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MODELS FOR DISCOURSE MARKERS

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Models for discourse markers

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1. Introduction

Several theories have been proposed for the treatment of pronouns in discourse, the most well known are discourse representation theory (DRT, Kamp 1981) and dynamic Montague grammar (DMG, Groenendijk and Stokhof 1989, 1990). To a certain degree these approaches are the same. Some formal logical representation is constructed in a systematic way from a discourse. In this process each term introduces a discourse marker in the logical representation, and during the process of building the representation somehow the links between the different discourse markers are laid. The main difference is that Groenendijk and Stokhof work in the compositional Montagovian tradition, whereas Kamp presents a unorthodox approach in which representations are essential.

In this paper we will formalize certain properties of discourse markers by means of a formal restriction on models: 'the 'update postulate'. The main issue of this paper is the question whether a set-theoretical model can be constructed which satisfies this postulate. It seems reasonable to assume that the question arise in both in discourse representation theory and in dynamic Montague grammar. The difference between the two approaches becomes evident when the answers to the question are considered. The first model that we will construct turns out to be acceptable for only one of the theories. Then a second, more complex model will be build. Finally some general characteristics of models satisfying the postulate are presented.

2. The update postulate

Below we will make three observations concerning discourse markers. These observations will below be formalized as 'the update postulate'.

i) In the discourse

A man enters Mary's room. He smiles.

the *he* in the second sentence introduces a discourse marker, say dm_3. This discourse marker gets as interpretation the man who entered Mary's room. For every individual in the model it is possible think of a situation (of a possible world) in which he enters Mary's room. Hence, under suitable circumstances, any individual can be the value of dm_3. Generally formulated, a discourse marker of type τ, can have any object of type τ as value.

ii) Consider the discourse

A man enters Mary's room . He smiles. A woman enters Mary's room.
She smiles. She sees him.

In the first sentence a discourse marker is introduced for *a man*, and the discourse marker of the second sentence gets as interpretation this man. Third and fourth sentence sentence introduce new discourse markers. But the *him* in the last sentence is again the man who entered. The interpretation of the discourse markers referring to the man from are not changed by the process of interpreting the discourse markers in the third and fourth sentence. Generally formulated, the interpretation of a discourse marker, leaves the interpretation of all other discourse markers untouched.

iii) There are discourse markers of any type. For instance, the *she* in the second sentence of the discourse

Miss Universe is well known. She is elected every year.

does not say the one and the same individual is elected every year. It rather says that the property of being elected every year is a property of the individual concept *Miss Universe*. So the *she* introduces a discourse marker of type <s,e>. And the *it* in

John loves Mary. He likes it.

denotes the property of loving Mary . Hence the *it* introduces a discourse marker of type <s,<e,t>>.

The above observations give a motivation for

the update postulate

For each discourse marker dm_τ each object d of type τ , and each state s, there is a state s' such that

$[dm_\tau]_{s'} = d,$

whereas for all other discourse markers dm' holds

$[dm']_{s'} = [dm']_s.$

Although Kamp (1981) only deals with discourse markers of type e, the approach can in principle be extended with discourse markers of other types. Then the update postulate can be incorporated in that theory.

The update postulate originates from Janssen & Van Emde Boas 1977, where it is introduced for describing a phenomenon in the semantics of programming languages. The assignment statement is an instruction in programming languages that gives a value to an identifier. This value can be any value of the appropriate type, and other identifiers remain unchanged by such an assignment statement. So the update postulate formalizes the semantics effects of such assignments on identifiers. For more details, see Janssen & Van Emde Boas 1981 or Janssen 1986.

3. The problem

The combination of the update postulate with discourse markers of type $<s,\tau>$ gives rise to a fundamental problem. Let us consider the simplest case: type $<s,e>$. The update postulate would require that for discourse marker *dm*, each function $d \in D_{<s,e>}$ and each state $s \in S$ there is a state $s' \in S$ such that $[dm]_{s'} = d$. As a matter of fact, the number of functions in D_e^S is $|D_e|^{|S|}$, so this is the number of possible values for the right hand side of the equality $[dm]_{s'} = d$. For the left hand side this equals $|S|$. Elementary set theory learns us that $|S| < |D_e|^{|S|}$. For instance, if $|S| = 10$ and $|D_e| = 2$ then $|D_e|^{|S|} = 2^{10} = 1024$. So there are much more values than states. This also holds for the infinite case. If S is countable and $|D_e|$ is finite, then is $|D_e|^{|S|}$

uncountable. This means that neither in the finite case, nor in the infinite case there are enough states to have a state for each value in D_e^S .

This raises the question whether it is possible at all to satisfy the update postulate, and whether dynamic Montague grammar and discourse representation theory have models at all.

There is, however, hope for a positive answer. The update postulate requires that for each value in $D_{<s,e>}$ there is a state in which a given discourse marker has that value. Consequently, there have to be more states than elements in $D_{<s,e>}$, hence $|S| \leq |D_{<s,e>}|$. Since set theory learns us that $|S| < |D_e|^{|S|}$, we might conclude that $D_{<s,e>} \subset D_e^S$. This means that we have to work with a generalized model in the sense of Henkin 1950, see also Gallin 1975.

The idea to use a generalized model does not solve the fundamental problem, since we do not know yet whether a generalized model exists that satisfies the update postulate. It gives us a direction in which a solution might be found. And that is what will be done in the next sections.

The above discussion gives us the following heuristics . If we would take the set S as primitive, then the set D_e^S has hardly any structure. In such a situation it is difficult to indicate some subset as $D_{<s,e>}$. Therefore we will not take S as primitive, but build it from values for discourse markers.

The same problem as described above arises in the semantics of programming languages. Pointers are identifiers which have identifiers as values. Such pointers can be considered as functions of type <s, <s,e>> and the above above problem arises for assignments to pointers. The solution of Janssen & van Emde Boas 1977 can, however not be used in the situation considered here.

4. Presentation of a simple model

In this section we will consider a simple, but not ideal, model satisfying the update postulate, and in the next section a richer model. It is for two reasons useful to present it the simple model. Firstly, it is satisfactory as a model for discourse representation theory. Secondly, it is useful as a preparation for the richer model that will be presented in the next section.

The model formalizes the following two observations concerning discourse markers:

i.) The interpretation of constants (like *John* or *walk*) is independent of the interpretation of discourse markers. For instance, in the discourse

A man enters. He smiles.

the meaning of *smiles* is independent of the person referred to by the discourse marker introduced by *he*.

ii) The interpretation of a discourse marker is fully determined by the previous discourse. In particular, if a discourse marker is of the type of type **e**, then it is associated with some entity that is introduced in the previous discourse. Hence, given the previous discourse, the value associated with a discourse marker is a value from the ordinary model (i.e. the model for sentences with discourse markers). Below, two examples will illustrate this point. Consider the discourse

A man enters. He cries.

The discourse marker that corresponds with the *he* in the second sentence refers to an entity that is introduced by the term *A man* in the first sentence, and that entity would also arise in the standard Montague model for the sentence *A man enters* (i.e in the model without discourse markers).

The second example is

John loves Mary. It is a pleasant feeling

The *it* in the second sentence refers to a property (loving Mary) that arises in the standard model.

These observations lead us to the following construction. We start with the Montagovian model for sentences without discourse markers. Then we enrich the reference points (possible world with time index) with information concerning the values of discourse markers in that world. Finally we build a model for discourses in which we use these enriched possible worlds as states in our model for the discourse markers.

Following Montague 1973, we start with the following three primitive sets

E the set of basic entities

I the set of possible worlds

J the set of moments of time

From these sets we firstly build a standard Montague frame with Montague-Domains:

$MD_e = E$

$MD_t = \{1,0\}$

$MD_{<s,a>} = MD_a{}^{I \times J}$

$MD_{<a,b>} = MD_b{}^{MD_a}$

A model for Intensional Logic with Discourse Markers is defined as follows:

$$S = \Pi_\tau \Pi_{\mathbf{N}} (MD_\tau) \times I \times J$$

Hence an $s \in S$ is a triple (d,i,j), where

$$d = d_{\tau_1,1}, d_{\tau_1,2}, \ldots. d_{\tau_2,1}, d_{\tau_2,2}, \ldots d_{\tau_3,1}, d_{\tau_3,2} \ldots$$

So d is an infinite series of values: for each discourse marker there is a corresponding value of the appropriate type.

We are now prepared to define the domains D_τ.

$D_e = E$

$D_t = \{0,1\}$

$D_{<a,b>} = D_b{}^{D_a}$

$D_{<s,a>} \subset D_a{}^S$, more in particular it are those functions that are independent of the information aspect of the state. So it are functions that correspond with functions in the Montague Domains . Formally

$D_{<s,a>} =$

$\{f \mid \text{there is a } g \in MD_{<s,a>} \text{ such that for all } (w,d) \in S \text{ holds } f(w,d) = g(w)\}.$

The interpretation of discourse markers proceeds by means of the information component of the state, whereas the ordinary constants get Montague-like interpretations depending only on I and J (e.g. the interpretation of *walk* is not depending on the information aspect of the state).

We firstly introduce the (Montague - like) interpretation of ordinary constants, viz. a function MF: CON_τ x I x J --> D_τ . The function F interpreting the constantsand discourse markers in our model is then defined as follows:

F: CON_τ x S --> D_τ and F has the following two properties

1. for alle normal constants c_τ: $F(c_\tau, (d,i,j)) = MF(c_\tau, i,j)$
2. for all discourse markers dm_{τ_n, i_n}: $F(dm_{\tau_n, i_n}, (d,i,j))$ = the n-projection of d.

This model satisfies the update postulate: by changing the n-th component of the state to the given value, we obtain a new state in which discourse marker dm_{τ_n, i_n} gets that value, whereas all other constants have the same value as in the original state.

5 Discussion of the simple model

If the logical representation of a discourse is evaluated with respect to the simple model, then the desired results are obtained. The references for discourse markers are found, and the correct truth-values are assigned to the sentences of the discourse. The simple model yields correct outcomes for completed discourses and even for (abruptly) discontinued discourses.

However, this model is not sufficient for dynamic Montague grammar because of the principle of compositionality of meaning. That principle (generalized from sentences to discourses) states the *meaning* of a discourse is build from the *meanings* of the sentences of the discourse. This will be illustrated by an example. Consider the following discourse.

John enters. He smiles.

In dynamic Montague grammar it is required that the sentence *He smiles* has a meaning of its own, i.e. independently of the discourse in which it occurs. It must be possible to interpret the logical representation (with its discourse marker) even if the previous discourse is unknown. The meaning of *he smiles* will be some function from states to truth values. In particular it will be a function that varies from state to state, depending on the referent in such states of the discourse marker corresponding with *he*. Unfortunately such functions are not in the simple model. Therefore the simple model is unsuitable for dynamic Montague grammar.

The situation is different for discourse Representation theory . The interpretation of the presented discourse proceeds in that theory roughly as follows. For the discourse marker in the representation of the second sentence, a suitable discourse marker is to be found in the representation of the previous discourse. Only when these connections are laid, the model-theoretic interpretation can take place. The details of this process depend on the representation of the previous discourse. Therefore, the representation itself of the first sentence has to be available when interpreting the second sentence, and not its meaning (some abstract function). If the representation of the previous discourse is not available, then the interpretation of the last sentence cannot take place.

The above discussion explains that there is in discourse representation theory no need for assigning a meaning to the representation of a discourse sentence like *He smiles*. Hence no functions are needed which vary from state to state, and which depend on the referent in such states of the discourse marker corresponding with *he*. Therefore the simple model might be suitable for DRT whereas it is, due to compositionality, not suitable for DMG.

In discourse representation theory the logical *representation* of a discourse is build step by step from the *representations* for the sentences of the discourse. So one might consider this as a form of compositionality, viz. as compositionality of *representations*. But discourse representation theory does not aim at semantic compositionality. A model theoretic evaluation can only be performed on a final discourse representation. One might try to change DRT in such a way that it would obey the principle of compositionality of meaning. As a consequence the meaning of a discourse would then not be obtained from interaction of representations but from interaction of meanings. In such a situation the discourse representations would play the same role as the translations in Montague grammar, viz. figuring as representations of meanings, but playing no essential role. Then the representations could, in principle, be omitted (just as is the case with the IL-translations in Montague grammar). But in such a situation DRT would loose one of its essential features since it would not longer be a theory of discourse *representations*. As a matter of fact, dynamic Montague grammar started out as a reformulation of discourse representation theory that obeys the principle of semantic compositionality.

We may summarize this discussion as follows. Since DRT is a theory of representations, the simple model is a suitable model satisfying the update postulate. This model is not suitable for DMG because that theory, being a theory of meaning, aims at semantic compositionality.

6 A richer model

In this section a model will be presented that is richer than the model from the previous sections. Before doing so, we will make some observations concerning discourse markers that are of heuristic value.

Consider the following discourse

Mary enters. John loves her. He likes it.

This discourse expresses (in its most likely reading) that John likes loving Mary. The pronoun *her* in the second sentence introduces a discourse marker of entity type, say d_3 (which gets as interpretation Mary). The pronouns *he* and *it* in the third sentence introduce the discourse markers, say d_4 and d_5 of entity type and of property type respectively. Discourse marker d_4 is of course associated with *John*, and d_5 with the property expressed by *love her* in the previous sentence. So to d_5 is assigned, due to the interpretation of d_3 the property 'loving Mary'.

In this example we can make the following two observations.

i) In each stage of the discourse only a finite number of discourse markers is relevant. After the second sentence this number is one, after the last sentence three.

ii) Each of the relevant discourse markers is in finitely many steps connected with basic interpretations (i.e. interpretations which do not involve discourse markers). The d_5 is after two steps associated basic information (or after some larger number, depending on the precise definition of the notion 'step').

We can summarize the above by 'the amount of discourse information is finite, and the complexity of this information is finite'.

A model for dynamic montague grammar will be build in which the above observations are formalized. We will firstly introduce a series $DD_{\tau,m}$ of **D**omains **D**epending on the first m discourse markers. From these series we will make a generalized model in which only functions arise which depend on a finite number of discourse markers. The model will obey the update postulate, and allow for a compositional interpretation of the discourses we have considered, but in another respects it seems to be not completely satisfactory, see discussion at the end.

Step 1 <u>The series</u> $DD_{\tau,m}$

The first step in the construction of the model is the introduction of a a sequence of all discourse markers:

$dm_{\tau_1,i_1}, dm_{\tau_2,i_2}, \ldots, dm_{\tau_n,i_n}, \ldots$

Now we define for all m

$DD_{e,m} = E$ and $DD_{t,m} = \{0,1\}$

And by induction we define

$DD_{<a,b>,m+1} = \{f \in DD_{b,m}{}^{DD_{a,m}} : \text{f depends on } dm_{\tau_m,i_m}\}$

$DD_{<s,a>,m+1} = \{f \in DD_a{}^{\Pi_{1\leq n\leq m} DD_{\tau_n} \times I \times J} : \text{f depends on } dm_{\tau_m,i_m}\}.$

By the condition 'f depends on dm_{τ_m,i_m}' we require that the m-th argument is relevant for determining the value of f. In other words, that f varies with its m-th argument. This condition is not essential for the construction of the model, but it eliminates a lot of ambiguities. The formal version of this requirement is:

there are $d_1 \in DD_{\tau_1}, \ldots, d_{m-1} \in DD_{\tau_{m-1}}$ and $a_1, a_2 \in DD_{\tau_m}$ such that

$f(d_1, d_2, \ldots, d_{m-1}, a_1) \neq f(d_1, d_2, \ldots, d_{m-1}, a_2).$

Finally we define

$DD_\tau = \cup_m DD_{\tau_m}$

<u>Step 2 the model</u>

We define

$S = \Pi_{1\leq n} DD_{\tau_n} \times I \times J$, $D_e = E$, and $DD_t = \{0,1\}$.

$D_{<s,a>} = \{f \in D_a{}^S : \text{f depends only on the first m discourse markers}\}$

or, more formally

$D_{<s,a>} = \{f \in D_a{}^S : \exists\, m\ \exists\, g \in DD_{<s,a>,m} : \text{f is an extension of g to infinity tuples}\}$

We call f an extension of g to infinity tuples as arguments if for all

$d_{m+i} \in D_{\tau_{m+i}}$ holds that $f(<w,d_1,..d_m,d_{m+1}, ..,d_{m+i},...>) = g(<w,d_1,..d_m>)$.

$D_{<a,b>} =$

$\{f \in D_b{}^{D_a}$: the argument of f depends only on the first m discourse markers, and so does the value of f$\}$

or, more formally,

$\{f \in D_b{}^{D_a} : \exists\, m\ \exists\, g \in DD_{<a,b>,m}$: f is an extension of g to infinity tuples$\}$.

This extension has to take place (in argument or value of f) when there are subtypes of the form $<s,\tau>$.

Step 3 <u>The discourse markers</u>

As last step we will define the interpretation of discourse markers, and show that he model we satisfies the update postulate. The idea is that the interpretation of discourse marker dm_{τ_n,i_n} in state s is the n-th coordinate of the state. However, in case these coordinates are of an intensional or a functional type, then they are defined for a finite number of elements only. Therefore, the interpretation of a discourse marker is defined as the extension of such a function to an infinite tuple of arguments.

<u>Definition</u> $[dm_{\tau_n,i_n}]_s$ = n-th coordinate of s, extended to infinite tuples

Now the update postulate holds as will be explained below.

Let be given discourse marker dm_i, state s, and value $d \in D_{\tau_n}$. Then we find the state s' required by the update postulate as follows.

In case that τ_n=e or τ_n=t we change the n-th coordinate of s into d, and obtain in this way state s'. Consider next the case that $\tau_n = <s,a>$ for some type a. Then we know $d \in D_{<s,a>}$, so $\exists\, m\ \exists\, g \in DD_{<s,a>,m}$ such that d is an extension of g to infinity tuples. We obtain s' from s by changing the n-th coordinate into g. Finally we consider the case that $\tau_n = <a,b>$ for some types a and b. Then we know that $\exists\, m\ \exists\, g \in DD_{<a,b>,m}$: f is an extension of g to infinity tuples. Also in this case we change n-th coordinate into g.

Thus we have obtained a state s' in which the given discourse marker has the given value, and in which all other discourse markers have kept their original value. So the update postulate is satisfied.

This model is, however, in one respect not satisfactory . There is not a single identity function of type $\langle\langle s,\tau\rangle,\langle s,\tau\rangle\rangle$. According to the definition of $D_{\langle\langle s,\tau\rangle,\langle s,\tau,\rangle\rangle}$ we have an identity function that is defined for objects which depend on the first 5 discourse markers, one which depends on the first 6 discourse markers and so on. So there are an infinite number of identity functions of this type (and for certain other types as well), but not the one we would expect. I thank M. van den Berg for bringing this point to my attention.

7 General results.

It is striking to see that both models have the same structure; viz. cartesian product of values for discourse markers with the other parameters. In this section we will show that all models satisfying the update postulate are somehow of this nature.

<u>Definition</u>

Let DM be a set of discourse markers. Let M with S as set of states, be a model which satisfies the update postulate for all discourse markers in DM. By $\equiv_{DM}$ we understand the equivalence relation of having the same value for all discourse markers in DM.

<u>Theorem 1</u>

$S/_{\equiv\{dm_n\}}$ is isomorphic with D_{τ_n}.

<u>Proof</u>

Let f be a mapping from $S/_{\equiv dm_n}$ to D_{τ_n}. Then f is injective because states with the same value for dm_n are identified under the equivalence relation. The mapping is surjective due to the update postulate which requires that dm_n can take all values.

<u>End of Proof</u>

If we define $\equiv dm_{n1,n2}$ as having the same values for discourse markers dm_{n1} and dm_{n2}, then it is easy to see that

$S /_{\equiv dm_{n1,n2}}$ is isomorphic with $D_{\tau_{n1}} \times D_{\tau_{n2}}$

Theorem 2

Let DM be finite. Then is $S_{/\equiv DM}$ is isomorphic with $\Pi_{dm_n \in DM} D_{\tau_n}$.

Proof

Analogous to theorem 1.

Several special cases of this theorem are presented in Priatelj 1987.

Remark

The result of theorem 2 does not generalize to the case that DM is countably finite. Of course, the full product $\Pi_{dm_n \in DM} D_{\tau_n}$ is a correct model satisfying the update postulate. But also models are possible in which not the full product is used. This observation and the following model are due to P. van Emde Boas.

Let $s = (d_1, d_2, \ldots d_i \ldots)$ be an element of $\Pi_{dm_n \in DM} D_{\tau_n}$.

Define $S^{\sim}$ =

$$\bigcup_{D \text{ is a finite subset of DM}} \Pi_{dm_n \in D} (\underline{\text{if}}\ \tau_n \in D\ \underline{\text{then}}\ D_n\ \underline{\text{else}}\ \{d_n\})$$

All elements in $S^{\sim}$ have the property that they differ in finitely many coordinates from s. The update postulate is satisfied because it requires the change of one component, thus yielding another state that differs in only one coordinate from s.

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