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LOGICS FOR BELIEF DEPENDENCE

Zisheng Huang

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(Department of Mathematics and Computer Science)
Plantage Muidergracht 24
1018TV Amsterdam

Faculteit der Wijsbegeerte
(Department of Philosophy)
Nieuwe Doelenstraat 15
1012CP Amsterdam

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Zisheng Huang
Department of Mathemtics and Computer Science
University of Amsterdam

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Logics for Belief Dependence

Zhisheng Huang
Department of Mathematics and Computer Science
University of Amsterdam
1018TV Amsterdam, The Netherlands

Abstract

In this paper, we investigate the theoretical foundations of belief dependence in multiple agent environment, where agents may rely on someone else about their beliefs or knowledge. Several logics for belief dependence are introduced and studied. First of all, we try to formalize the problem of belief dependence in the framework of general epistemic logics, by which we will argue that general epistemic logic is not appropriate to formalize the problem of belief dependence. Then, based on an approach which is similar to Fagin and Halpern's general awareness logic, we present the second logic for belief dependence, which is called a syntactic approach. The third logic is an adapted possible world logic, where sub-beliefs are directly introduced in the models.

Contents

1 Introduction

One of the important topics of research in logics of computer science and artificial intelligence is to study the problem of reasoning about knowledge, especially, in multiple agent environment. Recently reasoning about knowledge in multiple agent environment has found many applications such as distributed knowledge-bases, communication and cooperation for multi agents planning[1,2,3,4,6,7,8,9,16,17,19].

However, little attention has been paid to study the problem of belief dependence in multiple agent environment, where agents may rely on someone else about their beliefs and knowledge. As is well known, in multiple agent environment, it is frequently beneficial to enable agents to communicate their knowledge or beliefs among agents, because these agents generally may have limited resources, or may lack computation capability for some specified problems or facts.

Although there have been attempts to study the problem of the communication of belief and knowledge among agents [7,8,9,19], the existing formalisms generally focus on the problem of communication, in which some main features about belief dependence, such as suspicion, indirect dependence, are rarely formalized. In this paper, we would like to develop a formal theory of belief dependence which serves as a foundation for understanding rational behaviour of artificial agents in multiple agent environment. Moreover, we expect that the proposed formalism would be expressive and natural enough to specify knowledge and belief passing and dependence among artificial agents, which can be found some applications in those relevant fields such as knowledge acquisition, machine learning, human-computer interaction, distributed artificial intelligence and distributed network systems.

In this paper, first of all, we would like to examine the problem of belief dependence in depth, and discuss main notions concerning belief dependence. Then, we will provide some syntactic considerations about the logics for belief dependence. Some axiom systems are introduced. Furthermore, we will study the semantics models of the logics. Several model approaches are provided and studies. The first approach is based on the general epistemic logic, because we would like to formalize the problem in the general epistemic logic framework. However, we will argue that the approach is not appropriate enough to formalize the problem. The second approach is based essentially on a syntactic approach, which is somewhat similar to Fagin and Halpern's general awareness logic. The third approach is an adapted possible world one, where sub-beliefs are directly introduced in the models. Finally, we will make summaries about those three approaches.

2 The Problem of Belief Dependence

2.1 Compartmentalized Information and Incorporated Information

Just like human beings, artificial agents (computers, knowledge bases, robots, and processes) get information from someone else, and then assimilate the information. From the viewpoint of reasoning, sometimes we call them agents' knowledge or beliefs. In the existing approaches to formalize the procedure of artificial agents' information assimilation, others' knowledge and beliefs are simply accepted or refused, which are handled with by different strategies. Because others' knowledge and beliefs are often contradict each other, many parts of the information may be refused. In order to solve the problem, a natural approach is to introduce the notion of probability-based beliefs, by which an agent may have contradict beliefs, because they can be indexed by different probabilities. However, as far as I know, there exist no strong psychological evidence which shows that it is necessary to use the notion of probability in human cognitive activities. An alternative approach is to introduce the notion of *society of minds*[5]. Formally, the notion of "society of minds" means that each agent possesses its own cluster of beliefs, which may contradict each other. Each cluster of beliefs is connected with each mind frame. However, if accepted information is simply separated in different mind frames, it is hard to say that agents can assimilate others' knowledge efficiently and can enlarge his knowledge and beliefs.

In the studies of incorporating new information into existing world knowledge of human beings, cognitive psychologists make a distinction between compartmentalized information and incorporated information. As Potts et al. point out in[18]:

> ...it is unlikely that subjects in most psychology experiments incorporate the new information they learn into their existing body of world knowledge. Though they certainly use their existing world knowledge to help comprehend the new material, the resulting amalgam of new information, and the existing world knowledge used to understand it, is isolated as a unit unto itself: it is *compartmentalized.*

We also believe that an appropriate procedure to assimilate others' knowledge and beliefs should pass the following two phases: compartmentalized information and incorporated information. Formally, *compartmentalized information* are those fragments of information which are accepted and remembered as isolated beliefs which are somewhat different from those beliefs are completely believed. Whereas *incorporated information* consists of those beliefs are completely believed by the agents.

2.2 Some Syntactic Considerations for Logics of Belief Dependence

There are some important and fundamental notions in logics for belief dependence. First of all, there is the general notion about knowledge and beliefs. Therefore, in our logics for belief dependence, general epistemic and doxastic operators are used to represent agents' knowledge and beliefs. For the sake of convenience, just like those in general epistemic logics, we use $L_i\varphi$ to represent that agent i knows or believes the formula φ. As is well known, L is interpreted as an epistemic operator, if the logic system is a **S5** system, whereas L is a doxastic operator if the system is a weak **S5** system.

In the existing epistemic logics, agents generally make no distinction among sources of those knowledge and beliefs. However, in real life, human beings seem not to be so naive. When peoples get information from outside, they generally keep in minds about the sources of information, They know from whom the information comes at the first phase of information assimilation, although they may finally forget these sources at all. Sometimes they even may make appraisal of agents who send the information to him. We call the phenomenon in which agents track sources of information *source indexing.*

In order to formalize the compartmentalized information and source indexing, in the logics of belief dependence, a natural strategy is to introduce a *compartment modal operator* $L_{i,j}$. Intuitively, we can give $L_{i,j}\varphi$ an interpretation: "agent i believes φ due to agent j". From the point of view of minds society, $L_{i,j}\varphi$ can be more intuitively interpreated as "agent i believes φ on the mind frame indexed j". Sometimes we call $L_{i,j}\varphi$ agent i's *sub-belief*, and $L_{i,j}$ is called *sub-belief operator.* $L_{i,i}\varphi$ naturally means that agent i believes φ, which semantically corresponds to the modal operator for knowledge and beliefs in general epistemic logics. Sometimes we use $L_i\varphi$ as an abbreviation of $L_{i,i}\varphi$.

Both sub-beliefs and general beliefs have close relationships with the truth and falsity of beliefs. Sometimes we need a neutral[1] modal operator $D_{i,j}$ for belief dependence logics. $D_{i,j}$ is called *dependent operator*, or alternatively *rely-on operator.* Intuitively, we can give $D_{i,j}\varphi$ a number of interpretations:"agent i relies on agent j about the formula φ", "agent i depends on agent j about believing φ", "agent j is the credible advisor of agent i about φ", even specially in distributed process networks, "processor i can obtain the knowledge about φ from processor j". Independently of this part of the interpretation, which reflects two natures of the agents and their interaction, the dependent operator can be understood in two different ways regarding its epistemic status. One possibility is *explicit dependence*, which says that belief dependence is explicitly known by believers. In other words, that means the axiom $D_{i,j}\varphi \rightarrow L_iD_{i,j}\varphi$ holds. The other one is implicit dependence, in which believers do not necessarily know their dependence.

However, it should be noted that $L_{i,i}\varphi$ is not necessarily equal to $L_i\varphi$. As is well known, general epistemic logics suffer from the problem of logical omniscience. The so-called *logical omniscience* means that agents are assumed to be intelligent that they must know all valid formulas, and that their knowledge is closed under implication, so that if an agent knows p, and that p implies q, then the agent must also know q. However, in computer science, even in real life, agents are not such ideal reasoners. In order to provide a more realistic representation of human reasoning, there are various attempts to deal with the problem of logical omniscience[6,12,13,17]. In [17], Levesque first presents the notions of *explicit belief* and *implicit belief.* Explicit beliefs are those beliefs an agent actually has, whereas implicit beliefs consist of all of the logical consequences of an agent's explicit beliefs. In [6], Fagin and Halpern point out that 'lack of awareness" is one of sources of logical omniscience. They argue that one cannot say he knows p or does not know p if p is a concept he is completely unaware of. In order to solve the problem of awareness, Fagin and Halpern offer a solution in which one can decide on a metalevel what formulas an agent is supposed to be aware of. In their general awareness logic, implicit beliefs are represented as $L_i\varphi$, whereas explicit beliefs are defined as $L_i\varphi \wedge A_i\varphi$, where $A_i\varphi$ means that agent i is aware of φ. In [12], we argue that the notion of belief dependence can be viewed as an intuitive

[1] Because we consider the axiom $D_{i,j}\varphi \equiv D_{i,j}\neg\varphi$ as a fundamental axiom about $D_{i,j}$

extension to the notion of awareness, since we can define $A_i\varphi \equiv \exists j D_{i,j}\varphi$. This means that agent i is aware of φ if and only if agent i believes in himself about φ or agent i could get the truth of the formula φ by consulting his adviser about φ. Moreover, at least, we can define $A_i\varphi \equiv D_{i,i}\varphi$, therefore, $L_{i,i}\varphi$ is not necessarily equal to $L_i\varphi$. From the point of view of explicit beliefs and implicit beliefs, $L_i\varphi$ can be interpret as implicit belief, whereas $L_{i,i}\varphi$ can be interpret as explicit beliefs if we define $L_{i,i}\varphi \equiv D_{i,i}\varphi \wedge L_i\varphi$.

Supposed we have a set $\mathbf{A_n}$ of n agents, and a set Ψ_0 of primitive propositions, the language $\mathbf{L}$ for belief dependence logics is the minimal set of formulas closed the following syntactic rules:

(i) $\mathbf{true} \in \mathbf{L}$

(ii) $p \in \Psi_0 \Rightarrow p \in \mathbf{L}$

(iii) $\varphi \in \mathbf{L}, \psi \in \mathbf{L} \Rightarrow \varphi \wedge \psi \in \mathbf{L}$,

(iv) $\varphi \in \mathbf{L} \Rightarrow \neg\varphi \in \mathbf{L}$,

(v) $\varphi \in \mathbf{L}, i \in \mathbf{A_n} \Rightarrow L_i\varphi \in \mathbf{L}$

(vi) $\varphi \in \mathbf{L}, i, j \in \mathbf{A_n} \Rightarrow L_{i,j}\varphi \in \mathbf{L}$

(vii) $\varphi \in \mathbf{L}, i, j \in \mathbf{A_n} \Rightarrow D_{i,j}\varphi \in \mathbf{L}$

Logical connectives such as $\rightarrow$ and $\vee$ are defined in terms of $\neg$ and $\wedge$ as usual, and **false** is an abbreviation of $\neg$**true**.

In some special belief dependence logics, among the three belief dependence modal operators, some may be defined by others. For example, the sub-belief modal operator can be defined by the general epistemic operator and the dependent operator, i.e. $L_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j}\varphi \wedge L_j\varphi$, if we suppose that the communications between agents are reliable, and every teller is honest. Moreover, sometimes we may view the general epistemic operator as a kind of special sub-epistemic operator, i.e. $L_i\varphi \stackrel{\text{def}}{=} L_{i,i}\varphi$. Therefore, sometimes we need some sub-language for belief dependence logics. We define the language $\mathbf{L_D}$ as the minimal set of formulas closed by the syntactic rules (i),(ii),(iii),(iv),(v), and (vii). Moreover, the language $\mathbf{L_L}$ is defined by the rules (i),(ii),(iii),(iv),(v) and the language $\mathbf{L_{Lij}}$ is defined by the rules (i),(ii),(iii),(iv), and (vi).

2.3 General Scenario

We have argued that an appropriate procedure for formalizing information assimilation should pass two phases: compartmentalized and incorporated information. In the logics for belief dependence, compartmentalized information corresponds to sub-beliefs $L_{i,j}\varphi$ for agent i. Whereas incorporated information corresponds to general beliefs of agent i, namely, $L_i\varphi$.

For multiple agent environment, we assume that some primitive rely-on relations about some propositions among those agents can be decided on the metalevel. We call the assumption *initial role-knowledge assumption.* We believe that the assumption is appropriate and intuitive. That is because, in multiple agent environment, some agents have to possess some minimal knowledge about someone else, in order to guarantee their communications. In many application situations, primitive rely-on relations are easy to be modeled, because primitive rely-on relations have no relationship with the process how the agents solve the conflicts between their own beliefs and new information. In other words, in a reliable communication network, assuming that agents are honest, no-doubt and something more,

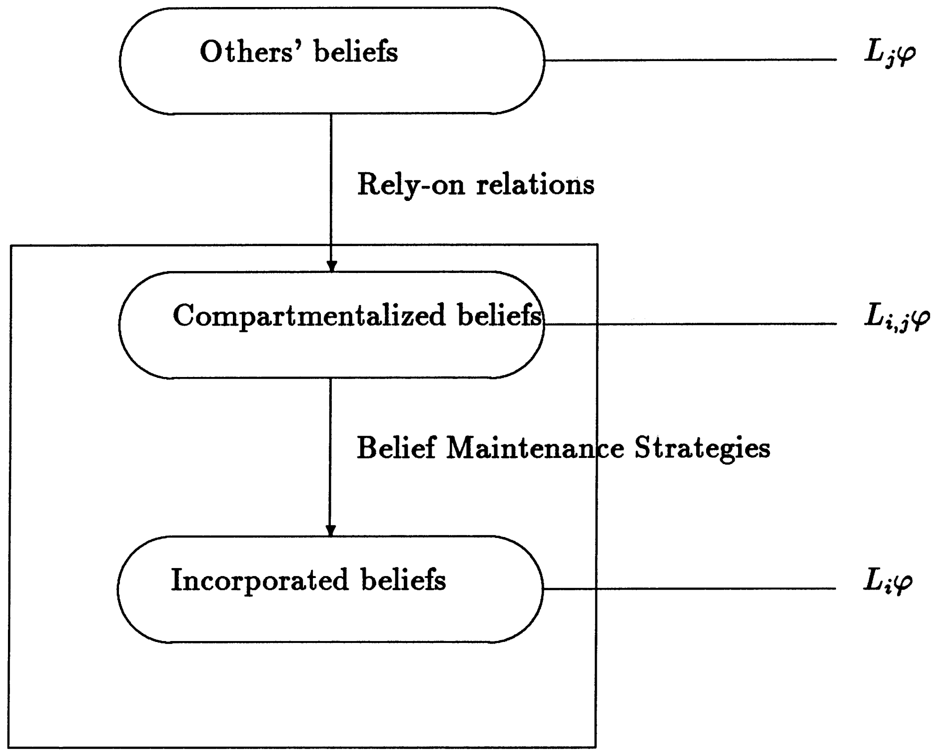

Figure 1: General Scenario

primitive rely-on relations often collapse into primitive communication relations, which turns them into observable entities.

Therefore, based on the primitive rely-on relations, we can capture a complete knowledge about agents' sub-beliefs by using the logics for belief dependence. Furthermore, based on the complete information concerning agents' sub-beliefs, we can figure out some agents' appraisal information about others. In the next section we will propose some role-appraisal axioms such as "fool believer", and "stubborn believer". Based on these role-appraisal information, it is possible to determine some rational belief maintenance strategies, by which we can figure out whether and how compartmentalized beliefs can be assimilated into the incorporated beliefs for some agents. However, in this paper, we would like to focus on the formalism concerning the first phase of information assimilation. That is, we will focus on the problem how the complete sub-belief and the complete rely-on relations can be captured, basing on the primitive rely-on relations. As far as the second phase of information assimilation is concerned, we will discuss the problem in the further papers[14]. The general scenario about the formalism of belief dependence is shown in the figure.

3 Formalizing Belief Dependence

3.1 Belief Dependence Systems Based on Epistemic Operator and Dependent Operator

In this subsection, first of all, we would like to present a belief dependence axiom system, basing on general epistemic operator and dependent operator. Naturally, weak-S5 system remains to be the subsystem of belief dependence system. Here is a logic system for belief dependence, which is called **L5$^-$+D4** system:

Axioms:

(L1) All instances of propositional tautologies.
(L2) $L_i\varphi \wedge L_i(\varphi \rightarrow \psi) \rightarrow L_i\psi$.
(L3) $\neg L_i$**false**.
(L4) $L_i\varphi \rightarrow L_iL_i\varphi$.
(L5) $\neg L_i\varphi \rightarrow L_i\neg L_i\varphi$.

The axioms above consist of a weak-S5 modal logic system. Moreover, we select the following axioms as axioms about dependent operator:

(D1) $D_{i,j}\varphi \equiv D_{i,j}\neg\varphi$.
(Neutral axiom. Rely on someone else about φ iff rely on about the negation of φ. It seems to be the most fundamental axiom for dependent operator.)
(D2) $D_{i,j}\varphi \wedge D_{i,j}(\varphi \rightarrow \psi) \rightarrow D_{i,j}\varphi$.
(Closure under implication, for dependent operator, closing under implication is intuitive.)
(D3) $D_{i,j}\varphi \wedge D_{i,j}\psi \rightarrow D_{i,j}(\varphi \wedge \psi)$.
(Closure under conjunction. Because we index sub-beliefs simply by agent name, this requires that beliefs which come from the same agent should be consistent. Therefore, we consider the axiom a reasonable one.)
(D4) $D_{i,j}\varphi \rightarrow L_iD_{i,j}\varphi$.
(Positive explicit dependent axiom. As it is argued above, the axiom means that dependency is explicitly known by believer.)

Rules of Inference:

(R1) $\vdash \varphi, \vdash \varphi \rightarrow \psi \Rightarrow \vdash \psi$.
(R2) $\vdash \varphi \Rightarrow \vdash L_i\varphi$.

So far we have not present any axiom concerning sun-belief operator $L_{i,j}$ in the logic system **L5$^-$+D4**. If we suppose that the communications in the system are reliable and every agent is honest, then a plausible definition about the sub-belief operator can be represented as follows:

Definitions:

(Lijdf) $L_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$.

3.2 Belief Dependence System Based on Sub-belief Operator

Based on the sub-belief operator, we also can present logic systems for belief dependence. The following axiom system is called to be a **Lij5$^-$+D** belief dependence logic system:

Axioms

(L1) All instances of propositional tautologies.
(Lij2) $L_{i,j}\varphi \wedge L_{i,j}(\varphi \rightarrow \psi) \rightarrow L_{i,j}\psi$.
(Just like those in general epistemic logics, sub-beliefs are closed under logical implication.)

(Lij3) $\neg L_{i,j}\mathbf{false}$.
(This axiom means that an agent never believe the false fact from someone else, including himself.)
(Lij4) $L_{i,j}\varphi \rightarrow L_i L_{i,j}\varphi$.
(Positive introspective axiom for sub-beliefs.)
(Lij5) $\neg L_{i,j}\varphi \rightarrow L_i \neg L_{i,j}\varphi$.
(Negative introspective axiom for sub-beliefs.)

Rules
(R1) $\vdash \varphi, \vdash \varphi \rightarrow \psi \Rightarrow \vdash \psi$.
(RLij) $\vdash \varphi \rightarrow \vdash L_{i,j}\varphi$.
(Remark that there is an alternative (RLii), namely, $\vdash \varphi \rightarrow \vdash L_{i,i}\varphi$, which is plausible but does not lead to the class of models treated further down in the paper.)
Definitions
(Ddf) $D_{i,j}\varphi \stackrel{\text{def}}{=} L_{i,j}\varphi \vee L_{i,j}\neg\varphi$.
(If agent i believes φ or believes $\neg\varphi$ from agent j, then this means that agent i rely on agent j about φ.)
(Ldf) $L_i\varphi \stackrel{\text{def}}{=} L_{i,i}\varphi$.
(This definition means that we make no distinction between implicit beliefs and explicit beliefs.)

Theorem 3.1 *The logic system* $\mathbf{L5^-{+}D4}$ *is a subsystem of the logic system* $\mathbf{Lij5^-{+}D}$. *That is,* $\mathbf{L5^-{+}D4} \subset \mathbf{Lij5^-{+}D}$.

Proof:
(a) The axioms concerning the modal operator L_i, namely, axiom(L2)-(L5), directly come from their corresponding axioms in the logic system $\mathbf{Lij5^-{+}D}$, because the modal operator $L_{i,j}$ subsumes the modal operator $L_{i,i}$, and $L_{i,i}$ is equal to the modal operator L_i by the definition (Ldf).
(b) According to the definition (Ddf), neutral axiom (D1), namely,
$D_{i,j}\varphi \equiv D_{i,j}\neg\varphi$, is evident, because $L_{i,j}$ is closed under logical equivalence, namely, $L_{i,j}\varphi \wedge (\varphi \equiv \psi) \Rightarrow L_{i,j}\psi$.
(c) Closure under implication, namely,
(D2) $D_{i,j}\varphi \wedge D_{i,j}(\varphi \rightarrow \psi) \rightarrow D_{i,j}\psi$.
$D_{i,j}\varphi \wedge D_{i,j}(\varphi \rightarrow \psi)$
$\Rightarrow (L_{i,j}\varphi \vee L_{i,j}\neg\varphi) \wedge (L_{i,j}(\varphi \rightarrow \psi) \vee L_{i,j}\neg(\varphi \rightarrow \psi))$
$\Rightarrow L_{i,j}(\varphi \wedge (\varphi \rightarrow \psi)) \vee L_{i,j}(\varphi \wedge \neg(\varphi \rightarrow \psi)) \vee L_{i,j}\neg\varphi \wedge L_{i,j}(\varphi \rightarrow \psi) \vee L_{i,j}(\neg\varphi \wedge \neg(\varphi \rightarrow \psi))$
$\Rightarrow (L_{i,j}\varphi \wedge L_{i,j}\psi) \vee (L_{i,j}\varphi \wedge L_{i,j}\neg\psi) \vee (L_{i,j}\neg\varphi \wedge L_{i,j}\neg\psi)$
$\Rightarrow L_{i,j}\psi \vee L_{i,j}\neg\psi$
$\Rightarrow D_{i,j}\psi$.
(d) Closure under conjunction, namely,
(D3) $D_{i,j}\varphi \wedge D_{i,j}\psi \rightarrow D_{i,j}(\varphi \wedge \psi)$.
$D_{i,j}\varphi \wedge D_{i,j}\psi$
$\Rightarrow (L_{i,j}\varphi \vee L_{i,j}\neg\varphi) \wedge (L_{i,j}\psi \vee L_{i,j}\neg\psi)$
$\Rightarrow L_{i,j}(\varphi \wedge \psi) \vee L_{i,j}(\neg\varphi \wedge \psi) \vee L_{i,j}(\varphi \wedge \neg\psi) \vee L_{i,j}(\neg\varphi \wedge \neg\psi)$
$\Rightarrow L_{i,j}(\varphi \wedge \psi) \vee L_{i,j}\neg(\varphi \wedge \psi)$

(Because $L_{i,j}(\neg\varphi \wedge \psi) \rightarrow L_{i,j}\neg(\varphi \wedge \psi)$, $L_{i,j}(\varphi \wedge \neg\psi) \rightarrow L_{i,j}\neg(\varphi \wedge \psi)$, and$L_{i,j}(\neg\varphi \wedge \neg\psi) \rightarrow L_{i,j}\neg(\varphi \wedge \psi)$)
$\Rightarrow D_{i,j}(\varphi \wedge \psi)$.
(e) Positive explicit dependence, namely,
(D4) $D_{i,j}\varphi \rightarrow L_i D_{i,j}\varphi$.
$D_{i,j}\varphi \Rightarrow L_{i,j}\varphi \vee L_{i,j}\neg\varphi$
$\Rightarrow L_{i,i}L_{i,j}\varphi \vee L_{i,i}L_{i,j}\neg\varphi$
$\Rightarrow L_{i,i}(L_{i,j}\varphi \vee L_{i,j}\neg\varphi)$
$\Rightarrow L_i D_{i,j}\varphi$
(f) Rules of Inference,
The rule (RLij) implies the rule (R2) because of the definition (Ldf)[2]. $\square$

3.3 Formalizing Suspicion and Other Features

Based on the three modal operators concerning belief dependence, namely, the general epistemic operator L_i, the sub-belief operator $L_{i,j}$, and the dependent operator $D_{i,j}$, we can formalize many important and interesting features about belief dependence. The following axioms can be some candidates for formalizing belief dependence.
(a) No-doubt Axiom
$L_{i,j}\varphi \rightarrow L_i L_j \varphi$.
(*Whatever come from someone else is believed to be true.*)
We know that $L_{i,j}\varphi$ is not necessarily equal to $L_i L_j \varphi$. However, in the no-doubt belief dependence system, the sub-belief $L_{i,j}\varphi$ implies $L_i L_j \varphi$.
(b) Honesty Axiom
$L_{i,j}\varphi \rightarrow L_j \varphi$.
(*Sub-beliefs are actually teller's beliefs.*)
Therefore, if we select the definition $L_{i,j}\varphi \stackrel{def}{=} D_{i,j}\varphi \wedge L_j\varphi$, then this means that in the system every agent is honest.
(c) Negative Explicit Dependent Axiom
$\neg D_{i,j}\varphi \rightarrow L_i \neg D_{i,j}\varphi$.
(*If agent i does not rely on agent j about φ, then agent i will know the fact.*)
(d) Consultation Axiom
$D_{i,j}\varphi \rightarrow L_j D_{i,j}\varphi$.
(*Agent i asks for the information about φ from agent j, and believes what is told. Therefore, agent j knows his relied on.*)
(e) Confidence Axiom
$L_i\varphi \wedge D_{i,j}\varphi \rightarrow L_i L_j \varphi$.
(*Agent i believes his dependent beliefs are actually true.*)
(f) Fool Believer Axiom
$L_i\varphi \rightarrow \exists j L_{i,j}\varphi \ \ (j \neq i)$[3].
(*All of his beliefs come from someone else.*)
(g) Stubborn Believer Axiom
$L_{i,j}\varphi \rightarrow L_i\varphi$.

[2] Note that the rule (RLii) suffices for this inference.

[3] Although we do not introduce any quantifer and equality in the language $\mathbf{L}$, however, because we generally consider a finite agent set, say $\mathbf{A_n} = \{i_1, ..., i_n\}$, the formula $\exists j L_{i_1,j}\varphi \ (j \neq i_l)$ can be viewed to be an abbreviation for the formula $L_{i_1,i_1}\varphi \vee ... \vee L_{i_1,i_{l-1}}\varphi \vee L_{i_1,i_{l+1}}\varphi \vee ... \vee L_{i_1,i_n}\varphi$

(*He never believes those come from someone else.*)
(h) Communicative Agent Axiom
$L_i\varphi \rightarrow \exists j L_{j,i}\varphi \quad (j \neq i)$.
(*All of his beliefs are believed by someone else.*)
(i) Cautious Believer Axiom
$L_{i,j}\varphi \rightarrow \exists k L_{i,k}\varphi (k \neq j)$.
(*He believes those which is believed by more than two agents.*)

Moreover, based on those operators, we can formalize the notion of suspicion as follows:
$\text{Suspect}_i\varphi \stackrel{def}{=} (\exists j)(L_{i,j}\varphi \wedge \neg L_i L_j \varphi)$.
(Agent i suspects φ if and only if there exists some agent j such that agent i believes φ from j, but agent i does not believe that agent j believes φ.)

Propositions 3.1 *For the system* **Lij5$^-$+D***:*
(a)$Suspect_i\varphi \rightarrow L_i Suspect_i\varphi$.
(If agent i suspects φ, then he can know his suspicion.)
(b)$\neg Suspect_i\varphi \rightarrow L_i\neg Suspect_i\varphi$.
(If agent i does not suspect φ, then he knows that fact.)

Proof:(a)$\text{Suspect}_i\varphi \equiv (\exists j)(L_{i,j}\varphi \wedge \neg L_i L_j\varphi)$
$\Rightarrow L_{i,j}\varphi \wedge \neg L_i L_j\varphi \Rightarrow L_i L_{i,j}\varphi \wedge L_i \neg L_i L_j\varphi$
$\Rightarrow L_i(L_{i,j}\varphi \wedge \neg L_i L_j\varphi) \Rightarrow L_i\text{Suspect}_i\varphi$.
(b)$\neg\text{Suspect}_i\varphi \equiv (\forall j)(\neg L_{i,j}\varphi \vee L_i L_j\varphi)$
$\Rightarrow (\forall j)(L_i\neg L_{i,j}\varphi \vee L_i L_i L_j\varphi) \Rightarrow L_i((\forall j)(\neg L_{i,j}\varphi \vee L_i L_j\varphi))$
$\Rightarrow L_i\neg\text{Suspect}_i\varphi$.

3.4 Formalizing Indirect Dependence

In multiple agent environment, knowledge and beliefs may be transitive among agents. Therefore, we would like to extend the definition of dependent beliefs into indirect dependent beliefs as follows:
We define that $D^+_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j_1}\varphi \wedge D_{j_1,j_2}\varphi \wedge ... \wedge D_{j_m,j}\varphi, \ (i \neq j_1)$, and
$D^*_{i,j}\varphi \stackrel{\text{def}}{=} D^+_{i,j}\varphi \vee D_{i,j}\varphi$.
We have the following propositions:

Propositions 3.2 *(Transitivity of Indirect Dependence)*
(a) $D^*_{i,j}\varphi \wedge D^*_{j,k}\varphi \rightarrow D^*_{i,k}\varphi$.
(b) $D^+_{i,j}\varphi \wedge D^+_{j,k}\varphi \rightarrow D^+_{i,k}\varphi$.
More generally, we have:
(c) for any $x, y, z \in \{*, +\}$ $(i \neq j), (j \neq k)$,
$D^x_{i,j}\varphi \wedge D^y_{j,k}\varphi \rightarrow D^z_{i,k}\varphi$.

We also would like to define indirect sub-beliefs as follows:
$L^1_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j}\varphi \wedge L_j\varphi$
$L^m_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j'}\varphi \wedge L^{m-1}_{j',j}\varphi$.
$L^*_{i,j}\varphi \stackrel{\text{def}}{=} [\vee^n_{k=1}]L^k_{i,j}\varphi$.

From the definitions above, we can easily show the following propositions:

Propositions 3.3
(a) Coincidence
$L^*_{i,j}\varphi \equiv D^*_{i,j}\varphi \wedge L_j\varphi$.
(b) Consistence
$L^*_{i,j}\varphi \rightarrow \neg L^*_{i,j}\neg\varphi$.
Proof $L^*_{i,j}\varphi \Rightarrow D^*_{i,j}\varphi \wedge L_j\varphi$
$\Rightarrow L_j\varphi \Rightarrow \neg L_j\neg\varphi$
$\Rightarrow \neg L_j\neg\varphi \vee \neg D^*_{i,j}\neg\varphi \Rightarrow \neg L^*_{i,j}\neg\varphi$.
(c) Same-source-propagation
$D^*_{i,k}\varphi \wedge L^*_{j,k}\varphi \rightarrow L^*_{i,k}\varphi$.
(d) Strong-consistence
$L^*_{i,j}\neg\varphi \rightarrow (\forall k)(\neg L^*_{k,j}\varphi)$.
(e) No-same-source-assertion
$L^*_{i,j}\varphi \wedge \neg L^*_{k,j}\varphi \rightarrow \neg D^*_{k,j}\varphi$.
Proof $L^*_{i,j}\varphi \wedge \neg L^*_{k,j}\varphi \Rightarrow D^*_{i,j}\varphi \wedge L_j\varphi \wedge (\neg D^*_{k,j}\varphi \vee \neg L_j\varphi)$
$\Rightarrow D^*_{i,j}\varphi \wedge L_j\varphi \wedge \neg D^*_{k,j}\varphi \Rightarrow \neg D^*_{k,j}\varphi$.

4 Semantics Models of Belief Dependence

4.1 L-Model of Belief Dependence: An Approach Based on General Epistemic Logic

In this section, we try to define the dependent operator by general doxastic and epistemic operator, by which we can study the problem of belief dependence in the general epistemic logics framework. $D_{i,j}\varphi$ means that agent i relies on agent j about believing φ . Formally, there might exist many different interpretations about the dependent operator. In other words, there are many semantically interpretations about the meaning of "rely on". Here are some of definitions:
(Ddf1) $D_{i,j}\varphi \stackrel{\text{def}}{=} (L_j\varphi \rightarrow L_i\varphi) \wedge (L_j\neg\varphi \rightarrow L_i\neg\varphi)$.
(If agent j believes φ, so does agent i; if agent j believes φ is false, agent i believes φ is false as well.)
(Ddf1') $D_{i,j}\varphi \stackrel{\text{def}}{=} (L_j\varphi \equiv L_i\varphi)$.
(If agent j believes φ, so does agent i; if agent j does not believe φ, neither does agent i)
(Ddf2) $D_{i,j}\varphi \stackrel{\text{def}}{=} L_i(L_j\varphi \rightarrow L_i\varphi) \wedge L_i(L_j\neg\varphi \rightarrow L_i\neg\varphi)$.
(Agent i believes that if agent j believes φ, then so does agent i, agent j believes its false, so does agent i.)
(Ddf2') $D_{i,j}\varphi \stackrel{\text{def}}{=} L_i(L_j\varphi \equiv L_i\varphi)$.
(Agent i believes that agent j believes φ iff agent i believes φ).
(Ddf3) $D_{i,j}\varphi \stackrel{\text{def}}{=} (L_iL_j\varphi \rightarrow L_i\varphi) \wedge (L_iL_j\neg\varphi \rightarrow L_i\neg\varphi)$.
(If agent i believes that agent j believes φ, then agent i will believe it; if agent i believes agent j believes φ is false, then agent i will also believe that φ is false.)

Of those definitions, (Ddf2) and (Ddf2') are the definitions of explicit dependence, because they say that agent i believes the dependent relation. Whereas

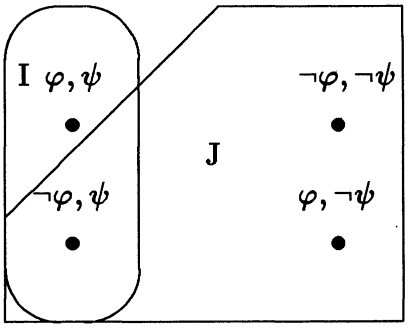

Figure 2: Satisfibility for (Formula 2)

other definitions are implicit. Moreover, (Ddf1) seems to be a simple one, but it is completely implicit. (Ddf3) can be viewed as a semi-implicit one since agent i's dependent beliefs depend on parts of its own beliefs. (Ddf1') is a symmetric definition. However, dependent relations are not intuitively symmetric. Although (Ddf2') is not symmetric, "≡" still make the definition is too strong. Therefore, we view the definitions (Ddf1), (Ddf2), and (Ddf3) are more reasonable and acceptable.

For those three definitions (Ddf1), (Ddf2), and (Ddf3), we know that the neutral axiom (D1), namely, $D_{i,j}\varphi \equiv D_{i,j}\neg\varphi$, holds in any epistemic logics systems. Moreover, we naturally expect that the closure under conjunction axiom will hold for those definitions. Unfortunely, we have the following result.

Claim 4.1 *In any possible world semantic model for the epistemic operator L_i, $D_{i,j}\varphi \wedge D_{i,j}\psi \wedge \neg D_{i,j}(\varphi \wedge \psi)$ is satisfiable if $D_{i,j}\varphi$ is defined by (Ddf1), (Ddf2), or (Ddf3).*

Proof: For the definition (Ddf1),
$D_{i,j}\varphi \wedge D_{i,j}\psi \wedge \neg D_{i,j}(\varphi \wedge \psi)$
$\equiv (L_j\varphi \rightarrow L_i\varphi) \wedge (L_j\neg\varphi \rightarrow L_i\neg\varphi) \wedge (L_j\psi \rightarrow L_i\psi) \wedge (L_j\neg\psi \rightarrow L_i\neg\psi)$
$\wedge \neg((L_j(\varphi \wedge \psi) \rightarrow L_i(\varphi \wedge \psi)) \wedge (L_j\neg(\varphi \wedge \psi) \rightarrow L_i\neg(\varphi \wedge \psi)))$
$\equiv (L_j\varphi \rightarrow L_i\varphi) \wedge (L_j\neg\varphi \rightarrow L_i\neg\varphi) \wedge (L_j\psi \rightarrow L_i\psi) \wedge (L_j\neg\varphi \rightarrow L_i\neg\psi)$
$\wedge ((L_j(\varphi \wedge \psi) \wedge \neg L_i(\varphi \wedge \psi) \vee L_j(\neg\varphi \vee \neg\psi) \wedge \neg L_i(\neg\varphi \vee \neg\psi))$ (Formula 1)
Moreover, let (Formula 2) be the formula $\neg L_j\varphi \wedge \neg L_j\neg\varphi \wedge \neg L_j\psi \wedge \neg L_j\neg\psi \wedge \neg L_i(\neg\varphi \vee \neg\psi) \wedge L_j(\neg\varphi \vee \neg\psi)$.
We know that if (Formula 2) is satisfiable, then so is (Formula 1), because we have:
$\neg L_j\varphi \Rightarrow (L_j\varphi \rightarrow L_i\varphi)$
$\neg L_j\neg\varphi \Rightarrow (L_j\neg\varphi \rightarrow L_i\neg\varphi)$
$\neg L_j\psi \Rightarrow L_j\psi \rightarrow L_i\psi$
$\neg L_j\neg\psi \Rightarrow L_j\neg\psi \rightarrow L_i\psi$
$L_j(\neg\varphi \vee \neg\psi) \wedge \neg L_i(\neg\varphi \vee \neg\psi) \Rightarrow L_j(\neg\varphi \vee \neg\psi) \wedge \neg L_i(\neg\varphi \vee \neg\psi)$.
It is easy to show that (Formula 2) is satisfiable. One of the cases is shown in the figure. The cases of (Ddf2) and (Ddf3) can be similarly shown. □

From the above argument, we know that general epistemic logics are not an appropriate mean to formalize the problem of belief dependence, since some intuitive properties such closure under conjunction cannot be formalized efficiently. However, in order to make a comparison with the other semantic models which would be studies in the next subsections, we would like to include the semantic model of epistemic logic as a kind of model for belief dependence, although it is a weak one, which does not explicitly represent the belief dependence at all. For the sake of notation consistency, we therefore have the following definition:

Definition 4.1 *(Belief Dependence L-model)*
A belief dependence L-model is a tuple $M = (S, \pi, \mathcal{L})$
where S is a set of states, $\pi(s,.)$ is a truth assignment for each state $s \in S$, and $\mathcal{L} : \mathbf{A_n} \to 2^{S\times S}$, which consists of n binary accessibility relations on S.

4.2 D-Model of Belief Dependence: A Syntactic Approach

We have known that sub-beliefs can be defined directly from the dependent operator and the general epistemic operator, namely, $L_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$. Therefore, to formulate belief dependence, naturally, an approach is to add dependent structure to general Kripke model of epistemic logics. The approach is similar to Fagin and Halpern's general awareness logic[6]. The general idea is that one can decide on a metalevel what formulas each agent is supposed to rely on others. By this approach, what we can do is to introduce dependent formula sets for each agent and each state, namely, formula sets $\mathcal{D}(i,j,s)$. The formula $\varphi \in \mathcal{D}(i,j,s)$ means that agent i relies on agent j about the formula φ. Therefore, we call it a syntactic approach.

In [12], we have presented a syntactic approach about modelling of belief dependence. However, in [12], the added dependent structure is a dependent function $\mathcal{D}_i : \mathbf{L_D} \times S \to \mathbf{An} \cup \{\lambda\}$. $\mathcal{D}_i(\varphi, s) = j$ means that in the state s agent i relies on agent j about φ, where λ means nobody. The dependent function requires each agent has only one credible advisor for each formula in each state, which seems not to be a flexible formalism for modelling belief dependence. In this paper, we would like to extend the dependent structure to formula sets, which allows that each agent has more than one credible advisor. Formally, we have the following definition:

Definition 4.2 *(Belief dependence D-model)*
A belief dependence D-model is a tuple $M = (S, \pi, \mathcal{L}, \mathcal{D})$
where S is a set of states, $\pi(s,.)$ is a truth assignment for each state $s \in S$, and $\mathcal{L} : \mathbf{A_n} \to 2^{S\times S}$, which consists of n binary accessibility relations on S, $\mathcal{D} : \mathbf{A_n} \times \mathbf{A_n} \times S \to 2^{\mathbf{L_D}}$.

The truth relation $\models$ is defined inductively as follows:

$M, s \models p$,	where p is a primitive proposition, iff $\pi(s,p) =$true,
$M, s \models \neg\varphi$	iff $M, s \not\models \varphi$
$M, s \models \varphi_1 \wedge \varphi_2$	iff $M, s \models \varphi_1 \wedge M, s \models \varphi_2$,
$M, s \models L_i\varphi$	iff $M, t \models \varphi$ for all t such $(s,t) \in \mathcal{L}(i)$
$M, s \models D_{i,j}\varphi$	iff $\varphi \in \mathcal{D}(i,j,s)$.

We say a formula φ is *valid in structure* M if $M, s \models \varphi$ for all possible worlds s in M; φ is *satisfiable in* M if $M, s \models \varphi$ for some possible worlds in M. We say φ is *valid* if it is valid in all structures; φ is *satisfiable* if it is satisfiable in some structure.

For D-models, we define sub-beliefs as $L_{i,j}\varphi \stackrel{\text{def}}{=} D_{i,j}\varphi \wedge L_j\varphi$, which means that system is honest because the honesty axiom $L_{i,j}\varphi \rightarrow L_j\varphi$ holds.

In the definition about D-model of belief dependence, we have placed no any restriction on the dependent formula sets. To capture certain properties for belief dependence, we may well to add some restrictions on the dependent formula sets. Some typical restrictions we may want to add to $\mathcal{D}(i,j,s)$ can be expressed by some closure properties under the logical connectives and modal operators.

Definition 4.3 *A dependent formula set* $\mathcal{D}(i,j,s)$ *is said to be:*
(i) closed under negation, iff $\varphi \in \mathcal{D}(i,j,s) \leftrightarrow \neg\varphi \in \mathcal{D}(i,j,s)$.
(b) closed under conjunction, iff $\varphi \in \mathcal{D}(i,j,s) \wedge \psi \in \mathcal{D}(i,j,s) \rightarrow (\varphi \wedge \psi) \in \mathcal{D}(i,j,s)$.
(c) decomposable under conjunction, iff $\varphi \wedge \psi \in \mathcal{D}(i,j,s) \rightarrow \varphi, \psi \in \mathcal{D}(i,j,s)$.
(d) closed under implication, iff $\varphi \in \mathcal{D}(i,j,s) \wedge (\varphi \rightarrow \psi) \in \mathcal{D}(i,j,s) \rightarrow \psi \in \mathcal{D}(i,j,s)$.

Definition 4.4 *A D-model for belief dependence* $M = (S, \pi, \mathcal{L}, \mathcal{D})$ *is an* **L5$^-$+D4** *D-model, if it satisfies the following conditions:*
(a) Each accessibility relation $\mathcal{L}(i,s)$ *is serial, transitive, and Euclidean,*
(b) Each dependent formula set $\mathcal{D}(i,j,s)$ *is closed under negation, implication, and conjunction,*
(c) For any formula φ*, if* $\varphi \in \mathcal{D}(i,j,s)$*, then* $\varphi \in \mathcal{D}(i,j,t)$ *for all of states* t *such that* $< s,t > \in \mathcal{L}_i$.

In order to show that soundness and completeness of system **L5$^-$+D4** for **L5$^-$+D4** D-models, we can use the standard techniques, namely, techniques of canonical structures[6,12,15]. First, we need the following definitions: A formula p is *consistent* (with respect to an axiom system) if $\neg p$ is not provable. A finite set $\{p_1, ..., p_k\}$ is consistent exactly if the formula $p_1 \wedge ... \wedge p_k$ is consistent. An infinite set of formulae is consistent if every finite subset of it is consistent. A set F of formulae is a *maximal consistent set* if it is consistent and any strict superset is inconsistent. As it is pointed out in [6], using standard techniques of propositional reasoning we can show:

Lemma 4.1 *In any axiom system that includes (L1) and (R1):*
(1) Any consistent set can be extended to a maximal consistent set.
(2) If F *is a maximal consistent set, then for all formulas* φ *and* ψ*:*
(2.a) either $\varphi \in F$ *or* $\neg\varphi \in F$,
(2.b) $\varphi \wedge \psi \in F$ *iff* $\varphi \in F$ *and* $\psi \in F$,
(2.c) if $\varphi \in F$ *and* $\varphi \rightarrow \psi$*, then* $\psi \in F$,
(2.d) if φ *is provable, then* $\varphi \in F$.

Theorem 4.1 L5$^-$+D4 *belief dependence systems are sound and complete for any* **L5$^-$+D4** *D-model.*

Proof:. Soundness is evident. For the completeness, we must show every valid formula is provable. Equivalently, we should show that every consistent formula is satisfiable. A canonical structure Mc is constructed as follows:
$Mc = (S, \pi, \mathcal{L}c, \mathcal{D}c)$
where
$S = \{s_v | V \text{ is a maximal consistent set}\}$,
$\pi(s_v, p) =$true, if $p \in V$; false, if $p \notin V$,
$\mathcal{L}c(i) = \{< s_v, s_w > | L_i^-(V) \subseteq W\}$, for any $i \in \mathbf{A_n}$
where $L_i^-(s_v) \stackrel{\text{def}}{=} \{\varphi | L_i\varphi \in V\}$
$\mathcal{D}c(i, j, s_v) = \{\varphi | D_{i,j}\varphi \in V\}$.

First, we show that Mc is an $\mathbf{L5^-}$+$\mathbf{D4}$ D-model. Axioms (L3), (L4), and (L5) guarantee that $\mathcal{L}c(i)$ is serial, transitive, and Euclidean. As for as the dependence formula set $\mathcal{D}c$ is concerned, we have:
$\varphi \in \mathcal{D}c(i, j, s_v) \Rightarrow D_{i,j}\varphi \in V$ (By the definition of Mc)
$\Rightarrow D_{i,j}\neg\varphi \in V$ (By axiom (D1) and lemma (2.c))
$\Rightarrow \neg\varphi \in \mathcal{D}c(i, j, s_v)$ (by the definition of Mc)
Therefore, the dependence formula sets are closed under negation. The cases concerning closure under conjunction and implication can be similarly shown. Moreover, for any formula φ,
$\varphi \in \mathcal{D}c(i, j, s_v) \Rightarrow D_{i,j}\varphi \in V$ (By the definition of Mc)
$\Rightarrow L_iD_{i,j}\varphi \in V$ (By axiom (D4) and lemma (2.c))
$\Rightarrow D_{i,j}\varphi \in W$ for all W such that $< s_v, s_w > \in \mathcal{L}c(i)$ (By the definition of Mc)
$\varphi \in \mathcal{D}c(i, j, s_w)$ for all s_w such that $< s_v, s_w > \in \mathcal{L}c(i)$.
Therefore, Mc is an $\mathbf{L5^-}$+$\mathbf{D4}$ D-model.

In order to show every formula φ is satisfiable, we should show $\varphi \in V \Leftrightarrow Mc, s_v \models \varphi$. we can show that by induction on the structure of formulas as follows:
(a) φ is a primitive proposition,
$\varphi \in V \Leftrightarrow \pi(s_v, \varphi) =$true (By the definition of Mc)
$\Leftrightarrow Mc, s_v \models \varphi$ (By the definition of $\models$)
(b) $\varphi \wedge \psi$,
$\varphi \wedge \psi \in V \Leftrightarrow \varphi, \psi \in V$ (By the lemma(2.b))
$\Leftrightarrow Mc, s_v \models \varphi, \psi$ (Induction Assumption)
$\Leftrightarrow Mc, s_v \models \varphi \wedge \psi$ (The definition of $\models$)
(c) $\neg\varphi$,
$\neg\varphi \in V \Leftrightarrow \varphi \notin V$ (By the definition of consistent set)
$\Leftrightarrow Mc, s_v \not\models \varphi$ (Induction Assumption)
$\Leftrightarrow Mc, s_v \models \neg\varphi$ (The definition of $\models$)
(d) $L_i\varphi$,
$L_i\varphi \in V \Leftrightarrow \varphi \in W$ for all W such that $< s_v, s_w > \in \mathcal{L}c(i)$ (By the definition of Mc)
$\Leftrightarrow Mc, s_w \models \varphi$ (Induction assumption)
$\Leftrightarrow Mc, s_v \models L_i\varphi$ (The definition of $\models$)
(e) $D_{i,j}\varphi$,
$D_{i,j}\varphi \in V \Leftrightarrow \varphi \in \mathcal{D}c(i, j, s_v)$ (By the definition of Mc)
$\Leftrightarrow Mc, s_v \models D_{i,j}\varphi$ (By the definition of $\models$)
Therefore, for any φ, $\varphi \in V$ iff $Mc, s_v \models \varphi$. $\square$

4.3 Lij-Model: An Adapted Possible World approach

D-models of belief dependence is a syntactic approach, which does not somewhat coincide with possible world semantics for epistemic logics. Moreover, L-models of belief dependence, which are based on general epistemic logics, suffer from the problem that the dependent operator can not be intuitively handled with. Therefore, in this section, we would like to present a third logic for belief dependence. The ideas behind the third logic are to adapt possible world semantics for modelling belief dependence by directly introduction of sub-belief structures. Formally, we have the following definition:

Definition 4.5 *(Belief dependence Lij-model)*

A belief dependence Lij-model is a tuple $M = (S, \pi, \mathcal{L})$ where S is a set of states, $\pi(s, .)$ is a truth assignment for each state $s \in S$, and $\mathcal{L} : \mathbf{A_n} \times \mathbf{A_n} \rightarrow 2^{S \times S}$, which consists of $n \times n$ binary accessibility relations on S.

The relation $\models$ is similarly defined inductively as follows:

$M, s \models p$, where p is a primitive proposition, iff $\pi(s, p)$ =true,

$M, s \models \neg\varphi$ iff $M, s \not\models \varphi$

$M, s \models \varphi_1 \wedge \varphi_2$ iff $M, s \models \varphi_1 \wedge M, s \models \varphi_2$,

$M, s \models L_{i,j}\varphi$ iff $M, t \models \varphi$ for all t such $(s, t) \in \mathcal{L}(i, j)$.

$L_{i,j}\varphi$ means that due to agent j, agent i believes the formula φ. In Lij-models, we intuitively consider $L_{i,i}\varphi$ as its general epistemic interpretation, namely, $L_i\varphi$. Just like the cases in epistemic logics, sometimes we hope that the axiom $L_i\varphi \rightarrow L_iL_i\varphi$ holds. Similarly, for sub-beliefs, we generally hope that the axiom $L_{i,j}\varphi \rightarrow L_iL_{i,j}\varphi$ holds. In order to formulate those properties, first of all, we need the following definitions:

Definition 4.6 *(Left-closed accessibility relations)*
For any Lij model $M = (S, \pi, \mathcal{L})$, an accessibility relation $\mathcal{L}(i, j)$ is a left-closed relation, if $\mathcal{L}(i, i) \circ \mathcal{L}(i, j) \subseteq \mathcal{L}(i, j)$ holds.

Propositions 4.1 *For any Lij model in which every accessibility relation is left-closed, the axiom $L_{i,j}\varphi \rightarrow L_iL_{i,j}\varphi$ holds.*

Definition 4.7 *(Almost-Euclidean accessibility relations)*
For any Lij-model $M = (s, \pi, \mathcal{L})$, an accessibility relation $\mathcal{L}(i, j)$ is an almost-Euclidean relation, if $(t, u) \in \mathcal{L}(i, j)$ whenever $(s, u) \in \mathcal{L}(i, j)$ and $(s, t) \in \mathcal{L}(i, i)$.

Propositions 4.2 *For any Lij-model in which every accessibility relation is almost-Euclidean, the axiom $\neg L_{i,j}\varphi \rightarrow L_{i,i}\neg L_{i,j}\varphi$ holds.*

Definition 4.8 *(Identity-closed accessibility relations)*
For any Lij-model $M = (S, \pi, \mathcal{L})$, an accessibility relation $\mathcal{L}(i, j)$ is an identity-closed relation, if $\mathcal{L}(i, i) \circ \mathcal{L}(j, j) \subseteq \mathcal{L}(i, j)$ holds.

Propositions 4.3 *For any Lij-model in which every accessibility relation is left-closed, the axiom $L_{i,j} \rightarrow L_{i,i}L_{j,j}\varphi$ holds.*

Definition 4.9 *For any accessibility relation $R \subseteq S \times S$, R is said to be:*
(i) a **D**-*relation, if it is serial, namely, for each $s \in S$ there is some $t \in S$ such that $(s,t) \in R$.*
(ii) a **4**-*relation, if it is transitive, namely, if $(s,u) \in R$ whenever $(s,t) \in R$ and $(t,u) \in R$.*
(iii) a **5**-*relation, if it is Euclidean, namely, if $(t,u) \in R$ whenever $(s,t) \in R$ and $(s,u) \in R$.*
(iv) a **4***-*relation, if it is a left-closed relation.*
(v) a **4'**-*relation, if it is an identity-closed relation.*
(vi) a **5***-*relation, if it is an almost-Euclidean relation .*

Definition 4.10 *(***D4*5*** *Lij-model)*
An Lij-model $M = (S, \pi, \mathcal{L})$ is a **D4*5*** *Lij-model, if every accessibility relation on S is serial, left-closed, and almost-Euclidean.*

Theorem 4.2 **Lij5$^-$+D** *belief dependence logics are sound and complete for any* **D4*5*** *Lij-model.*

Proof. Soundness is evident, and completeness can be proved in an analogous fashion to the theorem about the **L5$^-$+D4** systems. We define the canonical structure $Mc = (S, \pi, \mathcal{L}c)$as follows:

$S = \{s_v | V \text{ is a maximal consistent set}\}$,
$\pi(s_v, p) =$true, if $p \in V$; false, if $p \notin V$,
$\mathcal{L}c(i,j) = \{(s_v, s_w) | L^-_{i,j}(V) \subseteq W\}$.
where $L^-_{i,j}(V) \stackrel{\text{def}}{=} \{\varphi | L_{i,j}\varphi \in V\}$.

First, we show that Mc is a **D4*5*** Lij-model. Axiom (Lij3) guarantees every $\mathcal{L}c(i,j)$ is serial. For any $(s_v, s_w) \in \mathcal{L}c(i,i)$, and $(s_w, s_{w'}) \in \mathcal{L}c(i,j)$,
We have $L^-_{i,i}(V) \subseteq W$ and $L^-_{i,j} \subseteq W'$.
$L_{i,j}\varphi \in V \Rightarrow L_i L_{i,j} \in V$ (Axiom(Lij4) and lemma(2.c))
$\Rightarrow L_{i,j}\varphi \in W$ $\;(L^-_{i,i}(V) \subseteq W)$
$\Rightarrow \varphi \in W'$ $\;(L^-_{i,j}(W) \subseteq W')$

Therefore, every accessibility relation is a **4***-relation. Furthermore, for any $(s_v, s_w) \in \mathcal{L}c(i,j)$, and $(s_v, s_{w'}) \in \mathcal{L}c(i,i)$,
We have $L^-_{i,j}(V) \subseteq W$ and $L^-_{i,i}(V) \subseteq W'$.
$L_{i,j}\varphi \in W' \Rightarrow L_{i,i}L_{i,j}\varphi \in V$ $\;(L^-_{i,i}(V) \subseteq W')$
$\Rightarrow \neg L_{i,i}\neg L_{i,j}\varphi \in V$ (Axiom $L_{i,i}\psi \rightarrow \neg L_{i,i}\neg\psi$)
$\Rightarrow L_{i,j}\varphi \in V$ (Axiom $(Lij5)$)
$\Rightarrow \varphi \in W$ $\;(L^-_{i,j}(V) \subseteq W)$
Therefore, every accessibility relation is a **5***-relation, i.e., Mc is a **D4*5*** Lij-model. Moreover, we can show that $Mc, s_v \models \varphi$ iff $\varphi \in V$.$\Box$

5 Conclusions

We have proposed several approaches for belief dependence logics. All of those approaches can capture certain properties concerning belief dependence. There might exist many different criteria to appraise those approaches. We suggest some main criteria as follows:

Approaches	*Efficiency*	*Coherence*	*Avoidance of Logical Omniscience*
D-model	Yes	No	Yes
L-model	No	Yes	No
Lij-model	Yes	Yes	No

Figure 3: Summaries about Approaches

i) *Efficiency Adequacy*: Approaches can efficiently formalize the fundamental features such as closure, suspicion, indirect dependence, role-appraisal.
ii) *Coherence Adequacy*: Approaches can be captured intuitively, in which semantics models should be naturally connected.
iii) *Avoidance of Logical Omniscience*: Approaches do not suffer from the problem of logical omniscience.

The approach about D-model is based on a syntactic strategy, which is amalgamated with general possible world approach. The approach concerning L-model actually is a general epistemic logics approach, which fail to capture some important features of dependent operator. The approach of Lij-model seems to be a more reasonable and acceptable one, since which can capture many intuitive properties concerning the dependent operator, although the approach suffers from the problem of logical omniscience, just like those in general epistemic logics. The comparison is shown in the figure.

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