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NOMINAL TENSE LOGIC

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Nominal Tense Logic*

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This paper presents a simple method of incorporating temporal reference into Priorean tense logic. A new sort of atomic symbol — ***nominals*** — is introduced to languages of tense logic. These new symbols, distinguishable from the ordinary sort of atom, combine with other symbols in the usual way to form wffs. All else remains the same; the syntactic change involved could hardly be simpler. These languages are interpreted on frames as usual, except that we stipulate that nominals only take the value 'true' at precisely one point in any frame. Nominals can be thought of as instantaneous propositions, and the instant at which a nominal is true is the instant it names. Alternatively, and probably more usefully, we can think of nominals as a mechanism that allows Reichenbach's [21] and Prior's views on tense to be incorporated in single framework; nominals pick out Reichenbachian reference times.

While we briefly mention the possible wider relevance of these systems in the concluding remarks, the main aim of this paper is to discuss the logical properties of these languages of Nominal Tense Logic. It turns out that this simple sorting mechanism has a considerable effect on tensed languages: several important classes of frames not standardly definable — for example, irreflexive frames, discrete frames, and the integers — become definable and give rise to a new range of tense logics.

The paper is structured as follows. After presenting the basic concepts we turn to model theory, considering several examples of the increased expressive capability and its effect on preservation results. Two standard preservation results are lost: nominal validity is not preserved under the formation of either disjoint unions or p-morphic images. In the former case we give a necessary and sufficient condition for validity preservation, and in the latter, a sufficient condition. We then turn to axiomatics, and give two axiomatisations of the minimal logic. The characteristic axiom schemas needed — NOM or SWEEP — are presented as encapsulations of ***path equations***. We observe that either axiomatisation will suffice for minimal nominal *modal* logic as well. We then give two rather more abstemious axiomatisations of the minimal nominal tense logic, using weaker schemas NOM_W and SWEEP_W, and prove that neither of the new axiomatisations is strong enough for modal languages; both weakened schemas usefully exploit tense logic's bidirectional operators. In general, the temporal setting

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is the more natural one for nominals, both intuitively and formally. Following this we turn to extensions of the minimal logic. We sketch how Krister Segerberg's bulldozing technique can be applied to yield completeness results for many classes of frames of interest, including the rationals, the integers, and the natural numbers. We then note that because of the new expressive powers of our languages, most new logics of interest routinely lack the finite frame property. However Segerberg's theorem does *not* hold in nominal tense logic: it is possible for a logic to possess the finite model property while lacking the finite frame property. We sketch how to exploit this using a filtration argument, and thus establish decidability results for a number of logics.

When I began the work reported here I believed the idea of using nominals to be a novel one; in fact they had already been discussed on several occasions by Arthur Prior. In [18, Appendix B] he considers the difficulties of incorporating such entities into tense logic; in [20, Chapters 2 and 3] he applies 'egocentric logic' to the semantics of personal pronouns, and in [19] he analyses the semantics of 'now' with their help. Somewhat later, Robert Bull axiomatised a tense logic with an additional S5 modality in which nominals appear as variables over times which can be bound by quantifiers; we briefly mention Bull's work later. More recently, I was told of the (ongoing) work of a group of logicians who have been using nominals in intensional logics for some time.[1] For example, in [17], Passy and Tinchev introduce nominals to Propositional Dynamic Logic, and in [7] Gargov, Passy and Tinchev use them (in several variants) in languages of modal logic.[2] The authors' concerns are close to my own and some interesting comparisons can be made. Firstly, the axiomatisation of the minimal modal logic given in [7] is rather different from mine. My (modal) axiomatisation, couched in terms of 'path exploration' is simpler, and, I think, more intuitively appealing. In the case of extensions of the minimal logic, Gargov, Passy and Tinchev utilise an 'infinitary rule' *COV* that could lead to completeness proofs for logics not amenable to the bulldozing methods I have explored. These matters are discussed below.

1 Preliminaries

By a language of Nominal Tense Logic (NTL) $\mathcal{L}$ is meant a selection of two disjoint, countably infinite collections of symbols: $NOM_{\mathcal{L}} = \{i, j, k, \ldots\}$, the ***nominals*** of the language, and $VAR_{\mathcal{L}} = \{p, q, r, \ldots\}$, the ***variables*** of the language. The elements of $NOM_{\mathcal{L}} \cup VAR_{\mathcal{L}}$ are called ***atoms***. By $\mathcal{L}$-wffs or sentences are meant the strings formed by combining atoms in the usual way with $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\neg$, F, P, H and G. In short, a language of NTL looks just like an ordinary language of tense logic, save for the atomic level: there we have two sorts of atom. We talk of purely nominal, purely Priorean, and mixed wffs; these are wffs containing only nominals, only variables, or a mixture of the two respectively. For example, $i \rightarrow Fi$ is purely nominal; $FFp \rightarrow Fp$ purely Priorean; and $F(i \wedge p) \rightarrow Fp$ mixed. We often call a language of standard tense logic — that is, a language without nominals — a purely Priorean language. We use n as a metavariable across nominals, and ϕ, ψ and so on, as metavariables across arbitrary wffs. We use the usual syntactic machinery of tense logic; most importantly, by $deg(\phi)$ is meant the number of logical connectives in ϕ. Also useful is temporal depth; by $td(\phi)$ is meant the maximal level of embedding of tense operators in ϕ. The ***mirror image*** of

[1]I am grateful to Johan van Benthem who first drew my attention to this work; to Solomon Passy, who kindly sent me a copy of [7]; and Kit Fine who gave me several other of the group's papers. I would like to emphasize that this group's work on nominals preceded mine.

[2]A still more recent paper by Gargov and Goranko [8] has further extended this work.

a wff ϕ is formed by simultaneously replacing every F by P and G by H; and vice versa.

The semantics of these languages is given in terms of frames and models. As usual, by a *frame* $\mathbf{T}$ is meant a pair $\langle T, < \rangle$ consisting of a nonempty carrier set T and a binary relation $<$ on T.[3] The elements of T are called points. By a *model* $\mathbf{M}$ is meant a pair $\langle \mathbf{T}, V \rangle$ where $\mathbf{T}$ is a frame, and V a *valuation* on $\mathbf{T}$. It is only in the definition of what it is to be a valuation that the semantics of NTL differs from that of standard tense logic. As usual, a valuation on $\mathbf{T}$ is a mapping from the atoms of our language to $Pow(T)$, but we place a restriction on the subsets of T that nominals may be assigned. *Nominals must always be assigned singleton subsets of a frame.* A mapping from the atoms to $Pow(t)$ that does not obey this constraint is *not* a valuation. As usual, variables can denote arbitrary subsets of T. With this one change made, everything proceeds as in standard tense logic. In particular, we define the truth of a wff ϕ at a point t of a model $\mathbf{M}$, $\mathbf{M} \models \phi[t]$ in the usual fashion. Derived concepts — such as validity on a frame or validity simpliciter — are defined standardly, and the usual notation is used.

Obvious analogs of simple results for purely Priorean languages hold for languages of NTL; for example, isomorphic frames are equivalent. Another useful result is the following, which gives each formula a 'horizon', a limit past which it cannot see. Let $\mathbf{T} = \langle T, < \rangle$ be a frame and $t \in T$. By $S_n(\mathbf{T}, t)$, the *n-hull around t*, is meant the set of all points of $\mathbf{T}$ that are related in n steps to t. The Horizon Lemma states that for any frame $\mathbf{T}$ and any two valuations V, V' on $\mathbf{T}$ such that $V(a) \cap S_n(\mathbf{T}, t) = V'(a) \cap S_n(\mathbf{T}, t)$ for all atoms a, $\langle \mathbf{T}, V \rangle \models \phi[t]$ iff $\langle \mathbf{T}, V' \rangle \models \phi[t]$, for all ϕ such that $td(\phi) \leq n$.[4]

We will frequently talk of *paths*. By a path through a frame $\langle T, < \rangle$ is meant any *finite* sequence of elements of T such that for every pair t_m, t_{m+1} of the sequence, either $t_m < t_{m+1}$, or $t_{m+1} < t_m$. That is, a path through a frame is a sequence of moves both forward and backward in time. Sometimes to emphasize the bidirectionality of the concept we refer to paths as zig-zag paths. By the length of a path is meant the sequence length. A frame is *connected* iff there is a path between any two of its point.

Filtration theory [23] adapts straightforwardly to languages of NTL. The usual results can be proved; in particular, the standard argument yields that the validities of NTL form a recursive set. In complete contrast, the method of *unravelling* [22, pages 124–127] fails totally. Unravelling turns a (Priorean) model based on a frame of arbitrary structure into an equivalent (Priorean) model based on a tree. Among other things this shows that the purely Priorean validities on the class of all frames are precisely the same as the purely Priorean validities on the class of intransitive frames: purely Priorean languages cannot 'see' intransitivity. However there is a purely nominal wff valid on precisely the intransitive frames, namely, $FFi \rightarrow \neg Fi$; unravelling destroys structure that nominals can see. Somewhat more abstractly, as we shall see later, languages with nominals can state 'path equations' on frames. Unravelling systematically destroys path equations.

We later briefly discuss languages of nominal modal logic (NML). The definition of their syntax and semantics is the expected one; again we have two sorts of atom, and again nominals denote singleton subsets of frames. By $\Diamond^n$ and $\Box^n$ are meant n length unbroken sequences

[3]Frames will be denoted by letters in bold font — $\mathbf{T}, \mathbf{T}', \mathbf{T}_4, \mathbf{S}''$ etc. Their carrier sets will be referred to by the same symbol in mathematical font — T, T', T_4, S'' etc. If a frame, say $\mathbf{T_1}$, has been referred to in discussion, subsequent uses of T_1 and $<_1$ refers to the carrier set of the frame and the frame's relation respectively. Similarly, if reference has been made to a pair $\langle S_2, <_2 \rangle$, subsequent uses of $\mathbf{S_2}$ refer to this frame. Four familiar frames are $\mathbf{Z} = \langle Z, < \rangle$, $\mathbf{N} = \langle N, < \rangle$, $\mathbf{Q} = \langle Q, < \rangle$, and $\mathbf{R} = \langle R, < \rangle$; the integers, natural numbers, rationals, and reals respectively, in their usual orderings.

[4]For a more precise statement of this lemma see [2, page 29].

of $\Diamond$ or $\Box$ operators respectively. By a *modal path* through a frame $\langle T, < \rangle$ is meant a finite sequence of elements of T such that for all pairs t_m and t_{m+1} of the sequence, $t_m < t_{m+1}$. Note that modal paths are unidirectional.

To conclude this section I'll mention a natural addition to languages of NTL: adding a further operator L with the semantics $\mathbf{M} \models L\phi[t]$ iff for all points t' in the model $\mathbf{M} \models \phi[t']$. That is, L is an $S5$ operator meaning 'everywhere' or 'everywhen'. Although space precludes discussing this extension here, a brief remark should make it clear why L is useful in languages with nominals. Fundamentally this is because no matter what relational structure a model carries, using L allows us to jump to the point named by a nominal and proceed with evaluation there. For example, $L(i \rightarrow \phi)$ shifts to the point named by i and tests the condition ϕ there. (For this reason in [4] the operator is called 'the shifter'.)

The addition of L increases the expressive power of languages with nominals, and in many cases allows very concise axiomatisations to be given. Some results concerning L exist in the literature as every previous person who has considered nominals has also discussed the operator: all the papers cited in the introduction do so. Moreover, in a recent manuscript [9], Goranko and Passy investigate in detail the effect of introducing the shifter into standard (nominal free) modal languages, and note that:

> The prime stimulus for considering the universal modality has come up in the context of the *proper names for the possible worlds* [9, page 22]

Finally, in my thesis [4, Chapter 6], tensed languages utilising this operator are considered, and decidability results which appear to be new are proved.

2 Model Theory

We say an NTL formula ϕ *defines* a class of frames $\mathcal{T}$ iff: $\mathbf{T} \models \phi$ iff $\mathbf{T} \in \mathcal{T}$. For example, we have just noted that $FFi \rightarrow \neg Fi$ defines the intransitive frames. Note that if ϕ defines $\mathcal{T}$ and ψ defines $\mathcal{T}'$, then $\phi \wedge \psi$ defines $\mathcal{T} \cap \mathcal{T}'$. For further discussion of definability in standard tensed and modal languages see [1], or [2].

None of the following classes of frames are definable in a purely Priorean language: the irreflexive, asymmetric, antisymmetric, trichotomous, (right) directed or (right) discrete frames. Furthermore, neither are the partial orders (POs), strict partial orders (SPOs), total orders (TOs), or strict total orders (STOs). It is straightforward to verify that each of the first six classes of frames is defined by the purely nominal wff given:

$$
\begin{array}{ll}
i \rightarrow \neg Fi & \forall x \neg(x < x) \\
i \rightarrow \neg FFi & \forall xy(x < y \;\rightarrow\; \neg y < x) \\
i \rightarrow G(Fi \rightarrow i) & \forall xy((x < y \;\wedge\; y < x) \;\rightarrow\; x = y) \\
Pi \vee i \vee Fi & \forall xy(x < y \;\vee\; x = y \;\vee\; y < x) \\
FPi & \forall xy \exists z(x < z \;\wedge\; y < z) \\
i \rightarrow (F\top \rightarrow FHH\neg i) & \forall xy(x < y \;\rightarrow\; \exists z(x < z \;\wedge\; \neg \exists w(x < w < z)))
\end{array}
$$

(Corresponding to right directedness and discreteness are left directedness and discreteness, defined in the obvious way by mirror images. We here regard $\top$ as shorthand for $i \vee \neg i$, and $\perp$ as $\neg\top$.)

A quick check then reveals that $FFi \rightarrow Fi$ defines transitivity, and $i \rightarrow Fi$ defines reflexivity,[5] and thus by conjoining wffs from the above list we can define the classes of POs,

[5]Previously we only knew that $FFp \rightarrow Fp$ defined transitivity, and $p \rightarrow Fp$ reflexivity. In these cases the

SPOs, TOs and STOs. For example, define ϕ^L to be $(i \rightarrow \neg Fi) \wedge (Pi \vee i \vee Fi) \wedge (FFi \rightarrow Fi.)$ This purely nominal wff defines the STOs.

With the aid of purely Priorean formulas we can do better; we can define both the integers and the natural numbers up to isomorphism. Define ϕ^Z to be:

$$\phi^L \wedge (H(Hp \rightarrow p) \rightarrow (PHp \rightarrow Hp)) \wedge (G(Gp \rightarrow p) \rightarrow (FGp \rightarrow Gp))$$

We then have that $\mathbf{T} \models \phi^Z$ iff $\mathbf{T} \cong \mathbf{Z}$. To see this note that in [1, page 163] van Benthem shows that the two purely Priorean conjuncts of ϕ^Z define $\mathbf{Z}$ on the class of *connected strict partial orders*. But ϕ^L restricts us to this class.

Now define ϕ^N to be:

$$(H(Hp \rightarrow p) \rightarrow Hp) \wedge (G(Gp \rightarrow p) \rightarrow (FGp \rightarrow Gp)) \wedge F\top \wedge (Pi \vee i \vee Fi).$$

Again by appeal to a result of van Benthem's we have $\mathbf{T} \models \phi^N$ iff $\mathbf{T} \cong \mathbf{N}$.

It is important to note that both ϕ^Z and ϕ^N are *mixed* sentences. We will shortly see that only classes of frames expressible in a certain first order language $\mathbf{L}_0$ are definable using purely nominal sentences; thus we know that no purely nominal sentence can uniquely define these structures. Further, van Benthem's results concerning the definability of these structures in Priorean languages are 'best possible' results for purely Priorean languages, as the preservation of purely Priorean validity under the formation of p-morphic images and disjoint unions prevents the definition of either $\mathbf{Z}$ or $\mathbf{N}$ using just variables. The mixture of nominals and variables is necessary.

All initial segments of $\mathbf{N}$ are also definable. (They are not in a purely Priorean language.) Define ϕ^{L^n} to be

$$\phi^L \wedge G^n\bot \wedge (F^{n-1}\top \vee PF^{n-1}\top),$$

where $n \in N$ such that $n \geq 1$. Then $\mathbf{T} \models \phi^{L^n}$ iff $\mathbf{T}$ is a STO of length exactly n. Note that only nominals are used.

Next, in languages with nominals we can demand that every point has exactly n successors. This is not something that can be done with purely Priorean languages. In purely Priorean languages we can insist that every point has *at most n* successors, as the following encoding of the Pigeonhole Principle shows:

$$\bigwedge_{1\leq\alpha\leq n+1} Fa_\alpha \rightarrow \bigvee_{1\leq\alpha\leq n;\ 2\leq\beta\leq n+1;\ \alpha\leq\beta} F(a_\alpha \wedge a_\beta),$$

where the a_α are distinct *atoms* — either variables, nominals or a mixture will work. However we cannot demand that every point has *at least n* successors. With nominals, however, we need merely write down:

$$F\top \wedge (\bigwedge_{1\leq\alpha<n} Fi_\alpha \rightarrow F \bigwedge_{1\leq\alpha<n} \neg i_\alpha)$$

where the i_α are distinct nominals.

What can we say of a more general nature? For purely Priorean languages there are four classic validity preservation results: validity is preserved under the formation of generated subframes, disjoint unions, and p-morphic images; and anti-preserved under the formation of

uniform substitution of nominals for variables gave rise to a formula defining the same class, but this by no means always occurs. In general, purely Priorean formulas give rise to second order conditions on the frame ordering; purely nominal formula *always* give rise to first order conditions.

ultrafilter extensions.[6] Given that mixed languages are more expressive than purely Priorean ones, we might expect that one or more of these preservation results will fail. Indeed, for such languages only the generated subframe and ultrafilter extension results still hold.

The two unchanged results are rather dull. Anti-preservation of validity under ultrafilter extensions remains because $ue(V)(i)$ will contain only the principle ultrafilter generated by $V(i)$, for every nominal i and every valuation V; thus $ue(V)$ assigns singletons to nominals and is a valuation. With this noted, the usual proof of the anti-preservation result proceeds unchanged. In the generated subframe case we need to be a little careful in formulating what we mean by a generated submodel of $\langle \mathbf{T}, V\rangle$ — not every pair $\langle \mathbf{S}, V{\downarrow}_S\rangle$, where $\mathbf{S}$ is a generated subframe of $\mathbf{T}$ and $V{\downarrow}_S$ the restriction of V to $\mathbf{S}$, is a model as $V{\downarrow}_S$ may assign $\emptyset$ to nominals — but we need merely confine our attention to pairs where this does not happen. The usual induction then gives a generated submodel theorem for languages with nominals; and as an immediate corollary we have that validity is transmitted from any frame to its generated subframes.

The two results that fail are more interesting. For Priorean languages we have that given an indexed collection of frames $\{\mathbf{T}_m : m \in M\}$, if for all $m \in M$ $\mathbf{T}_m \models \phi$, then $\biguplus \mathbf{T}_m \models \phi$. An immediate consequence of this result is that Priorean languages cannot define the universal relation $\forall xy(x < y)$. Another obvious consequence is that connectedness is not definable in a Priorean language; indeed something stronger holds — no purely Priorean definable class of frames consists solely of connected frames.

For languages containing nominals the preservation result no longer holds. An immediate counterexample is given by the class of trichotomous frames, defined by $Pi \vee i \vee Fi$. Another is provided by the class of (right or left) directed frames. Yet another is given by the universal relation; this condition *is* definable using nominals, by Fi. Note that each member of these newly definable classes is a connected frame: in languages with nominals some classes of frames consisting solely of connected frames *are* definable.

Now, although the disjoint union preservation result fails for languages with nominals, a little reflection shows that it 'only just' fails. Suppose we have two frames $\mathbf{T}_1$ and $\mathbf{T}_2$ on each of which ϕ is valid. To keep things simple suppose ϕ contains occurrences of only one nominal, say i. We know that we cannot conclude that $\mathbf{T}_1 \uplus \mathbf{T}_2 \models \phi$, but why not? The reason is that in any valuation on $\mathbf{T}_1 \uplus \mathbf{T}_2$, on one of the components, say $\mathbf{T}_1$, i will be false everywhere. This is a situation that the validity of ϕ on the component frames simply gives us no information about: in any valuation on either frame i is true *somewhere*.

But suppose we knew something more: namely that not only was ϕ valid on each frame, but $\phi[\perp/i]$ was also. Then, intuitively, we *would* have the information needed to guarantee validity on the disjoint union: the validity of the new formula blocks the possibility that i being false everywhere in a component will cause trouble. This is indeed the case: indeed, not only is the condition ***sufficient***, it is also ***necessary*** as long as the disjoint union is not trivial — that is, as long as at least two frames are stuck together.

To state the result in full generality we need merely extend the above intuitions to the case where ϕ contains many different nominals. Essentially all we need to do is account for all the different ways the nominals can be 'dealt out' — like cards from a pack — to the 'players' — the components of the disjoint union.[7] That is, we must take into account all possible uniform substitutions of $\perp$ for nominals in ϕ. Let $S^{\perp}(\phi)$ be the (finite) set of sentences

[6] We assume the standard definitions of these concepts for tensed languages in what follows; see [1] for details.

[7] A particular deal, of course, is just a valuation.

consisting of precisely all the possible sentences obtainable by uniformly substituting $\perp$ for nominals occurring in ϕ, including the null substitution.[8] Let $\phi^{\perp}$ denote the conjunction of these sentences. Then we have:

Theorem 2.1 *Let $\{\mathbf{T}_m : m \in M\}$ be a family of frames such that card$(M) \geq 2$. Then:*

$$\biguplus \mathbf{T}_m \models \phi \textit{ iff } \forall m \in M \ \mathbf{T}_m \models \phi^{\perp}$$

for all wffs ϕ.

Proof:

A straightforward argument using the generated submodel result. Use the fact that nominals assigned points outside a generated subframe $\mathbf{S}$ behave like $\perp$ on $\mathbf{S}$. □

While p-morphisms preserve validity for Priorean languages, they do not do so for languages with nominals. There are many obvious counterexamples. Note that the unique function from $\mathbf{Z}$ to the singleton reflexive frame $\langle\{0\}, \{\langle 0, 0\rangle\}\rangle$ is a p-morphism; but both $i \rightarrow \neg Fi$ and $i \rightarrow \neg FFi$ are valid on $\mathbf{Z}$ and invalid on the singleton reflexive loop. A pretty p-morphism is constructed in [1, pages 160–161]. The source frame is discrete, the target frame indiscrete, thus demonstrating that discreteness is not Priorean definable. But we know that discreteness is definable with nominals, hence van Benthem's construction provides yet another counterexample. Finally, consider n-branching trees of depth ω. All points in such trees have precisely n successors, and we know that for all $n \in N$ we can write an expression in nominals valid on all frames with branch factor n. But the mapping from n branching trees of depth ω to the natural numbers under the successor relation, $\mathbf{N}^S$, which associates with each node its depth is a p-morphism and thus for all $n \geq 2$ we have an example of the non-transmission of nominal validity to p-morphic images.

For *models* however, the p-morphic link is the correct one. That is, if f is a p-morphism from $\mathbf{M}_s = \langle \mathbf{S}, V_s\rangle$ to $\mathbf{M}_t = \langle \mathbf{T}, V_t\rangle$, then we still have that

$$\mathbf{M}_s \models \phi[s] \text{ iff } \mathbf{M}_t \models \phi[f(s)],$$

for all $s \in S$ and all wffs ϕ, as the usual induction on $deg(\phi)$ shows.[9] Note why we cannot derive from this the usual validity preservation result. Suppose f is a p-morphism from $\mathbf{S}$ to $\mathbf{T}$. Just because a valuation V falsifies some formula ϕ on the target frame $\mathbf{T}$, we cannot necessarily transfer the falsifying valuation to $\mathbf{S}$; $f^{-1}[V(i)]$ may not be a singleton subset of the source frame and thus won't always yield a valuation. We do however have the following sufficient condition for a p-morphism to preserve validity. Call a p-morphism n-separating if it never maps distinct points s and s' of the source frame connected by a path of length n or less to the same point in the target frame. Then the following lemma is straightforward by appeal to the Horizon Lemma:

Lemma 2.1 *Let ϕ be a wff such that $td(\phi) = n \geq 1$, and f a $(2n+1)$-separating surjective p-morphism from $\mathbf{S}$ to $\mathbf{T}$. Then $\mathbf{S} \models \phi$ implies $\mathbf{T} \models \phi$.* □

Let us turn to the correspondence between languages of NTL and classical languages. Following [2] we define $\mathbf{L}_0$ to be a first order language with identity that contains precisely one non-logical symbol, a binary relation symbol '<'. Note that any frame is a structure for this

[8] For example, $S^{\perp}(i \wedge Fj) = \{i \wedge Fj,\ i \wedge F\perp,\ \perp \wedge Fj,\ \perp \wedge F\perp\}$.

[9] The usual strengthening also holds: two models linked by a *zigzag relation*, in the sense of [3, page 12], are equivalent.

language. Now, only $\mathbf{L}_0$ expressible classes of frame are definable by purely nominal sentences. To see this note that to deal with nominals we need merely augment the ***standard translation*** [1, page 151] of tensed languages into classical languages by adding the clause that the standard translation of any nominal i, $ST(i)$, is to be the $\mathbf{L}_0$-wff $x_i = t$. (Here x_i is the $\mathbf{L}_0$ variable designated as corresponding to the nominal i, and t the $\mathbf{L}_0$ variable representing the point of evaluation.) Now saying that a purely nominal formula ϕ is valid on a frame $\mathbf{T}$ is equivalent to saying that $\forall t \forall x_{i_1} \cdots x_{i_n} ST(\phi)$ is true in any first order model based on the structure $\mathbf{T}$, where the $x_{i_1}, \ldots, x_{i_n}$ correspond to all the nominals in ϕ. But $\forall t \forall x_{i_1} \cdots x_{i_n} ST(\phi)$ is a first order sentence, in fact an $\mathbf{L}_0$ sentence. With purely Priorean languages we need second order quantification when we talk about validity — variables correspond to predicates. With nominals matters are simpler. This translation immediately yields a number of results: that nominal validity is r.e., compactness and Lowenheim-Skolem theorems, and so on. It further shows that ***frame consequence*** [10] is also an r.e. relation for purely nominal sets of sentences, as for such sentences $\Sigma \models_f \phi$ iff $ST(\Sigma) \models ST(\phi)$; frame consequence $\models_f$ has been reduced to the r.e. relation of first order consequence, $\models$.

3 The Minimal Logic

The minimal logic can be axiomatized by the addition of either of two schemas to K_t, the usual axiomatisation of the minimal Priorean tense logic. The schemas are called the NOM and SWEEP schemas, and to present them we need a little notation. For any language $\mathcal{L}$ let an ***existential tense*** be any unbroken sequence of Ps and Fs. The sequence may contain both Ps and Fs, and we regard the null sequence $\triangle$ as an an existential tense.[11] We normally use E, E', and so on as metavariables across existential tenses. By a ***universal tense*** is meant any unbroken, possibly mixed, sequence of Gs and Hs, including the null sequence; A, A', and so on are used as metavariables over universal tenses. In the following two schemas, n is a metavariable across nominals, and ϕ and ψ are metavariables across arbitrary wffs.

$$\begin{array}{ll} \textbf{NOM} & E(n \wedge \phi) \wedge E'(n \wedge \psi) \rightarrow E(n \wedge \phi \wedge \psi) \\ \textbf{SWEEP} & E(n \wedge \phi) \rightarrow A(n \rightarrow \phi) \end{array}$$

Let's instantiate the NOM schema in i and consider what it says:

$$E(i \wedge \phi) \wedge E'(i \wedge \psi) \rightarrow E(i \wedge \phi \wedge \psi).$$

Think of the points of a model as boxes holding items of information. Suppose we are standing at a point t in some frame $\mathbf{T}$ and we know that both $E(i \wedge \phi)$ and $E'(i \wedge \psi)$ are true. This means we know that if we follow a certain zig-zag path from t, (the one coded up by E), we can get to a box marked i and containing the information ϕ; and that if we follow another possibly different path from t, (the one coded up by E') we get to another box, also marked i, and containing the information ψ. *But there is only one box marked i.* Hence this single box contains both the information ϕ and the information ψ, and the paths coded for by E and E' lead to the same point. This is precisely what the consequent of NOM gives us. In a nutshell, the NOM schema consists of all the path equations that must be satisfied in any model.

[10] We say a wff ϕ is a frame consequence of a set of wffs Σ iff whenever Σ is *valid* on a frame $\mathbf{T}$, so is ϕ. This relation is not r.e. for purely Priorean languages; see [26].

[11] Thus $FPFPPFPP$, F and $PPPP$ are existential tenses; $PFFGPP$ *isn't* because it contains a universal operator, G.

Let K_{nt} be the axiomatisation obtained by adjoining to K_t either of these schemas. We wish to show that K_{nt} captures the minimal logic for languages of NTL.[12] The soundness of either schema is immediate. Perhaps the neatest way to show completeness is to adapt a method originally due to David Makinson [14], and applied to tense logic in [15]. This is an elegant method and always yields a countable model; however it requires several preliminary definitions, and so we sketch instead an argument that uses generated subframes of the canonical Henkin frame $\mathbf{H}^{K_{nt}}$.[13] Note, however, that this method may yield an uncountable model.

But why use generated subframes of $\mathbf{H}^{K_{nt}}$? Why not build the usual 'canonical model' using the whole of $\mathbf{H}^{K_{nt}}$ and the 'natural valuation'? In fact we *cannot* do this: the 'natural mapping' V from the atoms of our language to H defined by $V(a) = \{h \in H : a \in h\}$ is *not* a valuation as each nominal occurs in more than one point of H. By restricting ourselves to generated subframes of $\mathbf{H}^{K_{nt}}$, however, we will be able to build a valuation from the natural mapping. So, given a consistent set of sentences Σ, take the subframe of $\mathbf{H}^{K_{nt}}$ generated by Σ^{∞}. The key lemma is:

Lemma 3.1 (Unique Occurrence Lemma) *Let* $\mathbf{H}^{\Sigma} = \langle H^{\Sigma}, <_h \rangle$ *be the subframe of* $\mathbf{H}^{K_{nt}}$ *generated by* Σ^{∞}. *Then for all* $h, h' \in H^{\Sigma}$, *and every nominal* i, *if* $i \in h$ *and* $i \in h'$ *then* $h = h'$.

Proof:

Suppose there are two distinct points h, $h' \in H^{\Sigma}$ that contain the same nominal i. As they are distinct MCS there is some wff ϕ that distinguishes them, so suppose $i \wedge \phi \in h$ and $i \wedge \neg\phi \in h'$. Now as $\mathbf{H}^{\Sigma}$ is generated from Σ^{∞}, there is a path from Σ^{∞} to h, and a path from Σ^{∞} to h'. By appeal to tense logical lemmas we can thus show that there are existential tenses E and E' such that both $E(i \wedge \phi)$ and $E'(i \wedge \neg\phi) \in \Sigma^{\infty}$. But by NOM this means that $E(i \wedge \phi \wedge \neg\phi) \in \Sigma^{\infty}$, and thus, by tense logic, we have $E\bot \in \Sigma^{\infty}$. But this is impossible as Σ^{∞} is consistent. (A similar argument works for SWEEP.) $\square$

Now it can happen that not all nominals of our language appear in some $h \in H^{\Sigma}$ — for example, for any choice of i the consistent set of sentences $\Sigma = \{\neg Ei : E \text{ is an existential tense}\}$ 'forces i out' of the subframe generated by Σ^{∞} — but this is easy to fix. Simply adjoin a new point h^{∞} to $\mathbf{H}^{\Sigma}$ that is unrelated to any other point, and define a new mapping V_n^+ that is identical to V_n, save only that where V_n assigns $\emptyset$ to some nominal i, V_N^+ assigns $\{h^{\infty}\}$ to the same nominal. Clearly V_n^+ is a valuation. The usual induction then shows that $\langle \mathbf{H}^{\Sigma}, V_n^+ \rangle \models \Sigma[\Sigma^{\infty}]$ and we have our completeness result.

It is also clear that the above proof yields a completeness result for languages of nominal modal logic. The modal analogs of existential and universal tenses are unbroken (possibly null) sequences of $\Diamond$s, and of $\Box$s respectively. With the E and A metavariables read in this fashion we have that either $K + NOM$ or $K + SWEEP$ axiomatises the minimal nominal modal logic, where K is the usual axiomatisation of minimal normal modal logic. We refer to either axiomatisation as K_{nm}.

[12] In what follows we assume the usual definitions (such as those of *consistency* and *maximal consistent sets of sentences* (MCS)), and all the usual tense logical lemmas needed in Henkin proofs; see [6] or [15] for further details. Note in particular that Lindenbaum's Lemma holds. We further assume that the wffs of our language have been standardly ordered; by Σ^{∞} we mean the Lindenbaum expansion of a consistent set of sentences Σ with respect to this standard ordering.

[13] By the canonical Henkin frame for K_{nt} is meant the frame $\mathbf{H}^{K_{nt}} = \langle H, <_h \rangle$, where H consists of all and only the K_{nt} MCSs; and for all h, $h' \in H$, $h <_h h'$ iff for all wffs ϕ, $G\phi \in h$ implies $\phi \in h'$.

Let us re-examine the proof of the Unique Occurrence Lemma; a little reflection shows that we can do rather better. In the above proof we made use of three distinct points, h, h' and Σ^∞; and two different paths. But we could have just used a 'two point argument': given h and h' as described above there must be a path from one to the other — we needn't bring the generating point Σ^∞ explicitly into the proof at all. But once this is observed it becomes clear that we don't need all the instances of either NOM or SWEEP to guarantee completeness; the instances of the following two weakened forms will suffice:

$$\begin{array}{ll} \textbf{NOM}_w & n \wedge E(n \wedge \phi) \rightarrow \phi \\ \textbf{SWEEP}_w & (n \wedge \phi) \rightarrow A(n \rightarrow \phi) \end{array}$$

To see this, we sketch a proof of a Unique Occurrence Lemma from the new axiomatic bases. We treat the case for SWEEP$_W$. Let our assumptions and notation be as before. Suppose two point h and h' in $\mathbf{H}^\Sigma$ contain the same nominal i. As $\mathbf{H}^\Sigma$ is generated from a single point Σ^∞ it is connected, and thus there is a path between h and h'. Let $A^{(h \rightarrow h')}$ be the universal tense that corresponds to the path as seen from h. (That is, starting at h we traverse the path until we reach h', writing down a G for every move forward in time, and H for every move backwards.) As all instances of SWEEP$_W$ occur in h, then in particular we have that

$$i \wedge \phi \rightarrow A^{(h \rightarrow h')}(i \rightarrow \phi) \ \in h.$$

But as $i \in h$, then for all $\phi \in h$ we have that $A^{(h \rightarrow h')}(i \rightarrow \phi) \ \in h$. But then by the usual tense logical lemmas we have that $i \rightarrow \phi \in h'$, and as $i \in h'$ we have that $\phi \in h'$. As h and h' are MCS this means that $h = h'$. Thus we have an improved completeness result.

However note that this improvement does *not* hold for modal languages. Intuitively, we have to use a 'three point argument' in modal languages as in such languages we can never look back. The 'two point argument' is the perogative of tense logic. It is straightforward to turn this intuition into a proof that neither $K + \text{NOM}_W$ nor $K + \text{SWEEP}_W$ suffices to axiomatise the minimal nominal modal logic. We will proceed by finding a semantical property which distinguishes the derivable from the non-derivable wffs. The first step is to define:

Definition 3.1 *Let* $\mathbf{T}$ *be a frame and* t *and* t' *be distinct elements of* T. *We say* t *and* t' *are a* separated pair *iff there is no modal path from* t *to* t', *and no modal path from* t' *to* t. *A frame is said to* separated *iff it contains at least one separated pair.* □

(Note that we talked of *modal* paths, not zig-zag paths, in the above definition.) We now change the interpretation of modal languages with nominals. Let $\mathcal{L}$ be any language of nominal modal logic. In the *separated interpretation* for $\mathcal{L}$ we define *separated valuations* on separated frames; in each separated valuation every nominal denotes exactly two distinct points, t and t', where t and t' are a separated pair. Everything else is as usual: variables denote arbitrary subsets of such frames and the non-atomic sentences are evaluated as usual. We say that an $\mathcal{L}$-wff ϕ is s-valid iff it is valid in any separated interpretation on any separated frame. Clearly both $K + \text{NOM}_W$ and $K + \text{SWEEP}_W$ are sound with respect to this interpretation; everything provable from either basis is s-valid. However it is easy to falsify instances of both the NOM and SWEEP schemas. Let $\mathbf{T}$ be the frame $\langle\{-1, 0, 1\}, \{\langle 0, -1\rangle, \langle 0, 1\rangle\}\rangle$. Clearly -1 and 1 are a separated pair. Let V be any valuation that assigns $\{-1, 1\}$ to i, and $\{1\}$ to p. Then both an instance of NOM, $\Diamond(i \wedge p) \wedge \Diamond(i \wedge \neg p) \rightarrow \Diamond(i \wedge p \wedge \neg p)$, and an instance of SWEEP, $\Diamond(i \wedge p) \rightarrow \Box(i \rightarrow p)$, are false at 0 and thus cannot be derived from the weakened basis.

In passing, there's some simple observations we can make about the impact the addition of nominals has on the Henkin frame of the minimal normal modal logic.[14] Suppose $\mathcal{L}$ is a standard language of modal logic; that is, $\mathcal{L}$ has a countably infinite set of variables and no nominals. Let K be the minimal normal logic in $\mathcal{L}$ and $\mathbf{H}^K$ its canonical frame. The following facts about $\mathbf{H}^K$ are well known: $\mathbf{H}^K$ is left directed, point generated, and indeed *strongly generated*. By this last is meant that there exists an $h \in H^K$ such that for all $h' \in H^K$, $h <_h h'$; from h we can get to any other point in one step. These properties follow from the the fact that K admits the *Law of Disjunction* (LOD): $\vdash_L \Box\phi_1 \vee \ldots \vee \Box\phi_n$ implies $\vdash \phi_m$, for some m such that $1 \leq m \leq n$.[15]

The minimal nominal modal logic, however, does not admit LOD. Note that $\mathbf{H}^{K_{nm}}$ cannot be left directed as no MCS h can precede both $\{i \wedge \phi\}^\infty$ and $\{i \wedge \neg\phi\}^\infty$; hence LOD cannot hold. This example also shows that $\mathbf{H}^{K_{nm}}$ cannot be strongly generated. In fact, it can't even be generated: for arbitrary existential modalities $\Diamond^n$ and $\Diamond^m$, $\Diamond^n(i \wedge \phi) \wedge \Diamond^m(i \wedge \neg\phi)$ is inconsistent, and thus no MCS h can precede both $\{i \wedge \phi\}^\infty$ and $\{i \wedge \neg\phi\}^\infty$, no matter how many steps intervene. The only obvious thing we can say about the structure of $\mathbf{H}^{K_{nm}}$ derives from the following observation: one special case of LOD is unaffected by the addition of nominals: $\vdash_{K_{nm}} \phi$ iff $\vdash_{K_{nm}} \Box\phi$, and thus $\mathbf{H}^{K_{nm}}$ is left unbounded.

Let us now consider the Gargov, Passy and Tinchev axiomatisation of the minimal modal logic for languages with nominals. They first define necessity and possibility forms:[16]

Definition 3.2 *Let $\mathcal{L}$ be a language of* NTL, *\$ be a new entity distinct from any $\mathcal{L}$ wff or symbol, and θ be a wff of $\mathcal{L}$. Then the* necessity forms *of $\mathcal{L}$, are the elements of the smallest set $\Box$-form such that:*

$$\begin{array}{l} \$ \in \Box\text{-}form \\ L \in \Box\text{-}form \text{ implies } \theta \rightarrow L \in \Box\text{-}form \\ L \in \Box\text{-}form \text{ implies } \Box L \in \Box\text{-}form; \end{array}$$

and the possibility forms *of $\mathcal{L}$, are the elements of the smallest set $\Diamond$-form such that:*

$$\begin{array}{l} \$ \in \Diamond\text{-}form \\ L \in \Diamond\text{-}form \text{ implies } \theta \wedge L \in \Diamond\text{-}form \\ L \in \Diamond\text{-}form \text{ implies } \Diamond L \in \Diamond\text{-}form. \end{array}$$

If ψ is any wff of $\mathcal{L}$, and L and M are $\Box$-forms and $\Diamond$-forms respectively, then by $L(\psi)$ and $M(\psi)$ are meant the $\mathcal{L}$-wffs obtained by replacing the (unique) occurrence of \$ in L and M respectively by ψ. $\Box$

They then axiomatise the minimal logic for languages of weak NML by adding to the usual axioms of the minimal modal logic K all instances of the following schema:

$$\mathbf{Ax_N} \qquad M(n \wedge \phi) \rightarrow L(n \rightarrow \phi),$$

where L and M are metavariables over $\Box$-forms and $\Diamond$-forms respectively. They prove completeness by a three point argument on generated subframes of the $\mathbf{H}^{K_{nm}}$.

[14]By way of contrast, there's not much we can say about the structure of the minimal tense logical Henkin frame beyond the fact that it's big and disconnected; the bidirectional operators homogenise the frame.

[15]For a discussion of why these properties follow from LOD, see [13] or [10]. Minimal tense logic does not admit LOD; see [3, page 11].

[16]In the following definition the use of L as a $\Box$-form has nothing to do with the shifter operator L mentioned in the preliminary section.

The form of the Ax_N schema is superficially reminiscent of that of SWEEP, but the M and the L don't range over universal and existential modalities but over the more complex $\Box-$ and $\Diamond-$ forms. Thus for fixed i and ϕ the consequents of Ax_N include all entries in the following infinite matrix:

$$\begin{array}{llll}
(i \to \phi) & \Box(i \to \phi) & \Box\Box(i \to \phi) & \cdots \\
\phi \to (i \to \phi) & \Box(\phi \to (i \to \phi)) & \Box\Box(\phi \to (i \to \phi)) & \cdots \\
\phi \to (\phi \to (i \to \phi)) & \Box(\phi \to (\phi \to (i \to \phi))) & \Box\Box(\phi \to (\phi \to (i \to \phi))) & \cdots \\
\vdots & \vdots & \vdots & \vdots \\
\phi \to \Box(i \to \phi) & \Box(\phi \to \Box(i \to \phi)) & \Box\Box(\phi \to \Box(i \to \phi)) & \cdots \\
\vdots & \vdots & \vdots & \vdots
\end{array}$$

The antecedents of Ax_N, again for fixed i and ϕ, consists of all entries in the matrix obtained from that above by replacing $\Box$ by $\Diamond$ and $\to$ by $\wedge$. Note that for fixed i and ϕ the SWEEP schema consists merely of conditionals formed from the first row of each of the above matrices. The simpler SWEEP_W schema that suffices for tense logic essentially consists, for fixed i and ϕ, of only the single wff occurring in the top left entry of the second matrix — $i \wedge \phi$ — as antecedent; and as consequents just the wffs in the first row of the above matrix. Thinking in terms of paths and path equations is a simpler way of adding nominals to modal (and especially tensed) languages.

We conclude this section by noting some theorems of the minimal tense logic. Firstly, nominals interact strongly with universal tenses; Hi and Gi can only be true under 'end conditions', hence both the following 'end effects' $i \wedge Gi \wedge F\phi \to \phi$ and $Gi \wedge F\psi \to G\psi$ are theorems. Note that if we replace i by p in the above the resulting wffs are ***not*** Priorean valid. Next, suppose t is a point and that there is a path P that leads away from t but eventually returns there. Then a 'reverse journey' exists: we could traverse P in the reverse direction and still get back to t. In NTL we can talk about such reverse journeys; we can't in standard languages as we cannot uniquely mark the starting point. To display the relevant theorem we first need to define the ***transposition*** E^T of an existential tense E. By this is meant the existential tense formed by reversing the sequence of tenses in E and forming the mirror image.[17] If an existential tense E codes a path between points t and t' as seen by an observer at t, then E^T codes the same path as viewed by an observer at t'.[18] The theorem asserting the existence of reverse journeys can now be given: $i \wedge Ei \to E^T i$. Again note that if we replace i by p we do not get a Priorean validity. Finally note that if we can break off a journey in the middle, pick up a piece of data, and then continue round, we can do the same thing backwards: $i \wedge E_1(\psi \wedge E_2 i) \to E_2^T(\psi \wedge E_1^T i)$. This schema is called the Stopover Schema and it is useful in deriving 'mirror image schema' in extensions of the minimal logic.

4 Extensions of K_{nt}

We now sketch completeness and decidability results for some more interesting classes of frames. Adding to K_{nt} as axioms all instances of $FF\phi \to F\phi$ (4), $\phi \to F\phi$ (T), $P\top \wedge F\top$ (D), or the *Lin* schema, which consists of the conjunction of

$$F\phi \wedge F\psi \to (F(\phi \wedge F\psi) \vee F(\psi \wedge F\phi) \vee F(\phi \wedge \psi)) \qquad (RLin),$$

[17] For example, $(PPFPF)^T = PFPFF$.

[18] Note that tense transpositions are an intrinsically *tense* logical concept; they have no correlate in modal languages — as we have seen, real path exploration requires bidirectional operator pairs.

with its mirror image $LLin$, yields Henkin frames that are transitive, reflexive, both right and left unbounded, and locally linear respectively; hence generating Henkin models and adding isolated points to complete the valuation gives an immediate crop of completeness results. But this is all familiar territory; what happens when we add schemas corresponding to newly definable conditions, such as irreflexivity or antisymmetry?

Adding as axioms all instances of $n \rightarrow \neg Fn$ (I), or $n \rightarrow G(Fn \rightarrow n)$ $(Anti)$, does *not* yield irreflexive or antisymmetric Henkin frames; some points don't contain nominals, and we cannot guarantee that such points have the desired property. However points in these frames containing nominals are well behaved. In particular, in the Henkin frame of any extension of $K_{nt}I$, all points containing nominals are irreflexive; and in any extension of $K_{nt}Anti$, no point containing a nominal is in a proper cluster — the proofs are immediate.[19] Now suppose we are working with some such extension of K_{nt} and that by our 'generate and add an isolated point' method we have built a model $\mathbf{M}^H$ verifying our original set of sentences Σ. As $\mathbf{M}^H$ is not guaranteed to have the correct structure we must find a 'rectification' technique to transform it into an equivalent model of the desired form. In doing this we must take care that our rectification method does not destroy the Unique Occurrence property of $\mathbf{M}^H$; we want 'local' methods that only need alter the points not containing nominals. Fortunately, some standard methods work this way.

The simplest example is provided by I $(= K_{nt}I)$. This is complete with respect to the irreflexive frames. To prove this we need merely observe that by stretching apart reflexive points t of $\mathbf{M}^H$ into two points s and s', stipulating that $s < s'$, $s' < s$, $s \not< s$ and $s' \not< s'$, and insisting that all points which preceded t now precede both s and s', and so on, we form a new irreflexive frame $\mathbf{S}$. By our previous remark, no point containing a nominal was reflexive, hence no such point was stretched apart; thus the obvious mapping V^s of atoms to $Pow(S)$ *is* a valuation. As $\mathbf{M}^H$ is a p-morphic image of $\langle \mathbf{S}, V^s \rangle$, the models are equivalent and we are through.

For logics of frames that are both transitive and irreflexive we apply *heavy* bulldozing. For example, $I4$ is complete with respect to the SPOs. This is proved by bulldozing *all* clusters of $\mathbf{M}^H$, embedding them into (say) $\mathbf{Z}$ or $\mathbf{Q}$; points containing nominals aren't in clusters and hence aren't bulldozed. For logics of frames that are transitive, reflexive, and antisymmetric we *lightly* bulldoze. For example, PO $(= K_{nt}4TAnti)$ is complete with respect to the POs. To see this, bulldoze all and only the proper clusters of $\mathbf{M}^H$, embedding them into (say) $\langle Z, \leq \rangle$ or $\langle Q, \leq \rangle$. Note that in this axiomatisation we only added the 'forward looking' schema that define antisymmetry; the mirror images are also valid on the POs and hence must be provable. A formal proof can be displayed by making use of the relevant instance of the Stopover Schema.

The logics of linear frames have some pleasant properties. Observe that we *never* need to add an isolated point after the generation process. For all such logics we add as axioms all instances of the schema defining trichotomy, $Pn \vee n \vee Fn$ (Tr), and its inclusion prevents nominals being 'driven out'; the troublesome sets of sentences $\{\neg Ei : E \text{ is an existential tense}\}$ are no longer consistent. We note that $I4LinTr$ is complete with respect to the STOs, and $POLinTr$ is complete with respect to the TOs; this is clear by the relevant forms of bulldozing.

But we can do better; it is possible to give finite axiomatisations for the logics of linear frames. Define $\Box_t\phi$ to be $H\phi \wedge \phi \wedge G\phi$. Then $(i \wedge \phi) \rightarrow \Box_t(i \rightarrow \phi)$, which is just SWEEP_W

[19]By a *cluster* C of a frame $\langle T, < \rangle$ is meant any $C \subseteq T$ such that $C^2 \cap <$ is an equivalence relation, and for no proper superset C' of C is this true. A cluster is *proper* if it contains at least two points, and *simple* otherwise. In what follows we assume the reader is familiar with 'cluster manipulation' completeness methods as presented in [23], especially the bulldozing technique.

with the universal tenses replaced by $\Box_t$, can replace NOM$_W$ or SWEEP$_W$ or whatever we're using in K_{nt}. With this done we can give a finite axiomatisation of the logics of linear frames by using axioms together with a rule of substitution instead of axiom schemas. Note that this rule of substitution must only replace nominals by other nominals. Moreover it is straightforward to axiomatise $\mathbf{Q}$, $\mathbf{Z}$, $\mathbf{N}$ and $\mathbf{R}$. In fact all we need do is take their normal axiomatisations in K_t, and add NOM$_W$ — or the new finite schema — plus I and Tr. The proof methods given in [23] still work.

What about decidability? Note that because of the new expressive powers of our languages many obvious logics lack the *finite frame property*. For example consider $I4D$. Any class of frames on which all its axioms are valid must consist solely of unbounded SPOs, hence no finite frame can validate its axioms and $I4D$ does not have the finite frame property. In such cases there is an apparent impediment to establishing decidability by the familiar 'search through finite structures' argument — but there is an interesting loophole. Although $I4D$ does not have the finite frame property it does have the *finite model property*. That is, it is possible to define a class of finite *models* $\mathcal{M}$ such that $\vdash_{I4D} \phi$ iff $\mathbf{M} \models \phi$, for all $\mathbf{M} \in \mathcal{M}$. Note that the loophole we are exploiting does not exist in standard languages: a well known theorem of Segerberg's states that any classical modal logic has the finite model property iff it has the finite frame property [24, page 33]. Thus the analog of Segerberg's theorem does not hold in NTL as $I4D$ is a counterexample.

What are these classes of models? Simply the finite members of the most obvious class of models to which the unrectified Henkin models $\mathbf{M}^H$ produced in the course of proving completeness belong. Let's consider the logic $I4$. Call $\mathbf{T} = \langle T, < \rangle$ an *irreflexivity containing* frame iff there is a $t \in T$ such that $t \not< t$. Call a valuation V on such a frame $\mathbf{T}$ *irreflexivity respecting* iff $t \in V(i)$ is irreflexive for all nominals i. That is, irreflexivity respecting valuations are valuations on frames containing irreflexive points that send all nominals to irreflexive points. We call $\mathbf{M} = \langle \mathbf{T}, V \rangle$ an I_1 *model* iff $\mathbf{T}$ is an irreflexivity containing frame and V an irreflexivity respecting valuation on $\mathbf{T}$. The class of all I_1 models is called $\mathcal{M}(I_1)$. It is clear from the Henkin proofs sketched above that $I4$ is sound and complete with respect to the class of all transitive I_1 models. That is, $\vdash_{I4} \phi$ iff $\mathbf{M} \models \phi$, for all $\mathbf{M} \in \mathcal{M}(I_1) \cap \mathcal{M}(Tran)$. It is now straightforward to use filtrations to show that $I4$ has the finite model property with respect to $\mathcal{M}(I_1) \cap \mathcal{M}(Tran)$. That is: $\vdash_{I4} \phi$ iff $\mathbf{M} \models \phi$ for all *finite* $\mathbf{M} \in \mathcal{M}(I_1) \cap \mathcal{M}(Tran)$.

Soundness is immediate. For the reverse direction we know that given an $I4$-consistent sentence ϕ we can find an $\mathbf{M} \in \mathcal{M}(I_1) \cap \mathcal{M}(Tran)$ such that $\mathbf{M} \models \phi[t]$, at some point t. (The usual (unbulldozed) Henkin model $\mathbf{M}^H$ suffices.) Now, if ϕ contains occurrences of nominals, define Σ^- to be

$$\{\phi\} \cup \{i \to \neg Fi : i \text{ occurs in } \phi\};$$

while if ϕ contains no occurrences of nominals choose any nominal — say i — and define Σ^- to be $\{\phi\} \cup \{i \to \neg Fi\}$. Let Σ be the smallest set of wffs containing Σ^- that is closed under subformulas. Form a transitive filtration $\mathbf{M}^f$ of $\mathbf{M}$ through Σ such that for all nominals $j \notin \Sigma$, $V_f(j) = V_f(i)$, for some nominal $i \in \Sigma$. By the Filtration Theorem $\mathbf{M}^f \models \phi[E(t)]$. But $\mathbf{M}^f$ is a model in the required class: clearly it is finite, because Σ is a finite set of sentences; and it is transitive because we took a transitive filtration. Moreover $\mathbf{M}^f$ *does* contain irreflexive points, and all nominals are assigned irreflexive points in this filtration. To see this, note that it follows from the usual definition of transitive filtrations that:

$$\exists\phi(F\phi \in \Sigma \;\&\; \mathbf{M} \models \phi[t] \;\&\; \mathbf{M} \not\models F\phi[t]) \text{ implies } E(t) \not<_f E(t).$$

But for all nominals $i \in \Sigma$ — and there is always at least one — $Fi \in \Sigma$. Further, as our original model was in $\mathcal{M}(I_1)$, $\mathbf{M} \models i[t]$ means $\mathbf{M} \not\models Fi[t]$, and thus for all such points

t, $E(t) \not< E(t)$. This means that all points in the filtration $\mathbf{M}^f$ denoted by nominals are irreflexive, and we have our result. An immediate corollary is that $I4$ is decidable.

Decidability for other extensions of $I4$ follow from this basic result.[20] In particular, $I4D$ is decidable because our usual Henkin completeness proof establishes that $\vdash_{I4} \phi$ iff $\mathbf{M} \models \phi$, for all unbounded $\mathbf{M} \in \mathcal{M}(I_1) \cap \mathcal{M}(Tran)$. As filtrations inherit unboundedness, the filtration described above for $I4$ establishes the finite model property for $I4D$ relative to this class of models, and decidability follows. Thus we have a tool that works for many of the logics of interest above $I4$. Moreover, in similar fashion it is simple to show analogous results for many logics above PO; the reader is referred to Chapter 5 of my thesis [4] for details. In passing, these methods can be used to establish the decidability of the nominal tense logic of $\mathbf{Q}$, but won't work for the logics of $\mathbf{N}$, $\mathbf{Z}$ or $\mathbf{R}$. (The easy soundness direction will not go through for these logics.) However Rabin-Gabbay techniques [6] can be used to show their decidability.

It should be clear that the classic cluster manipulation techniques of Segerberg — Henkin frame generation, bulldozing, filtration and related techniques — can be applied in the new setting to yield answers to many basic questions; nonetheless it is interesting to look for alternatives. There seem to be two possibilities. One is that explored by Gargov, Passy and Tinchev [7] who introduce an 'infinitary rule' COV to their nominal modal languages.[21] With its aid, in many cases they are able to build models in which every point contains a nominal. Obviously such a technique is very powerful; to take a simple example of where it could be useful, consider extending K_{nt} by adding all instances of the schemas defining right and left discreteness together with the 'discreteness' schema of Priorean logic, [6]. Without making additional assumptions of linearity and transitivity it is hard to make progress by cluster methods. However if we could drive a nominal into every world, we would have a completeness result because the discreteness schema is well behaved on points containing nominals. There are many other areas such a rule could be useful. For example, when considering interval based logics augmented by nominals, as in done in [4, Chapter 8], cluster manipulation techniques rapidly become difficult to apply, and the use of such a rule may be invaluable. Even so, once we have used the COV rule to build a model it is natural to enquire whether its use can be eliminated. The completeness results sketched above show that in some important cases this can be done; and combining these elimination results with the decidability results we have noted above, we have decidability results for certain axiomatisations employing COV.

A second method that seems interesting is to try to obtain a general conservativity result with respect to Bull's [5] system. Then one could use the quantifiers and S5 modality to tailor a theory in a language with new 'witness' nominals which would guarantee that the right sort of frame was produced during the generation process. Gargov, Passy and Tinchev report such a conservativity result with respect to a different 'quantified nominal' language in [7].

5 Concluding remarks

Does NTL lead anywhere? I believe that it is a prototype for a more general strategy of sorting in intensional languages.[22]

[20] Indeed the weaker logic I must also be decidable. It has the finite model property with respect to $\mathcal{M}(I_1)$.

[21] As the authors point out, although COV is often expressed as an infinitary rule of inference, it can be reduced to finitary form. The more recent paper by Gargov and Goranko [8] contains a more detailed discussion of the COV rule and its variants.

[22] Many of my ideas on sorting emerged in discussions with Jerry Seligman, and owe a lot to his work on *perspectives*, *constraints* and *classification* in Situation Theory; see [25].

The basic system of NTL just outlined provides useful logical analyses of temporal expressions in natural language; with some simple extensions it can be greatly improved. Two such extensions are ***relativisation to context*** and ***interval nominals***. Firstly, by adjoining a set of primitive contexts to frames, each assigned a time, one can introduce special nominals which behave like such locutions as 'now', 'today', and 'tomorrow'. The basic ideas are those of [11] and [12], except that these items are no longer operators but new sorts of names for times. It is straightforward to add further typical temporal referring expressions, such as dates, and these systems provide a clean model of the basic facts about temporal reference and its interaction with tense. Secondly, one can introduce interval nominals. The term is self-explanatory; these are atoms whose interpretation is constrained to be an interval, or time period. Note that unlike the strategy of ordinary interval logics, this gives us a device in our object language for talking about events that take time; for example we can refer to an interval and insist that an event took place throughout it. Now for present purposes the details of these adaptations aren't particularly important; what should be noted is the basic device used throughout. In each case we introduced a different sorts of atom into our object language whose interpretation was constrained in some fashion. Each sort of atom is the bearer of a different sort of referential information — we can 'read off' some information just by looking at its sort — and yet all this information is combined in a regular fashion by our usual connectives and operators.

It is tempting to try and carry this sorting over to the variables, the non-referential part of our languages, as well. Ever since (at least) the work of Vendler [27], such distinctions as states, processes, culminated processes and punctual events have been routinely invoked to explicate the semantics of temporal expressions. (This particular classification is due to Moens [16].) So why not attempt to build these distinctions into tense logic? Again we introduce different sorts of variable, each of which is constrained in its interpretation to subsets of frames which can model the type of information under consideration. The picture that emerges is of a temporal ontology that is neither a naked temporal flow (frames), nor a flow sloppily dressed (models), but a carefully groomed compromise that reflects our intuitions about event structure.

The idea of constraining the interpretation of variables in intensional languages is not new; it's the idea underlying general frames. What makes the idea interesting here is that we have differing restrictions, syntactically marked in our object languages. As we have seen with nominals, this gives rise to sublanguages with differing logical properties; the task of charting their behaviour seems worthwhile.

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