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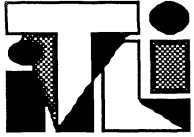
**TRADITIONAL LOGICIANS AND  
DE MORGAN'S EXAMPLE**

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## Traditional Logicians and De Morgan's Example

### Introduction

The pre-fregean logic of terms is usually identified with a finite set of monadic first-order schemata. This is one reason why the historical antecedents of one simple but pervasive proof-procedure have fallen into oblivion. We are referring to the rules  $R_1$ ,  $R_2$  (stated below in schematic form) which generate arguments involving multiple generality or relational expressions :

$R_1$ :     Every X is Y     . . . X . . .  
    . . . Y . . .  
 Providing X obeys condition  $C_1$   
 in the context . . . X . . .

$R_2$ :     Every X is Y     . . . Y . . .  
    . . . X . . .  
 Providing Y obeys condition  $C_2$   
 in the context . . Y . . . .

The pervasiveness of this procedure is apparent from the fact that three modern systems ( cf. Van Benthem 1986, Sommers 1982, Suppes 1979 ), developed independently of each other, contain instances of  $R_1$  and  $R_2$ . These versions of the rules differ in the way the conditions  $C_1$ ,  $C_2$  are formulated and motivated.<sup>1</sup> But within each system arguments can be generated which defy the recognition power of monadic logic, without having to resort to the familiar procedure of translation into predicate logic and subsequent manipulation of first-order formulas. These systems are intended as an account of inference which stays close to the grammatical form of the sentences involved. This, of

course, deviates from the most familiar strategy. Suppose, for instance, that *Every tail of a horse is the tail of a horse* is a valid sentence and that the second occurrence of *horse* therein obeys condition  $C_1$ .

Then, by  $R_1$ , the following sequence

Every horse is an animal      Every tail of a horse is the tail of a horse  
Every tail of a horse is the tail of an animal

constitutes a non-trivial justification of the well-known example of De Morgan:

Every horse is an animal  
Every tail of a horse is the tail of an animal

These modern systems deserve separate treatment and in a future paper we shall consider them from a systematic point of view. The present paper is an historical investigation into the pre-fregean formulations of the rules  $R_1$ ,  $R_2$  given by Ockham, Leibniz and De Morgan. We use De Morgan's example to illustrate the strength of the old versions of the inference rules and to test their soundness. This simple argument is commonly seen as an intuitively valid argument which exposes the weakness of traditional logic. De Morgan himself is supposed to be the first logician who was conscious of the validity of arguments involving relational expressions, and his relational logic is supposed to have been developed in order to explain this validity. This paper consists of three sections. In §1 we outline the context in which De Morgan introduced arguments involving multiple generality and relational expressions. There we shall see that his recognition of these arguments does not at all point to his later relational logic, but instead exemplifies the approach outlined in the first paragraph. In §2 we shall consider Leibniz' use of  $R_1$  in order to justify certain arguments, essentially similar to De Morgan's example, which had been put forward

by J. Jungius (1587-1657) as non-syllogistic but valid. In §3 we shall consider the oldest versions of  $R_1$  and  $R_2$  which we have as yet been able to find: Ockham's formulation of them in terms of the medieval supposition theory.

## §1. De Morgan

### 1.1. The General Setting

In his *Formal Logic* (De Morgan 1847) De Morgan hoped to frame a logical notation within which he could picture the logical form of arguments more clearly.<sup>2</sup> The arguments he had in mind included not only instances of classical schemata. As a matter of fact, he extended the range of application of logic by simply increasing its expressive power: in addition to the usual simple ones, compound terms, his logical language included negative, conjunctive and disjunctive terms.

This increase did pay off. On the systematic side, the negative terms introduced the notion of a universe of discourse into logic; while the interplay of the new terms suggested the so-called laws of De Morgan. On the practical side, De Morgan was able to build up a large reservoir of valid non-syllogistic schemata. Hence, by reference to them, the validity of more arguments could now be settled than was previously the case. Examples of simple schemata which escape the bounds of the expressive power of traditional logic, but which do not elude De Morgan's notation are:<sup>3</sup>

$$\frac{\text{Every X is Y} \quad \text{Every X is W}}{\text{Every X is Y and W}}$$

$$\frac{\text{Every X is Y and W}}{\text{Every X is Y and every X is W}}$$

It was in the course of this syntactic enterprise that De Morgan spoke of compound terms consisting of generality and relational expressions, the so-called relatives as *tail of a horse*. Nevertheless, we seek their formal counterparts in *Formal Logic* in vain. In fact it took him some years to devise a symbolism in which relatives could be expressed.<sup>4</sup> In the meantime his treatment of relative arguments (arguments that rest on manipulations inside the relatives) had to differ from the schematic approach, since he had no schemata with the required form at his disposal.<sup>5</sup>

Wanting a logic strong enough to include relative arguments but lacking a suitable formal representation strategy, De Morgan chose a direct approach. First, he formulated versions of  $R_1$  and  $R_2$  which can be directly applied to natural language sentences. Afterwards he could count an argument  $A_1 \dots A_n \Rightarrow B$  as valid, if the conclusion,  $B$ , follows from the premisses  $A_1 \dots A_n$  in accordance with these rules. De Morgan claimed his rules to be syllogistically complete, in the sense that all instances of traditional syllogisms are generated by them.<sup>6</sup> As we shall see, however, De Morgan did forget to check whether his rules were sound, that is, whether or not they generate arguments with true premisses and false conclusions.

## 1.2. DE MORGAN'S PROBLEM

Before stating the rules, De Morgan wished to make clear why we need them in the first place. Clearly, he is reasoning against the logician who holds the syllogistic fullness thesis :

Any valid argument is (an instance of) a classical  
immediate inference or (an instance of ) a classical

sylllogism or is reducible to one of these by accepted classical moves.<sup>7</sup>

De Morgan devised two simple arguments which meant trouble for this idea that syllogistic logic accounts for all valid reasoning:<sup>8</sup>

- (1)     Man is animal  
           The head of a man is the head of an animal
- (2)     Every man is an animal  
           He who kills a man kills an animal

It is supposed to be obvious that neither argument is an instance of any classical scheme. De Morgan spent a little more time arguing that they are not reducible to syllogisms either. This point is worked out with reference to the second argument. De Morgan considered this argument to be equivalent to the sequence:

- (3)     Every man is an animal     Some one kills a man  
   Some one kills an animal

But this move fails to yield a syllogism, for the last two sentences are not categorical. We can, however, substitute other sentences for them which have the same import and the required syllogistic form. We thus obtain the following sequence of categorical sentences:

- (4)     Every man is an animal     Some one is the killer of a man  
   Some one is the killer of an animal

According to the syllogistic standards of validity, however, this new argument unfortunately does not count as valid. Because of the missing middle there is no expression occurring twice in the premisses; hence we have an invalid syllogism. It is true that the expression

*man* occurs in both premisses, but in the second one it has no independent occurrence; hereby *man* cannot be seen as the syllogistic missing link between the extremes of the conclusion.

The usual interpretation of this passage from *Formal Logic* is that De Morgan wanted to show that traditional logic cannot handle relative arguments. It is of some importance to make two qualifications in this regard:

i. De Morgan himself spoke of compound expressions in general as a problem for the fullness thesis.<sup>9</sup> And even though he actually formulated his two arguments with relational expressions, the point he made can equally well be made without them. The following one does just that and for the same reasons as the original argument:

Every horse is an animal      Some brown horse runs  
Some brown animal runs

ii. In the light of De Morgan's proposed solution, traditional logic cannot really be seen as the true target of his criticism. It is rather the fullness thesis which he tried to prove wanting. Hence, if we look upon traditional logic not as a finite set of valid schemata, but instead as a set of a rules generating valid arguments, De Morgan's examples prompt a further question: can these rules perhaps be interpreted in such a way that they generate those arguments too? De Morgan's answer was that they indeed can, and we shall next see which kind of interpretation he offered.

### 1.3. DE MORGAN'S SOLUTION

Having thus disposed of the *fullness thesis*, De Morgan began on his main task of stating rules of inference which would take care of the intractable relative arguments,



in particular arguments (1)-(4). The rules he gave are neither unproblematic in their applications nor felicitously worded: <sup>10</sup>

D<sub>1</sub>      The genus may take the place of the species  
               when some of the species is mentioned.

D<sub>2</sub>      The species may take the place of the genus  
               when all the genus is spoken of.

But they do seem up to the work required of them. For instance, with the aid of D<sub>2</sub> we can explain the acceptability of this argument:

A man sees every animal

A man sees every horse

To achieve that goal we may resort to semantical facts. We know as a matter of fact that ANIMAL, the denotation of *animal*, is the genus of HORSE, the denotation of *horse*, and by the same token that HORSE is species of ANIMAL. We therefore use D<sub>2</sub> in order to substitute *horse* for *animal* in the given premiss; in doing this we reach the desired sentence as conclusion.

On the face of it, De Morgan's wording of the rules seems to imply that we will not be able to escape resorting to such underlying semantic facts. If so, his natural logic flies into the face of a logical principle; for it would rule out valid arguments with false premisses. For instance, consider the following argument:

Every animal is a horse      A man sees every horse

A man sees every animal

This argument is valid but the actual genus - species relationship which exists between the denotations of *horse* and *animal* precludes the rules saying this.

We know however that De Morgan explicitly adhered to the principle that a valid argument could have false premisses,<sup>11</sup> so we will have to bring our interpretation of the rules in line with this principle. So from now on, we will no longer require the relevant denotations to behave as genus and species. We will just assume that they do. To achieve this we exploit the fact that universal affirmative sentences like *Every S is P*, are employed to assert (truly or falsely) that S is species of P, and that P genus van S.<sup>12</sup> This is why universal sentences are an essential element in the arguments which De Morgan's rules generate.<sup>13</sup>

De Morgan called the first of his rules,  $D_1$ , a version of the *dictum de Omni*.<sup>14</sup>

This classical dictum (which lacks a standard formulation) has regularly been thought of as the central syllogistic principle. Central in all the versions of this dictum is the role assigned to universal sentences. Given the assertion *every S is P* and the further information *R is S*, all versions of the dictum entitle us to infer that R is also P. And that is exactly what  $D_1$  is supposed to do. Unlike some other writers, however, De Morgan avoided stating that his dictum de Omni is the only inference rule needed in the generation of all the classical syllogisms. By itself  $D_1$  yields the following sequences as valid syllogisms:

Every M is P      Some S is M

Some S is P

Every M is P      Every S is M

Every S is P

The rule  $D_2$  is the mirror image of the *dictum de Omni*. The dictum justifies the substitution of an expression with a supposedly more encompassing denotation, for another one with a supposedly less encompassing denotation. According to  $D_2$ , however, we may substitute an expression with a supposedly less encompassing denotation, for another expression with a supposedly more encompassing denotation. Hence  $D_2$  yields these syllogisms

$$\frac{\text{Every S is M} \quad \text{No M is P}}{\text{No S is P}}$$

$$\frac{\text{Every P is M} \quad \text{Some S is not M}}{\text{Some S is not P}}$$

This is, of course, not the only thing which De Morgan demanded of his rules. He also tried to generalize the old dictum and its mirror image beyond the categorical fragment: they were to form the basis for the justification of arguments involving relational expressions and multiple generality. For instance, the application of the rules to the two-premiss arguments (3) and (4) is quite direct. The universal sentence *Every man is an animal* lays down a genus-species relationship and the second premiss constitutes the context in which, in accordance with  $D_1$ , the substitution may be made.

But it is rather disappointing that we are in the dark about how he coped with his own original arguments (1) and (2). We have isolated the contribution of the universal sentence given as premiss in both arguments.<sup>15</sup> We know for sure that substitutions have to occur. But what we do not know is in which sentences the substitutions are to be carried out. De Morgan does not indicate this explicitly.

As we pointed out in the introduction, modern writers dealing with De Morgan's example from the perspective of natural logic often introduce a "tautological" premiss in which the substitution takes place.<sup>16</sup> They clearly exploit the idea that if

$A, B \Rightarrow C$  is a valid inference and  $A$  is a valid formula, i.e. true under every interpretation, then  $B \Rightarrow C$  has to be valid.

So let us suppose that De Morgan had the modern strategy in mind and consider argument (2). Choose as tautological premiss *He who kills a man kills a man*. And now apply  $D_1$  to the second occurrence of *man* in this sentence, using the information conveyed by *Every man is an animal* about the genus-species relationship. We then see that the sought-after conclusion, *He who kills a man kills an animal*, does indeed follow from the given premiss together with the tautological sentence.

It goes without saying that the validity of (1) and the so-called De Morgan's example can be established in the same manner from  $D_1$ , with the aid of suitably chosen tautological premisses.

#### 1.4. Shortcomings

As illustrated above, De Morgan's logic seems stronger than syllogistic logic, since the validity of some arguments involving multiple generality can be justified with the aid of the substitution rules. There are, however, a few problems. In the quotation of  $D_1$  and  $D_2$  we have underlined the when-clauses, which place certain restrictions on the substitutions. It is not sufficient that the denotation of the relevant expressions be given as genus and species. It is just as important that the expressions themselves be used in a particular way in the sentence in which the substitution is made. Before substituting one expression for another, we have to be certain that they obey the when-clauses. This means that we must be certain the "genus [is] being spoken universally of " in one case and that "some of the species [is] being mentioned " in the other.

Up now we have been working under the assumption that the expressions of our examples fulfil those restrictions. This is a simplification since we have not yet given any criterion which could be used to determine if such is the case. Once more we are in the



try to interpret them in terms of the traditional doctrine of distribution..<sup>19</sup> This is not all too farfetched, since De Morgan himself identifies the expressions *universally spoken* of and *distributed*..<sup>20</sup> With the backing of his own identification, we can re-word De Morgan's rules in the following fashion:

- D<sub>1</sub>     Every X is Y     ... X ...  
                  .. Y ...  
                  Providing X occurs non-distributively in  
                  the context ... X ...
- D<sub>2</sub>     Every X is Y     ... Y ...  
                  .. X ...  
                  Providing Y occurs distributively in  
                  the context ... Y ...

But this move turns out to be ineffectual. The distribution doctrine only says that *kills a man* has two different values within the tautological premiss; it says nothing at all about the distribution value of *man* therein. If we want to complement De Morgan's rules with the distribution doctrine, then this doctrine will itself have to be extended so as to include the elements of compound expressions.

As a matter of fact, this extension of the doctrine is somehow implicit in our unimaginative criteria C<sub>1</sub> and C<sub>2</sub>. In their formulation, the combinatorial characterization of distribution was abandoned, trusting the expressions of generality to settle the distribution values. However, *every* and *a* fail to give the required information. They certainly do play a role, but so does the position of the expressions. What we really need is a systematic way of computing distribution values, starting from basic expressions and using distribution valuations induced by the expressions of generality 21. De Morgan himself does not, however, appear to have recognized the need for such systematic procedure.

## § 2 LEIBNIZ

### 2.1. The inferences a rectis ad obliqua

As we pointed out in the introduction, Leibniz was concerned with arguments essentially similar to the De Morgan example. Leibniz' interest in them was aroused by the work of the Hamburg logician Joachim Jungius. Jungius' *Logica Hamburgensis* is generally seen as one of few 17th century logical texts which deserve any attention, mainly because he recognized certain non-syllogistic patterns of inference which yield valid arguments:

1. The inferences *a compositis ad divisa* <sup>23</sup>

Plato est philosophus eloquens

Plato est philosophus

Plato est philosophus eloquens

Plato est eloquens

2. The inferences *a divisis ad composita* <sup>23</sup>

Omnis planeta per zodiacum movetur      Omnis planeta est stella

Omnis planeta est stella quae per zodiacum movetur

3. The inferences *per inversionem relationis* <sup>24</sup>

Salomon est filius Davidis

David est pater Salomonis

David est pater Salomonis

Salomon est filius Davidis

4. The syllogisms *ex obliquis* <sup>25</sup>
5. The inferences *a rectis ad obliqua* <sup>23</sup>

We already encountered the first two patterns when we were commenting on De Morgan's syntactical expansion of traditional logic, although our representation used the universal determiner while Jungius employs a proper name. Later on we shall consider the syllogisms *ex obliqui*. Meanwhile it is sufficient to observe that De Morgan's arguments (3) and (4) are subsumed by this oblique syllogism.<sup>26</sup>

The inferences *a rectis ad obliqua*, on the other hand, are now of greater importance. These are immediate inferences whose premiss contains expressions in the nominative case (for instance *man* and *animal* in *Every man is an animal*) whereas in the conclusion these expressions are transferred into one of the non-nominative cases by some *notio respectiva* (for instance, the notion of killing transferred the expressions *man* and *animal* into the accusative case in *He who kills a man kills an animal*). The argument which Jungius used to illustrate this pattern is a variant of De Morgan's second argument:

Omnis circulus est figura  
 Quicumque circulum describit figuram describit

Jungius never tried to explain, however, why those patterns yield valid inferences; he simply took for granted that they do. Leibniz, on the other hand, considered this lack of justification a gap which had to be filled. Notorious is his treatment of the inferences *per inversionem relationis*; <sup>27</sup> Dummett's assertion that " Leibniz failed to tackle the problem of multiple generality " indicates that his treatment of the inferences *a rectis ad obliqua*, although since long available, has been unnoticed. <sup>28</sup> We shall see that Leibniz failed to tackle the problem in all its comprehensiveness but that he did strive to give non-



trivial demonstrations of some arguments commonly considered to involve multiple generality and calling for first-order procedures.

## 2.2. Leibniz on the inferences *a rectis ad obliqua*

In 'Specimen demonstratae consequentiae a rectis ad obliqua', <sup>29</sup> Leibniz outlined a strategy with the purpose of giving a syntactical demonstration of inferences *a rectis ad obliqua*. To this end he laid down primitive rules of inference and showed that according to these rules, the conclusion follows from the premisses. This strategy depends on the principle of substitutivity of equivalents, and also on a syntactical generalization of the dictum de Omni. To get some insight in Leibniz' method we shall apply it to this Latin version of De Morgan's example:

Omnis equus est animal

Omnis cauda equi est cauda animalis

Leibniz' rules are the following:

- L<sub>1</sub>      Esse prædicatum in propositione universali affirmativa, idem est, ac salva veritate loco subjecti substitui posse in omnia alia propositione affirmativa, ubi subjectum illud prædicati vice fungitur. Exempli causa: quia graphice est ars, si habemus *rem quæ est graphice*, substituere poterimus *rem quæ est ars*.
- L<sub>2</sub>      Obliquo speciali aequipollet obliquos generalis cum speciali recto, ideo sibi mutuo substitui possunt. Verbi gratia, pro termino *qui discit graphicem* substitui potest, *qui discit rem quæ est graphicem*. Et contra, pro termino *qui discit rem quæ est graphicem* substitui potest *qui discit graphicem*.

$L_1$  says that given the sentence *Omnis S est P*,  $P$  is substitutable for  $S$  in any affirmative sentence  $A$  in which  $S$  occurs as predicate; clearly Leibniz allowed  $A$  to be a relative sentence. Therefore it should be evident that Leibniz' generalization of the *Dictum de Omni* does not affect the kinds of expression which may be involved in the substitutions; it allows instead for a new syntactical context of substitutions: relative sentences.

It is thus not possible to apply  $L_1$  to expressions in one of the oblique cases directly, since it only concerns expressions in the nominative case. The task which remains to be accomplish is to bring oblique expressions into the range of the dictum:  $L_2$  says then that if  $X$  is an expression with  $Y$  as one of its non-nominative cases, then  $Y$  is equivalent to the complex expression *R quae est X*, where  $R$  is the word *res* in the same case as  $Y$ . For instance, *equi* and *animalis* are equivalent to *rei quae est equus* and *rei quae est animal*. In virtue of this equivalence, oblique expressions can be brought into the scope of  $L_1$ . To do this Leibniz must appeal to the principle of substitutivity of equivalents and this is what he does in saying that  $Y$  and the complex expression *R quae est X* are substitutable for each other.

Now we proceed to work out the derivation of *Omnis cauda equi est cauda animalis* from *Omnis equus est animal*, making use of a tautological premise:

1.	Omnis equus est animal.	P
2.	Omnis cauda equi est cauda equi	P
3.	Omnis cauda equi est cauda rei quae est equus	from 2, $L_2$
4.	Omnis cauda equi est cauda rei quae est animal	from 1, 3, $L_1$
5.	Omnis cauda equi est cauda animalis	from 4, $L_2$

### 2.3. The short-comings

Leibniz' strategy is to some extent more elegant than the solution which De Morgan advanced; moreover it is applicable and this is more than what we can say of De Morgan's proposal. However, Leibniz treatment of the inferences *a rectis ad obliqua* unfortunately does overlook a few things:

i. The only constraint Leibniz imposed upon  $L_1$  is that  $S$  must occur as predicate in the context of substitution. If he had limited himself to the standard categorical sentences, then he could, by implication, have derived a constraint. For in this case  $S$  has to appear non-distributively: the predicates of affirmative categorical sentences occur, per definition, in this way.<sup>30</sup> But Leibniz went beyond the categorical fragment by permitting certain relative sentences to be equivalent to oblique expressions and thereby making these substitutable for each other. Without undergoing a generalization, the traditional doctrine of distribution, however, does not predict which distribution values *equus* have in *Omnis cauda equi est cauda rei quae est equus*, since *equus* is here neither predicate nor subject of any categorical sentence.

ii. The lack of constraint on  $L_1$  becomes a problem when we look at  $L_2$ . Here, there is no mention of contexts where the non-nominative expression may occur, and no mention of contexts in which the substitution of the complex expression for the oblique expression should not be carried out.

As a result of Leibniz' overlooking those points his rules are unsound. The following sequence constructed in accordance with  $L_2$  and  $L_1$  proves this fact:

1.	Omnis cauda equi est cauda equi.	P
2.	Omnis cauda equi est cauda equi.	P
3.	Omnis cauda rei quae est equus est cauda equi.	from 2, $L_2$
4.	Omnis cauda rei quae est animal est cauda equi.	from 1,3, $L_1$
5.	Omnis cauda animalis est cauda equi.	from 4, $L_2$

### §.3. OCKHAM

#### 3.1. Suppositio and Distribution

The short-comings of Leibniz' strategy and De Morgan's proposal show that their generalizations of the *Dictum de Omni*, (their instantiation of our schematic  $R_1$ ) have to be supplemented: in the first case with effective constraints restricting the context of substitution, and in the second with effective definitions of this context. The distribution doctrine stops short of yielding those needed features because it is restricted to categorical sentences only; furthermore, it sees all categorical expressions as logically simple even when they are syntactical complex. Of course, there is no point in criticizing the distribution theory for not assigning distribution values to the components of complex expressions. Such assignment is meaningful only when it might be required for the recognition of the validity of arguments containing complex expressions.<sup>31</sup>

The doctrine of distribution, however, is considered to be a simplification, or even worse, as an impoverished version of medieval supposition theory.<sup>32</sup> With the help of this theory, medieval logicians were able to handle arguments which lie beyond the scope of first-order monadic logic. We have tried to find whether arguments similar to De Morgan's example were treated in terms of this theory. We have already stated that Ockham formulated inference rules which may be seen as precluding the rules offered by De Morgan, i.e. which may be seen as instantiations of our abstract couple  $R_1, R_2$ . We shall see presently that he formulated his rules in terms of supposition theory. Because of this, we give a short description of some aspects of this theory. Of course, we do not pursue the supposition theory in all its complexity (and its richness), taking instead the supposition assignments as primitive.<sup>33</sup> For convenience's sake we also consider only two

syncategorematical expressions, *omnis* and *non*, and we shall call all transitive verbs *copula*.

Regarding the assignments they induce, our two syncategorematical expressions differ from each other in one important way: *non* creates a context in which any categorical expression occurring to the right of *non* has supposition *confusa et distributiva*, whereas such supposition is said to adhere only to the expressions to which *omnis* is adjoined directly. When, in an affirmative sentence, an expression *X* occurs after *omnis* and the copula, it is said that *X* has supposition *confusa tantum*. There is, nevertheless, one respect in which they behave similarly: any expression not occurring after them is characterized as having supposition *determinata*.

Let us recapitulate this definitional matter in connection with these classical sentences:

1. *Omnis homo est animal.*
2. *Non homo est animal.*
3. *Homo est animal.*
4. *Homo non est animal.*

From the definitions it follows that supposition *confusa et distributiva* belongs to the occurrences of *homo* in 1,2 and likewise to *animal* in 2,3. Supposition *determinata*, on the other hand, is possessed by *homo* in 3,4. But *animal* has supposition *confusa tantum* in 1.<sup>34</sup>

It will be clear that within the categorical fragment supposition *confusa et distributiva* adheres to the expressions which according to the distribution doctrine occur distributively and conversely. It is also the case that any expression having supposition *determinata* occurs non-distributively. But the inverse does not follow: *animal* appears in 1

non-distributively according to the distribution doctrine and has supposition *confusa tantum* according to the medieval theory.

This distinction between supposition *confusa tantum* and *determinata* which distribution theory obliterates, gains importance when we look at sentences and arguments not belonging to the categorical framework. For instance, the distribution doctrine is unable to distinguish, in pure distribution terms, between the two occurrences of *asinum* in the apparently equivalent sentences:

(6) *Asinum omnis homo videt.*

(7) *Omnis homo videt asinum*

According to a generalized distribution assignment, both occurrences of *asinum* in (6) and (7) are non-distributive. Supposition theory allows for this negative characterization but goes a step further, saying that *asinum* has supposition *determinata* in (6) and supposition *confusa tantum* in (7). Moreover, medieval logicians formulated an inference rule based on this distinction: from a sentence having an expression *X* with suppositio *determinata* (say (6) and *asinum* therein) we can confidently move to another one, differing from the first in that *X* occurs now with supposition *confusa tantum* (thus (7)). The converse inference, however, was explicitly rejected; situations were described showing that moving the other way around could be moving from the true into the false.<sup>35</sup>

### 3.2. Ockham's rules

Our intention in the above paragraph was only to provide a background against which Ockham's instantiation of  $R_1$  and  $R_2$  can be understood, since they are couched in terms of supposition theory. His version of the rules are the following:

O<sub>1</sub> Ab inferiori ad superius sine distributione, sive illud superius supponat confusa tantum sive determinata, est consequentia bona.<sup>36</sup>

O<sub>2</sub> A superiori distributo ad inferius distributum est bona consequentia.<sup>37</sup>

The examples which illustrate the working of these rules make clear that they rest on universal sentences: the subject of such sentence is called the *inferior* and the predicate the *superior* expression. Thus, given a sentence of the form *Omnis S est P* and a context . . . *P* . . . in which *P* occurs with supposition confusa et distributiva, we can derive, in accordance with O<sub>2</sub>, . . . *S* . . . .<sup>38</sup> For instance, the following sequence is sanctioned by O<sub>2</sub>

Omnis equus est animal                      Omne animal videt hominem  
Omnis equus videt hominem

Similarly, when we have a context . . . *S* . . . in which *S* appears with supposition confusa tantum or determinata, we may conclude, in accordance with O<sub>1</sub>, . . . *P* . . . .<sup>39</sup> So the rule sanctions this argument:

Omnis equus est animal                      Equus videt hominem  
Animal videt hominem

Strictly speaking, De Morgan's rules are not applicable to these arguments because we lack effective criteria telling us whether we are entitled to use them or not. Ockham's rules, on the other hand, do not have the same short-comings. They are formulated in terms of

supposition theory and this doctrine yields criteria determining which of the rules, if any, has to be applied. Consequently, Ockham's rules seem to form a more promising mechanism for handling De Morgan's relative arguments.

### 3.3. Syllogisms *ex obliquis*

It is worth emphasizing that not all early logicians would be inclined to reject the validity of the arguments (3) and (4) on account of the fact that they are not standard syllogisms. Some logicians could see those arguments as valid, a fortiori well-formed, syllogisms *ex obliquis*. These are two-premiss arguments in which transitive verbs may play the role of the copula and in which oblique expressions are allowed to appear as (part of) syllogistic terms. The existence of these syllogisms was already acknowledged by Aristotle himself,<sup>39</sup> and as we pointed out earlier, the fact that Jungius recognized them has contributed to his reputation.

Ockham himself listed a great number of oblique syllogisms,<sup>40</sup> claiming that the Dictum de Omni is the logical principle governing their validity. As a matter of fact, we must remark that Ockham did not resort to  $O_1$  or  $O_2$  in justifying oblique syllogisms: the version of the dictum he employed in this connection does not contain any mention of supposition.

But the fact that Ockham did not use his rules for the generation of oblique syllogisms, need not to stop us from doing so. The reason for this is that we are not primarily interested in his treatment of this kind of syllogisms, but in stressing the relative superiority of  $O_1$  and  $O_2$  to De Morgan's rules. Consider this Latin version of (3):



Omnis homo est animal      Aliquis necat homine

Aliquis necat animal

We have in this case a universal sentence giving the superior-inferior characterization of *homo* and *animal*. Furthermore, in the other premiss *hominem* occurs with suppositio distributiva. Thus, in accordance with  $O_1$  the substitution of *animal* for *hominem* yields the given conclusion.

Someone might object to this demonstration because the expression *homo* and not *hominem* is initially given in the superior-inferior relationship. But Ockham himself allows for the possibility of having the inferior expression in one of the oblique cases in the context of substitution and, as a result of that, also the superior after the substitution has taken place.<sup>41</sup>

### 3.4. Ockham and De Morgan's Example

De Morgan introduced (3) as the syllogistic expansion of his second argument. We have shown that Ockham knew of inference rules which may be used to justify that argument. Furthermore, we have pointed out that Ockham would recognize the validity of (3), although his justification of it might be different from ours.

However, we have not yet touched the question whether  $O_1$  could equally well be used in reference to the original argument 2. Neither have we faced the question whether Ockham could recognize the validity of arguments as (1) and (2).

We shall treat these matters using the Latin version of De Morgan's example employed in the consideration of Leibniz' strategy. So let the premisses

- (8)      Omnis equus est animal.
- (9)      Omnis cauda equi est cauda equi.

be given. We have already seen what 8 has to do. But in order to apply  $O_1$  or  $O_2$  to (9) we need to know which supposition belongs to the occurrences of *equi* therein. We certainly know that the complex expression *cauda equi* has suppositio confusa et distributiva in its first and suppositio confusa tantum in its second occurrence. But the question which interests us is the supposition of the expressions which make up the complex one. Ockham's answer is conclusive: neither *equi* of *cauda* has supposition in (9).<sup>42</sup> Supposition, he says, adheres to the extremes of a sentence and not to the expressions making up subjects of predicates.

Ockham did not make clear why we should deny any supposition to the elements of *cauda equi*. But we can, to some extent following Buridan's remarks on this question, try to understand his motivation.<sup>43</sup> We take first the soundness of  $O_1$  and  $O_2$  as given. Therefore, if one of those rules, assisted by certain assumptions  $A_1 . . . A_n$ , yields an invalid argument, then we conclude that at least

one of the  $A_i$ 's has to be rejected. Consider now

(10) *Omnis asinus logici currit.*

Suppose that both *asinus* and *logici* have supposition in (10). Then we have three possibilities: they have either supposition determinata or supposition confusa tantum or supposition confusa et distributiva.

However, neither of them could have supposition determinata since both *asinus* and *logici* occur after *omnis*. But *logici* might have supposition confusa tantum. Thus, according to  $O_1$ , the following sequence would be valid:

- (11) Omnis logicus est homo \_\_\_\_\_ Omnis asinus logici currit  
 Omnis asinus hominis currit

But it is not: let every donkey owned a logician be running; let every logician be a man and let some donkey owned by a non-logician be resting. Then it follows that both premisses are true and the conclusion false, i.e. (11) is invalid. Therefore, *logici* cannot have suppositio confusa tantum in (10).

So, if *logici* has any supposition in (10), it must be supposition confusa et distributiva. This latter supposition has to be attributed to *asinus* in that sentence because *omnis* is adjoined to it directly. Hence, the next argument would be valid according to  $O_2$ :

- (12) Omnis asinus albus est asinus \_\_\_\_\_ Omnis asinus logici currit  
 Omnis asinus albus logici currit

We accept this argument as valid. Ockham and Buridan, however, did not. Ockham introduced the convention that an affirmative sentence with empty subject is false.<sup>44</sup>

From this point of view, we might describe a situation confutating the claim that (12) is valid. Let, again, every logician's donkey be running; let also white donkeys exist, all of which are wild. Then there is no white donkey belonging to a logician thus *asinus albus logici* is an empty expression and this makes the conclusion false, according to Ockham's convention. So (12) turns out to be invalid.

At this point, Ockham and Buridan might reject the supposition claim made in behalf of *asinus* or the convention on affirmative sentences. Clearly, both of them abandoned the supposition claim. So, *asinus* has no supposition in (10). The claim that *logici* has suppositio confusa et distributiva in (10) can be discredited in the same way. The conclusion is then that neither *logici* nor *asinus* has supposition in *Omnis asinus logici currit*.

Strictly speaking those contributions only show that *logici* and *asinus* have no supposition when they make up the subject of a universal affirmative sentence. Consequently, in the view of Ockham, *cauda* and *equi* lack any supposition in their first occurrence in (9). But we do not know which kind of considerations brought Ockham to deny them any supposition in their second occurrence as well.

If Ockham had let the supposition theory speak for itself, it would have attributed *suppositio confusa tantum* to *equi* in its second occurrence. Applying  $O_1$  to (8) and (9) he would get as conclusion *Omnis cauda equi est cauda animalis*. But the two premiss argument

(13)     Omnis equus est animal     Omnis cauda equi est cauda equi  
                   Omnis cauda equi est cauda animalis

would be as far as Ockham would get. It was not open to him treat (9) as a ladder that could be kicked away after having reached the conclusion, thus obtaining:

(14)     Omnis equus est animal  
                   Omnis cauda equi est cauda animalis

And this certainly is not due to the lack of metalogical backing. There is not much anachronism in using a rule of Buridan which, to the modern reader, allows the transition from (13) to (14):

Ad quamcumque propositionem cum aliqua necessaria sibi  
 apposita ... sequitur aliqua conclusio ad eadem propositionem solam  
 sequitur eadem conclusio, sine appositione illius necessaria. . . <sup>45</sup>

The real trouble is that (9), for Ockham nor for Buridan, cannot count as a necessarily proposition: Ockham's convention allows it to be false.

#### 4. Conclusion

Ockham's convention throws new light on the background of the argument

- (15)     Every horse is an animal  
           Every tail of a horse is the tail of an animal

the validity of which we take for granted as De Morgan himself would have done. According to that convention, (15) has to be considered as formally invalid, in the usual sense that there is an interpretation under which the premisses are true and the conclusion false: let every horse be an animal; let no horse have a tail, then *tail of a horse* would be empty and, hereby, *every tail of a horse is the tail of a horse* would be false.

We see thus in which way De Morgan's rejection of the syllogistic fullness thesis could fail to impress a holder of it. If (15) is not formally valid, then the fact that it is not reducible to syllogism cannot be regarded as proving the inadequacy of the thesis. This point was brought home to De Morgan by Mansel; this logician tried to prove that De Morgan's relative arguments are not formally valid. To do this he devised an argument, essentially similar to

- (16)     Every guinea pig is an animal  
           Every tail of a guinea pig is the tail of an animal

which had to play the counter-example role: given that guinea pigs do not have tails it follows, on Ockham's convention, that the premiss is true and the conclusion false.<sup>46</sup>

In his replay to this criticism, De Morgan abandoned Ockham's convention. However, he did not adopt the modern interpretation of universal categorical sentences, making the conclusion of (16) trivially true. He abandoned instead the principle of bivalence in reference to sentences containing empty expressions. On the schema *The tail of X is*, he said

A guinea pig, for instance, puts this proposition out of the pale of assertion, and equally out of that of denial; the tail of non tailed animal is beyond us.<sup>47</sup>

To sustain the validity of his non-monadic arguments, he seems to resort to a new view on formal validity: if the premisses are true and the conclusion has a truth-value, then this conclusion must be true.<sup>48</sup>

Mansel's criticism and De Morgan's answer show that relative expressions as *tail of a horse*, *killer of a man*, because of their possible emptiness, force a non-commensurable alternative to the syllogistic. In order to accept or deny the validity of arguments involving this kind of expression, some traditional principle will have to be abandoned. By implication, both Mansel and De Morgan had to deny the general validity of *Every X is X*. In contrast to this, the modern interpretation seems a less deep revision of the traditional framework, for if *X* is empty, the law of thought *Every X is X* is trivially true.

## NOTES

1. Van Benthem formulates  $C_1, C_2$  in terms of the generalized quantifier perspective; Sommers states them as generalizations of traditional distribution theory and Suppes bases the condition on a model-theoretic semantics for context-free grammars.  
Van Benthem 1986, §6; Sommers 1982 §7;  
Suppes 1979, pp 383-389.
  
2. The traditional categorical sentences are symbolized as follows:
 

Every S is P	→	S)P
No S is P	→	S.P
Some S's are P's	→	SP
Some S's are not P's	→	S:P

De Morgan also used lower case letters for negative terms. The conjunctive term *X and Y* is rendered as *XY* or *X-Y* whereas the disjunctive *X or Y* is rendered as *X,Y*.

De Morgan 1847, p. 60
  
3. De Morgan said that both arguments are equivalent:  
"Accordingly  $X)P + X)Q = X) P-Q$  is not a syllogism, nor even an inference, but only the assertion of our right to use at our pleasure either one of two ways of saying the same thing.  
De Morgan op.cit. p. 117.
  
4. De Morgan 1966,pp 208-222.
  
5. "There is another process which is often necessary . . . involving . . . a transformation which is neither done by syllogism, nor immediately reducible to it. It is the substitution in a compound phrase, of the name of the genus for that of the species, when the use of the name is particular. For example, *man is animal*, therefore *the head of a man is the head of an animal* is inference but no syllogism.  
De Morgan 1847 p. 114.

6. "If this postulate (De Morgan's rules, VS) be applied to the unstrengthened forms of the Aristotelian Syllogism, it will be seen that all which contain A are immediate application of it and all the other easily derived."  
De Morgan op.cit. p. 115
7. "Observing that every inference was frequently declared to be reducible to syllogism, with no exception unless in the case of mere transformations, as in the deduction of *No X is Y* from *No Y is X*, I gave a challenge in my work on formal logic, to deduce syllogistically from *Every man is an animal*, *Every head of a man is the head of an animal*."  
De Morgan 1966, p.29.
8. De Morgan 1847 p. 114.
9. See note 5. He also said that his rules are applicable "not only in syllogisms, but in all the ramifications of the description of a complex term. Thus for *men who are not European*, may be substituted *animals who are not English*."  
De Morgan op.cit. p. 115.
10. De Morgan op. cit. pp. 114-115; De Morgan 1966 pp. 29-30.
11. "Thus when we say that all men will die, and that all men are rational beings; and then infer that some rational beings will die, the logical truth of this sentence is the same whereas it is true or false that men are mortal and rational."  
De Morgan 1847, p.1.
12. This is De Morgan's own interpretation: "when  $X \supset Y$ , the relation of  $X$  to  $Y$  is well understood as that of species to genus."  
De Morgan op cit. p.75.



13. Seen as instances of  $R_1$  and  $R_2$ , De Morgan's rules have this form:

$D_1$     Every X is Y . . . . X . . . .  
           . . . Y . . . .

Providing some of the denotation of X is spoken  
of in . . . X . . . .

$D_2$     Every X is Y . . . . Y . . . .  
           . . . X . . . .

Providing all of the denotation of Y is spoken  
of in . . . Y . . . .

14. The most traditional wording of the Dictum de Omni, supposed to be found in Aristotle's work, is "what is predicated of any whole is predicated of any part of that whole." Van Benthem gives this interpretation of the dictum: "Likewise upward monotonicity reflects a central classical type of argument called the Dictum de Omni; Whatever is true of every X is true of what is X. What this terse formulation comes to, in our terminology, is this. If every X is Y and X occurs in upward monotone position in some statement . . . X . . . , then the same statement holds for Y: . . . Y . . . .  
Kneale & Kneale 1962, p. 79; Van Benthem 1986, p. 111
15. We identify, as De Morgan did, *man is animal* with *every man is an animal*.
16. Van Benthem 1986 p.111; Sommers 1982 p. 389;  
Suppes 1979 p. 189 .
17. "In such propositions as *Every X is Y, Some Xs are Y* &c., X is called the subject and Y the predicate. It is obvious that the words of the proposition point out whether the subject is spoken of universally or partially, but not so of the predicate."  
De Morgan 1847 p.6

18. The modern instantiations of  $R_1$  and  $R_2$  are more carefully worded, they permit the needed differentiation between the occurrences of the same expression in a context as *Every X is Y*.
19. The description Prior gave of distribution suggests a connection between the traditional doctrine of distribution and De Morgan's version of the conditions  $C_1$  and  $C_2$ : "It is often said . . . that a distributed refers to all and an undistributed term to only a part, of its extension . . . What the traditional writers were trying to express seems to be something of the following sort: a term  $I$  is distributed in a proposition  $f(I)$  if and only if it is replaceable in  $f(I)$ , without loss of truth, by any term 'falling under it' in the way that a species falls under a genus."  
Prior 1967.
20. "Logical writers generally give the name of distributed subjects or predicates to those which are spoken universally."  
De Morgan 1847 p.6.  
" It is usual in modern works to say that a term which is universally spoken of is distributed. . . The manner in which the subject is spoken of is expressed; as to the predicate, it is universal in negatives but particular in affirmatives."  
De Morgan 1966 p.6.
21. Sommers 1982 contains an example of a systematic extension of distribution assignments far beyond the categorical fragment.
22. "Ungleich tiefer und überhaupt die bedeutendste Logik des 17. Jahrhunderts, ist die Logica Hamburgensis des Joachim Jungius."  
Scholz 1931, p. 41.
23. Jungius 1957 p. 122.
24. Jungius op. cit. p.89.

25. Jungius op. cit. p. 151-154
26. See, for instance, Buridan's rejection of the idea that the middle term has to be subject or predicate in the premisses: "Deinde. . . supponamus quod aliquando in syllogizando ex obliquis non oportet quod extremitas syllogistica vel medium syllogisticum sit extremitas alicuis praemissae . . . . Verbi gratia, bonus est syllogismus *Homo omnem equum est videns; Brunellus est equus; ergo Homo Brunellum est videns*. In hoc autem syllogismo iste terminus *equus* est medium, qui nec est subjectum nec praedicatum in maiore propositione."  
Buridan 1976 p. 100.
27. Kneale & Kneale 1962 pp. 324-25.
28. Dummett 1973 p.8.
29. Leibniz 1768 vi p. 38-9; Leibniz 1966 p.80.
30. Sommers 1982 p. 143.
31. Geach keeps asking such assignment when it is clearly irrelevant. Geach 1968 p.16.
32. One of the simplifications of our description is that by supposition theory we understand only a fragment of it, the so-called *suppositio personalis*.
33. The justification for our 'categorical reading' of supposition is this sequence of quotations:"propositio universalis est illa . . . in qua subiicitur terminus communis sine signo universali cum negatione praecedente, propter tales: *Non aliquis homo currit, Non Homo est animal*."  
Ockham 1954 p.222  
  
". . . in omni propositione universali affirmativa et negativa. . . stat subjectum confuse et distributive."

Ockahm 1951 p. 206

" . . . in omni tali universali negativa praedicatum stat confuse et distributive."

Ockham loc.cit.

". . . quando negatio determinans compositionem principalem praecedit praedicatum, stat confuse et distributive, sicut in ista *homo non est animal*, li *animal* stat confuse et distributive, sed *homo* stat determinate."

Ockham 1951 loc. cit.

"Verbi gratia in ista *homo est animal* nullum signum universale addit, nec negatio. . . ideo uterque terminus supponit determinate."

Ockham op.cit. p. 192

". . . quod quando terminus communis sequitur signum universale affirmativum mediate, tunc stat confuse tantum, hoc est semper in universali affirmativa praedicatum supponit confuse tantum, sicut in ista *Omnis homo est animal* .

Ockham op.cit. p. 204

35. Kneale & Kneale 1962 p. 259

36. Ockham 1951 p. 274

37. Quoted in Bird 1961, p. 69.

38. According to our abstract format  $O_1$  and  $O_2$  are like this:

$O_1$  Omnis S est P . . . . S . . . .

. . . P . . . .

Providing S occurs in . . . S . . . .

with supposition non-distributiva.

O<sub>2</sub> Omnis S est P . . . P . . .

. . . S . . .

Providing P occurs in . . . P . . . with  
supposition confusa et distributiva.

- 39, Buridan's view on the role of the middle term in the premisses is precluded by this passage from the *Analytica Priora*: "That the first term belongs to the middle, and the middle to the extreme, must not be understood in the sense that they can always be predicated of one another or that the first term will be predicated of the middle in the same way as the middle is predicated of the last term." "It happens sometimes that the first term is stated of the middle, but the middle is not stated of the first term, e.g. if wisdom is knowledge, and wisdom is of the good, then conclusion is that there is knowledge of the good."  
Aristotle Book I, 36.
40. Ockham 1954 pp. 351-2; p. 360; pp. 365-7.
41. "Tamen aliquando consequentia valet, quia aliquando non possunt tales partes ordinari secundum superius et inferius nisi etiam tota extrema sic ordinentur vel possunt sic ordinari; sicut patet hic *homo albus-animal album; vzidens hominem-zidens animal.*"  
Ockham 1951 p. 188-9.
42. "Solum categorema quod est extremum propositionis . . . supponit personaliter."  
Ockham op .cit. p. 188.  
" Per illam particulam *extremum propositionis* excluditur pars extremi, quantumcumque sit nomen et categorema. Sicut hic *homo albus est animal nec homo, nec albus* supponit set totum extremum supponit."  
Ockham loc. cit.
43. Buridan 1976 pp. 98-9.

44. "As far as presently known the first logician to consider the question of existential import or to propose a tenable theory of it was William of Ockham, who holds that the affirmative categorical propositions are false and the negative true when the subject term is empty."  
Church 1965 p. 420.
45. Buridan 1976 p. 36.
46. De Morgan 1966 pp. 252-3.
47. De Morgan loc.cit.
48. "Again, let X be an existing animal, it follows that the tail of X is the tail of an animal. Is this consequence formal or material? Formal, because this is true whatever a tail may be, so long as there is a tail; and it cannot be refused assertion except when X has no tail." De Morgan loc. cit.  
See also Merrill 1977.