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THE SCHOENMAKERS PARADOX: ITS SOLUTION IN A BELIEF DEPENDENCE FRAMEWORK

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The Schoenmakers Paradox: Its Solution in a Belief Dependence Framework

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Abstract

In [16], Schoenmakers raised an important problem concerning knowledge acquisition. There may arise some unacceptable but hard-perceivable results in knowledge bases if the knowledge bases assimilate information from multiple expert sources. In this paper, we re-examine the Schoenmakers paradox in the framework of belief dependence for multiple agent environments. The notion of safe-assimilating is introduced to capture a better understanding the problem. Based on the logic of belief dependence, we introduce an *almost safely and soundly assimilating* operation, which is argued to be reasonable and acceptable strategy for the problem.

1 The Schoenmakers Paradox

In [16], Schoenmakers raises an interesting problem concerning knowledge acquisition from multiple expert sources. There may arise some unreasonable but hard-perceivable results if knowledge bases assimilate information from multiple expert sources. The problem, called the *Schoenmakers paradox* below, can be expressed by the following simple story:

> Once upon a time an wise but strictly formal judge heard two witnesses. They spoke to him on separate occasions. Witness w1 honestly stated that he was convinced that proposition P was true; witness w2 honestly stated that he was convinced that the implication $P \rightarrow Q$ was true. Nothing else was said or heard. The judge did not notice any inconsistency so he accepted both statements and concluded that Q had to be true. When the witnesses heard about his conclusion they were shocked because both witnesses were (still) convcined that Q was false. However, they were too late to prevent the execution of the verdict.

As Schoenmakers pointed out in [16], in the above story, no one could be blamed, neither the witnesses, nor the accused, and even not the judge. The witnesses cannot be blamed, even though they both knew, but did not tell, that Q was false. It is unrealistic to expect that witnesses will tell everything they know, notwithstanding their legal obligation to do so. One cannot blame the judge, since he had no reason for doubt; he knew that his knowledge was not inconsistent, and his reasoning was correct.

Of course one might blame the judge because he did not interrogate the witnesses about Q and did not confront them with his conclusion. In other words, judges should fully interact with their witnesses. For the judge this is indeed a possibility. However, as Schoenmakers points out, in the case of a knowledge base which assimilates information from multiple expert sources, this knowledge base may derive conclusions without making errors, and still the conclusion may turn out to be completely unreasonable. Moreover, it is unrealistic to require that the knowledge base system confronts all of their experts with all conclusions that can be or have been derived. Therefore, in [16], Schoenmakers concludes as follows:

> Intelligent database systems may behave perfectly in splendid isolation, operating on one world without inconsistencies, but even when they are consistent they may produce unacceptable results when operating on the information that is accessible in a community of such systems. Their results will be acceptable, most of the time, but nobody knows when.

In [3], the problem, and some of its extensions specifically involving juridical expert systems, are considered. The conclusion is once more that the development of juridical expert systems – expert systems for rending judgement based on evidence – is a far more difficult task than the development of systems in other domains of comparable complexity as law, because of the peculiarly interactive nature of the juridical process and the necessity of such a high level of interaction in order to protect the rights of the accused.

In this paper, we try to examine the Schoenmakers paradox within the framework of logic of belief dependence. This is a logic that is developed to serve as a foundation for understanding rational behavior involving the knowledge and belief communication and the assimilation of information[9, 11, 12]. We will argue that a plausible solution for the Schoenmakers paradox can be based on the framework of belief dependence.

The organization of this paper is as follows: In section 2, we will sketch the main notions concerning the logic of belief dependence, and introduce a logic system which is suitable to formalize the problem. Next, in section 3, we will formalize some requirements for the knowledge assimilation problem involving multiple expert sources, and introduce the notion of safe assimilation. It will turn out to be difficult to construct an update operation satisfying these requirements. Therefore in section 4 we propose an alternative for the notion of safe-assimilation, and present the needed fragment of an update operation having the required properties. Its impact on the story is described: it is shown that the unreasonability of the situation after the judge has passed his verdict is of temporary nature; once the two witnesses have understood the base for the judge's reasoning they will be both convinced after all that Q indeed was true, and the whole country will be convinced that the judge was very wise indeed. Section 5 contains our conclusion.

2 Logics for Belief Dependence

As is well known, in a multiple agent environment, it is frequently beneficial to enable agents to communicate their knowledge or beliefs among each other. Under such circumstances some agents may rely on someone else about their beliefs or knowledge. We called this phenomenon *belief dependence.* In [9, 11, 12], we present a formal theory for belief dependence which is expected to serve as a foundation for understanding rational behavior of artificial agents in multiple agent environments. In this section, we present the main notions from this theory.

Our logic involves in the first place the general notions of knowledge and belief, which are the equivalents of those notions in epistemic and doxastic logic. In our logic for belief dependence, we generally use $L_i\varphi$ to represent the fact that agent i knows or believes the formula φ. As is well known, the modal operator L represents an epistemic operator, when the logic is an **S5** system, whereas L is a doxatic operator if the logic is a weak **S5** system.

There exists a second important notion used for reasoning about dependent knowledge and beliefs; this notion is called the *dependent operator*, or alternatively *rely-on relation*, and it is denoted by $D_{i,j}$. Intuitively, we can give $D_{i,j}\varphi$ a number of different interpretations: "agent i relies on agent j about the formula φ", "agent i depends on agent j about believing φ", or even more specifically, "agent j is the credible advisor of agent i about φ".

In the communication of knowledge and belief among agents, agents do not necessarily view knowledge and belief accepted from other agents as their own knowledge, even though they may originally have asked for such information. In terms of cognitive psychology, these beliefs are compartmentalized[15]. In logics for belief dependence, we therefore introduce a *compartment operator*, or alternatively called a *sub-belief operator*, written $L_{i,j}$. Intuitively, $L_{i,j}\varphi$ can be read "agent i believes φ due to agent j". From the viewpoint of *minds society*, $L_{i,j}\varphi$ can be more intuitively interpreted as "agent i believes or knows φ on the mind frame indexed j. Consequently, as argued for in [11], we claim that an appropriate procedure for formalizing information assimilation should involve both phases: compartmentalization and incorporation of information. *Compartmentalized information* are those fragments of information which are accepted and remembered as isolated beliefs and which are treated somewhat different from the beliefs that are completely believed, whereas *incorporated information* consists of those beliefs that are completely believed by the agents. In the logic for belief dependence, compartmentalized information is modelled by sub-beliefs $L_{i,j}\varphi$ for agent i, whereas incorporated information corresponds to general beliefs of agent i, namely, $L_i\varphi$.

For multiple agent environment, we assume that some primitive rely-on relations among those agents for some propositions have been decided on the metalevel. We call this assumption the *initial role-knowledge assumption.* We believe that this assumption is appropriate and intuitive, because, in a multiple agent environment, some agents must possess some minimal knowledge about their partners in order to guarantee that they communicate at all. In a reliable communication network, assuming that agents are honest, no-doubt and something more [11], primitive rely-on relations often collapse into primitive communication relations, an this turns them into observable entities.

Based on the primitive rely-on relations, we can capture a complete knowledge about agents' sub-beliefs by using the logic for belief dependence. Using the complete information concerning agents' sub-beliefs, we can figure out some agents' appraisal information about others. Moreover, given this appraisal information, it becomes possible to determine some rational belief-maintenance strategies, by which we can compute whether and how compartmentalized beliefs can be assimilated into the incorporated beliefs.

In [11], we focus on the formalism describing the first phase of information assimilation. This paper involves the problem of determining how the complete sub-belief and the complete rely-on relations can be captured, using the primitive rely-on relations. In [12], we concentrate on the second phase of information assimilation, in particular with the situation where new information is inconsistent with agents' beliefs and knowledge. The Schoenmakers paradox however shows that even if new information is not inconsistent with agents' beliefs and knowledge, the agents still may reach some unreasonable state. Therefore, the work in this paper can be viewed once more as work which focuses on the second phase of information assimilation for multiple agent environment. Our general

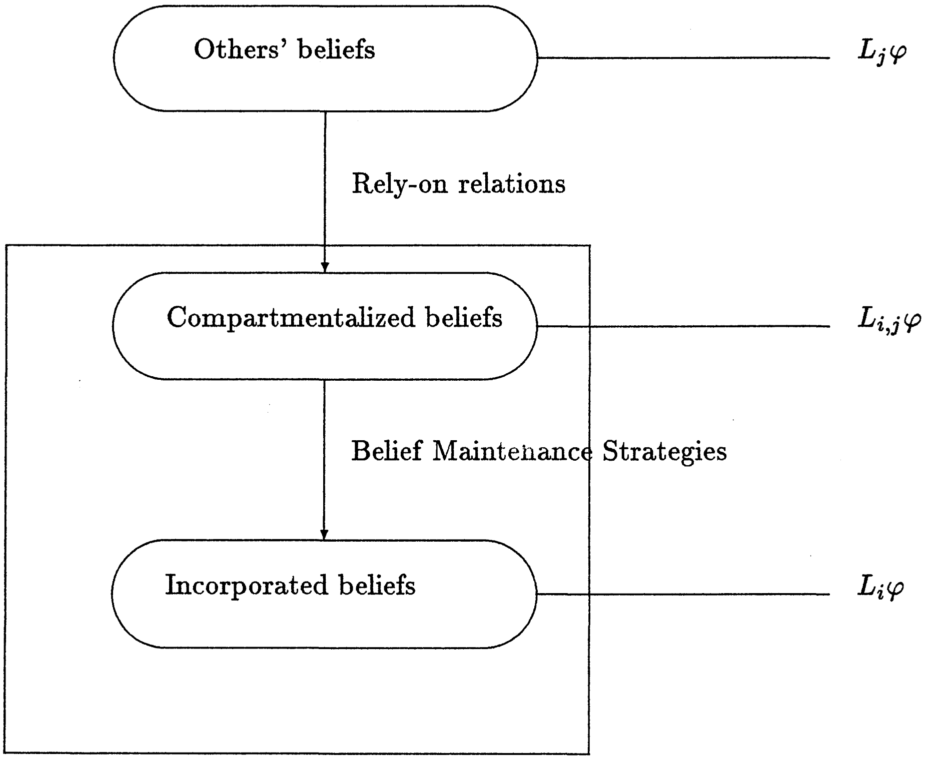

Figure 1: General Scenario

scenario about the formalism of belief dependence is illustrated by the figure above.

There are many different logic systems to formalize the problem of belief dependence. In this paper, for the studies of Schoenmakers paradox, we select the logic system called **Lij5+D5** .

Axioms:

(D1) $D_{i,j}\varphi \equiv D_{i,j}\neg\varphi$.

(Neutral axiom. It is the most fundamental axiom for dependent operator.)

(D2) $D_{i,j}\varphi \wedge D_{i,j}(\varphi \rightarrow \psi) \rightarrow D_{i,j}\psi$.

(Closure under implication, for the dependent operator, which is acceptable for the studies of the problem.)

(D3) $D_{i,j}\varphi \wedge D_{i,j}\psi \rightarrow D_{i,j}(\varphi \wedge \psi)$.

(Closure under conjunction. This shows that beliefs which come from the same agent should be consistent.).

(D4) $D_{i,j}\varphi \rightarrow L_i D_{i,j}\varphi$.

(Positive explicit dependent axiom. In fact, in the section 4, we will make the far stronger assumption that the rely-on relations are common knowledge among agents.)

(D5) $\neg D_{i,j}\varphi \rightarrow L_i\neg D_{i,j}\varphi$.

(Negative explicit dependent axiom.)

(L1) All instances of propositional tautologies.

(Which are fundamental for most logic systems)

(Lij2) $L_{i,j}\varphi \wedge L_{i,j}(\varphi \rightarrow \psi) \rightarrow L_{i,j}\psi)$.

(Sub-beliefs are closed under logic implication)

(Lij3) $\neg L_{i,j}\mathbf{false}$.

(Belief axiom, which means that the sub-beliefs are beliefs as well.)

(Lij4) $L_{i,j}\varphi \rightarrow L_i L_{i,j}\varphi$.
(Positive introspection axiom for sub-belief.)
(Lij5) $\neg L_{i,j}\varphi \rightarrow L_i \neg L_{i,j}\varphi$.
(Negative introspection axiom for sub-belief.)

(Lij-DL) $Li, j\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$.
(where we assume that if agent i relys on agent j about φ, and j believes φ, then i would believe φ (due to j). Moreover, this means that the communication is reliable.)

Rules:
(R1) $\vdash \varphi, \vdash \varphi \rightarrow \psi \Rightarrow \vdash \psi$.
(RLij) $\vdash \varphi \Rightarrow \vdash L_{i,j}\varphi$.

Definition:
(Ldf) $L_i\varphi \overset{def}{=} L_{i,i}\varphi$.

For the system **Lij5+D5**, we have the following propositions:

Proposition 2.1
(Lij∧) $L_{i,j}\varphi \wedge L_{i,j}\psi \equiv L_{i,j}(\varphi \wedge \psi)$.
(Lij¬) $L_{i,j}\varphi \rightarrow \neg L_{i,j}\neg\varphi$.
(Lij∨) $L_{i,j}\varphi \vee L_{i,j}\psi \rightarrow L_{i,j}(\varphi \vee \psi)$.

For the belief maintenance operation during the second phase of the information assimilation, we introduce the notion of the *belief maintenance model*, which is an ordered couple $< \mathbf{K}, \triangle >$ such that $\mathbf{K}$ is a set of belief sets and $\triangle : \mathbf{K} \times \mathrm{Sent}(L) \rightarrow \mathbf{K}$ is a function assigning a maintenance operation $\triangle(K, A)$ to any belief set $K \in \mathbf{K}$ and any L-sentence A. We shall write alternatively $K\triangle\varphi$ to represent $\triangle(K, \varphi)$.

Let K be the knowledge set, we define $L_{i,j}^{-}(K) \overset{def}{=} \{\psi | L_{i,j}\psi \in K\}$, which denotes the set of agent i's sub-belief indexed j. In order to define a belief maintenance operation, sometimes we use the following three kinds of update operations [6]: expansion $+$, revision $\dot{+}$, and $\dot{-}$.

3 Formalizing the Problem in Belief Dependence Framework

In this section, we would like to formalize the knowledge assimilation problem concerning multiple expert sources. We restrict ourself to the case in which there is only one agent who assimilates information and two agents which serve as expert sources. Therefore, we will consider an agent set $\mathbf{A}_3 = \{a, w_1, w_2\}$, where a denotes the agent who assimilates the information, and w_1, w_2 denotes the two agents who offers the information. We will call w_1 and w_2 *source agents*. The results can be easily extended to those cases with more than three agents.

The main goal in this paper is to determine a safe information assimilation strategy for agent a. The so-called safe strategy informally can be described to be a strategy for which there arise never results which are derived from the new information accepted from the source agents but which violate the belief of the all involved source agents. In this section we present a formal definition for such a safe assimilation operator. However, defining such a notion and constructing an operation satisfying the definition are different tasks, and we must concede that so far we have not founded a construction of such an operation.

Instead we will introduce some less demanding safety conditions in section 4, for which we know how to construct one.

First we need some relative definitions.

In the first place, we need the notion of *combined belief* which describes the set of formulas which can be derived from the combination of two or more agents' beliefs under the belief maintenance operation. Let K be the knowledge base and $\triangle$ be a belief maintenance operation, the formula $L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi$ means that agent i may believe φ by assimilating new information ψ from agent j_1 and ψ' from agent j_2. Formally, we have the following definition.

Definition 3.1 *For the knowledge base* K,
$L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi \overset{def}{=} \varphi \in (L^-_i(K)\triangle(L_{i,j_1}\psi \wedge L_{i,j_2}\psi')) \wedge (\psi \wedge \psi' \to \varphi)$.

For the safe operation, the derived results are compatible with the knowledge of both the source agents. For the strongly safe operation, the derived beliefs of agent i are at least supported by one source agent. Formally, we have:

Definition 3.2 *The belief maintenance operation* $\triangle$ *is said to be a safe one for agent* i*, if the following axiom is satisfied:*
for any ψ, ψ'*, and* φ, $\neg L_i\varphi \wedge L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi \Rightarrow \neg(L_{j_1}\neg\varphi \wedge L_{j_2}\neg\varphi)$.

In the above definition, we check the safety only on those formulas which are not originally believed by the agent i. In other words, it does not matter whether φ is safe or not if φ originally is already believed by agent i.

Definition 3.3 *The belief maintenance operation* $\triangle$ *is said to be a strongly safe one for agent* i*, if the following axiom is satisfied:*
for any ψ, ψ'*, and* φ, $\neg L_i\varphi \wedge L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi \Rightarrow L_{j_1}\varphi \vee L_{j_2}\varphi$.

Proposition 3.1 *If a belief maintenance operation* $\triangle$ *is strongly safe, then* $\triangle$ *is safe as well.*

However, for the above definition concerning safe or strongly safe operation, in order to keep the beliefs safe, one may have to refuse most parts of the new information, up to the point of not assimilating anything from others. This is expressed by the simple proposition.

Proposition 3.2 *The belief maintenance operation* $\triangle_{id}$*, which is defined as* $\triangle_{id}(K,\varphi) = K$ *for any* K, φ*, is a strongly safe operation.*

Proof: for any ψ, ψ'
$\neg L_i\varphi \wedge L^{\triangle_{id},\psi,\psi'}_{i,\{j_1,j_2\}}\varphi$
$\Rightarrow \neg L_i\varphi \wedge (\varphi \in L^-_i(K)\triangle_{id}(L_{i,j_1}\psi \wedge L_{i,j_2}\psi)) \wedge (\psi \wedge \psi' \to \varphi))$ (By the definition of $L^{\triangle,\psi,\psi'-}_{i,\{j_1,j_2\}}$)
$\Rightarrow \neg L_i\varphi \wedge \varphi \in L^-_i(K)\triangle_{id}(L_{i,j_1}\psi \wedge L_{i,j_2}\psi')$
$\Rightarrow \neg L_i\varphi \wedge \varphi \in L^-_i(K)$ (By the definition about $\triangle_{id}$)
$\Rightarrow \neg L_i\varphi \wedge L_i\varphi \Rightarrow$**false**$\Rightarrow (L_{j_1}\varphi \vee L_{j_2}\varphi)$.
Therefore, the operation $\triangle_{id}$ is strongly safe. Naturally, it is safe as well. □

Clearly, a reasonable and acceptable operation should be able to assimilate new information as closely as possible. We will call such an operation a *safely and soundly assimilating operation*, or alternatively an SSA operation.

Definition 3.4 *A belief maintenance operation* $\triangle$ *is a* φ*-assimilating operation (for agent* i*), if* $\neg L_i\varphi \wedge L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi$ *holds.*

Definition 3.5 *The operation* $\triangle$ *is an assimilating operation if there exists a* φ *such that* $\triangle$ *is* φ*-assimilation operation.*

Definition 3.6 $\triangle$ *is a safe assimilating operation if it is safe and assimilating;* $\triangle$ *is a strongly safe assimilating operation if it is strongly safe and assimilating.*

Definition 3.7 $L^{\triangle,\psi,\psi'-}_{i,\{j_1,j_2\}}(K) \overset{def}{=} \{\varphi | L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi$ *holds for the knowledge base* $K\}$.

Definition 3.8 $\triangle$ *is a safely and soundly assimilating operation if* $\triangle$ *is safe, and there exist no any other safe operation* $\triangle'$ *such that* $L^{\triangle,\psi,\psi'-}_{i,\{j_1,j_2\}}(K) \subset L^{\triangle',\psi,\psi'-}_{i,\{j_1,j_2\}}(K)$ *holds for some* K*, and for any* ψ, ψ'.

In most practical situations, it is difficult to describe a safe and soundly assimilating operation, since capturing the operation seems to largely depend on the information about others' beliefs. Agents do not generally possess enough knowledge about others' beliefs. Fortunately, we shall argue that, in the logics for belief dependence, the initial role knowledge assumption offers an alternative approach to solve the problem.

4 Almost-safely and Soundly-assimilating Strategies

The initial role-knowledge assumption says that some primitive rely-on relations among the agents have been decided on the metalevel. This suggests the possibility to make fully use of this information to capture an acceptable strategy for safe belief assimilating. This requires however a change of the original definition concerning safety.

In the original definition, we require that the derived beliefs did not contradict the beliefs of the source agents. An almost safe operation is one for which all of the derived beliefs are supported by the possible beliefs of the source agents obtainable by exchanging their knowledge. In other words, since we assume that the source agents rely on each other, it is also reasonable to assume that there exist beliefs which could have been produced, had they been given the opportunity to exchange their knowledge and beliefs before interacting with agent a. Our operation can be considered to be almost safe if the produced beliefs would be supported by those potential beliefs of the original agents.

Furthermore, we must extend the definition of sub-beliefs to one where agents can extend their sub-beliefs by combining their sub-belief with their own belief as long as the combination still is consistent. Therefore, we introduce the following extensive sub-belief assumption:

(Extensive Sub-belief Assumption)

(ESA) $L_{i,j}\varphi \wedge L_i\psi \wedge \neg L_{i,j}\neg\psi \rightarrow L_{i,j}(\varphi \wedge \psi)$

(If agent i believes that φ in the mind frame indexed j, and agent i originally believes ψ, and ψ are not unconsistent with his sub-beliefs indexed j, then agent i would believe φ and ψ in the mind frame indexed j.)

It should be noted that the notion of the extensive sub-belief is different from that of the combined belief, since the former requires that the combined belief should be not inconsistent with the original ones, whereas the latter has no such requirement. Moreover, under the extensive sub-belief assumption, the definition is not simply as $L_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$ as before. However, one of the implications, which can be denoted by this rule:

(DLLij) $D_{i,j}\varphi \wedge L_j\varphi \rightarrow L_{i,j}\varphi$

remains valid. Based on the extensive assumption, the notions of almost safety, and almost safely and soundly-assimilation, can be defined as follows:

Definition 4.1 *A belief maintenance operation $\triangle$ is an almost-safe operation (for agent i) if the following axiom is satisfied:*
for any ψ, ψ', φ, $\neg L_i\varphi \wedge L^{\triangle,\psi,\psi'}_{i,\{j_1,j_2\}}\varphi \Rightarrow L_{j_1,j_2}\varphi \wedge L_{j_2,j_1}\varphi$.

Definition 4.2 *$\triangle$ is an almost-safely and soundly-assimilating operation (for agent i), called an ASSA operation, for short, if it is almost safe and there exist no any almost-safe operation $\triangle'$ such that $L^{\triangle,\psi,\psi'-}_{i,\{j_1,j_2\}}(K) \subset L^{\triangle',\psi,\psi'-}_{i,\{j_1,j_2\}}(K)$ holds for some K, and for any ψ, ψ'.*

In the following, we would like to introduce an ASSA operation, based on belief dependence logic. We suppose that basic logic system is the system which consists of **Lij5+D5** by changing (Lij-DL) into (DLLij), and adding the following additional axioms. First, we assume that all agent's own beliefs (not including sub-beliefs) are true. Therefore, we have the following axioms.
(L2′) $L_{i,i}\varphi \to \varphi$.
Specially, from (L2′) we have $L_iD_{j,k}\varphi \to D_{j,k}\varphi$,
Moreover, we assume:
(CD+) $D_{i,j}\varphi \to L_kD_{i,j}\varphi$.
(CD−) $\neg D_{i,j}\varphi \to L_k\neg D_{i,j}\varphi$.
The axioms (CD+), (CD−), (L2′), (Lij4) and (Lij5) mean that the rely-on relations are common knowledge among the agents.

We will use a set of rules to describe the definition of a belief maintenance operation for the agent i, which forms are like $\varphi \wedge ... \wedge \varphi' \Rightarrow L_i^-(K)\triangle\psi = L_i^-(K)\theta\psi'$
where $\varphi, ..., \varphi' \in L_i^-(K)$, and $\psi \in K, \psi' \in$ the language $\mathbf{L}, \theta \in \{\dot{+}, \dot{-}, +\}$.
We will focus on the special case in which the agent w_1 offers the new information P, and agent w_2 offers the new information $P \to Q$, where P, Q are primitive proposition. Furthermore, the implication $P \to Q$ is not simply defined as $\neg(P \wedge \neg Q)$. We view $P \to Q$ as an independent conditional, like those which are studied in [14, 17, 7].

The Definition of Operation $\triangle_{assa_1}$ (for Agent a):
(a) $D_{w_1,w_2}(P \to Q) \wedge \neg D_{w_1,w_2}P \wedge D_{w_2,w_1}P \wedge \neg D_{w_2,w_1}(P \to Q)$
$\Rightarrow L_a^-(K)\triangle_{assa1}L_{a,w_1}P \wedge L_{a,w_2}(P \to Q) = L_a^-(K)\dot{+}P \wedge (P \to Q)$.

(b) $D_{w_2,w_1}P \wedge \neg D_{w_2,w_1}(P \to Q) \wedge D_{w_1,w_2}P \wedge L_{w_2}P \wedge D_{w_1,w_2}(P \to Q)$
$\Rightarrow L_a^-(K)\triangle_{assa1}L_{a,w_1}P \wedge L_{a,w_2}(P \to Q) = L_a^-(K)\dot{+}P \wedge (P \to Q)$.

(c) $D_{w_1,w_2}(P \to Q) \wedge \neg D_{w_1,w_2}P \wedge D_{w_2,w_1}P \wedge D_{w_2,w_1}(P \to Q) \wedge L_{w_1}(P \to Q)$
$\Rightarrow L_a^-(K)\triangle_{assa1}L_{a,w_1}P \wedge L_{a,w_2}(P \to Q) = L_a^-(K)\dot{+}P \wedge (P \to Q)$.

(d) $D_{w_1,w_2}P \wedge L_{w_2}P \wedge D_{w_1,w_2}(P \to Q) \wedge D_{w_2,w_1}P \wedge D_{w_2,w_1}(P \to Q) \wedge L_{w_1}(P \to Q)$
$\Rightarrow L_a^-(K)\triangle_{assa1}L_{a,w_1}P \wedge L_{a,w_2}(P \to Q) = L_a^-(K)\dot{+}P \wedge (P \to Q)$.

Of the above four cases, case(a) is representative for the problem of Schoenmakers paradox, since we need no further information about source agents' beliefs other than the general information about the rely-on relations among agents. Case(b), case(c) and case (d) deal with the situation where agent a may have collected some information about the source agents' beliefs. Although these situations are not representative for our problem, handling those situation is necessary for obtaining a complete operation.

Theorem 4.1 *The operation $\triangle_{assa1}$ is an ASSA operation.*

Proof: First of all, we would like to show that the operation $\triangle_{assal}$ is almost safe.
For case (a):
(a.1) $D_{w_1,w_2}(P \to Q) \wedge \neg D_{w_1,w_2}P \wedge L_{w_1}P \wedge L_{w_2}(P \to Q)$ (By the definition $\triangle_{assa_1}$)
$\Rightarrow L_{w_1,w_2}(P \to Q) \wedge L_{w_1}P \wedge \neg D_{w_1,w_2}P$ (By axiom (DLLij))
$\Rightarrow L_{w_1,w_2}(P \to Q) \wedge L_{w_1}P \wedge \neg L_{w_1,w_2}\neg P$ (By (DLLij) and (D1))
$\Rightarrow L_{w_1,w_2}(P \wedge (P \to Q))$ (By (ESA))
$\Rightarrow L_{w_1,w_2}P \wedge L_{w_1,w_2}(P \to Q) \wedge L_{w_1,w_2}Q$ (By (Lij$\wedge$))
Moreover, we have,
(a.2) $D_{w_2,w_1}P \wedge \neg D_{w_2,w_1}(P \to Q) \wedge L_{w_1}P \wedge L_{w_2}(P \to Q)$ (By the definition $\triangle_{assal}$)
$\Rightarrow L_{w_2,w_1}P \wedge L_{w_2}(P \to Q) \wedge \neg D_{w_2,w_1}(P \to Q)$
$\Rightarrow L_{w_2,w_1}P \wedge L_{w_2}(P \to Q) \wedge \neg L_{w_2,w_1}\neg(P \to Q)$
$\Rightarrow L_{w_2,w_1}P \wedge L_{w_2,w_1}(P \to Q)$
$\Rightarrow L_{w_2,w_1}P \wedge L_{w_2,w_1}(P \to Q) \wedge L_{w_2,w_1}Q$.
Therefore, Case (a)$\Rightarrow$ $(L_{w_1,w_2}P \wedge L_{w_2,w_1}P) \wedge (L_{w_1,w_2}(P \to Q) \wedge L_{w_2,w_1}(P \to Q)) \wedge (L_{w_1,w_2}Q \wedge L_{w_2,w_1}Q)$
This means that the operation $\triangle_{assal}$is safe under case(a).
For case (b):
(b.1) $D_{w_1,w_2}P \wedge L_{w_2}P \wedge D_{w_1,w_2}(P \to Q) \wedge L_{w_2}(P \to Q)$ (By the definition of the operation)
$\Rightarrow L_{w_1,w_2}P \wedge L_{w_1,w_2}(P \to Q)$ (By (DDLij))
$\Rightarrow L_{w_1,w_2}P \wedge L_{w_1,w_2}(P \to Q) \wedge L_{w_1,w_2}Q$ (By Lij$\wedge$)
Furthermore, by the result of the above (a.2), we can conclude that the operation is almost safe in case (b).
For case (c):
(c.1) $D_{w_1,w_2}P \wedge D_{w_2,w_1}(P \to Q) \wedge L_{w_1}(P \to Q) \wedge L_{w_2}P$ (By the definition of the operation)
$\Rightarrow L_{w_1,w_2}P \wedge L_{w_1,w_2}(P \to Q) \wedge L_{w_1,w_2}Q$.
Similarly, we also can show that the operation is almost safe in case (c) by the results of (a.1) and (c.1). Moreover, for case (d), by the results of (c.1) and (b.1), we can show that the operation is almost safe. Therefore, we have that the operation is almost safe.

Furthermore, we have to show that the operation is soundly-assimilating, as a matter of fact, that is equal to showing that $(L_{w_1,w_2}(P \wedge (P \to Q)) \wedge L_{w_2,w_1}(P \wedge (P \to Q)) \Rightarrow$Case (a)$\vee$Case (b)$\vee$Case (c)$\vee$Case (d).

From the extended definition about the sub-beliefs and the primitiveness of P and Q, we have $L_{w_1,w_2}(P \wedge (P \to Q))$
$\Rightarrow D_{w_1,w_2}(P \wedge (P \to Q)) \wedge L_{w_2}P \wedge L_{w_2}(P \to Q) \vee L_{w_1}P \wedge L_{w_1,w_2}(P \to Q) \wedge \neg L_{w_1,w_2}\neg P$
$\Rightarrow$Case (b.1)$\vee$ Case (a.1).
Similarly, we have that $L_{w_2,w_1}(P \wedge (P \to Q)) \Rightarrow$Case(c.1) $\vee$ Case(a.2).
Therefore, we have $L_{w_2,w_1}(P \wedge (P \to Q)) \wedge (L_{w_1,w_2}(P \wedge (P \to Q)) \Rightarrow$Case (a)$\vee$Case (b)$\vee$Case (c)$\vee$Case (d)
That is, the operation $\triangle_{assal}$ is soundly assimilating. Moreover, the operation is an ASSA operation. □

Application of the above ideas leads to a new appreciation of the Schoenmakers paradox. Assuming that the judge draws his conclusion based on an ASSA, we find that the unacceptability of the state of affairs as indicated by the story only is a temporary stage in the process of exchanging information and incorporation of beliefs. One possible scenario for the continuation of the story is presented below:

When the judge was told that P was true by the witness $w1$ and that the conditional $P \rightarrow Q$ was true by the witness $w2$, the judge had to figure out whether these assertions could be accepted together. However, the judge had good reasons for not asking the witnesses for more information about their knowledge. The judge based his decision on his knowledge concerning the rely-on relation. He knew that the witness $w1$ was the only authority concerning the statement P, and that witness $w2$ was the only authority concerning the conditional $P \rightarrow Q$. Moreover, this information was common knowledge among the two witnesses and himself. Therefore, the judge could safely arrive to the conclusion Q was true, and ordered to execute the verdict. Still, both witnesses, $w1$ and $w2$, came forward and claimed that Q was false. Then the judge patiently told witness $w1$ about the witness $w2'$s belief, helding that $P \rightarrow Q$ was true. Because the witness $w1$ accepted that $w2$ was the authority on the conditional $P \rightarrow Q$, $w1$ accepted this assertion, and had to agree with the judge. A similar thing happened with witness $w2$. The judge told witness $w2$ about $w1$'s belief, that is, that P was true. The witness $w2$ also had to agree with the judge's verdict, since $w2$ accepted that the $w1$ was the authority about P.

5 Conclusions

In order to solve the Schoenmakers paradox, we have proposed a plausible analysis in the framework of logic for belief dependence. The main new notion is that of an almost-safely operation, by which we can describe the potential beliefs which may be produced when agents, can communicate their knowledge and beliefs. The proposed strategy opens the possibility of capturing an almost-safely and soundly-assimilating operation, which is intuitive and acceptable. Moreover, we believe that the proposed strategy also offers the possibility to apply belief dependence in fields such as knowledge acquisitions, knowledge bases management, and user models. So far we have not succeded in defining a non trivial example of an update operation satisfying all our requirements. However, for a plausible analysis of the original paradox, we have proceeded on an ad hoc basis, and we have shown that there exist reasonable scenarios for embedding the paradoxical story; these scenarios moreover are based on our original target notions.

References

[1] Jon Doyle, A Society of Mind-Multiple Perspectives, Reasoned Assumptions, and Virtual Copies, *Proceedings of the 8th Inter. Joint Conference on AI*, Vol.I, 1983, 309-314.

[2] Fagin, R. F. and Halpern, J. Y., Belief, Awareness, and Limited Reasoning, in: *Artificial Intelligence,* 34 (1988) 39-76.

[3] Joseph S. Fulda, The Logic of Expert Judging Systems and the Rights of the Accused, *AI and Society: The Journal of Human and Machine Intelligence*, 1988.

[4] Peter Gärdenfors, *Knowledge in Flux-Modeling the Dynamics of Epistemic States*, MIT Press, Cambridge, Mass., 1988.

[5] Peter Gärdenfors, The Dynamics of Belief Systems: Foundations vs. Coherence Theories, *Revue Internationale de Philosophie*, (1)1990, 24-46.

[6] Peter Gärdenfors and David Makinson, Revision of knowledge systems using epistemic entrenchment. In M. Vardi,(ed), *Proceedings of the 2nd Conference on Theoretical Aspects of Reasoning about Knowledge*, Morgan Kaufmann, Los Altos, CA.,1988.

[7] M. L. Ginsberg, Counterfactuals, *Artificial Intelligence*, 30(1986), 35-79.

[8] J. R. Hobbs and R. C. Moore, *Formal Theories of the Commonsense World.* Ablex, Norwood, N.J., 1985.

[9] Zhisheng Huang, Dependency of Belief in Distributed Systems, in: M. Stokhof and L. Torenvliet (eds.) *Proceedings of the Seventh Amsterdam Colloquium*, 637-662, 1990. Also available in: ITLI LP-89-09, University of Amsterdam, 1989.

[10] Zhisheng Huang and Karen Kwast, Awareness, Negation and Logical Omniscience, in: J. van Eijck (ed.), *Proceedings of European Workshop on Logics in Artificial Intelligence*, (JELIA'90), Springer Lecture Notes in Artificial Intelligence, LCS vol. 478 (1991), pp. 282-300.

[11] Zhisheng Huang, Logics for Belief Dependence, to appear in: *Proceedings of the 1990 Workshop on Computer Science Logic*(CSL'90). Also available in: Institute for Language, Logic and Information, Preprint LP-90-13, University of Amsterdam, 1990.

[12] Zhisheng Huang and Peter van Emde Boas, Belief Dependence, Revision and Persistence, in preparation.

[13] Nebel, B., *Reasoning and Revision in Hybrid Representation Systems*, Lecture Notes in Computer Science 422, 1990.

[14] Donald Nute, Conditional Logic, in D. Gabby and F. Guenthner (eds.), *Handbook of Philosophical Logic*, Vol.II, 387-439, 1986.

[15] Potts, G., John, M., and Kirson, D., Incorporating New Information into Existing World Knowledge, *Cognitive Psychology* 21,(1989), 303-333.

[16] W.J. Schoenmakers, A Problem in Knowledge Acquisition, SIGART Newsletter. 95(1986), 56-57.

[17] Elizabeth C. Traugott, Alice T. Meulen, Judy S. Reilly, Charles A. Ferguson,(eds.), *On Conditionals*, Cambridge University Press, 1986.

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