Coherence Preservation: A Threat to Probabilistic Measures of Coherence

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written by

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Abstract

This thesis proposes a new requirement that probabilistic measures of coherence should ideally satisfy. This requirement is called *'coherence preservation'*. Probabilistic measures of coherence build on the idea that coherence is the mutual support between elements of a set. Using the requirement of coherence preservation, one may reevaluate mainstream probabilistic coherence measures, and draw the conclusion that all these measures fail to capture certain aspects of our intuitive understanding of coherence.

We begin with a review of different probabilistic coherence measures. Next, we extend our survey with a proof for the non-existence of a truth-conducive coherence measure, and we discuss various follow-up attempts of saving coherence. By presenting the requirement of coherence preservation, it can be shown that in some cases, the degree of coherence of a set decreases when the set is extended with a proposition which confirms every element of the set. Based on this observation, we can show that attempts of saving coherence leads to counterintuitive results. One should therefore look for a different way of characterizing coherence, which better captures the non-quantitative aspect of this notion.

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Chapter 1

Introduction

Coherence is one of the most, if not the most, important notions in contemporary epistemology. With this notion, one can give accounts to a variety of issues in epistemology including epistemic justification, reliability of information sources, and confirmation of theories. Because of the numerous potential usages of coherence, philosophers have long been trying to gain a deep and thorough understanding of this perplexing notion, so as to provide a solid ground for further applications of it. The approach of characterizing coherence is to provide a probabilistic measure which allows one to calculate the degree of coherence of a set. Every specific way of measuring coherence formally represents a specific conception of coherence. If one can find a coherence measure which generates results that are in perfect accordance to our intuition, it can be taken as the proper probabilistic definition of coherence, which philosophers may develop applications for. The primary concern of this thesis is to show that mainstream probabilistic measures of coherence all violate a simple but crucial new requirement of coherence, and hence fail to correctly represent our ordinary understanding of coherence.

Early attempts to define coherence are made by Blanshard (1939), A.C. Ewing (1934), C.I. Lewis (1946) and Laurence BonJour (1985). By contemplating on the nature of coherence, these authors provide fine-grained conceptual analyses of coherence in a non-formal fashion, and apply this notion in different fields. The most prominent application of coherence is to explain the notion of epistemic justification. Instead of taking the concept of belief (satisfying certain properties) as the foundation of knowledge, some epistemologists suggest to give an account of epistemic justification in terms of coherence. This view is called *coherentism*. A belief is justified, as coherentists claim, if it is included in a coherent belief set, since every element in such a set supports and is supported by some other elements. This claim sounds more plausible than other views concerning epistemic justification, and hence was widely accepted.

Several authors have investigated the criteria of truth-conduciveness of coherence. Here we see different camps: Klein and Warfield (1994, 1996) argue that the notion of coherence, understood in terms of probabilistic reasoning, is not a truth-conducive notion. This observation poses a serious threat to supporters of Coherentism. Since truthfulness is often considered as an essential ingredient of knowledge, if coherence is indeed not truth-conducive, it definitely cannot be used as a proper explanation of epistemic justification.

On the other hand, there are also attempts to show that there are measures which are truth conducive, epistemologists (Shogenji 1999, Olsson 2002, Fitelson 2003, Douven and Meijs 2007, Roche 2013) provide a variety of ways to measure coherence in terms of probability. If it can be shown by any of these measures that the more coherent a set is, the more likely the set is true, the notion of coherence could be saved from Klein and Warfield's criticism, and be accepted as a plausible account of epistemic justification.

In order to survey the question whether it is possible to find a truth-conducive coherence measure, Bovens and Hartmann (2003) construct a model of information gathering. By taking the reliability of information sources as a parameter, they prove that there does not exist any coherence measure which is truth-conducive. With the model, they derive the following result: Suppose there are two sets with different degrees of coherence, one of them may be more likely to be true given that the reliability of information sources is high, and less likely to be true when the reliability of information sources is low. Hence, the degree of coherence of a set is not positively correlated with its probability.

The above mentioned result of Bovens and Hartmann poses a serious threat to supporters of coherentism. Given that the primary function of coherence is to explain the nature of epistemic justification, if coherence, as represented by probabilistic measures, can never be truth-conducive, this notion becomes valueless. To save the notion of coherence and prove that it has some other usages, epistemologists provide applications of this notion, and claim that although coherence is not truth-conducive, it may still play an important role in contemporary epistemology.

An attempt made by Olsson and Schubert (2007) is to show that coherence, as characterized by Shogenji's measure, is a *reliability-conducive* notion. In a specific scenario, the coherence of a set of propositions is positively correlated with the reliability of sources providing these propositions. If a set is highly coherent, we can infer that the information sources of this set are highly reliable. Since the sources are reliable, this set of propositions is quite likely to be true.

Another attempt Dietrich and Moretti (2005) proposed is that coherence can be proved as a *confirmation-conducive* notion. If a set is highly coherent, a piece of evidence confirming an element of that set also confirms all other elements of that set. Moretti (2007) further proves the reverse that if a set is highly coherent and involves an element which confirms a proposition, other elements in the set also confirms that proposition. These attempts show that although coherence is not truth-conducive, it can be *indirectly* truth-conducive. Hence, it may still account for epistemic justification.

There are also some minor attempts of saving the notion of coherence. An important one made by Glass (2007) is that coherence can be taken as a way of ranking scientific explanations. Given a proposition, if one wants to compare the goodness of several competing explanations for that proposition, one may measure the degree of coherence between the proposition and its explanation, and rank the explanations according to their coherence with the proposition in question. Apart from showing that coherence is indirectly truth-conducive, Glass proves that coherence has pragmatic value in scientific practices,

Each attempt of saving the notion of coherence is based on a certain coherence measure, which reflects a specific understanding of coherence. If these proofs are correct, the notion of coherence can again account for epistemic justification and other related issues. However, all these measures violate the intuitive requirement of *coherence preservation* which states that for any set of propositions, when extended with a proposition confirming every element of it, the set should become more coherent. Since all mainstream coherence measures violate this requirement, they fail to capture our ordinary understanding of coherence. As a result, the notion that is studied by all these approaches is not the notion people commonly understand as coherence. Therefore, coherence may still be an valueless notion, and coherentism is again in great danger.

In the following chapters, I will first introduce the mainstream probabilistic definitions of coherence and briefly discuss if they correctly capture our ordinary understanding of coherence. After reviewing these coherence measures, I will explain the way Bovens and Hartmann derive the result that coherence in not truth-conducive, and go on to discuss various attempts of saving coherence. In the end, I will present the requirement of *coherence preservation*, which shows that most coherence measures fail to capture our intuitive understanding of coherence. This discovery indicates that certain features are still missing in the current approach. Hence we raise the question and ask epistemologists to reflect upon our coherence preservation requirement and to take it into account when proposing new measures of coherence.

Chapter 2

Measuring Coherence

2.1 The notion of coherence

Philosophers have been trying to clarify the intriguing nature of epistemic justification for ages. Since justification is traditionally regarded as a necessary condition for knowledge,[1] without an explicit explanation of how beliefs are justified, people cannot tell whether a belief could possibly be taken as knowledge. In order to characterize the nature of knowledge, a proper account of justification is called for.

A natural explanation is to say that a belief b is justified if it can be inferred, either by induction or deduction, from some other beliefs $b_1, ..., b_n$. With this explanation, we can further derive a requirement that in order to justify a belief b, all its justifiers $b_1, ..., b_n$ must be already justified. Without this requirement, we would have to accept the claim that a belief could sometimes be justified by a set of unjustified beliefs, which is intuitively unacceptable. Again, for $b_1, ..., b_n$ to be justified, there needs to be another set of beliefs $b'_1, ..., b'_m$ which justifies each $b_1, ..., b_n$. Following this line of thought, justification can be regarded as a tree-like structure. Each member in the structure is justified by its successors, and justifies its predecessors.

A question immediately follows from this picture: at which point does the chain of justification come to an end? If, for every justifying belief, we need another justified belief to justify it, the chain of justification would extend infinitely and become a vicious regress, which

[1] Recent study in *knowledge first epistemology* (Williamson 2000) suggests that the attempt to analyze knowledge is mistakenly oriented. Nevertheless, it does not undermine the current project. The primary concern here is to evaluate different formal definitions of coherence, and judge if any of them is appropriate. Although pursuit of a formal definition of coherence originates from the debate on epistemic justification, the notion of coherence, as epistemologists have shown, has its own value. Thus, the search of a proper formal definition of coherence can be separated from the discourse of epistemic justification.

is undesirable for epistemologists. If epistemic justification is a regress, it would be impossible for one to ascertain whether a belief is justified, for the chain of justification of that belief has not, and will not come to an end. That is, if this view is adopted, one can never eliminate the possibility that the chain involves an unjustified belief. Therefore, infinite regress cannot be a proper explanation of how the chain of epistemic justification ends.

There are two possible views concerning how justification could be done. One may either claim that the chain of justification stops at a certain point, or claim that the chain circles back to itself. To adopt the former claim, one will have to argue that the stopping points have certain special property, and hence need not be justified inferentially because of that special property. In other words, there needs to be certain kind of entities that can be taken as the *foundation* which justifies other beliefs but need not be justified. Since this view emphasizes the existence of a foundation of knowledge, it is called *foundationalism*.

Foundationalists have to answer two fundamental questions: What is the foundation of knowledge? How does the foundation correlate to, and further justify other beliefs? Some foundationalists suggest to take sensory experience as the foundation of epistemic justification, for it does not need to be justified by inference, and thus satisfies the requirement for being the foundation of knowledge. However, sensory experience is not propositional, which means that it is categorically different from beliefs people have. In other words, there is a conceptual gap between sensory experience and propositional beliefs. Without an explanation of how sensory experience interacts with beliefs, it remains unclear how can it be taken as the foundation of a belief system. To adopt foundationalism, one needs to either give a proper account of how sensory experience is connected to beliefs, or take some other entities (other than sensory experience) as the foundation of knowledge.

Apart from foundationalism, an alternative is to claim that the chain of justification circles back to itself. That is, beliefs in the chain are justified by some other beliefs in the same chain. All the beliefs form a system, where each member of the system ts supported by some members, and also supporting some other members. Adopting such view, both justifying and justified objects in the chain are beliefs. Thus, there is no conceptual gap between justifying and justified beliefs. This view is called *coherentism* of epistemic justification. Roche (2013) provides a sophisticated characterization of this view:

Definition 2.1.1. Circular Chain of Implication (CCI)

An agent's belief in p is justified only if:

1. The agent's belief in p is implied (deductively or inductively) by certain of her other beliefs, which themselves are implied by certain of her other beliefs, and so on.

2. The chain of evidential support circles back around at some point and does not continue *ad infinitum* with new belief after new belief.

By claiming that the chain of justification circles back to itself, coherentists provide an explanation of how the chain of justification ends. Compared with other possibilities, coherentism seems quite reasonable.

With Roche's explanation of coherentism, one might still ask: what is the nature of coherence? Claiming that elements of a coherent set support other elements merely provides us with a rough idea about what the notion of coherence really is. Without a thorough characterization of coherence, we are unable to ascertain whether coherentism, compared with foundationalism, is indeed a better explanation for epistemic justification.

An approach to gain a better understanding of the notion of coherence is to see how epistemologists measure the degree of coherence of a set. In the following sections, I will review several accounts of coherence, and focus on a variety of probabilistic measures of coherence that have been proposed. With a overview of these measures, one may have a clear idea of how coherence is characterized in terms of probability.

2.2 Traditional accounts of coherence

In *Idealism: A Critical Survey* (1934), A. C. Ewing provides the following account of coherence: A set is coherent if every belief in it *logically follows*[2] from all other beliefs in the set taken together, namely the conjunction of all other elements in the set. Consider the belief set $\{b_1, b_2, b_1 \wedge b_2\}$. Since $b_1 \wedge b_2$ follows from $\{b_1, b_2\}$, b_1 follows from $\{b_2, b_1 \wedge b_2\}$, b_2 follows from $\{b_1, b_1 \wedge b_2\}$, this set is coherent under Ewing's definition.

Ewing's definition of coherence is apparently too strong. We can have a coherent set of logically unconnected[3] beliefs.

Example 2.2.1. In F. Scott Fitzgerald's novel *The Great Gatsby*, the narrator and main character Nick Carraway has the following set of beliefs:

(b_1) Jay Gatsby has a mansion.

(b_2) Jay Gatsby has an enormous garden.

(b_3) Jay Gatsby has a gorgeous car.

[2] Although Ewing does not provide a clear definition of what 'logically follows' mean, we can infer from his examples that what Ewing has in mind is actually the entailment relation.

[3] Here 'logically unconnected' means that one belief cannot be deduced from another.

b_1, b_2 and b_3 are not logically connected, hence, the set $\{b_1, b_2, b_3\}$ is incoherent if we follow Ewing's definition. Intuitively, using our everyday understanding of coherence, this set seems to be coherent. All three beliefs indicate that Jay Gatsby is pretty wealthy. One can hence conclude that Ewing's definition of coherence violates our ordinary understanding of coherence for being too strict.

C. I. Lewis (1946) provides a different definition for coherence[4]. He claims that if a set $S = \{b_1, ..., b_n\}$ is coherent, then for any b_i which is an element of S, if all other elements in S are assumed as true, the probability of b_i raises. That is, the probability of b_i conditional on $S \backslash \{b_i\}$ is greater than the unconditional probability of b_i.[5] This definition of coherence has two significant advantages. First, it is not as strict as Ewing's definition, for the notion Lewis uses is *'raising probability'*, rather than the much stronger 'logically follows'. Second, explain with probability allows people to decide if a set of *partial beliefs* is coherent, while Ewing's definition can only judge whether a set of *full beliefs* is coherent.

Convincing as it seems, Lewis' definition of coherence is still far from satisfactory. Lewis takes raise of probability of a single belief as the criterion for coherence, but neglects the fact that coherence can also be a relation between subsets of a set. Given a set $S = \{b_1, ..., b_n\}$, we cannot tell if its subset $\{b_1, ..., b_k\}$ coheres with another subset $\{b_k, ..., b_n\}$. Another deficiency of Lewis' definition is that although probability is involved in Lewis' definition, the notion of coherence so characterized is still a qualitative, rather than a quantitative one. With this definition, we can only tell if a set is coherent, but cannot compare the coherence of different set. Hence, Lewis' definition is not good enough for coherentists.

BonJour (1985) proposes a set of 'coherence criteria' which characterizes the notion of coherence in a more subtle way:

1. A system of beliefs is coherent only if it is logically consistent.

2. A system of beliefs is coherent in proportion to its degree of probabilistic consistency.

3. The coherence of a system of beliefs is increased by the presence of inferential connections between its component beliefs and increased in proportion to the number and strength of such connections.

[4]In the original text, Lewis calls it *congruence*, which has been generally taken as identical to coherence.

[5]Chisholm (1977) provides a definition of coherence which is similar to the one Lewis proposed, which says 'a set of propositions S is coherent just if S is a set of two or more propositions each of which is such that the conjunction of all the others tends to confirm it and is logically independent of it.' The disadvantages of this definition is also similar to problems of Lewis' definition.

4. The coherence of a system of beliefs is diminished to the extent to which it is divided into subsystems of beliefs which are relatively unconnected to each other by inferential connections.

5. The coherence of a system of beliefs is decreased in proportion to the presence of unexplained anomalies in the believed content of the system.

These criteria emphasize that the essence of coherence is the *inferential connection* between beliefs in a set. Also, they reflect the idea that coherence can be understood as a matter of degree. However, since the way of measuring coherence is not mentioned in these criteria, BonJour's definition of coherence still does not provide people a way to compare the degree of coherence between different sets.

2.3 Coherence and truth-conduciveness

Although traditional definitions of coherence are all too rough, some of them correctly point out that coherence can be characterized in terms of probability. Based on the idea that probability and coherence are correlated, Klein and Warfield (1994, 1996) derive a rather striking result which undermines coherentism of epistemic justification.

Since all traditional analyses of knowledge takes truth as an essential ingredient, for coherence to be taken as a correct explanation of epistemic justification, it has to be truth-conducive. That is, given that a belief set S is more coherent than another belief set S', S should more likely to be true than S'. If coherence is not truth-conducive, supporters of coherentism will have to admit that a justified belief is no more likely to be true than an unjustified one, which is highly undesirable.

In order to disprove coherentism, Klein and Warfield claim that *coherence is not truth-conducive.* A belief set with a high degree of coherence, compared with a less coherent set, is less likely to be true. Their idea can be illustrated by two propositions:

1. Any set of beliefs S is more likely to be true than any other set of beliefs $S \cup \{b_i, ..., b_j\}$, given that at least one element in $\{b_i, ..., b_j\}$ is not entailed by S and does not have an objective probability of 1.

2. To increase the coherence of a set of beliefs S, one may add a belief which is not entailed by S and does not have an objective probability of 1.

Other things being equal, people tend to consider a larger belief set as more coherent than a smaller one, i.e. a belief set can be made more coherent by adding beliefs to it. But on the other hand, adding beliefs to a set may make the set less likely to be true, given that the beliefs are not absolutely true.[6] Consider the example given in section 2.2. The set $\{b_1, b_2\}$ is less coherent than $\{b_1, b_2, b_3\}$. But since it is possible that b_3 is false, the probability that all elements of $\{b_1, b_2, b_3\}$ are true is lower than the probability that both $\{b_1, b_2\}$ are true.

The two propositions, taken as premises, allow Klein and Warfield to derive the result that *coherence is not truth-conducive.* If higher degree of coherence does not guarantee greater likelihood of truth, coherence cannot be a proper explanation for epistemic justification. They thereby conclude that coherentists have two options: either give up the idea of explaining justification in terms of coherence, or admit that epistemic justification is not truth-conducive.

2.4 Shogenji's coherence measure

In order to argue against Klein and Warfield's criticism of coherentism, Shogenji (1999) provides a probabilistic coherence measure to show that coherence *per se* is truth-conducive. Given a belief set $S = \{b_1, ..., b_n\}$ and a probability function $Pr(\cdot)$ which follows Kolmogorov's axioms:

(Non-negatvity)	$Pr(b_i) \geq 0$ for all $b_i \in S$
(Normalization)	Given b_i is a logical truth, $Pr(b_i) = 1$.
(Finite Additivity)	$Pr(b_i \vee b_j) = Pr(b_i) + Pr(b_j)$ given that b_i and b_j are pairwise independent.

Shogenji defines a way to measure coherence of binary sets:

Definition 2.4.1. Shogenji's pairwise coherence measure

Given any two beliefs b_1, b_2 and a probability function $Pr(\cdot)$, the degree of coherence of $\{b_1, b_2\}$ is measured as:

$$C_{Sh}(\{b_1, b_2\}) \stackrel{\text{def}}{=\!=} \frac{Pr(b_1 \wedge b_2)}{Pr(b_1)Pr(b_2)}$$

This definition represents our ordinary idea of coherence that the more likely two beliefs are true or false *together*, the more coherent they are. If the denominator $Pr(b_1)Pr(b_2)$ is held fixed, the greater extent b_1 and b_2 overlap, the more coherent $\{b_1, b_2\}$ is. To measure the degree of coherence of more than two beliefs, this measure can be generalized as:

Definition 2.4.2. Shogenji's coherence measure

$$C_{Sh}(\{b_1, ..., b_n\}) \stackrel{\text{def}}{=\!=} \frac{Pr(b_1 \wedge ... \wedge b_n)}{Pr(b_1)...Pr(b_n)}$$

[6] An absolutely true belief is tautologous which does not provide any non-trivial information, and thus cannot lead to greater degree of coherence when added to a belief set.

This generalized measure retains an important merit of the original measure: it is sensitive to the size of the belief set being measured. A belief set with bigger size is more likely to be of high degree of coherence. This captures our intuitive idea that, for any two belief sets, if the degree of agreement between elements of the two sets are the same, the one which has more elements should be considered as more coherent. It is natural to think this way, for if one compares two belief sets which are of different size, it is less likely for elements of the bigger set to agree with each other. Therefore, when comparing two belief sets with the same degree of agreement, the one with greater size should be rendered with greater coherence. This feature of coherence is captured by Shogenji's measure, which can be illustrated by the following example:

Example 2.4.1. Given two belief sets $A = \{a_1, ..., a_i\}$ and $B = \{b_1, ..., b_j\}$, suppose that $i > j$, $Pr(a_1 \wedge ... \wedge a_i)$ is equivalent to $Pr(b_1 \wedge ... \wedge b_j)$ and for every a_n which is an element of A, $Pr(a_n)$ is smaller than 1. According to the given premises, the denominator of $C_{Sh}(\{a_1, ..., a_i\})$ is smaller than the denominator of $C_{Sh}(\{b_1, ..., b_j\})$. Hence, the degree of coherence of A is greater than the degree of coherence of B under Shogenji's measure, namely that

$$C_{Sh}(\{a_1, ...a_i\}) = \frac{Pr(a_1 \wedge ... \wedge a_i)}{Pr(a_1)...Pr(a_i)} > \frac{Pr(b_1 \wedge ... \wedge b_j)}{Pr(b_1)...Pr(b_j)} = C_{Sh}(\{b_1, ..., b_j\})$$

Another factor which needs to be considered in measuring coherence is the specificity of elements of a belief set. Two highly specific beliefs, compared with two general ones, are less likely to agree with each other. This point can be illustrated by the following example:

Example 2.4.2. Consider two pairs of beliefs concerning the same subject matter but with different specificity:

(a_1) Gatsby lives in New York.

(a_2) Gatsby attended college.

(b_1) Gatsby lives on Long Island in New York.

(b_2) Gatsby attended Trinity College, Oxford.

In this example, b_1 implies a_1 and b_2 implies a_2, therefore, $Pr(b_1) < Pr(a_1)$, $Pr(b_2) < Pr(a_2)$. It can thus be derived that $Pr(a_1)Pr(a_2)$ is greater than $Pr(b_1)Pr(b_2)$, which implies that the denominator of $C_{Sh}(\{a_1, a_2\})$ is greater than the denominator of $C_{Sh}(\{b_1, b_2\})$. If $Pr(a_1 \wedge a_2)$ is equivalent to $Pr(b_1 \wedge b_2)$, $C_{Sh}(\{b_1, b_2\})$ would be greater than $C_{Sh}(\{a_1, a_2\})$, which is in accordance with our ordinary understanding of coherence.

Shogenji calls the size and specificity of a belief set its *total individual strength*, and points out that given two belief sets with the same total individual strength, a coherent set, compared with a less coherent one, is more likely to be true. Suppose there are two belief sets $I = \{i_1, i_2\}$ and $J = \{j_1, j_2\}$. If $Pr(i_1)Pr(i_2) = Pr(j_1)Pr(j_2)$ and $Pr(i_1 \wedge i_2) > Pr(j_1 \wedge j_2)$, the degree of coherence of I will be greater than the degree of coherence of J. Since the degree of agreement between i_1 and i_2 is greater, I is more likely to be true than J. Arguing this way, Shogenji defends the view that coherence is a truth-conducive notion.

In spite of its plausibility, many people propose serious challenges to the Shogenji measure. Akiba (2000) points out that the Shogenji measure is vulnerable to *the problem of falsity-conduciveness* and *the problem of conjunction*. Given two beliefs b_1 and b_2, if b_1 entails b_2, the pairwise coherence between b_1 and b_2 is

$$C_{Sh}(\{b_1, b_2\}) = \frac{Pr(b_1 \wedge b_2)}{Pr(b1)Pr(b_2)} = \frac{Pr(b_1)}{Pr(b1)Pr(b_2)} = \frac{1}{Pr(b_2)}$$

In this case, $Pr(b_2)$ is negatively correlated with $C_{Sh}(\{b_1, b_2\})$ such that when $Pr(b_2)$ decreases, $C_{Sh}(\{b_1, b_2\})$ increases. Since being less probable leads to greater coherence according to Shogenji's measure, it does not follow that coherence is truth-conducive.

Another problem of Shogenji's measure can be shown by the following example:

Example 2.4.3. When throwing a dice, one may have three different beliefs:

b_1: The dice will come up two.

b_2: The dice will come up an even number less than six.

b_3: The dice will come up an even number.

Akiba claims that given b_1 entails both b_2 and b_3, the degree of coherence of $\{b_1, b_2\}$ should be the same as $\{b_1, b_3\}$. For any arbitrary set of beliefs $\{p_1, p_2, p_3\}$, if belief p_1 entails two other beliefs p_2 and p_3, the degree of coherence between p_1 and p_2 should be equivalent to the degree of coherence between p_1 and p_3. But in this case, $C_{Sh}(\{(b_1, b_2\})$ is 3, whereas $C_{Sh}(\{b_1, b_3\})$ is 2. Hence, the outcome of Shogenji's measure fails to capture our intuition of coherence in some occasions.

Akiba further points out that if one measures the coherence of a singleton belief set with Shogenji's measure, the degree of coherence will always be 1, which is supposed to be a high degree of coherence, for a belief (i.e. a singleton belief set) is perfectly coherent with itself. If we take two independent beliefs b_1, b_2 and measure the coherence of the singleton set $\{b_1 \wedge b_2\}$, the degree of coherence of $C_{Sh}(\{b_1 \wedge b_2\})$ would also be 1, which is another counterintuitive result given the assumption that b_1 and b_2 are independent. For these reasons, Akiba concludes that Shogenji's measure is an inadequate measure for coherence.

Shogenji (2001) denies all Akiba's criticisms. The fact that lower probability leads to greater coherence, according to Shogenji, does not really pose a threat to his measure. What Shogenji intends to show with his measure is exactly that lower probability, which is equivalent to higher specificity, leads to greater coherence, Akiba's criticism does not show that Shogenji's measure is falsity-conducive, but instead reveals the fact that the degree of coherence raises when the specificity of beliefs is greater. Hence, in debating whether coherence is truth-conducive, this factor should be fixed. Akiba understands Shogenji's measure in a straightforward way and thus neglects the underlying motivation, which results in an incorrect criticism.

As for the dice case, Shogenji provides an example to show that pairs of beliefs with entailment relation can differ in coherence.

Example 2.4.4. Consider the following beliefs:

p_1 : The fossil was deposited 64-to-66 million years ago.

p_2 : The fossil was deposited 63-to-67 million years ago.

p_3 : The fossil was deposited more than 10 years ago.

p_1 entails both p_2 and p_3, but intuitively, the set $\{p_1, p_2\}$ is more coherent than $\{p_1, p_3\}$. Following this line of thought, it should be acceptable that in Akiba's example, the degree of coherence of $\{b_1, b_2\}$ differs from the degree of coherence of $\{b_1, b_3\}$.

The problem concerning the coherence of the conjunction of two individual beliefs does not apply either. If coherence is taken as a relation between beliefs, rather than the property of a single belief, claiming that a belief is of maximum coherence, in this sense, is nonsensical. Therefore, Akiba's example does not really show that Shogenji's measure is incorrect.

Two more serious problems for Shogenji's measure are the *depth problem* and *problem of Irrelevant Addition*. Fitelson (2003) points out that Shogenji's measure does not take into account the coherence of subsets of a belief set. Given a belief set with n elements, Shogenji's measure can only calculate its n-wise coherence, but not k-wise coherence for any $k < n$. However, it is quite common for belief sets to be incoherent as a whole, but partially highly coherent. Failing to capture the mixed nature of coherence is definitely a shortcoming of Shogenji's measure. Consider the following example Schupbach (2011) provides:

Example 2.4.5. Police investigators caught eight robbery suspects, each of them are equally likely to have committed the crime. Three independent witnesses claimed that they have seen the criminal. In the first case, the witnesses provide the set of testimonies:

w_1: The criminal was either suspect 1, 2 or 3.

w_2: The criminal was either suspect 1, 3 or 4.

w_3: The criminal was either suspect 1, 2 or 4.

In the second case, the witnesses provide:

w_1': The criminal was either suspect 1, 2 or 3.

w_2': The criminal was either suspect 1, 4 or 5.

w_3': The criminal was either suspect 1, 6 or 7.

Intuitively, the set of testimonies in the first case is more coherent than the testimonies in the second case. But with Shogenji's measure, the coherence of E is equivalent to the coherence of E':

$$C_{Sh}(E) = \frac{Pr(w_1 \wedge w_2 \wedge w_3)}{Pr(w_1)Pr(w_2)Pr(w_3)} = \frac{Pr(w_1' \wedge w_2' \wedge w_3')}{Pr(w_1')Pr(w_2')Pr(w_3')} = C_{Sh}(E')$$

It can thus be seen that Shogenji fails to measure the 'sub-coherence' of belief sets, and hence it leads to strange results. This is the so-called *depth problem*.

The problem of irrelevant addition states that if a belief which is totally irrelevant to a set S is added to S, the degree of coherence of that set remains the same, which also violates our ordinary understanding of coherence.

Example 2.4.6. In the robbery example, suppose a witness by accident provides another testimony w_4: 'It is raining in Paris now'. If we add w_4 to E, the degree of coherence of the new set $E \cup \{w_4\}$ is:

$$C_{Sh}(E \cup \{w_4\}) = \frac{Pr(w_1 \wedge w_2 \wedge w_3)Pr(w_4)}{Pr(w_1)Pr(w_2)Pr(w_3)Pr(w_4)} = \frac{Pr(w_1 \wedge w_2 \wedge w_3)}{Pr(w_1)Pr(w_2)Pr(w_3)}$$

which is again equivalent to the degree of coherence of E.

With Shogenji's measure, no matter how many irrelevant beliefs are added to a belief set, as long as they are independent, the degree of coherence of the set will not change, which is highly counterintuitive. When a set is extended with independent propositions, people normally consider the new set as less coherent then the original set. Because of these two serious shortcomings, Shogenji's measure cannot be adopted as an ideal coherence measure. Coherentists need to propose a different measure to show that coherence is truth-conducive.

2.5 Olsson's coherence measure

Olsson (2002) criticized Shogenji's measure for being specificity sensitive. If a coherence measure is specificity sensitive, the degree of coherence of a set would be bounded by the specificity of its elements, according to that measure. This deficiency can be illustrated by a simple example. Suppose there are four beliefs b_1, b_2, b'_1 and b'_2 such that $Pr(b_1) = Pr(b_2) = 0.5$, $Pr(b'_1) = Pr(b'_2) = 0.3$. The degree of coherence of $\{b_1, b_2\}$, according to Shogenji's measure, is

$$C_{Sh}(\{b_1, b_2\}) = \frac{Pr(b_1 \wedge b_2)}{Pr(b_1)Pr(b_2)} = \frac{Pr(b_1 \wedge b_2)}{0.25}$$

Since $Pr(b_1) = Pr(b_2) = 0.5$, when b_1 and b_2 coincide perfectly, $\{b_1, b_2\}$ has maximal degree of coherence $\frac{0.5}{0.25} = 2$. On the other hand, the maximal degree of coherence of $\{b'_1, b'_2\}$ is $\frac{10}{3}$, which is greater than 2. If we suppose that both $\{b_1, b_2\}$ and $\{b'_1, b'_2\}$ are maximally coherent, $\{b_1, b_2\}$ will be rendered a degree of coherence lower than $\{b'_1, b'_2\}$ simply because b_1 and b_2 are more probable than b'_1 and b'_2. Such result is undesirable, for we can imagine cases in which the set $\{b'_1, b'_2\}$ is only of neutral coherence, yet still more coherent than a perfectly coherent but more probable set $\{b_1, b_2\}$.

The underlying problem is that Shogenji's measure does not have a *maximal value*. No matter how coherent a belief set is, there exist some other sets that are more coherent. Hence, a set of logically equivalent beliefs, which is supposedly the most coherent set that can possible be perceived, is not judged as maximally coherent.

Aware of the shortcomings of Shogenji's measure, Olsson provides another coherence measure which is free from these problems:

Definition 2.5.1. Olsson's coherence measure

Given a set $S = \{b_1, ..., b_n\}$, the degree of coherence of S is:

$$C_O(S) \overset{\text{def}}{=\!=} \frac{Pr(b_1 \wedge ... \wedge b_n)}{Pr(b_1 \vee ... \vee b_n)}$$

With Olsson's measure, the degree of coherence of a belief set is no longer bounded by the probability of elements in the set, but takes $[0, 1]$ as range. For a set of beliefs which do not agree on anything, the set has minimal degree of coherence, while a belief set $\{b_1, ..., b_n\}$ is maximally coherent when $Pr(b_1 \wedge ... \wedge b_n)$ equals to $Pr(b_1 \vee ... \vee b_n)$.

Also, Olsson's measure is free from the problem of irrelevant addition. Suppose there are two belief sets $S = \{b_1, b_2\}$ and $S' = \{b_1, b_2, b_3\}$. Given that b_3 is irrelevant to $\{b_1, b_2\}$, the denominator of $C_O(S')$ is greater than the denominator of $C_O(S)$, and hence

$$C_O(S) = \frac{Pr(b_1 \wedge b_2)}{Pr(b_1 \vee b_2)} > \frac{Pr(b_1 \wedge b_2 \wedge b_3)}{Pr(b_1 \vee b_2)Pr(b_3)} = C_O(S')$$

With Olsson's measure, adding irrelevant beliefs leads to a decrease in coherence. Thus, Olsson's measure is better than the Shogenji measure.

Siebel (2005) points out that under Olsson's measure, adding necessary truths to a set makes the set less coherent. A belief set $\{b_1, b_2\}$ becomes less coherent if extended with a necessary truth, say b_t. That is,

$$C_O(\{b_1, b_2\}) = \frac{Pr(b_1 \wedge b_2)}{Pr(b_1 \vee b_2)} > \frac{Pr(b_1 \wedge b_2 \wedge b_t)}{Pr(b_1 \vee b_2 \vee b_t)} = \frac{Pr(b_1 \wedge b_2)}{Pr(b_1 \vee b_2 \vee b_t)} = C_O(\{b_1, b_2, b_t\})$$

When extended with a necessary truth b_t which is irrelevant to b_1 and b_2, $Pr(b_1 \wedge b_2)$ remains the same, while $Pr(b_1 \vee b_2 \vee b_3)$ increases. Therefore, adding b_t lowers the degree of coherence of the original set.

Siebel's criticism is quite unconvincing. Given a belief set $\{b_1, ..., b_n\}$, if one adds a necessary truth which is irrelevant to all elements of that set, it is intuitive to think that the new set is less coherent than the original one. Take the robbery case in section 1.4 for example. Suppose that a witness provides the testimony

w_4 : Five plus seven equals twelve.

Since this testimony is totally irrelevant to the robbery, it should not be regarded as coherent with the original set of testimonies. According to Olsson's measure, the degree of coherence of $\{w_1, w_2, w_3, w_4\}$ is less than the degree of coherence of $\{w_1, w_2, w_3\}$, which correctly captures this idea. Hence, the point Siebel criticized should be taken as an advantage, rather than a shortcoming.

The real problem of Olsson's measure is its *size-insensitiveness*. Recall that by the term *total strength*, Shogenji refers to both the specificity and size of a belief set. Consider two belief sets $B = \{b_1, b_2\}$ and $B' = \{b'_1, ..., b'_{100}\}$. If $Pr(b_1 \wedge b_2) = Pr(b'_1 \wedge ... \wedge b'_{100})$ and $Pr(b_1 \vee b_2) = Pr(b'_1 \vee ... \vee b'_{100})$, according to Olsson's measure, the degree of coherence of B is equivalent to B'. This result is quite dubious. With other things being equal, people tend to take sets with greater size as more coherent. We can illustrate this with a revised version of the robbery example:

Example 2.5.1. Police investigators caught eight suspects for a robbery, each of them are equally likely to have committed the crime. In the first scenario, there are two independent witnesses who claimed that they have seen the suspect and provided the following set of testimonies:

w_1: The criminal was either suspect 1, 2 or 3.

w_2: The criminal was either suspect 1, 3 or 4.

In the second scenario, there are one hundred witnesses who claimed that they have seen the suspect and provided the following set of testimonies:

w_{1-50}: The criminal was either suspect 1, 2 or 3.

w_{51-100}: The criminal was either suspect 1, 3 or 4.

Intuitively, the set of testimonies in the second scenario is more coherent than in the first scenario, for the size of the set of testimonies is much larger than the set of testimonies in the first scenario. But according to Olsson's measure, they are equally coherent.

In measuring coherence, Shogenji involves the total strength of a set, while Olsson does not. If we accept the requirement that a coherence measure should be insensitive to the specificity of beliefs but sensitive to the size of belief set, both Shogenji and Olsson's measure fail to be proper. Coherentists need to provide other ways of measuring coherence.

2.6 Fitelson's coherence measure

Being aware of the deficiencies of Shogenji's measure, Fitelson (2003, 2004) propose a coherence measure based on the notion of *mutual confirmation*. It is generally accepted that coherence is the mutual support between the elements of a set. With this idea, it is intuitive to take the degree of coherence of a set as the average degree of confirmation between all elements in that set.

To construct a measure which captures the notion of coherence as confirmation, Fitelson first introduces a two-place function $F(X, Y)$[7] which measures the degree a belief Y[8] confirms another belief X, defined as:

Definition 2.6.1. Measure for support

Given any two beliefs[9] X and Y and a probability function $Pr(\cdot)$, the degree that Y confirms X, denoted by $F(X, Y)$, is defined as:

[7] This function is a modification of the measure of factual support which Kemeny and Oppenheim (1952) propose.

[8] Here X and Y can also be sets. We Can just take the conjunction of all elements of a set as a single belief, and measure it in the way suggested.

[9] As noted, they can also be belief sets.

$$F(X,Y) \overset{\text{def}}{=\!=} \begin{cases} \dfrac{Pr(Y|X) - Pr(Y|\neg X)}{Pr(Y|X) + Pr(Y|\neg X)} & \text{if } Y \text{ does not entail } X \text{ and } Y \text{ does not entail } X \\ 1 & \text{if } X \text{ entail } Y \text{ and } X \text{ is not inconsistent} \\ -1 & \text{if } Y \text{ entails } \neg X \end{cases}$$

With this function, Fitelson defines his coherence measure as follows:

Definition 2.6.2. Fitelson's coherence measure

Suppose S is a belief set $\{b_1, ..., b_n\}$. The degree of coherence of S is defined as:

$$C_F(S) \overset{\text{def}}{=\!=} \frac{1}{[\![M]\!]} \sum_{\langle X,Y\rangle \in M} F(\bigwedge X, \bigwedge Y)$$

where M is the set of all pairs of non-overlapping subsets of S defined as $\{\langle X, Y\rangle | X, Y \in (\wp(S)/\varnothing) \wedge X \cap Y = \varnothing\}$ and $[\![M]\!]$ is the cardinality of M.

In a belief set S, every $X \in \wp(S\backslash\varnothing)$ is confirmed or disconfirmed by another subset $Y \in \wp(S\backslash\varnothing)$. By averaging the degree each $X \in \wp(S)$ is confirmed or disconfirmed by every other non-empty element of $\wp(S)$, one may measure the strength of mutual confirmation among all the subsets in S, and take this value as the degree of coherence of S. With a simple example, we can see how this measure works. Take a belief set $S = \{b_1, b_2, b_3\}$. According to the definition given, M equals to:

$$\{\langle b_1, b_2\rangle, \langle b_1, b_3\rangle, \langle b_1, b_2 \wedge b_3\rangle, \langle b_2, b_1\rangle, \langle b_2, b_3\rangle, \langle b_2, b_1 \wedge b_3\rangle, \langle b_3, b_1\rangle, \langle b_3, b_2\rangle, \langle b_3, b_1 \wedge b_2\rangle,$$
$$\langle b_1 \wedge b_2, b_3\rangle, \langle b_1 \wedge b_3, b_2\rangle, \langle b_2 \wedge b_3, b_1\rangle\}$$

We measure the degree of coherence of S by averaging the degree of confirmation of every pair in M.

This measure is free from the depth problem. Given any set, the degrees of coherence of all subsets of it are taken into account with Fitelson's measure. Take the robbery case in section 1.4 for example. Recall that $E = \{w_1, w_2, w_3\}$. The degree of coherence is the average of the set $\{F(w_1, w_2),\ F(w_1, w_3),\ F(w_2, w_1),\ F(w_2, w_3),\ F(w_3, w_1),\ F(w_3, w_2),\ F(w_1, w_2 \wedge w_3),\ F(w_2, w_1 \wedge w_3),\ F(w_3, w_1 \wedge w_2),\ F(w_1 \wedge w_2, w_3),\ F(w_1 \wedge w_3, w_2),\ F(w_2 \wedge w_3, w_1)\}$. With the function $F(X,Y)$ defined above, we can derive that

$$F(w_1, w_2) = F(w_1, w_3) = F(w_2, w_1) = F(w_2, w_3) = F(w_3, w_1) = F(w_3, w_2) = \frac{7}{13}$$

$$F(w_1, w_2 \wedge w_3) = F(w_2, w_3 \wedge w_1) = F(w_1, w_3 \wedge w_2) = \frac{7}{13}$$

$$F(w_1 \wedge w_2, w_3) = F(w_2 \wedge w_3, w_!) = F(w_1 \wedge w_3, w_2) = 1$$

Hence, $C_F(E)$ is $\frac{17}{26}$. On the other hand, for $E = \{w'_1, w'_2, w'_3\}$,

$$F(w'_1, w'_2) = F(w'_1, w'_3) = F(w'_2, w'_1) = F(w'_2, w'_3) = F(w'_3, w'_1) = F(w'_3, w'_2) = \frac{-1}{11}$$

$$F(w_1, w_2 \wedge w_3) = F(w_2, w_3 \wedge w_1) = F(w_1, w_3 \wedge w_2) = 1$$

$$F(w_1 \wedge w_2, w_3) = F(w_2 \wedge w_3, w_!) = F(w_1 \wedge w_3, w_2) = \frac{5}{9}$$

We may derive that $C_F(E')$ is $\frac{34}{99}$, which is lower than $C_F(E)$. Fitelson's measure correctly reflects our intuition that E is more coherent than E'.

Fitelson's measure is also immune to the problem of irrelevant additions. Since irrelevant beliefs do not confirm any belief in a set, adding them would reduce the degree of confirmation between subsets, and further reduce the degree of coherence of the whole set. Moreover, Fitelson's measure has a maximal value for perfectly coherent belief sets, while Shogenji's measure does not. That is, for two different but both perfectly coherent belief sets, Fitelson's measure renders them with equal coherence.

Fitelson's measure is quite plausible, since it is based on the idea that the coherence of a belief set is the confirmation between the elements of that set. However, Bovens and Hartmann (2003) provide an example to cast doubt on Fitelson's coherence measure:

Example 2.6.1. Imagine two criminal scenarios: in the first one, there are 100 suspects, 6 of them play chess, 6 of them are from the Trobriand island, only one of the suspects is a Trobriand chess player. The coherence of the belief set $S = \{$The culprit is a chess player, The culprit is a Trobriand$\}$, according to Fitelson's measure, is approximately 0.52[10]. In the second case, among 100 suspects, there are 85 rugby players, 85 people from Uganda and 80 rugby players are from Uganda. The coherence of the set $S' = \{$ The culprit is a rugby player, The culprit is from Uganda$\}$ is 0.48[11]. The overlapping part between elements of S' is greater than the overlapping part between elements of S, but the coherence of S is greater than S'.

This result again violates our intuitive idea of coherence, for we normally consider the first case as more coherent. As a result, we need to search for some other coherence measures which better captures our intuitive idea of coherence.

[10] Given that $C =$'the culprit is a chess player', $T =$'the culprit is a Trobriand'.
$F(C,T) = F(T,C) = \dfrac{\frac{1}{6} - \frac{5}{94}}{\frac{1}{6} + \frac{5}{94}} = \frac{16}{31}$. $C_F(\{T,C\}) = \frac{16}{31} \times 2 \div 2 = \frac{16}{31} \approx 0.52$.

[11] Given that $R =$'The culprit is a rugby player', $U =$'The culprit is from Uganda'.
$F(U,R) = F(R,U) = \dfrac{\frac{80}{85} - \frac{1}{3}}{\frac{80}{85} + \frac{1}{3}} = \frac{31}{65}, C_F(\{U,R\}) = \frac{31}{65} \times 2 \div 2 \approx 0.48$.

2.7 Douven and Meijs' measure

Douven and Meijs (2007) provide a scheme for confirmation-based coherence measures which, similar to Fitelson's measure, takes the degree of coherence of a set S as the average degree of mutual confirmation between all subsets of S. With their scheme, it is possible to generate many different measures simply by plugging in different confirmation measures.

They first introduce three major types of confirmation measures: the *difference measure*, *ratio measure* and *likelihood measure*.

Definition 2.7.1. Confirmation measures

Given a probability function $Pr(\cdot)$, the degree of a belief Y's confirmation to X can be measured in the following ways:

Difference measure: $d(X,Y) \overset{\text{def}}{=\!=} Pr(X|Y) - Pr(X)$

Ratio measure: $r(X,Y) \overset{\text{def}}{=\!=} \dfrac{Pr(X|Y)}{Pr(X)}$

Likelihood measure: $l(X,Y) \overset{\text{def}}{=\!=} \dfrac{Pr(X|Y)}{Pr(X|\neg Y)}$

These confirmation measures can be generalized to measure the degree of confirmation between sets:

Definition 2.7.2. Confirmation between sets

The degree a set S' confirms another set S can be measured as:

Difference measure: $d(S,S') \overset{\text{def}}{=\!=} Pr(\bigwedge S|\bigwedge S') - Pr(\bigwedge S)$

Ratio measure: $r(S,S') \overset{\text{def}}{=\!=} \dfrac{Pr(\bigwedge S|\bigwedge S')}{Pr(\bigwedge S)}$

Likelihood measure: $l(S,S') \overset{\text{def}}{=\!=} \dfrac{Pr(\bigwedge S|\bigwedge S')}{Pr(\bigwedge S|\overline{\bigwedge S'})}$

Let d, r, l stand respectively for these three measures, and let m be the variable for measures. Define $[S]$ as $\{\langle S', S''\rangle | S', S'' \subset S \backslash \{\varnothing\} \wedge S' \cap S'' = \varnothing\}$, namely the set of pairs of non-empty, non-overlapping subsets of S, we can establish the following scheme of coherence measures:

Definition 2.7.3. Scheme for coherence measure

Given a set $S = \{b_1, ..., b_n\}$. With an ordering $\langle \hat{S}_1, ..., \hat{S}_{[\![S]\!]}\rangle$ of members of $[S]$, the degree of coherence of S is given by the function

$$C_m(S) \overset{\text{def}}{=\!=} \frac{\sum_{i=1}^{[\![S]\!]} m(\hat{S}_i)}{[\![S]\!]}$$

for $m \in \{d, r, l\}$.

For example, given a set $S^* = \{P_1, P_2\}$, the degree of coherence of S under the difference measure is

$$C_d(S^*) = \frac{d(P_1, P_2) + d(P_2, P_1)}{[\![S]\!]} = \frac{Pr(P_1|P_2) - Pr(P_1) + Pr(P_2|P_1) - Pr(P_2)}{2}$$

Douven and Meijs (2007: p.417) claim that C_d is the least problematic coherence measure. To show this, they provide several test cases:

Example 2.7.1. Consider the following scenarios:

Case 1. A murder happened in a city with 10,000,000 inhabitants. 1,059 among them are Japanese, 1059 among them own Samurai swords while only 9 of them are Japanese owning Samurai swords.

Case 2. A murder happened on a street with 100 inhabitants. 10 of them are Japanese, 10 of them own Samurai swords, and 9 of them are Japanese who own Samurai swords.

Let J stand for the belief 'The murderer is Japanese' and O for the belief 'The murderer owns a Samurai sword.' Degrees of coherence of $S = \{J, O\}$ under different coherence measures in two cases are as follows:

	Case 1.	Case 2.
C_{Sh}	80.3	9
C_O	0.0043	0.818
C_F	0.97559	0.97561
C_d	0.0084	0.8
C_r	80.3	9
C_l	80.9	81

The intuition is that coherence of S in case 1 should be much greater than the coherence of S in case 1. C_{Sh}, C_F, C_r, C_l all fail to capture this intuition. C_F and C_l renders S with similar degree of coherence in both cases. C_r and C_{Sh} renders S with greater coherence in case 1 than in case 2. Only C_d and C_O correctly represent the great difference between coherence of S in case 1 and case 2.

Another example, originally provided by Bovens and Hartmann (2003), shows that Olsson's measure leads to an unacceptable result:

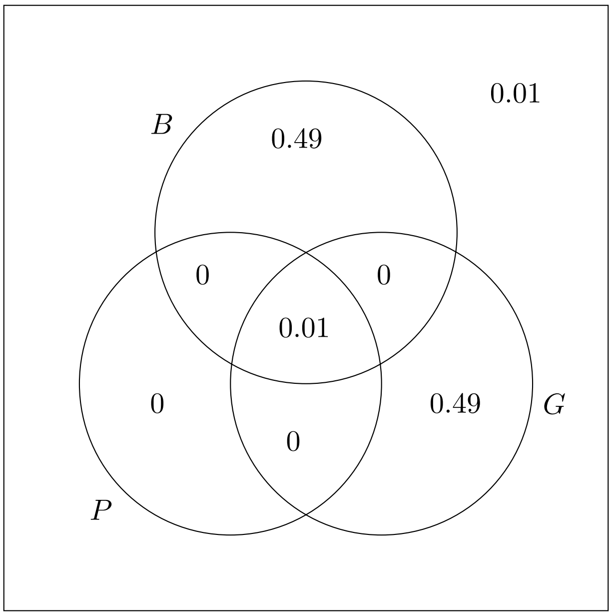
Example 2.7.2. Consider two sets $S = \{B, G\}$ and $S' = \{B, G, P\}$ such that

B : Our pet is a bird.

G : Our pet is a ground dweller.

P : Our pet is a penguin.

Given the probability distribution represented in the following diagram:



Intuitively, S' is more coherent than S. However, under C_O, the degree of coherence of S is $\frac{0.01}{0.99}$ which is equivalent to the degree of coherence of S', while $C_d(S)$ reflects a difference between the coherence of S and S', and therefore correctly captures the intuition that S' is more coherent than S.

With these examples, Duoven and Meijs (2007) show that C_d is the only coherence measure which does not generate unacceptable outcomes, and hence should be taken as the correct coherence measure.

Roche (2013) provides a variant to Douven and Meijs' coherence measure. He criticized that although C_d is free from problems of other coherence measures, it generates unacceptable results for other cases. Consider the following scenario:

Example 2.7.3. Supose there are 10 suspects of committing a murder. Each of the suspects has equal probability of 0.1 of being the murderer. 6 of them have committed both pickpocketing and robbery, 2 of them have only committed pickpocketing and another 2 committed only robbery. Let $S^* = \{r, p\}$ and

r : The murderer has committed robbery.

$p:$ The murderer has committed pickpocketing.

The coherence of S^* is $\dfrac{d(r,p)+d(p,r)}{2} = -0.05$. That is, C_d indicates that S^* is incoherent, which violates our intuition that S^* is pretty coherent.

To avoid this problem, Roche suggests to measure coherence with a confirmation measure which differs from d, r, l:

$$R(X,Y) \overset{\text{def}}{=\!=} \begin{cases} Pr(X|Y) & \text{if } X \text{ does not entail } Y \text{ and } X \text{ does not entail } \neg Y. \\ 1 & \text{if } X \text{ entails } Y \text{ and } X \text{ is consistent.} \\ 0 & \text{if } X \text{ entails } \neg Y. \end{cases}$$

By plugging R in Douven and Meijs' scheme, we may obtain Roche's coherence measure C_R which is:

$$C_R(S) \overset{\text{def}}{=\!=} \frac{\sum_{i=1}^{[\![S]\!]} a(\hat{S}_i)}{[\![S]\!]}$$

It is easy to check that this measure is invulnerable to all the problematic cases for other confirmation-based coherence measures. Hence, Roche claims that C_R is an ideal way for measuring coherence.

2.8 Revisiting the agreement measures of coherence

Shogenji and Olsson's measures are quite different from measures generated with Douvan and Meijs' scheme. The former type of measures focus on the *agreement* between beliefs in a set. The latter type of measures, on the other hand, take the confirmation between beliefs in a set as the primary factor. We may thus call Shogenji and Olsson's measures the *agreement* measures, and others the *confirmation* measures of coherence.

Agreement measures, compared with confirmation based measures, have a huge disadvantage for being insensitive to the coherence of subsets of the set being measured. That is, in measuring the coherence of a set S, agreement measures do not take into account the degree of coherence of any $S_i \subseteq S$. Recall the problems that threat Shogenji's measure, the most important ones are the *depth* problem and the problem of *irrelevant addition.* The first reveals the fact that for a set S with cardinality i, Shogenji's measure fails to show any k-wise coherence for any $k < i$. As a result, Shogenji's measure may fail to correctly represent our intuitive ranking of coherence in certain occasions. The second problem, namely the problem of irrelevant additions, shows that when a set is extended with irrelevant beliefs, the degree of coherence of that very set remains the same under Shogenji's measure. Olsson's measure is free from the problem of irrelevant additions, but still suffers from the depth problem.

It can be observed that both problems stem from the *subset-insensitivity* of agreement measures. If, when measuring the coherence of a set S, agreement measures are sensitive to the coherence of subsets of S, the depth problem can be solved. Similarly, since the degree of coherence between a single belief and a totaly irrelevant belief is low, being subset sensitive can also solve the problem of irrelevant addition.

With this underlying thought, Schupbach (2011) provides a refined version of Shogenji's measure which is sensitive to the coherence of subsets. He first defines the k-wise coherence of a set under Shogenji's measure as the following:

Definition 2.8.1. k-wise coherence with Shogenji's measure

For a set $S = \{b_1, ..., b_n\}$, $[S]^k$ represents the set of all subsets of S with k elements. Given an ordering $\langle \tilde{S_1}, ..., \tilde{S_m} \rangle$ of the members of $[S]^k$, the degree of *k-wise* coherence of S is measured as:

$$C^k(S) \stackrel{\text{def}}{=\!=} \frac{\sum_{i=1}^{m} s(\tilde{S_i})}{m}$$

in which m is the number of elements in $[S]^k$ and $s(S)$ is the logarithm[12] of Shogenji's generalized coherence measure, namely:

$$s(S) \stackrel{\text{def}}{=\!=} log\left(\frac{Pr(b_1 \wedge ... \wedge b_n)}{Pr(b_1)...Pr(b_n)}\right)$$

With k-wise coherence, we can define the coherence of a set by giving a weigh vector to each k and obtain a coherence measure, namely

Definition 2.8.2. Generalized Shogenji's measure

Given a set $S = \{b_1, ..., b_n\}$ and a weight vector $\langle w_1, ..., w_{n-1} \rangle$ which assigns weights to k-wise coherence for every k such that $\sum_{i=1}^{n-1} w_i = 1$, the degree of coherence is measured as

$$C(S) \stackrel{\text{def}}{=\!=} \sum_{i=1}^{n-1} w_i C^{i+1}(S)$$

With this scheme, we can define different coherence measures by changing the value of the weight vectors. The simplest one is generated by assigning equal weight to all k-wise coherence:

Definition 2.8.3. Straight Average

[12]Schupbach takes the logarithm of Shogenji's measure for sake of simplicity.

$$C_{SA}(S) \stackrel{\text{def}}{=\!=} \frac{\sum_{k=2}^{n} C^k(S)}{n-1}$$

We can define another measure which assigns greater weight to *k-wise* coherence when k is distant from n.

Definition 2.8.4. Deeper Decreasing

Let the scheme assign decreasing weights to decreasing k as:

$$w_i = \frac{i}{(n-1)+(n-2)+...+1} = \frac{2i}{n(n-1)}$$

The degree of coherence of $S = \{b_1, ..., b_n\}$ is

$$C_{DD}(S) \stackrel{\text{def}}{=\!=} \textstyle\sum_{i=1}^{n-1} \displaystyle\frac{2i}{n(n-1)} C^{i+1}(S) = \frac{\sum_{i=1}^{n-1} iC^{i+1}(S)}{n(n-1)/2}$$

On the other hand, we can also define a measure which assigns greater weight to *k-wise* coherence when k is close to n:

Definition 2.8.5. Deeper Increasing

Let the scheme assign increasing weights to decreasing k as:

$$w_i = \frac{n-i}{(n-1)+(n-2)+...+1} = \frac{2(n-1)}{n(n-1)}$$

The degree of coherence is thus measured as

$$C_{DI}(S) \stackrel{\text{def}}{=\!=} \textstyle\sum_{i=1}^{n-1} \displaystyle\frac{2(n-1)}{n(n-1)} C^{i+1}(S) = \frac{\sum_{i=1}^{n-1}(n-i)C^{i+1}(S)}{n(n-1)}/2$$

All three different measures are free from the depth problem, for they all take the coherence of subsets of a set into account while measuring coherence. C_{SA} and C_{DI} are also free from the problem of irrelevant addition.[13] Revising this way, Schupbach saves Shogenji's measure.

Olsson's measure can also be refined to be *subset-sensitive* similarly. Meijs (2006) provides a refined version of Olsson's measure with the scheme for coherence measures proposed by Douven and Meijs:

Definition 2.8.6. Generalized Olsson's measure

Let $[S]_1$ be the set of all subsets of S with cardinality greater than 1, and $[\![S]\!]_1$ denote the cardinality of $[S]_1$. Given a set $S = \{b_1, ..., b_i\}$. With an ordering $\langle \hat{S}_1, ..., \hat{S}_{[\![S]\!]_1} \rangle$ of members of $[S]_1$, the degree of coherence of S is given by the function:

$$C_{O^*}(S) \stackrel{\text{def}}{=\!=} \frac{\sum_{i=1}^{[\![S]\!]_1} o(\hat{S}_i)}{[\![S]\!]}$$

[13] C_{DD} is similar to the original C_{Sh} that it assigns less wight to smaller subsets that are small, hence, C_{DD} is still vulnerable to the problem of irrelevant addition.

in which $o(S) = \dfrac{Pr(\bigwedge S)}{Pr(\bigvee S)}$

This measure is slightly different from the measures generated with Douven and Meijs' original scheme in the respect that it does not measure the confirmation between subsets, but measures coherence by averaging the coherence of each subset. Hence, the order of elements of $[S]_1$ does not really matter. We can, of course, also generalize Olsson's measure in the way Schupbach generalized Shogenji's measure, and assign different weights to subsets of different cardinality.

With Schupbach and Meijs' revision, agreement measures are made subset sensitive, and hence could again be candidates for a suitable coherence measure.

2.9 Summary of chapter two

Each of the coherence measures surveyed in this chapter has its own special advantage, and stands for a specific conception of coherence. By checking if a coherence measure generates counterintuitive results, one can see if certain conceptions of coherence is fallacious, and gradually approach an ideal coherence measure which leads to the least amount of unacceptable results. However, according to the information gathering model established by Bovens and Hartmann (2003), there is no truth-conducive coherence measure, which means that even if we can find a perfect coherence measure which does not generate any counterintuitive consequence, the attempt to explain epistemic justification in terms of coherence is doomed to fail.

Chapter 3

New Ideals for Coherence

3.1 Impossibility results and the pursuit of new epistemic ideal

Bovens and Hartmann (2003, pp.10-22) prove the significant *impossibility results* which show that there is no truth-conducive coherence measure. Given that the primary function of coherence is to account for epistemic justification, if coherence is not truth-conducive, knowing that a set is more coherent than another does not provide us with any epistemically useful information. Hence, the impossibility results motivate epistemologists to search for another epistemic ideal which coherence may be conducive of. If this ideal does exist, coherence may still be regarded as an important notion in epistemology, that is, knowing that a set is coherent allows us to infer that the set conforms to an epistemic ideal.

The primary concern of this chapter is to demonstrate how Bovens and Hartmann prove the impossibility results, and introduce the follow-up attempts to search for a new epistemic ideal.

3.2 The impossibility results

Recall that the original purpose of finding a proper probabilistic coherence measure is to show that coherence is a truth-conducive notion in a quantitative manner, which is the central tenet of *Bayesian Coherentism*. Assume that an information set[1] $S = \{R_1, ..., R_n\}$ is given by n independent and partially reliable sources. Let $\mathbf{S}$ be the set of all such information sets, Bayesian Coherentism can be defined by the following two claims:

[1]Traditionally, philosophers tend to take coherence as a property of belief sets, since the primary function of coherence is to account for epistemic justification. In *Bayesian Epistemology* (Bovens and Hartmann 2003), the authors use the term *information* instead of beliefs. Here I follow this usage to avoid unnecessary misunderstanding of Bovens and Hartmann's framework.

Definition 3.2.1. Bayesian Coherentism

(BC1) For all information sets $S, S' \in \mathbf{S}$, if S is no less coherent than S', then our degree of confidence that the content of S is true is no less than our degree of confidence that the content of S' is true, *ceteris paribus*.

(BC2) A coherence ordering over $\mathbf{S}$ is fully determined by the probabilistic features of the information sets contained in $\mathbf{S}$.

If Bayesian Coherentism is correct, a highly coherent set is more likely to be true than a less coherent set. Hence, by proving Bayesian Coherentism, the attempt to explain epistemic justification with the notion of coherence can be formally supported.

One way to check if Bayesian Coherentism holds is to find counterexamples to it. If there exists an information set which, in comparison with another set, is more coherent but less likely to be true, Bayesian Coherentism can be falsified. To find this desired counterexample, Bovens and Hartmann (2003, pp.14-19) construct an information gathering model which allows us to calculate the change of probability of an information set after receiving new information from a group of partially reliable sources. With this model, they prove the existence of pairs of information sets (k, k') such that k has greater probability when the reliability of information sources is within a certain interval, while k' has greater probability in other occasions. From (BC2), we know that given any ideal coherence measure, either the coherence of k is greater than k' or the other way round. Bovens and Hartmann hence conclude that there is no coherence measure which guarantees that a set with greater coherence, compared with a less coherent one, is always more likely to be true. In this section, I will introduce their information gathering model, and explain how they derive the so-called *impossibility results* with this model (Bovens and Hartmann 2003: pp.10-22).

The first step for constructing this information gathering model is to measure the reliability of information sources. Suppose there are n independent and partially reliable sources. Each source i provides a piece of information R_i. The information set in question is thus $\{R_1, ..., R_n\}$. Let R_i be a *fact variable*, and $REPR_i$ a *report variable* which can take either $REPR_i$ or $\neg REPR_i$ as value. $REPR_i$ stands for the proposition that after consulting the proper source, there is a report that R_i is the case, while $\neg REPR_i$ stands for the contrary that, after consulting a proper source, there is no report saying that R_i is the case.

An intuitive way to model the reliability of sources is to compare the number of true reports with the number of false reports. Given a probability distribution $Pr(\cdot)$ over the set $\{R_1, ..., R_n, REPR_i, ..., REPR_n\}$ which satisfies the constraint that information sources are mutually independent and partially reliable, we can define two parameters p_i and q_i as:

$$p_i \stackrel{\text{def}}{=\!=} Pr(REPR_i|R_i) \ ; \ q_i \stackrel{\text{def}}{=\!=} Pr(REPR_i|\neg R_i)$$

p_i is the probability that source i makes a positive report for a obtaining fact, which is the probability that p_i reports correctly, while q_i is the probability that i reports incorrectly. We call p_i the *true-positive rate*, and q_i the *false-positive rate* of i. Being fully reliable, a witness would not make any false report. Therefore, the false-positive rate of that witness is 0. On the other hand, a fully unreliable witness would have $p_i = q_i$, which means that the witness reports randomly. Since we have assumed that all the sources in question are partially reliable, we stipulate that $p_i > q_i > 0$. For sake of simplicity, we further assume that $p_i = p$ and $q_i = q$, namely all sources have equal reliability. We can then define the parameter of reliability of information sources r in terms of q and p:

$$r = 1 - \frac{q}{p}$$

We further define the *weight vector* for an information set:

Definition 3.2.2. Weight vector

Let a_i stands for the sum of joint probabilities of all combinations of i negative and $n - i$ positive occurrences of $R_1, ..., R_n$. The weight vector of an information set is $\langle a_1, ..., a_n \rangle$.

For instance, given an information set $\{R_1, R_2, R_3\}$, a_2 is the sum of probabilities of $\{\neg R_1, \neg R_2, R_3\}$, $\{R_1, \neg R_2, \neg R_3\}$ and $\{\neg R_1, R_2, \neg R_3\}$.

Let the function $Pr^*(\cdot)$ represent the posterior probability after receiving the reports from sources, that is:

$$Pr^*(R_1, ..., R_n) = Pr(R_1, ..., R_n|REPR_1, ..., REPR_n)$$

We can calculate posterior probability with the parameters defined:

Definition 3.2.3. Posterior probability

$$Pr^*(R_1, ..., R_n) = \frac{a_o}{\sum_{i=0}^{n} a_i (1 - r)^i}$$

This formula calculates the posterior probability of an information set after updated with the report of a group of sources. The denominator represents the probability of all cases in which i sources are reporting incorrectly. For example, $a_1(1 - r)^1$ of the information set $\{R_1, R_2, R_3\}$ is the sum of probability of $\{\neg R_1, R_2, R_3\}$, $\{R_1, \neg R_2, R_3\}$ and $\{R_1, R_2, \neg R_3\}$ times the probability that one of the sources reports incorrectly, which is $(1-r)^1$. By summing up $a_i(1-r)^i$, all possible cases are taken into consideration. We can thus calculate the posterior probability of the information set after updated with the reliability of sources.

If we can find a pair of information sets (k, k') for which the posterior probability of k is greater than k' when r is below a certain threshold, while the posterior probability of k' is greater than k when r is above that threshold, then it can be shown that Bayesian Coherentism is false, for greater coherence does not guarantee greater probability.

Proposition 3.2.1. Counterexample to Bayesian Coherentism

Consider information sets k with the weight vector $\langle a_0, a_1, a_2, a_3\rangle = \langle 0.05, 0.3, 0.1, 0.55\rangle$ and k' with $\langle a'_0, a'_1, a'_2, a'_3\rangle = \langle 0.05, 0.2, 0.7, 0.05\rangle$. Suppose the coherence of k is greater than k', given $r \in (0.8, 1)$, the posterior probability of k' is greater than the posterior probability of k. Suppose otherwise that the coherence of k' is greater than k, given $r \in (0, 0.8)$, the posterior probability of k is greater than the posterior probability of k'. For instance, take $r = 0.9$,

$$Pr^*(k) = \frac{0.05}{0.05 + 0.3(1-0.9) + 0.1(1-r)^2 + 0.55(1-0.9)^3} = \frac{0.05}{0.08065} \approx 0.62$$
$$Pr^*(k') = \frac{0.05}{0.05 + 0.2(1-0.9) + 0.7(1-0.9)^2 + 0.05(1-0.9)^3} = \frac{0.05}{0.07705} \approx 0.65$$

In this case, $Pr^*(k') > Pr^*(k)$. But assuming $r = 0.5$, the posterior probability is:

$$Pr^*(k) = \frac{0.05}{0.05 + 0.3(1-0.5) + 0.1(1-0.5)^2 + 0.55(1-0.5)^3} = \frac{0.05}{0.29375} \approx 0.17$$
$$Pr^*(k') = \frac{0.05}{0.05 + 0.2(1-0.5) + 0.7(1-0.5)^2 + 0.05(1-0.5)^3} = \frac{0.05}{0.33125} \approx 0.15$$

In this case, $Pr^*(k) > Pr^*(k')$. Thus, the pair (k, k') can be taken as an example which falsifies the claim that an information set with greater coherence also have greater likelihood of truth. This is what Bovens and Hartmann call the *impossibility results*. It immediately follows that the search for a truth-conducive coherence measure can never be accomplished in this setting.

3.3 The Bovens-Hartmann measure

The impossibility results pose serious threat to Bayesian Coherentism. To solve this problem, Bovens and Hartmann suggest (2003, p.22) to revise (BC2) and adopt a weaker version of Bayesian Coherentism. According to (BC2), a coherence ordering is fully determined by the probabilistic features of the sets in **S**. It can be divided into two parts:

(BC2$_a$) The binary relation of '...being no less coherent than' over **S** is fully determined by the probabilistic features of the information sets contained in **S**.

($BC2_b$) The binary relation of '...being no less coherent than' is a total ordering.

Instead of a total ordering, as ($BC2_b$) states, we can claim that there exists a *quasi-ordering* of coherence of information sets in $\mathbf{S}$. That is, to evade the problem, we have to abandon the idea that every pair of information sets in $\mathbf{S}$ are comparable. Formally speaking, let $\succeq$ stand for the binary relation of 'being no less coherent than', the following condition should be met for a proper coherence measure:

> For all $S, S' \in \mathbf{S}$, if $S = \{R_1, ..., R_n\}$, $S' = \{R'_1, ..., R'_n\}$ and $Pr(R_1, ..., R_n) = a_0 = a'_0 = Pr(R'_1, ..., R'_n)$, then $S \succeq S'$ iff $Pr^*(R_1, ..., R_n) \geq Pr^*(R'_1, ..., R'_n)$ for all values of the reliability parameter $r \in (0, 1)$.

With this condition, cases violating the original (BC2) can be excluded, which validates weak Bayesian Coherentism.

Although excluding problematic cases may save Bayesian Coherentism, this solution has an obvious deficiency. With this condition, one can only compare information sets of equal size. For an ideal coherence measure, we expect it to be more flexible, which would allow us to compare between information sets of unequal size. Therefore, a more general coherence measure is called for.

Instead of measuring coherence of a set with agreement or confirmation between its elements, Bovens and Hartmann take a different approach. Their idea is that coherence should be defined in terms of its primary function, which is *boost of confidence* (Bovens and Hartmann 2003, pp.28-39 Ch.2). Given two information sets, people tend to have greater confidence in the one which is more coherent. Thus, boost of confidenceis one the defining features of coherence, and should be taken as the core factor in measuring the degree of coherence of an information set. To formally define boost of confidence, we can take it as the ratio between prior and posterior probability of an information set, namely:

Definition 3.3.1. Boost of confidence

$$b(\{R_1, ..., R_n\}) \stackrel{\text{def}}{=\!=} \frac{Pr^*(R_1, ..., R_n)}{Pr(R_1, ..., R_n)}$$

That is, if a set is more coherent than another, the probability of it raises significantly when updated with reports that are equally reliable.

However, boost of confidence alone is insufficient to be taken as a degree of coherence, for it is still determined by the reliability of information sources, which is a factor that should be ruled out while measuring the coherence of an information set. If the coherence of an information set depends on the reliability of sources, we may have two information sets which are identical in content, but different in coherence, and that is intuitively unacceptable. To eliminate the influence of reliability, the boost of an information set should be compared with a fixed reference point. An ideal reference point to compare with, as Bovens and Hartmann (2003, p.35) suggest, is the *maximal boost of confidence*, namely the boost of confidence upon receiving the same set of information in a maximally coherent form. This idea can be better illustrated with a more concrete example. Consider a specific information set S which has certain degree of coherence and thus leads to certain degree of boost of confidence. Suppose that S is maximally coherent, it leads to the maximal boost of confidence. The ratio between the maximal degree of boost and actual degree of boost shows originally how coherent S is. If we want to compare the degree of coherence of two different information sets S_1 and S_2, we can keep the reliability of sources fixed, and calculate the ratio of actual boost to maximal boost of both S_1 and S_2. By comparing the ratios, one can compare the degree of coherence of two information sets.

To compare the coherence of two different sets, one needs to define the maximal coherence of an information set, which leads to maximal boost of confidence. For a maximally coherent set of information with n elements, the weight vector is $\langle a_0, 0, ..., 0, a_n \rangle$, that is, elements $\{R_1, ..., R_n\}$ of the information set are either true altogether or false altogether. The probability of cases like $\{R_1, \neg R_2, ..., \neg R_n\}$ or $\{\neg R_1, R_2, ..., R_n\}$ is 0. Maximal posterior probability can thus be defined as:

Definition 3.3.2. Maximal posterior probability

$$Pr^{max*}(R_1, ..., R_n) = \frac{a_o}{a_0 + a_n(1-r)^n}$$

Given $Pr^{max*}(\cdot)$, the maximal possible boost of a set $\{R_1, ..., R_n\}$ is:

Definition 3.3.3. Maximal boost of confidence

$$b^{max}(\{R_1, ..., R_n\}) \stackrel{\text{def}}{=\!=} \frac{Pr^{max*}(R_1, ...R_n)}{Pr(R_1, ..., R_n)}$$

With $b^{max}(\cdot)$, we can further define a measure comparing the actual and maximal boost of a belief set.

$$c_r(\{R_1, ..., R_n\}) \stackrel{\text{def}}{=\!=} \frac{b(\{R_1, ..., R_n\})}{b^{max}(\{R_1, ..., R_n\})} = \frac{Pr^*(R_1, ..., R_n)}{Pr^{max*}(R_1, ..., R_n)} = \frac{a_0 + (1-a_0)(1-r)^n}{\sum_{i=0}^{n} a_i(1-r)^i}.$$

c_r measures the ratio of the actual boost to the maximal boost. However, c_r still involves the reliability parameter r which has to be separated from the degree of coherence. To get this around, we need to further define a difference function

$$f_r(S, S') = c_r(S) - c_r(S')$$

Given the same reliability of sources, the difference function compares the boost of two different information sets. That is, given a fixed r, S is more coherent than S' implies that $c_r(S)$ is greater than $c_r(S')$. The binary relation *no less coherent than*, namely the relation represented by $\succeq$, is thus defined as

Definition 3.3.4. Comparing coherence

For two information sets $S, S' \in \mathbf{S}, S \succeq S'$ iff $f_r(S, S') \geq 0$ for all values of $r \in (0, 1)$.

With this definition, we can compare the coherence of different sets with the difference function $f_r(\cdot, \cdot)$. It is to be noticed that, first, since cases that lead to the impossibility results are ruled out by the condition $f_r(S, S') > 0$, the *no less coherent than* relation is just a quasi-ordering, rather than a total ordering. Second, Bovens and Hartmann do not provide an absolute measure for the precise degree of coherence which assigns a specific degree to every information set, but only a way to compare the coherence of different sets. This measure excludes the problematic cases, and is compatible with weak Bayesian Coherentism.

3.4 Douven and Meijs' revision

Although the Bovens-Hartmann measure is invulnerable to the impossibility results and thus should be taken as an ideal way of comparing coherence, Douven and Meijs (2005) find pairs of information sets that are excluded by the constraint specified in Bovens and Hartmann's work (2003, p.36).

Example 3.4.1. Consider the following case: Kate is taking a flight which has 0.04 probability of flying to the North Pole, 0.49 of flying to the South Pole and 0.47 of flying to New Zealand. The probability of Kate seeing a penguin in the South Pole is $\frac{10}{49}$, while the probability of the same event is $\frac{1}{47}$ in New Zealand and 0 in the North Pole. When Kate arrives at the destination, she does see an animal, but cannot make sure if it is a penguin. Short after that, she receives two sets of information:

$S_1 =$ {The animal you saw is a penguin, You are in the South pole}

$S_2 =$ {The animal you saw is a penguin, You are in the North pole}

By the Bovens-Hartmann measure, we can compare the coherence of S_1 and S_2 as:

$$f_r(S_1, S_2) = \frac{0.1 + 0.9(1-r)^2}{0.1 + 0.4(1-r) + 0.5(1-r)^2} - \frac{(1-r)^2}{0.15(1-r) + 0.85(1-r)^2}$$

Assume that $r = 0.5$, we may derive:

$$f_{0.5}(S_1, S_2) = \frac{0.1 + 0.9(0.25)}{0.1 + 0.4(0.5) + 0.5(0.25)} - \frac{0.25}{0.15(0.5) + 0.85(0.25)} < 0$$

With the Bovens-Hartmann measure, $f_{0.5} < 1$, which implies that S_1 and S_2 are incomparable and should be excluded from our coherence quasi-ordering. This result is pretty counter-intuitive. With the assumption that the probability of seeing a penguin in the North Pole is 0, we know that S_2 is far less coherent than S_1.

Meijs (2007) provides other cases that are intuitively comparable in coherence but excluded by Bovens and Hartmann's constraint.

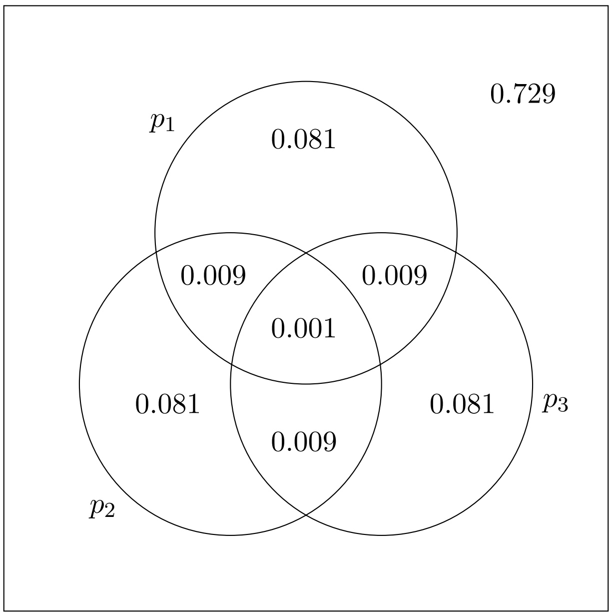
Example 3.4.2. Consider the following two sets:

$$S^I = \{\{p_1 : \text{That dog is brown}\}, \{p_2 : \text{It is raining outside}\}, \{p_3 : \text{Tokyo is the capital of Japan}\}\}$$

$$S^P = \{\{p_4 : \text{This bird is black}\}, \{p_5 : \text{This bird is a crow}\}, \{p_6 : \text{This bird has a life-long mate}\}\}$$

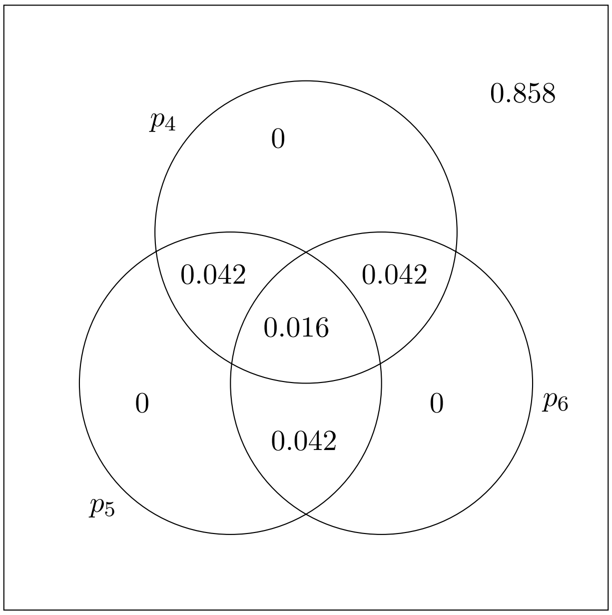
Elements in S^I are mutually independent, while elements in S^P support each other. Therefore, S^P is obviously more coherent than S^I. Suppose the probability of elements in S^I is distributed as shown in figure 1:

Figure 1: probabilities of set S^I



Further assume that the probability of elements in S^P is distributed as:

Figure 2: probabilities of set S^P



The difference function $f_r(S^P, S^I)$, given the two probability distributions, is as follows:

$$\frac{0.016 + 0.984(1-r)^3}{0.016 + 0.126(1-r) + 0.858(1-r)^3} - \frac{0.001 + 0.999(1-r)^3}{0.001 + 0.027(1-r) + 0.243(1-r)^2 + 0.729(1-r)^3}$$

Assuming $r = 0.5$, $f_{0.5}(S^P, S^I) < 0$. This case is also excluded by the Bovens-Hartmann measure, which is again counter-intuitive.

The problem of Bovens and Hartmann's framework, as Meijs (2007, p.3) sees, is the claim that a set is maximally coherent when all its elements are equivalent. If we adopt this requirement, adding any proposition to a maximally coherent set does not raise its coherence. This result may lead to an unacceptable consequence. Consider a maximally coherent set $S = \{p_1, p_2, p_3\}$. If, from an independent and partially reliable source, one receives a proposition p_4 which is equivalent to $p_1 \wedge p_2 \wedge p_3$. Adding p_4 to S does raise one's confidence in the set S. But according to Bovens and Hartmann, adding p_4 does not raise the coherence of S, for it is already maximally coherent. Therefore, there is some factor other than coherence which leads to boost of confidence. If we want to keep the core idea of the Bovens-Hartmann measure that coherence is to be measured in terms of boost of confidence, the maximality requirement should be abandoned, that is, there should not exist a maximal value of coherence.

Instead of comparing the actual boost of a set with its maximal possible boost, Meijs (2007, pp.9-13) suggests to compare the actual boost with the minimal boost, which is the boost of confidence of an independent set. The boost of confidence of an independent set is solely determined by the reliability of sources and the unconditional probabilities of information, and can be used as a reference point representing *neutral* coherence. Put it formally, given an information set $R = \{R_1, ..., R_n\}$, we can construct another set $R^I = \{R^I_1, ..., R^I_n\}$, and assume that $\forall R^I_i, R^I_j \in R^I, R^I_i \cap R^I_j = \varnothing$.

Let $b_{ind}(R)$ be the boost of confidence of R^I, the coherence of R can be measured as:

$$c^I_r(R) \stackrel{\text{def}}{=\!=} \frac{b(R_1 \wedge ... \wedge R_n)}{b_{ind}(R_1 \wedge ... \wedge R_n)}$$

We can then define the difference function in a similar way:

$$f^I_r(R, R') = c^I_r(R) - c^I_r(R')$$

Meijs (2007, pp.11-13) shows that this revised measure keeps the core idea of the Bovens-Hartmann measure, but is free from its counter-intuitive outcomes. Hence, the revised Bovens-Hartmann measure should be used as an appropriate measure for us to compare the coherence between different sets.

3.5 Saving coherence

With Meijs' revision of the Bovens-Hartmann measure, we do have a measure which allows us to compare degrees of coherence between different information sets without counterintuitive outcomes. However, having an acceptable way of measuring coherence does not undermine the impossibility results any bit. The fact that coherence is not a truth-conducive notion remains intact, which means that knowing a set S is more coherent than another set S' does not imply that S is more likely to be a proper justification of knowledge than S'. Since the primary function of coherence is to account for epistemic justification, As long as the impossibility results hold, people do not have any good reason to make comparison of coherence between different sets.

To save the notion of coherence, one needs to provide new epistemic ideals other than truth. If it could be shown that coherence is conducive to these new ideals, it may still be regarded as an important notion in epistemology. Some epistemologists (Olsson and Schubert 2007, Schubert 2012a) claim that coherence is *reliability-conducive*, namely that when an agent finds an information set highly coherent, that agent would consider the information sources of this set highly reliable. This idea is intuitively persuasive. Imagine that a group of witnesses is making reports about a certain fact. If the witnesses are unreliable, they would make testimonies which differ from what they observe. Since there are many non-factual reports one could make concerning a single fact, it is quite likely that a set of testimonies provided by unreliable witnesses is incoherent. One can hence infer that when a group of witnesses provide a set of highly coherent information, these witnesses are more likely to be reliable.

Some other philosophers (Dietrich and Moretti 2005, Moretti 2007) claim that coherence is *confirmation-conducive*. That is, if a piece of evidence confirms a single proposition in a set which is sufficiently coherent, this piece of evidence also confirms other propositions in the set. In other words, confirmation of certain proposition in a sufficiently coherent set *'transmits'* to other propositions in that set. The idea that coherence is confirmation-conducive is also plausible. A coherent information set indicates certain fact in reality. As a result, a piece of evidence confirming an element of that set must be related to that very fact the information set is concerned with, and therefore confirms other elements in the set.

The attempts of proving that coherence is reliability an confirmation-conducive show that although coherence is not truth-conducive, it can be conducive to other epistemic ideals, and hence be *indirectly* truth-conducive. A higher degree of reliability of sources, as well as a higher degree of confirmation, implies greater likelihood of being true. If coherence is indeed reliability or confirmation-conducive, this notion may regain its place in contemporary epistemology. There are also some minor attempt to save coherence, such as taking coherence as characterizing *best explanation* (Glass 2007).

3.6 Coherence as a reliability-conducive notion

The idea that coherence is *reliability-conducive* can be traced back to C. I. Lewis (1946), where he states:

> For any one of these reports, taken singly, the extent to which it confirms what is reported may be slight. And antecedently, the probability of what is reported may also be small. But congruence of the reports establishes a high probability of what they agree upon, by principles of probability determination which are familiar: on any other hypothesis than that of truth-telling, this agreement is highly unlikely; the story any one false witness might tell being one out of so very large a number of equally possible choices. (p.246)

If a set of testimonies provided by a group of witnesses is coherent, we may infer that all the witnesses are highly reliable, reporting correctly of what they observe. This idea can be illustrated by considering the opposite situation in which the witnesses are unreliable. Suppose that a group of independent unreliable witnesses are making reports concerning a specific event. Being unreliable, the witnesses make incorrect reports which deviate from the fact they observe. Since non-factual testimonies largely outnumbers factual testimonies, the chance of the testimonies being incoherent would be much greater than the chance of testimonies being coherent.

Olsson and Schubert (2007) prove that although there does not exist any coherence measure which is truth-conducive, there are coherence measures that are *reliability-conducive* in a specific scenario, such that for any pair of information sets S, S', if the degree of coherence of S is greater than S', the information sources of S are more reliable than S'.

To justify this claim, we must first define the notion of *reliability-conduciveness*. As preliminary, we need to introduce the *basic Lewis scenario* which involves two *equivalent* evidence[2]. Let H stand for the hypothesis in question, E_i be the evidence that source i asserts that H is true, R_i be the proposition that source i is reliable and U_i stands for the proposition that i is unreliable. A basic Lewis scenario is defined as follows:

Definition 3.6.1. A basic Lewis scenario is a pair $(\mathbf{S}, \mathbf{P})$ where $\mathbf{S} = \{\langle E_1, H\rangle, \langle E_2, H\rangle\}$ and $\mathbf{P}$ is a class of probability distributions defined on the algebra generated by propositions $E_1, E_2, R_1, R_2, U_1, U_2$ and H such that $Pr \in \mathbf{P}$ if and only if:

(i) $Pr(R_i) + Pr(U_i) = 1$

(ii) $0 < Pr(H) < 1$

(iii) $Pr(E_1|H, R_1) = 1 = Pr(E_2|H, R_2)$

(iv) $Pr(E_1|\neg H, R_1) = 0 = Pr(E_2|\neg H, R_2)$

(v) $Pr(E_1|H, U_1) = Pr(H) = Pr(E_2|H, U_2)$

(vi) $Pr(E_1|\neg H, U_1) = Pr(H) = Pr(E_2|\neg H, U_2)$

(vii) $Pr(R_i|H) = Pr(R_i) = Pr(R_i|\neg H)$

(viii) $Pr(U_i|H) = Pr(U_i) = Pr(U_i|\neg H)$

(ix) $0 < Pr(R_1) = Pr(R_2) < 1$

(i) states that for any information source, its reliability profile involves only reliability and unreliability. (ii) states that the hypothesis is neither certainly true, nor certainly false. By (iii) and (iv), the probability of a reliable source making correct report is 1, while the probability of a reliable source reporting incorrectly is 0. On the other hand, (v) and (vi) state that the probability of H is not affected by the proposition provided by an unreliable source. That is, the probability of the source reporting that H is true, given that the source is unreliable and H is indeed true, it the same as the probability of H. (vii) and (viii) indicates that the reliability or unreliability of a source is not affected by the hypothesis. By (ix), the truth of sources being reliable is neither certainly true nor certainly false.

[2]Here the term evidence simply refers to the proposition provided by certain witness. The terms belief, information and proposition, in the current context, are used interchangeably. In the following sections, I take *proposition* as the standard term for this kind of linguistic entity.

We can further define *informativeness* and *reliability-conduciveness* of a coherence measure:

Definition 3.6.2. Informativeness

A coherence measure C is *informative* in a basic Lewis scenario ($\mathbf{S}$,$\mathbf{P}$) if and only if there are $Pr, Pr' \in \mathbf{P}$ such that $C_{Pr}(S) \neq C_{Pr'}(S)$.

Definition 3.6.3. Reliability-conduciveness

A coherence measure C is *reliability-conducive ceteris paribus* in a basic Lewis scenario ($\mathbf{S}, \mathbf{P}$) if and only if: if $C_{Pr}(S) > C_{Pr'}(S)$, then $Pr(R_i|E_1, E_2) > Pr'(R_i|E_1, E_2)$ for all $Pr, Pr' \in \mathbf{P}$ such that $Pr(R_i) = Pr'(R_i)$.

To judge if a coherence measure is reliability-conducive, we need a way to compute the change of reliability after receiving evidence from multiple sources. Bovens and Hartmann (2003) suggest to calculate change of reliability in the following way: in a single source case, let E be the propositional variable which takes either the presence or absence of evidence as value, R be the variable that the source is reliable or unreliable and H the truth or falsity of an hypothesis. Knowing that a source is unreliable, the evidence provided by that source does not influence the probability of the hypothesis in question. Therefore, we can first assume:

$$Pr(H|\neg R) = Pr(H|E, \neg R)$$

With this assumption, Bovens and Hartmann (2003, p.57) derive the *randomization parameter* a:

$$Pr(E|H, \neg R) = Pr(E|\neg H, \neg R) = a$$

If the source is unreliable, it provides positive report of the hypothesis randomly. a stands for the probability of an unreliable source making positive report. Next, we assume that:

$$Pr(E|H, R) = 1 \text{ and } Pr(E|\neg H, R) = 0$$

A reliable source always makes positive report when the hypothesis is true, and always makes negative report when the hypothesis is false. We assume that H and R are independent, namely the reliability of a source does not vary with the probability of hypothesis.

Let ρ be the reliability parameter of sources $Pr(R)$, and h the probability of hypothesis $Pr(H)$, we can compute the posterior reliability $Pr^*(R)$ as

$$Pr^*(R) = Pr(R|E) = \frac{Pr(R, E)}{Pr(E)}$$

By expansion, it is equivalent to

$$\frac{\Sigma_H Pr(H,R,E)}{\Sigma_{H,R} Pr(H,R,E)}$$

which, by the chain rule, is

$$\frac{\Sigma_H Pr(E|H,R)Pr(H|R)Pr(R)}{\Sigma_{H,R} Pr(E|H,R)Pr(H|R)Pr(R)}$$

Given the assumption that R and H are independent variables, we can derive that

$$\frac{\Sigma_H Pr(E|H,R)Pr(H)Pr(R)}{\Sigma_{H,R} Pr(E|H,R)Pr(H)Pr(R)}$$

Hence, the posterior reliability is

$$Pr^*(R) = \frac{h\rho + 0 \cdot \rho(1-h)}{h\rho + 0 \cdot \rho(1-h) + ah(1-\rho) + a(1-\rho)(1-h)} = \frac{h\rho}{h\rho + a(1-\rho)}$$

The difference between prior and posterior reliability ΔREL, then, is:

$$Pr^*(R) - Pr(R) = \frac{h\rho}{h\rho + a(1-\rho)} - \rho = (h-a)\frac{\rho(1-\rho)}{h\rho + a(1-\rho)}$$

This formula computes the posterior reliability in a case where a single piece of evidence is given.

We can generalize it for cases with multiple sources. Suppose there are n sources, we want to know the value of:

$$Pr^{*n}(R_i) = Pr(R_i|E_1, ..., E_n)$$

As in single source cases, we need several assumptions concerning the independence of sources. First, the value of the report source i provides depends only on the reliability of i and the truth-value of hypothesis. That is, E_i is independent of all other reports and the reliability of all other sources. Second, the reliability of a source i is independent of the reliability of other sources and the truth-value of hypothesis.

We can define the characteristic of sources in the same way as in the single source case:

$$Pr(E_i|H, \neg R_i) = Pr(E_i|\neg H, \neg R_i) = a$$

$$Pr(E_i|H, R_i) = 1 \text{ and } Pr(E_i|\neg H, R_i) = 0$$

$$Pr(R_i) = \rho$$

and introduce another parameter x which represents the likelihood of a single report:

$$x = \frac{Pr(E_i|\neg H)}{Pr(E_i|H)} = \frac{a(1-\rho)}{\rho + a(1-\rho)}$$

The numerator of x is the probability that the source is unreliable, and the denominator is the probability that the source gives the correct report, which is the sum of the probability that the source is reliable and the probability that the source is unreliable but by chance gives the correct report.

Given these parameters, the posterior reliability can be defined as:

$$Pr^{*n}(R_i) = Pr(R_i|E_1, ..., E_n) = \frac{h(1-x)}{h + x^n(1-h)}$$

We can observe that $Pr^{*n}(R_i)$ is a strictly decreasing function of the prior probability of the hypothesis,[3] namely the prior probability of hypothesis decreases when the reliability of source raises. Hence, for a coherence measure C to be reliability-conducive, C has to be a strictly decreasing function of the prior probability of the hypothesis. In other words, if C is reliability-conducive, when the degree of coherence raises and every other factors are fixed, the prior probability of hypothesis decreases.

With this observation, we may check which coherence measure is reliability-conducive in a basic Lewis scenario. Recall that given a set of propositions $S = \{P_1, ..., P_n\}$, Shogenji measures its coherence as:

$$C_{Sh}(\{P_1, ..., P_n\}) = \frac{Pr(P_1 \wedge ... \wedge P_n)}{Pr(P_1)...Pr(P_n)}$$

Given a basic Lewis scenario, if we measure the coherence of two equivalent propositions E_1 and E_2, the result is:

$$C_{Sh}(\{E_1, E_2\}) = \frac{Pr(E_1 \wedge E_2)}{Pr(E_1)Pr(E_2)} = \frac{1}{Pr(E_1)}$$

Measured with C_{Sh}, the lower the prior probability of E_1 is, the greater degree of coherence $\{E_1, E_2\}$ is and *vice versa*. Since Shogenji's measure is a strictly increasing function of the number of propositions, and a strictly decreasing function of the prior probability of hypothesis, it is indeed reliability-conducive. On the other hand, if we measure two equivalent propositions E_1 and E_2 with Olsson's measure, the result would be:

$$C_O(\{E_1, E_2\}) = \frac{Pr(E_1 \wedge E_2)}{Pr(E_1 \vee E_2)} = 1$$

In this scenario, the result of Olsson's measure is a constant. The degree of coherence does not raise when the prior probability decreases. Hence, Olsson's measure is not reliability-conducive in this scenario.

[3] This result can be easily observed in a case with two information sources. Given that $n = 2$, $\rho^* = \frac{(\rho-\rho^2)(\rho^2+h-h\rho^2)-(\rho^2+\rho h-h\rho^2)(1-\rho^2)}{(\rho^2+h-h\rho^2)^2}$. The numerator $\rho^2(\rho-1)$ is strictly decreasing function when $1 > \rho > 0$. Since the denominator is always positive, ρ^* is a strictly decreasing function.

Schubert (2012a) further proved that Shogenji's measure is the only reliability-conducive coherence measure in a more general scenario with more than two equivalent evidence.

Definition 3.6.4. A *scenario with equivalent evidence* is a pair $(\mathbf{S}, \mathbf{P})$ where $\mathbf{S} = \{S_n | S_n = \{\langle E_1, H\rangle, ..., \langle E_n, H\rangle\}$ and $n \geq 2\}$ and $\mathbf{P}$ is a class of probability distributions such that $S_n \in \mathbf{S}$, $Pr \in \mathbf{P}$ if and only if:

(i) $Pr(H|E_i, R_i) = 1$ for $i = 1, ..., n$.

(ii) $Pr(H|E_i, \neg R_i) = Pr(H|\neg R_i)$ for i=1,...,n.

(iii) $0 < Pr(H) < 1$

(iv) $0 < Pr(R_i|E_i) < 1$ for $i = 1, ..., n$.

(v) $E_i \perp\!\!\!\perp E_1, R_1, ..., E_{i-1}, R_{i-1}, E_{i+1}, R_{i+1}, ..., E_n, R_n | R_i, H$ for $i = 1, ..., n$.

(vi) $R_i \perp\!\!\!\perp R_1, ..., R_{i-1}, R_{i+1}, ..., R_n, H$ for $i = 1, ..., n$.

(vii) $Pr(R_i|E_i) = Pr(R_j|E_j)$ for $i, j = 1, ..., n$.

Since the number of evidence is no longer fixed, this scenario is more general than the basic Lewis scenario. These conditions are similar to the conditions in a basic Lewis scenario. (i) states that the probability of hypothesis, conditional on the fact that the information source i is reliable and the fact that evidence i provides supports H, is 1. (ii) states that evidence provided by an unreliable source does not affect the probability of the hypothesis. (iii) is the condition that the hypothesis is neither absolutely true nor absolutely false, while (iv) is the condition that given the evidence i provided, a source i is neither absolutely reliable nor absolutely unreliable. (v) is an independence assumption concerning E_i, which states that the evidence from source i is independent from other evidence and the reliability of other sources. Similarly, (vi) says that the reliability of i is independent from the reliability of other sources. The last condition (vii) says that the reliability of a source i, given the evidence provided by it, equals to the reliability of any other source j given the evidence j provided.

Given this scenario, we can define reliability-conduciveness in a more subtle way.

Definition 3.6.5. Specificity informativeness

A coherence measure C is *specificity informative* in a scenario $(\mathbf{S}, \mathbf{P})$ with a fixed number of equivalent evidence if and only if for all $S_n \in \mathbf{S}$ there are $Pr, Pr' \in \mathbf{P}$ such that $C_{Pr}(S_n) \neq C_{Pr'}(S_n)$.

Definition 3.6.6. Reliability-conduciveness with fixed number of evidence

A coherence measure C is *reliability-conducive ceteris paribus* in a scenario $(\mathbf{S}, \mathbf{P})$ with a fixed number of equivalent evidence if and only if: if $C_{Pr}(S_j) > C_{Pr'}(S_k)$ and $j \neq k$, then $Pr(R_i|E_1, ..., E_j) > Pr'(R_i|E_1, ..., E_k)$ for all $Pr, Pr' \in \mathbf{P}$ such that $Pr(R_i|E_i) = Pr'(R_i|E_i)$.

Definition 3.6.7. Size informativeness

A coherence measure C is *size informative* in a scenario with equivalent evidence $(\mathbf{S}, \mathbf{P})$ if and only if for all $Pr, Pr' \in \mathbf{P}$ such that $Pr(H) = Pr'(H)$ there are $S_j, S_k \in \mathbf{S}$ where $j \neq k$ such that $C_{Pr}(S_j) \neq C_{Pr'}(S_k)$.

Definition 3.6.8. Reliability-conduciveness with fixed specificity

A coherence measure C is *reliability-conducive ceteris paribus* in a scenario with equivalent evidence $(\mathbf{S},\mathbf{P})$ where the prior probability of the hypothesis is held fixed if and only if: if $C_{Pr}(S_j) > C_{Pr'}(S_k)$ and $j \neq k$, then $Pr(R_i|E_1, ..., E_n) > Pr'(R_i|E_1, ..., E_m)$ for all $Pr, Pr' \in \mathbf{P}$ such that $Pr(R_i|E_i) = Pr'(R_i|E_i)$ and $Pr(H) = Pr'(H)$.

Similar to the case in a basic Lewis scenario, a coherence measure C is specifically informative and reliability-conducive *ceteris paribus* in this scenario if and only if C is a decreasing function of the prior probability of hypothesis. On the other hand, for C to be size informative and reliability-conducive, C needs to be an increasing function of the number of evidence.

It can be seen that Shogenji's measure satisfies both conditions. Given a set of evidence $\{E_1, ..., E_n\}$, the coherence of the set, according to Shogenji's measure, is:

$$C_{Sh}(\{E_1, ..., E_n\}) = \frac{Pr(E_1 \wedge ... \wedge E_n)}{Pr(E_1)...Pr(E_n)} = \frac{Pr(E_n)}{Pr(E_1)...Pr(E_n)} = Pr(E_1)^{1-n}$$

which is an increasing function of n. Also, given the number of evidence fixed, the less probable E_n is, the greater $C_{Sh}(\{E_1, ..., E_n\})$ is. We can hence claim that Shogenji's measure is reliability-conducive in the scenario.

Shogenji's measure is actually the only reliability-conducive coherence measure in this scenario. To prove this claim, we need to show that every other coherence measure fails to satisfy the conditions proposed. It can be easily shown that Olsson's measure is not a strictly increasing function of the number of evidence, in the scenario with equivalent evidence, for a set $E = \{E_1, ..., E_n\}$ such that all $E_i \in E$ are equivalent, its degree of coherence, as measured in the way Olsson suggests, does not differ from any other set $E^* = \{E_1, ..., E_m\}$ such that all $E_j \in E^*$ are equivalent and $n \neq m$. Olsson's measure is not an increasing function of the number of evidence, and hence is not reliability-conducive.

All other coherence measures besides Olsson's can be generated with Douvens and Meijs' scheme. As described in section 1.7, this scheme is defined as follows:

Definition 3.6.9. Douven and Meijs' scheme of coherence measure

$$C_m(S) \stackrel{\text{def}}{=\!=} \frac{\sum_{i=1}^{[\![S]\!]} m(\hat{S}_i)}{[\![S]\!]}$$

in which $[S]$ is the set of pairs of non-overlapping subsets of S, $\langle \hat{S}_1, ..., \hat{S}_{[\![S]\!]} \rangle$ is an ordering of elements of $[S]$, and $[\![S]\!]$ is the cardinality of S. In a scenario of equivalent evidence, since $S = \{b_1, ..., b_n\} = \{b_1\}$, there does not exist any pair of non-overlapping subsets of S, which means that $[S]$ is empty and cannot be measured with this scheme. Hence, we need to make a slight modification by taking S as an ordered set $\langle b_1, ..., b_n \rangle$.

With this scheme, we can generate many coherence measures by plugging in different confirmation measures for $m(\cdot)$. In the scenario given, suppose that A is the proposition the agents agree on and provide as evidence. Since all the evidence in the scenario are equivalent, every subset of S is equivalent to A. Hence, $C_m(S)$ is equivalent to $m(\langle A, A \rangle)$, namely in this scenario, all results generated from these coherence measures are just the confirmation between A and itself. Suppose we add one additional evidence A^* to the set S, since $A^* = A$ by assumption, the degree of coherence would be equivalent to $m(\langle A, A \rangle)$. We can thus conclude that all the measures generated from this scheme are not size-informative, for adding new pieces of evidence does not raise the coherence of this set. With this simple proof, it can be inferred that all the measures generated with this scheme are not reliability-conducive. Hence, Shogenji's measure is the only reliability-conducive coherence measure.

Schubert (2011) also examines if any other coherence measure is reliability-conducive in a scenario with two non-equivalent evidence. The scenario he considers is slightly different from the scenario with equivalent evidence. In the new scenario $(\mathbf{S}, \mathbf{P})$, the number of evidence is fixed, namely $\mathbf{S} = \{\langle E_1, H_1 \rangle, \langle E_2, H_2 \rangle\}$. Given such scenario, Schubert proves that the posterior probability of reliability $Pr(R_i|E_1, E_2)$ is a strictly increasing function of $\dfrac{Pr(E_1, E_2)}{Pr(E_1)Pr(E_2)}$. As an immediate result, for a coherence measure to be reliability-conducive, it also has to be an strictly increasing function of this formula.

Schubert introduces three different coherence measures:

Definition 3.6.10. Confirmation-based coherence measures

$$\text{Ratio measure: } S_r(E_1, E_2) \stackrel{\text{def}}{=\!=} \frac{Pr(E_1|E_2)}{Pr(E_1)}$$

$$\text{Log-ratio measure: } S_{lr}(E_1, E_2) \stackrel{\text{def}}{=\!=} log\frac{Pr(E_1|E_2)}{Pr(E_1)}$$

$$\text{Fnch's measure: } S_{Fi}(E_1, E_2) \stackrel{\text{def}}{=\!=} \frac{Pr(E_2|E_1) - Pr(E_2)}{Pr(E_2)}$$

If we plug these confirmation measures in Douven and Meijs' scheme of coherence measure, we may obtain three coherence measures C_r, C_{lr} and C_{Fi}:

$$\text{Ratio-based coherence measure: } C_r(E_1, E_2) \stackrel{\text{def}}{=\!=} \frac{Pr(E_1, E_2)}{Pr(E_1)Pr(E_2)}$$

Log-ratio-based coherence measure: $C_{lr}(E_1, E_2) \stackrel{\text{def}}{=\!=} log\dfrac{Pr(E_1, E_2)}{Pr(E_1)Pr(E_2)}$

Finch-based coherence measure: $C_{Fi}(E_1, E_2) \stackrel{\text{def}}{=\!=} \dfrac{Pr(E_!, E_2)}{Pr(E_1)Pr(E_2)} - 1$

All three measures satisfy the condition of reliability-conduciveness. With simple observation, one may see that all three coherence measures are ordinally equivalent to Shogenji's measure. This result is not surprising at all, since the formula $\dfrac{Pr(E_1 \wedge E_2)}{Pr(E_1)Pr(E_2)}$ is equivalent to the way Shogenji measures the coherence of $\{E_1, E_2\}$.

We have examined whether coherence is reliability-conducive in three different scenarios, including scenario with two equivalent evidence, scenario with n equivalent evidence and scenario with two different pieces of evidence. In all these scenarios, Shogenji's measure correctly represents coherence as a reliability-conducive measure. Thus, it seems that Shogenji's measure does indeed secures the importance of coherence by showing that coherence is indeed reliability-conducive.

However, Schubert (2012b) proves that no coherence measure is reliability-conducive in a more general scenario. In a scenario with different evidence ($\mathbf{S}$,$\mathbf{P}$) such that $\mathbf{S} = \{S_n | S_n = \{\langle E_1, H_1, R_1\rangle, ..., \langle E_n, H_n, R_n\rangle\}$ and $n = 2 \vee n = 3\}$, it can be shown that there are cases in which a set S is more coherent than another set S', but the sources of S are less reliable than S'.

Schubert first proves two equations concerning the relation between reliability of sources $Pr(R_i|E_i, ..., E_n)$ and probability of hypothesis in question $Pr(H_i)$:

$$Pr(R_1|E_1, E_2) = \frac{C_{Sh}(\{H_1, H_2\}) + x}{C_{Sh}(\{H_1, H_2\}) + 2x + x^2}$$

$Pr(R_1|E_1, E_2, E_3) =$

$$\frac{C_{Sh}(\{H_1, H_2, H_3\}) + x(C_{Sh}(\{H_1, H_2\}) + C_{Sh}(\{H_1, H_3\})) + x^2}{C_{Sh}(\{H_1, H_2, H_3\}) + x(C_{Sh}(\{H_1, H_2\}) + C_{Sh}(\{H_1, H_3\}) + C_{Sh}(\{H_2.H_3\})) + 3x^2 + x^3}$$

in which

$$m = Pr(R_i|E_i);\ x = \frac{1-m}{m};\ C_{Sh}(\{H_1, ..., H_n\}) = \frac{Pr(H_1 \wedge ... \wedge H_n)}{Pr(H_1)...Pr(H_n)}$$

With these two equations, it can be shown that there are two sets of equivalent evidence $\{H_1, H_2\}$ and $\{H_1', H_2', H_3'\}$ such that for some $Pr \in \mathbf{P}$, $Pr(R_i|E_1, E_2) > Pr(R_i|E_1', E_2', E_3')$ and for some other $Pr' \in \mathbf{P}$, $Pr(R_i|E_1', E_2', E_3') > Pr(R_i|E_1, E_2)$. That is, there are cases which demonstrates that Shogenji's measure is not reliability-conducive in the scenario.

Proposition 3.6.1. Shogenji's measure is not reliability-conducive

Given the probability distribution:

$$Pr(H_1) = Pr(H_2) = 0.4$$

$$Pr(H'_1) = Pr(H'_2) = Pr(H'_3) = 0.5$$

We can derive that

$$C_{Sh}(\{H_1, H_2\}) = \frac{0.4}{0.4 \times 0.4} = \frac{5}{2};\ C_{Sh}(\{H'_1, H'_2, H'_3\}) = \frac{0.5}{0.25} = 2$$

The probability of R_1 given E_1, E_2 and the probability of R'_1 given E'_1, E'_2, E'_3 will then be

$$Pr(R_1|E_1, E_2) = \frac{\frac{5}{2} + x}{\frac{5}{2} + 2x + x^2}$$

$$Pr(R'_1|E'_1, E'_2, E'_3) = \frac{4 + 4x + x^2}{4 + 6x + 3x^2 + x^3}$$

When $m = \frac{1}{2}$,

$$Pr(R'_1|E'_1, E'_2, E'_3) \approx 0.643 > Pr(R_1|E_1, E_2) \approx 0.636$$

but when $m = \frac{1}{4}$,

$$Pr(R_1|E_1, E_2) \approx 0.44 > Pr(R'_1|E'_1, E'_2, E'_3) \approx 0.329.$$

Although $C_{Sh}(\{H_1, H_2\}) > C_{Sh}(\{H'_1, H'_2, H'_3\})$, the reliability of sources of the set $\{H'_1, H'_2, H'_3\}$ is greater than the reliability of courses for $\{H_1, H_2\}$ under certain occasions. We can therefore conclude that C_{Sh} is not reliability-conducive in this scenario.

Schubert proves in a similar way that there are two sets with the same number of evidence $\{H_1, H_2, H_3\}$ and $\{H'_1, H'_2, H'_3\}$ such that, given a probability distribution $Pr^* \in \mathbf{P}$, $Pr^*(R_i|E_i) = Pr^*(R_i|E'_i)$ for all i and $Pr^*(R_i|E_1, E_2, E_3) > Pr^*(R'_i|E'_1, E'_2, E'_3)$ and for some other $Pr^{**} \in \mathbf{P}$, $Pr^{**}(R_i|E'_1, E'_2) > Pr^{**}(R_i|E_1, E_2, E_3)$. Thus, it can be proved that Shogenji's measure is reliability-conducive only in a really restricted scenario. The claim that coherence is a reliability-conducive notion, regretfully, is not a general result.

3.7 Coherence as a confirmation-conducive notion

Another approach of saving coherence is to argue that it is *confirmation-conducive* (Dietrich and Moretti 2005, Moretti 2007). They prove that a coherent set, compared with a relatively incoherent one, is more likely to be confirmed. This idea comes from the intuition that when a set is coherent, its elements 'hang together' well, namely that elements in the set have similar contents. Therefore, if there is a piece of evidence confirming an element of a highly coherent set, this piece of evidence should also confirms other elements of that set. Following this line of thought, a piece of evidence confirming an element of a coherent set is supposed to confirm other elements of the set, and also the conjunction of them. In other words, the confirmation of an element can be *transmitted* to other elements of the set.

The correlation between coherence and confirmation transmission can be illustrated by a concrete example. It sounds pretty natural to say that if a piece of evidence confirms two elements of a set, it should also confirm the conjunction of them. Unfortunately, it is not always the case. A piece of evidence e confirming two hypotheses may fail to confirm their conjunction. This fact can be shown by the following example:

Example 3.7.1. Non-transmitting confirmation

(H_1) Alex is a football fan living in Manchester.

(H_2) Alex is a supporter of Manchester United F.C..

(H_3) Alex is a supporter of Manchester City F.C..

Suppose there is a piece of evidence E:

(E) Alex has a mansion in Manchester with a football field.

E confirms H_1, H_2 and H_3 respectively, that is, for all H_i such that $i = 1, 2, 3$, $Pr(H_i|E)$ is greater than $Pr(H_i)$. However, the probability of $H_1 \wedge H_2 \wedge H_3$ conditional on E is much lower than its prior probability, for it is highly improbable that both H_2 and H_3 are true.

The reason E does not support $H_1 \wedge H_2 \wedge H_3$ is that H_2 and H_3 do not cohere well with each other. As a result, the probability for them to be true together is rather low. That is to say, whether the conjunction of a set of elements can be confirmed with the same piece of evidence depends on how coherent they are. The notion of coherence, understood this way, can be taken as representing the relevance between propositions. With this underlying idea, Dietrich and Moretti (2005) define *confirmation transmission* and *confirmation transmission to the conjunction* as the following:

Definition 3.7.1. Confirmation transmission (CT)

For any formulae E, H such that E confirms H, there exists a non-trivial[4] coherence threshold $c_{E,H} \in \mathbb{R}$ such that for any set $S \in \mathbf{S}$ containing H with coherence $C(S) \geq c_{E,H}$, E confirms each member of S.

[4]Non-triviality here means that the threshold is strictly less than the maximal degree of coherence of the measure.

Definition 3.7.2. Confirmation transmission to the Conjunction (CTC)

For any formulae E, H such that E confirms H, there exists a coherence threshold $c_{E,H} \in \mathbb{R}$ such that for any set $S \in \mathbf{S}$ containing H with coherence $C(S) \geq c_{E,H}$, E confirms $\bigwedge_{H^* \in S} H^*$.

If there exists a coherence measure which satisfies both (CT) and (CTC), it can be derived that the notion of coherence, as characterized by the measure, is a confirmation-conducive notion such that greater degree of coherence leads to stronger confirmation.

With the idea that coherence provides a different way to characterize relevance between propositions, we can further define a weaker notion of confirmation transmission by restricting to binary sets:

Definition 3.7.3. Weak confirmation transmission (CT*)

For any formulae E, H such that E confirms H, there exists a coherence threshold $c_{E,H} \in \mathbb{R}$ such that for any formula H^* with $Pr(H^*) > 0$ and $C(\{H, H^*\}) > c_{E,H}$, E confirms H^*.

(CT*) is weaker than (CT) and (CTC) since the number of elements in the set being confirmed is fixed. That is, when the number of elements of a set is greater than two, a coherence measure satisfying only (CT*) may fail to show that coherence is confirmation-conducive.

It is to be noted that the relevance so construed is a non-deductive relation. Suppose that a coherence measure C satisfies both (CT) and (CTC), given a set $H : \{H_1, ..., H_n\}$ and a piece of evidence E which confirms H_1. If the degree of coherence of H is higher than certain threshold c_{E,H_1}, although $\bigwedge_{H_i \in H} H_i$ is not a logical consequence of H_1, E still confirms $\bigwedge_{H_i \in H} H_i$. With the notion of coherence, it can be shown that there is an indirect, non-deductive relation between H_i and $\bigwedge_{H_i \in H} H_i$.

Based on the idea of confirmation transmission, Moretti (2007) further defines two properties *evidence gathering* and *conjunctive evidence gathering*:

Definition 3.7.4. Evidence gathering (EG)

For any formulae E and H such that E confirms H, there exists a coherence threshold $c_{E,H} \in \mathbb{R}$ such that for any set $S \in \mathbf{S}$ containing E with coherence $C(S) \geq c_{E,H}$, each member of S confirms H.

Definition 3.7.5. Conjunctive evidence gathering (CEG)

For any formulae E and H such that E confirms H, there exists a coherence threshold $c_{E,H} \in \mathbb{R}$ such that for any set $S \in \mathbf{S}$ containing E with degree of coherence $C(S) \geq c_{E,H}$, the conjunction $\bigwedge_{E^* \in S} E^*$ confirms H.

It is easy to see that (EG) and (CEG) are converses of (CT) and (CTC). (EG) states that given E confirms H, if the set S containing E is sufficiently coherent, other elements of S also confirm

H, while (CEG) states that given the same premise, the conjunction of elements of S confirms H. Moretti further show that a coherence measure satisfies (EG) if and only if it satisfies (CT), and satisfies (CEG) if and only if it satisfies (CTC).

Dietrich and Moretti prove that Olsson's coherence measure satisfies (CT), (CTC), (EG) and (CEG) with the threshold $\frac{1}{1+Pr(E|H)-Pr(E)}$, while Shogenji's measure does not satisfy any of them.[5] The interesting case is Fitelson's measure C_F which satisfies (CT*) with the threshold $\frac{1}{1+Pr(E\wedge H)-Pr(E)Pr(H)}$, but fails to satisfy (CT) and (CTC). Dietrich and Moretti provide a sketchy proof which illustrates the reason. When the number of equivalent hypotheses of a set tends to infinity, the coherence of a set, under Fitelson's measures C_F, tends to 1. Given this fact, suppose $n > m$, a set with n equivalent propositions is more coherent than a set with m equivalent propositions under C_F. Assume there is a set of hypotheses $S = \{H_1, ...H_n\}$ and a piece of evidence E which confirms only H_1. If $C_F(S) < c_{E,H_1}$, it will turn out that E does not confirm any other H_i in S apart from H_1. However, one may raise $C_F(S)$ simply by adding another set of hypotheses $S^* = \{H_1^*, ..., H_m^*\}$ to S such that every element H_i^* in S^* is equivalent to H_n. When m tends to infinity, $C_F(S \cup S^*)$ tends to 1, and thus will be greater than c_{E,H_1} at some point. Since every element in S^* is equivalent to H_n which is not confirmed by E, it is not the case that every element of the new set $S \cup S^*$ is confirmed by E. Hence, Fitelson's measure violates (CT) and (CTC). As for (CT*), since the size of the set being measured is fixed, this problem does not occur.

The proof which shows that Fttelson's measure is not confirmation-conducive provides an important insight concerning confirmation and reliability-conduciveness. With the fact that C_F does not satisfy (CT) and (CTC), we may infer that any size-informative coherence measure is not confirmation-conducive. For any set of proposition S and a coherence measure C, if adding propositions that are equivalent to some elements in S makes S more coherent under C, C is not confirmation-conducive, The reason is pretty straightforward. Given a set $\{H_1, H_2\}$, if a piece of evidence E confirms H_1 but not H_2, adding other hypotheses which are equivalent to H_1 would not make E confirm H_2. But if C is size-informative, adding hypotheses equivalent to H_i does raise the coherence of $\{H_1, H_2\}$. Hence, size-informativeness is incompatible with confirmation-conduciveness. On the other hand, for a coherence measure C to be reliability-conducive in any scenario, C has to be size-informative. The idea of reliability-conduciveness is that when a group of information sources provide a set of highly coherent evidence, these sources are considered to be highly reliable. Thus, when the number of sources providing the same piece of evidence increases, the degree of coherence should raise, which indicates that the

[5]For a detailed proof, see Dietrich and Moretti (2005)

information sources are more reliable.

In sum, confirmation transmission and evidence gathering provide us with a different way to check if a piece of evidence confirms certain hypotheses. Therefore, we may conclude that the notion of coherence has practical importance for scientists.

3.8 Inference to the most coherent explanation

An important issue in philosophy of science is the study of *abduction*, which is also called *inference to the best* (IBE). The primary purpose of scientific practice is to provide a correct explanation for a phenomena. In search of a correct explanation, scientists often encounter a situation where multiple explanations are possible. To find the correct explanation, a natural solution is to choose the *best* explanation among all possibilities as the correct one.

A question immediately follows: How do we judge the goodness of an explanation? We need a method to tell which explanation should be taken as the best one. There are three quantitative approaches to characterize the goodness of an explanation. The simplest one is to take the hypothesis with *maximal likelihood* (ML) as the best, namely that for a given piece of evidence E, the hypothesis which leads to the greatest posterior probability of E should be taken as the best explanation. That is, suppose there are n competing hypotheses $\{H_1, ..., H_n\}$ and a piece of evidence E, we should compare the probability of E conditional on every H_i, and take H_j as the best explanation if $Pr(E|H_j) = max(Pr(E|H_1), ..., Pr(E|H_n))$. This approach seems to be convincing at first glance. Given that E is a fact, the best explanation is the one which overlaps with E to the greatest extent among all competing hypotheses.

Intuitive as it seems, the maximal likelihood approach is regarded as incorrect because of the *base rate fallacy*. The base rate fallacy states that when provided with general and specific information of a single fact, people tend to ignore the former, and reason with the more specific information. For example, with the information that Alex eats curry everyday, one would naturally consider Ales as more likely to be an Indian than an non-Indian. But since the number of non-Indian people is greater than the number of Indian people, conditional on the information that Alex eats curry everyday, the actual probability of Alex being an non-Indian is greater than the probability of Alex being an Indian. This type of reasoning neglects the influence of prior probability of propositions. Similarly, adopting the explanation which leads to greatest posterior probability of E may also be fallacious. Suppose that two hypotheses H_1, H_2 leads to different posterior probability of E such that $Pr(E|H_1) > Pr(E|H_2)$, the hypothesis with greater posterior probability, according to ML, should be taken as the better explanation. But if $Pr(H_2) > Pr(H_1)$, the better explanation may actually be H_2. Hence, ML is fallacious.

Another approach is to take the *most probable explanation* (MPE) as the best explanation, namely that when presented with a set of hypotheses $\{H_1, ..., H_n\}$, pick the hypothesis H_i such that $Pr(H_1|E) = max(Pr(H_1|E), ..., Pr(H_n|E))$. According to MPE, the prior probability of hypotheses does influences the evaluation of explanations. Hence, MPE is free from the problem of ML. However, because of exactly the same reason, MPE is fallacious. This approach put too much emphasis on the prior probability of hypotheses. Suppose there are two competing hypotheses H_1, H_2 and a piece of evidence E such that $Pr(H_1) > Pr(H_1|E) > Pr(H_2|E) > Pr(H_2)$. Since the probability of H_1 given E is lower than its prior probability, while the probability of H_2 raises when given E, people tend to take H_2 as a better explanation of E. But MPE generates the opposite result that H_1 is the best explanation, which is quite implausible.

An alternative to ML and MPE is the *conservative Bayesian* (CB) approach, which takes an explanation H_1 as better than H_2 with regard to E if and only if $Pr(E|H_1) > Pr(E|H_2)$ and $Pr(H_1) > Pr(H_2)$. Judging this way, the best explanation need to has both greatest prior probability and also be the most probable. This approach can be regarded as a revised version of MPE which rules out problematic cases in which the prior probability is too low. Since it does not provide us a total ordering over explanations, it is a *conservative* approach. Unfortunately, this approach is still far from satisfactory. We may think of cases which is the best according to ML and MPE, but excluded by CB.[6] As a compromise between ML and MPE, this kind of cases should not occur.

Due to the failure of all three approaches, Glass (2007) proposes to characterize the notion of best explanation in terms of coherence, namely to take the best explanation as the one which coheres with evidence to the greatest extent. He finds out that Olsson's measure can be transformed into a combination of ML and MPE, namely that

Definition 3.8.1. Olsson's coherence measure (C_O)

$$C_O(\{H_i, E\}) = \frac{Pr(H_i \wedge E)}{Pr(H_i \vee E)} = (\frac{1}{Pr(E|H_i)} + \frac{1}{Pr(H_i|E)} - 1)^{-1}$$

Glass shows that this approach, like CB, retains both the merits of ML and MPE, and is free from the deficiency of CB. It generates the same result as ML and MPE when ML and MPE

[6] We can show this by considering a simple instance. Given two hypotheses H_1 and H_2. and a piece of evidence E such that $Pr(H_1) = 0.5$, $Pr(H_2) = 0.6$, $Pr(E \wedge H_1) = 0.3$ and $Pr(E \wedge H_2) = 0.2$. In this case, $Pr(H_1|E) > Pr(H_2|E)$ and $Pr(E|H_1) > Pr(E|H_1)$, hence, both ML and MPE rank H_1 as better than H_2. But since $Pr(H_2) > Pr(H_1)$, this case is excluded by CB. Hence, CB disagrees with ML and MPE. Since the motivation of proposing CB is to make a compromise between ML and MBP, the result that CB disagrees with both in certain circumstance is pretty strange.

agrees with each other. The advantage of this approach can be illustrated by the following example:

Example 3.8.1. Different accounts of best explanation

A farmer finds out that some of his sheep died (E) and wants to figure out why. There are two possible explanations: either some of the sheep have hoof-and-mouth disease (H_1), or the living conditions are too bad for them (H_2). Suppose that $P(E) = 0.1$, we may have the following four cases.

Case 1. $Pr(H_1) = 0.05$, $Pr(H_2) = 0.1$, $Pr(E|H_1) = 0.5$, $Pr(E|H_2) = 1$

Case 2. $Pr(H_1) = 0.1$, $Pr(H_2) = 0.05$, $Pr(E|H_1) = 0.25$, $Pr(E|H_2) = 1$

Case 3. $Pr(H_1) = 0.36$, $Pr(H_2) = 0.02$, $Pr(E|H_1) = 0.15$, $Pr(E|H_2) = 1$

Case 4. $Pr(H_1) = 0.1$, $Pr(H_2) = 0.01$, $Pr(E|H_1) = 0.75$, $Pr(E|H_2) = 1$

Case 1: The probability of sheep having hoof-and-mouth disease is 5%, the probability that they live in a bad environment is 10%. The probability that the sheep died because of the disease is 50%, while sheep living in bad environment leads to a 100% death rate. Suppose there are two hundred sheep, since $Pr(E)$ is 0.1, the total deaths is twenty. Intuitively, we would accept H_2 as the best explanation, for all twenty dead sheep lived in a bad environment, all four approaches suggests that H_2 is the real cause, which is in accordance with the intuition.

Case 2: The probability of the sheep having hoof-and-mouth disease is 10%, the probability that they live in a bad environment is 5%. The probability that the sheep died because of the disease is 25%, while sheep living in a bad environment are all definitely going to die. In this case, $Pr(H_1|E) = 0.25$, $Pr(H_2|E) = 0.5$, $C_O(H_1, E) = \frac{1}{7}$, $C_O(H_2, E) = \frac{1}{5}$. We may infer that ML, MPE and C_O all indicate that H_2 is the best explanation, while CB excludes this case.

Case 3: The probability of the sheep having hoof-and-mouth disease is 36%, while the probability that they live in a bad environment is 2%. The probability that the sheep died because of the disease is 15%, while the sheep living in a bad environment are all definitely going to die. In this case,

$$Pr(E|H_1) = 0.15 < Pr(E|H_2) = 1$$

$$Pr(H_1|E) = 0.54 > Pr(H_2|E) = 0.2$$

$$C_O(H_1, E) = \frac{27}{203} < C_O(H_2, E) = \frac{1}{5}$$

ML and C_O both take H_2 as better, while MPE ranks H_1 higher than H_2. In this case, the reason for $Pr(H_2|E)$ to be greater than $Pr(H_1|E)$ is that the prior probability of H_2 is much greater than H_1, which does not imply that H_2 is a better explanation for E. Hence, ML and C_O is more convincing.

Case 4: 10% of the sheep have hoof-and-mouth disease, 1% of them live in a bad environment. 75% among the sheep with a disease died, and all those sheep living in a bad environment died. We can derive that

$$Pr(E|H_1) = 0.75 < Pr(E|H_2) = 1$$

$$Pr(H_1|E) = 0.75 > Pr(H_2|E) = 0.1$$

$$C_O(H_1, E) = \frac{3}{5} > C_O(H_2, E) = \frac{1}{10}$$

In this case, C_O agrees with MPE that H_1 is a better explanation than H_2. Since $Pr(H_1|E)$ is much greater than $Pr(H_2|E)$ not simply because of having prior probability, H_1 is intuitively a better explanation.

With this illustrating case, Glass claims that C_O has the advantage of both ML and MPE, and generates results that fit better with our intuition concerning best explanation. Therefore, coherence, as characterized with Olsson's measure C_O, provides a goodness ranking of explanations.

3.9 Summary of chapter three

In this chapter, we have seen how Bovens and Hartmann prove that the search of a truth-conducive probabilistic coherence measure is doomed to fail. To save the notion of coherence, philosophers provide several applications of this notion other than explaining epistemic justification. If it is true that the notion of coherence is useful to deal with these aspects, it may still be regarded as an important notion is epistemology.

Although some of the attempts are successful, there are still hidden problems of the general approach of saving coherence. The fact that is revealed by these attempts is not that coherence is useful, but rather that coherence, as characterized by specific probabilistic measures, is useful. If one can prove that a coherence measure is conducive to an epistemic ideal but violates some of our intuitive understanding of coherence, it would be doubtful whether the notion, as represented by the measure, should be accepted as identical to coherence. That is to say, if a coherence measure is intuitively incorrect but conducive to an epistemic ideal, then either our intuition is wrong, or the notion represented by this measure is not coherence. The primary purpose of next chapter, therefore, is to review this approach with a new requirement for coherence measures.

Chapter 4

Coherence and Confirmation

4.1 A new requirement

As introduced in the previous chapters, there are many probabilistic coherence measures, each has different features and can be used for a variety of purposes. For instance, if one wants to ascertain whether a piece of evidence E confirming a proposition P_1 also confirms another proposition P_2, one may calculate the degree of coherence between P_1 and P_2 with Olsson's measure. If the degree of coherence between them turns out to be above a certain threshold, one can draw the conclusion that E also confirms P_2. Also, to judge if a scientific explanation is better than another according to some pieces of evidence, one may calculate the coherence between the evidence and each explanation, and pick the explanation which coheres with the evidence to the greatest extent. In spite of being non-truth-conducive, coherence can still be a valuable notion in contemporary epistemology and philosophy of science.

However, having practical merits does not guarantee that the coherence measures proposed so far are correct. They may still violate intuitive requirements of coherence, and hence result in counterintuitive consequences. *Coherence preservation* is a requirement of this kind which poses a serious threat to these probabilistic coherence measures.

It is generally accepted that coherence is the *mutual support* between the elements of a set. If every element in a set supports some other elements in the set, the set should be regarded as highly coherent. It is then natural to think that for any set of propositions, if extended with a proposition which confirms every element of that set, the degree of coherence of the new set should be greater than, or at least equal to the coherence of the original set. In other words, the degree of coherence of a set should be *preserved* when the set is confirmed. We call this requirement *coherence preservation.*

The requirement of coherence preservation is baaed on the same idea as BonJour's (1985) third coherence criteria which states that 'The coherence of a system of beliefs is increased by the presence of inferential connections between its component beliefs and increased in proportion to the number and strength of such connections.' If we interpret what BonJour means by *inferential connection* as *confirmation relation*, it is then pretty natural to accept the idea that the coherence of a set should be preserved when every element of it is confirmed.

To provide a formal definition of coherence preservation, we must first clarify the notion of confirmation. A widely accepted probabilistic definition of confirmation is: a proposition H is confirmed by another proposition E if and only if the probability of H conditional on E is greater than the prior probability of H, namely:

Definition 4.1.1. Confirmation

Given a probability distribution $Pr(\cdot)$, a proposition E *confirms* another proposition H if and only if $Pr(H|E) > Pr(H)$.

With this formal definition of confirmation, we can derive the following requirement for a coherence measure:

Definition 4.1.2. Coherence preservation (CP)

Given a set of propositions $S = \{P_1, ..., P_n\}$ and a proposition E such that for all $P_i \in S$, $Pr(P_i|E) > Pr(P_i)$. A coherence measure C is *coherence preserving* if and only if $C(S \cup \{E\}) \geq C(S)$.

That is, if a coherence measure satisfies (CP), the degree of coherence it assigns to $S \cup \{E\}$ would be greater than the degree it assigns to S.

Based on (CP), we can further derive another requirement called *coherence preservation to the conjunction*:

Definition 4.1.3. Coherence preservation to the conjunction (CPC)

Given a set of propositions $S = \{P_1, ..., P_n\}$ and a proposition E such that for every $P_i \in S$, $Pr(P_i|E) > Pr(P_i)$ and $Pr(P_1 \wedge ... \wedge P_n|E) > Pr(P_1 \wedge ... \wedge P_n)$. A coherence measure C is *coherence preserving to the conjunction* if and only if $C(S \cup \{E\}) \geq C(S)$.

We can easily see that violating (CPC) implies violating (CP), but not the other way round.

Given the ordinary understanding of coherence, it is natural to consider (CP) and as an appropriate requirement for coherence measures. If a coherence measure C fails to satisfy (CP), C fails to capture the intuition that coherence is the mutual support between a set of elements. Surprisingly, most mainstream coherence measures do not conform to (CP). The primary concern of this chapter is to reevaluate different coherence measures with this new requirement, and further discuss the results of this observation.

4.2 Agreement measures for coherence are not coherence preserving

As introduced in chapter two, two of the most prominent agreement measures of coherence are Shogenji and Olsson's measures. Unfortunately, both measures fail to satisfy (CP). To show that a coherence measure C violates (CP), it suffices to provide a simple counterexample in which a proposition E confirms every element of a set S but $C(S) > C(S \cup \{E\})$.

Proposition 4.2.1. Shogneji's measure does not satisfy (CP)

Assume there is a set of propositions $\{H_1.H_2\}$ and a proposition E confirming both H_1 and H_2, i.e. $Pr(H_1|E) > Pr(H_1)$, $Pr(H_2|E) > Pr(H_2)$. Suppose that the probability of H_1, H_2 and E are distributed as follows:

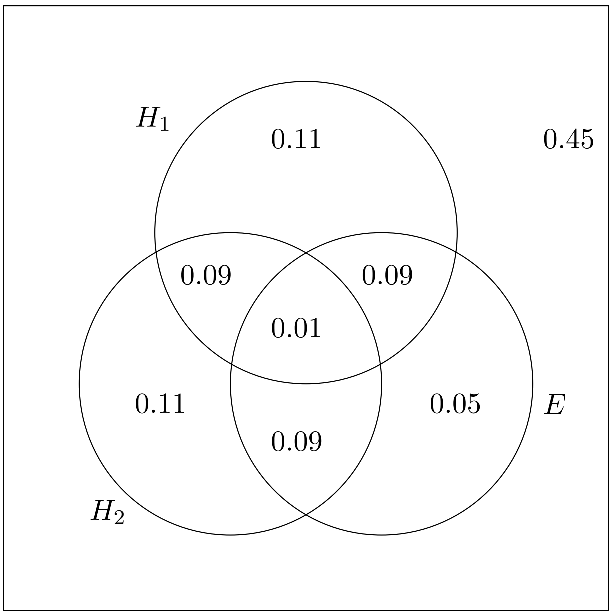


Figure 1.

In this case,

$$Pr(H_1) = Pr(H_2) = 0.3$$

$$Pr(E) = 0.24$$

$$Pr(H_1|E) = Pr(H_2|E) = \frac{5}{12} \approx 0.41$$

Since $Pr(H_1|E) > Pr(H_1)$ and $Pr(H_2|E) > Pr(H_2)$, E confirms both H_1 and H_2.

The coherence of $\{H_1, H_2\}$ and $\{H_1, H_2, E\}$ are measured as:

$$C_{Sh}(\{H_1, H_2\}) = \frac{Pr(H_1 \wedge H_2)}{Pr(H_1)Pr(H_2)} = \frac{10}{9}$$

$$C_{Sh}(\{H_1, H_2, E\}) = \frac{Pr(H_1 \wedge H_2 \wedge E)}{Pr(H_1)Pr(H_2)Pr(E)} = \frac{25}{54}$$

Given this probability distribution, although E confirms both H_1 and H_2, the degree of coherence of $\{H_1, H_2\}$ is greater than the coherence of $\{H_1, H_2, E\}$. We may hence conclude that Shogenji's measure violates (CP).

The revised Shogenji's measure proposed by Schupbach (2011) also fails to satisfy this requirement.

Proposition 4.2.2. The revised Shogenji measures do not satisfy (CP)

Schupbach provides three revised Shogenji measures: Straight Average C_{SA}, Deeper Increasing C_{DI} and Deeper Decreasing C_{DD}. With the probability distribution in Figure 1, we can show that all three measures violate (CP). To derive the result, we need first calculate the pairwise and three-wise coherence of $\{H_1, H_2, E\}$.

$$C^2(\{H_1, H_2, E\}) = \frac{1}{3}(log(\frac{Pr(H_1 \wedge H_2)}{Pr(H_1)Pr(H_2)}) + log(\frac{Pr(H_1 \wedge E)}{Pr(H_1)Pr(E)}) + log(\frac{Pr(H_2 \wedge E)}{Pr(H_2)Pr(E)})) \approx 0.11$$
$$C^3(\{H_1, H_2, E\}) = log(\frac{25}{54}) = -0.334$$

Measured with C_{SA}, the degree of coherence of $\{H_1, H_2, E\}$ is:

$$C_{SA}(\{H_1, H_2, E\}) = \frac{1}{2}(C^2(\{H_1, H_2, E\}) + C^3(\{H_1, H_2, E\})) = \frac{0.11 - 0.334}{2} \approx -0.11$$

With C_{DD}, the coherence of $\{H_1, H_2, E\}$ is

$$C_{DD}(\{H_1, H_2, E\}) = \frac{2}{6}C^2(\{H_1, H_2, E\}) + \frac{4}{6}C^3(\{H_1, H_2, E\}) = -0.186$$

with C_{DI}, the degree of coherence of $\{H_1, H_2, E\}$ is

$$C_{DI} = \frac{4}{6}C^2(\{H_1, H_2, E\}) + \frac{2}{6}C^3(\{H_1, H_2, E\}) = -0.038$$

The coherence of $\{H_1, H_2\}$ under these revised measures is just the logarithm of its degree of coherence with the original measure:

$$C^2(\{H_1, H_2\}) = log(\frac{Pr(H_1 \wedge H_2)}{Pr(H_1)Pr(H_2)}) = 0.045$$

Since $C^2(\{H_1, H_2\})$ is greater than $C_{SA}(\{H_1, H_2, E\})$, $C_{DD}(\{H_1, H_2, E\})$ and $C_{DI}(\{H_1, H_2, E\})$, we can conclude that the revised Shogenji measures also violate (CP).

Similarly, Olsson's measure fails to meet this requirement.

Proposition 4.2.3. Olsson's measure does not satisfy (CP) and (CPC)

Consider the following probability distribution:

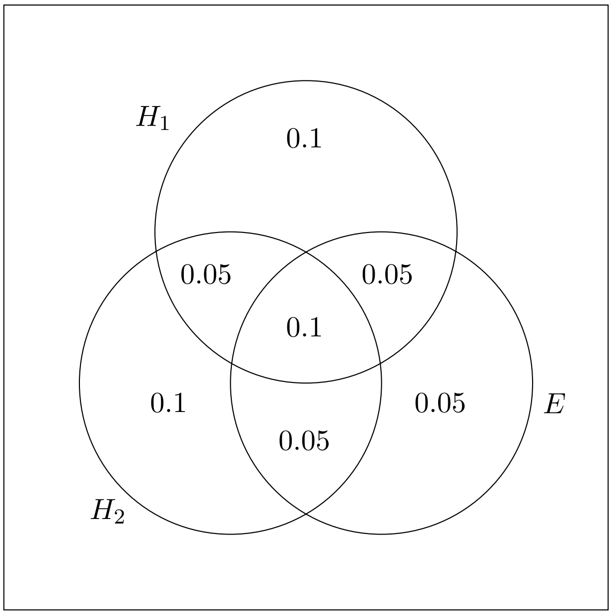


Figure 2.

Given this probability distribution, we can derive that

$$Pr(H_1|E) = 0.6 > 0.3 = Pr(H_1)$$

$$Pr(H_2|E) = 0.6 > 0.3 = Pr(H_2)$$

$$Pr(H_1 \wedge H_2|E) = 0.4 > 0.15 = Pr(H_1 \wedge H_2)$$

$$C_O(\{H_1, H_2\}) = \frac{1}{3} > \frac{1}{5} = C_O(\{H_1, H_2, E\})$$

Hence, although E confirms both H_1 and H_2, the degree of coherence of $\{H_1, H_2\}$ is greater than $\{H_1, H_2, E\}$. Since $Pr(H_1 \wedge H_2|E) > Pr(E)$, namely that E also confirms the conjunction of H_1 and H_2, Olsson's measure also violates (CPC).

The revised Olsson measure, proposed by Meijs (2006), also violates (CP), which can be shown with the probability distribution in Figure 2.

Proposition 4.2.4. The revised Olsson's measure violates (CP)

Given three propositions H_1, H_2 and E and the probability distribution in Figure 2, the degree of coherence of $\{H_1, H_2\}$, according to the revised Olsson's measure C_{O^*}, is:

$$C_{O^*}(\{H_1, H_2\}) = \frac{Pr(H_1, \wedge H_2)}{Pr(H_1 \vee H_2)} = \frac{15}{45} = \frac{1}{3}$$

The coherence of $\{H_1, H_2, E\}$, measured with C_{O^*}, is

$$\frac{o(H_1,H_2)+o(H_1,E)+o(H_2,E)+o(H_1,H_2,E)}{4} = \frac{(\frac{1}{3}+\frac{3}{8}+\frac{3}{8}+\frac{1}{5})}{4} \approx 0.321$$

$C_{O^*}(\{H_1, H_2, E\})$ is lower than $C_{O^*}(\{H_1, H_2\})$, therefore, the revised Olsson measure violates (CP).

4.3 Confirmation based measures of coherence are not coherence preserving

Similar to agreement measures, confirmation based measures of coherence fail to satisfy (CP). To prove this, we first list the measures that should be considered. With Douven and Meijs' scheme, we can generate many different coherence measures by plugging in different confirmation measures. Recall that the scheme is written as follows:

$$C_m(S) \overset{\text{def}}{=\!=} \frac{\sum_{i=1}^{[\![S]\!]} m(\hat{S}_i)}{[\![S]\!]}$$

The degree of coherence of a set is the average degree of confirmation between every subset of the set being measured. There are many confirmation measures that can be plugged in this scheme:

Definition 4.3.1. Confirmation measures

(Carnap 1962)	$D(E,H) =$	$Pr(H \mid E) - Pr(H)$
(Christensen 1999)	$S(E,H) =$	$Pr(H \mid E) - Pr(H \mid \neg E)$
(Mortimer 1988)	$M(E,H) =$	$Pr(E \mid H) - Pr(E)$
(Nozick 1981)	$N(E,H) =$	$Pr(E \mid H) - Pr(E \mid \neg H)$
(Carnap 1962)	$C(E,H) =$	$Pr(E \wedge H) - Pr(E)Pr(H)$
(Finch 1960)	$F_i(E,H) =$	$\frac{Pr(H \mid E)}{Pr(H)} - 1$
(Rips 2001)	$R_i(E,H) =$	$1 - \frac{Pr(\neg H \mid E)}{Pr(\neg H)}$
(Kemeny&Oppenheim 1952)	$L(E,H) =$	$\frac{Pr(E \mid H) - Pr(E \mid \neg H)}{Pr(E \mid H) + Pr(E \mid \neg H)}$
(Keynes 1921)	$K(E,H) =$	$log(\frac{Pr(H \mid E)}{Pr(H)})$
(Good 1984)	$G(E,H) =$	$log(\frac{Pr(E \mid H)}{Pr(E \mid \neg H)})$

To show that the measures generated with Douven and Meijs' scheme violates (CP), it suffices to provide a counterexample for each C_m in which $m \in \{D, S, M, N, C, F_i, R_i, L, K, G\}$.

Proposition 4.3.1. C_m violates (CP)

Given a set $\{H_1, H_2\}$ and a piece of evidence E such that $Pr(H_1|E) > Pr(H_1)$ and $Pr(H_2|E) > Pr(H_2)$. The degree of coherence of $\{H_1, H_2\}$, according to C_m, is

$$C_m(\{H_1, H_2\}) = \frac{m(H_1, H_2) + m(H_2, H_1)}{2}$$

while the degree of coherence of $\{H_1, H_2, E\}$ is

$$C_m(\{H_1, H_2, E\}) = \frac{1}{12}\left(\begin{array}{l} m(H_1, H_2) + m(H_1, E) + m(H_1, H_2 \wedge E) \\ +m(H_2, H_1) + m(H_2, E) + m(H_2, H_1 \wedge E) \\ +m(E, H_1) + m(E, H_2) + m(E, H_1 \wedge H_2) \\ +m(H_1 \wedge H_2, E) + m(H_1 \wedge E, H_2) + m(H_2 \wedge E, H_1) \end{array}\right)$$

With these two formulas, we may begin to check if the coherence measures generated from Douven and Meijs' scheme satisfy (CP).

Case 1: $m(E, H) = D(E, H) = Pr(H \mid E) - Pr(H)$

Given the same probability distribution as in Figure 1, we may derive the following results:

$$D(H_1, H_2) = D(H_2, H_1) = Pr(H_1|H_2) - Pr(H_1) = \frac{1}{3} - \frac{3}{10} = \frac{1}{30}$$

$$D(H_1, E) = D(H_2, E) = Pr(E|H_1) - Pr(E) = Pr(E|H_2) - Pr(E) = \frac{6}{75}$$

$$D(H_1, H_2 \wedge E) = D(H_2, H_1 \wedge E) = Pr(H_2 \wedge E|H_1) - Pr(H_2 \wedge E) = -\frac{1}{15}$$

$$D(E, H_1) = D(E, H_2) = Pr(H_1|E) - Pr(H_1) = Pr(H_2|E) - Pr(H_2) = \frac{7}{60}$$

$$D(E, H_1 \wedge H_2) = Pr(H_1 \wedge H_2|E) - Pr(H_1 \wedge H_2) = -\frac{7}{120}$$

$$D(H_1 \wedge H_2, E) = Pr(E|H_1 \wedge H_2) - Pr(E) = -\frac{7}{50}$$

$$D(H_1 \wedge E, H_2) = D(H_2 \wedge E, H_1) = Pr(H_1|H_2 \wedge E) - Pr(H_1) = -\frac{1}{5}$$

Hence,

$$C_D(\{H_1, H_2\}) = \frac{1}{2}(D(H_1, H_2) + D(H_2, H_1)) \approx 0.03$$

$$C_D(\{H_1, H_2, E\}) \approx -0.02$$

The degree of coherence of $\{H_1, H_2, E\}$ is lower than the coherence of $\{H_1, H_2\}$

Case 2: $m(E, H) = S(E, H) = Pr(H \mid E) - Pr(H \mid \neg E)$

$$S(H_1, H_2) = S(H_2, H_1) = Pr(H_2|H_1) - Pr(H_2|\neg H_1) = \frac{1}{3} - \frac{2}{7} = \frac{1}{21}$$

$$S(H_1, E) = S(H_2, E) = Pr(E|H_1) - Pr(E|\neg H_1) = \frac{15}{2}$$

$$S(H_1, H_2 \wedge E) = S(H_2, H_1 \wedge E) = Pr(H_2 \wedge E|H_1) - Pr(H_2 \wedge E|\neg H_1) = -\frac{2}{21}$$

$$S(E, H_1) = S(E, H_2) = Pr(H_1|E) - Pr(H_1|\neg E) = \frac{35}{228}$$

$$S(E, H_1 \wedge H_2) = Pr(H_1 \wedge H_2|E) - Pr(H_1 \wedge H_2|\neg E) = -\frac{35}{456}$$

$$S(H_1 \wedge H_2, E) = Pr(E|H_1 \wedge H_2) - Pr(E|\neg(H_1 \wedge H_2)) = \frac{2}{45}$$

$$S(H_1 \wedge E, H_2) = S(H_2 \wedge E, H_1) = Pr(H_2|H_1 \wedge E) = -\frac{101}{90}$$

We can hence derive that $C_S(\{H_1, H_2\}) = \frac{1}{2}(S(H_1, H_2) + S(H_2, H_1)) \approx 0.05$. On the other hand, $C_S(\{H_1, H_2, E\}) \approx -0.15$. Again, $C_S(\{H_1, H_2\})$ is greater than $C_S(\{H_1, H_2, E\})$.

Case 3: $m(E, H) = M(E, H) = Pr(E \mid H) - Pr(E)$

Similar to Case 1.

Case 4: $m(E, H) = N(E, H) = Pr(E \mid H) - Pr(E \mid \neg H)$

Similar to Case 2.

Case 5: $m(E, H) = C(E, H) = Pr(E \wedge H) - Pr(E)Pr(H)$

$$C(H_1, H_2) = C(H_2, H_1) = Pr(H_1 \wedge H_2) - Pr(H_1)Pr(H_2) = 0.01$$

$$C(H_1, E) = C(H_2, E) = C(E, H_1) = C(E, H_2) = 0.1 - 0.072 = 0.028$$

$$C(H_1, H_2 \wedge E) = C(H_2 \wedge E, H_1) = C(H_2, H_1 \wedge E) = C(H_1 \wedge E, H_2) = -0.02$$

$$C(H_1 \wedge H_2, E) = C(E, H_1 \wedge H_2) = -0.062$$

Given this probability distribution, $C_C(\{H_1, H_2\}) = 0.01$, $C_C(\{H_1, H_2, E\}) = -0.006$.

Case 6: $m(E, H) = F_i(E, H) = \dfrac{Pr(H \mid E)}{Pr(H)} - 1$

$$F_i(H_1, H_2) = F_i(H_2, H_1) = \frac{Pr(H_1|H_2)}{Pr(H_1)} - 1 = \frac{10}{9} - 1 = \frac{1}{9}$$

$$F_i(H_1, E) = F_i(H_2, E) = \frac{Pr(E|H_1)}{Pr(E)} - 1 = \frac{25}{18} - 1 = \frac{7}{18}$$

$$F_i(H_1, H_2 \wedge E) = F_i(H_2, H_1 \wedge E) = \frac{Pr(H_2 \wedge E|H_1)}{Pr(H_2 \wedge E)} - 1 = -\frac{2}{3}$$

$$F_i(E, H_1) = F(E, H_2) = \frac{Pr(H_1|E)}{Pr(H_1)} - 1 = \frac{7}{18}$$

$$F_i(E, H_1 \wedge H_2) = \frac{Pr(H_1 \wedge H_2|E)}{Pr(H_1 \wedge H_2)} - 1 = -\frac{7}{12}$$

$$F_i(H_1 \wedge H_2, E) = \frac{Pr(E|H_1 \wedge H_2)}{Pr(E)} - 1 = -\frac{7}{12}$$

$$F_i(H_1 \wedge E, H_2) = F_i(H_2 \wedge E, H_1) = \frac{Pr(H_1|H_2 \wedge E)}{Pr(H_1)} - 1 = -\frac{1}{2}$$

$C_{F_i}(\{H_1, H_2\}) \approx 0.111$, $C_{F_i}(\{H_1, H_2, E\}) \approx -0.143$

Case 7: $m(E, H) = R_i(E, H) = 1 - \dfrac{Pr(\neg H \mid E)}{Pr(\neg H)}$

$$R_i(H_1, H_2) = R_i(H_2, H_1) = 1 - \frac{Pr(\neg H_1|H_2)}{Pr(\neg H_1)} = \frac{1}{21}$$

$$R_i(H_1, E) = R_i(H_2, E) = 1 - \frac{Pr(\neg E|H_1)}{Pr(\neg E)} = \frac{7}{57}$$

$$R_i(H_1, H_2 \wedge E) = R_i(H_2, H_1 \wedge E) = 1 - \frac{Pr(\neg(H_1 \wedge E)|H_2)}{Pr(\neg(H_1 \wedge E))} = \frac{-2}{27}$$

$$R_i(E, H_1) = R_i(E, H_2) = 1 - \frac{Pr(\neg H_1|E)}{Pr(\neg H_1)} = \frac{1}{6}$$

$$R_i(E, H_1 \wedge H_2) = 1 - \frac{Pr(\neg(H_1 \wedge H_2|E))}{Pr(\neg(H_1 \wedge H_2))} = \frac{-7}{108}$$

$$R_i(H_1 \wedge H_2, E) = \frac{Pr(\neg E|H_1 \wedge H_2)}{Pr(\neg E)} = -\frac{7}{38}$$

$$R_i(H_1 \wedge E, H_2) = R(H_2 \wedge E, H_1) = \frac{Pr(\neg H_1|H_2 \wedge E)}{Pr(\neg H_1)} = -\frac{2}{7}$$

$C_R(\{H_1, H_2\}) \approx 0.05$; $C_R(\{H_1, H_2, E\} \approx -0.024$

Case 8: $m(E, H) = L(E, H) = \dfrac{Pr(E \mid H) - Pr(E \mid \neg H)}{Pr(E \mid H) + Pr(E \mid \neg H)}$

$$L(H_1, H_2) = L(H_2, H_1) = \frac{Pr(H_2|H_1) - Pr(H_2|\neg H_1)}{Pr(H_2|H_1) + Pr(H_2|\neg H_1)} = \frac{1}{13}$$

$$L(H_1, E) = L(H_2, E) = \frac{Pr(H_1|E) - Pr(H_1|\neg E)}{Pr(H_1|E) + Pr(H_1|\neg E)} = \frac{7}{31}$$

$$L(H_1, H_2 \wedge E) = L(H_2, H_1 \wedge E) = \frac{Pr(H_1|H_2 \wedge E) - Pr(H_1|\neg(H_2 \wedge E))}{Pr(H_1|H_2 \wedge E) + Pr(H_1|\neg(H_2 \wedge E)} = -\frac{101}{119}$$

$$L(E, H_1) = L(E, H_2) = \frac{Pr(E|H_1) - Pr(E|\neg H_1)}{Pr(E|H_1) + Pr(E|\neg H_1)} = \frac{1}{4}$$

$$L(E, H_1 \wedge H_2) = \frac{Pr(E|H_1 \wedge H_2) - Pr(E|\neg(H_1 \wedge H_2))}{Pr(E|H_1 \wedge H_2) + Pr(E|\neg(H_1 \wedge H_2))} = \frac{2}{7}$$

$$L(H_1 \wedge H_2, E) = \frac{Pr(H_1 \wedge H_2|E) - Pr(H_1 \wedge H_2|\neg E)}{Pr(H_1 \wedge H_2|E) + Pr(H_1 \wedge H_2|\neg E)} = -\frac{1}{7}$$

$$L(H_1 \wedge E, H_2) = L(H_2 \wedge E, H_1) = \frac{Pr(H_1 \wedge E|H_2) - Pr(H_1 \wedge E|\neg H_2)}{Pr(H_1 \wedge E|H_2) + Pr(H_1 \wedge E|\neg H_2)} = -\frac{2}{21}$$

$C_L(\{H_1, H_2\}) \approx 0.076$; $C_L(\{H_1, H_2, E\}) \approx -0.053$.

Case 9: $m(E, h) = K(E, H) = log(\frac{Pr(h \mid E)}{Pr(h)})$

$$K(H_1, H_2) = K(H_2, H_1) = log(\frac{Pr(H_1|H_2)}{Pr(H_1)}) = log\frac{10}{9}$$

$$K(H_1, E) = K(H_2, E) = log\frac{Pr(E|H_2)}{Pr(E)} = log\frac{25}{18}$$

$$K(H_1, H_2 \wedge E) = K(H_2, H_1 \wedge E) = log\frac{Pr(H_2 \wedge E|H_1)}{Pr(H_2 \wedge E)} = log\frac{10}{3}$$

$$K(E, H_1) = K(E, H_2) = log\frac{Pr(H_1|E)}{Pr(H_1)} = log\frac{25}{18}$$

$$K(E, H_1 \wedge H_2) = log\frac{Pr(H_1 \wedge H_2|E)}{Pr(H_1 \wedge H_2)} = log\frac{5}{12}$$

$$K(H_1 \wedge H_2, E) = log\frac{Pr(E|H_1 \wedge H_2)}{Pr(E)} = log\frac{5}{12}$$

$$K(H_1 \wedge E, H_2) = K(H_2 \wedge E, H_1) = log\frac{Pr(H_1|H_2 \wedge E)}{Pr(H_1)} = log\frac{1}{3}$$

$C_K(\{H_1, H_2\}) \approx 0.0457$ $C_K(\{H_1, H_2, E\}) \approx -0.0067$

Case 10: $m(E, h) = G(E, h) = log(\frac{Pr(E \mid H)}{Pr(E \mid \neg H)})$

$$G(H_1, H_2) = G(H_2, H_1) = log(\frac{Pr(H_1|H_2)}{Pr(H_1|\neg H_1)}) = log\frac{7}{6}$$

$$G(H_1, E) = G(H_2, E) = log\frac{Pr(H_1|E)}{Pr(H_1|\neg E)} = log\frac{19}{12}$$

$$G(H_1, H_2 \wedge E) = G(H_2, H_1 \wedge E) = log\frac{Pr(H_1|H_2 \wedge E)}{Pr(H_1|\neg(H_2 \wedge E))} = log\frac{9}{110}$$

$$G(E, H_1) = G(E, H_2) = log\frac{Pr(E|H_2)}{Pr(E|\neg H_2)} = log\frac{5}{3}$$

$$G(E, H_1 \wedge H_2) = log\frac{Pr(E|H_1 \wedge H_2)}{Pr(E|\neg(H_1 \wedge H_2))} = log\frac{9}{5}$$

$$G(H_1 \wedge H_2, E) = log\frac{Pr(H_1 \wedge H_2|E)}{Pr(H_1 \wedge H_2|\neg E)} = log\frac{95}{27}$$

$$K(H_1 \wedge E, H_2) = K(H_2 \wedge E, H_1) = log\frac{Pr(H_1|H_2 \wedge E)}{Pr(H_1|\neg(H_2 \wedge E))} = log\frac{9}{110}$$

$C_F(\{H_1, H_2\}) = 0.066$; $C_G(\{H_1, H_2, E\}) \approx -0.214$

The coherence measure Roche (2013) proposed also violates (CP). Roche suggests to plug in the function

$$R(X,Y) \stackrel{\text{def}}{=\!=} \begin{cases} Pr(X|Y) & \text{if } e \not\models h \text{ and } e \not\models \neg h \\ 1 & \text{if } e \models h \text{ and } e \not\models \bot \\ 0 & \text{if } e \models \neg h \end{cases}$$

Given the same probability distribution,

$$R(H_1, H_2) = R(H_2, H_1) = Pr(H_1|H_2) = \frac{1}{3}$$

$$R(H_1, E) = R(H_2, E) = Pr(H_1|E) = \frac{5}{12}$$

$$R(H_1, H_2 \wedge E) = R(H_2, H_1 \wedge E) = Pr(H_1|H_2 \wedge E) = \frac{1}{10}$$

$$R(E, H_1) = R(E, H_2) = Pr(E|H_1) = \frac{1}{3}$$

$$R(E, H_1 \wedge H_2) = Pr(E|H_1 \wedge H_2) = \frac{1}{10}$$

$$R(H_1 \wedge H_2, E) = Pr(H_1 \wedge H_2|E) = \frac{1}{24}$$

$$R(H_1 \wedge E, H_2) = R(H_2 \wedge E, H_1) = Pr(H_2 \wedge E|H_1) = \frac{1}{30}$$

The coherence of $\{H_1, H_2\}$ is $\frac{1}{3}$, while the coherence of $\{H_1, H_2, E\}$ is is approximately 0.216. Again, the coherence of $\{H_1, H_2, E\}$ is lower than the degree of coherence of $\{H_1, H_2\}$

With the probability distribution in Figure 1, it can be shown that all these confirmation based measures of coherence fail to satisfy (CP). Although not exhaustive, the instance shows that most mainstream coherence measures fail to satisfy (CP).

4.4 Undesirable results of violating (CP)

If we agree that (CP) is an intuitive requirement of coherence, the fact that most probabilistic coherence measures violate (CP) implies that these measures fail to represent our intuitive idea of coherence correctly. Therefore, various applications of coherence which are derived from these coherence measures may lead to problematic results.

An example can be found with the attempt of showing that coherence is confirmation-conducive. As introduced in chapter three, Moretti (2007) proves that Olsson's measure is one and the only measure which satisfies *confirmation transmission* (CT) and *evidence gathering* (EG). Under Olsson's measure, given that a proposition E_1 confirms P, if a set of propositions E which contains E_1 is sufficiently coherent, all other elements in that set also confirm P. Unfortunately, since Olsson's measure violates (CP), it may happen that adding propositions confirming every element of E makes the set less coherent, and this further leads to the consequence that E no longer confirms P. In other words, the set may lose an important feature when extended with confirming propositions. This problem can be illustrated with the following example.

Suppose that symptom E_1 is a sign for disease D. When a medical laboratory scientist finds out that a patient has symptom E_1, it is quite likely that the patient has D. That is, the probability of patient having disease D conditional on the presence of E_1 is greater than the probability of a patient having D. Assume there is another symptom E_2 which usually comes with E_1, the claim that the patient has E_1 is highly coherent with the claim that the patient has E_2. By further assuming that the degree of coherence between E_1 and E_2, according to the Olsson measure, is above the threshold for evidence gathering, given the fact that E_1 confirms D, it can be inferred that E_2 also confirms D. However, since Olsson's measure violates (CP), it is possible that there exists another symptom E_3 which confirms both E_1 and E_2, but the set of symptoms $\{E_1, E_2, E_3\}$ does not confirm that the patient has disease D. That is, it may happen that a symptom E_3 indicates that the patient does have symptom E_1 and symptom E_2, but the collection of all three symptoms does not indicate that the patient has disease D.

The attempt to rank scientific explanations in terms of coherence has the same problem. Suppose there are several competing theories $T_1, ..., T_n$ which explain a certain phenomenon P. One way of evaluating competing theories is to measure the extent each T_i coheres with P, and rank them accordingly. That is to say, the goodness of an explanation is measured as the degree of coherence between the explanation with the explanandum. It has been proved that Olsson's measure captures the idea of mainstream probabilistic accounts of goodness of explanation, and hence can be adopted as the proper measure for ranking different explanations. Nonetheless, since Olsson's measure does not satisfy (CPC), it may happen that the best explanation fails to be the best when it is further confirmed. Suppose that among a set of competing theories $T_1, ..., T_n$, T_k coheres with a phenomenon P to the greatest extent, namely that $max(C_O(T_1, P), ..., C_O(T_n, P)) = C_O(T_k, P)$. With the idea that the best explanation is the most coherent explanation, T_k is the best explanation. Assume that scientists discover a new piece of evidence E which confirms both T_k and P, namely that $Pr(T_k|E) > Pr(T_k)$, $Pr(P|E) > Pr(P)$ and $Pr(T_k \wedge P|E) > Pr(T_k \wedge P)$. Since Olsson's measure violates (CPC), it is possible that adding E to $T_k \cup \{P\}$ makes $T_k \cup \{P\}$ less coherent. Thus, T_k might no longer be the theory which best coheres with P when extended with E. That is, a piece of evidence E makes T_k and the explanandum P more probable, but makes T_k less good as an explanation of P. Taking coherence as a way of evaluating explanations leads to the unacceptable consequence that, in some cases, goodness of an explanation is negatively correlated with the truthfulness of that explanation.

With these two examples, we can see that violating (CP) leads to undesirable results. The reason for these counterintuitive results here is that the notion of coherence, as formally characterized by these measures, is not truth-conducive. When a set evidence for a certain claim is confirmed, we take these evidence as more likely to justify that claim. Also, when an explanation of a hypothesis is confirmed, we take that explanation as more likely to be the correct explanation. However, since coherence is non-truth-conducive. These approaches of proving that coherence is valuable result in counterintuitive consequences.

We can now locate (CP) in the debate concerning whether coherence is a truth-conducive notion. Klein and Warfiled (1994, 1996) first claim that adding a proposition to a set may make the set more coherent, but may also make it less likely to be true. Thus, coherence is not truth-conducive. Bovens and Hartmann (2003, pp.19-22) prove a more specific result that there is no truth-conducive coherence measure. What is shown by (CP) is that, when a set is extended with a confirming proposition and hence made more likely to be true, the degree of coherence of that set may be lower than the original set according to most probabilistic measures of coherence. Examining coherence measures with (CP) can thus be regarded as a more direct way of showing that these coherence measures are not truth-conducive. One no longer needs to accept Bovens and Hartmann's complicated model to see this point.

Because of the above mentioned reasons, attempts of saving coherence do not seem successful. Although it may be true that the notion of coherence, as characterized by these probabilistic measures, is correlated to other notions in epistemology, it is not yet in accordance with our intuitive understanding of coherence, and leads to unacceptable results. To solve this problem, one has to either give up the attempt to show that coherence is a useful notion with these probabilistic measures, or revise the probabilistic measures, so as to avoid the violation of (CP).

4.5 Avoiding violation of (CP)

To see whether it is possible to solve this problem, one must first find the reason why these measures violate (CP). For agreement measures of coherence, the answer is quite simple. Agreement measures take the degree of coherence of a set as the agreement of all elements of that set. Therefore, if a proposition confirms every element of a set, but agrees with all elements to a lesser extent than the agreement between elements in the original set, the degree of coherence of the new set would be lower than the original set. Consider the simplest case with three propositions H_1, H_2, E and an agreement measure of coherence C. If the agreement between H_1 and H_2 is greater than the degree of agreement between H_1, H_2 and E, $C(\{H_1, H_2\})$ would be greater than $C(\{H_1, H_2, E\})$, which is an example showing that C violates (CP). The fact that E confirms both H_1 and H_2 does not imply that E agrees with H_1 and H_2 to a greater extent than the agreement between H_1 and H_2. Hence, adding propositions which confirms every element does not necessarily lead to greater coherence, according to agreement measures of coherence..

For confirmation based measures of coherence, the reason is similar. Since confirmation based measures take coherence as the average degree of confirmation between every pair of elements of a set, when extended with a proposition which confirms every element to an extent less than the mutual confirmation between elements of the original set, the coherence of the new set decreases. Consider again the simple case with three propositions H_1, H_2 and E. If the average degree of mutual confirmation between H_1 and H_2 is greater than the average degree of mutual confirmation between H_1, H_2 and E, the coherence of $\{H_1, H_2, E\}$ would be lower than the coherence of $\{H_1, H_2\}$, as measured with confirmation based measures.

We can thus see why probabilistic coherence measures violate (CP). (CP) concerns whether a proposition which confirms a set makes that set more coherent, but does not take how strong that confirmation is into account. Whether a proposition confirms another is not a matter of degree, but simply a yes or no question. Probabilistic measures cannot capture the non-quantitative aspect of our intuitive understanding of coherence, hence fail to satisfy (CP) and are deemed counterintuitive.

With the cause of this problem clarified, the next step is to think of ways to save these coherence measures. It can be seen that it is impossible to save Olsson's measure from (CP). Recall that Olsson measures the coherence of a set as the ratio of the probability of the conjunction of all elements to the probability of the disjunction of all elements of that set. Measuring coherence this way, adding any other propositions that are not entailed by elements of the set either leads to an increase of the denominator, or a decrease of the nominator. That is to say, adding any proposition that is not entailed by a set always leads to a decrease in its coherence, no matter if the added proposition confirms the elements or not. Hence, Olsson's measure can never be saved from the threat of (CP).

For Shogenji's measure, the situation is better. Since Shogenji's measure is size-informative, a set can be made more coherent when extended with new propositions. It is thus possible to find a condition which rules out cases violating (CP). A natural solution is to set a threshold for incoming propositions. For a set S, if a proposition P confirms every element of S to a sufficiently high degree, adding P would not lead to a decrease in the coherence of S, and hence would not be a case violating (CP). We can consider a simple case with two propositions $S = \{P_1, P_2\}$:

Example 4.5.1. Suppose there is a set $S = \{P_1, P_2\}$ and a proposition P^* such that P^* confirms both P_1 and P_2, i.e. $Pr(P_1|P^*) > Pr(P_1)$ and $Pr(P_2|P^*) > Pr(P_2)$. With Shogenji's measure, the coherence of S is $\dfrac{Pr(P_1 \wedge P_2)}{Pr(P_1)Pr(P_2)}$, while the coherence of $S \cup \{P^*\}$ is $\dfrac{Pr(P_1 \wedge P_2 \wedge P^*)}{Pr(P_1)Pr(P_2)Pr(P^*)}$. To guarantee that cases violating (CP) do not occur, it suffices to set a condition such that

$$\frac{Pr(P_1 \wedge P_2 \wedge P^*)}{Pr(P_1)Pr(P_2)Pr(P^*)} > \frac{Pr(P_1 \wedge P_2)}{Pr(P_1)Pr(P_2)}$$

which allows us to derive that

$$\frac{Pr(P_1 \wedge P_2 \wedge P^*)}{Pr(P^*)} = Pr(P_1 \wedge P_2|P^*) > Pr(P_1 \wedge P_2)$$

That is, for a case with two propositions, if the incoming proposition confirms the conjunction of both propositions, the result would not violate (CP).

We can further generalize the result to find a condition for Shogenji's measure to satisfy (CP). Given a set $S = \{P_1, ..., P_n\}$ and a proposition P^* confirming every $P_i \in S$, we can derive that

$$C_{Sh}(S) = \frac{Pr(P_1 \wedge ... \wedge P_n)}{Pr(P_1)...Pr(P_n)} \; ; \; C_{Sh}(S \cup \{P^8\}) = \frac{Pr(P_1 \wedge ... \wedge P_n \wedge P^*)}{Pr(P_1)...Pr(P_n)Pr(P^*)}$$

To guarantee that $C_{Sh}(S \cup \{P^*\}) > C_{Sh}(S)$, the following condition must be satisfied:

$$\frac{Pr(P_1 \wedge ... \wedge P_n \wedge P^*)}{Pr(P_1)...Pr(P_n)Pr(P^*)} > \frac{Pr(P_1 \wedge ... \wedge P_n)}{Pr(P_1)...Pr(P_n)}$$

This condition is equivalent to

$$\frac{Pr(P_1 \wedge ... \wedge P_n \wedge P^*)}{Pr(P^*)} = Pr(P_1 \wedge ... \wedge P_n|P^*) > Pr(P_1 \wedge ... \wedge P_n)$$

That is, if P^* confirms the conjunction of all elements in S, the coherence of $S \cup \{P^*\}$ would be greater than the coherence of S. We can hence drive the following condition:

Proposition 4.5.1. Given a set $S = \{P_1, ..., P_n\}$ and a proposition P^* which confirms every element of S, i.e. $Pr(P_i|P^*) > Pr(P_i)$ for all $P_i \in S$. if P^* confirms $\bigwedge S$, the degree of coherence of $S \cup \{P^*\}$ would be greater than the degree of coherence of S, as measured with Shogenji's measure.

With this condition, Shogenji's measure would not generate problematic cases violating (CP).

Saving confirmation measures of coherence from (CP) is a more complicated task. A possible solution is to give up the idea of averaging the degree of confirmation, and take the sum of degrees of mutual confirmation in a set as its coherence. If coherence is measured in this way, it is guaranteed that adding a proposition which confirms every element of a set leads to a greater degree of coherence. However, this solution has a significant shortcoming. If coherence is just the sum of all degrees of mutual confirmation of a set, it may happen that a large set of weakly correlated propositions is more coherent than a small but perfectly coherent, which is unacceptable. That is, the idea of averaging the degree of mutual confirmation between every pair of elements cannot be abandoned.

Another possible solution is to find a threshold for incoming propositions so as to rule out cases violating (CP). For example, given a set $\{P_1.P_2\}$, one may simply set a restriction that any incoming proposition should confirm both P_1 and P_2 to a extent greater than the mutual confirmation between P_1 and P_2. However, the threshold for each measure is highly dependent on the specific probability distribution of each set. It is therefore very hard to derive a systematic way to find the desired threshold for incoming confirming propositions.

Nevertheless, the strongest one can easily be found. For any set of propositions $S = \{P_1, ..., P_n\}$, the degree of coherence of S is the average degree of confirmation between every element. If one adds a proposition P^* such that, for every element S' of the power set of S, the degree P^* confirms S' is greater than the average degree of confirmation between elements of S, then the coherence of $S \cup \{P^*\}$ would definitely not be lower than the coherence of S.

Proposition 4.5.2. The strongest threshold to rule out cases violating (CP)

Given a confirmation based measure of coherence $C_m(\cdot)$ and a set of propositions $S = \{P_1, ..., P_n\}$ and a proposition P^* such that $Pr(P_i|P^*) > Pr(P_i)$ for every $P_i \in S$. If for every $S' \in \wp(S)$, $m(P^*, S') > C_m(S)$, then $C_m(S \cup \{P^*\}) > C_m(S)$.

Although proposition 4.5.2 does rule out cases violating (CP), it is way too strong. One can find cases in which the added proposition violates this condition, yet still conforms to (CP). Moreover, it can be considered *ad hoc* to solve this problem by setting up thresholds, for both confirmation and agreement measures. Indeed, it is possible to find a threshold for each coherence measure in every case, and claim that for a set S, only propositions that confirm element of S to a certain degree are allowed to be added to S. With this threshold, we can guarantee that (CP) would not be violated. The problem of this solution is that there is no reason for people to set up such a threshold. In measuring coherence, one simply picks several propositions, and apply coherence measures to the arbitrary set of propositions. There is no reason for us to set up a restriction and claim that certain ways of picking elements to measure are not allowed. Without a convincing reason, it would be rather strange to claim that certain kind of sets, namely those violate (CP), are illegitimate and cannot be measured. Putting it in another way. For an arbitrary set S and a coherence measure C, we can set a threshold such that for any proposition P which confirms every element of S above this threshold, $C(S \cup \{P\}) > C(S)$. But since one can still measure the coherence of the union of S and another proposition P^* which confirms elements of S to a degree lower than this threshold, it makes no sense to set up a threshold. A more promising way of saving coherence measures is to define a more sophisticated measure of coherence which takes the number of confirmation relations as a factor.

Chapter 5

Conclusion and Future Work

The primary purpose of this thesis is to show that all mainstream coherence measures violate a natural requirement, and hence fail to correctly capture our intuitive understanding of coherence. We first introduced the traditional accounts of coherence to illustrate the ordinary understanding of coherence, and further surveyed different probabilistic measures which capture the notion of coherence with formal tools. After making a complete investigation of the mainstream probabilistic measures, we demonstrated Bovens and Hartmann's proof for the impossibility results. On the basis of a model of information gathering, they show that it is impossible to find a truth-conducive probabilistic coherence measure. The probability of a set depends on the reliability of sources, rather than the coherence of that set. Given the fact that there does not exist any truth-conducive coherence measure, one needs to find other epistemic ideals to show that the notion of coherence is important. However, although coherence measures are conducive to some other measures, they may still generate unacceptable results because of violating a new intuitive requirement *coherence preservation*. We examined a number of coherence measures taking into account the requirement of coherence preservation, we have shown that most measures fail to capture an important non-quantitative aspect of coherence, and hence cannot correctly represent our intuitive understanding of coherence.

There are several significant conclusions that can be drawn from this discovery. First, cases showing that coherence measures violate the condition of coherence preservation (CP) provide reassurance of the claim that these probabilistic coherence measures are not truth-conducive. It is Klein and Warfield (1994, 1996) who first propose an argument against the truth-conduciveness of coherence, and ignite the whole search for an ideal coherence measure. Bovens and Hartmann's (2003) work, as a follow-up attempt, shows in a complicated way that a truth-conducive coherence measure does not exist. On the basis of the requirement (CP), the fact that coherence measures are not truth-conducive can be proved in a simpler way. One does not need to accept Bovens and Hartmann's perplexing assumptions, but can still derive similar results indicating that most coherence measures are not truth-conducive. In other words, although not as exhaustive as Bovens and Hartmann's work, (CP) is a more straightforward way to prove that most coherence measures are not truth-conducive. Besides, (CP) is a problem for most current coherence measures,

Second, the result that probabilistic measures fail to correctly represent coherence again casts doubt on the claim that coherence is a useful notion in epistemology. After Bovens and Hartmann's striking proof that there does not exist any truth-conducive coherence measure, philosophers have been trying to save the notion of coherence by showing that it is conducive to other epistemic ideals. Unfortunately, although it has been proven that coherence is a notion which can be applied to a wide range of issues, there is a strong reason to believe that these attempts are not successful. The notion of coherence that is proved useful is characterized by problematic measures, hence it may not be what we generally want to accept as coherence. The validity of arguments showing the practical value of coherence, therefore, should be reevaluated.

Third, since the hope of finding a coherence measure which satisfies (CP) looks pretty frail, epistemologists might need to reconsider if the approach of measuring coherence in terms of probability is mistakenly oriented. Perhaps the notion of coherence should better be understood in a different fashion. It would be interesting to combine the probabilistic approach with other qualitative approaches, and derive a definition which is in better accordance with our intuitive idea of coherence.

There are several interesting directions that one can explore further. One may keep trying to prove more positive results with these measures, and further show that the notion of coherence, as characterized by these measures, is pragmatically valuable. If it is indeed highly beneficial to adopt this notion as coherence, violating (CP) might be an acceptable shortcoming.

Another possible approach is to combine probabilistic coherence measures with other approaches of characterizing coherence. A recent study[1] suggests to define coherence in terms of epistemic utility, which says that a belief set is coherent if the epistemic utility one obtains with that belief set would never be weakly dominated. If one can prove that the notion of coherence, as characterized by certain probabilistic measures, is conducive to epistemic utility such as accuracy, probabilistic measures can again be used as an important approach of defining coherence.

[1] A group of formal epistemologists including Branden Fitelson, Kenny Easwaran, David McCarthy, James Joyce characterize coherence in terms of decision theory. This approach provides us a different perspective to understand the notion of coherence. See Joyce (2009).

One can also keep working on the idea of measuring coherence in terms of confirmation, and propose a coherence measure which is free from violating (CP). In order to get rid of the problems caused by (CP), one might need to take the number of confirmation relations in a set into account. That is, if the non-quantitative aspect could be taken as a factor while measuring coherence, it would be possible to construct a coherence measure which is free from violating (CP).

In sum, the requirement of coherence preservation indicates that the non-quantitative aspect of our normal understanding of coherence is not represented by probabilistic coherence measures. As a result, the approach of characterizing coherence only in terms of probability is not optimal and leads to the mentioned problems. To solve this problem, we would ask epistemologists to either propose a new probabilistic measure which better captures the notion of coherence, or to combine the probabilistic approach with other formal approaches, including logic designed to reason about the belief dynamics of agents.

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