Institute for Language, Logic and Information

# MODAL LOGIC
# AS A THEORY OF INFORMATION

Johan van Benthem

ITLI Prepublication Series
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Received September 1989

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June 1989

## 1 Introduction

In recent years, there has been a growing feeling that the days of Modal Logic are numbered. After its flourishing in the sixties and seventies, the enterprise is now succumbing to joint attacks coming from newer 'information-oriented' and 'dynamic' paradigms for logical research. Now, leaving behind the ontological extravagances of the 'science of possible worlds' can hardly be counted a great loss. But then, this was merely a dominant ideology concerning a framework which itself admits of many possible uses: in particular, uses having to do with the structure and processing of information. One indication of the latter potential has been known for a long time, namely the possible worlds semantics of intuitionistic, or more generally, constructive systems of logic. The purpose of this note is to elaborate this point more systematically, investigating the nature of Modal Logic as a theory of information structure, and eventually even of information flow.

What is needed for such an investigation is a liberal conception of our enterprise, encompassing both richer semantic structures and richer formal languages than the proud but poor austerity of the $\Box, \Diamond$ notation. In a sense, our thesis would be that ordinary modal or intuitionistic logic have always had the power of being theories of information - but they somehow failed to realize their full potential. One reason for this lies in the usual direction of thought in modal semantics. Starting from a given formalism, one searched for appropriate modelings, without entering the next stage of what should be an iterative process: from the point of view of our models, is the original language the most appropriate one, or should we re-design it? This point is brought home most clearly, perhaps, in the case of intuitionistic logic. Given that we can model this system succesfully in possible world models thought of as patterns of information stages, the obvious next question ought to be this: whether the original set of logical constants employed in intuitionistic logic is really expressive enough to say all that we would like to say about such information patterns.

It is of interest to observe that there are other areas of Intensional Logic where research has been more varied and liberal in the above sense. In particular, this is true for temporal logic, where there has always been an emphasis on two directions of thought: not just modelling some prior formalism, but also finding appropriate descriptions of temporal structures which are of intrinsic interest. For instance, temporal logic has been a laboratory for experimentation with progressively richer structures (pure orders $(T, <)$ , metric structures $(T, <, +)$ allowing addition) as well as increasingly expressive formalisms (from tenses "past", "future" to complex connectives like "since", "until"). This development will serve as a lead in what follows.

## 2 The Pattern of Information States

The simplest kind of structure that is relevant here is that of <u>partial orders</u> of information states under possible 'growth':

$$(W, \subseteq) .$$

The simplest appropriate language is that of a modal propositional logic, with evaluation on models $M = (W, \subseteq, V)$ having a valuation $V$ for the proposition letters spread over the state pattern:

$$M \models \Box\varphi[w] \text{ iff } M \models \varphi[v] \ \underline{\text{for all}}\ v \supseteq w .$$

The minimal logic valid here is S4 , or S4 plus the 'induction axiom' Grz if one restricts attention to finite partial orders.

Now, let us analyze this situation more systematically, following an idea of Dov Gabbay. As is well-known, the above modal language can be translated into a standard first-order one involving a binary relation $R$ as well as unary predicates $P$ corresponding to the proposition letters $p$ . Next, the point is that the first-order formulas needed for this are special in several ways. For instance, all their quantifiers occur 'restricted' to $R$ . But most significantly, they can make do with a very small number of distinct variables:

<u>two variables</u> suffice.

To see this, translate proposition letters as $Px$ or $Py$ as the case may be, respect Boolean operations as usual, and treat the modality as follows:

$$\tau(\Box\varphi) \;=\; \forall y\,(x \subseteq y \rightarrow \tau(\varphi))$$

if $y$ is the free variable in $\tau(\varphi)$ (otherwise, use $\forall x\,(y \subseteq x \rightarrow \tau(\varphi))$ ).

**Example.** The translation of $\Box\Diamond\Box p$ need not become

$$\forall y\,(x \subseteq y \rightarrow \exists z\,(y \subseteq z \wedge \forall u\,(z \subseteq u \rightarrow Pu))),$$

as usual. It can be written more economically, but equivalently, as

$$\forall y\,(x \subseteq y \rightarrow \exists x\,(y \subseteq x \wedge \forall y\,(x \subseteq y \rightarrow Py))).$$

Thus, ordinary modal logic is part of a 2-variable fragment of a first-order language over information structures. And what Gabbay has shown is that, in general, there is an effective one-to-one correspondence between finite intensional operator formalisms and fixed k-variable fragments of first-order logic.

Here is a description of the first two steps in the resulting hierarchy.

Fix some model $M$.

**Proposition.** All operations of the form $\lambda x \cdot \varphi(x, A_1, \ldots, A_n)$ with $\varphi$ first-order, employing only *one* variable are definable by Boolean combination of the $A_i$.

With *two* variables, the following set of modal operators is functionally complete:

$$\overset{\rightarrow}{\Diamond}{}^{+} p \;:\; \lambda x \cdot \exists y\,(x \subsetneqq y \wedge Py)$$

$$\overset{\leftarrow}{\Diamond}{}^{+} p \;:\; \lambda x \cdot \exists y\,(y \subsetneqq x \wedge Py)$$

$$I\,p \;:\; \lambda x \cdot \exists y\,(x \not\subseteq y \wedge y \not\subseteq x \wedge Py).$$

For the full first-order language, however, no finite functionally complete set of operators exists.

**Proof.** With one variable $(x)$. Subformulas of the form $\exists x\alpha$ are closed and hence have a fixed truth value. Thus, a Boolean combination remains.

With two variables. Consider innermost subformulas starting with, say, a quantifier $\exists y$. Such formulas $\exists y\,\alpha(y, x)$ may be rewritten as Boolean combinations of cases

$$\exists y \bigwedge \{(\neg)\, x \subseteq y,\ (\neg) y \subseteq x,\ (\neg) Py\}$$

using general logic. But then, by the partial order axioms, the above three possibilities clearly suffice.

Finally, that no general functional completeness result holds over a class of models allowing arbitrary finite widths of branching, may be shown using the method of Ehrenfeucht Games with Pebbling presented in Immerman & Kozen 1987.

Thus, one part of the art in Modal Logic is to locate suitably expressive fragments which still admit of an enlightening operator analysis.

But, there is more to the study of information structures. Our statements of interest will often have further special semantic characteristics.

A first example of this has to do with 'search'. It seems intuitively plausible that deciding the truth or falsity of some statement $\varphi$ at an information state $w$ involves only surveying states 'accessible' from $w$ via successive steps in the growth pattern $\subseteq$. This constraint is embodied in the well-known "Generation Theorem" stating that modal formulas are invariant, at any state $w$, between evaluation in a full model $M$ and evaluation in the smallest $\subseteq$-closed submodel 'generated' by $w$.

Another example has to do with an issue which deserves more interest in semantics generally. It is one thing to introduce a notion of information model. But, we should also provide some criterion of identity, telling us when two models can be considered 'equivalent'. Now, one candidate has a strong backing in contemporary computer science (but also, e.g., in set theory), namely <u>bisimulation</u>. As it happens, this notion already existed in Modal Logic, be it under a different name:

A relation $C$ between two models $M1$, $M2$ is a <u>zigzag</u> if it satisfies the following conditions:

1 if $w_1Cw_2$ then $w_1$, $w_2$ carry the same valuation on proposition letters

2a if $w_1Cw_2$, $w_1 \subseteq v_1$, then there exists $v_2$ such that $w_2 \subseteq v_2$, $v_1Cv_2$

2b analogously, in the other direction.

Note that, e.g., the identity is a zigzag relation between any model and its generated submodels.

Now, modal formulas $\varphi$ are <u>invariant</u> for zigzag relations, in the sense that

$$\text{if } w_1Cw_2, \text{ then } M1 \models \varphi[w_1] \text{ iff } M2 \models \varphi[w_2].$$

And this property is characteristic for the modal formalism (van Benthem 1985):

**Theorem.** A first-order formula $\varphi = \varphi(x)$ is (equivalent to) a translation of a modal formula if and only if it is invariant for zigzag relations.

**Proof.** (Sketch) 'Only if'. By a straightforward induction on modal formulas, where the back-and-forth clauses take care of the modality.

'If'. Let $m(\varphi)$ be the set of modal consequences of $\varphi$. We prove that, conversely,

$$m(\varphi) \models \varphi,$$

from which the desired definability follows by compactness.

So, let $M \models m(\varphi)[w]$ . By a standard model-theoretic argument, we find N, v satisfying

- $N \models \varphi[v]$ ,
- (M,w), (N,v) verify the same modal formulas .

Now, take any two countably saturated elementary extensions of M,N : say, $M^*$ , $N^*$ .
In such saturated models, the following relation defines a zigzag:

" $(M^*, x)$ verifies the same modal formulas as $(N^*, y)$ " .

But then, we have, successively:

$N \models \varphi[v]$ , $N^* \models \varphi[v]$ , $M^* \models \varphi[w]$ (!)
and hence $M \models \varphi[w]$ .

**Example.** Finite Models.
Since finite models M are always saturated, the above is illustrated more concretely there.
For finite M, N , we even have a general equivalence between the following two assertions:

1. (M,w) , (N,v) verify the same modal formulas ,
2. some zigzag between M and N connects w to v .

**Question.** Can we improve the above theorem so as to remove the restriction to first-order formulas, without merely introducing the usual closure conditions for first-orderness (using ultraproducts)?

What we have seen so far are the characteristic semantic properties of the modal formalism - and thus, what will have to be abandoned in richer logics of information structures.

On the other hand, we can also specialize even further, restricting attention to modal formulas with very special semantic behaviour. A prime example here is that of persistence:

which formulas $\varphi$ have the property that always
if $M \models \varphi[w]$ and $w \sqsubseteq v$ , then $M \models \varphi[v]$ ?

This is a well-known property of intuitionistic formulas, and indeed we can make the following simple

**Observation.** The persistent modal formulas are exactly those definable using the intuitionistic connectives $\wedge$ , $\vee$ , $\rightarrow$ , $\perp$
(with each proposition letter occurring in the scope of some $\rightarrow$ ) .

**Proof.** In one direction, intuitionistic formulas $\varphi \to \psi$ (i.e., $\Box(\varphi \to \psi)$ in the modal reading) are persistent, and so are their compounds with $\wedge$, $\vee$ and $\bot$.

Conversely, let $\varphi$ be persistent: and hence equivalent to $\Box\varphi$.
Now, rewrite $\varphi$ using some well-known equivalences to a form as described.
The key observation here is that any formula $\Box\alpha$ is equivalent to some form

$$\Box \bigwedge\bigvee\{(\neg)p, (\neg)\Box\beta\},$$

and hence to

$$\bigwedge\Box \bigvee\{(\neg)p, (\neg)\Box\beta\},$$

i.e., to a conjunction of forms

$$\Box(\bigwedge\{p, \Box\beta\} \to \bigvee\{p, \Box\beta\}$$

(with the falsum $\bot$ used for empty disjunctions).

Thus, we can study various kinds of 'informative behaviour' of propositions in a modal setting.

## 3 Richer Formalisms

There are several kinds of statement that one would like to make about information structures which go beyond the resources of the standard modal language. For instance, when 'updating' an information state with the proposition $\varphi$, the new relevant states would be those where $\varphi$ has become true for the first time: and this requires a comparison with a 3-configuration of the initial state, the first $\varphi$-state, and those in between. Thus, in addition to succession, <u>betweenness</u> along the ordering becomes important.

A modal language which is appropriate at this level introduces an operator "until" as follows:

$$M \models U\varphi\psi[w] \quad \text{iff} \quad \underline{\text{there exists some}}\ v \sqsupseteq w\ \underline{\text{with}}\ M \models \varphi[v]$$
$$\underline{\text{and for all}}\ u\ \underline{\text{with}}\ w \sqsubseteq u \sqsubsetneq v : M \models \psi[u]\,.$$

For instance, the formula

$$U(\varphi \wedge \psi)\neg\varphi$$

expresses that $\psi$ holds at some updated $\varphi$-state.
This language is appropriate for describing the behaviour of programs over time (cf. Goldblatt 1987), involving properties like 'safety', 'liveness' or 'absence of unsolicited response'. Thus, it also seems useful as a description language for information processing in a more general sense. Here is one further illustration.

**Example.** Necessary and Sufficient Conditions.
To say that $\varphi$ is a <u>sufficient</u> condition for $\psi$ may be rendered, at least to a reasonable approximation, within the basic modal logic:

$$\Box(\varphi \to \Diamond\psi) .$$

This is in fact what computer scientists call a liveness property. To say that $\varphi$ is a <u>necessary</u> condition for $\psi$ involves inspection of the past of $\psi$ states. A first attempt might read

$$\Box(\psi \to P\varphi) ,$$

where "P" is the past analogue of "$\Diamond$". But, we want only past occurrences within the future of the initial point of evaluation, and hence, the appropriate formalization is this:

$$\neg U\psi\neg\varphi .$$

For the logic of this enriched modal formalism, one may consult Goldblatt 1987 or Burgess 1982. (But note that some of their axioms depend on <u>linearity</u> of the underlying ordering: which is of course not assumed here.)

As has been noted already, the underlying first-order language of this enriched formalism now employs 3 variables, witness a typical translation clause like that for "until":

$$\tau(Upq) \;=\; \exists y(x \subseteq y \wedge Py \wedge \forall z(x \subseteq z \subsetneq y \to Qz)) .$$

What about an invariance analysis for the relevant formulas?

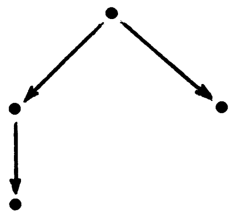
One plausible candidate here is the following enriched notion of bisimulation or zigzag: it satisfies all earlier clauses, as well as

3a if $w_1Cw_2$, $v_1Cv_2$ and $w_1 \subseteq u_1 \subseteq v_1$,
then there exists some $u_2$ with $u_1Cu_2$, $w_2 \subseteq u_2 \subseteq v_2$,

3b analogously in the opposite direction .

Thus, the strengthened bisimulation also has to respect betweenness. This gives us additional power of discrimination:

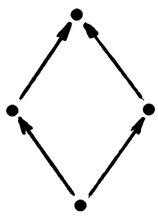
**Example.** Finer Distinctions.
The following two frames are bisimulation equivalent, but do not admit of a strengthened bisimulation:

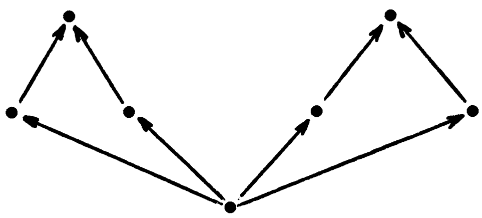


Yet, there are even 3-variable patterns which are still not detected under this mode of identification, such as the position of being a common successor of two states:

$$x \subseteq z \wedge y \subseteq z \,.$$

Here is a graphical illustration:





There is an obvious strengthened bisimulation between these frames: and yet only the one on the left validates

$$\forall x \forall y \forall z\, ((x \subseteq y \wedge x \subseteq z) \rightarrow \exists u (y \subseteq u \wedge z \subseteq u)) \,.$$

Now, it is easy to see that all statements in the Until-formalism are invariant for strengthened bisimulations. Nevertheless, this notion does not quite 'fit' - as becomes clear with some technical difficulties in adapting the earlier characterization result of Section 2. A better candidate would be a bit weaker, namely

3a' if $w_1Cw_2$ and $w_1 \subseteq v_1$, then there exists some $v_2$ with $w_2 \subseteq v_2$, $v_1Cv_2$ and for each $u_2$ with $w_2 \subseteq u_2 \subseteq v_2$ there is a matching $u_1$ such that $w_1 \subseteq u_1 \subseteq v_1$, $u_1Cu_2$.

3b' analogously for the other direction.

But again, the earlier method of proof does not seem to transfer easily.

Without going into technical details, the core difficulty appears to be this: we are trying to characterize a 3-variable fragment by focussing on its formulas with one free variable. This seems somewhat inappropriate, and it would be much easier to work with a formalism allowing formulas both with one free variable (<u>properties</u> of states) and with two free variables, expressing binary <u>relations</u> over states. As we shall see later on, there is in fact a good deal of independent motivation for making such a move (see Section 6), and some further results will be presented there.

For the moment, it may suffice to observe that the present turn has its roots in ordinary Modal Logic too: witness the early interest in so-called 'multi-dimensional' modal logics having various indices of evaluation simultaneously.

**Digression**: D-Logic.

On the issue of expressive strength, it may be worth pointing out that there exists a relatively simple addition to the modal language which removes some of the most basic failures of expressive power. The idea (due to Gargov et al. 1987, Koymans 1989) is to introduce a difference operator D, whose semantic truth condition reads

$M \models D\varphi[w]$ iff $M \models \varphi[v]$ <u>for some</u> $v \neq w$ .

This is again a modality, satisfying the usual Distribution axiom

$D(\varphi \vee \psi) \leftrightarrow D\varphi \vee D\psi$ ,

as well as principles of symmetry and 'pseudo-transivity'.

Adding D to the basic modal language will already allow us to define all <u>universal</u> first-order properties of the order $\sqsubseteq$ . In particular, its <u>anti-symmetry</u>, which is beyond the resources of the basic formalism, now is expressed as follows:

$\forall x \forall y((x \sqsubseteq y \wedge y \sqsubseteq x) \rightarrow y=x)$ :

$(p \wedge \neg Dp \wedge \Diamond(q \wedge \Diamond p)) \rightarrow q$ .

So, there are many options for increasing the strength of our modal formalisms, as measured against the underlying first-order description language for information structures.

We conclude with two general remarks, one concerning axiomatization, the other concerning expressive power. First, at least on the universe of partial orders, <u>recursive axiomatizability</u> of our modal logics, enriched or not, is never in doubt. For, validity of any modal formula $\varphi$ will be equivalent to the validity of its first-order transcription on all partial orders: and the latter consequence problem is RE . The art is, of course, to find enlightening purely modal axiomatizations.

Next, viewing matters from the perspective of the full first-order language allows us to raise some interesting general questions. For instance, what <u>is</u> a 'modality'? There are more or less restrictive answers here, but one candidate would be this:

A <u>modality</u> is any function on sets defined by some first-order schema $\lambda x \cdot \varphi(x, A_1, \ldots, A_n)$ which is <u>continuous</u> in the sense of commuting with arbitrary unions of its arguments $A_i$ $(1 \leq i \leq n)$ .

Continuity expresses a requirement of 'local computability': as may also be seen from the following syntactic characterization.

A first-order formula $\varphi$ is distributive if it has been constructed from conjunctions of atoms $A_iy$ in which no predicate $A_i$ occurs more than once, and arbitrary formulas in which no $A_j$ occurs $(1 \leq j \leq n)$ using only $\vee, \exists$.

A typical example, of course, is the modality $\Diamond$ itself:

$$\lambda x \cdot \exists y\ (x \subseteq y \wedge Ay)\ .$$

**Proposition.** A first-order formula defines a modality if and only if it is equivalent to a distributive form.

**Proof.** (This argument is a simplification of a much more complex ancestor. It is due to Peter Aczel.)

All distributive forms define continuous functions. In one direction, this is clear from the positive occurrence of all $A_i$, and the resulting semantic monotonicity. In the other, from a union to at least one of its members, the statement is clear for formulas of the form

$$\exists \bar{x} : \bigwedge\!\!\!\bigwedge\{\ A_ix,\ \text{wholly } A_j\text{-free formulas}\ \} \qquad \text{(no iterated } A_i)$$

as well as disjunctions of these. But, all distributive formulas can be brought into this shape.

Conversely, suppose that $\varphi$ defines a continuous operation. Then, there is a distributive equivalent, as shown in the following special case. Let $\varphi = \varphi(A_1, A_2)$. We have the following semantic consequence, distinguishing cases as to (non-)emptiness of arguments:

$$\begin{aligned} \varphi \models\ & \exists x_1 \exists x_2 (A_1x_1 \wedge A_2x_2 \wedge [\lambda y \cdot y = x_1/A_1, \lambda y \cdot y = x_2/A_2]\varphi) \\ & \vee \\ & \exists x_1 (A_1x_1 \wedge [\lambda y \cdot y = x_1/A_1, \bot/ A_2]\varphi) \\ & \vee \\ & \exists x_2 (A_2x_2 \wedge [\bot/A_1, \lambda y \cdot y = x_2/A_2]\varphi) \\ & \vee \\ & [\bot/A_1, \bot/A_2]\varphi\ . \end{aligned}$$

This consequence uses the downward half of continuity (observing that each denotation $[[A_1]], [[A_2]]$ is the union of its singleton subsets): that it is actually an equivalence, follows from the upward half (being monotonicity).

**Remark.** This argument does not go through if we merely require the usual Distribution Axiom, which expresses commutation with finite unions of arguments. For instance, on linear orders, the following principle is finitely, but not fully continuous in A :

$$\forall x \exists y\,(x<y \wedge Ay)\,.$$

What would be an appropriate syntactic characterization in this case?

Other questions arising at this level of generality have to do with the distinction between monadic and polyadic modalities. For instance, which modalities are genuinely binary, resisting decomposition into a Boolean compound of unary modalities? Here is one answer, concerning general operators.

**Proposition.** The binary operator Until is not definable in terms of unary modalities.

**Proof.** Consider the rational numbers $(Q, <)$ with the following valuation:

$$V(p) \;=\; \{(n, n{+}1) \mid n \in N,\ n \text{ is even}\} \cup \{1\}\,.$$

The non-definability follows from three observations:

(1) the formula $U\neg pp$ holds in all open p-intervals, but not in the right-closed $(0,1]$.

(2) all points in $V(p)$ verify the same formulas in the propositional tense logic on $\mathbf{Q}$ with operators F ("future") and P ("past"), and so do all those in $N\text{-}V(p)$. (In particular, the number 1 has no distinguishing features here.)

(3) all unary modalities on $\mathbf{Q}$ are definable in its tense logic. For, consider distributive forms. It suffices to look at disjuncts $\exists y(\alpha(x,y) \wedge Py)$, where $\alpha$ is a P-free first-order formula in $<$. But, because of the homogeneity of the rational order, the latter reduce to disjunctions of cases $x<y$, $x>y$, $x=y$ : which are all covered by the F, P formalism.

Other examples of genuine binary notions occur, e.g., with the earlier 'minimal' updating. Saying that all future $\varphi$ are $\psi$ can be done with a unary modality:

$$\Box(\varphi \rightarrow \psi)\,.$$

But, saying that all first $\varphi$ in the future are $\psi$ amounts to an essentially binary connection of the form

$$\text{'}\mu(\varphi,\ \psi)\text{'}\,.$$

**Remark.** On the latter view, binary modalities become more like generalized quantifiers over sets of states - and might be profitably studied as such. In general, of course, this would take us outside of the first-order representation language on information structures. But, that is an interesting transition anyway:

Does the modal logic of information also need higher-order truth conditions eventually?

## 4 Richer Information Structures

In a theory of information, one central notion will be that of <u>addition</u> of pieces of evidence. Thus, the mere partial orders of the preceding Sections will become upper <u>lattices</u>, having suprema in their parial order:

$$(W, \subseteq, +) .$$

This perspective is familiar, e.g., from another information-oriented research program, namely Relevance Logic (cf. Dunn 1985).

If we want to take advantage of this richer structure, suitable modalities will have to be introduced exploiting it. Here is one obvious candidate:

$$M \models \varphi \cup \psi\, [x] \quad \text{iff} \quad \text{there exist } y, z \text{ with } x = y+z$$
$$\text{such that } M \models \varphi[y]\,,\ M \models \psi[z]\,.$$

This will be useful, in particular, once we start exploring different notions of information-oriented <u>consequence</u>. In the standard setting, we put

- $\varphi_1, \ldots, \varphi_n \models \psi$

if, in all information models $M$, at each state $x$,

$$M \models \varphi_1[x] \ \underline{\text{and}}\ \ldots\ \underline{\text{and}}\ M \models \varphi_n[x] \quad \underline{\text{implies}} \quad M \models \psi[x]\,.$$

Thus, there is an equivalence with ordinary conjunction of premises:

$$\varphi_1 \wedge \ldots \wedge \varphi_n \models \psi\,.$$

But, in many recent proposals oriented toward information processing (as reviewed in van Benthem 1989A), we think of successive addition of the information supplied by the premises:

- $\varphi_1, \ldots, \varphi_n \models^{*} \psi$

if, in all information models $M$, at all states $x_1,\ldots,x_n$,

$$M \models \varphi_1[x_1] \ \underline{\text{and}}\ \ldots\ \underline{\text{and}}\ M \models \varphi_n[x_n] \quad \underline{\text{implies}} \quad M \models \psi[x_1+ \ldots +x_n]\,.$$

The latter notion thus becomes equivalent with

$$\varphi_1 \cup \ldots \cup \varphi_n \models \psi\,.$$

For the purpose of comparison with the earlier Sections, however, we shall step back now, and return to the original partial orders. Instead of demanding the existence of suprema, we shall merely take them as they come. In fact, absence of a lowest upper bound for some states might model the surely not unreasonable assumption that they contain conflicting information.

Then, the truth condition for the addition modality becomes:

$M \models \varphi \sqcup \psi\ [x]$ iff there exist $y, z$ such that

$x = \sup(y,z)$, $M \models \varphi[y]$, $M \models \psi[z]$.

When spelt out, this takes us into the 4 variable fragment of our first-order language:

$\exists y \exists z\ (y \sqsubseteq x \wedge z \sqsubseteq x \wedge \neg\exists u\ (y \sqsubseteq u \wedge z \sqsubseteq u \wedge u \sqsubsetneq x) \wedge \varphi(y) \wedge \psi(z))$.

Moreover, this fragment may be analyzed in terms of semantic invariances as before. For instance, call a bisimulation <u>strong</u> if it respects suprema in the following sense:

4a if $w_1 C w_2$ and $w_1 = \sup(u_1,v_1)$, then there exist $u_2$, $v_2$ such that $w_2 = \sup(u_2, v_2)$ and $u_1 C u_2$, $v_1 C v_2$,

4b and vice versa.

Here is a small illustration of the characterization results which may be obtained here:

**Proposition.** Two finite models $(M,w)$, $(N,v)$ are strongly bisimulation equivalent if and only if they verify the same extended modal formulas (of the $\Diamond$, $\sqcup$ language) in their roots.

**Proof.** The 'only if' direction is a simple induction.

As for 'if': here is the crucial observation. Suppose that $x, y$ verify the same modal formulas, and $x = \sup(u,z)$. Assume that there are no $s, t$ with $y = \sup(s,t)$ with both $u, s$ and $z, t$ verifying the same modal formulas. For each of these finitely many cases, then, pick some formula $\alpha$ with $u \models \alpha$, $s \not\models \alpha$ or $\beta$ with $z \models \beta$, $z \not\models \beta$. Then, $x, y$ disagree on the modal formula $\bigwedge\alpha \sqcup \bigwedge\beta$.

In a modal language like this, one can develop a more elaborate calculus of special types of 'informative content'. For instance, not just 'persistent' propositions will be of interest now, but also, e.g., 'additive' ones, satisfying the condition

<u>if</u> $M \models \varphi[x]$ <u>and</u> $M \models \varphi[y]$ <u>and</u> $z = \sup(x,y)$,

<u>then</u> $M \models \varphi[z]$.

For instance, which syntactic forms guarantee this behaviour, starting from atomic propositions already having it?

One example are the persistent formulas: but are there others involving $\sqcup$ ?

**Remark.** This type of question has strong formal analogies with calculi of mass terms or temporal aspect in the semantics of natural language, where persistence and additivity are fundamental notions (cf. Krifka 1989).

Finally, it should be noted that we can also rearrange the modal perspective, so as to make addition, and hence $\sqcup$, the central modality, rather than some binary order among information states. In that case, the abstract pattern is just this:

There is a distributive binary modality $\sqcup$,

which induces a <u>ternary</u> relation for its evaluation:

$M \models \varphi \sqcup \psi\,[x]$ iff <u>there exist</u> y, z <u>with</u> Ryzx <u>such that</u> $M \models \varphi[y]$, $M \models \psi[z]$.

This modal logic too has a perfectly ordinary development, as may be seen in van Benthem 1989 B, which uses it to model some modal aspects of Relation Algebra (where R stands for <u>composition</u> of relations, and the objects in W for 'transition arrows').

Here are some example of expressive power in this formalism. The formula

$$(\varphi \sqcup \psi) \sqcup \chi \to \varphi \sqcup (\psi \sqcup \chi)$$

corresponds on frames to the requirement of associativity in the following form:

$$\forall x \forall y \forall z \forall u \forall v\colon (Ryzx \wedge Ruvy) \to \exists s\colon (Rusx \wedge Rvzs)\ .$$

Likewise,

$$\varphi \sqcup \varphi \to \varphi$$

expresses idempotence:

$$\forall x \forall y\colon Ryyx \to x=y\ .$$

Having the ordinary modalities present after all, we can also enforce connections such as the following:

$$\varphi \sqcup \psi \ \to\ P\varphi \wedge P\psi$$

(with P for "past", as before) defines that

$$\forall x \forall y \forall z\colon Ryzx \to (y \subseteq x\ \wedge\ z \subseteq x)\ .$$

Behind these examples, there even lies a general definability result (see again the above reference):

**Theorem.** All modal principles of the following form define first-order conditions on $R, \subseteq$ (which are effectively obtainable from them):
$\varphi \to \psi$, with $\varphi$ a compound of proposition letters, $\vee$, $\wedge$, $\sqcup$, $\Diamond$, P and $\psi$ an arbitrary formula in which each proposition letter occurs only positively.

## 5 Other Directions of Information Processing

Up till now, modal languages of information structures have been mainly 'forward-directed', looking at future information states. Only occasionally, some 'backward-looking' operators from tense logic made their appearance.

In a general theory of information, however, both directions of search through information patterns will be essential. After all, many important constructions involve surveying the 'epistemic past', or even back-and-forth movement.

**Example.** Conditionals.
A popular folklore account of possibly counterfactual conditionals runs like this:
"Assume the antecedent. Or, if this is not consistently possible, go back to the first stage where it was still possible: then see if the consequent always follows from the antecedent".
In a past-time version of the earlier "until" language, adding a dual "since" (S), this would read as follows:

$$\neg S\,(\Diamond\varphi \wedge \neg\Box(\varphi\to\psi)\,,\ \neg\Diamond\varphi)\,.$$ 

Another illustration is the recent work on epistemic operations changing knowledge states. Here, revision is just as important as addition. For instance, one can view our modal logics as an alternative to the semantic theory of such epistemic operations as developed in Gärdenfors 1987.

**Example.** Addition and Subtraction.
A knowledge state updated by $\varphi$ validates just those $\psi$ which satisfy

$$\neg U(\varphi \wedge \neg\psi, \neg\varphi)\,.$$

A knowledge state 'downdated' by $\varphi$ validates just those $\psi$ which satisfy

$$\neg S(\neg\varphi \wedge \neg\psi, \varphi)\,.$$

Our modal logic by itself then already forms a systematic theory of updating and revision, with their interactions. (Cf., e.g., van Benthem 1989C on such issues as whether an update followed by a downdate with respect to the same proposition cancels out.)

Finally, the notion of 'addition' of information in the binary sense of Section 4 also admits of an obvious downward dual: we may just as well consider infima in the information ordering, giving rise to a dual modality $\varphi\sqcap\psi$ expressing a kind of generalized disjunction.

# 6 Dynamics of Information Flow

Various topics treated so far have a 'dynamic' flavour, referring not just to what is true at single information states, but also to transitions between such states. In cognition, we are not just describing what is true, but also giving <u>instructions</u> as to getting from one cognitive state to another. Thus, from many different angles, there is a growing contemporary interest in what might be called the programming aspects of information flow. As one current slogan has it:

'Natural Language is a Programming Language for Cognition'.

What this move brings with it is a change from 'static' propositions to 'dynamic' ones, serving as instructions for updating information states. This is the point of view defended, e.g., in Discourse Representation Theory or other current dynamic semantics.

Our perspective here will be rather be one of co-existence, having both traditional and more dynamic propositions. In terms of a standard intensional type theory, we have

type (s,t) : classical propositions

type (s,(s,t)) : propositions as programs ,

together with appropriate operations on these.

The two systems will have different flavours. In the realm of propositions as programs, important operations for compounding will be notions of <u>control</u>: such as

$\bullet$ : sequential <u>composition</u>

$\cup$ : (indeterministic) <u>choice</u> .

And the logical paradigm for the latter is not so much Boolean Algebra, as Relation Algebra. Still, the Boolean operations make sense in type (s,(s,t)) too: and hence we expect (and find) a rather richer structure of logical constants in current systems of dynamic semantics than was usual in standard logic.

To the modal logician, this perspective will be familiar from the area of Dynamic Logic, but now viewed as a theory of cognitive computation, rather than calculation with mechanical devices. (See Harel 1984 for a survey.)

Now, one important feature of dynamic logics is precisely the <u>interaction</u> between statements and programs, as effected by various cross-categorial operators:

<u>test</u> takes statements $\varphi$ to programs $\varphi?$ ,

<u>modality</u> $\Diamond$ takes programs $\pi$ to operators $<\pi>$ on statements .

And more simply, one can think of a fix-point operator

$$\Delta(R) = \{x \mid (x,x) \in R\}$$

which takes cognitive programs to their 'truth set': i.e., those states where they have no effect.

It is this interplay which is involved, e.g., in so-called 'correctness statements' for programs $\pi$ :

$$\varphi \rightarrow [\pi]\psi$$

('from $\varphi$-states, program $\pi$ always takes you to $\psi$-states'). And a similar interplay seems useful in the study of information processing and cognition.

Thus, the world of dynamic logic for information processing looks like this:

| | | |
|---|---|---|
| | static projections $\xleftarrow{\qquad}$ | |
| static logical operators $\circlearrowright$ (s,t) | | (s,(s,t)) $\circlearrowleft$ dynamic logical operators |
| | dynamic modes $\xrightarrow{\qquad}$ | |

Note in particular the two transformation arrows in the types ((s,(s,t)), (s,t)) and ((s,t), (s,(s,t))) . From right to left, we are interested in static 'projections' of dynamic relations, such as the above <u>diagonal</u> operator $\Delta$ , or the usual projections to <u>domain</u> or <u>range</u>. From left to right, we are concerned with various dynamic 'modes' which may be acquired by a standard proposition: it can serve as a content for a variety of dynamic activities such as <u>testing</u>, <u>updating</u>, <u>downdating</u> (and the latter both in 'liberal' or 'minimal' variants).

To define the latter, we have to endow the base domain $D_s$ with at least the partial order structure of earlier Sections. Then we can set, e.g.,

$$\begin{array}{lcl}
?\varphi & = & \{(x,x) \mid \varphi(x)\} \\
\mathrm{ADD}\varphi & = & \{(x,y) \mid x \subseteq y \ \&\ \varphi(y)\} \\
\mathrm{MIN}\varphi & = & \{(x,y) \mid y \subseteq x \ \&\ \neg\varphi(y)\} \\
\mathrm{UPD}\varphi & = & \{(x,y) \mid x \subseteq y \ \&\ \varphi(y) \ \&\ \neg\exists z\colon\ x \subseteq z \subsetneq y \ \&\ \varphi(z)\} \\
\mathrm{DOWND}\varphi & = & \{(x,y) \mid y \subseteq x \ \&\ \neg\varphi(y) \ \&\ \neg\exists z\colon\ y \subsetneq z \subseteq x \ \&\ \neg\varphi(z)\}\ .
\end{array}$$

And Modal Logic of information flow will now be the dynamic logic of structures and operators such as these.

Of course, in this enterprise, the questions of crucial interest to us need not be the standard technical concerns inherited from our founding fathers - as will be illustrated in what follows.

**Example.** Characteristic Properties of Modes.
In van Benthem 1989C, a folklore view is studied where information states are sets of Tarski structures, ordered by a preference relation $<$ . Then, two modes of taking a new proposition are distinguished:

'straight addition': $A \mapsto A \cap MOD(\varphi)$

'minimal addition': $A \mapsto \mu(A \cap MOD(\varphi))$ ,

where $\mu$ picks out the <-minimal ('most preferred') models in a class of structures. The latter notion turns out to have the following characteristic formal properties (in a self-explanatory modal notation):

$$\mu p \rightarrow p$$
$$\mu(p \vee q) \rightarrow \mu p \vee \mu q$$
$$\mu p \wedge \mu q \rightarrow \mu(p \vee q) .$$

Now, given any information state $x$ , the above modes suggest a notion of minimization too:

$$\mu(\varphi) = \{y \mid x \subseteq y \ \&\ \varphi(y) \ \&\ \neg\exists z: x \subseteq z \subsetneq y \ \&\ \varphi(z)\} .$$

E.g, UPD amounts to some minimized version of ADD . And, the above three principles give the essentials of this process.

Here is another non-standard concern which comes to the fore now:

What is an appropriate choice of logical constants?

Disregarding the $\subseteq$-structure for the moment, there is one general set-theoretic notion which makes sense as a constraint on logicality across arbitrary types, viz.

invariance for permutations of the base domain $D_s$ .

What this means is that, given any permutation $\pi$ of $D_s$ , an operation $f$ on $(s,t)$ propositions should commute with it:

$$f(\pi[P] \quad = \quad \pi[f(P)] ,$$

and likewise with more than one argument, or with operations on $(s,(s,t))$ . The general notion is analyzed in van Benthem 1986, 1988B, with outcomes such as the following:

- the only permutation-invariant operators on $(s,t)$ type propositions are the Boolean ones,
- on the type $(s,(s,t))$ , such operators include all the usual notions of Relation Algebra (in particular, all Booleans, composition, diagnonal and converse).

On top of this, one can then study the effects of further important constraints. For instance, we can determine all permutation-invariant operators of the above 'transformer' types which also respect inferential structure: in that they are Boolean homomorphisms.

**Proposition.** The only permutation-invariant Boolean homomorphism in type ((s,(s,t)), ((s,t)) is the earlier 'reflexive' operator $\Delta$ .

In type ((s,t), (s,(s,t))) the only two examples are the functions

$$\lambda P_s \cdot \lambda x_s \cdot \lambda y_s \cdot\ P(x_s)$$

$$\lambda P_s \cdot \lambda x_s \cdot \lambda y_s \cdot\ P(y_s)\ .$$

**Proof.** It is shown in the above references that Boolean homomorphisms in any type ((a,t), (b,t)) correspond one-to-one with ordinary functions of type (b,a) . What this implies in the present case is that we need only search for permutation-invariant items in types $(s,\ s \cdot s)$ and $(s \cdot s,\ s)$, respectively.

And these are just

$$\lambda x_s \cdot\ \langle x,x \rangle$$

and

$$\lambda x_{s \cdot s} \cdot\ \pi_{left}(x)$$

$$\lambda x_{s \cdot s} \cdot\ \pi_{right}(x)\ :$$

which explains the above outcome.

Note that not even the earlier test operator qualified here: it is permutation-invariant, but not a Boolean homomorphism. ( $?\neg\varphi$ is not equal to $-?\ \varphi$ .) Nevertheless, it does retain the useful property of <u>continuity</u> introduced already in Section 3: i.e., it respects arbitrary unions of its arguments – and hence it may be computed 'locally' on its singleton arguments. Conversely, permutation-invariant continuous projections which are not homomorphisms include the earlier operations of 'domain' and 'range' of binary relations.

In the modal setting, this style of analysis will shift somewhat. Significant modal operators need not be invariant for all permutations of information states: after all, the latter may destroy relevant information about their $\subseteq$ ordering pattern. But, what they should be invariant for are those permutations of $D_s$ which preserve the ordering structure: i.e., the <u>inclusion automorphisms</u>.

Moreover, it makes sense to generalize various elements in the earlier modal analysis to the present type-theoretical framework. One important example is the notion of <u>bisimulation invariance</u> introduced in Section 2. We would like to express, e.g., that certain higher modal operations as considered here are bisimulation invariant.

Here are some illustrations.

**Example.** Bisimulation in Higher Types.
Let C be a zigzag relation between two models M1 , M2 on domains $D^1_s$ , $D^2_s$ .

Intuitively, two propositions $\varphi_1$ , $\varphi_2$ correspond under C if always

$w_1Cw_2$ implies that $\varphi_1(w_1)$ iff $\varphi_2(w_2)$ .

And then, an operator on propositions is bisimulation invariant if it takes C- corresponding propositions to C-corresponding propositions. Note that the usual 'p-Morphism Lemma' states just this fact for propositional operators which are definable in the standard modal formalism.

To take another type of modal operator, we have bisimulation invariance for the following 'modal projection' of dynamic propositions:

$\lambda R_{(s,(s,t))}\bullet\ \lambda x_s\bullet\ \exists y_s{\sqsupseteq}x_s\bullet\ R(x,y)$ .

What this means is that, for each pair R1, R2 of C-corresponding relations [i.e., whenever $w_1Cw_2$, $v_1Cv_2$, then $R1(w_1,v_1)$ iff $R2(w_2,v_2)$ ], their modal projections are C-corresponding propositions as above.

A proper generalization of this notion goes as follows.
Starting from some relation C between $M_1$ and $M_2$, we define a family of relations $\{C_a\}_{a\in TYPE}$ between objects in corresponding type domains in the hierarchies built on $M_1$ and $M_2$:

type s : $w_1\, C_s\, w_2$ iff $w_1\, C\, w_2$

type t : $x\, C_t\, y$ iff $x{=}y$

type (a,b) : $f\, C_{(a,b)}\, g$ iff for all x, y such that $x\, C_a\, y$ : $f(x)\, C_b\, g(y)$ .

For instance, for the types (s,t) and (s,(s,t)) , this coincides with the above notion of 'C-correspondence'. Now, an expression E in any type a is <u>bisimulation</u> <u>invariant</u> if its denotation, viewed as a function from models to a-type objects in the corresponding type hierarchy has the following property:

for all bisimulations C between models $M_1$ , $M_2$ ,

$[[E]](M_1)\ C_a\ [[E]](M_2)$ .

Again, this fits the above examples of modalities.

And in fact, we have the following general result.

**Proposition.** Any closed typed lambda calculus term containing Boolean parameters as well as restricted quantification of the forms $\exists y_s \sqsubseteq x_s$ , $\exists y_s \sqsupseteq x_s$ defines a bisimulation invariant expression.

**Proof.** By induction on the construction of such lambda terms, it is easily proved that the following stronger assertion holds:

If term $\tau_a$ has the free variables $x_1, ..., x_n$,
and $A_1$, $A_2$ are assignments in $M_1$, $M_2$, respectively,
such that $A_1(x_i)\, C_{a_i}\, A_2(x_i)$ (where $a_i$ is the type of $x_i$),
then $[[\tau]](M_1, A_1)\, C_a\, [[\tau]](M_2, A_2)$.

Of course, not all important modal constructions pass this test: after all, it was too restrictive even for ordinary Modal Logic eventually. For instance, even a propositional mode like ADD is not bisimulation-invariant: as zigzag relations will not preserve inclusion in a suitably strict fashion. Another counter-example is a relational operator like composition. But then, as before, we can introduce stronger notions of invariance to deal with such cases. These will not be pursued here, however:

The point of the present technical excursion has been merely to show how some of the central notions of Modal Logic can be lifted to a more general type-theoretic setting.

Finally, let us return to a general perspective from the earlier Sections. Contrary to what many people seem to believe, the shift toward dynamic formats does not mean an essential break with earlier static formalisms at the meta-level. For, the behaviour of programs can always be described 'statically' in terms of transition predicates having the proper arity. Indeed,

Dynamic Logics can always be reduced to Classical Ones .

Many specific reductions in special cases support this generalization. Thus, the issue is rather more subtle: dynamic formalisms are not beyond the scope of classical systems, but they provide an intrinsically import new perspective on questions which would not easily come to the fore otherwise.

In this perspective, let us look again at the first-order description language of information structures. As was stated already in Section 3, this formalism provides for one-place propositions (via formulas with one free variable), but also for two-place relations (two free variables), etcetera. Now, we can also extend the scope of the earlier investigation of suitable modal fragments and their semantic characteristics to this wider formalism. Here is an example, inspired by the 3-variable analysis of Section 3: which also seems to be the proper level of complexity for formulating many significant dynamic operations.

First, we extend the notion of zigzag relation in a perhaps unexpected direction:

A <u>trisimulation</u> between two models M1, M2 is a relation C between individuals in M1, M2 , but also between ordered pairs and between ordered triples of such objects which are linearly ordered by $\subseteq$, satisfying the following conditions:

1 if $w_1$ C $w_2$ , then $w_1$, $w_2$ verify the same proposition letters

2 if C relates two objects of length smaller than 3 , then they satisfy the back-and-forth property with respect to extension to longer linear sequences (in particular, with length 2 , this allows both 'extension at the ends' and 'interpolation')

3 if two pairs or triples are related by R , then so are their restrictions to subsequences of lower length.

A trisimulation may be viewed as a correspondence between linear searches that could be performed in two information patterns M1, M2 . The point of this definition is, amongst others, that it induces a notion of <u>invariance</u> for formulas having up to three free variables, in an obvious manner. And for instance, it is not hard to see that all transcriptions of formulas from the modal language with "Until" (and "Since" ) added are invariant for trisimulations. The effect of this will be seen in the following result.

**Theorem.** The trisimulation invariant first-order formulas $\phi = \phi\ (x)$ are precisely those which are definable from unary atoms using $\neg, \wedge, \vee$ as well as restricted quantifiers $\exists y\ (x \subseteq y \wedge \alpha(x,y))$ , $\exists y\ (y \subseteq x \wedge \alpha(x,y))$ , $\exists z\ (x \subseteq z \subseteq y \wedge \alpha(x,y,z))$ , $\exists z\ (x \subseteq y \subseteq z \wedge \alpha(x,y,z))$ , $\exists z\ (z \subseteq x \subseteq y \wedge \alpha(x,y,z))$ , modulo logical equivalence.

**Proof.** This may be shown essentially as in the proof of the characterization theorem for basic modal formulas in Section 2. In one direction, matching sequences in the relation C verify the same formulas from the language displayed (having the appropriate number of free variables): where the special forms of quantification ensure that only linear sequences need be considered. For the converse, a trisimulation between two saturated structures may be defined essentially as was done before for bisimulations: this time, in terms of pairs of sequences (up to length 3 ) satisfying the same formulas of our special language. Again, the above quantificational patterns of 'succession' and 'betweenness' are just what is needed to obtain the required back-and-forth properties.

These unary formulas can all be written using 3 variables in all, as is easily seen by inspection of their syntax. Nevertheless, there is a subtlety here. For, already with binary relations defined by formulas $\phi(x,y)$ , more variables may occur essentially. A trivial example is the following trisimulation invariant pattern:

$$x \not\subseteq y \wedge y \not\subseteq x \wedge \alpha(x,y) \;, \qquad \text{for arbitrary first-order formulas } \alpha \,.$$

This complication can be met in several ways. First, we shall be mostly interested in those cases where the free variables $x$, $y$ represent states encountered during some process of search through an information pattern. But then, their relative positions will be connected somehow. For instance, very often, $y$ will follow $x$ in the inclusion order. Accordingly, the question is rather what the binary formula $\phi(x,y)$ looks like in those cases where $x \subseteq y$ . And it may be shown that, for this relativized form of definability, the forms of definition listed in the above theorem suffice. (Of course, the connection between the positions of $x$ and $y$ may also be more complex: arising, e.g., from a 'zigzagging' search through the information pattern, choosing $\subseteq$-successors or -predecessors.)
But also, there is a good deal of independent logical interest to the 3-variable formalism: which possesses an interesting characteristic semantic invariance that will be determined separately below.

Now, by the Gabbay analysis, there must also be a finite functionally complete set of 'modal' operators for the dynamic logic of 3 state variables. Such a set will not be written out explicitly here. Let us merely note some useful candidates for inclusion:

- Booleans both on unary and binary predicates
- Relational algebra operations of composition, converse and diagonal
  (these are complete for a 3-variable first-order language for manipulating binary relations: as was already observed by Tarski)
- Unary modalities F, P on unary and binary predicates
- A modality of "Betweenness" taking unary predicates $\phi$ to binary ones:
  $$\lambda x \cdot \lambda y \cdot \exists z\, (x \subseteq z \subseteq y \wedge \phi(z)) \;,$$
  as well as some book-keeping operators for introducing or removing argument places.

In such a formalism again, one can pursue all the earlier semantic concerns.
For instance, which syntactic forms of definition will guarantee which desired semantic behaviour? To take a specific example:

Which modal schemata define binary relations among information states that are <u>progressive</u>, in the sense of being included in $\subseteq$ ?

This would seem to be one obvious dynamic counterpart to the determination , in the unary case, of all upward persistent statements. No answer will be given here - but note, e.g., that the progressive relations are closed under Boolean $\wedge$, $\vee$ and relational $\bullet$ , as well as modal P .

With this example, we conclude our exploration of the basic model theory of a dynamic modal logic of information.

**Digression.** There is also a perspective from Abstract Model Theory behind the above notion of a trisimulation. For a full first-order language, we have the well-known concept of partial isomorphism between models, being the existence of a family of finite partial isomorphisms between their domains, satisfying the Back and Forth properties. Now, for k-variable fragments, this notion may be restricted to partial isomorphisms of length at most k ( 'k-partial isomorphism' ). In fact, the basic modal language needs only length 2 : where the maximum length is not even involved in the back-and-forth process. Thus, zigzag relations could be thought of as coupling individual objects only. Likewise, with trisimulations, the relevant length is 3 : but, the 'action' occurs only at lengths 1 and 2 .

Here are some useful model-theoretic observations about these notions, demonstrated for the case $k = 3$ (but the outcomes are completely general) :

- Formulas $\phi = \phi(x,y,z)$ constructed using only the variables x,y,z are invariant for 3-partial isomorphism in the following sense:

Let $\mathbb{P}$ be a family of partial isomorphisms of length at most 3 establishing 3-partial isomorphism between two models M1 and M2 . Any pair of matching sequences in $\mathbb{P}$ will give $\phi$ the same truth value in both models.

But, there is also a converse:

- Any formula $\phi = \phi(x,y,z)$ in the full first-order language (possibly employing other bound variables besides x,y,z ) which is invariant for 3-partial isomorphism is logically equivalent to a formula constructed using these three variables only.

The first assertion follows by a straightforward induction on the relevant class of formulas. Moreover, the second assertion may be proved essentially like the characterization of the modal fragment in Section 2: starting from two models which are elementarily equivalent with respect to three-variable formulas, one finds saturated elementary extensions which are 3-partially isomorphic via their pairs of sequences up to length 3 verifying the same type in this restricted language.

Together, these two assertions provide a complete model-theoretic characterization of the 3-variable fragment of a full first-order language (and of k-variable fragments in the general case).

As an application of this analysis, the well-known Functional Completeness of the 3-variable fragment of a monadic first-order language over linear orders (due to Kamp and Gabbay) may be understood as follows. Between such structures, 3-partial isomorphism implies full partial isomorphism (by a simple argument resting on linearity): and hence, any first-order formula $\phi(x,y,z)$ on these models is already definable by one from the relevant 3-variable fragment.

Moreover, the various 'extension patterns' needed to induce the back-and-forth properties up to length $k$ yield an obvious choice for a functionally complete set of operators in a corresponding variable-free modal notation.

Finally, it should be observed that the relation between the preceding notion and bisimulation or trisimulation invariance is not wholly straightforward. Of course, trisimulation is a relative of 3-partial isomorphism where attention has been restricted to linear sequences and comparisons between unary predicates at corresponding states only. But, in general, trisimulation invariance need not imply invariance for 3-partial isomorphism, or vice versa. What we have, by a simple argument, is only the former implication for formulas having one free variable: which explains the outcome in the earlier characterization theorem.

## 7 Conclusion

The main purpose of this paper has been to investigate, and advertize, the prospects of Modal Logic, in a suitably liberal version of its research program, as a theory of information structure and information flow.

Up until now, the main motivation for having such applications at all have stemmed from natural language semantics and cognitive science generally. But perhaps, this perspective may eventually affect even the core areas of standard logic and mathematical foundations.

## 8 **Appendix:** Cognitive Programming

If we are to take the dynamic perspective on information processing seriously, then we shall have to pay attention to actual algorithms and procedures in our logical semantics. And indeed, there are some ways of introducing such concerns into Logic, mainly using tools from Automata Theory.

One natural question is whether the earlier logical constants on information models admit of a procedural explication, in terms of instructions for searching through the information pattern. Such explications have been given for logical quantifiers in van

Benthem 1986, using 'semantic automata' surveying the universe of relevant individuals in some arbitrary order. But in the present case, search should probably proceed along the built-in relation of inclusion among information states. Semantic automata which are suitable for this kind of task have been studied, e.g., in the model theory of Temporal Logic (cf. Thomas 1989) - but also, for more general linguistic purposes, in van Benthem 1988C. Here is an example of the latter kind of approach.

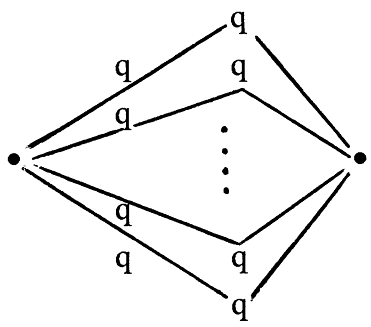
Let us assume that our information models are <u>finite</u> <u>trees</u>, with truth or falsity of atomic propositions indicated at their nodes. Our automata will work progressively upward, processing a node only when all its children have been processed. The core machine is a set of instructions which first reads the atomic information on the current node, then determines which routine to run on the set of state markers left on its children at the end of some previous cycle, and following the outcome of that, prints another such state marker on the current node. In the simplest case, the relevant routine will be a finite state transducer.

For instance, to check whether some extension of the distinguished node (i.e., the top node of the tree) has property $q$ , the machine will print some suitable state marker $\mathbb{q}$ on q-nodes, which gets passed on to their parents, and so on, until the top. (The central routine here just checks an existential quantifier on children.) This is merely one illustration of a general phenomenon (the relevant theorem is in van Benthem 1989C):

> All forward-looking basic modal properties of trees can be computed
> in this way by finite state procedures.

But not just unary modalities like $\Diamond$ can be computed in this format, also essentially binary ones like $Upq$ . (Here, the idea is to mark nodes having $p$ , and then to pass up some special state marker on their parents having $q$ , etcetera.) Thus, the whole modal hierarchy on information models can also be analyzed in terms of machine instructions for this kind of search.

Nevertheless, this perspective is not fully satisfactory. For one thing, the 'bottom up' direction of search does not reflect our intuitions concerning inspection of truth conditions, which rather seem to work 'top-down'. Still, this is not essential - and we can rework the above into a top-down set of recursive instructions for checking our desired semantic properties. The problem is rather that information models need be neither finite nor tree-like: and we may have to inspect, e.g., non-well-founded graphs. Or even assuming well-foundedness, checking for $Upq$ may involve non-linear inspection of patterns like the following:




Traversing just one single unbroken q-path up to a p-state is not enough.

So, we have to conclude here with a

**Question**: What would be an appropriate kind of graph automaton on information models?

But, there are also other ways in which such procedural considerations can enter. Notably, there is the issue of programming cognitive transitions. Given some relation among information states, can we write an explicit program in our dynamic logic effecting just these transitions? Or conversely, what is the class of relations expressible by various classes of modal programs?

Here, we can still consider a great variety of programs: arising by choices of control instructions (finitary ones, or also infinitary) and of basic actions. To be more concrete, one could take all the regular operations:

$;\quad \cup\quad *$

and allow two kinds of atomic action:

a test ? on a proposition letter ,

a move S or P to some successor or predecessor of the current node .

This allows us to program, e.g., the following transitions on <u>trees</u>:

$S^*;\ q?\quad :\quad \lambda x\cdot\lambda y\cdot\ x\subseteq y \wedge qy$

$(\neg q?\ ;\ S)^*;\ q?\quad :\quad \lambda x\cdot\lambda y\cdot\ x\subseteq y \wedge qy \wedge \forall z\ (x\subseteq z\subsetneq y \to \neg qz)$ .

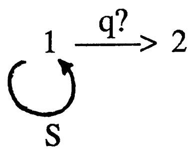
But in general, we also obtain non-first-order relations, say, demanding an odd number of intermediate q positions for an "Until pq" statement. (First-orderness is guaranteed when we abandon the infinitary iteration $^*$ : using, e.g., only the earlier operations of Relation Algebra. But in addition, some simple, 'acyclic' programming structures involving $^*$ will be first-order too.)

Without going into details here, it may be observed that the present analysis does suggest a rather interesting kind of (non-deterministic) finite automaton matching the above programs which has some Turing machine-like behaviour. Its format will be this:

from state plus test on propositional atoms

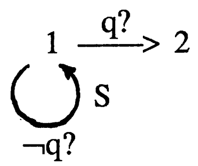
to move ( S, P or "stay" ) plus new state .

Here are the diagrams for the above two examples:




:

| | | | | | |
|---|---|---|---|---|---|
| 1 | q? | ⇒ | – | 2 | (accepting) |
| 1 | TRUE | ⇒ | S | 1 | |




:

| | | | | | |
|---|---|---|---|---|---|
| 1 | q? | ⇒ | – | 2 | (accepting) |
| 1 | ¬q? | ⇒ | S | 1 | |

But one can easily vary. For instance, the next machine will search for q in the whole information pattern generated from the current node:

| | | | | | |
|---|---|---|---|---|---|
| 1 | q? | ⇒ | – | 2 | (accepting) |
| 1 | TRUE | ⇒ | S | 1 | |
| 1 | TRUE | ⇒ | P | 1 . | |

Finally, a more realistic theory would have automata not just testing facts at fixed information states, but also e.g. have them construct new states, by adding individuals or facts. Basic actions would then include such instructions as

"create x such that ..." ,

"see to it that Px becomes true" .

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