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Dependency of Belief in Distributed Systems

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ABSTRACT

This paper investigates the problem of dependency of belief in distributed environments, where an agent may take others' beliefs as its own beliefs. Kripke models for dependency of belief in distributed systems are presented, The notion of dependency of belief can be viewed as an intuitive extension to the notion of awareness, which is generally introduced in the systems of knowledge and belief. Synchronous distributed systems with dependency of belief are formalized. The corresponding logic system, S5Dn, is offered.

1. Introduction

A distributed system consists of a collection of agents are connected by a communication network. These agents may be processes, humans, or robots, which generally have limited resources. Reasoning about knowledge in distributed systems has been viewed as an important topic of investigation for analyzing distributed systems. In recent years, reasoning about knowledge in distributed environment has found many applications such as distributed knowledgebases, communication and cooperation for multiagent planning in artificial intelligence and knowledge engineering.[BG88]

In distributed environments, it is frequently beneficial to enable agents to communicate their knowledges or beliefs among agents, because these agents generally may have limit resources, or may lack computation capability for some specified problems or facts. This means that in such distributed environments some agents may take others' knowledges or

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beliefs as their own knowledges or beliefs. In real lifes, the phenomenon is well known. We call the processes in which some agents can take others' beliefs as their own beliefs the dependency of belief. In this paper, we investigate the problem of dependency of belief in distributed environments.

Based on the classical possible worlds model for knowledge and belief, we present a general model for distributed systems with dependency of belief, in which some functions of dependency are introduced. We also find that notions of dependency of belief in distributed environments can be more intuitively as an extension to the notion of awareness, which is generally introduced in the logic system of belief and knowledge. That is because awareness of an agent not only includes that the agent can figure out the truth by himself, but also includes that the agent knows from which other agent the truth would be obtained correctly. Whereas the dependency of belief can be implemented only in distributed environments.

For synchronous distributed systems, we offer the corresponding formalism of dependency of belief. The system of logic with dependency of belief, called S5Dn, is provided, which formalizes synchronous distributed systems with dependency of belief in which some special properties hold.

Our work has somewhat close relationship with the work of [HL85], [FV86] and [FH88a]. In [HL85], Halpern and Fagin present a formal model that captures the interaction between knowledge, action, and communication in distributed systems. They extend the standard notion of protocol by defining knowledge-based protocols, ones in which a processor's action may explicitly depends on its knowledge. They also present the notion of honest protocol, where one only sends messages that it knows to be true. In [FV86], Fagin and Vardi characterize knowledge and implicit knowledge that are attainable in distributed systems. Implicit knowledge is the knowledge that can be obtained by pooling together the knowledge of a group. In [FH88a], Fagin and Halpern introduce and study several new logics for belief and knowledge. In these logics, the set of beliefs of an agent dose not necessarily beliefs of agents with limited reasoning capabilities. Especially, in [FH88a], a logic of general awareness is offered.

In distributed environments with dependency of belief, an agent only take others' true beliefs as its own belief. Therefore, the system with dependency of belief also can be viewed to be honest. However, our work is quite different from their work. First of all, our models for dependency of belief are introduced not only based on distributed protocols but also based on the general multi-agents environments. This means that the formalism we offer also can be used in general relevant fields such as Philosophy and Linguistics. Moreover, our systems for dependency of belief allow that nested modalities for knowledge and belief. In [FV86], $Ki\varphi \rightarrow I\varphi$ (whatever each individual agent knows is implicit knowledge) is an implicit knowledge axiom. We do not consider this axiom as a suitable axiom for systems of belief. Because an agent may disbelieve what other agents believe at all. In a distributed system, an agent may disbelieve what other agents honestly tell. Therefore, in this paper, we introduce a dependence function for system of belief, which can avoid agents' credulity.

The organization of this paper is as follows: The section " General Model for Knowledge in Distributed Systems" provides a general overview of fundamental notions and relevant work concerned with the study for dependency of belief. In the section 3, first, we examine the notion of dependency and its relationship with awareness, then a general Kripke model for dependency of belief is offered. The section 4 examines some properties of the system for dependency of belief. In the section 5, we provide a Kripke model based on synchronous distributed systems with dependency of belief. The section 6 contains the system S5Dn of logic for dependency of belief, which formalizes the synchronous distributed system with direct dependency of belief. In the section 7, we summarize the study of dependency of belief, and offer some future researches.

2. General Model for Knowledge in Distributed Systems

2.1 The Classical Kripke Model for Knowledge and Belief

In this section we briefly review the possible worlds semantics for knowledge and belief. Suppose we consider a distributed system which consists of n agents, say 1,...,n, and we have a set Φ_0 of primitive propositions about which we wish to reason. In order to formalize the reasoning about knowledge in distributed systems, we use a modal propositional logic, which consists of the standard connectives such as $\wedge$, $\vee$, and $\sim$, and some modal operators L1,...,Ln. A formula such as $Li\varphi$ is read "agent i believes φ."

We give semantics to these formulas by means of Kripke structures, which formalize the intuitions behind possible worlds. A Kripke structure for knowledge for n agents is a tuple $(S, \pi, L_1,\ldots,L_n)$, where S is a set of states, $\pi(p,s)$ is a truth assignment to the primitive propositions of Φ_0 for each state $s \in S$, and Li, $i=1,\ldots,n$, are binary relations on

S which is serial, transitive, and Euclidean. A relation R is *serial* if for each $s \in S$ there is some $t \in S$ such that $(s,t) \in R$; R is *transitive* if $(s,u) \in R$ whenever $(s,t) \in R$ and $(t,u) \in R$; R is *Euclidean* if $(t,u) \in R$ whenever $(s,t) \in R$ and $(s,u) \in R$., The relation L_i is intended to capture the possibility relation according to agents i: $(s,t) \in L_i$ if in world s agent i considers t a possible world.

We now assign truth values to formulas at a state in a structure. We write $M,s \models \varphi$ if the formula φ is true at state s in Kripke structure M.

$M,s \models p$,	where p is a primitive proposition, iff $\pi(p,s)$=true
$M,s \models \neg\varphi$	iff $M,s \not\models \varphi$,
$M,s \models \varphi \wedge \psi$	iff $M,s \models \varphi$ and $M,s \models \psi$,
$M,s \models Li\varphi$	iff $M,t \models \varphi$ for all t such that $(s,t) \in L_i$.

We say a formula φ is *valid in structure M* if $(M,s) \models \varphi$ for all states s in M; φ is *satisfiable in M* if $(M,s) \models \varphi$ for some states in M. We say φ is *valid* if it is valid in all Kripke structures; φ is *satisfiable* if it is satisfiable in some Kripke structure.

The logic of belief above is characterized by the following axiom system, called weak S5 or KD45.

(L1) All instances of propositional tautologies.
(L2) $L_i\, \varphi \wedge L_i(\varphi \rightarrow \psi) \rightarrow L_i\psi$.
(L3) $\neg L_i(\textbf{false})$.
(L4) $L_i\varphi \rightarrow L_iL_i\varphi$.
(L5) $\neg L_i\varphi \rightarrow L_i\neg L_i\varphi$.
(R1) $\vdash\varphi, \vdash\varphi\rightarrow\psi \Rightarrow \vdash\psi$.
(R2) $\vdash\varphi \Rightarrow \vdash Li\varphi$.

If an axiom stronger than (L3),
(L3') $L_i\varphi \rightarrow \varphi$,
is added, the resulting system is called S5 and the corresponding Kripke structure are reflexive, transitive, and symmetric, i.e. the relations become equivalence relations.The classical modal logic of knowledge is S5.

2.2 Distributed Protocols

A distributed system consists of a collection of agents, alternatively called processes.These processes run some protocol.Formally, we can definite a distributed protocol as follows,(cf. [FI86] for the details).

$P=\langle n,Q,I,\tau\rangle$,

It consists of a number n of processes, a set Q of local states, a set $I \in Q^n$ of initial global states, and a next move relation $\tau \subseteq Q^n \times Q^n$ on global states.

For any protocol P, the τ-reachable global states RP of P is the set of all global states we can reach by starting in I and taking any number of τ steps. Only the reachable global states can occur in a run of P. A *run* of P is a sequence of global states, which describes a possible execution of a system over time. For $p \in Q^n$ a global state and i an agent, we write $[p]_i$ to denote the i-th component of p. We also define a binary relation $p \sim i\ q$, for $i=1,\ldots,n$,and $p,q \in Qn$ iff $[p]_i=[q]_i$, i.e. process i has the same local state at the global states p and q.

The Kripke model for distributed system with distributed protocol P is

$K=\langle RP,\pi,\sim 1,\ldots,\sim n\rangle$

where π is a truth assignment to the primitive propositions of Φ_0 at every global states $p \in RP$.

Based on Kripke models for distributed system,for a language of the system of knowledge and belief, the relation " $\models$" can similarly be defined as that in the subsection 2.1.

The general definition above for distributed protocols can precisely formalize the synchronous distributed environments, However, in which the distributed protocols lack local physical clocks. In [HF85], in order to capture the subtle interaction between knowledge, action and communication in distributed system, Halpern and Fagin assume a distributed system where processors communicate over links and have local physical clocks. A deterministic protocol is defined to be one where the messages sent by a processor depend only on its initial state, its message history, and its physical clock reading. A message history for processor p is a record of all the messages p has sent or received up to a certain point in real time. It seems to be a flexible one for the formalization of asynchronous distributed system with dependency of belief. Inasmuch as we only investigate the case of synchronous distributed systems in this paper, we assume a distributed system without local physical clock.

2.3 Logic of Awareness

Possible worlds semantics for knowledge and belief do not appropriate for modelling human reasoning since they suffer from the problem of logical omniscience. So-called *logical omniscience* means that agents are assumed to be intelligence that they must know all valid formulas, and that their knowledge is closed under implication, so that if an agent knows p, and that p implies q, then the agent must also know q.

However, in real life people are not such ideal reasoners. In order to provide a more realistic representation of human reasoning, various attempts to deal with this problem have been proposed. In [FH88a], Fagin and Halpern pointed out that 'lack of awareness' is one of sources of logical omniscience. They argue that one cannot say he knows p or doesn't know p if p is a concept he is completely unaware of. Another source of logical omniscience is that people are resource-bounded. The problem of resource-bounded means that any agent does not necessarily know all relevant facts , and that agents may lack the complete computational resources to reason all consequences of the facts they know.

In order to solve the problem of awareness, Fagin and Halpern offer a solution in which one can decide on a metalevel what formulas an agent is supposed to be aware of. In [FH88a], they provide a logic of general awareness.

A Kripke structure for general awareness is a tuple $M=(S,\pi,A_1,\dots,A_n,B_1,\dots,B_n)$, where S is a set of states, $\pi(s,\cdot)$ is a truth assignment for each state $s \in S$, and B_i is a serial, transitive, Euclidean relation on S for each agent i, and $Ai(s)$ to be an arbitrary set of formulas.

A relation $\models$, is defined inductively as follows:

$M,s \models$ **true**,	
$M,s \models p$,	where p is a primitive proposition, iff $\pi(s,p)$=true,
$M,s \models \neg\varphi$	iff $M,s \not\models \varphi$,
$M,s \models \varphi_1 \wedge \varphi_2$	iff $M,s \models \varphi_1$ and $M,s \models \varphi_2$,
$M,s \models A_i\varphi$	iff $\varphi \in Ai(s)$,
$M,s \models L_i\varphi$	iff $M,t \models \varphi$ for all t such that $(s,t) \in B_i$,

$M,s \models B_i\varphi$ iff $\varphi \in Ai(s)$ and $M,t \models \varphi$ for all t such $(s,t) \in B_i$.

$A_i\varphi$ is read "agent i is aware of φ", it also can be interpreted as agent i is able to figure out the truth of φ. $B_i\varphi$ is read "agent i explicitly believes φ". $B_i\varphi \equiv L_i\varphi \wedge A_i\varphi$ means that no agents can have explicit beliefs about formulas they are not aware of.

In [HMe88],van der Hoek and Meyer introduce a notion of principles, or prejudices, which is as dual of the notion of awareness. The notion of principle can model "reasoning against the facts". While awareness can prevent an agent from believing φ, that he would believe if he were logical omniscience, principles even enable him to believe $\neg\varphi$ in such a case.

3. General Model for Dependency of Belief

3.1 Dependency Functions

It is well known, in real life, people often consult other people and take other people's beliefs as their own beliefs. In a distributed system with n agents (processes or persons), if agent i take a formula φ, which is believed by agent j, as its own belief, that seems to mean that agent i think that the agent j is his adviser about φ. Of course, agent i may think that the agent j is its adviser in a special field of knowledge which consists of φ and other more formulas. It seems to be necessary to classify some specified formulas into some fields of knowledge $\Psi_1,\ldots,\Psi_m$.

On the other hand, agent might have two or more credible advisers about a special formula φ. Unfortunately, those credible advisers' opinion may be conflicting. It seems to require to impose a hierarchical structure on the agents with respects to their advisers and beliefs, even more generally, fields of knowledge, in order to solving the problem of the conflicting of advisers.

However, no matter what field of knowledge a formula φ will belong to, and no matter how many advisers agent i would have about the formula φ, in our opinion, agent i could eventually decide who the most credible adviser would be about the formula φ. We assume that the problems above have been solved in the metalevel, which results in the simplicity of dependency structures. We only introduce a function *Di*, for each agent i, that determines which agent would be the most credible adviser and be consulted by agent i about formula φ in the possible world s. For example, $Di(\varphi,s)=j$ means that agent j is the most credible adviser for agent i about the formula φ in the possible world s. If j=i, that mean agent i only believe in himself about formula φ. Of course, for most formulas, agent i even does not know who the most credible adviser would be (including himself). Therefore, we introduce a special symbol "λ" in dependency functions.The special symbol "λ" means that nobody. thus, $Di(\varphi,s)=\lambda$ means that agent i has no credible agents about formulaφ, even does not believe in himself about formula φ. Let Ψ be the set of all formulas and S be the set of possible worlds.

$$Di: \Psi\times S \rightarrow \{1,2,...,n\}\cup\{\lambda\}.$$

Intuitively, we can give $Di(\varphi,s)=j$ a number of interpretations: "agent i depends on agent j about believing φ in the state s", "agent j is the credible adviser of agent i about φ in the state s", even simplely, "agent i asks agent j about φ in the state s". Specially in distributed process networks, "processor i can obtain the knowledge about φ from processor j in the state s", or "processor i receives a responsive message about φ from processor j in the state s".

3.2 Awareness Functions based on Dependency Functions

In [FH88a], Fagin and Halpern give the formula $A_i\varphi$ a number of interpretations: "i is aware of φ", "i is able to figure out the truth of φ" and when reasoning about knowledge bases "i is able to compute the truth of φ within time T". In order to express intuitively the necessary of awareness function in belief and knowledge of logics, they introduce an example cornerning Bantu tribesman: One can imagine the puzzled from on a Bantu tribesman's face when asked if he knows that personal computer prices are going down!

We find that awareness function can be intuitively introduced based on dependency functions in belief and knowledge of logics. Aiφ can be defined as $Di(\varphi,s)=j$ and $j\neq\lambda$. This means that agent i is aware of φ if and only if agent i believe in himself about φ or agent i could get the truth of formula φ by consulting his credible adviser about φ. As concerns intuitiveness of awareness functions, there seems to be no problem that Aiφ holds if agent i can figure out the truth of formula φ by himself, i.e. $Di(\varphi,s)=i$. In the case of $Di(\varphi,s)=j$ where $j\neq i$ and $j\neq\lambda$, we argue that agent i also can be said to be aware of φ. One can image an example about doctors and patients. When a patient is asked if he knows he is hypothyroid, he can say that he will be aware of hypothyroid because he can ask his doctor about that.

Therefore, we can formally define the awareness function based on dependency function as follows:

$A_i(\varphi, s)$ holds in the possible world s if and only if $Di(\varphi, s) \neq \lambda$ in the Kripke model for dependency of belief.

3.3 Logic of Dependency Beliefs

In this subsection we shall define the language in which we shall express our notion and notations about the logic of belief with dependency function first, then the semantics of the language will be given based on a Kripke structure with extension of dependency functions.

Suppose we will deal with the n agents case, the language **L** is the minimal set of formulas closed under the following rules:

(i) a set Φ_0 of primitive propositions is included in **L**,
(ii) if $\varphi, \psi \in$ **L** then $\varphi \wedge \psi \in$ **L**,
(iii) if $\varphi \in$ **L** then $\neg\varphi \in$ **L**,
(iv) if $\varphi \in$ **L** then $L_i\varphi \in$ **L**, where $i \in \{1,2,...,n\}$
(v) if $\varphi \in$ **L** then $D_{i,j}\varphi \in$ **L**, where $i,j \in \{1,2,...,n\}$.

We define $\varphi \vee \psi \equiv \neg(\neg\varphi \wedge \neg\psi)$, $(\varphi \rightarrow \psi) \equiv (\neg\varphi \vee \psi)$. $Li\varphi$ can be read as "agent i know formula φ", $D_{i,j}\varphi$ can be said as "agent j is the most credible adviser about φ" or "agent i depends on agent j about the belief of φ", etc.. See the subsection 3.1 for the details.

A Kripke structure for dependency beliefs is a tuple M
$M=(S,\pi,D_1,...,D_n,L_1,...,L_n)$
where S is a set of possible worlds, π is a truth assignment to the primitive propositions for each possible world $s \in S$, and Li is an equivalence relation on S for i=1,...,n. D_i is a function from each possible world and each formula in the language **L** into the set $\{1,...n,\lambda\}$.

The relation $\models$ is defined inductively as follows:

M,s $\models$**true**,	where **true** is a special formula included in **L**,
M,s $\models$p,	where p is a primitive proposition, iff $\pi(s,p)$=true,

$M,s \models \neg\varphi$ iff $M,s \not\models \varphi$,

$M,s \models \varphi_1 \wedge \varphi_2$ iff $M,s \models \varphi_1$ and $M,s \models \varphi_2$,

$M,s \models L_i\varphi$ iff $M,t \models \varphi$ for all t such that $(s,t) \in L_i$,

$M,s \models D_{i,j}\varphi$ iff $Di(\varphi,s)=j$.

Furthermore, we can define awareness function based on dependency function about belief: $M,s \models A_i\varphi$ iff $Di(\varphi,s) \neq \lambda$. Similar to Fagin and Halpern's logics of general awareness, we can also define explicit beliefs $B_i\varphi \equiv A_i\varphi \wedge L_i\varphi$.

Logical connectives such as $\rightarrow$ and $\vee$ are defined in terms of $\neg$ and $\wedge$ as usual. **false** to be an abbreviation of $\neg$**true**.

In the case of multi-agents, beliefs may be transitive among agents. Therefore, we extend the definition of dependency beliefs into indirect dependency beliefs as follows:

$M,s \models D^*_{i,j}\varphi$ iff for some j1,...,jm, $M,s \models D_{i,j1}\varphi \wedge D_{j1,j2}\varphi \wedge \ldots \wedge D_{jm,j}\varphi \wedge D_{j,j}\varphi$.

$D^*_{i,j}\varphi$ is read "agent i indirectly depends on agent j about φ",or "agent i relies on the agent j about φ". The sequence <i,j1,j2,...,jm,j> is said to be a chain of dependency belief. Notice that, in the definition above, the last agent in a chain of dependency belief is one who only believes in himself about φ.

We define also $I^*_{i,j}\varphi \equiv D^*_{i,j}\varphi \wedge L_j\varphi$, where $I^*_{i,j}\varphi$ is read "agent i learns from agent j that φ is true".

From the definitions above, we can easily show the following propositions:

Proposition 3.1

(a) *Confidence*

$D^*_{i,j}\varphi \rightarrow \neg D^*_{j,k}\varphi$ ($k \neq j$).

(b) *Uniqueness*

$D^*_{i,j}\varphi \rightarrow \neg D^*_{i,k}\varphi$ ($k \neq j$).

(c) *Quasi-transitivity*

$D_{i,j}\varphi \wedge D^*_{j,k}\varphi \rightarrow D^*_{i,k}\varphi$.

Proposition 3.2

(a) *Same-source- propagation*

$D^*_{i,k}\varphi \wedge I^*_{j,k}\varphi \rightarrow I^*_{i,k}\varphi.$

(b) *Strong-consistence*

$I^*_{i,j}\neg\varphi \rightarrow (\forall k)(\neg I^*_{k,j}\varphi).$

(c) *No-same-source-assertion*

$I^*_{i,j}\varphi \wedge \neg I^*_{k,j} \rightarrow \neg D^*_{k,j}\varphi.$

4. Some Properties about Functions of Dependency Belief

In order to examine and express pricisely the characteristics of distributed system with dependency belief, based on the functions of dependency belief in Kripke structure for logic of dependency belief, two kinds of important binary relations would be introduced. One is the *dependency order relation* $Ra[\varphi,s]$ on the set $\{1,2,...,n\}$ for each formulas φ in every state s, which is defined as follows:

$i\ Ra[\varphi,s]\ j$ iff $D_i(\varphi,s)=j$

The another kind of relations is the *formula relation* $Rf[i,s]$ on the set of all formulas in the language L for each agent i in every state s, which is introduced as follows:

$\varphi\ Rf[i,s]\ \psi$ iff $D_i(\varphi,s)=D_i(\psi,s)$.

Obviously, the formula relation is an equivalence relation. For each formula relation $Rf[i,s]$ and any formula φ,ψ, formula relation $Rf[i,s]$ is said to be *closed under the logical connective* $\neg$ if $\varphi\ Rf[i,s]\ \neg\varphi$. $Rf[i,s]$ is closed under $\wedge$ if $\varphi\wedge\psi\ Rf[i,s]\ \psi$ whenever $\varphi\ Rf[i,s]\ \psi$. $Rf[i,s]$ is closed under $\rightarrow$ if $\varphi\ Rf[i,s]\ \psi$ whenever $\varphi\ Rf[i,s]\ \varphi\rightarrow\psi$. $Rf[i,s]$ is said to be *closed under modal operators* D, L, if $\varphi\ Rf[i,s]\ D_{i,j}\varphi$ and $\varphi\ Rf[i,s]\ L_j\varphi$ for the unique j and any formula φ.

In the section 3 about the definition about logic of dependency belief, we have placed no restrictions on the set of formulas that an agent depends on the other agents, similar to the situation about Awareness in [FH88],we may well to add some restrictions to the dependency functions to capture certain properties. Some typical restrictions we may want to add to D_i can be expressed by some close properties of the corresponding formula relation under some logical connectives.In order to capture a notion of dependency generated by a set of primitive propositions, formula relations are closed under the logical connectives $\wedge, \neg$ and modal operator $D_{i,j}$ are necessary.

In the distributed system with dependency belief, since the messages of belief are send and received by a communication network, in order to guarantee the communication, an important property is deadlock freedom. This means that any agent can finally obtain the messages it wants. The property of deadlock freedom can be expressed by the following axiom:

(DF) $D_{i,j}\varphi \rightarrow (\exists\kappa)\, D^{*}_{i,k}\, \varphi.$

An order relation R is called to be *reflexive in the limit,*if the following property is hold by R:

$$(\forall t,s)(t\ R\ s \rightarrow (\exists s')(s\ R^*\ s' \wedge s'\ R\ s')) \qquad (1)$$

The axiom of deadlock freedom needs that every dependency order relation ,which is introduced from the functions of dependency belief, is reflexive in the limit, in any state.

Specially, in this paper , we are interested in the special system of dependency belief for which the simplicity property of reflexive in the limit holds. The simplicity property of reflexive in the limit is:

$$(\forall t,s)(t\ R\ s \rightarrow s\ R\ s). \qquad (2)$$

The relation R is called to be *almost reflexive* if the property above holds. The corresponding axiom is:

(DF') $D_{i,j}\varphi \rightarrow D_{j,j}\, \varphi.$

The system of logic about dependency belief with the axiom (DF') has close relationship with synchronous distributed system with dependency belief,which is examined in the sections 5 and 6.

In distributed systems with dependency of belief, an interesting property is that $I^{*}_{i,j}\varphi \wedge L_i\neg\varphi$ is satisfiable, This means that an agent may implicitly believe that it originally does not believe. Therefore,it raises an important problem of the system of dependency belief, which can be called the problem of *credibility of consulting*. It requires that in the system never make any agent believed in what he originally does not believe. This property can be expressed by the following axiom:

(CC) $I^{*}_{i,j}\varphi \rightarrow \neg L_i \neg \varphi$.

5. Kripke Model of Dependency Belief Based on Distributed Protocols

In this section, we examine Kripke models of dependency belief based on distributed protocols, specially, we study the models based on synchronous protocols in which every processor sends a message to every other processor during each round. In [FI86], One way to model the synchronous protocols is given.[1] Each processors' local state consists of an n-tuple, in which the i-th entry of j's state is the value of the message sent from i to j during the previous round. That is done as follows:for all processors,i,j, and for all global states p,q,r,s,if $<p,q>$ and $<r,s>$ are in τ and if processor i has the same state in p as in τ, then the i-th component of processor j's state is the same in q as in s.

As far as dependency beliefs are concerned, in a synchronous distributed system, we expand the synchronous distributed protocols with functions of dependency beliefs. One way to model this is to append a special message which specifies the function of dependency belief to each normal message. Let L be the language of logics of dependency belief for a distributed system. The formulas in the language L can be enumerated. Let Ψ be the set of all formulas in the language L,i.e. $\Psi=\{\varphi_1,\varphi_2,\ldots,\varphi_m,\ldots\}$.

For a synchronous distributed system, in every global state, each processor receives messages which encode the function of dependency belief and the truth of corresponding formulas from any other processors.Since there exist communication chains among processors in a distributed system,some messages may be retarded. Therefore, in encoded messages, we introduce a special value "**waiting**" , which denotes that required information is retarded, and a value "**undefined**", which means that corresponding formula is not depended on. Formally, the set of the encoded messages of dependency functions is a subset of the set of the functions $\{F | F:\Psi \rightarrow$ {**true, false, waiting, undefined**}}, where Ψ is the set of all formulas. Let D be the set of encoded messages of dependency functions and their corresponding values,i.e. $D \subseteq \{F | F:\Psi \rightarrow$ {**true, false, waiting, undefined**}}, and $Q' =\{m_1,m_2,\ldots,m_k,\ldots\}$ be the set of normal messages as in the synchronous distributed protocols without dependency beliefs. Let Q be the set of local states in a synchronous distributed system with dependency belief , thus, Q is a subset of

[1] *Halpern and Fagin provide a different definition: R is a synchronous system if for all processes i and points (r, m), (r', m') in R, if (r, m) ~i (r', m'), then m=m'.* [HF89]

$(Q'\times D)^n$. It means that a local state in a synchronous distributed system consists of n components of normal communication message appended with encoded message which specifies the partial function of dependency beliefs and their corresponding values.

In order to formalize precisely the Kripke model of logic for dependency belief in a synchronous distributed system, we have the following definitions:

For the set Q of local states , and a global state $p \in Q^n$,

$[p]_i$'s definition is as previous,i.e, $[p]_i$ is the i-th component of global state p,

$[p]_i^j$ is the j-th component of local state $[p]_i$,it means that message send by processor j to processor i in the global state,

Let Qc be the set of all components of local states,i.e. $Qc=\{[p]_i^j \mid i,j \in \{1,2,...,n\}$ and $p\in Q^n\}$. For any $r=<q',d> \in Qc$ where $q' \in Q'$ and $d \in D$, we definite:

Π_M: $Qc \rightarrow Q'$ $\quad \Pi_M(r)=q'$.

Π_D: $Qc \rightarrow D$ $\quad \Pi_D(r)=d$.

$\Psi([p]_i^j)=df\ \{\varphi \mid \Pi_D([p]_i^j)(\varphi) \in \{\mathbf{true,false,waiting}\}\}$.

$\Psi([p]_i^j)$ denotes the formula set about which agent i depends on agent j in global state p.

$\Sigma M([p]i)=df <\Pi_M([p]_i^1),...,\Pi_M([p]_i^n)>$, which denotes all of normal messages that agent i receives in state p. For a synchronous distributed system with dependency of belief, we assume that two global states are indiscernible for an agent so long as it receives the same normal messages in the two states, although the encoded dependency mesages may be different.

For a synchronous distributed system with a set Q' of normal communication message and a set D of encoded messages which specify the function of dependency beliefs among n agents and the values of corresponding formulas, and a truth assignment π for each primitive proposition, the synchronous distributed protocols P is a tuple as follows:

$P=<n,Q,I,\tau>$

where $Q \subseteq (Q' \times D)^n$ is a set of local states,$\tau \subseteq Q^n \times Q^n$ is a next move relation, $I \subseteq Q^n$ is a set of initial global states, and the following conditions are satisfied for the synchronous distributed protocol P with dependency belief :

(1) *Deadlock freedom.* It requires that no two agents are implicitly depended on each other about believing any formula. In order to satisfy this condition, a naive strategy is to introduce a hierarchy among agents.

(2) *Dependency Uniqueness.* It requires that any agent has only one adviser for each formula.

Formally, the two conditions above can be expressed as follows:

for any $i,j \in \{1,\dots,n\}$, $p \in Q^n$, $\varphi \in \Psi$,

(1) Deadlock freedom

(1.a) $\Pi_D([p]_i^j)(\varphi)=\mathbf{true} \rightarrow (\exists k)(\Pi_D([p]_j^k)(\varphi)=\mathbf{true}) \vee \Pi_D([p]_j^j)(\varphi)=\mathbf{true}$.

(1.b) $\Pi_D([p]_i^j)(\varphi)=\mathbf{false} \rightarrow (\exists k)(\Pi_D([p]_j^k)(\varphi)=\mathbf{false}) \vee \Pi_D([p]_j^j)(\varphi)=\mathbf{false}$.

(1.c) $\Pi_D([p]_i^j)(\varphi)=\mathbf{waiting} \rightarrow (\exists k)(\Pi_D([p]_j^k)(\varphi)=\mathbf{waiting} \vee \Pi_D([p]_j^k)(\varphi)=\mathbf{true} \vee \Pi_D([p]_j^k)(\varphi)=\mathbf{false})$.

(2) Dependency Uniqueness

$\forall j_1,j_2(\Psi([p]_i^{j1}) \cap \Psi([p]_i^{j2}) \neq \varnothing \rightarrow j_1=j_2)$.

Remarks: The conditions (1.a), (1.b) and (1.c) guarantee the property of reflexive in the limit holds in the synchronous distributed system, which will be shown in the theorem 5.3

Based on a synchronous distributed protocol with dependency beliefs, we can introduce an equivalence relation $\sim i$ and a function Di of dependency belief, for each processor i, in every global state. That can be done as follows:

for any global states p, q, and processor i

$p \sim_i q$ iff $\Sigma M([p]_i)=\Sigma M([q]_i)$.

$$Di(\varphi,p)=\begin{cases} j & \text{if } \varphi \in \Psi([p]_i^j). \\ \lambda & \text{else} \end{cases}$$

Now,based on a synchronous distributed protocol $P=< n,Q,I,\tau >$, we have a Kripke model K for synchronous distributed system with dependency belief as follows:

$K=(S,\pi,D_1,\ldots,D_n,\sim1,\sim2,\ldots,\sim n)$

where $S \subseteq Q^n$ is a set of global states,

π is a truth assignment for each primitive proposition in every global state,

D_i are functions of dependency belief introduced based the synchronous distributed protocol,

$\sim i$ are equivalence relations on global states,which is introduced based the synchronous distributed protocol.

For a Kripke structure K which is introduced based on a synchronous distributed system P, and the language L of logic of dependency belief, relation "|=" is defined that as in the section 3.

Proposition 5.1

For a synchronous distributed protocol P with dependency of belief $P=\langle n,Q,I,\tau\rangle$ with the set Q' of normal messages and the set D of encoded message, and a Kripke structure which is introduced based on P, the following property holds:

$K,p \models L_j\varphi$ if $\Pi_D([p]_j^j)(\varphi)=$**true.**

Proposition 5.2

$(\exists i,j)I_{i,j}^{*}\varphi$ iff $(\exists p)(\exists k)(\Pi_D([p]_k^k)(\varphi)=$**true**$)$.

Theorem 5.3

In a synchronous distributed protocol, let Ra be the dependency order relation introduced from the function of dependency in its corresponding Kripke structure Ra$[\varphi,p]$ *is reflexive in the limit , for any formula φ and any global state p.*

Proof.(Sketch)

For any agent i, j,

$i\ Ra[\varphi,p]\ j \Rightarrow D_i(\varphi,p)=j \Rightarrow \varphi \in \Psi([p]_i^j)$

$\Rightarrow \Pi_D([p]_i^j)(\varphi) \in \{$**true, false, waiting**$\}$

$$\Pi_D([p]_i^j)(\varphi)=\textbf{true} \Rightarrow (\exists k_1k_2...k_l)(\ \Pi_D([p]_j^{k_1})(\varphi)=\textbf{true} \wedge \ ... \wedge \ \Pi_D([p]_{k_l}^{k_l})(\varphi)=\textbf{true}$$

(Condition (1.a))

Similarly, based on Condition (1.b), we also can show that the property holds where the value is "**false**". Based on Conditions (1.a), (1.b) and (1.c),the property holds where the value is "**waiting**".

Therefore, Ra[φ,p] is reflexive in the limit.

For distributed systems with dependency of belief, an important feature is to allow the communication about nested belief and knowledge. The feature has close relationship with the notion of common knowledge, which generally is introduced in knowledge systems. In [HM84],Halpern and Moses argue that while common knowledge is desirable, it is unattainable in many realistic distributed systems. For example, if communication is not guarantied, then common knowledge is not attainable. In [HM84], they introduce various relaxations of common knowledge that are attainable in many cases of interest. In this paper, we assume that the communication is guarantied. Meanwhile, we argue that common knowledge is attainable by ask and answer among agents in distributed systems with dependency of belief, because in which the conditions of deadlock freedom and some dependence constraints are satisfied. we call the common knowledge which is attained by ask nested belief by ask.

For example, there exist three agents i, j, and k in a distributed system. At the first time, i may ask k if a formula p is true, and j may ask k if a formula q is true, where p and q are formulas that do not concern any belief modalities. Now agent k is able to tell agents i and j about its knowledge. At the second time, i may ask k if L_jq is true. Now agent k is able to tell the agent i that L_jq is true, because agent j has known L_kq, and agent j believe what agent k has told. Therefore, p and q can be considered as the common knowledge among agents i,j and k. Of course, at the first time, i may ask k if L_jq is true, then agent k can tell agent i "waiting" for the answer till the second time. As a matter of fact, no waiting, agent k even can directly answer the question as long as agent k certainly knows that agent j can ask k about q. That suggests a hiearchical modality constraint on dependency function, in order to enable nested beliefs by ask to be attainable. The constraint axiom is expressed as follows:

(C1) $D_{i,j}L_k\varphi \rightarrow D_{k,j}\varphi$. (Dependence constraint)

Notice that in the example above we confuse L_jq and L_kq. As a matter of fact , Although agent j believe in k about q, the meanings L_jq and L_kq still are different. In order to enable common knowledge to be attainable, the condition of credibility of consulting should be satisfied. But, in realistic distributed systems, the condition seems to be impossible. However, we can introduce a weaker condition, *no-doubt agreement,* which says that any agent never doubt what its adviser tells. In the concrete, it means that agents never intend to compare the diffirences between its explicit beliefs and its implicit beliefs. Therefore, based on the no-doubt agreement,we can consider that L_jq and L_kq have the same meanings approximately. The corresponding axiom is as follows:

(C2) $D_{i,j}\varphi \rightarrow (L_i\varphi \leftrightarrow L_j\varphi)$. (no-doubt agreement)

As mentioned above, the dependency of belief means ask and answer among agents in distributed systems. However, in synchronous distributed protocols with dependency belief which are proposed above, there no exist any explicit expression about ask and answer. That is because we assume the dependency function is known by both consulting and consulted agents in each round. For synronous distributed systems, in each round,the communication about dependcy of belief expresses not only the information of ask, but also the information of answer.

In a synchronous distributed system, every processor sends and received a message for every other processor in each round. Therefore, in order to simplify the relationship of communication of dependency belief among processors, it is reasonable to suppose that every processor can directly communicates with any other processor about dependency belief in a synchronous distributed system.This means that there are no any processor need any other processor as median processor for the communication of dependency belief. Under this circumstance, the property of simplified deadlock freedom holds,i.e., $D_{i,j}\varphi \rightarrow D_{j,j}\varphi$. We call the synchronous distributed system with dependency belief in which the property of simplified deadlock freedom holds the synchronous distributed system with directly dependency belief. In the section 6, we introduce a system of logic, called S5Dn, which formalizes the synchronous distributed system with directly dependency of belief.

6. The System of S5Dn

Let $M=(S,\pi,D_1,...,D_n,L_1,...,L_n)$ Be a Kripke structure with dependency of belief. M is an S5Dn structure, If M satisfies the following conditions:

(1) every dependency order relation introduced from Di,i=1,...,n, is almost reflexive.

(2) every formula relation introduced from Di,i=1,...,n, is closed under logical connectives, $\wedge,\neg$, and modal operators D, L.

(3) $Di(\varphi,s)=j \rightarrow Di(\varphi,t)=j \wedge Di(\varphi,t')=j$ for any t and t' such that $(s,t) \in L_i$ and $(s,t') \in L_j$.

In synchronous distributed system with directly dependency belief, because the property $D_{i,j}\varphi \rightarrow D_{j,j}\varphi$ holds, we need the definition of explicit belief with directly dependency belief . This needs to add the following definition on the standard model

$M,s \models I_{i,j}\varphi$ iff $D_i(\varphi,s)=j$ and $M,t \models \varphi$ for all t such that $<s,t> \in L_j$.

As a matter of fact, $I_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$. Now, In S5Dn system, $I^*_{i,j}\varphi$ reduces to $I_{i,j}\varphi$

The formal system S5Dn,which consists of the following axioms and rules:

Axioms:

(L1) All instances of propositional tautologies.
(L2) $L_i\varphi \wedge L_i(\varphi\rightarrow\psi)\rightarrow L_i\psi$.
(L3) $L_i\varphi\rightarrow\varphi$.
(L4) $L_i\varphi\rightarrow L_iL_i\varphi$.
(L5) $\neg L_i\varphi\rightarrow L_i\neg L_i\varphi$.
(D$\neg$) $D_{i,j}\varphi\equiv D_{i,j}\neg\varphi$.
(D$\wedge$) $D_{i,j}(\varphi\wedge\psi) \equiv D_{i,j}\varphi\wedge D_{i,j}\psi$.
(DD) $D_{i,j}\varphi\equiv D_{i,j}D_{i,j}\varphi$.
(DL) $D_{i,j}\varphi\equiv D_{i,j}L_j\varphi$.
(LD) $D_{i,j}\varphi\rightarrow L_iD_{i,j}\varphi$.
(LD') $D_{i,j}\varphi\rightarrow L_jD_{i,j}\varphi$.
(DF') $D_{i,j}\varphi\rightarrow D_{j,j}\varphi$.
(DU) $D_{i,j}\varphi \rightarrow\neg D_{i,k}\varphi$ $(k \neq j)$.

Axioms (L1)-(L5) guarantee that the system about modal operator L_i is a system of logic of Knowledge,i.e., S5 system.Axiom ($D\neg$) expresses that an agent's dependency of beliefs is closed under negation. Axiom ($D\wedge$) means that closed under conjunction. Axiom (DD) says that an agent 's consulting should be agreed with the consulted agent. Axiom (DL) means that if an agent consults another agent about a formula, then he also should consult the another agent about his knowledge concerning the formula.Axiom (LD) and Axiom (LD') express that every agent knows their consulting and consulted. Axiom (DF') says that the property of simplified deadlock freedom holds. Axiom (DU) guarantees the uniqueness of dependency.

Rules:

(R1) $\vdash\varphi, \vdash\varphi\rightarrow\psi \Rightarrow \vdash\psi$.
(RI) $\vdash\varphi \Rightarrow \vdash L_i\varphi$.

For distributed systems, the common knowledge play an important role in reasoning about knowledge. Intuitively, common knowledge can be viewed as the state of knowledge where every one knows, everyone knows that everyone knows,etc.. Generally, In order to extend the system of logics so that one can reason about common knowledge, modal operators E_G and C_G are introduced, where G is a subset if $\{1,...,n\}$, $E_G\varphi$ is read "everyone in the group G know φ", and C_G is read "φ is common knowledge among the group G".

$(M,s) \models E_G\varphi$ iff $(M,s) \models L_i\varphi$ for all $i \in G$

$(M,s) \models C_G\varphi$ iff $(M,s) \models E^k_G\varphi$ for all k >=1,where $E^1_G\varphi$ is an abbreviation for $E_G\varphi$, and $E^{k+1}_G\varphi$ is an abbreviation for $E_GE^k_G\varphi$.

From axioms (LD), (LD'), (DL), and the definitions about common knowledge above, we have the following propositions:

Proposition 6.1
(1) $\vdash D_{i,j}\varphi \rightarrow E_{\{i,j\}}D_{i,j}\varphi$.
(2) $\vdash D_{i,j}\varphi \rightarrow C_{\{i,j\}}D_{i,j}\varphi$.
(3) $\vdash D_{i,j}\varphi \rightarrow C_{\{i,j\}}D_{i,j}L_j\varphi$.
(4) $\vdash D_{i,j}\varphi \wedge D_{i,j}(\varphi\rightarrow\psi) \rightarrow D_{i,j}\psi$.

(5) $\vdash D_{i,j}\varphi \equiv D_{i,j}I_{i,j}\varphi$.

We introduce the definition of awareness $A_i\varphi \equiv D_{i,1}\varphi \vee D_{i,2}\varphi \vee \ldots \vee D_{i,n}\varphi$.

Proposition 6.2
(1) $\vdash A_i\varphi \wedge A_i(\varphi \to \psi) \to A_i\psi$.
(2) $\vdash A_i(\varphi \wedge \psi) \to A_i\varphi \wedge A_i\psi$.
(3) $\vdash A_i\varphi \to A_iA_i\varphi$.
(4) $\vdash A_i\varphi \to L_iA_i\varphi$.

Proposition 6.3
(1) $\vdash I_{i,j}\varphi \wedge I_{i,j}(\varphi \to \psi) \to I_{i,j}\psi$.
(2) $\vdash I_{i,j}\varphi \to I_{i,j}I_{i,j}\varphi$.
(3) $\vdash \neg I_{i,j}\varphi \wedge D_{i,j}\varphi \to I_{i,j}\neg I_{i,j}\varphi$.
(4) $\vdash I_{i,j}\varphi \to I_{i,j}L_j\varphi$.

Proof.

(1) $\vdash I_{i,j}\varphi \wedge I_{i,j}(\varphi \to \psi) \equiv D_{i,j}\varphi \wedge L_j\varphi \wedge D_{i,j}(\varphi \to \psi) \wedge L_j(\varphi \to \psi)$
$\to D_{i,j}\psi \wedge L_j\psi \equiv I_{i,j}\psi$.

(2) $\vdash I_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi \to D_{i,j}D_{i,j}\varphi \wedge L_j\varphi$
$\to D_{i,j}D_{i,j}\varphi \wedge D_{i,j}L_j\varphi \wedge L_j\varphi \wedge D_{i,j}\varphi$
$\to D_{i,j}(D_{i,j}\varphi \wedge L_j\varphi) \wedge L_j(D_{i,j}\varphi \wedge L_j\varphi)$
$\to I_{i,j}I_{i,j}\varphi$.

(3) $\vdash \neg I_{i,j}\varphi \wedge D_{i,j}\varphi \to \neg I_{i,j}\varphi \wedge D_{i,j}I_{i,j}\varphi \to \neg (D_{i,j}\varphi \wedge L_j\varphi) \wedge D_{i,j}\varphi \wedge D_{i,j}L_j\varphi$
$\to \neg L_j\varphi \wedge D_{i,j}\neg I_{i,j}\varphi \to L_j\neg I_{i,j}\varphi \wedge D_{i,j}\neg I_{i,j}\varphi \to I_{i,j}\neg I_{i,j}\varphi$.

(4) evident.

In order to show that soundness and completeness of the system S5Dn, we use the standard techniques (cf. [FH88a],[MH88],[HC68]). First, we need the following definitions: A formula p is *consistent* (with respect to an axiom system) if $\neg p$ is not provable. A finite set $\{p_1,\ldots,p_k\}$ is consistent exactly if the formula $p_1 \wedge \ldots \wedge p_k$ is consistent. An infinite set of formulas is consistent if every finite subset of it is consistent. A set F of formulas is a *maximal consistence set* if it is consistent and any strict superset is inconsistent. As pointed out in [FH88a], using standard techniques of propositional reasoning we can show

Lemma 6.4 *In any axiom system that includes (L1) and (R1):*

(1) Any consistent set can be extended to a maximal consistent set.

(2) If F is a maximal consistent set, then for all formulas φ and ψ:

(a) either $\varphi \in F$ or $\neg\varphi \in F$,

(b) $\varphi \wedge \psi \in F$ iff $\varphi \in F$ and $\psi \in F$,

(c) if $\varphi \in F$ and $\varphi \rightarrow \psi \in F$, then $\psi \in F$,

(d) if φ is provable, then $\varphi \in F$.

We also can easily show:

Lemma 6.5 (Uniqueness of Dependency) In any axiom system that includes (L1), (R1) and (DU), if F is a maximal consistent set, then for all formula φ, the following property holds:

if $D_{i,j}\varphi \in F$, then $D_{i,k}\varphi \notin F$, for any k such that $k \neq j$.

Theorem 6.6 (Soundness and Completeness) *The axioms system of S5Dn is sound and complete for any structure of S5Dn.*

Proof. Soundness is evident. For the completeness, we must show every valid formula is provable. Equivalently, we can show that every consistent formula is satisfiable. A canonical structure Mc is constructed as follows:

$$Mc=(S,\pi,D_1,\ldots,D_n,L_1,\ldots,L_n)$$

where

$S=\{s_V \mid V \text{ is a maximal consistent set}\}$,

$$\pi(s_V,p)=\begin{cases}\textbf{true} & \text{if } p \in V\\ \textbf{false} & \text{if } p \notin V\end{cases}$$

$$Di(\varphi,s_V)=\begin{cases}j & \text{if } D_{i,j}\varphi \in V\\ \lambda & \text{if } D_{i,j}\varphi \notin V\end{cases}$$

Lemma 6.5 guarantees the construct of D_i is valid.

$Li=\{(s_V,s_W) \mid \{\varphi \mid L_i\varphi \in V\} \subseteq W\}$.

First, we show that Mc is an S5Dn structure. Axioms (L3), (L4), and (L5) guarantee the *Li* are equivalence relations. Axiom (DF') express that every dependency order relation is directly reflexive. Axioms (D$\neg$), (D$\wedge$), (DD), and (DL) enable every formula relation is closed under $\neg$, $\wedge$, D, and L. Axioms (LD) and (LD') guarantee the condition (3) in the definition of S5Dn structure is satisfied. In order to show every formula φ is satisfiable, we should show $\varphi \in V \rightarrow Mc,s_V \models \varphi$ for any maximal consistent set V. Because of Maximality of V, we can easily show by induction on the structure of formulas that $\varphi \in V$ iff $Mc,s_V \models \varphi$.

7. Conclusions

We have examined the problem of dependency of belief, and presented a general Kripke model for dependency of belief. Also, we have argued that the notion of dependency of belief can be viewed as an intuitive extension to the notion of awareness. We expect the aspects of general awareness can be more intuitively captured by the logics of dependency about beliefs in distributed environments.

Moreover, we have offered a Kripke model for dependency of belief, which is based on the synchronous distributed protocols with dependency of belief. The system of S5Dn we offer formalizes synchronous distributed systems with dependency of belief in which some simplified properties hold. Of course, an interesting topic is to formalize asynchronous distributed system with dependency of belief. Intuitively, we feel that a subtle formalism about asynchronous distributed systems with dependency of belief seems to need the temporal logics as tools, because there exist the physical local clocks in the standard asynchronous distributed systems .

Another interesting direction is to impose some different frames on the functions of dependency, which allow temporal changes of dependency. In such distributed environments actions of agents seems to be more flexible, even more intelligent.

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