m(m_i, f_j)$ and $u_f(m_i, f_j)$ that is needed to be obtained before m_i and f_j would start a relationship with one another. Note that all agents have the same pickiness.

6.4.1 Inserting a cost of being in a relationship

We assume that this pickiness results from the fact that there is a cost for being in a relationship; agents in a relationship meet less potential partners than they would if they were single. This means that saying "no" to a potential partner that is not very much preferred by an agent when single, could be a good decision. This is the case because it increases the chance that this agent meets a potential partner that is preferred more later on.

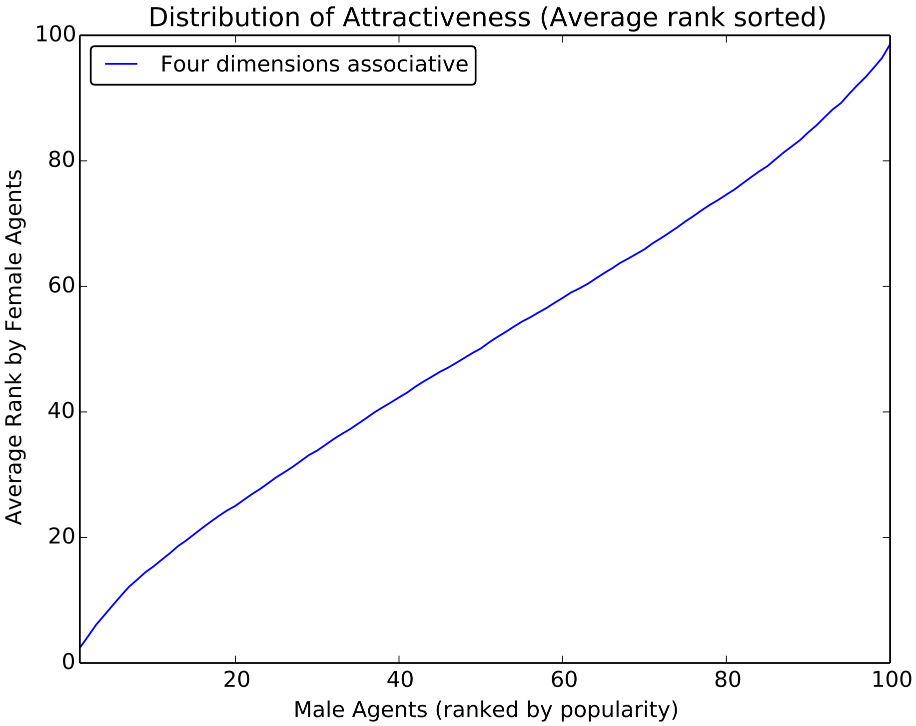


Fig. 6.12 The average ranks of agents by the agents of the other side of the market per rank, from most popular to least popular over 200 instances for 100 male and 100 female agents and four-dimensional weighted preferences

When we will insert a cost of being in a relationship, by having agents meet less potential partners when they are in one, we give agents interesting incentives to not pair up with just any agent during the simulations. We introduce a relationship cost factor $r \in \mathcal{N}$, which makes a male agent m_i and a female agent f_j that are in a relationship r times less likely to be part of D_m and D_f which means that they are r times less likely to "date" new agents and thus have the possibility to find a better partner every dateround that they are in a relationship. For both sides of the market we count every round how many agents are in a relationship (rm for male agents and rf for female agents) and how many are single (sm for male agents and sf for female agents). For the number of agents num_l in D_f and D_m at dateround l, and thus the number of dates we now have $num_l = min(sm + \frac{rm}{r}, sf + \frac{rf}{r})$. We then construct a list of all male agents and a list of all female agents to which we add $r-1$ times all single males and single females respectively. We randomly shuffle this list and we let the first members of the male and the female list date. After each date we remove all occurrences of the dating pair from this list, to make sure they do not date twice in a dateround, before having the new first male and female agent date. This way agents meet each other randomly and agents in

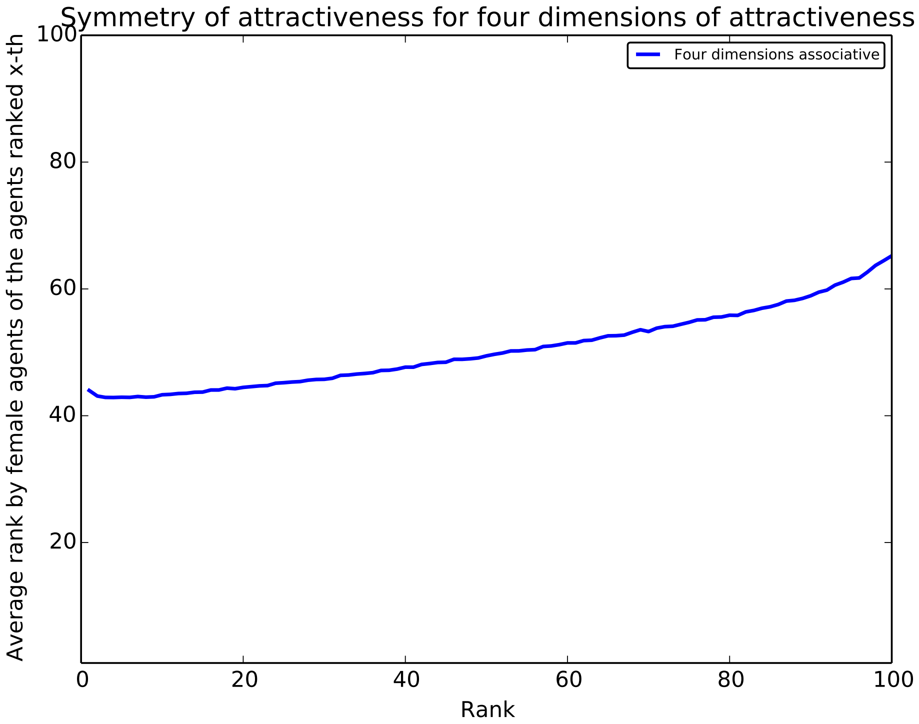


Fig. 6.13 The average rank the female agents assign to the male agents that rank them first, second and so on. Average over 200 instances for 100 male and 100 female agents for four-dimensional weighted preferences

a relationship are *r* times less likely to have a date than single agents. After inserting this cost of a relationship it is of course no longer the case that all agents date another agent every dateround. Note that by introducing pickiness we can no longer guarantee that our model converges towards the stable matching obtained in the Deferred Acceptance Algorithm. However if we take into account the fact that agents will only match with agents that score above a certain treshold, we could incorporate this treshold in the Deferred Acceptance Algorithm and find stable matchings in which not all agents have to be matched. Since the proof of Diamantoudi et al. [4] also holds for partial preference, we have that our model will eventually converge stable matchings (that might be incomplete) when we have partial preferences.

6.4.2 Dynamic pickiness

Agents seek to maximize the preference score they assign to the agent they are in a relationship with at the end of a simulation. Even with positive value for r, it therefore seems trivial that agents would obtain higher utilities by accepting any agent when single in the last dateround, since there is no possibility to find a better partner after this round. For earlier daterounds however it could be better for agents to not just start a new relationship with any agent, since the relationship cost factor r lowers the chances for an agent to find a better partner when in a relationship. The earlier in the simulations the better it seems to be for agents to be picky in starting a relationship when single. We therefore want agents to have a pickiness that varies during the simulation. Although there are endless possibilities for defining the pickiness of agents we reduce this dynamic pickiness to functions with only two parameters. We know that at the end of the simulation all agents are best off by accepting any partner. It is also reasonable to expect that agents will become less picky during the simulation (especially in case all agents have the same pickiness). The two parameters we thus introduce are $sp \in \mathscr{R}$, the starting pickiness, and $q \in \mathscr{N}$ the exponent by which this pickiness decreases. For every agent we thus have that the dynamic pickiness dp_l at dateround l out of k daterounds is defined the following way: $dp_l = sp - sp * (\frac{l}{k})^q$ with $0 \leq sp \leq 10$ and thus $0 \leq dp_l \leq 10$. We thus obtain the following update rules with dynamic pickiness: If $\{m_i, f_j\} \in E$ of G_l then agents m_i, currently in a relationship with f_p, and f_j currently in a relationship with m_p, date at dateround l and if $u_m(m_i, f_j) > u_m(m_i, f_p)$ and $u_m(m_i, f_j) > dp_l$ and $u_f(m_i, f_j) > u_f(m_p, f_j)$ and $u_f(m_i, f_j) > dp_l$, where $P(f_j, l-1) = m_p$, $P(m_p, l-1) = f_j$, $P(m_i, l-1) = f_p$ and $P(f_p, l-1) = m_i$, then $P(m_i, l) = f_j$, $P(f_j, l) = m_i$, $\neg P(m_i, l) = f_p$, $\neg P(f_p, l) = m_i$, $\neg P(m_p, l) = f_j$, $\neg P(f_j, l) = m_p$. Note that that in our model agent's behaviour is not influenced by the moment a dateround took place during a simulation. With dynamic pickiness we extend our model, such that agents behave differently at different moments during the simulation.

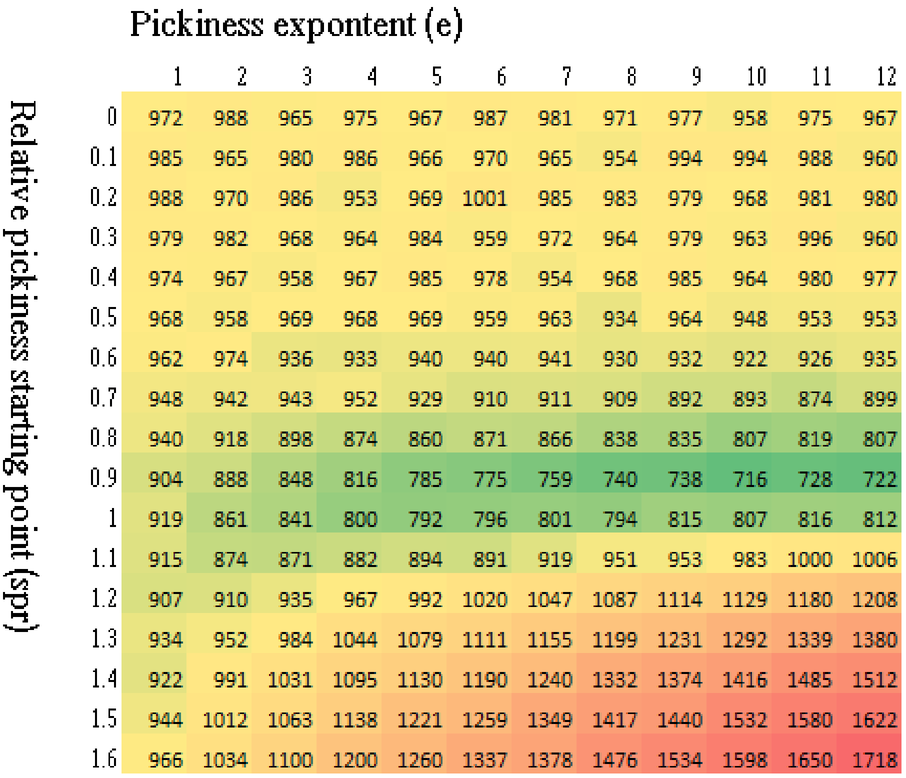
6.4.3 Relative dynamic pickiness

When there is some objectivity involved, like in the multi-dimensional weighted model, agents who are relatively popular are more likely to be proposed to. In that case it seems to make more sense to have pickiness dependent of the popularity of an agent. We therefore introduce relative dynamic pickiness at dateround 1 pr_l as a certain percentage of the estimated attractiveness of an agent. Note that we do need agents to be aware of their attractiveness in some sense to be able to have relative pickiness. We now have that for every agent the relative dynamic pickiness at dateround l out of k daterounds is $pr_l = spr - spr * (\frac{l}{k})^q$ with $0 \leq spr \leq 1$ and thus $0 \leq pr_l \leq 1$. This relative dynamic pickiness is the percentage of their own attractiveness estimate an agent has as a treshold for starting a new relationship. For the four-dimensional weighted model we introduced e_{m_i} and e_{f_j} as the attractiveness estimates of male agent m_i and female agent f_j, based on the average of their scores on the four dimensions. With relative dynamic pickiness we obtain the following update rules: If $\{m_i, f_j\} \in E$ of G_l then agents m_i and f_j date at dateround l. If $u_m(m_i, f_j) > u_m(m_i, f_p)$ and $u_m(m_i, f_j) > (pr_l * e_{m_i})$ and $u_f(m_i, f_j) > u_f(m_p, f_j)$ and $u_f(m_i, f_j) > (pr_l * e_{f_j})$, where $P(f_j, l-1) = m_p$, $P(m_p, l-1) = f_j$, $P(m_i, l-1) = f_p$ and $P(f_p, l-1) = m_i$, then $P(m_i, l) = f_j$, $P(f_j, l) = m_i$, $\neg P(m_i, l) = f_p$, $\neg P(f_p, l) = m_i$, $\neg P(m_p, l) = f_j$, $\neg P(f_j, l) = m_p$. We thus will have that agents only start a relationship with an agent, if they both find this agent more attractive than their current partner and if they assign a utility to being in a relationship with this potential partner that is higher than a certain percentage of their attractiveness estimate. Here the attractiveness estimate is the average of the scores they obtain on the four dimensions of attractiveness. This pickiness percentage is based on the pickiness starting point $0 \leq spr \leq 1$, the exponent by which the pickiness decreases during the simulation q, and the dateround of the simulation. Note that since pickiness decreases during the simulations, agents never leave their partner because of pickiness. Pickiness in our model thus only affects agents when they are single.

6.5 Simulations with relative dynamic pickiness with multi-dimensional preferences

To simulate a real-world two-sided matching market we need to find an appropriate value for the relationship cost factor r. For now we will just set r equal to 3. We now examine the effects of different relative dynamic pickiness starting points spr and different pickiness exponents q on the stability, payoffs and the number of agents in a relationship at the end of a simulation. We will use four-dimensional weighted preferences.

We run simulations with 100 male and 100 female agents for 80 daterounds with relationship cost factor $r = 3$ and with relative dynamic pickiness. We let values of the relative pickiness starting point spr range from 0 to 160% with steps of 10%. We let the values of the pickiness exponent q increase from 1 (linear decrease of pickiness) to 12 with steps of 1. Just as we did for p and s in the previous chapter we run simulations for every possible



Pickiness expontent (e)

Relative pickiness starting point (spr)

	1	2	3	4	5	6	7	8	9	10	11	12
0	972	988	965	975	967	987	981	971	977	958	975	967
0.1	985	965	980	986	966	970	965	954	994	994	988	960
0.2	988	970	986	953	969	1001	985	983	979	968	981	980
0.3	979	982	968	964	984	959	972	964	979	963	996	960
0.4	974	967	958	967	985	978	954	968	985	964	980	977
0.5	968	958	969	968	969	959	963	934	964	948	953	953
0.6	962	974	936	933	940	940	941	930	932	922	926	935
0.7	948	942	943	952	929	910	911	909	892	893	874	899
0.8	940	918	898	874	860	871	866	838	835	807	819	807
0.9	904	888	848	816	785	775	759	740	738	716	728	722
1	919	861	841	800	792	796	801	794	815	807	816	812
1.1	915	874	871	882	894	891	919	951	953	983	1000	1006
1.2	907	910	935	967	992	1020	1047	1087	1114	1129	1180	1208
1.3	934	952	984	1044	1079	1111	1155	1199	1231	1292	1339	1380
1.4	922	991	1031	1095	1130	1190	1240	1332	1374	1416	1485	1512
1.5	944	1012	1063	1138	1221	1259	1349	1417	1440	1532	1580	1622
1.6	966	1034	1100	1200	1260	1337	1378	1476	1534	1598	1650	1718

Fig. 6.14 Average number of blocking pairs with four-dimensional weighted preferences for different relative dynamic pickiness settings after 80 daterounds over 200 instances for 100 male and 100 female agents

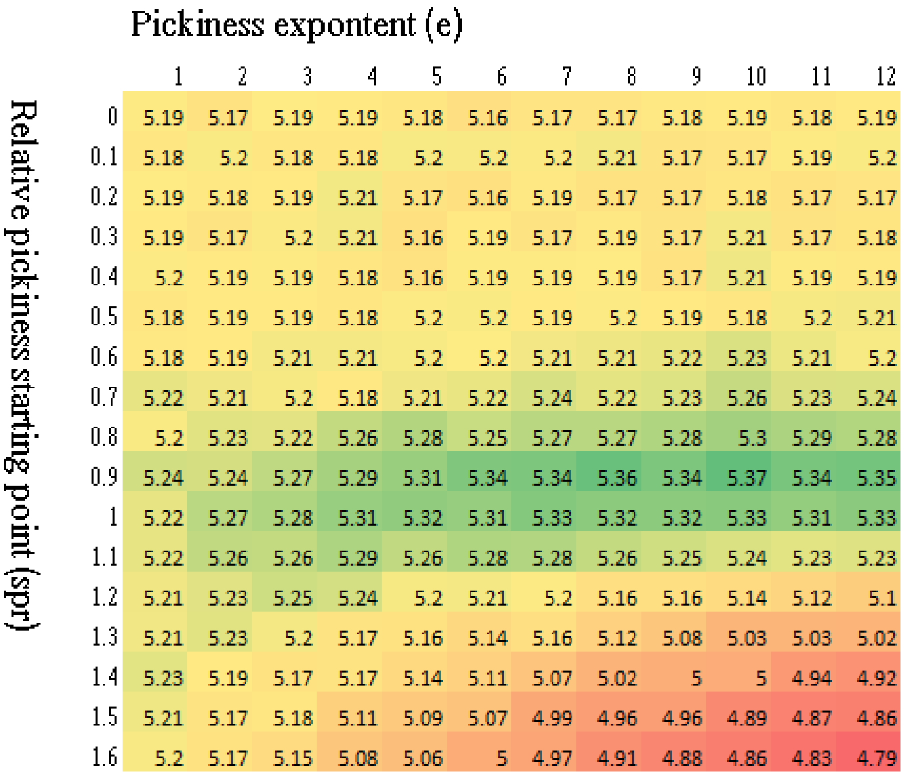
combination of the values of *spr* and *q*.

We find that the resulting matchings after 80 daterounds have the most agents in a relationship with 91% (Figure 6.16), are most stable with 716 blocking pairs (Figure 6.14) and yield the highest payoff with 5.37 (Figure 6.14) for pickiness starting point $spr = 0.9$ and pickiness exponent $q = 10$.

In (Figure 6.17) we see the relative dynamic pickiness during the simulation plotted for $spr = 0.9$ and $q = 10$, which yielded optimal outcomes for 100 male and 100 female agents for 80 dateround simulations.

6.6 Value of our model

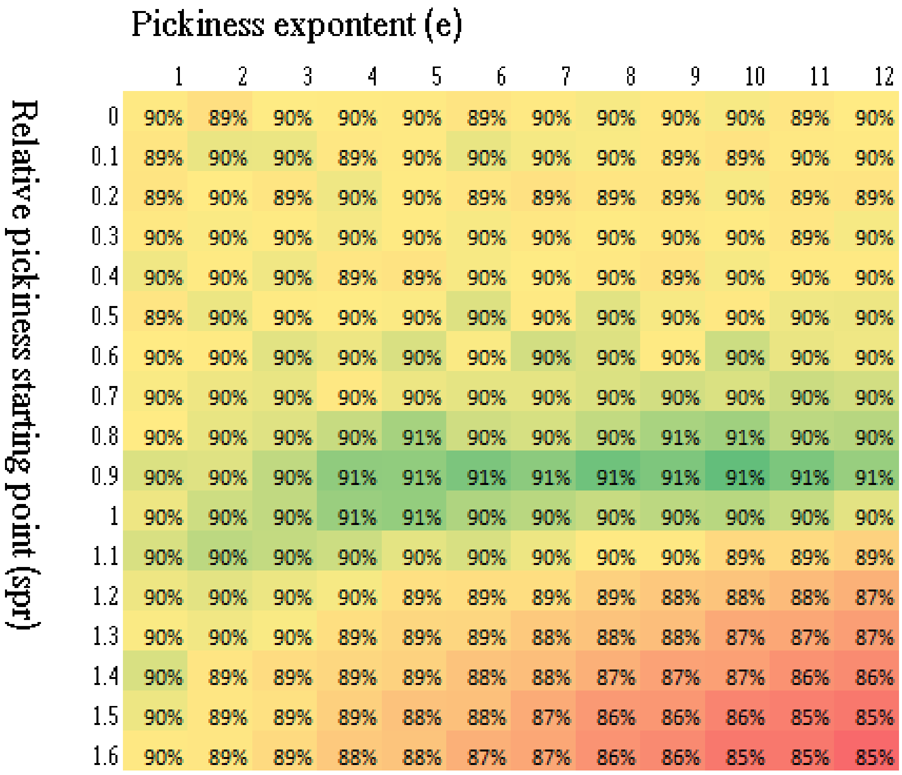
Extensions of our model could possibly be of great value for better understanding and improving real-world two-sided matching markets. The first step in using our model to



	1	2	3	4	5	6	7	8	9	10	11	12
0	5.19	5.17	5.19	5.19	5.18	5.16	5.17	5.17	5.18	5.19	5.18	5.19
0.1	5.18	5.2	5.18	5.18	5.2	5.2	5.2	5.21	5.17	5.17	5.19	5.2
0.2	5.19	5.18	5.19	5.21	5.17	5.16	5.19	5.17	5.17	5.18	5.17	5.17
0.3	5.19	5.17	5.2	5.21	5.16	5.19	5.17	5.19	5.17	5.21	5.17	5.18
0.4	5.2	5.19	5.19	5.18	5.16	5.19	5.19	5.19	5.17	5.21	5.19	5.19
0.5	5.18	5.19	5.19	5.18	5.2	5.2	5.19	5.2	5.19	5.18	5.2	5.21
0.6	5.18	5.19	5.21	5.21	5.2	5.2	5.21	5.21	5.22	5.23	5.21	5.2
0.7	5.22	5.21	5.2	5.18	5.21	5.22	5.24	5.22	5.23	5.26	5.23	5.24
0.8	5.2	5.23	5.22	5.26	5.28	5.25	5.27	5.27	5.28	5.3	5.29	5.28
0.9	5.24	5.24	5.27	5.29	5.31	5.34	5.34	5.36	5.34	5.37	5.34	5.35
1	5.22	5.27	5.28	5.31	5.32	5.31	5.33	5.32	5.32	5.33	5.31	5.33
1.1	5.22	5.26	5.26	5.29	5.26	5.28	5.28	5.26	5.25	5.24	5.23	5.23
1.2	5.21	5.23	5.25	5.24	5.2	5.21	5.2	5.16	5.16	5.14	5.12	5.1
1.3	5.21	5.23	5.2	5.17	5.16	5.14	5.16	5.12	5.08	5.03	5.03	5.02
1.4	5.23	5.19	5.17	5.17	5.14	5.11	5.07	5.02	5	5	4.94	4.92
1.5	5.21	5.17	5.18	5.11	5.09	5.07	4.99	4.96	4.96	4.89	4.87	4.86
1.6	5.2	5.17	5.15	5.08	5.06	5	4.97	4.91	4.88	4.86	4.83	4.79

Fig. 6.15 Average payoffs with four-dimensional weighted preferences for different relative dynamic pickiness settings after 80 daterounds over 200 instances for 100 male and 100 female agents

simulate real-world matching markets, is to use empirical data to choose the right parameter settings that influence the preferences and the size of the market. Following, we need to find the right pickiness settings. We validate these settings by checking whether agents in our model indeed behave like the agents in the two-sided matching market we want to simulate. We could validate this by examining whether the number of agents that is matched and the distribution of switches during the simulation is similar in our model as in the real-world market. Our model could for example be used to better understand and improve real-world two-sided markets like the job market. Imagine we set the parameters such that agents behave just like agents in the job market, in the sense that the number of people that have a job and the distribution of job switches is similar. We can now investigate what would happen if we change certain parameter settings. We could for example investigate what would happen if we would increase the number of jobs or applicants available. We could also investigate what effect making a small number of jobs (or employees) very popular, has on the number of people that have a job and the overall satisfaction of people that have a job. We could also



Pickiness expontent (e)

Relative pickiness starting point (spr)

	1	2	3	4	5	6	7	8	9	10	11	12
0	90%	89%	90%	90%	90%	89%	90%	90%	90%	90%	89%	90%
0.1	89%	90%	90%	89%	90%	90%	90%	90%	89%	89%	90%	90%
0.2	89%	90%	89%	90%	90%	89%	89%	89%	89%	90%	89%	89%
0.3	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%	89%	90%
0.4	90%	90%	90%	89%	89%	90%	90%	90%	89%	90%	90%	90%
0.5	89%	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%
0.6	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%
0.7	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%	90%
0.8	90%	90%	90%	90%	91%	90%	90%	90%	91%	91%	90%	90%
0.9	90%	90%	90%	91%	91%	91%	91%	91%	91%	91%	91%	91%
1	90%	90%	90%	91%	91%	90%	90%	90%	90%	90%	90%	90%
1.1	90%	90%	90%	90%	90%	90%	90%	90%	90%	89%	89%	89%
1.2	90%	90%	90%	90%	89%	89%	89%	89%	88%	88%	88%	87%
1.3	90%	90%	90%	89%	89%	89%	88%	88%	88%	87%	87%	87%
1.4	90%	89%	89%	89%	89%	88%	88%	87%	87%	87%	86%	86%
1.5	90%	89%	89%	89%	88%	88%	87%	86%	86%	86%	85%	85%
1.6	90%	89%	89%	88%	88%	87%	87%	86%	86%	85%	85%	85%

Fig. 6.16 Average percentage of agents in a relationship with four-dimensional weighted preferences for different relative dynamic pickiness settings after 80 daterounds over 200 instances for 100 male and 100 female agents

simulate policy changes, where people who have a job obtain more possibilities to look for a better job.

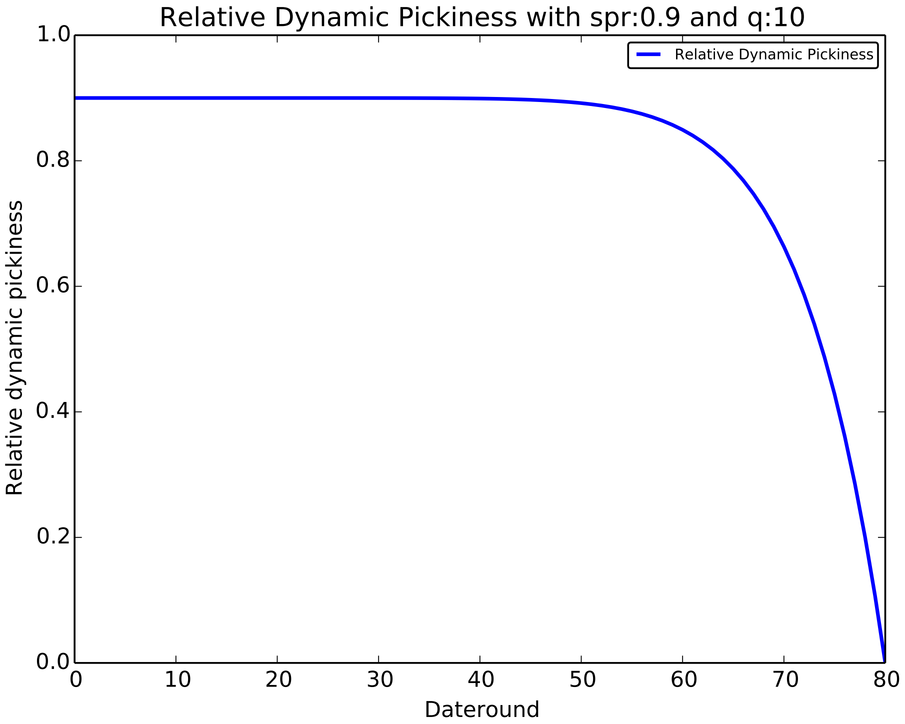


Fig. 6.17 The relative dynamic pickiness of agents during simulations consisting of 80 daterounds when $sp = 0.9$ and $q = 10$

Chapter 7

Discussion

Our model ultimately converges towards a stable matching, and does so with on average higher average payoffs (and lower average ranks) than would be the case on average in the Deferred Acceptance Algorithm (once with male agents as proposing party and once with female agents as proposing party) or for a randomly selected stable matching. Another benefit of our model is that it converges towards stable matchings that are more egalitarian than those obtained in the Deferred Acceptance Algorithm. In this chapter we raise some points for discussion that might arise when we use our model for solving instances of the Stable Marriage Problem. Furthermore we discuss some issues that might come with using, and further extending, our model for simulating real-world two-sided matching markets.

7.1 Discussion on using our model for solving the Stable Marriage Problem

There seem to be other methods for finding stable matchings for instances of the Stable Marriage Problem that we did not compare with our model. As further mentioned in Chapter 9 it might be the case that having randomly selected pairs of agents meet each other sequentially might result in higher payoffs or in any other way more wanted results. In Chapter 5 we investigated how many times stable matchings in which the average rank agents assign to their partner is minimal. Irving et al. devised an algorithm, which for any instance of the Stable Marriage Problem finds this stable matching in polynomial time [7]. It might be the case that such efficient algorithms also exist for finding the stable matchings that yield the highest average payoff or that are most egalitarian.

Although it enables us to pick the most wanted stable matching, checking the possibly very large number of possible matchings ($n!$ for n male and n female agents, and $\frac{n!}{(n-m)!}$ if we have unequal number of male and female agents and m is the size of the smallest side of the market and n the size of the largest side of the market) takes an enormous amount of time, already for a moderate number of agents. On the other hand, the Deferred Acceptance Algorithm is a very efficient procedure for finding a stable matching, that are optimal for one side of the market. In the same way the algorithm of Irving et al. very efficiently gives us the stable matching with the lowest average rank [7]. Due to the stochastic elements it takes much longer in our model to obtain stable matchings than is the case in these algorithms. Furthermore we need to check actively whether a stable matching is obtained, while this is not needed in these algorithms. Which method of finding stable matchings is most optimal seems to be dependent on the resources, the market and what characteristics in the resulting matchings are more wanted. Algorithms can be very useful to efficiently find stable matchings for which one wanted characteristic (like the lowest average ranking) is guaranteed. When we have small markets or enormous computing time it might be best to check all possible matchings and pick the most wanted stable one. Although it takes some more time to obtain the stable matching than in the algorithms of Shapley and Gale and Irving et al., our model might be most suitable for finding the most wanted stable matching when multiple characteristics are wanted in the resulting matchings.

7.2 Discussion on using our model for simulating real-world two-sided markets

Our model still seems to be highly stylized, and possibly not suitable yet for simulating complex real-world markets. In this section follow some of those points, for which further research is suggested in Chapter 9.

Unbalanced markets

Although our model allows for it, we have not investigated markets with different number of male and female agents. From Ashlagi et al. we however know that in unbalanced instances of the Stable Marriage Problem (thus with different sizes of the two sides of the market) the number of stable matchings drastically decrease [2]. They furthermore show that the resulting stable matching for unbalanced instances of the Stable Marriage Problem strongly favor the small side of the market. This seems to be an important point of discussion since we cannot expect many real-world two-sided matching markets to be balanced.

Daterounds need authority

Our model differs from most other stochastic models since agents meet maximally one agent from the other side of the market in consecutive daterounds. Next to the fact that it complicated the proof that our model converges towards stable matchings, it might also make our model a little less suitable for modeling two-sided markets for which no central authority exists to clear the market. This is the case since some sort of authority seems needed to make sure agents only meet maximally one other agent every dateround.

Potential of model to represent real-world markets

Our model might be too stylized for modeling some real-world markets since the utility of agents is only based on the partner they are matched with after the last dateround. Furthermore we have all agents use the same strategy, which might to simplistic to simulate the complex behaviour of agents in most real-world two-sided markets. In Chapter 9 we will elaborate more on these points and suggest further extensions of our model to tackle these issues.

7.3 Discussion on research method

Focus on averages

In our model we have been focusing on average results. It could however be the case that the payoffs or the number of blocking pairs for agents within an instance varied a lot, and in different ways for different parameter settings. It could also be the case, that for different instances of the Stable Marriage Problem very different results were obtained. It could furthermore be the case that there for example are certain correlations between results found in the Deferred Acceptance Algorithm and in our stochastic model for specific instances. Other than objectivity and symmetry we did not investigate the effects of other characteristics of the preferences of the agents, like for example the existence of cycles.

Chapter 8

Conclusion

We have constructed a stochastic model for simulating real-world two-sided matching markets. In our model we run simulations for solving instances of the Stable Marriage Problem, and compare the results to the optimal stable matchings resulting from the Deferred Acceptance Algorithm devised by Shapley and Gale [6]. As opposed to other stochastic matching methods, in our model all agents randomly meet one agent of the other side of the market every dateround. Simulations in our model consist of multiple daterounds. When a pair of agents forms a blocking pair, and thus both agents prefer each other over the agent they are currently matched with, they start a relationship. In our model agents seek to maximize the payoff they assign to the agent they are matched with after the final dateround.

Based on a proof of Diamantoudi et al. [4] we have shown that our model will eventually always converge towards a stable matching. We then randomly generated different instances of the Stable Marriage Problem, with two parameters that influenced the randomly generated preferences. The first is the preference agreement factor p and the second is the symmetry factor s. We investigated the effects of changing the values of p and s and found that it takes longest to obtain a stable matching for instances of the Stable Marriage Problem with completely random (and thus subjective and asymmetric) preferences generated by setting $p = 0$ and $s = 0$. We found that in those cases it is most likely that our model converges to a stable matching that is different from the stable matchings resulting from the Deferred Acceptance Algorithm. We furthermore found that for all parameter settings, our model on average obtained higher payoffs than the stable matchings resulting from the Deferred Acceptance Algorithm, but with the highest difference for subjective $(p = 0)$ preferences that were not symmetric $(s = 0)$. We then generated 450 instances of the Stable Marriage Problem with $p = 0$ and $s = 0$ and 10 male and 10 female agents. For these instances we checked all matchings and found that per instance there were on average 3.02 stable matchings. We then ran 500 simulations in our model for every instance and found that our model on average yields both higher payoffs and better average ranks than would be the case when we would either run the Deferred Acceptance Algorithm or when we would randomly pick a stable matching. The good results of our model can be accounted to the fact that suboptimal matchings exist that yield better results than the optimal stable matchings are relatively more likely to be obtained in our model. Furthermore we find that our model yields stable matchings that are more egalitarian in the sense that the differences between the two sides of the market are on average lower than in a randomly selected stable matching and lower than in the optimal stable matchings resulting from the Deferred Acceptance Algorithm.

Thereafter, we extended our model such that it can be used for simulating real-world two-sided matching markets by introducing a more advanced method for investigating preference agreement and symmetry in attractiveness. We also introduced a cost for being in a relationship and pickiness, a lower threshold for starting a relationship when single. We furthermore introduced a multi-dimensional model for generating preferences which we used for investigating the optimal relative dynamic pickiness, where agents let their threshold for starting a relationship be dependent of their estimated attractiveness and the progress of the simulation.

Chapter 9

Further research

9.1 Simulations

Unequal number of agents

As mentioned in Chapter it is very important to investigate what happens when we have different numbers of male and female agents. Especially when using our model to simulate real-world two-sided markets it can be very interesting to investigate what happens when we have different number of male and female agents.

Partial preferences

Our model allows agents to prefer being single over being matched with specific agents. We could let agents assign a baseline utility to being single, and investigate the effects of changing this baseline on the utilities that are obtained and the time it takes to converge to a stable matching. As a reference for starting a new relationship or staying with their current partner agents could use absolute values as lower payoff limits for starting a relationship. For example an agent would only start a relationship with an agent when they assign a score to them higher than 5. Note that we did extend our model with pickiness, such that agents did not just started a relationship with any agent they met during the relationship. We however used this absolute pickiness as a heuristic agents could use for obtaining better payoffs when there was not enough time to converge to a stable matching. Here we assumed that agents wanted to maximize the payoff they assigned to their partner at the last dateround, which was always higher than the payoff they assigned to being single. We extended pickiness directly to dynamic pickiness, and thus did not do simulations in our model and the Deferred Acceptance Algorithm with partial preferences.

Comparing our model to other models of stochastic matching

As mentioned in Chapter 7 it might very well be the case that other stochastic matching models converge to stable matchings with higher payoffs (or in any other sense more wanted results) and possibly do so in less time. To find out whether our model could really be valuable as a matching procedure we need to investigate the difference between the results of our model and other methods of matching. Comparing our model to the uniform random process, where pairs of agents randomly meet each other sequentially and satisfy a blocking pair when they form one, seems most important, since this method is very similar to our model but might need less computing time.

Cumulative utility

In our model the utility of an agent was equal to the payoff this agent assigned to being in a relationship with the agent he was paired with at the end of the simulation at dateround k. It could however also be interesting to investigate cumulative (or average) utility where agents m_j and f_j have payoffs in simulations consisting of k daterounds expressed by: $\pi_{m_j} = \sum_{i=1}^{k} u_m(m_j, P(m_j, i))$ and $\pi_{f_j} = \sum_{i=1}^{k} u_f(P(f_j, i), f_j)$. This could for example be interesting when we want to model real-world markets where agents also seek to be matched with a preferred agent during the matching period. If we would for example want to model the job market, where people want to have a job they like during all of their career, but keep looking for better jobs, a simulation would model a persons career. Here agents want to have a job that is as good as possible for the longest amount of time.

Pickiness with upper bound and sweet spot

It could also be better for agents to say "no" to starting a new relationship with a new agent because this new agent could be too attractive and therefore be likely to leave later on in the simulation (note that this is only the case when there is some objectivity in preferences). This means we should also introduce an upper limit of attractiveness of a partner, and a sweet spot. Agents would then not start a relationship with an agent when they score below this upper limit and agents are seeking a partner closest to their sweet spot value. Just as for the pickiness in our model we could have this upper bound and sweet spot change during the simulation and relative to their attractiveness estimate.

Heterogeneity in preferences and strategies

We could adjust our model such that different sides of the market behave in different ways. For example in the case of people looking for a romantic partner we could imagine that males have a distribution of attractiveness different than that of the females. It could also very well be the case that different side of the markets have different pickiness. Also within one side of the market it could very well be the case that different agents employ different strategies in the sense of pickiness. We experimented with an evolutionary competition where agents start with randomly generated pickiness strategies, and successful strategies were more likely to be present during the next simulation. Since the pickiness strategy of an agent influences the outcomes of other agents, this might lead to interesting results that are definitely worth further investigating.

Empirical Data

If we want to use our model for simulating real-world markets, it is in the first place essential to use empirical data to set the right parameter settings in our model. Furthermore, it is crucial to compare the results obtained in our model with those obtained in real-world markets. Although we fully acknowledge its importance for further research, we decided not to focus on using empirical data at this early stage.

9.2 Formal proofs of important results

We have shown with extensive simulations that both suboptimal stable matchings that yield higher average payoffs and suboptimal stable matchings that yield more egalitarian payoffs are more likely to be obtained in our model. It would be very valuable and interesting to formally prove that this is indeed the case.

References

[1] Ackermann, H., Goldberg, P. W., Mirrokni, V. S., Röglin, H., and Vöcking, B. (2011). Uncoordinated two-sided matching markets. *SIAM J. Comput.*, pages 92–106.

[2] Ashlagi, I., Kanoria, Y., and Leshno, J. D. (2015). Unbalanced random matching markets: The stark effect of competition. *Unpublished mimeo.*

[3] Biro, P. and Norman, G. (20013). Analysis of stochastic matching markets. *International Journal of Game Theory*, 42(4):1021–1040.

[4] Diamantoudi, E., Miyagawa, E., and Xue, L. (2004). Random paths to stability in the roommate problem. *Games and Economic Behavior*, 48(1):18 – 28.

[5] Ferguson, T. S. (1989). Who solved the secretary problem? *Statistical Science*, 4(3):294–296.

[6] Gale, D. and Shapley, L. (1962). College admissions and the stability of marriage. *American Mathematical Monthly*, 69(3):9–14.

[7] Irving, R., Leather, P., and Gusfield, D. (1987). An efficient algorithm for the optimal stable marriage. *Journal of the ACM*, 34:532–544.

[8] Knuth, D. (1976). *Marriages stables et leurs relations avec d'autres problèmes combinatoires.* Les Presses de l'Université de Montréal, Montreal.

[9] Lindley, D. (1961). Dynamic programming and decision theory. *Appl. Statist.*, 10:39–51.

[10] Roth, A. (1982). The economics of matching: Stability and incentives. *Mathematics of Operations Research*, 7:617–628.

[11] Roth, A. (1984). The evolution of the labor market for medical interns and residents: A case study in game theory. *Journal of Political Economy*, 92:991–1016.

[12] Roth, A., Sönmez, T., and Ünver, M. U. (2004). Kidney exchange. *Quarterly Journal of Economics*, 119(2):457–488.

[13] Roth, A. and VandeVate, J. (1990). Random paths to stability in two-sided matching. *Econometrica*, pages 1475–1480.

[14] Tamura, A. (1993). Transformation from arbitrary matchings to stable matchings. *Journal of Combinatorial Theory, Series A*, 62(2):310 – 323.

[15] Tan, J. and Su, W. (1993). On the divorce digraph of the stable marriage problem. *Unpublished manuscript.*